X86 instruction semantics and basic block symbolic execution

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Abstract

This AFP entry provides semantics for roughly 120 different X86-64 assembly instructions. These instructions include various moves, arithmetic/logical operations, jumps, call/return, SIMD extensions and others. External functions are supported by allowing a user to provide custom semantics for these calls. Floating-point operations are mapped to uninterpreted functions. The model provides semantics for register aliassing and a byte-level little-endian memory model. The semantics are purposefully incomplete, but overapproximative. For example, the precise effect of flags may be undefined for certain instructions, or instructions may simply have no semantics at all. In those cases, the semantics are mapped to universally quantified uninterpreted terms from a locale. Second, this entry provides a method to symbolic execution of basic blocks. The method, called "se_step" (for: symbolic execution step) fetches an instruction and updates the current symbolic state while keeping track of assumptions made over the memory model. A key component is a set of theorems that prove how reads from memory resolve after writes have occured. Thirdly, this entry provides a parser that allows the user to copy-paste the output of the standard disassembly tool objdump into Isabelle/HOL. Several examples are supplied: a couple small and explanatory examples, functions from the word count program, the floating-point modulo function from FDLIBM, the GLIBC strlen function and the CoreUtils SHA256 implementation.

Contents

1 Bit and byte-level theorems

theory *BitByte*
 imports *Main Word-Lib.Syntax-Bundles Word-Lib.Bit-Shifts-Infix-Syntax Word-Lib.Bitwise*
begin

1.1 Basics

unbundle *bit-operations-syntax*
unbundle *bit-projection-infix-syntax*

definition *take-bits* :: *nat* ⇒ *nat* ⇒ *'a::len word* ⇒ *'a::len word* (‹⟨-,-⟩-› *51*) — little-endian
 where *take-bits l h w* ≡ (*w* >> *l*) *AND mask* (*h*−*l*)

⟨*l*,*h*⟩*w* takes a subset of bits from word *w*, from low (inclusive) to high (exclusive). For example, ⟨*2*::*nat*,*5*::*nat*⟩*28*::*8 word* = (*7*::*8 word*).

definition *take-byte* :: *nat* ⇒ *'a::len word* ⇒ *8word* — little-endian
 where *take-byte n w* ≡ *ucast* (⟨*n*∗*8*,*n*∗*8*+*8*⟩*w*)

take-byte n w takes the nth byte from word *w*. For example, *take-byte 1* ((*42*::*16 word*) << (*8*::*nat*)) = (*42*::*8 word*).

definition *overwrite* :: *nat* ⇒ *nat* ⇒ *'a::len word* ⇒ *'a::len word* ⇒ *'a::len word*
 where *overwrite l h w0 w1* ≡ (((⟨*h*,*LENGTH*(*'a*)⟩*w0*) << *h*) *OR* ((⟨*l*,*h*⟩*w1*) << *l*) *OR* (⟨*0*,*l*⟩*w0*)

overwrite l h w0 w1 overwrites low (inclusive) to high (exclusive) bits in word *w0* with bits from word *w1*. For example, *overwrite* (*2*::*nat*) (*4*::*nat*) (*28*::*8 word*) (*227*::*8 word*) = (*16*::*8 word*).

We prove some theorems about taking the nth bit/byte of an operation. These are useful to prove equaltiy between different words, by first applying rule ($\bigwedge n.\ n < size\ ?u \Longrightarrow ?u\ !!\ n = ?v\ !!\ n) \Longrightarrow ?u = ?v$.

lemma *bit-take-bits-iff* [*bit-simps*]:
 ‹$((\langle l,h\rangle w)\ !!\ n \longleftrightarrow n < LENGTH('a) \wedge n < h - l \wedge w\ !!\ (n + l)$› **for** *w* :: ‹*'a::len word*›

by (*simp add*: *take-bits-def bit-simps ac-simps*)

lemma *bit-take-byte-iff* [*bit-simps*]:
‹*take-byte m w* !! $n \longleftrightarrow n < LENGTH('a) \wedge n < 8 \wedge w$!! $(n + m * 8)$› **for** *w* :: ‹*'a*::*len word*›
by (*auto simp add*: *take-byte-def bit-simps*)

lemma *bit-overwrite-iff* [*bit-simps*]:
‹*overwrite l h w0 w1* !! $n \longleftrightarrow n < LENGTH('a) \wedge$
(*if* $l \leq n \wedge n < h$ *then w1 else w0*) !! *n*›
for *w0 w1* :: ‹*'a*::*len word*›
by (*auto simp add*: *overwrite-def bit-simps*)

lemma *nth-takebits*:
fixes *w* :: *'a*::*len word*
shows $(\langle l,h\rangle w)$!! $n =$ (*if* $n < LENGTH('a) \wedge n < h - l$ *then w* !! $(n + l)$ *else False*)
by (*auto simp add*: *bit-simps*)

lemma *nth-takebyte*:
fixes *w* :: *'a*::*len word*
shows *take-byte* (*n div 8*) *w* !! (*n mod 8*) = (*if n mod 8* < *LENGTH*(*'a*) *then w*!!*n else False*)
by (*simp add*: *bit-simps*)

lemma *nth-take-byte-overwrite*:
fixes *v v'* :: *'a*::*len word*
shows *take-byte n* (*overwrite l h v v'*) !! *i* = (*if* $i + n * 8 < l \vee i + n * 8 \geq h$ *then take-byte n v* !! *i else take-byte n v'* !! *i*)
by (*auto simp add*: *bit-simps dest*: *bit-imp-le-length*)

lemma *nth-bitNOT*:
fixes *a* :: *'a*::*len word*
shows (*NOT a*) !! $n \longleftrightarrow$ (*if* $n < LENGTH('a)$ *then* $\neg$(*a* !! *n*) *else False*)
by (*simp add*: *bit-simps*)

Various simplification rules

lemma *ucast-take-bits*:
fixes *w* :: *'a*::*len word*
assumes $h = LENGTH('b)$
and $LENGTH('b) \leq LENGTH('a)$
shows *ucast* $(\langle 0,h\rangle w)$ = (*ucast w* ::*'b* :: *len word*)
apply (*rule bit-word-eqI*)
using *assms*
apply (*simp add*: *bit-simps*)
done

lemma *take-bits-ucast*:
fixes *w* :: *'b*::*len word*

assumes $h = LENGTH('b)$
shows $(\langle 0,h\rangle\ (ucast\ w ::'a :: len\ word)) = (ucast\ w ::'a :: len\ word)$
apply (*rule bit-word-eqI*)
using *assms*
apply (*auto simp add*: *bit-simps dest*: *bit-imp-le-length*)
done

lemma *take-bits-take-bits*:
fixes $w :: 'a::len\ word$
shows $(\langle l,h\rangle(\langle l',h'\rangle w)) = (if\ min\ LENGTH('a)\ h \geq h' - l'\ then\ \langle l+l',h'\rangle w\ else\ (\langle l+l',l'+min\ LENGTH('a)\ h\rangle w))$
apply (*rule bit-word-eqI*)
apply (*simp add*: *bit-simps ac-simps*)
apply *auto*
done

lemma *take-bits-overwrite*:
shows $\langle l,h\rangle(overwrite\ l\ h\ w0\ w1) = \langle l,h\rangle w1$
apply (*rule bit-word-eqI*)
apply (*simp add*: *bit-simps ac-simps*)
apply (*auto dest*: *bit-imp-le-length*)
done

lemma *overwrite-0-take-bits-0*:
shows $overwrite\ 0\ h\ (\langle 0,h\rangle w0)\ w1 = \langle 0,h\rangle w1$
apply (*rule bit-word-eqI*)
apply (*simp add*: *bit-simps ac-simps*)
done

lemma *take-byte-shiftlr-256*:
fixes $v :: 256\ word$
assumes $m \leq n$
shows $take\text{-}byte\ n\ (v << m*8) = (if\ (n+1)*8 \leq 256\ then\ take\text{-}byte\ (n-m)\ v\ else\ 0)$
apply (*rule bit-word-eqI*)
using *assms*
apply (*simp add*: *bit-simps*)
apply (*simp add*: *algebra-simps*)
done

1.2 Take_Bits and arithmetic

This definition is based on $to\text{-}bl\ (?x + ?y) = rev\ (foldr\ (\lambda(x, y)\ res\ car.\ xor3\ x\ y\ car\ \#\ res\ (carry\ x\ y\ car))\ (rev\ (zip\ (to\text{-}bl\ ?x)\ (to\text{-}bl\ ?y)))\ (\lambda\text{-}.\ [])\ False)$, which formulates addition as bitwise operations using *xor3* and *carry*.

definition $bitwise\text{-}add :: (bool \times bool)\ list \Rightarrow bool \Rightarrow bool\ list$
where $bitwise\text{-}add\ x\ c \equiv foldr\ (\lambda(x, y)\ res\ car.\ xor3\ x\ y\ car\ \#\ res\ (carry\ x\ y\ car))\ x\ (\lambda\text{-}.\ [])\ c$

lemma *length-foldr-bitwise-add*:
shows *length* (*bitwise-add x c*) = *length x*
unfolding *bitwise-add-def*
by(*induct x arbitrary*: *c*) *auto*

This is the "heart" of the proof: bitwise addition of two appended zipped lists can be expressed as two consecutive bitwise additions. Here, I need to make the assumption that the final carry is False.

lemma *bitwise-add-append*:
assumes $x = [] \vee \neg carry\ (fst\ (last\ x))\ (snd\ (last\ x))\ True$
shows $bitwise\text{-}add\ (x\ @\ y)\ (x \neq [] \wedge c) = bitwise\text{-}add\ x\ (x \neq [] \wedge c)\ @\ bitwise\text{-}add\ y\ False$
using *assms*
unfolding *bitwise-add-def*
by(*induct x arbitrary*: *c*) (*auto simp add*: *case-prod-unfold xor3-def carry-def split*: *if-split-asm*)

lemma *bitwise-add-take-append*:
shows *take* (*length x*) (*bitwise-add* (*x* @ *y*) *c*) = *bitwise-add x c*
unfolding *bitwise-add-def*
by(*induct x arbitrary*: *c*) (*auto simp add*: *case-prod-unfold xor3-def carry-def split*: *if-split-asm*)

lemma *bitwise-add-zero*:
shows *bitwise-add* (*replicate n* (*False*, *False*)) *False* = *replicate n False*
unfolding *bitwise-add-def*
by(*induct n*) (*auto simp add*: *xor3-def carry-def*)

lemma *bitwise-add-take*:
shows *take n* (*bitwise-add x c*) = *bitwise-add* (*take n x*) *c*
unfolding *bitwise-add-def*
by (*induct n arbitrary*: *x c*,*auto*)
(*metis append-take-drop-id bitwise-add-def bitwise-add-take-append diff-is-0-eq′ length-foldr-bitwise-add length-take nat-le-linear rev-min-pm1 take-all*)

lemma *fst-hd-drop-zip*:
assumes *n* < *length x*
and *length x* = *length y*
shows *fst* (*hd* (*drop n* (*zip x y*))) = *hd* (*drop n x*)
using *assms*
by (*induct x arbitrary*: *n y*,*auto*)
(*metis* (*no-types*, *lifting*) *Cons-nth-drop-Suc drop-zip fst-conv length-Cons list.sel*(*1*) *zip-Cons-Cons*)

lemma *snd-hd-drop-zip*:
assumes *n* < *length x*
and *length x* = *length y*

shows *snd* (*hd* (*drop n* (*zip x y*))) = *hd* (*drop n y*)
using *assms*
by (*induct x arbitrary*: *n y*,*auto*)
(*metis* (*no-types*, *lifting*) *Cons-nth-drop-Suc drop-zip snd-conv length-Cons list.sel*(*1*) *zip-Cons-Cons*)

Ucasting of $a + b$ can be rewritten to taking bits of a and b.

lemma *ucast-plus*:
fixes *a b* :: *'a*::*len word*
assumes *LENGTH*(*'a*) > *LENGTH*(*'b*)
shows (*ucast* (*a* + *b*) ::*'b*::*len word*) = (*ucast a* + *ucast b*::*'b*::*len word*)
proof−
have *to-bl* (*ucast* (*a* + *b*) ::*'b*::*len word*) = *to-bl* (*ucast a* + *ucast b*::*'b*::*len word*)
using *assms*
apply (*auto simp add*: *to-bl-ucast to-bl-plus-carry word-rep-drop length-foldr-bitwise-add drop-zip*[*symmetric*] *rev-drop bitwise-add-def simp del*: *foldr-replicate foldr-append*)
apply (*simp only*: *bitwise-add-def*[*symmetric*] *length-foldr-bitwise-add*)
by (*auto simp add*: *drop-take bitwise-add-take*[*symmetric*] *rev-take length-foldr-bitwise-add*)
thus *?thesis*
using *word-bl.Rep-eqD*
by *blast*
qed

lemma *ucast-uminus*:
fixes *a b* :: *'a*::*len word*
assumes *LENGTH*(*'a*) > *LENGTH*(*'b*)
shows *ucast* (− *a*) = (− *ucast a* :: *'b*::*len word*)
apply (*subst twos-complement*)+
apply (*subst word-succ-p1*)+
apply (*subst ucast-plus*)
apply (*rule assms*)
apply *simp*
apply (*rule word-eqI*)
apply (*auto simp add*: *word-size nth-ucast nth-bitNOT*)
using *assms order.strict-trans*
by *blast*

lemma *ucast-minus*:
fixes *a b* :: *'a*::*len word*
assumes *LENGTH*(*'a*) > *LENGTH*(*'b*)
shows (*ucast* (*a* − *b*) ::*'b*::*len word*) = (*ucast a* − *ucast b*::*'b*::*len word*)
using *ucast-plus*[*OF assms*,*of a* −*b*] *ucast-uminus*[*OF assms*,*of b*]
by *auto*

lemma *to-bl-takebits*:
fixes *a* :: *'a*::*len word*
shows *to-bl* (⟨*0*,*h*⟩*a*) = *replicate* (*LENGTH*(*'a*) − *h*) *False* @ *drop* (*LENGTH*(*'a*) − *h*) (*to-bl a*)
apply (*auto simp add*: *take-bits-def bl-word-and to-bl-mask*)

apply (*rule nth-equalityI*)
by (*auto simp add*: *min-def nth-append*)

All simplification rules that are used during symbolic execution.

lemmas *BitByte-simps* = *ucast-plus ucast-minus ucast-uminus take-bits-overwrite take-bits-take-bits*
ucast-take-bits overwrite-0-take-bits-0 mask-eq-exp-minus-1
ucast-down-ucast-id is-down take-bits-ucast ucast-up-ucast-id is-up

Simplification for immediate (numeral) values.

lemmas *take-bits-numeral*[*simp*] = *take-bits-def*[*of - - numeral n*] **for** *n*
lemmas *take-bits-num0*[*simp*] = *take-bits-def*[*of - - 0*] **for** *n*
lemmas *take-bits-num1*[*simp*] = *take-bits-def*[*of - - 1*] **for** *n*
lemmas *overwrite-numeral-numeral*[*simp*] = *overwrite-def*[*of - - numeral n numeral m*] **for** *n m*
lemmas *overwrite-num0-numeral*[*simp*] = *overwrite-def*[*of - - 0 numeral m*] **for** *n m*
lemmas *overwrite-numeral-num0*[*simp*] = *overwrite-def*[*of - - numeral m 0*] **for** *n m*
lemmas *overwrite-numeral-00*[*simp*] = *overwrite-def*[*of - - 0 0*]

end

2 Memory-related theorems

theory *Memory*
imports *BitByte*
begin

context
fixes *dummy-type* :: *'a::len*
begin

primrec *read-bytes* :: (*'a word* ⇒ *8 word*) ⇒ *'a word* ⇒ *nat* ⇒ *8word list*
where *read-bytes m a 0* = []
| *read-bytes m a* (*Suc n*) = *m* (*a* + *of-nat n*) # *read-bytes m a n*

Read bytes from memory. Memory is represented by a term of *64 word* ⇒ *8 word*. Given an address *a*::*64 word* and a size *n*, retrieve the bytes in the order they are stored in memory.

definition *region-addresses* :: *'a word* ⇒ *nat* ⇒ *'a word set*
where *region-addresses a si* ≡ {*a'* . ∃ *i* < *si* . *a'* = *a* + *of-nat* (*si* − *i* − *1*)}

The set of addresses belonging to a region starting at address *a* of *si* bytes.

definition *region-overflow* :: *'a word* ⇒ *nat* ⇒ *bool*
where *region-overflow a si* ≡ *unat a* + *si* ≥ *2^LENGTH*(*'a*)

An overflow occurs if the address plus the size is greater equal $(2::'b)^{64}$

definition *enclosed* :: $'a$ *word* $\Rightarrow$ *nat* $\Rightarrow$ $'a$ *word* $\Rightarrow$ *nat* $\Rightarrow$ *bool*
where *enclosed* a' si' *a si* $\equiv$ *unat a* + *si* < *2^LENGTH*($'a$) $\wedge$ *unat a* $\leq$ *unat* a' $\wedge$ *unat* a' + si' $\leq$ *unat a* + *si*

A region is enclosed in another if its *local.region-addresses* is a subset of the other.

definition *separate* :: $'a$ *word* $\Rightarrow$ *nat* $\Rightarrow$ $'a$ *word* $\Rightarrow$ *nat* $\Rightarrow$ *bool*
where *separate a si* a' si' $\equiv$ *si* $\neq$*0* $\wedge$ si' $\neq$ *0* $\wedge$ *region-addresses a si* $\cap$ *region-addresses* a' si' = {}

A region is separate from another if they do not overlap.

lemma *region-addresses-iff*: a' $\in$ *region-addresses a si* $\longleftrightarrow$ *unat* (a' − *a*) < *si*
apply (*auto simp add*: *region-addresses-def unsigned-of-nat*)
apply (*metis* (*no-types*, *opaque-lifting*) *Suc-leI bot-nat-0.extremum diff-less gr0-conv-Suc le-unat-uoi nat-le-linear of-nat-Suc of-nat-diff order-le-less-trans*)
by (*metis Suc-diff-Suc add.commute diff-Suc-less diff-add-cancel diff-diff-cancel dual-order.strict-trans order-less-imp-le unat-0 unat-gt-0 word-unat.Rep-inverse*)

lemma *notin-region-addresses*:
assumes *x* $\notin$ *region-addresses a si*
shows *unat x* < *unat a* $\vee$ *unat a* + *si* $\leq$ *unat x*
by (*metis assms add.commute less-diff-conv2 not-le-imp-less region-addresses-iff unat-sub-if'*)

lemma *notin-region-addresses-sub*:
assumes *x* $\notin$ *region-addresses a si*
shows *unat* (*x* − a') < *unat* (*a* − a') $\vee$ *unat* (*a* − a') + *si* $\leq$ *unat* (*x* − a')
using *assms notin-region-addresses region-addresses-iff* **by** *auto*

lemma *region-addresses-eq-empty-iff*: *region-addresses a si* = {} $\longleftrightarrow$ *si* = *0*
by (*metis region-addresses-iff add-diff-cancel-left' ex-in-conv neq0-conv not-less0 unsigned-0*)

lemma *length-read-bytes*:
shows *length* (*read-bytes m a si*) = *si*
by (*induct si*,*auto*)

lemma *nth-read-bytes*:
assumes *n* < *si*
shows *read-bytes m a si* ! *n* = *m* (*a* + *of-nat* (*si* − *1* − *n*))
using *assms*
apply (*induct si arbitrary*: *n*,*auto*)
subgoal for *si n*
by(*cases n*,*auto*)
done

Writing to memory occurs via function *override-on*. In case of enclosure, reading bytes from memory overridden on a set of region addresses can be simplified to reading bytes from the overwritten memory only. In case of separation, reading bytes from overridden memory can be simplified to reading from the original memory.

lemma *read-bytes-override-on-enclosed*:
assumes $offset' \leq offset$
  and $si' \leq si$
  and $unat\ offset + si' \leq si + unat\ offset'$
 shows $read\text{-}bytes\ (override\text{-}on\ m\ m'\ (region\text{-}addresses\ (a - offset)\ si))\ (a - offset')\ si' = read\text{-}bytes\ m'\ (a - offset')\ si'$
proof−
 {
  fix i
  assume 1: $i < si'$
  let $?i = (si + i + unat\ offset') - unat\ offset - si'$

  have $i + unat\ offset' < si' + unat\ offset$
   using 1 $assms(1)$
   by *unat-arith*
  hence 2: $si + i + unat\ offset' - (unat\ offset + si') < si$
   using $diff\text{-}less[of\ (si' + unat\ offset - i) - unat\ offset'\ si]$ $assms(3)$
   by *auto*
  moreover
  have $of\text{-}nat\ (si' - Suc\ i) - offset' = of\text{-}nat\ (si - Suc\ ?i) - offset$
   using *assms 1 2* **by** (*auto simp add*: *of-nat-diff*)
  ultimately
  have $\exists\ i' < si.\ of\text{-}nat\ (si' - Suc\ i) - offset' = of\text{-}nat\ (si - Suc\ i') - offset$
   by *auto*
 }
 note $1 = this$
 show *?thesis*
  apply (*rule nth-equalityI*)
  using *1*
 by (*auto simp*: *length-read-bytes nth-read-bytes override-on-def region-addresses-def*
   simp flip: *of-nat-diff*)
qed

lemmas *read-bytes-override-on* = *read-bytes-override-on-enclosed*[**where** *offset=0* **and** *offset′=0*,*simplified*]

lemma *read-bytes-override-on-enclosed-plus*:
 assumes $unat\ offset + si' \leq si$
  and $si \leq 2\hat{\ }LENGTH('a)$
 shows $read\text{-}bytes\ (override\text{-}on\ m\ m'\ (region\text{-}addresses\ a\ si))\ (offset+a)\ si' = read\text{-}bytes\ m'\ (offset+a)\ si'$
proof−
 {
  fix i
  have $i < si' \Longrightarrow \exists\ i' < si.\ offset + (of\text{-}nat\ (si' - Suc\ i)::'a\ word) = of\text{-}nat\ (si - Suc\ i')$
   apply (*rule exI*[*of* $-si - si' + i - unat\ offset$])

using *assms* **by** (*auto simp add*: *of-nat-diff*)
}
note *1 = this*
show *?thesis*
apply (*rule nth-equalityI*)
using *assms 1*
by (*auto simp*: *override-on-def length-read-bytes nth-read-bytes region-addresses-def simp flip*: *of-nat-diff*)
qed

lemma *read-bytes-override-on-separate*:
assumes *separate a si a′ si′*
shows *read-bytes* (*override-on m m′* (*region-addresses a si*)) *a′ si′* = *read-bytes m a′ si′*
apply (*rule nth-equalityI*)
using *assms*
by (*auto simp*: *length-read-bytes nth-read-bytes override-on-def separate-def region-addresses-def*
simp flip: *of-nat-diff*)

Bytes are are written to memory one-by-one, then read by *local.read-bytes* producing a list of bytes. That list is concatenated again using *word-rcat*. Writing *si* bytes of word *w* into memory, reading the byte-list and concatenating again produces *si* bytes of the original word.

lemma *word-rcat-read-bytes-enclosed*:
fixes *w* :: *′b*::*len word*
assumes $LENGTH('b) \leq 2\hat{\ }LENGTH('a)$
and *unat offset* + *si* $\leq 2\hat{\ }LENGTH('a)$
shows *word-rcat* (*read-bytes* (*λa′*. *take-byte* (*unat* (*a′* − *a*)) *w*) (*a* + *offset*) *si*) = ⟨*unat offset* ∗ *8*,(*unat offset* + *si*) ∗ *8*⟩*w*
apply (*rule word-eqI*)
using *assms*
apply (*auto simp add*: *test-bit-rcat word-size length-read-bytes rev-nth nth-read-bytes unat-of-nat nth-takebyte unat-word-ariths*)
apply (*auto simp add*: *take-byte-def nth-ucast nth-takebits take-bit-eq-mod split*: *if-split-asm*)[*1*]
apply (*auto simp add*: *nth-takebits split*: *if-split-asm*)[*1*]
apply (*auto simp add*: *take-byte-def nth-ucast nth-takebits split*: *if-split-asm*)[*1*]
by (*auto simp add*: *rev-nth length-read-bytes take-byte-def nth-ucast nth-takebits nth-read-bytes unat-word-ariths unat-of-nat take-bit-eq-mod split*: *if-split-asm*)

lemmas *word-rcat-read-bytes* = *word-rcat-read-bytes-enclosed*[**where** *offset*=*0*,*simplified*]

The following theorems allow reasoning over enclosure and separation, for example as linear arithmetic.

lemma *enclosed-spec*:
assumes *enclosed*: *enclosed a′ si′ a si*
and *x-in*: *x* ∈ *region-addresses a′ si′*

shows $x \in$ *region-addresses a si*
proof $-$
from *x-in* **have** *unat* $(x - a') < si'$
using *region-addresses-iff* **by** *blast*
with *enclosed* **have** *unat* $(x - a) < si$
unfolding *enclosed-def* **by** (*auto simp add*: *unat-sub-if'* *split*: *if-split-asm*)
then show *?thesis*
using *region-addresses-iff* **by** *blast*
qed

lemma *address-in-enclosed-region-as-linarith*:
assumes *enclosed a' si' a si*
and $x \in$ *region-addresses a' si'*
shows $a \leq x \wedge a' \leq x \wedge x < a' +$ *of-nat si'* $\wedge x < a +$ *of-nat si*
using *assms*
by (*auto simp add*: *enclosed-def region-addresses-def word-le-nat-alt word-less-nat-alt unat-of-nat unat-word-ariths unat-sub-if' take-bit-eq-mod*)

lemma *address-of-enclosed-region-ge*:
assumes *enclosed a' si' a si*
shows $a' \geq a$
using *assms word-le-nat-alt*
by (*auto simp add*: *enclosed-def*)

lemma *address-in-enclosed-region*:
assumes *enclosed a' si' a si*
and $x \in$ *region-addresses a' si'*
shows *unat* $(x - a) \geq$ *unat* $(a' - a) \wedge$ *unat* $(a' - a) + si' >$ *unat* $(x - a) \wedge$ *unat* $(x - a) < si$
by (*smt* (*verit*, *ccfv-threshold*) *Memory.enclosed-def Memory.region-addresses-iff address-in-enclosed-region-as-linarith assms*(*1*) *assms*(*2*) *diff-diff-add le-add-diff-inverse mcs*(*4*) *nat-add-left-cancel-less no-ulen-sub of-nat-add un-ui-le unat-mono unat-of-nat-eq unat-sub unsigned-less word-sub-le-iff*)

lemma *enclosed-minus-minus*:
fixes *a* :: *'a word*
assumes *offset* $\geq$ *offset'*
and *unat offset* $-$ *si* $\leq$ *unat offset'* $-$ *si'*
and *unat offset'* $\geq$ *si'*
and *unat offset* $\geq$ *si*
and $a \geq$ *offset*
shows *enclosed* $(a -$ *offset'*$)$ *si'* $(a -$ *offset*$)$ *si*
proof$-$
have *unat offset'* $\leq$ *unat a*
using *assms*(*1*,*5*)
by *unat-arith*
thus *?thesis*
using *assms*

apply (*auto simp add*: *enclosed-def unat-sub-if-size word-size*)
apply *unat-arith*
using *diff-le-mono2*[*of unat offset* $-$ *si unat offset*$'$ $-$ *si*$'$ *unat a*]
apply (*auto simp add*: *enclosed-def unat-sub-if-size word-size*)
by *unat-arith+*
qed

lemma *enclosed-plus*:
fixes a :: $'a$ *word*
assumes $si' < si$
and $unat\ a + si < 2\verb|^|LENGTH('a)$
shows *enclosed a si*$'$ *a si*
using *assms*
by (*auto simp add*: *enclosed-def*)

lemma *separate-symm*: *separate a si a*$'$ *si*$'$ = *separate a*$'$ *si*$'$ *a si*
by (*metis inf.commute separate-def*)

lemma *separate-iff*: *separate a si a*$'$ *si*$'$ $\longleftrightarrow si > 0 \wedge si' > 0 \wedge unat\ (a' - a) \geq si \wedge unat\ (a - a') \geq si'$
proof
assume *assm*: *separate a si a*$'$ *si*$'$
have $unat\ (a' - a) \geq si$ **if** *separate a si a*$'$ *si*$'$ **for** *a si a*$'$ *si*$'$
proof (*rule ccontr*)
assume $\neg\ unat\ (a' - a) \geq si$
then have $a' \in$ *region-addresses a si*
by (*simp add*: *region-addresses-iff*)
moreover from *that* **have** $a' \in$ *region-addresses a*$'$ *si*$'$
using *region-addresses-iff separate-def* **by** *auto*
ultimately have $\neg$*separate a si a*$'$ *si*$'$
by (*meson disjoint-iff separate-def*)
with *that* **show** *False*
by *blast*
qed
with *assm* **have** $unat\ (a' - a) \geq si$ **and** $unat\ (a - a') \geq si'$
using *separate-symm* **by** *auto*
with *assm* **show** $si > 0 \wedge si' > 0 \wedge unat\ (a' - a) \geq si \wedge unat\ (a - a') \geq si'$
using *separate-def* **by** *auto*
next
assume *assm*: $si > 0 \wedge si' > 0 \wedge unat\ (a' - a) \geq si \wedge unat\ (a - a') \geq si'$
then have $unat\ (x - a') \geq si'$ **if** $unat\ (x - a) < si$ **for** x
using *that* **apply** (*auto simp add*: *unat-sub-if*$'$ *split*: *if-split-asm*)
apply (*meson Nat.le-diff-conv2 add-increasing le-less-trans less-imp-le-nat unsigned-greater-eq unsigned-less*)
by (*smt* (*verit*) *Nat.le-diff-conv2 add-leD2 le-less-trans linorder-not-less nat-le-linear unat-lt2p*)
then have *region-addresses a si* $\cap$ *region-addresses a*$'$ *si*$'$ = {}
by (*simp add*: *region-addresses-iff disjoint-iff leD*)
with *assm* **show** *separate a si a*$'$ *si*$'$

by (*simp add*: *separate-def*)
qed

lemma *separate-as-linarith*:
assumes $\neg$*region-overflow a si*
and $\neg$*region-overflow a′ si′*
shows *separate a si a′ si′* $\longleftrightarrow$ *0* $<$ *si* $\wedge$ *0* $<$ *si′* $\wedge$ (*a* $+$ *of-nat si* $\leq$ *a′* $\vee$ *a′* $+$ *of-nat si′* $\leq$ *a*)
(**is** *?lhs* $\longleftrightarrow$ *?rhs*)
proof
assume *?lhs* **then show** *?rhs*
by (*meson separate-iff le-less le-plus not-le-imp-less word-of-nat-le*)
next
have $*$: *separate a si a′ si′*
if *si* $>$ *0* **and** *si′* $>$ *0* **and** *a* $+$ *of-nat si* $\leq$ *a′*
and $\neg$*region-overflow a si* **and** $\neg$*region-overflow a′ si′*
for *a si a′ si′*
proof $-$
from *that* **have** *unat a* $+$ *si* $<$ *2^LENGTH(′a)* **and** *unat a′* $+$ *si′* $<$ *2^LENGTH(′a)*
by (*meson not-le region-overflow-def*)+
have *x* $<$ *a* $+$ *of-nat si* **if** *x* $\in$ *region-addresses a si* **for** *x*
by (*smt* (*verit*) *Abs-fnat-hom-add* ‹*unat a* $+$ *si* $<$ *2 ^LENGTH(′a)*› *add.commute dual-order.trans le-add1 less-diff-conv2 not-less region-addresses-iff that unat-of-nat-len unat-sub-if′ word-less-iff-unsigned word-unat.Rep-inverse*)
moreover have *x* $\geq$ *a′* **if** *x* $\in$ *region-addresses a′ si′* **for** *x*
using *address-in-enclosed-region-as-linarith enclosed-def* ‹*unat a′* $+$ *si′* $<$ *2 ^LENGTH(′a)*› *that* **by** *blast*
ultimately show *?thesis*
using *separate-def that* **by** *fastforce*
qed
assume *?rhs* **then show** *?lhs*
by (*meson* $*$ *assms separate-symm*)
qed

Compute separation in case the addresses and sizes are immediate values.

lemmas *separate-as-linarith-numeral* [*simp*] =
separate-as-linarith [*of numeral a*::′*a word numeral si numeral a′*::′*a word numeral si′*] **for** *a si a′ si′*
lemmas *separate-as-linarith-numeral-1* [*simp*] =
separate-as-linarith [*of numeral a*::′*a word numeral si numeral a′*::′*a word Suc 0*] **for** *a si a′*
lemmas *separate-as-linarith-numeral1-* [*simp*] =
separate-as-linarith [*of numeral a*::′*a word Suc 0 numeral a′*::′*a word numeral si′*] **for** *a a′ si′*
lemmas *separate-as-linarith-numeral11* [*simp*] =
separate-as-linarith [*of numeral a*::′*a word Suc 0 numeral a′*::′*a word Suc 0*] **for** *a a′*
lemmas *region-overflow-numeral*[*simp*] =
region-overflow-def [*of numeral a*::′*a word numeral si*] **for** *a si*

lemmas *region-overflow-numeral1* [*simp*] =
region-overflow-def [*of numeral a*::'*a word Suc 0*] **for** *a*

lemma *separate-plus-none*:
assumes $si' \leq$ *unat offset*
and $0 < si$
and $0 < si'$
and *unat offset* + *si* $\leq$ *2^LENGTH*('*a*)
shows *separate* (*offset* + *a*) *si a si'*
using *assms* **apply** (*auto simp add*: *separate-iff*)
by (*smt* (*verit*) *Nat.diff-diff-right add.commute add-leD1 diff-0 diff-is-0-eq diff-zero not-gr-zero unat-sub-if' unsigned-0*)

lemmas *unat-minus* = *unat-sub-if'*[*of 0*,*simplified*]

lemma *separate-minus-minus'*:
assumes $si \neq 0$
and $si' \neq 0$
and *unat offset* $\geq$ *si*
and *unat offset'* $\geq$ *si'*
and *unat offset* − *si* $\geq$ *unat offset'*
shows *separate* (*a* − *offset*) *si* (*a* − *offset'*) *si'*
using *assms* **apply** (*auto simp add*: *separate-iff*)
apply (*metis Nat.le-diff-conv2 add.commute add-leD2 unat-sub-if'*)
by (*smt* (*verit*) *add.commute add-diff-cancel-right' diff-add-cancel diff-le-self diff-less less-le-trans nat-le-linear not-le unat-sub-if'*)

lemma *separate-minus-minus*:
assumes $si \neq 0$
and $si' \neq 0$
and *unat offset* $\geq$ *si*
and *unat offset'* $\geq$ *si'*
and *unat offset* − *si* $\geq$ *unat offset'* $\vee$ *unat offset'* − *si'* $\geq$ *unat offset*
shows *separate* (*a* − *offset*) *si* (*a* − *offset'*) *si'*
by (*meson assms separate-minus-minus' separate-symm*)

lemma *separate-minus-none*:
assumes $si \neq 0$
and $si' \neq 0$
and *unat offset* $\geq$ *si*
and $si' \leq$ *2^LENGTH*('*a*) − *unat offset*
shows *separate* (*a* − *offset*) *si a si'*
proof −
have $0 < si$ **and** $0 < si'$
using *assms*(*1*,*2*) **by** *blast+*
moreover have ¬ *offset* $\leq$ *0*
using *assms*(*1*) *assms*(*3*) **by** *fastforce*
ultimately show *?thesis*

by (*simp add*: *assms*(*3*) *assms*(*4*) *local.unat-minus separate-iff unsigned-eq-0-iff*)
qed

The following theorems are used during symbolic execution to determine whether two regions are separate.

lemmas *separate-simps* = *separate-plus-none separate-minus-none separate-minus-minus*

end
end

3 Concrete state and instructions

theory *State*
imports *Main Memory*
begin

A state consists of registers, memory, flags and a rip. Some design considerations here:

- All register values are 256 bits. We could also distinguish 64 bits registers, 128 registers etc. That would increase complexity in proofs and datastructures. The cost of using 256 everywhere is that a goal typically will have some casted 64 bits values.
- The instruction pointer RIP is a special 64-bit register outside of the normal register set.
- Strings are used for registers and flags. We would prefer an enumerative datatype, however, that would be extremely slow since there are roughly 100 register names.

```
record state =
  regs  :: string  ⇒ 256word
  mem   :: 64 word ⇒ 8 word
  flags :: string  ⇒ bool
  rip   :: 64 word

definition real-reg :: string ⇒ bool × string × nat × nat
  where real-reg reg ≡
  — TODO: xmm, ymm, etc.
  case reg of
  — rip
    ''rip''  ⇒ (True,  ''rip'', 0,64)
  — rax,rbx,rcx,rdx
  | ''rax''  ⇒ (True,  ''rax'', 0,64)
  | ''eax''  ⇒ (True,  ''rax'', 0,32)
  | ''ax''   ⇒ (False, ''rax'', 0,16)
  | ''ah''   ⇒ (False, ''rax'', 8,16)
```

| $''al''$ $\Rightarrow$ (*False*, $''rax''$, *0*,*8*)
| $''rbx''$ $\Rightarrow$ (*True*, $''rbx''$, *0*,*64*)
| $''ebx''$ $\Rightarrow$ (*True*, $''rbx''$, *0*,*32*)
| $''bx''$ $\Rightarrow$ (*False*, $''rbx''$, *0*,*16*)
| $''bh''$ $\Rightarrow$ (*False*, $''rbx''$, *8*,*16*)
| $''bl''$ $\Rightarrow$ (*False*, $''rbx''$, *0*,*8*)
| $''rcx''$ $\Rightarrow$ (*True*, $''rcx''$, *0*,*64*)
| $''ecx''$ $\Rightarrow$ (*True*, $''rcx''$, *0*,*32*)
| $''cx''$ $\Rightarrow$ (*False*, $''rcx''$, *0*,*16*)
| $''ch''$ $\Rightarrow$ (*False*, $''rcx''$, *8*,*16*)
| $''cl''$ $\Rightarrow$ (*False*, $''rcx''$, *0*,*8*)
| $''rdx''$ $\Rightarrow$ (*True*, $''rdx''$, *0*,*64*)
| $''edx''$ $\Rightarrow$ (*True*, $''rdx''$, *0*,*32*)
| $''dx''$ $\Rightarrow$ (*False*, $''rdx''$, *0*,*16*)
| $''dh''$ $\Rightarrow$ (*False*, $''rdx''$, *8*,*16*)
| $''dl''$ $\Rightarrow$ (*False*, $''rdx''$, *0*,*8*)
— RBP, RSP
| $''rbp''$ $\Rightarrow$ (*True*, $''rbp''$, *0*,*64*)
| $''ebp''$ $\Rightarrow$ (*True*, $''rbp''$, *0*,*32*)
| $''bp''$ $\Rightarrow$ (*False*, $''rbp''$, *0*,*16*)
| $''bpl''$ $\Rightarrow$ (*False*, $''rbp''$, *0*,*8*)
| $''rsp''$ $\Rightarrow$ (*True*, $''rsp''$, *0*,*64*)
| $''esp''$ $\Rightarrow$ (*True*, $''rsp''$, *0*,*32*)
| $''sp''$ $\Rightarrow$ (*False*, $''rsp''$, *0*,*16*)
| $''spl''$ $\Rightarrow$ (*False*, $''rsp''$, *0*,*8*)
— RDI, RSI, R8 to R15
| $''rdi''$ $\Rightarrow$ (*True*, $''rdi''$, *0*,*64*)
| $''edi''$ $\Rightarrow$ (*True*, $''rdi''$, *0*,*32*)
| $''di''$ $\Rightarrow$ (*False*, $''rdi''$, *0*,*16*)
| $''dil''$ $\Rightarrow$ (*False*, $''rdi''$, *0*,*8*)
| $''rsi''$ $\Rightarrow$ (*True*, $''rsi''$, *0*,*64*)
| $''esi''$ $\Rightarrow$ (*True*, $''rsi''$, *0*,*32*)
| $''si''$ $\Rightarrow$ (*False*, $''rsi''$, *0*,*16*)
| $''sil''$ $\Rightarrow$ (*False*, $''rsi''$, *0*,*8*)
| $''r15''$ $\Rightarrow$ (*True*, $''r15''$, *0*,*64*)
| $''r15d''$ $\Rightarrow$ (*True*, $''r15''$, *0*,*32*)
| $''r15w''$ $\Rightarrow$ (*False*, $''r15''$, *0*,*16*)
| $''r15b''$ $\Rightarrow$ (*False*, $''r15''$, *0*,*8*)
| $''r14''$ $\Rightarrow$ (*True*, $''r14''$, *0*,*64*)
| $''r14d''$ $\Rightarrow$ (*True*, $''r14''$, *0*,*32*)
| $''r14w''$ $\Rightarrow$ (*False*, $''r14''$, *0*,*16*)
| $''r14b''$ $\Rightarrow$ (*False*, $''r14''$, *0*,*8*)
| $''r13''$ $\Rightarrow$ (*True*, $''r13''$, *0*,*64*)
| $''r13d''$ $\Rightarrow$ (*True*, $''r13''$, *0*,*32*)
| $''r13w''$ $\Rightarrow$ (*False*, $''r13''$, *0*,*16*)
| $''r13b''$ $\Rightarrow$ (*False*, $''r13''$, *0*,*8*)
| $''r12''$ $\Rightarrow$ (*True*, $''r12''$, *0*,*64*)
| $''r12d''$ $\Rightarrow$ (*True*, $''r12''$, *0*,*32*)
| $''r12w''$ $\Rightarrow$ (*False*, $''r12''$, *0*,*16*)

```
| ''r12b'' ⇒ (False, ''r12'', 0,8)
| ''r11'' ⇒ (True, ''r11'', 0,64)
| ''r11d'' ⇒ (True, ''r11'', 0,32)
| ''r11w'' ⇒ (False, ''r11'', 0,16)
| ''r11b'' ⇒ (False, ''r11'', 0,8)
| ''r10'' ⇒ (True, ''r10'', 0,64)
| ''r10d'' ⇒ (True, ''r10'', 0,32)
| ''r10w'' ⇒ (False, ''r10'', 0,16)
| ''r10b'' ⇒ (False, ''r10'', 0,8)
| ''r9'' ⇒ (True, ''r9'' , 0,64)
| ''r9d'' ⇒ (True, ''r9'' , 0,32)
| ''r9w'' ⇒ (False, ''r9'' , 0,16)
| ''r9b'' ⇒ (False, ''r9'' , 0,8)
| ''r8'' ⇒ (True, ''r8'' , 0,64)
| ''r8d'' ⇒ (True, ''r8'' , 0,32)
| ''r8w'' ⇒ (False, ''r8'' , 0,16)
| ''r8b'' ⇒ (False, ''r8'' , 0,8)
— xmm
| ''xmm0'' ⇒ (True, ''xmm0'' , 0,128)
| ''xmm1'' ⇒ (True, ''xmm1'' , 0,128)
| ''xmm2'' ⇒ (True, ''xmm2'' , 0,128)
| ''xmm3'' ⇒ (True, ''xmm3'' , 0,128)
| ''xmm4'' ⇒ (True, ''xmm4'' , 0,128)
| ''xmm5'' ⇒ (True, ''xmm5'' , 0,128)
| ''xmm6'' ⇒ (True, ''xmm6'' , 0,128)
| ''xmm7'' ⇒ (True, ''xmm7'' , 0,128)
| ''xmm8'' ⇒ (True, ''xmm8'' , 0,128)
| ''xmm9'' ⇒ (True, ''xmm9'' , 0,128)
| ''xmm10'' ⇒ (True, ''xmm10'' , 0,128)
| ''xmm11'' ⇒ (True, ''xmm11'' , 0,128)
| ''xmm12'' ⇒ (True, ''xmm12'' , 0,128)
| ''xmm13'' ⇒ (True, ''xmm13'' , 0,128)
| ''xmm14'' ⇒ (True, ''xmm14'' , 0,128)
| ''xmm15'' ⇒ (True, ''xmm15'' , 0,128)
```

x86 has register aliassing. For example, register EAX is the lower 32 bits of register RAX. This function map register aliasses to the "real" register. For example:

real-reg ''ah'' = (*False*, *''rax''*, *8*, *16*).

This means that register AH is the second byte (bits 8 to 16) of register RAX. The bool *False* indicates that writing to AH does not overwrite the remainder of RAX.

real-reg ''eax'' = (*True*, *''rax''*, *0*, *32*).

Register EAX is the lower 4 bytes of RAX. Writing to EAX means overwritting the remainder of RAX with zeroes.

definition *reg-size* :: *string* ⇒ *nat* — in bytes
where *reg-size reg* ≡ *let* (-,-,*l*,*h*) = *real-reg reg in* (*h* − *l*) *div 8*

We now define functions for reading and writing from state.

definition *reg-read* :: *state* ⇒ *string* ⇒ *256 word*
  where *reg-read* σ *reg* ≡
    if reg = ''*rip*'' *then ucast* (*rip* σ) *else*
    if reg = '''' *then 0 else* — happens if no base register is used in an address
    let (-,*r*,*l*,*h*) = *real-reg reg in*
      ⟨*l*,*h*⟩(*regs* σ *r*)

primrec *fromBool* :: *bool* ⇒ *'a* :: *len word*
  where
    fromBool True = *1*
  | *fromBool False* = *0*

definition *flag-read* :: *state* ⇒ *string* ⇒ *256 word*
  where *flag-read* σ *flag* ≡ *fromBool* (*flags* σ *flag*)

definition *mem-read* :: *state* ⇒ *64 word* ⇒ *nat* ⇒ *256 word*
  where *mem-read* σ *a si* ≡ *word-rcat* (*read-bytes* (*mem* σ) *a si*)

Doing state-updates occur through a tiny deeply embedded language of state updates. This allows us to reason over state updates through theorems.

datatype *StateUpdate* =
    RegUpdate string 256 word — Write value to register
  | *FlagUpdate string* ⇒ *bool* — Update all flags at once
  | *RipUpdate 64 word* — Update instruction pointer with address
  | *MemUpdate 64 word nat 256 word* — Write a number of bytes of a value to the address

primrec *state-update*
  where
    state-update (*RegUpdate reg val*) = (λ σ . σ⦇*regs* := (*regs* σ)(*reg* := *val*)⦈)
  | *state-update* (*FlagUpdate val*) = (λ σ . σ⦇*flags* := *val*⦈)
  | *state-update* (*RipUpdate a*) = (λ σ . σ⦇*rip* := *a*⦈)
  | *state-update* (*MemUpdate a si val*) = (λ σ .
      let new = (λ *a'* . *take-byte* (*unat* (*a'* − *a*)) *val*) *in*
       σ⦇*mem* := *override-on* (*mem* σ) *new* (*region-addresses a si*)⦈)

abbreviation *RegUpdateSyntax* (‹- :=$_r$ -› *30*)
  where *RegUpdateSyntax reg val* ≡ *RegUpdate reg val*
abbreviation *MemUpdateSyntax* (‹⟦-,-⟧ :=$_m$ -› *30*)
  where *MemUpdateSyntax a si val* ≡ *MemUpdate a si val*
abbreviation *FlagUpdateSyntax* (‹*setFlags*›)
  where *FlagUpdateSyntax val* ≡ *FlagUpdate val*
abbreviation *RipUpdateSyntax* (‹*setRip*›)
  where *RipUpdateSyntax val* ≡ *RipUpdate val*

Executes a write to a register in terms of the tiny deeply embedded language above.

definition *reg-write*

where *reg-write reg val* $\sigma \equiv$
let (b,r,l,h) = *real-reg reg*;
curr-val = *reg-read* σ *r*;
new-val = *if b then val else overwrite l h curr-val val in*
state-update (*RegUpdate r new-val*) σ

A datatype for operands of instructions.

datatype *Operand* =
Imm 256 word
| *Reg string*
| *Flag string*
| *Mem nat 64 word string string nat*
— size offset base-reg index-reg scale

abbreviation *mem-op-no-offset-no-index* :: *string* ⇒ (*64 word* × *string* × *string* × *nat*) (‹[-]$_1$› *40*)
where *mem-op-no-offset-no-index r* ≡ (*0*,*r*,[],*0*)

abbreviation *mem-op-no-index* :: *64 word* ⇒ *string* ⇒ (*64 word* × *string* × *string* × *nat*) (‹[- + -]$_2$› *40*)
where *mem-op-no-index offset r* ≡ (*offset*,*r*,[],*0*)

abbreviation *mem-op* :: *64 word* ⇒ *string* ⇒ *string* ⇒ *nat* ⇒ (*64 word* × *string* × *string* × *nat*) (‹[- + - + - ∗ -]$_3$› *40*)
where *mem-op offset r index scale*≡ (*offset*,*r*,*index*,*scale*)

definition *ymm-ptr* (‹*YMMWORD PTR* -›)
where *YMMWORD PTR x* ≡ *case x of* (*offset*,*base*,*index*,*scale*) ⇒ *Mem 32 offset base index scale*

definition *xmm-ptr* (‹*XMMWORD PTR* -›)
where *XMMWORD PTR x* ≡ *case x of* (*offset*,*base*,*index*,*scale*) ⇒ *Mem 16 offset base index scale*

definition *qword-ptr* (‹*QWORD PTR* -›)
where *QWORD PTR x* ≡ *case x of* (*offset*,*base*,*index*,*scale*) ⇒ *Mem 8 offset base index scale*

definition *dword-ptr* (‹*DWORD PTR* -›)
where *DWORD PTR x* ≡ *case x of* (*offset*,*base*,*index*,*scale*) ⇒ *Mem 4 offset base index scale*

definition *word-ptr* (‹*WORD PTR* -›)
where *WORD PTR x* ≡ *case x of* (*offset*,*base*,*index*,*scale*) ⇒ *Mem 2 offset base index scale*

definition *byte-ptr* (‹*BYTE PTR* -›)
where *BYTE PTR x* ≡ *case x of* (*offset*,*base*,*index*,*scale*) ⇒ *Mem 1 offset base index scale*

primrec (*nonexhaustive*) *operand-size* :: *Operand* ⇒ *nat* — in bytes
where
operand-size (*Reg r*) = *reg-size r*
| *operand-size* (*Mem si - - - -*) = *si*

fun *resolve-address* :: *state* ⇒ *64 word* ⇒ *char list* ⇒ *char list* ⇒ *nat* ⇒ *64 word*
where *resolve-address* σ *offset base index scale* =
(*let i* = *ucast* (*reg-read* σ *index*);
b = *ucast* (*reg-read* σ *base*) *in*
offset + *b* + *of-nat scale*∗*i*)

primrec *operand-read* :: *state* ⇒ *Operand* ⇒ *256 word*
where
operand-read σ (*Imm i*) = *i*
| *operand-read* σ (*Reg r*) = *reg-read* σ *r*
| *operand-read* σ (*Flag f*) = *flag-read* σ *f*
| *operand-read* σ (*Mem si offset base index scale*) =
(*let a* = *resolve-address* σ *offset base index scale in*
mem-read σ *a si*
)

primrec *state-with-updates* :: *state* ⇒ *StateUpdate list* ⇒ *state* (**infixl** ‹*with*› *66*)
where
σ *with* [] = σ
| (σ *with* (*f*#*fs*)) = *state-update f* (σ *with fs*)

primrec (*nonexhaustive*) *operand-write* :: *Operand* ⇒ *256word* ⇒ *state* ⇒ *state*
where
operand-write (*Reg r*) *v* σ = *reg-write r v* σ
| *operand-write* (*Mem si offset base index scale*) *v* σ =
(*let i* = *ucast* (*reg-read* σ *index*);
b = *ucast* (*reg-read* σ *base*);
a = *offset* + *b* + *of-nat scale*∗*i in*
σ *with* [$[\![a,si]\!] :=_m v$]
)

The following theorems simplify reading from state parts after doing updates to other state parts.

lemma *regs-reg-write*:
shows *regs* (σ *with* (($r :=_r w$)#*updates*)) r' = (*if* $r{=}r'$ *then w else regs* (σ *with updates*) r')
by (*induct updates arbitrary*: σ, *auto simp add*: *case-prod-unfold Let-def*)

lemma *regs-mem-write*:
shows *regs* (σ *with* (($[\![a,si]\!] :=_m v$)#*updates*)) *r* = *regs* (σ *with updates*) *r*
by (*induct updates arbitrary*: σ, *auto*)

lemma *regs-flag-write*:
  shows *regs* (σ *with* ((*setFlags v*)#*updates*)) *r* = *regs* (σ *with updates*) *r*
  by (*induct updates arbitrary*: σ, *auto*)

lemma *regs-rip-write*:
  shows *regs* (σ *with* ((*setRip a*)#*updates*)) *f* = *regs* (σ *with updates*) *f*
  by (*auto*)

lemma *flag-read-reg-write*:
  shows *flag-read* (σ *with* (($r :=_r w$)#*updates*)) *f* = *flag-read* (σ *with updates*) *f*
  by (*induct updates arbitrary*: σ, *auto simp add*: *flag-read-def*)

lemma *flag-read-mem-write*:
  shows *flag-read* (σ *with* (($[\![a,si]\!] :=_m v$)#*updates*)) *f* = *flag-read* (σ *with updates*) *f*
  by (*induct updates arbitrary*: σ, *auto simp add*: *flag-read-def*)

lemma *flag-read-flag-write*:
  shows *flag-read* (σ *with* ((*setFlags v*)#*updates*)) = *fromBool o v*
  by (*induct updates arbitrary*: σ, *auto simp add*: *flag-read-def*)

lemma *flag-read-rip-write*:
  shows *flag-read* (σ *with* ((*setRip a*)#*updates*)) *f* = *flag-read* (σ *with updates*) *f*
  by (*auto simp add*: *flag-read-def*)

lemma *mem-read-reg-write*:
  shows *mem-read* (σ *with* (($r :=_r w$)#*updates*)) *a si* = *mem-read* (σ *with updates*) *a si*
  by (*auto simp add*: *mem-read-def read-bytes-def*)

lemma *mem-read-flag-write*:
  shows *mem-read* (σ *with* ((*setFlags v*)#*updates*)) *a si* = *mem-read* (σ *with updates*) *a si*
  by (*auto simp add*: *mem-read-def read-bytes-def*)

lemma *mem-read-rip-write*:
  shows *mem-read* (σ *with* ((*setRip a'*)#*updates*)) *a si* = *mem-read* (σ *with updates*) *a si*
  by (*auto simp add*: *mem-read-def read-bytes-def*)

lemma *mem-read-mem-write-alias*:
  assumes $si' \le si$
    and $si \le 2\hat{}64$
  shows *mem-read* (σ *with* (($[\![a,si]\!] :=_m v$)#*updates*)) *a si'* = $\langle 0,si'*8\rangle$ *v*
  using *assms*
  by (*auto simp add*: *mem-read-def word-rcat-read-bytes read-bytes-override-on-enclosed*[**where** *offset=0* **and** *offset'=0*,*simplified*])

lemma *mem-read-mem-write-separate*:
  assumes *separate a si a′ si′*
shows *mem-read* (σ *with* (($[\![a,si]\!] :=_m v$)#*updates*)) *a′ si′* = *mem-read* (σ *with updates*) *a′ si′*
  using *assms*
  by (*auto simp add*: *mem-read-def read-bytes-override-on-separate*)

lemma *mem-read-mem-write-enclosed-minus*:
  assumes $offset' \leq offset$
    and $si' \leq si$
    and *unat* $(offset - offset') + si' < 2^{64}$
    and *unat offset* $+ si' \leq si +$ *unat offset′*
  shows *mem-read* (σ *with* (($[\![a - offset,si]\!] :=_m v$)#*updates*)) $(a - offset')$ si' = $\langle$*unat* $(offset - offset') * 8$,*unat* $(offset - offset') * 8 + si' * 8\rangle v$
  using *assms*
  by (*auto simp add*: *mem-read-def read-bytes-override-on-enclosed word-rcat-read-bytes-enclosed*[*of offset* − *offset′ si′ a* − *offset v*,*simplified*])

lemma *mem-read-mem-write-enclosed-plus*:
assumes *unat offset* $+ si' \leq si$
    and $si < 2 \mathbin{\char`\^} 64$
  shows *mem-read* (σ *with* (($[\![a,si]\!] :=_m v$)#*updates*)) $(offset + a)$ si' = $\langle$*unat offset* $* 8$,(*unat offset* $+ si') * 8\rangle v$
  using *assms*
  apply (*auto simp add*: *mem-read-def read-bytes-override-on-enclosed-plus*)
  using *word-rcat-read-bytes-enclosed*[*of offset si′ a v*]
  by *auto* (*simp add*: *add.commute*)

lemma *mem-read-mem-write-enclosed-plus2*:
assumes *unat offset* $+ si' \leq si$
    and $si < 2 \mathbin{\char`\^} 64$
  shows *mem-read* (σ *with* (($[\![a,si]\!] :=_m v$)#*updates*)) $(a + offset)$ si' = $\langle$*unat offset* $* 8$,(*unat offset* $+ si') * 8\rangle v$
  using *mem-read-mem-write-enclosed-plus*[*OF assms*]
  by (*auto simp add*: *add.commute*)

lemma *mem-read-mem-write-enclosed-numeral*[*simp*]:
assumes *unat* (*numeral a′* − *numeral a*::*64 word*) + (*numeral si′*::*nat*) ≤ *numeral si*
    and *numeral a′* ≥ (*numeral a*::*64 word*)
    and *numeral si* < ($2 \mathbin{\char`\^} 64$::*nat*)
  shows *mem-read* (σ *with* (($[\![numeral\ a,numeral\ si]\!] :=_m v$)#*updates*)) (*numeral a′*) (*numeral si′*) = $\langle$*unat* (*numeral a′* − (*numeral a*::*64 word*)) $* 8$,(*unat* (*numeral a′* − (*numeral a*::*64 word*)) + (*numeral si′*)) $* 8\rangle v$
proof−
  have *1*: *numeral a* + (*numeral a′* − *numeral a*) = (*numeral a′*::*64 word*)
    using *assms*(*2*) **by** (*metis add.commute diff-add-cancel*)
  thus *?thesis*

using *mem-read-mem-write-enclosed-plus2*[*of numeral a' − numeral a numeral si' numeral si σ numeral a v updates*,*OF assms*(*1*,*3*)]
by *auto*
qed

lemma *mem-read-mem-write-enclosed-numeral1-*[*simp*]:
assumes *unat* (*numeral a' − numeral a*::*64 word*) + (*numeral si'*::*nat*) ≤ *Suc 0*
and *numeral a'* ≥ (*numeral a*::*64 word*)
shows *mem-read* (*σ with* ((⟦*numeral a*,*Suc 0*⟧ :=$_m$ *v*)#*updates*)) (*numeral a'*) (*numeral si'*) = ⟨*unat* (*numeral a'* − (*numeral a*::*64 word*)) ∗ *8*,(*unat* (*numeral a'* − (*numeral a*::*64 word*)) + (*numeral si'*)) ∗ *8*⟩*v*
proof−
have *1*: *numeral a* + (*numeral a'* − *numeral a*) = (*numeral a'*::*64 word*)
using *assms*(*2*) **by** (*metis add.commute diff-add-cancel*)
thus *?thesis*
using *mem-read-mem-write-enclosed-plus2*[*of numeral a' − numeral a numeral si' Suc 0 σ numeral a v updates*,*OF assms*(*1*)]
by *auto*
qed

lemma *mem-read-mem-write-enclosed-numeral-1*[*simp*]:
assumes *unat* (*numeral a' − numeral a*::*64 word*) + (*Suc 0*) ≤ *numeral si*
and *numeral a'* ≥ (*numeral a*::*64 word*)
and *numeral si* < (*2* ^ *64*::*nat*)
shows *mem-read* (*σ with* ((⟦*numeral a*,*numeral si*⟧ :=$_m$ *v*)#*updates*)) (*numeral a'*) (*Suc 0*) = ⟨*unat* (*numeral a'* − (*numeral a*::*64 word*)) ∗ *8*,(*unat* (*numeral a'* − (*numeral a*::*64 word*)) + (*Suc 0*)) ∗ *8*⟩*v*
proof−
have *1*: *numeral a* + (*numeral a'* − *numeral a*) = (*numeral a'*::*64 word*)
using *assms*(*2*) **by** (*metis add.commute diff-add-cancel*)
thus *?thesis*
using *mem-read-mem-write-enclosed-plus2*[*of numeral a' − numeral a Suc 0 numeral si σ numeral a v updates*,*OF assms*(*1*,*3*)]
by *auto*
qed

lemma *mem-read-mem-write-enclosed-numeral11*[*simp*]:
assumes *unat* (*numeral a' − numeral a*::*64 word*) + (*Suc 0*) ≤ *Suc 0*
and *numeral a'* ≥ (*numeral a*::*64 word*)
shows *mem-read* (*σ with* ((⟦*numeral a*,*Suc 0*⟧ :=$_m$ *v*)#*updates*)) (*numeral a'*) (*Suc 0*) = ⟨*unat* (*numeral a'* − (*numeral a*::*64 word*)) ∗ *8*,(*unat* (*numeral a'* − (*numeral a*::*64 word*)) + (*Suc 0*)) ∗ *8*⟩*v*
proof−
have *1*: *numeral a* + (*numeral a'* − *numeral a*) = (*numeral a'*::*64 word*)
using *assms*(*2*) **by** (*metis add.commute diff-add-cancel*)
thus *?thesis*
using *mem-read-mem-write-enclosed-plus2*[*of numeral a' − numeral a Suc 0 Suc 0 σ numeral a v updates*,*OF assms*(*1*)]

by *auto*
qed

lemma *rip-reg-write*[*simp*]:
shows *rip* (σ *with* ((r $:=_r$ v)#*updates*)) = *rip* (σ *with updates*)
by (*auto simp add*: *case-prod-unfold Let-def*)

lemma *rip-flag-write*[*simp*]:
shows *rip* (σ *with* ((*setFlags v*)#*updates*)) = *rip* (σ *with updates*)
by (*auto*)

lemma *rip-mem-write*[*simp*]:
shows *rip* (σ *with* (([[*a*,*si*]] $:=_m$ v)#*updates*)) = *rip* (σ *with updates*)
by (*auto*)

lemma *rip-rip-write*[*simp*]:
shows *rip* (σ *with* ((*setRip a*)#*updates*)) = *a*
by (*auto*)

lemma *with-with*:
shows (σ *with updates*) *with updates'* = σ *with* (*updates'* @ *updates*)
by (*induct updates' arbitrary*: σ,*auto*)

lemma *add-state-update-to-list*:
shows *state-update upd* (σ *with updates*) = σ *with* (*upd*#*updates*)
by *auto*

The updates performed to a state are ordered: memoery, registers, flags, rip. This function is basically insertion sort. Moreover, consecutive updates to the same register are removed.

fun *insert-state-update*
where
insert-state-update (*setRip a*) (*setRip a'*#*updates*) = *insert-state-update* (*setRip a*) *updates*
| *insert-state-update* (*setRip a*) (*setFlags v*#*updates*) = *setFlags v* # (*insert-state-update* (*setRip a*) *updates*)
| *insert-state-update* (*setRip a*) ((r $:=_r$ v)#*updates*) = (r $:=_r$ v) # (*insert-state-update* (*setRip a*) *updates*)
| *insert-state-update* (*setRip a*) (([[*a'*,*si*]] $:=_m$ v)#*updates*) = ([[*a'*,*si*]] $:=_m$ v) # (*insert-state-update* (*setRip a*) *updates*)

| *insert-state-update* (*setFlags v*) (*setFlags v'*#*updates*) = *insert-state-update* (*setFlags v*) *updates*
| *insert-state-update* (*setFlags v*) ((r $:=_r$ v')#*updates*) = (r $:=_r$ v') # *insert-state-update* (*setFlags v*) *updates*
| *insert-state-update* (*setFlags v*) (([[*a'*,*si*]] $:=_m$ v')#*updates*) = ([[*a'*,*si*]] $:=_m$ v') # *insert-state-update* (*setFlags v*) *updates*

| *insert-state-update* $((r :=_r v))$ $((r' :=_r v')$#*updates*$)$ = (*if* $r = r'$ *then insert-state-update* $(r :=_r v)$ *updates else* $(r' :=_r v')$#*insert-state-update* $(r :=_r v)$ *updates*)
| *insert-state-update* $((r :=_r v))$ $((\llbracket a',si \rrbracket :=_m v')$#*updates*$)$ = $(\llbracket a',si \rrbracket :=_m v')$ # *insert-state-update* $(r :=_r v)$ *updates*

| *insert-state-update upd updates* = *upd* # *updates*

fun *clean*
where
clean [] = []
| *clean* [*upd*] = [*upd*]
| *clean* (*upd*#*upd'*#*updates*) = *insert-state-update upd* (*clean* (*upd'*#*updates*))

lemma *insert-state-update*:
shows σ *with* (*insert-state-update upd updates*) = σ *with* (*upd* # *updates*)
by (*induct updates rule*: *insert-state-update.induct*,*auto simp add*: *fun-upd-twist*)

lemma *clean-state-updates*:
shows σ *with* (*clean updates*) = σ *with updates*
by (*induct updates rule*: *clean.induct*,*auto simp add*: *insert-state-update*)

The set of simplification rules used during symbolic execution.

lemmas *state-simps* =
qword-ptr-def dword-ptr-def word-ptr-def byte-ptr-def reg-size-def
reg-write-def real-reg-def reg-read-def

regs-rip-write regs-mem-write regs-reg-write regs-flag-write
flag-read-reg-write flag-read-mem-write flag-read-rip-write flag-read-flag-write
mem-read-reg-write mem-read-flag-write mem-read-rip-write
mem-read-mem-write-alias mem-read-mem-write-separate
mem-read-mem-write-enclosed-minus mem-read-mem-write-enclosed-plus mem-read-mem-write-enclosed-pl

with-with add-state-update-to-list

declare *state-with-updates.simps*(*2*)[*simp del*]
declare *state-update.simps*[*simp del*]

end

4 Instruction Semantics

theory *X86-InstructionSemantics*
imports *State*
begin

A datatype for storing instructions. Note that we add a special kind of meta-instruction, called ExternalFunc. A call to an external function can manually be mapped to a manually supplied state transformation function.

datatype I =
  Instr string Operand option Operand option Operand option 64 word
  | *ExternalFunc state* $\Rightarrow$ *state*

A datatype for the result of floating point comparisons.

datatype *FP-Order* = *FP-Unordered* | *FP-GT* | *FP-LT* | *FP-EQ*

abbreviation *instr-next i* $\equiv$ *case i of* (*Instr - - - - a'*) $\Rightarrow$ *a'*

locale *unknowns* =
  fixes *unknown-addsd* :: *64 word* $\Rightarrow$ *64 word* $\Rightarrow$ *64 word*
    and *unknown-subsd* :: *64 word* $\Rightarrow$ *64 word* $\Rightarrow$ *64 word*
    and *unknown-mulsd* :: *64 word* $\Rightarrow$ *64 word* $\Rightarrow$ *64 word*
    and *unknown-divsd* :: *64 word* $\Rightarrow$ *64 word* $\Rightarrow$ *64 word*
    and *unknown-ucomisd* :: *64 word* $\Rightarrow$ *64 word* $\Rightarrow$ *FP-Order*
    and *unknown-semantics* :: $I \Rightarrow$ *state* $\Rightarrow$ *state*
    and *unknown-flags* :: *string* $\Rightarrow$ *string* $\Rightarrow$ *bool*
begin

The semantics below are intended to be overapproximative and incomplete. This is achieved using locale "unknowns". Any place where semantics is *not* modelled, it is mapped to a universally quantified uninterpreted function from that locale. We do not make use of *undefined*, since that could be used to prove that the semantics of two undefined behaviors are equivalent. For example:

- Only a subset of instructions has semantics. In case of an unknown instruction i, the function *semantics* below will result in *unknown-semantics i*.
- Not all flags have been defined. In case a flag is read whose semantics is not defined below, the read will resolve to *unknown-flags i f*. Note that if the semantics of an instruction do not set flags, an overapproximative semantics such as below imply that the instruction indeed does not modify flags. In order words, if we were uncertain we would assign unknown values to flags.
- Not all operations have been defined. For example, floating points operations have no executable semantics, but are mapped to uninterpreted functions such as *unknown-addsd*.

Moves

definition *semantics-MOV* :: *Operand* $\Rightarrow$ *Operand* $\Rightarrow$ *state* $\Rightarrow$ *state*
  where *semantics-MOV op1 op2* σ $\equiv$
      let src = *operand-read* σ *op2 in*

operand-write op1 src σ

abbreviation *MOV*
where *MOV op1 op2* ≡ *Instr ''mov'' (Some op1) (Some op2) None*

abbreviation *MOVABS*
where *MOVABS op1 op2* ≡ *Instr ''movabs'' (Some op1) (Some op2) None*

abbreviation *MOVAPS*
where *MOVAPS op1 op2* ≡ *Instr ''movaps'' (Some op1) (Some op2) None*

abbreviation *MOVZX*
where *MOVZX op1 op2* ≡ *Instr ''movzx'' (Some op1) (Some op2) None*

abbreviation *MOVDQU*
where *MOVDQU op1 op2* ≡ *Instr ''movdqu'' (Some op1) (Some op2) None*

definition *semantics-MOVD* :: *Operand* ⇒ *Operand* ⇒ *state* ⇒ *state*
where *semantics-MOVD op1 op2 σ* ≡
let src = ucast(operand-read σ op2)::32word in
operand-write op1 (ucast src) σ

abbreviation *MOVD*
where *MOVD op1 op2* ≡ *Instr ''movd'' (Some op1) (Some op2) None*

fun *isXMM* :: *Operand* ⇒ *bool*
where *isXMM (Reg r) = (take 3 r = ''xmm'')*
| *isXMM - = False*

definition *semantics-MOVSD* :: *Operand* ⇒ *Operand* ⇒ *state* ⇒ *state*
where *semantics-MOVSD op1 op2 σ* ≡
if isXMM op1 ∧ isXMM op2 then
let src = ⟨0,64⟩operand-read σ op2;
dst = ⟨64,128⟩operand-read σ op1 in
operand-write op1 (overwrite 0 64 dst src) σ
else
let src = ⟨0,64⟩operand-read σ op2 in
operand-write op1 src σ

abbreviation *MOVSD*
where *MOVSD op1 op2* ≡ *Instr ''movsd'' (Some op1) (Some op2) None*

abbreviation *MOVQ*
where *MOVQ op1 op2* ≡ *Instr ''movq'' (Some op1) (Some op2) None*

lea/push/pop/call/ret/leave

definition *semantics-LEA* :: *Operand* ⇒ *Operand* ⇒ *state* ⇒ *state*
where *semantics-LEA op1 op2 σ* ≡
case op2 of Mem si offset base index scale ⇒

operand-write op1 (*ucast* (*resolve-address* σ *offset base index scale*)) σ

abbreviation *LEA*
where *LEA op1 op2* $\equiv$ *Instr ''lea''* (*Some op1*) (*Some op2*) *None*

definition *semantics-PUSH* :: *Operand* $\Rightarrow$ *state* $\Rightarrow$ *state*
where *semantics-PUSH op1* σ $\equiv$
let src = *operand-read* σ *op1*;
si = *operand-size op1*;
rsp = *ucast* (*ucast*(*reg-read* σ *''rsp''*) − *of-nat si* :: *64 word*) *in*
operand-write (*QWORD PTR* [*''rsp''*]$_1$) *src* (*operand-write* (*Reg ''rsp''*) *rsp* σ)

abbreviation *PUSH*
where *PUSH op1* $\equiv$ *Instr ''push''* (*Some op1*) *None None*

definition *semantics-POP* :: *Operand* $\Rightarrow$ *state* $\Rightarrow$ *state*
where *semantics-POP op1* σ $\equiv$
let si = *operand-size op1*;
src = *operand-read* σ (*QWORD PTR* [*''rsp''*]$_1$);
rsp = *ucast* (*ucast*(*reg-read* σ *''rsp''*) + *of-nat si*::*64 word*) *in*
operand-write op1 src (*operand-write* (*Reg ''rsp''*) *rsp* σ)

abbreviation *POP*
where *POP op1* $\equiv$ *Instr ''pop''* (*Some op1*) *None None*

definition *semantics-CALL* :: *Operand* $\Rightarrow$ *state* $\Rightarrow$ *state*
where *semantics-CALL op1* σ $\equiv$
let src = *ucast* (*operand-read* σ *op1*) *in*
(*state-update* (*setRip src*) *o semantics-PUSH* (*Reg ''rip''*)) σ

definition *semantics-RET* :: *state* $\Rightarrow$ *state*
where *semantics-RET* σ $\equiv$
let a = *ucast* (*operand-read* σ (*QWORD PTR* [*''rsp''*]$_1$));
rsp = *ucast* (*reg-read* σ *''rsp''*) + *8* :: *64 word in*
(*state-update* (*setRip a*) *o operand-write* (*Reg ''rsp''*) (*ucast rsp*)) σ

abbreviation *RET*
where *RET* $\equiv$ *Instr ''ret'' None None None*

definition *semantics-LEAVE* :: *state* $\Rightarrow$ *state*
where *semantics-LEAVE* $\equiv$ *semantics-POP* (*Reg ''rbp''*) *o semantics-MOV* (*Reg ''rsp''*) (*Reg ''rbp''*)

abbreviation *LEAVE*
where *LEAVE op1* $\equiv$ *Instr ''pop''* (*Some op1*) *None None*

Generic operators

definition *unop* :: (*'a* ::*len word* $\Rightarrow$ *'a*::*len word*) $\Rightarrow$

$('a::len\ word\ \Rightarrow string \Rightarrow bool) \Rightarrow$
$Operand \Rightarrow state \Rightarrow state$
where $unop\ f\ g\ op1\ \sigma \equiv$
$let\ si\ = operand\text{-}size\ op1;$
$dst = ucast\ (operand\text{-}read\ \sigma\ op1)::'a::len\ word\ in$
$operand\text{-}write\ op1\ (ucast\ (f\ dst))\ (\sigma\ with\ [setFlags\ (g\ dst)])$

definition $binop :: ('a::len\ word \Rightarrow 'a\ ::len\ word \Rightarrow 'a::len\ word) \Rightarrow$
$('a::len\ word \Rightarrow 'a::len\ word\ \Rightarrow string \Rightarrow bool) \Rightarrow$
$Operand \Rightarrow Operand \Rightarrow state \Rightarrow state$
where $binop\ f\ g\ op1\ op2\ \sigma \equiv$
$let\ dst = ucast\ (operand\text{-}read\ \sigma\ op1)::'a::len\ word;$
$src = ucast\ (operand\text{-}read\ \sigma\ op2)::'a::len\ word\ in$
$operand\text{-}write\ op1\ (ucast\ (f\ dst\ src))\ (\sigma\ with\ [setFlags\ (g\ dst\ src)])$

definition $unop\text{-}no\text{-}flags :: ('a\ ::len\ word \Rightarrow 'a::len\ word) \Rightarrow Operand \Rightarrow state \Rightarrow$
$state$
where $unop\text{-}no\text{-}flags\ f\ op1\ \sigma \equiv$
$let\ dst = ucast\ (operand\text{-}read\ \sigma\ op1)::'a::len\ word\ in$
$operand\text{-}write\ op1\ (ucast\ (f\ dst))\ \sigma$

definition $binop\text{-}flags :: ('a::len\ word \Rightarrow 'a::len\ word\ \Rightarrow string \Rightarrow bool) \Rightarrow$
$Operand \Rightarrow Operand \Rightarrow state \Rightarrow state$
where $binop\text{-}flags\ g\ op1\ op2\ \sigma \equiv$
$let\ si\ = operand\text{-}size\ op1;$
$dst = ucast\ (operand\text{-}read\ \sigma\ op1)::'a::len\ word;$
$src = ucast\ (operand\text{-}read\ \sigma\ op2)::'a::len\ word\ in$
$\sigma\ with\ [setFlags\ (g\ dst\ src)]$

definition $binop\text{-}no\text{-}flags :: ('a::len\ word \Rightarrow 'a\ ::len\ word \Rightarrow 'a::len\ word) \Rightarrow$
$Operand \Rightarrow Operand \Rightarrow state \Rightarrow state$
where $binop\text{-}no\text{-}flags\ f\ op1\ op2\ \sigma \equiv$
$let\ si\ = operand\text{-}size\ op1;$
$dst = ucast\ (operand\text{-}read\ \sigma\ op1)::'a::len\ word;$
$src = ucast\ (operand\text{-}read\ \sigma\ op2)::'a::len\ word\ in$
$operand\text{-}write\ op1\ (ucast\ (f\ dst\ src))\ \sigma$

definition $binop\text{-}XMM :: (64\ word \Rightarrow 64\ word \Rightarrow 64\ word) \Rightarrow Operand \Rightarrow Operand$
$\Rightarrow state \Rightarrow state$
where $binop\text{-}XMM\ f\ op1\ op2\ \sigma \equiv$
$let\ dst = ucast\ (operand\text{-}read\ \sigma\ op1)::64word;$
$src = ucast\ (operand\text{-}read\ \sigma\ op2)::64word\ in$
$operand\text{-}write\ op1\ (ucast\ (overwrite\ 0\ 64\ dst\ (f\ dst\ src)))\ \sigma$

Arithmetic

definition $ADD\text{-}flags :: 'a::len\ word \Rightarrow 'a::len\ word \Rightarrow string \Rightarrow bool$
where $ADD\text{-}flags\ w0\ w1\ flag \equiv case\ flag\ of$
$''zf'' \Rightarrow w0 + w1 = 0$
$|\ ''cf'' \Rightarrow unat\ w0 + unat\ w1 \geq 2\widehat{\ }(LENGTH('a))$

| ''of'' ⇒ (w0 <s 0 ⟷ w1 <s 0) ∧ ¬(w0 <s 0 ⟷ w0+w1 <s 0)
| ''sf'' ⇒ w0 + w1 <s 0
| f ⇒ unknown-flags ''ADD'' f

definition *semantics-ADD* :: *Operand* ⇒ *Operand* ⇒ *state* ⇒ *state*
where *semantics-ADD op1* ≡
if operand-size op1 = 32 then binop ((+)::256 word ⇒ - ⇒ -) ADD-flags op1
else if operand-size op1 = 16 then binop ((+)::128 word ⇒ - ⇒ -) ADD-flags op1
else if operand-size op1 = 8 then binop ((+)::64 word ⇒ - ⇒ -) ADD-flags op1
else if operand-size op1 = 4 then binop ((+)::32 word ⇒ - ⇒ -) ADD-flags op1
else if operand-size op1 = 2 then binop ((+)::16 word ⇒ - ⇒ -) ADD-flags op1
else if operand-size op1 = 1 then binop ((+)::8 word ⇒ - ⇒ -) ADD-flags op1
else undefined

abbreviation *ADD*
where *ADD op1 op2* ≡ *Instr ''add'' (Some op1) (Some op2) None*

definition *INC-flags* :: *256 word* ⇒ (*'a::len word* ⇒ *string* ⇒ *bool*)
where *INC-flags cf w0 flag* ≡ *case flag of*
''zf'' ⇒ w0 + 1 = 0
| ''cf'' ⇒ cf ≠ 0
| ''of'' ⇒ 0 <=s w0 ∧ w0+1 <s 0
| ''sf'' ⇒ w0 + 1 <s 0
| f ⇒ unknown-flags ''INC'' f

definition *semantics-INC* :: *Operand* ⇒ *state* ⇒ *state*
where *semantics-INC op1 σ* ≡
let cf = flag-read σ ''cf'' in
if operand-size op1 = 32 then unop ((+) (1::256 word)) (INC-flags cf) op1 σ
else if operand-size op1 = 16 then unop ((+) (1::128 word)) (INC-flags cf) op1 σ
else if operand-size op1 = 8 then unop ((+) (1::64 word)) (INC-flags cf) op1 σ
else if operand-size op1 = 4 then unop ((+) (1::32 word)) (INC-flags cf) op1 σ
else if operand-size op1 = 2 then unop ((+) (1::16 word)) (INC-flags cf) op1 σ
else if operand-size op1 = 1 then unop ((+) (1::8 word)) (INC-flags cf) op1 σ
else undefined

abbreviation *INC*
  where *INC op1* ≡ *Instr ''inc'' (Some op1) None None*

definition *DEC-flags* :: *256 word* ⇒ (*'a::len word* ⇒ *string* ⇒ *bool*)
  where *DEC-flags cf w0 flag* ≡ *case flag of*
    ''zf'' ⇒ *w0 = 1*
  | *''cf''* ⇒ *cf* ≠ *0*
  | *''of''* ⇒ *w0 <s 0* ∧ *0 <=s w0 − 1*
  | *''sf''* ⇒ *w0 − 1 <s 0*
  | *f*   ⇒ *unknown-flags ''DEC'' f*

definition *semantics-DEC* :: *Operand* ⇒ *state* ⇒ *state*
  where *semantics-DEC op1 σ* ≡
    let cf = flag-read σ ''cf'' in
          if operand-size op1 = 32 then unop (λ w . w − 1::256 word) (DEC-flags cf) op1 σ
      else if operand-size op1 = 16 then unop (λ w . w − 1::128 word) (DEC-flags cf) op1 σ
      else if operand-size op1 = 8 then unop (λ w . w − 1::64 word) (DEC-flags cf) op1 σ
      else if operand-size op1 = 4 then unop (λ w . w − 1::32 word) (DEC-flags cf) op1 σ
      else if operand-size op1 = 2 then unop (λ w . w − 1::16 word) (DEC-flags cf) op1 σ
      else if operand-size op1 = 1 then unop (λ w . w − 1::8   word) (DEC-flags cf) op1 σ
      else undefined

abbreviation *DEC*
  where *DEC op1* ≡ *Instr ''dec'' (Some op1) None None*

definition *NEG-flags* :: (*'a::len word* ⇒ *string* ⇒ *bool*)
  where *NEG-flags w0 flag* ≡ *case flag of*
    ''zf'' ⇒ *w0 = 0*
  | *''cf''* ⇒ *w0* ≠ *0*
  | *''sf''* ⇒ − *w0 <s 0*
  | *''of''* ⇒ *msb* (− *w0*) ∧ *msb w0*
  | *f*   ⇒ *unknown-flags ''NEG'' f*

definition *semantics-NEG* :: *Operand* ⇒ *state* ⇒ *state*
  where *semantics-NEG op1 σ* ≡
          if operand-size op1 = 32 then unop (λ w0 . − (w0::256 word)) NEG-flags op1 σ
      else if operand-size op1 = 16 then unop (λ w0 . − (w0::128 word)) NEG-flags op1 σ
      else if operand-size op1 = 8 then unop (λ w0 . − (w0::64 word)) NEG-flags op1 σ
      else if operand-size op1 = 4 then unop (λ w0 . − (w0::32 word)) NEG-flags

op1 σ
  else if operand-size op1 = 2 then unop (λ w0 . − (w0::16 word)) NEG-flags op1 σ
  else if operand-size op1 = 1 then unop (λ w0 . − (w0::8 word)) NEG-flags op1 σ
  else undefined

abbreviation *NEG*
  where *NEG op1 ≡ Instr ''neg'' (Some op1) None None*

definition *SUB-flags :: 'a::len word ⇒ 'a::len word ⇒ string ⇒ bool*
  where *SUB-flags w0 w1 flag ≡ case flag of*
    ''zf'' ⇒ w0 = w1
  | ''cf'' ⇒ w0 < w1
  | ''sf'' ⇒ w0 − w1 <s 0
  | ''of'' ⇒ (msb w0 ≠ msb w1) ∧ (msb (w0 − w1) = msb w1)
  | f ⇒ unknown-flags ''SUB'' f

definition *semantics-SUB :: Operand ⇒ Operand ⇒ state ⇒ state*
  where *semantics-SUB op1 ≡*
    if operand-size op1 = 32 then binop ((−)::256 word ⇒ - ⇒ -) SUB-flags op1
  else if operand-size op1 = 16 then binop ((−)::128 word ⇒ - ⇒ -) SUB-flags op1
  else if operand-size op1 = 8 then binop ((−)::64 word ⇒ - ⇒ -) SUB-flags op1
  else if operand-size op1 = 4 then binop ((−)::32 word ⇒ - ⇒ -) SUB-flags op1
  else if operand-size op1 = 2 then binop ((−)::16 word ⇒ - ⇒ -) SUB-flags op1
  else if operand-size op1 = 1 then binop ((−)::8 word ⇒ - ⇒ -) SUB-flags op1
  else undefined

abbreviation *SUB*
  where *SUB op1 op2 ≡ Instr ''sub'' (Some op1) (Some op2) None*

definition *sbb :: 'b::len word ⇒ 'a::len word ⇒ 'a word ⇒ 'a word*
  where *sbb cf dst src ≡ dst − (src + ucast cf)*

definition *SBB-flags :: 'b::len word ⇒ 'a::len word ⇒ 'a::len word ⇒ string ⇒ bool*
  where *SBB-flags cf dst src flag ≡ case flag of*
    ''zf'' ⇒ sbb cf dst src = 0
  | ''cf'' ⇒ dst < src + ucast cf
  | ''sf'' ⇒ sbb cf dst src <s 0
  | ''of'' ⇒ (msb dst ≠ msb (src + ucast cf)) ∧ (msb (sbb cf dst src) = msb (src + ucast cf))
  | f ⇒ unknown-flags ''SBB'' f

definition *semantics-SBB* :: *Operand* ⇒ *Operand* ⇒ *state* ⇒ *state*
where *semantics-SBB op1 op2 σ* ≡
let cf = flag-read σ ''cf'' in
if operand-size op1 = 32 then binop (sbb cf::256 word ⇒ - ⇒ -) (SBB-flags cf) op1 op2 σ
else if operand-size op1 = 16 then binop (sbb cf::128 word ⇒ - ⇒ -) (SBB-flags cf) op1 op2 σ
else if operand-size op1 = 8 then binop (sbb cf::64 word ⇒ - ⇒ -) (SBB-flags cf) op1 op2 σ
else if operand-size op1 = 4 then binop (sbb cf::32 word ⇒ - ⇒ -) (SBB-flags cf) op1 op2 σ
else if operand-size op1 = 2 then binop (sbb cf::16 word ⇒ - ⇒ -) (SBB-flags cf) op1 op2 σ
else if operand-size op1 = 1 then binop (sbb cf::8 word ⇒ - ⇒ -) (SBB-flags cf) op1 op2 σ
else undefined

abbreviation *SBB*
where *SBB op1 op2* ≡ *Instr ''sbb'' (Some op1) (Some op2) None*

definition *adc* :: *'b::len word* ⇒ *'a::len word* ⇒ *'a word* ⇒ *'a word*
where *adc cf dst src* ≡ *dst + (src + ucast cf)*

definition *ADC-flags* :: *'b::len word* ⇒ *'a::len word* ⇒ *'a::len word* ⇒ *string* ⇒ *bool*
where *ADC-flags cf dst src flag* ≡ *case flag of*
''zf'' ⇒ *adc cf dst src = 0*
| *''cf''* ⇒ *unat dst + unat src + unat cf* ≥ *2^(LENGTH('a))*
| *''of''* ⇒ *(dst <s 0* ⟷ *src + ucast cf <s 0)* ∧ ¬*(dst <s 0* ⟷ *adc cf dst src <s 0)*
| *''sf''* ⇒ *adc cf dst src <s 0*
| *f* ⇒ *unknown-flags ''ADC'' f*

definition *semantics-ADC* :: *Operand* ⇒ *Operand* ⇒ *state* ⇒ *state*
where *semantics-ADC op1 op2 σ* ≡
let cf = flag-read σ ''cf'' in
if operand-size op1 = 32 then binop (adc cf::256 word ⇒ - ⇒ -) (ADC-flags cf) op1 op2 σ
else if operand-size op1 = 16 then binop (adc cf::128 word ⇒ - ⇒ -) (ADC-flags cf) op1 op2 σ
else if operand-size op1 = 8 then binop (adc cf::64 word ⇒ - ⇒ -) (ADC-flags cf) op1 op2 σ
else if operand-size op1 = 4 then binop (adc cf::32 word ⇒ - ⇒ -) (ADC-flags cf) op1 op2 σ
else if operand-size op1 = 2 then binop (adc cf::16 word ⇒ - ⇒ -) (ADC-flags cf) op1 op2 σ
else if operand-size op1 = 1 then binop (adc cf::8 word ⇒ - ⇒ -) (ADC-flags

cf) *op1 op2* σ
else undefined

abbreviation *ADC*
where *ADC op1 op2* $\equiv$ *Instr ''adc'' (Some op1) (Some op2) None*

definition *write-MUL-result* :: *string* $\Rightarrow$ *string* $\Rightarrow$ *'a::len word* $\Rightarrow$ - $\Rightarrow$ *state* $\Rightarrow$ *state*
where *write-MUL-result rh rl result flgs* σ $\equiv$
let si = LENGTH('a) div 2 in
operand-write (Reg rh) (ucast ($\langle$*si*,*2*$*$*si*$\rangle$*result*))
(*operand-write (Reg rl) (ucast* ($\langle$*0*,*si*$\rangle$*result*))
(σ *with [setFlags flgs]*)))

definition *MUL-flags* :: *'a::len word* $\Rightarrow$ *string* $\Rightarrow$ *bool*
where *MUL-flags result flag* $\equiv$ *case flag of*
''cf'' $\Rightarrow$ ($\langle$*LENGTH('a) div 2*,*LENGTH('a)*$\rangle$*result*) $\neq$ *0*
| *''of''* $\Rightarrow$ ($\langle$*LENGTH('a) div 2*,*LENGTH('a)*$\rangle$*result*) $\neq$ *0*
| *f* $\Rightarrow$ *unknown-flags ''MUL'' f*

definition *IMUL-flags* :: *'a::len word* $\Rightarrow$ *string* $\Rightarrow$ *bool*
where *IMUL-flags result flag* $\equiv$ *case flag of*
''cf'' $\Rightarrow$ ($\langle$*LENGTH('a) div 2*,*LENGTH('a)*$\rangle$*result*) $\neq$ (*if result* !! (*LENGTH('a) div 2* $-$ *1*) *then 2^(LENGTH('a) div 2)*$-$*1 else 0*)
| *''of''* $\Rightarrow$ ($\langle$*LENGTH('a) div 2*,*LENGTH('a)*$\rangle$*result*) $\neq$ (*if result* !! (*LENGTH('a) div 2* $-$ *1*) *then 2^(LENGTH('a) div 2)*$-$*1 else 0*)
| *f* $\Rightarrow$ *unknown-flags ''IMUL'' f*

definition *unop-MUL* :: *'a::len itself* $\Rightarrow$ *bool* $\Rightarrow$ *string* $\Rightarrow$ *Operand* $\Rightarrow$ *state* $\Rightarrow$ *state*
where *unop-MUL - signd op1-reg op2* σ $\equiv$
let cast = (if signd then scast else ucast);
dst = cast (operand-read σ *(Reg op1-reg))::'a::len word*;
src = cast (operand-read σ *op2)::'a::len word*;
prod = dst $*$ *src*;
flgs = (if signd then IMUL-flags else MUL-flags) prod in
if LENGTH('a) = 16 then
write-MUL-result ''ah'' op1-reg prod flgs σ
else if LENGTH('a) = 32 then
write-MUL-result ''dx'' op1-reg prod flgs σ
else if LENGTH('a) = 64 then
write-MUL-result ''edx'' op1-reg prod flgs σ
else if LENGTH('a) = 128 then
write-MUL-result ''rdx'' op1-reg prod flgs σ
else

undefined

definition *semantics-MUL* :: *Operand* ⇒ *state* ⇒ *state*
where *semantics-MUL op2* ≡
if operand-size op2 = 8 then unop-MUL TYPE(128) False ''rax'' op2
else if operand-size op2 = 4 then unop-MUL TYPE(64) False ''eax'' op2
else if operand-size op2 = 2 then unop-MUL TYPE(32) False ''ax'' op2
else if operand-size op2 = 1 then unop-MUL TYPE(16) False ''al'' op2
else undefined

abbreviation *MUL*
where *MUL op1* ≡ *Instr ''mul'' (Some op1) None None*

definition *semantics-IMUL1* :: *Operand* ⇒ *state* ⇒ *state*
where *semantics-IMUL1 op2* ≡
if operand-size op2 = 8 then unop-MUL TYPE(128) True ''rax'' op2
else if operand-size op2 = 4 then unop-MUL TYPE(64) True ''eax'' op2
else if operand-size op2 = 2 then unop-MUL TYPE(32) True ''ax'' op2
else if operand-size op2 = 1 then unop-MUL TYPE(16) True ''al'' op2
else undefined

abbreviation *IMUL1*
where *IMUL1 op1* ≡ *Instr ''imul'' (Some op1) None None*

definition *ternop-IMUL* :: *'a::len itself* ⇒ *Operand* ⇒ *Operand* ⇒ *Operand* ⇒ *state* ⇒ *state*
where *ternop-IMUL - op1 op2 op3 σ* ≡
let src1 = scast (operand-read σ op2)::'a::len word;
src2 = scast (operand-read σ op3)::'a::len word;
prod = src1 ∗ src2;
flgs = IMUL-flags prod in
(operand-write op1 (ucast (⟨0,LENGTH('a) div 2⟩prod))
(σ with [setFlags flgs]))

definition *semantics-IMUL2* :: *Operand* ⇒ *Operand* ⇒ *state* ⇒ *state*
where *semantics-IMUL2 op1 op2* ≡
if operand-size op1 = 8 then ternop-IMUL TYPE(128) op1 op1 op2
else if operand-size op1 = 4 then ternop-IMUL TYPE(64) op1 op1 op2
else if operand-size op1 = 2 then ternop-IMUL TYPE(32) op1 op1 op2
else if operand-size op1 = 1 then ternop-IMUL TYPE(16) op1 op1 op2
else undefined

abbreviation *IMUL2*
where *IMUL2 op1 op2* ≡ *Instr ''imul'' (Some op1) (Some op2) None*

definition *semantics-IMUL3* :: *Operand* ⇒ *Operand* ⇒ *Operand* ⇒ *state* ⇒ *state*
where *semantics-IMUL3 op1 op2 op3* ≡
if operand-size op1 = 8 then ternop-IMUL TYPE(128) op1 op2 op3
else if operand-size op1 = 4 then ternop-IMUL TYPE(64) op1 op2 op3

else if operand-size op1 = 2 then ternop-IMUL TYPE(32) op1 op2 op3
else if operand-size op1 = 1 then ternop-IMUL TYPE(16) op1 op2 op3
else undefined

abbreviation *IMUL3*
where *IMUL3 op1 op2 op3 ≡ Instr ''imul'' (Some op1) (Some op2) (Some op3)*

definition *SHL-flags :: nat ⇒ ('a::len word ⇒ string ⇒ bool)*
where *SHL-flags n dst flag ≡ case flag of*
''cf'' ⇒ dst !! (LENGTH('a) − n)
| ''of'' ⇒ dst !! (LENGTH('a) − n − 1) ≠ dst !! (LENGTH('a) − n)
| ''zf'' ⇒ (dst << n) = 0
| ''sf'' ⇒ dst !! (LENGTH('a) − n − 1)
| f ⇒ unknown-flags ''SHL'' f

definition *semantics-SHL :: Operand ⇒ Operand ⇒ state ⇒ state*
where *semantics-SHL op1 op2 σ ≡*
let src = unat (operand-read σ op2) in
if operand-size op1 = 32 then unop (λ w . w << src::256 word) (SHL-flags src) op1 σ
else if operand-size op1 = 16 then unop (λ w . w << src::128 word) (SHL-flags src) op1 σ
else if operand-size op1 = 8 then unop (λ w . w << src::64 word) (SHL-flags src) op1 σ
else if operand-size op1 = 4 then unop (λ w . w << src::32 word) (SHL-flags src) op1 σ
else if operand-size op1 = 2 then unop (λ w . w << src::16 word) (SHL-flags src) op1 σ
else if operand-size op1 = 1 then unop (λ w . w << src::8 word) (SHL-flags src) op1 σ
else undefined

abbreviation *SHL*
where *SHL op1 op2 ≡ Instr ''shl'' (Some op1) (Some op2) None*

abbreviation *SAL*
where *SAL op1 op2 ≡ Instr ''sal'' (Some op1) (Some op2) None*

definition *SHR-flags :: nat ⇒ ('a::len word ⇒ string ⇒ bool)*
where *SHR-flags n dst flag ≡ case flag of*
''cf'' ⇒ dst !! (n − 1)
| ''of'' ⇒ msb dst
| ''zf'' ⇒ (dst >> n) = 0
| f ⇒ unknown-flags ''SHR'' f

definition *semantics-SHR :: Operand ⇒ Operand ⇒ state ⇒ state*
where *semantics-SHR op1 op2 σ ≡*
let src = unat (operand-read σ op2) in
if operand-size op1 = 32 then unop (λ w . w >> src::256 word) (SHR-flags

src) *op1* σ
  else if operand-size op1 = *16 then unop* (λ *w* . *w* >> *src*::*128 word*) (*SHR-flags src*) *op1* σ
  else if operand-size op1 = *8 then unop* (λ *w* . *w* >> *src*::*64 word*) (*SHR-flags src*) *op1* σ
  else if operand-size op1 = *4 then unop* (λ *w* . *w* >> *src*::*32 word*) (*SHR-flags src*) *op1* σ
  else if operand-size op1 = *2 then unop* (λ *w* . *w* >> *src*::*16 word*) (*SHR-flags src*) *op1* σ
  else if operand-size op1 = *1 then unop* (λ *w* . *w* >> *src*::*8   word*) (*SHR-flags src*) *op1* σ
  else undefined

abbreviation *SHR*
  where *SHR op1 op2* ≡ *Instr* ''*shr*'' (*Some op1*) (*Some op2*) *None*

definition *SAR-flags* :: *nat* ⇒ ('*a*::*len word* ⇒ *string* ⇒ *bool*)
  where *SAR-flags n dst flag* ≡ *case flag of*
    ''*cf*'' ⇒ *dst* !! (*n* − *1*)
   | ''*of*'' ⇒ *False*
   | ''*zf*'' ⇒ (*dst* >>> *n*) = *0*
   | *f*   ⇒ *unknown-flags* ''*SAR*'' *f*

definition *semantics-SAR* :: *Operand* ⇒ *Operand* ⇒ *state* ⇒ *state*
  where *semantics-SAR op1 op2* σ ≡
   let src = *unat* (*operand-read* σ *op2*) *in*
    if operand-size op1 = *32 then unop* (λ *w* . *w* >>> *src*::*256 word*) (*SAR-flags src*) *op1* σ
    else if operand-size op1 = *16 then unop* (λ *w* . *w* >>> *src*::*128 word*) (*SAR-flags src*) *op1* σ
  else if operand-size op1 = *8 then unop* (λ *w* . *w* >>> *src*::*64 word*) (*SAR-flags src*) *op1* σ
  else if operand-size op1 = *4 then unop* (λ *w* . *w* >>> *src*::*32 word*) (*SAR-flags src*) *op1* σ
  else if operand-size op1 = *2 then unop* (λ *w* . *w* >>> *src*::*16 word*) (*SAR-flags src*) *op1* σ
  else if operand-size op1 = *1 then unop* (λ *w* . *w* >>> *src*::*8   word*) (*SAR-flags src*) *op1* σ
  else undefined

abbreviation *SAR*
  where *SAR op1 op2* ≡ *Instr* ''*sar*'' (*Some op1*) (*Some op2*) *None*

definition *shld* :: '*b*::*len itself* ⇒ *nat* ⇒ '*a*::*len word* ⇒ '*a word* ⇒ '*a word*
  where *shld* - *n dst src* ≡
   let dstsrc = (*ucast dst* << *LENGTH*('*a*)) *OR* (*ucast src* :: '*b word*);
     shifted = ⟨*LENGTH*('*a*),*LENGTH*('*a*)∗*2*⟩(*dstsrc* << *n*) *in*

ucast shifted

definition *SHLD-flags* :: *'b::len itself* ⇒ *nat* ⇒ (*'a::len word* ⇒ *'a::len word* ⇒ *string* ⇒ *bool*)
where *SHLD-flags b n src dst flag* ≡ *case flag of*
''cf'' ⇒ *dst* !! (*LENGTH*(*'a*) − *n*)
| *''of''* ⇒ *dst* !! (*LENGTH*(*'a*) − *n* − *1*) ≠ *dst* !! (*LENGTH*(*'a*) − *n*)
| *''zf''* ⇒ *shld b n dst src* = *0*
| *''sf''* ⇒ *dst* !! (*LENGTH*(*'a*) − *n* − *1*) — msb (shld n dst src)
| *f* ⇒ *unknown-flags ''SHLD'' f*

definition *semantics-SHLD* :: *Operand* ⇒ *Operand* ⇒ *Operand* ⇒ *state* ⇒ *state*
where *semantics-SHLD op1 op2 op3 σ* ≡
let src2 = *unat* (*operand-read σ op3*) *in*
if operand-size op1 = *32 then binop* (*shld* (*TYPE*(*512*)) *src2* ::*256 word* ⇒ *-* ⇒ *-*) (*SHLD-flags* (*TYPE*(*512*)) *src2*) *op1 op2 σ*
else if operand-size op1 = *16 then binop* (*shld* (*TYPE*(*256*)) *src2* ::*128 word* ⇒ *-* ⇒ *-*) (*SHLD-flags* (*TYPE*(*256*)) *src2*) *op1 op2 σ*
else if operand-size op1 = *8 then binop* (*shld* (*TYPE*(*128*)) *src2* ::*64 word* ⇒ *-* ⇒ *-*) (*SHLD-flags* (*TYPE*(*128*)) *src2*) *op1 op2 σ*
else if operand-size op1 = *4 then binop* (*shld* (*TYPE*(*64*)) *src2* ::*32 word* ⇒ *-* ⇒ *-*) (*SHLD-flags* (*TYPE*(*64*)) *src2*) *op1 op2 σ*
else if operand-size op1 = *2 then binop* (*shld* (*TYPE*(*32*)) *src2* ::*16 word* ⇒ *-* ⇒ *-*) (*SHLD-flags* (*TYPE*(*32*)) *src2*) *op1 op2 σ*
else if operand-size op1 = *1 then binop* (*shld* (*TYPE*(*16*)) *src2* ::*8 word* ⇒ *-* ⇒ *-*) (*SHLD-flags* (*TYPE*(*16*)) *src2*) *op1 op2 σ*
else undefined

definition *ROL-flags* :: *nat* ⇒ (*'a::len word* ⇒ *string* ⇒ *bool*)
where *ROL-flags n dst flag* ≡ *case flag of*
''cf'' ⇒ *dst* !! (*LENGTH*(*'a*) − *n*)
| *''of''* ⇒ *dst* !! (*LENGTH*(*'a*) − *n* − *1*) ≠ *dst* !! (*LENGTH*(*'a*) − *n*)
| *f* ⇒ *unknown-flags ''ROL'' f*

definition *semantics-ROL* :: *Operand* ⇒ *Operand* ⇒ *state* ⇒ *state*
where *semantics-ROL op1 op2 σ* ≡
let src = *unat* (*operand-read σ op2*) *in*
if operand-size op1 = *32 then unop* (*word-rotl src*::*256 word*⇒*-*) (*ROL-flags src*) *op1 σ*
else if operand-size op1 = *16 then unop* (*word-rotl src*::*128 word*⇒*-*) (*ROL-flags src*) *op1 σ*
else if operand-size op1 = *8 then unop* (*word-rotl src*::*64 word*⇒*-*) (*ROL-flags src*) *op1 σ*
else if operand-size op1 = *4 then unop* (*word-rotl src*::*32 word*⇒*-*) (*ROL-flags src*) *op1 σ*
else if operand-size op1 = *2 then unop* (*word-rotl src*::*16 word*⇒*-*) (*ROL-flags src*) *op1 σ*

else if operand-size op1 = 1 then unop (word-rotl src::8 word⇒-) (ROL-flags src) op1 σ
else undefined

abbreviation *ROL*
where *ROL op1 op2 ≡ Instr ''rol'' (Some op1) (Some op2) None*

definition *ROR-flags* :: *nat ⇒ ('a::len word ⇒ string ⇒ bool)*
where *ROR-flags n dst flag ≡ case flag of*
''cf'' ⇒ dst !! (n − 1)
| ''of'' ⇒ msb (word-rotr n dst) ≠ (word-rotr n dst !! (LENGTH('a)−2))
| f ⇒ unknown-flags ''ROR'' f

definition *semantics-ROR* :: *Operand ⇒ Operand ⇒ state ⇒ state*
where *semantics-ROR op1 op2 σ ≡*
let src = unat (operand-read σ op2) in
if operand-size op1 = 32 then unop (word-rotr src::256 word⇒-) (ROR-flags src) op1 σ
else if operand-size op1 = 16 then unop (word-rotr src::128 word⇒-) (ROR-flags src) op1 σ
else if operand-size op1 = 8 then unop (word-rotr src::64 word⇒-) (ROR-flags src) op1 σ
else if operand-size op1 = 4 then unop (word-rotr src::32 word⇒-) (ROR-flags src) op1 σ
else if operand-size op1 = 2 then unop (word-rotr src::16 word⇒-) (ROR-flags src) op1 σ
else if operand-size op1 = 1 then unop (word-rotr src::8 word⇒-) (ROR-flags src) op1 σ
else undefined

abbreviation *ROR*
where *ROR op1 op2 ≡ Instr ''ror'' (Some op1) (Some op2) None*

flag-related

definition *semantics-CMP* :: *Operand ⇒ Operand ⇒ state ⇒ state*
where *semantics-CMP op1 ≡*
if operand-size op1 = 32 then binop-flags (SUB-flags::256 word ⇒ - ⇒ - ⇒ -) op1
else if operand-size op1 = 16 then binop-flags (SUB-flags::128 word ⇒ - ⇒ - ⇒ -) op1
else if operand-size op1 = 8 then binop-flags (SUB-flags::64 word ⇒ - ⇒ - ⇒ -) op1
else if operand-size op1 = 4 then binop-flags (SUB-flags::32 word ⇒ - ⇒ - ⇒ -) op1
else if operand-size op1 = 2 then binop-flags (SUB-flags::16 word ⇒ - ⇒ - ⇒ -) op1
else if operand-size op1 = 1 then binop-flags (SUB-flags::8 word ⇒ - ⇒ - ⇒ -) op1
else undefined

abbreviation *CMP*
  where *CMP op1 op2* ≡ *Instr ''cmp'' (Some op1) (Some op2) None*

definition *logic-flags* :: (*'a::len word* ⇒ *'a::len word* ⇒ *'a::len word*) ⇒ *'a::len word* ⇒ *'a::len word* ⇒ *string* ⇒ *bool*
  where *logic-flags logic-op w0 w1 flag* ≡ *case flag of*
    ''zf'' ⇒ *logic-op w0 w1* = *0*
  | *''cf''* ⇒ *False*
  | *''of''* ⇒ *False*
  | *''sf''* ⇒ *msb (logic-op w0 w1)*
  | *f*   ⇒ *unknown-flags ''logic'' f*

definition *semantics-TEST* :: *Operand* ⇒ *Operand* ⇒ *state* ⇒ *state*
  where *semantics-TEST op1* ≡
      if operand-size op1 = 32 then binop-flags (logic-flags ((AND)::256 word ⇒ - ⇒ -)) op1
    else if operand-size op1 = 16 then binop-flags (logic-flags ((AND)::128 word ⇒ - ⇒ -)) op1
    else if operand-size op1 = 8 then binop-flags (logic-flags ((AND)::64 word ⇒ - ⇒ -)) op1
    else if operand-size op1 = 4 then binop-flags (logic-flags ((AND)::32 word ⇒ - ⇒ -)) op1
    else if operand-size op1 = 2 then binop-flags (logic-flags ((AND)::16 word ⇒ - ⇒ -)) op1
    else if operand-size op1 = 1 then binop-flags (logic-flags ((AND)::8   word ⇒ - ⇒ -)) op1
    else undefined

abbreviation *TEST*
  where *TEST op1 op2* ≡ *Instr ''test'' (Some op1) (Some op2) None*

sign extension

definition *mov-sign-extension* :: (*'a::len*) *itself* ⇒ (*'b::len*) *itself* ⇒ *Operand* ⇒ *Operand* ⇒ *state* ⇒ *state*
  where *mov-sign-extension - - op1 op2 σ* ≡
      let src = ucast (operand-read σ op2)::'b word in
        operand-write op1 (ucast (scast src::'a word)) σ

definition *semantics-MOVSXD* :: *Operand* ⇒ *Operand* ⇒ *state* ⇒ *state*
  where *semantics-MOVSXD op1 op2* ≡
    if (operand-size op1, operand-size op2) = (8,4) then
      mov-sign-extension (TYPE(64)) (TYPE(32)) op1 op2
    else if (operand-size op1, operand-size op2) = (8,2) then
      mov-sign-extension (TYPE(64)) (TYPE(16)) op1 op2
    else if (operand-size op1, operand-size op2) = (8,1) then
      mov-sign-extension (TYPE(64)) (TYPE(8)) op1 op2
    else if (operand-size op1, operand-size op2) = (4,2) then
      mov-sign-extension (TYPE(32)) (TYPE(16)) op1 op2

else if (*operand-size op1*, *operand-size op2*) = (*4*,*1*) *then*
mov-sign-extension (*TYPE*(*32*)) (*TYPE*(*8*)) *op1 op2*
else if (*operand-size op1*, *operand-size op2*) = (*2*,*1*) *then*
mov-sign-extension (*TYPE*(*16*)) (*TYPE*(*8*)) *op1 op2*
else
undefined

abbreviation *MOVSXD*
where *MOVSXD op1 op2* ≡ *Instr ''movsxd''* (*Some op1*) (*Some op2*) *None*

abbreviation *MOVSX*
where *MOVSX op1 op2* ≡ *Instr ''movsx''* (*Some op1*) (*Some op2*) *None*

definition *semantics-CDQE* :: *state* ⇒ *state*
where *semantics-CDQE* ≡ *semantics-MOVSXD* (*Reg ''rax''*) (*Reg ''eax''*)

abbreviation *CDQE*
where *CDQE* ≡ *Instr ''cdqe'' None None None*

definition *semantics-CDQ* :: *state* ⇒ *state*
where *semantics-CDQ* σ ≡
let src = *ucast* (*operand-read* σ (*Reg ''eax''*)) :: *32 word in*
operand-write (*Reg ''edx''*) (*ucast* (⟨*32*,*64*⟩(*scast src*::*64 word*))) σ

abbreviation *CDQ*
where *CDQ* ≡ *Instr ''cdq'' None None None*

definition *semantics-CQO* :: *state* ⇒ *state*
where *semantics-CQO* σ ≡
let src = *ucast* (*operand-read* σ (*Reg ''rax''*)) :: *64 word in*
operand-write (*Reg ''rdx''*) (*ucast* (⟨*64*,*128*⟩(*scast src*::*128 word*))) σ

abbreviation *CQO*
where *CQO* ≡ *Instr ''cqo'' None None None*

logic

definition *semantics-AND* :: *Operand* ⇒ *Operand* ⇒ *state* ⇒ *state*
where *semantics-AND op1 op2* σ ≡
if operand-size op1 = *32 then binop* ((*AND*)::*256 word* ⇒ *-* ⇒ *-*) (*logic-flags* ((*AND*)::*256 word* ⇒ *-* ⇒ *-*)) *op1 op2* σ
else if operand-size op1 = *16 then binop* ((*AND*)::*128 word* ⇒ *-* ⇒ *-*) (*logic-flags* ((*AND*)::*128 word* ⇒ *-* ⇒ *-*)) *op1 op2* σ
else if operand-size op1 = *8 then binop* ((*AND*)::*64 word* ⇒ *-* ⇒ *-*) (*logic-flags* ((*AND*)::*64 word* ⇒ *-* ⇒ *-*)) *op1 op2* σ
else if operand-size op1 = *4 then binop* ((*AND*)::*32 word* ⇒ *-* ⇒ *-*) (*logic-flags* ((*AND*)::*32 word* ⇒ *-* ⇒ *-*)) *op1 op2* σ
else if operand-size op1 = *2 then binop* ((*AND*)::*16 word* ⇒ *-* ⇒ *-*) (*logic-flags* ((*AND*)::*16 word* ⇒ *-* ⇒ *-*)) *op1 op2* σ
else if operand-size op1 = *1 then binop* ((*AND*)::*8 word* ⇒ *-* ⇒ *-*) (*logic-flags*

$((AND)::8 \quad word \Rightarrow \text{-} \Rightarrow \text{-}))\ op1\ op2\ \sigma$
else undefined

abbreviation *AND′*
where $AND'\ op1\ op2 \equiv Instr\ ''and''\ (Some\ op1)\ (Some\ op2)\ None$

definition *semantics-OR* :: $Operand \Rightarrow Operand \Rightarrow state \Rightarrow state$
where *semantics-OR op1 op2* $\sigma \equiv$
if operand-size op1 = 32 then binop $((OR)::256\ word \Rightarrow \text{-} \Rightarrow \text{-})$ (*logic-flags* $((OR)::256\ word \Rightarrow \text{-} \Rightarrow \text{-}))\ op1\ op2\ \sigma$
else if operand-size op1 = 16 then binop $((OR)::128\ word \Rightarrow \text{-} \Rightarrow \text{-})$ (*logic-flags* $((OR)::128\ word \Rightarrow \text{-} \Rightarrow \text{-}))\ op1\ op2\ \sigma$
else if operand-size op1 = 8 then binop $((OR)::64\ word \Rightarrow \text{-} \Rightarrow \text{-})$ (*logic-flags* $((OR)::64\ word \Rightarrow \text{-} \Rightarrow \text{-}))\ op1\ op2\ \sigma$
else if operand-size op1 = 4 then binop $((OR)::32\ word \Rightarrow \text{-} \Rightarrow \text{-})$ (*logic-flags* $((OR)::32\ word \Rightarrow \text{-} \Rightarrow \text{-}))\ op1\ op2\ \sigma$
else if operand-size op1 = 2 then binop $((OR)::16\ word \Rightarrow \text{-} \Rightarrow \text{-})$ (*logic-flags* $((OR)::16\ word \Rightarrow \text{-} \Rightarrow \text{-}))\ op1\ op2\ \sigma$
else if operand-size op1 = 1 then binop $((OR)::8\ word \Rightarrow \text{-} \Rightarrow \text{-})$ (*logic-flags* $((OR)::8\ word \Rightarrow \text{-} \Rightarrow \text{-}))\ op1\ op2\ \sigma$
else undefined

abbreviation *OR′*
where $OR'\ op1\ op2 \equiv Instr\ ''or''\ (Some\ op1)\ (Some\ op2)\ None$

definition *semantics-XOR* :: $Operand \Rightarrow Operand \Rightarrow state \Rightarrow state$
where *semantics-XOR op1 op2* $\sigma \equiv$
if operand-size op1 = 32 then binop $((XOR)::256\ word \Rightarrow \text{-} \Rightarrow \text{-})$ (*logic-flags* $((XOR)::256\ word \Rightarrow \text{-} \Rightarrow \text{-}))\ op1\ op2\ \sigma$
else if operand-size op1 = 16 then binop $((XOR)::128\ word \Rightarrow \text{-} \Rightarrow \text{-})$ (*logic-flags* $((XOR)::128\ word \Rightarrow \text{-} \Rightarrow \text{-}))\ op1\ op2\ \sigma$
else if operand-size op1 = 8 then binop $((XOR)::64\ word \Rightarrow \text{-} \Rightarrow \text{-})$ (*logic-flags* $((XOR)::64\ word \Rightarrow \text{-} \Rightarrow \text{-}))\ op1\ op2\ \sigma$
else if operand-size op1 = 4 then binop $((XOR)::32\ word \Rightarrow \text{-} \Rightarrow \text{-})$ (*logic-flags* $((XOR)::32\ word \Rightarrow \text{-} \Rightarrow \text{-}))\ op1\ op2\ \sigma$
else if operand-size op1 = 2 then binop $((XOR)::16\ word \Rightarrow \text{-} \Rightarrow \text{-})$ (*logic-flags* $((XOR)::16\ word \Rightarrow \text{-} \Rightarrow \text{-}))\ op1\ op2\ \sigma$
else if operand-size op1 = 1 then binop $((XOR)::8\ word \Rightarrow \text{-} \Rightarrow \text{-})$ (*logic-flags* $((XOR)::8\ word \Rightarrow \text{-} \Rightarrow \text{-}))\ op1\ op2\ \sigma$
else undefined

abbreviation *XOR′*
where $XOR'\ op1\ op2 \equiv Instr\ ''xor''\ (Some\ op1)\ (Some\ op2)\ None$

definition *semantics-XORPS* :: $Operand \Rightarrow Operand \Rightarrow state \Rightarrow state$
where *semantics-XORPS op1* $\equiv$
if operand-size op1 = 32 then binop-no-flags $((XOR)::256\ word \Rightarrow \text{-} \Rightarrow \text{-})$ *op1*
else if operand-size op1 = 16 then binop-no-flags $((XOR)::128\ word \Rightarrow \text{-} \Rightarrow$

-) *op1*
else if operand-size op1 = 8 then binop-no-flags ((XOR)::64 word ⇒ - ⇒ -) op1
else if operand-size op1 = 4 then binop-no-flags ((XOR)::32 word ⇒ - ⇒ -) op1
else if operand-size op1 = 2 then binop-no-flags ((XOR)::16 word ⇒ - ⇒ -) op1
else if operand-size op1 = 1 then binop-no-flags ((XOR)::8 word ⇒ - ⇒ -) op1
else undefined

abbreviation *XORPS*
where *XORPS op1 op2 ≡ Instr ''xorps'' (Some op1) (Some op2) None*

definition *semantics-NOT* :: *Operand ⇒ state ⇒ state*
where *semantics-NOT op1 σ ≡*
if operand-size op1 = 32 then unop-no-flags (not::256 word⇒-) op1 σ
else if operand-size op1 = 16 then unop-no-flags (not::128 word⇒-) op1 σ
else if operand-size op1 = 8 then unop-no-flags (not::64 word⇒-) op1 σ
else if operand-size op1 = 4 then unop-no-flags (not::32 word⇒-) op1 σ
else if operand-size op1 = 2 then unop-no-flags (not::16 word⇒-) op1 σ
else if operand-size op1 = 1 then unop-no-flags (not::8 word⇒-) op1 σ
else undefined

abbreviation *NOT'*
where *NOT' op1 ≡ Instr ''not'' (Some op1) None None*

jumps

datatype *FlagExpr = Flag string | FE-NOT FlagExpr | FE-AND FlagExpr FlagExpr | FE-OR FlagExpr FlagExpr | FE-EQ FlagExpr FlagExpr*

primrec *readFlagExpr* :: *FlagExpr ⇒ state ⇒ bool*
where
readFlagExpr (Flag f) σ = (flag-read σ f = 1)
| readFlagExpr (FE-NOT fe) σ = (¬readFlagExpr fe σ)
| readFlagExpr (FE-AND fe0 fe1) σ = (readFlagExpr fe0 σ ∧ readFlagExpr fe1 σ)
| readFlagExpr (FE-OR fe0 fe1) σ = (readFlagExpr fe0 σ ∨ readFlagExpr fe1 σ)
| readFlagExpr (FE-EQ fe0 fe1) σ = (readFlagExpr fe0 σ ⟷ readFlagExpr fe1 σ)

definition *semantics-cond-jump* :: *FlagExpr ⇒ 64 word ⇒ state ⇒ state*
where *semantics-cond-jump fe a σ ≡*
let fv = readFlagExpr fe σ in
if fv then state-update (setRip a) σ else σ

definition *semantics-JMP* :: *Operand ⇒ state ⇒ state*
where *semantics-JMP op1 σ ≡*
let a = ucast (operand-read σ op1) in

state-update (*setRip a*) σ

abbreviation *JMP*
  where *JMP op1* ≡ *Instr ''jmp''* (*Some op1*) *None None*

definition *semantics-JO* :: *Operand* ⇒ *state* ⇒ *state*
  where *semantics-JO op1* σ ≡
    let a = *ucast* (*operand-read* σ *op1*) *in*
      semantics-cond-jump (*Flag ''of''*) *a* σ

abbreviation *JO*
  where *JO op1* ≡ *Instr ''jo''* (*Some op1*) *None None*

definition *semantics-JNO* :: *Operand* ⇒ *state* ⇒ *state*
  where *semantics-JNO op1* σ ≡
    let a = *ucast* (*operand-read* σ *op1*) *in*
      semantics-cond-jump (*FE-NOT* (*Flag ''of''*)) *a* σ

abbreviation *JNO*
  where *JNO op1* ≡ *Instr ''jno''* (*Some op1*) *None None*

definition *semantics-JS* :: *Operand* ⇒ *state* ⇒ *state*
  where *semantics-JS op1* σ ≡
    let a = *ucast* (*operand-read* σ *op1*) *in*
      semantics-cond-jump (*Flag ''sf''*) *a* σ

abbreviation *JS*
  where *JS op1* ≡ *Instr ''js''* (*Some op1*) *None None*

definition *semantics-JNS* :: *Operand* ⇒ *state* ⇒ *state*
  where *semantics-JNS op1* σ ≡
    let a = *ucast* (*operand-read* σ *op1*) *in*
      semantics-cond-jump (*FE-NOT* (*Flag ''sf''*)) *a* σ

abbreviation *JNS*
  where *JNS op1* ≡ *Instr ''jns''* (*Some op1*) *None None*

definition *semantics-JE* :: *Operand* ⇒ *state* ⇒ *state*
  where *semantics-JE op1* σ ≡
    let a = *ucast* (*operand-read* σ *op1*) *in*
      semantics-cond-jump (*Flag ''zf''*) *a* σ

abbreviation *JE*
  where *JE op1* ≡ *Instr ''je''* (*Some op1*) *None None*

abbreviation *JZ*
  where *JZ op1* ≡ *Instr ''jz''* (*Some op1*) *None None*

definition *semantics-JNE* :: *Operand* ⇒ *state* ⇒ *state*

where *semantics-JNE op1* $\sigma \equiv$
let a = *ucast* (*operand-read* σ *op1*) *in*
semantics-cond-jump (*FE-NOT* (*Flag* ''*zf*'')) *a* σ

abbreviation *JNE*
where *JNE op1* $\equiv$ *Instr* ''*jne*'' (*Some op1*) *None None*

abbreviation *JNZ*
where *JNZ op1* $\equiv$ *Instr* ''*jnz*'' (*Some op1*) *None None*

definition *semantics-JB* :: *Operand* $\Rightarrow$ *state* $\Rightarrow$ *state*
where *semantics-JB op1* $\sigma \equiv$
let a = *ucast* (*operand-read* σ *op1*) *in*
semantics-cond-jump (*Flag* ''*cf*'') *a* σ

abbreviation *JB*
where *JB op1* $\equiv$ *Instr* ''*jb*'' (*Some op1*) *None None*

abbreviation *JNAE*
where *JNAE op1* $\equiv$ *Instr* ''*jnae*'' (*Some op1*) *None None*

abbreviation *JC*
where *JC op1* $\equiv$ *Instr* ''*jc*'' (*Some op1*) *None None*

definition *semantics-JNB* :: *Operand* $\Rightarrow$ *state* $\Rightarrow$ *state*
where *semantics-JNB op1* $\sigma \equiv$
let a = *ucast* (*operand-read* σ *op1*) *in*
semantics-cond-jump (*FE-NOT* (*Flag* ''*cf*'')) *a* σ

abbreviation *JNB*
where *JNB op1* $\equiv$ *Instr* ''*jnb*'' (*Some op1*) *None None*

abbreviation *JAE*
where *JAE op1* $\equiv$ *Instr* ''*jae*'' (*Some op1*) *None None*

abbreviation *JNC*
where *JNC op1* $\equiv$ *Instr* ''*jnc*'' (*Some op1*) *None None*

definition *semantics-JBE* :: *Operand* $\Rightarrow$ *state* $\Rightarrow$ *state*
where *semantics-JBE op1* $\sigma \equiv$
let a = *ucast* (*operand-read* σ *op1*) *in*
semantics-cond-jump (*FE-OR* (*Flag* ''*cf*'') (*Flag* ''*zf*'')) *a* σ

abbreviation *JBE*
where *JBE op1* $\equiv$ *Instr* ''*jbe*'' (*Some op1*) *None None*

abbreviation *JNA*
where *JNA op1* $\equiv$ *Instr* ''*jna*'' (*Some op1*) *None None*

definition *semantics-JA* :: *Operand* ⇒ *state* ⇒ *state*
where *semantics-JA op1 σ* ≡
let a = *ucast* (*operand-read σ op1*) *in*
semantics-cond-jump (*FE-AND* (*FE-NOT* (*Flag ''cf''*)) (*FE-NOT* (*Flag ''zf''*))) *a σ*

abbreviation *JA*
where *JA op1* ≡ *Instr ''ja''* (*Some op1*) *None None*

abbreviation *JNBE*
where *JNBE op1* ≡ *Instr ''jnbe''* (*Some op1*) *None None*

definition *semantics-JL* :: *Operand* ⇒ *state* ⇒ *state*
where *semantics-JL op1 σ* ≡
let a = *ucast* (*operand-read σ op1*) *in*
semantics-cond-jump (*FE-NOT* (*FE-EQ* (*Flag ''sf''*) (*Flag ''of''*))) *a σ*

abbreviation *JL*
where *JL op1* ≡ *Instr ''jl''* (*Some op1*) *None None*

abbreviation *JNGE*
where *JNGE op1* ≡ *Instr ''jnge''* (*Some op1*) *None None*

definition *semantics-JGE* :: *Operand* ⇒ *state* ⇒ *state*
where *semantics-JGE op1 σ* ≡
let a = *ucast* (*operand-read σ op1*) *in*
semantics-cond-jump (*FE-EQ* (*Flag ''sf''*) (*Flag ''of''*)) *a σ*

abbreviation *JGE*
where *JGE op1* ≡ *Instr ''jge''* (*Some op1*) *None None*

abbreviation *JNL*
where *JNL op1* ≡ *Instr ''jnl''* (*Some op1*) *None None*

definition *semantics-JLE* :: *Operand* ⇒ *state* ⇒ *state*
where *semantics-JLE op1 σ* ≡
let a = *ucast* (*operand-read σ op1*) *in*
semantics-cond-jump (*FE-OR* (*Flag ''zf''*) (*FE-NOT* (*FE-EQ* (*Flag ''sf''*) (*Flag ''of''*)))) *a σ*

abbreviation *JLE*
where *JLE op1* ≡ *Instr ''jle''* (*Some op1*) *None None*

abbreviation *JNG*
where *JNG op1* ≡ *Instr ''jng''* (*Some op1*) *None None*

definition *semantics-JG* :: *Operand* ⇒ *state* ⇒ *state*
where *semantics-JG op1 σ* ≡
let a = *ucast* (*operand-read σ op1*) *in*

semantics-cond-jump (*FE-AND* (*FE-NOT* (*Flag* ''*zf*'')) (*FE-EQ* (*Flag* ''*sf*'') (*Flag* ''*of*''))) *a* σ

abbreviation *JG*
where *JG op1* ≡ *Instr* ''*jg*'' (*Some op1*) *None None*

abbreviation *JNLE*
where *JNLE op1* ≡ *Instr* ''*jnle*'' (*Some op1*) *None None*

setXX

definition *semantics-setXX* :: *FlagExpr* ⇒ *Operand* ⇒ *state* ⇒ *state*
where *semantics-setXX fe op1* σ ≡
let fv = *readFlagExpr fe* σ *in*
operand-write op1 (*fromBool fv*) σ

abbreviation *semantics-SETO* :: *Operand* ⇒ *state* ⇒ *state*
where *semantics-SETO* ≡ *semantics-setXX* (*Flag* ''*of*'')

abbreviation *SETO*
where *SETO op1* ≡ *Instr* ''*seto*'' (*Some op1*) *None None*

abbreviation *semantics-SETNO* :: *Operand* ⇒ *state* ⇒ *state*
where *semantics-SETNO* ≡ *semantics-setXX* (*FE-NOT* (*Flag* ''*of*''))

abbreviation *SETNO*
where *SETNO op1* ≡ *Instr* ''*setno*'' (*Some op1*) *None None*

abbreviation *semantics-SETS* :: *Operand* ⇒ *state* ⇒ *state*
where *semantics-SETS* ≡ *semantics-setXX* (*Flag* ''*sf*'')

abbreviation *SETS*
where *SETS op1* ≡ *Instr* ''*sets*'' (*Some op1*) *None None*

abbreviation *semantics-SETNS* :: *Operand* ⇒ *state* ⇒ *state*
where *semantics-SETNS* ≡ *semantics-setXX* (*FE-NOT* (*Flag* ''*sf*''))

abbreviation *SETNS*
where *SETNS op1* ≡ *Instr* ''*setns*'' (*Some op1*) *None None*

abbreviation *semantics-SETE* :: *Operand* ⇒ *state* ⇒ *state*
where *semantics-SETE* ≡ *semantics-setXX* (*Flag* ''*zf*'')

abbreviation *SETE*
where *SETE op1* ≡ *Instr* ''*sete*'' (*Some op1*) *None None*

abbreviation *SETZ*
where *SETZ op1* ≡ *Instr* ''*setz*'' (*Some op1*) *None None*

abbreviation *semantics-SETNE* :: *Operand* ⇒ *state* ⇒ *state*

where *semantics-SETNE* ≡ *semantics-setXX* (*FE-NOT* (*Flag* ''*zf*''))

abbreviation *SETNE*
where *SETNE op1* ≡ *Instr* ''*setne*'' (*Some op1*) *None None*

abbreviation *SETNZ*
where *SETNZ op1* ≡ *Instr* ''*setnz*'' (*Some op1*) *None None*

abbreviation *semantics-SETB* :: *Operand* ⇒ *state* ⇒ *state*
where *semantics-SETB* ≡ *semantics-setXX* (*Flag* ''*cf*'')

abbreviation *SETB*
where *SETB op1* ≡ *Instr* ''*setb*'' (*Some op1*) *None None*

abbreviation *SETNAE*
where *SETNAE op1* ≡ *Instr* ''*setnae*'' (*Some op1*) *None None*

abbreviation *SETC*
where *SETC op1* ≡ *Instr* ''*setc*'' (*Some op1*) *None None*

abbreviation *semantics-SETNB* :: *Operand* ⇒ *state* ⇒ *state*
where *semantics-SETNB* ≡ *semantics-setXX* (*FE-NOT* (*Flag* ''*cf*''))

abbreviation *SETNB*
where *SETNB op1* ≡ *Instr* ''*setnb*'' (*Some op1*) *None None*

abbreviation *SETAE*
where *SETAE op1* ≡ *Instr* ''*setae*'' (*Some op1*) *None None*

abbreviation *SETNC*
where *SETNC op1* ≡ *Instr* ''*setnc*'' (*Some op1*) *None None*

abbreviation *semantics-SETBE* :: *Operand* ⇒ *state* ⇒ *state*
where *semantics-SETBE* ≡ *semantics-setXX* (*FE-OR* (*Flag* ''*cf*'') (*Flag* ''*zf*''))

abbreviation *SETBE*
where *SETBE op1* ≡ *Instr* ''*setbe*'' (*Some op1*) *None None*

abbreviation *SETNA*
where *SETNA op1* ≡ *Instr* ''*setna*'' (*Some op1*) *None None*

abbreviation *semantics-SETA* :: *Operand* ⇒ *state* ⇒ *state*
where *semantics-SETA* ≡ *semantics-setXX* (*FE-AND* (*FE-NOT* (*Flag* ''*cf*''))
(*FE-NOT* (*Flag* ''*zf*'')))

abbreviation *SETA*
where *SETA op1* ≡ *Instr* ''*seta*'' (*Some op1*) *None None*

abbreviation *SETNBE*

where *SETNBE op1* ≡ *Instr ''setnbe'' (Some op1) None None*

abbreviation *semantics-SETL* :: *Operand* ⇒ *state* ⇒ *state*
where *semantics-SETL* ≡ *semantics-setXX (FE-NOT (FE-EQ (Flag ''sf'') (Flag ''of'')))*

abbreviation *SETL*
where *SETL op1* ≡ *Instr ''setl'' (Some op1) None None*

abbreviation *SETNGE*
where *SETNGE op1* ≡ *Instr ''setnge'' (Some op1) None None*

abbreviation *semantics-SETGE* :: *Operand* ⇒ *state* ⇒ *state*
where *semantics-SETGE* ≡ *semantics-setXX (FE-EQ (Flag ''sf'') (Flag ''of''))*

abbreviation *SETGE*
where *SETGE op1* ≡ *Instr ''setge'' (Some op1) None None*

abbreviation *SETNL*
where *SETNL op1* ≡ *Instr ''setnl'' (Some op1) None None*

abbreviation *semantics-SETLE* :: *Operand* ⇒ *state* ⇒ *state*
where *semantics-SETLE* ≡ *semantics-setXX (FE-OR (Flag ''zf'') (FE-NOT (FE-EQ (Flag ''sf'') (Flag ''of''))))*

abbreviation *SETLE*
where *SETLE op1* ≡ *Instr ''setle'' (Some op1) None None*

abbreviation *SETNG*
where *SETNG op1* ≡ *Instr ''setng'' (Some op1) None None*

abbreviation *semantics-SETG* :: *Operand* ⇒ *state* ⇒ *state*
where *semantics-SETG* ≡ *semantics-setXX (FE-AND (FE-NOT (Flag ''zf'')) (FE-EQ (Flag ''sf'') (Flag ''of'')))*

abbreviation *SETG*
where *SETG op1* ≡ *Instr ''setg'' (Some op1) None None*

abbreviation *SETNLE*
where *SETNLE op1* ≡ *Instr ''setnle'' (Some op1) None None*

conditional moves

primrec *cmov*
where
cmov True dst src = src
| *cmov False dst src = dst*

definition *semantics-CMOV* :: *FlagExpr* ⇒ *Operand* ⇒ *Operand* ⇒ *state* ⇒ *state*
where *semantics-CMOV fe op1 op2* σ ≡

let fv = readFlagExpr fe σ;
dst = operand-read σ op1;
src = operand-read σ op2 in
operand-write op1 (cmov fv dst src) σ

abbreviation *semantics-CMOVNE ≡ semantics-CMOV (FE-NOT (Flag ''zf''))*

abbreviation *CMOVNE*
where *CMOVNE op1 op2 ≡ Instr ''movne'' (Some op1) (Some op2) None*

abbreviation *semantics-CMOVNS ≡ semantics-CMOV (FE-NOT (Flag ''sf''))*

abbreviation *CMOVNS*
where *CMOVNS op1 op2 ≡ Instr ''movns'' (Some op1) (Some op2) None*

Floating Point

definition *semantics-ADDSD :: Operand ⇒ Operand ⇒ state ⇒ state*
where *semantics-ADDSD ≡ binop-XMM unknown-addsd*

definition *semantics-SUBSD :: Operand ⇒ Operand ⇒ state ⇒ state*
where *semantics-SUBSD ≡ binop-XMM unknown-subsd*

definition *semantics-MULSD :: Operand ⇒ Operand ⇒ state ⇒ state*
where *semantics-MULSD ≡ binop-XMM unknown-mulsd*

definition *semantics-DIVSD :: Operand ⇒ Operand ⇒ state ⇒ state*
where *semantics-DIVSD ≡ binop-XMM unknown-divsd*

definition *UCOMISD-flags :: 64 word ⇒ 64 word ⇒ string ⇒ bool*
where *UCOMISD-flags w0 w1 f ≡*
if f ∈ {''zf'',''pf'',''cf''} then case unknown-ucomisd w0 w1 of
FP-Unordered ⇒ True
| FP-GT ⇒ False
| FP-LT ⇒ f = ''cf''
| FP-EQ ⇒ f = ''zf''
else
unknown-flags ''UCOMISD'' f

definition *semantics-UCOMISD :: Operand ⇒ Operand ⇒ state ⇒ state*
where *semantics-UCOMISD ≡ binop-flags UCOMISD-flags*

abbreviation *ADDSD*
where *ADDSD op1 op2 ≡ Instr ''addsd'' (Some op1) (Some op2) None*

abbreviation *SUBSD*
where *SUBSD op1 op2 ≡ Instr ''subsd'' (Some op1) (Some op2) None*

abbreviation *MULSD*

where *MULSD op1 op2* ≡ *Instr ''mulsd'' (Some op1) (Some op2) None*

abbreviation *DIVSD*
where *DIVSD op1 op2* ≡ *Instr ''divsd'' (Some op1) (Some op2) None*

abbreviation *UCOMISD*
where *UCOMISD op1 op2* ≡ *Instr ''ucomisd'' (Some op1) (Some op2) None*

definition *simd-32-128* :: (*32 word* ⇒ *32 word* ⇒ *32 word*) ⇒ *128 word* ⇒ *128 word* ⇒ *128 word*
where *simd-32-128 f dst src* ≡
((*ucast* (⟨*0*,*32*⟩(*f* (*ucast* (⟨*96*,*128*⟩*dst*)) (*ucast* (⟨*96*,*128*⟩*src*))))) << *96*)
OR
((*ucast* (⟨*0*,*32*⟩(*f* (*ucast* (⟨*64*,*96*⟩*dst*)) (*ucast* (⟨*64*,*96*⟩*src*))))) << *64*)
OR
((*ucast* (⟨*0*,*32*⟩(*f* (*ucast* (⟨*32*,*64*⟩*dst*)) (*ucast* (⟨*32*,*64*⟩*src*))))) << *32*)
OR
(*ucast* (⟨*0*,*32*⟩(*f* (*ucast* (⟨*0*,*32*⟩*dst*)) (*ucast* (⟨*0*,*32*⟩*src*)))))

abbreviation *semantics-PADDD* :: *Operand* ⇒ *Operand* ⇒ *state* ⇒ *state*
where *semantics-PADDD* ≡ *binop-no-flags* (*simd-32-128* (+))

abbreviation *PADDD*
where *PADDD op1 op2* ≡ *Instr ''paddd'' (Some op1) (Some op2) None*

definition *pshufd* :: *128 word* ⇒ *8 word* ⇒ *128 word*
where *pshufd src n* ≡ (((⟨*0*,*32*⟩(*src* >> (*unat* (⟨*6*,*8*⟩*n*)**32*))) << *96*) *OR*
(((⟨*0*,*32*⟩(*src* >> (*unat* (⟨*4*,*6*⟩*n*)**32*))) << *64*) *OR*
(((⟨*0*,*32*⟩(*src* >> (*unat* (⟨*2*,*4*⟩*n*)**32*))) << *32*) *OR*
(((⟨*0*,*32*⟩(*src* >> (*unat* (⟨*0*,*2*⟩*n*)**32*))))

lemmas *pshufd-numeral*[*simp*] = *pshufd-def*[*of numeral n*] **for** *n*
lemmas *pshufd-0*[*simp*] = *pshufd-def*[*of 0*]
lemmas *pshufd-1*[*simp*] = *pshufd-def*[*of 1*]

definition *semantics-PSHUFD* :: *Operand* ⇒ *Operand* ⇒ *Operand* ⇒ *state* ⇒ *state*
where *semantics-PSHUFD op1 op2 op3 σ* ≡
let src = *ucast* (*operand-read σ op2*);
n = *ucast* (*operand-read σ op3*) *in*
operand-write op1 (*ucast* (*pshufd src n*)) *σ*

abbreviation *PSHUFD*
where *PSHUFD op1 op2 op3* ≡ *Instr ''pshufd'' op1 op2 op3*

definition *semantics-PEXTRD* :: *Operand* ⇒ *Operand* ⇒ *Operand* ⇒ *state* ⇒ *state*
  where *semantics-PEXTRD op1 op2 op3 σ* ≡
    let src = *operand-read σ op2*;
      n = *unat* (*operand-read σ op3*) *mod 4 in*
      operand-write op1 (*ucast* ((⟨*0*,*32*⟩(*src* >> *n*∗*32*)))) *σ*

abbreviation *PEXTRD*
  where *PEXTRD op1 op2 op3* ≡ *Instr ''pextrd'' op1 op2 op3*

definition *semantics-PINSRD* :: *Operand* ⇒ *Operand* ⇒ *Operand* ⇒ *state* ⇒ *state*
  where *semantics-PINSRD op1 op2 op3 σ* ≡
    let dst = *ucast* (*operand-read σ op1*)::*128 word*;
      src = *ucast* (*operand-read σ op2*)::*128 word*;
      n = *unat* (*operand-read σ op3*) *mod 4*;
      m = *0xFFFFFFFF* << (*n* ∗ *32*) :: *128 word*;
      t = (*src* << (*n* ∗*32*)) *AND m in*
      operand-write op1 (*ucast* ((*dst AND NOT m*) *OR t*)) *σ*

abbreviation *PINSRD*
  where *PINSRD op1 op2 op3* ≡ *Instr ''pinsrd'' op1 op2 op3*

definition *bswap* :: *32 word* ⇒ *32 word*
  where *bswap w* ≡ (((⟨*0*,*8*⟩*w*) << *24*) *OR* (((⟨*8*,*16*⟩*w*) << *16*) *OR* (((⟨*16*,*24*⟩*w*) << *8*) *OR* (⟨*24*,*32*⟩*w*)

lemmas *bswap-numeral*[*simp*] = *bswap-def*[*of numeral n*] **for** *n*
lemmas *bswap-0*[*simp*] = *bswap-def*[*of 0*]
lemmas *bswap-1*[*simp*] = *bswap-def*[*of 1*]

definition *semantics-BSWAP* :: *Operand* ⇒ *state* ⇒ *state*
  where *semantics-BSWAP* ≡ *unop-no-flags bswap*

abbreviation *BSWAP*
  where *BSWAP op1* ≡ *Instr ''bswap'' op1 None None*

definition *semantics-NOP* :: *state* ⇒ *state*
  where *semantics-NOP* ≡ *id*

abbreviation *NOP0*
  where *NOP0* ≡ *Instr ''nop'' None None None*

abbreviation *NOP1*
where *NOP1 op1* ≡ *Instr ''nop'' (Some op1) None None*

abbreviation *NOP2*
where *NOP2 op1 op2* ≡ *Instr ''nop'' (Some op1) (Some op2) None*

abbreviation *NOP3*
where *NOP3 op1 op2 op3* ≡ *Instr ''nop'' (Some op1) (Some op2) (Some op3)*

definition *semantics*
where *semantics i* ≡
case i of
(*Instr ''mov''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-MOV op1 op2*
| (*Instr ''movabs''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-MOV op1 op2*
| (*Instr ''movaps''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-MOV op1 op2*
| (*Instr ''movdqu''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-MOV op1 op2*
| (*Instr ''movd''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-MOVD op1 op2*
| (*Instr ''movzx''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-MOV op1 op2*
| (*Instr ''movsd''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-MOVSD op1 op2*
| (*Instr ''movq''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-MOVSD op1 op2*
| (*Instr ''lea''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-LEA op1 op2*
| (*Instr ''push''* (*Some op1*) - - -) ⇒ *semantics-PUSH op1*
| (*Instr ''pop''* (*Some op1*) - - -) ⇒ *semantics-POP op1*
| (*Instr ''ret''* - - - -) ⇒ *semantics-RET*
| (*Instr ''call''* (*Some op1*) - - -) ⇒ *semantics-CALL op1*
| (*Instr ''leave''* - - - -) ⇒ *semantics-LEAVE*
— arithmetic
| (*Instr ''add''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-ADD op1 op2*
| (*Instr ''inc''* (*Some op1*) - - -) ⇒ *semantics-INC op1*
| (*Instr ''dec''* (*Some op1*) - - -) ⇒ *semantics-DEC op1*
| (*Instr ''neg''* (*Some op1*) - - -) ⇒ *semantics-NEG op1*
| (*Instr ''sub''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-SUB op1 op2*
| (*Instr ''sbb''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-SBB op1 op2*
| (*Instr ''adc''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-ADC op1 op2*
| (*Instr ''mul''* (*Some op1*) - - -) ⇒ *semantics-MUL op1*
| (*Instr ''imul''* (*Some op1*) *None* - -) ⇒ *semantics-IMUL1 op1*
| (*Instr ''imul''* (*Some op1*) (*Some op2*) *None* -) ⇒ *semantics-IMUL2 op1 op2*

| (*Instr ''imul''* (*Some op1*) (*Some op2*) (*Some op3*) -) ⇒ *semantics-IMUL3 op1 op2 op3*
| (*Instr ''shl''* (*Some op1*) (*Some op2*) *None* -) ⇒ *semantics-SHL op1 op2*
| (*Instr ''sal''* (*Some op1*) (*Some op2*) *None* -) ⇒ *semantics-SHL op1 op2*
| (*Instr ''shr''* (*Some op1*) (*Some op2*) *None* -) ⇒ *semantics-SHR op1 op2*
| (*Instr ''sar''* (*Some op1*) (*Some op2*) *None* -) ⇒ *semantics-SAR op1 op2*
| (*Instr ''shld''* (*Some op1*) (*Some op2*) (*Some op3*) -) ⇒ *semantics-SHLD op1 op2 op3*
| (*Instr ''rol''* (*Some op1*) (*Some op2*) *None* -) ⇒ *semantics-ROL op1 op2*
| (*Instr ''ror''* (*Some op1*) (*Some op2*) *None* -) ⇒ *semantics-ROR op1 op2*
— flag-related
| (*Instr ''cmp''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-CMP op1 op2*
| (*Instr ''test''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-TEST op1 op2*
— sign-extension
| (*Instr ''movsxd''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-MOVSXD op1 op2*
| (*Instr ''movsx''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-MOVSXD op1 op2*
| (*Instr ''cdqe''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-CDQE*
| (*Instr ''cdq''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-CDQ*
| (*Instr ''cqo''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-CQO*
— logic
| (*Instr ''and''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-AND op1 op2*
| (*Instr ''or''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-OR op1 op2*
| (*Instr ''xor''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-XOR op1 op2*
| (*Instr ''xorps''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-XORPS op1 op2*
| (*Instr ''not''* (*Some op1*) - - -) ⇒ *semantics-NOT op1*
— jumps
| (*Instr ''jmp''* (*Some op1*) *None* - -) ⇒ *semantics-JMP op1*
| (*Instr ''jo''* (*Some op1*) *None* - -) ⇒ *semantics-JO op1*
| (*Instr ''jno''* (*Some op1*) *None* - -) ⇒ *semantics-JNO op1*
| (*Instr ''js''* (*Some op1*) *None* - -) ⇒ *semantics-JS op1*
| (*Instr ''jns''* (*Some op1*) *None* - -) ⇒ *semantics-JNS op1*
| (*Instr ''je''* (*Some op1*) *None* - -) ⇒ *semantics-JE op1*
| (*Instr ''jz''* (*Some op1*) *None* - -) ⇒ *semantics-JE op1*
| (*Instr ''jne''* (*Some op1*) *None* - -) ⇒ *semantics-JNE op1*
| (*Instr ''jnz''* (*Some op1*) *None* - -) ⇒ *semantics-JNE op1*
| (*Instr ''jb''* (*Some op1*) *None* - -) ⇒ *semantics-JB op1*
| (*Instr ''jnae''* (*Some op1*) *None* - -) ⇒ *semantics-JB op1*
| (*Instr ''jc''* (*Some op1*) *None* - -) ⇒ *semantics-JB op1*

| (*Instr ''jnb''* (*Some op1*) *None* - -) ⇒ *semantics-JNB op1*
| (*Instr ''jae''* (*Some op1*) *None* - -) ⇒ *semantics-JNB op1*
| (*Instr ''jnc''* (*Some op1*) *None* - -) ⇒ *semantics-JNB op1*
| (*Instr ''jbe''* (*Some op1*) *None* - -) ⇒ *semantics-JBE op1*
| (*Instr ''jna''* (*Some op1*) *None* - -) ⇒ *semantics-JBE op1*
| (*Instr ''ja''* (*Some op1*) *None* - -) ⇒ *semantics-JA op1*
| (*Instr ''jnbe''* (*Some op1*) *None* - -) ⇒ *semantics-JA op1*
| (*Instr ''jl''* (*Some op1*) *None* - -) ⇒ *semantics-JL op1*
| (*Instr ''jnge''* (*Some op1*) *None* - -) ⇒ *semantics-JL op1*
| (*Instr ''jge''* (*Some op1*) *None* - -) ⇒ *semantics-JGE op1*
| (*Instr ''jnl''* (*Some op1*) *None* - -) ⇒ *semantics-JGE op1*
| (*Instr ''jle''* (*Some op1*) *None* - -) ⇒ *semantics-JLE op1*
| (*Instr ''jng''* (*Some op1*) *None* - -) ⇒ *semantics-JLE op1*
| (*Instr ''jg''* (*Some op1*) *None* - -) ⇒ *semantics-JG op1*
| (*Instr ''jnle''* (*Some op1*) *None* - -) ⇒ *semantics-JG op1*
— setXX
| (*Instr ''seto''* (*Some op1*) *None* - -) ⇒ *semantics-SETO op1*
| (*Instr ''setno''* (*Some op1*) *None* - -) ⇒ *semantics-SETNO op1*
| (*Instr ''sets''* (*Some op1*) *None* - -) ⇒ *semantics-SETS op1*
| (*Instr ''setns''* (*Some op1*) *None* - -) ⇒ *semantics-SETNS op1*
| (*Instr ''sete''* (*Some op1*) *None* - -) ⇒ *semantics-SETE op1*
| (*Instr ''setz''* (*Some op1*) *None* - -) ⇒ *semantics-SETE op1*
| (*Instr ''setne''* (*Some op1*) *None* - -) ⇒ *semantics-SETNE op1*
| (*Instr ''setnz''* (*Some op1*) *None* - -) ⇒ *semantics-SETNE op1*
| (*Instr ''setb''* (*Some op1*) *None* - -) ⇒ *semantics-SETB op1*
| (*Instr ''setnae''* (*Some op1*) *None* - -) ⇒ *semantics-SETB op1*
| (*Instr ''setc''* (*Some op1*) *None* - -) ⇒ *semantics-SETB op1*
| (*Instr ''setnb''* (*Some op1*) *None* - -) ⇒ *semantics-SETNB op1*
| (*Instr ''setae''* (*Some op1*) *None* - -) ⇒ *semantics-SETNB op1*
| (*Instr ''setnc''* (*Some op1*) *None* - -) ⇒ *semantics-SETNB op1*
| (*Instr ''setbe''* (*Some op1*) *None* - -) ⇒ *semantics-SETBE op1*
| (*Instr ''setna''* (*Some op1*) *None* - -) ⇒ *semantics-SETBE op1*
| (*Instr ''seta''* (*Some op1*) *None* - -) ⇒ *semantics-SETA op1*
| (*Instr ''setnbe''* (*Some op1*) *None* - -) ⇒ *semantics-SETA op1*
| (*Instr ''setl''* (*Some op1*) *None* - -) ⇒ *semantics-SETL op1*
| (*Instr ''setnge''* (*Some op1*) *None* - -) ⇒ *semantics-SETL op1*
| (*Instr ''setge''* (*Some op1*) *None* - -) ⇒ *semantics-SETGE op1*
| (*Instr ''setnl''* (*Some op1*) *None* - -) ⇒ *semantics-SETGE op1*
| (*Instr ''setle''* (*Some op1*) *None* - -) ⇒ *semantics-SETLE op1*
| (*Instr ''setng''* (*Some op1*) *None* - -) ⇒ *semantics-SETLE op1*
| (*Instr ''setg''* (*Some op1*) *None* - -) ⇒ *semantics-SETG op1*
| (*Instr ''setnle''* (*Some op1*) *None* - -) ⇒ *semantics-SETG op1*
— conditional moves
| (*Instr ''cmovne''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-CMOVNE op1 op2*
| (*Instr ''cmovns''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-CMOVNS op1 op2*
— floating point (double)
| (*Instr ''addsd''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-ADDSD op1*

op2
| (*Instr ''subsd''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-SUBSD op1 op2*
| (*Instr ''mulsd''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-MULSD op1 op2*
| (*Instr ''divsd''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-DIVSD op1 op2*
| (*Instr ''ucomisd''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-UCOMISD op1 op2*
— simd
| (*Instr ''paddd''* (*Some op1*) (*Some op2*) - -) ⇒ *semantics-PADDD op1 op2*
| (*Instr ''pshufd''* (*Some op1*) (*Some op2*) (*Some op3*) -) ⇒ *semantics-PSHUFD op1 op2 op3*
| (*Instr ''pextrd''* (*Some op1*) (*Some op2*) (*Some op3*) -) ⇒ *semantics-PEXTRD op1 op2 op3*
| (*Instr ''pinsrd''* (*Some op1*) (*Some op2*) (*Some op3*) -) ⇒ *semantics-PINSRD op1 op2 op3*
— remainder
| (*Instr ''nop''* - - - -) ⇒ *semantics-NOP*
| (*Instr ''bswap''* (*Some op1*) - - -) ⇒ *semantics-BSWAP op1*
— external function
| (*ExternalFunc f*) ⇒ *f*
| *i* ⇒ *unknown-semantics i*

A step function. In X86. the RIP register is incremented before the instruction is executed. This is important, e.g., when RIP is used in a jump address.

definition *step* :: *I* ⇒ *state* ⇒ *state*
where *step i σ* ≡
let σ' = σ with [*setRip* (*instr-next i*)] *in*
semantics i σ'

All simplification rules used during symbolic execution.

lemmas *semantics-simps* =
Let-def unop-def unop-no-flags-def binop-def binop-flags-def binop-no-flags-def binop-XMM-def
semantics-def mov-sign-extension-def simd-32-128-def
write-MUL-result-def unop-MUL-def ternop-IMUL-def sbb-def adc-def shld-def

semantics-MOV-def semantics-MOVSD-def semantics-MOVD-def semantics-CMOV-def
semantics-LEA-def semantics-PUSH-def semantics-POP-def
semantics-RET-def semantics-CALL-def semantics-LEAVE-def
semantics-ADD-def semantics-INC-def semantics-DEC-def semantics-NEG-def semantics-SUB-def
semantics-SBB-def semantics-ADC-def
semantics-MUL-def semantics-IMUL1-def semantics-IMUL2-def semantics-IMUL3-def
semantics-SHL-def semantics-SHR-def semantics-SAR-def semantics-SHLD-def

semantics-ROL-def semantics-ROR-def
semantics-CMP-def semantics-TEST-def
semantics-MOVSXD-def semantics-CDQE-def semantics-CDQ-def semantics-CQO-def
semantics-AND-def semantics-OR-def semantics-XOR-def semantics-XORPS-def semantics-NOT-def
semantics-cond-jump-def semantics-JMP-def
semantics-JO-def semantics-JNO-def semantics-JS-def semantics-JNS-def
semantics-JE-def semantics-JNE-def semantics-JB-def semantics-JNB-def
semantics-JBE-def semantics-JA-def semantics-JL-def semantics-JGE-def
semantics-JLE-def semantics-JG-def
semantics-setXX-def
semantics-ADDSD-def semantics-SUBSD-def semantics-MULSD-def semantics-DIVSD-def semantics-UCOMISD-def
semantics-NOP-def semantics-BSWAP-def semantics-PSHUFD-def semantics-PEXTRD-def semantics-PINSRD-def

SUB-flags-def ADD-flags-def INC-flags-def DEC-flags-def NEG-flags-def MUL-flags-def IMUL-flags-def
SHL-flags-def SHR-flags-def SAR-flags-def SHLD-flags-def logic-flags-def UCOMISD-flags-def

end
end

5 Removing superfluous memory writes

theory *StateCleanUp*
imports *State HOL−Eisbach.Eisbach*
begin

definition *assumptions* ≡ *id*

We are going to make schematic theorems of the form:

$$assumptions ?A \implies \ldots$$

The assumptions will be generated on-the-fly. The seemingly weird lemmas below achieves that.

lemma *assumptions-impI*:
assumes *assumptions* (*P* ∧ *A*)
shows *P*
using *assms*
by (*auto simp add*: *assumptions-def*)

lemma *assumptions-conjE*:
shows *assumptions* (*P* ∧ *A*) ⟷ *P* ∧ *assumptions A*
by (*auto simp add*: *assumptions-def*)

lemma *assumptionsI*:
  shows *assumptions True*
  by (*auto simp add*: *assumptions-def*)

Consider two consecutive memory updates. If they write to the same memory locations, only one of these need to be kept. We formulate an Eisbach method to that end.

Returns true if two states are equal except for a specific memory region.

definition *eq-except-for-mem* :: *state* $\Rightarrow$ *state* $\Rightarrow$ *64 word* $\Rightarrow$ *nat* $\Rightarrow$ *256 word* $\Rightarrow$ *bool* $\Rightarrow$ *bool*
  where *eq-except-for-mem* σ σ' *a si v b* $\equiv$ σ *with* $[[\![a,si]\!] :=_m v]$ = (*if b then* σ' *else* σ' *with* $[[\![a,si]\!] :=_m v]$)

lemma *eefm-start*:
  assumes *eq-except-for-mem* (σ *with updates*) (σ *with updates'*) *a si v b*
  shows (σ *with* $(([\![a,si]\!] :=_m v)\#updates)$) = (*if b then* σ *with updates' else* σ *with* $(([\![a,si]\!] :=_m v)\#updates')$)
  using *assms*
  by (*auto simp add*: *eq-except-for-mem-def region-addresses-def state-with-updates.simps*(*2*) *state-update.simps*)

lemma *eefm-clean-mem*:
assumes $si' \leq si$
    and *eq-except-for-mem* (σ *with updates*) (σ *with updates'*) *a si v b*
  shows *eq-except-for-mem* (σ *with* $(([\![a,si']\!] :=_m v')\#updates)$) ($\sigma$ *with updates'*) *a si v b*
  using *assms*
  apply (*auto simp add*: *eq-except-for-mem-def state-with-updates.simps*(*2*) *state-update.simps*)
  subgoal
    apply (*cases* σ *with updates*;*cases* σ *with updates'*;*cases* σ)
    by (*auto simp add*: *override-on-def region-addresses-iff*)
  apply (*cases* σ *with updates*;*cases* σ *with updates'*)
  apply (*auto simp add*: *override-on-def region-addresses-iff*)
  apply (*rule ext*)
  apply (*auto split*: *if-splits*)
  by *meson*

method *eefm-clean-mem* = (*rule eefm-clean-mem*, (*simp* (*no-asm*);*fail*))

lemma *eefm-clean-mem-enclosed*:
  assumes $si < 32$
      and *enclosed* a' si' *a si*
      and *eq-except-for-mem* (σ *with updates*) (σ *with updates'*) a' si' v' *b*
  shows *eq-except-for-mem* (σ *with* $(([\![a,si]\!] :=_m v)\#updates)$)
                          (σ *with* $(([\![a,si]\!] :=_m$ *overwrite* $(8 * unat\ (a' - a))\ (8 * unat\ (a' - a) + 8*si')\ v\ (v' << unat\ (a'-a)*8))\#updates')$)

```
                    a' si' v' True
proof(cases b)
  case True
  thus ?thesis
    using assms(3)
   apply (auto simp add: eq-except-for-mem-def state-with-updates.simps(2) state-update.simps)
    apply (cases σ with updates;cases σ with updates')
    apply (auto simp add: override-on-def)
    apply (rule ext)
    apply (auto)

    apply (rule word-eqI)
    subgoal premises prems for - - - - x n
    proof-
      have 1: unat (x − a) − unat (a' − a) = unat(x − a')
        using address-in-enclosed-region-as-linarith[OF assms(2),of x]
              address-of-enclosed-region-ge[OF assms(2)]
              word-le-nat-alt prems(4−5)
        by (auto simp add: unat-sub-if-size word-size)
      thus ?thesis
      using prems address-in-enclosed-region[of a' si' a si x,OF assms(2)] assms(1)
        by (auto simp add: word-size take-byte-shiftlr-256 nth-take-byte-overwrite)
    qed

    apply (rule word-eqI)
    subgoal for - - - - x n
      using notin-region-addresses-sub[of x a' si' a]
      by (auto simp add: word-size nth-take-byte-overwrite)
    done
next
  case False
  thus ?thesis
    using assms(3)
   apply (auto simp add: eq-except-for-mem-def state-with-updates.simps(2) state-update.simps)
    apply (cases σ with updates;cases σ with updates')
    apply (auto simp add: override-on-def)
    apply (rule ext)
    using enclosed-spec[OF assms(2)]
    apply (auto)

    subgoal premises prems for - - - - - x
    proof-
      have 1: unat (x − a) − unat (a' − a) = unat(x − a')
        using address-in-enclosed-region-as-linarith[OF assms(2),of x]
              address-of-enclosed-region-ge[OF assms(2)]
              word-le-nat-alt prems(6)
        by (auto simp add: unat-sub-if-size word-size)
      show ?thesis
        apply (rule word-eqI)
```

using *1 prems address-in-enclosed-region*[*of a′ si′ a si x*,*OF assms*(*2*)] *assms*(*1*)
by (*auto simp add*: *word-size take-byte-shiftlr-256 nth-take-byte-overwrite*)
qed

subgoal for - - - - - *x*
apply (*rule word-eqI*)
using *notin-region-addresses-sub*[*of x a′ si′ a*]
by (*auto simp add*: *word-size nth-take-byte-overwrite*)
by *meson*
qed

lemmas *eefm-clean-mem-enclosed-plus* = *eefm-clean-mem-enclosed*[*OF* - *enclosed-plus*, *simplified*]
lemmas *eefm-clean-mem-enclosed-minus-numeral* = *eefm-clean-mem-enclosed*[*OF* - *enclosed-minus-minus*, *of* - *numeral n numeral m*] **for** *n m*

method *eefm-clean-mem-enclosed-plus* =
(*rule eefm-clean-mem-enclosed-plus*, (
(*simp* (*no-asm*);*fail*), (*simp* (*no-asm*);*fail*),
(
(*simp* (*no-asm-simp*);*fail*) |
(*rule assumptions-impI*[*of* - + - < *18446744073709551616*],*simp* (*no-asm-simp*),*subst* (*asm*) *assumptions-conjE*))
)
)

method *eefm-clean-mem-enclosed-minus-numeral* =
(*rule eefm-clean-mem-enclosed-minus-numeral*, (
(*simp* (*no-asm*);*fail*), (*simp* (*no-asm*);*fail*), (*simp* (*no-asm*);*fail*), (*simp* (*no-asm*);*fail*), (*simp* (*no-asm*);*fail*),
(
(*simp* (*no-asm-simp*);*fail*) |
(*rule assumptions-impI*[*of* - ≤ -],*simp* (*no-asm-simp*),*subst* (*asm*) *assumptions-conjE*))
)
)

lemma *eefm-next-mem*:
assumes *separate a si a′ si′*
and *eq-except-for-mem* (*σ with updates*) (*σ with updates′*) *a si v b*
shows *eq-except-for-mem* (*σ with* ((⟦*a′*,*si′*⟧ $:=_m$ *v′*)#*updates*)) (*σ with* ((⟦*a′*,*si′*⟧ $:=_m$ *v′*)#*updates′*)) *a si v b*
using *assms*
apply (*auto simp add*: *eq-except-for-mem-def override-on-def separate-def state-with-updates.simps*(*2*) *state-update.simps*)
apply (*cases σ with updates*;*cases σ with updates′*)
apply (*auto simp add*: *override-on-def*)

apply (*rule ext*)
apply (*auto*)
apply (*cases σ with updates*;*cases σ with updates′*)
apply (*auto simp add*: *override-on-def*)
apply (*rule ext*)
apply (*auto*)
by (*metis select-convs*(*2*))

method *eefm-next-mem* =
(*rule eefm-next-mem*,
((*simp* (*no-asm-simp*) *add*: *separate-simps state-simps*; *fail*) |
(*rule assumptions-impI*[*of separate - - - -*],*simp* (*no-asm-simp*),*subst* (*asm*) *assumptions-conjE*))
)

lemma *eefm-end*:
shows *eq-except-for-mem* (*σ with updates*) (*σ with updates*) *a si v False*
by (*auto simp add*: *eq-except-for-mem-def*)

We need a tactic exactly like "subst" but that applies to the outer most term.

ML-file ‹*MySubst.ML*›

The following method takes a goal with state with symbolic state updates. It first applies *eq-except-for-mem* (*?σ with ?updates*) (*?σ with ?updates′*) *?a ?si ?v ?b* $\Longrightarrow$ *?σ with* (($[\![?a,?si]\!] :=_m ?v$) # *?updates*) = (*if ?b then ?σ with ?updates′ else ?σ with* (($[\![?a,?si]\!] :=_m ?v$) # *?updates′*)), considering the outer-most memory update to some region $[\![a, si]\!]$. A list *updates′* is generated that produces a similar state except for region $[\![a, si]\!]$. The list thus can have fewer updates since any update to a region that is enclosed in region $[\![a, si]\!]$ can be removed. Consecutively, this method applies rules to determine whether a state update can be kept, merged or removed. It may add assumptions to the goal, that assume no overflow.

method *clean-up-mem* = (
mysubst eefm-start,
(*eefm-clean-mem* | *eefm-clean-mem-enclosed-plus* | *eefm-clean-mem-enclosed-minus-numeral* | *eefm-next-mem*)+,
rule eefm-end,
simp (*no-asm*),
((*match* **premises in** *A*[*thin*]: *assumptions* (*?A* ∧ *?B*) ⇒ ‹*cut-tac A*;*subst* (*asm*) *assumptions-conjE*, *erule conjE* ›)+)*?*
)

The method above applies to one state update. This method can be repeated as long as modifies the goal, as it always makes the goal smaller. The method below repeats a given method until the goal is unchanged. In deterministic fashion (a la the REPEAT_DETERM tactic).

method-setup *repeat-until-unchanged* = ‹

```
  Method.text-closure >>
   (fn text => fn ctxt => fn using =>
         — parse the method supplied as parameter
     let val ctxt-tac = Method.evaluate-runtime text ctxt using
         fun repeat-until-unchanged (ctxt,st) =
           case Seq.pull (ctxt-tac (ctxt,st)) of
                SOME (Seq.Result (ctxt',st'), -) =>
                  if Thm.eq-thm (st, st') then
                    Seq.make-results (Seq.succeed (ctxt',st'))
                  else
                    repeat-until-unchanged (ctxt',st')
              | - => Seq.make-results (Seq.succeed (ctxt,st))
     in
       repeat-until-unchanged
     end)
›
```

method *clean-up* = *repeat-until-unchanged clean-up-mem*

end

6 A symbolic execution engine

theory *SymbolicExecution*
  imports *X86-InstructionSemantics StateCleanUp*
begin

definition *eq* (**infixl** ‹≜› *50*)
  where (≜) ≡ (=)

context *unknowns*
begin

inductive *run* :: *64 word* ⇒ (*64 word* ⇒ *I*) ⇒ *state* ⇒ *state* ⇒ *bool*
  where
    $rip\ \sigma = a_f \Longrightarrow fetch\ (rip\ \sigma) = i \Longrightarrow \sigma' \triangleq step\ i\ \sigma \Longrightarrow run\ a_f\ fetch\ \sigma\ \sigma'$
  | $rip\ \sigma \neq a_f \Longrightarrow fetch\ (rip\ \sigma) = i \Longrightarrow run\ a_f\ fetch\ (step\ i\ \sigma)\ \sigma' \Longrightarrow run\ a_f\ fetch\ \sigma\ \sigma'$

method *fetch-and-execute* = (
  ((*rule run.intros*(*2*),(*simp* (*no-asm*) *add*: *state-simps*;*fail*))
   | (*rule run.intros*(*1*),(*simp* (*no-asm*) *add*: *state-simps*;*fail*))),
  (*simp* (*no-asm*) *add*: *state-simps*), — fetch instruction
  (*simp* (*no-asm-simp*) *add*: *step-def semantics-simps state-simps BitByte-simps*),

— simplification
  (*subst clean-state-updates*[*symmetric*],*simp* (*no-asm*)) — cleaning up
)

method *resolve-mem-reads* = (
  subst mem-read-mem-write-separate,
  ((*simp* (*no-asm-simp*) *add*: *separate-simps state-simps*; *fail*)
    | (*rule assumptions-impI*,*simp* (*no-asm-simp*),*subst* (*asm*) *assumptions-conjE*, *erule conjE*)),
 (*simp* (*no-asm-simp*) *add*: *semantics-simps state-simps BitByte-simps separate-simps*)?
)

method *se-step* =
  fetch-and-execute,
  ((*resolve-mem-reads*)+;(*subst clean-state-updates*[*symmetric*],*simp* (*no-asm*)))?,
  clean-up

method *se-step-no-clean* =
  fetch-and-execute,
  ((*resolve-mem-reads*)+;(*subst clean-state-updates*[*symmetric*],*simp* (*no-asm*)))?

end

abbreviation *RSP0*
  where *RSP0* σ ≡ *regs* σ ''*rsp*''
abbreviation *RBP0*
  where *RBP0* σ ≡ *regs* σ ''*rbp*''
abbreviation *RAX0*
  where *RAX0* σ ≡ *regs* σ ''*rax*''
abbreviation *RBX0*
  where *RBX0* σ ≡ *regs* σ ''*rbx*''
abbreviation *RCX0*
  where *RCX0* σ ≡ *regs* σ ''*rcx*''
abbreviation *RDX0*
  where *RDX0* σ ≡ *regs* σ ''*rdx*''
abbreviation *RDI0*
  where *RDI0* σ ≡ *regs* σ ''*rdi*''
abbreviation *RSI0*
  where *RSI0* σ ≡ *regs* σ ''*rsi*''
abbreviation *R150*
  where *R150* σ ≡ *regs* σ ''*r15*''
abbreviation *R140*
  where *R140* σ ≡ *regs* σ ''*r14*''
abbreviation *R130*
  where *R130* σ ≡ *regs* σ ''*r13*''
abbreviation *R120*
  where *R120* σ ≡ *regs* σ ''*r12*''

abbreviation *R110*
  where *R110* $\sigma \equiv$ *regs* σ *''r11''*
abbreviation *R100*
  where *R100* $\sigma \equiv$ *regs* σ *''r10''*
abbreviation *R90*
  where *R90* $\sigma \equiv$ *regs* σ *''r9''*
abbreviation *R80*
  where *R80* $\sigma \equiv$ *regs* σ *''r8''*

Repeat a command up to n times, in deterministic fashion (a la the REPEAT_DETERM tactic).

```
method-setup repeat-n = ‹
  Scan.lift Parse.nat -- Method.text-closure >>
    (fn (n,text) => fn ctxt => fn using =>
      let
        val ctxt-tac = Method.evaluate-runtime text ctxt using;
        fun repeat-n 0 ctxt-st = Seq.make-results (Seq.succeed ctxt-st)
          | repeat-n i ctxt-st = case Seq.pull (ctxt-tac ctxt-st) of
                          SOME (Seq.Result ctxt-st', -) => repeat-n (i−1) ctxt-st'
                        | - => Seq.make-results (Seq.succeed ctxt-st)
      in
        repeat-n n
      end)
›
```

end

7 Small examples

theory *Examples*
  imports *SymbolicExecution*
begin

context *unknowns*
begin

A simple hand-crafted example showing some basic instructions.

```
schematic-goal example1:
  assumes[simp]: fetch 0x0  = PUSH (Reg ''rbp'') 1
    and[simp]: fetch 0x1  = SUB  (Reg ''rsp'') (Imm 30) 2
    and[simp]: fetch 0x2  = MOV  (QWORD PTR [22 + ''rsp'']₂) (Imm 42) 3
    and[simp]: fetch 0x3  = MOV  (QWORD PTR [30 + ''rsp'']₂) (Imm 43) 4
    and[simp]: fetch 0x4  = ADD  (Reg ''rsp'') (Imm 30) 5
    and[simp]: fetch 0x5  = POP  (Reg ''rbp'') 6
    and[simp]: fetch 0x6  = RET 1
   shows run 0x6 fetch (σ with [setRip 0x0]) ?σ'
  apply se-step+
  apply (subst eq-def,simp)
  done
```

thm *example1*

Demonstrates little-endian memory and register-aliasing

```
RAX +    0     1     2      3      4      5      6      7
      | FF  | EE  | DD   | CC   | BB   | AA   | 00   | 00   |

EDI := 0xCCDDEEFF
EBX := 0xAABB
RCX := 0xAABBCCDDAABB
```

schematic-goal *example2*:
assumes[*simp*]: *fetch 0x0* $= MOV$ (*QWORD PTR* [*''rax''*]$_1$) (*Imm 0xAABBCCDDEEFF*) *1*
and[*simp*]: *fetch 0x1* $= MOV$ (*Reg ''edi''*) (*DWORD PTR* [*''rax''*]$_1$) *2*
and[*simp*]: *fetch 0x2* $= MOV$ (*Reg ''ebx''*) (*DWORD PTR* [*4* + *''rax''*]$_2$) *3*
and[*simp*]: *fetch 0x3* $= MOV$ (*Reg ''rcx''*) (*QWORD PTR* [*''rax''*]$_1$) *4*
and[*simp*]: *fetch 0x4* $= MOV$ (*Reg ''cx''*) (*WORD PTR* [*4* + *''rax''*]$_2$) *5*
shows *run 0x4 fetch* (σ *with* [*setRip 0x0*]) *?*σ'
apply *se-step+*
apply (*subst eq-def*,*simp*)
done

thm *example2*

This example show how assumptions over regions are generated. Since no relation over rax and rbx is known in the initial state, they will be assumed to be separate by default.

schematic-goal *example3*:
assumes[*simp*]: *fetch 0x0* $= MOV$ (*QWORD PTR* [*''rax''*]$_1$) (*Imm 0xAABBCCDDEEFF*) *1*
and[*simp*]: *fetch 0x1* $= MOV$ (*QWORD PTR* [*''rbx''*]$_1$) (*Imm 0x112233445566*) *2*
and[*simp*]: *fetch 0x2* $= MOV$ (*Reg ''rcx''*) (*DWORD PTR* [*2* + *''rax''*]$_2$) *3*
and[*simp*]: *fetch 0x3* $= MOV$ (*Reg ''cx''*) (*WORD PTR* [*4* + *''rbx''*]$_2$) *4*
and[*simp*]: *fetch 0x4* $= MOV$ (*Reg ''cl''*) (*BYTE PTR* [*''rax''*]$_1$) *5*
shows *assumptions ?A* $\Longrightarrow$ *run 0x4 fetch* (σ *with* [*setRip 0x0*]) *?*σ'
apply *se-step+*
apply (*subst eq-def*,*simp*)
done

thm *example3*

end

end

8 Parser

theory *X86-Parse*
imports *X86-InstructionSemantics*
keywords *x86-64-parser* :: *thy-decl*
begin

ML-file ‹*X86-Parse.ML*›

end

9 Example: word count program from GNU

theory *Example-WC*
imports *SymbolicExecution X86-Parse*
begin

The wordcount (wc) program, specifically, the functions getword and counter. We compiled the source code found here:

https://www.gnu.org/software/cflow/manual/html_node/Source-of-wc-command.html

The source code is also found in the directory `./examples/wc`.
The assembly below has been obtained by running in `./examples/wc`:

```
gcc wc.c -o wc
objdump -M intel -d --no-show-raw-insn wc
```

This example:

- contains a lot of memory accesses and demonstrates how a memory model is generated through assumptions;
- contains external function calls and demonstrates how to deal with that.

First, we define definitions named "EXTERNAL_FUNCTION_*" for each external function. The definitions are added to the simplifier.

We model a C file (of C-type "FILE") as a pointer to a part of memory that contains the contents. We assume the contents are 0-terminated.

Function `feof` takes as input (via `rdi`) a FILE*. It reads one byte from **rdi, i.e., the next byte of the file. It returns true iff the byte equals 0.

Function `fopen` writes into memory both 1.) the contents of a file (the string "Hello"), and 2.) a pointer to the beginning of that file. It returns a pointer to that pointer.

Function `getc` reads the next byte of the FILE (same is feof) and increments the pointer.

Function `isword` simply returns true, and functions report and fclose simply do nothing.

context *unknowns*
begin

definition *EXTERNAL-FUNCTION-feof* :: *state* ⇒ *state*
  where *EXTERNAL-FUNCTION-feof* $\sigma \equiv$
    let ptr = ucast (operand-read σ *(QWORD PTR [''rdi'']*$_1$*));*
      val = mem-read σ *ptr 1 in*
    (semantics-RET o
     semantics-MOV (Reg ''eax'') (Imm (fromBool (val = 0))))
     σ

declare *EXTERNAL-FUNCTION-feof-def* [*simp*]

definition *EXTERNAL-FUNCTION--IO-getc* :: *state* ⇒ *state*
  where *EXTERNAL-FUNCTION--IO-getc* $\sigma \equiv$
    let ptr = ucast (operand-read σ *(QWORD PTR [''rdi'']*$_1$*));*
      val = mem-read σ *ptr 1 in*
    (semantics-RET o
      semantics-MOV (Reg ''rax'') (Imm (if val = 0 then -1 *else val)) o*
      semantics-INC (QWORD PTR [''rdi'']$_1$*))*
      σ

declare *EXTERNAL-FUNCTION--IO-getc-def* [*simp*]

definition *EXTERNAL-FUNCTION-fopen* σ =
  semantics-RET (σ *with [''rax''* $:=_r$ *100,*
      $[\![100,8]\!]$ $:=_m$ *108,*
      $[\![108,6]\!]$ $:=_m$ *0x006E6C6C6548])*

declare *EXTERNAL-FUNCTION-fopen-def* [*simp*]

definition *EXTERNAL-FUNCTION-isword* :: *state* ⇒ *state*
where *EXTERNAL-FUNCTION-isword* = *operand-write* (*Reg ''rax''*) *1 o semantics-RET*

declare *EXTERNAL-FUNCTION-isword-def* [*simp*]

definition *EXTERNAL-FUNCTION-fclose* :: *state* ⇒ *state*
where *EXTERNAL-FUNCTION-fclose* = *semantics-RET*

declare *EXTERNAL-FUNCTION-fclose-def* [*simp*]

definition *EXTERNAL-FUNCTION-report* :: *state* ⇒ *state*
where *EXTERNAL-FUNCTION-report* = *semantics-RET*

declare *EXTERNAL-FUNCTION-report-def* [*simp*]

end

Below, one can see that, e.g. 810 denotes an external call (see address 0xc1b). For each external call, we replace the actual .got.plt section with a special instruction *ExternalFunc* followed by a name. These special instructions are interpreted as executing the related definition above.

context *unknowns*
begin
x86-64-parser *wc-objdump* ‹

```
7d0: EXTERNAL-FUNCTION fclose
810: EXTERNAL-FUNCTION feof
820: EXTERNAL-FUNCTION -IO-getc
830: EXTERNAL-FUNCTION fopen
bd5: EXTERNAL-FUNCTION isword
b93: EXTERNAL-FUNCTION report

c01: push   rbp
c02: mov    rbp,rsp
c05: sub    rsp,0x20
c09: mov    QWORD PTR [rbp−0x18],rdi
c0d: mov    DWORD PTR [rbp−0x4],0x0
c14: mov    rax,QWORD PTR [rbp−0x18]
c18: mov    rdi,rax
c1b: call   810 <feof@plt>
c20: test   eax,eax
c22: je     c7d <getword+0x7c>
c24: mov    eax,0x0
c29: jmp    cf1 <getword+0xf0>
c2e: mov    eax,DWORD PTR [rbp−0x8]
c31: movzx  eax,al
c34: mov    edi,eax
c36: call   bd5 <isword>
```

```
c3b: test    eax,eax
c3d: je      c53 <getword+0x52>
c3f: mov     rax,QWORD PTR [rip+0x201402]        # 202048 <wcount>
c46: add     rax,0x1
c4a: mov     QWORD PTR [rip+0x2013f7],rax        # 202048 <wcount>
c51: jmp     c92 <getword+0x91>
c53: mov     rax,QWORD PTR [rip+0x2013f6]        # 202050 <ccount>
c5a: add     rax,0x1
c5e: mov     QWORD PTR [rip+0x2013eb],rax        # 202050 <ccount>
c65: cmp     DWORD PTR [rbp-0x8],0xa
c69: jne     c7d <getword+0x7c>
c6b: mov     rax,QWORD PTR [rip+0x2013e6]        # 202058 <lcount>
c72: add     rax,0x1
c76: mov     QWORD PTR [rip+0x2013db],rax        # 202058 <lcount>
c7d: mov     rax,QWORD PTR [rbp-0x18]
c81: mov     rdi,rax
c84: call    820 <-IO-getc@plt>
c89: mov     DWORD PTR [rbp-0x8],eax
c8c: cmp     DWORD PTR [rbp-0x8],0xffffffff
c90: jne     c2e <getword+0x2d>
c92: jmp     cde <getword+0xdd>
c94: mov     rax,QWORD PTR [rip+0x2013b5]        # 202050 <ccount>
c9b: add     rax,0x1
c9f: mov     QWORD PTR [rip+0x2013aa],rax        # 202050 <ccount>
ca6: cmp     DWORD PTR [rbp-0x8],0xa
caa: jne     cbe <getword+0xbd>
cac: mov     rax,QWORD PTR [rip+0x2013a5]        # 202058 <lcount>
cb3: add     rax,0x1
cb7: mov     QWORD PTR [rip+0x20139a],rax        # 202058 <lcount>
cbe: mov     eax,DWORD PTR [rbp-0x8]
cc1: movzx   eax,al
cc4: mov     edi,eax
cc6: call    bd5 <isword>
ccb: test    eax,eax
ccd: je      ce6 <getword+0xe5>
ccf: mov     rax,QWORD PTR [rbp-0x18]
cd3: mov     rdi,rax
cd6: call    820 <-IO-getc@plt>
cdb: mov     DWORD PTR [rbp-0x8],eax
cde: cmp     DWORD PTR [rbp-0x8],0xffffffff
ce2: jne     c94 <getword+0x93>
ce4: jmp     ce7 <getword+0xe6>
ce6: nop
ce7: cmp     DWORD PTR [rbp-0x8],0xffffffff
ceb: setne   al
cee: movzx   eax,al
cf1: leave
cf2: ret
```

```
cf3: push   rbp
cf4: mov    rbp,rsp
cf7: sub    rsp,0x20
cfb: mov    QWORD PTR [rbp−0x18],rdi
cff: mov    rax,QWORD PTR [rbp−0x18]
d03: lea    rsi,[rip+0x1ff]        # f09 <-IO-stdin-used+0x19>
d0a: mov    rdi,rax
d0d: call   830 <fopen@plt>
d12: mov    QWORD PTR [rbp−0x8],rax
d16: cmp    QWORD PTR [rbp−0x8],0x0
d1b: jne    d35 <counter+0x42>
d1d: mov    rax,QWORD PTR [rbp−0x18]
d21: mov    rsi,rax
d24: lea    rdi,[rip+0x1e0]        # f0b <-IO-stdin-used+0x1b>
d2b: mov    eax,0x0
d30: call   ac6 <perrf>
d35: mov    QWORD PTR [rip+0x201318],0x0        # 202058 <lcount>
d40: mov    rax,QWORD PTR [rip+0x201311]        # 202058 <lcount>
d47: mov    QWORD PTR [rip+0x2012fa],rax        # 202048 <wcount>
d4e: mov    rax,QWORD PTR [rip+0x2012f3]        # 202048 <wcount>
d55: mov    QWORD PTR [rip+0x2012f4],rax        # 202050 <ccount>
d5c: nop
d5d: mov    rax,QWORD PTR [rbp−0x8]
d61: mov    rdi,rax
d64: call   c01 <getword>
d69: test   eax,eax
d6b: jne    d5d <counter+0x6a>
d6d: mov    rax,QWORD PTR [rbp−0x8]
d71: mov    rdi,rax
d74: call   7d0 <fclose@plt>
d79: mov    rcx,QWORD PTR [rip+0x2012d8]        # 202058 <lcount>
d80: mov    rdx,QWORD PTR [rip+0x2012c1]        # 202048 <wcount>
d87: mov    rsi,QWORD PTR [rip+0x2012c2]        # 202050 <ccount>
d8e: mov    rax,QWORD PTR [rbp−0x18]
d92: mov    rdi,rax
d95: call   b93 <report>
d9a: mov    rdx,QWORD PTR [rip+0x20128f]        # 202030 <total-ccount>
da1: mov    rax,QWORD PTR [rip+0x2012a8]        # 202050 <ccount>
da8: add    rax,rdx
dab: mov    QWORD PTR [rip+0x20127e],rax        # 202030 <total-ccount>
db2: mov    rdx,QWORD PTR [rip+0x20127f]        # 202038 <total-wcount>
db9: mov    rax,QWORD PTR [rip+0x201288]        # 202048 <wcount>
dc0: add    rax,rdx
dc3: mov    QWORD PTR [rip+0x20126e],rax        # 202038 <total-wcount>
dca: mov    rdx,QWORD PTR [rip+0x20126f]        # 202040 <total-lcount>
dd1: mov    rax,QWORD PTR [rip+0x201280]        # 202058 <lcount>
dd8: add    rax,rdx
ddb: mov    QWORD PTR [rip+0x20125e],rax        # 202040 <total-lcount>
```

de2: *nop*
de3: *leave*
de4: *ret*
›
end

context *wc-objdump*
begin
find-theorems *fetch*

Note: this theorems takes roughly 15 minutes to prove.

schematic-goal *counter*:
assumes $\sigma_I = \sigma$ *with* [*setRip 0xcf3*]
shows *assumptions ?A* $\Longrightarrow$ *run 0xde4 fetch* σ_I *?*σ'
apply (*subst assms*)

apply (*repeat-n 8 se-step*)
— rip = 0x830 (2096), calling fopen
apply *se-step*

apply (*repeat-n 12 se-step*)
— rip = 0xc01 (3073), calling getword
apply *se-step*

apply (*repeat-n 13 se-step*)
— rip = 0xc84 (2080), calling getc
apply *se-step*

apply (*repeat-n 32 se-step*)
— rip = 0xc84 (2080), calling getc
apply *se-step*

apply (*repeat-n 18 se-step*)
— rip = 0xc84 (2080), calling getc
apply *se-step*

apply (*repeat-n 18 se-step*)
— rip = 0xc84 (2080), calling getc
apply *se-step*

apply (*repeat-n 18 se-step*)
— rip = 0xc84 (2080), calling getc
apply *se-step*

apply (*repeat-n 18 se-step*)
— rip = 0xc84 (2080), calling getc

apply *se-step*

apply (*repeat-n 9 se-step*)

apply (*repeat-n 5 se-step*)
— rip = 0x7d0 (2000), calling fclose
apply *se-step*

apply (*repeat-n 6 se-step*)
— rip = 0xb93 (2963), calling report
apply *se-step*

apply (*repeat-n 15 se-step*)
apply (*subst eq-def*,*simp*)
done

thm *counter*

The file opened by "fopen" contains the zero-terminated string "Hello": 0x006E6C6C6548. After each call of getc, register RAX contains the read characters' ASCII code.

After termination, we can see the contents of the following global variables, set by function getword:

```
Word count:      wcount = 1      (0x202048 = 2105416)
Character count: ccount = 5      (0x202050 = 2105424)
Line count:      lcount = lcount (0x202058 = 2105432)
```

The totals accumulate to:

```
Word count:      total_wcount = total_wcount + 5 (0x202038 = 2105400)
Character count: total_ccount = total_ccount + 5 (0x202030 = 2105392)
Line count:      total_lcount = total_lcount     (0x202040 = 2105408)
```

end
end
[1]

References

[1] T. Nipkow, L. Paulson, and M. Wenzel. *Isabelle/HOL — A Proof Assistant for Higher-Order Logic*, volume 2283. 2002. http://www.in.tum.de/~nipkow/LNCS2283/.