Haskell’s Show-Class in Isabelle/HOL

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Abstract

We implemented a type-class for pretty-printing, similar to Haskell’s Show-class [1]. Moreover, we provide instantiations for Isabelle/HOL’s standard types like B, prod, sum, N, Z, and Q. It is further possible, to automatically derive “to-string” functions for arbitrary user defined datatypes similar to Haskell’s “deriving Show”.

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1 Converting Arbitrary Values to Readable Strings

A type class similar to Haskell’s Show class, allowing for constant-time concatenation of strings using function composition.

theory Show
imports Main

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Deriving Generator-Aux
Deriving Derive-Manager

begin

type-synonym
shows = string ⇒ string
— show-functions with precedence

type-synonym
'\text{a} \text{ showsp} = \text{nat} ⇒ '\text{a} ⇒ shows

1.1 The Show-Law

The ”show law”, \text{shows-prec} p x (r @ s) = \text{shows-prec} p x r @ s, states that show-functions do not temper with or depend on output produced so far.

named-theorems show-law-simps (simplification rules for proving the show law)
named-theorems show-law-intros (introduction rules for proving the show law)

definition show-law :: 'a showsp ⇒ 'a ⇒ bool
where

show-law s x ←→ (∀ p y z. s p x (y @ z) = s p x y @ z)

lemma show-lawI:

show-law s x =⇒ (\forall p y z. s p x (y @ z) = s p x y @ z) =⇒ show-law s x
⟨proof⟩

lemma show-lawE:

show-law s x =⇒ (show-law (s' p x (y @ z)) = show-law (s p x y @ z) =⇒ P) =⇒ P
⟨proof⟩

lemma show-lawD:

show-law s x =⇒ s p x (y @ z) = s p x y @ z
⟨proof⟩

class show =

fixes shows-prec :: 'a showsp
and shows-list :: 'a list ⇒ shows
assumes shows-prec-append [show-law-simps]: shows-prec p x (r @ s) = shows-prec p x r @ s
and shows-list-append [show-law-simps]: shows-list xs (r @ s) = shows-list xs r @ s

begin

abbreviation shows x ≡ shows-prec 0 x
abbreviation show x ≡ shows x'''

end

Convert a string to a show-function that simply prepends the string unchanged.
**definition** shows-string :: string \(\Rightarrow\) shows

**where**

shows-string = (⟨⟩)

**lemma** shows-string-append [show-law-simps]:

shows-string \(x\) (\(r @ s\)) = shows-string \(x r @ s\)

\(\langle\text{proof}\rangle\)

**fun** shows-sep :: (\(a \Rightarrow\) shows) \(\Rightarrow\) shows \(\Rightarrow\) \(\langle\text{proof}\rangle\)

**definition** shows-sep :: (\(a \Rightarrow\) string) \(\Rightarrow\) shows

**where**

\(\text{shows-sep} s s [\vec{x}] = \text{shows-string} \langle\vec{x}\rangle\)
\(\text{shows-sep} s s [x] = s x\)
\(\text{shows-sep} s s (\vec{x} \# \vec{y}) = s x \circ \text{shows-sep} s s \vec{y}\)

\(\langle\text{proof}\rangle\)

**lemma** shows-sep-map:

\(\text{shows-sep} f s \circ \text{map} g \vec{x} = \text{shows-sep} (f \circ g) s \vec{x}\)

\(\langle\text{proof}\rangle\)

**definition** shows-list-gen :: (\(a \Rightarrow\) shows) \(\Rightarrow\) string \(\Rightarrow\) string \(\Rightarrow\) string \(\Rightarrow\) \(\langle\text{proof}\rangle\)

**where**

\(\text{shows-list-gen} \vec{e} \vec{s} \vec{r} \vec{x} =\)
\(\text{if} \ \vec{x} = [\vec{e}] \ \text{then}\)
\(\text{shows-string} \vec{e}\)
\(\text{else}\)
\(\text{shows-string} \vec{e} \circ \text{shows-sep} \vec{s} \vec{r} \vec{x}\)

\(\langle\text{proof}\rangle\)

**lemma** shows-list-gen-append [show-law-simps]:

\(\text{shows-list-gen} \vec{e} \vec{s} \vec{r} \vec{x} = \text{shows-list-gen} \vec{e} \vec{s} \vec{r} \vec{x}\)

\(\langle\text{proof}\rangle\)

**definition** pshowsp-list :: nat \(\Rightarrow\) shows list \(\Rightarrow\) shows

**where**

\(\text{pshowsp-list} \vec{p} \vec{x} = \text{shows-list-gen} \text{id} \vec{p} \vec{x}\)

\(\text{code del}\)

\(\text{showsp-list} \vec{s} \vec{p} = \text{pshowsp-list} \vec{p} \vec{p} \vec{m} \vec{s} \vec{p}\)

\(\langle\text{proof}\rangle\)

**definition** showsp-list :: (\(a \Rightarrow\) showsp) \(\Rightarrow\) nat \(\Rightarrow\) \(\langle\text{proof}\rangle\)

**where**

\(\text{showsp-list} \vec{a} \vec{p} = \text{pshowsp-list} \vec{p} \vec{p} \vec{m}\)

\(\langle\text{proof}\rangle\)
lemma showsp-list-code [code]:
  shows-list s p xs = shows-list-gen (s 0) "" """" "" "" xs
  ⟨proof⟩

lemma show-law-list [show-law-intros]:
  (\x. x ∈ set xs → show-law s x) → show-law (shows-list s) xs
  ⟨proof⟩

lemma showsp-list-append [show-law-simps]:
  (\p y z. \x ∈ set xs. s p x (y @ z) = s p x y @ z) →
  shows-list s p xs (y @ z) = shows-list s p xs y @ z
  ⟨proof⟩

1.2 Show-Functions for Characters and Strings

instantiation char :: show begin

definition shows-prec p (c::char) = (#) c

definition shows-list (cs::string) = shows-string cs

instance ⟨proof⟩

end

definition shows-nl = shows (CHR "\")

definition shows-space = shows (CHR "\"

definition shows-paren s = shows (CHR "\') o s o shows (CHR "]\")

definition shows-quote s = shows (CHR 0x27) o s o shows (CHR 0x27)

abbreviation apply-if b s ≡ (if b then s else id) — conditional function application

Parenthesize only if precedence is greater than 0.

definition shows-pl (p::nat) = apply-if (p > 0) (shows (CHR "\'))

definition shows-pr (p::nat) = apply-if (p > 0) (shows (CHR "]\'))

lemma shows-nl-append [show-law-simps]: shows-nl (x @ y) = shows-nl x @ y and

shows-space-append [show-law-simps]: shows-space (x @ y) = shows-space x @ y and

shows-paren-append [show-law-simps]: shows-paren (x @ y) = shows-paren s x @ y and

shows-quote-append [show-law-simps]: shows-quote (x @ y) = shows-quote s x @ y and

shows-pl-append [show-law-simps]: shows-pl p (x @ y) = shows-pl p x @ y and

shows-pr-append [show-law-simps]: shows-pr p (x @ y) = shows-pr p x @ y
  ⟨proof⟩

4
lemma o-append:
\( (\forall x y. f (x \append y) = f x \append y) \implies g (x \append y) = g x \append y \implies (f \circ g) (x \append y) = (f \circ g) x \append y \)  
\( \langle \text{proof} \rangle \)

instantiation list :: (show) show
begin

definition shows-prec (p :: nat) (xs :: 'a list) = shows-list xs
definition shows-list (xss :: 'a list list) = showsp-list shows-prec 0 xss

instance  
\( \langle \text{proof} \rangle \)
end

definition shows-lines :: 'a::show list ⇒ shows  
where
shows-lines = shows-sep shows shows-nl

definition shows-many :: 'a::show list ⇒ shows  
where
shows-many = shows-sep shows id

definition shows-words :: 'a::show list ⇒ shows  
where
shows-words = shows-sep shows shows-space

lemma shows-lines-append [show-law-simps]:
shows-lines xs (r @ s) = shows-lines xs r @ s  
\( \langle \text{proof} \rangle \)

lemma shows-many-append [show-law-simps]:
shows-many xs (r @ s) = shows-many xs r @ s  
\( \langle \text{proof} \rangle \)

lemma shows-words-append [show-law-simps]:
shows-words xs (r @ s) = shows-words xs r @ s  
\( \langle \text{proof} \rangle \)

lemma shows-foldr-append [show-law-simps]:  
assumes \( \forall r s. \forall x \in \text{set} \; \text{xs}. \; \text{showx} x \; (r \append s) = \text{showx} x \; r \append s \)
shows foldr showx xs (r @ s) = foldr showx xs r @ s  
\( \langle \text{proof} \rangle \)

lemma shows-sep-cong [fundef-cong]:  
assumes xs = ys and \( \forall x. \; x \in \text{set} \; \text{ys} \implies f x = g x \)
\textbf{shows} shows-sep f sep xs = shows-sep g sep ys
\langle proof \rangle

\textbf{lemma} shows-list-gen-cong [fundef-cong]:
\textbf{assumes} xs = ys and \( \forall x \in \text{set } ys \implies f x = g x \)
\textbf{shows} shows-list-gen f e l sep r xs = shows-list-gen g e l sep r ys
\langle proof \rangle

\textbf{lemma} showsp-list-cong [fundef-cong]:
xs = ys \implies p = q \implies \langle \forall p. x \in \text{set } ys \implies f p x = g p x \rangle \implies \text{showsp-list } f p \text{ xs} = \text{showsp-list } g q \text{ ys}
\langle proof \rangle

\textbf{abbreviation} (input) shows-cons :: string \Rightarrow shows \Rightarrow shows (infixr ++ #+ 10)
\textbf{where}
s ++ p \equiv \text{shows-string } s \circ p

\textbf{abbreviation} (input) shows-append :: shows \Rightarrow shows \Rightarrow shows (infixr +@ + 10)
\textbf{where}
s +@ p \equiv s \circ p

\textbf{instantiation} String.lITERAL :: show
begin

definition shows-prec-literal :: nat \Rightarrow String.lITERAL \Rightarrow string \Rightarrow string
\textbf{where} shows-prec p s = \text{shows-string } (\text{String.exp} \text{plode } s)

definition shows-list-literal :: String.lITERAL list \Rightarrow string \Rightarrow string
\textbf{where} shows-list ss = \text{shows-string } (\text{concat } (\text{map } \text{String.exp} \text{plode } ss))

\textbf{lemma} shows-list-literal-code [code]:
shows-list = \text{foldr } (\lambda s. \text{shows-string } (\text{String.exp} \text{plode } s))
\langle proof \rangle

\textbf{instance} \langle proof \rangle
end

Don’t use Haskell’s existing ”Show” class for code-generation, since it is not compatible to the formalized class.

\textbf{code-reserved} Haskell Show
end

\section{Instances of the Show Class for Standard Types}

\textbf{theory} Show-Instances
\textbf{imports}
Show
HOL.Rat

begin

definition showsp-unit :: unit showsp
where
  showsp-unit p x = shows-string "()"

lemma show-law-unit [show-law-intros]:
  show-law showsp-unit x
  ⟨proof⟩

abbreviation showsp-char :: char showsp
where
  showsp-char ≡ shows-prec

lemma show-law-char [show-law-intros]:
  show-law showsp-char x
  ⟨proof⟩

primrec showsp-bool :: bool showsp
where
  showsp-bool p True = shows-string "True"
  showsp-bool p False = shows-string "False"

lemma show-law-bool [show-law-intros]:
  show-law showsp-bool x
  ⟨proof⟩

primrec pshowsp-prod :: (shows × shows) showsp
where
  pshowsp-prod p (x, y) = shows-string "(" o s1 1 x o shows-string ", " o s2 1 y o shows-string ")"

-definition showsp-prod :: 'a showsp ⇒ 'b showsp ⇒ ('a × 'b) showsp
where
  [code del]: showsp-prod s1 s2 p = pshowsp-prod p o map-prod (s1 1) (s2 1)

lemma showsp-prod-simps [simp, code]:
  shows-string "(" o s1 1 x o shows-string ", " o s2 1 y o shows-string ")"
  ⟨proof⟩

lemma show-law-prod [show-law-intros]:
  (\x. x ∈ Basic-BNFs.fsts y ⇒ show-law s1 x) ⇒
  (\x. x ∈ Basic-BNFs.snds y ⇒ show-law s2 x) ⇒
  show-law (showsp-prod s1 s2) y
  ⟨proof⟩
fun string-of-digit :: nat ⇒ string
where
  string-of-digit n =
  (if n = 0 then "0"·
    else if n = 1 then "1"·
    else if n = 2 then "2"·
    else if n = 3 then "3"·
    else if n = 4 then "4"·
    else if n = 5 then "5"·
    else if n = 6 then "6"·
    else if n = 7 then "7"·
    else if n = 8 then "8"·
    else "9")

fun showsp-nat :: nat showsp
where
  showsp-nat p n =
  (if n < 10 then shows-string (string-of-digit n)
    else showsp-nat p (n div 10) o shows-string (string-of-digit (n mod 10)))
declare showsp-nat.simps [simp del]

lemma show-law-nat [show-law-intros]:
  show-law showsp-nat n
  ⟨proof⟩

lemma showsp-nat-append [show-law-simps]:
  showsp-nat p n (x @ y) = showsp-nat p n x @ y
  ⟨proof⟩

definition showsp-int :: int showsp
where
  showsp-int p i =
  (if i < 0 then shows-string "−" o showsp-int p (nat (− i)) else showsp-int p (nat i))

lemma show-law-int [show-law-intros]:
  show-law showsp-int i
  ⟨proof⟩

lemma showsp-int-append [show-law-simps]:
  showsp-int p i (x @ y) = showsp-int p i x @ y
  ⟨proof⟩

definition showsp-rat :: rat showsp
where
  showsp-rat p x =
  (case quotient-of x of (d, n) ⇒
    if n = 1 then showsp-int p d else showsp-int p d o shows-string "/" o showsp-int p n)
lemma show-law-rat [show-law-intros]:
  show-law showsp-rat r
⟨proof⟩

lemma showsp-rat-append [show-law-simps]:
  showsp-rat p r (x @ y) = showsp-rat p r x @ y
⟨proof⟩

Automatic show functions are not used for unit, prod, and numbers: for unit and prod, we do not want to display "Unity" and "Pair"; for nat, we do not want to display "Suc (Suc (... (Suc 0) ...))"; and neither int nor rat are datatypes.
⟨ML⟩
derive show option sum prod unit bool nat int rat

export-code
  shows-prec :: 'a::show option showsp
  shows-prec :: ('a::show, 'b::show) sum showsp
  shows-prec :: ('a::show × 'b::show) showsp
  shows-prec :: unit showsp
  shows-prec :: char showsp
  shows-prec :: bool showsp
  shows-prec :: nat showsp
  shows-prec :: int showsp
  shows-prec :: rat showsp
  checking
end

2.1 Displaying Polynomials

We define a method which converts polynomials to strings and registers it in the Show class.

theory Show-Poly
imports
  Show-Instances
  HOL-Computational-Algebra.Polynomial
begin

fun show-factor :: nat ⇒ string where
  show-factor 0 = []
| show-factor (Suc 0) = "x"
| show-factor n = "x" ^ n @ show n

fun show-coeff-factor where
  show-coeff-factor c n = (if n = 0 then show c else if c = 1 then show-factor n else show c @ show-coeff-factor n)
fun show-poly-main :: nat ⇒ 'a :: {zero,one,show} list ⇒ string where
  show-poly-main - [] = "0"
| show-poly-main n [c] = show-coeff-factor c n
| show-poly-main n (c # cs) = (if c = 0 then show-poly-main (Suc n) cs else
  show-coeff-factor c n @ " + " @ show-poly-main (Suc n) cs)

definition show-poly :: 'a :: {zero,one,show} poly ⇒ string where
  show-poly p = show-poly-main 0 (coeffs p)

definition showsp-poly :: 'a :: {zero,one,show} poly showsp
  where
    showsp-poly p x = shows-string (show-poly x)

instantiation poly :: ({show,one,zero}) show
begin

  definition shows-prec p (x :: 'a poly) = showsp-poly p x
  definition shows-list (ps :: 'a poly list) = showsp-list shows-prec 0 ps

  lemma show-law-poly [show-law-simps]:
    shows-prec p (a :: 'a poly) (r @ s) = shows-prec p a r @ s
    ⟨proof⟩

  instance ⟨proof⟩

end

end

3 Show for Real Numbers – Interface

We just demand that there is some function from reals to string and register
this as show-function. Implementations are available in one of the theories
Show-Real-Impl and ../Algebraic-Numbers/Show-Real-....

theory Show-Real
imports
  HOL.Real
  Show
begin

consts show-real :: real ⇒ string

definition showsp-real :: real showsp
  where
    showsp-real p x y = (show-real x @ y)
lemma show-law-real [show-law-intros]:
show-law showsp-real r
⟨proof⟩

lemma showsp-real-append [show-law-simps]:
showsp-real p r (x ⊕ y) = showsp-real p r x ⊕ y
⟨proof⟩

⟨ML⟩

derive show real
end

4 Show for Complex Numbers

We print complex numbers as real and imaginary parts. Note that by transitivity, this theory demands that an implementations for show-real is available, e.g., by using one of the theories Show-Real-Impl or ../Algebraic-Numbers/Show-Real-....

theory Show-Complex
imports
  HOL.Complex
  Show-Real
begin

definition show-complex x = (let r = Re x; i = Im x in
if (i = 0) then show-real r else if r = 0 then show-real i @ "i" else
"(" @ show-real r @ "+" @ show-real i @ ")"
)
definition showsp-complex :: complex showsp
where
  showsp-complex p x y = (show-complex x ⊕ y)

lemma show-law-complex [show-law-intros]:
show-law showsp-complex r
⟨proof⟩

lemma showsp-complex-append [show-law-simps]:
showsp-complex p r (x ⊕ y) = showsp-complex p r x ⊕ y
⟨proof⟩

⟨ML⟩

derive show complex
5 Show Implementation for Real Numbers via Rational Numbers

We just provide an implementation for show of real numbers where we assume that real numbers are implemented via rational numbers.

theory Show-Real-Impl
imports
  Show-Real
  Show-Instances
begin

  We now define show-real.

overloading show-real ≡ show-real
begin
  definition show-real
    where show-real x ≡
      (if (∃ y. x = Ratreal y) then show (THE y. x = Ratreal y) else "Irrational")
  end

lemma show-real-code[code]: show-real (Ratreal x) = show x
  ⟨proof⟩
end

References