Haskell’s Show-Class in Isabelle/HOL*

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Abstract

We implemented a type-class for pretty-printing, similar to Haskell’s Show-class [1]. Moreover, we provide instantiations for Isabelle/HOL’s standard types like \( \mathbb{B} \), \( \text{prod} \), \( \text{sum} \), \( \mathbb{N} \), \( \mathbb{Z} \), and \( \mathbb{Q} \). It is further possible, to automatically derive “to-string” functions for arbitrary user defined datatypes similar to Haskell’s “deriving Show”.

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1 Converting Arbitrary Values to Readable Strings

A type class similar to Haskell’s Show class, allowing for constant-time concatenation of strings using function composition.

theory Show
imports
  Main

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Deriving Generator-Aux

Deriving Derive-Manager

begin

type-synonym
  shows = string ⇒ string

— show-functions with precedence

type-synonym
  'a showsp = nat ⇒ 'a ⇒ shows

1.1 The Show-Law

The "show law", shows-prec p x (r @ s) = shows-prec p x r @ s, states that show-functions do not temper with or depend on output produced so far.

named-theorems show-law-simps (simplification rules for proving the show law)

named-theorems show-law-intros (introduction rules for proving the show law)

definition show-law :: 'a showsp ⇒ 'a ⇒ bool

where
  show-law s x ←→ (∀ p y z. s p x (y @ z) = s p x y @ z)

lemma show-lawI:
  (∀ p y z. s p x (y @ z) = s p x y @ z) ⇒ show-law s x
  by (simp add: show-law-def)

lemma show-lawE:
  show-law s x ⇒ (s p x (y @ z) = s p x y @ z) ⇒ P ⇒ P
  by (auto simp: show-law-def)

lemma show-lawD:
  show-law s x ⇒ s p x (y @ z) = s p x y @ z
  by (blast elim: show-lawE)

class show =

  fixes shows-prec :: 'a showsp
  and shows-list :: 'a list ⇒ shows

  assumes shows-prec-append [show-law-simps]: shows-prec p x (r @ s) = shows-prec p x r @ s
  and shows-list-append [show-law-simps]: shows-list xs (r @ s) = shows-list xs r @ s

begin

abbreviation shows x ≡ shows-prec 0 x

abbreviation show x ≡ shows x ""

end

Convert a string to a show-function that simply prepends the string unchanged.
definition shows-string :: string ⇒ shows
where
  shows-string = (@)

lemma shows-string-append [show-law-simps]:
  shows-string x (r @ s) = shows-string x r @ s
by (simp add: shows-string-def)

fun shows-sep :: ('a ⇒ shows) ⇒ shows ⇒ 'a list ⇒ shows
where
  shows-sep s sep [] = shows-string "" |
  shows-sep s sep [x] = s x |
  shows-sep s sep (x#xs) = s x o sep o shows-sep s sep xs

lemma shows-sep-append [show-law-simps]:
  assumes
    \forall r s. \forall x ∈ set xs. showsx x (r @ s) = showsx x r @ s
    and \forall r s. sep (r @ s) = sep r @ s
  shows
    shows-sep showsx sep xs (r @ s) = shows-sep showsx sep xs r @ s
using assms
proof (induct xs)
case (Cons x xs) then show ?case by (cases xs) (simp-all)
qed (simp add: show-law-simps)

lemma shows-sep-map:
  shows-sep f sep (map g xs) = shows-sep (f o g) sep xs
by (induct xs) (simp, case_tac xs, simp-all)

definition shows-list-gen :: ('a ⇒ shows) ⇒ string ⇒ string ⇒ string ⇒ string ⇒ 'a list ⇒ shows
where
  shows-list-gen showsx e l s r xs =
    (if xs = [] then shows-string e
     else shows-string l o shows-sep showsx (shows-string s) xs o shows-string r)

lemma shows-list-gen-append [show-law-simps]:
  assumes
    \forall r s. \forall x ∈ set xs. showsx x (r @ s) = showsx x r @ s
  shows
    shows-list-gen showsx e l sep r xs (s @ t) = shows-list-gen showsx e l sep r xs s @ t
using assms by (cases xs) (simp-all add: shows-list-gen-def show-law-simps)

lemma shows-list-gen-map:
  shows-list-gen f e l sep r (map g xs) = shows-list-gen (f o g) e l sep r xs
by (simp-all add: shows-list-gen-def shows-sep-map)

definition pshowsp-list :: nat ⇒ shows list ⇒ shows
where
  pshowsp-list p xs = shows-list-gen id "" "" "" "" "" "" "" xs
definition showsp-list :: 'a showsp ⇒ nat ⇒ 'a list ⇒ shows
where
[code del]: showsp-list s p = pshowsp-list p o map (s 0)

lemma showsp-list-code [code]:
showsp-list s p xs = shows-list-gen (s 0) "" "" "" "" xs
by (simp add: showsp-list-def pshowsp-list-def shows-list-gen-map)

lemma show-law-list [show-law-intros]:
(∀ x ∈ set xs ⇒ show-law s x) ⇒ show-law (showsp-list s) xs
by (simp add: show-law-def showsp-list-code show-law-simps)

lemma showsp-list-append [show-law-simps]:
(∀ p y z. ∀ x ∈ set xs. s p x (y @ z) = s p x y @ z) ⇒
showsp-list s p xs (y @ z) = showsp-list s p xs y @ z
by (simp add: show-law-simps showsp-list-def pshowsp-list-def)

1.2 Show-Functions for Characters and Strings

instantiation char :: show begin

definition shows-prec p (c::char) = (#) c

definition shows-list (cs::string) = shows-string cs
instance
  by standard (simp-all add: shows-prec-char-def shows-list-char-def show-law-simps)
end

definition shows-nl = shows (CHR "\n"")
definition shows-space = shows (CHR " ")
definition shows-paren s = shows (CHR "(" o s o shows (CHR ")")
definition shows-quote s = shows (CHR 0x27) o s o shows (CHR 0x27)
abbreviation apply-if b s ≡ (if b then s else id) — conditional function application

Parenthesize only if precedence is greater than 0.

definition shows-pl (p::nat) = apply-if (p > 0) (shows (CHR "("))
definition shows-pr (p::nat) = apply-if (p > 0) (shows (CHR ")")

lemma shows-nl-append [show-law-simps]: shows-nl (x @ y) = shows-nl x @ y and
shows-space-append [show-law-simps]: shows-space (x @ y) = shows-space x @ y and
shows-paren-append [show-law-simps]:
  (∀ x y. s (x @ y) = s x @ y) ⇒ shows-paren s (x @ y) = shows-paren s x @ y and
shows-quote-append [show-law-simps]:
  (∀ x y. s (x @ y) = s x @ y) ⇒ shows-quote s (x @ y) = shows-quote s x @ y and
shows-pl-append [show-law-simps]: shows-pl p (x @ y) = shows-pl p x @ y and
shows-pr-append [show-law-simps]: shows-pr p (x @ y) = shows-pr p x @ y
by (simp-all add: shows-pl-def shows-space-def shows-paren-def shows-quote-def
shows-pr-def show-law-simps)

lemma o-append:
(\(\forall x y. f(x @ y) = f x @ y\) \(\implies\) \(g(x @ y) = g x @ y \implies (f\ o\ g)(x @ y) = (f\ o\ g)\ x @ y\)
by simp

ML-file ⟨show-generator.ML⟩

local-setup ⟨\begin{aligned} &\text{Show-Generator.register-foreign-partial-and-full-showsp} @\{\text{type-name list}\} 0 \\
& @\{\text{term pshowsp-list}\} (\text{SOME} @\{\text{thm pshowsp-list-def}\}) \\
& @\{\text{term map}\} (\text{SOME} @\{\text{thm map-comp}\}) [\text{true}] @\{\text{thm show-law-list}\} \end{aligned}⟩

instantiation list :: (show) show
begin
definition shows-prec :: (p :: nat) (xs :: 'a list) = shows-list xs
definition shows-list :: (xs :: 'a list list) = shows-sp-list shows-prec 0 xs

instance
by standard (simp-all add: show-law-simps shows-prec-list-def shows-list-list-def)
end
definition shows-lines :: 'a::show list \(\Rightarrow\) shows
where
shows-lines = shows-sep shows shows-nl
definition shows-many :: 'a::show list \(\Rightarrow\) shows
where
shows-many = shows-sep shows id
definition shows-words :: 'a::show list \(\Rightarrow\) shows
where
shows-words = shows-sep shows shows-space

lemma shows-lines-append [show-law-simps]:
shows-lines xs (r @ s) = shows-lines xs r @ s
by (simp add: shows-lines-def show-law-simps)

lemma shows-many-append [show-law-simps]:
shows-many xs (r @ s) = shows-many xs r @ s
by (simp add: shows-many-def show-law-simps)
lemma shows-words-append [show-law-simps]:
  shows-words xs (r @ s) = shows-words xs r @ s
by (simp add: shows-words-def show-law-simps)

text
lemma shows-foldr-append [show-law-simps]:
  assumes \( \forall r, s. \forall x \in \text{set } xs. \text{show}_x(x r @ s) = \text{show}_x(xs r @ s) \)
  shows foldr showx xs (r @ s) = foldr showx xs r @ s
using assms by (induct xs) (simp-all)

lemma shows-sep-cong [fundef-cong]:
  assumes xs = ys and \( \forall x \in \text{set } ys. \text{f } x = \text{g } x \)
  shows shows-sep f sep xs = shows-sep g sep ys
using assms proof (induct ys arbitrary: xs)
  case (Cons y ys)
  then show ?case by (cases ys) simp-all
qed simp

abbreviation (input) shows-cons :: string \Rightarrow show \Rightarrow shows (infixr +#+ 10)
where
  s +#+ p \equiv shows-string s \circ p

abbreviation (input) shows-append :: shows \Rightarrow show \Rightarrow shows (infixr +@+ 10)
where
  s +@+ p \equiv s \circ p

instantiation String.literal :: show
begin

definition shows-prec-literal :: nat \Rightarrow String.literal \Rightarrow string \Rightarrow string
where
  shows-prec p s = shows-string (String.explode s)

definition shows-list-literal :: String.literal list \Rightarrow string \Rightarrow string
where
  shows-list ss = shows-string (concat (map String.explode ss))

lemma shows-list-literal-code [code]:
  shows-list = foldr (\lambda s. shows-string (String.explode s))
proof
  fix ss
shows-list \texttt{ss} = \texttt{foldr} (\lambda \texttt{s}. \texttt{shows-string} (\texttt{String.explode} \texttt{s})) \texttt{ss}
by (induct \texttt{ss}) (simp-all add: shows-list-literal-def shows-string-def)
qed

instance by standard
(simp-all add: shows-prec-literal-def shows-list-literal-def shows-string-def)
end

Don’t use Haskell’s existing ”Show” class for code-generation, since it is not compatible to the formalized class.

code-reserved Haskell Show
end

2 Instances of the Show Class for Standard Types

theory Show-Instances
imports
Show
HOL.Rat
begin

definition showsp-unit :: unit showsp
where
  showsp-unit \texttt{p} \texttt{x} = \texttt{shows-string} "()"

lemma show-law-unit [show-law-intros]:
  show-law showsp-unit \texttt{x}
by (rule show-lawI) (simp add: showsp-unit-def show-law-simps)

abbreviation showsp-char :: char showsp
where
  showsp-char \equiv shows-prec

lemma show-law-char [show-law-intros]:
  show-law showsp-char \texttt{x}
by (rule show-lawI) (simp add: show-law-simps)

primrec showsp-bool :: bool showsp
where
  showsp-bool \texttt{p} \texttt{True} = \texttt{shows-string} ""True"
  showsp-bool \texttt{p} \texttt{False} = \texttt{shows-string} "False"

lemma show-law-bool [show-law-intros]:
  show-law showsp-bool \texttt{x}
by (rule show-lawI, cases \texttt{x}) (simp-all add: show-law-simps)

primrec pshowsprod :: (shows \times shows) showsp
where
\[ p\text{showsp-prod } p (x, y) = \text{shows-string } "'\text{o } o \text{ shows-string } ',' \text{ o } y \text{ o shows-string } "'\)" \]

**definition** showsp-prod :: 'a showsp ⇒ 'b showsp ⇒ ('a × 'b) showsp

**where**

[code del]: showsp-prod s1 s2 p = p\text{showsp-prod } p o \text{map-prod } (s1 1) (s2 1)

**lemma** showsp-prod-simps [simp, code]:

\[ \text{showsp-prod } s1 s2 p (x, y) = \text{shows-string } "'\text{o } s1 1 x \text{ o shows-string } ',' \text{ o } s2 1 y \text{ o shows-string } "'\]

by (simp add: showsp-prod-def)

**lemma** show-law-prod [show-law-intros]:

\[ (\forall x, x \in \text{Basic-BNFs} \Rightarrow \text{fsts } y \Rightarrow \text{show-law } s1 x) \Rightarrow \]

\[ (\forall x, x \in \text{Basic-BNFs} \Rightarrow \text{snds } y \Rightarrow \text{show-law } s2 x) \Rightarrow \]

\[ \text{show-law } (\text{showsp-prod } s1 s2) y \]

**proof** (induct y)

case (Pair x y)

note \* = Pair [unfolded prod-set-simps]

show ?case

by (rule show-lawI)

(auto simp del: o-apply intro: o-append intro: show-lawD \* simp: show-law-simps)

qed

**fun** string-of-digit :: nat ⇒ string

**where**

\[ \text{string-of-digit } n = \]

\[ (\text{if } n = 0 \text{ then } "'0" \text{ else if } n = 1 \text{ then } "'1" \text{ else if } n = 2 \text{ then } "'2" \text{ else if } n = 3 \text{ then } "'3" \text{ else if } n = 4 \text{ then } "'4" \text{ else if } n = 5 \text{ then } "'5" \text{ else if } n = 6 \text{ then } "'6" \text{ else if } n = 7 \text{ then } "'7" \text{ else if } n = 8 \text{ then } "'8" \text{ else } "'9") \]

**fun** showsp-nat :: nat showsp

**where**

\[ \text{showsp-nat } p n = \]

\[ (\text{if } n < 10 \text{ then } \text{shows-string } (\text{string-of-digit } n) \text{ else } \text{showsp-nat } p (n \text{ div } 10) o \text{ shows-string } (\text{string-of-digit } (n \text{ mod } 10))) \]

**declare** showsp-nat.simps [simp del]

**lemma** show-law-nat [show-law-intros]:

\[ \text{show-law } \text{showsp-nat } n \]
by (rule show-lawI, induct n rule: nat-less-induct) (simp add: show-law-simps showsp-nat-simps)

lemma showsp-nat-append [show-law-simps]:
  showsp-nat p n (x @ y) = showsp-nat p n x @ y
by (intro show-lawD show-law-intros)

definition showsp-int :: int showsp
where
  showsp-int p i =
  (if i < 0 then shows-string "−" o showsp-nat p (nat (− i)) else showsp-nat p (nat i))

lemma show-law-int [show-law-intros]:
  show-law showsp-int i
by (rule show-lawI, cases i < 0) (simp-all add: showsp-int-def show-law-simps)

lemma showsp-int-append [show-law-simps]:
  showsp-int p i (x @ y) = showsp-int p i x @ y
by (intro show-lawD show-law-intros)

definition showsp-rat :: rat showsp
where
  showsp-rat p x =
  (case quotient-of x of (d, n) ⇒
    if n = 1 then showsp-int p d else showsp-int p d o shows-string "/" o showsp-int p n)

lemma show-law-rat [show-law-intros]:
  show-law showsp-rat r
by (rule show-lawI, cases quotient-of r) (simp add: showsp-rat-def show-law-simps)

lemma showsp-rat-append [show-law-simps]:
  showsp-rat p r (x @ y) = showsp-rat p r x @ y
by (intro show-lawD show-law-intros)

Automatic show functions are not used for unit, prod, and numbers: for unit and prod, we do not want to display "Unity" and "Pair"; for nat, we do not want to display "Suc (Suc (... (Suc 0) ...))"; and neither int nor rat are datatypes.

local-setup :
  Show-Generator.register-foreign-partial-and-full-showsp @\{type-name prod\} 0
    @\{term pshowsp-prod\}
    @\{term showsp-prod\} (SOME @\{thm showsp-prod-def\})
    @\{term map-prod\} (SOME @\{thm prod.map-comp\}) [true, true]
    @\{thm show-law-prod\}
  #=> Show-Generator.register-foreign-showsp @\{typ unit\} @\{term showsp-unit\}
    @\{thm show-law-unit\}
  #=> Show-Generator.register-foreign-showsp @\{typ bool\} @\{term showsp-bool\}
\[\text{derive} \quad \text{show} \quad \text{option} \quad \text{sum} \quad \text{prod} \quad \text{unit} \quad \text{bool} \quad \text{nat} \quad \text{rat} \]

\section{2.1 Displaying Polynomials}

We define a method which converts polynomials to strings and registers it in the Show class.

\textbf{theory} Show-Poly

\textbf{imports}

Show-Instances

\textit{HOL\textendash Computational-Algebra}.Polynomial

\textbf{begin}

\textbf{fun} show-factor :: nat \Rightarrow string \textbf{where}

\quad show-factor 0 = \[\]

\quad \mid \text{show-factor} (Suc 0) = "x"

\quad \mid \text{show-factor} n = "x^n" \circ \text{show} \ n

\textbf{fun} show-coeff-factor \textbf{where}

\quad show-coeff-factor \ c \ n \ n = (\text{if } n = 0 \text{ then show } c \text{ else if } c = 1 \text{ then show-factor } n \text{ else show } c \circ \text{show-factor } n)

\textbf{fun} show-poly-main :: nat \Rightarrow 'a :: \{\text{zero,one,show}\} \text{ list} \Rightarrow string \textbf{where}

\quad show-poly-main - \[\] = "0"

\quad \mid \text{show-poly-main} \ n \ [c] = \text{show-coeff-factor } c \ n
| show-poly-main n (c # cs) = (if c = 0 then show-poly-main (Suc n) cs else show-coeff-factor c n \@ "" @ show-poly-main (Suc n) cs)

definition show-poly :: 'a :: {zero,one,show}poly \Rightarrow string where
  show-poly p = show-poly-main 0 (coeffs p)

definition showsp-poly :: 'a :: {zero,one,show}poly showsp where
  showsp-poly p x = shows-string (show-poly x)

instantiation poly :: (\{show,one,zero\}) show
begin

definition shows-prec p (x :: 'a poly) = showsp-poly p x
definition shows-list (ps :: 'a poly list) = showsp-list shows-prec 0 ps

lemma show-law-poly [show-law-simps]:
  shows-prec p (a :: 'a poly) (r \@ s) = shows-prec p a r \@ s
by (simp add: shows-prec-poly-def showsp-poly-def show-law-simps)

instance by standard (auto simp: shows-list-poly-def show-law-simps)

end

end

3 Show for Real Numbers – Interface

We just demand that there is some function from reals to string and register this as show-function. Implementations are available in one of the theories Show-Real-Impl and ..\Algebraic-Numbers\Show-Real-.....

theory Show-Real
imports
  HOL.Real
  Show
begin

consts show-real :: real \Rightarrow string

definition showsp-real :: real showsp where
  showsp-real p x y = (show-real x \@ y)

lemma show-law-real [show-law-intros]:
  show-law showsp-real r
by (rule show-lawI) (simp add: showsp-real-def show-law-simps)
lemma showsp-real-append [show-law-simps]:
  showsp-real p r (x @ y) = showsp-real p r x @ y
by (intro show-lawD show-law-intros)

local-setup ⟨
  Show-Generator.register-foreign-showsp @{typ real} @{term showsp-real} @{thm show-law-real}
⟩

derive show real
end

4 Show for Complex Numbers

We print complex numbers as real and imaginary parts. Note that by
transitivity, this theory demands that an implementations for show-real is
available, e.g., by using one of the theories Show-Real-Impl or ../Algebraic-
Numbers/Show-Real-....

theory Show-Complex
imports
  HOL.Complex
  Show-Real
begin

definition show-complex x = (let r = Re x; i = Im x in
  if (i = 0) then show-real r else if
  r = 0 then show-real i @ "i" else
  "(" @ show-real r @ "+" @ show-real i @ "i")")

definition showsp-complex :: complex showsp
where
  showsp-complex p x y = (show-complex x @ y)

lemma show-law-complex [show-law-intros]:
  show-law showsp-complex r
by (rule show-lawI) (simp add: showsp-complex-def show-law-simps)

lemma showsp-complex-append [show-law-simps]:
  showsp-complex p r (x @ y) = showsp-complex p r x @ y
by (intro show-lawD show-law-intros)

local-setup ⟨
  Show-Generator.register-foreign-showsp @{typ complex} @{term showsp-complex} @{thm show-law-complex}
⟩
}
derive show complex
end

5 Show Implementation for Real Numbers via Rational Numbers

We just provide an implementation for show of real numbers where we assume that real numbers are implemented via rational numbers.

theory Show-Real-Impl
imports Show-Real Show-Instances
begin

We now define show-real.

overloading show-real ≡ show-real
begin
definition show-real
where show-real x ≡
(if (∃ y. x = Ratreal y) then show (THE y. x = Ratreal y) else "Irrational")
end

lemma show-real-code[code]: show-real (Ratreal x) = show x
unfolding show-real-def by auto
end

References