

Formal Network Models and Their Application to Firewall Policies (UPF-Firewall)

Achim D. Brucker* Lukas Brügger[†] Burkhardt Wolff[‡]

*Department of Computer Science, The University of Sheffield, Sheffield, UK
a.brucker@sheffield.ac.uk

Information Security, ETH Zurich, 8092 Zurich, Switzerland
Lukas.A.Bruegger@gmail.com

[‡]Univ. Paris-Sud, Laboratoire LRI, UMR8623, 91405 Orsay, France France
burkhart.wolff@lri.fr

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Abstract

We present a formal model of network protocols and their application to modeling firewall policies. The formalization is based on the *Unified Policy Framework* (UPF). The formalization was originally developed with for generating test cases for testing the security configuration actual firewall and router (middle-boxes) using HOL-TestGen. Our work focuses on modeling application level protocols on top of tcp/ip.

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1 Introduction

1.1 Motivation

Because of its connected life, the modern world is increasingly depending on secure implementations and configurations of network infrastructures. As building blocks of the latter, firewalls are playing a central role in ensuring the overall *security* of networked applications.

Firewalls, routers applying network-address-translation (NAT) and similar networking systems suffer from the same quality problems as other complex software. Jennifer Rexford mentioned in her keynote at POPL 2012 that high-end firewalls consist of more than 20 million lines of code comprising components written in Ada as well as LISP. However, the testing techniques discussed here are of wider interest to all network infrastructure operators that need to ensure the security and reliability of their infrastructures across system changes such as system upgrades or hardware replacements. This is because firewalls and routers are active network elements that can filter and rewrite network traffic based on configurable rules. The *configuration* by appropriate rule sets implements a security policy or links networks together.

Thus, it is, firstly, important to test both the implementation of a firewall and, secondly, the correct configuration for each use. To address this problem, we model firewall policies formally in Isabelle/HOL. This formalization is based on the Unified Policy Framework (UPF) [6]. This formalization allows to express access control policies on the network level using a combinator-based language that is close to textbook-style specifications of firewall rules. To actually test the implementation as well as the configuration of a firewall, we use HOL-TestGen [1, 2, 5] to generate test cases that can be used to validate the compliance of real network middleboxes (e.g., firewalls, routers). In this document, we focus on the Isabelle formalization of network access control policies. For details of the overall approach, we refer the reader elsewhere [7]

1.2 The Unified Policy Framework (UPF)

Our formalization of firewall policies is based on the Unified Policy Framework (UPF). In this section, we briefly introduce UPF, for all details we refer the reader to) [6].

UPF is a generic framework for policy modeling with the primary goal of being used for test case generation. The interested reader is referred to [4] for an application of UPF to large scale access control policies in the health care domain; a comprehensive treatment is also contained in the reference manual coming with the distribution on the HOL-TestGen website (<http://www.brucker.ch/projects/hol-testgen/>). UPF is based on

the following four principles:

1. policies are represented as *functions* (rather than relations),
2. policy combination avoids conflicts by construction,
3. the decision type is three-valued (allow, deny, undefined),
4. the output type does not only contain the decision but also a ‘slot’ for arbitrary result data.

Formally, the concept of a policy is specified as a partial function from some input to a decision value and additional some output. *Partial* functions are used because elementary policies are described by partial system behavior, which are glued together by operators such as function override and functional composition.

$$\text{type_synonym } \alpha \mapsto \beta = \alpha \multimap \beta \text{ decision}$$

where the enumeration type decision is

$$\text{datatype } \alpha \text{ decision} = \text{allow } \alpha \mid \text{deny } \alpha$$

As policies are partial functions or ‘maps’, the notions of a *domain* $\text{dom } p :: \alpha \multimap \beta \Rightarrow \alpha \text{ set}$ and a *range* $\text{ran } p :: [\alpha \multimap \beta] \Rightarrow \beta \text{ set}$ can be inherited from the Isabelle library.

Inspired by the Z notation [8], there is the concept of *domain restriction* $_ \triangleleft _$ and *range restriction* $_ \triangleright _$, defined as:

$$\begin{aligned} \text{definition } _ \triangleleft _ &:: \alpha \text{ set} \Rightarrow \alpha \mapsto \beta \Rightarrow \alpha \mapsto \beta \\ \text{where } S \triangleleft p &= \lambda x. \text{ if } x \in S \text{ then } p x \text{ else } \perp \\ \text{definition } _ \triangleright _ &:: \alpha \mapsto \beta \Rightarrow \beta \text{ decision set} \Rightarrow \alpha \mapsto \beta \\ \text{where } p \triangleright S &= \lambda x. \text{ if } (\text{the } (p x)) \in S \text{ then } p x \text{ else } \perp \end{aligned}$$

The operator ‘the’ strips off the Some, if it exists. Otherwise the range restriction is underspecified.

There are many operators that change the result of applying the policy to a particular element. The essential one is the *update*:

$$p(x \mapsto t) = \lambda y. \text{ if } y = x \text{ then } [t] \text{ else } p y$$

Next, there are three categories of elementary policies in UPF, relating to the three possible decision values:

- The empty policy; undefined for all elements: $\emptyset = \lambda x. \perp$
- A policy allowing everything, written as $A_f f$, or A_U if the additional output is unit (defined as $\lambda x. [\text{allow}()]$).
- A policy denying everything, written as $D_f f$, or D_U if the additional output is unit.

The most often used approach to define individual rules is to define a rule as a refinement of one of the elementary policies, by using a domain restriction. As an example,

$$\{(Alice, obj1, read)\} \triangleleft A_U$$

Finally, rules can be combined to policies in three different ways:

- Override operators: used for policies of the same type, written as $_ \oplus_i _$.
- Parallel combination operators: used for the parallel composition of policies of potentially different type, written as $_ \otimes_i _$.
- Sequential combination operators: used for the sequential composition of policies of potentially different type, written as $_ \circ_i _$.

All three combinators exist in four variants, depending on how the decisions of the constituent policies are to be combined. For example, the $_ \otimes_2 _$ operator is the parallel combination operator where the decision of the second policy is used.

Several interesting algebraic properties are proved for UPF operators. For example, distributivity

$$(P_1 \oplus P_2) \otimes P_3 = (P_1 \otimes P_3) \oplus (P_2 \otimes P_3)$$

Other UPF concepts are introduced in this paper on-the-fly when needed.

2 UPF Firewall

theory

UPF – Firewall

imports

PacketFilter / PacketFilter

NAT / NAT

FWNormalisation / FWNormalisation

StatefulFW / StatefulFW

begin

This is the main entry point for specifications of firewall policies.

end

2.1 Network Models

theory

NetworkModels

imports

DatatypeAddress

DatatypePort

IntegerAddress

IntegerPort

IntegerPort-TCPUDP

IPv4

IPv4-TCPUDP

begin

One can think of many different possible address representations. In this distribution, we include seven different variants:

- *DatatypeAddress*: Three explicitly named addresses, which build up a network consisting of three disjunct subnetworks. I.e. there are no overlaps and there is no way to distinguish between individual hosts within a network.
- *DatatypePort*: An address is a pair, with the first element being the same as above, and the second being a port number modelled as an *Integer*¹.

¹For technical reasons, we always use Integers instead of Naturals. As a consequence, the (test) specifications have to be adjusted to eliminate negative numbers.

- `adr_i`: An address in an Integer.
- `adr_ip`: An address is a pair of an Integer and a port (which is again an Integer).
- `adr_ipp`: An address is a triple consisting of two Integers modelling the IP address and the port number, and the specification of the network protocol
- `IPv4`: An address is a pair. The first element is a four-tuple of Integers, modelling an IPv4 address, the second element is an Integer denoting the port number.
- `IPv4_TCPUDP`: The same as above, but including additionally the specification of the network protocol.

The theories of each of the networks are relatively small. It suffices to provide the required types, a couple of lemmas, and - if required - a definition for the source and destination ports of a packet.

end

2.1.1 Packets and Networks

theory

NetworkCore

imports

Main

begin

In networks based e.g. on TCP/IP, a message from A to B is encapsulated in *packets*, which contain the content of the message and routing information. The routing information mainly contains its source and its destination address.

In the case of stateless packet filters, a firewall bases its decision upon this routing information and, in the stateful case, on the content. Thus, we model a packet as a four-tuple of the mentioned elements, together with an id field.

The ID is an integer:

type-synonym *id* = *int*

To enable different representations of addresses (e.g. IPv4 and IPv6, with or without ports), we model them as an unconstrained type class and directly provide several instances:

class *adr*

type-synonym $'\alpha$ *src* = $'\alpha$

type-synonym $'\alpha$ *dest* = $'\alpha$

instance *int* :: *adr* ..

instance *nat* :: *adr* ..

instance *fun* :: (*adr,adr*) *adr* ..

instance *prod* :: (*adr,adr*) *adr* ..

The content is also specified with an unconstrained generic type:

type-synonym *'β content* = *'β*

For applications where the concrete representation of the content field does not matter (usually the case for stateless packet filters), we provide a default type which can be used in those cases:

datatype *DummyContent* = *data*

Finally, a packet is:

type-synonym (*'α,'β*) *packet* = *id* × *'α src* × *'α dest* × *'β content*

Protocols (e.g. http) are not modelled explicitly. In the case of stateless packet filters, they are only visible by the destination port of a packet, which are modelled as part of the address. Additionally, stateful firewalls often determine the protocol by the content of a packet.

definition *src* :: (*'α::adr,'β*) *packet* ⇒ *'α*

where *src* = *fst o snd*

Port numbers (which are part of an address) are also modelled in a generic way. The integers and the naturals are typical representations of port numbers.

class *port*

instance *int* :: *port* ..

instance *nat* :: *port* ..

instance *fun* :: (*port,port*) *port* ..

instance *prod* :: (*port,port*) *port* ..

A packet therefore has two parameters, the first being the address, the second the content. For the sake of simplicity, we do not allow to have a different address representation format for the source and the destination of a packet.

To access the different parts of a packet directly, we define a couple of projectors:

definition *id* :: (*'α::adr,'β*) *packet* ⇒ *id*

where *id* = *fst*

definition *dest* :: (*'α::adr,'β*) *packet* ⇒ *'α dest*

where *dest* = *fst o snd o snd*

definition *content* :: (*'α::adr,'β*) *packet* ⇒ *'β content*

where *content* = *snd o snd o snd*

datatype *protocol* = *tcp* | *udp*

lemma *either*: $\llbracket a \neq tcp; a \neq udp \rrbracket \implies False$
by (*case-tac a, simp-all*)

lemma *either2*[*simp*]: $(a \neq tcp) = (a = udp)$
by (*case-tac a, simp-all*)

lemma *either3*[*simp*]: $(a \neq udp) = (a = tcp)$
by (*case-tac a, simp-all*)

The following two constants give the source and destination port number of a packet. Address representations using port numbers need to provide a definition for these types.

consts *src-port* :: $('\alpha::adr, '\beta) packet \Rightarrow '\gamma::port$
consts *dest-port* :: $(''\alpha::adr, '\beta) packet \Rightarrow '\gamma::port$
consts *src-protocol* :: $(''\alpha::adr, '\beta) packet \Rightarrow protocol$
consts *dest-protocol* :: $(''\alpha::adr, '\beta) packet \Rightarrow protocol$

A subnetwork (or simply a network) is a set of sets of addresses.

type-synonym $'\alpha net = '\alpha set set$

The relation *in_subnet* (\sqsubset) checks if an address is in a specific network.

definition

in_subnet :: $'\alpha::adr \Rightarrow '\alpha net \Rightarrow bool$ (**infixl** \sqsubset 100) **where**
in_subnet $a S = (\exists s \in S. a \in s)$

The following lemmas will be useful later.

lemma *in_subnet*:
 $(a, e) \sqsubset \{\{(x1, y). P x1 y\}\} = P a e$
by (*simp add: in_subnet-def*)

lemma *src-in_subnet*:
 $src(q, (a, e), r, t) \sqsubset \{\{(x1, y). P x1 y\}\} = P a e$
by (*simp add: in_subnet-def in_subnet src-def*)

lemma *dest-in_subnet*:
 $dest(q, r, ((a), e), t) \sqsubset \{\{(x1, y). P x1 y\}\} = P a e$
by (*simp add: in_subnet-def in_subnet dest-def*)

Address models should provide a definition for the following constant, returning a network consisting of the input address only.

consts *subnet-of* :: $'\alpha::adr \Rightarrow '\alpha net$

lemmas *packet-defs = in_subnet-def id-def content-def src-def dest-def*

end

2.1.2 Datatype Addresses

theory

DatatypeAddress

imports

NetworkCore

begin

A theory describing a network consisting of three subnetworks. Hosts within a network are not distinguished.

datatype *DatatypeAddress* = *dmz-adr* | *intranet-adr* | *internet-adr*

definition

dmz::DatatypeAddress net where

dmz = $\{\{dmz-adr\}\}$

definition

intranet::DatatypeAddress net where

intranet = $\{\{intranet-adr\}\}$

definition

internet::DatatypeAddress net where

internet = $\{\{internet-adr\}\}$

end

2.1.3 Datatype Addresses with Ports

theory

DatatypePort

imports

NetworkCore

begin

A theory describing a network consisting of three subnetworks, including port numbers modelled as Integers. Hosts within a network are not distinguished.

datatype *DatatypeAddress* = *dmz-adr* | *intranet-adr* | *internet-adr*

type-synonym

port = *int*

type-synonym

DatatypePort = (*DatatypeAddress* × *port*)

instance *DatatypeAddress* :: *adr* ..

definition

dmz::DatatypePort net where

```

    dmz = {{{(a,b). a = dmz-adr}}}
definition
    intranet::DatatypePort net where
    intranet = {{{(a,b). a = intranet-adr}}}
definition
    internet::DatatypePort net where
    internet = {{{(a,b). a = internet-adr}}}

overloading src-port-datatype ≡ src-port :: ('α::adr,'β) packet ⇒ 'γ::port
begin
definition
    src-port-datatype (x::(DatatypePort,'β) packet) ≡ (snd o fst o snd) x
end

overloading dest-port-datatype ≡ dest-port :: ('α::adr,'β) packet ⇒ 'γ::port
begin
definition
    dest-port-datatype (x::(DatatypePort,'β) packet) ≡(snd o fst o snd o snd) x
end

overloading subnet-of-datatype ≡ subnet-of :: 'α::adr ⇒ 'α net
begin
definition
    subnet-of-datatype (x::DatatypePort) ≡ {{{(a,b::int). a = fst x}}}
end

lemma src-port : src-port ((a,x,d,e)::(DatatypePort,'β) packet) = snd x
    by (simp add: src-port-datatype-def in-subnet)

lemma dest-port : dest-port ((a,d,x,e)::(DatatypePort,'β) packet) = snd x
    by (simp add: dest-port-datatype-def in-subnet)

lemmas DatatypePortLemmas = src-port dest-port src-port-datatype-def
    dest-port-datatype-def

end

```

2.1.4 Integer Addresses

```

theory
    IntegerAddress
imports
    NetworkCore
begin

```

A theory where addresses are modelled as Integers.

type-synonym

adr_i = int

end

2.1.5 Integer Addresses with Ports

theory

IntegerPort

imports

NetworkCore

begin

A theory describing addresses which are modelled as a pair of Integers - the first being the host address, the second the port number.

type-synonym

address = int

type-synonym

port = int

type-synonym

adr_{ip} = address × port

overloading *src-port-int* \equiv *src-port* :: ('α::adr,'β) packet \Rightarrow 'γ::port

begin

definition

src-port-int (*x*::(adr_{ip},β) packet) \equiv (snd o fst o snd) *x*

end

overloading *dest-port-int* \equiv *dest-port* :: ('α::adr,'β) packet \Rightarrow 'γ::port

begin

definition

dest-port-int (*x*::(adr_{ip},β) packet) \equiv (snd o fst o snd o snd) *x*

end

overloading *subnet-of-int* \equiv *subnet-of* :: 'α::adr \Rightarrow 'α net

begin

definition

subnet-of-int (*x*::(adr_{ip})) \equiv {(a,b::int). a = fst *x*}

end

lemma *src-port*: *src-port* (a,x::adr_{ip},d,e) = snd *x*

```

    by (simp add: src-port-int-def in-subnet)

lemma dest-port: dest-port (a,d,x::adrip,e) = snd x
  by (simp add: dest-port-int-def in-subnet)

lemmas adripLemmas = src-port dest-port src-port-int-def dest-port-int-def

end

```

2.1.6 Integer Addresses with Ports and Protocols

theory

IntegerPort-TCPUDP

imports

NetworkCore

begin

A theory describing addresses which are modelled as a pair of Integers - the first being the host address, the second the port number.

type-synonym

address = int

type-synonym

port = int

type-synonym

adr_{ipp} = address × port × protocol

instance protocol :: adr ..

overloading *src-port-int-TCPUDP* ≡ *src-port* :: ('α::adr,'β) packet ⇒ 'γ::port

begin

definition

src-port-int-TCPUDP (x::(adr_{ipp}, 'β) packet) ≡ (fst o snd o fst o snd) x

end

overloading *dest-port-int-TCPUDP* ≡ *dest-port* :: ('α::adr,'β) packet ⇒ 'γ::port

begin

definition

dest-port-int-TCPUDP (x::(adr_{ipp}, 'β) packet) ≡ (fst o snd o fst o snd o snd) x

end

overloading *subnet-of-int-TCPUDP* ≡ *subnet-of* :: 'α::adr ⇒ 'α net

begin

definition

subnet-of-int-TCPUDP ($x::(adr_{ipp})$) $\equiv \{(a,b,c). a = fst\ x\}::adr_{ipp}\ net$
end

overloading *src-protocol-int-TCPUDP* $\equiv src\ protocol :: ('\alpha::adr, '\beta)\ packet \Rightarrow protocol$

begin**definition**

src-protocol-int-TCPUDP ($x::(adr_{ipp}, '\beta)\ packet$) $\equiv (snd\ o\ snd\ o\ fst\ o\ snd)\ x$
end

overloading *dest-protocol-int-TCPUDP* $\equiv dest\ protocol :: ('\alpha::adr, '\beta)\ packet \Rightarrow proto\ col$

begin**definition**

dest-protocol-int-TCPUDP ($x::(adr_{ipp}, '\beta)\ packet$) $\equiv (snd\ o\ snd\ o\ fst\ o\ snd\ o\ snd)\ x$
end

lemma *src-port*: $src\ port\ (a, x::adr_{ipp}, d, e) = fst\ (snd\ x)$

by (*simp add: src-port-int-TCPUDP-def in-subnet*)

lemma *dest-port*: $dest\ port\ (a, d, x::adr_{ipp}, e) = fst\ (snd\ x)$

by (*simp add: dest-port-int-TCPUDP-def in-subnet*)

Common test constraints:

definition *port-positive* $:: (adr_{ipp}, '\beta)\ packet \Rightarrow bool$ **where**

port-positive $x = (dest\ port\ x > (0::port))$

definition *fix-values* $:: (adr_{ipp}, DummyContent)\ packet \Rightarrow bool$ **where**

fix-values $x = (src\ port\ x = (1::port) \wedge src\ protocol\ x = udp \wedge content\ x = data \wedge id\ x = 1)$

lemmas *adr_{ipp}Lemmas* = *src-port dest-port src-port-int-TCPUDP-def dest-port-int-TCPUDP-def*

src-protocol-int-TCPUDP-def dest-protocol-int-TCPUDP-def subnet-of-int-TCPUDP-def

lemmas *adr_{ipp}TestConstraints* = *port-positive-def fix-values-def*

end

2.1.7 Formalizing IPv4 Addresses

theory

```

IPv4
imports
  NetworkCore
begin

  A theory describing IPv4 addresses with ports. The host address is a four-tuple of
  Integers, the port number is a single Integer.

type-synonym
  ipv4-ip = (int × int × int × int)

type-synonym
  port = int

type-synonym
  ipv4 = (ipv4-ip × port)

overloading src-port-ipv4 ≡ src-port :: ('α::adr, 'β) packet ⇒ 'γ::port
begin
definition
  src-port-ipv4 (x::(ipv4, 'β) packet) ≡ (snd o fst o snd) x
end

overloading dest-port-ipv4 ≡ dest-port :: ('α::adr, 'β) packet ⇒ 'γ::port
begin
definition
  dest-port-ipv4 (x::(ipv4, 'β) packet) ≡ (snd o fst o snd o snd) x
end

overloading subnet-of-ipv4 ≡ subnet-of :: 'α::adr ⇒ 'α net
begin
definition
  subnet-of-ipv4 (x::ipv4) ≡ { {(a, b::int). a = fst x} }
end

definition subnet-of-ip :: ipv4-ip ⇒ ipv4 net
  where subnet-of-ip ip = { {(a, b). (a = ip)} }

lemma src-port: src-port (a, (x::ipv4), d, e) = snd x
  by (simp add: src-port-ipv4-def in-subnet)

lemma dest-port: dest-port (a, d, (x::ipv4), e) = snd x
  by (simp add: dest-port-ipv4-def in-subnet)

```

lemmas *IPv4Lemmas* = *src-port dest-port src-port-ipv4-def dest-port-ipv4-def*

end

2.1.8 IPv4 with Ports and Protocols

theory

IPv4-TCPUDP

imports *IPv4*

begin

type-synonym

ipv4-TCPUDP = (*ipv4-ip* × *port* × *protocol*)

instance *protocol* :: *adr* ..

overloading *src-port-ipv4-TCPUDP* ≡ *src-port* :: ('α::*adr*, 'β) *packet* ⇒ 'γ::*port*

begin

definition

src-port-ipv4-TCPUDP (*x*::(*ipv4-TCPUDP*, 'β) *packet*) ≡ (*fst o snd o fst o snd*) *x*

end

overloading *dest-port-ipv4-TCPUDP* ≡ *dest-port* :: ('α::*adr*, 'β) *packet* ⇒ 'γ::*port*

begin

definition

dest-port-ipv4-TCPUDP (*x*::(*ipv4-TCPUDP*, 'β) *packet*) ≡ (*fst o snd o fst o snd o snd*) *x*

end

overloading *subnet-of-ipv4-TCPUDP* ≡ *subnet-of* :: 'α::*adr* ⇒ 'α *net*

begin

definition

subnet-of-ipv4-TCPUDP (*x*::*ipv4-TCPUDP*) ≡ { {(*a*, *b*). *a* = *fst x* } }::(*ipv4-TCPUDP net*)

end

overloading *dest-protocol-ipv4-TCPUDP* ≡ *dest-protocol* :: ('α::*adr*, 'β) *packet* ⇒ *protocol*

begin

definition

dest-protocol-ipv4-TCPUDP (*x*::(*ipv4-TCPUDP*, 'β) *packet*) ≡ (*snd o snd o fst o snd o snd*) *x*

end

definition *subnet-of-ip* :: *ipv4-ip* \Rightarrow *ipv4-TCPUDP net*
where *subnet-of-ip ip* = $\{\{(a,b). (a = ip)\}\}$

lemma *src-port*: *src-port* (*a*, (*x*::*ipv4-TCPUDP*), *d*, *e*) = *fst* (*snd x*)
by (*simp add: src-port-ipv4-TCPUDP-def in-subnet*)

lemma *dest-port*: *dest-port* (*a*, *d*, (*x*::*ipv4-TCPUDP*), *e*) = *fst* (*snd x*)
by (*simp add: dest-port-ipv4-TCPUDP-def in-subnet*)

lemmas *Ipv4-TCPUDPLemmas* = *src-port dest-port src-port-ipv4-TCPUDP-def*
dest-port-ipv4-TCPUDP-def
dest-protocol-ipv4-TCPUDP-def subnet-of-ipv4-TCPUDP-def
end

2.2 Network Policies: Packet Filter

theory
PacketFilter
imports
NetworkModels
ProtocolPortCombinators
Ports
begin
end

2.2.1 Policy Core

theory
PolicyCore
imports
NetworkCore
UPF.UPF
begin

A policy is seen as a partial mapping from packet to packet out.

type-synonym ($'\alpha$, $'\beta$) *FWPolicy* = ($'\alpha$, $'\beta$) *packet* \mapsto *unit*

When combining several rules, the firewall is supposed to apply the first matching one. In our setting this means the first rule which maps the packet in question to *Some* (*packet out*). This is exactly what happens when using the map-add operator (*rule1 ++ rule2*). The only difference is that the rules must be given in reverse order.

The constant *p-accept* is *True* iff the policy accepts the packet.

definition
p-accept :: ($'\alpha$, $'\beta$) *packet* \Rightarrow ($'\alpha$, $'\beta$) *FWPolicy* \Rightarrow *bool* **where**

$p\text{-accept } p \text{ pol} = (\text{pol } p = [\text{allow } ()])$

end

2.2.2 Policy Combinators

theory

PolicyCombinators

imports

PolicyCore

begin

In order to ease the specification of a concrete policy, we define some combinators. Using these combinators, the specification of a policy gets very easy, and can be done similarly as in tools like IPTables.

definition

$\text{allow-all-from} :: 'a::\text{adr net} \Rightarrow (('a, 'b) \text{ packet} \mapsto \text{unit}) \text{ where}$
 $\text{allow-all-from src-net} = \{pa. \text{src } pa \sqsubset \text{src-net}\} \triangleleft A_U$

definition

$\text{deny-all-from} :: 'a::\text{adr net} \Rightarrow (('a, 'b) \text{ packet} \mapsto \text{unit}) \text{ where}$
 $\text{deny-all-from src-net} = \{pa. \text{src } pa \sqsubset \text{src-net}\} \triangleleft D_U$

definition

$\text{allow-all-to} :: 'a::\text{adr net} \Rightarrow (('a, 'b) \text{ packet} \mapsto \text{unit}) \text{ where}$
 $\text{allow-all-to dest-net} = \{pa. \text{dest } pa \sqsubset \text{dest-net}\} \triangleleft A_U$

definition

$\text{deny-all-to} :: 'a::\text{adr net} \Rightarrow (('a, 'b) \text{ packet} \mapsto \text{unit}) \text{ where}$
 $\text{deny-all-to dest-net} = \{pa. \text{dest } pa \sqsubset \text{dest-net}\} \triangleleft D_U$

definition

$\text{allow-all-from-to} :: 'a::\text{adr net} \Rightarrow 'a::\text{adr net} \Rightarrow (('a, 'b) \text{ packet} \mapsto \text{unit}) \text{ where}$
 $\text{allow-all-from-to src-net dest-net} =$
 $\{pa. \text{src } pa \sqsubset \text{src-net} \wedge \text{dest } pa \sqsubset \text{dest-net}\} \triangleleft A_U$

definition

$\text{deny-all-from-to} :: 'a::\text{adr net} \Rightarrow 'a::\text{adr net} \Rightarrow (('a, 'b) \text{ packet} \mapsto \text{unit}) \text{ where}$
 $\text{deny-all-from-to src-net dest-net} = \{pa. \text{src } pa \sqsubset \text{src-net} \wedge \text{dest } pa \sqsubset \text{dest-net}\} \triangleleft D_U$

All these combinators and the default rules are put into one single lemma called *PolicyCombinators* to facilitate proving over policies.

lemmas *PolicyCombinators* = *allow-all-from-def deny-all-from-def*

allow-all-to-def deny-all-to-def allow-all-from-to-def

deny-all-from-to-def UPFDefs

end

2.2.3 Policy Combinators with Ports

theory

PortCombinators

imports

PolicyCombinators

begin

This theory defines policy combinators for those network models which have ports. They are provided in addition to the the ones defined in the PolicyCombinators theory.

This theory requires from the network models a definition for the two following constants:

- $src_port :: ('\alpha, '\beta)packet \Rightarrow (' \gamma :: port)$
- $dest_port :: (' \alpha, '\beta)packet \Rightarrow (' \gamma :: port)$

definition

$allow_all_from_port :: '\alpha :: adr\ net \Rightarrow (' \gamma :: port) \Rightarrow ((' \alpha, '\beta) packet \mapsto unit)$ **where**
 $allow_all_from_port\ src_net\ s_port = \{pa.\ src_port\ pa = s_port\} \triangleleft allow_all_from\ src_net$

definition

$deny_all_from_port :: '\alpha :: adr\ net \Rightarrow (' \gamma :: port) \Rightarrow ((' \alpha, '\beta) packet \mapsto unit)$ **where**
 $deny_all_from_port\ src_net\ s_port = \{pa.\ src_port\ pa = s_port\} \triangleleft deny_all_from\ src_net$

definition

$allow_all_to_port :: '\alpha :: adr\ net \Rightarrow (' \gamma :: port) \Rightarrow ((' \alpha, '\beta) packet \mapsto unit)$ **where**
 $allow_all_to_port\ dest_net\ d_port = \{pa.\ dest_port\ pa = d_port\} \triangleleft allow_all_to\ dest_net$

definition

$deny_all_to_port :: '\alpha :: adr\ net \Rightarrow (' \gamma :: port) \Rightarrow ((' \alpha, '\beta) packet \mapsto unit)$ **where**
 $deny_all_to_port\ dest_net\ d_port = \{pa.\ dest_port\ pa = d_port\} \triangleleft deny_all_to\ dest_net$

definition

$allow_all_from_port_to :: '\alpha :: adr\ net \Rightarrow (' \gamma :: port) \Rightarrow '\alpha :: adr\ net \Rightarrow ((' \alpha, '\beta) packet \mapsto unit)$
where
 $allow_all_from_port_to\ src_net\ s_port\ dest_net$
 $= \{pa.\ src_port\ pa = s_port\} \triangleleft allow_all_from_to\ src_net\ dest_net$

definition

deny-all-from-port-to :: 'α::adr net ⇒ 'γ::port ⇒ 'α::adr net ⇒ (('α,'β) packet ↦ unit)
where

deny-all-from-port-to src-net s-port dest-net
= {pa. src-port pa = s-port} ◁ *deny-all-from-to src-net dest-net*

definition

allow-all-from-port-to-port :: 'α::adr net ⇒ 'γ::port ⇒ 'α::adr net ⇒ 'γ::port ⇒
(('α,'β) packet ↦ unit) **where**

allow-all-from-port-to-port src-net s-port dest-net d-port =
{pa. dest-port pa = d-port} ◁ *allow-all-from-port-to src-net s-port dest-net*

definition

deny-all-from-port-to-port :: 'α::adr net ⇒ 'γ::port ⇒ 'α::adr net ⇒
'γ::port ⇒ (('α,'β) packet ↦ unit) **where**

deny-all-from-port-to-port src-net s-port dest-net d-port =
{pa. dest-port pa = d-port} ◁ *deny-all-from-port-to src-net s-port dest-net*

definition

allow-all-from-to-port :: 'α::adr net ⇒ 'α::adr net ⇒
'γ::port ⇒ (('α,'β) packet ↦ unit) **where**

allow-all-from-to-port src-net dest-net d-port =
{pa. dest-port pa = d-port} ◁ *allow-all-from-to src-net dest-net*

definition

deny-all-from-to-port :: 'α::adr net ⇒ 'α::adr net ⇒ 'γ::port ⇒
(('α,'β) packet ↦ unit) **where**

deny-all-from-to-port src-net dest-net d-port =
{pa. dest-port pa = d-port} ◁ *deny-all-from-to src-net dest-net*

definition

allow-from-port-to :: 'γ::port ⇒ 'α::adr net ⇒ 'α::adr net ⇒ (('α,'β) packet ↦ unit)
where

allow-from-port-to port src-net dest-net =
{pa. src-port pa = port} ◁ *allow-all-from-to src-net dest-net*

definition

deny-from-port-to :: 'γ::port ⇒ 'α::adr net ⇒ 'α::adr net ⇒ (('α,'β) packet ↦ unit)
where

deny-from-port-to port src-net dest-net =
{pa. src-port pa = port} ◁ *deny-all-from-to src-net dest-net*

definition

allow-from-to-port :: 'γ::port ⇒ 'α::adr net ⇒ 'α::adr net ⇒ (('α,'β) packet ↦ unit)
where

allow-from-to-port port src-net dest-net =
{pa. dest-port pa = port} ◁ allow-all-from-to src-net dest-net

definition

deny-from-to-port :: 'γ::port ⇒ 'α::adr net ⇒ 'α::adr net ⇒ (('α,'β) packet ↦ unit)

where

deny-from-to-port port src-net dest-net =
{pa. dest-port pa = port} ◁ deny-all-from-to src-net dest-net

definition

allow-from-ports-to :: 'γ::port set ⇒ 'α::adr net ⇒ 'α::adr net ⇒
*(('α,'β) packet ↦ unit) **where***

allow-from-ports-to ports src-net dest-net =
{pa. src-port pa ∈ ports} ◁ allow-all-from-to src-net dest-net

definition

allow-from-to-ports :: 'γ::port set ⇒ 'α::adr net ⇒ 'α::adr net ⇒
*(('α,'β) packet ↦ unit) **where***

allow-from-to-ports ports src-net dest-net =
{pa. dest-port pa ∈ ports} ◁ allow-all-from-to src-net dest-net

definition

deny-from-ports-to :: 'γ::port set ⇒ 'α::adr net ⇒ 'α::adr net ⇒
*(('α,'β) packet ↦ unit) **where***

deny-from-ports-to ports src-net dest-net =
{pa. src-port pa ∈ ports} ◁ deny-all-from-to src-net dest-net

definition

deny-from-to-ports :: 'γ::port set ⇒ 'α::adr net ⇒ 'α::adr net ⇒
*(('α,'β) packet ↦ unit) **where***

deny-from-to-ports ports src-net dest-net =
{pa. dest-port pa ∈ ports} ◁ deny-all-from-to src-net dest-net

definition

allow-all-from-port-tos:: 'α::adr net ⇒ ('γ::port) set ⇒ 'α::adr net ⇒ (('α,'β) packet
↦ unit)

where

allow-all-from-port-tos src-net s-port dest-net
= {pa. dest-port pa ∈ s-port} ◁ allow-all-from-to src-net dest-net

As before, we put all the rules into one lemma called PortCombinators to ease writing later.

lemmas *PortCombinatorsCore =*

allow-all-from-port-def deny-all-from-port-def allow-all-to-port-def


```

deny-all-to-port-def allow-all-from-to-port-def
deny-all-from-to-port-def
allow-from-ports-to-def allow-from-to-ports-def
deny-from-ports-to-def deny-from-to-ports-def
allow-all-from-port-to-def deny-all-from-port-to-def
allow-from-port-to-def allow-from-to-port-def deny-from-to-port-def
deny-from-port-to-def allow-all-from-port-to-def

```

lemmas *PortCombinators* = *PortCombinatorsCore* *PolicyCombinators*

end

2.2.4 Policy Combinators with Ports and Protocols

theory

ProtocolPortCombinators

imports

PortCombinators

begin

This theory defines policy combinators for those network models which have ports. They are provided in addition to the the ones defined in the *PolicyCombinators* theory.

This theory requires from the network models a definition for the two following constants:

- $src_port :: ('\alpha, '\beta)packet \Rightarrow (' \gamma :: port)$
- $dest_port :: (' \alpha, '\beta)packet \Rightarrow (' \gamma :: port)$

definition

$allow_all_from_port_prot :: protocol \Rightarrow '\alpha :: adr\ net \Rightarrow (' \gamma :: port) \Rightarrow ((' \alpha, '\beta) packet \mapsto unit)$ **where**

$allow_all_from_port_prot\ p\ src_net\ s_port =$
 $\{pa. dest_protocol\ pa = p\} \triangleleft allow_all_from_port\ src_net\ s_port$

definition

$deny_all_from_port_prot :: protocol \Rightarrow '\alpha :: adr\ net \Rightarrow '\gamma :: port \Rightarrow ((' \alpha, '\beta) packet \mapsto unit)$ **where**

$deny_all_from_port_prot\ p\ src_net\ s_port =$
 $\{pa. dest_protocol\ pa = p\} \triangleleft deny_all_from_port\ src_net\ s_port$

definition

$allow_all_to_port_prot :: protocol \Rightarrow '\alpha :: adr\ net \Rightarrow '\gamma :: port \Rightarrow ((' \alpha, '\beta) packet \mapsto unit)$

where

$allow_all_to_port_prot\ p\ dest_net\ d_port =$

$\{pa. \text{ dest-protocol } pa = p\} \triangleleft \text{ allow-all-to-port dest-net d-port}$

definition

$\text{deny-all-to-port-prot} :: \text{protocol} \Rightarrow 'a::\text{adr net} \Rightarrow 'b::\text{port} \Rightarrow (('a, 'b) \text{ packet} \mapsto \text{unit})$

where

$\text{deny-all-to-port-prot } p \text{ dest-net d-port} =$
 $\{pa. \text{ dest-protocol } pa = p\} \triangleleft \text{ deny-all-to-port dest-net d-port}$

definition

$\text{allow-all-from-port-to-prot} :: \text{protocol} \Rightarrow 'a::\text{adr net} \Rightarrow 'b::\text{port} \Rightarrow 'c::\text{adr net} \Rightarrow (('a, 'b) \text{ packet} \mapsto \text{unit})$

where

$\text{allow-all-from-port-to-prot } p \text{ src-net s-port dest-net} =$
 $\{pa. \text{ dest-protocol } pa = p\} \triangleleft \text{ allow-all-from-port-to src-net s-port dest-net}$

definition

$\text{deny-all-from-port-to-prot} :: \text{protocol} \Rightarrow 'a::\text{adr net} \Rightarrow 'b::\text{port} \Rightarrow 'c::\text{adr net} \Rightarrow (('a, 'b) \text{ packet} \mapsto \text{unit})$

where

$\text{deny-all-from-port-to-prot } p \text{ src-net s-port dest-net} =$
 $\{pa. \text{ dest-protocol } pa = p\} \triangleleft \text{ deny-all-from-port-to src-net s-port dest-net}$

definition

$\text{allow-all-from-port-to-port-prot} :: \text{protocol} \Rightarrow 'a::\text{adr net} \Rightarrow 'b::\text{port} \Rightarrow 'c::\text{adr net} \Rightarrow 'd::\text{port} \Rightarrow$

$(('a, 'b) \text{ packet} \mapsto \text{unit})$ **where**

$\text{allow-all-from-port-to-port-prot } p \text{ src-net s-port dest-net d-port} =$
 $\{pa. \text{ dest-protocol } pa = p\} \triangleleft \text{ allow-all-from-port-to-port src-net s-port dest-net d-port}$

definition

$\text{deny-all-from-port-to-port-prot} :: \text{protocol} \Rightarrow 'a::\text{adr net} \Rightarrow 'b::\text{port} \Rightarrow 'c::\text{adr net} \Rightarrow 'd::\text{port} \Rightarrow$

$'e::\text{port} \Rightarrow (('a, 'b) \text{ packet} \mapsto \text{unit})$ **where**

$\text{deny-all-from-port-to-port-prot } p \text{ src-net s-port dest-net d-port} =$
 $\{pa. \text{ dest-protocol } pa = p\} \triangleleft \text{ deny-all-from-port-to-port src-net s-port dest-net d-port}$

definition

$\text{allow-all-from-to-port-prot} :: \text{protocol} \Rightarrow 'a::\text{adr net} \Rightarrow 'b::\text{adr net} \Rightarrow 'c::\text{port} \Rightarrow (('a, 'b) \text{ packet} \mapsto \text{unit})$ **where**

$\text{allow-all-from-to-port-prot } p \text{ src-net dest-net d-port} =$
 $\{pa. \text{ dest-protocol } pa = p\} \triangleleft \text{ allow-all-from-to-port src-net dest-net d-port}$

definition

$deny-all-from-to-port-prot \quad :: \text{protocol} \Rightarrow 'a::\text{adr net} \Rightarrow 'a::\text{adr net} \Rightarrow 'c::\text{port} \Rightarrow$
 $(('a, 'b) \text{ packet} \mapsto \text{unit}) \textbf{ where}$
 $deny-all-from-to-port-prot \ p \ \text{src-net} \ \text{dest-net} \ \text{d-port} =$
 $\{pa. \ \text{dest-protocol} \ pa = p\} \triangleleft deny-all-from-to-port \ \text{src-net} \ \text{dest-net} \ \text{d-port}$

definition

$allow-from-port-to-prot \quad :: \text{protocol} \Rightarrow 'c::\text{port} \Rightarrow 'a::\text{adr net} \Rightarrow 'a::\text{adr net} \Rightarrow (('a, 'b)$
 $\text{packet} \mapsto \text{unit})$

where

$allow-from-port-to-prot \ p \ \text{port} \ \text{src-net} \ \text{dest-net} =$
 $\{pa. \ \text{dest-protocol} \ pa = p\} \triangleleft allow-from-port-to \ \text{port} \ \text{src-net} \ \text{dest-net}$

definition

$deny-from-port-to-prot \quad :: \text{protocol} \Rightarrow 'c::\text{port} \Rightarrow 'a::\text{adr net} \Rightarrow 'a::\text{adr net} \Rightarrow (('a, 'b)$
 $\text{packet} \mapsto \text{unit})$

where

$deny-from-port-to-prot \ p \ \text{port} \ \text{src-net} \ \text{dest-net} =$
 $\{pa. \ \text{dest-protocol} \ pa = p\} \triangleleft deny-from-port-to \ \text{port} \ \text{src-net} \ \text{dest-net}$

definition

$allow-from-to-port-prot \quad :: \text{protocol} \Rightarrow 'c::\text{port} \Rightarrow 'a::\text{adr net} \Rightarrow 'a::\text{adr net} \Rightarrow (('a, 'b)$
 $\text{packet} \mapsto \text{unit})$

where

$allow-from-to-port-prot \ p \ \text{port} \ \text{src-net} \ \text{dest-net} =$
 $\{pa. \ \text{dest-protocol} \ pa = p\} \triangleleft allow-from-to-port \ \text{port} \ \text{src-net} \ \text{dest-net}$

definition

$deny-from-to-port-prot \quad :: \text{protocol} \Rightarrow 'c::\text{port} \Rightarrow 'a::\text{adr net} \Rightarrow 'a::\text{adr net} \Rightarrow (('a, 'b)$
 $\text{packet} \mapsto \text{unit})$

where

$deny-from-to-port-prot \ p \ \text{port} \ \text{src-net} \ \text{dest-net} =$
 $\{pa. \ \text{dest-protocol} \ pa = p\} \triangleleft deny-from-to-port \ \text{port} \ \text{src-net} \ \text{dest-net}$

definition

$allow-from-ports-to-prot \quad :: \text{protocol} \Rightarrow 'c::\text{port set} \Rightarrow 'a::\text{adr net} \Rightarrow 'a::\text{adr net} \Rightarrow$
 $(('a, 'b) \text{ packet} \mapsto \text{unit}) \textbf{ where}$

$allow-from-ports-to-prot \ p \ \text{ports} \ \text{src-net} \ \text{dest-net} =$
 $\{pa. \ \text{dest-protocol} \ pa = p\} \triangleleft allow-from-ports-to \ \text{ports} \ \text{src-net} \ \text{dest-net}$

definition

$allow-from-to-ports-prot \quad :: \text{protocol} \Rightarrow 'c::\text{port set} \Rightarrow 'a::\text{adr net} \Rightarrow 'a::\text{adr net} \Rightarrow$
 $(('a, 'b) \text{ packet} \mapsto \text{unit}) \textbf{ where}$

$allow-from-to-ports-prot \ p \ \text{ports} \ \text{src-net} \ \text{dest-net} =$
 $\{pa. \ \text{dest-protocol} \ pa = p\} \triangleleft allow-from-to-ports \ \text{ports} \ \text{src-net} \ \text{dest-net}$

definition

deny-from-ports-to-prot :: *protocol* => ' γ ::*port set* => ' α ::*adr net* => ' α ::*adr net* =>
 ((α, β) *packet* \mapsto *unit*) **where**
deny-from-ports-to-prot *p ports src-net dest-net* =
 {*pa. dest-protocol pa = p*} \triangleleft *deny-from-ports-to ports src-net dest-net*

definition

deny-from-to-ports-prot :: *protocol* => ' γ ::*port set* => ' α ::*adr net* => ' α ::*adr net* =>
 ((α, β) *packet* \mapsto *unit*) **where**
deny-from-to-ports-prot *p ports src-net dest-net* =
 {*pa. dest-protocol pa = p*} \triangleleft *deny-from-to-ports ports src-net dest-net*

As before, we put all the rules into one lemma to ease writing later.

lemmas *ProtocolCombinatorsCore* =

allow-all-from-port-prot-def deny-all-from-port-prot-def allow-all-to-port-prot-def
deny-all-to-port-prot-def allow-all-from-to-port-prot-def
deny-all-from-to-port-prot-def
allow-from-ports-to-prot-def allow-from-to-ports-prot-def
deny-from-ports-to-prot-def deny-from-to-ports-prot-def
allow-all-from-port-to-prot-def deny-all-from-port-to-prot-def
allow-from-port-to-prot-def allow-from-to-port-prot-def deny-from-to-port-prot-def
deny-from-port-to-prot-def

lemmas *ProtocolCombinators* = *PortCombinators.PortCombinators ProtocolCombinatorsCore*

end

2.2.5 Ports

theory *Ports*

imports

Main

begin

This theory can be used if we want to specify the port numbers by names denoting their default Integer values. If you want to use them, please add *Ports* to the simplifier.

definition *http::int* **where** *http* = 80

lemma *http1*: $x \neq 80 \implies x \neq \text{http}$
by (*simp add: http-def*)

lemma *http2*: $x \neq 80 \implies \text{http} \neq x$

by (simp add: http-def)

definition smtp::int where smtp = 25

lemma smtp1: $x \neq 25 \implies x \neq \text{smtp}$
by (simp add: smtp-def)

lemma smtp2: $x \neq 25 \implies \text{smtp} \neq x$
by (simp add: smtp-def)

definition ftp::int where ftp = 21

lemma ftp1: $x \neq 21 \implies x \neq \text{ftp}$
by (simp add: ftp-def)

lemma ftp2: $x \neq 21 \implies \text{ftp} \neq x$
by (simp add: ftp-def)

And so on for all desired port numbers.

lemmas Ports = http1 http2 ftp1 ftp2 smtp1 smtp2

end

2.2.6 Network Address Translation

theory

NAT

imports

../PacketFilter/PacketFilter

begin

definition src2pool :: $'\alpha \text{ set} \Rightarrow ('\alpha::\text{adr}, '\beta) \text{ packet} \Rightarrow (''\alpha, '\beta) \text{ packet set}$ **where**
src2pool t = $(\lambda p. (\{(i,s,d,da). (i = \text{id } p \wedge s \in t \wedge d = \text{dest } p \wedge da = \text{content } p)\}))$

definition src2poolAP **where**
src2poolAP t = $A_f (\text{src2pool } t)$

definition srcNat2pool :: $'\alpha \text{ set} \Rightarrow '\alpha \text{ set} \Rightarrow (''\alpha::\text{adr}, '\beta) \text{ packet} \mapsto (''\alpha, '\beta) \text{ packet set}$
where

srcNat2pool srcs transl = $\{x. \text{src } x \in \text{srcs}\} \triangleleft (\text{src2poolAP } \text{transl})$

definition src2poolPort :: $\text{int set} \Rightarrow (\text{adr}_{ip}, '\beta) \text{ packet} \Rightarrow (\text{adr}_{ip}, '\beta) \text{ packet set}$ **where**
src2poolPort t = $(\lambda p. (\{(i,(s1,s2),(d1,d2),da).$

$$(i = id\ p \wedge s1 \in t \wedge s2 = (snd\ (src\ p)) \wedge d1 = (fst\ (dest\ p)) \wedge d2 = snd\ (dest\ p) \wedge da = content\ p))$$

definition *src2poolPort-Protocol* :: *int set* \Rightarrow (*adr_{ipp}*,[']*β*) *packet* \Rightarrow (*adr_{ipp}*,[']*β*) *packet set* **where**

$$\begin{aligned} src2poolPort-Protocol\ t &= (\lambda\ p.\ (\{(i,(s1,s2,s3),(d1,d2,d3),\ da).\} \\ (i = id\ p \wedge s1 \in t \wedge s2 = (fst\ (snd\ (src\ p))) \wedge s3 = snd\ (snd\ (src\ p)) \wedge \\ (d1,d2,d3) = dest\ p \wedge da = content\ p)\})) \end{aligned}$$

definition *srcNat2pool-IntPort* :: *address set* \Rightarrow *address set* \Rightarrow (*adr_{ip}*,[']*β*) *packet* \mapsto (*adr_{ip}*,[']*β*) *packet set* **where**

$$\begin{aligned} srcNat2pool-IntPort\ srcs\ transl &= \\ \{x.\ fst\ (src\ x) \in srcs\} &\triangleleft (A_f\ (src2poolPort\ transl)) \end{aligned}$$

definition *srcNat2pool-IntProtocolPort* :: *int set* \Rightarrow *int set* \Rightarrow (*adr_{ipp}*,[']*β*) *packet* \mapsto (*adr_{ipp}*,[']*β*) *packet set* **where**

$$\begin{aligned} srcNat2pool-IntProtocolPort\ srcs\ transl &= \\ \{x.\ (fst\ (src\ x)) \in srcs\} &\triangleleft (A_f\ (src2poolPort-Protocol\ transl)) \end{aligned}$$

definition *srcPat2poolPort-t* :: *int set* \Rightarrow (*adr_{ip}*,[']*β*) *packet* \Rightarrow (*adr_{ip}*,[']*β*) *packet set* **where**

$$\begin{aligned} srcPat2poolPort-t\ t &= (\lambda\ p.\ (\{(i,(s1,s2),(d1,d2),da).\} \\ (i = id\ p \wedge s1 \in t \wedge d1 = (fst\ (dest\ p)) \wedge d2 = snd\ (dest\ p) \wedge da = content\ p)\})) \end{aligned}$$

definition *srcPat2poolPort-Protocol-t* :: *int set* \Rightarrow (*adr_{ipp}*,[']*β*) *packet* \Rightarrow (*adr_{ipp}*,[']*β*) *packet set* **where**

$$\begin{aligned} srcPat2poolPort-Protocol-t\ t &= (\lambda\ p.\ (\{(i,(s1,s2,s3),(d1,d2,d3),da).\} \\ (i = id\ p \wedge s1 \in t \wedge s3 = src-protocol\ p \wedge (d1,d2,d3) = dest\ p \wedge da = content\ p)\})) \end{aligned}$$

definition *srcPat2pool-IntPort* :: *int set* \Rightarrow *int set* \Rightarrow (*adr_{ip}*,[']*β*) *packet* \mapsto (*adr_{ip}*,[']*β*) *packet set* **where**

$$\begin{aligned} srcPat2pool-IntPort\ srcs\ transl &= \\ \{x.\ (fst\ (src\ x)) \in srcs\} &\triangleleft (A_f\ (srcPat2poolPort-t\ transl)) \end{aligned}$$

definition *srcPat2pool-IntProtocol* ::

$$int\ set \Rightarrow int\ set \Rightarrow (adr_{ipp},'\beta)\ packet \mapsto (adr_{ipp},'\beta)\ packet\ set\ \mathbf{where}$$

$$\begin{aligned} srcPat2pool-IntProtocol\ srcs\ transl &= \\ \{x.\ (fst\ (src\ x)) \in srcs\} &\triangleleft (A_f\ (srcPat2poolPort-Protocol-t\ transl)) \end{aligned}$$

The following lemmas are used for achieving a normalized output format of packages after applying NAT. This is used, e.g., by our firewall execution tool.

lemma *datasimp*: $\{(i, (s1, s2, s3), aba).\}$

$\forall a aa b ba. aba = ((a, aa, b), ba) \longrightarrow i = i1 \wedge s1 = i101 \wedge$
 $s3 = iudp \wedge a = i110 \wedge aa = X606X3 \wedge b = X607X4 \wedge ba$
 $= data\}$
 $= \{(i, (s1, s2, s3), aba).$
 $i = i1 \wedge s1 = i101 \wedge s3 = iudp \wedge (\lambda ((a,aa,b),ba). a = i110 \wedge aa =$
 $X606X3 \wedge$
 $b = X607X4 \wedge ba = data) aba\}$
by auto

lemma datasimp2: $\{(i, (s1, s2, s3), aba).$
 $\forall a aa b ba. aba = ((a, aa, b), ba) \longrightarrow i = i1 \wedge s1 = i132 \wedge s3 = iudp$
 \wedge
 $s2 = i1 \wedge a = i110 \wedge aa = i4 \wedge b = iudp \wedge ba = data\}$
 $= \{(i, (s1, s2, s3), aba).$
 $i = i1 \wedge s1 = i132 \wedge s3 = iudp \wedge s2 = i1 \wedge (\lambda ((a,aa,b),ba). a =$
 $i110 \wedge$
 $aa = i4 \wedge b = iudp \wedge ba = data) aba\}$
by auto

lemma datasimp3: $\{(i, (s1, s2, s3), aba).$
 $\forall a aa b ba. aba = ((a, aa, b), ba) \longrightarrow i = i1 \wedge i115 < s1 \wedge s1 <$
 $i124 \wedge$
 $s3 = iudp \wedge s2 = ii1 \wedge a = i110 \wedge aa = i3 \wedge b = itcp \wedge ba =$
 $data\}$
 $= \{(i, (s1, s2, s3), aba).$
 $i = i1 \wedge i115 < s1 \wedge s1 < i124 \wedge s3 = iudp \wedge s2 = ii1 \wedge$
 $(\lambda ((a,aa,b),ba). a = i110 \ \& \ aa = i3 \ \& \ b = itcp \ \& \ ba = data) aba\}$
by auto

lemma datasimp4: $\{(i, (s1, s2, s3), aba).$
 $\forall a aa b ba. aba = ((a, aa, b), ba) \longrightarrow i = i1 \wedge s1 = i132 \wedge s3 = iudp$
 \wedge
 $s2 = ii1 \wedge a = i110 \wedge aa = i7 \wedge b = itcp \wedge ba = data\}$
 $= \{(i, (s1, s2, s3), aba).$
 $i = i1 \wedge s1 = i132 \wedge s3 = iudp \wedge s2 = ii1 \wedge$
 $(\lambda ((a,aa,b),ba). a = i110 \wedge aa = i7 \wedge b = itcp \wedge ba = data) aba\}$
by auto

lemma datasimp5: $\{(i, (s1, s2, s3), aba).$
 $i = i1 \wedge s1 = i101 \wedge s3 = iudp \wedge (\lambda ((a,aa,b),ba). a = i110 \wedge aa =$
 $X606X3 \wedge$
 $b = X607X4 \wedge ba = data) aba\}$
 $= \{(i, (s1, s2, s3), (a,aa,b),ba).$
 $i = i1 \wedge s1 = i101 \wedge s3 = iudp \wedge a = i110 \wedge aa = X606X3 \wedge$

```

      b = X607X4 ∧ ba = data}
by auto

lemma datasimp6: {(i, (s1, s2, s3), aba).
  i = i1 ∧ s1 = i132 ∧ s3 = iudp ∧ s2 = i1 ∧
  (λ ((a,aa,b),ba). a = i110 ∧ aa = i4 ∧ b = iudp ∧ ba = data) aba}
= {(i, (s1, s2, s3), (a,aa,b),ba).
  i = i1 ∧ s1 = i132 ∧ s3 = iudp ∧ s2 = i1 ∧ a = i110 ∧
  aa = i4 ∧ b = iudp ∧ ba = data}

by auto

lemma datasimp7: {(i, (s1, s2, s3), aba).
  i = i1 ∧ i115 < s1 ∧ s1 < i124 ∧ s3 = iudp ∧ s2 = ii1 ∧
  (λ ((a,aa,b),ba). a = i110 ∧ aa = i3 ∧ b = itcp ∧ ba = data) aba}
= {(i, (s1, s2, s3), (a,aa,b),ba).
  i = i1 ∧ i115 < s1 ∧ s1 < i124 ∧ s3 = iudp ∧ s2 = ii1
  ∧ a = i110 ∧ aa = i3 ∧ b = itcp ∧ ba = data}

by auto

lemma datasimp8: {(i, (s1, s2, s3), aba). i = i1 ∧ s1 = i132 ∧ s3 = iudp ∧ s2 = ii1
  ∧
  (λ ((a,aa,b),ba). a = i110 ∧ aa = i7 ∧ b = itcp ∧ ba = data) aba}
= {(i, (s1, s2, s3), (a,aa,b),ba). i = i1 ∧ s1 = i132 ∧ s3 = iudp
  ∧ s2 = ii1 ∧ a = i110 ∧ aa = i7 ∧ b = itcp ∧ ba = data}

by auto

lemmas datasimps = datasimp datasimp2 datasimp3 datasimp4
  datasimp5 datasimp6 datasimp7 datasimp8

lemmas NATLemmas = src2pool-def src2poolPort-def
  src2poolPort-Protocol-def src2poolAP-def srcNat2pool-def
  srcNat2pool-IntProtocolPort-def srcNat2pool-IntPort-def
  srcPat2poolPort-t-def srcPat2poolPort-Protocol-t-def
  srcPat2pool-IntPort-def srcPat2pool-IntProtocol-def
end

```

2.3 Firewall Policy Normalisation

```

theory
  FWNormalisation
imports
  NormalisationIPPProofs
  ElementaryRules

```


begin

end

2.3.1 Policy Normalisation: Core Definitions

theory

FWNormalisationCore

imports

../PacketFilter/PacketFilter

begin

This theory contains all the definitions used for policy normalisation as described in [3, 7].

The normalisation procedure transforms policies into semantically equivalent ones which are “easier” to test. It is organized into nine phases. We impose the following two restrictions on the input policies:

- Each policy must contain a **DenyAll** rule. If this restriction were to be lifted, the **insertDenies** phase would have to be adjusted accordingly.
- For each pair of networks n_1 and n_2 , the networks are either disjoint or equal. If this restriction were to be lifted, we would need some additional phases before the start of the normalisation procedure presented below. This rule would split single rules into several by splitting up the networks such that they are all pairwise disjoint or equal. Such a transformation is clearly semantics-preserving and the condition would hold after these phases.

As a result, the procedure generates a list of policies, in which:

- each element of the list contains a policy which completely specifies the blocking behavior between two networks, and
- there are no shadowed rules.

This result is desirable since the test case generation for rules between networks A and B is independent of the rules that specify the behavior for traffic flowing between networks C and D . Thus, the different segments of the policy can be processed individually. The normalization procedure does not aim to minimize the number of rules. While it does remove unnecessary ones, it also adds new ones, enabling a policy to be split into several independent parts.

Policy transformations are functions that map policies to policies. We decided to represent policy transformations as *syntactic rules*; this choice paves the way for expressing the entire normalisation process inside HOL by functions manipulating abstract policy syntax.

Basics

We define a very simple policy language:

```
datatype (' $\alpha$ , ' $\beta$ ) Combinators =  
  DenyAll  
  | DenyAllFromTo ' $\alpha$  ' $\alpha$   
  | AllowPortFromTo ' $\alpha$  ' $\alpha$  ' $\beta$   
  | Conc ((' $\alpha$ , ' $\beta$ ) Combinators) ((' $\alpha$ , ' $\beta$ ) Combinators) (infixr  $\oplus$  80)
```

And define the semantic interpretation of it. For technical reasons, we fix here the type to policies over IntegerPort addresses. However, we could easily provide definitions for other address types as well, using a generic constants for the type definition and a primitive recursive definition for each desired address model.

Auxiliary definitions and functions.

This section defines several functions which are useful later for the combinators, invariants, and proofs.

```
fun srcNet where  
  srcNet (DenyAllFromTo  $x$   $y$ ) =  $x$   
| srcNet (AllowPortFromTo  $x$   $y$   $p$ ) =  $x$   
| srcNet DenyAll = undefined  
| srcNet ( $v \oplus va$ ) = undefined
```

```
fun destNet where  
  destNet (DenyAllFromTo  $x$   $y$ ) =  $y$   
| destNet (AllowPortFromTo  $x$   $y$   $p$ ) =  $y$   
| destNet DenyAll = undefined  
| destNet ( $v \oplus va$ ) = undefined
```

```
fun srcnets where  
  srcnets DenyAll = []  
| srcnets (DenyAllFromTo  $x$   $y$ ) = [ $x$ ]  
| srcnets (AllowPortFromTo  $x$   $y$   $p$ ) = [ $x$ ]  
| (srcnets ( $x \oplus y$ )) = (srcnets  $x$ )@(srcnets  $y$ )
```

```
fun destnets where  
  destnets DenyAll = []  
| destnets (DenyAllFromTo  $x$   $y$ ) = [ $y$ ]  
| destnets (AllowPortFromTo  $x$   $y$   $p$ ) = [ $y$ ]  
| (destnets ( $x \oplus y$ )) = (destnets  $x$ )@(destnets  $y$ )
```

```
fun (sequential) net-list-aux where  
  net-list-aux [] = []
```

```

|net-list-aux (DenyAll#xs) = net-list-aux xs
|net-list-aux ((DenyAllFromTo x y)#xs) = x#y#(net-list-aux xs)
|net-list-aux ((AllowPortFromTo x y p)#xs) = x#y#(net-list-aux xs)
|net-list-aux ((x⊕y)#xs) = (net-list-aux [x])@(net-list-aux [y])@(net-list-aux xs)

```

```

fun net-list where net-list p = remdups (net-list-aux p)

```

```

definition bothNets where bothNets x = (zip (srcnets x) (destnets x))

```

```

fun (sequential) normBothNets where
  normBothNets ((a,b)#xs) = (if ((b,a) ∈ set xs) ∨ (a,b) ∈ set xs)
    then (normBothNets xs)
    else (a,b)#(normBothNets xs)
|normBothNets x = x

```

```

fun makeSets where
  makeSets ((a,b)#xs) = ({a,b}#(makeSets xs))
|makeSets [] = []

```

```

fun bothNet where
  bothNet DenyAll = {}
|bothNet (DenyAllFromTo a b) = {a,b}
|bothNet (AllowPortFromTo a b p) = {a,b}
|bothNet (v ⊕ va) = undefined

```

Nets_List provides from a list of rules a list where the entries are the appearing sets of source and destination network of each rule.

```

definition Nets-List
where
  Nets-List x = makeSets (normBothNets (bothNets x))

```

```

fun (sequential) first-srcNet where
  first-srcNet (x⊕y) = first-srcNet x
| first-srcNet x = srcNet x

```

```

fun (sequential) first-destNet where
  first-destNet (x⊕y) = first-destNet x
| first-destNet x = destNet x

```

```

fun (sequential) first-bothNet where
  first-bothNet (x⊕y) = first-bothNet x
|first-bothNet x = bothNet x

```

```

fun (sequential) in-list where

```

in-list DenyAll $l = True$
in-list $x\ l = (bothNet\ x \in\ set\ l)$

fun *all-in-list* **where**
all-in-list $[]\ l = True$
all-in-list $(x\#\!xs)\ l = (in-list\ x\ l \wedge all-in-list\ xs\ l)$

fun (*sequential*) *member* **where**
member $a\ (x\oplus\!xs) = ((member\ a\ x) \vee (member\ a\ xs))$
member $a\ x = (a = x)$

fun *sdnets* **where**
sdnets *DenyAll* $= \{\}$
sdnets (*DenyAllFromTo* $a\ b$) $= \{(a,b)\}$
sdnets (*AllowPortFromTo* $a\ b\ c$) $= \{(a,b)\}$
sdnets $(a \oplus b) = sdnets\ a \cup sdnets\ b$

definition *packet-Nets* **where** *packet-Nets* $x\ a\ b = ((src\ x \sqsubset a \wedge dest\ x \sqsubset b) \vee (src\ x \sqsubset b \wedge dest\ x \sqsubset a))$

definition *subnetsOfAdr* **where** *subnetsOfAdr* $a = \{x.\ a \sqsubset x\}$

definition *fst-set* **where** *fst-set* $s = \{a.\ \exists\ b.\ (a,b) \in s\}$

definition *snd-set* **where** *snd-set* $s = \{a.\ \exists\ b.\ (b,a) \in s\}$

fun *memberP* **where**
memberP $r\ (x\#\!xs) = (member\ r\ x \vee memberP\ r\ xs)$
memberP $r\ [] = False$

fun *firstList* **where**
firstList $(x\#\!xs) = (first-bothNet\ x)$
firstList $[] = \{\}$

Invariants

If there is a DenyAll, it is at the first position

fun *wellformed-policy1*:: ($'\alpha,\ ' \beta$) *Combinators*) *list* $\Rightarrow\ bool$ **where**
wellformed-policy1 $[] = True$
wellformed-policy1 $(x\#\!xs) = (DenyAll \notin (set\ xs))$

There is a DenyAll at the first position

fun *wellformed-policy1-strong*:: ($'\alpha,\ ' \beta$) *Combinators*) *list* $\Rightarrow\ bool$
where

wellformed-policy1-strong [] = *False*
| *wellformed-policy1-strong* (*x#xs*) = (*x=DenyAll* \wedge (*DenyAll* \notin (*set xs*)))

All two networks are either disjoint or equal.

definition *netsDistinct* **where** *netsDistinct* *a b* = (\neg ($\exists x. x \sqsubset a \wedge x \sqsubset b$))

definition *twoNetsDistinct* **where**

twoNetsDistinct *a b c d* = (*netsDistinct* *a c* \vee *netsDistinct* *b d*)

definition *allNetsDistinct* **where**

allNetsDistinct *p* = ($\forall a b. (a \neq b \wedge a \in \text{set } (\text{net-list } p) \wedge b \in \text{set } (\text{net-list } p)) \longrightarrow \text{netsDistinct } a b$)

definition *disjSD-2* **where**

disjSD-2 *x y* = ($\forall a b c d. ((a,b) \in \text{sdnets } x \wedge (c,d) \in \text{sdnets } y \longrightarrow (\text{twoNetsDistinct } a b c d \wedge \text{twoNetsDistinct } a b d c))$)

The policy is given as a list of single rules.

fun *singleCombinators* **where**

singleCombinators [] = *True*

| *singleCombinators* (*(x⊕y)#xs*) = *False*

| *singleCombinators* (*x#xs*) = *singleCombinators xs*

definition *onlyTwoNets* **where**

onlyTwoNets *x* = ($(\exists a b. (\text{sdnets } x = \{(a,b)\})) \vee (\exists a b. \text{sdnets } x = \{(a,b),(b,a)\})$)

Each entry of the list contains rules between two networks only.

fun *OnlyTwoNets* **where**

OnlyTwoNets (*DenyAll#xs*) = *OnlyTwoNets xs*

| *OnlyTwoNets* (*x#xs*) = (*onlyTwoNets x* \wedge *OnlyTwoNets xs*)

| *OnlyTwoNets* [] = *True*

fun *noDenyAll* **where**

noDenyAll (*x#xs*) = ($(\neg \text{member } \text{DenyAll } x) \wedge \text{noDenyAll } xs$)

| *noDenyAll* [] = *True*

fun *noDenyAll1* **where**

noDenyAll1 (*DenyAll#xs*) = *noDenyAll xs*

| *noDenyAll1* *xs* = *noDenyAll xs*

fun *separated* **where**

separated (*x#xs*) = ($(\forall s. s \in \text{set } xs \longrightarrow \text{disjSD-2 } x s) \wedge \text{separated } xs$)

| *separated* [] = *True*

fun *NetsCollected* **where**

NetsCollected ($x\#xs$) = (((*first-bothNet* $x \neq \text{firstList } xs$) \longrightarrow
($\forall a \in \text{set } xs. \text{first-bothNet } x \neq \text{first-bothNet } a$) \wedge *NetsCollected* (xs))
| *NetsCollected* [] = *True*

fun *NetsCollected2* **where**

NetsCollected2 ($x\#xs$) = ($xs = [] \vee (\text{first-bothNet } x \neq \text{firstList } xs \wedge$
NetsCollected2 $xs)$)
| *NetsCollected2* [] = *True*

Transformations

The following two functions transform a policy into a list of single rules and vice-versa (by staying on the combinator level).

fun *policy2list*::('α, 'β) *Combinators* \Rightarrow

((('α, 'β) *Combinators*) *list* **where**
policy2list ($x \oplus y$) = (*concat* [(*policy2list* x),(*policy2list* y)])
| *policy2list* $x = [x]$

fun *list2FWpolicy*::('α, 'β) *Combinators* *list* \Rightarrow

((('α, 'β) *Combinators*) **where**
list2FWpolicy [] = *undefined*
| *list2FWpolicy* ($x\#[]$) = x
| *list2FWpolicy* ($x\#y$) = $x \oplus (\text{list2FWpolicy } y)$

Remove all the rules appearing before a *DenyAll*. There are two alternative versions.

fun *removeShadowRules1* **where**

removeShadowRules1 ($x\#xs$) = (*if* (*DenyAll* $\in \text{set } xs$)
then ((*removeShadowRules1* xs))
else $x\#xs$)
| *removeShadowRules1* [] = []

fun *removeShadowRules1-alternative-rev* **where**

removeShadowRules1-alternative-rev [] = []
| *removeShadowRules1-alternative-rev* (*DenyAll* $\#xs$) = [*DenyAll*]
| *removeShadowRules1-alternative-rev* [x] = [x]
| *removeShadowRules1-alternative-rev* ($x\#xs$)=
 $x\#(\text{removeShadowRules1-alternative-rev } xs)$

definition *removeShadowRules1-alternative* **where**

removeShadowRules1-alternative $p =$
rev (*removeShadowRules1-alternative-rev* (*rev* p))

Remove all the rules which allow a port, but are shadowed by a deny between these subnets.

fun *removeShadowRules2*:: (('α, 'β) Combinators) list ⇒
 (('α, 'β) Combinators) list

where

(*removeShadowRules2* ((*AllowPortFromTo* x y p)#z)) =
 (if (((*DenyAllFromTo* x y) ∈ set z))
 then ((*removeShadowRules2* z))
 else (((*AllowPortFromTo* x y p)#(*removeShadowRules2* z))))
 | *removeShadowRules2* (x#y) = x#(*removeShadowRules2* y)
 | *removeShadowRules2* [] = []

Sorting a policies: We first need to define an ordering on rules. This ordering depends on the *Nets_List* of a policy.

fun *smaller* :: ('α, 'β) Combinators ⇒
 ('α, 'β) Combinators ⇒
 (('α) set) list ⇒ bool

where

smaller DenyAll x l = True
 | *smaller* x *DenyAll* l = False
 | *smaller* x y l =
 ((x = y) ∨ (if (bothNet x) = (bothNet y) then
 (case y of (*DenyAllFromTo* a b) ⇒ (x = *DenyAllFromTo* b a)
 | - ⇒ True)
 else
 (position (bothNet x) l <= position (bothNet y) l)))

We provide two different sorting algorithms: Quick Sort (*qsort*) and Insertion Sort (*sort*)

fun *qsort* **where**

qsort [] l = []
 | *qsort* (x#xs) l = (*qsort* [y←xs. ¬ (smaller x y l)] l) @ [x] @ (*qsort* [y←xs. smaller x y l] l)

lemma *qsort-permutes*:

set (*qsort* xs l) = set xs
apply (*induct* xs l rule: *qsort.induct*)
by (*auto*)

lemma *set-qsort* [*simp*]: set (*qsort* xs l) = set xs

by (*simp* add: *qsort-permutes*)

fun *insort* **where**

insort a [] l = [a]
 | *insort* a (x#xs) l = (if (smaller a x l) then a#x#xs else x#(*insort* a xs l))

fun sort where

sort [] l = []
| sort (x#xs) l = insert x (sort xs l) l

fun sorted where

sorted [] l = True
| sorted [x] l = True
| sorted (x#y#zs) l = (smaller x y l \wedge sorted (y#zs) l)

fun separate where

separate (DenyAll#x) = DenyAll#(separate x)
| separate (x#y#z) = (if (first-bothNet x = first-bothNet y)
then (separate ((x \oplus y)#z))
else (x#(separate(y#z))))
| separate x = x

Insert the DenyAllFromTo rules, such that traffic between two networks can be tested individually.

fun insertDenies where

insertDenies (x#xs) = (case x of DenyAll \Rightarrow (DenyAll#(insertDenies xs))
| - \Rightarrow (DenyAllFromTo (first-srcNet x) (first-destNet x) \oplus
(DenyAllFromTo (first-destNet x) (first-srcNet x)) \oplus x)#
(insertDenies xs))
| insertDenies [] = []

Remove duplicate rules. This is especially necessary as insertDenies might have inserted duplicate rules. The second function is supposed to work on a list of policies. Only rules which are duplicated within the same policy are removed.

fun removeDuplicates where

removeDuplicates (x \oplus xs) = (if member x xs then (removeDuplicates xs)
else x \oplus (removeDuplicates xs))
| removeDuplicates x = x

fun removeAllDuplicates where

removeAllDuplicates (x#xs) = ((removeDuplicates (x))#(removeAllDuplicates xs))
| removeAllDuplicates x = x

Insert a DenyAll at the beginning of a policy.

fun insertDeny where

insertDeny (DenyAll#xs) = DenyAll#xs
| insertDeny xs = DenyAll#xs

definition sort' p l = sort l p

definition $qsort' p l = qsort l p$

declare $dom\text{-}eq\text{-}empty\text{-}conv$ [*simp del*]

fun $list2policyR::('α, 'β) Combinators$ $list \Rightarrow$
 $(('α, 'β) Combinators)$ **where**
 $list2policyR (x\#\[]) = x$
 $| list2policyR (x\#y) = (list2policyR y) \oplus x$
 $| list2policyR [] = undefined$

We provide the definitions for two address representations.

IntPort

fun $C :: (adr_{ip} net, port) Combinators \Rightarrow (adr_{ip}, DummyContent) packet \mapsto unit$
where
 $C DenyAll = deny\text{-}all$
 $| C (DenyAllFromTo x y) = deny\text{-}all\text{-}from\text{-}to x y$
 $| C (AllowPortFromTo x y p) = allow\text{-}from\text{-}to\text{-}port p x y$
 $| C (x \oplus y) = C x ++ C y$

fun $CRotate :: (adr_{ip} net, port) Combinators \Rightarrow (adr_{ip}, DummyContent) packet \mapsto unit$
where
 $CRotate DenyAll = C DenyAll$
 $| CRotate (DenyAllFromTo x y) = C (DenyAllFromTo x y)$
 $| CRotate (AllowPortFromTo x y p) = C (AllowPortFromTo x y p)$
 $| CRotate (x \oplus y) = ((CRotate y) ++ ((CRotate x)))$

fun $rotatePolicy$ **where**
 $rotatePolicy DenyAll = DenyAll$
 $| rotatePolicy (DenyAllFromTo a b) = DenyAllFromTo a b$
 $| rotatePolicy (AllowPortFromTo a b p) = AllowPortFromTo a b p$
 $| rotatePolicy (a \oplus b) = (rotatePolicy b) \oplus (rotatePolicy a)$

lemma $check: rev (policy2list (rotatePolicy p)) = policy2list p$
apply (*induct p*)
by (*simp-all*)

All rules appearing at the left of a `DenyAllFromTo`, have disjunct domains from it (except `DenyAll`).

fun (*sequential*) $wellformed\text{-}policy2$ **where**
 $wellformed\text{-}policy2 [] = True$
 $| wellformed\text{-}policy2 (DenyAll\#xs) = wellformed\text{-}policy2 xs$
 $| wellformed\text{-}policy2 (x\#xs) = ((\forall c a b. c = DenyAllFromTo a b \wedge c \in set xs \longrightarrow$

$$\text{Map.dom } (C x) \cap \text{Map.dom } (C c) = \{\} \wedge \text{wellformed-policy2 } xs$$

An allow rule is disjunct with all rules appearing at the right of it. This invariant is not necessary as it is a consequence from others, but facilitates some proofs.

fun (*sequential*) *wellformed-policy3*::((*adr_{ip}* *net*, *port*) *Combinators*) *list* \Rightarrow *bool* **where**
wellformed-policy3 [] = *True*
| *wellformed-policy3* ((*AllowPortFromTo* *a b p*)#*xs*) = ((\forall *r*. *r* \in *set xs* \longrightarrow
 $\text{dom } (C r) \cap \text{dom } (C (\text{AllowPortFromTo } a b p)) = \{\}$) \wedge *wellformed-policy3 xs*)
| *wellformed-policy3* (*x*#*xs*) = *wellformed-policy3 xs*

definition

normalize' *p* = (*removeAllDuplicates* *o insertDenies* *o separate* *o*
(*sort'* (*Nets-List p*)) *o removeShadowRules2* *o remdups* *o*
(*rm-MT-rules C*) *o insertDeny* *o removeShadowRules1* *o*
policy2list) *p*

definition

normalizeQ' *p* = (*removeAllDuplicates* *o insertDenies* *o separate* *o*
(*qsort'* (*Nets-List p*)) *o removeShadowRules2* *o remdups* *o*
(*rm-MT-rules C*) *o insertDeny* *o removeShadowRules1* *o*
policy2list) *p*

definition *normalize* ::

(*adr_{ip}* *net*, *port*) *Combinators* \Rightarrow
(*adr_{ip}* *net*, *port*) *Combinators list*

where

normalize p = (*removeAllDuplicates* (*insertDenies* (*separate* (*sort*
(*removeShadowRules2* (*remdups* ((*rm-MT-rules C*) (*insertDeny*
(*removeShadowRules1* (*policy2list p*)))))) ((*Nets-List p*))))))

definition

normalize-manual-order p l = *removeAllDuplicates* (*insertDenies* (*separate*
(*sort* (*removeShadowRules2* (*remdups* ((*rm-MT-rules C*) (*insertDeny*
(*removeShadowRules1* (*policy2list p*)))))) ((*l*))))))

definition *normalizeQ* ::

(*adr_{ip}* *net*, *port*) *Combinators* \Rightarrow
(*adr_{ip}* *net*, *port*) *Combinators list*

where

normalizeQ p = (*removeAllDuplicates* (*insertDenies* (*separate* (*qsort*
(*removeShadowRules2* (*remdups* ((*rm-MT-rules C*) (*insertDeny*
(*removeShadowRules1* (*policy2list p*)))))) ((*Nets-List p*))))))

definition

normalize-manual-orderQ $p\ l = \text{removeAllDuplicates } (\text{insertDenies } (\text{separate } (\text{qsort } (\text{removeShadowRules2 } (\text{remdups } ((\text{rm-MT-rules } C) (\text{insertDeny } (\text{removeShadowRules1 } (\text{policy2list } p)))))) (l))))))$

Of course, *normalize* is equal to *normalize'*, the latter looks nicer though.

lemma *normalize = normalize'*

by (*rule ext, simp add: normalize-def normalize'-def sort'-def*)

declare *C.simps* [*simp del*]

TCP_UDP_IntegerPort

fun *Cp* :: (*adr_{ipp} net, protocol × port*) *Combinators* ⇒
 (*adr_{ipp}, DummyContent*) *packet* ↦ *unit*

where

Cp DenyAll = *deny-all*
 | *Cp (DenyAllFromTo x y)* = *deny-all-from-to x y*
 | *Cp (AllowPortFromTo x y p)* = *allow-from-to-port-prot (fst p) (snd p) x y*
 | *Cp (x ⊕ y)* = *Cp x ++ Cp y*

fun *Dp* :: (*adr_{ipp} net, protocol × port*) *Combinators* ⇒
 (*adr_{ipp}, DummyContent*) *packet* ↦ *unit*

where

Dp DenyAll = *Cp DenyAll*
 | *Dp (DenyAllFromTo x y)* = *Cp (DenyAllFromTo x y)*
 | *Dp (AllowPortFromTo x y p)* = *Cp (AllowPortFromTo x y p)*
 | *Dp (x ⊕ y)* = *Cp (y ⊕ x)*

All rules appearing at the left of a *DenyAllFromTo*, have disjunct domains from it (except *DenyAll*).

fun (*sequential*) *wellformed-policy2Pr* **where**

wellformed-policy2Pr [] = *True*
 | *wellformed-policy2Pr (DenyAll#xs)* = *wellformed-policy2Pr xs*
 | *wellformed-policy2Pr (x#xs)* = ((∀ *c a b. c = DenyAllFromTo a b ∧ c ∈ set xs* →
 Map.dom (Cp x) ∩ Map.dom (Cp c) = {}) ∧ *wellformed-policy2Pr xs*)

An allow rule is disjunct with all rules appearing at the right of it. This invariant is not necessary as it is a consequence from others, but facilitates some proofs.

fun (*sequential*) *wellformed-policy3Pr*::(*adr_{ipp} net, protocol × port*) *Combinators*) *list* ⇒ *bool* **where**

wellformed-policy3Pr [] = *True*
 | *wellformed-policy3Pr ((AllowPortFromTo a b p)#xs)* = ((∀ *r. r ∈ set xs* →

$dom (Cp\ r) \cap dom (Cp\ (AllowPortFromTo\ a\ b\ p)) = \{\}$ \wedge *wellformed-policy3Pr*
xs)
 $|$ *wellformed-policy3Pr* ($x\#xs$) = *wellformed-policy3Pr* *xs*

definition

normalizePr' :: (*adr_{ipp}* *net*, *protocol* \times *port*) *Combinators*
 \Rightarrow (*adr_{ipp}* *net*, *protocol* \times *port*) *Combinators list* **where**
normalizePr' *p* = (*removeAllDuplicates* *o* *insertDenies* *o* *separate* *o*
(sort' (Nets-List p)) *o* *removeShadowRules2* *o* *remdups* *o*
(rm-MT-rules Cp) *o* *insertDeny* *o* *removeShadowRules1* *o*
policy2list) *p*

definition *normalizePr* ::

(*adr_{ipp}* *net*, *protocol* \times *port*) *Combinators*
 \Rightarrow (*adr_{ipp}* *net*, *protocol* \times *port*) *Combinators list* **where**
normalizePr *p* = (*removeAllDuplicates* (*insertDenies* (*separate* (*sort*
(*removeShadowRules2* (*remdups* ((*rm-MT-rules Cp*) (*insertDeny*
(*removeShadowRules1* (*policy2list p*)))))) ((*Nets-List p*))))))

definition

normalize-manual-orderPr *p l* = *removeAllDuplicates* (*insertDenies* (*separate*
(*sort* (*removeShadowRules2* (*remdups* ((*rm-MT-rules Cp*) (*insertDeny*
(*removeShadowRules1* (*policy2list p*)))))) ((*l*))))))

definition

normalizePrQ' :: (*adr_{ipp}* *net*, *protocol* \times *port*) *Combinators*
 \Rightarrow (*adr_{ipp}* *net*, *protocol* \times *port*) *Combinators list* **where**
normalizePrQ' *p* = (*removeAllDuplicates* *o* *insertDenies* *o* *separate* *o*
(*qsort' (Nets-List p)*) *o* *removeShadowRules2* *o* *remdups* *o*
(rm-MT-rules Cp) *o* *insertDeny* *o* *removeShadowRules1* *o*
policy2list) *p*

definition *normalizePrQ* ::

(*adr_{ipp}* *net*, *protocol* \times *port*) *Combinators*
 \Rightarrow (*adr_{ipp}* *net*, *protocol* \times *port*) *Combinators list* **where**
normalizePrQ *p* = (*removeAllDuplicates* (*insertDenies* (*separate* (*qsort*
(*removeShadowRules2* (*remdups* ((*rm-MT-rules Cp*) (*insertDeny*
(*removeShadowRules1* (*policy2list p*)))))) ((*Nets-List p*))))))

definition

normalize-manual-orderPrQ *p l* = *removeAllDuplicates* (*insertDenies* (*separate*
(*qsort* (*removeShadowRules2* (*remdups* ((*rm-MT-rules Cp*) (*insertDeny*
(*removeShadowRules1* (*policy2list p*)))))) ((*l*))))))

Of course, `normalize` is equal to `normalize'`, the latter looks nicer though.

lemma `normalizePr = normalizePr'`

by (`rule ext`, `simp add: normalizePr-def normalizePr'-def sort'-def`)

The following definition helps in creating the test specification for the individual parts of a normalized policy.

definition `makeFUTPr` **where**

```
makeFUTPr FUT p x n =
  (packet-Nets x (fst (normBothNets (bothNets p)!n))
    (snd(normBothNets (bothNets p)!n)) →
  FUT x = Cp ((normalizePr p)!Suc n) x
```

declare `Cp.simps` [`simp del`]

lemmas `PLemmas = C.simps Cp.simps dom-def PolicyCombinators.PolicyCombinators`

`PortCombinators.PortCombinatorsCore aux`

`ProtocolPortCombinators.ProtocolCombinatorsCore src-def dest-def in-subnet-def`

`adrippLemmas adrippLemmas`

lemma `aux: $\llbracket x \neq a; y \neq b; (x \neq y \wedge x \neq b) \vee (a \neq b \wedge a \neq y) \rrbracket \implies \{x, a\} \neq \{y, b\}$`
by (`auto`)

lemma `aux2: $\{a, b\} = \{b, a\}$`

by `auto`

end

2.3.2 Normalisation Proofs (Generic)

theory

`NormalisationGenericProofs`

imports

`FWNormalisationCore`

begin

This theory contains the generic proofs of the normalisation procedure, i.e. those which are independent from the concrete semantical interpretation function.

lemma `domNMT: $\text{dom } X \neq \{\} \implies X \neq \emptyset$`

by `auto`

lemma `denyNMT: $\text{deny-all} \neq \emptyset$`

apply (`rule domNMT`)

by (`simp add: deny-all-def dom-def`)

lemma *wellformed-policy1-charn*[*rule-format*]:
wellformed-policy1 $p \longrightarrow \text{DenyAll} \in \text{set } p \longrightarrow (\exists p'. p = \text{DenyAll} \# p' \wedge \text{DenyAll} \notin \text{set } p')$
by (*induct* $p, \text{simp-all}$)

lemma *singleCombinatorsConc*: *singleCombinators* $(x \# xs) \Longrightarrow \text{singleCombinators } xs$
by (*case-tac* $x, \text{simp-all}$)

lemma *aux0-0*: *singleCombinators* $x \Longrightarrow \neg (\exists a b. (a \oplus b) \in \text{set } x)$
apply (*induct* $x, \text{simp-all}$)
subgoal for $a b$
by (*case-tac* $a, \text{simp-all}$)
done

lemma *aux0-4*: $(a \in \text{set } x \vee a \in \text{set } y) = (a \in \text{set } (x @ y))$
by *auto*

lemma *aux0-1*: $\llbracket \text{singleCombinators } xs; \text{singleCombinators } [x] \rrbracket \Longrightarrow \text{singleCombinators } (x \# xs)$
by (*case-tac* $x, \text{simp-all}$)

lemma *aux0-6*: $\llbracket \text{singleCombinators } xs; \neg (\exists a b. x = a \oplus b) \rrbracket \Longrightarrow \text{singleCombinators } (x \# xs)$
apply (*rule* *aux0-1, simp-all*)
apply (*case-tac* $x, \text{simp-all}$)
done

lemma *aux0-5*: $\neg (\exists a b. (a \oplus b) \in \text{set } x) \Longrightarrow \text{singleCombinators } x$
apply (*induct* x)
apply *simp-all*
by (*metis* *aux0-6*)

lemma *ANDConc*[*rule-format*]: *allNetsDistinct* $(a \# p) \longrightarrow \text{allNetsDistinct } (p)$
apply (*simp* *add: allNetsDistinct-def*)
apply (*case-tac* a)
by *simp-all*

lemma *aux6*: *twoNetsDistinct* $a1 a2 a b \Longrightarrow \text{dom } (\text{deny-all-from-to } a1 a2) \cap \text{dom } (\text{deny-all-from-to } a b) = \{\}$
by (*auto* *simp: twoNetsDistinct-def netsDistinct-def src-def dest-def in-subnet-def PolicyCombinators.PolicyCombinators dom-def*)

lemma *aux5*[*rule-format*]: $(\text{DenyAllFromTo } a b) \in \text{set } p \longrightarrow a \in \text{set } (\text{net-list } p)$

by (*rule net-list-aux.induct,simp-all*)

lemma *aux5a*[*rule-format*]: $(DenyAllFromTo\ b\ a) \in set\ p \longrightarrow a \in set\ (net-list\ p)$
by (*rule net-list-aux.induct,simp-all*)

lemma *aux5c*[*rule-format*]:
 $(AllowPortFromTo\ a\ b\ po) \in set\ p \longrightarrow a \in set\ (net-list\ p)$
by (*rule net-list-aux.induct,simp-all*)

lemma *aux5d*[*rule-format*]:
 $(AllowPortFromTo\ b\ a\ po) \in set\ p \longrightarrow a \in set\ (net-list\ p)$
by (*rule net-list-aux.induct,simp-all*)

lemma *aux10*[*rule-format*]: $a \in set\ (net-list\ p) \longrightarrow a \in set\ (net-list-aux\ p)$
by *simp*

lemma *srcInNetListaux*[*simp*]:
 $\llbracket x \in set\ p; singleCombinators[x]; x \neq DenyAll \rrbracket \Longrightarrow srcNet\ x \in set\ (net-list-aux\ p)$
apply (*induct p*)
apply *simp-all*
subgoal for *a p*
apply (*case-tac x = a, simp-all*)
apply (*case-tac a, simp-all*)
apply (*case-tac a, simp-all*)
done
done

lemma *destInNetListaux*[*simp*]:
 $\llbracket x \in set\ p; singleCombinators[x]; x \neq DenyAll \rrbracket \Longrightarrow destNet\ x \in set\ (net-list-aux\ p)$
apply (*induct p*)
apply *simp-all*
subgoal for *a p*
apply (*case-tac x = a, simp-all*)
apply (*case-tac a, simp-all*)
apply (*case-tac a, simp-all*)
done
done

lemma *tND1*: $\llbracket allNetsDistinct\ p; x \in set\ p; y \in set\ p; a = srcNet\ x;$
 $b = destNet\ x; c = srcNet\ y; d = destNet\ y; a \neq c;$
 $singleCombinators[x]; x \neq DenyAll; singleCombinators[y];$
 $y \neq DenyAll \rrbracket \Longrightarrow twoNetsDistinct\ a\ b\ c\ d$
by (*simp add: allNetsDistinct-def twoNetsDistinct-def*)

lemma *tND2*: $\llbracket \text{allNetsDistinct } p; x \in \text{set } p; y \in \text{set } p; a = \text{srcNet } x;$
 $b = \text{destNet } x; c = \text{srcNet } y; d = \text{destNet } y; b \neq d;$
 $\text{singleCombinators}[x]; x \neq \text{DenyAll}; \text{singleCombinators}[y];$
 $y \neq \text{DenyAll} \rrbracket \implies \text{twoNetsDistinct } a \ b \ c \ d$
by (*simp add: allNetsDistinct-def twoNetsDistinct-def*)

lemma *tND*: $\llbracket \text{allNetsDistinct } p; x \in \text{set } p; y \in \text{set } p; a = \text{srcNet } x;$
 $b = \text{destNet } x; c = \text{srcNet } y; d = \text{destNet } y; a \neq c \vee b \neq d;$
 $\text{singleCombinators}[x]; x \neq \text{DenyAll}; \text{singleCombinators}[y]; y \neq \text{DenyAll} \rrbracket$
 $\implies \text{twoNetsDistinct } a \ b \ c \ d$
apply (*case-tac a \neq c, simp-all*)
apply (*erule-tac x = x and y = y in tND1, simp-all*)
apply (*erule-tac x = x and y = y in tND2, simp-all*)
done

lemma *aux7*: $\llbracket \text{DenyAllFromTo } a \ b \in \text{set } p; \text{allNetsDistinct } ((\text{DenyAllFromTo } c \ d) \# p);$
 $a \neq c \vee b \neq d \rrbracket \implies \text{twoNetsDistinct } a \ b \ c \ d$
apply (*erule-tac x = DenyAllFromTo a b and y = DenyAllFromTo c d in tND*)
by *simp-all*

lemma *aux7a*: $\llbracket \text{DenyAllFromTo } a \ b \in \text{set } p;$
 $\text{allNetsDistinct } ((\text{AllowPortFromTo } c \ d \ \text{po}) \# p); a \neq c \vee b \neq d \rrbracket \implies$
 $\text{twoNetsDistinct } a \ b \ c \ d$
apply (*erule-tac x = DenyAllFromTo a b and*
 $y = \text{AllowPortFromTo } c \ d \ \text{po}$ **in** *tND*)
by *simp-all*

lemma *nDComm*: **assumes** *ab*: *netsDistinct a b* **shows** *ba*: *netsDistinct b a*
apply (*insert ab*)
by (*auto simp: netsDistinct-def in-subnet-def*)

lemma *tNDComm*:
assumes *abcd*: *twoNetsDistinct a b c d* **shows** *twoNetsDistinct c d a b*
apply (*insert abcd*)
by (*metis twoNetsDistinct-def nDComm*)

lemma *aux[rule-format]*: $a \in \text{set } (\text{removeShadowRules2 } p) \longrightarrow a \in \text{set } p$
apply (*case-tac a*)
by (*rule removeShadowRules2.induct, simp-all*)⁺

lemma *aux12*: $\llbracket a \in x; b \notin x \rrbracket \implies a \neq b$
by *auto*

lemma *ND0aux1[rule-format]*: $\text{DenyAllFromTo } x \ y \in \text{set } b \implies$

$x \in \text{set } (\text{net-list-aux } b)$

by (*metis aux5 net-list.simps set-remdups*)

lemma *ND0aux2*[*rule-format*]: *DenyAllFromTo* $x y \in \text{set } b \implies$
 $y \in \text{set } (\text{net-list-aux } b)$
by (*metis aux5a net-list.simps set-remdups*)

lemma *ND0aux3*[*rule-format*]: *AllowPortFromTo* $x y p \in \text{set } b \implies$
 $x \in \text{set } (\text{net-list-aux } b)$
by (*metis aux5c net-list.simps set-remdups*)

lemma *ND0aux4*[*rule-format*]: *AllowPortFromTo* $x y p \in \text{set } b \implies$
 $y \in \text{set } (\text{net-list-aux } b)$
by (*metis aux5d net-list.simps set-remdups*)

lemma *aNDSubsetaux*[*rule-format*]: *singleCombinators* $a \longrightarrow \text{set } a \subseteq \text{set } b \longrightarrow$
 $\text{set } (\text{net-list-aux } a) \subseteq \text{set } (\text{net-list-aux } b)$

apply (*induct a*)
apply (*simp-all*)
apply (*clarify*)
apply (*drule mp, erule singleCombinatorsConc*)
subgoal for $a a' x$
apply (*case-tac a*)
apply (*simp-all add: contra-subsetD*)
apply (*metis contra-subsetD*)
apply (*metis ND0aux1 ND0aux2 contra-subsetD*)
apply (*metis ND0aux3 ND0aux4 contra-subsetD*)
done
done

lemma *aNDSetsEqaux*[*rule-format*]: *singleCombinators* $a \longrightarrow \text{singleCombinators } b \longrightarrow$
 $\text{set } a = \text{set } b \longrightarrow \text{set } (\text{net-list-aux } a) = \text{set } (\text{net-list-aux } b)$

apply (*rule impI*)
apply (*rule equalityI*)
apply (*rule aNDSubsetaux, simp-all*)
done

lemma *aNDSubset*: $\llbracket \text{singleCombinators } a; \text{set } a \subseteq \text{set } b; \text{allNetsDistinct } b \rrbracket \implies$
 $\text{allNetsDistinct } a$

apply (*simp add: allNetsDistinct-def*)
apply (*rule allI*)
apply (*rule impI*)
subgoal for $x y$

```

apply (drule-tac  $x = x$  in spec, drule-tac  $x = y$  in spec)
using aNDSubsetaux by blast
done

lemma aNDSetsEq:  $\llbracket \text{singleCombinators } a; \text{singleCombinators } b; \text{set } a = \text{set } b; \text{allNetsDistinct } b \rrbracket \implies \text{allNetsDistinct } a$ 
apply (simp add: allNetsDistinct-def)
apply (rule allI)+
apply (rule impI)+
subgoal for  $x$   $y$ 
  apply (drule-tac  $x = x$  in spec, drule-tac  $x = y$  in spec)
  using aNDSetsEqaux by auto
done

lemma SCConca:  $\llbracket \text{singleCombinators } p; \text{singleCombinators } [a] \rrbracket \implies \text{singleCombinators } (a\#p)$ 
by(metis aux0-1)

lemma aux3[simp]:  $\llbracket \text{singleCombinators } p; \text{singleCombinators } [a]; \text{allNetsDistinct } (a\#p) \rrbracket \implies \text{allNetsDistinct } (a\#a\#p)$ 
by (metis aNDSetsEq aux0-1 insert-absorb2 list.set(2))

lemma wp1-aux1a[rule-format]:  $xs \neq [] \longrightarrow \text{wellformed-policy1-strong } (xs @ [x]) \longrightarrow \text{wellformed-policy1-strong } xs$ 
by (induct  $xs$ ,simp-all)

lemma wp1alternative-RS1[rule-format]:  $\text{DenyAll} \in \text{set } p \longrightarrow \text{wellformed-policy1-strong } (\text{removeShadowRules1 } p)$ 
by (induct  $p$ ,simp-all)

lemma wellformed-eq:  $\text{DenyAll} \in \text{set } p \longrightarrow ((\text{wellformed-policy1 } p) = (\text{wellformed-policy1-strong } p))$ 
by (induct  $p$ ,simp-all)

lemma set-insort:  $\text{set}(\text{insort } x \text{ } xs \ l) = \text{insert } x (\text{set } xs)$ 
by (induct  $xs$ ) auto

lemma set-sort[simp]:  $\text{set}(\text{sort } xs \ l) = \text{set } xs$ 
by (induct  $xs$ ) (simp-all add:set-insort)

lemma set-sortQ:  $\text{set}(\text{qsort } xs \ l) = \text{set } xs$ 
by simp

```

lemma *aux79*[*rule-format*]: $y \in \text{set } (\text{insort } x \ a \ l) \longrightarrow y \neq x \longrightarrow y \in \text{set } a$
apply (*induct a*)
by *auto*

lemma *aux80*: $\llbracket y \notin \text{set } p; y \neq x \rrbracket \Longrightarrow y \notin \text{set } (\text{insort } x \ (\text{sort } p \ l) \ l)$
apply (*metis aux79 set-sort*)
done

lemma *WP1Conca*: $\text{DenyAll} \notin \text{set } p \Longrightarrow \text{wellformed-policy1 } (a\#p)$
by (*case-tac a,simp-all*)

lemma *saux*[*simp*]: $(\text{insort } \text{DenyAll } p \ l) = \text{DenyAll}\#p$
by (*induct-tac p,simp-all*)

lemma *saux3*[*rule-format*]: $\text{DenyAllFromTo } a \ b \in \text{set list} \longrightarrow$
 $\text{DenyAllFromTo } c \ d \notin \text{set list} \longrightarrow (a \neq c) \vee (b \neq d)$
by *blast*

lemma *waux2*[*rule-format*]: $(\text{DenyAll} \notin \text{set } xs) \longrightarrow \text{wellformed-policy1 } xs$
by (*induct-tac xs,simp-all*)

lemma *waux3*[*rule-format*]: $\llbracket x \neq a; x \notin \text{set } p \rrbracket \Longrightarrow x \notin \text{set } (\text{insort } a \ p \ l)$
by (*metis aux79*)

lemma *wellformed1-sorted-aux*[*rule-format*]: $\text{wellformed-policy1 } (x\#p) \Longrightarrow$
 $\text{wellformed-policy1 } (\text{insort } x \ p \ l)$
by (*metis NormalisationGenericProofs.set-insort list.set(2) saux waux2 wellformed-eq*
 $\text{wellformed-policy1-strong.simps(2)}$)

lemma *wellformed1-sorted-auxQ*[*rule-format*]: $\text{wellformed-policy1 } (p) \Longrightarrow$
 $\text{wellformed-policy1 } (\text{qsort } p \ l)$

proof (*induct p*)
case *Nil* **show** *?case* **by** *simp*
next
case (*Cons a S*) **then show** *?case*
apply *simp-all*
apply (*cases a,simp-all*)
by (*metis Combinators.simps append-Cons append-Nil qsort.simps(2) set-ConsD*
 set-qsort waux2)
qed

lemma *SR1Subset*: $set (removeShadowRules1 p) \subseteq set p$
apply (*induct-tac* *p*, *simp-all*)
subgoal for *x xs*
apply (*case-tac* *x*, *simp-all*)
apply(*auto*)
done
done

lemma *SCSubset*[*rule-format*]: $singleCombinators b \longrightarrow set a \subseteq set b \longrightarrow singleCombinators a$

proof (*induct a*)
case *Nil* **thus** ?*case* **by** *simp*
next
case (*Cons x xs*) **thus** ?*case*
by (*meson aux0-0 aux0-5 subsetCE*)
qed

lemma *setInsert*[*simp*]: $set list \subseteq insert a (set list)$
by *auto*

lemma *SC-RS1*[*rule-format, simp*]: $singleCombinators p \longrightarrow allNetsDistinct p \longrightarrow singleCombinators (removeShadowRules1 p)$
apply (*induct-tac* *p*)
apply *simp-all*
using *ANDConc singleCombinatorsConc* **by** *blast*

lemma *RS2Set*[*rule-format*]: $set (removeShadowRules2 p) \subseteq set p$
apply(*induct p*, *simp-all*)
subgoal for *a as*
apply(*case-tac a*)
by(*auto*)
done

lemma *WP1*: $a \notin set list \implies a \notin set (removeShadowRules2 list)$
using *RS2Set* [*of list*] **by** *blast*

lemma *denyAllDom*[*simp*]: $x \in dom (deny-all)$
by (*simp add: UPFDefs(24) domI*)

lemma *lCdom2*: $(list2FWpolicy (a @ (b @ c))) = (list2FWpolicy ((a@b)@c))$
by *auto*

lemma *SCConcEnd*: $singleCombinators (xs @ [xa]) \implies singleCombinators xs$

apply (*induct xs, simp-all*)
subgoal for *a as*
by (*case-tac a, simp-all*)
done

lemma *list2FWpolicyconc*[*rule-format*]: $a \neq [] \longrightarrow$
 $(\text{list2FWpolicy } (xa \# a)) = (xa) \oplus (\text{list2FWpolicy } a)$
by (*induct a, simp-all*)

lemma *wp1n-tl* [*rule-format*]: *wellformed-policy1-strong p* \longrightarrow
 $p = (\text{DenyAll}\#(\text{tl } p))$
by (*induct p, simp-all*)

lemma *foo2*: $a \notin \text{set } ps \implies$
 $a \notin \text{set } ss \implies$
 $\text{set } p = \text{set } s \implies$
 $p = (a\#(ps)) \implies$
 $s = (a\#ss) \implies$
 $\text{set } (ps) = \text{set } (ss)$
by *auto*

lemma *SCnotConc*[*rule-format, simp*]: $a \oplus b \in \text{set } p \longrightarrow \text{singleCombinators } p \longrightarrow \text{False}$
apply (*induct p, simp-all*)
subgoal for *p ps*
by(*case-tac p, simp-all*)
done

lemma *auxx8*: *removeShadowRules1-alternative-rev* $[x] = [x]$
by (*case-tac x, simp-all*)

lemma *RS1End*[*rule-format*]: $x \neq \text{DenyAll} \longrightarrow \text{removeShadowRules1 } (xs @ [x]) =$
 $(\text{removeShadowRules1 } xs)@[x]$
by (*induct-tac xs, simp-all*)

lemma *aux114*: $x \neq \text{DenyAll} \implies \text{removeShadowRules1-alternative-rev } (x\#xs) =$
 $x\#(\text{removeShadowRules1-alternative-rev } xs)$
apply (*induct-tac xs*)
apply (*auto simp: auxx8*)
by (*case-tac x, simp-all*)

lemma *aux115*[*rule-format*]: $x \neq \text{DenyAll} \implies \text{removeShadowRules1-alternative } (xs@[x])$
 $= (\text{removeShadowRules1-alternative } xs)@[x]$

apply (*simp add: removeShadowRules1-alternative-def aux114*)
done

lemma *RS1-DA*[*simp*]: *removeShadowRules1 (xs @ [DenyAll]) = [DenyAll]*
by (*induct-tac xs, simp-all*)

lemma *rSR1-eq: removeShadowRules1-alternative = removeShadowRules1*
apply (*rule ext*)
apply (*simp add: removeShadowRules1-alternative-def*)
subgoal for *x*
apply (*rule-tac xs = x in rev-induct*)
apply *simp-all*
subgoal for *x xs*
apply (*case-tac x = DenyAll, simp-all*)
apply (*metis RS1End aux114 rev.simps(2)*)
done
done
done

lemma *domInterMT*[*rule-format*]: $\llbracket \text{dom } a \cap \text{dom } b = \{\}; x \in \text{dom } a \rrbracket \implies x \notin \text{dom } b$
by *auto*

lemma *domComm*: $\text{dom } a \cap \text{dom } b = \text{dom } b \cap \text{dom } a$
by *auto*

lemma *r-not-DA-in-tl*[*rule-format*]:
 $\text{wellformed-policy1-strong } p \longrightarrow a \in \text{set } p \longrightarrow a \neq \text{DenyAll} \longrightarrow a \in \text{set } (\text{tl } p)$
by (*induct p, simp-all*)

lemma *wp1-aux1aa*[*rule-format*]: $\text{wellformed-policy1-strong } p \longrightarrow \text{DenyAll} \in \text{set } p$
by (*induct p, simp-all*)

lemma *mauxa*: $(\exists r. a \ b = \lfloor r \rfloor) = (a \ b \neq \perp)$
by *auto*

lemma *l2p-aux*[*rule-format*]: $\text{list} \neq [] \longrightarrow$
 $\text{list2FWpolicy } (a \ \# \ \text{list}) = a \ \oplus (\text{list2FWpolicy } \text{list})$
by (*induct list, simp-all*)

lemma *l2p-aux2*[*rule-format*]: $\text{list} = [] \implies \text{list2FWpolicy } (a \ \# \ \text{list}) = a$
by *simp*

lemma *aux7aa*:
assumes *1 : AllowPortFromTo a b poo* $\in \text{set } p$

```

and 2 : allNetsDistinct ((AllowPortFromTo c d po) # p)
and 3 :  $a \neq c \vee b \neq d$ 
shows twoNetsDistinct a b c d (is ?H)
proof(cases a  $\neq$  c) print-cases
  case True assume *:a  $\neq$  c show ?H
    by (meson 1 2 True allNetsDistinct-def aux5c list.set-intros(1)
      list.set-intros(2) twoNetsDistinct-def)
  next
    case False assume *:(a  $\neq$  c) show twoNetsDistinct a b c d
      by (meson 1 2 3 False allNetsDistinct-def aux5d list.set-intros(1)
        list.set-intros(2) twoNetsDistinct-def)
qed

```

```

lemma ANDConcEnd: [allNetsDistinct (xs @ [xa]); singleCombinators xs]  $\implies$ 
  allNetsDistinct xs
by (rule aNDSubset, auto)

```

```

lemma WP1ConcEnd[rule-format]:
  wellformed-policy1 (xs@[xa])  $\longrightarrow$  wellformed-policy1 xs
by (induct xs, simp-all)

```

```

lemma NDComm: netsDistinct a b = netsDistinct b a
by (auto simp: netsDistinct-def in-subnet-def)

```

```

lemma wellformed1-sorted[simp]:
  assumes wp1: wellformed-policy1 p
  shows wellformed-policy1 (sort p l)
proof (cases p)
  case Nil thus ?thesis by simp
next
  case (Cons x xs) thus ?thesis
  proof (cases x = DenyAll)
    case True thus ?thesis using assms Cons by simp
  next
    case False thus ?thesis using assms
      by (metis Cons set-sort False waux2 wellformed-eq
        wellformed-policy1-strong.simps(2))
  qed
qed

```

```

lemma wellformed1-sortedQ[simp]:
  assumes wp1: wellformed-policy1 p

```

```

  shows      wellformed-policy1 (qsort p l)
proof (cases p)
  case Nil thus ?thesis by simp
next
  case (Cons x xs) thus ?thesis
proof (cases x = DenyAll)
  case True thus ?thesis using assms Cons by simp
next
  case False thus ?thesis using assms
  by (metis Cons set-qsort False wauw2 wellformed-eq
    wellformed-policy1-strong.simps(2))
qed
qed

lemma SC1[simp]: singleCombinators p  $\implies$  singleCombinators (removeShadowRules1 p)
  by (erule SCSubset) (rule SR1Subset)

lemma SC2[simp]: singleCombinators p  $\implies$  singleCombinators (removeShadowRules2 p)
  by (erule SCSubset) (rule RS2Set)

lemma SC3[simp]: singleCombinators p  $\implies$  singleCombinators (sort p l)
  by (erule SCSubset) simp

lemma SC3Q[simp]: singleCombinators p  $\implies$  singleCombinators (qsort p l)
  by (erule SCSubset) simp

lemma aND-RS1[simp]:  $\llbracket$ singleCombinators p; allNetsDistinct p $\rrbracket \implies$ 
  allNetsDistinct (removeShadowRules1 p)
  apply (rule aNDSubset)
  apply (erule SC-RS1, simp-all)
  apply (rule SR1Subset)
  done

lemma aND-RS2[simp]:  $\llbracket$ singleCombinators p; allNetsDistinct p $\rrbracket \implies$ 
  allNetsDistinct (removeShadowRules2 p)
  apply (rule aNDSubset)
  apply (erule SC2, simp-all)
  apply (rule RS2Set)
  done

lemma aND-sort[simp]:  $\llbracket$ singleCombinators p; allNetsDistinct p $\rrbracket \implies$ 
  allNetsDistinct (sort p l)

```


apply (*rule aNDSubset*)
by (*erule SC3, simp-all*)

lemma *aND-sortQ*[*simp*]: $\llbracket \text{singleCombinators } p; \text{allNetsDistinct } p \rrbracket \implies$
 $\text{allNetsDistinct } (\text{qsort } p \ l)$

apply (*rule aNDSubset*)
by (*erule SC3Q, simp-all*)

lemma *inRS2*[*rule-format, simp*]: $x \notin \text{set } p \longrightarrow x \notin \text{set } (\text{removeShadowRules2 } p)$

apply (*insert RS2Set [of p]*)
by *blast*

lemma *distinct-RS2*[*rule-format, simp*]: $\text{distinct } p \longrightarrow$

$\text{distinct } (\text{removeShadowRules2 } p)$

apply (*induct p*)
apply *simp-all*
apply *clarify*
subgoal for *a p*
apply (*case-tac a*)
by *auto*
done

lemma *setPaireq*: $\{x, y\} = \{a, b\} \implies x = a \wedge y = b \vee x = b \wedge y = a$

by (*metis doubleton-eq-iff*)

lemma *position-positive*[*rule-format*]: $a \in \text{set } l \longrightarrow \text{position } a \ l > 0$

by (*induct l, simp-all*)

lemma *pos-noteq*[*rule-format*]:

$a \in \text{set } l \longrightarrow b \in \text{set } l \longrightarrow c \in \text{set } l \longrightarrow$

$a \neq b \longrightarrow \text{position } a \ l \leq \text{position } b \ l \longrightarrow \text{position } b \ l \leq \text{position } c \ l \longrightarrow$

$a \neq c$

proof(*induct l*)

case *Nil show ?case by simp*

next

case (*Cons a R*) **show** *?case*

by (*metis (no-types, lifting) Cons.hyps One-nat-def Suc-le-mono le-antisym*

length-greater-0-conv list.size(3) nat.inject position.simps(2)

position-positive set-ConsD)

qed

lemma *setPair-noteq*: $\{a, b\} \neq \{c, d\} \implies \neg ((a = c) \wedge (b = d))$

by auto

lemma *setPair-noteq-allow*: $\{a,b\} \neq \{c,d\} \implies \neg ((a = c) \wedge (b = d) \wedge P)$

by auto

lemma *order-trans*:

$\llbracket \text{in-list } x \ l; \text{ in-list } y \ l; \text{ in-list } z \ l; \text{ singleCombinators } [x];$
 $\text{singleCombinators } [y]; \text{ singleCombinators } [z]; \text{ smaller } x \ y \ l; \text{ smaller } y \ z \ l \rrbracket \implies$
 $\text{smaller } x \ z \ l$

apply (*case-tac* *x*, *simp-all*)
apply (*case-tac* *z*, *simp-all*)
 apply (*case-tac* *y*, *simp-all*)
 apply (*case-tac* *y*, *simp-all*)
 apply (*rule conjI*|*rule impI*)
 apply (*simp add: setPaireq*)
 apply (*rule conjI*|*rule impI*)
 apply (*simp-all split: if-splits*)
 apply *metis*
 apply *auto*[1]
 apply (*simp add: setPaireq*)
 apply (*rule impI*,*case-tac* *y*, *simp-all*)
 apply (*simp-all split: if-splits*, *metis*,*simp-all add: setPair-noteq setPair-noteq-allow*)
apply (*case-tac* *z*, *simp-all*)
 apply (*case-tac* *y*, *simp-all*)
 apply (*case-tac* *y*, *simp-all*)
 apply (*intro impI*|*rule conjI*)
 apply (*simp-all split: if-splits*)
 apply (*simp add: setPair-noteq*)
 apply (*erule pos-noteq*, *simp-all*)
 apply *auto*[1]
 apply (*rule conjI*,*simp add: setPair-noteq-allow*)
 apply (*erule pos-noteq*, *simp-all*)
 apply *auto*[1]
apply (*rule impI*,*rule disjI2*)
apply (*case-tac* *y*, *simp-all split: if-splits*)
 apply *metis*
 apply (*simp-all add: setPair-noteq-allow*)
done

lemma *sortedConcStart*[*rule-format*]:

$\text{sorted } (a \ \# \ aa \ \# \ p) \ l \longrightarrow \text{in-list } a \ l \longrightarrow \text{in-list } aa \ l \longrightarrow \text{all-in-list } p \ l \longrightarrow$
 $\text{singleCombinators } [a] \longrightarrow \text{singleCombinators } [aa] \longrightarrow \text{singleCombinators } p \longrightarrow$
 $\text{sorted } (a\#p) \ l$
apply (*induct* *p*)

```

apply simp-all
apply (rule impI)
apply simp
apply (rule-tac y = aa in order-trans)
  apply simp-all
subgoal for p ps
  apply (case-tac p, simp-all)
  done
done

```

```

lemma singleCombinatorsStart[simp]: singleCombinators (x#xs) ==>
  singleCombinators [x]
by (case-tac x, simp-all)

```

```

lemma sorted-is-smaller[rule-format]:
  sorted (a # p) l -> in-list a l -> in-list b l -> all-in-list p l ->
  singleCombinators [a] -> singleCombinators p -> b ∈ set p -> smaller a b l
apply (induct p)
apply (auto intro: singleCombinatorsConc sortedConcStart)
done

```

```

lemma sortedConcEnd[rule-format]: sorted (a # p) l -> in-list a l ->
  all-in-list p l -> singleCombinators [a] ->
  singleCombinators p -> sorted p l
apply (induct p)
apply (auto intro: singleCombinatorsConc sortedConcStart)
done

```

```

lemma in-set-in-list[rule-format]: a ∈ set p -> all-in-list p l -> in-list a l
by (induct p auto)

```

```

lemma sorted-Consb[rule-format]:
  all-in-list (x#xs) l -> singleCombinators (x#xs) ->
  (sorted xs l & (∀ y ∈ set xs. smaller x y l)) -> (sorted (x#xs) l)
apply(induct xs arbitrary: x)
apply (auto simp: order-trans)
done

```

```

lemma sorted-Cons:  $\llbracket$ all-in-list (x#xs) l; singleCombinators (x#xs) $\rrbracket ==>$ 
  (sorted xs l & (∀ y ∈ set xs. smaller x y l)) = (sorted (x#xs) l)
apply auto
apply (rule sorted-Consb, simp-all)
apply (metis singleCombinatorsConc singleCombinatorsStart sortedConcEnd)

```

apply (*erule sorted-is-smaller*)
apply (*auto intro: singleCombinatorsConc singleCombinatorsStart in-set-in-list*)
done

lemma *smaller-antisym*: $\llbracket \neg \text{smaller } a \ b \ l; \text{in-list } a \ l; \text{in-list } b \ l; \text{singleCombinators}[a]; \text{singleCombinators } [b] \rrbracket \implies \text{smaller } b \ a \ l$

apply (*case-tac a*)
apply *simp-all*
apply (*case-tac b*)
apply *simp-all*
apply (*simp-all split: if-splits*)
apply (*rule setPaireq*)
apply *simp*
apply (*case-tac b*)
apply *simp-all*
apply (*simp-all split: if-splits*)
done

lemma *set-insort-insert*: $\text{set } (\text{insort } x \ xs \ l) \subseteq \text{insert } x \ (\text{set } xs)$
by (*induct xs*) *auto*

lemma *all-in-listSubset[rule-format]*: $\text{all-in-list } b \ l \longrightarrow \text{singleCombinators } a \longrightarrow \text{set } a \subseteq \text{set } b \longrightarrow \text{all-in-list } a \ l$
by (*induct-tac a*) (*auto intro: in-set-in-list singleCombinatorsConc*)

lemma *singleCombinators-insort*: $\llbracket \text{singleCombinators } [x]; \text{singleCombinators } xs \rrbracket \implies \text{singleCombinators } (\text{insort } x \ xs \ l)$
by (*metis NormalisationGenericProofs.set-insort aux0-0 aux0-1 aux0-5 list.simps(15)*)

lemma *all-in-list-insort*: $\llbracket \text{all-in-list } xs \ l; \text{singleCombinators } (x\#xs); \text{in-list } x \ l \rrbracket \implies \text{all-in-list } (\text{insort } x \ xs \ l) \ l$
apply (*rule-tac b = x#xs in all-in-listSubset*)
apply *simp-all*
apply (*metis singleCombinatorsConc singleCombinatorsStart singleCombinators-insort*)
apply (*rule set-insort-insert*)
done

lemma *sorted-ConsA*: $\llbracket \text{all-in-list } (x\#xs) \ l; \text{singleCombinators } (x\#xs) \rrbracket \implies (\text{sorted } (x\#xs) \ l) = (\text{sorted } xs \ l \ \& \ (\forall y \in \text{set } xs. \text{smaller } x \ y \ l))$
by (*metis sorted-Cons*)

lemma *is-in-insort*: $y \in \text{set } xs \implies y \in \text{set } (\text{insort } x \ xs \ l)$

by (simp add: NormalisationGenericProofs.set-insort)

```
lemma sorted-insorta[rule-format]:
  assumes 1 : sorted (insort x xs l) l
  and 2 : all-in-list (x#xs) l
  and 3 : all-in-list (x#xs) l
  and 4 : distinct (x#xs)
  and 5 : singleCombinators [x]
  and 6 : singleCombinators xs
  shows sorted xs l
proof (insert 1 2 3 4 5 6, induct xs)
  case Nil show ?case by simp
next
  case (Cons a xs)
  then show ?case
  apply simp
  apply (auto intro: is-in-insort sorted-ConsA set-insort singleCombinators-insort
    singleCombinatorsConc sortedConcEnd all-in-list-insort) [1]
  apply (cases smaller x a l, simp-all)
  by (metis NormalisationGenericProofs.set-insort NormalisationGener-
icProofs.sorted-Cons
  all-in-list.simps(2) all-in-list-insort aux0-1 insert-iff singleCombinatorsConc
  singleCombinatorsStart singleCombinators-insort)
qed
```

```
lemma sorted-insortb[rule-format]:
  sorted xs l  $\longrightarrow$  all-in-list (x#xs) l  $\longrightarrow$  distinct (x#xs)  $\longrightarrow$ 
  singleCombinators [x]  $\longrightarrow$  singleCombinators xs  $\longrightarrow$  sorted (insort x xs l) l
proof (induct xs)
  case Nil show ?case by simp-all
next
  case (Cons a xs)
  have * : sorted (a # xs) l  $\implies$  all-in-list (x # a # xs) l  $\implies$ 
    distinct (x # a # xs)  $\implies$  singleCombinators [x]  $\implies$ 
    singleCombinators (a # xs)  $\implies$  sorted (insort x xs l) l
  apply (insert Cons.hyps, simp-all)
  apply (metis sorted-Cons all-in-list.simps(2) singleCombinatorsConc)
  done
show ?case
  apply (insert Cons.hyps)
  apply (rule impI)+
  apply (insert *, auto intro!: sorted-Consb)
proof (rule-tac b = x#xs in all-in-listSubset)
```

```

show in-list x l  $\implies$  all-in-list xs l  $\implies$  all-in-list (x # xs) l
  by simp-all
next
show singleCombinators [x]  $\implies$ 
      singleCombinators (a # xs)  $\implies$ 
      FWNormalisationCore.sorted (FWNormalisationCore.insort x xs l) l
 $\implies$ 
      singleCombinators (FWNormalisationCore.insort x xs l)
apply (rule singleCombinators-insort, simp-all)
by (erule singleCombinatorsConc)
next
show set (FWNormalisationCore.insort x xs l)  $\subseteq$  set (x # xs)
  using NormalisationGenericProofs.set-insort-insert by auto
next
show singleCombinators [x]  $\implies$ 
      singleCombinators (a # xs)  $\implies$ 
      singleCombinators (a # FWNormalisationCore.insort x xs l)
  by (metis SCConca singleCombinatorsConc singleCombinatorsStart
      singleCombinators-insort)
next
fix y
show FWNormalisationCore.sorted (a # xs) l  $\implies$ 
      singleCombinators [x]  $\implies$  singleCombinators (a # xs)  $\implies$ 
      in-list x l  $\implies$  in-list a l  $\implies$  all-in-list xs l  $\implies$ 
       $\neg$  smaller x a l  $\implies$   $y \in \text{set } (\text{FWNormalisationCore.insort } x \text{ xs } l) \implies$ 
      smaller a y l
  by (metis NormalisationGenericProofs.set-insort in-set-in-list insert-iff
      singleCombinatorsConc singleCombinatorsStart smaller-antisym
      sorted-is-smaller)
qed
qed

```

lemma *sorted-insort*:

```

[[all-in-list (x#xs) l; distinct(x#xs); singleCombinators [x];
  singleCombinators xs]]  $\implies$ 
  sorted (insort x xs l) l = sorted xs l

```

by (*auto intro: sorted-insorta sorted-insortb*)

lemma *distinct-insort*: *distinct* (*insort* x xs l) = (x \notin *set* xs \wedge *distinct* xs)

by(*induct* xs)(*auto simp:set-insort*)

lemma *distinct-sort[simp]*: *distinct* (*sort* xs l) = *distinct* xs

by(*induct* xs)(*simp-all add:distinct-insort*)

lemma *sort-is-sorted*[rule-format]:
all-in-list p l \longrightarrow *distinct p* \longrightarrow *singleCombinators p* \longrightarrow *sorted (sort p l) l*
apply (*induct p*)
apply *simp*
by (*metis (no-types, lifting) NormalisationGenericProofs.distinct-sort*
NormalisationGenericProofs.set-sort SC3 all-in-list.simps(2) all-in-listSubset
distinct.simps(2) set-subset-Cons singleCombinatorsConc singleCombinatorsStart
sort.simps(2) sorted-insortb)

lemma *smaller-sym*[rule-format]: *all-in-list [a] l* \longrightarrow *smaller a a l*
by (*case-tac a, simp-all*)

lemma *SC-sublist*[rule-format]:
singleCombinators xs \implies *singleCombinators (qsort [y \leftarrow xs. P y] l)*
by (*auto intro: SCSubset*)

lemma *all-in-list-sublist*[rule-format]:
singleCombinators xs \longrightarrow *all-in-list xs l* \longrightarrow *all-in-list (qsort [y \leftarrow xs. P y] l) l*
by (*auto intro: all-in-listSubset SC-sublist*)

lemma *SC-sublist2*[rule-format]:
singleCombinators xs \longrightarrow *singleCombinators ([y \leftarrow xs. P y])*
by (*auto intro: SCSubset*)

lemma *all-in-list-sublist2*[rule-format]:
singleCombinators xs \longrightarrow *all-in-list xs l* \longrightarrow *all-in-list ([y \leftarrow xs. P y]) l*
by (*auto intro: all-in-listSubset SC-sublist2*)

lemma *all-in-listAppend*[rule-format]:
all-in-list (xs) l \longrightarrow *all-in-list (ys) l* \longrightarrow *all-in-list (xs @ ys) l*
by (*induct xs*) *simp-all*

lemma *distinct-sortQ*[rule-format]:
singleCombinators xs \longrightarrow *all-in-list xs l* \longrightarrow *distinct xs* \longrightarrow *distinct (qsort xs l)*
apply (*induct xs l rule: qsort.induct*)
apply *simp*
apply (*auto simp: SC-sublist2 singleCombinatorsConc all-in-list-sublist2*)
done

lemma *singleCombinatorsAppend*[rule-format]:
singleCombinators (xs) \longrightarrow *singleCombinators (ys)* \longrightarrow *singleCombinators (xs @ ys)*
apply (*induct xs, auto*)
subgoal for a as

```

apply (case-tac a,simp-all)
done
subgoal for a as
  apply (case-tac a,simp-all)
  done
done

```

```

lemma sorted-append1[rule-format]:
  all-in-list xs l  $\longrightarrow$  singleCombinators xs  $\longrightarrow$ 
  all-in-list ys l  $\longrightarrow$  singleCombinators ys  $\longrightarrow$ 
  (sorted (xs@ys) l  $\longrightarrow$ 
  (sorted xs l & sorted ys l & ( $\forall x \in \text{set } xs. \forall y \in \text{set } ys. \text{smaller } x \ y \ l$ )))
  apply(induct xs)
  apply(simp-all)
  by (metis NormalisationGenericProofs.sorted-Cons all-in-list.simps(2)
  all-in-listAppend aux0-1
  aux0-4 singleCombinatorsAppend singleCombinatorsConc singleCombinatorsStart)

```

```

lemma sorted-append2[rule-format]:
  all-in-list xs l  $\longrightarrow$  singleCombinators xs  $\longrightarrow$ 
  all-in-list ys l  $\longrightarrow$  singleCombinators ys  $\longrightarrow$ 
  (sorted xs l & sorted ys l & ( $\forall x \in \text{set } xs. \forall y \in \text{set } ys. \text{smaller } x \ y \ l$ ))  $\longrightarrow$ 
  (sorted (xs@ys) l)
  apply (induct xs)
  apply simp-all
  by (metis NormalisationGenericProofs.sorted-Cons all-in-list.simps(2)
  all-in-listAppend aux0-1
  aux0-4 singleCombinatorsAppend singleCombinatorsConc singleCombinatorsStart)

```

```

lemma sorted-append[rule-format]:
  all-in-list xs l  $\longrightarrow$  singleCombinators xs  $\longrightarrow$ 
  all-in-list ys l  $\longrightarrow$  singleCombinators ys  $\longrightarrow$ 
  (sorted (xs@ys) l) =
  (sorted xs l & sorted ys l & ( $\forall x \in \text{set } xs. \forall y \in \text{set } ys. \text{smaller } x \ y \ l$ ))
  apply (rule impI)+
  apply (rule iffI)
  apply (rule sorted-append1,simp-all)
  apply (rule sorted-append2,simp-all)
done

```

```

lemma sort-is-sortedQ[rule-format]:
  all-in-list p l  $\longrightarrow$  singleCombinators p  $\longrightarrow$  sorted (qsort p l) l
proof (induct p l rule: qsort.induct) print-cases
  case 1 show ?case by simp

```



```

next
case 2 fix x::('a,'b) Combinators fix xs::('a,'b) Combinators list fix l
show all-in-list [y←xs . ¬ smaller x y l] l →
  singleCombinators [y←xs . ¬ smaller x y l] →
  sorted (qsort [y←xs . ¬ smaller x y l] l) l ⇒
  all-in-list [y←xs . smaller x y l] l →
  singleCombinators [y←xs . smaller x y l] →
  sorted (qsort [y←xs . smaller x y l] l) l ⇒
  all-in-list(x#xs) l → singleCombinators(x#xs) → sorted (qsort(x#xs)
l) l
  apply (intro impI)
  apply (simp-all add: SC-sublist all-in-list-sublist all-in-list-sublist2
    singleCombinatorsConc SC-sublist2)
proof (subst sorted-append)
  show in-list x l ∧ all-in-list xs l ⇒
    singleCombinators (x # xs) ⇒
    all-in-list (qsort [y←xs . ¬ smaller x y l] l) l
  by (metis all-in-list-sublist singleCombinatorsConc)
next
show in-list x l ∧ all-in-list xs l ⇒
  singleCombinators (x # xs) ⇒
  singleCombinators (qsort [y←xs . ¬ smaller x y l] l)
  apply (auto simp: SC-sublist all-in-list-sublist SC-sublist2
    all-in-list-sublist2 sorted-Cons sorted-append not-le)
  apply (metis SC3Q SC-sublist2 singleCombinatorsConc)
done
next
show sorted (qsort [y←xs . ¬ smaller x y l] l) l ⇒
  sorted (qsort [y←xs . smaller x y l] l) l ⇒
  in-list x l ∧ all-in-list xs l ⇒ singleCombinators (x # xs) ⇒
  all-in-list (x # qsort [y←xs . smaller x y l] l) l
  using all-in-list.simps(2) all-in-list-sublist singleCombinatorsConc by blast
next
show sorted (qsort [y←xs . smaller x y l] l) l ⇒
  in-list x l ∧ all-in-list xs l ⇒ singleCombinators (x # xs) ⇒
  singleCombinators (x # qsort [y←xs . smaller x y l] l)
  using SC-sublist aux0-1 singleCombinatorsConc singleCombinatorsStart by blast
next
show sorted (qsort [y←xs . ¬ smaller x y l] l) l ⇒
  sorted (qsort [y←xs . smaller x y l] l) l ⇒
  in-list x l ∧ all-in-list xs l ⇒
  singleCombinators (x # xs) ⇒
  FWNormalisationCore.sorted (qsort [y←xs . ¬ smaller x y l] l) l ∧
  FWNormalisationCore.sorted (x # qsort [y←xs . smaller x y l] l) l ∧

```

$(\forall x' \in \text{set } (\text{qsort } [y \leftarrow xs . \neg \text{smaller } x \ y \ l] \ l).$
 $\forall y \in \text{set } (x \ \# \ \text{qsort } [y \leftarrow xs . \text{smaller } x \ y \ l] \ l). \text{smaller } x' \ y \ l)$

apply (*auto*)[1]
apply (*metis* (*mono-tags*, *lifting*) *SC-sublist all-in-list.simps(2)*
all-in-list-sublist aux0-1 mem-Collect-eq set-filter set-qsort
singleCombinatorsConc singleCombinatorsStart sorted-Consb)
apply (*metis* *aux0-0 aux0-6 in-set-in-list singleCombinatorsConc*
singleCombinatorsStart smaller-antisym)
by (*metis* (*no-types*, *lifting*) *NormalisationGenericProofs.order-trans aux0-0*
aux0-6 in-set-in-list
singleCombinatorsConc singleCombinatorsStart smaller-antisym)
qed
qed

lemma *inSet-not-MT*: $a \in \text{set } p \implies p \neq []$
by *auto*

lemma *RS1n-assoc*:
 $x \neq \text{DenyAll} \implies \text{removeShadowRules1-alternative } xs \ @ \ [x] =$
 $\text{removeShadowRules1-alternative } (xs \ @ \ [x])$
by (*simp add: removeShadowRules1-alternative-def aux114*)

lemma *RS1n-nMT[rule-format,simp]*: $p \neq [] \longrightarrow \text{removeShadowRules1-alternative } p \neq []$
apply (*simp add: removeShadowRules1-alternative-def*)
apply (*rule-tac xs = p in rev-induct, simp-all*)
subgoal for $x \ xs$
apply (*case-tac xs = [], simp-all*)
apply (*case-tac x, simp-all*)
apply (*rule-tac xs = xs in rev-induct, simp-all*)
apply (*case-tac x, simp-all*)
done
done

lemma *RS1N-DA[simp]*: $\text{removeShadowRules1-alternative } (a \ @ \ [\text{DenyAll}]) = [\text{DenyAll}]$
by (*simp add: removeShadowRules1-alternative-def*)

lemma *WP1n-DA-notinSet[rule-format]*: $\text{wellformed-policy1-strong } p \longrightarrow$
 $\text{DenyAll} \notin \text{set } (tl \ p)$
by (*induct p*) (*simp-all*)

lemma *mt-sym*: $\text{dom } a \cap \text{dom } b = \{\} \implies \text{dom } b \cap \text{dom } a = \{\}$
by *auto*

lemma *DAnotTL*[rule-format]:
 $xs \neq [] \longrightarrow \text{wellformed-policy1 } (xs @ [DenyAll]) \longrightarrow \text{False}$
by (*induct xs, simp-all*)

lemma *AND-tl*[rule-format]: $\text{allNetsDistinct } (p) \longrightarrow \text{allNetsDistinct } (tl\ p)$
apply (*induct p, simp-all*)
by (*auto intro: ANDConc*)

lemma *distinct-tl*[rule-format]: $\text{distinct } p \longrightarrow \text{distinct } (tl\ p)$
by (*induct p, simp-all*)

lemma *SC-tl*[rule-format]: $\text{singleCombinators } (p) \longrightarrow \text{singleCombinators } (tl\ p)$
by (*induct p, simp-all*) (*auto intro: singleCombinatorsConc*)

lemma *Conc-not-MT*: $p = x\#xs \implies p \neq []$
by *auto*

lemma *wp1-tl*[rule-format]:
 $p \neq [] \wedge \text{wellformed-policy1 } p \longrightarrow \text{wellformed-policy1 } (tl\ p)$
by (*induct p*) (*auto intro: waux2*)

lemma *wp1-eq*[rule-format]:
 $\text{wellformed-policy1-strong } p \implies \text{wellformed-policy1 } p$
apply (*case-tac DenyAll \in set p*)
apply (*subst wellformed-eq*)
apply (*auto elim: waux2*)
done

lemma *wellformed1-alternative-sorted*:
 $\text{wellformed-policy1-strong } p \implies \text{wellformed-policy1-strong } (\text{sort } p\ l)$
by (*case-tac p, simp-all*)

lemma *wp1n-RS2*[rule-format]:
 $\text{wellformed-policy1-strong } p \longrightarrow \text{wellformed-policy1-strong } (\text{removeShadowRules2 } p)$
by (*induct p, simp-all*)

lemma *RS2-NMT*[rule-format]: $p \neq [] \longrightarrow \text{removeShadowRules2 } p \neq []$
apply (*induct p, simp-all*)
subgoal for *a p*
apply (*case-tac p \neq [], simp-all*)
apply (*case-tac a, simp-all*)
done

done

lemma *wp1-alternative-not-mt[simp]*: *wellformed-policy1-strong* $p \implies p \neq []$
by *auto*

lemma *AIL1[rule-format,simp]*: *all-in-list* $p\ l \longrightarrow$
all-in-list (*removeShadowRules1* p) l
by (*induct-tac* p , *simp-all*)

lemma *wp1ID*: *wellformed-policy1-strong* (*insertDeny* (*removeShadowRules1* p))
apply (*induct* p , *simp-all*)
subgoal for $a\ p$
apply (*case-tac* a , *simp-all*)
done
done

lemma *dRD[simp]*: *distinct* (*remdups* p)
by *simp*

lemma *AILrd[rule-format,simp]*: *all-in-list* $p\ l \longrightarrow$ *all-in-list* (*remdups* p) l
by (*induct* p , *simp-all*)

lemma *AILiD[rule-format,simp]*: *all-in-list* $p\ l \longrightarrow$ *all-in-list* (*insertDeny* p) l
apply (*induct* p , *simp-all*)
apply (*rule impI*, *simp*)
subgoal for $a\ p$
apply (*case-tac* a , *simp-all*)
done
done

lemma *SCRd[rule-format,simp]*: *singleCombinators* $p \longrightarrow$ *singleCombinators*(*remdups* p)
apply (*induct* p , *simp-all*)
subgoal for $a\ p$
apply (*case-tac* a , *simp-all*)
done
done

lemma *SCRiD[rule-format,simp]*: *singleCombinators* $p \longrightarrow$
singleCombinators(*insertDeny* p)
apply (*induct* p , *simp-all*)
subgoal for $a\ p$
apply (*case-tac* a , *simp-all*)
done
done

lemma *WP1rd*[*rule-format,simp*]:

wellformed-policy1-strong $p \longrightarrow$ *wellformed-policy1-strong* (*remdups* p)
by (*induct* p , *simp-all*)

lemma *ANDrd*[*rule-format,simp*]:

singleCombinators $p \longrightarrow$ *allNetsDistinct* $p \longrightarrow$ *allNetsDistinct* (*remdups* p)
apply (*rule impI*)
apply (*rule-tac* $b = p$ **in** *aNDSubset*)
apply *simp-all*
done

lemma *ANDiD*[*rule-format,simp*]:

allNetsDistinct $p \longrightarrow$ *allNetsDistinct* (*insertDeny* p)
apply (*induct* p , *simp-all*)
apply (*simp add: allNetsDistinct-def*)
apply (*auto intro: ANDConc*)
subgoal for a p
apply (*case-tac* a ,*simp-all add: allNetsDistinct-def*)
done
done

lemma *mr-iD*[*rule-format*]:

wellformed-policy1-strong $p \longrightarrow$ *matching-rule* x $p =$ *matching-rule* x (*insertDeny* p)
by (*induct* p , *simp-all*)

lemma *WP1iD*[*rule-format,simp*]: *wellformed-policy1-strong* $p \longrightarrow$

wellformed-policy1-strong (*insertDeny* p)
by (*induct* p , *simp-all*)

lemma *DAiniD*: *DenyAll* \in *set* (*insertDeny* p)

apply (*induct* p , *simp-all*)
subgoal for a p
apply(*case-tac* a , *simp-all*)
done
done

lemma *p2lNmt*: *policy2list* $p \neq []$

by (*rule policy2list.induct*, *simp-all*)

lemma *AIL2*[*rule-format,simp*]:

all-in-list p $l \longrightarrow$ *all-in-list* (*removeShadowRules2* p) l
apply (*induct-tac* p , *simp-all*)
subgoal for a p

```

apply(case-tac a, simp-all)
done
done

```

```

lemma SCConc: singleCombinators x  $\implies$  singleCombinators y  $\implies$  singleCombinators
(x@y)
apply (rule aux0-5)
apply (metis aux0-0 aux0-4)
done

```

```

lemma SCp2l: singleCombinators (policy2list p)
by (induct-tac p) (auto intro: SCConc)

```

```

lemma subnetAux:  $Dd \cap A \neq \{\}$   $\implies A \subseteq B \implies Dd \cap B \neq \{\}$ 
by auto

```

```

lemma soadisj:  $x \in \text{subnetsOfAdr } a \implies y \in \text{subnetsOfAdr } a \implies \neg \text{netsDistinct } x y$ 
by(simp add: subnetsOfAdr-def netsDistinct-def, auto)

```

```

lemma not-member:  $\neg \text{member } a (x \oplus y) \implies \neg \text{member } a x$ 
by auto

```

```

lemma soadisj2:  $(\forall a x y. x \in \text{subnetsOfAdr } a \wedge y \in \text{subnetsOfAdr } a \longrightarrow \neg \text{netsDistinct } x y)$ 
by (simp add: subnetsOfAdr-def netsDistinct-def, auto)

```

```

lemma ndFalse1:
 $(\forall a b c d. (a,b) \in A \wedge (c,d) \in B \longrightarrow \text{netsDistinct } a c) \implies$ 
 $\exists (a, b) \in A. a \in \text{subnetsOfAdr } D \implies$ 
 $\exists (a, b) \in B. a \in \text{subnetsOfAdr } D \implies \text{False}$ 
apply (auto simp: soadisj)
using soadisj2 by blast

```

```

lemma ndFalse2:  $(\forall a b c d. (a,b) \in A \wedge (c,d) \in B \longrightarrow \text{netsDistinct } b d) \implies$ 
 $\exists (a, b) \in A. b \in \text{subnetsOfAdr } D \implies$ 
 $\exists (a, b) \in B. b \in \text{subnetsOfAdr } D \implies \text{False}$ 
apply (auto simp: soadisj)
using soadisj2 by blast

```

```

lemma tndFalse:  $(\forall a b c d. (a,b) \in A \wedge (c,d) \in B \longrightarrow \text{twoNetsDistinct } a b c d) \implies$ 
 $\exists (a, b) \in A. a \in \text{subnetsOfAdr } (D::('a::\text{adr})) \wedge b \in \text{subnetsOfAdr } (F::'a) \implies$ 
 $\exists (a, b) \in B. a \in \text{subnetsOfAdr } D \wedge b \in \text{subnetsOfAdr } F$ 
 $\implies \text{False}$ 
apply (simp add: twoNetsDistinct-def)

```

```

apply (auto simp: ndFalse1 ndFalse2)
apply (metis soadisj)
done

```

```

lemma sepnMT[rule-format]:  $p \neq [] \longrightarrow (\text{separate } p) \neq []$ 
by (induct p rule: separate.induct) simp-all

```

```

lemma sepDA[rule-format]:  $\text{DenyAll} \notin \text{set } p \longrightarrow \text{DenyAll} \notin \text{set } (\text{separate } p)$ 
by (induct p rule: separate.induct) simp-all

```

```

lemma setnMT:  $\text{set } a = \text{set } b \implies a \neq [] \implies b \neq []$ 
by auto

```

```

lemma sortnMT:  $p \neq [] \implies \text{sort } p \ l \neq []$ 
by (metis set-sort setnMT)

```

```

lemma idNMT[rule-format]:  $p \neq [] \longrightarrow \text{insertDenies } p \neq []$ 
apply (induct p, simp-all)
subgoal for a p
  apply(case-tac a, simp-all)
  done
done

```

```

lemma OTNoTN[rule-format]:  $\text{OnlyTwoNets } p \longrightarrow x \neq \text{DenyAll} \longrightarrow x \in \text{set } p \longrightarrow$ 
 $\text{onlyTwoNets } x$ 
apply (induct p, simp-all, rename-tac a p)
apply (intro impI conjI, simp)
subgoal for a p
  apply(case-tac a, simp-all)
  done
subgoal for a p
  apply(case-tac a, simp-all)
  done
done

```

```

lemma first-isIn[rule-format]:  $\neg \text{member } \text{DenyAll } x \longrightarrow (\text{first-srcNet } x, \text{first-destNet } x)$ 
 $\in \text{sdnets } x$ 
by (induct x, case-tac x, simp-all)

```

```

lemma sdnets2:
   $\exists a \ b. \text{sdnets } x = \{(a, b), (b, a)\} \implies \neg \text{member } \text{DenyAll } x \implies$ 
 $\text{sdnets } x = \{(\text{first-srcNet } x, \text{first-destNet } x), (\text{first-destNet } x, \text{first-srcNet } x)\}$ 
proof -
  have * :  $\exists a \ b. \text{sdnets } x = \{(a, b), (b, a)\} \implies \neg \text{member } \text{DenyAll } x$ 

```

$\implies (first\text{-}srcNet\ x, first\text{-}destNet\ x) \in sdnets\ x$
by (*erule first-isIn*)
show $\exists a\ b. sdnets\ x = \{(a, b), (b, a)\} \implies \neg member\ DenyAll\ x \implies$
 $sdnets\ x = \{(first\text{-}srcNet\ x, first\text{-}destNet\ x), (first\text{-}destNet\ x, first\text{-}srcNet\ x)\}$
using * **by** *auto*
qed

lemma *alternativelistconc1*[*rule-format*]:
 $a \in set\ (net\text{-}list\text{-}aux\ [x]) \longrightarrow a \in set\ (net\text{-}list\text{-}aux\ [x,y])$
by (*induct x, simp-all*)

lemma *alternativelistconc2*[*rule-format*]:
 $a \in set\ (net\text{-}list\text{-}aux\ [x]) \longrightarrow a \in set\ (net\text{-}list\text{-}aux\ [y,x])$
by (*induct y, simp-all*)

lemma *noDA*[*rule-format*]:
 $noDenyAll\ xs \longrightarrow s \in set\ xs \longrightarrow \neg member\ DenyAll\ s$
by (*induct xs, simp-all*)

lemma *isInAlternativeList*:
 $(aa \in set\ (net\text{-}list\text{-}aux\ [a]) \vee aa \in set\ (net\text{-}list\text{-}aux\ p)) \implies aa \in set\ (net\text{-}list\text{-}aux\ (a\ \# p))$
by (*case-tac a, simp-all*)

lemma *netlistaux*:
 $x \in set\ (net\text{-}list\text{-}aux\ (a\ \# p)) \implies x \in set\ (net\text{-}list\text{-}aux\ ([a]) \vee x \in set\ (net\text{-}list\text{-}aux\ p))$
apply (*case-tac x \in set (net-list-aux [a]), simp-all*)
apply (*case-tac a, simp-all*)
done

lemma *firstInNet*[*rule-format*]:
 $\neg member\ DenyAll\ a \longrightarrow first\text{-}destNet\ a \in set\ (net\text{-}list\text{-}aux\ (a\ \# p))$
apply (*rule Combinators.induct, simp-all*)
apply (*metis netlistaux*)
done

lemma *firstInNeta*[*rule-format*]:
 $\neg member\ DenyAll\ a \longrightarrow first\text{-}srcNet\ a \in set\ (net\text{-}list\text{-}aux\ (a\ \# p))$
apply (*rule Combinators.induct, simp-all*)
apply (*metis netlistaux*)
done

lemma *disjComm*: $disjSD-2\ a\ b \implies disjSD-2\ b\ a$
apply (*simp add: disjSD-2-def*)
apply (*intro allI impI conjI*)
using *tNDComm apply blast*
by (*meson tNDComm twoNetsDistinct-def*)

lemma *disjSD2aux*:
 $disjSD-2\ a\ b \implies \neg\ member\ DenyAll\ a \implies \neg\ member\ DenyAll\ b \implies$
 $disjSD-2\ (DenyAllFromTo\ (first-srcNet\ a)\ (first-destNet\ a) \oplus$
 $DenyAllFromTo\ (first-destNet\ a)\ (first-srcNet\ a) \oplus a)$
 b
apply (*drule disjComm, rule disjComm*)
apply (*simp add: disjSD-2-def*)
using *first-isIn by blast*

lemma *noDA1eq*[*rule-format*]: $noDenyAll\ p \longrightarrow noDenyAll1\ p$
apply (*induct p, simp, rename-tac a p*)
subgoal for *a p*
apply(*case-tac a, simp-all*)
done
done

lemma *noDA1C*[*rule-format*]: $noDenyAll1\ (a\#\ p) \longrightarrow noDenyAll1\ p$
by (*case-tac a, simp-all, rule impI, rule noDA1eq, simp*)**+**

lemma *disjSD-2IDa*:
 $disjSD-2\ x\ y \implies$
 $\neg\ member\ DenyAll\ x \implies$
 $\neg\ member\ DenyAll\ y \implies$
 $a = first-srcNet\ x \implies$
 $b = first-destNet\ x \implies$
 $disjSD-2\ (DenyAllFromTo\ a\ b \oplus DenyAllFromTo\ b\ a \oplus x)\ y$
by(*simp add:disjSD2aux*)

lemma *noDAID*[*rule-format*]: $noDenyAll\ p \longrightarrow noDenyAll\ (insertDenies\ p)$
apply (*induct p, simp-all*)
subgoal for *a p*
apply(*case-tac a, simp-all*)
done
done

lemma *isInIDo*[*rule-format*]:
 $noDenyAll\ p \longrightarrow s \in set\ (insertDenies\ p) \longrightarrow$
 $(\exists!\ a.\ s = (DenyAllFromTo\ (first-srcNet\ a)\ (first-destNet\ a)) \oplus$

$(DenyAllFromTo (first-destNet a) (first-srcNet a)) \oplus a \wedge a \in set p)$
apply (*induct p, simp, rename-tac a p*)
subgoal for a p
apply (*case-tac a = DenyAll, simp*)
apply (*case-tac a, auto*)
done
done

lemma *id-aux1*[*rule-format*]: $DenyAllFromTo (first-srcNet s) (first-destNet s) \oplus$
 $DenyAllFromTo (first-destNet s) (first-srcNet s) \oplus s \in set (insertDenies p)$
 $\longrightarrow s \in set p$
apply (*induct p, simp-all, rename-tac a p*)
subgoal for a p
apply(*case-tac a, simp-all*)
done
done

lemma *id-aux2*:
 $noDenyAll p \implies$
 $\forall s. s \in set p \longrightarrow disjSD-2 a s \implies$
 $\neg member DenyAll a \implies$
 $DenyAllFromTo (first-srcNet s) (first-destNet s) \oplus$
 $DenyAllFromTo (first-destNet s) (first-srcNet s) \oplus s \in set (insertDenies p) \implies$
 $disjSD-2 a (DenyAllFromTo (first-srcNet s) (first-destNet s) \oplus$
 $DenyAllFromTo (first-destNet s) (first-srcNet s) \oplus s)$
by (*metis disjComm disjSD2aux isInIDo noDA*)

lemma *id-aux4*[*rule-format*]:
 $noDenyAll p \implies \forall s. s \in set p \longrightarrow disjSD-2 a s \implies$
 $s \in set (insertDenies p) \implies \neg member DenyAll a \implies$
 $disjSD-2 a s$
apply (*subgoal-tac $\exists a. s =$*
 $DenyAllFromTo (first-srcNet a) (first-destNet a) \oplus$
 $DenyAllFromTo (first-destNet a) (first-srcNet a) \oplus a \wedge$
 $a \in set p)$
apply (*drule-tac Q = disjSD-2 a s in exE, simp-all, rule id-aux2, simp-all*)
using *isInIDo* **by** *blast*

lemma *sepNetsID*[*rule-format*]:
 $noDenyAll1 p \longrightarrow separated p \longrightarrow separated (insertDenies p)$
apply (*induct p, simp*)
apply (*rename-tac a p, auto*)
using *noDA1C* **apply** *blast*
subgoal for a p

apply (*case-tac a = DenyAll, auto*)
apply (*simp add: disjSD-2-def*)
apply (*case-tac a, auto*)
apply (*rule disjSD-2IDa, simp-all, rule id-aux4, simp-all, metis noDA noDAID*)+
done
done

lemma *aNDDA*[*rule-format*]: $allNetsDistinct\ p \longrightarrow allNetsDistinct(DenyAll\ \#p)$
by (*case-tac p, auto simp: allNetsDistinct-def*)

lemma *OTNConc*[*rule-format*]: $OnlyTwoNets\ (y\ \#z) \longrightarrow OnlyTwoNets\ z$
by (*case-tac y, simp-all*)

lemma *first-bothNetsd*: $\neg member\ DenyAll\ x \Longrightarrow first-bothNet\ x = \{first-srcNet\ x, first-destNet\ x\}$
by (*induct x*) *simp-all*

lemma *bNaux*:
 $\neg member\ DenyAll\ x \Longrightarrow \neg member\ DenyAll\ y \Longrightarrow$
 $first-bothNet\ x = first-bothNet\ y \Longrightarrow$
 $\{first-srcNet\ x, first-destNet\ x\} = \{first-srcNet\ y, first-destNet\ y\}$
by (*simp add: first-bothNetsd*)

lemma *setPair*: $\{a, b\} = \{a, d\} \Longrightarrow b = d$
by (*metis setPaireq*)

lemma *setPair1*: $\{a, b\} = \{d, a\} \Longrightarrow b = d$
by (*metis Un-empty-right Un-insert-right insert-absorb2 setPaireq*)

lemma *setPair4*: $\{a, b\} = \{c, d\} \Longrightarrow a \neq c \Longrightarrow a = d$
by *auto*

lemma *otnaux1*: $\{x, y, x, y\} = \{x, y\}$
by *auto*

lemma *OTNIDaux4*: $\{x, y, x\} = \{y, x\}$
by *auto*

lemma *setPair5*: $\{a, b\} = \{c, d\} \Longrightarrow a \neq c \Longrightarrow a = d$
by *auto*

lemma *otnaux*:
 $\llbracket first-bothNet\ x = first-bothNet\ y; \neg member\ DenyAll\ x; \neg member\ DenyAll\ y;$
 $onlyTwoNets\ y; onlyTwoNets\ x \rrbracket \Longrightarrow$

```

onlyTwoNets ( $x \oplus y$ )
apply (simp add: onlyTwoNets-def)
apply (subgoal-tac {first-srcNet  $x$ , first-destNet  $x$ } =
           {first-srcNet  $y$ , first-destNet  $y$ }})
apply (case-tac ( $\exists a b. \textit{sdnets } y = \{(a, b)\}$ ))
apply simp-all
apply (case-tac ( $\exists a b. \textit{sdnets } x = \{(a, b)\}$ ))
apply simp-all
apply (subgoal-tac sdnets  $x = \{(first-srcNet } x, first-destNet } x)\}$ )
apply (subgoal-tac sdnets  $y = \{(first-srcNet } y, first-destNet } y)\}$ )
apply simp
apply (case-tac first-srcNet  $x = first-srcNet } y$ )
apply simp-all
apply (rule disjI1)
apply (rule setPair)
apply simp
apply (subgoal-tac first-srcNet  $x = first-destNet } y$ )
apply simp
apply (subgoal-tac first-destNet  $x = first-srcNet } y$ )
apply simp
apply (rule-tac  $x = first-srcNet } y$  in exI, rule-tac  $x = first-destNet } y$  in exI, simp)
apply (rule setPair1)
apply simp
apply (rule setPair4)
apply simp-all
apply (metis first-isIn singletonE)
apply (metis first-isIn singletonE)
apply (subgoal-tac sdnets  $x = \{(first-srcNet } x, first-destNet } x),$ 
           (first-destNet } x, first-srcNet } x)\})
apply (subgoal-tac sdnets  $y = \{(first-srcNet } y, first-destNet } y)\}$ )
apply simp
apply (case-tac first-srcNet  $x = first-srcNet } y$ )
apply simp-all
apply (subgoal-tac first-destNet  $x = first-destNet } y$ )
apply simp
apply (rule setPair)
apply simp
apply (subgoal-tac first-srcNet  $x = first-destNet } y$ )
apply simp
apply (subgoal-tac first-destNet  $x = first-srcNet } y$ )
apply simp
apply (rule-tac  $x = first-srcNet } y$  in exI, rule-tac  $x = first-destNet } y$  in exI)
apply (metis OTNIDaux4 insert-commute )
apply (rule setPair1)

```

```

    apply simp
  apply (rule setPair5)
  apply assumption
  apply simp
  apply (metis first-isIn singletonE)
  apply (rule sdnets2)
  apply simp-all
  apply (case-tac ( $\exists a b. \text{sdnets } x = \{(a, b)\}$ ))
  apply simp-all
  apply (subgoal-tac  $\text{sdnets } x = \{(\text{first-srcNet } x, \text{first-destNet } x)\}$ )
  apply (subgoal-tac  $\text{sdnets } y = \{(\text{first-srcNet } y, \text{first-destNet } y),$ 
     $(\text{first-destNet } y, \text{first-srcNet } y)\}$ )

  apply simp
  apply (case-tac  $\text{first-srcNet } x = \text{first-srcNet } y$ )
  apply simp-all
  apply (subgoal-tac  $\text{first-destNet } x = \text{first-destNet } y$ )
  apply simp
  apply (rule-tac  $x = \text{first-srcNet } y$  in  $exI$ , rule-tac  $x = \text{first-destNet } y$  in  $exI$ )
  apply (metis OTNIDaux4 insert-commute )
  apply (rule setPair)
  apply simp
  apply (subgoal-tac  $\text{first-srcNet } x = \text{first-destNet } y$ )
  apply simp
  apply (subgoal-tac  $\text{first-destNet } x = \text{first-srcNet } y$ )
  apply simp
  apply (rule setPair1)
  apply simp
  apply (rule setPair4)
  apply assumption
  apply simp
  apply (rule sdnets2)
  apply simp
  apply simp
  apply (metis singletonE first-isIn)
  apply (subgoal-tac  $\text{sdnets } x = \{(\text{first-srcNet } x, \text{first-destNet } x),$ 
     $(\text{first-destNet } x, \text{first-srcNet } x)\}$ )
  apply (subgoal-tac  $\text{sdnets } y = \{(\text{first-srcNet } y, \text{first-destNet } y),$ 
     $(\text{first-destNet } y, \text{first-srcNet } y)\}$ )

  apply simp
  apply (case-tac  $\text{first-srcNet } x = \text{first-srcNet } y$ )
  apply simp-all
  apply (subgoal-tac  $\text{first-destNet } x = \text{first-destNet } y$ )
  apply simp
  apply (rule-tac  $x = \text{first-srcNet } y$  in  $exI$ , rule-tac  $x = \text{first-destNet } y$  in  $exI$ )

```

```

  apply (rule otnaux1)
  apply (rule setPair)
  apply simp
  apply (subgoal-tac first-srcNet x = first-destNet y)
  apply simp
  apply (subgoal-tac first-destNet x = first-srcNet y)
  apply simp
  apply (rule-tac x =first-srcNet y in exI, rule-tac x = first-destNet y in exI)
  apply (metis OTNIDaux4 insert-commute)
  apply (rule setPair1)
  apply simp
  apply (rule setPair4)
  apply assumption
  apply simp
  apply (rule sdnets2,simp-all)+
  apply (rule bNaux, simp-all)
done

```

lemma *OTNSepaux*:

```

  onlyTwoNets (a ⊕ y) ∧ OnlyTwoNets z → OnlyTwoNets (separate (a ⊕ y # z)) ⇒
  ¬ member DenyAll a ⇒ ¬ member DenyAll y ⇒
  noDenyAll z ⇒ onlyTwoNets a ⇒ OnlyTwoNets (y # z) ⇒ first-bothNet a =
first-bothNet y ⇒
  OnlyTwoNets (separate (a ⊕ y # z))
  apply (drule mp)
  apply simp-all
  apply (rule conjI)
  apply (rule otnaux)
  apply simp-all
  apply (rule-tac p = (y # z) in OTNoTN)
  apply simp-all
  apply (metis member.simps(2))
  apply (simp add: onlyTwoNets-def)
  apply (rule-tac y = y in OTNConc,simp)
done

```

lemma *OTNSEp[rule-format]*:

```

  noDenyAll1 p → OnlyTwoNets p → OnlyTwoNets (separate p)
  apply (induct p rule: separate.induct)
  by (simp-all add: OTNSepaux noDA1eq)

```

lemma *nda[rule-format]*:

```

  singleCombinators (a#p) → noDenyAll p → noDenyAll1 (a # p)
  apply (induct p,simp-all)

```

apply (*case-tac a, simp-all*)+
done

lemma *nDAcharn*[*rule-format*]: $\text{noDenyAll } p = (\forall r \in \text{set } p. \neg \text{member } \text{DenyAll } r)$
by (*induct p, simp-all*)

lemma *nDAeqSet*: $\text{set } p = \text{set } s \implies \text{noDenyAll } p = \text{noDenyAll } s$
by (*simp add: nDAcharn*)

lemma *nDASCaux*[*rule-format*]:
 $\text{DenyAll } \notin \text{set } p \longrightarrow \text{singleCombinators } p \longrightarrow r \in \text{set } p \longrightarrow \neg \text{member } \text{DenyAll } r$
apply (*case-tac r, simp-all*)
using *SCnotConc* **by** *blast*

lemma *nDASC*[*rule-format*]:
 $\text{wellformed-policy1 } p \longrightarrow \text{singleCombinators } p \longrightarrow \text{noDenyAll1 } p$
apply (*induct p, simp-all*)
using *nDASCaux nDAcharn nda singleCombinatorsConc* **by** *blast*

lemma *noDAAll*[*rule-format*]: $\text{noDenyAll } p = (\neg \text{memberP } \text{DenyAll } p)$
by (*induct p*) *simp-all*

lemma *memberPsep*[*symmetric*]: $\text{memberP } x p = \text{memberP } x (\text{separate } p)$
by (*induct p rule: separate.induct*) *simp-all*

lemma *noDAsep*[*rule-format*]: $\text{noDenyAll } p \implies \text{noDenyAll } (\text{separate } p)$
by (*simp add: noDAAll, subst memberPsep, simp*)

lemma *noDA1sep*[*rule-format*]: $\text{noDenyAll1 } p \longrightarrow \text{noDenyAll1 } (\text{separate } p)$
by (*induct p rule: separate.induct, simp-all add: noDAsep*)

lemma *isInAlternativeLista*:
 $(aa \in \text{set } (\text{net-list-aux } [a])) \implies aa \in \text{set } (\text{net-list-aux } (a \# p))$
by (*case-tac a, auto*)

lemma *isInAlternativeListb*:
 $(aa \in \text{set } (\text{net-list-aux } p)) \implies aa \in \text{set } (\text{net-list-aux } (a \# p))$
by (*case-tac a, simp-all*)

lemma *ANDSepaux*: $\text{allNetsDistinct } (x \# y \# z) \implies \text{allNetsDistinct } (x \oplus y \# z)$
apply (*simp add: allNetsDistinct-def*)
apply (*intro allI impI, rename-tac a b*)
subgoal for *a b*

```

apply (drule-tac  $x = a$  in spec, drule-tac  $x = b$  in spec)
by (meson isInAlternativeList)
done

```

lemma *netlistalternativeSeparateaux*:

```

net-list-aux [y] @ net-list-aux  $z = \text{net-list-aux } (y \# z)$ 
by (case-tac y, simp-all)

```

lemma *netlistalternativeSeparate*: *net-list-aux* $p = \text{net-list-aux } (\text{separate } p)$

```

by (induct p rule:separate.induct, simp-all) (simp-all add: netlistalternativeSeparateaux)

```

lemma *ANDSepaux2*:

```

allNetsDistinct( $x \# y \# z$ )  $\implies \text{allNetsDistinct}(\text{separate}(y \# z)) \implies \text{allNetsDistinct}(x \# \text{separate}(y \# z))$ 

```

```

apply (simp add: allNetsDistinct-def)

```

```

by (metis isInAlternativeList netlistalternativeSeparate netlistaux)

```

lemma *ANDSep[rule-format]*: *allNetsDistinct* $p \longrightarrow \text{allNetsDistinct}(\text{separate } p)$

```

apply (induct p rule: separate.induct, simp-all)

```

```

apply (metis ANDConc aNDDA)

```

```

apply (metis ANDConc ANDSepaux ANDSepaux2)

```

```

apply (metis ANDConc ANDSepaux ANDSepaux2)

```

```

apply (metis ANDConc ANDSepaux ANDSepaux2)

```

```

done

```

lemma *wp1-alternativesep[rule-format]*:

```

wellformed-policy1-strong  $p \longrightarrow \text{wellformed-policy1-strong } (\text{separate } p)$ 

```

```

by (metis sepDA separate.simps(1) wellformed-policy1-strong.simps(2) wp1n-tl)

```

lemma *noDAsort[rule-format]*: *noDenyAll1* $p \longrightarrow \text{noDenyAll1 } (\text{sort } p \ l)$

```

apply (case-tac p, simp-all)

```

```

subgoal for a as

```

```

apply (case-tac  $a = \text{DenyAll}$ , auto)

```

```

using NormalisationGenericProofs.set-sort nDAeqSet apply blast

```

```

proof –

```

```

fix a::('a,'b)Combinators fix list

```

```

have  $* : a \neq \text{DenyAll} \implies \text{noDenyAll1 } (a \# \text{list}) \implies \text{noDenyAll } (a \# \text{list})$  by
(case-tac a, simp-all)

```

```

show  $a \neq \text{DenyAll} \implies \text{noDenyAll1 } (a \# \text{list}) \implies \text{noDenyAll1 } (\text{insort } a (\text{sort } \text{list } l) \ l)$ 

```

```

apply(cases insort  $a (\text{sort } \text{list } l) \ l$ , simp-all)

```

```

by (metis  $*$  NormalisationGenericProofs.set-insort NormalisationGener-)

```


icProofs.set-sort

list.simps(15) nDAeqSet noDA1eq

qed
done

lemma *OTNSC*[*rule-format*]: *singleCombinators p* \longrightarrow *OnlyTwoNets p*

apply (*induct p,simp-all*)
apply (*rename-tac a p*)
apply (*rule impI,drule mp*)
apply (*erule singleCombinatorsConc*)
subgoal for a b
apply (*case-tac a, simp-all*)
apply (*simp add: onlyTwoNets-def*)
done
done

lemma *fMTaux*: \neg *member DenyAll x* \implies *first-bothNet x* \neq $\{\}$

by (*metis first-bothNetsd insert-commute insert-not-empty*)

lemma *fl2*[*rule-format*]: *firstList (separate p)* = *firstList p*

by (*rule separate.induct*) *simp-all*

lemma *fl3*[*rule-format*]: *NetsCollected p* \longrightarrow (*first-bothNet x* \neq *firstList p* \longrightarrow

$(\forall a \in \text{set } p. \text{first-bothNet } x \neq \text{first-bothNet } a) \longrightarrow$ *NetsCollected (x\#p)*

by (*induct p*) *simp-all*

lemma *sortedConc*[*rule-format*]: *sorted (a \# p) l* \longrightarrow *sorted p l*

by (*induct p*) *simp-all*

lemma *smalleraux2*:

$\{a,b\} \in \text{set } l \implies \{c,d\} \in \text{set } l \implies \{a,b\} \neq \{c,d\} \implies$
smaller (DenyAllFromTo a b) (DenyAllFromTo c d) l \implies
 \neg *smaller (DenyAllFromTo c d) (DenyAllFromTo a b) l*
by (*metis bothNet.simps(2) pos-noteq smaller.simps(5)*)

lemma *smalleraux2a*:

$\{a,b\} \in \text{set } l \implies \{c,d\} \in \text{set } l \implies \{a,b\} \neq \{c,d\} \implies$
smaller (DenyAllFromTo a b) (AllowPortFromTo c d p) l \implies
 \neg *smaller (AllowPortFromTo c d p) (DenyAllFromTo a b) l*
by (*simp*) (*metis eq-imp-le pos-noteq*)

lemma *smalleraux2b*:

$\{a,b\} \in \text{set } l \implies \{c,d\} \in \text{set } l \implies \{a,b\} \neq \{c,d\} \implies y = \text{DenyAllFromTo } a \ b \implies$

$smaller (AllowPortFromTo\ c\ d\ p)\ y\ l \implies$
 $\neg\ smaller\ y\ (AllowPortFromTo\ c\ d\ p)\ l$
by (*simp*) (*metis eq-imp-le pos-noteq*)

lemma *smalleraux2c*:

$\{a,b\} \in set\ l \implies \{c,d\} \in set\ l \implies \{a,b\} \neq \{c,d\} \implies y = AllowPortFromTo\ a\ b\ q \implies$
 $smaller (AllowPortFromTo\ c\ d\ p)\ y\ l \implies \neg\ smaller\ y\ (AllowPortFromTo\ c\ d\ p)\ l$
by (*simp*) (*metis pos-noteq*)

lemma *smalleraux3*:

assumes $x \in set\ l$ **and** $y \in set\ l$ **and** $x \neq y$ **and** $x = bothNet\ a$ **and** $y = bothNet\ b$
and $smaller\ a\ b\ l$ **and** *singleCombinators* [a] **and** *singleCombinators* [b]
shows $\neg\ smaller\ b\ a\ l$
proof (*cases a*)
case *DenyAll* **thus** ?thesis **using** *assms* **by** (*case-tac b, simp-all*)
next
case (*DenyAllFromTo c d*) **thus** ?thesis
proof (*cases b*)
case *DenyAll* **thus** ?thesis **using** *assms DenyAll DenyAllFromTo* **by** *simp*
next
case (*DenyAllFromTo e f*) **thus** ?thesis **using** *assms DenyAllFromTo*
by (*metis DenyAllFromTo* $\langle a = DenyAllFromTo\ c\ d \rangle$ *bothNet.simps(2)* *smaller-
aux2*)
next
case (*AllowPortFromTo e f g*) **thus** ?thesis
using *assms DenyAllFromTo AllowPortFromTo* **by** *simp* (*metis eq-imp-le pos-noteq*)
next
case (*Conc e f*) **thus** ?thesis **using** *assms* **by** *simp*
qed
next
case (*AllowPortFromTo c d p*) **thus** ?thesis
proof (*cases b*)
case *DenyAll* **thus** ?thesis **using** *assms AllowPortFromTo DenyAll* **by** *simp*
next
case (*DenyAllFromTo e f*) **thus** ?thesis
using *assms* **by** *simp* (*metis AllowPortFromTo DenyAllFromTo bothNet.simps(3)*
smalleraux2a)
next
case (*AllowPortFromTo e f g*) **thus** ?thesis
using *assms* **by** (*simp*) (*metis AllowPortFromTo* $\langle a = AllowPortFromTo\ c\ d \rangle$
bothNet.simps(3) *smalleraux2c*)
next
case (*Conc e f*) **thus** ?thesis **using** *assms* **by** *simp*
qed

next
case (*Conc c d*) **thus** *?thesis using assms by simp*
qed

lemma *smalleraux3a*:

$a \neq \text{DenyAll} \implies b \neq \text{DenyAll} \implies \text{in-list } b \ l \implies \text{in-list } a \ l \implies$
 $\text{bothNet } a \neq \text{bothNet } b \implies \text{smaller } a \ b \ l \implies \text{singleCombinators } [a] \implies$
 $\text{singleCombinators } [b] \implies \neg \text{smaller } b \ a \ l$
apply (*rule smalleraux3, simp-all*)
apply (*case-tac a, simp-all*)
apply (*case-tac b, simp-all*)
done

lemma *posaux[rule-format]*: *position a l < position b l \longrightarrow a \neq b*
by (*induct l, simp-all*)

lemma *posaux6[rule-format]*:

$a \in \text{set } l \longrightarrow b \in \text{set } l \longrightarrow a \neq b \longrightarrow \text{position } a \ l \neq \text{position } b \ l$
by (*induct l*) (*simp-all add: position-positive*)

lemma *notSmallerTransaux[rule-format]*:

$x \neq \text{DenyAll} \implies r \neq \text{DenyAll} \implies$
 $\text{singleCombinators } [x] \implies \text{singleCombinators } [y] \implies \text{singleCombinators } [r] \implies$
 $\neg \text{smaller } y \ x \ l \implies \text{smaller } x \ y \ l \implies \text{smaller } x \ r \ l \implies \text{smaller } y \ r \ l \implies$
 $\text{in-list } x \ l \implies \text{in-list } y \ l \implies \text{in-list } r \ l \implies \neg \text{smaller } r \ x \ l$
by (*metis order-trans*)

lemma *notSmallerTrans[rule-format]*:

$x \neq \text{DenyAll} \longrightarrow r \neq \text{DenyAll} \longrightarrow \text{singleCombinators } (x\#y\#z) \longrightarrow$
 $\neg \text{smaller } y \ x \ l \longrightarrow \text{sorted } (x\#y\#z) \ l \longrightarrow r \in \text{set } z \longrightarrow$
 $\text{all-in-list } (x\#y\#z) \ l \longrightarrow \neg \text{smaller } r \ x \ l$
apply (*rule impI*)
apply (*rule notSmallerTransaux, simp-all*)
apply (*metis singleCombinatorsConc singleCombinatorsStart*)
apply (*metis SCSubset equalityE remdups.simps(2) set-remdups*
singleCombinatorsConc singleCombinatorsStart)
apply *metis*
apply (*metis sorted.simps(3) in-set-in-list singleCombinatorsConc*
singleCombinatorsStart sortedConcStart sorted-is-smaller)
apply (*metis sorted-Cons all-in-list.simps(2)*
singleCombinatorsConc)
apply (*metis, metis in-set-in-list*)
done

lemma *NCSaux1*[*rule-format*]:
 $noDenyAll\ p \longrightarrow \{x, y\} \in set\ l \longrightarrow all-in-list\ p\ l \longrightarrow singleCombinators\ p \longrightarrow$
 $sorted\ (DenyAllFromTo\ x\ y\ \#)\ p\ l \longrightarrow \{x, y\} \neq firstList\ p \longrightarrow$
 $DenyAllFromTo\ u\ v \in set\ p \longrightarrow \{x, y\} \neq \{u, v\}$

proof (*cases p*)
case *Nil* **thus** *?thesis* **by** *simp*
next
case (*Cons a list*)
then show *?thesis* **apply** *simp*
apply (*intro impI conjI*)
apply (*metis bothNet.simps(2) first-bothNet.simps(3)*)
proof –
assume *1*: $\{x, y\} \in set\ l$ **and** *2*: $in-list\ a\ l \wedge all-in-list\ list\ l$
and *3*: $singleCombinators\ (a\ \#)\ list$
and *4*: $smaller\ (DenyAllFromTo\ x\ y)\ a\ l \wedge sorted\ (a\ \#)\ list\ l$
and *5*: $DenyAllFromTo\ u\ v \in set\ list$
and *6*: $\neg member\ DenyAll\ a \wedge noDenyAll\ list$
have ***: $smaller\ ((DenyAllFromTo\ x\ y)::('a, 'b)\ Combinators)\ (DenyAllFromTo\ u\ v)\ l$
apply (*insert 1 2 3 4 5, rule-tac y = a in order-trans, simp-all*)
using *in-set-in-list* **apply** *fastforce*
by (*simp add: sorted-ConsA*)

have ****: $\{x, y\} \neq first-bothNet\ a \implies$
 $\neg smaller\ ((DenyAllFromTo\ u\ v)::('a, 'b)\ Combinators)\ (DenyAllFromTo\ x\ y)\ l$
apply (*insert 1 2 3 4 5 6,*
rule-tac y = a and z = list in notSmallerTrans,
simp-all del: smaller.simps)
apply (*rule smalleraux3a, simp-all del: smaller.simps*)
apply (*case-tac a, simp-all del: smaller.simps*)
by (*metis aux0-0 first-bothNet.elims list.set-intros(1)*)
show $\{x, y\} \neq first-bothNet\ a \implies \{x, y\} \neq \{u, v\}$
using *3 * *** **by** *force*
qed
qed

lemma *posaux3*[*rule-format*]: $a \in set\ l \longrightarrow b \in set\ l \longrightarrow a \neq b \longrightarrow position\ a\ l \neq$
 $position\ b\ l$
apply (*induct l, auto*)
by(*metis position-positive*)+

lemma *posaux4*[*rule-format*]:
 $singleCombinators\ [a] \longrightarrow a \neq DenyAll \longrightarrow b \neq DenyAll \longrightarrow in-list\ a\ l \longrightarrow in-list\ b\ l$

\longrightarrow
smaller a b l \longrightarrow *x = (bothNet a)* \longrightarrow *y = (bothNet b)* \longrightarrow
position x l <= position y l
proof (*cases a*)
case *DenyAll* **then show** *?thesis* **by simp**
next
case (*DenyAllFromTo c d*) **thus** *?thesis*
proof (*cases b*)
case *DenyAll* **thus** *?thesis* **by simp**
next
case (*DenyAllFromTo e f*) **thus** *?thesis* **using** *DenyAllFromTo*
by (*auto simp: eq-imp-le <a = DenyAllFromTo c d>*)
next
case (*AllowPortFromTo e f p*) **thus** *?thesis* **using** *<a = DenyAllFromTo c d>* **by**
simp
next
case (*Conc e f*) **thus** *?thesis* **using** *Conc <a = DenyAllFromTo c d>* **by simp**
qed
next
case (*AllowPortFromTo c d p*) **thus** *?thesis*
proof (*cases b*)
case *DenyAll* **thus** *?thesis* **by simp**
next
case (*DenyAllFromTo e f*) **thus** *?thesis* **using** *AllowPortFromTo* **by simp**
next
case (*AllowPortFromTo e f p2*) **thus** *?thesis* **using** *<a = AllowPortFromTo c d p>*
by simp
next
case (*Conc e f*) **thus** *?thesis* **using** *AllowPortFromTo* **by simp**
qed
next
case (*Conc c d*) **thus** *?thesis* **by simp**
qed

lemma *NCSaux2[rule-format]:*

noDenyAll p \longrightarrow *{a, b} ∈ set l* \longrightarrow *all-in-list p l* \longrightarrow *singleCombinators p* \longrightarrow
sorted (DenyAllFromTo a b # p) l \longrightarrow *{a, b} ≠ firstList p* \longrightarrow
AllowPortFromTo u v w ∈ set p \longrightarrow *{a, b} ≠ {u, v}*

proof (*cases p*)

case *Nil* **then show** *?thesis* **by simp**

next

case (*Cons aa list*)

have $*$: *{a, b} ∈ set l* \implies *in-list aa l* \wedge *all-in-list list l* \implies

singleCombinators (aa # list) \implies *AllowPortFromTo u v w ∈ set list*

⇒

```
      smaller (DenyAllFromTo a b) aa l ∧ sorted (aa # list) l ⇒
      smaller (DenyAllFromTo a b) (AllowPortFromTo u v w) l
apply (rule-tac y = aa in order-trans,simp-all del: smaller.simps)
using in-set-in-list apply fastforce
      using NormalisationGenericProofs.sorted-Cons all-in-list.simps(2) by blast
```

```
have **: AllowPortFromTo u v w ∈ set list ⇒
      in-list aa l ⇒ all-in-list list l ⇒
      in-list (AllowPortFromTo u v w) l
apply (rule-tac p = list in in-set-in-list)
apply simp-all
done
assume p = aa # list
then show ?thesis
apply simp
apply (intro impI conjI,hypsubst, simp)
apply (subgoal-tac smaller (DenyAllFromTo a b) (AllowPortFromTo u v w) l)
apply (subgoal-tac ¬ smaller (AllowPortFromTo u v w) (DenyAllFromTo a b) l)
apply (rule-tac l = l in posaux)
apply (rule-tac y = position (first-bothNet aa) l in basic-trans-rules(22))
apply (simp-all split: if-splits)
apply (case-tac aa, simp-all)
subgoal for x x'
apply (case-tac a = x ∧ b = x', simp-all)
apply (case-tac a = x, simp-all)
apply (simp add: order.not-eq-order-implies-strict posaux6)
apply (simp add: order.not-eq-order-implies-strict posaux6)
done
apply (simp add: order.not-eq-order-implies-strict posaux6)
apply (rule basic-trans-rules(18))
apply (rule-tac a = DenyAllFromTo a b and b = aa in posaux4, simp-all)
apply (case-tac aa,simp-all)
apply (case-tac aa, simp-all)
apply (rule posaux3, simp-all)
apply (case-tac aa, simp-all)
apply (rule-tac a = aa and b = AllowPortFromTo u v w in posaux4, simp-all)
apply (case-tac aa,simp-all)
apply (rule-tac p = list in sorted-is-smaller, simp-all)
apply (case-tac aa, simp-all)
apply (case-tac aa, simp-all)
apply (rule-tac a = aa and b = AllowPortFromTo u v w in posaux4, simp-all)
apply (case-tac aa,simp-all)
using ** apply auto[1]
```

```

    apply (metis all-in-list.simps(2) sorted-Cons)
    apply (case-tac aa, simp-all)
    apply (metis ** bothNet.simps(3) in-list.simps(3) posaux6)
    using * by force
qed

```

lemma *NCSaux3*[rule-format]:

```

noDenyAll p → {a, b} ∈ set l → all-in-list p l → singleCombinators p →
sorted (AllowPortFromTo a b w # p) l → {a, b} ≠ firstList p →
DenyAllFromTo u v ∈ set p → {a, b} ≠ {u, v}

```

```

apply (case-tac p, simp-all, intro impI conjI, hypsubst, simp)

```

proof –

```

fix aa::('a, 'b) Combinators fix list::('a, 'b) Combinators list

```

```

assume 1 : ¬ member DenyAll aa ∧ noDenyAll list and 2: {a, b} ∈ set l

```

```

and 3 : in-list aa l ∧ all-in-list list l and 4: singleCombinators (aa # list)

```

```

and 5 : smaller (AllowPortFromTo a b w) aa l ∧ sorted (aa # list) l

```

```

and 6 : {a, b} ≠ first-bothNet aa and 7: DenyAllFromTo u v ∈ set list

```

```

have *: ¬ smaller (DenyAllFromTo u v) (AllowPortFromTo a b w) l

```

```

apply (insert 1 2 3 4 5 6 7, rule-tac y = aa and z = list in notSmallerTrans)

```

```

    apply (simp-all del: smaller.simps)

```

```

    apply (rule smalleraux3a, simp-all del: smaller.simps)

```

```

    apply (case-tac aa, simp-all del: smaller.simps)

```

```

    apply (case-tac aa, simp-all del: smaller.simps)

```

```

done

```

```

have **: smaller (AllowPortFromTo a b w) (DenyAllFromTo u v) l

```

```

apply (insert 1 2 3 4 5 6 7, rule-tac y = aa in order-trans, simp-all del: smaller.simps)

```

```

    apply (subgoal-tac in-list (DenyAllFromTo u v) l, simp)

```

```

    apply (rule-tac p = list in in-set-in-list, simp-all)

```

```

    apply (rule-tac p = list in sorted-is-smaller, simp-all del: smaller.simps)

```

```

    apply (subgoal-tac in-list (DenyAllFromTo u v) l, simp)

```

```

    apply (rule-tac p = list in in-set-in-list, simp-all)

```

```

    apply (erule singleCombinatorsConc)

```

```

done

```

```

show {a, b} ≠ {u, v} by (insert ** , simp split: if-splits)

```

qed

lemma *NCSaux4*[rule-format]:

```

noDenyAll p → {a, b} ∈ set l → all-in-list p l → singleCombinators p →
sorted (AllowPortFromTo a b c # p) l → {a, b} ≠ firstList p →

```

```

AllowPortFromTo u v w ∈ set p → {a, b} ≠ {u, v}

```

```

apply (cases p, simp-all)

```

```

apply (intro impI conjI)

```

```

    apply (hypsubst, simp-all)

```

proof –

```

fix aa::('a, 'b) Combinators fix list::('a, 'b) Combinators list
assume 1 :  $\neg$  member DenyAll aa  $\wedge$  noDenyAll list and 2:  $\{a, b\} \in$  set l
  and 3 : in-list aa l  $\wedge$  all-in-list list l and 4: singleCombinators (aa # list)
  and 5 : smaller (AllowPortFromTo a b c) aa l  $\wedge$  sorted (aa # list) l
  and 6 :  $\{a, b\} \neq$  first-bothNet aa and 7: AllowPortFromTo u v w  $\in$  set list
have *:  $\neg$  smaller (AllowPortFromTo u v w) (AllowPortFromTo a b c) l
  apply (insert 1 2 3 4 5 6 7, rule-tac y = aa and z = list in notSmallerTrans)
    apply (simp-all del: smaller.simps)
  apply (rule smallerAux3a, simp-all del: smaller.simps)
  apply (case-tac aa, simp-all del: smaller.simps)
  apply (case-tac aa, simp-all del: smaller.simps)
done
have **: smaller (AllowPortFromTo a b c) (AllowPortFromTo u v w) l
  apply(insert 1 2 3 4 5 6 7)
  apply (case-tac aa, simp-all del: smaller.simps)
  apply (rule-tac y = aa in order-trans, simp-all del: smaller.simps)
  apply (subgoal-tac in-list (AllowPortFromTo u v w) l, simp)
  apply (rule-tac p = list in in-set-in-list, simp)
  apply (case-tac aa, simp-all del: smaller.simps)
  apply (rule-tac p = list in sorted-is-smaller, simp-all del: smaller.simps)
  apply (subgoal-tac in-list (AllowPortFromTo u v w) l, simp)
  apply (rule-tac p = list in in-set-in-list, simp, simp)
  apply (rule-tac y = aa in order-trans, simp-all del: smaller.simps)
  apply (subgoal-tac in-list (AllowPortFromTo u v w) l, simp)
  using in-set-in-list apply blast
  by (metis all-in-list.simps(2) bothNet.simps(3) in-list.simps(3)
    singleCombinators.simps(5) sorted-ConsA)
show  $\{a, b\} \neq \{u, v\}$  by (insert * **, simp-all split: if-splits)
qed

lemma NetsCollectedSorted[rule-format]:
  noDenyAll1 p  $\longrightarrow$  all-in-list p l  $\longrightarrow$  singleCombinators p  $\longrightarrow$  sorted p l  $\longrightarrow$  NetsCol-
  lected p
  apply (induct p)
  apply simp
  apply (intro impI, drule mp, erule noDA1C, drule mp, simp)
  apply (drule mp, erule singleCombinatorsConc)
  apply (drule mp, erule sortedConc)
proof –
  fix a:: ('a, 'b) Combinators fix p:: ('a, 'b) Combinators list
  assume 1: noDenyAll1 (a # p) and 2: all-in-list (a # p) l
  and 3: singleCombinators (a # p) and 4: sorted (a # p) l and 5: NetsCollected
  p
  show NetsCollected (a # p)

```



```

apply(insert 1 2 3 4 5, rule fl3)
  apply(simp, rename-tac aa)
proof (cases a)
  case DenyAll
  fix aa::('a, 'b) Combinators
  assume 6: aa ∈ set p
  show first-bothNet a ≠ first-bothNet aa
    apply(insert 1 2 3 4 5 6 ⟨a = DenyAll⟩, simp-all)
    using fMTaux noDA by blast
next
  case (DenyAllFromTo x21 x22)
  fix aa::('a, 'b) Combinators
  assume 6: first-bothNet a ≠ firstList p and 7 :aa ∈ set p
  show first-bothNet a ≠ first-bothNet aa
    apply(insert 1 2 3 4 5 6 7 ⟨a = DenyAllFromTo x21 x22⟩)
    apply(case-tac aa, simp-all)
    apply (meson NCSaux1)
    apply (meson NCSaux2)
    using SCnotConc by auto[1]
next
  case (AllowPortFromTo x31 x32 x33)
  fix aa::('a, 'b) Combinators
  assume 6: first-bothNet a ≠ firstList p and 7 :aa ∈ set p
  show first-bothNet a ≠ first-bothNet aa
    apply(insert 1 2 3 4 6 7 ⟨a = AllowPortFromTo x31 x32 x33⟩)
    apply(case-tac aa, simp-all)
    apply (meson NCSaux3)
    apply (meson NCSaux4)
    using SCnotConc by auto
next
  case (Conc x41 x42)
  fix aa::('a, 'b) Combinators
  show first-bothNet a ≠ first-bothNet aa
    by(insert 3 4 ⟨a = x41 ⊕ x42⟩,simp)
qed
qed

```

```

lemma NetsCollectedSort: distinct p ⇒ noDenyAll1 p ⇒ all-in-list p l ⇒
  singleCombinators p ⇒ NetsCollected (sort p l)
  apply (rule-tac l = l in NetsCollectedSorted,rule noDAsort, simp-all)
  apply (rule-tac b=p in all-in-listSubset)
  by (auto intro: sort-is-sorted)

```

```

lemma fBNsep[rule-format]: (∀ a∈set z. {b,c} ≠ first-bothNet a) →

```

$(\forall a \in \text{set } (\text{separate } z). \{b, c\} \neq \text{first-bothNet } a)$

apply (*induct* *z* *rule*: *separate.induct*, *simp*)
by (*rule* *impI*, *simp*)⁺

lemma *fBNsep1*[*rule-format*]: $(\forall a \in \text{set } z. \text{first-bothNet } x \neq \text{first-bothNet } a) \longrightarrow$
 $(\forall a \in \text{set } (\text{separate } z). \text{first-bothNet } x \neq \text{first-bothNet } a)$
apply (*induct* *z* *rule*: *separate.induct*, *simp*)
by (*rule* *impI*, *simp*)⁺

lemma *NetsCollectedSepauxa*:
 $\{b, c\} \neq \text{firstList } z \implies \text{noDenyAll1 } z \implies \forall a \in \text{set } z. \{b, c\} \neq \text{first-bothNet } a \implies \text{NetsCollected } z \implies$
 $\text{NetsCollected } (\text{separate } z) \implies \{b, c\} \neq \text{firstList } (\text{separate } z) \implies a \in \text{set } (\text{separate } z) \implies$
 $\{b, c\} \neq \text{first-bothNet } a$
by (*rule* *fBNsep*) *simp-all*

lemma *NetsCollectedSepaux*:
 $\text{first-bothNet } (x::('a, 'b)\text{Combinators}) \neq \text{first-bothNet } y \implies \neg \text{member } \text{DenyAll } y \wedge \text{noDenyAll } z \implies$
 $(\forall a \in \text{set } z. \text{first-bothNet } x \neq \text{first-bothNet } a) \wedge \text{NetsCollected } (y \# z) \implies$
 $\text{NetsCollected } (\text{separate } (y \# z)) \implies \text{first-bothNet } x \neq \text{firstList } (\text{separate } (y \# z))$
 \implies
 $a \in \text{set } (\text{separate } (y \# z)) \implies$
 $\text{first-bothNet } (x::('a, 'b)\text{Combinators}) \neq \text{first-bothNet } (a::('a, 'b)\text{Combinators})$
by (*rule* *fBNsep1*) *auto*

lemma *NetsCollectedSep*[*rule-format*]:
 $\text{noDenyAll1 } p \longrightarrow \text{NetsCollected } p \longrightarrow \text{NetsCollected } (\text{separate } p)$
proof (*induct* *p* *rule*: *separate.induct*, *simp-all*, *goal-cases*)
fix *x*::('a, 'b) *Combinators* *list*
case 1 then show *?case*
by (*metis* *fMTaux* *noDA* *noDA1eq* *noDAsep*)
next
fix *v* *va* *y* **fix** *z*::('a, 'b) *Combinators* *list*
case 2 then show *?case*
apply (*intro* *conjI* *impI*, *simp*)
apply (*metis* *NetsCollectedSepaux* *fl3* *noDA1eq* *noDenyAll.simps(1)*)
by (*metis* *noDA1eq* *noDenyAll.simps(1)*)
next
fix *v* *va* *vb* *y* **fix** *z*::('a, 'b) *Combinators* *list*
case 3 then show *?case*
apply (*intro* *conjI* *impI*)

```

    apply (metis NetsCollectedSepaux fl3 noDA1eq noDenyAll.simps(1))
  by (metis noDA1eq noDenyAll.simps(1))
next
fix v va y fix z::('a, 'b) Combinators list
case 4 then show ?case
  by (metis NetsCollectedSepaux fl3 noDA1eq noDenyAll.simps(1))
qed

```

lemma *OTNaur*:

```

onlyTwoNets a  $\implies$   $\neg$  member DenyAll a  $\implies$  (x,y)  $\in$  sdnets a  $\implies$ 
(x = first-srcNet a  $\wedge$  y = first-destNet a)  $\vee$  (x = first-destNet a  $\wedge$  y = first-srcNet
a)
  apply (case-tac (x = first-srcNet a  $\wedge$  y = first-destNet a),simp-all add: onlyT-
woNets-def)
  apply (case-tac ( $\exists$  aa b. sdnets a = {(aa, b)}), simp-all)
  apply (subgoal-tac sdnets a = {(first-srcNet a,first-destNet a)}, simp-all)
  apply (metis singletonE first-isIn)
  apply (subgoal-tac sdnets a = {(first-srcNet a,first-destNet a),(first-destNet a,
first-srcNet a)})
  by(auto intro!: sdnets2)

```

lemma *sdnets-charn*: *onlyTwoNets a \implies \neg member DenyAll a \implies*

```

sdnets a = {(first-srcNet a,first-destNet a)}  $\vee$ 
sdnets a = {(first-srcNet a, first-destNet a),(first-destNet a, first-srcNet a)}
  apply (case-tac sdnets a = {(first-srcNet a, first-destNet a)}, simp-all add: onlyT-
woNets-def)
  apply (case-tac ( $\exists$  aa b. sdnets a = {(aa, b)}), simp-all)
  apply (metis singletonE first-isIn)
  apply (subgoal-tac sdnets a = {(first-srcNet a,first-destNet a),(first-destNet
a,first-srcNet a)})
  by( auto intro!: sdnets2)

```

lemma *first-bothNet-charn*[*rule-format*]:

```

 $\neg$  member DenyAll a  $\implies$  first-bothNet a = {first-srcNet a, first-destNet a}
  by (induct a) simp-all

```

lemma *sdnets-noteq*:

```

onlyTwoNets a  $\implies$  onlyTwoNets aa  $\implies$  first-bothNet a  $\neq$  first-bothNet aa  $\implies$ 
 $\neg$  member DenyAll a  $\implies$   $\neg$  member DenyAll aa  $\implies$  sdnets a  $\neq$  sdnets aa
  apply (insert sdnets-charn [of a])
  apply (insert sdnets-charn [of aa])
  apply (insert first-bothNet-charn [of a])
  apply (insert first-bothNet-charn [of aa])

```

by(*metis OTNaux first-isIn insert-absorb2 insert-commute*)

lemma *fbn-noteq*:

onlyTwoNets a \implies *onlyTwoNets aa* \implies *first-bothNet a* \neq *first-bothNet aa* \implies
 \neg *member DenyAll a* \implies \neg *member DenyAll aa* \implies *allNetsDistinct [a, aa]* \implies
first-srcNet a \neq *first-srcNet aa* \vee *first-srcNet a* \neq *first-destNet aa* \vee
first-destNet a \neq *first-srcNet aa* \vee *first-destNet a* \neq *first-destNet aa*
apply (*insert sdnets-charn [of a]*)
apply (*insert sdnets-charn [of aa]*)
by (*metis first-bothNet-charn*)

lemma *NCisSD2aux*:

assumes *1*: *onlyTwoNets a* **and** *2*: *onlyTwoNets aa* **and** *3*: *first-bothNet a* \neq
first-bothNet aa

and *4*: \neg *member DenyAll a* **and** *5*: \neg *member DenyAll aa* **and** *6*: *allNetsDistinct*
[a, aa]

shows *disjSD-2 a aa*

apply (*insert 1 2 3 4 5 6*)

apply (*simp add: disjSD-2-def*)

apply (*intro allI impI*)

apply (*insert sdnets-charn [of a] sdnets-charn [of aa], simp*)

apply (*insert sdnets-noteq [of a aa] fbn-noteq [of a aa], simp*)

apply (*simp add: allNetsDistinct-def twoNetsDistinct-def*)

proof –

fix *ab b c d*

assume *7*: $\forall ab b. ab \neq b \wedge ab \in \text{set}(\text{net-list-aux}[a,aa]) \wedge b \in \text{set}(\text{net-list-aux}[a,aa]) \longrightarrow$
netsDistinct ab b

and *8*: $(ab, b) \in \text{sdnets } a \wedge (c, d) \in \text{sdnets } aa$

and *9*: $\text{sdnets } a = \{(first-srcNet a, first-destNet a)\} \vee$

$\text{sdnets } a = \{(first-srcNet a, first-destNet a), (first-destNet a, first-srcNet a)\}$

and *10*: $\text{sdnets } aa = \{(first-srcNet aa, first-destNet aa)\} \vee$

$\text{sdnets } aa = \{(first-srcNet aa, first-destNet aa), (first-destNet aa, first-srcNet$

aa)\}

and *11*: $\text{sdnets } a \neq \text{sdnets } aa$

and *12*: $first-destNet a = first-srcNet aa \longrightarrow first-srcNet a = first-destNet aa \longrightarrow$
 $first-destNet aa \neq first-srcNet aa$

show $(\text{netsDistinct } ab c \vee \text{netsDistinct } b d) \wedge (\text{netsDistinct } ab d \vee \text{netsDistinct } b c)$

proof (*rule conjI*)

show $\text{netsDistinct } ab c \vee \text{netsDistinct } b d$

apply(*insert 7 8 9 10 11 12*)

apply (*cases sdnets a = \{(first-srcNet a, first-destNet a)\}*)

apply (*cases sdnets aa = \{(first-srcNet aa, first-destNet aa)\}, simp-all*)

```

apply (metis 4 5 firstInNeta firstInNet alternativelistconc2)
apply (case-tac (c = first-srcNet aa  $\wedge$  d = first-destNet aa), simp-all)
apply (case-tac (first-srcNet a)  $\neq$  (first-srcNet aa), simp-all)
  apply (metis 4 5 firstInNeta alternativelistconc2)
apply (subgoal-tac first-destNet a  $\neq$  first-destNet aa)
  apply (metis 4 5 firstInNet alternativelistconc2)
apply (metis 3 4 5 first-bothNetsd)
apply (case-tac (first-destNet aa)  $\neq$  (first-srcNet a), simp-all)
apply (metis 4 5 firstInNeta firstInNet alternativelistconc2)
apply (case-tac first-destNet aa  $\neq$  first-destNet a, simp-all)
apply (subgoal-tac first-srcNet aa  $\neq$  first-destNet a)
  apply (metis 4 5 firstInNeta firstInNet alternativelistconc2)
apply (metis 3 4 5 first-bothNetsd insert-commute)
apply (metis 5 firstInNeta firstInNet alternativelistconc2)
apply (case-tac (c = first-srcNet aa  $\wedge$  d = first-destNet aa), simp-all)
apply (case-tac (ab = first-srcNet a  $\wedge$  b = first-destNet a), simp-all)
apply (case-tac (first-srcNet a)  $\neq$  (first-srcNet aa), simp-all)
  apply (metis 4 5 firstInNeta alternativelistconc2)
apply (subgoal-tac first-destNet a  $\neq$  first-destNet aa)
  apply (metis 4 5 firstInNet alternativelistconc2)
apply (metis 3 4 5 first-bothNetsd )
apply (case-tac (first-destNet aa)  $\neq$  (first-srcNet a), simp-all)
apply (metis 4 5 firstInNeta firstInNet alternativelistconc2)
apply (case-tac first-destNet aa  $\neq$  first-destNet a, simp)
apply (subgoal-tac first-srcNet aa  $\neq$  first-destNet a)
  apply (metis 4 5 firstInNeta firstInNet alternativelistconc2)
apply (metis 3 4 5 first-bothNetsd insert-commute)
apply (metis)
proof -
  assume 14 : (ab = first-srcNet a  $\wedge$  b = first-destNet a  $\vee$  ab = first-destNet a  $\wedge$ 
b = first-srcNet a)  $\wedge$  (c, d)  $\in$  sdnets aa
    and 15 : sdnets a = {(first-srcNet a, first-destNet a), (first-destNet a,
first-srcNet a)}
      and 16 : sdnets aa = {(first-srcNet aa, first-destNet aa)}  $\vee$  sdnets aa =
{(first-srcNet aa, first-destNet aa), (first-destNet aa, first-srcNet aa)}
      and 17 : {(first-srcNet a, first-destNet a), (first-destNet a, first-srcNet a)}  $\neq$ 
sdnets aa
      and 18 : first-destNet a = first-srcNet aa  $\longrightarrow$  first-srcNet a = first-destNet
aa  $\longrightarrow$  first-destNet aa  $\neq$  first-srcNet aa
      and 19 : first-destNet a  $\neq$  first-srcNet a
      and 20 : c = first-srcNet aa  $\longrightarrow$  d  $\neq$  first-destNet aa
show netsDistinct ab c  $\vee$  netsDistinct b d
apply (case-tac (ab = first-srcNet a  $\wedge$  b = first-destNet a), simp-all)
apply (case-tac c = first-srcNet aa, simp-all)

```

```

  apply (metis 2 5 14 20 OTNaux)
  apply (subgoal-tac c = first-destNet aa, simp)
  apply (subgoal-tac d = first-srcNet aa, simp)
  apply (case-tac (first-srcNet a) ≠ (first-destNet aa), simp-all)
  apply (metis 4 5 7 firstInNeta firstInNet alternativelistconc2)
  apply (subgoal-tac first-destNet a ≠ first-srcNet aa)
  apply (metis 4 5 7 firstInNeta firstInNet alternativelistconc2)
  apply (metis 3 4 5 first-bothNetsd insert-commute)
  apply (metis 2 5 14 OTNaux)
  apply (metis 2 5 14 OTNaux)
  apply (case-tac c = first-srcNet aa, simp-all)
  apply (metis 2 5 14 20 OTNaux)
  apply (subgoal-tac c = first-destNet aa, simp)
  apply (subgoal-tac d = first-srcNet aa, simp)
  apply (case-tac (first-destNet a) ≠ (first-destNet aa), simp-all)
  apply (metis 4 5 7 14 firstInNet alternativelistconc2)
  apply (subgoal-tac first-srcNet a ≠ first-srcNet aa)
  apply (metis 4 5 7 14 firstInNeta alternativelistconc2)
  apply (metis 3 4 5 first-bothNetsd insert-commute)
  apply (metis 2 5 14 OTNaux)
  apply (metis 2 5 14 OTNaux)
  done
qed
next
show netsDistinct ab d ∨ netsDistinct b c
  apply (insert 1 2 3 4 5 6 7 8 9 10 11 12)
  apply (cases sdnets a = {(first-srcNet a, first-destNet a)})
  apply (cases sdnets aa = {(first-srcNet aa, first-destNet aa)}, simp-all)
  apply (case-tac (c = first-srcNet aa ∧ d = first-destNet aa), simp-all)
  apply (case-tac (first-srcNet a) ≠ (first-destNet aa), simp-all)
  apply (metis firstInNeta firstInNet alternativelistconc2)
  apply (subgoal-tac first-destNet a ≠ first-srcNet aa)
  apply (metis firstInNeta firstInNet alternativelistconc2)
  apply (metis first-bothNetsd insert-commute)
  apply (case-tac (c = first-srcNet aa ∧ d = first-destNet aa), simp-all)
  apply (case-tac (ab = first-srcNet a ∧ b = first-destNet a), simp-all)
  apply (case-tac (first-destNet a) ≠ (first-srcNet aa), simp-all)
  apply (metis firstInNeta firstInNet alternativelistconc2)
  apply (subgoal-tac first-srcNet a ≠ first-destNet aa)
  apply (metis firstInNeta firstInNet alternativelistconc2)
  apply (metis first-bothNetsd insert-commute)
  apply (case-tac (first-srcNet aa) ≠ (first-srcNet a), simp-all)
  apply (metis firstInNeta alternativelistconc2)
  apply (case-tac first-destNet aa ≠ first-destNet a, simp-all)

```

apply (*metis firstInNet alternativelistconc2*)
apply (*metis first-bothNetsd*)
proof –
assume 13: $\forall ab b. ab \neq b \wedge ab \in \text{set}(\text{net-list-aux}[a,aa]) \wedge b \in \text{set}(\text{net-list-aux}[a,aa])$
 $\longrightarrow \text{netsDistinct } ab \ b$
and 14 : $(ab = \text{first-srcNet } a \wedge b = \text{first-destNet } a \vee$
 $ab = \text{first-destNet } a \wedge b = \text{first-srcNet } a) \wedge (c, d) \in \text{sdnets } aa$
and 15 : $\text{sdnets } a = \{(\text{first-srcNet } a, \text{first-destNet } a),$
 $(\text{first-destNet } a, \text{first-srcNet } a)\}$
and 16 : $\text{sdnets } aa = \{(\text{first-srcNet } aa, \text{first-destNet } aa)\} \vee$
 $\text{sdnets } aa = \{(\text{first-srcNet } aa, \text{first-destNet } aa),$
 $(\text{first-destNet } aa, \text{first-srcNet } aa)\}$
and 17 : $\{(\text{first-srcNet } a, \text{first-destNet } a),$
 $(\text{first-destNet } a, \text{first-srcNet } a)\} \neq \text{sdnets } aa$
show $\text{first-destNet } a \neq \text{first-srcNet } a \implies \text{netsDistinct } ab \ d \vee \text{netsDistinct } b \ c$
apply (*insert 1 2 3 4 5 6 13 14 15 16 17*)
apply (*cases sdnets aa = {(first-srcNet aa, first-destNet aa)}, simp-all*)
apply (*case-tac (c = first-srcNet aa \wedge d = first-destNet aa), simp-all*)
apply (*case-tac (ab = first-srcNet a \wedge b = first-destNet a), simp-all*)
apply (*case-tac (first-destNet a) \neq (first-srcNet aa), simp-all*)
apply (*metis firstInNeta firstInNet alternativelistconc2*)
apply (*subgoal-tac first-srcNet a \neq first-destNet aa*)
apply (*metis firstInNeta firstInNet alternativelistconc2*)
apply (*metis first-bothNetsd insert-commute*)
apply (*case-tac (first-srcNet aa) \neq (first-srcNet a), simp-all*)
apply (*metis firstInNeta alternativelistconc2*)
apply (*case-tac first-destNet aa \neq first-destNet a, simp-all*)
apply (*metis firstInNet alternativelistconc2*)
apply (*metis first-bothNetsd*)
proof –
assume 20: $\{(\text{first-srcNet } a, \text{first-destNet } a), (\text{first-destNet } a, \text{first-srcNet } a)\} \neq$
 $\{(\text{first-srcNet } aa, \text{first-destNet } aa), (\text{first-destNet } aa, \text{first-srcNet } aa)\}$
and 21: $\text{first-destNet } a \neq \text{first-srcNet } a$
show $\text{netsDistinct } ab \ d \vee \text{netsDistinct } b \ c$
apply (*case-tac (c = first-srcNet aa \wedge d = first-destNet aa), simp-all*)
apply (*case-tac (ab = first-srcNet a \wedge b = first-destNet a), simp-all*)
apply (*case-tac (first-destNet a) \neq (first-srcNet aa), simp-all*)
apply (*metis 4 5 7 firstInNeta firstInNet alternativelistconc2*)
apply (*subgoal-tac first-srcNet a \neq first-destNet aa*)
apply (*metis 4 5 7 firstInNeta firstInNet alternativelistconc2*)
apply (*metis 20 insert-commute*)
apply (*case-tac (first-srcNet aa) \neq (first-srcNet a), simp-all*)
apply (*metis 4 5 13 14 firstInNeta alternativelistconc2*)
apply (*case-tac first-destNet aa \neq first-destNet a, simp-all*)

```

    apply (metis 4 5 13 14 firstInNet alternativelistconc2)
  apply (case-tac (ab = first-srcNet a ∧ b = first-destNet a), simp-all)
  apply (case-tac (first-destNet a) ≠ (first-srcNet aa), simp-all)
  apply (metis 20)
  apply (subgoal-tac first-srcNet a ≠ first-srcNet aa)
  apply (metis 20)
  apply (metis 21)
  apply (case-tac (first-srcNet aa) ≠ (first-destNet a))
  apply (metis (no-types, lifting) 2 3 4 5 7 14 OTNaux
    firstInNet firstInNeta first-bothNetsd isInAlternativeList)
  by (metis 2 4 5 7 20 14 OTNaux doubleton-eq-iff firstInNet
    firstInNeta isInAlternativeList)
qed
qed
qed
qed

lemma ANDaux3[rule-format]:
  y ∈ set xs → a ∈ set (net-list-aux [y]) → a ∈ set (net-list-aux xs)
  by (induct xs) (simp-all add: isInAlternativeList)

lemma ANDaux2:
  allNetsDistinct (x # xs) ⇒ y ∈ set xs ⇒ allNetsDistinct [x,y]
  apply (simp add: allNetsDistinct-def)
  by (meson ANDaux3 isInAlternativeList netlistaux)

lemma NCisSD2[rule-format]:
  ¬ member DenyAll a ⇒ OnlyTwoNets (a#p) ⇒
  NetsCollected2 (a # p) ⇒ NetsCollected (a#p) ⇒
  noDenyAll ( p) ⇒ allNetsDistinct (a # p) ⇒ s ∈ set p ⇒
  disjSD-2 a s
  by (metis ANDaux2 FWNnormalisationCore.member.simps(2) NCisSD2aux NetsCol-
    lected.simps(1)
    NetsCollected2.simps(1) OTNConc OTNoTN empty-iff empty-set list.set-intros(1)
    noDA)

lemma separatedNC[rule-format]:
  OnlyTwoNets p → NetsCollected2 p → NetsCollected p → noDenyAll1 p →
  allNetsDistinct p → separated p
proof (induct p, simp-all, rename-tac a b, case-tac a = DenyAll, simp-all, goal-cases)
  fix a fix p::('a set set, 'b) Combinators list
  show OnlyTwoNets p → NetsCollected2 p → NetsCollected p → noDenyAll1 p
  →

```


$allNetsDistinct\ p \longrightarrow separated\ p \implies a \neq DenyAll \implies OnlyTwoNets\ (a \# p)$
 \longrightarrow
 $first\ bothNet\ a \neq firstList\ p \wedge NetsCollected2\ p \longrightarrow$
 $(\forall aa \in set\ p.\ first\ bothNet\ a \neq first\ bothNet\ aa) \wedge NetsCollected\ p \longrightarrow$
 $noDenyAll1\ (a \# p) \longrightarrow allNetsDistinct\ (a \# p) \longrightarrow (\forall s.\ s \in set\ p \longrightarrow$
 $disjSD-2\ a\ s) \wedge separated\ p$
apply (*intro impI, drule mp, erule OTNConc, drule mp*)
apply (*case-tac p, simp-all*)
apply (*drule mp, erule noDA1C, intro conjI allI impI NCisSD2, simp-all*)
apply (*case-tac a, simp-all*)
apply (*case-tac a, simp-all*)
using *ANDConc by auto*
next
fix *a::('a set set, 'b) Combinators* **fix** *p::('a set set, 'b) Combinators list*
show $OnlyTwoNets\ p \longrightarrow NetsCollected2\ p \longrightarrow NetsCollected\ p \longrightarrow noDenyAll1\ p$
 \longrightarrow
 $allNetsDistinct\ p \longrightarrow separated\ p \implies a = DenyAll \implies OnlyTwoNets\ p \longrightarrow$
 $\{\} \neq firstList\ p \wedge NetsCollected2\ p \longrightarrow (\forall a \in set\ p.\ \{\} \neq first\ bothNet$
 $a) \wedge NetsCollected\ p \longrightarrow$
 $noDenyAll\ p \longrightarrow allNetsDistinct\ (DenyAll \# p) \longrightarrow$
 $(\forall s.\ s \in set\ p \longrightarrow disjSD-2\ DenyAll\ s) \wedge separated\ p$
by (*simp add: ANDConc disjSD-2-def noDA1eq*)
qed
lemma *separatedNC'*[*rule-format*]:
 $OnlyTwoNets\ p \longrightarrow NetsCollected2\ p \longrightarrow NetsCollected\ p \longrightarrow noDenyAll1\ p \longrightarrow$
 $allNetsDistinct\ p \longrightarrow separated\ p$
proof (*induct p*)
case *Nil* **show** *?case by simp*
next
case (*Cons a p*) **then show** *?case*
apply *simp*
proof (*cases a = DenyAll*) **print-cases**
case *True*
then show $OnlyTwoNets\ (a \# p) \longrightarrow first\ bothNet\ a \neq firstList\ p \wedge NetsCollected2$
 $p \longrightarrow$
 $(\forall aa \in set\ p.\ first\ bothNet\ a \neq first\ bothNet\ aa) \wedge NetsCollected\ p \longrightarrow$
 $noDenyAll1\ (a \# p) \longrightarrow allNetsDistinct\ (a \# p) \longrightarrow$
 $(\forall s.\ s \in set\ p \longrightarrow disjSD-2\ a\ s) \wedge separated\ p$
apply(*insert Cons.hyps <a = DenyAll>*)
apply (*intro impI, drule mp, erule OTNConc, drule mp*)
apply (*case-tac p, simp-all*)
apply (*case-tac a, simp-all*)
apply (*case-tac a, simp-all*)

by (*simp add: ANDConc disjSD-2-def noDA1eq*)
next
case *False*
then show *OnlyTwoNets (a # p) → first-bothNet a ≠ firstList p ∧ NetsCollected2 p →*
 $(\forall aa \in \text{set } p. \text{first-bothNet } a \neq \text{first-bothNet } aa) \wedge \text{NetsCollected } p \rightarrow$
 $\text{noDenyAll1 } (a \# p) \rightarrow \text{allNetsDistinct } (a \# p) \rightarrow (\forall s. s \in \text{set } p$
 \rightarrow
 $\text{disjSD-2 } a \ s) \wedge \text{separated } p$
apply(*insert Cons.hyps ⟨a ≠ DenyAll⟩*)
by (*metis NetsCollected.simps(1) NetsCollected2.simps(1) separated.simps(1) separatedNC*)
qed
qed

lemma *NC2Sep[rule-format]: noDenyAll1 p → NetsCollected2 (separate p)*
proof (*induct p rule: separate.induct, simp-all, goal-cases*)
fix *x :: ('a, 'b) Combinators list*
case 1 then show *?case*
by (*metis fMTaux firstList.simps(1) fl2 noDA1eq noDenyAll.elims(2) separate.simps(5)*)
next
fix *v va fix y:: ('a, 'b) Combinators fix z*
case 2 then show *?case*
by (*simp add: fl2 noDA1eq*)
next
fix *v va vb fix y:: ('a, 'b) Combinators fix z*
case 3 then show *?case*
by (*simp add: fl2 noDA1eq*)
next
fix *v va fix y:: ('a, 'b) Combinators fix z*
case 4 then show *?case*
by (*simp add: fl2 noDA1eq*)
qed

lemma *separatedSep[rule-format]:*
 $\text{OnlyTwoNets } p \rightarrow \text{NetsCollected2 } p \rightarrow \text{NetsCollected } p \rightarrow$
 $\text{noDenyAll1 } p \rightarrow \text{allNetsDistinct } p \rightarrow \text{separated } (\text{separate } p)$
by (*simp add: ANDSep NC2Sep NetsCollectedSep OTNSEp noDA1sep separatedNC*)

lemma *rADnMT[rule-format]: p ≠ [] → removeAllDuplicates p ≠ []*
by (*induct p*) *simp-all*

lemma *remDupsNMT*[*rule-format*]: $p \neq [] \longrightarrow \text{remdups } p \neq []$
by (*metis remdups-eq-nil-iff*)

lemma *sets-distinct1*: $(n::\text{int}) \neq m \implies \{(a,b). a = n\} \neq \{(a,b). a = m\}$
by *auto*

lemma *sets-distinct2*: $(m::\text{int}) \neq n \implies \{(a,b). a = n\} \neq \{(a,b). a = m\}$
by *auto*

lemma *sets-distinct5*: $(n::\text{int}) < m \implies \{(a,b). a = n\} \neq \{(a,b). a = m\}$
by *auto*

lemma *sets-distinct6*: $(m::\text{int}) < n \implies \{(a,b). a = n\} \neq \{(a,b). a = m\}$
by *auto*
end

2.3.3 Normalisation Proofs: Integer Port

theory

NormalisationIntegerPortProof

imports

NormalisationGenericProofs

begin

Normalisation proofs which are specific to the IntegerPort address representation.

lemma *ConcAssoc*: $C((A \oplus B) \oplus D) = C(A \oplus (B \oplus D))$
by (*simp add: C.simps*)

lemma *aux26*[*simp*]: $\text{twoNetsDistinct } a \ b \ c \ d \implies$
 $\text{dom } (C (\text{AllowPortFromTo } a \ b \ p)) \cap \text{dom } (C (\text{DenyAllFromTo } c \ d)) = \{\}$
apply (*auto simp: PLemmas twoNetsDistinct-def netsDistinct-def*)[1]
by *auto*

lemma *wp2-aux*[*rule-format*]: $\text{wellformed-policy2 } (xs \ @ \ [x]) \longrightarrow$
 $\text{wellformed-policy2 } xs$

apply (*induct xs, simp-all*)

subgoal for $a \ xs$

apply (*case-tac a, simp-all*)

done

done

lemma *Cdom2*: $x \in \text{dom}(C \ b) \implies C \ (a \oplus \ b) \ x = (C \ b) \ x$
by (*auto simp: C.simps*)

lemma *wp2Conc*[*rule-format*]: *wellformed-policy2* (*x#xs*) \implies *wellformed-policy2* *xs*
by (*case-tac x,simp-all*)

lemma *DAimpliesMR-E*[*rule-format*]: *DenyAll* \in *set p* \longrightarrow
 $(\exists r. \text{applied-rule-rev } C \ x \ p = \text{Some } r)$
apply (*simp add: applied-rule-rev-def*)
apply (*rule-tac xs = p in rev-induct, simp-all*)
by (*metis C.simps(1) denyAllDom*)

lemma *DAimplieMR*[*rule-format*]: *DenyAll* \in *set p* \implies *applied-rule-rev* *C x p* \neq *None*
by (*auto intro: DAimpliesMR-E*)

lemma *MRList1*[*rule-format*]: $x \in \text{dom } (C \ a) \implies \text{applied-rule-rev } C \ x \ (b@[a]) = \text{Some } a$
by (*simp add: applied-rule-rev-def*)

lemma *MRList2*: $x \in \text{dom } (C \ a) \implies \text{applied-rule-rev } C \ x \ (c@b@[a]) = \text{Some } a$
by (*simp add: applied-rule-rev-def*)

lemma *MRList3*:
 $x \notin \text{dom } (C \ xa) \implies \text{applied-rule-rev } C \ x \ (a \ @ \ b \ \# \ xs \ @ \ [xa]) = \text{applied-rule-rev } C \ x \ (a \ @ \ b \ \# \ xs)$
by (*simp add: applied-rule-rev-def*)

lemma *CConcEnd*[*rule-format*]:
 $C \ a \ x = \text{Some } y \longrightarrow C \ (\text{list2FWpolicy } (xs \ @ \ [a])) \ x = \text{Some } y$
(is ?P xs)
apply (*rule-tac P = ?P in list2FWpolicy.induct*)
by (*simp-all add:C.simps*)

lemma *CConcStartaux*: $C \ a \ x = \text{None} \implies (C \ aa \ ++ \ C \ a) \ x = C \ aa \ x$
by (*simp add: PLemmas*)

lemma *CConcStart*[*rule-format*]:
 $xs \neq [] \longrightarrow C \ a \ x = \text{None} \longrightarrow C \ (\text{list2FWpolicy } (xs \ @ \ [a])) \ x = C \ (\text{list2FWpolicy } xs) \ x$
apply (*rule list2FWpolicy.induct*)
by (*simp-all add: PLemmas*)

lemma *mrNnt*[*simp*]: *applied-rule-rev* *C x p* = *Some a* $\implies p \neq []$
apply (*simp add: applied-rule-rev-def*)
by *auto*

lemma *mr-is-C*[*rule-format*]:

applied-rule-rev $C\ x\ p = \text{Some } a \longrightarrow C\ (\text{list2FWpolicy } (p))\ x = C\ a\ x$

apply (*simp add: applied-rule-rev-def*)

apply (*rule rev-induct, auto*)

apply (*metis CConcEnd*)

apply (*metis CConcEnd*)

by (*metis CConcStart applied-rule-rev-def mrNnt option.exhaust*)

lemma *CConcStart2*:

$p \neq [] \implies x \notin \text{dom } (C\ a) \implies C\ (\text{list2FWpolicy } (p\ @\ [a]))\ x = C\ (\text{list2FWpolicy } p)\ x$

by (*erule CConcStart, simp add: PLemmas*)

lemma *CConcEnd1*:

$q\ @\ p \neq [] \implies x \notin \text{dom } (C\ a) \implies C\ (\text{list2FWpolicy } (q\ @\ p\ @\ [a]))\ x = C\ (\text{list2FWpolicy } (q\ @\ p))\ x$

apply (*subst lCdom2*)

by (*rule CConcStart2, simp-all*)

lemma *CConcEnd2*[*rule-format*]:

$x \in \text{dom } (C\ a) \longrightarrow C\ (\text{list2FWpolicy } (xs\ @\ [a]))\ x = C\ a\ x\ (\text{is } ?P\ xs)$

apply (*rule-tac P = ?P in list2FWpolicy.induct*)

by (*auto simp:C.simps*)

lemma *bar3*:

$x \in \text{dom } (C\ (\text{list2FWpolicy } (xs\ @\ [xa]))) \implies x \in \text{dom } (C\ (\text{list2FWpolicy } xs)) \vee x \in \text{dom } (C\ xa)$

by *auto (metis CConcStart eq-Nil-appendI l2p-aux2 option.simps(3))*

lemma *CeqEnd*[*rule-format, simp*]:

$a \neq [] \longrightarrow x \in \text{dom } (C\ (\text{list2FWpolicy } a)) \longrightarrow C\ (\text{list2FWpolicy } (b\ @\ a))\ x = (C\ (\text{list2FWpolicy } a))\ x$

apply (*rule rev-induct, simp-all*)

subgoal for *xa xs*

apply (*case-tac xs $\neq []$, simp-all*)

apply (*case-tac $x \in \text{dom } (C\ xa)$*)

apply (*metis CConcEnd2 MRList2 mr-is-C*)

apply (*metis CConcEnd1 CConcStart2 Nil-is-append-conv bar3*)

apply (*metis MRList2 eq-Nil-appendI mr-is-C*)

done

done

lemma *CConcStartA*[*rule-format, simp*]:

$x \in \text{dom } (C a) \longrightarrow x \in \text{dom } (C (\text{list2FWpolicy } (a \# b)))$ (is ?P b)
apply (rule-tac P = ?P in list2FWpolicy.induct)
apply (simp-all add: C.simps)
done

lemma domConc:

$x \in \text{dom } (C (\text{list2FWpolicy } b)) \implies b \neq [] \implies x \in \text{dom } (C (\text{list2FWpolicy } (a @ b)))$
by (auto simp: PLemmas)

lemma CeqStart[rule-format,simp]:

$x \notin \text{dom}(C(\text{list2FWpolicy } a)) \longrightarrow a \neq [] \longrightarrow b \neq [] \longrightarrow C(\text{list2FWpolicy}(b@a)) x =$
 $(C(\text{list2FWpolicy } b)) x$
apply (rule list2FWpolicy.induct,simp-all)
apply (auto simp: list2FWpolicyconc PLemmas)
done

lemma C-eq-if-mr-eq2:

$\text{applied-rule-rev } C x a = [r] \implies$
 $\text{applied-rule-rev } C x b = [r] \implies a \neq [] \implies b \neq [] \implies$
 $C (\text{list2FWpolicy } a) x = C (\text{list2FWpolicy } b) x$

by (metis mr-is-C)

lemma nMRtoNone[rule-format]:

$p \neq [] \longrightarrow \text{applied-rule-rev } C x p = \text{None} \longrightarrow C (\text{list2FWpolicy } p) x = \text{None}$
apply (rule rev-induct, simp-all)
subgoal for xa xs
apply (case-tac xs = [], simp-all)
by (simp-all add: applied-rule-rev-def dom-def)
done

lemma C-eq-if-mr-eq:

$\text{applied-rule-rev } C x b = \text{applied-rule-rev } C x a \implies a \neq [] \implies b \neq [] \implies$
 $C (\text{list2FWpolicy } a) x = C (\text{list2FWpolicy } b) x$
apply (cases applied-rule-rev C x a = None, simp-all)
apply (subst nMRtoNone,simp-all)
apply (subst nMRtoNone, simp-all)
by (auto intro: C-eq-if-mr-eq2)

lemma notmatching-notdom: $\text{applied-rule-rev } C x (p@[a]) \neq \text{Some } a \implies x \notin \text{dom } (C a)$

by (simp add: applied-rule-rev-def split: if-splits)

lemma foo3a[rule-format]:

$\text{applied-rule-rev } C x (a@[b]@c) = \text{Some } b \longrightarrow r \in \text{set } c \longrightarrow b \notin \text{set } c \longrightarrow x \notin \text{dom}$

$(C\ r)$
apply (*rule rev-induct*)
apply (*simp-all*)
apply (*intro impI conjI, simp*)
subgoal for $xa\ xs$
apply (*rule-tac p = a @ b # xs in notmatching-notdom, simp-all*)
done
by (*metis MRList2 MRList3 append-Cons option.inject*)

lemma *foo3D*:

$wellformed-policy1\ p \implies p = DenyAll \# ps \implies$
 $applied-rule-rev\ C\ x\ p = [DenyAll] \implies r \in set\ ps \implies x \notin dom\ (C\ r)$
by (*rule-tac a = [] and b = DenyAll and c = ps in foo3a, simp-all*)

lemma *foo4*[*rule-format*]:

$set\ p = set\ s \wedge (\forall\ r. r \in set\ p \longrightarrow x \notin dom\ (C\ r)) \longrightarrow (\forall\ r. r \in set\ s \longrightarrow x \notin dom\ (C\ r))$
by *simp*

lemma *foo5b*[*rule-format*]:

$x \in dom\ (C\ b) \longrightarrow (\forall\ r. r \in set\ c \longrightarrow x \notin dom\ (C\ r)) \longrightarrow applied-rule-rev\ C\ x\ (b\#c)$
 $= Some\ b$
apply (*simp add: applied-rule-rev-def*)
apply (*rule-tac xs = c in rev-induct, simp-all*)
done

lemma *mr-first*:

$x \in dom\ (C\ b) \implies \forall r. r \in set\ c \longrightarrow x \notin dom\ (C\ r) \implies s = b \# c \implies applied-rule-rev\ C\ x\ s = [b]$
by (*simp add: foo5b*)

lemma *mr-charn*[*rule-format*]:

$a \in set\ p \longrightarrow (x \in dom\ (C\ a)) \longrightarrow (\forall\ r. r \in set\ p \wedge x \in dom\ (C\ r) \longrightarrow r = a)$
 \longrightarrow
 $applied-rule-rev\ C\ x\ p = Some\ a$
unfolding *applied-rule-rev-def*
apply (*rule-tac xs = p in rev-induct*)
apply (*simp*)
by (*safe, auto*)

lemma *foo8*:

$\forall r. r \in set\ p \wedge x \in dom\ (C\ r) \longrightarrow r = a \implies set\ p = set\ s \implies$
 $\forall r. r \in set\ s \wedge x \in dom\ (C\ r) \longrightarrow r = a$
by *auto*

lemma *mrConcEnd*[*rule-format*]:
applied-rule-rev C x (b # p) = Some a \longrightarrow *a* \neq *b* \longrightarrow *applied-rule-rev C x p = Some a*

apply (*simp add: applied-rule-rev-def*)
by (*rule-tac xs = p in rev-induct, auto*)

lemma *wp3tl*[*rule-format*]: *wellformed-policy3 p* \longrightarrow *wellformed-policy3 (tl p)*

apply (*induct p, simp-all*)
subgoal for *a p*
apply (*case-tac a, simp-all*)
done
done

lemma *wp3Conc*[*rule-format*]: *wellformed-policy3 (a#p)* \longrightarrow *wellformed-policy3 p*
by (*induct p, simp-all, case-tac a, simp-all*)

lemma *foo98*[*rule-format*]:

applied-rule-rev C x (aa # p) = Some a \longrightarrow *x* \in *dom (C r)* \longrightarrow *r* \in *set p* \longrightarrow *a* \in *set p*

unfolding *applied-rule-rev-def*
apply (*rule rev-induct, simp-all*)
subgoal for *xa xs*
apply (*case-tac r = xa, simp-all*)
done
done

lemma *mrMTNone*[*simp*]: *applied-rule-rev C x [] = None*
by (*simp add: applied-rule-rev-def*)

lemma *DAAux*[*simp*]: *x* \in *dom (C DenyAll)*
by (*simp add: dom-def PolicyCombinators.PolicyCombinators C.simps*)

lemma *mrSet*[*rule-format*]: *applied-rule-rev C x p = Some r* \longrightarrow *r* \in *set p*

unfolding *applied-rule-rev-def*
by (*rule-tac xs=p in rev-induct, simp-all*)

lemma *mr-not-Conc*: *singleCombinators p* \implies *applied-rule-rev C x p* \neq *Some (a \oplus b)*

apply (*auto simp: mrSet*)
apply (*drule mrSet*)
apply (*erule SCnotConc, simp*)
done

lemma *foo25*[*rule-format*]: *wellformed-policy3* ($p@[x]$) \longrightarrow *wellformed-policy3* p
by (*induct* p , *simp-all*, *case-tac* a , *simp-all*)

lemma *mr-in-dom*[*rule-format*]: *applied-rule-rev* C x $p = \text{Some } a \longrightarrow x \in \text{dom } (C a)$
apply (*rule-tac* $xs = p$ **in** *rev-induct*)
by (*auto simp: applied-rule-rev-def*)

lemma *wp3EndMT*[*rule-format*]:
wellformed-policy3 ($p@[xs]$) \longrightarrow *AllowPortFromTo* a b $po \in \text{set } p \longrightarrow$
 $\text{dom } (C (\text{AllowPortFromTo } a b po)) \cap \text{dom } (C xs) = \{\}$
apply (*induct* p , *simp-all*)
apply (*intro impI, drule mp, erule wp3Conc*)
by *clarify auto*

lemma *foo29*: $\llbracket \text{dom } (C a) \neq \{\}; \text{dom } (C a) \cap \text{dom } (C b) = \{\} \rrbracket \Longrightarrow a \neq b$ **by** *auto*

lemma *foo28*:
AllowPortFromTo a b $po \in \text{set } p \Longrightarrow \text{dom } (C (\text{AllowPortFromTo } a b po)) \neq \{\} \Longrightarrow$
wellformed-policy3 ($p @ [x]$) $\Longrightarrow x \neq \text{AllowPortFromTo } a b po$
by (*metis foo29 C.simps(3) wp3EndMT*)

lemma *foo28a*[*rule-format*]: $x \in \text{dom } (C a) \Longrightarrow \text{dom } (C a) \neq \{\}$ **by** *auto*

lemma *allow-deny-dom*[*simp*]:
 $\text{dom } (C (\text{AllowPortFromTo } a b po)) \subseteq \text{dom } (C (\text{DenyAllFromTo } a b))$
by (*simp-all add: twoNetsDistinct-def netsDistinct-def PLemmas*) *auto*

lemma *DenyAllowDisj*:
 $\text{dom } (C (\text{AllowPortFromTo } a b p)) \neq \{\} \Longrightarrow$
 $\text{dom } (C (\text{DenyAllFromTo } a b)) \cap \text{dom } (C (\text{AllowPortFromTo } a b p)) \neq \{\}$
by (*metis Int-absorb1 allow-deny-dom*)

lemma *foo31*:
 $\forall r. r \in \text{set } p \wedge x \in \text{dom } (C r) \longrightarrow$
 $r = \text{AllowPortFromTo } a b po \vee r = \text{DenyAllFromTo } a b \vee r = \text{DenyAll} \Longrightarrow$
 $\text{set } p = \text{set } s \Longrightarrow$
 $\forall r. r \in \text{set } s \wedge x \in \text{dom } (C r) \longrightarrow r = \text{AllowPortFromTo } a b po \vee r = \text{DenyAllFromTo } a b \vee r = \text{DenyAll}$
by *auto*

lemma *wp1-axa*:

wellformed-policy1-strong $p \implies (\exists r. \text{applied-rule-rev } C \ x \ p = \text{Some } r)$
apply (*rule DAimpliesMR-E*)
by (*erule wp1-aux1aa*)

lemma *deny-dom[simp]*:
 $\text{twoNetsDistinct } a \ b \ c \ d \implies \text{dom } (C \ (\text{DenyAllFromTo } a \ b)) \cap \text{dom } (C \ (\text{DenyAllFromTo } c \ d)) = \{\}$
apply (*simp add: C.simps*)
by (*erule aux6*)

lemma *domTrans*: $\text{dom } a \subseteq \text{dom } b \implies \text{dom } b \cap \text{dom } c = \{\} \implies \text{dom } a \cap \text{dom } c = \{\}$ **by** *auto*

lemma *DomInterAllowsMT*:
 $\text{twoNetsDistinct } a \ b \ c \ d \implies$
 $\text{dom } (C \ (\text{AllowPortFromTo } a \ b \ p)) \cap \text{dom } (C \ (\text{AllowPortFromTo } c \ d \ po)) = \{\}$
apply (*case-tac p = po, simp-all*)
apply (*rule-tac b = C (DenyAllFromTo a b) in domTrans, simp-all*)
apply (*metis domComm aux26 tNDComm*)
by (*simp add: twoNetsDistinct-def netsDistinct-def PLemmas*) *auto*

lemma *DomInterAllowsMT-Ports*:
 $p \neq po \implies \text{dom } (C \ (\text{AllowPortFromTo } a \ b \ p)) \cap \text{dom } (C \ (\text{AllowPortFromTo } c \ d \ po)) = \{\}$
by (*simp add: twoNetsDistinct-def netsDistinct-def PLemmas*) *auto*

lemma *wellformed-policy3-charn[rule-format]*:
 $\text{singleCombinators } p \longrightarrow \text{distinct } p \longrightarrow \text{allNetsDistinct } p \longrightarrow$
 $\text{wellformed-policy1 } p \longrightarrow \text{wellformed-policy2 } p \longrightarrow \text{wellformed-policy3 } p$
apply (*induct-tac p*)
apply *simp-all*
apply (*auto intro: singleCombinatorsConc ANDConc waux2 wp2Conc*)
subgoal for *a list*
apply (*case-tac a, simp-all, clarify*)
apply (*metis C.elims DomInterAllowsMT DomInterAllowsMT-Ports aux0-0 aux7aa inf-commute*)
done
done

lemma *DistinctNetsDenyAllow*:
 $\text{DenyAllFromTo } b \ c \in \text{set } p \implies$
 $\text{AllowPortFromTo } a \ d \ po \in \text{set } p \implies$

```

allNetsDistinct p  $\implies$  dom (C (DenyAllFromTo b c))  $\cap$  dom (C (AllowPortFromTo a
d po))  $\neq$  {}  $\implies$ 
  b = a  $\wedge$  c = d
unfolding allNetsDistinct-def
apply (frule-tac x = b in spec)
apply (drule-tac x = d in spec)
apply (drule-tac x = a in spec)
apply (drule-tac x = c in spec)
apply (simp,metis Int-commute ND0aux1 ND0aux3 NDCComm aux26 twoNetsDis-
tinct-def ND0aux2 ND0aux4)
done

```

lemma *DistinctNetsAllowAllow*:

```

AllowPortFromTo b c po  $\in$  set p  $\implies$ 
  AllowPortFromTo a d po  $\in$  set p  $\implies$ 
    allNetsDistinct p  $\implies$ 
      dom (C (AllowPortFromTo b c po))  $\cap$  dom (C (AllowPortFromTo a d po))  $\neq$  {}
 $\implies$ 
  b = a  $\wedge$  c = d  $\wedge$  po = po
unfolding allNetsDistinct-def
apply (frule-tac x = b in spec)
apply (drule-tac x = d in spec)
apply (drule-tac x = a in spec)
apply (drule-tac x = c in spec)
apply (simp,metis DomInterAllowsMT DomInterAllowsMT-Ports ND0aux3 ND0aux4
NDCComm
  twoNetsDistinct-def)
done

```

lemma *WP2RS2[*simp*]*:

```

singleCombinators p  $\implies$  distinct p  $\implies$  allNetsDistinct p  $\implies$ 
  wellformed-policy2 (removeShadowRules2 p)
proof (induct p)
  case Nil thus ?case by simp
next
  case (Cons x xs)
  have wp-xs: wellformed-policy2 (removeShadowRules2 xs)
  by (metis Cons ANDConc distinct.simps(2) singleCombinatorsConc)
  show ?case
  proof (cases x)
  case DenyAll thus ?thesis using wp-xs by simp
next
  case (DenyAllFromTo a b) thus ?thesis
  using wp-xs Cons by (simp,metis DenyAllFromTo aux aux7 tNDCComm deny-dom)

```

```

next
  case (AllowPortFromTo a b p) thus ?thesis
    using wp-xs by (simp, metis aux26 AllowPortFromTo Cons(4) aux aux7a tND-
Comm)
  next
    case (Conc a b) thus ?thesis
      by (metis Conc Cons(2) singleCombinators.simps(2))
qed
qed

```

lemma AD-aux:

```

AllowPortFromTo a b po ∈ set p ⇒ DenyAllFromTo c d ∈ set p ⇒
allNetsDistinct p ⇒ singleCombinators p ⇒ a ≠ c ∨ b ≠ d ⇒
dom (C (AllowPortFromTo a b po)) ∩ dom (C (DenyAllFromTo c d)) = {}
by (rule aux26, rule-tac x = AllowPortFromTo a b po and y = DenyAllFromTo c d in
tND, auto)

```

lemma sorted-WP2[rule-format]: $sorted\ p\ l \longrightarrow all\text{-in-list}\ p\ l \longrightarrow distinct\ p \longrightarrow$
 $allNetsDistinct\ p \longrightarrow singleCombinators\ p \longrightarrow wellformed\text{-policy2}\ p$

```

proof (induct p)
  case Nil thus ?case by simp
next
  case (Cons a p) thus ?case
  proof (cases a)
    case DenyAll thus ?thesis using Cons
      by (auto intro: ANDConc singleCombinatorsConc sortedConcEnd)
  next
    case (DenyAllFromTo c d) thus ?thesis using Cons
      apply simp
      apply (intro impI conjI allI)
      apply (rule deny-dom)
      apply (auto intro: aux7 tNDComm ANDConc singleCombinatorsConc sorted-
ConcEnd)
      done
  next
    case (AllowPortFromTo c d e) thus ?thesis using Cons
      apply simp
      apply (intro impI conjI allI aux26)
      apply (rule-tac x = AllowPortFromTo c d e and y = DenyAllFromTo aa b in
tND)
        apply (assumption, simp-all)
      apply (subgoal-tac smaller (AllowPortFromTo c d e) (DenyAllFromTo aa b) l)
      apply (simp split: if-splits)
      apply metis

```

```

    apply (erule sorted-is-smaller, simp-all)
    apply (metis bothNet.simps(2) in-list.simps(2) in-set-in-list)
  by (auto intro: aux7 tNDComm ANDConc singleCombinatorsConc sortedConcEnd)
next
  case (Conc a b) thus ?thesis using Cons by simp
qed
qed

```

```

lemma wellformed2-sorted[simp]:
  all-in-list p l  $\implies$  distinct p  $\implies$  allNetsDistinct p  $\implies$ 
  singleCombinators p  $\implies$  wellformed-policy2 (sort p l)
  apply (rule sorted-WP2,erule sort-is-sorted, simp-all)
  apply (auto elim: all-in-listSubset intro: SC3 singleCombinatorsConc sorted-insort)
done

```

```

lemma wellformed2-sortedQ[simp]:  $\llbracket$ all-in-list p l; distinct p; allNetsDistinct p;
  singleCombinators p $\rrbracket \implies$  wellformed-policy2 (qsort p l)
  apply (rule sorted-WP2,erule sort-is-sortedQ, simp-all)
  apply (auto elim: all-in-listSubset intro: SC3Q singleCombinatorsConc distinct-sortQ)
done

```

```

lemma C-DenyAll[simp]: C (list2FWpolicy (xs @ [DenyAll])) x = Some (deny ())
  by (auto simp: PLemmas)

```

```

lemma C-eq-RS1n:
  C(list2FWpolicy (removeShadowRules1-alternative p)) = C(list2FWpolicy p)
proof (cases p)print-cases
  case Nil then show ?thesis apply(simp-all)
  by (metis list2FWpolicy.simps(1) rSR1-eq removeShadowRules1.simps(2))
next
  case (Cons x list) show ?thesis
  apply (rule rev-induct)
  apply (metis rSR1-eq removeShadowRules1.simps(2))
  subgoal for x xs
  apply (case-tac xs = [], simp-all)
  unfolding removeShadowRules1-alternative-def
  apply (case-tac x, simp-all)
  by (metis (no-types, opaque-lifting) CConcEnd2 CConcStart C-DenyAll RS1n-nMT
aux114
  domIff removeShadowRules1-alternative-def
  removeShadowRules1-alternative-rev.simps(2) rev.simps(2))
  done
qed

```

lemma *C-eq-RS1*[*simp*]:

$p \neq [] \implies C(\text{list2FWpolicy } (\text{removeShadowRules1 } p)) = C(\text{list2FWpolicy } p)$
by (*metis rSR1-eq C-eq-RS1n*)

lemma *EX-MR-aux*[*rule-format*]:

$\text{applied-rule-rev } C \ x \ (\text{DenyAll } \# \ p) \neq \text{Some } \text{DenyAll} \implies (\exists y. \text{applied-rule-rev } C \ x \ p = \text{Some } y)$
apply (*simp add: applied-rule-rev-def*)
apply (*rule-tac xs = p in rev-induct, simp-all*)
done

lemma *EX-MR* :

$\text{applied-rule-rev } C \ x \ p \neq [\text{DenyAll}] \implies p = \text{DenyAll } \# \ ps \implies$
 $\text{applied-rule-rev } C \ x \ p = \text{applied-rule-rev } C \ x \ ps$
apply *auto*
apply (*subgoal-tac applied-rule-rev C x (DenyAll#ps) ≠ None, auto*)
apply (*metis mrConcEnd*)
by (*metis DAimpliesMR-E list.sel(1) hd-in-set list.simps(3) not-Some-eq*)

lemma *mr-not-DA*:

$\text{wellformed-policy1-strong } s \implies$
 $\text{applied-rule-rev } C \ x \ p = [\text{DenyAllFromTo } a \ ab] \implies \text{set } p = \text{set } s \implies$
 $\text{applied-rule-rev } C \ x \ s \neq [\text{DenyAll}]$
apply (*subst wp1n-tl, simp-all*)
apply (*subgoal-tac x ∈ dom (C (DenyAllFromTo a ab))*)
apply (*subgoal-tac DenyAllFromTo a ab ∈ set (tl s)*)
apply (*metis wp1n-tl foo98 wellformed-policy1-strong.simps(2)*)
using *mrSet r-not-DA-in-tl* **apply** *blast*
by (*simp add: mr-in-dom*)

lemma *domsMT-notND-DD*:

$\text{dom } (C \ (\text{DenyAllFromTo } a \ b)) \cap \text{dom } (C \ (\text{DenyAllFromTo } c \ d)) \neq \{\} \implies \neg \text{nets-Distinct } a \ c$
using *deny-dom twoNetsDistinct-def* **by** *blast*

lemma *domsMT-notND-DD2*:

$\text{dom } (C \ (\text{DenyAllFromTo } a \ b)) \cap \text{dom } (C \ (\text{DenyAllFromTo } c \ d)) \neq \{\} \implies \neg \text{nets-Distinct } b \ d$
using *deny-dom twoNetsDistinct-def* **by** *blast*

lemma *domsMT-notND-DD3*:

$x \in \text{dom } (C \ (\text{DenyAllFromTo } a \ b)) \implies x \in \text{dom } (C \ (\text{DenyAllFromTo } c \ d)) \implies \neg \text{nets-Distinct } a \ c$
by(*auto intro!: domsMT-notND-DD*)

lemma *domsMT-notND-DD4*:
 $x \in \text{dom} (C (\text{DenyAllFromTo } a \ b)) \implies x \in \text{dom} (C (\text{DenyAllFromTo } c \ d)) \implies \neg$
netsDistinct $b \ d$
by (*auto intro!*:*domsMT-notND-DD2*)

lemma *NetsEq-if-sameP-DD*:
 $\text{allNetsDistinct } p \implies u \in \text{set } p \implies v \in \text{set } p \implies u = \text{DenyAllFromTo } a \ b \implies$
 $v = \text{DenyAllFromTo } c \ d \implies x \in \text{dom} (C \ u) \implies x \in \text{dom} (C \ v) \implies a = c \wedge b = d$
apply (*simp add: allNetsDistinct-def*)
by (*metis ND0aux1 ND0aux2 domsMT-notND-DD3 domsMT-notND-DD4*)

lemma *rule-charn1*:
assumes *aND*: *allNetsDistinct* p
and *mr-is-allow*: *applied-rule-rev* $C \ x \ p = \text{Some} (\text{AllowPortFromTo } a \ b \ po)$
and *SC*: *singleCombinators* p
and *inp*: $r \in \text{set } p$
and *inDom*: $x \in \text{dom} (C \ r)$
shows $(r = \text{AllowPortFromTo } a \ b \ po \vee r = \text{DenyAllFromTo } a \ b \vee r = \text{DenyAll})$
proof (*cases* r)
case *DenyAll* **show** *?thesis* **by** (*metis DenyAll*)
next
case (*DenyAllFromTo* $x \ y$) **show** *?thesis*
by (*metis AD-aux DenyAllFromTo SC aND domInterMT inDom inp mrSet*
mr-in-dom mr-is-allow)
next
case (*AllowPortFromTo* $x \ y \ b$) **show** *?thesis*
by (*metis (no-types, lifting) AllowPortFromTo DistinctNetsAllowAllow aND dom-*
InterMT
inDom inp mrSet mr-in-dom mr-is-allow)
next
case (*Conc* $x \ y$) **thus** *?thesis* **using** *assms* **by** (*metis aux0-0*)
qed

lemma *none-MT-rulessubset[rule-format]*:
 $\text{none-MT-rules } C \ a \longrightarrow \text{set } b \subseteq \text{set } a \longrightarrow \text{none-MT-rules } C \ b$
by (*induct* $b, \text{simp-all}$) (*metis notMTnMT*)

lemma *nMTSort*: $\text{none-MT-rules } C \ p \implies \text{none-MT-rules } C \ (\text{sort } p \ l)$
by (*metis set-sort nMTeqSet*)

lemma *nMTSortQ*: $\text{none-MT-rules } C \ p \implies \text{none-MT-rules } C \ (q\text{sort } p \ l)$
by (*metis set-sortQ nMTeqSet*)

lemma *wp3char*[*rule-format*]:
 $none\text{-}MT\text{-}rules\ C\ xs\ \wedge\ C\ (AllowPortFromTo\ a\ b\ po)=\emptyset\ \wedge\ well\text{-}formed\text{-}policy3(xs@[DenyAllFromTo\ a\ b])\ \longrightarrow$
 $AllowPortFromTo\ a\ b\ po\ \notin\ set\ xs$
apply (*induct xs, simp-all*)
by (*metis domNMT wp3Conc*)

lemma *wp3charn*[*rule-format*]:
assumes *domAllow*: $dom\ (C\ (AllowPortFromTo\ a\ b\ po))\ \neq\ \{\}$
and *wp3*: $wellformed\text{-}policy3\ (xs\ @\ [DenyAllFromTo\ a\ b])$
shows $AllowPortFromTo\ a\ b\ po\ \notin\ set\ xs$
apply (*insert assms*)
proof (*induct xs*)
case Nil show ?case by simp
next
case (Cons x xs) show ?case using Cons
by (*simp, auto intro: wp3Conc*) (*auto simp: DenyAllowDisj domAllow*)
qed

lemma *rule-charn2*:
assumes *aND*: $allNetsDistinct\ p$
and *wp1*: $wellformed\text{-}policy1\ p$
and *SC*: $singleCombinators\ p$
and *wp3*: $wellformed\text{-}policy3\ p$
and *allow-in-list*: $AllowPortFromTo\ c\ d\ po\ \in\ set\ p$
and *x-in-dom-allow*: $x\ \in\ dom\ (C\ (AllowPortFromTo\ c\ d\ po))$
shows $applied\text{-}rule\text{-}rev\ C\ x\ p = Some\ (AllowPortFromTo\ c\ d\ po)$
proof (*insert assms, induct p rule: rev-induct*)
case Nil show ?case using Nil by simp
next
case (snoc y ys)
show ?case using snoc
apply (*case-tac y = (AllowPortFromTo c d po), simp-all*)
apply (*simp add: applied-rule-rev-def*)
apply (*subgoal-tac ys \neq []*)
apply (*subgoal-tac applied-rule-rev C x ys = Some (AllowPortFromTo c d po)*)
defer 1
apply (*metis ANDConcEnd SCConcEnd WP1ConcEnd foo25*)
apply (*metis inSet-not-MT*)
proof (*cases y*)
case DenyAll thus ?thesis using DenyAll snoc
apply simp
by (*metis DAnotTL DenyAll inSet-not-MT policy2list.simps(2)*)
next


```

case (DenyAllFromTo a b) thus ?thesis using snoc apply simp
  apply (simp-all add: applied-rule-rev-def)
  apply (rule conjI)
  apply (metis domInterMT wp3EndMT)
  apply (rule impI)
  by (metis ANDConcEnd DenyAllFromTo SCConcEnd WP1ConcEnd foo25)
next
case (AllowPortFromTo a1 a2 b) thus ?thesis
  using AllowPortFromTo snoc apply simp
  apply (simp-all add: applied-rule-rev-def)
  apply (rule conjI)
  apply (metis domInterMT wp3EndMT)
  by (metis ANDConcEnd AllowPortFromTo SCConcEnd WP1ConcEnd foo25
x-in-dom-allow)
next
case (Conc a b) thus ?thesis using Conc snoc apply simp
  by (metis Conc aux0-0 in-set-conv-decomp)
qed
qed

```

lemma rule-charn3:

```

wellformed-policy1 p  $\implies$  allNetsDistinct p  $\implies$  singleCombinators p  $\implies$ 
wellformed-policy3 p  $\implies$  applied-rule-rev C x p = [DenyAllFromTo c d]  $\implies$ 
AllowPortFromTo a b po  $\in$  set p  $\implies$  x  $\notin$  dom (C (AllowPortFromTo a b po))
by (clarify, auto simp: rule-charn2 dom-def)

```

lemma rule-charn4:

```

assumes wp1: wellformed-policy1 p
  and aND: allNetsDistinct p
  and SC: singleCombinators p
  and wp3: wellformed-policy3 p
  and DA: DenyAll  $\notin$  set p
  and mr: applied-rule-rev C x p = Some (DenyAllFromTo a b)
  and rinp: r  $\in$  set p
  and xindom: x  $\in$  dom (C r)
shows r = DenyAllFromTo a b
proof (cases r)
case DenyAll thus ?thesis using DenyAll assms by simp
next
case (DenyAllFromTo c d) thus ?thesis using assms apply simp
  apply (erule-tac x = x and p = p and v = (DenyAllFromTo a b) and
    u = (DenyAllFromTo c d) in NetsEq-if-sameP-DD)
  apply simp-all
  apply (erule mrSet)

```

```

    by (erule mr-in-dom)
next
case (AllowPortFromTo c d e) thus ?thesis using assms apply simp
  apply (subgoal-tac  $x \notin \text{dom} (C (AllowPortFromTo c d e))$ )
  apply simp
  apply (rule-tac  $p = p$  in rule-charn3)
  by (auto intro: SCnotConc)
next
case (Conc a b) thus ?thesis using assms apply simp
  by (metis Conc aux0-0)
qed

```

lemma foo31a:

```

 $\forall r. r \in \text{set } p \wedge x \in \text{dom} (C r) \longrightarrow r = \text{AllowPortFromTo } a b \text{ po} \vee r = \text{DenyAllFromTo } a b \vee r = \text{DenyAll} \implies$ 
   $\text{set } p = \text{set } s \implies r \in \text{set } s \implies x \in \text{dom} (C r) \implies$ 
   $r = \text{AllowPortFromTo } a b \text{ po} \vee r = \text{DenyAllFromTo } a b \vee r = \text{DenyAll}$ 
  by auto

```

lemma aux4 [rule-format]:

```

  applied-rule-rev C x (a#p) = Some a  $\longrightarrow a \notin \text{set } (p) \longrightarrow \text{applied-rule-rev } C x p = \text{None}$ 
  apply (rule rev-induct,simp-all)
  by (metis aux0-4 empty-iff empty-set insert-iff list.simps(15) mrSet mreq-end3)

```

lemma mrDA-tl:

```

  assumes mr-DA: applied-rule-rev C x p = Some DenyAll
  and wp1n: wellformed-policy1-strong p
  shows applied-rule-rev C x (tl p) = None
  apply (rule aux4 [where a = DenyAll])
  apply (metis wp1n-tl mr-DA wp1n)
  by (metis WP1n-DA-notinSet wp1n)

```

lemma rule-charnDAFT:

```

  wellformed-policy1-strong p  $\implies \text{allNetsDistinct } p \implies \text{singleCombinators } p \implies$ 
  wellformed-policy3 p  $\implies \text{applied-rule-rev } C x p = \lfloor \text{DenyAllFromTo } a b \rfloor \implies r \in \text{set } (tl p) \implies$ 
   $x \in \text{dom} (C r) \implies r = \text{DenyAllFromTo } a b$ 
  apply (subgoal-tac  $p = \text{DenyAll}\#(tl p)$ )
  apply (metis AND-tl Combinators.distinct(1) SC-tl list.sel(3) mrConcEnd rule-charn4 waux2 wellformed-policy1-charn wp1-aux1aa wp1-eq wp3tl)
  using wp1n-tl by blast

```

lemma mrDenyAll-is-unique:

$\llbracket \text{wellformed-policy1-strong } p; \text{ applied-rule-rev } C \ x \ p = \text{Some } \text{DenyAll};$
 $r \in \text{set } (tl \ p) \rrbracket \implies x \notin \text{dom } (C \ r)$
apply (*rule-tac* $a = []$ **and** $b = \text{DenyAll}$ **and** $c = tl \ p$ **in** *foo3a*, *simp-all*)
apply (*metis wp1n-tl*)
by (*metis WP1n-DA-notinSet*)

theorem *C-eq-Sets-mr*:

assumes *sets-eq*: $\text{set } p = \text{set } s$
and *SC*: *singleCombinators* p
and *wp1-p*: *wellformed-policy1-strong* p
and *wp1-s*: *wellformed-policy1-strong* s
and *wp3-p*: *wellformed-policy3* p
and *wp3-s*: *wellformed-policy3* s
and *aND*: *allNetsDistinct* p
shows $\text{applied-rule-rev } C \ x \ p = \text{applied-rule-rev } C \ x \ s$
proof (*cases applied-rule-rev C x p*)
case *None*
have *DA*: $\text{DenyAll} \in \text{set } p$ **using** *wp1-p* **by** (*auto simp: wp1-aux1aa*)
have *notDA*: $\text{DenyAll} \notin \text{set } p$ **using** *None* **by** (*auto simp: DAimplieMR*)
thus *?thesis* **using** *DA* **by** (*contradiction*)
next
case (*Some y*) **thus** *?thesis*
proof (*cases y*)
have *tl-p*: $p = \text{DenyAll}\#(tl \ p)$ **by** (*metis wp1-p wp1n-tl*)
have *tl-s*: $s = \text{DenyAll}\#(tl \ s)$ **by** (*metis wp1-s wp1n-tl*)
have *tl-eq*: $\text{set } (tl \ p) = \text{set } (tl \ s)$
by (*metis list.sel(3) WP1n-DA-notinSet sets-eq foo2*
wellformed-policy1-charn wp1-aux1aa wp1-eq wp1-p wp1-s)
{ case *DenyAll*
have *mr-p-is-DenyAll*: $\text{applied-rule-rev } C \ x \ p = \text{Some } \text{DenyAll}$
by (*simp add: DenyAll Some*)
hence *x-notin-tl-p*: $\forall r. r \in \text{set } (tl \ p) \longrightarrow x \notin \text{dom } (C \ r)$ **using** *wp1-p*
by (*auto simp: mrDenyAll-is-unique*)
hence *x-notin-tl-s*: $\forall r. r \in \text{set } (tl \ s) \longrightarrow x \notin \text{dom } (C \ r)$ **using** *tl-eq*
by *auto*
hence *mr-s-is-DenyAll*: $\text{applied-rule-rev } C \ x \ s = \text{Some } \text{DenyAll}$ **using** *tl-s*
by (*auto simp: mr-first*)
thus *?thesis* **using** *mr-p-is-DenyAll* **by** *simp*
}
{case (*DenyAllFromTo a b*)
have *mr-p-is-DAFT*: $\text{applied-rule-rev } C \ x \ p = \text{Some } (\text{DenyAllFromTo } a \ b)$
by (*simp add: DenyAllFromTo Some*)
have *DA-notin-tl*: $\text{DenyAll} \notin \text{set } (tl \ p)$
by (*metis WP1n-DA-notinSet wp1-p*)

have *mr-tl-p*: *applied-rule-rev C x p = applied-rule-rev C x (tl p)*
by (*metis Combinators.simps(4) DenyAllFromTo Some mrConcEnd tl-p*)
have *dom-tl-p*: $\bigwedge r. r \in \text{set } (tl\ p) \wedge x \in \text{dom } (C\ r) \implies r = (\text{DenyAllFromTo } a\ b)$
using *wp1-p aND SC wp3-p mr-p-is-DAFT*
by (*auto simp: rule-charnDAFT*)
hence *dom-tl-s*: $\bigwedge r. r \in \text{set } (tl\ s) \wedge x \in \text{dom } (C\ r) \implies r = (\text{DenyAllFromTo } a\ b)$
b)
using *tl-eq* **by** *auto*
have *DAFT-in-tl-s*: *DenyAllFromTo a b ∈ set (tl s) using mr-tl-p*
by (*metis DenyAllFromTo mrSet mr-p-is-DAFT tl-eq*)
have *x-in-dom-DAFT*: *x ∈ dom (C (DenyAllFromTo a b))*
by (*metis mr-p-is-DAFT DenyAllFromTo mr-in-dom*)
hence *mr-tl-s-is-DAFT*: *applied-rule-rev C x (tl s) = Some (DenyAllFromTo a b)*
using *DAFT-in-tl-s dom-tl-s* **by** (*metis mr-charn*)
hence *mr-s-is-DAFT*: *applied-rule-rev C x s = Some (DenyAllFromTo a b)*
using *tl-s*
by (*metis DA-notin-tl DenyAllFromTo EX-MR mrDA-tl not-Some-eq tl-eq wellformed-policy1-strong.simps(2)*)
thus *?thesis using mr-p-is-DAFT by simp*
}

{**case** (*AllowPortFromTo a b c*)
have *wp1s*: *wellformed-policy1 s* **by** (*metis wp1-eq wp1-s*)
have *mr-p-is-A*: *applied-rule-rev C x p = Some (AllowPortFromTo a b c)*
by (*simp add: AllowPortFromTo Some*)
hence *A-in-s*: *AllowPortFromTo a b c ∈ set s* **using** *sets-eq*
by (*auto intro: mrSet*)
have *x-in-dom-A*: *x ∈ dom (C (AllowPortFromTo a b c))*
by (*metis mr-p-is-A AllowPortFromTo mr-in-dom*)
have *SCs*: *singleCombinators s* **using** *SC sets-eq*
by (*auto intro: SCSubset*)
hence *ANDs*: *allNetsDistinct s* **using** *aND sets-eq SC*
by (*auto intro: aNDSetsEq*)
hence *mr-s-is-A*: *applied-rule-rev C x s = Some (AllowPortFromTo a b c)*
using *A-in-s wp1s mr-p-is-A aND SCs wp3-s x-in-dom-A*
by (*simp add: rule-charn2*)
thus *?thesis using mr-p-is-A by simp*
}

case (*Conc a b*) **thus** *?thesis by (metis Some mr-not-Conc SC)*
qed
qed

lemma *C-eq-Sets*:

singleCombinators p \implies *wellformed-policy1-strong p* \implies *wellformed-policy1-strong s* \implies

$wellformed_policy3\ p \implies wellformed_policy3\ s \implies allNetsDistinct\ p \implies set\ p = set\ s \implies$
 $C\ (list2FWpolicy\ p)\ x = C\ (list2FWpolicy\ s)\ x$
by(*auto intro: C-eq-iff-mr-eq C-eq-Sets-mr [symmetric]*)

lemma *C-eq-sorted*:

$distinct\ p \implies all-in-list\ p\ l \implies singleCombinators\ p \implies wellformed_policy1-strong\ p \implies$
 $wellformed_policy3\ p \implies allNetsDistinct\ p \implies$
 $C\ (list2FWpolicy\ (FWNormalisationCore.sort\ p\ l)) = C\ (list2FWpolicy\ p)$
apply (*rule ext*)
by (*auto intro: C-eq-Sets simp: nMTSort wellformed1-alternative-sorted wellformed-policy3-charn wp1-eq*)

lemma *C-eq-sortedQ*:

$distinct\ p \implies all-in-list\ p\ l \implies singleCombinators\ p \implies wellformed_policy1-strong\ p \implies$
 $wellformed_policy3\ p \implies allNetsDistinct\ p \implies$
 $C\ (list2FWpolicy\ (qsort\ p\ l)) = C\ (list2FWpolicy\ p)$
apply (*rule ext*)
apply (*auto intro!: C-eq-Sets simp: nMTSortQ wellformed1-alternative-sorted distinct-sortQ wellformed-policy3-charn wp1-eq*)
by (*metis set-qsort wellformed1-sortedQ wellformed-eq wp1-aux1aa*)

lemma *C-eq-RS2-mr: applied-rule-rev C x (removeShadowRules2 p) = applied-rule-rev C x p*

proof (*induct p*)

case *Nil* **thus** *?case* **by** *simp*

next

case (*Cons y ys*) **thus** *?case*

proof (*cases ys = []*)

case *True* **thus** *?thesis* **by** (*cases y, simp-all*)

next

case *False* **thus** *?thesis*

proof (*cases y*)

case *DenyAll* **thus** *?thesis* **by** (*simp, metis Cons DenyAll mreq-end2*)

next

case (*DenyAllFromTo a b*) **thus** *?thesis*

by (*simp, metis Cons DenyAllFromTo mreq-end2*)

next

case (*AllowPortFromTo a b p*) **thus** *?thesis*

proof (*cases DenyAllFromTo a b ∈ set ys*)

case *True* **thus** *?thesis* **using** *AllowPortFromTo Cons*

```

apply (cases applied-rule-rev C x ys = None, simp-all)
apply (subgoal-tac x ∉ dom (C (AllowPortFromTo a b p)))
apply (subst mrconcNone, simp-all)
apply (simp add: applied-rule-rev-def )
apply (rule contra-subsetD [OF allow-deny-dom])
apply (erule mrNoneMT, simp)
apply (metis AllowPortFromTo mrconc)
done
next
  case False thus ?thesis using False Cons AllowPortFromTo
  by (simp, metis AllowPortFromTo Cons mreq-end2) qed
next
  case (Conc a b) thus ?thesis
  by (metis Cons mreq-end2 removeShadowRules2.simps(4))
qed
qed
qed

```

lemma *C-eq-None*[rule-format]:

```

p ≠ [] --> applied-rule-rev C x p = None → C (list2FWpolicy p) x = None
apply (simp add: applied-rule-rev-def)
apply (rule rev-induct, simp-all)
apply (intro impI, simp)
subgoal for xa xs
  apply (case-tac xs ≠ [])
  apply (simp-all add: dom-def)
done
done

```

lemma *C-eq-None2*:

```

a ≠ [] ⇒ b ≠ [] ⇒ applied-rule-rev C x a = ⊥ ⇒ applied-rule-rev C x b = ⊥ ⇒
C (list2FWpolicy a) x = C (list2FWpolicy b) x
by (auto simp: C-eq-None)

```

lemma *C-eq-RS2*:

```

wellformed-policy1-strong p ⇒ C (list2FWpolicy (removeShadowRules2 p)) = C
(list2FWpolicy p)
apply (rule ext)
by (metis C-eq-RS2-mr C-eq-if-mr-eq wellformed-policy1-strong.simps(1) wp1n-RS2)

```

lemma *none-MT-rulesRS2*:

```

none-MT-rules C p ⇒ none-MT-rules C (removeShadowRules2 p)
by (auto simp: RS2Set none-MT-rulessubset)

```

lemma *CconcNone*:

$\text{dom } (C a) = \{\} \implies p \neq [] \implies C (\text{list2FWpolicy } (a \# p)) x = C (\text{list2FWpolicy } p) x$
apply (*case-tac* $p = []$, *simp-all*)
apply (*case-tac* $x \in \text{dom } (C (\text{list2FWpolicy}(p)))$)
apply (*metis* *Cdom2 list2FWpolicyconc*)
apply (*metis* *C.simps(4) map-add-dom-app-simps(2) inSet-not-MT list2FWpolicy-conc set-empty2*)
done

lemma *none-MT-rulesrd*[*rule-format*]:

$\text{none-MT-rules } C p \longrightarrow \text{none-MT-rules } C (\text{remdups } p)$
by (*induct* p , *simp-all*)

lemma *DARS3*[*rule-format*]:

$\text{DenyAll} \notin \text{set } p \longrightarrow \text{DenyAll} \notin \text{set } (\text{rm-MT-rules } C p)$
by (*induct* p , *simp-all*)

lemma *DAnMT*: $\text{dom } (C \text{DenyAll}) \neq \{\}$

by (*simp* *add: dom-def C.simps PolicyCombinators.PolicyCombinators*)

lemma *DAnMT2*: $C \text{DenyAll} \neq \text{Map.empty}$

by (*metis* *DAAux dom-eq-empty-conv empty-iff*)

lemma *wp1n-RS3*[*rule-format, simp*]:

$\text{wellformed-policy1-strong } p \longrightarrow \text{wellformed-policy1-strong } (\text{rm-MT-rules } C p)$
by (*induct* p , *simp-all* *add: DARS3 DAnMT*)

lemma *AILRS3*[*rule-format, simp*]:

$\text{all-in-list } p l \longrightarrow \text{all-in-list } (\text{rm-MT-rules } C p) l$
by (*induct* p , *simp-all*)

lemma *SCRS3*[*rule-format, simp*]:

$\text{singleCombinators } p \longrightarrow \text{singleCombinators}(\text{rm-MT-rules } C p)$
apply (*induct* p , *simp-all*)
subgoal for $a p$
apply(*case-tac* a , *simp-all*)
done
done

lemma *RS3subset*: $\text{set } (\text{rm-MT-rules } C p) \subseteq \text{set } p$

by (*induct* p , *auto*)

lemma *ANDRS3*[*simp*]:

$\text{singleCombinators } p \implies \text{allNetsDistinct } p \implies \text{allNetsDistinct } (\text{rm-MT-rules } C p)$

using *RS3subset SCRS3 aNDSsubset* **by** *blast*

lemma *nlpaux*: $x \notin \text{dom} (C b) \implies C (a \oplus b) x = C a x$
by (*metis C.simps(4) map-add-dom-app-simps(3)*)

lemma *notindom*[*rule-format*]:
 $a \in \text{set } p \longrightarrow x \notin \text{dom} (C (\text{list2FWpolicy } p)) \longrightarrow x \notin \text{dom} (C a)$
apply (*induct p, simp-all*)
by (*metis CConcStartA Cdom2 domIff empty-iff empty-set l2p-aux*)

lemma *C-eq-rd*[*rule-format*]:
 $p \neq [] \implies C (\text{list2FWpolicy} (\text{remdups } p)) = C (\text{list2FWpolicy } p)$
proof (*rule ext, induct p*)
case *Nil* **thus** *?case* **by** *simp*
next
case (*Cons y ys*) **thus** *?case*
proof (*cases ys = []*)
case *True* **thus** *?thesis* **by** *simp*
next
case *False* **thus** *?thesis* **using** *Cons*
apply (*simp*) **apply** (*rule conjI, rule impI*)
apply (*cases x \in dom (C (list2FWpolicy ys))*)
apply (*metis Cdom2 False list2FWpolicyconc*)
apply (*metis False domIff list2FWpolicyconc nlpaux notindom*)
apply (*rule impI*)
apply (*cases x \in dom (C (list2FWpolicy ys))*)
apply (*subgoal-tac x \in dom (C (list2FWpolicy (remdups ys)))*)
apply (*metis Cdom2 False list2FWpolicyconc remdups-eq-nil-iff*)
apply (*metis domIff*)
apply (*subgoal-tac x \notin dom (C (list2FWpolicy (remdups ys)))*)
apply (*metis False list2FWpolicyconc nlpaux remdups-eq-nil-iff*)
apply (*metis domIff*)
done
qed
qed

lemma *nMT-domMT*:
 $\neg \text{not-MT } C p \implies p \neq [] \implies r \notin \text{dom} (C (\text{list2FWpolicy } p))$
proof (*induct p*)
case *Nil* **thus** *?case* **by** *simp*
next
case (*Cons x xs*) **thus** *?case*
apply (*simp split: if-splits*)
apply (*cases xs = [], simp-all*)


```

    by (metis CconcNone domIff)
qed

lemma C-eq-RS3-aux[rule-format]:
  not-MT C p  $\implies$  C (list2FWpolicy p) x = C (list2FWpolicy (rm-MT-rules C p)) x
proof (induct p)
  case Nil thus ?case by simp
next
  case (Cons y ys)
  thus ?case
proof (cases not-MT C ys)
  case True thus ?thesis using Cons
    apply (simp) apply (rule conjI, rule impI, simp)
    apply (metis CconcNone True not-MTimpnotMT)
    apply (rule impI, simp)
    apply (cases x  $\in$  dom (C (list2FWpolicy ys)))
    apply (subgoal-tac x  $\in$  dom (C (list2FWpolicy (rm-MT-rules C ys))))
    apply (metis Cdom2 NMPPrm l2p-aux not-MTimpnotMT)
    apply (simp add: domIff)
    apply (subgoal-tac x  $\notin$  dom (C (list2FWpolicy (rm-MT-rules C ys))))
    apply (metis l2p-aux l2p-aux2 nlpaux)
    apply (metis domIff)
  done
next
  case False thus ?thesis using Cons False
proof (cases ys = [])
  case True thus ?thesis using Cons by (simp) (rule impI, simp)
next
  case False thus ?thesis
    using Cons False  $\langle \neg$  not-MT C ys  $\rangle$  apply (simp)
    by (metis SR3nMT l2p-aux list2FWpolicy.simps(2) nMT-domMT nlpaux)
qed
qed
qed

lemma C-eq-id:
  wellformed-policy1-strong p  $\implies$  C(list2FWpolicy (insertDeny p)) = C (list2FWpolicy
p)
  by (rule ext) (auto intro: C-eq-if-mr-eq elim: mr-iD)

lemma C-eq-RS3:
  not-MT C p  $\implies$  C(list2FWpolicy (rm-MT-rules C p)) = C (list2FWpolicy p)
  by (rule ext) (erule C-eq-RS3-aux[symmetric])

```

lemma *NMPrd*[*rule-format*]: *not-MT C p* \longrightarrow *not-MT C (remdups p)*
by (*induct p*) (*auto simp: NMPcharn*)

lemma *NMPDA*[*rule-format*]: *DenyAll \in set p* \longrightarrow *not-MT C p*
by (*induct p*, *simp-all add: DAnMT*)

lemma *NMPiD*[*rule-format*]: *not-MT C (insertDeny p)*
by (*simp add: DAiniD NMPDA*)

lemma *list2FWpolicy2list*[*rule-format*]: *C (list2FWpolicy(policy2list p)) = (C p)*
apply (*rule ext*)
apply (*induct-tac p, simp-all*)
by (*metis (no-types, lifting) Cdom2 CeqEnd CeqStart domIff nlpaux p2lNmt*)

lemmas *C-eq-Lemmas = none-MT-rulesRS2 none-MT-rulesrd SCp2l wp1n-RS2
wp1ID NMPiD wp1-eq
wp1alternative-RS1 p2lNmt list2FWpolicy2list wellformed-policy3-charn
wauux2*

lemmas *C-eq-subst-Lemmas = C-eq-sorted C-eq-sortedQ C-eq-RS2 C-eq-rd C-eq-RS3
C-eq-id*

lemma *C-eq-All-untilSorted*:

*DenyAll \in set(policy2list p) \implies all-in-list(policy2list p) l \implies allNetsDistinct(policy2list
p) \implies*

*C (list2FWpolicy
(FWNormalisationCore.sort
(removeShadowRules2 (remdups (rm-MT-rules C
(insertDeny (removeShadowRules1 (policy2list p)))))) l)) =
C p*

apply (*subst C-eq-sorted,simp-all add: C-eq-Lemmas*)

apply (*subst C-eq-RS2,simp-all add: C-eq-Lemmas*)

apply (*subst C-eq-rd,simp-all add: C-eq-Lemmas*)

apply (*subst C-eq-RS3,simp-all add: C-eq-Lemmas*)

apply (*subst C-eq-id,simp-all add: C-eq-Lemmas*)

done

lemma *C-eq-All-untilSortedQ*:

*DenyAll \in set(policy2list p) \implies all-in-list(policy2list p) l \implies allNetsDistinct(policy2list
p) \implies*

*C (list2FWpolicy
(qsort (removeShadowRules2 (remdups (rm-MT-rules C
(insertDeny (removeShadowRules1 (policy2list p)))))) l)) =
C p*

```

apply (subst C-eq-sortedQ,simp-all add: C-eq-Lemmas)
apply (subst C-eq-RS2,simp-all add: C-eq-Lemmas)
apply (subst C-eq-rd,simp-all add: C-eq-Lemmas)
apply (subst C-eq-RS3,simp-all add: C-eq-Lemmas)
apply (subst C-eq-id,simp-all add: C-eq-Lemmas)
done

```

lemma *C-eq-All-untilSorted-withSimps*:

```

DenyAll∈set(policy2list p) ⇒ all-in-list(policy2list p) l ⇒ allNetsDistinct (policy2list
p) ⇒
  C (list2FWpolicy
    (FWNormalisationCore.sort
      (removeShadowRules2 (remdups (rm-MT-rules C
        (insertDeny (removeShadowRules1 (policy2list p)))))) l)) =
  C p
by (simp-all add: C-eq-Lemmas C-eq-subst-Lemmas)

```

lemma *C-eq-All-untilSorted-withSimpsQ*:

```

DenyAll∈set(policy2list p) ⇒ all-in-list(policy2list p) l ⇒ allNetsDistinct(policy2list
p) ⇒
  C (list2FWpolicy
    (qsort (removeShadowRules2 (remdups (rm-MT-rules C
      (insertDeny (removeShadowRules1 (policy2list p)))))) l)) =
  C p
by (simp-all add: C-eq-Lemmas C-eq-subst-Lemmas)

```

lemma *InDomConc[rule-format]*:

```

p ≠ [] → x ∈ dom (C (list2FWpolicy (p))) → x ∈ dom (C (list2FWpolicy (a#p)))
apply (induct p, simp-all)
subgoal for a' p
  apply (case-tac p = [], simp-all add: dom-def C.simps)
  done
done

```

lemma *not-in-member[rule-format]*: member a b → x ∉ dom (C b) → x ∉ dom (C a)

```

by (induct b) (simp-all add: dom-def C.simps)

```

lemma *src-in-sdnets[rule-format]*:

```

¬ member DenyAll x → p ∈ dom (C x) → subnetsOfAdr (src p) ∩ (fst-set (sdnets
x)) ≠ {}
apply (induct rule: Combinators.induct)
  apply (simp-all add: fst-set-def subnetsOfAdr-def PLemmas fst-set-def)

```

```

apply (intro impI)
subgoal for x1 x2
  apply (case-tac p ∈ dom (C x2))
  apply (rule subnetAux)
  apply (auto simp: PLemmas fst-set-def)
done
done

```

```

lemma dest-in-sdnets[rule-format]:
   $\neg \text{member DenyAll } x \longrightarrow p \in \text{dom } (C \ x) \longrightarrow \text{subnetsOfAdr } (\text{dest } p) \cap (\text{snd-set } (\text{sdnets } x)) \neq \{\}$ 
  apply (induct rule: Combinators.induct)
  apply (simp-all add: snd-set-def subnetsOfAdr-def PLemmas)
  apply (intro impI)
  apply (simp add: snd-set-def)
  subgoal for x1 x2
    apply (case-tac p ∈ dom (C x2))
    apply (rule subnetAux)
    apply (auto simp: PLemmas)
  done
done

```

```

lemma sdnets-in-subnets[rule-format]:
   $p \in \text{dom } (C \ x) \longrightarrow \neg \text{member DenyAll } x \longrightarrow$ 
   $(\exists (a,b) \in \text{sdnets } x. a \in \text{subnetsOfAdr } (\text{src } p) \wedge b \in \text{subnetsOfAdr } (\text{dest } p))$ 
  apply (rule Combinators.induct)
  apply (simp-all add: PLemmas subnetsOfAdr-def)
  apply (intro impI, simp)
  subgoal for x1 x2
    apply (case-tac p ∈ dom (C (x2)))
    apply (auto simp: PLemmas subnetsOfAdr-def)
  done
done

```

```

lemma disjSD-no-p-in-both[rule-format]:
   $\text{disjSD-2 } x \ y \Longrightarrow \neg \text{member DenyAll } x \Longrightarrow \neg \text{member DenyAll } y \Longrightarrow p \in \text{dom}(C \ x)$ 
 $\Longrightarrow p \in \text{dom}(C \ y) \Longrightarrow$ 
  False
  apply (rule-tac A = sdnets x and B = sdnets y and D = src p and F = dest p in tndFalse)
  by (auto simp: dest-in-sdnets src-in-sdnets sdnets-in-subnets disjSD-2-def)

```

```

lemma list2FWpolicy-eq:
   $zs \neq [] \Longrightarrow C (\text{list2FWpolicy } (x \oplus y \# z)) \ p = C (x \oplus \text{list2FWpolicy } (y \# z)) \ p$ 

```

by (metis ConcAssoc l2p-aux list2FWpolicy.simps(2))

lemma dom-sep[rule-format]:

$x \in \text{dom} (C (\text{list2FWpolicy } p)) \longrightarrow x \in \text{dom} (C (\text{list2FWpolicy}(\text{separate } p)))$

proof (induct p rule: separate.induct, simp-all, goal-cases)

case (1 v va y z) then show ?case

apply (intro conjI impI)

apply (simp, drule mp)

apply (case-tac $x \in \text{dom} (C (\text{DenyAllFromTo } v \text{ va}))$)

apply (metis CConcStartA domIff l2p-aux2 list2FWpolicyconc not-Cons-self)

apply (metis Conc-not-MT domIff list2FWpolicy-eq, simp)

by (metis InDomConc domIff list.simps(3) list2FWpolicyconc nlpaux sepnMT)

next

case (2 v va vb y z)

assume * : $\{v, va\} = \text{first-bothNet } y \Longrightarrow$

$x \in \text{dom} (C (\text{list2FWpolicy} (\text{AllowPortFromTo } v \text{ va } vb \oplus y \# z))) \longrightarrow$

$x \in \text{dom} (C (\text{list2FWpolicy} (\text{separate} (\text{AllowPortFromTo } v \text{ va } vb \oplus y \#$

$z))))$

and **: $\{v, va\} \neq \text{first-bothNet } y \Longrightarrow$

$x \in \text{dom}(C(\text{list2FWpolicy}(y\#z))) \longrightarrow x \in \text{dom}$

$(C(\text{list2FWpolicy}(\text{separate}(y\#z))))$

show ?case

apply (insert * **, rule impI | rule conjI)+

apply (simp, case-tac $x \in \text{dom} (C (\text{AllowPortFromTo } v \text{ va } vb))$)

apply (metis CConcStartA domIff l2p-aux2 list2FWpolicyconc not-Cons-self)

apply (subgoal-tac $x \in \text{dom} (C (\text{list2FWpolicy} (y \# z)))$)

apply (metis CConcStartA Cdom2 domIff l2p-aux2 list2FWpolicyconc nlpaux)

apply (simp add: dom-def C.simps)

apply (intro impI, simp-all)

apply (case-tac $x \in \text{dom} (C (\text{AllowPortFromTo } v \text{ va } vb)), \text{simp-all}$)

by (metis Cdom2 domIff l2p-aux list2FWpolicy.simps(3) nlpaux sepnMT)

next

case (3 v va y z)

assume * : $(\text{first-bothNet } v = \text{first-bothNet } y \Longrightarrow$

$x \in \text{dom} (C (\text{list2FWpolicy} ((v \oplus va) \oplus y \# z))) \longrightarrow$

$x \in \text{dom} (C (\text{list2FWpolicy} (\text{separate} ((v \oplus va) \oplus y \# z))))$)

and **: $(\text{first-bothNet } v \neq \text{first-bothNet } y \Longrightarrow$

$x \in \text{dom}(C(\text{list2FWpolicy}(y\#z))) \longrightarrow x \in \text{dom} (C (\text{list2FWpolicy} (\text{separate}$

$(y \# z))))$)

show ?case

apply (insert * **, rule conjI | rule impI)+

apply (simp, drule mp)

apply (case-tac $x \in \text{dom} (C ((v \oplus va)))$)

apply (metis C.simps(4) CConcStartA ConcAssoc domIff list2FWpolicy2list

$list2FWpolicyconc\ p2lNmt$
apply $simp-all$
apply ($metis\ Conc-not-MT\ domIff\ list2FWpolicy-eq$)
by ($metis\ CConcStartA\ Conc-not-MT\ InDomConc\ domIff\ nlpaux\ sepnMT$)
qed

lemma $domdConcStart[rule-format]$:
 $x \in dom\ (C\ (list2FWpolicy\ (a\ \#b))) \longrightarrow x \notin dom\ (C\ (list2FWpolicy\ b)) \longrightarrow x \in dom\ (C\ (a))$
by ($induct\ b,\ simp-all$) ($auto\ simp:\ PLemmas$)

lemma $sep-dom2-aux$:
 $x \in dom\ (C\ (list2FWpolicy\ (a\ \oplus\ y\ \#z))) \implies x \in dom\ (C\ (a\ \oplus\ list2FWpolicy\ (y\ \#z)))$
by ($auto$)[1] ($metis\ list2FWpolicy-eq\ p2lNmt$)

lemma $sep-dom2-aux2$:
 $x \in dom\ (C\ (list2FWpolicy\ (separate\ (y\ \#z)))) \longrightarrow x \in dom\ (C\ (list2FWpolicy\ (y\ \#z))) \implies$
 $x \in dom\ (C\ (list2FWpolicy\ (a\ \#separate\ (y\ \#z)))) \implies x \in dom\ (C\ (list2FWpolicy\ (a\ \oplus\ y\ \#z)))$
by ($metis\ CConcStartA\ InDomConc\ domdConcStart\ list.simps(2)\ list2FWpolicy.simps(2)\ list2FWpolicyconc$)

lemma $sep-dom2[rule-format]$:
 $x \in dom\ (C\ (list2FWpolicy\ (separate\ p))) \longrightarrow x \in dom\ (C\ (list2FWpolicy\ (p)))$
by ($rule\ separate.induct$) ($simp-all\ add:\ sep-dom2-aux\ sep-dom2-aux2$)

lemma $sepDom$: $dom\ (C\ (list2FWpolicy\ p)) = dom\ (C\ (list2FWpolicy\ (separate\ p)))$
apply ($rule\ equalityI$)
by ($rule\ subsetI,\ (erule\ dom-sep|erule\ sep-dom2))+$

lemma $C-eq-s-ext[rule-format]$:
 $p \neq [] \longrightarrow C\ (list2FWpolicy\ (separate\ p))\ a = C\ (list2FWpolicy\ p)\ a$
proof ($induct\ rule:\ separate.induct,\ goal-cases$)
case ($1\ x$) **thus** $?case$
apply $simp$
apply ($cases\ x = []$)
apply ($metis\ l2p-aux2\ separate.simps(5)$)
apply $simp$
apply ($cases\ a \in dom\ (C\ (list2FWpolicy\ x))$)
apply ($subgoal-tac\ a \in dom\ (C\ (list2FWpolicy\ (separate\ x)))$)
apply ($metis\ Cdom2\ list2FWpolicyconc\ sepDom\ sepnMT$)
apply ($metis\ sepDom$)

```

apply (subgoal-tac a  $\notin$  dom (C (list2FWpolicy (separate x))))
apply (subst list2FWpolicyconc,simp add: sepnMT)
apply (subst list2FWpolicyconc,simp add: sepnMT)
apply (metis nlpaux sepDom)
apply (metis sepDom)
done
next
case (2 v va y z) thus ?case
apply (cases z = [], simp-all)
apply (rule conjI|rule impI|simp)+
apply (subst list2FWpolicyconc)
apply (metis not-Cons-self sepnMT)
apply (metis C.simps(4) CConcStartaux Cdom2 domIff)
apply (rule conjI|rule impI|simp)+
apply (erule list2FWpolicy-eq)
apply (rule impI, simp)
apply (subst list2FWpolicyconc)
apply (metis list.simps(2) sepnMT)
by (metis C.simps(4) CConcStartaux Cdom2 domIff)
next
case (3 v va vb y z) thus ?case
apply (cases z = [], simp-all)
apply (rule conjI|rule impI|simp)+
apply (subst list2FWpolicyconc)
apply (metis not-Cons-self sepnMT)
apply (metis C.simps(4) CConcStartaux Cdom2 domIff)
apply (rule conjI|rule impI|simp)+
apply (erule list2FWpolicy-eq)
apply (rule impI, simp)
apply (subst list2FWpolicyconc)
apply (metis list.simps(2) sepnMT)
by (metis C.simps(4) CConcStartaux Cdom2 domIff)
next
case (4 v va y z) thus ?case
apply (cases z = [], simp-all)
apply (rule conjI|rule impI|simp)+
apply (subst list2FWpolicyconc)
apply (metis not-Cons-self sepnMT)
apply (metis C.simps(4) CConcStartaux Cdom2 domIff)
apply (rule conjI|rule impI|simp)+
apply (erule list2FWpolicy-eq)
apply (rule impI, simp)
apply (subst list2FWpolicyconc)
apply (metis list.simps(2) sepnMT)

```

by (*metis C.simps(4) CConcStartaux Cdom2 domIff*)
 next
 case 5 thus ?case by *simp*
 next
 case 6 thus ?case by *simp*
 next
 case 7 thus ?case by *simp*
 next
 case 8 thus ?case by *simp*
 qed

lemma *C-eq-s*:

$p \neq [] \implies C (\text{list2FWpolicy } (\text{separate } p)) = C (\text{list2FWpolicy } p)$
 apply (*rule ext*) using *C-eq-s-ext* by *blast*

lemma *sortnMTQ*: $p \neq [] \implies \text{qsort } p \ l \neq []$
 by (*metis set-sortQ setnMT*)

lemmas *C-eq-Lemmas-sep* =

C-eq-Lemmas sortnMT sortnMTQ RS2-NMT NMPrd not-MTimpnotMT

lemma *C-eq-until-separated*:

$\text{DenyAll} \in \text{set}(\text{policy2list } p) \implies \text{all-in-list}(\text{policy2list } p)l \implies \text{allNetsDistinct}$
 $(\text{policy2list } p) \implies$
 $C (\text{list2FWpolicy}$
 $(\text{separate}$
 $(\text{FWNormalisationCore.sort}$
 $(\text{removeShadowRules2 } (\text{remdups } (\text{rm-MT-rules } C$
 $(\text{insertDeny } (\text{removeShadowRules1 } (\text{policy2list } p)))))) l))) =$
 $C p$
 by (*simp add: C-eq-All-untilSorted-withSimps C-eq-s wellformed1-alternative-sorted*
wp1ID wp1n-RS2)

lemma *C-eq-until-separatedQ*:

$\text{DenyAll} \in \text{set}(\text{policy2list } p) \implies \text{all-in-list}(\text{policy2list } p)l \implies \text{allNetsDistinct}(\text{policy2list}$
 $p) \implies$
 $C (\text{list2FWpolicy}$
 $(\text{separate } (\text{qsort } (\text{removeShadowRules2 } (\text{remdups } (\text{rm-MT-rules } C$
 $(\text{insertDeny } (\text{removeShadowRules1 } (\text{policy2list } p)))))) l))) =$
 $C p$
 by (*simp add: C-eq-All-untilSorted-withSimpsQ C-eq-s sortnMTQ wp1ID wp1n-RS2*)

lemma *domID[rule-format]*: $p \neq [] \wedge x \in \text{dom}(C(\text{list2FWpolicy } p)) \implies$
 $x \in \text{dom} (C(\text{list2FWpolicy}(\text{insertDenies } p)))$


```

proof(induct p)
  case Nil then show ?case by simp
next
  case (Cons a p) then show ?case
  proof(cases p=[],goal-cases)
    case 1 then show ?case
      apply(simp) apply(rule impI)
      apply (cases a, simp-all)
      apply (simp-all add: C.simps dom-def)+
      by auto
  next
  case 2 then show ?case
  proof(cases x ∈ dom(C(list2FWpolicy p)), goal-cases)
    case 1 then show ?case
      apply simp apply (rule impI)
      apply (cases a, simp-all)
      using InDomConc idNMT apply blast
      apply (rule InDomConc, simp-all add: idNMT)+
      done
  next
  case 2 then show ?case
    apply simp apply (rule impI)
  proof(cases x ∈ dom (C (list2FWpolicy (insertDenies p))), goal-cases)
    case 1 then show ?case
    proof(induct a)
      case DenyAll then show ?case by simp
    next
      case (DenyAllFromTo src dest) then show ?case
        apply simp by( rule InDomConc, simp add: idNMT)
    next
      case (AllowPortFromTo src dest port) then show ?case
        apply simp by(rule InDomConc, simp add: idNMT)
    next
      case (Conc - -) then show ?case
        apply simp by(rule InDomConc, simp add: idNMT)
    qed
  next
  case 2 then show ?case
  proof (induct a)
    case DenyAll then show ?case by simp
  next
    case (DenyAllFromTo src dest) then show ?case
      by(simp,metis domIff CConcStartA list2FWpolicyconc nlpaux Cdom2)
  next

```

```

    case (AllowPortFromTo src dest port) then show ?case
      by (simp,metis domIff CConcStartA list2FWpolicyconc nlpaux Cdom2)
  next
    case (Conc - -) then show ?case
      by (simp,metis CConcStartA Cdom2 domIff domdConcStart)
  qed
  qed
  qed
  qed
  qed

```

lemma *DA-is-deny*:

```

  x ∈ dom (C (DenyAllFromTo a b ⊕ DenyAllFromTo b a ⊕ DenyAllFromTo a b)) ⇒
  C (DenyAllFromTo a b ⊕ DenyAllFromTo b a ⊕ DenyAllFromTo a b) x = Some (deny ())
  apply (case-tac x ∈ dom (C (DenyAllFromTo a b)))
  apply (simp-all add: PLemmas)
  apply (simp-all split: if-splits)
  done

```

lemma *iDdomAux*[*rule-format*]:

```

  p ≠ [] → x ∉ dom (C (list2FWpolicy p)) →
  x ∈ dom (C (list2FWpolicy (insertDenies p))) →
  C (list2FWpolicy (insertDenies p)) x = Some (deny ())
  proof (induct p)
  case Nil thus ?case by simp
  next
  case (Cons y ys) thus ?case
  proof (cases y)
  case DenyAll then show ?thesis by simp
  next
  case (DenyAllFromTo a b) then show ?thesis using DenyAllFromTo Cons
    apply simp
    apply (intro impI)
  proof (cases ys = [], goal-cases)
  case 1 then show ?case by (simp add: DA-is-deny)
  next
  case 2 then show ?case
    apply simp
    apply (drule mp)
    apply (metis DenyAllFromTo InDomConc)
    apply (cases x ∈ dom (C (list2FWpolicy (insertDenies ys))), simp-all)
    apply (metis Cdom2 DenyAllFromTo idNMT list2FWpolicyconc)
    apply (subgoal-tac C (list2FWpolicy (DenyAllFromTo a b ⊕

```

```

DenyAllFromTo b a  $\oplus$  DenyAllFromTo a b#insertDenies ys))
x =
C ((DenyAllFromTo a b  $\oplus$  DenyAllFromTo b a  $\oplus$  DenyAllFromTo
a b)) x )
  apply simp
  apply (rule DA-is-deny)
  apply (metis DenyAllFromTo domdConcStart)
  apply (metis DenyAllFromTo l2p-aux2 list2FWpolicyconc nlpaux)
  done
qed
next
case (AllowPortFromTo a b c) then show ?thesis using Cons AllowPortFromTo
proof (cases ys = [], goal-cases)
  case 1 then show ?case
    apply simp
    apply (intro impI)
    apply (subgoal-tac x  $\in$  dom (C (DenyAllFromTo a b  $\oplus$  DenyAllFromTo b a)))
    apply (simp-all add: PLemmas)
    apply (simp split: if-splits, auto)
    done
  next
  case 2 then show ?case
    apply simp
    apply (intro impI)
    apply (drule mp)
    apply (metis AllowPortFromTo InDomConc)
    apply (cases x  $\in$  dom (C (list2FWpolicy (insertDenies ys))))
    apply simp-all
    apply (metis AllowPortFromTo Cdom2 idNMT list2FWpolicyconc)
    apply (subgoal-tac C (list2FWpolicy (DenyAllFromTo a b  $\oplus$ 
DenyAllFromTo b a  $\oplus$  AllowPortFromTo a b c#insertDenies
ys)) x =
C ((DenyAllFromTo a b  $\oplus$  DenyAllFromTo b a)) x )
      apply simp
      defer 1
      apply (metis AllowPortFromTo CConcStartA ConcAssoc idNMT list2FWpoli-
cyconc nlpaux)
      apply (simp add: PLemmas, simp split: if-splits, auto)
      done
    qed
  next
  case (Conc a b) then show ?thesis
proof (cases ys = [], goal-cases)
  case 1 then show ?case

```

```

apply simp
apply (rule impI)+
apply (subgoal-tac x ∈ dom (C (DenyAllFromTo (first-srcNet a)
      (first-destNet a) ⊕ DenyAllFromTo (first-destNet a) (first-srcNet
a))))))
  apply (simp-all add: PLemmas)
  apply (simp split: if-splits, auto)
  done
next
case 2 then show ?case
  apply simp
  apply (intro impI)
  apply (cases x ∈ dom (C (list2FWpolicy (insertDenies ys))))
  apply (metis Cdom2 Conc Cons InDomConc idNMT list2FWpolicyconc)
  apply (subgoal-tac C (list2FWpolicy (DenyAllFromTo (first-srcNet a)
      (first-destNet a) ⊕ DenyAllFromTo (first-destNet a) (first-srcNet
a)
      ⊕ a ⊕ b#insertDenies ys)) x =
      C ((DenyAllFromTo (first-srcNet a) (first-destNet a) ⊕
      DenyAllFromTo (first-destNet a) (first-srcNet a) ⊕ a ⊕
b)) x)
  apply simp
  defer 1
  apply (metis Conc l2p-aux2 list2FWpolicyconc nlpaux)
  apply (subgoal-tac C((DenyAllFromTo (first-srcNet a)
      (first-destNet a) ⊕ DenyAllFromTo (first-destNet a)
      (first-srcNet a) ⊕ a ⊕ b)) x =
      C((DenyAllFromTo (first-srcNet a) (first-destNet a) ⊕
      DenyAllFromTo (first-destNet a) (first-srcNet a))) x )
  apply simp
  defer 1
  apply (metis CConcStartA Conc ConcAssoc nlpaux)
  apply (simp add: PLemmas, simp split: if-splits, auto)
  done
qed
qed
qed

```

lemma *iD-isD*[*rule-format*]:

```

p ≠ [] → x ∉ dom (C (list2FWpolicy p)) →
C (DenyAll ⊕ list2FWpolicy (insertDenies p)) x = C DenyAll x
apply (case-tac x ∈ dom (C (list2FWpolicy (insertDenies p))))
apply (simp add: Cdom2 PLemmas(1) deny-all-def iDdomAux)

```

by (simp add: nlpaux)

lemma *inDomConc*: $\llbracket x \notin \text{dom } (C a); x \notin \text{dom } (C (\text{list2FWpolicy } p)) \rrbracket \implies$
 $x \notin \text{dom } (C (\text{list2FWpolicy}(a\#p)))$

by (metis domdConcStart)

lemma *domsdisj*[rule-format]:

$p \neq [] \longrightarrow (\forall x s. s \in \text{set } p \wedge x \in \text{dom } (C A) \longrightarrow x \notin \text{dom } (C s)) \longrightarrow y \in \text{dom } (C$
 $A) \longrightarrow$

$y \notin \text{dom } (C (\text{list2FWpolicy } p))$

proof (induct p)

case Nil show ?case by simp

next

case (Cons a p) then show ?case

apply (case-tac p = [])

apply fastforce

by (meson domdConcStart list.set-intros(1) list.set-intros(2))

qed

lemma *isSepaux*:

$p \neq [] \implies \text{noDenyAll } (a \# p) \implies \text{separated } (a \# p) \implies$

$x \in \text{dom } (C (\text{DenyAllFromTo } (\text{first-srcNet } a) (\text{first-destNet } a) \oplus$

$\text{DenyAllFromTo } (\text{first-destNet } a) (\text{first-srcNet } a) \oplus a)) \implies$

$x \notin \text{dom } (C (\text{list2FWpolicy } p))$

apply (rule-tac A = ($\text{DenyAllFromTo } (\text{first-srcNet } a) (\text{first-destNet } a) \oplus$

$\text{DenyAllFromTo } (\text{first-destNet } a) (\text{first-srcNet } a) \oplus a)$ in domsdisj)

apply simp-all

by (metis Combinators.distinct(1) FWNormalisationCore.member.simps(1)

$\text{FWNormalisationCore.member.simps}(3)$ disjSD2aux disjSD-no-p-in-both noDA)

lemma *none-MT-rulessep*[rule-format]: *none-MT-rules* C p \longrightarrow *none-MT-rules* C
(separate p)

apply(induct p rule: separate.induct)

by (simp-all add: C.simps map-add-le-mapE map-le-antisym)

lemma *dom-id*:

$\text{noDenyAll}(a\#p) \implies \text{separated}(a\#p) \implies p \neq [] \implies x \notin \text{dom}(C(\text{list2FWpolicy } p)) \implies$
 $x \in \text{dom } (C a) \implies$

$x \notin \text{dom } (C (\text{list2FWpolicy } (\text{insertDenies } p)))$

apply (rule-tac a = a in isSepaux, simp-all)

using idNMT **apply** blast

using noDAID **apply** blast

using id-aux4 noDA1eq sepNetsID **apply** blast

by (*metis list.set-intros(1) list.set-intros(2) list2FWpolicy.simps(2) list2FWpolicy.simps(3) notindom*)

lemma *C-eq-iD-aux2*[*rule-format*]:

noDenyAll1 p \longrightarrow *separated p* \longrightarrow *p* \neq [] \longrightarrow *x* \in *dom* (*C* (*list2FWpolicy p*)) \longrightarrow
C(*list2FWpolicy* (*insertDenies p*)) *x* = *C*(*list2FWpolicy p*) *x*

proof (*induct p*)

case *Nil* **thus** ?*case* **by** *simp*

next

case (*Cons y ys*) **thus** ?*case* **using** *Cons*

proof (*cases y*)

case *DenyAll* **thus** ?*thesis* **using** *Cons DenyAll* **apply** *simp*

apply (*case-tac ys* = [], *simp-all*)

apply (*case-tac x* \in *dom* (*C* (*list2FWpolicy ys*)), *simp-all*)

apply (*metis Cdom2 domID idNMT list2FWpolicyconc noDA1eq*)

apply (*metis DenyAll iD-isD idNMT list2FWpolicyconc nlpaux*)

done

next

case (*DenyAllFromTo a b*) **thus** ?*thesis* **using** *Cons* **apply** *simp*

apply (*rule impI|rule allI|rule conjI|simp*)+

apply (*case-tac ys* = [], *simp-all*)

apply (*metis Cdom2 ConcAssoc DenyAllFromTo*)

apply (*case-tac x* \in *dom* (*C* (*list2FWpolicy ys*)), *simp-all*)

apply (*simp add: Cdom2 domID idNMT l2p-aux noDA1eq*)

apply (*case-tac x* \in *dom* (*C* (*list2FWpolicy* (*insertDenies ys*))))

apply (*meson Combinators.distinct(1) FWNORMALISATIONCORE.member.simps(3)*)

dom-id domdConcStart

noDenyAll.simps(1) separated.simps(1))

by (*metis Cdom2 DenyAllFromTo domIff dom-def domdConcStart l2p-aux l2p-aux2 nlpaux*)

next

case (*AllowPortFromTo a b c*) **thus** ?*thesis*

using *AllowPortFromTo Cons* **apply** *simp*

apply (*rule impI|rule allI|rule conjI|simp*)+

apply (*case-tac ys* = [], *simp-all*)

apply (*metis Cdom2 ConcAssoc AllowPortFromTo*)

apply (*case-tac x* \in *dom* (*C* (*list2FWpolicy ys*)), *simp-all*)

apply (*simp add: Cdom2 domID idNMT list2FWpolicyconc noDA1eq*)

apply (*case-tac x* \in *dom* (*C* (*list2FWpolicy* (*insertDenies ys*))))

apply (*meson Combinators.distinct(3) FWNORMALISATIONCORE.member.simps(4)*)

dom-id domdConcStart noDenyAll.simps(1) separated.simps(1))

by (*metis Cdom2 ConcAssoc l2p-aux list2FWpolicy.simps(2) nlpaux*)

next

case (*Conc a b*) **thus** ?*thesis* **using** *Cons Conc*

```

apply simp
apply (rule impI|rule allI|rule conjI|simp)+
apply (case-tac ys = [], simp-all)
apply (metis Cdom2 ConcAssoc Conc)
apply (case-tac x ∈ dom (C (list2FWpolicy ys)),simp-all)
apply (simp add: Cdom2 domID idNMT list2FWpolicyconc noDA1eq)
apply (case-tac x ∈ dom (C (a ⊕ b)))
apply (case-tac x ∉ dom (C (list2FWpolicy (insertDenies ys))),simp-all)
apply (simp add: Cdom2 domIff idNMT list2FWpolicyconc nlpaux)
apply (metis FWNormalisationCore.member.simps(1) dom-id noDenyAll.simps(1))
separated.simps(1))
by (simp add: inDomConc)
qed
qed

```

lemma *C-eq-iD*:

```

separated p ⇒ noDenyAll1 p ⇒ wellformed-policy1-strong p ⇒
C (list2FWpolicy (insertDenies p)) = C (list2FWpolicy p)
by (rule ext) (metis CConcStartA C-eq-iD-aux2 DAAux wp1-alternative-not-mt)
wp1n-tl)

```

lemma *noDAsortQ[rule-format]*: *noDenyAll1 p → noDenyAll1 (qsort p l)*

```

apply (case-tac p,simp-all, rename-tac a list)
subgoal for a list
apply (case-tac a = DenyAll,simp-all)
using nDAeqSet set-sortQ apply blast
apply (rule impI,rule noDA1eq)
apply (subgoal-tac noDenyAll (a#list))
apply (metis append-Cons append-Nil nDAeqSet qsort.simps(2) set-sortQ)
by (case-tac a, simp-all)
done

```

lemma *NetsCollectedSortQ*:

```

distinct p ⇒ noDenyAll1 p ⇒ all-in-list p l ⇒ singleCombinators p ⇒
NetsCollected (qsort p l)
by (metis NetsCollectedSorted SC3Q all-in-list.elims(2) all-in-list.simps(1))
all-in-list.simps(2))
all-in-listAppend all-in-list-sublist noDAsortQ qsort.simps(1) qsort.simps(2)
singleCombinatorsConc sort-is-sortedQ)

```

lemmas *CLemmas = nMTSort nMTSortQ none-MT-rulesRS2 none-MT-rulesrd*
noDAsort noDAsortQ nDASC wp1-eq wp1ID
SCp2l ANDSep wp1n-RS2

*OTNSEp OTNSC noDA1sep wp1-alternativesep wellformed-eq
wellformed1-alternative-sorted*

lemmas *C-eqLemmas-id = CLemmas NC2Sep NetsCollectedSep
NetsCollectedSort NetsCollectedSortQ separatedNC*

lemma *C-eq-Until-InsertDenies:*

*DenyAll ∈ set(policy2list p) ⇒ all-in-list(policy2list p)l ⇒ allNetsDistinct(policy2list
p) ⇒*
*C (list2FWpolicy
(insertDenies
(separate
(FWNormalisationCore.sort
(removeShadowRules2 (remdups (rm-MT-rules C
(insertDeny (removeShadowRules1 (policy2list p)))))) l)))) =*
C p
apply (*subst C-eq-iD,simp-all add: C-eqLemmas-id*)
apply (*rule C-eq-until-separated, simp-all*)
done

lemma *C-eq-Until-InsertDeniesQ:*

*DenyAll ∈ set(policy2list p) ⇒ all-in-list(policy2list p)l ⇒ allNetsDistinct(policy2list
p) ⇒*
*C(list2FWpolicy
(insertDenies
(separate (qsort (removeShadowRules2 (remdups (rm-MT-rules C
(insertDeny (removeShadowRules1 (policy2list p)))))) l)))) =*
C p
apply (*subst C-eq-iD,simp-all add: C-eqLemmas-id*)
apply (*metis WP1rd set-qsort wellformed1-sortedQ wellformed-eq wp1ID
wp1-alternativesep wp1-aux1aa wp1n-RS2 wp1n-RS3*)
by (*rule C-eq-until-separatedQ, simp-all*)

lemma *C-eq-RD-aux[rule-format]: C (p) x = C (removeDuplicates p) x*

apply (*induct p,simp-all*)
by (*metis Cdom2 domIff nlpaux not-in-member*)

lemma *C-eq-RAD-aux[rule-format]:*

p ≠ [] → C (list2FWpolicy p) x = C (list2FWpolicy (removeAllDuplicates p)) x
proof (*induct p*)
case Nil show ?case by simp
next
case (Cons a p) show ?case

apply (*case-tac* $p = []$, *simp-all*)
apply (*metis* *C-eq-RD-aux*)
apply (*subst* *list2FWpolicyconc*, *simp*)
apply (*case-tac* $x \in \text{dom} (C (\text{list2FWpolicy } p))$)
apply (*simp add*: *Cdom2 Cons.hyphs domIff l2p-aux rADnMT*)
by (*metis* *C-eq-RD-aux Cons.hyphs domIff list2FWpolicyconc nlpaux rADnMT*)
qed

lemma *C-eq-RAD*:

$p \neq [] \implies C (\text{list2FWpolicy } p) = C (\text{list2FWpolicy } (\text{removeAllDuplicates } p))$
by (*rule ext,erule C-eq-RAD-aux*)

lemma *C-eq-compile*:

$\text{DenyAll} \in \text{set}(\text{policy2list } p) \implies \text{all-in-list}(\text{policy2list } p)l \implies \text{allNetsDistinct}(\text{policy2list } p) \implies$

$C (\text{list2FWpolicy } (\text{removeAllDuplicates } (\text{insertDenies } (\text{separate } (\text{FWNormalisationCore.sort } (\text{removeShadowRules2 } (\text{remdups } (\text{rm-MT-rules } C (\text{insertDeny } (\text{removeShadowRules1 } (\text{policy2list } p)))))) l)))))) =$
 $C p$

apply (*subst* *C-eq-RAD[symmetric]*)
apply (*rule idNMT, simp add*: *C-eqLemmas-id*)
by (*rule C-eq-Until-InsertDenies, simp-all*)

lemma *C-eq-compileQ*:

$\text{DenyAll} \in \text{set}(\text{policy2list } p) \implies \text{all-in-list}(\text{policy2list } p)l \implies \text{allNetsDistinct}(\text{policy2list } p) \implies$

$C (\text{list2FWpolicy } (\text{removeAllDuplicates } (\text{insertDenies } (\text{separate } (\text{qsort } (\text{removeShadowRules2 } (\text{remdups } (\text{rm-MT-rules } C (\text{insertDeny } (\text{removeShadowRules1 } (\text{policy2list } p)))))) l)))))) =$
 $C p$

apply (*subst* *C-eq-RAD[symmetric],rule idNMT*)
apply (*metis* *WP1rd sepnMT sortnMTQ wellformed-policy1-strong.simps(1) wp1ID wp1n-RS2 wp1n-RS3*)
by (*rule C-eq-Until-InsertDeniesQ, simp-all*)

lemma *C-eq-normalize*:

$\text{DenyAll} \in \text{set} (\text{policy2list } p) \implies \text{allNetsDistinct} (\text{policy2list } p) \implies \text{all-in-list}(\text{policy2list } p)(\text{Nets-List } p) \implies$

$C (\text{list2FWpolicy } (\text{normalize } p)) = C p$
unfolding *normalize-def*
by (*simp add: C-eq-compile*)

lemma *C-eq-normalizeQ*:

$\text{DenyAll} \in \text{set } (\text{policy2list } p) \implies \text{allNetsDistinct } (\text{policy2list } p) \implies$
 $\text{all-in-list } (\text{policy2list } p) (\text{Nets-List } p) \implies$
 $C (\text{list2FWpolicy } (\text{normalizeQ } p)) = C p$
by (*simp add: normalizeQ-def C-eq-compileQ*)

lemma *domSubset3*: $\text{dom } (C (\text{DenyAll} \oplus x)) = \text{dom } (C (\text{DenyAll}))$

by (*simp add: PLemmas split-tupled-all split: option.splits*)

lemma *domSubset4*:

$\text{dom } (C (\text{DenyAllFromTo } x y \oplus \text{DenyAllFromTo } y x \oplus \text{AllowPortFromTo } x y \text{ dn})) =$
 $\text{dom } (C (\text{DenyAllFromTo } x y \oplus \text{DenyAllFromTo } y x))$
by (*auto simp: PLemmas split: option.splits decision.splits*)

lemma *domSubset5*:

$\text{dom } (C (\text{DenyAllFromTo } x y \oplus \text{DenyAllFromTo } y x \oplus \text{AllowPortFromTo } y x \text{ dn})) =$
 $\text{dom } (C (\text{DenyAllFromTo } x y \oplus \text{DenyAllFromTo } y x))$
by (*auto simp: PLemmas split: option.splits decision.splits*)

lemma *domSubset1*:

$\text{dom } (C (\text{DenyAllFromTo } \text{one two} \oplus \text{DenyAllFromTo } \text{two one} \oplus \text{AllowPortFromTo } \text{one two dn} \oplus x)) =$
 $\text{dom } (C (\text{DenyAllFromTo } \text{one two} \oplus \text{DenyAllFromTo } \text{two one} \oplus x))$
by (*simp add: PLemmas split: option.splits decision.splits*) (*auto simp: allow-all-def deny-all-def*)

lemma *domSubset2*:

$\text{dom } (C (\text{DenyAllFromTo } \text{one two} \oplus \text{DenyAllFromTo } \text{two one} \oplus \text{AllowPortFromTo } \text{two one dn} \oplus x)) =$
 $\text{dom } (C (\text{DenyAllFromTo } \text{one two} \oplus \text{DenyAllFromTo } \text{two one} \oplus x))$
by (*simp add: PLemmas split: option.splits decision.splits*) (*auto simp: allow-all-def deny-all-def*)

lemma *ConcAssoc2*: $C (X \oplus Y \oplus ((A \oplus B) \oplus D)) = C (X \oplus Y \oplus A \oplus B \oplus D)$

by (*simp add: C.simps*)

lemma *ConcAssoc3*: $C (X \oplus ((Y \oplus A) \oplus D)) = C (X \oplus Y \oplus A \oplus D)$

by (*simp add: C.simps*)

lemma *RS3-NMT[rule-format]*:

$DenyAll \in set\ p \longrightarrow rm\text{-}MT\text{-}rules\ C\ p \neq []$
by (*induct-tac p*) (*simp-all add: PLemmas*)

lemma *norm-notMT*: $DenyAll \in set\ (policy2list\ p) \implies normalize\ p \neq []$
by (*simp add: DAiniD RS2-NMT RS3-NMT idNMT normalize-def rADnMT sepnMT sortnMT*)

lemma *norm-notMTQ*: $DenyAll \in set\ (policy2list\ p) \implies normalizeQ\ p \neq []$
by (*simp add: DAiniD RS2-NMT RS3-NMT idNMT normalizeQ-def rADnMT sepnMT sortnMTQ*)

lemmas *domDA = NormalisationIntegerPortProof.domSubset3*

lemmas *domain-reasoning = domDA ConcAssoc2 domSubset1 domSubset2*
domSubset3 domSubset4 domSubset5 domSubsetDistr1
domSubsetDistr2 domSubsetDistrA domSubsetDistrD coerc-assoc
ConcAssoc

ConcAssoc3

The following lemmas help with the normalisation

lemma *list2policyR-Start*[*rule-format*]: $p \in dom\ (C\ a) \longrightarrow$
 $C\ (list2policyR\ (a\ \# list))\ p = C\ a\ p$
by (*induct a # list rule:list2policyR.induct*) (*auto simp: C.simps dom-def map-add-def*)

lemma *list2policyR-End*: $p \notin dom\ (C\ a) \implies$
 $C\ (list2policyR\ (a\ \# list))\ p = (C\ a\ \oplus\ list2policy\ (map\ C\ list))\ p$
by (*rule list2policyR.induct*)
(simp-all add: C.simps dom-def map-add-def list2policy-def split: option.splits)

lemma *l2polR-eq-el*[*rule-format*]:
 $N \neq [] \longrightarrow C(list2policyR\ N)\ p = (list2policy\ (map\ C\ N))\ p$

proof (*induct N*)

case Nil show ?case **by** (*simp-all add: list2policy-def*)

next

case (Cons a N) then show ?case

apply (*case-tac p ∈ dom (C a),simp-all add: domStart list2policy-def*)

apply (*rule list2policyR-Start, simp-all*)

apply (*rule list2policyR.induct, simp-all*)

apply (*simp-all add: C.simps dom-def map-add-def*)

apply (*simp split: option.splits*)

done

qed

lemma *l2polR-eq*:

$N \neq [] \implies C(\text{list2policyR } N) = (\text{list2policy } (\text{map } C \ N))$
by (*auto simp: list2policy-def l2polR-eq-el*)

lemma *list2FWpolicys-eq-el[rule-format]*:

$\text{Filter} \neq [] \longrightarrow C(\text{list2policyR } \text{Filter}) \ p = C(\text{list2FWpolicy } (\text{rev } \text{Filter})) \ p$

proof (*induct Filter*) **print-cases**

case Nil show ?case by (*simp*)

next

case (Cons a list) then show ?case

apply *simp-all*

apply (*case-tac list = [], simp-all*)

apply (*case-tac p ∈ dom (C a), simp-all*)

apply (*rule list2policyR-Start, simp-all*)

by (*metis C.simps(4) l2polR-eq list2policyR-End nlpaux*)

qed

lemma *list2FWpolicys-eq*:

$\text{Filter} \neq [] \implies C(\text{list2policyR } \text{Filter}) = C(\text{list2FWpolicy } (\text{rev } \text{Filter}))$
by (*rule ext, erule list2FWpolicys-eq-el*)

lemma *list2FWpolicys-eq-sym*:

$\text{Filter} \neq [] \implies C(\text{list2policyR } (\text{rev } \text{Filter})) = C(\text{list2FWpolicy } \text{Filter})$
by (*metis list2FWpolicys-eq rev-is-Nil-conv rev-rev-ident*)

lemma *p-eq[rule-format]*:

$p \neq [] \longrightarrow \text{list2policy } (\text{map } C \ (\text{rev } p)) = C(\text{list2FWpolicy } p)$

by (*metis l2polR-eq list2FWpolicys-eq-sym rev.simps(1) rev-rev-ident*)

lemma *p-eq2[rule-format]*:

$\text{normalize } x \neq [] \longrightarrow C(\text{list2FWpolicy}(\text{normalize } x)) = C \ x \longrightarrow$

$\text{list2policy}(\text{map } C \ (\text{rev}(\text{normalize } x))) = C \ x$

by (*simp add: p-eq*)

lemma *p-eq2Q[rule-format]*:

$\text{normalizeQ } x \neq [] \longrightarrow C(\text{list2FWpolicy } (\text{normalizeQ } x)) = C \ x \longrightarrow$

$\text{list2policy } (\text{map } C \ (\text{rev } (\text{normalizeQ } x))) = C \ x$

by (*simp add: p-eq*)

lemma *list2listNMT[rule-format]*: $x \neq [] \longrightarrow \text{map sem } x \neq []$

by (*case-tac x*) *simp-all*

lemma *Norm-Distr2*:

$r \ o\text{-f } ((P \otimes_2 (\text{list2policy } Q)) \ o \ d) = (\text{list2policy } ((P \otimes_L Q) (\otimes_2) \ r \ d))$

by (rule ext, rule Norm-Distr-2)

lemma NATDistr:

$N \neq [] \implies F = C (\text{list2policyR } N) \implies$
 $(\lambda(x, y). x) \circ_f (\text{NAT} \otimes_2 F \circ (\lambda x. (x, x))) =$
 $\text{list2policy} ((\text{NAT} \otimes_L \text{map } C N) (\otimes_2) (\lambda(x, y). x) (\lambda x. (x, x)))$
apply (simp add: l2polR-eq)
apply (rule ext)
apply (rule Norm-Distr-2)
done

lemma C-eq-normalize-manual:

$\text{DenyAll} \in \text{set}(\text{policy2list } p) \implies \text{allNetsDistinct}(\text{policy2list } p) \implies \text{all-in-list}(\text{policy2list } p) l \implies$
 $C (\text{list2FWpolicy} (\text{normalize-manual-order } p l)) = C p$
by (simp add: normalize-manual-order-def C-eq-compile)

lemma p-eq2-manualQ[rule-format]:

$\text{normalize-manual-orderQ } x l \neq [] \longrightarrow C(\text{list2FWpolicy} (\text{normalize-manual-orderQ } x l)) = C x \longrightarrow$
 $\text{list2policy} (\text{map } C (\text{rev} (\text{normalize-manual-orderQ } x l))) = C x$
by (simp add: p-eq)

lemma norm-notMT-manualQ: $\text{DenyAll} \in \text{set} (\text{policy2list } p) \implies \text{normalize-manual-orderQ } p l \neq []$

by (simp add: DAiniD RS2-NMT RS3-NMT idNMT normalize-manual-orderQ-def rADnMT sepnMT sortnMTQ)

lemma C-eq-normalize-manualQ:

$\text{DenyAll} \in \text{set}(\text{policy2list } p) \implies \text{allNetsDistinct}(\text{policy2list } p) \implies \text{all-in-list}(\text{policy2list } p) l \implies$
 $C (\text{list2FWpolicy} (\text{normalize-manual-orderQ } p l)) = C p$
by (simp add: normalize-manual-orderQ-def C-eq-compileQ)

lemma p-eq2-manual[rule-format]:

$\text{normalize-manual-order } x l \neq [] \longrightarrow C (\text{list2FWpolicy} (\text{normalize-manual-order } x l)) = C x \longrightarrow$
 $\text{list2policy} (\text{map } C (\text{rev} (\text{normalize-manual-order } x l))) = C x$
by (simp add: p-eq)

lemma norm-notMT-manual: $\text{DenyAll} \in \text{set} (\text{policy2list } p) \implies \text{normalize-manual-order } p l \neq []$

by (simp add: RS2-NMT idNMT normalize-manual-order-def rADnMT sepnMT sortnMT wp1ID)

As an example, how this theorems can be used for a concrete normalisation instantiation.

lemma *normalizeNAT*:

$DenyAll \in set (policy2list Filter) \implies allNetsDistinct (policy2list Filter) \implies$
 $all-in-list (policy2list Filter) (Nets-List Filter) \implies$
 $(\lambda(x, y). x) \circ_f (NAT \otimes_2 C Filter \circ (\lambda x. (x, x))) =$
 $list2policy ((NAT \otimes_L map C (rev (FWNormalisationCore.normalize Filter)))) (\otimes_2)$
 $(\lambda(x, y). x) (\lambda x. (x, x))$
by (*simp add: C-eq-normalize NATDistr list2FWpolicys-eq-sym norm-notMT*)

lemma *domSimpl[simp]*: $dom (C (A \oplus DenyAll)) = dom (C (DenyAll))$

by (*simp add: PLemmas*)

The followin theorems can be applied when prepending the usual normalisation with an additional step and using another semantical interpretation function. This is a general recipe which can be applied whenever one needs to combine several normalisation strategies.

lemma *CRotate-eq-rotateC*: $CRotate p = C (rotatePolicy p)$

by (*induct p rule: rotatePolicy.induct (simp-all add: C.simps map-add-def)*)

lemma *DAinRotate*:

$DenyAll \in set (policy2list p) \implies DenyAll \in set (policy2list (rotatePolicy p))$

apply (*induct p, simp-all*)

subgoal for *p1 p2*

apply (*case-tac DenyAll \in set (policy2list p1), simp-all*)

done

done

lemma *DAUniv*: $dom (CRotate (P \oplus DenyAll)) = UNIV$

by (*metis CRotate.simps(1) CRotate.simps(4) CRotate-eq-rotateC DAAux PLemmas(4) UNIV-eq-I domSubset3*)

lemma *p-eq2R[rule-format]*:

$normalize (rotatePolicy x) \neq [] \longrightarrow C(list2FWpolicy(normalize (rotatePolicy x))) =$
 $CRotate x \longrightarrow$

$list2policy (map C (rev (normalize (rotatePolicy x)))) = CRotate x$

by (*simp add: p-eq*)

lemma *C-eq-normalizeRotate*:

$DenyAll \in set (policy2list p) \implies allNetsDistinct (policy2list (rotatePolicy p)) \implies$

$all-in-list (policy2list (rotatePolicy p)) (Nets-List (rotatePolicy p)) \implies$

$C (list2FWpolicy$
 $(removeAllDuplicates$

```

    (insertDenies
      (separate
        (sort(removeShadowRules2(remdups(rm-MT-rules C
          (insertDeny(removeShadowRules1(policy2list(rotatePolicy p)))))))
          (Nets-List (rotatePolicy p)))))) =
    CRotate p
  by (simp add: CRotate-eq-rotateC C-eq-compile DAinRotate)

```

lemma *C-eq-normalizeRotate2*:

```

  DenyAll ∈ set (policy2list p) ⇒
  allNetsDistinct (policy2list (rotatePolicy p)) ⇒
  all-in-list (policy2list (rotatePolicy p)) (Nets-List (rotatePolicy p)) ⇒
  C (list2FWpolicy (FWNormalisationCore.normalize (rotatePolicy p))) = CRotate p
  by (simp add: normalize-def, erule C-eq-normalizeRotate,simp-all)

```

end

2.3.4 Normalisation Proofs: Integer Protocol

theory

NormalisationIPPProofs

imports

NormalisationIntegerPortProof

begin

Normalisation proofs which are specific to the IntegerProtocol address representation.

lemma *ConcAssoc*: $Cp((A \oplus B) \oplus D) = Cp(A \oplus (B \oplus D))$

by (*simp add: Cp.simps*)

lemma *aux26*[*simp*]:

$twoNetsDistinct\ a\ b\ c\ d \implies dom\ (Cp\ (AllowPortFromTo\ a\ b\ p)) \cap dom\ (Cp\ (DenyAllFromTo\ c\ d)) = \{\}$

by(*auto simp:twoNetsDistinct-def netsDistinct-def PLemmas, auto*)

lemma *wp2-aux*[*rule-format*]:

$wellformed-policy2Pr\ (xs\ @\ [x]) \longrightarrow wellformed-policy2Pr\ xs$

apply(*induct xs, simp-all*)

subgoal for *a as*

apply(*case-tac a, simp-all*)

done

done

lemma *Cdom2*: $x \in dom(Cp\ b) \implies Cp\ (a \oplus b)\ x = (Cp\ b)\ x$

by (*auto simp: Cp.simps*)

lemma *wp2Conc*[*rule-format*]: *wellformed-policy2Pr* (*x#xs*) \implies *wellformed-policy2Pr* *xs*

by (*case-tac x,simp-all*)

lemma *DAimpliesMR-E*[*rule-format*]: *DenyAll* \in *set p* \longrightarrow

(\exists *r*. *applied-rule-rev Cp x p = Some r*)

apply (*simp add: applied-rule-rev-def*)

apply (*rule-tac xs = p in rev-induct, simp-all*)

by (*metis Cp.simps(1) denyAllDom*)

lemma *DAimplieMR*[*rule-format*]: *DenyAll* \in *set p* \implies *applied-rule-rev Cp x p \neq None*

by (*auto intro: DAimpliesMR-E*)

lemma *MRList1*[*rule-format*]: *x* \in *dom (Cp a)* \implies *applied-rule-rev Cp x (b@[a]) = Some a*

by (*simp add: applied-rule-rev-def*)

lemma *MRList2*: *x* \in *dom (Cp a)* \implies *applied-rule-rev Cp x (c@b@[a]) = Some a*

by (*simp add: applied-rule-rev-def*)

lemma *MRList3*:

x \notin *dom(Cp xa)* \implies *applied-rule-rev Cp x (a@b#xs@[xa]) = applied-rule-rev Cp x (a @ b # xs)*

by (*simp add: applied-rule-rev-def*)

lemma *CConcEnd*[*rule-format*]:

Cp a x = Some y \longrightarrow *Cp (list2FWpolicy (xs @ [a])) x = Some y* (**is** *?P xs*)

apply (*rule-tac P = ?P in list2FWpolicy.induct*)

by (*simp-all add:Cp.simps*)

lemma *CConcStartaux*: *Cp a x = None* \implies (*Cp aa ++ Cp a*) *x = Cp aa x*

by (*simp add: PLemmas*)

lemma *CConcStart*[*rule-format*]:

xs \neq [] \longrightarrow *Cp a x = None* \longrightarrow *Cp (list2FWpolicy (xs @ [a])) x = Cp (list2FWpolicy xs) x*

by (*rule list2FWpolicy.induct*) (*simp-all add: PLemmas*)

lemma *mrNnt*[*simp*]: *applied-rule-rev Cp x p = Some a* \implies *p \neq []*

by (*simp add: applied-rule-rev-def*)(*auto*)

lemma *mr-is-C*[*rule-format*]:

applied-rule-rev Cp x p = Some a \longrightarrow *Cp (list2FWpolicy (p)) x = Cp a x*

apply (*simp add: applied-rule-rev-def*)
apply (*rule rev-induct, simp-all, safe*)
apply (*metis CConcEnd*)
apply (*metis CConcEnd*)
by (*metis CConcStart applied-rule-rev-def mrNnt option.exhaust*)

lemma *CConcStart2*:

$p \neq [] \implies x \notin \text{dom} (Cp\ a) \implies Cp(\text{list2FWpolicy}\ (p@[a]))\ x = Cp\ (\text{list2FWpolicy}\ p)\ x$
by (*erule CConcStart, simp add: PLemmas*)

lemma *CConcEnd1*:

$q@p \neq [] \implies x \notin \text{dom} (Cp\ a) \implies Cp(\text{list2FWpolicy}\ (q@p@[a]))\ x = Cp\ (\text{list2FWpolicy}\ (q@p))\ x$
by (*subst lCdom2*) (*rule CConcStart2, simp-all*)

lemma *CConcEnd2*[*rule-format*]:

$x \in \text{dom} (Cp\ a) \longrightarrow Cp\ (\text{list2FWpolicy}\ (xs\ @\ [a]))\ x = Cp\ a\ x$ (**is** $?P\ xs$)
by (*rule-tac P = ?P in list2FWpolicy.induct*) (*auto simp: Cp.simps*)

lemma *bar3*:

$x \in \text{dom} (Cp\ (\text{list2FWpolicy}\ (xs\ @\ [xa]))) \implies x \in \text{dom} (Cp\ (\text{list2FWpolicy}\ xs)) \vee x \in \text{dom} (Cp\ xa)$
by *auto* (*metis CConcStart eq-Nil-appendI l2p-aux2 option.simps(3)*)

lemma *CeqEnd*[*rule-format, simp*]:

$a \neq [] \longrightarrow x \in \text{dom} (Cp(\text{list2FWpolicy}\ a)) \longrightarrow Cp(\text{list2FWpolicy}\ (b@a))\ x = Cp(\text{list2FWpolicy}\ a)\ x$

proof (*induct rule: rev-induct*)

case Nil show ?case by simp

next

case (snoc xa xs) show ?case

apply (*case-tac xs \neq [], simp-all*)

apply (*case-tac x \in dom (Cp xa)*)

apply (*metis CConcEnd2 MRList2 mr-is-C*)

apply (*metis snoc.hyps CConcEnd1 CConcStart2 Nil-is-append-conv bar3*)

by (*metis MRList2 eq-Nil-appendI mr-is-C*)

qed

lemma *CConcStartA*[*rule-format, simp*]:

$x \in \text{dom} (Cp\ a) \longrightarrow x \in \text{dom} (Cp\ (\text{list2FWpolicy}\ (a\ \# \ b)))$ (**is** $?P\ b$)

by (*rule-tac P = ?P in list2FWpolicy.induct*) (*simp-all add: Cp.simps*)

lemma *domConc*:

$x \in \text{dom} (Cp\ (\text{list2FWpolicy}\ b)) \implies b \neq [] \implies x \in \text{dom} (Cp\ (\text{list2FWpolicy}\ (a@b)))$

by (*auto simp: PLemmas*)

lemma *CeqStart*[*rule-format,simp*]:

$x \notin \text{dom} (Cp (list2FWpolicy a)) \longrightarrow a \neq [] \longrightarrow b \neq [] \longrightarrow$
 $Cp (list2FWpolicy (b@a)) x = (Cp (list2FWpolicy b)) x$

by (*rule list2FWpolicy.induct,simp-all*) (*auto simp: list2FWpolicyconc PLemmas*)

lemma *C-eq-if-mr-eq2*:

$\text{applied-rule-rev } Cp x a = \text{Some } r \Longrightarrow \text{applied-rule-rev } Cp x b = \text{Some } r \Longrightarrow a \neq [] \Longrightarrow$
 $b \neq [] \Longrightarrow$

$(Cp (list2FWpolicy a)) x = (Cp (list2FWpolicy b)) x$

by (*metis mr-is-C*)

lemma *nMRtoNone*[*rule-format*]:

$p \neq [] \longrightarrow \text{applied-rule-rev } Cp x p = \text{None} \longrightarrow Cp (list2FWpolicy p) x = \text{None}$

proof (*induct rule: rev-induct*)

case Nil show ?case by simp

next

case (snoc xa xs) show ?case

apply (*case-tac xs = [], simp-all*)

by (*simp-all add: snoc.hyps applied-rule-rev-def dom-def*)

qed

lemma *C-eq-if-mr-eq*:

$\text{applied-rule-rev } Cp x b = \text{applied-rule-rev } Cp x a \Longrightarrow a \neq [] \Longrightarrow b \neq [] \Longrightarrow$
 $(Cp (list2FWpolicy a)) x = (Cp (list2FWpolicy b)) x$

apply (*cases applied-rule-rev Cp x a = None, simp-all*)

apply (*subst nMRtoNone,simp-all*)

apply (*subst nMRtoNone,simp-all*)

by (*auto intro: C-eq-if-mr-eq2*)

lemma *notmatching-notdom*:

$\text{applied-rule-rev } Cp x (p@[a]) \neq \text{Some } a \Longrightarrow x \notin \text{dom} (Cp a)$

by (*simp add: applied-rule-rev-def split: if-splits*)

lemma *foo3a*[*rule-format*]:

$\text{applied-rule-rev } Cp x (a@[b]@c) = \text{Some } b \longrightarrow r \in \text{set } c \longrightarrow b \notin \text{set } c \longrightarrow x \notin \text{dom}$
 $(Cp r)$

proof (*induct rule: rev-induct*)

case Nil show ?case by simp

next

case (snoc xa xs) show ?case

apply *simp-all*

apply (*rule impI|rule conjI|simp*)+

apply (*rule-tac* $p = a @ b \# xs$ **in** *notmatching-notdom,simp-all*)
by (*metis Cons-eq-appendI NormalisationIPPProofs.MRList2 NormalisationIPPProofs.MRList3*
append-Nil option.inject snoc.hypos)
qed

lemma *foo3D*:

wellformed-policy1 $p \implies p = \text{DenyAll} \# ps \implies \text{applied-rule-rev } Cp \ x \ p = \text{Some } \text{DenyAll}$
 $\implies r \in \text{set } ps \implies$
 $x \notin \text{dom } (Cp \ r)$
by (*rule-tac* $a = []$ **and** $b = \text{DenyAll}$ **and** $c = ps$ **in** *foo3a, simp-all*)

lemma *foo4*[*rule-format*]:

$\text{set } p = \text{set } s \wedge (\forall r. r \in \text{set } p \longrightarrow x \notin \text{dom } (Cp \ r)) \longrightarrow (\forall r. r \in \text{set } s \longrightarrow x \notin \text{dom } (Cp \ r))$
by *simp*

lemma *foo5b*[*rule-format*]:

$x \in \text{dom } (Cp \ b) \longrightarrow (\forall r. r \in \text{set } c \longrightarrow x \notin \text{dom } (Cp \ r)) \longrightarrow \text{applied-rule-rev } Cp \ x \ (b \# c) = \text{Some } b$
apply (*simp add: applied-rule-rev-def*)
apply (*rule-tac* $xs = c$ **in** *rev-induct, simp-all*)
done

lemma *mr-first*:

$x \in \text{dom } (Cp \ b) \implies (\forall r. r \in \text{set } c \longrightarrow x \notin \text{dom } (Cp \ r)) \implies s = b \# c \implies$
 $\text{applied-rule-rev } Cp \ x \ s = \text{Some } b$
by (*simp add: foo5b*)

lemma *mr-charn*[*rule-format*]:

$a \in \text{set } p \longrightarrow (x \in \text{dom } (Cp \ a)) \longrightarrow (\forall r. r \in \text{set } p \wedge x \in \text{dom } (Cp \ r) \longrightarrow r = a)$
 \longrightarrow
 $\text{applied-rule-rev } Cp \ x \ p = \text{Some } a$
apply(*rule-tac* $xs = p$ **in** *rev-induct*)
apply(*simp-all only:applied-rule-rev-def*)
apply(*simp,safe,simp-all*)
by(*auto*)

lemma *foo8*:

$\forall r. r \in \text{set } p \wedge x \in \text{dom } (Cp \ r) \longrightarrow r = a \implies \text{set } p = \text{set } s \implies$
 $\forall r. r \in \text{set } s \wedge x \in \text{dom } (Cp \ r) \longrightarrow r = a$
by *auto*

lemma *mrConcEnd*[*rule-format*]:

applied-rule-rev Cp x (b # p) = Some a \longrightarrow *a \neq b* \longrightarrow *applied-rule-rev Cp x p = Some a*

apply (*simp add: applied-rule-rev-def*)
apply (*rule-tac xs = p in rev-induct, simp-all*)
by auto

lemma *wp3tl*[*rule-format*]: *wellformed-policy3Pr p* \longrightarrow *wellformed-policy3Pr (tl p)*

apply (*induct p, simp-all*)
subgoal for a as
apply(*case-tac a, simp-all*)
done
done

lemma *wp3Conc*[*rule-format*]: *wellformed-policy3Pr (a#p)* \longrightarrow *wellformed-policy3Pr p*
by (*induct p, simp-all, case-tac a, simp-all*)

lemma *foo98*[*rule-format*]:

applied-rule-rev Cp x (aa # p) = Some a \longrightarrow *x \in dom (Cp r)* \longrightarrow *r \in set p* \longrightarrow *a \in set p*

unfolding *applied-rule-rev-def*
proof (*induct rule: rev-induct*)
case Nil show ?case by simp
next
case (snoc xa xs) then show ?case
by simp-all (case-tac r = xa, simp-all)
qed

lemma *mrMTNone*[*simp*]: *applied-rule-rev Cp x [] = None*

by (*simp add: applied-rule-rev-def*)

lemma *DAAux*[*simp*]: *x \in dom (Cp DenyAll)*

by (*simp add: dom-def PolicyCombinators.PolicyCombinators Cp.simps*)

lemma *mrSet*[*rule-format*]: *applied-rule-rev Cp x p = Some r* \longrightarrow *r \in set p*

unfolding *applied-rule-rev-def*
by (*rule-tac xs=p in rev-induct*) *simp-all*

lemma *mr-not-Conc*: *singleCombinators p* \implies *applied-rule-rev Cp x p \neq Some (a \oplus b)*

by (*auto simp: mrSet dest: mrSet elim: SCnotConc*)

lemma *foo25*[*rule-format*]: *wellformed-policy3Pr (p@[x])* \longrightarrow *wellformed-policy3Pr p*

apply(*induct p, simp-all*)
subgoal for a p
 apply(*case-tac a, simp-all*)
 done
done

lemma mr-in-dom[*rule-format*]: *applied-rule-rev Cp x p = Some a* \longrightarrow $x \in \text{dom } (Cp a)$
by (*rule-tac xs = p in rev-induct*) (*auto simp: applied-rule-rev-def*)

lemma wp3EndMT[*rule-format*]:
wellformed-policy3Pr (p@[xs]) \longrightarrow *AllowPortFromTo a b po* \in *set p* \longrightarrow
 $\text{dom } (Cp (\text{AllowPortFromTo } a \ b \ po)) \cap \text{dom } (Cp \ xs) = \{\}$
apply (*induct p, simp-all*)
by (*metis NormalisationIPPPProofs.wp3Conc aux0-4 inf-commute list.set-intros(1)*
wellformed-policy3Pr.simps(2))

lemma foo29: $\text{dom } (Cp \ a) \neq \{\}$ \implies $\text{dom } (Cp \ a) \cap \text{dom } (Cp \ b) = \{\}$ \implies $a \neq b$
by auto

lemma foo28:
AllowPortFromTo a b po \in *set p* \implies $\text{dom}(Cp(\text{AllowPortFromTo } a \ b \ po)) \neq \{\}$ \implies
(wellformed-policy3Pr(p@[x])) \implies
 $x \neq \text{AllowPortFromTo } a \ b \ po$
by (*metis foo29 Cp.simps(3) wp3EndMT*)

lemma foo28a[*rule-format*]: $x \in \text{dom } (Cp \ a) \implies \text{dom } (Cp \ a) \neq \{\}$
by auto

lemma allow-deny-dom[*simp*]:
 $\text{dom } (Cp (\text{AllowPortFromTo } a \ b \ po)) \subseteq \text{dom } (Cp (\text{DenyAllFromTo } a \ b))$
by (*simp-all add: twoNetsDistinct-def netsDistinct-def PLemmas*) *auto*

lemma DenyAllowDisj:
 $\text{dom } (Cp (\text{AllowPortFromTo } a \ b \ p)) \neq \{\} \implies$
 $\text{dom } (Cp (\text{DenyAllFromTo } a \ b)) \cap \text{dom } (Cp (\text{AllowPortFromTo } a \ b \ p)) \neq \{\}$
by (*metis Int-absorb1 allow-deny-dom*)

lemma foo31:
 $\forall r. r \in \text{set } p \wedge x \in \text{dom } (Cp \ r) \longrightarrow$
 $(r = \text{AllowPortFromTo } a \ b \ po \vee r = \text{DenyAllFromTo } a \ b \vee r = \text{DenyAll}) \implies$
 $\text{set } p = \text{set } s \implies$
 $(\forall r. r \in \text{set } s \wedge x \in \text{dom}(Cp \ r) \longrightarrow r = \text{AllowPortFromTo } a \ b \ po \vee r = \text{DenyAllFromTo}$
 $a \ b \vee r = \text{DenyAll})$

by auto

lemma *wp1-auxa*: *wellformed-policy1-strong* $p \implies (\exists r. \text{applied-rule-rev } Cp \ x \ p = \text{Some } r)$

apply (*rule DAimpliesMR-E*)

by (*erule wp1-aux1aa*)

lemma *deny-dom[simp]*:

$\text{twoNetsDistinct } a \ b \ c \ d \implies \text{dom } (Cp \ (\text{DenyAllFromTo } a \ b)) \cap \text{dom } (Cp \ (\text{DenyAllFromTo } c \ d)) = \{\}$

by (*simp add: Cp.simps*) (*erule aux6*)

lemma *domTrans*: $\llbracket \text{dom } a \subseteq \text{dom } b; \text{dom}(b) \cap \text{dom}(c) = \{\} \rrbracket \implies \text{dom}(a) \cap \text{dom}(c) = \{\}$

by auto

lemma *DomInterAllowsMT*:

$\text{twoNetsDistinct } a \ b \ c \ d \implies \text{dom } (Cp(\text{AllowPortFromTo } a \ b \ p)) \cap \text{dom}(Cp(\text{AllowPortFromTo } c \ d \ po)) = \{\}$

apply (*case-tac p = po, simp-all*)

apply (*rule-tac b = Cp (DenyAllFromTo a b) in domTrans, simp-all*)

apply (*metis domComm aux26 tNDComm*)

apply (*simp add: twoNetsDistinct-def netsDistinct-def PLemmas*)

by (*auto simp: prod-eqI*)

lemma *DomInterAllowsMT-Ports*:

$p \neq po \implies \text{dom } (Cp \ (\text{AllowPortFromTo } a \ b \ p)) \cap \text{dom } (Cp \ (\text{AllowPortFromTo } c \ d \ po)) = \{\}$

apply (*simp add: twoNetsDistinct-def netsDistinct-def PLemmas*)

by (*auto simp: prod-eqI*)

lemma *wellformed-policy3-charn[rule-format]*:

$\text{singleCombinators } p \longrightarrow \text{distinct } p \longrightarrow \text{allNetsDistinct } p \longrightarrow$

$\text{wellformed-policy1 } p \longrightarrow \text{wellformed-policy2Pr } p \longrightarrow \text{wellformed-policy3Pr } p$

proof (*induct p*)

case Nil show ?case by simp

next

case (Cons a p) then show ?case

apply (*auto intro: singleCombinatorsConc ANDConc waux2 wp2Conc*)

apply (*case-tac a, simp-all, clarify*)

subgoal for a b c d r

apply (*case-tac r, simp-all*)

apply (*metis Int-commute*)

apply (*metis DomInterAllowsMT aux7aa DomInterAllowsMT-Ports*)

```

    apply (metis aux0-0 )
  done
done
qed

```

lemma *DistinctNetsDenyAllow*:

```

  DenyAllFromTo b c ∈ set p ⇒ AllowPortFromTo a d po ∈ set p ⇒ allNetsDistinct
  p ⇒
  dom (Cp (DenyAllFromTo b c)) ∩ dom (Cp (AllowPortFromTo a d po)) ≠ {} ⇒
  b = a ∧ c = d
  apply (simp add: allNetsDistinct-def)
  apply (frule-tac x = b in spec)
  apply (drule-tac x = d in spec)
  apply (drule-tac x = a in spec)
  apply (drule-tac x = c in spec)
  apply (metis Int-commute ND0aux1 ND0aux3 NDComm aux26 twoNetsDistinct-def
  ND0aux2 ND0aux4)
done

```

lemma *DistinctNetsAllowAllow*:

```

  AllowPortFromTo b c poo ∈ set p ⇒ AllowPortFromTo a d po ∈ set p ⇒
  allNetsDistinct p ⇒ dom(Cp(AllowPortFromTo b c poo)) ∩
  dom(Cp(AllowPortFromTo a d po)) ≠ {} ⇒
  b = a ∧ c = d ∧ poo = po
  apply (simp add: allNetsDistinct-def)
  apply (frule-tac x = b in spec)
  apply (drule-tac x = d in spec)
  apply (drule-tac x = a in spec)
  apply (drule-tac x = c in spec)
  apply (metis DomInterAllowsMT DomInterAllowsMT-Ports ND0aux3 ND0aux4 ND-
  Comm twoNetsDistinct-def)
done

```

lemma *WP2RS2[simp]*:

```

  singleCombinators p ⇒ distinct p ⇒ allNetsDistinct p ⇒
  wellformed-policy2Pr (removeShadowRules2 p)
proof (induct p)
  case Nil
  then show ?case by simp
next
  case (Cons x xs)
  have wp-xs: wellformed-policy2Pr (removeShadowRules2 xs)
  by (metis Cons ANDConc distinct.simps(2) singleCombinatorsConc)
  show ?case

```

```

proof (cases x)
  case DenyAll thus ?thesis using wp-xs by simp
next
  case (DenyAllFromTo a b) thus ?thesis
    using wp-xs Cons
    by (simp,metis DenyAllFromTo aux aux7 tNDComm deny-dom)
next
  case (AllowPortFromTo a b p) thus ?thesis
    using wp-xs
    by (simp, metis aux26 AllowPortFromTo Cons(4) aux aux7a tNDComm)
next
  case (Conc a b) thus ?thesis
    by (metis Conc Cons(2) singleCombinators.simps(2))
qed
qed

lemma AD-aux:
  AllowPortFromTo a b po ∈ set p ⇒ DenyAllFromTo c d ∈ set p ⇒
  allNetsDistinct p ⇒ singleCombinators p ⇒ a ≠ c ∨ b ≠ d ⇒
  dom (Cp (AllowPortFromTo a b po)) ∩ dom (Cp (DenyAllFromTo c d)) = {}
  by (rule aux26,rule-tac x = AllowPortFromTo a b po and y = DenyAllFromTo c d in
  tND) auto

lemma sorted-WP2[rule-format]:
  sorted p l → all-in-list p l → distinct p → allNetsDistinct p → singleCombinators
  p →
  wellformed-policy2Pr p
proof (induct p)
  case Nil thus ?case by simp
next
  case (Cons a p) thus ?case
  proof (cases a)
    case DenyAll thus ?thesis
      using Cons by (auto intro: ANDConc singleCombinatorsConc sortedConcEnd)
  next
    case (DenyAllFromTo c d) thus ?thesis
      using Cons apply (simp, intro impI conjI allI impI deny-dom)
      by (auto intro: aux7 tNDComm ANDConc singleCombinatorsConc sortedConcEnd)

  next
    case (AllowPortFromTo c d e) thus ?thesis
      using Cons apply simp
      apply (intro impI conjI allI, rename-tac aa b)
      apply (rule aux26)

```



```

subgoal for aa b
  apply (rule-tac x = AllowPortFromTo c d e and y = DenyAllFromTo aa b in
tND,
    assumption,simp-all)
  apply (subgoal-tac smaller (AllowPortFromTo c d e) (DenyAllFromTo aa b) l)
  apply (simp split: if-splits)
  apply metis
  apply (erule sorted-is-smaller, simp-all)
  apply (metis bothNet.simps(2) in-list.simps(2) in-set-in-list)
  done
  by (auto intro: aux7 tNDComm ANDConc singleCombinatorsConc sortedConcEnd)
next
  case (Conc a b) thus ?thesis using Cons by simp
qed
qed

```

```

lemma wellformed2-sorted[simp]:
  all-in-list p l  $\implies$  distinct p  $\implies$  allNetsDistinct p  $\implies$  singleCombinators p  $\implies$ 
  wellformed-policy2Pr (sort p l)
  by (metis distinct-sort set-sort sorted-WP2 SC3 aND-sort all-in-listSubset order-refl
sort-is-sorted)

```

```

lemma wellformed2-sortedQ[simp]:
  all-in-list p l  $\implies$  distinct p  $\implies$  allNetsDistinct p  $\implies$  singleCombinators p  $\implies$ 
  wellformed-policy2Pr (qsort p l)
  by (metis sorted-WP2 SC3Q aND-sortQ all-in-listSubset distinct-sortQ set-sortQ
sort-is-sortedQ subsetI)

```

```

lemma C-DenyAll[simp]: Cp (list2FWpolicy (xs @ [DenyAll])) x = Some (deny ())
  by (auto simp: PLemmas)

```

```

lemma C-eq-RS1n:
  Cp(list2FWpolicy (removeShadowRules1-alternative p)) = Cp(list2FWpolicy p)
proof (cases p)
  case Nil then show ?thesis
    by (simp, metis list2FWpolicy.simps(1) rSR1-eq removeShadowRules1.simps(2))
next
  case (Cons a list) then show ?thesis
    apply (hypsubst, simp)
    apply (thin-tac p = a # list)
proof (induct rule: rev-induct)
  case Nil show ?case by (metis rSR1-eq removeShadowRules1.simps(2))
next
  case (snoc x xs) show ?case

```

```

apply (case-tac xs = [], simp-all)
apply (simp add: removeShadowRules1-alternative-def)
apply (insert snoc.hyps, case-tac x, simp-all)
apply (rule ext, rename-tac xa)
apply (case-tac x = DenyAll, simp-all add: PLemmas)
apply (rule-tac t = removeShadowRules1-alternative (xs @ [x]) and
  s = (removeShadowRules1-alternative xs)@[x] in subst)
apply (erule RS1n-assoc)
subgoal for a
  apply (case-tac a ∈ dom (Cp x), simp-all)
  done
done
qed
qed

```

```

lemma C-eq-RS1[simp]:
  p ≠ [] ⇒ Cp(list2FWpolicy (removeShadowRules1 p)) = Cp(list2FWpolicy p)
  by (metis rSR1-eq C-eq-RS1n)

```

```

lemma EX-MR-aux[rule-format]:
  applied-rule-rev Cp x (DenyAll # p) ≠ Some DenyAll ⇒ (∃ y. applied-rule-rev Cp x
  p = Some y)
  by (simp add: applied-rule-rev-def) (rule-tac xs = p in rev-induct, simp-all)

```

```

lemma EX-MR :
  applied-rule-rev Cp x p ≠ (Some DenyAll) ⇒ p = DenyAll#ps ⇒
  (applied-rule-rev Cp x p = applied-rule-rev Cp x ps)
  apply (auto, subgoal-tac applied-rule-rev Cp x (DenyAll#ps) ≠ None, auto)
  apply (metis mrConcEnd)
  by (metis DAimpliesMR-E list.sel(1) hd-in-set list.simps(3) not-Some-eq)

```

```

lemma mr-not-DA:
  wellformed-policy1-strong s ⇒ applied-rule-rev Cp x p = Some (DenyAllFromTo a
  ab) ⇒
  set p = set s ⇒ applied-rule-rev Cp x s ≠ Some DenyAll
  apply (subst wp1n-tl, simp-all)
  by (metis (mono-tags, lifting) Combinators.distinct(1) foo98
  mrSet mr-in-dom WP1n-DA-notinSet set-ConsD wp1n-tl)

```

```

lemma domsMT-notND-DD:
  dom (Cp (DenyAllFromTo a b)) ∩ dom (Cp (DenyAllFromTo c d)) ≠ {} ⇒ ¬
  netsDistinct a c
  by (erule contrapos-nn) (simp add: Cp.simps aux6 twoNetsDistinct-def)

```

lemma *domsMT-notND-DD2*:

$dom (Cp (DenyAllFromTo a b)) \cap dom (Cp (DenyAllFromTo c d)) \neq \{\} \implies \neg netsDistinct b d$

by (*erule contrapos- \neg*) (*simp add: Cp.simps aux6 twoNetsDistinct-def*)

lemma *domsMT-notND-DD3*:

$x \in dom (Cp (DenyAllFromTo a b)) \implies x \in dom (Cp (DenyAllFromTo c d)) \implies \neg netsDistinct a c$

by (*auto intro!: domsMT-notND-DD*)

lemma *domsMT-notND-DD4*:

$x \in dom (Cp (DenyAllFromTo a b)) \implies x \in dom (Cp (DenyAllFromTo c d)) \implies \neg netsDistinct b d$

by (*auto intro!: domsMT-notND-DD2*)

lemma *NetsEq-if-sameP-DD*:

$allNetsDistinct p \implies u \in set p \implies v \in set p \implies u = (DenyAllFromTo a b) \implies v = (DenyAllFromTo c d) \implies x \in dom (Cp (u)) \implies x \in dom (Cp (v)) \implies a = c \wedge b = d$

unfolding *allNetsDistinct-def*

by (*simp*)(*metis allNetsDistinct-def ND0aux1 ND0aux2 domsMT-notND-DD3 domsMT-notND-DD4*)

lemma *rule-charn1*:

assumes *aND* : *allNetsDistinct p*

and *mr-is-allow* : *applied-rule-rev Cp x p = Some (AllowPortFromTo a b po)*

and *SC* : *singleCombinators p*

and *inp* : $r \in set p$

and *inDom* : $x \in dom (Cp r)$

shows $(r = AllowPortFromTo a b po \vee r = DenyAllFromTo a b \vee r = DenyAll)$

proof (*cases r*)

case *DenyAll* **show** *?thesis* **by** (*metis DenyAll*)

next

case (*DenyAllFromTo x y*) **show** *?thesis*

by (*metis DenyAllFromTo NormalisationIPPProofs.AD-aux NormalisationIPPProofs.mrSet*

NormalisationIPPProofs.mr-in-dom SC aND domInterMT inDom inp mr-is-allow)

next

case (*AllowPortFromTo x y b*) **show** *?thesis*

by (*metis (mono-tags, lifting) AllowPortFromTo NormalisationIPPProofs.DistinctNetsAllowAllow*

NormalisationIPPProofs.mrSet NormalisationIPPProofs.mr-in-dom aND dom-InterMT inDom

inp mr-is-allow)

```

next
  case (Conc x y) thus ?thesis using assms by (metis aux0-0)
qed

lemma none-MT-rulessubset[rule-format]:
  none-MT-rules Cp a  $\longrightarrow$  set b  $\subseteq$  set a  $\longrightarrow$  none-MT-rules Cp b
  by (induct b, simp-all) (metis notMTnMT)

lemma nMTSort: none-MT-rules Cp p  $\implies$  none-MT-rules Cp (sort p l)
  by (metis set-sort nMTEqSet)

lemma nMTSortQ: none-MT-rules Cp p  $\implies$  none-MT-rules Cp (qsort p l)
  by (metis set-sortQ nMTEqSet)

lemma wp3char[rule-format]: none-MT-rules Cp xs  $\wedge$  Cp (AllowPortFromTo a b po)
= Map.empty  $\wedge$ 
      wellformed-policy3Pr (xs @ [DenyAllFromTo a b])  $\longrightarrow$ 
      AllowPortFromTo a b po  $\notin$  set xs
  by (induct xs, simp-all) (metis domNMT wp3Conc)

lemma wp3charn[rule-format]:
  assumes domAllow: dom (Cp (AllowPortFromTo a b po))  $\neq$  {}
    and wp3: wellformed-policy3Pr (xs @ [DenyAllFromTo a b])
  shows allowNotInList: AllowPortFromTo a b po  $\notin$  set xs
  apply (insert assms)
proof (induct xs)
  case Nil show ?case by simp
next
  case (Cons x xs) show ?case using Cons
    by (simp, auto intro: wp3Conc) (auto simp: DenyAllowDisj domAllow)
qed

lemma rule-charn2:
  assumes aND: allNetsDistinct p
    and wp1: wellformed-policy1 p
    and SC: singleCombinators p
    and wp3: wellformed-policy3Pr p
    and allow-in-list: AllowPortFromTo c d po  $\in$  set p
    and x-in-dom-allow: x  $\in$  dom (Cp (AllowPortFromTo c d po))
  shows applied-rule-rev Cp x p = Some (AllowPortFromTo c d po)
proof (insert assms, induct p rule: rev-induct)
  case Nil show ?case using Nil by simp
next
  case (snoc y ys) show ?case using snoc

```

```

apply simp
apply (case-tac y = (AllowPortFromTo c d po))
  apply (simp add: applied-rule-rev-def)
apply simp-all
apply (subgoal-tac ys ≠ [])
  apply (subgoal-tac applied-rule-rev Cp x ys = Some (AllowPortFromTo c d po))
  defer 1
  apply (metis ANDConcEnd SCConcEnd WP1ConcEnd foo25)
  apply (metis inSet-not-MT)
proof (cases y)
  case DenyAll thus ?thesis using DenyAll snoc
    apply simp
    by (metis DAnotTL DenyAll inSet-not-MT policy2list.simps(2))
  next
    case (DenyAllFromTo a b) thus ?thesis using snoc apply simp
      apply (simp-all add: applied-rule-rev-def)
      apply (rule conjI)
      apply (metis domInterMT wp3EndMT)
      apply (rule impI)
      by (metis ANDConcEnd DenyAllFromTo SCConcEnd WP1ConcEnd foo25)
    next
      case (AllowPortFromTo a1 a2 b) thus ?thesis using AllowPortFromTo snoc apply
simp
        apply (simp-all add: applied-rule-rev-def)
        apply (rule conjI)
        apply (metis domInterMT wp3EndMT)
        by (metis ANDConcEnd AllowPortFromTo SCConcEnd WP1ConcEnd foo25
x-in-dom-allow)
      next
        case (Conc a b) thus ?thesis
          using Conc snoc apply simp
          by (metis Conc aux0-0 in-set-conv-decomp)
        qed
      qed

```

lemma *rule-charn3*:

wellformed-policy1 p \implies *allNetsDistinct p* \implies *singleCombinators p* \implies
wellformed-policy3Pr p \implies *applied-rule-rev Cp x p* = *Some* (*DenyAllFromTo c d*) \implies

AllowPortFromTo a b po ∈ *set p* \implies *x* ∉ *dom* (*Cp* (*AllowPortFromTo a b po*))
by (*clarify*) (*simp* *add*: *NormalisationIPPPProofs.rule-charn2 domI*)

lemma *rule-charn4*:

assumes *wp1*: *wellformed-policy1 p*

```

and   aND:   allNetsDistinct p
and   SC:    singleCombinators p
and   wp3:   wellformed-policy3Pr p
and   DA:    DenyAll  $\notin$  set p
and   mr:    applied-rule-rev Cp x p = Some (DenyAllFromTo a b)
and   rinp:  r  $\in$  set p
and   xindom: x  $\in$  dom (Cp r)
shows r = DenyAllFromTo a b
proof (cases r)
case DenyAll thus ?thesis using DenyAll assms by simp
next
case (DenyAllFromTo c d) thus ?thesis
using assms apply simp
apply (erule-tac x = x and p = p and v = (DenyAllFromTo a b) and
      u = (DenyAllFromTo c d) in NetsEq-if-sameP-DD, simp-all)
apply (erule mrSet)
by (erule mr-in-dom)
next
case (AllowPortFromTo c d e) thus ?thesis
using assms apply simp
apply (subgoal-tac x  $\notin$  dom (Cp (AllowPortFromTo c d e)), simp)
by (rule-tac p = p in rule-charn3, auto intro: SCnotConc)
next
case (Conc a b) thus ?thesis
using assms apply simp
by (metis Conc aux0-0)
qed

```

lemma foo31a:

```

( $\forall$  r. r  $\in$  set p  $\wedge$  x  $\in$  dom (Cp r)  $\longrightarrow$ 
  (r = AllowPortFromTo a b po  $\vee$  r = DenyAllFromTo a b  $\vee$  r = DenyAll))  $\implies$ 
  set p = set s  $\implies$  r  $\in$  set s  $\implies$  x  $\in$  dom (Cp r)  $\implies$ 
  (r = AllowPortFromTo a b po  $\vee$  r = DenyAllFromTo a b  $\vee$  r = DenyAll)
by auto

```

lemma aux4[rule-format]:

```

applied-rule-rev Cp x (a#p) = Some a  $\longrightarrow$  a  $\notin$  set (p)  $\longrightarrow$  applied-rule-rev Cp x p =
None
by (rule rev-induct, simp-all) (intro impI, simp add: applied-rule-rev-def split: if-splits)

```

lemma mrDA-tl:

```

assumes mr-DA: applied-rule-rev Cp x p = Some DenyAll
and     wp1n: wellformed-policy1-strong p
shows   applied-rule-rev Cp x (tl p) = None

```

apply (*rule aux4* [**where** $a = \text{DenyAll}$])
apply (*metis wp1n-tl mr-DA wp1n*)
by (*metis WP1n-DA-notinSet wp1n*)

lemma *rule-charnDAFT*:

wellformed-policy1-strong $p \implies \text{allNetsDistinct } p \implies \text{singleCombinators } p \implies$
wellformed-policy3Pr $p \implies \text{applied-rule-rev } Cp \ x \ p = \text{Some } (\text{DenyAllFromTo } a \ b)$
 \implies
 $r \in \text{set } (tl \ p) \implies x \in \text{dom } (Cp \ r) \implies$
 $r = \text{DenyAllFromTo } a \ b$
apply (*subgoal-tac* $p = \text{DenyAll}\#(tl \ p)$)
apply (*metis (no-types, lifting) ANDConc Combinators.distinct(1) Normalisation-IPPProofs.mrConcEnd*
NormalisationIPPProofs.rule-charn4 NormalisationIPPProofs.wp3Conc
WP1n-DA-notinSet
singleCombinatorsConc waux2)
using *wp1n-tl* **by** *auto*

lemma *mrDenyAll-is-unique*:

wellformed-policy1-strong $p \implies \text{applied-rule-rev } Cp \ x \ p = \text{Some } \text{DenyAll} \implies r \in \text{set}$
 $(tl \ p) \implies$
 $x \notin \text{dom } (Cp \ r)$
apply (*rule-tac* $a = []$ **and** $b = \text{DenyAll}$ **and** $c = tl \ p$ **in** *foo3a, simp-all*)
apply (*metis wp1n-tl*)
by (*metis WP1n-DA-notinSet*)

theorem *C-eq-Sets-mr*:

assumes *sets-eq*: $\text{set } p = \text{set } s$
and *SC*: *singleCombinators* p
and *wp1-p*: *wellformed-policy1-strong* p
and *wp1-s*: *wellformed-policy1-strong* s
and *wp3-p*: *wellformed-policy3Pr* p
and *wp3-s*: *wellformed-policy3Pr* s
and *aND*: *allNetsDistinct* p
shows $\text{applied-rule-rev } Cp \ x \ p = \text{applied-rule-rev } Cp \ x \ s$
proof (*cases applied-rule-rev Cp x p*)
case *None*
have *DA*: $\text{DenyAll} \in \text{set } p$ **using** *wp1-p* **by** (*auto simp: wp1-aux1aa*)
have *notDA*: $\text{DenyAll} \notin \text{set } p$ **using** *None* **by** (*auto simp: DAimplieMR*)
thus *?thesis* **using** *DA* **by** (*contradiction*)
next
case (*Some y*) **thus** *?thesis*
proof (*cases y*)
have *tl-p*: $p = \text{DenyAll}\#(tl \ p)$ **by** (*metis wp1-p wp1n-tl*)

```

have tl-s:  $s = \text{DenyAll}\#(\text{tl } s)$  by (metis wp1-s wp1n-tl)
have tl-eq:  $\text{set } (\text{tl } p) = \text{set } (\text{tl } s)$ 
  by (metis list.sel(3) WP1n-DA-notinSet sets-eq foo2
    wellformed-policy1-charn wp1-aux1aa wp1-eq wp1-p wp1-s)
{
  case DenyAll
  have mr-p-is-DenyAll: applied-rule-rev Cp x p = Some DenyAll
    by (simp add: DenyAll Some)
  hence x-notin-tl-p:  $\forall r. r \in \text{set } (\text{tl } p) \longrightarrow x \notin \text{dom } (\text{Cp } r)$  using wp1-p
    by (auto simp: mrDenyAll-is-unique)
  hence x-notin-tl-s:  $\forall r. r \in \text{set } (\text{tl } s) \longrightarrow x \notin \text{dom } (\text{Cp } r)$  using tl-eq
    by auto
  hence mr-s-is-DenyAll: applied-rule-rev Cp x s = Some DenyAll using tl-s
    by (auto simp: mr-first)
  thus ?thesis using mr-p-is-DenyAll by simp
next
  case (DenyAllFromTo a b)
  have mr-p-is-DAFT: applied-rule-rev Cp x p = Some (DenyAllFromTo a b)
    by (simp add: DenyAllFromTo Some)
  have DA-notin-tl:  $\text{DenyAll} \notin \text{set } (\text{tl } p)$ 
    by (metis WP1n-DA-notinSet wp1-p)
  have mr-tl-p: applied-rule-rev Cp x p = applied-rule-rev Cp x (tl p)
    by (metis Combinators.simps(4) DenyAllFromTo Some mrConcEnd tl-p)
  have dom-tl-p:  $\bigwedge r. r \in \text{set } (\text{tl } p) \wedge x \in \text{dom } (\text{Cp } r) \implies$ 
     $r = (\text{DenyAllFromTo } a \ b)$ 
    using wp1-p aND SC wp3-p mr-p-is-DAFT
    by (auto simp: rule-charnDAFT)
  hence dom-tl-s:  $\bigwedge r. r \in \text{set } (\text{tl } s) \wedge x \in \text{dom } (\text{Cp } r) \implies$ 
     $r = (\text{DenyAllFromTo } a \ b)$ 
    using tl-eq by auto
  have DAFT-in-tl-s:  $\text{DenyAllFromTo } a \ b \in \text{set } (\text{tl } s)$  using mr-tl-p
    by (metis DenyAllFromTo mrSet mr-p-is-DAFT tl-eq)
  have x-in-dom-DAFT:  $x \in \text{dom } (\text{Cp } (\text{DenyAllFromTo } a \ b))$ 
    by (metis mr-p-is-DAFT DenyAllFromTo mr-in-dom)
  hence mr-tl-s-is-DAFT: applied-rule-rev Cp x (tl s) = Some (DenyAllFromTo a b)
    using DAFT-in-tl-s dom-tl-s by (metis mr-charn)
  hence mr-s-is-DAFT: applied-rule-rev Cp x s = Some (DenyAllFromTo a b)
    using tl-s
    by (metis DA-notin-tl DenyAllFromTo EX-MR mrDA-tl
      not-Some-eq tl-eq wellformed-policy1-strong.simps(2))
  thus ?thesis using mr-p-is-DAFT by simp
next
  case (AllowPortFromTo a b c)
  have wp1s: wellformed-policy1 s by (metis wp1-eq wp1-s)

```



```

have mr-p-is-A: applied-rule-rev Cp x p = Some (AllowPortFromTo a b c)
  by (simp add: AllowPortFromTo Some)
hence A-in-s: AllowPortFromTo a b c ∈ set s using sets-eq
  by (auto intro: mrSet)
have x-in-dom-A: x ∈ dom (Cp (AllowPortFromTo a b c))
  by (metis mr-p-is-A AllowPortFromTo mr-in-dom)
have SCs: singleCombinators s using SC sets-eq
  by (auto intro: SCSubset)
hence ANDs: allNetsDistinct s using aND sets-eq SC
  by (auto intro: aNDSetsEq)
hence mr-s-is-A: applied-rule-rev Cp x s = Some (AllowPortFromTo a b c)
  using A-in-s wp1s mr-p-is-A aND SCs wp3-s x-in-dom-A
  by (simp add: rule-charn2)
thus ?thesis using mr-p-is-A by simp
}
next
  case (Conc a b) thus ?thesis by (metis Some mr-not-Conc SC)
qed
qed

```

lemma *C-eq-Sets*:

```

singleCombinators p ⇒ wellformed-policy1-strong p ⇒ wellformed-policy1-strong s
⇒
wellformed-policy3Pr p ⇒ wellformed-policy3Pr s ⇒ allNetsDistinct p ⇒ set p =
set s ⇒
Cp (list2FWpolicy p) x = Cp (list2FWpolicy s) x
by (metis C-eq-Sets-mr C-eq-if-mr-eq wellformed-policy1-strong.simps(1))

```

lemma *C-eq-sorted*:

```

distinct p ⇒ all-in-list p l ⇒ singleCombinators p ⇒
wellformed-policy1-strong p ⇒ wellformed-policy3Pr p ⇒ allNetsDistinct p ⇒
Cp (list2FWpolicy (sort p l)) = Cp (list2FWpolicy p)
by (rule ext)
(meson distinct-sort set-sort C-eq-Sets wellformed2-sorted wellformed-policy3-charn
SC3 aND-sort
wellformed1-alternative-sorted wp1-eq)

```

lemma *C-eq-sortedQ*:

```

distinct p ⇒ all-in-list p l ⇒ singleCombinators p ⇒
wellformed-policy1-strong p ⇒ wellformed-policy3Pr p ⇒ allNetsDistinct p ⇒
Cp (list2FWpolicy (qsort p l)) = Cp (list2FWpolicy p)
by (rule ext)
(metis C-eq-Sets wellformed2-sortedQ wellformed-policy3-charn SC3Q aND-sortQ
distinct-sortQ)

```

set-sortQ wellformed1-sorted-auxQ wellformed-eq wp1-aux1aa)

lemma *C-eq-RS2-mr: applied-rule-rev Cp x (removeShadowRules2 p)= applied-rule-rev Cp x p*

proof (*induct p*)
 case *Nil* **thus** ?*case* **by** *simp*
next
 case (*Cons y ys*) **thus** ?*case*
proof (*cases ys = []*)
 case *True* **thus** ?*thesis* **by** (*cases y, simp-all*)
next
 case *False* **thus** ?*thesis*
proof (*cases y*)
 case *DenyAll* **thus** ?*thesis* **by** (*simp, metis Cons DenyAll mreq-end2*)
next
 case (*DenyAllFromTo a b*) **thus** ?*thesis* **by** (*simp, metis Cons DenyAllFromTo mreq-end2*)
next
 case (*AllowPortFromTo a b p*) **thus** ?*thesis*
proof (*cases DenyAllFromTo a b ∈ set ys*)
 case *True* **thus** ?*thesis* **using** *AllowPortFromTo Cons*
 apply (*cases applied-rule-rev Cp x ys = None, simp-all*)
 apply (*subgoal-tac x ∉ dom (Cp (AllowPortFromTo a b p))*)
 apply (*subst mrconcNone, simp-all*)
 apply (*simp add: applied-rule-rev-def*)
 apply (*rule contra-subsetD [OF allow-deny-dom]*)
 apply (*erule mrNoneMT, simp*)
 apply (*metis AllowPortFromTo mrconc*)
 done
next
 case *False* **thus** ?*thesis* **using** *False Cons AllowPortFromTo*
by (*simp, metis AllowPortFromTo Cons mreq-end2*) **qed**
next
 case (*Conc a b*) **thus** ?*thesis*
by (*metis Cons mreq-end2 removeShadowRules2.simps(4)*)
qed
qed
qed

lemma *C-eq-None[rule-format]:*

p ≠ [] → applied-rule-rev Cp x p = None → Cp (list2FWpolicy p) x = None

unfolding *applied-rule-rev-def*

proof(*induct rule: rev-induct*)

case *Nil* **show** ?*case* **by** *simp*

```

next
  case (snoc xa xs) show ?case
    apply (insert snoc.hyps, intro impI, simp)
    apply (case-tac xs ≠ [])
    apply (metis CConcStart2 option.simps(3))
    by (metis append-Nil domIff l2p-aux2 option.distinct(1))
qed

lemma C-eq-None2:
  a ≠ [] ⇒ b ≠ [] ⇒ applied-rule-rev Cp x a = None ⇒ applied-rule-rev Cp x b
= None ⇒
  (Cp (list2FWpolicy a)) x = (Cp (list2FWpolicy b)) x
  by (auto simp: C-eq-None)

lemma C-eq-RS2:
  wellformed-policy1-strong p ⇒
  Cp (list2FWpolicy (removeShadowRules2 p)) = Cp (list2FWpolicy p)
  apply (rule ext)
  by (metis C-eq-RS2-mr C-eq-if-mr-eq RS2-NMT wp1-alternative-not-mt)

lemma none-MT-rulesRS2: none-MT-rules Cp p ⇒ none-MT-rules Cp
(removeShadowRules2 p)
  by (auto simp: RS2Set none-MT-rulessubset)

lemma CconcNone:
  dom (Cp a) = {} ⇒ p ≠ [] ⇒ Cp (list2FWpolicy (a # p)) x = Cp (list2FWpolicy
p) x
  apply (case-tac p = [], simp-all)
  apply (case-tac x ∈ dom (Cp (list2FWpolicy(p))))
  apply (metis Cdom2 list2FWpolicyconc)
  apply (metis Cp.simps(4) map-add-dom-app-simps(2) inSet-not-MT list2FWpolicy-
conc set-empty2)
  done

lemma none-MT-rulesrd[rule-format]: none-MT-rules Cp p → none-MT-rules Cp
(remdups p)
  by (induct p, simp-all)

lemma DARS3[rule-format]: DenyAll ∉ set p → DenyAll ∉ set (rm-MT-rules Cp p)
  by (induct p, simp-all)

lemma DAnMT: dom (Cp DenyAll) ≠ {}
  by (simp add: dom-def Cp.simps PolicyCombinators.PolicyCombinators)

```

lemma *DAnMT2*: $Cp \text{ DenyAll} \neq \text{Map.empty}$
 by (*metis* *DAAux* *dom-eq-empty-conv* *empty-iff*)

lemma *wp1n-RS3*[*rule-format,simp*]:
 $\text{wellformed-policy1-strong } p \longrightarrow \text{wellformed-policy1-strong } (\text{rm-MT-rules } Cp \ p)$
apply (*induct* *p*, *simp-all*)
apply (*rule conjI* | *rule impI* | *simp*) +
apply (*metis* *DAnMT*)
apply (*metis* *DARS3*)
done

lemma *AILRS3*[*rule-format,simp*]:
 $\text{all-in-list } p \ l \longrightarrow \text{all-in-list } (\text{rm-MT-rules } Cp \ p) \ l$
 by (*induct* *p*, *simp-all*)

lemma *SCRS3*[*rule-format,simp*]:
 $\text{singleCombinators } p \longrightarrow \text{singleCombinators}(\text{rm-MT-rules } Cp \ p)$
apply (*induct* *p*, *simp-all*)
subgoal for *a* *p*
apply(*case-tac* *a*, *simp-all*)
done
done

lemma *RS3subset*: $\text{set } (\text{rm-MT-rules } Cp \ p) \subseteq \text{set } p$
 by (*induct* *p*, *auto*)

lemma *ANDRS3*[*simp*]:
 $\text{singleCombinators } p \Longrightarrow \text{allNetsDistinct } p \Longrightarrow \text{allNetsDistinct } (\text{rm-MT-rules } Cp \ p)$
 by (*rule-tac* $b = p$ **in** *aNDSsubset*, *simp-all* *add:RS3subset*)

lemma *nlpaux*: $x \notin \text{dom } (Cp \ b) \Longrightarrow Cp \ (a \oplus b) \ x = Cp \ a \ x$
 by (*metis* *Cp.simps*(4) *map-add-dom-app-simps*(3))

lemma *notindom*[*rule-format*]:
 $a \in \text{set } p \longrightarrow x \notin \text{dom } (Cp \ (\text{list2FWpolicy } p)) \longrightarrow x \notin \text{dom } (Cp \ a)$
proof (*induct* *p*)
case *Nil* **show** *?case* **by** *simp*
next
case (*Cons* *a* *p*) **then show** *?case*
apply (*simp-all,intro* *conjI* *impI*)
apply (*metis* *CConcStartA*)
apply *simp*
apply (*metis* *Cdom2* *List.set-simps*(2) *domIff* *insert-absorb* *list.simps*(2) *list2FW-policyconc* *set-empty*)

done
qed

lemma *C-eq-rd*[*rule-format*]:

$p \neq [] \implies Cp (list2FWpolicy (remdups p)) = Cp (list2FWpolicy p)$

proof (*rule ext*, *induct p*)

case *Nil* **thus** ?*case* **by** *simp*

next

case (*Cons y ys*) **thus** ?*case*

proof (*cases ys = []*)

case *True* **thus** ?*thesis* **by** *simp*

next

case *False* **thus** ?*thesis*

using *Cons* **apply** *simp*

apply (*intro conjI impI*)

apply (*metis Cdom2 nlpaux notindom domIff l2p-aux*)

by (*metis (no-types, lifting) Cdom2 nlpaux domIff l2p-aux remDupsNMT*)

qed

qed

lemma *nMT-domMT*:

$\neg \text{not-MT } Cp \ p \implies p \neq [] \implies r \notin \text{dom } (Cp (list2FWpolicy p))$

proof (*induct p*)

case *Nil* **thus** ?*case* **by** *simp*

next

case (*Cons x xs*) **thus** ?*case*

apply (*simp split: if-splits*)

apply (*cases xs = [], simp-all*)

by (*metis CconcNone domIff*)

qed

lemma *C-eq-RS3-aux*[*rule-format*]:

$\text{not-MT } Cp \ p \implies Cp (list2FWpolicy p) \ x = Cp (list2FWpolicy (rm-MT-rules Cp p))$

x

proof (*induct p*)

case *Nil* **thus** ?*case* **by** *simp*

next

case (*Cons y ys*) **thus** ?*case*

proof (*cases not-MT Cp ys*)

case *True* **thus** ?*thesis*

using *Cons* **apply** *simp*

apply (*intro conjI impI, simp*)

apply (*metis CconcNone True not-MTimpnotMT*)

apply (*cases x \in dom (Cp (list2FWpolicy ys))*)

```

    apply (subgoal-tac x ∈ dom (Cp (list2FWpolicy (rm-MT-rules Cp ys))))
      apply (metis (mono-tags) Cons-eq-appendI NMPPrm CeqEnd append-Nil
not-MTimpnotMT)
    apply (simp add: domIff)
    apply (subgoal-tac x ∉ dom (Cp (list2FWpolicy (rm-MT-rules Cp ys))))
    apply (metis l2p-aux l2p-aux2 nlpaux)
    by (metis domIff)
next
case False thus ?thesis
  using Cons False
proof (cases ys = [])
  case True thus ?thesis using Cons by (simp) (rule impI, simp)
next
case False thus ?thesis
  using Cons False <¬ not-MT Cp ys> apply (simp)
  apply (intro conjI impI | simp)+
  apply (subgoal-tac rm-MT-rules Cp ys = [])
  apply (subgoal-tac x ∉ dom (Cp (list2FWpolicy ys)))
  apply simp-all
  apply (metis l2p-aux nlpaux)
  apply (erule nMT-domMT, simp-all)
  by (metis SR3nMT)
qed
qed
qed

```

lemma *C-eq-id*:

```

  wellformed-policy1-strong p ⇒ Cp(list2FWpolicy (insertDeny p)) = Cp
(list2FWpolicy p)
  by (rule ext) (metis insertDeny.simps(1) wp1n-tl)

```

lemma *C-eq-RS3*:

```

  not-MT Cp p ⇒ Cp(list2FWpolicy (rm-MT-rules Cp p)) = Cp (list2FWpolicy p)
  by (rule ext) (erule C-eq-RS3-aux[symmetric])

```

lemma *NMPrd[rule-format]*: $\text{not-MT } Cp \ p \longrightarrow \text{not-MT } Cp \ (\text{remdups } p)$

```

  by (induct p, simp-all) (auto simp: NMPcharn)

```

lemma *NMPDA[rule-format]*: $\text{DenyAll} \in \text{set } p \longrightarrow \text{not-MT } Cp \ p$

```

  by (induct p, simp-all add: DAnMT)

```

lemma *NMPiD[rule-format]*: $\text{not-MT } Cp \ (\text{insertDeny } p)$

```

  by (insert DAiniD [of p]) (erule NMPDA)

```

lemma *list2FWpolicy2list*[*rule-format*]:

```
Cp (list2FWpolicy(policy2list p)) = (Cp p)
apply (rule ext)
apply (induct-tac p, simp-all)
subgoal for x x1 x2
  apply (case-tac x  $\in$  dom (Cp (x2)))
  apply (metis Cdom2 CeqEnd domIff p2lNmt)
  apply (metis CeqStart domIff p2lNmt nlpaux)
done
done
```

lemmas *C-eq-Lemmas* = *none-MT-rulesRS2* *none-MT-rulesrd* *SCp2l* *wp1n-RS2*
wp1ID *NMPiD* *waux2*
wp1alternative-RS1 *p2lNmt* *list2FWpolicy2list* *wellformed-policy3-charn*
wp1-eq

lemmas *C-eq-subst-Lemmas* = *C-eq-sorted* *C-eq-sortedQ* *C-eq-RS2* *C-eq-rd* *C-eq-RS3*
C-eq-id

lemma *C-eq-All-untilSorted*:

DenyAll \in *set*(*policy2list* *p*) \implies *all-in-list*(*policy2list* *p*) *l* \implies *allNetsDistinct*(*policy2list*
p) \implies

Cp(*list2FWpolicy* (*sort* (*removeShadowRules2* (*remdups* (*rm-MT-rules* *Cp*
(*insertDeny*

(*removeShadowRules1* (*policy2list* *p*)))))) *l*) =

Cp *p*

```
apply (subst C-eq-sorted,simp-all add: C-eq-Lemmas)
apply (subst C-eq-RS2,simp-all add: C-eq-Lemmas)
apply (subst C-eq-rd,simp-all add: C-eq-Lemmas)
apply (subst C-eq-RS3,simp-all add: C-eq-Lemmas)
apply (subst C-eq-id,simp-all add: C-eq-Lemmas)
done
```

lemma *C-eq-All-untilSortedQ*:

DenyAll \in *set*(*policy2list* *p*) \implies *all-in-list*(*policy2list* *p*) *l* \implies *allNetsDis-*
tinct(*policy2list* *p*) \implies

Cp(*list2FWpolicy* (*qsort* (*removeShadowRules2* (*remdups* (*rm-MT-rules* *Cp*
(*insertDeny*

(*removeShadowRules1* (*policy2list* *p*)))))) *l*) =

Cp *p*

```
apply (subst C-eq-sortedQ,simp-all add: C-eq-Lemmas)
apply (subst C-eq-RS2,simp-all add: C-eq-Lemmas)
apply (subst C-eq-rd,simp-all add: C-eq-Lemmas)
apply (subst C-eq-RS3,simp-all add: C-eq-Lemmas)
```

apply (*subst C-eq-id,simp-all add: C-eq-Lemmas*)
done

lemma *C-eq-All-untilSorted-withSimps*:

DenyAll \in *set (policy2list p)* \implies *all-in-list (policy2list p) l* \implies
allNetsDistinct (policy2list p) \implies
Cp(list2FWpolicy(sort(removeShadowRules2(remdups(rm-MT-rules Cp (insertDeny
(removeShadowRules1 (policy2list p)))))) l)) =
Cp p
by (*simp-all add: C-eq-Lemmas C-eq-subst-Lemmas*)

lemma *C-eq-All-untilSorted-withSimpsQ*:

DenyAll \in *set (policy2list p)* \implies *all-in-list (policy2list p) l* \implies
allNetsDistinct (policy2list p) \implies
Cp(list2FWpolicy(qsort(removeShadowRules2(remdups(rm-MT-rules Cp (insertDeny
(removeShadowRules1 (policy2list p)))))) l)) =
Cp p
by (*simp-all add: C-eq-Lemmas C-eq-subst-Lemmas*)

lemma *InDomConc[rule-format]*: $p \neq [] \longrightarrow x \in \text{dom} (Cp (list2FWpolicy (p))) \longrightarrow$
 $x \in \text{dom} (Cp (list2FWpolicy (a\#p)))$

apply (*induct p, simp-all*)
subgoal for *a p*
apply(*case-tac p = [],simp-all add: dom-def Cp.simps*)
done
done

lemma *not-in-member[rule-format]*: $\text{member } a \ b \longrightarrow x \notin \text{dom} (Cp \ b) \longrightarrow x \notin \text{dom} (Cp$
 $a)$

by (*induct b*)(*simp-all add: dom-def Cp.simps*)

lemma *src-in-sdnets[rule-format]*:

$\neg \text{member } DenyAll \ x \longrightarrow p \in \text{dom} (Cp \ x) \longrightarrow \text{subnetsOfAdr} (\text{src } p) \cap (\text{fst-set} (\text{sdnets}$
 $x)) \neq \{\}$

apply (*induct rule: Combinators.induct*)
apply (*simp-all add: fst-set-def subnetsOfAdr-def PLemmas, rename-tac x1 x2*)
apply (*intro impI*)
apply (*simp add: fst-set-def*)
subgoal for *x1 x2*
apply (*case-tac p* \in *dom (Cp x2)*)
apply (*rule subnetAux*)
apply (*auto simp: PLemmas*)
done
done

lemma *dest-in-sdnets*[rule-format]:
 $\neg \text{member DenyAll } x \longrightarrow p \in \text{dom } (Cp \ x) \longrightarrow \text{subnetsOfAdr } (\text{dest } p) \cap (\text{snd-set } (\text{sdnets } x)) \neq \{\}$
apply (*induct rule: Combinators.induct*)
apply (*simp-all add: snd-set-def subnetsOfAdr-def PLemmas, rename-tac x1 x2*)
apply (*intro impI,simp add: snd-set-def*)
subgoal for *x1 x2*
apply (*case-tac p ∈ dom (Cp x2)*)
apply (*rule subnetAux*)
apply (*auto simp: PLemmas*)
done
done

lemma *sdnets-in-subnets*[rule-format]:
 $p \in \text{dom } (Cp \ x) \longrightarrow \neg \text{member DenyAll } x \longrightarrow$
 $(\exists (a,b) \in \text{sdnets } x. a \in \text{subnetsOfAdr } (\text{src } p) \wedge b \in \text{subnetsOfAdr } (\text{dest } p))$
apply (*rule Combinators.induct*)
apply (*simp-all add: PLemmas subnetsOfAdr-def*)
apply (*intro impI, simp*)
subgoal for *x1 x2*
apply (*case-tac p ∈ dom (Cp (x2))*)
apply (*auto simp: PLemmas subnetsOfAdr-def*)
done
done

lemma *disjSD-no-p-in-both*[rule-format]:
 $\llbracket \text{disjSD-2 } x \ y; \neg \text{member DenyAll } x; \neg \text{member DenyAll } y; p \in \text{dom } (Cp \ x); p \in \text{dom } (Cp \ y) \rrbracket \Longrightarrow \text{False}$
apply (*rule-tac A = sdnets x and B = sdnets y and D = src p and F = dest p in tndFalse*)
by (*auto simp: dest-in-sdnets src-in-sdnets sdnets-in-subnets disjSD-2-def*)

lemma *list2FWpolicy-eq*:
 $zs \neq [] \Longrightarrow Cp \ (\text{list2FWpolicy } (x \oplus y \ \# \ z)) \ p = Cp \ (x \oplus \text{list2FWpolicy } (y \ \# \ z)) \ p$
by (*metis ConcAssoc l2p-aux list2FWpolicy.simps(2)*)

lemma *dom-sep*[rule-format]:
 $x \in \text{dom } (Cp \ (\text{list2FWpolicy } p)) \longrightarrow x \in \text{dom } (Cp \ (\text{list2FWpolicy}(\text{separate } p)))$
proof (*induct p rule: separate.induct,simp-all, goal-cases*)
case (*1 v va y z*) **then show** *?case*
apply (*intro conjI impI*)
apply (*simp,drule mp*)
apply (*case-tac x ∈ dom (Cp (DenyAllFromTo v va))*)

```

    apply (metis CConcStartA domIff l2p-aux2 list2FWpolicyconc not-Cons-self )
    apply (subgoal-tac x ∈ dom (Cp (list2FWpolicy (y #z))))
    apply (metis CConcStartA Cdom2 domIff l2p-aux2 list2FWpolicyconc nlpaux)
    apply (subgoal-tac x ∈ dom (Cp (list2FWpolicy ((DenyAllFromTo v va)#y#z))))
    apply (simp add: dom-def Cp.simps,simp-all)
    apply (case-tac x ∈ dom (Cp (DenyAllFromTo v va)), simp-all)
    apply (subgoal-tac x ∈ dom (Cp (list2FWpolicy (y #z))))
    apply (metis InDomConc sepnMT list.simps(2))
    apply (subgoal-tac x ∈ dom (Cp (list2FWpolicy ((DenyAllFromTo v va)#y#z))))
    by (simp-all add: dom-def Cp.simps)
next
case (2 v va vb y z) then show ?case
  apply (intro impI conjI,simp)
  apply (case-tac x ∈ dom (Cp (AllowPortFromTo v va vb)))
  apply (metis CConcStartA domIff l2p-aux2 list2FWpolicyconc not-Cons-self )
  apply (subgoal-tac x ∈ dom (Cp (list2FWpolicy (y #z))))
  apply (metis CConcStartA Cdom2 InDomConc domIff l2p-aux2 list2FWpolicyconc
nlpaux)
  apply (simp add: dom-def Cp.simps, simp-all)
  apply (case-tac x ∈ dom (Cp (AllowPortFromTo v va vb)), simp-all)
  apply (subgoal-tac x ∈ dom (Cp (list2FWpolicy (y #z))),simp)
  apply (metis Conc-not-MT InDomConc sepnMT)
  apply (metis domIff nlpaux)
done
next
case (3 v va y z) then show ?case
  apply (intro conjI impI,simp)
  apply (drule mp)
  apply (case-tac x ∈ dom (Cp ((v ⊕ va))))
  apply (metis Cp.simps(4) CConcStartA ConcAssoc domIff list2FWpolicy2list
list2FWpolicyconc p2lNmt)
  defer 1
  apply simp-all
  apply (case-tac x ∈ dom (Cp ((v ⊕ va))),simp-all)
  apply (drule mp)
  apply (simp add: Cp.simps dom-def)
  apply (metis InDomConc list.simps(2)sepnMT)
  apply (subgoal-tac x ∈ dom (Cp (list2FWpolicy (y#z))))
  apply (case-tac x ∈ dom (Cp y),simp-all)
  apply (metis CConcStartA Cdom2 ConcAssoc domIff)
  apply (metis InDomConc domIff l2p-aux2 list2FWpolicyconc nlpaux)
  apply (case-tac x ∈ dom (Cp y),simp-all)
  by (metis domIff nlpaux)
qed

```

lemma domdConcStart[rule-format]:
 $x \in \text{dom} (\text{Cp} (\text{list2FWpolicy} (a\#b))) \longrightarrow x \notin \text{dom} (\text{Cp} (\text{list2FWpolicy} b)) \longrightarrow x \in \text{dom} (\text{Cp} (a))$
by (*induct b, simp-all*) (*auto simp: PLemmas*)

lemma sep-dom2-aux:
 $x \in \text{dom} (\text{Cp} (\text{list2FWpolicy} (a \oplus y \# z))) \implies x \in \text{dom} (\text{Cp} (a \oplus \text{list2FWpolicy} (y \# z)))$
by auto (*metis list2FWpolicy-eq p2lNmt*)

lemma sep-dom2-aux2:
 $(x \in \text{dom} (\text{Cp} (\text{list2FWpolicy} (\text{separate} (y \# z)))) \longrightarrow x \in \text{dom} (\text{Cp} (\text{list2FWpolicy} (y \# z)))) \implies$
 $x \in \text{dom} (\text{Cp} (\text{list2FWpolicy} (a \# \text{separate} (y \# z)))) \implies$
 $x \in \text{dom} (\text{Cp} (\text{list2FWpolicy} (a \oplus y \# z)))$
by (*metis CConcStartA InDomConc domdConcStart list.simps(2) list2FWpolicy.simps(2) list2FWpolicyconc*)

lemma sep-dom2[rule-format]:
 $x \in \text{dom} (\text{Cp} (\text{list2FWpolicy} (\text{separate} p))) \longrightarrow x \in \text{dom} (\text{Cp} (\text{list2FWpolicy} (p)))$
by (*rule separate.induct*) (*simp-all add: sep-dom2-aux sep-dom2-aux2*)

lemma sepDom: $\text{dom} (\text{Cp} (\text{list2FWpolicy} p)) = \text{dom} (\text{Cp} (\text{list2FWpolicy} (\text{separate} p)))$
by (*rule equalityI*) (*rule subsetI, (erule dom-sep|erule sep-dom2)*)⁺

lemma C-eq-s-ext[rule-format]:
 $p \neq [] \longrightarrow \text{Cp} (\text{list2FWpolicy} (\text{separate} p)) a = \text{Cp} (\text{list2FWpolicy} p) a$

proof (*induct rule: separate.induct, goal-cases*)

case (*1 x*) **thus** *?case*
apply (*cases x = [], simp-all*)
apply (*cases a \in \text{dom} (\text{Cp} (\text{list2FWpolicy} x))*)
apply (*subgoal-tac a \in \text{dom} (\text{Cp} (\text{list2FWpolicy} (\text{separate} x)))*)
apply (*metis Cdom2 list2FWpolicyconc sepDom sepnMT*)
apply (*metis sepDom*)
by (*metis nlpaux sepDom list2FWpolicyconc sepnMT*)

next

case (*2 v va y z*) **thus** *?case*
apply (*cases z = [], simp-all*)
apply (*intro conjI impI|simp*)⁺
apply (*simp add: PLemmas(8) UPFDefs(8) list2FWpolicyconc sepnMT*)
by (*metis (mono-tags, lifting) Conc-not-MT Cdom2 list2FWpolicy-eq nlpaux sepDom l2p-aux sepnMT*)

next

```

case (3 v va vb y z) thus ?case
  apply (cases z = [], simp-all)
  apply (simp add: PLemmas(8) UPFDefs(8) list2FWpolicyconc sepnMT)
  by (metis (no-types, opaque-lifting) Conc-not-MT Cdom2 nlpaux domIff l2p-aux
sepnMT)
next
case (4 v va y z) thus ?case
  apply (cases z = [], simp-all)
  apply (simp add: PLemmas(8) UPFDefs(8) l2p-aux sepnMT)
  by (metis (no-types, lifting) ConcAssoc PLemmas(8) UPFDefs(8) list.distinct(1)
list2FWpolicyconc sepnMT)
next
case 5 thus ?case by simp
next
case 6 thus ?case by simp
next
case 7 thus ?case by simp
next
case 8 thus ?case by simp
qed

```

lemma *C-eq-s*: $p \neq [] \implies C_p (\text{list2FWpolicy } (\text{separate } p)) = C_p (\text{list2FWpolicy } p)$
by (rule ext) (simp add: C-eq-s-ext)

lemmas *sortnMTQ* = *NormalisationIntegerPortProof.C-eq-Lemmas-sep(14)*

lemmas *C-eq-Lemmas-sep* = *C-eq-Lemmas sortnMT sortnMTQ RS2-NMT NMPrd not-MTimpnotMT*

lemma *C-eq-until-separated*:

$\text{DenyAll} \in \text{set} (\text{policy2list } p) \implies \text{all-in-list} (\text{policy2list } p) l \implies \text{allNetsDistinct} (\text{policy2list } p) \implies$

$C_p (\text{list2FWpolicy } (\text{separate } (\text{sort } (\text{removeShadowRules2 } (\text{remdups } (\text{rm-MT-rules } C_p$
 $(\text{insertDeny } (\text{removeShadowRules1 } (\text{policy2list } p)))))) l))) =$

$C_p p$

by (simp add: C-eq-All-untilSorted-withSimps C-eq-s wellformed1-alternative-sorted wp1ID wp1n-RS2)

lemma *C-eq-until-separatedQ*:

$\text{DenyAll} \in \text{set} (\text{policy2list } p) \implies \text{all-in-list} (\text{policy2list } p) l \implies$

$\text{allNetsDistinct} (\text{policy2list } p) \implies$

$C_p (\text{list2FWpolicy} (\text{separate} (\text{qsort} ($
 $\text{removeShadowRules2} (\text{remdups } (\text{rm-MT-rules } C_p$
 $(\text{insertDeny } (\text{removeShadowRules1 } (\text{policy2list } p)))))) l))) =$

$C_p p$

by (simp add: C-eq-All-untilSorted-withSimpsQ C-eq-s setnMT wp1ID wp1n-RS2)

lemma domID[rule-format]:

$p \neq [] \wedge x \in \text{dom}(Cp(\text{list2FWpolicy } p)) \longrightarrow x \in \text{dom}(Cp(\text{list2FWpolicy}(\text{insertDenies } p)))$

proof(induct p)

case Nil then show ?case by simp

next

case (Cons a p) then show ?case

proof(cases p=[], goal-cases)

case 1 then show ?case

apply(simp) apply(rule impI)

apply (cases a, simp-all)

apply (simp-all add: Cp.simps dom-def)+

by auto

next

case 2 then show ?case

proof(cases $x \in \text{dom}(Cp(\text{list2FWpolicy } p))$, goal-cases)

case 1 then show ?case

apply simp apply (rule impI)

apply (cases a, simp-all)

apply (metis InDomConc idNMT)

apply (rule InDomConc, simp-all add: idNMT)+

done

next

case 2 then show ?case

apply simp apply (rule impI)

proof(cases $x \in \text{dom}(Cp(\text{list2FWpolicy}(\text{insertDenies } p)))$, goal-cases)

case 1 then show ?case

proof(induct a)

case DenyAll then show ?case by simp

next

case (DenyAllFromTo src dest) then show ?case

by simp (rule InDomConc, simp add: idNMT)

next

case (AllowPortFromTo src dest port) then show ?case

by simp (rule InDomConc, simp add: idNMT)

next

case (Conc - -) then show ?case

by simp(rule InDomConc, simp add: idNMT)

qed

next

case 2 then show ?case

proof (induct a)

```

    case DenyAll then show ?case by simp
next
case (DenyAllFromTo src dest) then show ?case
  by (simp, metis domIff CConcStartA list2FWpolicyconc nlpaux Cdom2)
next
case (AllowPortFromTo src dest port) then show ?case
  by (simp, metis domIff CConcStartA list2FWpolicyconc nlpaux Cdom2)
next
case (Conc - -) then show ?case
  by simp (metis CConcStartA Cdom2 Conc(5) ConcAssoc domIff domdConc-
Start)
  qed
  qed
  qed
  qed
qed

```

lemma *DA-is-deny*:

```

x ∈ dom (Cp (DenyAllFromTo a b ⊕ DenyAllFromTo b a ⊕ DenyAllFromTo a b))
⇒
Cp (DenyAllFromTo a b ⊕ DenyAllFromTo b a ⊕ DenyAllFromTo a b) x = Some (deny
())
  by (case-tac x ∈ dom (Cp (DenyAllFromTo a b))) (simp-all add: PLemmas split:
if-splits)

```

lemma *iDdomAux*[rule-format]:

```

p ≠ [] → x ∉ dom (Cp (list2FWpolicy p)) →
x ∈ dom (Cp (list2FWpolicy (insertDenies p))) →
Cp (list2FWpolicy (insertDenies p)) x = Some (deny ())
proof (induct p)
  case Nil thus ?case by simp
next
  case (Cons y ys) thus ?case
proof (cases y)
  case DenyAll then show ?thesis by simp
next
  case (DenyAllFromTo a b) then show ?thesis
    using DenyAllFromTo Cons apply simp
    apply (rule impI)+
proof (cases ys = [], goal-cases)
  case 1 then show ?case by (simp add: DA-is-deny)
next
  case 2 then show ?case
    apply simp

```

```

apply (drule mp)
apply (metis DenyAllFromTo InDomConc )
apply (cases x ∈ dom (Cp (list2FWpolicy (insertDenies ys))),simp-all)
apply (metis Cdom2 DenyAllFromTo idNMT list2FWpolicyconc)
apply (subgoal-tac Cp (list2FWpolicy (DenyAllFromTo a b ⊕
      DenyAllFromTo b a ⊕ DenyAllFromTo a b#insertDenies
ys)) x =
      Cp ((DenyAllFromTo a b ⊕ DenyAllFromTo b a ⊕
DenyAllFromTo a b)) x )
apply (metis DA-is-deny DenyAllFromTo domdConcStart)
apply (metis DenyAllFromTo l2p-aux2 list2FWpolicyconc nlpaux)
done
qed
next
case (AllowPortFromTo a b c) then show ?thesis using Cons AllowPortFromTo
proof (cases ys = [], goal-cases)
case 1 then show ?case
apply (simp,intro impI)
apply (subgoal-tac x ∈ dom (Cp (DenyAllFromTo a b ⊕ DenyAllFromTo b a)))
apply (auto simp: PLemmas split: if-splits)
done
next
case 2 then show ?case
apply (simp, intro impI)
apply (drule mp)
apply (metis AllowPortFromTo InDomConc)
apply (cases x ∈ dom (Cp (list2FWpolicy (insertDenies ys))),simp-all)
apply (metis AllowPortFromTo Cdom2 idNMT list2FWpolicyconc)
apply (subgoal-tac Cp (list2FWpolicy (DenyAllFromTo a b ⊕
      DenyAllFromTo b a ⊕
      AllowPortFromTo a b c#insertDenies ys)) x =
      Cp ((DenyAllFromTo a b ⊕ DenyAllFromTo b a)) x )
apply (auto simp: PLemmas split: if-splits)[1]
by (metis AllowPortFromTo CConcStartA ConcAssoc idNMT list2FWpolicyconc
nlpaux)
qed
next
case (Conc a b) then show ?thesis
proof (cases ys = [], goal-cases)
case 1 then show ?case
apply(simp,intro impI)
apply (subgoal-tac x ∈ dom (Cp (DenyAllFromTo (first-srcNet a) (first-destNet
a) ⊕
      DenyAllFromTo (first-destNet a) (first-srcNet a))))

```

```

    by (auto simp: PLemmas split: if-splits)
  next
  case 2 then show ?case
    apply(simp,intro impI)
    apply(cases x ∈ dom (Cp (list2FWpolicy (insertDenies ys))))
    apply (metis Cdom2 Conc Cons InDomConc idNMT list2FWpolicyconc)
  apply (subgoal-tac Cp (list2FWpolicy (DenyAllFromTo (first-srcNet a)(first-destNet
a) ⊕
                                DenyAllFromTo (first-destNet a) (first-srcNet
a)⊕
                                a ⊕ b#insertDenies ys)) x =
    Cp ((DenyAllFromTo(first-srcNet a) (first-destNet a) ⊕
    DenyAllFromTo (first-destNet a)(first-srcNet a) ⊕ a ⊕
b)) x)
    apply simp
    defer 1
    apply (metis Conc l2p-aux2 list2FWpolicyconc nlpaux)
    apply (subgoal-tac Cp((DenyAllFromTo(first-srcNet a)(first-destNet a) ⊕
    DenyAllFromTo (first-destNet a)(first-srcNet a)⊕ a ⊕ b))
x =
                                Cp((DenyAllFromTo (first-srcNet a)(first-destNet a)⊕
    DenyAllFromTo (first-destNet a) (first-srcNet a))) x )
    apply simp
    defer 1
    apply (metis CConcStartA Conc ConcAssoc nlpaux)
    by (auto simp: PLemmas split: if-splits)
  qed
  qed
  qed

```

lemma *iD-isD*[rule-format]:

```

p ≠ [] → x ∉ dom (Cp (list2FWpolicy p)) →
Cp (DenyAll ⊕ list2FWpolicy (insertDenies p)) x = Cp DenyAll x
  apply (case-tac x ∈ dom (Cp (list2FWpolicy (insertDenies p))))
  apply (simp add: Cp.simps(1) Cdom2 iDdomAux deny-all-def)
  using NormalisationIPPProofs.nlpaux
  by blast

```

lemma *inDomConc*:

```

x ∉ dom (Cp a) ⇒ x ∉ dom (Cp (list2FWpolicy p)) ⇒ x ∉ dom (Cp
(list2FWpolicy(a#p)))
  by (metis domdConcStart)

```

lemma *domsdisj*[rule-format]:

$p \neq [] \longrightarrow (\forall x s. s \in \text{set } p \wedge x \in \text{dom } (Cp A) \longrightarrow x \notin \text{dom } (Cp s)) \longrightarrow y \in \text{dom } (Cp A) \longrightarrow$
 $y \notin \text{dom } (Cp (\text{list2FWpolicy } p))$
proof (*induct* p)
case *Nil* **show** *?case* **by** *simp*
next
case (*Cons* a p) **show** *?case*
apply (*case-tac* $p = []$, *simp*)
apply (*rule-tac* $x = y$ **in** *spec*)
apply (*simp* *add: split-tupled-all*)
by (*metis* *Cons.hyps inDomConc list.set-intros(1) list.set-intros(2)*)
qed

lemma *isSepaux*:

$p \neq [] \implies \text{noDenyAll } (a\#p) \implies \text{separated } (a \# p) \implies$
 $x \in \text{dom } (Cp (\text{DenyAllFromTo } (\text{first-srcNet } a) (\text{first-destNet } a) \oplus$
 $\text{DenyAllFromTo } (\text{first-destNet } a) (\text{first-srcNet } a) \oplus a)) \implies$
 $x \notin \text{dom } (Cp (\text{list2FWpolicy } p))$
apply (*rule-tac* $A = (\text{DenyAllFromTo } (\text{first-srcNet } a) (\text{first-destNet } a) \oplus$
 $\text{DenyAllFromTo } (\text{first-destNet } a) (\text{first-srcNet } a) \oplus a)$ **in** *domsdisj*,
simp-all)
apply (*rule* *notI*)
subgoal for xa s
apply (*rule-tac* $p = xa$ **and** $x = (\text{DenyAllFromTo } (\text{first-srcNet } a) (\text{first-destNet } a) \oplus$
 $\text{DenyAllFromTo } (\text{first-destNet } a) (\text{first-srcNet } a) \oplus a)$
and $y = s$ **in** *disjSD-no-p-in-both*, *simp-all*)
using *disjSD2aux noDA* **apply** *blast*
using *noDA*
by *blast*
done

lemma *none-MT-rulessep[rule-format]*: *none-MT-rules* Cp $p \longrightarrow$ *none-MT-rules* Cp (*separate* p)

by (*induct* p *rule: separate.induct*) (*simp-all* *add: Cp.simps map-add-le-mapE map-le-antisym*)

lemma *dom-id*:

$\text{noDenyAll } (a\#p) \implies \text{separated } (a\#p) \implies p \neq [] \implies$
 $x \notin \text{dom } (Cp (\text{list2FWpolicy } p)) \implies x \in \text{dom } (Cp (a)) \implies$
 $x \notin \text{dom } (Cp (\text{list2FWpolicy } (\text{insertDenies } p)))$
apply (*rule-tac* $a = a$ **in** *isSepaux*, *simp-all*)
using *idNMT* **apply** *blast*
using *noDAID* **apply** *blast*
using *id-aux4 noDA1eq sepNetsID* **apply** *blast*

by (simp add: NormalisationIPPProofs.Cdom2 domIff)

lemma *C-eq-iD-aux2*[rule-format]:

noDenyAll1 $p \longrightarrow \text{separated } p \longrightarrow p \neq [] \longrightarrow x \in \text{dom } (Cp (\text{list2FWpolicy } p)) \longrightarrow$
 $Cp(\text{list2FWpolicy } (\text{insertDenies } p)) \ x = Cp(\text{list2FWpolicy } p) \ x$

proof (induct *p*)

case *Nil* thus ?case by simp

next

case (Cons *y ys*) thus ?case

using Cons proof (cases *y*)

case *DenyAll* thus ?thesis using Cons *DenyAll* apply simp

apply (case-tac $ys = []$, simp-all)

apply (case-tac $x \in \text{dom } (Cp (\text{list2FWpolicy } ys))$, simp-all)

apply (metis Cdom2 domID idNMT list2FWpolicyconc noDA1eq)

apply (metis *DenyAll* iD-isD idNMT list2FWpolicyconc nlpaux)

done

next

case (*DenyAllFromTo a b*) thus ?thesis

using Cons apply simp

apply (rule impI|rule allI|rule conjI|simp)+

apply (case-tac $ys = []$, simp-all)

apply (metis Cdom2 ConcAssoc *DenyAllFromTo*)

apply (case-tac $x \in \text{dom } (Cp (\text{list2FWpolicy } ys))$, simp-all)

apply (drule mp)

apply (metis noDA1eq)

apply (case-tac $x \in \text{dom } (Cp (\text{list2FWpolicy } (\text{insertDenies } ys)))$)

apply (metis Cdom2 *DenyAllFromTo* idNMT list2FWpolicyconc)

apply (metis domID)

apply (case-tac $x \in \text{dom } (Cp (\text{list2FWpolicy } (\text{insertDenies } ys)))$)

apply (subgoal-tac $Cp (\text{list2FWpolicy } (\text{DenyAllFromTo } a \ b \ \oplus \ \text{DenyAllFromTo } b$

$a \ \oplus$

$\text{DenyAllFromTo } a \ b \ \# \ \text{insertDenies } ys)) \ x = \text{Some } (\text{deny } ()))$

apply simp-all

apply (subgoal-tac $Cp (\text{list2FWpolicy } (\text{DenyAllFromTo } a \ b \ \# \ ys)) \ x =$

$Cp ((\text{DenyAllFromTo } a \ b)) \ x$

apply (simp add: PLemmas, simp split: if-splits)

apply (metis list2FWpolicyconc nlpaux)

apply (metis Cdom2 *DenyAllFromTo* iD-isD iDdomAux idNMT list2FWpolicyconc)

apply (metis Cdom2 *DenyAllFromTo* domIff idNMT list2FWpolicyconc nlpaux)

done

next

case (*AllowPortFromTo a b c*) thus ?thesis

using *AllowPortFromTo* Cons apply simp

apply (rule impI|rule allI|rule conjI|simp)+

```

apply (case-tac ys = [], simp-all)
  apply (metis Cdom2 ConcAssoc AllowPortFromTo)
apply (case-tac x ∈ dom (Cp (list2FWpolicy ys)),simp-all)
  apply (drule mp)
    apply (metis noDA1eq)
  apply (case-tac x ∈ dom (Cp (list2FWpolicy (insertDenies ys))))
    apply (metis Cdom2 AllowPortFromTo idNMT list2FWpolicyconc)
  apply (metis domID)
apply (subgoal-tac x ∈ dom (Cp (AllowPortFromTo a b c)))
  apply (case-tac x ∉ dom (Cp (list2FWpolicy (insertDenies ys))), simp-all)
    apply (metis AllowPortFromTo Cdom2 ConcAssoc l2p-aux2 list2FWpolicyconc
nlpaux)
      apply (meson Combinators.distinct(3) FWNormalisationCore.member.simps(4)
NormalisationIPPProofs.dom-id noDenyAll.simps(1) separated.simps(1))
    apply (metis AllowPortFromTo domdConcStart)
  done
next
case (Conc a b) thus ?thesis
  using Cons Conc apply simp
  apply (intro impI allI conjI|simp)+
  apply (case-tac ys = [],simp-all)
    apply (metis Cdom2 ConcAssoc Conc)
  apply (case-tac x ∈ dom (Cp (list2FWpolicy ys)),simp-all)
    apply (drule mp)
      apply (metis noDA1eq)
    apply (case-tac x ∈ dom (Cp (a ⊕ b)))
      apply (case-tac x ∉ dom (Cp (list2FWpolicy (insertDenies ys))), simp-all)
        apply (subst list2FWpolicyconc)
          apply (rule idNMT, simp)
        apply (metis domID)
          apply (metis Cdom2 Conc idNMT list2FWpolicyconc)
        apply (metis Cdom2 Conc domIff idNMT list2FWpolicyconc )
    apply (case-tac x ∈ dom (Cp (a ⊕ b)))
      apply (case-tac x ∉ dom (Cp (list2FWpolicy (insertDenies ys))), simp-all)
        apply (subst list2FWpolicyconc)
          apply (rule idNMT, simp)
        apply (metis Cdom2 Conc ConcAssoc list2FWpolicyconc nlpaux)
      apply (metis (lifting, no-types) FWNormalisationCore.member.simps(1) Normal-
isationIPPProofs.dom-id noDenyAll.simps(1) separated.simps(1))
    apply (metis Conc domdConcStart)
  done
qed
qed

```

lemma *C-eq-iD*:

separated p \implies *noDenyAll1 p* \implies *wellformed-policy1-strong p* \implies
Cp(list2FWpolicy (insertDenies p)) = Cp (list2FWpolicy p)
by (*rule ext*) (*metis CConcStartA C-eq-iD-aux2 DAAux wp1-alternative-not-mt wp1n-tl*)

lemma *noDAsortQ[rule-format]*: *noDenyAll1 p* \longrightarrow *noDenyAll1 (qsort p l)*

proof (*cases p*)

case *Nil* **then show** *?thesis* **by** *simp*

next

case (*Cons a list*) **show** *?thesis*

apply (*insert* $\langle p = a \# list \rangle$, *simp-all*)

proof (*cases a = DenyAll*)

case *True*

assume $*$: *a = DenyAll*

show *noDenyAll1(a # list)* \longrightarrow

noDenyAll1(qsort[y \leftarrow list . \neg smaller a y l] l @ a # qsort [y \leftarrow list . smaller a y l] l)

using *noDAsortQ* **by** *fastforce*

next

case *False*

assume $*$: *a \neq DenyAll*

have $**$: *noDenyAll1 (a # list)* \implies *noDenyAll (a # list)* **by**(*case-tac a, simp-all add:**)

show *noDenyAll1(a # list)* \longrightarrow

noDenyAll1(qsort[y \leftarrow list . \neg smaller a y l] l @ a # qsort [y \leftarrow list . smaller a y l] l)

apply (*insert* $*$, *rule impI*)

apply (*rule noDA1eq*, *frule ***)

by (*metis append-Cons append-Nil nDAeqSet qsort.simps(2) set-sortQ*)

qed

qed

lemma *NetsCollectedSortQ*:

distinct p \implies *noDenyAll1 p* \implies *all-in-list p l* \implies *singleCombinators p* \implies

NetsCollected (qsort p l)

by(*metis C-eqLemmas-id(22)*)

lemmas *CLemmas* = *nMTSort nMTSortQ none-MT-rulesRS2 none-MT-rulesrd*

noDAsort noDAsortQ nDASC wp1-eq wp1IID SCp2l ANDSep wp1n-RS2

OTNSEp OTNSC noDA1sep wp1-alternativesep wellformed-eq

wellformed1-alternative-sorted

lemmas $C\text{-eqLemmas-id} = C\text{Lemmas } NC2Sep \text{ NetsCollectedSep}$
 $NetsCollectedSort \text{ NetsCollectedSortQ separatedNC}$

lemma $C\text{-eq-Until-InsertDenies}$:

$DenyAll \in set(policy2list \ p) \implies all\text{-in-list}(policy2list \ p) \ l \implies allNetsDistinct$
 $(policy2list \ p) \implies$
 $Cp \ (list2FWpolicy((insertDenies(separate(sort(removeShadowRules2$
 $(remdups(rm\text{-}MT\text{-}rules \ Cp \ (insertDeny \ (removeShadowRules1 \ (policy2list$
 $p)))))) \ l)))) =$
 $Cp \ p$
by $(subst \ C\text{-eq-id}, simp\text{-}all \ add: \ C\text{-eqLemmas-id}) \ (rule \ C\text{-eq-until-separated}, \ simp\text{-}all)$

lemma $C\text{-eq-Until-InsertDeniesQ}$:

$DenyAll \in set \ (policy2list \ p) \implies all\text{-in-list} \ (policy2list \ p) \ l \implies$
 $allNetsDistinct \ (policy2list \ p) \implies$
 $Cp \ (list2FWpolicy \ ((insertDenies \ (separate \ (qsort \ (removeShadowRules2$
 $(remdups \ (rm\text{-}MT\text{-}rules \ Cp \ (insertDeny \ (removeShadowRules1 \ (policy2list$
 $p)))))) \ l)))) =$
 $Cp \ p$
apply $(subst \ C\text{-eq-id}, \ simp\text{-}all \ add: \ C\text{-eqLemmas-id})$
apply $(metis \ WP1rd \ set\text{-}qsort \ wellformed1\text{-}sortedQ \ wellformed\text{-}eq \ wp1ID$
 $wp1\text{-}alternativesep$
 $wp1\text{-}aux1aa \ wp1n\text{-}RS2 \ wp1n\text{-}RS3)$
apply $(rule \ C\text{-eq-until-separatedQ})$
by $simp\text{-}all$

lemma $C\text{-eq-RD-aux}[rule\text{-}format]$: $Cp \ (p) \ x = Cp \ (removeDuplicates \ p) \ x$

apply $(induct \ p, \ simp\text{-}all)$
apply $(intro \ conjI \ impI)$
by $(metis \ Cdom2 \ domIff \ nlpaux \ not\text{-}in\text{-}member) \ (metis \ Cp.simps(4) \ CConcStartaux$
 $Cdom2 \ domIff)$

lemma $C\text{-eq-RAD-aux}[rule\text{-}format]$:

$p \neq [] \longrightarrow Cp \ (list2FWpolicy \ p) \ x = Cp \ (list2FWpolicy \ (removeAllDuplicates \ p)) \ x$
proof $(induct \ p)$
case Nil **show** $?case$ **by** $simp$
next
case $(Cons \ a \ p)$ **then show** $?case$
apply $simp\text{-}all$
apply $(case\text{-}tac \ p = [], \ simp\text{-}all)$
apply $(metis \ C\text{-eq-RD-aux})$
apply $(subst \ list2FWpolicyconc, \ simp)$
apply $(case\text{-}tac \ x \in dom \ (Cp \ (list2FWpolicy \ p)))$
apply $(subst \ list2FWpolicyconc)$
apply $(rule \ rADnMT, \ simp)$

apply (*subst Cdom2,simp*)
apply (*simp add: NormalisationIPPProofs.Cdom2 domIff*)
by (*metis C-eq-RD-aux nlpaux domIff list2FWpolicyconc rADnMT*)
qed

lemma C-eq-RAD:
 $p \neq [] \implies Cp (list2FWpolicy p) = Cp (list2FWpolicy (removeAllDuplicates p))$
by (*rule ext*) (*erule C-eq-RAD-aux*)

lemma C-eq-compile:
 $DenyAll \in set (policy2list p) \implies all-in-list (policy2list p) l \implies$
 $allNetsDistinct (policy2list p) \implies$
 $Cp (list2FWpolicy (removeAllDuplicates (insertDenies (separate$
 $(sort (removeShadowRules2 (remdups (rm-MT-rules Cp (insertDeny$
 $(removeShadowRules1 (policy2list p)))))) l)))) = Cp p$
by (*metis C-eq-RAD C-eq-Until-InsertDenies removeAllDuplicates.simps(2)*)

lemma C-eq-compileQ:
 $DenyAll \in set (policy2list p) \implies all-in-list (policy2list p) l \implies allNetsDis-$
 $tinct (policy2list p) \implies$
 $Cp (list2FWpolicy (removeAllDuplicates (insertDenies (separate (qsort$
 $(removeShadowRules2 (remdups (rm-MT-rules Cp (insertDeny$
 $(removeShadowRules1 (policy2list p)))))) l)))) = Cp p$
apply (*subst C-eq-RAD[symmetric]*)
apply (*rule idNMT*)
apply (*metis WP1rd sepnMT sortnMTQ wellformed-policy1-strong.simps(1) wp1ID*
 $wp1n-RS2 wp1n-RS3$)
apply (*rule C-eq-Until-InsertDeniesQ, simp-all*)
done

lemma C-eq-normalizePr:
 $DenyAll \in set (policy2list p) \implies allNetsDistinct (policy2list p) \implies$
 $all-in-list (policy2list p) (Nets-List p) \implies$
 $Cp (list2FWpolicy (normalizePr p)) = Cp p$
unfolding *normalizePrQ-def*
by (*simp add: C-eq-compile normalizePr-def*)

lemma C-eq-normalizePrQ:
 $DenyAll \in set (policy2list p) \implies allNetsDistinct (policy2list p) \implies$
 $all-in-list (policy2list p) (Nets-List p) \implies$
 $Cp (list2FWpolicy (normalizePrQ p)) = Cp p$
unfolding *normalizePrQ-def*
using *C-eq-compileQ by auto*

lemma *domSubset3*: $\text{dom } (Cp (DenyAll \oplus x)) = \text{dom } (Cp (DenyAll))$
by (*simp add: PLemmas split-tupled-all split: option.splits*)

lemma *domSubset4*:
 $\text{dom } (Cp (DenyAllFromTo x y \oplus DenyAllFromTo y x \oplus AllowPortFromTo x y dn)) =$
 $\text{dom } (Cp (DenyAllFromTo x y \oplus DenyAllFromTo y x))$
by (*simp add: PLemmas split: option.splits decision.splits*) *auto*

lemma *domSubset5*:
 $\text{dom } (Cp (DenyAllFromTo x y \oplus DenyAllFromTo y x \oplus AllowPortFromTo y x dn)) =$
 $\text{dom } (Cp (DenyAllFromTo x y \oplus DenyAllFromTo y x))$
by (*simp add: PLemmas split: option.splits decision.splits*) *auto*

lemma *domSubset1*:
 $\text{dom } (Cp (DenyAllFromTo one two \oplus DenyAllFromTo two one \oplus AllowPortFromTo one two dn \oplus x)) =$
 $\text{dom } (Cp (DenyAllFromTo one two \oplus DenyAllFromTo two one \oplus x))$
by (*simp add: PLemmas allow-all-def deny-all-def split: option.splits decision.splits*) *auto*

lemma *domSubset2*:
 $\text{dom } (Cp (DenyAllFromTo one two \oplus DenyAllFromTo two one \oplus AllowPortFromTo two one dn \oplus x)) =$
 $\text{dom } (Cp (DenyAllFromTo one two \oplus DenyAllFromTo two one \oplus x))$
by (*simp add: PLemmas allow-all-def deny-all-def split: option.splits decision.splits*) *auto*

lemma *ConcAssoc2*: $Cp (X \oplus Y \oplus ((A \oplus B) \oplus D)) = Cp (X \oplus Y \oplus A \oplus B \oplus D)$
by (*simp add: Cp.simps*)

lemma *ConcAssoc3*: $Cp (X \oplus ((Y \oplus A) \oplus D)) = Cp (X \oplus Y \oplus A \oplus D)$
by (*simp add: Cp.simps*)

lemma *RS3-NMT[rule-format]*: $DenyAll \in \text{set } p \longrightarrow$
 $\text{rm-MT-rules } Cp p \neq []$
by (*induct-tac p*) (*simp-all add: PLemmas*)

lemma *norm-notMT*: $DenyAll \in \text{set } (\text{policy2list } p) \implies \text{normalizePr } p \neq []$
unfolding *normalizePrQ-def*
by (*simp add: DAiniD RS3-NMT RS2-NMT idNMT normalizePr-def rADnMT sep-nMT sortnMT*)

lemma *norm-notMTQ*: $DenyAll \in set (policy2list p) \implies normalizePrQ p \neq []$
unfolding *normalizePrQ-def*
by (*simp add: DAiniD RS3-NMT sortnMTQ RS2-NMT idNMT rADnMT sepnMT*)

lemma *domDA*: $dom (Cp (DenyAll \oplus A)) = dom (Cp (DenyAll))$
by (*rule domSubset3*)

lemmas *domain-reasoningPr = domDA ConcAssoc2 domSubset1 domSubset2 domSubset3 domSubset4 domSubset5 domSubsetDistr1 domSubsetDistr2 domSubsetDistrA domSubsetDistrD coerc-assoc ConcAssoc ConcAssoc3*

The following lemmas help with the normalisation

lemma *list2policyR-Start[rule-format]*: $p \in dom (Cp a) \longrightarrow Cp (list2policyR (a \# list)) p = Cp a p$
by (*induct a \# list rule:list2policyR.induct*)
(auto simp: Cp.simps dom-def map-add-def)

lemma *list2policyR-End*: $p \notin dom (Cp a) \implies Cp (list2policyR (a \# list)) p = (Cp a \oplus list2policy (map Cp list)) p$
by (*rule list2policyR.induct*)
(simp-all add: Cp.simps dom-def map-add-def list2policy-def split: option.splits)

lemma *l2polR-eq-el[rule-format]*: $N \neq [] \longrightarrow Cp (list2policyR N) p = (list2policy (map Cp N)) p$

proof (*induct N*)

case *Nil show ?case by simp*

next

case (*Cons a p show ?case*)

apply (*insert Cons.hyps, simp-all add: list2policy-def*)

by (*metis list2policyR-End list2policyR-Start domStart list2policy-def*)

qed

lemma *l2polR-eq*:

$N \neq [] \implies Cp (list2policyR N) = (list2policy (map Cp N))$

by (*auto simp: list2policy-def l2polR-eq-el*)

lemma *list2FWpolicys-eq-el[rule-format]*:

$Filter \neq [] \longrightarrow Cp (list2policyR Filter) p = Cp (list2FWpolicy (rev Filter)) p$

apply (*induct-tac Filter*)

apply *simp-all*

subgoal for a list

apply (*case-tac list = []*)

apply *simp-all*


```

apply (case-tac  $p \in \text{dom } (Cp \ a)$ )
apply simp-all
apply (rule list2policyR-Start)
apply simp-all
apply (subgoal-tac  $Cp \ (\text{list2policyR } (a \ \# \ \text{list})) \ p = Cp \ (\text{list2policyR } \text{list}) \ p$ )
apply (subgoal-tac  $Cp \ (\text{list2FWpolicy } (\text{rev } \text{list} \ @ \ [a])) \ p = Cp \ (\text{list2FWpolicy } (\text{rev } \text{list})) \ p$ )
apply simp
apply (rule CConcStart2)
apply simp
apply simp
apply (case-tac list,simp-all)
apply (simp-all add: Cp.simps dom-def map-add-def)
done
done

```

lemma list2FWpolicys-eq:

```

Filter  $\neq [] \implies$ 
Cp (list2policyR Filter) = Cp (list2FWpolicy (rev Filter))
by (rule ext, erule list2FWpolicys-eq-el)

```

lemma list2FWpolicys-eq-sym:

```

Filter  $\neq [] \implies$ 
Cp (list2policyR (rev Filter)) = Cp (list2FWpolicy Filter)
by (metis list2FWpolicys-eq rev-is-Nil-conv rev-rev-ident)

```

lemma p-eq[rule-format]: $p \neq [] \longrightarrow$

```

list2policy (map Cp (rev p)) = Cp (list2FWpolicy p)
by (metis l2polR-eq list2FWpolicys-eq-sym rev.simps(1) rev-rev-ident)

```

lemma p-eq2[rule-format]: $\text{normalizePr } x \neq [] \longrightarrow$

```

Cp (list2FWpolicy (normalizePr x)) = Cp x  $\longrightarrow$ 
list2policy (map Cp (rev (normalizePr x))) = Cp x
by (simp add: p-eq)

```

lemma p-eq2Q[rule-format]: $\text{normalizePrQ } x \neq [] \longrightarrow$

```

Cp (list2FWpolicy (normalizePrQ x)) = Cp x  $\longrightarrow$ 
list2policy (map Cp (rev (normalizePrQ x))) = Cp x
by (simp add: p-eq)

```

lemma list2listNMT[rule-format]: $x \neq [] \longrightarrow \text{map sem } x \neq []$

```

by (case-tac x) (simp-all)

```

lemma Norm-Distr2:

$r \text{ o-f } ((P \otimes_2 (\text{list2policy } Q)) \text{ o } d) =$
 $(\text{list2policy } ((P \otimes_L Q) (\otimes_2) r d))$
by (*rule ext, rule Norm-Distr-2*)

lemma *NATDistr*:

$N \neq [] \implies F = \text{Cp } (\text{list2policyR } N) \implies$
 $((\lambda (x,y). x) \text{ o-f } ((\text{NAT } \otimes_2 F) \text{ o } (\lambda x. (x,x)))) =$
 $(\text{list2policy } (((\text{NAT } \otimes_L (\text{map } \text{Cp } N)) (\otimes_2)$
 $(\lambda (x,y). x) (\lambda x. (x,x))))))$
by (*simp add: l2polR-eq*) (*rule ext,rule Norm-Distr-2*)

lemma *C-eq-normalize-manual*:

$\text{DenyAll} \in \text{set } (\text{policy2list } p) \implies \text{allNetsDistinct } (\text{policy2list } p) \implies$
 $\text{all-in-list } (\text{policy2list } p) l \implies$
 $\text{Cp } (\text{list2FWpolicy } (\text{normalize-manual-orderPr } p l)) = \text{Cp } p$
unfolding *normalize-manual-orderPr-def*
by(*simp-all add: C-eq-compile*)

lemma *p-eq2-manualQ*[*rule-format*]:

$\text{normalize-manual-orderPrQ } x l \neq [] \longrightarrow$
 $\text{Cp } (\text{list2FWpolicy } (\text{normalize-manual-orderPrQ } x l)) = \text{Cp } x \longrightarrow$
 $\text{list2policy } (\text{map } \text{Cp } (\text{rev } (\text{normalize-manual-orderPrQ } x l))) = \text{Cp } x$
by (*simp add: p-eq*)

lemma *norm-notMT-manualQ*: $\text{DenyAll} \in \text{set } (\text{policy2list } p) \implies \text{normalize-manual-orderPrQ } p l \neq []$

by (*simp add: DAiniD RS3-NMT sortnMTQ RS2-NMT idNMT normalize-manual-orderPrQ-def rADnMT sepnMT*)

lemma *C-eq-normalizePr-manualQ*:

$\text{DenyAll} \in \text{set } (\text{policy2list } p) \implies$
 $\text{allNetsDistinct } (\text{policy2list } p) \implies$
 $\text{all-in-list } (\text{policy2list } p) l \implies$
 $\text{Cp } (\text{list2FWpolicy } (\text{normalize-manual-orderPrQ } p l)) = \text{Cp } p$
by (*simp add: normalize-manual-orderPrQ-def C-eq-compileQ*)

lemma *p-eq2-manual*[*rule-format*]: $\text{normalize-manual-orderPr } x l \neq [] \longrightarrow$

$\text{Cp } (\text{list2FWpolicy } (\text{normalize-manual-orderPr } x l)) = \text{Cp } x \longrightarrow$
 $\text{list2policy } (\text{map } \text{Cp } (\text{rev } (\text{normalize-manual-orderPr } x l))) = \text{Cp } x$
by (*simp add: p-eq*)

lemma *norm-notMT-manual*: $\text{DenyAll} \in \text{set } (\text{policy2list } p) \implies \text{normalize-manual-orderPr } p l \neq []$

unfolding *normalize-manual-orderPr-def*

by (simp add: idNMT rADnMT wellformed1-alternative-sorted wp1ID wp1-alternativesep wp1n-RS2)

As an example, how this theorems can be used for a concrete normalisation instantiation.

lemma *normalizePrNAT*:

DenyAll \in set (policy2list Filter) \implies
allNetsDistinct (policy2list Filter) \implies
all-in-list (policy2list Filter) (*Nets-List Filter*) \implies
 $((\lambda (x,y). x) \text{ o-f } (((\text{NAT} \otimes_2 \text{Cp Filter}) \text{ o } (\lambda x. (x,x)))))) =$
list2policy $((\text{NAT} \otimes_L (\text{map Cp } (\text{rev } (\text{normalizePr Filter})))) (\otimes_2) (\lambda (x,y). x) (\lambda x. (x,x)))$
 by (simp add: C-eq-normalizePr NATDistr list2FWpolicys-eq-sym norm-notMT)

lemma *domSimpl[simp]*: *dom* (Cp (A \oplus DenyAll)) = *dom* (Cp (DenyAll))

by (simp add: PLemmas)

end

2.4 Stateful Network Protocols

theory

StatefulFW

imports

FTPVOIP

begin

end

2.4.1 Stateful Protocols: Foundations

theory

StatefulCore

imports

../PacketFilter/PacketFilter

LTL-alike

begin

The simple system of a stateless packet filter is not enough to model all common real-world scenarios. Some protocols need further actions in order to be secured. A prominent example is the File Transfer Protocol (FTP), which is a popular means to move files across the Internet. It behaves quite differently from most other application layer protocols as it uses a two-way connection establishment which opens a dynamic port. A stateless packet filter would only have the possibility to either always open all the possible dynamic ports or not to allow that protocol at all. Neither of these options

is satisfactory. In the first case, all ports above 1024 would have to be opened which introduces a big security hole in the system, in the second case users wouldn't be very happy. A firewall which tracks the state of the TCP connections on a system does not help here either, as the opening and closing of the ports takes place on the application layer. Therefore, a firewall needs to have some knowledge of the application protocols being run and track the states of these protocols. We next model this behaviour.

The key point of our model is the idea that a policy remains the same as before: a mapping from packet to packet out. We still specify for every packet, based on its source and destination address, the expected action. The only thing that changes now is that this mapping is allowed to change over time. This indicates that our test data will not consist of single packets but rather of sequences thereof.

At first we hence need a state. It is a tuple from some memory to be refined later and the current policy.

type-synonym $(\alpha, \beta, \gamma) \text{ FWState} = \alpha \times ((\beta, \gamma) \text{ packet} \mapsto \text{unit})$

Having a state, we need of course some state transitions. Such a transition can happen every time a new packet arrives. State transitions can be modelled using a state-exception monad.

type-synonym $(\alpha, \beta, \gamma) \text{ FWStateTransitionP} =$
 $(\beta, \gamma) \text{ packet} \Rightarrow (((\beta, \gamma) \text{ packet} \mapsto \text{unit}) \text{ decision}, (\alpha, \beta, \gamma) \text{ FWState})$
 MON_{SE}

type-synonym $(\alpha, \beta, \gamma) \text{ FWStateTransition} =$
 $((\beta, \gamma) \text{ packet} \times (\alpha, \beta, \gamma) \text{ FWState}) \mapsto (\alpha, \beta, \gamma) \text{ FWState}$

The memory could be modelled as a list of accepted packets.

type-synonym $(\beta, \gamma) \text{ history} = (\beta, \gamma) \text{ packet list}$

fun *packet-with-id* **where**

packet-with-id [] $i = []$
 $| \text{packet-with-id } (x\#xs) \ i = (\text{if } \text{id } x = i \text{ then } (x\#(\text{packet-with-id } xs \ i)) \text{ else } (\text{packet-with-id } xs \ i))$

fun *ids1* **where**

ids1 $i \ (x\#xs) = (\text{id } x = i \wedge \text{ids1 } i \ xs)$
 $| \text{ids1 } i \ [] = \text{True}$

fun *ids* **where**

ids $a \ (x\#xs) = (\text{NetworkCore.id } x \in a \wedge \text{ids } a \ xs)$
 $| \text{ids } a \ [] = \text{True}$

definition *applyPolicy*:: $(i \times (i \mapsto o)) \mapsto o$

where $applyPolicy = (\lambda (x,z). z x)$

end

2.4.2 The File Transfer Protocol (ftp)

theory

FTP

imports

StatefulCore

begin

The protocol syntax

The File Transfer Protocol FTP is a well known example of a protocol which uses dynamic ports and is therefore a natural choice to use as an example for our model.

We model only a simplified version of the FTP protocol over IntegerPort addresses, still containing all messages that matter for our purposes. It consists of the following four messages:

1. *init*: The client contacts the server indicating his wish to get some data.
2. *ftp-port-request p*: The client, usually after having received an acknowledgement of the server, indicates a port number on which he wants to receive the data.
3. *ftp-ftp-data*: The server sends the requested data over the new channel. There might be an arbitrary number of such messages, including zero.
4. *ftp-close*: The client closes the connection. The dynamic port gets closed again.

The content field of a packet therefore now consists of either one of those four messages or a default one.

datatype $msg = ftp-init \mid ftp-port-request \ port \mid ftp-data \mid ftp-close \mid ftp-other$

We now also make use of the ID field of a packet. It is used as session ID and we make the assumption that they are all unique among different protocol runs.

At first, we need some predicates which check if a packet is a specific FTP message and has the correct session ID.

definition

$is-init :: id \Rightarrow (adr_{ip}, msg)packet \Rightarrow bool$ **where**
 $is-init = (\lambda i \ p. (id \ p = i \wedge content \ p = ftp-init))$

definition

$is-ftp-port-request :: id \Rightarrow port \Rightarrow (adr_{ip}, msg) packet \Rightarrow bool$ **where**
 $is-ftp-port-request = (\lambda i \ port \ p. (id \ p = i \wedge content \ p = ftp-port-request \ port))$

definition

is-ftp-data :: $id \Rightarrow (adr_{ip}, msg) \text{ packet} \Rightarrow \text{bool}$ **where**
is-ftp-data = $(\lambda i p. (id\ p = i \wedge \text{content}\ p = \text{ftp-data}))$

definition

is-ftp-close :: $id \Rightarrow (adr_{ip}, msg) \text{ packet} \Rightarrow \text{bool}$ **where**
is-ftp-close = $(\lambda i p. (id\ p = i \wedge \text{content}\ p = \text{ftp-close}))$

definition

port-open :: $(adr_{ip}, msg) \text{ history} \Rightarrow id \Rightarrow \text{port} \Rightarrow \text{bool}$ **where**
port-open = $(\lambda L a p. (\text{not-before}\ (is-ftp-close\ a)\ (is-ftp-port-request\ a\ p)\ L))$

definition

is-ftp-other :: $id \Rightarrow (adr_{ip}, msg) \text{ packet} \Rightarrow \text{bool}$ **where**
is-ftp-other = $(\lambda i p. (id\ p = i \wedge \text{content}\ p = \text{ftp-other}))$

fun are-ftp-other where

are-ftp-other $i\ (x\#\!xs) = (is-ftp-other\ i\ x \wedge are-ftp-other\ i\ xs)$
are-ftp-other $i\ [] = \text{True}$

The protocol policy specification

We now have to model the respective state transitions. It is important to note that state transitions themselves allow all packets which are allowed by the policy, not only those which are allowed by the protocol. Their only task is to change the policy. As an alternative, we could have decided that they only allow packets which follow the protocol (e.g. come on the correct ports), but this should in our view rather be reflected in the policy itself.

Of course, not every message changes the policy. In such cases, we do not have to model different cases, one is enough. In our example, only messages 2 and 4 need special transitions. The default says that if the policy accepts the packet, it is added to the history, otherwise it is simply dropped. The policy remains the same in both cases.

fun last-opened-port where

last-opened-port $i\ ((j,s,d,ftp-port-request\ p)\#\!xs) = (\text{if}\ i=j\ \text{then}\ p\ \text{else}\ \text{last-opened-port}\ i\ xs)$
last-opened-port $i\ (x\#\!xs) = \text{last-opened-port}\ i\ xs$
last-opened-port $x\ [] = \text{undefined}$

fun FTP-STA :: ((adr_{ip},msg) history, adr_{ip}, msg) FWStateTransition where

FTP-STA $((i,s,d,ftp-port-request\ pr), (log, pol)) =$
 $(\text{if}\ \text{before}(\text{Not}\ o\ is-ftp-close\ i)(is-init\ i)\ log \wedge$
 $\text{dest-port}\ (i,s,d,ftp-port-request\ pr) = (21::\text{port}))$

then Some (((i,s,d,ftp-port-request pr)#log,
 (allow-from-to-port pr (subnet-of d) (subnet-of s)) \oplus pol))
 else Some (((i,s,d,ftp-port-request pr)#log,pol)))

|FTP-STA ((i,s,d,ftp-close), (log,pol)) =
 (if (\exists p. port-open log i p) \wedge dest-port (i,s,d,ftp-close) = (21::port)
 then Some ((i,s,d,ftp-close)#log,
 deny-from-to-port (last-opened-port i log) (subnet-of d)(subnet-of s) \oplus
 pol)
 else Some (((i,s,d,ftp-close)#log, pol)))

|FTP-STA (p, s) = Some (p#(fst s),snd s)

fun FTP-STD :: ((adr_{ip},msg) history, adr_{ip}, msg) FWStateTransition
where FTP-STD (p,s) = Some s

definition TRPolicy :: (adr_{ip},msg)packet \times (adr_{ip},msg)history \times ((adr_{ip},msg)packet
 \mapsto unit)

\mapsto (unit \times (adr_{ip},msg)history \times ((adr_{ip},msg)packet \mapsto
 unit))

where TRPolicy = ((FTP-STA,FTP-STD) \otimes_{∇} applyPolicy) o
 ($\lambda(x,(y,z)).((x,z),(x,(y,z)))$)

definition TRPolicy_{Mon}

where TRPolicy_{Mon} = policy2MON(TRPolicy)

If required to contain the policy in the output

definition TRPolicy_{Mon}'

where TRPolicy_{Mon}' = policy2MON ((($\lambda(x,y,z). (z,(y,z))$) o-f TRPolicy))

Now we specify our test scenario in more detail. We could test:

- one correct FTP-Protocol run,
- several runs after another,
- several runs interleaved,
- an illegal protocol run, or
- several illegal protocol runs.

We only do the the simplest case here: one correct protocol run.

There are four different states which are modelled as a datatype.

datatype *ftp-states* = *S0* | *S1* | *S2* | *S3*

The following constant is *True* for all sets which are correct FTP runs for a given source and destination address, ID, and data-port number.

fun

is-ftp :: *ftp-states* \Rightarrow *adr_{ip}* \Rightarrow *adr_{ip}* \Rightarrow *id* \Rightarrow *port* \Rightarrow
 (*adr_{ip},msg*) *history* \Rightarrow *bool*

where

is-ftp *H c s i p* [] = (*H=S3*)
is-ftp *H c s i p* (*x#InL*) = (*snd s = 21* \wedge ((λ (*id,sr,de,co*). (((*id = i* \wedge (
 (*H=ftp-states.S2* \wedge *sr = c* \wedge *de = s* \wedge *co = ftp-init* \wedge *is-ftp S3 c s i p InL*) \vee
 (*H=ftp-states.S1* \wedge *sr = c* \wedge *de = s* \wedge *co = ftp-port-request p* \wedge *is-ftp S2 c s i p*
 InL) \vee
 (*H=ftp-states.S1* \wedge *sr = s* \wedge *de = (fst c,p)* \wedge *co= ftp-data* \wedge *is-ftp S1 c s i p InL*) \vee
 (*H=ftp-states.S0* \wedge *sr = c* \wedge *de = s* \wedge *co = ftp-close* \wedge *is-ftp S1 c s i p InL*))))))
 x))

definition *is-single-ftp-run* :: *adr_{ip}* *src* \Rightarrow *adr_{ip}* *dest* \Rightarrow *id* \Rightarrow *port* \Rightarrow (*adr_{ip},msg*)
history set

where *is-single-ftp-run s d i p* = {*x. (is-ftp S0 s d i p x)*}

The following constant then returns a set of all the historys which denote such a normal behaviour FTP run, again for a given source and destination address, ID, and data-port.

The following definition returns the set of all possible interleaving of two correct FTP protocol runs.

definition

ftp-2-interleaved :: *adr_{ip}* *src* \Rightarrow *adr_{ip}* *dest* \Rightarrow *id* \Rightarrow *port* \Rightarrow
 adr_{ip} *src* \Rightarrow *adr_{ip}* *dest* \Rightarrow *id* \Rightarrow *port* \Rightarrow
 (*adr_{ip},msg*) *history set* **where**
ftp-2-interleaved s1 d1 i1 p1 s2 d2 i2 p2 =
 {x. (*is-ftp S0 s1 d1 i1 p1 (packet-with-id x i1)*) \wedge
 (*is-ftp S0 s2 d2 i2 p2 (packet-with-id x i2)*)}

lemma *subnetOf-lemma*: (*a::int*) \neq (*c::int*) $\implies \forall x \in \text{subnet-of } (a, b::\text{port}). (c, d) \notin x$
by (*rule ballI, simp add: subnet-of-int-def*)

lemma *subnetOf-lemma2*: $\forall x \in \text{subnet-of } (a::\text{int}, b::\text{port}). (a, b) \in x$
by (*rule ballI, simp add: subnet-of-int-def*)

lemma *subnetOf-lemma3*: ($\exists x. x \in \text{subnet-of } (a::\text{int}, b::\text{port})$)
by (*rule exI, simp add: subnet-of-int-def*)


```

lemma subnetOf-lemma4:  $\exists x \in \text{subnet-of } (a::\text{int}, b::\text{port}). (a, c::\text{port}) \in x$ 
  by (rule bexI, simp-all add: subnet-of-int-def)

lemma port-open-lemma:  $\neg (Ex (\text{port-open } [] (x::\text{port})))$ 
  by (simp add: port-open-def)

lemmas FTPLemmas = TRPolicy-def applyPolicy-def policy2MON-def
  Let-def in-subnet-def src-def
  dest-def subnet-of-int-def
  is-init-def p-accept-def port-open-def is-ftp-data-def is-ftp-close-def
  is-ftp-port-request-def content-def PortCombinators
  exI subnetOf-lemma subnetOf-lemma2 subnetOf-lemma3 subnetOf-lemma4

  NetworkCore.id-def adripLemmas port-open-lemma
  bind-SE-def unit-SE-def valid-SE-def
end

```

2.4.3 FTP enriched with a security policy

theory

FTP-WithPolicy

imports

FTP

begin

FTP where the policy is part of the output.

definition *POL* :: $'a \Rightarrow 'a$ **where** *POL* $x = x$

Variant 2 takes the policy into the output

fun *FTP-STP* ::

$((id \rightarrow \text{port}), \text{adr}_{ip}, \text{msg}) \text{FWStateTransitionP}$

where

FTP-STP $(i,s,d,\text{ftp-port-request } pr) (\text{ports}, \text{policy}) =$
 (if *p-accept* $(i,s,d,\text{ftp-port-request } pr) \text{policy}$ then
 Some $(\text{allow } (POL ((\text{allow-from-to-port } pr (\text{subnet-of } d) (\text{subnet-of } s)) \oplus \text{policy})),$
 $((\text{ports}(i \mapsto pr)), (\text{allow-from-to-port } pr (\text{subnet-of } d) (\text{subnet-of } s))$
 $\oplus \text{policy}))$
 else $(\text{Some } (\text{deny } (POL \text{policy}), (\text{ports}, \text{policy}))))$

| *FTP-STP* $(i,s,d,\text{ftp-close}) (\text{ports}, \text{policy}) =$
 (if $(\text{p-accept } (i,s,d,\text{ftp-close}) \text{policy})$ then

```

case ports i of
  Some pr ⇒
    Some(allow (POL (deny-from-to-port pr (subnet-of d) (subnet-of s) ⊕ policy)),
          ports(i:=None),
          deny-from-to-port pr (subnet-of d) (subnet-of s) ⊕ policy)
|None ⇒Some(allow (POL policy), ports, policy)
      else Some (deny (POL policy), ports, policy)

|FTP-STP p x = (if p-accept p (snd x)
                then Some (allow (POL (snd x)),((fst x),snd x))
                else Some (deny (POL (snd x)),(fst x,snd x)))
end

```

2.4.4 A simple voice-over-ip model

```

theory VOIP
  imports StatefulCore
begin

```

After the FTP-Protocol which was rather simple we show the strength of the model with a more current and especially much more complicated example, namely Voice over IP (VoIP). VoIP is standardized by the ITU-T under the name H.323, which can be seen as an "umbrella standard" which aggregates standards for multimedia conferencing over packet-based networks. H.323 poses many problems to firewalls. These problems include:

- An H.323 call is made up of many different simultaneous connections.
- Most connections are made to dynamic ports.
- The addresses and port numbers are exchanged within the data stream of the next higher connection.
- Calls can be initiated from outside the firewall.

Again we only consider a simplified VoIP scenario with the following seven messages which are grouped into four subprotocols:

- Registration and Admission (H.225, port 1719): The caller contacts its gatekeeper with a call request. The gatekeeper either rejects or confirms the request, returning the address of the callee in the latter case.
 - Admission Request (ARQ)
 - Admission Reject (ARJ)
 - Admission Confirm (ACF) 'a
- Call Signaling (Q.931, port 1720) The caller and the callee agree on the dynamic ports over which the call will take place.

- Setup *port*
- Connect *port*
- Stream (dynamic ports). The call itself. In reality, several connections are used here.
- Fin (port 1720).

The two main differences to FTP are:

- In VoIP, we deal with three different entities: the caller, the callee, and the gatekeeper.
- We do not know in advance which entity will close the connection.

We model the protocol as seen from a firewall at the caller, namely we are not interested in the messages from the callee to its gatekeeper. Incoming calls are not modelled either, they would require a different set of state transitions.

The content of a packet now consists of one of the seven messages or a default one. It is parameterized with the type of the address that the gatekeeper returns.

```
datatype 'a voip-msg = ARQ
    | ACF 'a
    | ARJ
    | Setup port
    | Connect port
    | Stream
    | Fin
    | other
```

As before, we need operators which check if a packet contains a specific content and ID, respectively if such a packet has appeared in the trace.

definition

```
is-arq :: NetworkCore.id ⇒ ('a::adr, 'b voip-msg) packet ⇒ bool where
is-arq i p = (NetworkCore.id p = i ∧ content p = ARQ)
```

definition

```
is-fin :: id ⇒ ('a::adr, 'b voip-msg) packet ⇒ bool where
is-fin i p = (id p = i ∧ content p = Fin)
```

definition

```
is-connect :: id ⇒ port ⇒ ('a::adr, 'b voip-msg) packet ⇒ bool where
is-connect i port p = (id p = i ∧ content p = Connect port)
```

definition

$is-setup :: id \Rightarrow port \Rightarrow ('a::adr, 'b voip-msg) packet \Rightarrow bool$ **where**
 $is-setup\ i\ port\ p = (id\ p = i \wedge content\ p = Setup\ port)$

We need also an operator $ports-open$ to get access to the two dynamic ports.

definition

$ports-open :: id \Rightarrow port \times port \Rightarrow (adr_{ip}, 'a voip-msg) history \Rightarrow bool$ **where**
 $ports-open\ i\ p\ L = ((not-before\ (is-fin\ i)\ (is-setup\ i\ (fst\ p)))\ L) \wedge$
 $not-before\ (is-fin\ i)\ (is-connect\ i\ (snd\ p))\ L)$

As we do not know which entity closes the connection, we define an operator which checks if the closer is the caller.

fun

$src-is-initiator :: id \Rightarrow adr_{ip} \Rightarrow (adr_{ip}, 'b voip-msg) history \Rightarrow bool$ **where**
 $src-is-initiator\ i\ a\ [] = False$
 $|src-is-initiator\ i\ a\ (p\#S) = (((id\ p = i) \wedge$
 $(\exists\ port. content\ p = Setup\ port) \wedge$
 $((fst\ (src\ p) = fst\ a))) \vee$
 $(src-is-initiator\ i\ a\ S))$

The first state transition is for those messages which do not change the policy. In this scenario, this only happens for the Stream messages.

definition subnet-of-adr where

$subnet-of-adr\ x = \{(a,b). a = x\}$

fun VOIP-STA ::

$((adr_{ip}, address\ voip-msg) history, adr_{ip}, address\ voip-msg) FWStateTransition$
where

$VOIP-STA\ ((a,c,d,ARQ), (InL, policy)) =$
 $Some\ (((a,c,d, ARQ)\#InL,$
 $(allow-from-to-port\ (1719::port)(subnet-of\ d)\ (subnet-of\ c)) \oplus\ policy))$

$|VOIP-STA\ ((a,c,d,ARJ), (InL, policy)) =$
 $(if\ (not-before\ (is-fin\ a)\ (is-arq\ a)\ InL)$
 $then\ Some\ (((a,c,d,ARJ)\#InL,$
 $deny-from-to-port\ (14::port)\ (subnet-of\ c)\ (subnet-of\ d)) \oplus\ policy))$
 $else\ Some\ (((a,c,d,ARJ)\#InL,policy)))$

$|VOIP-STA\ ((a,c,d,ACF\ callee), (InL, policy)) =$
 $Some\ (((a,c,d,ACF\ callee)\#InL,$
 $allow-from-to-port\ (1720::port)\ (subnet-of-adr\ callee)\ (subnet-of\ d)) \oplus$
 $allow-from-to-port\ (1720::port)\ (subnet-of\ d)\ (subnet-of-adr\ callee)) \oplus$
 $deny-from-to-port\ (1719::port)\ (subnet-of\ d)\ (subnet-of\ c)) \oplus$

policy))

| *VOIP-STA* ((*a,c,d, Setup port*), (*InL, policy*)) =
 Some (((*a,c,d,Setup port*)#*InL*,
 allow-from-to-port port (subnet-of d) (subnet-of c) ⊕ policy))

| *VOIP-STA* ((*a,c,d, Connect port*), (*InL, policy*)) =
 Some (((*a,c,d,Connect port*)#*InL*,
 allow-from-to-port port (subnet-of d) (subnet-of c) ⊕ policy))

| *VOIP-STA* ((*a,c,d,Fin*), (*InL,policy*)) =
 (*if* ∃ *p1 p2. ports-open a (p1,p2) InL then* (
 (*if src-is-initiator a c InL*
 then (*Some* (((*a,c,d,Fin*)#*InL*,
 (*deny-from-to-port (1720::int) (subnet-of c) (subnet-of d)) ⊕*
 (*deny-from-to-port (snd (SOME p. ports-open a p InL))*
 (*subnet-of c) (subnet-of d)) ⊕*
 (*deny-from-to-port (fst (SOME p. ports-open a p InL))*
 (*subnet-of d) (subnet-of c)) ⊕ policy*)))

 else (*Some* (((*a,c,d,Fin*)#*InL*,
 (*deny-from-to-port (1720::int) (subnet-of c) (subnet-of d)) ⊕*
 (*deny-from-to-port (fst (SOME p. ports-open a p InL))*
 (*subnet-of c) (subnet-of d)) ⊕*
 (*deny-from-to-port (snd (SOME p. ports-open a p InL))*
 (*subnet-of d) (subnet-of c)) ⊕ policy*))))))

 else
 (*Some* (((*a,c,d,Fin*)#*InL,policy*))))))

| *VOIP-STA* (*p, (InL, policy)*) =
 Some ((*p#InL,policy*))

fun *VOIP-STD* **where**
 VOIP-STD (*p,s*) = *Some s*

definition *VOIP-TRPolicy* **where**

$VOIP-TRPolicy = policy2MON ($
 $((VOIP-STA, VOIP-STD) \otimes_{\nabla} applyPolicy) o (\lambda (x,(y,z)). ((x,z),(x,(y,z))))))$

For a full protocol run, six states are needed.

datatype $voip-states = S0 | S1 | S2 | S3 | S4 | S5$

The constant $is-voip$ checks if a trace corresponds to a legal VoIP protocol, given the IP-addresses of the three entities, the ID, and the two dynamic ports.

fun $is-voip :: voip-states \Rightarrow address \Rightarrow address \Rightarrow address \Rightarrow id \Rightarrow port \Rightarrow$
 $port \Rightarrow (adr_{ip}, address\ voip-msg)\ history \Rightarrow bool$

where

$is-voip\ H\ s\ d\ g\ i\ p1\ p2\ [] = (H = S5)$
 $| is-voip\ H\ s\ d\ g\ i\ p1\ p2\ (x\#\ InL) =$
 $((\lambda (id, sr, de, co).$
 $((id = i \wedge$
 $(H = S4 \wedge ((sr = (s,1719) \wedge de = (g,1719) \wedge co = ARQ \wedge$
 $is-voip\ S5\ s\ d\ g\ i\ p1\ p2\ InL)))) \vee$
 $(H = S0 \wedge sr = (g,1719) \wedge de = (s,1719) \wedge co = ARJ \wedge$
 $is-voip\ S4\ s\ d\ g\ i\ p1\ p2\ InL) \vee$
 $(H = S3 \wedge sr = (g,1719) \wedge de = (s,1719) \wedge co = ACF\ d \wedge$
 $is-voip\ S4\ s\ d\ g\ i\ p1\ p2\ InL) \vee$
 $(H = S2 \wedge sr = (s,1720) \wedge de = (d,1720) \wedge co = Setup\ p1 \wedge$
 $is-voip\ S3\ s\ d\ g\ i\ p1\ p2\ InL) \vee$
 $(H = S1 \wedge sr = (d,1720) \wedge de = (s,1720) \wedge co = Connect\ p2 \wedge$
 $is-voip\ S2\ s\ d\ g\ i\ p1\ p2\ InL) \vee$
 $(H = S1 \wedge sr = (s,p1) \wedge de = (d,p2) \wedge co = Stream \wedge$
 $is-voip\ S1\ s\ d\ g\ i\ p1\ p2\ InL) \vee$
 $(H = S1 \wedge sr = (d,p2) \wedge de = (s,p1) \wedge co = Stream \wedge$
 $is-voip\ S1\ s\ d\ g\ i\ p1\ p2\ InL) \vee$
 $(H = S0 \wedge sr = (d,1720) \wedge de = (s,1720) \wedge co = Fin \wedge$
 $is-voip\ S1\ s\ d\ g\ i\ p1\ p2\ InL) \vee$
 $(H = S0 \wedge sr = (s,1720) \wedge de = (d,1720) \wedge co = Fin \wedge$
 $is-voip\ S1\ s\ d\ g\ i\ p1\ p2\ InL))))))\ x)$

Finally, $NB-voip$ returns the set of protocol traces which correspond to a correct protocol run given the three addresses, the ID, and the two dynamic ports.

definition

$NB-voip :: address \Rightarrow address \Rightarrow address \Rightarrow id \Rightarrow port \Rightarrow port \Rightarrow$
 $(adr_{ip}, address\ voip-msg)\ history\ set\ \mathbf{where}$
 $NB-voip\ s\ d\ g\ i\ p1\ p2 = \{x. (is-voip\ S0\ s\ d\ g\ i\ p1\ p2\ x)\}$

end

2.4.5 FTP and VoIP Protocol

theory

FTPVOIP

imports

FTP-WithPolicy VOIP

begin

```
datatype ftpvoip = ARQ
  | ACF int
  | ARJ
  | Setup port
  | Connect port
  | Stream
  | Fin
  | ftp-init
  | ftp-port-request port
  | ftp-data
  | ftp-close
  | other
```

We now also make use of the ID field of a packet. It is used as session ID and we make the assumption that they are all unique among different protocol runs.

At first, we need some predicates which check if a packet is a specific FTP message and has the correct session ID.

definition

FTPVOIP-is-init :: $id \Rightarrow (adr_{ip}, ftpvoip) \text{ packet} \Rightarrow \text{bool}$ **where**
FTPVOIP-is-init = $(\lambda i p. (id\ p = i \wedge content\ p = ftp-init))$

definition

FTPVOIP-is-port-request :: $id \Rightarrow port \Rightarrow (adr_{ip}, ftpvoip) \text{ packet} \Rightarrow \text{bool}$ **where**
FTPVOIP-is-port-request = $(\lambda i\ port\ p. (id\ p = i \wedge content\ p = ftp-port-request\ port))$

definition

FTPVOIP-is-data :: $id \Rightarrow (adr_{ip}, ftpvoip) \text{ packet} \Rightarrow \text{bool}$ **where**
FTPVOIP-is-data = $(\lambda i\ p. (id\ p = i \wedge content\ p = ftp-data))$

definition

FTPVOIP-is-close :: $id \Rightarrow (adr_{ip}, ftpvoip) \text{ packet} \Rightarrow \text{bool}$ **where**
FTPVOIP-is-close = $(\lambda i\ p. (id\ p = i \wedge content\ p = ftp-close))$

definition

FTPVOIP-port-open :: $(adr_{ip}, ftpvoip) \text{ history} \Rightarrow id \Rightarrow port \Rightarrow \text{bool}$ **where**

$FTPVOIP\text{-}port\text{-}open = (\lambda L a p. (not\text{-}before (FTPVOIP\text{-}is\text{-}close i a) (FTPVOIP\text{-}is\text{-}port\text{-}request a p) L))$

definition

$FTPVOIP\text{-}is\text{-}other :: id \Rightarrow (adr_{ip}, ftpvoip) packet \Rightarrow bool$ **where**

$FTPVOIP\text{-}is\text{-}other = (\lambda i p. (id p = i \wedge content p = other))$

fun $FTPVOIP\text{-}are\text{-}other$ **where**

$FTPVOIP\text{-}are\text{-}other i (x\#xs) = (FTPVOIP\text{-}is\text{-}other i x \wedge FTPVOIP\text{-}are\text{-}other i xs)$

$|FTPVOIP\text{-}are\text{-}other i [] = True$

fun $last\text{-}opened\text{-}port$ **where**

$last\text{-}opened\text{-}port i ((j,s,d,ftp\text{-}port\text{-}request p)\#xs) = (if i=j then p else last\text{-}opened\text{-}port i xs)$

$|last\text{-}opened\text{-}port i (x\#xs) = last\text{-}opened\text{-}port i xs$

$|last\text{-}opened\text{-}port x [] = undefined$

fun $FTPVOIP\text{-}FTP\text{-}STA ::$

$((adr_{ip}, ftpvoip) history, adr_{ip}, ftpvoip) FWStateTransition$

where

$FTPVOIP\text{-}FTP\text{-}STA ((i,s,d,ftp\text{-}port\text{-}request pr), (InL, policy)) =$
 $(if not\text{-}before (FTPVOIP\text{-}is\text{-}close i) (FTPVOIP\text{-}is\text{-}init i) InL \wedge$
 $dest\text{-}port (i,s,d,ftp\text{-}port\text{-}request pr) = (21::port) then$
 $Some (((i,s,d,ftp\text{-}port\text{-}request pr)\#InL, policy ++$
 $(allow\text{-}from\text{-}to\text{-}port pr (subnet\text{-}of d) (subnet\text{-}of s))))$
 $else Some (((i,s,d,ftp\text{-}port\text{-}request pr)\#InL,policy)))$

$|FTPVOIP\text{-}FTP\text{-}STA ((i,s,d,ftp\text{-}close), (InL,policy)) =$
 $(if (\exists p. FTPVOIP\text{-}port\text{-}open InL i p) \wedge dest\text{-}port (i,s,d,ftp\text{-}close) = (21::port)$
 $then Some ((i,s,d,ftp\text{-}close)\#InL, policy ++$
 $deny\text{-}from\text{-}to\text{-}port (last\text{-}opened\text{-}port i InL) (subnet\text{-}of d) (subnet\text{-}of s))$
 $else Some (((i,s,d,ftp\text{-}close)\#InL, policy)))$

$|FTPVOIP\text{-}FTP\text{-}STA (p, s) = Some (p\#(fst s),snd s)$

fun $FTPVOIP\text{-}FTP\text{-}STD :: ((adr_{ip}, ftpvoip) history, adr_{ip}, ftpvoip) FWStateTransition$

where $FTPVOIP\text{-}FTP\text{-}STD (p,s) = Some s$

definition

$FTPVOIP-is-arq :: NetworkCore.id \Rightarrow ('a::adr, ftpvoip) packet \Rightarrow bool$ **where**
 $FTPVOIP-is-arq\ i\ p = (NetworkCore.id\ p = i \wedge content\ p = ARQ)$

definition

$FTPVOIP-is-fin :: id \Rightarrow ('a::adr, ftpvoip) packet \Rightarrow bool$ **where**
 $FTPVOIP-is-fin\ i\ p = (id\ p = i \wedge content\ p = Fin)$

definition

$FTPVOIP-is-connect :: id \Rightarrow port \Rightarrow ('a::adr, ftpvoip) packet \Rightarrow bool$ **where**
 $FTPVOIP-is-connect\ i\ port\ p = (id\ p = i \wedge content\ p = Connect\ port)$

definition

$FTPVOIP-is-setup :: id \Rightarrow port \Rightarrow ('a::adr, ftpvoip) packet \Rightarrow bool$ **where**
 $FTPVOIP-is-setup\ i\ port\ p = (id\ p = i \wedge content\ p = Setup\ port)$

We need also an operator *ports-open* to get access to the two dynamic ports.

definition

$FTPVOIP-ports-open :: id \Rightarrow port \times port \Rightarrow (adr_{ip}, ftpvoip) history \Rightarrow bool$ **where**
 $FTPVOIP-ports-open\ i\ p\ L = ((not-before\ (FTPVOIP-is-fin\ i)\ (FTPVOIP-is-setup\ i\ (fst\ p)))\ L) \wedge$
 $not-before\ (FTPVOIP-is-fin\ i)\ (FTPVOIP-is-connect\ i\ (snd\ p))$
 $L)$

As we do not know which entity closes the connection, we define an operator which checks if the closer is the caller.

fun

$FTPVOIP-src-is-initiator :: id \Rightarrow adr_{ip} \Rightarrow (adr_{ip}, ftpvoip) history \Rightarrow bool$ **where**
 $FTPVOIP-src-is-initiator\ i\ a\ [] = False$
 $|FTPVOIP-src-is-initiator\ i\ a\ (p\#S) = (((id\ p = i) \wedge$
 $(\exists\ port. content\ p = Setup\ port) \wedge$
 $((fst\ (src\ p) = fst\ a))) \vee$
 $(FTPVOIP-src-is-initiator\ i\ a\ S))$

definition $FTPVOIP-subnet-of-adr :: int \Rightarrow adr_{ip}\ net$ **where**

$FTPVOIP-subnet-of-adr\ x = \{(a,b). a = x\}$

fun $FTPVOIP-VOIP-STA ::$

$((adr_{ip}, ftpvoip) history, adr_{ip}, ftpvoip) FWStateTransition$
where
 $FTPVOIP-VOIP-STA\ ((a,c,d,ARQ), (InL, policy)) =$
 $Some\ (((a,c,d, ARQ)\#InL,$
 $(allow-from-to-port\ (1719::port)(subnet-of\ d)\ (subnet-of\ c)) \oplus policy))$

|*FTPVOIP-VOIP-STA* ((*a,c,d,ARJ*), (*InL, policy*)) =
 (*if* (*not-before* (*FTPVOIP-is-fin a*) (*FTPVOIP-is-arq a*) *InL*)
 then *Some* (((*a,c,d,ARJ*)#*InL*,
 deny-from-to-port (*14::port*) (*subnet-of c*) (*subnet-of d*) \oplus *policy*))
 else *Some* (((*a,c,d,ARJ*)#*InL,policy*)))

|*FTPVOIP-VOIP-STA* ((*a,c,d,ACF callee*), (*InL, policy*)) =
 Some (((*a,c,d,ACF callee*)#*InL*,
 allow-from-to-port (*1720::port*) (*subnet-of-adr callee*) (*subnet-of d*) \oplus
 allow-from-to-port (*1720::port*) (*subnet-of d*) (*subnet-of-adr callee*) \oplus
 deny-from-to-port (*1719::port*) (*subnet-of d*) (*subnet-of c*) \oplus
 policy))

|*FTPVOIP-VOIP-STA* ((*a,c,d, Setup port*), (*InL, policy*)) =
 Some (((*a,c,d, Setup port*)#*InL*,
 allow-from-to-port *port* (*subnet-of d*) (*subnet-of c*) \oplus *policy*))

|*FTPVOIP-VOIP-STA* ((*a,c,d, ftpvoip.Connect port*), (*InL, policy*)) =
 Some (((*a,c,d,ftpvoip.Connect port*)#*InL*,
 allow-from-to-port *port* (*subnet-of d*) (*subnet-of c*) \oplus *policy*))

|*FTPVOIP-VOIP-STA* ((*a,c,d,Fin*), (*InL,policy*)) =
 (*if* \exists *p1 p2. FTPVOIP-ports-open a (p1,p2) InL then* (
 (*if* *FTPVOIP-src-is-initiator a c InL*
 then (*Some* (((*a,c,d,Fin*)#*InL*,
 (*deny-from-to-port* (*1720::int*) (*subnet-of c*) (*subnet-of d*)) \oplus
 (*deny-from-to-port* (*snd* (*SOME p. FTPVOIP-ports-open a p InL*)
 (*subnet-of c*) (*subnet-of d*)) \oplus
 (*deny-from-to-port* (*fst* (*SOME p. FTPVOIP-ports-open a p InL*)
 (*subnet-of d*) (*subnet-of c*)) \oplus *policy*))))

else (*Some* (((*a,c,d,Fin*)#*InL*,
 (*deny-from-to-port* (*1720::int*) (*subnet-of c*) (*subnet-of d*)) \oplus
 (*deny-from-to-port* (*fst* (*SOME p. FTPVOIP-ports-open a p InL*)
 (*subnet-of c*) (*subnet-of d*)) \oplus
 (*deny-from-to-port* (*snd* (*SOME p. FTPVOIP-ports-open a p InL*)
 (*subnet-of d*) (*subnet-of c*)) \oplus *policy*))))))

else
 (*Some* (((*a,c,d,Fin*)#*InL,policy*))))

| *FTPVOIP-VOIP-STA* (*p*, (*InL, policy*)) =

Some ((p#InL,policy))

fun *FTPVOIP-VOIP-STD* ::
 ((*adr_{ip}*, *ftpvoip*) *history*, *adr_{ip}*, *ftpvoip*) *FWStateTransition*
where
FTPVOIP-VOIP-STD (*p,s*) = *Some s*

definition *FTP-VOIP-STA* :: ((*adr_{ip}*, *ftpvoip*) *history*, *adr_{ip}*, *ftpvoip*) *FWStateTransition*
where
FTP-VOIP-STA = (($\lambda(x,x).$ *Some x*) \circ_m ((*FTPVOIP-FTP-STA* \otimes_S *FTPVOIP-VOIP-STA* \circ ($\lambda(p,x).$ (*p,x,x*))))))

definition *FTP-VOIP-STD* :: ((*adr_{ip}*, *ftpvoip*) *history*, *adr_{ip}*, *ftpvoip*) *FWStateTransition*
where
FTP-VOIP-STD = ($\lambda(x,x).$ *Some x*) \circ_m ((*FTPVOIP-FTP-STD* \otimes_S *FTPVOIP-VOIP-STD* \circ ($\lambda(p,x).$ (*p,x,x*))))

definition *FTPVOIP-TRPolicy* **where**
FTPVOIP-TRPolicy = *policy2MON* (
 (((*FTP-VOIP-STA,FTP-VOIP-STD*) \otimes_{∇} *applyPolicy*) \circ ($\lambda(x,(y,z)).$
 ((*x,z*),(*x,(y,z)*))))))

lemmas *FTPVOIP-ST-simps* = *Let-def in-subnet-def src-def dest-def*
subnet-of-int-def id-def FTPVOIP-port-open-def
FTPVOIP-is-init-def FTPVOIP-is-data-def FTPVOIP-is-port-request-def FTPVOIP-is-close-def p-accept-def content-def PortCombinators exI
NetworkCore.id-def adr_{ip}Lemmas

datatype *ftp-states2* = *FS0* | *FS1* | *FS2* | *FS3*
datatype *voip-states2* = *V0* | *V1* | *V2* | *V3* | *V4* | *V5*

The constant *is-voip* checks if a trace corresponds to a legal VoIP protocol, given the IP-addresses of the three entities, the ID, and the two dynamic ports.

fun *FTPVOIP-is-voip* :: *voip-states2* \Rightarrow *address* \Rightarrow *address* \Rightarrow *address* \Rightarrow *id* \Rightarrow *port* \Rightarrow
port \Rightarrow (*adr_{ip}*, *ftpvoip*) *history* \Rightarrow *bool*
where
FTPVOIP-is-voip *H s d g i p1 p2* [] = (*H* = *V5*)
FTPVOIP-is-voip *H s d g i p1 p2* (*x#InL*) =
 ((($\lambda(id,sr,de,co).$
 (((*id* = *i* \wedge
 (*H* = *V4* \wedge ((*sr* = (*s,1719*) \wedge *de* = (*g,1719*) \wedge *co* = *ARQ* \wedge

$$\begin{aligned}
& \text{FTPVOIP-is-voip } V5 \text{ s d g i p1 p2 InL})) \vee \\
& (H = V0 \wedge sr = (g,1719) \wedge de = (s,1719) \wedge co = ARJ \wedge \\
& \quad \text{FTPVOIP-is-voip } V4 \text{ s d g i p1 p2 InL}) \vee \\
& (H = V3 \wedge sr = (g,1719) \wedge de = (s,1719) \wedge co = ACF \text{ d} \wedge \\
& \quad \text{FTPVOIP-is-voip } V4 \text{ s d g i p1 p2 InL}) \vee \\
& (H = V2 \wedge sr = (s,1720) \wedge de = (d,1720) \wedge co = Setup \text{ p1} \wedge \\
& \quad \text{FTPVOIP-is-voip } V3 \text{ s d g i p1 p2 InL}) \vee \\
& (H = V1 \wedge sr = (d,1720) \wedge de = (s,1720) \wedge co = Connect \text{ p2} \wedge \\
& \quad \text{FTPVOIP-is-voip } V2 \text{ s d g i p1 p2 InL}) \vee \\
& (H = V1 \wedge sr = (s,p1) \wedge de = (d,p2) \wedge co = Stream \wedge \\
& \quad \text{FTPVOIP-is-voip } V1 \text{ s d g i p1 p2 InL}) \vee \\
& (H = V1 \wedge sr = (d,p2) \wedge de = (s,p1) \wedge co = Stream \wedge \\
& \quad \text{FTPVOIP-is-voip } V1 \text{ s d g i p1 p2 InL}) \vee \\
& (H = V0 \wedge sr = (d,1720) \wedge de = (s,1720) \wedge co = Fin \wedge \\
& \quad \text{FTPVOIP-is-voip } V1 \text{ s d g i p1 p2 InL}) \vee \\
& (H = V0 \wedge sr = (s,1720) \wedge de = (d,1720) \wedge co = Fin \wedge \\
& \quad \text{FTPVOIP-is-voip } V1 \text{ s d g i p1 p2 InL)))))) x)
\end{aligned}$$

Finally, *NB-voip* returns the set of protocol traces which correspond to a correct protocol run given the three addresses, the ID, and the two dynamic ports.

definition

FTPVOIP-NB-voip :: *address* \Rightarrow *address* \Rightarrow *address* \Rightarrow *id* \Rightarrow *port* \Rightarrow *port* \Rightarrow
(*adr_{ip}*, *ftpvoip*) *history set* **where**
FTPVOIP-NB-voip *s d g i p1 p2* = {*x*. (*FTPVOIP-is-voip* *V0 s d g i p1 p2 x*)}

fun

FTPVOIP-is-ftp :: *ftp-states2* \Rightarrow *adr_{ip}* \Rightarrow *adr_{ip}* \Rightarrow *id* \Rightarrow *port* \Rightarrow
(*adr_{ip}*, *ftpvoip*) *history* \Rightarrow *bool*

where

FTPVOIP-is-ftp *H c s i p []* = (*H=FS3*)
|*FTPVOIP-is-ftp* *H c s i p (x#InL)* = (*snd s* = 21 \wedge ((λ (*id,sr,de,co*). (((*id* = *i* \wedge (
(*H=FS2* \wedge *sr* = *c* \wedge *de* = *s* \wedge *co* = *ftp-init* \wedge *FTPVOIP-is-ftp* *FS3 c s i p InL*) \vee
(*H=FS1* \wedge *sr* = *c* \wedge *de* = *s* \wedge *co* = *ftp-port-request p* \wedge *FTPVOIP-is-ftp* *FS2 c s i*
p InL) \vee
(*H=FS1* \wedge *sr* = *s* \wedge *de* = (*fst c,p*) \wedge *co* = *ftp-data* \wedge *FTPVOIP-is-ftp* *FS1 c s i p*
InL) \vee
(*H=FS0* \wedge *sr* = *c* \wedge *de* = *s* \wedge *co* = *ftp-close* \wedge *FTPVOIP-is-ftp* *FS1 c s i p InL*)
)))))) *x*)

definition

FTPVOIP-NB-ftp :: *adr_{ip}* *src* \Rightarrow *adr_{ip}* *dest* \Rightarrow *id* \Rightarrow *port* \Rightarrow (*adr_{ip}*, *ftpvoip*) *history*
set **where**
FTPVOIP-NB-ftp *s d i p* = {*x*. (*FTPVOIP-is-ftp* *FS0 s d i p x*)}

definition

ftp-voip-interleaved :: $adr_{ip} \ src \Rightarrow \ adr_{ip} \ dest \Rightarrow \ id \Rightarrow \ port \Rightarrow$
 $address \Rightarrow \ address \Rightarrow \ address \Rightarrow \ id \Rightarrow \ port \Rightarrow \ port \Rightarrow$
 $(adr_{ip}, \ ftpvoip) \ history \ set$

where

ftp-voip-interleaved $s1 \ d1 \ i1 \ p1 \ vs \ vd \ vg \ vi \ vp1 \ vp2 =$
 $\{x. (FTPVOIP-is-ftp \ FS0 \ s1 \ d1 \ i1 \ p1 \ (packet-with-id \ x \ i1)) \wedge$
 $(FTPVOIP-is-voip \ V0 \ vs \ vd \ vg \ vi \ vp1 \ vp2 \ (packet-with-id \ x \ vi))\}$

end

3 Examples

theory

Examples

imports

DMZ/DMZ

Voice-over-IP/Voice-over-IP

Transformation/Transformation

NAT-FW/NAT-FW

PersonalFirewall/PersonalFirewall

begin

end

3.1 A Simple DMZ Setup

theory

DMZ

imports

DMZDatatype

DMZInteger

begin

end

3.1.1 DMZ Datatype

theory

DMZDatatype

imports

../.. /UPF-Firewall

begin

This is the fourth scenario, slightly more complicated than the previous one, as we now also model specific servers within one network. Therefore, we could not use anymore the modelling using datatype synonym, but only use the one where an address is modelled as an integer (with ports).

Just for comparison, this theory is the same scenario with datatype synonym anyway, but with four distinct networks instead of one contained in another. As there is no corresponding network model included, we need to define a custom one.

datatype *Adr* = *Intranet* | *Internet* | *Mail* | *Web* | *DMZ*
instance *Adr::adr ..*
type-synonym *port* = *int*
type-synonym *Networks* = *Adr* × *port*

definition

intranet::Networks net where
intranet = { {(a,b). a= Intranet} }

definition

dmz :: Networks net where
dmz = { {(a,b). a= DMZ} }

definition

mail :: Networks net where
mail = { {(a,b). a=Mail} }

definition

web :: Networks net where
web = { {(a,b). a=Web} }

definition

internet :: Networks net where
internet = { {(a,b). a= Internet} }

definition

Intranet-mail-port :: (Networks ,DummyContent) FWPolicy where
Intranet-mail-port = (allow-from-ports-to {21::port,14} intranet mail)

definition

Intranet-Internet-port :: (Networks,DummyContent) FWPolicy where
Intranet-Internet-port = allow-from-ports-to {80::port,90} intranet internet

definition

Internet-web-port :: (Networks,DummyContent) FWPolicy where
Internet-web-port = (allow-from-ports-to {80::port,90} internet web)

definition

Internet-mail-port :: (Networks,DummyContent) FWPolicy where
Internet-mail-port = (allow-all-from-port-to internet (21::port) dmz)

definition

policyPort :: (Networks, DummyContent) FWPolicy where
policyPort = deny-all ++
Intranet-Internet-port ++
Intranet-mail-port ++
Internet-mail-port ++
Internet-web-port

We only want to create test cases which are sent between the three main networks: e.g. not between the mailservers and the dmz. Therefore, the constraint looks as follows.
%

definition

```
not-in-same-net :: (Networks, DummyContent) packet => bool where
not-in-same-net x = ((src x ⊆ internet → ¬ dest x ⊆ internet) ∧
                    (src x ⊆ intranet → ¬ dest x ⊆ intranet) ∧
                    (src x ⊆ dmz → ¬ dest x ⊆ dmz))
```

```
lemmas PolicyLemmas = dmz-def internet-def intranet-def mail-def web-def
Internet-web-port-def Internet-mail-port-def
Intranet-Internet-port-def Intranet-mail-port-def
src-def dest-def src-port dest-port in-subnet-def
```

end

3.1.2 DMZ: Integer

theory

```
DMZInteger
```

imports

```
../.. /UPF-Firewall
```

begin

This scenario is slightly more complicated than the SimpleDMZ one, as we now also model specific servers within one network. Therefore, we cannot use anymore the modelling using datatype synonym, but only use the one where an address is modelled as an integer (with ports).

The scenario is the following:

- Networks:
- Intranet (Company intern network)
 - DMZ (demilitarised zone, servers, etc), containing at least two distinct servers “mail” and “web”
 - Internet (“all others”)
- Policy:
- allow http(s) from Intranet to Internet
 - deny all traffic from Internet to Intranet
 - allow imaps and smtp from intranet to mailservers
 - allow smtp from Internet to mailservers
 - allow http(s) from Internet to webservers
 - deny everything else

definition

intranet :: *adr_{ip} net* **where**
intranet = { {(a,b) . (a > 1 ∧ a < 4) } }

definition

dmz :: *adr_{ip} net* **where**
dmz = { {(a,b) . (a > 6) ∧ (a < 11) } }

definition

mail :: *adr_{ip} net* **where**
mail = { {(a,b) . a = 7 } }

definition

web :: *adr_{ip} net* **where**
web = { {(a,b) . a = 8 } }

definition

internet :: *adr_{ip} net* **where**
internet = { {(a,b) . ¬ ((a > 1 ∧ a < 4) ∨ (a > 6) ∧ (a < 11)) } }

definition

Intranet-mail-port :: (*adr_{ip}, 'b*) *FWPolicy* **where**
Intranet-mail-port = (*allow-from-to-ports* {21::port,14} *intranet mail*)

definition

Intranet-Internet-port :: (*adr_{ip}, 'b*) *FWPolicy* **where**
Intranet-Internet-port = *allow-from-to-ports* {80::port,90} *intranet internet*

definition

Internet-web-port :: (*adr_{ip}, 'b*) *FWPolicy* **where**
Internet-web-port = (*allow-from-to-ports* {80::port,90} *internet web*)

definition

Internet-mail-port :: (*adr_{ip}, 'b*) *FWPolicy* **where**
Internet-mail-port = (*allow-all-from-port-to internet* (21::port) *dmz*)

definition

policyPort :: (*adr_{ip}, DummyContent*) *FWPolicy* **where**
policyPort = *deny-all* ++
Intranet-Internet-port ++
Intranet-mail-port ++
Internet-mail-port ++
Internet-web-port

We only want to create test cases which are sent between the three main networks: e.g. not between the mailservers and the dmz. Therefore, the constraint looks as follows.

definition

not-in-same-net :: (*adr_{ip}, DummyContent*) *packet* ⇒ *bool* **where**

$$\begin{aligned} \text{not-in-same-net } x = & ((\text{src } x \sqsubset \text{internet} \longrightarrow \neg \text{dest } x \sqsubset \text{internet}) \wedge \\ & (\text{src } x \sqsubset \text{intranet} \longrightarrow \neg \text{dest } x \sqsubset \text{intranet}) \wedge \\ & (\text{src } x \sqsubset \text{dmz} \longrightarrow \neg \text{dest } x \sqsubset \text{dmz})) \end{aligned}$$

```

lemmas PolicyLemmas = policyPort-def dmz-def internet-def intranet-def mail-def
web-def
  Intranet-Internet-port-def Intranet-mail-port-def Internet-web-port-def
  Internet-mail-port-def src-def dest-def IntegerPort.src-port
  in-subnet-def IntegerPort.dest-port

```

end

3.2 Personal Firewall

theory

PersonalFirewall

imports

PersonalFirewallInt

PersonalFirewallIpv4

PersonalFirewallDatatype

begin

end

3.2.1 Personal Firewall: Integer

theory

PersonalFirewallInt

imports

../UPF-Firewall

begin

The most basic firewall scenario; there is a personal PC on one side and the Internet on the other. There are two policies: the first one allows all traffic from the PC to the Internet and denies all coming into the PC. The second policy only allows specific ports from the PC. This scenario comes in three variants: the first one specifies the allowed protocols directly, the second together with their respective port numbers, the third one only with the port numbers.

Definitions of the subnets

definition

$PC :: (\text{adr}_{ip} \text{ net}) \text{ where}$
 $PC = \{\{(a,b). a = 3\}\}$

definition

Internet :: *adr_{ip}* net **where**
Internet = { {(a,b). ¬ (a = 3)} }

definition

not-in-same-net :: (*adr_{ip}*, *DummyContent*) *packet* ⇒ *bool* **where**
not-in-same-net *x* = ((*src* *x* ⊆ *PC* → *dest* *x* ⊆ *Internet*) ∧ (*src* *x* ⊆ *Internet* → *dest* *x* ⊆ *PC*))

Definitions of the policies

definition

strictPolicy :: (*adr_{ip}*, *DummyContent*) *FWPolicy* **where**
strictPolicy = *deny-all* ++ *allow-all-from-to* *PC* *Internet*

definition

PortPolicy :: (*adr_{ip}*, *DummyContent*) *FWPolicy* **where**
PortPolicy = *deny-all* ++ *allow-from-ports-to* {*http*,*smtp*,*ftp*} *PC* *Internet*

definition

PortPolicyBig :: (*adr_{ip}*, *DummyContent*) *FWPolicy* **where**
PortPolicyBig = *deny-all* ++
allow-from-port-to *http* *PC* *Internet* ++
allow-from-port-to *smtp* *PC* *Internet* ++
allow-from-port-to *ftp* *PC* *Internet*

lemmas *policyLemmas* = *strictPolicy-def* *PortPolicy-def* *PC-def*
Internet-def *PortPolicyBig-def* *src-def* *dest-def*
adr_{ip}Lemmas *content-def*
PortCombinators *in-subnet-def* *PortPolicyBig-def* *id-def*

declare *Ports* [*simp* *add*]

definition *wellformed-packet*::(*adr_{ip}*, *DummyContent*) *packet* ⇒ *bool* **where**
wellformed-packet *p* = (*content* *p* = *data*)

end

3.2.2 Personal Firewall IPv4

theory

PersonalFirewallIpv4

imports

../UPF-Firewall

begin

The most basic firewall scenario; there is a personal PC on one side and the Internet

on the other. There are two policies: the first one allows all traffic from the PC to the Internet and denies all coming into the PC. The second policy only allows specific ports from the PC. This scenario comes in three variants: the first one specifies the allowed protocols directly, the second together with their respective port numbers, the third one only with the port numbers.

Definitions of the subnets

definition

PC :: (*ipv4 net*) **where**

PC = $\{\{(a,b,c,d),e\}. a = 1 \wedge b = 3 \wedge c = 5 \wedge d = 2\}\}$

definition

Internet :: *ipv4 net* **where**

Internet = $\{\{(a,b,c,d),e\}. \neg (a = 1 \wedge b = 3 \wedge c = 5 \wedge d = 2)\}\}$

definition

not-in-same-net :: (*ipv4, DummyContent*) *packet* \Rightarrow *bool* **where**

not-in-same-net *x* = $((src\ x \sqsubset PC \longrightarrow dest\ x \sqsubset Internet) \wedge (src\ x \sqsubset Internet \longrightarrow dest\ x \sqsubset PC))$

Definitions of the policies

definition

strictPolicy :: (*ipv4, DummyContent*) *FWPolicy* **where**

strictPolicy = *deny-all ++ allow-all-from-to PC Internet*

definition

PortPolicy :: (*ipv4, DummyContent*) *FWPolicy* **where**

PortPolicy = *deny-all ++ allow-from-ports-to {80::port,24,21} PC Internet*

definition

PortPolicyBig :: (*ipv4, DummyContent*) *FWPolicy* **where**

PortPolicyBig = *deny-all ++ allow-from-port-to (80::port) PC Internet ++ allow-from-port-to (24::port) PC Internet ++ allow-from-port-to (21::port) PC Internet*

lemmas *policyLemmas* = *strictPolicy-def PortPolicy-def PC-def*

Internet-def PortPolicyBig-def src-def dest-def

IPv4.src-port

IPv4.dest-port PolicyCombinators

PortCombinators in-subnet-def PortPolicyBig-def

end

3.2.3 Personal Firewall: Datatype

theory

```

PersonalFirewallDatatype
imports
  ../UPF-Firewall
begin

```

The most basic firewall scenario; there is a personal PC on one side and the Internet on the other. There are two policies: the first one allows all traffic from the PC to the Internet and denies all coming into the PC. The second policy only allows specific ports from the PC. This scenario comes in three variants: the first one specifies the allowed protocols directly, the second together with their respective port numbers, the third one only with the port numbers.

```
datatype Adr = pc | internet
```

```
type-synonym DatatypeTwoNets = Adr × int
```

```
instance Adr::adr ..
```

definition

```
PC :: DatatypeTwoNets net where
PC = { {(a,b). a = pc }}
```

definition

```
Internet :: DatatypeTwoNets net where
Internet = { {(a,b). a = internet }}
```

definition

```
not-in-same-net :: (DatatypeTwoNets,DummyContent) packet ⇒ bool where
not-in-same-net x = ((src x ⊆ PC → dest x ⊆ Internet) ∧ (src x ⊆ Internet → dest x ⊆ PC))
```

Definitions of the policies

In fact, the short definitions wouldn't have to be written down - they are the automatically simplified versions of their big counterparts.

definition

```
strictPolicy :: (DatatypeTwoNets,DummyContent) FWPolicy where
strictPolicy = deny-all ++ allow-all-from-to PC Internet
```

definition

```
PortPolicy :: (DatatypeTwoNets,'b) FWPolicy where
PortPolicy = deny-all ++ allow-from-ports-to {80::port,24,21} PC Internet
```

definition

```
PortPolicyBig :: (DatatypeTwoNets,'b) FWPolicy where
PortPolicyBig =
```

```

allow-from-port-to (80::port) PC Internet ⊕
allow-from-port-to (24::port) PC Internet ⊕
allow-from-port-to (21::port) PC Internet ⊕
deny-all

```

```

lemmas policyLemmas = strictPolicy-def PortPolicy-def PC-def Internet-def PortPoli-
cyBig-def src-def
PolicyCombinators PortCombinators in-subnet-def

```

```
end
```

3.3 Demonstrating Policy Transformations

```

theory
Transformation
imports
Transformation01
Transformation02
begin
end

```

3.3.1 Transformation Example 1

```

theory
Transformation01
imports
../UPF-Firewall
begin

```

```

definition
FWLink :: adrip net where
FWLink = { {(a,b). a = 1} }

```

```

definition
any :: adrip net where
any = { {(a,b). a > 5} }

```

```

definition
i4 :: adrip net where
i4 = { {(a,b). a = 2 } }

```

```

definition
i27 :: adrip net where
i27 = { {(a,b). a = 3 } }

```

definition

eth-intern:: *adr_{ip} net* **where**
eth-intern = $\{\{(a,b). a = 4\}\}$

definition

eth-private:: *adr_{ip} net* **where**
eth-private = $\{\{(a,b). a = 5\}\}$

definition

MG2 :: (*adr_{ip} net,port*) *Combinators* **where**
MG2 = *AllowPortFromTo i27 any 1* \oplus
AllowPortFromTo i27 any 2 \oplus
AllowPortFromTo i27 any 3

definition

MG3 :: (*adr_{ip} net,port*) *Combinators* **where**
MG3 = *AllowPortFromTo any FWLink 1*

definition

MG4 :: (*adr_{ip} net,port*) *Combinators* **where**
MG4 = *AllowPortFromTo FWLink FWLink 4*

definition

MG7 :: (*adr_{ip} net,port*) *Combinators* **where**
MG7 = *AllowPortFromTo FWLink i4 6* \oplus
AllowPortFromTo FWLink i4 7

definition

MG8 :: (*adr_{ip} net,port*) *Combinators* **where**
MG8 = *AllowPortFromTo FWLink i4 6* \oplus
AllowPortFromTo FWLink i4 7

definition

DG3:: (*adr_{ip} net,port*) *Combinators* **where**
DG3 = *AllowPortFromTo any any 7*

definition

Policy = *DenyAll* \oplus *MG8* \oplus *MG7* \oplus *MG4* \oplus *MG3* \oplus *MG2* \oplus *DG3*

lemmas *PolicyLemmas* = *Policy-def*

FWLink-def

any-def
i27-def
i4-def
eth-intern-def
eth-private-def
MG2-def MG3-def MG4-def MG7-def MG8-def
DG3-def

lemmas *PolicyL* = *MG2-def MG3-def MG4-def MG7-def MG8-def DG3-def Policy-def*

definition

not-in-same-net :: (*adr_{ip}, DummyContent*) *packet* ⇒ *bool* **where**
not-in-same-net *x* = (((*src x* □ *i27*) → (¬ (*dest x* □ *i27*))) ∧
((*src x* □ *i4*) → (¬ (*dest x* □ *i4*))) ∧
((*src x* □ *eth-intern*) → (¬ (*dest x* □ *eth-intern*))) ∧
((*src x* □ *eth-private*) → (¬ (*dest x* □ *eth-private*))))

consts *fixID* :: *id*

consts *fixContent* :: *DummyContent*

definition *fixElements* *p* = (*id p* = *fixID* ∧ *content p* = *fixContent*)

lemmas *fixDefs* = *fixElements-def NetworkCore.id-def NetworkCore.content-def*

lemma *sets-distinct1*: (*n*::*int*) ≠ *m* ⇒ {(*a,b*). *a* = *n*} ≠ {(*a,b*). *a* = *m*}

by *auto*

lemma *sets-distinct2*: (*m*::*int*) ≠ *n* ⇒ {(*a,b*). *a* = *n*} ≠ {(*a,b*). *a* = *m*}

by *auto*

lemma *sets-distinct3*: {((*a*::*int*),(*b*::*int*)). *a* = *n*} ≠ {(*a,b*). *a* > *n*}

by *auto*

lemma *sets-distinct4*: {((*a*::*int*),(*b*::*int*)). *a* > *n*} ≠ {(*a,b*). *a* = *n*}

by *auto*

lemma *aux*: [*a* ∈ *c*; *a* ∉ *d*; *c* = *d*] ⇒ *False*

by *auto*

lemma *sets-distinct5*: (*s*::*int*) < *g* ⇒ {(*a*::*int*, *b*::*int*). *a* = *s*} ≠ {(*a*::*int*, *b*::*int*). *g* < *a*}

apply (*auto simp: sets-distinct3*)[1]

```

apply (subgoal-tac (s,4) ∈ {(a::int,b::int). a = (s)})
apply (subgoal-tac (s,4) ∉ {(a::int,b::int). g < a})
apply (erule aux)
apply assumption+
apply simp
by blast

```

```

lemma sets-distinct6: (s::int) < g ⇒ {(a::int, b::int). g < a} ≠ {(a::int, b::int). a
= s}
apply (rule not-sym)
apply (rule sets-distinct5)
by simp

```

```

lemma distinctNets: FWLink ≠ any ∧ FWLink ≠ i4 ∧ FWLink ≠ i27 ∧ FWLink
≠ eth-intern ∧ FWLink ≠ eth-private ∧
any ≠ FWLink ∧ any ≠ i4 ∧ any ≠ i27 ∧ any ≠ eth-intern ∧ any ≠ eth-private
∧ i4 ≠ FWLink ∧
i4 ≠ any ∧ i4 ≠ i27 ∧ i4 ≠ eth-intern ∧ i4 ≠ eth-private ∧ i27 ≠ FWLink ∧ i27
≠ any ∧
i27 ≠ i4 ∧ i27 ≠ eth-intern ∧ i27 ≠ eth-private ∧ eth-intern ≠ FWLink ∧ eth-intern
≠ any ∧
eth-intern ≠ i4 ∧ eth-intern ≠ i27 ∧ eth-intern ≠ eth-private ∧ eth-private ≠
FWLink ∧
eth-private ≠ any ∧ eth-private ≠ i4 ∧ eth-private ≠ i27 ∧ eth-private ≠ eth-intern
by (simp add: PolicyLemmas sets-distinct1 sets-distinct2 sets-distinct3 sets-distinct4
sets-distinct5 sets-distinct6)

```

```

lemma aux5: [x ≠ a; y≠b; (x ≠ y ∧ x ≠ b) ∨ (a ≠ b ∧ a ≠ y)] ⇒ {x,a} ≠ {y,b}
by auto

```

```

lemma aux2: {a,b} = {b,a}
by auto

```

```

lemma ANDex: allNetsDistinct (policy2list Policy)
apply (simp add: PolicyL allNetsDistinct-def distinctNets)
by (auto simp: PLemmas PolicyLemmas netsDistinct-def sets-distinct5 sets-distinct6)

```

```

fun (sequential) numberOfRules where
  numberOfRules (a⊕b) = numberOfRules a + numberOfRules b
|numberOfRules a = (1::int)

```

```

fun numberOfRulesList where
  numberOfRulesList (x#xs) = ((numberOfRules x)#(numberOfRulesList xs))
|numberOfRulesList [] = []

```

```

lemma all-in-list: all-in-list (policy2list Policy) (Nets-List Policy)
  apply (simp add: PolicyL)
  apply (unfold Nets-List-def)
  apply (unfold bothNets-def)
  apply (insert distinctNets)
  by simp

```

```

lemmas normalizeUnfold = normalize-def Policy-def Nets-List-def bothNets-def aux
aux2 bothNets-def

```

end

3.3.2 Transformation Example 2

theory

Transformation02

imports

../UPF-Firewall

begin

definition

FWLink :: *adr_{ip} net* **where**
FWLink = $\{\{(a,b). a = 1\}\}$

definition

any :: *adr_{ip} net* **where**
any = $\{\{(a,b). a > 5\}\}$

definition

i4-32:: *adr_{ip} net* **where**
i4-32 = $\{\{(a,b). a = 2\}\}$

definition

i10-32:: *adr_{ip} net* **where**
i10-32 = $\{\{(a,b). a = 3\}\}$

definition

eth-intern:: *adr_{ip} net* **where**
eth-intern = $\{\{(a,b). a = 4\}\}$

definition

eth-private:: *adr_{ip} net* **where**
eth-private = $\{\{(a,b). a = 5\}\}$

definition

$D1a :: (adr_{ip} \text{ net}, port) \text{ Combinators where}$
 $D1a = AllowPortFromTo \text{ eth-intern any } 1 \oplus$
 $AllowPortFromTo \text{ eth-intern any } 2$

definition

$D1b :: (adr_{ip} \text{ net}, port) \text{ Combinators where}$
 $D1b = AllowPortFromTo \text{ eth-private any } 1 \oplus$
 $AllowPortFromTo \text{ eth-private any } 2$

definition

$D2a :: (adr_{ip} \text{ net}, port) \text{ Combinators where}$
 $D2a = AllowPortFromTo \text{ any } i4\text{-}32 \ 21$

definition

$D2b :: (adr_{ip} \text{ net}, port) \text{ Combinators where}$
 $D2b = AllowPortFromTo \text{ any } i10\text{-}32 \ 21 \oplus$
 $AllowPortFromTo \text{ any } i10\text{-}32 \ 43$

definition

$Policy :: (adr_{ip} \text{ net}, port) \text{ Combinators where}$
 $Policy = DenyAll \oplus D2b \oplus D2a \oplus D1b \oplus D1a$

lemmas $PolicyLemmas = Policy\text{-def } D1a\text{-def } D1b\text{-def } D2a\text{-def } D2b\text{-def}$

lemmas $PolicyL = Policy\text{-def}$

$FWLink\text{-def}$
 $any\text{-def}$
 $i10\text{-}32\text{-def}$
 $i4\text{-}32\text{-def}$
 $eth\text{-intern}\text{-def}$
 $eth\text{-private}\text{-def}$
 $D1a\text{-def } D1b\text{-def } D2a\text{-def } D2b\text{-def}$

consts $fixID :: id$

consts $fixContent :: DummyContent$

definition $fixElements \ p = (id \ p = fixID \wedge content \ p = fixContent)$

lemmas $fixDefs = fixElements\text{-def } NetworkCore.id\text{-def } NetworkCore.content\text{-def}$

lemma $sets\text{-distinct1}: (n::int) \neq m \implies \{(a,b). a = n\} \neq \{(a,b). a = m\}$
by $auto$

lemma sets-distinct2: $(m::int) \neq n \implies \{(a,b). a = n\} \neq \{(a,b). a = m\}$
by auto

lemma sets-distinct3: $\{((a::int),(b::int)). a = n\} \neq \{(a,b). a > n\}$
by auto

lemma sets-distinct4: $\{((a::int),(b::int)). a > n\} \neq \{(a,b). a = n\}$
by auto

lemma aux: $\llbracket a \in c; a \notin d; c = d \rrbracket \implies False$
by auto

lemma sets-distinct5: $(s::int) < g \implies \{(a::int, b::int). a = s\} \neq \{(a::int, b::int). g < a\}$
apply (auto simp: sets-distinct3)
apply (subgoal-tac (s,4) $\in \{(a::int,b::int). a = (s)\}$)
apply (subgoal-tac (s,4) $\notin \{(a::int,b::int). g < a\}$)
apply (erule aux)
apply assumption+
apply simp
by blast

lemma sets-distinct6: $(s::int) < g \implies \{(a::int, b::int). g < a\} \neq \{(a::int, b::int). a = s\}$
apply (rule not-sym)
apply (rule sets-distinct5)
by simp

lemma distinctNets: $FWLink \neq any \wedge FWLink \neq i4-32 \wedge FWLink \neq i10-32 \wedge FWLink \neq eth-intern \wedge FWLink \neq eth-private \wedge any \neq FWLink \wedge any \neq i4-32 \wedge any \neq i10-32 \wedge any \neq eth-intern \wedge any \neq eth-private \wedge i4-32 \neq FWLink \wedge i4-32 \neq any \wedge i4-32 \neq i10-32 \wedge i4-32 \neq eth-intern \wedge i4-32 \neq eth-private \wedge i10-32 \neq FWLink \wedge i10-32 \neq any \wedge i10-32 \neq i4-32 \wedge i10-32 \neq eth-intern \wedge i10-32 \neq eth-private \wedge eth-intern \neq FWLink \wedge eth-intern \neq any \wedge eth-intern \neq i4-32 \wedge eth-intern \neq i10-32 \wedge eth-intern \neq eth-private \wedge eth-private \neq FWLink \wedge eth-private \neq any \wedge eth-private \neq i4-32 \wedge eth-private \neq i10-32 \wedge eth-private \neq eth-intern$
by (simp add: PolicyL sets-distinct1 sets-distinct2 sets-distinct3 sets-distinct4 sets-distinct5 sets-distinct6)

lemma aux5: $\llbracket x \neq a; y \neq b; (x \neq y \wedge x \neq b) \vee (a \neq b \wedge a \neq y) \rrbracket \implies \{x,a\} \neq \{y,b\}$
by auto

```

lemma aux2: {a,b} = {b,a}
  by auto

lemma ANDex: allNetsDistinct (policy2list Policy)
  apply (simp add: PolicyLemmas allNetsDistinct-def distinctNets)
  apply (simp add: PolicyL)
  by (auto simp: PLemmas PolicyL netsDistinct-def sets-distinct5 sets-distinct6
sets-distinct1
      sets-distinct2)

fun (sequential) numberOfRules where
  numberOfRules (a⊕b) = numberOfRules a + numberOfRules b
|numberOfRules a = (1::int)

fun numberOfRulesList where
  numberOfRulesList (x#xs) = ((numberOfRules x)#(numberOfRulesList xs))
|numberOfRulesList [] = []

lemma all-in-list: all-in-list (policy2list Policy) (Nets-List Policy)
  apply (simp add: PolicyLemmas)
  apply (unfold Nets-List-def)
  apply (unfold bothNets-def)
  apply (insert distinctNets)
  by simp

lemmas normalizeUnfold = normalize-def PolicyL Nets-List-def bothNets-def aux
aux2 bothNets-def sets-distinct1 sets-distinct2 sets-distinct3 sets-distinct4 sets-distinct5
sets-distinct6 aux5 aux2

end

```

3.4 Example: NAT

```

theory
  NAT-FW
  imports
    ../UPF-Firewall
begin

definition subnet1 :: adrip net where
  subnet1 = {{(d,e). d > 1 ∧ d < 256}}

definition subnet2 :: adrip net where

```

$subnet2 = \{(d,e). d > 500 \wedge d < 1256\}$

definition

$accross-subnets\ x \equiv$
 $((src\ x \sqsubset\ subnet1 \wedge (dest\ x \sqsubset\ subnet2)) \vee$
 $(src\ x \sqsubset\ subnet2 \wedge (dest\ x \sqsubset\ subnet1)))$

definition

$filter :: (adr_{ip}, DummyContent) FWPoly\ where$
 $filter = allow-from-port-to\ (1::port)\ subnet1\ subnet2\ ++$
 $allow-from-port-to\ (2::port)\ subnet1\ subnet2\ ++$
 $allow-from-port-to\ (3::port)\ subnet1\ subnet2\ ++\ deny-all$

definition

$nat-0\ where$
 $nat-0 = (A_f(\lambda x. \{x\}))$

lemmas $UnfoldPolicy0 = filter-def\ nat-0-def$

$NATLemmas$
 $ProtocolPortCombinators.ProtocolCombinators$
 $adr_{ip}Lemmas$
 $packet-defs\ accross-subnets-def$
 $subnet1-def\ subnet2-def$

lemmas $subnets = subnet1-def\ subnet2-def$

definition $Adr11 :: int\ set$

where $Adr11 = \{d. d > 2 \wedge d < 3\}$

definition $Adr21 :: int\ set\ where$

$Adr21 = \{d. d > 502 \wedge d < 503\}$

definition $nat-1\ where$

$nat-1 = nat-0\ ++\ (srcPat2pool-IntPort\ Adr11\ Adr21)$

definition $policy-1\ where$

$policy-1 = ((\lambda (x,y). x)\ o-f$
 $((nat-1 \otimes_2 filter)\ o\ (\lambda x. (x,x))))$

lemmas $UnfoldPolicy1 = UnfoldPolicy0\ nat-1-def\ Adr11-def\ Adr21-def\ policy-1-def$

definition $Adr12 :: int\ set$

where $Adr12 = \{d. d > 4 \wedge d < 6\}$

definition *Adr22* :: *int set* **where**
Adr22 = {*d*. *d* > 504 ∧ *d* < 506}

definition *nat-2* **where**
nat-2 = *nat-1* ++ (*srcPat2pool-IntPort* *Adr12* *Adr22*)

definition *policy-2* **where**
policy-2 = ((λ (*x,y*). *x*) o-*f*
((*nat-2* ⊗₂ *filter*) o (λ *x*. (*x,x*))))

lemmas *UnfoldPolicy2* = *UnfoldPolicy1* *nat-2-def* *Adr12-def* *Adr22-def* *policy-2-def*

definition *Adr13* :: *int set*
where *Adr13* = {*d*. *d* > 6 ∧ *d* < 9}

definition *Adr23* :: *int set* **where**
Adr23 = {*d*. *d* > 506 ∧ *d* < 509}

definition *nat-3* **where**
nat-3 = *nat-2* ++ (*srcPat2pool-IntPort* *Adr13* *Adr23*)

definition *policy-3* **where**
policy-3 = ((λ (*x,y*). *x*) o-*f*
((*nat-3* ⊗₂ *filter*) o (λ *x*. (*x,x*))))

lemmas *UnfoldPolicy3* = *UnfoldPolicy2* *nat-3-def* *Adr13-def* *Adr23-def* *policy-3-def*

definition *Adr14* :: *int set*
where *Adr14* = {*d*. *d* > 8 ∧ *d* < 12}

definition *Adr24* :: *int set* **where**
Adr24 = {*d*. *d* > 508 ∧ *d* < 512}

definition *nat-4* **where**
nat-4 = *nat-3* ++ (*srcPat2pool-IntPort* *Adr14* *Adr24*)

definition *policy-4* **where**
policy-4 = ((λ (*x,y*). *x*) o-*f*
((*nat-4* ⊗₂ *filter*) o (λ *x*. (*x,x*))))

lemmas *UnfoldPolicy4* = *UnfoldPolicy3* *nat-4-def* *Adr14-def* *Adr24-def* *policy-4-def*

definition *Adr15* :: *int set*
where *Adr15* = {*d*. *d* > 10 ∧ *d* < 15}

definition *Adr25* :: *int set* **where**
Adr25 = {*d*. *d* > 510 ∧ *d* < 515}

definition *nat-5* **where**
nat-5 = *nat-4* ++ (*srcPat2pool-IntPort* *Adr15* *Adr25*)

definition *policy-5* **where**
policy-5 = ((λ (*x,y*). *x*) o-f
((*nat-5* ⊗₂ *filter*) o (λ *x*. (*x,x*))))

lemmas *UnfoldPolicy5* = *UnfoldPolicy4* *nat-5-def* *Adr15-def* *Adr25-def* *policy-5-def*

definition *Adr16* :: *int set*
where *Adr16* = {*d*. *d* > 12 ∧ *d* < 18}

definition *Adr26* :: *int set* **where**
Adr26 = {*d*. *d* > 512 ∧ *d* < 518}

definition *nat-6* **where**
nat-6 = *nat-5* ++ (*srcPat2pool-IntPort* *Adr16* *Adr26*)

definition *policy-6* **where**
policy-6 = ((λ (*x,y*). *x*) o-f
((*nat-6* ⊗₂ *filter*) o (λ *x*. (*x,x*))))

lemmas *UnfoldPolicy6* = *UnfoldPolicy5* *nat-6-def* *Adr16-def* *Adr26-def* *policy-6-def*

definition *Adr17* :: *int set*
where *Adr17* = {*d*. *d* > 14 ∧ *d* < 21}

definition *Adr27* :: *int set* **where**
Adr27 = {*d*. *d* > 514 ∧ *d* < 521}

definition *nat-7* **where**
nat-7 = *nat-6* ++ (*srcPat2pool-IntPort* *Adr17* *Adr27*)

definition *policy-7* **where**
policy-7 = ((λ (*x,y*). *x*) o-f
((*nat-7* ⊗₂ *filter*) o (λ *x*. (*x,x*))))

lemmas *UnfoldPolicy7* = *UnfoldPolicy6* *nat-7-def* *Adr17-def* *Adr27-def* *policy-7-def*

definition *Adr18* :: *int set*

where $Adr18 = \{d. d > 16 \wedge d < 24\}$

definition $Adr28 :: int\ set$ **where**
 $Adr28 = \{d. d > 516 \wedge d < 524\}$

definition $nat-8$ **where**
 $nat-8 = nat-7 ++ (srcPat2pool-IntPort\ Adr18\ Adr28)$

definition $policy-8$ **where**
 $policy-8 = ((\lambda\ (x,y).\ x)\ o-f$
 $((nat-8\ \otimes_2\ filter)\ o\ (\lambda\ x.\ (x,x))))$

lemmas $UnfoldPolicy8 = UnfoldPolicy7\ nat-8-def\ Adr18-def\ Adr28-def\ policy-8-def$

definition $Adr19 :: int\ set$
where $Adr19 = \{d. d > 18 \wedge d < 27\}$

definition $Adr29 :: int\ set$ **where**
 $Adr29 = \{d. d > 518 \wedge d < 527\}$

definition $nat-9$ **where**
 $nat-9 = nat-8 ++ (srcPat2pool-IntPort\ Adr19\ Adr29)$

definition $policy-9$ **where**
 $policy-9 = ((\lambda\ (x,y).\ x)\ o-f$
 $((nat-9\ \otimes_2\ filter)\ o\ (\lambda\ x.\ (x,x))))$

lemmas $UnfoldPolicy9 = UnfoldPolicy8\ nat-9-def\ Adr19-def\ Adr29-def\ policy-9-def$

definition $Adr110 :: int\ set$
where $Adr110 = \{d. d > 20 \wedge d < 30\}$

definition $Adr210 :: int\ set$ **where**
 $Adr210 = \{d. d > 520 \wedge d < 530\}$

definition $nat-10$ **where**
 $nat-10 = nat-9 ++ (srcPat2pool-IntPort\ Adr110\ Adr210)$

definition $policy-10$ **where**
 $policy-10 = ((\lambda\ (x,y).\ x)\ o-f$
 $((nat-10\ \otimes_2\ filter)\ o\ (\lambda\ x.\ (x,x))))$

lemmas $UnfoldPolicy10 = UnfoldPolicy9\ nat-10-def\ Adr110-def\ Adr210-def\ policy-10-def$

end

3.5 Voice over IP

theory

Voice-over-IP

imports

../.. / UPF-Firewall

begin

In this theory we generate the test data for correct runs of the FTP protocol. As usual, we start with defining the networks and the policy. We use a rather simple policy which allows only FTP connections starting from the Intranet and going to the Internet, and deny everything else.

definition

intranet :: *adr_{ip} net* **where**
intranet = $\{\{(a,e) . a = 3\}\}$

definition

internet :: *adr_{ip} net* **where**
internet = $\{\{(a,c) . a > 4\}\}$

definition

gatekeeper :: *adr_{ip} net* **where**
gatekeeper = $\{\{(a,c) . a = 4\}\}$

definition

voip-policy :: (*adr_{ip}, address voip-msg*) *FWPolicy* **where**
voip-policy = *A_U*

The next two constants check if an address is in the Intranet or in the Internet respectively.

definition

is-in-intranet :: *address* \Rightarrow *bool* **where**
is-in-intranet *a* = (*a* = 3)

definition

is-gatekeeper :: *address* \Rightarrow *bool* **where**
is-gatekeeper *a* = (*a* = 4)

definition

is-in-internet :: *address* \Rightarrow *bool* **where**
is-in-internet *a* = (*a* > 4)

The next definition is our starting state: an empty trace and the just defined policy.

definition

$\sigma\text{-}0\text{-voip} :: (\text{adr}_{ip}, \text{address voip-msg}) \text{ history} \times$
 $(\text{adr}_{ip}, \text{address voip-msg}) \text{ FWPolicy}$

where

$\sigma\text{-}0\text{-voip} = ([], \text{voip-policy})$

Next we state the conditions we have on our trace: a normal behaviour FTP run from the intranet to some server in the internet on port 21.

definition $\text{accept-voip} :: (\text{adr}_{ip}, \text{address voip-msg}) \text{ history} \Rightarrow \text{bool}$ **where**

$\text{accept-voip } t = (\exists c s g i p1 p2. t \in \text{NB-voip } c s g i p1 p2 \wedge \text{is-in-intranet } c$
 $\wedge \text{is-in-internet } s$
 $\wedge \text{is-gatekeeper } g)$

fun packet-with-id **where**

$\text{packet-with-id } [] i = []$
 $|\text{packet-with-id } (x\#xs) i =$
 $(\text{if } id\ x = i \text{ then } (x\#(\text{packet-with-id } xs\ i)) \text{ else } (\text{packet-with-id } xs\ i))$

The depth of the test case generation corresponds to the maximal length of generated traces, 4 is the minimum to get a full FTP protocol run.

fun ids1 **where**

$\text{ids1 } i (x\#xs) = (id\ x = i \wedge \text{ids1 } i\ xs)$
 $|\text{ids1 } i [] = \text{True}$

lemmas $\text{ST-simps} = \text{Let-def valid-SE-def unit-SE-def bind-SE-def}$

$\text{subnet-of-int-def } p\text{-accept-def content-def}$
 $\text{is-in-intranet-def is-in-internet-def intranet-def internet-def exI}$
 $\text{subnetOf-lemma subnetOf-lemma2 subnetOf-lemma3 subnetOf-lemma4 voip-policy-def}$
 $\text{NetworkCore.id-def is-arq-def is-fin-def}$
 $\text{is-connect-def is-setup-def ports-open-def subnet-of-adr-def}$
 $\text{VOIP.NB-voip-def } \sigma\text{-}0\text{-voip-def } \text{PLemmas VOIP-TRPolicy-def}$
 $\text{policy2MON-def applyPolicy-def}$

end

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