A formal model for the SPARCv8 ISA and a proof of non-interference for the LEON3 processor

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Abstract

We formalise the SPARCv8 instruction set architecture (ISA) which is used in processors such as LEON3. Our formalisation can be specialised to any SPARCv8 CPU, here we use LEON3 as a running example. Our model covers the operational semantics for all the instructions in the integer unit of the SPARCv8 architecture and it supports Isabelle code export, which effectively turns the Isabelle model into a SPARCv8 CPU simulator. We prove the language-based non-interference property for the LEON3 processor.

Our model is based on deterministic monad, which is a modified version of the non-deterministic monad from NICTA/l4v. We also use the Word library developed by Jeremy Dawson and Gerwin Klein.

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1 SPARC V8 architecture CPU model

theory *Sparc-Types*
imports *Main ../lib/ WordDecl Word-Lib.Bit-Shifts-Infix-Syntax*
begin

The following type definitions are taken from David Sanan's definitions for SPARC machines.

type-synonym *machine-word = word32*
type-synonym *byte = word8*
type-synonym *phys-address = word36*

type-synonym *virtua-address = word32*
type-synonym *page-address = word24*
type-synonym *offset = word12*

type-synonym *table-entry = word8*

definition *page-size* :: *word32* **where** *page-size* ≡ *4096*

type-synonym *virtua-page-address = word20*
type-synonym *context-type = word8*

type-synonym *word-length-t1 = word-length8*
type-synonym *word-length-t2 = word-length6*
type-synonym *word-length-t3 = word-length6*
type-synonym *word-length-offset = word-length12*
type-synonym *word-length-page = word-length24*
type-synonym *word-length-phys-address = word-length36*
type-synonym *word-length-virtua-address = word-length32*
type-synonym *word-length-entry-type = word-length2*
type-synonym *word-length-machine-word = word-length32*

definition *length-machine-word* :: *nat*
where *length-machine-word* ≡ *LENGTH*(*word-length-machine-word*)

2 CPU Register Definitions

The definitions below come from the SPARC Architecture Manual, Version 8. The LEON3 processor has been certified SPARC V8 conformant (2005).

definition *leon3khz* ::*word32*
where
leon3khz ≡ *33000*

The following type definitions for MMU is taken from David Sanan's definitions for MMU.

The definitions below come from the UT699 LEON 3FT/SPARC V8 Microprocessor Functional Manual, Aeroflex, June 20, 2012, p35.

datatype *MMU-register*
= *CR* — Control Register
| *CTP* — ConText Pointer register
| *CNR* — Context Register
| *FTSR* — Fault Status Register
| *FAR* — Fault Address Register

lemma *MMU-register-induct*:
P CR ⟹ *P CTP* ⟹ *P CNR* ⟹ *P FTSR* ⟹ *P FAR*
⟹ *P x*
⟨*proof*⟩

lemma *UNIV-MMU-register* [*no-atp*]: *UNIV* = {*CR*, *CTP*, *CNR*, *FTSR*, *FAR*}
⟨*proof*⟩

instantiation *MMU-register* :: *enum* **begin**

definition *enum-MMU-register* = [*CR*, *CTP*, *CNR*, *FTSR*, *FAR*]

definition
enum-all-MMU-register P ⟷ *P CR* ∧ *P CTP* ∧ *P CNR* ∧ *P FTSR* ∧ *P FAR*
definition
enum-ex-MMU-register P ⟷ *P CR* ∨ *P CTP* ∨ *P CNR* ∨ *P FTSR* ∨ *P FAR*

instance ⟨*proof*⟩
end
type-synonym *MMU-context* = *MMU-register* ⇒ *machine-word*

PTE-flags is the last 8 bits of a PTE. See page 242 of SPARCv8 manual.

- C - bit 7
- M - bit 6,
- R - bit 5

- ACC - bit 4 2
- ET - bit 1 0.

type-synonym *PTE-flags* = *word8*

CPU-register datatype is an enumeration with the CPU registers defined in the SPARC V8 architecture.

datatype *CPU-register* =
  PSR — Processor State Register
| *WIM* — Window Invalid Mask
| *TBR* — Trap Base Register
| *Y* — Multiply/Divide Register
| *PC* — Program Counter
| *nPC* — next Program Counter
| *DTQ* — Deferred-Trap Queue
| *FSR* — Floating-Point State Register
| *FQ* — Floating-Point Deferred-Trap Queue
| *CSR* — Coprocessor State Register
| *CQ* — Coprocessor Deferred-Trap Queue

| *ASR word5* — Ancillary State Register

The following two functions are dummies since we will not use ASRs. Future formalisation may add more details to this.

context
  includes *bit-operations-syntax*
begin

definition *privileged-ASR* :: *word5* ⇒ *bool*
where
privileged-ASR r ≡ *False*

definition *illegal-instruction-ASR* :: *word5* ⇒ *bool*
where
illegal-instruction-ASR r ≡ *False*

definition *get-tt* :: *word32* ⇒ *word8*
where
get-tt tbr ≡
  ucast (((*AND*) *tbr 0b00000000000000000000111111110000*) >> *4*)

Write the tt field of the TBR register. Return the new value of TBR.

definition *write-tt* :: *word8* ⇒ *word32* ⇒ *word32*
where

```
write-tt new-tt-val tbr-val ≡
  let tmp = (AND) tbr-val 0b11111111111111111111000000001111 in
      (OR) tmp (((ucast new-tt-val)::word32) << 4)
```

Get the nth bit of WIM. This equals ((AND) WIM 2^n). N.B. the first bit of WIM is the 0th bit.

```
definition get-WIM-bit :: nat ⇒ word32 ⇒ word1
where
get-WIM-bit n wim ≡
  let mask = ((ucast (0b1::word1))::word32) << n in
  ucast (((AND) mask wim) >> n)


definition get-CWP :: word32 ⇒ word5
where
get-CWP psr ≡
  ucast ((AND) psr 0b00000000000000000000000000011111)


definition get-ET :: word32 ⇒ word1
where
get-ET psr ≡
  ucast (((AND) psr 0b00000000000000000000000000100000) >> 5)


definition get-PIL :: word32 ⇒ word4
where
get-PIL psr ≡
  ucast (((AND) psr 0b00000000000000000000111100000000) >> 8)


definition get-PS :: word32 ⇒ word1
where
get-PS psr ≡
  ucast (((AND) psr 0b00000000000000000000000001000000) >> 6)


definition get-S :: word32 ⇒ word1
where
get-S psr ≡
  ucast (((AND) psr 0b00000000000000000000000010000000) >> 7)
  if ((AND) psr (0b00000000000000000000000010000000::word32)) = 0 then 0
  else 1


definition get-icc-N :: word32 ⇒ word1
where
get-icc-N psr ≡
```

```
  ucast (((AND) psr 0b00000000100000000000000000000000) >> 23)


definition get-icc-Z :: word32 ⇒ word1
where
get-icc-Z psr ≡
  ucast (((AND) psr 0b00000000010000000000000000000000) >> 22)


definition get-icc-V :: word32 ⇒ word1
where
get-icc-V psr ≡
  ucast (((AND) psr 0b00000000001000000000000000000000) >> 21)


definition get-icc-C :: word32 ⇒ word1
where
get-icc-C psr ≡
  ucast (((AND) psr 0b00000000000100000000000000000000) >> 20)


definition update-S :: word1 ⇒ word32 ⇒ word32
where
update-S s-val psr-val ≡
  let tmp0 = (AND) psr-val 0b11111111111111111111111011111111 in
  (OR) tmp0 (((ucast s-val)::word32) << 7)
```

Update the CWP field of PSR. Return the new value of PSR.

```
definition update-CWP :: word5 ⇒ word32 ⇒ word32
where
update-CWP cwp-val psr-val ≡
  let tmp0 = (AND) psr-val (0b11111111111111111111111100000::word32);
      s-val = ((ucast (get-S psr-val))::word1)
  in
  if s-val = 0 then
   (AND) ((OR) tmp0 ((ucast cwp-val)::word32)) (0b11111111111111111111111101111111::word32)
  else
   (OR) ((OR) tmp0 ((ucast cwp-val)::word32)) (0b00000000000000000000000010000000::word32)
```

Update the the ET, CWP, and S fields of PSR. Return the new value of PSR.

```
definition update-PSR-rett :: word5 ⇒ word1 ⇒ word1 ⇒ word32 ⇒ word32
where
update-PSR-rett cwp-val et-val s-val psr-val ≡
  let tmp0 = (AND) psr-val 0b11111111111111111111111101000000;
      tmp1 = (OR) tmp0 ((ucast cwp-val)::word32);
      tmp2 = (OR) tmp1 (((ucast et-val)::word32) << 5);
```

```
    tmp3 = (OR) tmp2 (((ucast s-val)::word32) << 7)
  in
  tmp3
```

```
definition update-PSR-exe-trap :: word5 ⇒ word1 ⇒ word1 ⇒ word32 ⇒ word32
where
update-PSR-exe-trap cwp-val et-val ps-val psr-val ≡
  let tmp0 = (AND) psr-val 0b11111111111111111111111110000000;
      tmp1 = (OR) tmp0 ((ucast cwp-val)::word32);
      tmp2 = (OR) tmp1 (((ucast et-val)::word32) << 5);
      tmp3 = (OR) tmp2 (((ucast ps-val)::word32) << 6)
  in
  tmp3
```

Update the N, Z, V, C fields of PSR. Return the new value of PSR.

```
definition update-PSR-icc :: word1 ⇒ word1 ⇒ word1 ⇒ word1 ⇒ word32 ⇒
word32
where
update-PSR-icc n-val z-val v-val c-val psr-val ≡
  let
      n-val-32 = if n-val = 0 then 0
                 else      (0b00000000100000000000000000000000::word32);
      z-val-32 = if z-val = 0 then 0
                 else      (0b00000000010000000000000000000000::word32);
      v-val-32 = if v-val = 0 then 0
                 else      (0b00000000001000000000000000000000::word32);
      c-val-32 = if c-val = 0 then 0
                 else      (0b00000000000100000000000000000000::word32);
      tmp0 = (AND) psr-val (0b11111111100001111111111111111111::word32);
      tmp1 = (OR) tmp0 n-val-32;
      tmp2 = (OR) tmp1 z-val-32;
      tmp3 = (OR) tmp2 v-val-32;
      tmp4 = (OR) tmp3 c-val-32
  in
  tmp4
```

Update the ET, PIL fields of PSR. Return the new value of PSR.

```
definition update-PSR-et-pil :: word1 ⇒ word4 ⇒ word32 ⇒ word32
where
update-PSR-et-pil et pil psr-val ≡
  let tmp0 = (AND) psr-val 0b11111111111111111111000011011111;
      tmp1 = (OR) tmp0 (((ucast et)::word32) << 5);
      tmp2 = (OR) tmp1 (((ucast pil)::word32) << 8)
  in
  tmp2
```

end

SPARC V8 architecture is organized in windows of 32 user registers. The data stored in a register is defined as a 32 bits word *reg-type*:

type-synonym *reg-type* = *word32*

The access to the value of a CPU register of type *CPU-register* is defined by a total function *cpu-context*

type-synonym *cpu-context* = *CPU-register* ⇒ *reg-type*

User registers are defined with the type *user-reg* represented by a 5 bits word.

type-synonym *user-reg-type* = *word5*

definition *PSR-S* ::*reg-type*
where *PSR-S* ≡ *6*

Each window context is defined by a total function *window-context* from *user-register* to *reg-type* (32 bits word storing the actual value of the register).

type-synonym *window-context* = *user-reg-type* ⇒ *reg-type*

The number of windows is implementation dependent. The LEON architecture is composed of 16 different windows (a 4 bits word).

definition *NWINDOWS* :: *int*
where *NWINDOWS* ≡ *8*

Maximum number of windows is 32 in SPARCv8.

type-synonym ('*a*) *window-size* = '*a word*

Finally the user context is defined by another total function *user-context* from *window-size* to *window-context*. That is, the user context is a function taking as argument a register set window and a register within that window, and it returns the value stored in that user register.

type-synonym ('*a*) *user-context* = ('*a*) *window-size* ⇒ *window-context*

datatype *sys-reg* =
CCR — Cache control register
|*ICCR* — Instruction cache configuration register
|*DCCR* — Data cache configuration register

type-synonym *sys-context* = *sys-reg* ⇒ *reg-type*

The memory model is defined by a total function from 32 bits words to 8 bits words

type-synonym *asi-type* = *word8*

The memory is defined as a function from page address to page, which is also defined as a function from physical address to *machine-word*

type-synonym *mem-val-type* = *word8*
type-synonym *mem-context* = *asi-type* ⇒ *phys-address* ⇒ *mem-val-type option*

type-synonym *cache-tag* = *word20*
type-synonym *cache-line-size* = *word12*
type-synonym *cache-type* = (*cache-tag* × *cache-line-size*)
type-synonym *cache-context* = *cache-type* ⇒ *mem-val-type option*

The delayed-write pool generated from write state register instructions.

type-synonym *delayed-write-pool* = (*int* × *reg-type* × *CPU-register*) *list*

definition *DELAYNUM* :: *int*
where *DELAYNUM* ≡ *0*

Convert a set to a list.

definition *list-of-set* :: *'a set* ⇒ *'a list*
  where *list-of-set s* = (*SOME l*. *set l* = *s*)

lemma *set-list-of-set*: *finite s* ⟹ *set* (*list-of-set s*) = *s*
⟨*proof*⟩

type-synonym *ANNUL* = *bool*
type-synonym *RESET-TRAP* = *bool*
type-synonym *EXECUTE-MODE* = *bool*
type-synonym *RESET-MODE* = *bool*
type-synonym *ERROR-MODE* = *bool*
type-synonym *TICC-TRAP-TYPE* = *word7*
type-synonym *INTERRUPT-LEVEL* = *word3*
type-synonym *STORE-BARRIER-PENDING* = *bool*

The processor asserts this signal to ensure that the memory system will not process another SWAP or LDSTUB operation to the same memory byte.

type-synonym *pb-block-ldst-byte* = *virtua-address* ⇒ *bool*

The processor asserts this signal to ensure that the memory system will not process another SWAP or LDSTUB operation to the same memory word.

type-synonym *pb-block-ldst-word* = *virtua-address* ⇒ *bool*

record *sparc-state-var* =
annul:: *ANNUL*
resett:: *RESET-TRAP*
exe:: *EXECUTE-MODE*
reset:: *RESET-MODE*
err:: *ERROR-MODE*

ticc:: *TICC-TRAP-TYPE*
itrpt-lvl:: *INTERRUPT-LEVEL*
st-bar:: *STORE-BARRIER-PENDING*
atm-ldst-byte:: *pb-block-ldst-byte*
atm-ldst-word:: *pb-block-ldst-word*

definition *get-annul* :: *sparc-state-var* ⇒ *bool*
where *get-annul v* ≡ *annul v*

definition *get-reset-trap* :: *sparc-state-var* ⇒ *bool*
where *get-reset-trap v* ≡ *resett v*

definition *get-exe-mode* :: *sparc-state-var* ⇒ *bool*
where *get-exe-mode v* ≡ *exe v*

definition *get-reset-mode* :: *sparc-state-var* ⇒ *bool*
where *get-reset-mode v* ≡ *reset v*

definition *get-err-mode* :: *sparc-state-var* ⇒ *bool*
where *get-err-mode v* ≡ *err v*

definition *get-ticc-trap-type* :: *sparc-state-var* ⇒ *word7*
where *get-ticc-trap-type v* ≡ *ticc v*

definition *get-interrupt-level* :: *sparc-state-var* ⇒ *word3*
where *get-interrupt-level v* ≡ *itrpt-lvl v*

definition *get-store-barrier-pending* :: *sparc-state-var* ⇒ *bool*
where *get-store-barrier-pending v* ≡ *st-bar v*

definition *write-annul* :: *bool* ⇒ *sparc-state-var* ⇒ *sparc-state-var*
where *write-annul b v* ≡ *v*(|*annul* := *b*|)

definition *write-reset-trap* :: *bool* ⇒ *sparc-state-var* ⇒ *sparc-state-var*
where *write-reset-trap b v* ≡ *v*(|*resett* := *b*|)

definition *write-exe-mode* :: *bool* ⇒ *sparc-state-var* ⇒ *sparc-state-var*
where *write-exe-mode b v* ≡ *v*(|*exe* := *b*|)

definition *write-reset-mode* :: *bool* ⇒ *sparc-state-var* ⇒ *sparc-state-var*
where *write-reset-mode b v* ≡ *v*(|*reset* := *b*|)

definition *write-err-mode* :: *bool* ⇒ *sparc-state-var* ⇒ *sparc-state-var*
where *write-err-mode b v* ≡ *v*(|*err* := *b*|)

definition *write-ticc-trap-type* :: *word7* ⇒ *sparc-state-var* ⇒ *sparc-state-var*
where *write-ticc-trap-type w v* ≡ *v*(|*ticc* := *w*|)

definition *write-interrupt-level* :: *word3* ⇒ *sparc-state-var* ⇒ *sparc-state-var*

where *write-interrupt-level w v* ≡ *v*(|*itrpt-lvl* := *w*|)

definition *write-store-barrier-pending* :: *bool* ⇒ *sparc-state-var* ⇒ *sparc-state-var*
where *write-store-barrier-pending b v* ≡ *v*(|*st-bar* := *b*|)

context
  includes *bit-operations-syntax*
begin

Given a word7 value, find the highest bit, and fill the left bits to be the highest bit.

definition *sign-ext7*::*word7* ⇒ *word32*
where
sign-ext7 w ≡
  let highest-bit = ((*AND*) *w 0b1000000*) >> *6 in*
  if highest-bit = *0 then*
    (*ucast w*)::*word32*
  else (*OR*) ((*ucast w*)::*word32*) *0b11111111111111111111111110000000*

definition *zero-ext8* :: *word8* ⇒ *word32*
where
zero-ext8 w ≡ (*ucast w*)::*word32*

Given a word8 value, find the highest bit, and fill the left bits to be the highest bit.

definition *sign-ext8*::*word8* ⇒ *word32*
where
sign-ext8 w ≡
  let highest-bit = ((*AND*) *w 0b10000000*) >> *7 in*
  if highest-bit = *0 then*
    (*ucast w*)::*word32*
  else (*OR*) ((*ucast w*)::*word32*) *0b11111111111111111111111100000000*

Given a word13 value, find the highest bit, and fill the left bits to be the highest bit.

definition *sign-ext13*::*word13* ⇒ *word32*
where
sign-ext13 w ≡
  let highest-bit = ((*AND*) *w 0b1000000000000*) >> *12 in*
  if highest-bit = *0 then*
    (*ucast w*)::*word32*
  else (*OR*) ((*ucast w*)::*word32*) *0b11111111111111111110000000000000*

definition *zero-ext16* :: *word16* ⇒ *word32*

where
zero-ext16 w ≡ (*ucast w*)::*word32*

Given a word16 value, find the highest bit, and fill the left bits to be the highest bit.

definition *sign-ext16*::*word16* ⇒ *word32*
where
sign-ext16 w ≡
  let highest-bit = ((*AND*) *w 0b1000000000000000*) >> *15 in*
  if highest-bit = *0 then*
    (*ucast w*)::*word32*
  else (*OR*) ((*ucast w*)::*word32*) *0b11111111111111110000000000000000*

Given a word22 value, find the highest bit, and fill the left bits to tbe the highest bit.

definition *sign-ext22*::*word22* ⇒ *word32*
where
sign-ext22 w ≡
  let highest-bit = ((*AND*) *w 0b1000000000000000000000*) >> *21 in*
  if highest-bit = *0 then*
    (*ucast w*)::*word32*
  else (*OR*) ((*ucast w*)::*word32*) *0b11111111110000000000000000000000*

Given a word24 value, find the highest bit, and fill the left bits to tbe the highest bit.

definition *sign-ext24*::*word24* ⇒ *word32*
where
sign-ext24 w ≡
  let highest-bit = ((*AND*) *w 0b100000000000000000000000*) >> *23 in*
  if highest-bit = *0 then*
    (*ucast w*)::*word32*
  else (*OR*) ((*ucast w*)::*word32*) *0b11111111000000000000000000000000*

Operations to be defined. The SPARC V8 architecture is composed of the following set of instructions:

- Load Integer Instructions
- Load Floating-point Instructions
- Load Coprocessor Instructions
- Store Integer Instructions
- Store Floating-point Instructions

- Store Coprocessor Instructions
- Atomic Load-Store Unsigned Byte Instructions
- SWAP Register With Memory Instruction
- SETHI Instructions
- NOP Instruction
- Logical Instructions
- Shift Instructions
- Add Instructions
- Tagged Add Instructions
- Subtract Instructions
- Tagged Subtract Instructions
- Multiply Step Instruction
- Multiply Instructions
- Divide Instructions
- SAVE and RESTORE Instructions
- Branch on Integer Condition Codes Instructions
- Branch on Floating-point Condition Codes Instructions
- Branch on Coprocessor Condition Codes Instructions
- Call and Link Instruction
- Jump and Link Instruction
- Return from Trap Instruction
- Trap on Integer Condition Codes Instructions
- Read State Register Instructions
- Write State Register Instructions
- STBAR Instruction
- Unimplemented Instruction
- Flush Instruction Memory

- Floating-point Operate (FPop) Instructions
- Convert Integer to Floating point Instructions
- Convert Floating point to Integer Instructions
- Convert Between Floating-point Formats Instructions
- Floating-point Move Instructions
- Floating-point Square Root Instructions
- Floating-point Add and Subtract Instructions
- Floating-point Multiply and Divide Instructions
- Floating-point Compare Instructions
- Coprocessor Operate Instructions

The CALL instruction.

datatype *call-type* = *CALL* — Call and Link

The SETHI instruction.

datatype *sethi-type* = *SETHI* — Set High 22 bits of r Register

The NOP instruction.

datatype *nop-type* = *NOP* — No Operation

The Branch on integer condition codes instructions.

datatype *bicc-type* =
BE — Branch on Equal
| *BNE* — Branch on Not Equal
| *BGU* — Branch on Greater Unsigned
| *BLE* — Branch on Less or Equal
| *BL* — Branch on Less
| *BGE* — Branch on Greater or Equal
| *BNEG* — Branch on Negative
| *BG* — Branch on Greater
| *BCS* — Branch on Carry Set (Less than, Unsigned)
| *BLEU* — Branch on Less or Equal Unsigned
| *BCC* — Branch on Carry Clear (Greater than or Equal, Unsigned)
| *BA* — Branch Always
| *BN* — Branch Never — Added for unconditional branches
| *BPOS* — Branch on Positive
| *BVC* — Branch on Overflow Clear
| *BVS* — Branch on Overflow Set

Memory instructions. That is, load and store.

datatype *load-store-type* =
  LDSB — Load Signed Byte
| *LDUB* — Load Unsigned Byte
| *LDUBA* — Load Unsigned Byte from Alternate space
| *LDUH* — Load Unsigned Halfword
| *LD* — Load Word
| *LDA* — Load Word from Alternate space
| *LDD* — Load Doubleword
| *STB* — Store Byte
| *STH* — Store Halfword
| *ST* — Store Word
| *STA* — Store Word into Alternate space
| *STD* — Store Doubleword
| *LDSBA* — Load Signed Byte from Alternate space
| *LDSH* — Load Signed Halfword
| *LDSHA* — Load Signed Halfword from Alternate space
| *LDUHA* — Load Unsigned Halfword from Alternate space
| *LDDA* — Load Doubleword from Alternate space
| *STBA* — Store Byte into Alternate space
| *STHA* — Store Halfword into Alternate space
| *STDA* — Store Doubleword into Alternate space
| *LDSTUB* — Atomic Load Store Unsigned Byte
| *LDSTUBA* — Atomic Load Store Unsinged Byte in Alternate space
| *SWAP* — Swap r Register with Mmemory
| *SWAPA* — Swap r Register with Mmemory in Alternate space
| *FLUSH* — Flush Instruction Memory
| *STBAR* — Store Barrier

Arithmetic instructions.

datatype *arith-type* =
  ADD — Add
| *ADDcc* — Add and modify icc
| *ADDX* — Add with Carry
| *SUB* — Subtract
| *SUBcc* — Subtract and modify icc
| *SUBX* — Subtract with Carry
| *UMUL* — Unsigned Integer Multiply
| *SMUL* — Signed Integer Multiply
| *SMULcc* — Signed Integer Multiply and modify icc
| *UDIV* — Unsigned Integer Divide
| *UDIVcc* — Unsigned Integer Divide and modify icc
| *SDIV* — Signed Integer Divide
| *ADDXcc* — Add with Carry and modify icc
| *TADDcc* — Tagged Add and modify icc
| *TADDccTV* — Tagged Add and modify icc and Trap on overflow
| *SUBXcc* — Subtract with Carry and modify icc
| *TSUBcc* — Tagged Subtract and modify icc
| *TSUBccTV* — Tagged Subtract and modify icc and Trap on overflow
| *MULScc* — Multiply Step and modify icc

| *UMULcc* — Unsigned Integer Multiply and modify icc
| *SDIVcc* — Signed Integer Divide and modify icc

Logical instructions.

datatype *logic-type* =
ANDs — And
| *ANDcc* — And and modify icc
| *ANDN* — And Not
| *ANDNcc* — And Not and modify icc
| *ORs* — Inclusive-Or
| *ORcc* — Inclusive-Or and modify icc
| *ORN* — Inclusive Or Not
| *XORs* — Exclusive-Or
| *XNOR* — Exclusive-Nor
| *ORNcc* — Inclusive-Or Not and modify icc
| *XORcc* — Exclusive-Or and modify icc
| *XNORcc* — Exclusive-Nor and modify icc

Shift instructions.

datatype *shift-type* =
SLL — Shift Left Logical
| *SRL* — Shift Right Logical
| *SRA* — Shift Right Arithmetic

Other Control-transfer instructions.

datatype *ctrl-type* =
JMPL — Jump and Link
| *RETT* — Return from Trap
| *SAVE* — Save caller's window
| *RESTORE* — Restore caller's window

Access state registers instructions.

datatype *sreg-type* =
RDASR — Read Ancillary State Register
| *RDY* — Read Y Register
| *RDPSR* — Read Processor State Register
| *RDWIM* — Read Window Invalid Mask Register
| *RDTBR* — Read Trap Base Regiser
| *WRASR* — Write Ancillary State Register
| *WRY* — Write Y Register
| *WRPSR* — Write Processor State Register
| *WRWIM* — Write Window Invalid Mask Register
| *WRTBR* — Write Trap Base Register

Unimplemented instruction.

datatype *uimp-type* = *UNIMP* — Unimplemented

Trap on integer condition code instructions.

datatype *ticc-type* =
TA — Trap Always
| *TN* — Trap Never
| *TNE* — Trap on Not Equal
| *TE* — Trap on Equal
| *TG* — Trap on Greater
| *TLE* — Trap on Less or Equal
| *TGE* — Trap on Greater or Equal
| *TL* — Trap on Less
| *TGU* — Trap on Greater Unsigned
| *TLEU* — Trap on Less or Equal Unsigned
| *TCC* — Trap on Carry Clear (Greater than or Equal, Unsigned)
| *TCS* — Trap on Carry Set (Less Than, Unsigned)
| *TPOS* — Trap on Postive
| *TNEG* — Trap on Negative
| *TVC* — Trap on Overflow Clear
| *TVS* — Trap on Overflow Set

datatype *sparc-operation* =
call-type call-type
| *sethi-type sethi-type*
| *nop-type nop-type*
| *bicc-type bicc-type*
| *load-store-type load-store-type*
| *arith-type arith-type*
| *logic-type logic-type*
| *shift-type shift-type*
| *ctrl-type ctrl-type*
| *sreg-type sreg-type*
| *uimp-type uimp-type*
| *ticc-type ticc-type*

datatype *Trap* =
reset
|*data-store-error*
|*instruction-access-MMU-miss*
|*instruction-access-error*
|*r-register-access-error*
|*instruction-access-exception*
|*privileged-instruction*
|*illegal-instruction*
|*unimplemented-FLUSH*
|*watchpoint-detected*
|*fp-disabled*
|*cp-disabled*
|*window-overflow*
|*window-underflow*
|*mem-address-not-aligned*
|*fp-exception*

|*cp-exception*
|*data-access-error*
|*data-access-MMU-miss*
|*data-access-exception*
|*tag-overflow*
|*division-by-zero*
|*trap-instruction*
|*interrupt-level-n*

datatype *Exception* =
— The following are processor states that are not in the instruction model,
— but we MAY want to deal with these from hardware perspective.
~~*execute-mode*~~
~~*reset-mode*~~
~~*error-mode*~~
— The following are self-defined exceptions.
invalid-cond-f2
|*invalid-op2-f2*
|*illegal-instruction2* — when $i = 0$ for load/store not from alternate space
|*invalid-op3-f3-op11*
|*case-impossible*
|*invalid-op3-f3-op10*
|*invalid-op-f3*
|*unsupported-instruction*
|*fetch-instruction-error*
|*invalid-trap-cond*

end

end

theory *Lib*
imports *Main*
begin

lemma *hd-map-simp*:
$b \neq [] \Longrightarrow hd\ (map\ a\ b) = a\ (hd\ b)$
⟨*proof*⟩

lemma *tl-map-simp*:
$tl\ (map\ a\ b) = map\ a\ (tl\ b)$
⟨*proof*⟩

lemma *Collect-eq*:

$\{x.\ P\ x\} = \{x.\ Q\ x\} \longleftrightarrow (\forall x.\ P\ x = Q\ x)$
⟨*proof*⟩

lemma *iff-impI*: $[\![P \Longrightarrow Q = R]\!] \Longrightarrow (P \longrightarrow Q) = (P \longrightarrow R)$ ⟨*proof*⟩

definition
fun-app :: $('a \Rightarrow 'b) \Rightarrow 'a \Rightarrow 'b$ (**infixr** $ *10*) **where**
$f\ \$\ x \equiv f\ x$

declare *fun-app-def* [*iff*]

lemma *fun-app-cong*[*fundef-cong*]:
$[\![\ f\ x = f'\ x'\ ]\!] \Longrightarrow (f\ \$\ x) = (f'\ \$\ x')$
⟨*proof*⟩

lemma *fun-app-apply-cong*[*fundef-cong*]:
$f\ x\ y = f'\ x'\ y' \Longrightarrow (f\ \$\ x)\ y = (f'\ \$\ x')\ y'$
⟨*proof*⟩

lemma *if-apply-cong*[*fundef-cong*]:
$[\![\ P = P';\ x = x';\ P' \Longrightarrow f\ x' = f'\ x';\ \neg\ P' \Longrightarrow g\ x' = g'\ x'\ ]\!]$
$\Longrightarrow (\mathit{if}\ P\ \mathit{then}\ f\ \mathit{else}\ g)\ x = (\mathit{if}\ P'\ \mathit{then}\ f'\ \mathit{else}\ g')\ x'$
⟨*proof*⟩

abbreviation (*input*) *split* :: $('a \Rightarrow 'b \Rightarrow 'c) \Rightarrow 'a \times 'b \Rightarrow 'c$ **where**
split ≡ *case-prod*

lemma *split-apply-cong*[*fundef-cong*]:
$[\![\ f\ (\mathit{fst}\ p)\ (\mathit{snd}\ p)\ s = f'\ (\mathit{fst}\ p')\ (\mathit{snd}\ p')\ s'\ ]\!] \Longrightarrow \mathit{split}\ f\ p\ s = \mathit{split}\ f'\ p'\ s'$
⟨*proof*⟩

definition
pred-conj :: $('a \Rightarrow \mathit{bool}) \Rightarrow ('a \Rightarrow \mathit{bool}) \Rightarrow ('a \Rightarrow \mathit{bool})$ (**infixl** *and 35*)
where
pred-conj $P\ Q \equiv \lambda x.\ P\ x \wedge Q\ x$

definition
pred-disj :: $('a \Rightarrow \mathit{bool}) \Rightarrow ('a \Rightarrow \mathit{bool}) \Rightarrow ('a \Rightarrow \mathit{bool})$ (**infixl** *or 30*)
where
pred-disj $P\ Q \equiv \lambda x.\ P\ x \vee Q\ x$

definition
pred-neg :: $('a \Rightarrow \mathit{bool}) \Rightarrow ('a \Rightarrow \mathit{bool})$ (*not* - [*40*] *40*)
where
pred-neg $P \equiv \lambda x.\ \neg\ P\ x$

definition $K \equiv \lambda x\ y.\ x$

definition

zipWith :: $('a \Rightarrow 'b \Rightarrow 'c) \Rightarrow 'a\ list \Rightarrow 'b\ list \Rightarrow 'c\ list$ **where**
zipWith f xs ys $\equiv$ *map* (*split f*) (*zip xs ys*)

primrec
delete :: $'a \Rightarrow 'a\ list \Rightarrow 'a\ list$
where
delete y [] = []
| *delete y* (*x*#*xs*) = (*if y*=*x then xs else x* # *delete y xs*)

primrec
find :: $('a \Rightarrow bool) \Rightarrow 'a\ list \Rightarrow 'a\ option$
where
find f [] = *None*
| *find f* (*x* # *xs*) = (*if f x then Some x else find f xs*)

definition
swp f $\equiv \lambda x\ y.\ f\ y\ x$

primrec (*nonexhaustive*)
theRight :: $'a + 'b \Rightarrow 'b$ **where**
theRight (*Inr x*) = *x*

primrec (*nonexhaustive*)
theLeft :: $'a + 'b \Rightarrow 'a$ **where**
theLeft (*Inl x*) = *x*

definition
isLeft x $\equiv (\exists\, y.\ x = Inl\ y)$

definition
isRight x $\equiv (\exists\, y.\ x = Inr\ y)$

definition
const x $\equiv \lambda y.\ x$

lemma *tranclD2*:
$(x,\ y) \in R^+ \Longrightarrow \exists\, z.\ (x,\ z) \in R^* \wedge (z,\ y) \in R$
⟨*proof*⟩

lemma *linorder-min-same1* [*simp*]:
(*min y x* = *y*) = (*y* $\leq$ (*x*::'*a*::*linorder*))
⟨*proof*⟩

lemma *linorder-min-same2* [*simp*]:
(*min x y* = *y*) = (*y* $\leq$ (*x*::'*a*::*linorder*))
⟨*proof*⟩

A combinator for pairing up well-formed relations. The divisor function splits the population in halves, with the True half greater than the False

half, and the supplied relations control the order within the halves.

definition
  wf-sum :: $('a \Rightarrow bool) \Rightarrow ('a \times 'a)\ set \Rightarrow ('a \times 'a)\ set \Rightarrow ('a \times 'a)\ set$
where
  wf-sum divisor r r' ≡
    $(\{(x,\ y).\ \neg\ divisor\ x \wedge \neg\ divisor\ y\} \cap r')$
  ∪ $\{(x,\ y).\ \neg\ divisor\ x \wedge divisor\ y\}$
  ∪ $(\{(x,\ y).\ divisor\ x \wedge divisor\ y\} \cap r)$

lemma *wf-sum-wf*:
  ⟦ *wf r*; *wf r'* ⟧ ⟹ *wf* (*wf-sum divisor r r'*)
  ⟨*proof*⟩

abbreviation(*input*)
 option-map == *map-option*

lemmas *option-map-def* = *map-option-case*

lemma *False-implies-equals* [*simp*]:
  ((*False* ⟹ *P*) ⟹ *PROP Q*) ≡ *PROP Q*
  ⟨*proof*⟩

lemma *split-paired-Ball*:
  $(\forall x \in A.\ P\ x) = (\forall x\ y.\ (x,y) \in A \longrightarrow P\ (x,y))$
  ⟨*proof*⟩

lemma *split-paired-Bex*:
  $(\exists x \in A.\ P\ x) = (\exists x\ y.\ (x,y) \in A \wedge P\ (x,y))$
  ⟨*proof*⟩

end

theory *DetMonad*
imports *../Lib*
begin

State monads are used extensively in the seL4 specification. They are defined below.

3 The Monad

The basic type of the deterministic state monad with failure is very similar to the normal state monad. Instead of a pair consisting of result and new state, we return a pair coupled with a failure flag. The flag is *True* if the computation have failed. Conversely, if the flag is *False*, the computation resulting in the returned result have succeeded.

type-synonym $('s,'a)$ *det-monad* $= 's \Rightarrow ('a \times 's) \times bool$

The definition of fundamental monad functions *return* and *bind*. The monad function *return* x does not change the state, does not fail, and returns x.

definition
return :: $'a \Rightarrow ('s,'a)$ *det-monad* **where**
return $a \equiv \lambda s.\ ((a,s),False)$

The monad function *bind* f g, also written $f >>= g$, is the execution of f followed by the execution of g. The function g takes the result value *and* the result state of f as parameter. The definition says that the result of the combined operation is the result which is created by g applied to the result of f. The combined operation may have failed, if f may have failed or g may have failed on the result of f.

David Sanan and Zhe Hou: The original definition of bind is very inefficient when converted to executable code. Here we change it to a more efficient version for execution. The idea remains the same.

definition *h1 f s* $=$ *f s*
definition *h2 g fs* $=$ (*let* (*a*,*b*) $=$ *fst* (*fs*) *in g a b*)
definition *bind*:: $('s,\ 'a)$ *det-monad* $\Rightarrow$ $('a \Rightarrow ('s,\ 'b)$ *det-monad*$) \Rightarrow$
$('s,\ 'b)$ *det-monad* (**infixl** $>>= 60$)
where
bind f g $\equiv \lambda s.$ (
let fs $=$ *h1 f s*;
v $=$ *h2 g fs*
in
(*fst v*, (*snd v* $\vee$ *snd fs*)))

Sometimes it is convenient to write *bind* in reverse order.

abbreviation(*input*)
bind-rev :: $('c \Rightarrow ('a,\ 'b)$ *det-monad*$) \Rightarrow ('a,\ 'c)$ *det-monad* $\Rightarrow$
$('a,\ 'b)$ *det-monad* (**infixl** $=<< 60$) **where**
g $=<<$ *f* $\equiv$ *f* $>>=$ *g*

The basic accessor functions of the state monad. *get* returns the current state as result, does not fail, and does not change the state. *put s* returns nothing (*unit*), changes the current state to s and does not fail.

definition

get :: ('*s*,'*s*) *det-monad* **where**
get ≡ λ*s*. ((*s*,*s*), *False*)

definition
put :: '*s* ⇒ ('*s*, *unit*) *det-monad* **where**
put s ≡ λ-. (((),*s*), *False*)

3.1 Failure

The monad function that always fails. Returns the current state and sets the failure flag.

definition
fail :: '*a* ⇒ ('*s*, '*a*) *det-monad* **where**
fail a ≡ λ*s*. ((*a*,*s*), *True*)

Assertions: fail if the property P is not true

definition
assert :: *bool* ⇒ ('*a*, *unit*) *det-monad* **where**
assert P ≡ *if P then return* () *else fail* ()

An assertion that also can introspect the current state.

definition
state-assert :: ('*s* ⇒ *bool*) ⇒ ('*s*, *unit*) *det-monad*
where
state-assert P ≡ *get* >>= (λ*s*. *assert* (*P s*))

3.2 Generic functions on top of the state monad

Apply a function to the current state and return the result without changing the state.

definition
gets :: ('*s* ⇒ '*a*) ⇒ ('*s*, '*a*) *det-monad* **where**
gets f ≡ *get* >>= (λ*s*. *return* (*f s*))

Modify the current state using the function passed in.

definition
modify :: ('*s* ⇒ '*s*) ⇒ ('*s*, *unit*) *det-monad* **where**
modify f ≡ *get* >>= (λ*s*. *put* (*f s*))

lemma *simpler-gets-def*: *gets f* = (λ*s*. ((*f s*, *s*), *False*))
⟨*proof*⟩

lemma *simpler-modify-def*:
modify f = (λ*s*. (((), *f s*), *False*))
⟨*proof*⟩

Execute the given monad when the condition is true, return () otherwise.

definition
when1 :: *bool* ⇒ ('*s*, *unit*) *det-monad* ⇒
('*s*, *unit*) *det-monad* **where**
when1 P m ≡ *if P then m else return* ()

Execute the given monad unless the condition is true, return () otherwise.

definition
unless :: *bool* ⇒ ('*s*, *unit*) *det-monad* ⇒
('*s*, *unit*) *det-monad* **where**
unless P m ≡ *when1* (¬*P*) *m*

Perform a test on the current state, performing the left monad if the result is true or the right monad if the result is false.

definition
condition :: ('*s* ⇒ *bool*) ⇒ ('*s*, '*r*) *det-monad* ⇒ ('*s*, '*r*) *det-monad* ⇒ ('*s*, '*r*) *det-monad*
where
condition P L R ≡ λ*s*. *if* (*P s*) *then* (*L s*) *else* (*R s*)

notation (**output**)
condition ((*condition* (-)// (-)// (-)) [*1000*,*1000*,*1000*] *1000*)

3.3 The Monad Laws

Each monad satisfies at least the following three laws.

return is absorbed at the left of a (>>=), applying the return value directly:

lemma *return-bind* [*simp*]: (*return x* >>= *f*) = *f x*
⟨*proof*⟩

return is absorbed on the right of a (>>=)

lemma *bind-return* [*simp*]: (*m* >>= *return*) = *m*
⟨*proof*⟩

(>>=) is associative

lemma *bind-assoc*:
fixes *m* :: ('*a*,'*b*) *det-monad*
fixes *f* :: '*b* ⇒ ('*a*,'*c*) *det-monad*
fixes *g* :: '*c* ⇒ ('*a*,'*d*) *det-monad*
shows (*m* >>= *f*) >>= *g* = *m* >>= (λ*x*. *f x* >>= *g*)
⟨*proof*⟩

4 Adding Exceptions

The type ('*s*, '*a*) *det-monad* gives us determinism and failure. We now extend this monad with exceptional return values that abort normal execution, but can be handled explicitly. We use the sum type to indicate exceptions.

In $('s,\ 'e + 'a)\ det\text{-}monad$, $'s$ is the state, $'e$ is an exception, and $'a$ is a normal return value.

This new type itself forms a monad again. Since type classes in Isabelle are not powerful enough to express the class of monads, we provide new names for the *return* and (>>=) functions in this monad. We call them *returnOk* (for normal return values) and *bindE* (for composition). We also define *throwError* to return an exceptional value.

definition
returnOk :: $'a \Rightarrow ('s,\ 'e + 'a)\ det\text{-}monad$ **where**
returnOk $\equiv$ *return o Inr*

definition
throwError :: $'e \Rightarrow ('s,\ 'e + 'a)\ det\text{-}monad$ **where**
throwError $\equiv$ *return o Inl*

Lifting a function over the exception type: if the input is an exception, return that exception; otherwise continue execution.

definition
lift :: $('a \Rightarrow ('s,\ 'e + 'b)\ det\text{-}monad) \Rightarrow$
$'e + 'a \Rightarrow ('s,\ 'e + 'b)\ det\text{-}monad$
where
lift f v $\equiv$ *case v of Inl e* $\Rightarrow$ *throwError e*
| *Inr v'* $\Rightarrow$ *f v'*

The definition of (>>=) in the exception monad (new name *bindE*): the same as normal (>>=), but the right-hand side is skipped if the left-hand side produced an exception.

definition
bindE :: $('s,\ 'e + 'a)\ det\text{-}monad \Rightarrow$
$('a \Rightarrow ('s,\ 'e + 'b)\ det\text{-}monad) \Rightarrow$
$('s,\ 'e + 'b)\ det\text{-}monad$ (**infixl** >>=*E 60*)
where
bindE f g $\equiv$ *bind f* (*lift g*)

Lifting a normal deterministic monad into the exception monad is achieved by always returning its result as normal result and never throwing an exception.

definition
liftE :: $('s,'a)\ det\text{-}monad \Rightarrow ('s,\ 'e+'a)\ det\text{-}monad$
where
liftE f $\equiv$ *f* >>= $(\lambda r.\ return\ (Inr\ r))$

Since the underlying type and *return* function changed, we need new definitions for when and unless:

definition
whenE :: *bool* $\Rightarrow ('s,\ 'e + unit)\ det\text{-}monad \Rightarrow$

$('s, 'e + unit)$ *det-monad*
where
whenE P f $\equiv$ *if P then f else returnOk* ()

definition
unlessE :: *bool* $\Rightarrow$ $('s, 'e + unit)$ *det-monad* $\Rightarrow$
$('s, 'e + unit)$ *det-monad*
where
unlessE P f $\equiv$ *if P then returnOk* () *else f*

Throwing an exception when the parameter is *None*, otherwise returning v for *Some v*.

definition
throw-opt :: $'e \Rightarrow 'a$ *option* $\Rightarrow$ $('s, 'e + 'a)$ *det-monad* **where**
throw-opt ex x $\equiv$
case x of None $\Rightarrow$ *throwError ex* | *Some v* $\Rightarrow$ *returnOk v*

4.1 Monad Laws for the Exception Monad

More direct definition of *liftE*:

lemma *liftE-def2*:
liftE f = $(\lambda s.\ ((\lambda(v,s').\ (Inr\ v,\ s'))\ \ (fst\ (f\ s)),\ snd\ (f\ s)))$
⟨*proof*⟩

Left *returnOk* absorbtion over (>>=*E*):

lemma *returnOk-bindE* [*simp*]: (*returnOk x* >>=*E f*) = *f x*
⟨*proof*⟩

lemma *lift-return* [*simp*]:
lift (*return* $\circ$ *Inr*) = *return*
⟨*proof*⟩

Right *returnOk* absorbtion over (>>=*E*):

lemma *bindE-returnOk* [*simp*]: (*m* >>=*E returnOk*) = *m*
⟨*proof*⟩

Associativity of (>>=*E*):

lemma *bindE-assoc*:
(*m* >>=*E f*) >>=*E g* = *m* >>=*E* $(\lambda x.\ f\ x$ >>=*E g*)
⟨*proof*⟩

returnOk could also be defined via *liftE*:

lemma *returnOk-liftE*:
returnOk x = *liftE* (*return x*)
⟨*proof*⟩

Execution after throwing an exception is skipped:

lemma *throwError-bindE* [*simp*]:
(*throwError E >>=E f*) = *throwError E*
⟨*proof*⟩

5 Syntax

This section defines traditional Haskell-like do-syntax for the state monad in Isabelle.

5.1 Syntax for the Nondeterministic State Monad

We use *K-bind* to syntactically indicate the case where the return argument of the left side of a (>>=) is ignored

definition
K-bind-def [*iff*]: *K-bind* ≡ λ*x y*. *x*

nonterminal
dobinds **and** *dobind* **and** *nobind*

syntax
-dobind :: [*pttrn*, '*a*] => *dobind* ((- ←/ -) *10*)
:: *dobind* => *dobinds* (-)
-nobind :: '*a* => *dobind* (-)
-dobinds :: [*dobind*, *dobinds*] => *dobinds* ((-);//(-))

-do :: [*dobinds*, '*a*] => '*a* ((*do* ((-);//(-))//*od*) *100*)
translations
-do (*-dobinds b bs*) *e* == *-do b* (*-do bs e*)
-do (*-nobind b*) *e* == *b >>=* (*CONST K-bind e*)
do x ← *a*; *e od* == *a >>=* (λ*x*. *e*)

Syntax examples:

lemma *do x* ← *return 1*;
return (*2*::*nat*);
return x
od =
return 1 >>=
(λ*x*. *return* (*2*::*nat*) >>=
K-bind (*return x*))
⟨*proof*⟩

lemma *do x* ← *return 1*;
return 2;
return x
od = *return 1*
⟨*proof*⟩

5.2 Syntax for the Exception Monad

Since the exception monad is a different type, we need to syntactically distinguish it in the syntax. We use *doE*/*odE* for this, but can re-use most of the productions from *do*/*od* above.

syntax
-doE :: [*dobinds*, *'a*] => *'a* ((*doE* ((-);//(-))//*odE*) *100*)

translations
-doE (*-dobinds b bs*) *e* == *-doE b* (*-doE bs e*)
-doE (*-nobind b*) *e* == *b* >>=*E* (*CONST K-bind e*)
doE x ← *a*; *e odE* == *a* >>=*E* (λ*x*. *e*)

Syntax examples:

lemma *doE x* ← *returnOk 1*;
returnOk (*2*::*nat*);
returnOk x
odE =
returnOk 1 >>=*E*
(λ*x*. *returnOk* (*2*::*nat*) >>=*E*
K-bind (*returnOk x*))
⟨*proof*⟩

lemma *doE x* ← *returnOk 1*;
returnOk 2;
returnOk x
odE = *returnOk 1*
⟨*proof*⟩

6 Library of Monadic Functions and Combinators

Lifting a normal function into the monad type:

definition
liftM :: (*'a* ⇒ *'b*) ⇒ (*'s*,*'a*) *det-monad* ⇒ (*'s*, *'b*) *det-monad*
where
liftM f m ≡ *do x* ← *m*; *return* (*f x*) *od*

The same for the exception monad:

definition
liftME :: (*'a* ⇒ *'b*) ⇒ (*'s*,*'e*+*'a*) *det-monad* ⇒ (*'s*,*'e*+*'b*) *det-monad*
where
liftME f m ≡ *doE x* ← *m*; *returnOk* (*f x*) *odE*

Run a sequence of monads from left to right, ignoring return values.

definition
sequence-x :: (*'s*, *'a*) *det-monad list* ⇒ (*'s*, *unit*) *det-monad*
where

sequence-x xs ≡ *foldr* (λ*x y*. *x* >>= (λ-. *y*)) *xs* (*return* ())

Map a monadic function over a list by applying it to each element of the list from left to right, ignoring return values.

definition
mapM-x :: ('*a* ⇒ ('*s*,'*b*) *det-monad*) ⇒ '*a list* ⇒ ('*s*, *unit*) *det-monad*
where
mapM-x f xs ≡ *sequence-x* (*map f xs*)

Map a monadic function with two parameters over two lists, going through both lists simultaneously, left to right, ignoring return values.

definition
zipWithM-x :: ('*a* ⇒ '*b* ⇒ ('*s*,'*c*) *det-monad*) ⇒
'*a list* ⇒ '*b list* ⇒ ('*s*, *unit*) *det-monad*
where
zipWithM-x f xs ys ≡ *sequence-x* (*zipWith f xs ys*)

The same three functions as above, but returning a list of return values instead of *unit*

definition
sequence :: ('*s*, '*a*) *det-monad list* ⇒ ('*s*, '*a list*) *det-monad*
where
sequence xs ≡ *let mcons* = (λ*p q*. *p* >>= (λ*x*. *q* >>= (λ*y*. *return* (*x*#*y*))))
in foldr mcons xs (*return* [])

definition
mapM :: ('*a* ⇒ ('*s*,'*b*) *det-monad*) ⇒ '*a list* ⇒ ('*s*, '*b list*) *det-monad*
where
mapM f xs ≡ *sequence* (*map f xs*)

definition
zipWithM :: ('*a* ⇒ '*b* ⇒ ('*s*,'*c*) *det-monad*) ⇒
'*a list* ⇒ '*b list* ⇒ ('*s*, '*c list*) *det-monad*
where
zipWithM f xs ys ≡ *sequence* (*zipWith f xs ys*)

definition
foldM :: ('*b* ⇒ '*a* ⇒ ('*s*, '*a*) *det-monad*) ⇒ '*b list* ⇒ '*a* ⇒ ('*s*, '*a*) *det-monad*
where
foldM m xs a ≡ *foldr* (λ*p q*. *q* >>= *m p*) *xs* (*return a*)

The sequence and map functions above for the exception monad, with and without lists of return value

definition
sequenceE-x :: ('*s*, '*e*+'*a*) *det-monad list* ⇒ ('*s*, '*e*+*unit*) *det-monad*
where
sequenceE-x xs ≡ *foldr* (λ*x y*. *doE* - ← *x*; *y odE*) *xs* (*returnOk* ())

definition
mapME-x :: ('*a* ⇒ ('*s*,'*e*+'*b*) *det-monad*) ⇒ '*a list* ⇒
('*s*,'*e*+*unit*) *det-monad*
where
mapME-x f xs ≡ *sequenceE-x* (*map f xs*)

definition
sequenceE :: ('*s*, '*e*+'*a*) *det-monad list* ⇒ ('*s*, '*e*+'*a list*) *det-monad*
where
sequenceE xs ≡ *let mcons* = (λ*p q*. *p* >>=*E* (λ*x*. *q* >>=*E* (λ*y*. *returnOk* (*x*#*y*))))
in foldr mcons xs (*returnOk* [])

definition
mapME :: ('*a* ⇒ ('*s*,'*e*+'*b*) *det-monad*) ⇒ '*a list* ⇒
('*s*,'*e*+'*b list*) *det-monad*
where
mapME f xs ≡ *sequenceE* (*map f xs*)

Filtering a list using a monadic function as predicate:

primrec
filterM :: ('*a* ⇒ ('*s*, *bool*) *det-monad*) ⇒ '*a list* ⇒ ('*s*, '*a list*) *det-monad*
where
filterM P [] = *return* []
| *filterM P* (*x* # *xs*) = *do*
b ← *P x*;
ys ← *filterM P xs*;
return (*if b then* (*x* # *ys*) *else ys*)
od

7 Catching and Handling Exceptions

Turning an exception monad into a normal state monad by catching and handling any potential exceptions:

definition
catch :: ('*s*, '*e* + '*a*) *det-monad* ⇒
('*e* ⇒ ('*s*, '*a*) *det-monad*) ⇒
('*s*, '*a*) *det-monad* (**infix** <*catch*> *10*)
where
f <*catch*> *handler* ≡
do x ← *f*;
case x of
Inr b ⇒ *return b*
| *Inl e* ⇒ *handler e*
od

Handling exceptions, but staying in the exception monad. The handler may throw a type of exceptions different from the left side.

definition

```
handleE' :: ('s, 'e1 + 'a) det-monad ⇒
            ('e1 ⇒ ('s, 'e2 + 'a) det-monad) ⇒
            ('s, 'e2 + 'a) det-monad (infix <handle2> 10)
where
 f <handle2> handler ≡
  do
    v ← f;
    case v of
      Inl e ⇒ handler e
    | Inr v' ⇒ return (Inr v')
  od
```

A type restriction of the above that is used more commonly in practice: the exception handle (potentially) throws exception of the same type as the left-hand side.

```
definition
 handleE :: ('s, 'x + 'a) det-monad ⇒
            ('x ⇒ ('s, 'x + 'a) det-monad) ⇒
            ('s, 'x + 'a) det-monad (infix <handle> 10)
where
 handleE ≡ handleE'
```

Handling exceptions, and additionally providing a continuation if the left-hand side throws no exception:

```
definition
 handle-elseE :: ('s, 'e + 'a) det-monad ⇒
                 ('e ⇒ ('s, 'ee + 'b) det-monad) ⇒
                 ('a ⇒ ('s, 'ee + 'b) det-monad) ⇒
                 ('s, 'ee + 'b) det-monad
 (- <handle> - <else> - 10)
where
 f <handle> handler <else> continue ≡
  do v ← f;
  case v of Inl e  ⇒ handler e
          | Inr v' ⇒ continue v'
  od
```

8 Hoare Logic

8.1 Validity

This section defines a Hoare logic for partial correctness for the deterministic state monad as well as the exception monad. The logic talks only about the behaviour part of the monad and ignores the failure flag.

The logic is defined semantically. Rules work directly on the validity predicate.

In the deterministic state monad, validity is a triple of precondition, monad, and postcondition. The precondition is a function from state to bool (a state predicate), the postcondition is a function from return value to state to bool. A triple is valid if for all states that satisfy the precondition, all result values and result states that are returned by the monad satisfy the postcondition. Note that if the computation returns the empty set, the triple is trivially valid. This means *assert P* does not require us to prove that P holds, but rather allows us to assume P! Proving non-failure is done via separate predicate and calculus (see below).

definition
valid :: $('s \Rightarrow bool) \Rightarrow ('s,'a)$ *det-monad* $\Rightarrow ('a \Rightarrow 's \Rightarrow bool) \Rightarrow bool$
($\{\!|$-$|\!\}$/ - /$\{\!|$-$|\!\}$)
where
$\{\!|P|\!\}\ f\ \{\!|Q|\!\} \equiv \forall\, s.\ P\ s \longrightarrow (\forall\, r\ s'.\ ((r,s') = fst\ (f\ s) \longrightarrow Q\ r\ s'))$

Validity for the exception monad is similar and build on the standard validity above. Instead of one postcondition, we have two: one for normal and one for exceptional results.

definition
validE :: $('s \Rightarrow bool) \Rightarrow ('s,\ 'a + 'b)$ *det-monad* $\Rightarrow$
$('b \Rightarrow 's \Rightarrow bool) \Rightarrow$
$('a \Rightarrow 's \Rightarrow bool) \Rightarrow bool$
($\{\!|$-$|\!\}$/ - /($\{\!|$-$|\!\}$,/ $\{\!|$-$|\!\}$))
where
$\{\!|P|\!\}\ f\ \{\!|Q|\!\},\{\!|E|\!\} \equiv \{\!|P|\!\}\ f\ \{\!|\ \lambda v\ s.\ case\ v\ of\ Inr\ r \Rightarrow Q\ r\ s \mid Inl\ e \Rightarrow E\ e\ s\ |\!\}$

The following two instantiations are convenient to separate reasoning for exceptional and normal case.

definition
validE-R :: $('s \Rightarrow bool) \Rightarrow ('s,\ 'e + 'a)$ *det-monad* $\Rightarrow$
$('a \Rightarrow 's \Rightarrow bool) \Rightarrow bool$
($\{\!|$-$|\!\}$/ - /$\{\!|$-$|\!\}$, $-$)
where
$\{\!|P|\!\}\ f\ \{\!|Q|\!\},- \equiv validE\ P\ f\ Q\ (\lambda x\ y.\ True)$

definition
validE-E :: $('s \Rightarrow bool) \Rightarrow\ ('s,\ 'e + 'a)$ *det-monad* $\Rightarrow$
$('e \Rightarrow 's \Rightarrow bool) \Rightarrow bool$
($\{\!|$-$|\!\}$/ - /$-$, $\{\!|$-$|\!\}$)
where
$\{\!|P|\!\}\ f\ -,\{\!|Q|\!\} \equiv validE\ P\ f\ (\lambda x\ y.\ True)\ Q$

Abbreviations for trivial preconditions:

abbreviation(*input*)
top :: $'a \Rightarrow bool$ ($\top$)
where
$\top \equiv \lambda$-. *True*

abbreviation(*input*)
  bottom :: $'a \Rightarrow$ *bool* ($\bot$)
where
  $\bot \equiv \lambda$-. *False*

Abbreviations for trivial postconditions (taking two arguments):

abbreviation(*input*)
  toptop :: $'a \Rightarrow 'b \Rightarrow$ *bool* ($\top\top$)
where
 $\top\top \equiv \lambda$- -. *True*

abbreviation(*input*)
  botbot :: $'a \Rightarrow 'b \Rightarrow$ *bool* ($\bot\bot$)
where
 $\bot\bot \equiv \lambda$- -. *False*

Lifting $\wedge$ and $\vee$ over two arguments. Lifting $\wedge$ and $\vee$ over one argument is already defined (written *and* and *or*).

definition
  bipred-conj :: $('a \Rightarrow 'b \Rightarrow$ *bool*$) \Rightarrow ('a \Rightarrow 'b \Rightarrow$ *bool*$) \Rightarrow ('a \Rightarrow 'b \Rightarrow$ *bool*$)$
  (**infixl** *And 96*)
where
  bipred-conj $P\ Q \equiv \lambda x\ y.\ P\ x\ y \wedge Q\ x\ y$

definition
  bipred-disj :: $('a \Rightarrow 'b \Rightarrow$ *bool*$) \Rightarrow ('a \Rightarrow 'b \Rightarrow$ *bool*$) \Rightarrow ('a \Rightarrow 'b \Rightarrow$ *bool*$)$
  (**infixl** *Or 91*)
where
  bipred-disj $P\ Q \equiv \lambda x\ y.\ P\ x\ y \vee Q\ x\ y$

8.2 Determinism

A monad of type *det-monad* is deterministic iff it returns exactly one state and result and does not fail

definition
  det :: $('a,'s)$ *det-monad* $\Rightarrow$ *bool*
where
  det $f \equiv \forall s.\ \exists r.\ f\ s = (r,$*False*$)$

A deterministic *det-monad* can be turned into a normal state monad:

definition
  the-run-state :: $('s,'a)$ *det-monad* $\Rightarrow 's \Rightarrow 'a \times 's$
where
  the-run-state $M \equiv \lambda s.$ *THE* $s'.$ *fst* $(M\ s) = s'$

8.3 Non-Failure

With the failure flag, we can formulate non-failure separately from validity. A monad m does not fail under precondition P, if for no start state in that precondition it sets the failure flag.

definition
no-fail :: $('s \Rightarrow bool) \Rightarrow ('s,'a)$ *det-monad* $\Rightarrow$ *bool*
where
no-fail P m $\equiv \forall s.\ P\ s \longrightarrow \neg\ (snd\ (m\ s))$

It is often desired to prove non-failure and a Hoare triple simultaneously, as the reasoning is often similar. The following definitions allow such reasoning to take place.

definition
validNF ::$('s \Rightarrow bool) \Rightarrow ('s,'a)$ *det-monad* $\Rightarrow ('a \Rightarrow 's \Rightarrow bool) \Rightarrow$ *bool*
(⦃-⦄/ - /⦃-⦄!)
where
validNF P f Q $\equiv$ *valid P f Q* $\wedge$ *no-fail P f*

definition
validE-NF :: $('s \Rightarrow bool) \Rightarrow ('s,\ 'a + 'b)$ *det-monad* $\Rightarrow$
$('b \Rightarrow 's \Rightarrow bool) \Rightarrow$
$('a \Rightarrow 's \Rightarrow bool) \Rightarrow$ *bool*
(⦃-⦄/ - /(⦃-⦄,/ ⦃-⦄!))
where
validE-NF P f Q E $\equiv$ *validE P f Q E* $\wedge$ *no-fail P f*

lemma *validE-NF-alt-def*:
⦃ *P* ⦄ *B* ⦃ *Q* ⦄,⦃ *E* ⦄! = ⦃ *P* ⦄ *B* ⦃ $\lambda v\ s.$ *case v of Inl e* $\Rightarrow$ *E e s* | *Inr r* $\Rightarrow$ *Q r s* ⦄!
⟨*proof*⟩

9 Basic exception reasoning

The following predicates *no-throw* and *no-return* allow reasoning that functions in the exception monad either do no throw an exception or never return normally.

definition *no-throw P A* $\equiv$ ⦃ *P* ⦄ *A* ⦃ λ- -. *True* ⦄,⦃ λ- -. *False* ⦄

definition *no-return P A* $\equiv$ ⦃ *P* ⦄ *A* ⦃λ- -. *False*⦄,⦃λ- -. *True* ⦄

end

theory *DetMonadLemmas*

imports *DetMonad*
begin

10 General Lemmas Regarding the Deterministic State Monad

10.1 Congruence Rules for the Function Package

lemma *bind-cong*[*fundef-cong*]:
$[\![f = f'; \bigwedge v\ s\ s'.\ (v, s') = fst\ (f'\ s) \Longrightarrow g\ v\ s' = g'\ v\ s']\!] \Longrightarrow f >>= g = f' >>= g'$
⟨*proof*⟩

lemma *bind-apply-cong* [*fundef-cong*]:
$[\![f\ s = f'\ s'; \bigwedge rv\ st.\ (rv, st) = fst\ (f'\ s') \Longrightarrow g\ rv\ st = g'\ rv\ st]\!]$
$\Longrightarrow (f >>= g)\ s = (f' >>= g')\ s'$
⟨*proof*⟩

lemma *bindE-cong*[*fundef-cong*]:
$[\![M = M'; \bigwedge v\ s\ s'.\ (Inr\ v, s') = fst\ (M'\ s) \Longrightarrow N\ v\ s' = N'\ v\ s']\!] \Longrightarrow bindE\ M\ N = bindE\ M'\ N'$
⟨*proof*⟩

lemma *bindE-apply-cong*[*fundef-cong*]:
$[\![f\ s = f'\ s'; \bigwedge rv\ st.\ (Inr\ rv, st) = fst\ (f'\ s') \Longrightarrow g\ rv\ st = g'\ rv\ st]\!]$
$\Longrightarrow (f >>=E\ g)\ s = (f' >>=E\ g')\ s'$
⟨*proof*⟩

lemma *K-bind-apply-cong*[*fundef-cong*]:
$[\![f\ st = f'\ st']\!] \Longrightarrow K\text{-}bind\ f\ arg\ st = K\text{-}bind\ f'\ arg'\ st'$
⟨*proof*⟩

lemma *when-apply-cong*[*fundef-cong*]:
$[\![C = C'; s = s'; C' \Longrightarrow m\ s' = m'\ s']\!] \Longrightarrow whenE\ C\ m\ s = whenE\ C'\ m'\ s'$
⟨*proof*⟩

lemma *unless-apply-cong*[*fundef-cong*]:
$[\![C = C'; s = s'; \neg\ C' \Longrightarrow m\ s' = m'\ s']\!] \Longrightarrow unlessE\ C\ m\ s = unlessE\ C'\ m'\ s'$
⟨*proof*⟩

lemma *whenE-apply-cong*[*fundef-cong*]:
$[\![C = C'; s = s'; C' \Longrightarrow m\ s' = m'\ s']\!] \Longrightarrow whenE\ C\ m\ s = whenE\ C'\ m'\ s'$
⟨*proof*⟩

lemma *unlessE-apply-cong*[*fundef-cong*]:
$[\![C = C'; s = s'; \neg\ C' \Longrightarrow m\ s' = m'\ s']\!] \Longrightarrow unlessE\ C\ m\ s = unlessE\ C'\ m'\ s'$
⟨*proof*⟩

10.2 Simplifying Monads

lemma *nested-bind* [*simp*]:
do $x \leftarrow$ *do* $y \leftarrow f$; *return* $(g\ y)$ *od*; $h\ x$ *od* =
do $y \leftarrow f$; $h\ (g\ y)$ *od*
⟨*proof*⟩

lemma *assert-True* [*simp*]:
assert True >>= $f = f\ ()$
⟨*proof*⟩

lemma *when-True-bind* [*simp*]:
when1 True g >>= $f = g$ >>= f
⟨*proof*⟩

lemma *whenE-False-bind* [*simp*]:
whenE False g >>=E $f = f\ ()$
⟨*proof*⟩

lemma *whenE-True-bind* [*simp*]:
whenE True g >>=E $f = g$ >>=E f
⟨*proof*⟩

lemma *when-True* [*simp*]: *when1 True* $X = X$
⟨*proof*⟩

lemma *when-False* [*simp*]: *when1 False* $X =$ *return* ()
⟨*proof*⟩

lemma *unless-False* [*simp*]: *unless False* $X = X$
⟨*proof*⟩

lemma *unless-True* [*simp*]: *unless True* $X =$ *return* ()
⟨*proof*⟩

lemma *unlessE-whenE*:
unlessE $P =$ *whenE* $(\sim P)$
⟨*proof*⟩

lemma *unless-when*:
unless $P =$ *when1* $(\sim P)$
⟨*proof*⟩

lemma *gets-to-return* [*simp*]: *gets* $(\lambda s.\ v) =$ *return* v
⟨*proof*⟩

lemma *liftE-handleE'* [*simp*]: $((liftE\ a)$ <*handle2*> $b) = liftE\ a$
⟨*proof*⟩

lemma *liftE-handleE* [*simp*]: $((liftE\ a)$ <*handle*> $b) = liftE\ a$

⟨*proof*⟩

lemma *condition-split*:
$P\ (\mathit{condition}\ C\ a\ b\ s) = ((((C\ s) \longrightarrow P\ (a\ s)) \wedge (\neg\ (C\ s) \longrightarrow P\ (b\ s))))$
⟨*proof*⟩

lemma *condition-split-asm*:
$P\ (\mathit{condition}\ C\ a\ b\ s) = (\neg\ (C\ s \wedge \neg\ P\ (a\ s) \vee \neg\ C\ s \wedge \neg\ P\ (b\ s)))$
⟨*proof*⟩

lemmas *condition-splits* = *condition-split condition-split-asm*

lemma *condition-true-triv* [*simp*]:
$\mathit{condition}\ (\lambda\text{-}.\ \mathit{True})\ A\ B = A$
⟨*proof*⟩

lemma *condition-false-triv* [*simp*]:
$\mathit{condition}\ (\lambda\text{-}.\ \mathit{False})\ A\ B = B$
⟨*proof*⟩

lemma *condition-true*: $[\![\ P\ s\ ]\!] \Longrightarrow \mathit{condition}\ P\ A\ B\ s = A\ s$
⟨*proof*⟩

lemma *condition-false*: $[\![\ \neg\ P\ s\ ]\!] \Longrightarrow \mathit{condition}\ P\ A\ B\ s = B\ s$
⟨*proof*⟩

11 Low-level monadic reasoning

lemma *valid-make-schematic-post*:
$(\forall\, s0.\ \{\!|\ \lambda s.\ P\ s0\ s\ |\!\}\ f\ \{\!|\ \lambda rv\ s.\ Q\ s0\ rv\ s\ |\!\}) \Longrightarrow$
$\{\!|\ \lambda s.\ \exists\, s0.\ P\ s0\ s \wedge (\forall\, rv\ s'.\ Q\ s0\ rv\ s' \longrightarrow Q'\ rv\ s')\ |\!\}\ f\ \{\!|\ Q'\ |\!\}$
⟨*proof*⟩

lemma *validNF-make-schematic-post*:
$(\forall\, s0.\ \{\!|\ \lambda s.\ P\ s0\ s\ |\!\}\ f\ \{\!|\ \lambda rv\ s.\ Q\ s0\ rv\ s\ |\!\}!) \Longrightarrow$
$\{\!|\ \lambda s.\ \exists\, s0.\ P\ s0\ s \wedge (\forall\, rv\ s'.\ Q\ s0\ rv\ s' \longrightarrow Q'\ rv\ s')\ |\!\}\ f\ \{\!|\ Q'\ |\!\}!$
⟨*proof*⟩

lemma *validE-make-schematic-post*:
$(\forall\, s0.\ \{\!|\ \lambda s.\ P\ s0\ s\ |\!\}\ f\ \{\!|\ \lambda rv\ s.\ Q\ s0\ rv\ s\ |\!\}, \{\!|\ \lambda rv\ s.\ E\ s0\ rv\ s\ |\!\}) \Longrightarrow$
$\{\!|\ \lambda s.\ \exists\, s0.\ P\ s0\ s \wedge (\forall\, rv\ s'.\ Q\ s0\ rv\ s' \longrightarrow Q'\ rv\ s')$
$\quad \wedge (\forall\, rv\ s'.\ E\ s0\ rv\ s' \longrightarrow E'\ rv\ s')\ |\!\}\ f\ \{\!|\ Q'\ |\!\}, \{\!|\ E'\ |\!\}$
⟨*proof*⟩

lemma *validE-NF-make-schematic-post*:
$(\forall\, s0.\ \{\!|\ \lambda s.\ P\ s0\ s\ |\!\}\ f\ \{\!|\ \lambda rv\ s.\ Q\ s0\ rv\ s\ |\!\}, \{\!|\ \lambda rv\ s.\ E\ s0\ rv\ s\ |\!\}!) \Longrightarrow$
$\{\!|\ \lambda s.\ \exists\, s0.\ P\ s0\ s \wedge (\forall\, rv\ s'.\ Q\ s0\ rv\ s' \longrightarrow Q'\ rv\ s')$
$\quad \wedge (\forall\, rv\ s'.\ E\ s0\ rv\ s' \longrightarrow E'\ rv\ s')\ |\!\}\ f\ \{\!|\ Q'\ |\!\}, \{\!|\ E'\ |\!\}!$
⟨*proof*⟩

lemma *validNF-conjD1*: $\{\!|\ P\ |\!\}\ f\ \{\!|\ \lambda rv\ s.\ Q\ rv\ s \wedge Q'\ rv\ s\ |\!\}! \Longrightarrow \{\!|\ P\ |\!\}\ f\ \{\!|\ Q\ |\!\}!$
⟨*proof*⟩

lemma *validNF-conjD2*: $\{\!|\ P\ |\!\}\ f\ \{\!|\ \lambda rv\ s.\ Q\ rv\ s \wedge Q'\ rv\ s\ |\!\}! \Longrightarrow \{\!|\ P\ |\!\}\ f\ \{\!|\ Q'\ |\!\}!$
⟨*proof*⟩

lemma *exec-gets*:
$(gets\ f >>= m)\ s = m\ (f\ s)\ s$
⟨*proof*⟩

lemma *in-gets*:
$(r, s') = fst\ (gets\ f\ s) = (r = f\ s \wedge s' = s)$
⟨*proof*⟩

end

12 Register Operations

theory *RegistersOps*
imports *Main ../lib/ WordDecl Word-Lib.Bit-Shifts-Infix-Syntax*
begin

context
includes *bit-operations-syntax*
begin

This theory provides operations to get, set and clear bits in registers

13 Getting Fields

Get a field of type *'b word* starting at *index* from *addr* of type *'a word*

definition *get-field-from-word-a-b*:: *'a*::*len word* ⇒ *nat* ⇒ *'b*::*len word*
where
get-field-from-word-a-b addr index
≡ *let off* = (*size addr* − *LENGTH*('*b*))
in ucast ((*addr* << (*off*−*index*)) >> *off*)

Obtain, from addr of type *'a word*, another *'a word* containing the field of length *len* starting at *index* in *addr*.

definition *get-field-from-word-a-a*:: *'a*::*len word* ⇒ *nat* ⇒ *nat* ⇒ *'a*::*len word*
where
get-field-from-word-a-a addr index len
≡ (*addr* << (*size addr* − (*index*+*len*)) >> (*size addr* − *len*))

14 Setting Fields

Set the field of type *'b word* at *index* from *record* of type *'a word*.

definition *set-field* :: *'a::len word* ⇒ *'b::len word* ⇒ *nat* ⇒ *'a::len word*
where
set-field record field index
≡ *let mask*:: (*'a::len word*) = (*mask* (*size field*)) << *index*
in (*record AND* (*NOT mask*)) *OR* ((*ucast field*) << *index*)

15 Clearing Fields

Zero the *n* initial bits of *addr*.

definition *clear-n-bits*:: *'a::len word* ⇒ *nat* ⇒ *'a::len word*
where
clear-n-bits addr n ≡ *addr AND* (*NOT* (*mask n*))

Gets the natural value of a 32 bit mask

definition *get-nat-from-mask*::*word32* ⇒ *nat* ⇒ *nat* ⇒ (*word32* × *nat*)
where

get-nat-from-mask w m v ≡ *if* (*w AND* (*mask m*) =*0*) *then* (*w*>>*m*, *v*+*m*)
else (*w*,*m*)

definition *get-nat-from-mask32*::*word32*⇒ *nat*
where
get-nat-from-mask32 w ≡
if (*w*=*0*) *then len-of TYPE* (*word-length32*)
else
let (*w*,*res*) = *get-nat-from-mask w 16 0 in*
let (*w*,*res*)= *get-nat-from-mask w 8 res in*
let (*w*,*res*) = *get-nat-from-mask w 4 res in*
let (*w*,*res*) = *get-nat-from-mask w 2 res in*
let (*w*,*res*) = *get-nat-from-mask w 1 res in*
res

end

end

16 Memory Management Unit (MMU)

theory *MMU*
imports *Main RegistersOps Sparc-Types*
begin

17 MMU Sizing

We need some citation here for documentation about the MMU.

The MMU uses the Address Space Identifiers (ASI) to control memory access. ASI = 8, 10 are for user; ASI = 9, 11 are for supervisor.

17.1 MMU Types

type-synonym *word-PTE-flags* = *word8*
type-synonym *word-length-PTE-flags* = *word-length8*

17.2 MMU length values

Definitions for the length of the virtua address, page size, virtual translation tables indexes, virtual address offset and Page protection flags

definition *length-entry-type* :: *nat*
where *length-entry-type* ≡ *LENGTH*(*word-length-entry-type*)
definition *length-phys-address*:: *nat*
where *length-phys-address* ≡ *LENGTH*(*word-length-phys-address*)
definition *length-virtua-address*:: *nat*
where *length-virtua-address* ≡ *LENGTH*(*word-length-virtua-address*)
definition *length-page*:: *nat* **where** *length-page* ≡ *LENGTH*(*word-length-page*)
definition *length-t1*:: *nat* **where** *length-t1* ≡ *LENGTH*(*word-length-t1*)
definition *length-t2*:: *nat* **where** *length-t2* ≡ *LENGTH*(*word-length-t2*)
definition *length-t3*:: *nat* **where** *length-t3* ≡ *LENGTH*(*word-length-t3*)
definition *length-offset*:: *nat* **where** *length-offset* ≡ *LENGTH*(*word-length-offset*)
definition *length-PTE-flags* :: *nat* **where**
length-PTE-flags ≡ *LENGTH*(*word-length-PTE-flags*)

17.3 MMU index values

definition *va-t1-index* :: *nat* **where** *va-t1-index* ≡ *length-virtua-address* − *length-t1*
definition *va-t2-index* :: *nat* **where** *va-t2-index* ≡ *va-t1-index* − *length-t2*
definition *va-t3-index* :: *nat* **where** *va-t3-index* ≡ *va-t2-index* − *length-t3*
definition *va-offset-index* :: *nat* **where** *va-offset-index* ≡ *va-t3-index* − *length-offset*
definition *pa-page-index* :: *nat*
where *pa-page-index* ≡ *length-phys-address* − *length-page*
definition *pa-offset-index* :: *nat* **where**
pa-offset-index ≡ *pa-page-index* −*length-page*

18 MMU Definition

record *MMU-state* =
registers :: *MMU-context*

The following functions access MMU registers via addresses. See UT699LEON3FT manual page 35.

definition *mmu-reg-val*:: *MMU-state* ⇒ *virtua-address* ⇒ *machine-word option*
where *mmu-reg-val mmu-state addr* ≡
  if addr = 0x000 then — MMU control register
    Some ((*registers mmu-state*) *CR*)
  else if addr = 0x100 then — Context pointer register
    Some ((*registers mmu-state*) *CTP*)
  else if addr = 0x200 then — Context register
    Some ((*registers mmu-state*) *CNR*)
  else if addr = 0x300 then — Fault status register
    Some ((*registers mmu-state*) *FTSR*)
  else if addr = 0x400 then — Fault address register
    Some ((*registers mmu-state*) *FAR*)
  else None

definition *mmu-reg-mod*:: *MMU-state* ⇒ *virtua-address* ⇒ *machine-word* ⇒
  MMU-state option **where**
mmu-reg-mod mmu-state addr w ≡
  if addr = 0x000 then — MMU control register
    Some (*mmu-state*(|*registers* := (*registers mmu-state*)(*CR* := *w*)|))
  else if addr = 0x100 then — Context pointer register
    Some (*mmu-state*(|*registers* := (*registers mmu-state*)(*CTP* := *w*)|))
  else if addr = 0x200 then — Context register
    Some (*mmu-state*(|*registers* := (*registers mmu-state*)(*CNR* := *w*)|))
  else if addr = 0x300 then — Fault status register
    Some (*mmu-state*(|*registers* := (*registers mmu-state*)(*FTSR* := *w*)|))
  else if addr = 0x400 then — Fault address register
    Some (*mmu-state*(|*registers* := (*registers mmu-state*)(*FAR* := *w*)|))
  else None

19 Virtual Memory

19.1 MMU Auxiliary Definitions

definition *getCTPVal*:: *MMU-state* ⇒ *machine-word*
where *getCTPVal mmu* ≡ (*registers mmu*) *CTP*

definition *getCNRVal*::*MMU-state* ⇒ *machine-word*
where *getCNRVal mmu* ≡ (*registers mmu*) *CNR*

The physical context table address is got from the ConText Pointer register (CTP) and the Context Register (CNR) MMU registers. The CTP is shifted to align it with the physical address (36 bits) and we add the table index given on CNR. CTP is right shifted 2 bits, cast to phys address and left shifted 6 bytes to be aligned with the context register. CNR is 2 bits left shifted for alignment with the context table.

definition *compose-context-table-addr* :: *machine-word* ⇒*machine-word*
  ⇒ *phys-address*
where

compose-context-table-addr ctp cnr
  ≡ ((*ucast* (*ctp* >> *2*)) << *6*) + (*ucast cnr* << *2*)

19.2 Virtual Address Translation

Get the context table phys address from the MMU registers

definition *get-context-table-addr* :: *MMU-state* ⇒ *phys-address*
where
get-context-table-addr mmu
  ≡ *compose-context-table-addr* (*getCTPVal mmu*) (*getCNRVal mmu*)

definition *va-list-index* :: *nat list* **where**
va-list-index ≡ [*va-t1-index*,*va-t2-index*,*va-t3-index*,*0*]

definition *offset-index* :: *nat list* **where**
offset-index
  ≡ [*length-machine-word*
  , *length-machine-word*−*length-t1*
  , *length-machine-word*−*length-t1*−*length-t2*
  , *length-machine-word*−*length-t1*−*length-t2*−*length-t3*
  ]

definition *index-len-table* :: *nat list* **where** *index-len-table* ≡ [*8*,*6*,*6*,*0*]

definition *n-context-tables* :: *nat* **where** *n-context-tables* ≡ *3*

The following are basic physical memory read functions. At this level we don't need the write memory yet.

definition *mem-context-val*:: *asi-type* ⇒ *phys-address* ⇒
                mem-context ⇒ *mem-val-type option*
where
mem-context-val asi add m ≡
  let asi8 = *word-of-int 8*;
    r1 = *m asi add*
  in
  if r1 = *None then*
    m asi8 add
  else r1

context
  includes *bit-operations-syntax*
begin

Given an ASI (word8), an address (word32) addr, read the 32bit value from the memory addresses starting from address addr' where addr' = addr exception that the last two bits are 0's. That is, read the data from addr', addr'+1, addr'+2, addr'+3.

```
definition mem-context-val-w32 :: asi-type ⇒ phys-address ⇒
                                  mem-context ⇒ word32 option
where
mem-context-val-w32 asi addr m ≡
  let addr' = (AND) addr 0b11111111111111111111111111111100;
      addr0 = (OR) addr' 0b00000000000000000000000000000000;
      addr1 = (OR) addr' 0b00000000000000000000000000000001;
      addr2 = (OR) addr' 0b00000000000000000000000000000010;
      addr3 = (OR) addr' 0b00000000000000000000000000000011;
      r0 = mem-context-val asi addr0 m;
      r1 = mem-context-val asi addr1 m;
      r2 = mem-context-val asi addr2 m;
      r3 = mem-context-val asi addr3 m
  in
  if r0 = None ∨ r1 = None ∨ r2 = None ∨ r3 = None then
    None
  else
    let byte0 = case r0 of Some v ⇒ v;
        byte1 = case r1 of Some v ⇒ v;
        byte2 = case r2 of Some v ⇒ v;
        byte3 = case r3 of Some v ⇒ v
    in
    Some ((OR) ((OR) ((OR) ((ucast(byte0)) << 24)
                           ((ucast(byte1)) << 16))
                      ((ucast(byte2)) << 8))
                (ucast(byte3)))
```

get-addr-from-table browses the page description tables until it finds a PTE (bits==suc (suc 0).

If it is a PTE it aligns the 24 most significant bits of the entry with the most significant bits of the phys address and or-ed with the offset, which will vary depending on the entry level. In the case we are looking at the last table level (level 3), the offset is aligned to 0 otherwise it will be 2.

If the table entry is a PTD (bits== Suc 0), the index is obtained from the virtual address depending on the current level and or-ed with the PTD.

```
function ptd-lookup:: virtua-address ⇒ virtua-address ⇒
mem-context ⇒ nat ⇒ (phys-address × PTE-flags) option
where ptd-lookup va pt m lvl = (
  if lvl > 3 then None
  else
    let thislvl-offset = (
      if lvl = 1 then (ucast ((ucast (va >> 24))::word8))::word32
      else if lvl = 2 then (ucast ((ucast (va >> 18))::word6))::word32
      else (ucast ((ucast (va >> 12))::word6))::word32);
        thislvl-addr = (OR) pt thislvl-offset;
        thislvl-data = mem-context-val-w32 (word-of-int 9) (ucast thislvl-addr) m
    in
```

```
    case thislvl-data of
    Some v ⇒ (
      let et-val = (AND) v 0b00000000000000000000000000000011 in
      if et-val = 0 then — Invalid
        None
      else if et-val = 1 then — Page Table Descriptor
        let ptp = (AND) v 0b11111111111111111111111111111100 in
        ptd-lookup va ptp m (lvl+1)
      else if et-val = 2 then — Page Table Entry
        let ppn = (ucast (v >> 8))::word24;
            va-offset = (ucast ((ucast va)::word12))::word36
        in
        Some (((OR) (((ucast ppn)::word36) << 12) va-offset),
              ((ucast v)::word8))
      else — et-val = 3, reserved.
        None
    )
    |None ⇒ None)
```

⟨*proof*⟩
termination
⟨*proof*⟩

```
definition get-acc-flag:: PTE-flags ⇒ word3 where
get-acc-flag w8 ≡ (ucast (w8 >> 2))::word3

definition mmu-readable:: word3 ⇒ asi-type ⇒ bool where
mmu-readable f asi ≡
  if uint asi ∈ {8, 10} then
    if uint f ∈ {0,1,2,3,5} then True
    else False
  else if uint asi ∈ {9, 11} then
    if uint f ∈ {0,1,2,3,5,6,7} then True
    else False
  else False


definition mmu-writable:: word3 ⇒ asi-type ⇒ bool where
mmu-writable f asi ≡
  if uint asi ∈ {8, 10} then
    if uint f ∈ {1,3} then True
    else False
  else if uint asi ∈ {9, 11} then
    if uint f ∈ {1,3,5,7} then True
    else False
  else False


definition virt-to-phys :: virtua-address ⇒ MMU-state  ⇒ mem-context ⇒
```

```
                                    (phys-address × PTE-flags) option
where
 virt-to-phys va mmu m ≡
    let ctp-val = mmu-reg-val mmu (0x100);
        cnr-val = mmu-reg-val mmu (0x200);
        mmu-cr-val = (registers mmu) CR
    in
    if (AND) mmu-cr-val 1 ≠ 0 then — MMU enabled.
      case (ctp-val,cnr-val) of
      (Some v1, Some v2) ⇒
        let context-table-entry = (OR) ((v1 >> 11) << 11)
            (((AND) v2 0b00000000000000000000000111111111) << 2);
            context-table-data = mem-context-val-w32 (word-of-int 9)
              (ucast context-table-entry) m
        in (
        case context-table-data of
        Some lvl1-page-table ⇒
          ptd-lookup va lvl1-page-table m 1
        |None ⇒ None)
      |- ⇒ None
    else Some ((ucast va), ((0b11101111)::word8))
```

The below function gives the initial values of MMU registers. In particular, the MMU context register CR is 0 because: We don't know the bits for IMPL, VER, and SC; the bits for PSO are 0s because we use TSO; the reserved bits are 0s; we assume NF bits are 0s; and most importantly, the E bit is 0 because when the machine starts up, MMU is disabled. An initial boot procedure (bootloader or something like that) should configure the MMU and then enable it if the OS uses MMU.

definition *MMU-registers-init* :: *MMU-context*
where *MMU-registers-init r* ≡ *0*

definition *mmu-setup* :: *MMU-state*
where *mmu-setup* ≡ (|*registers*=*MMU-registers-init*|)

end

end

20 SPARC V8 state model

theory *Sparc-State*
imports *Main Sparc-Types ../lib/wp/DetMonadLemmas MMU*
begin

21 state as a function

record *cpu-cache* =
dcache:: *cache-context*
icache:: *cache-context*

The state *sparc-state* is defined as a tuple *cpu-context*, *user-context*, *mem-context*, defining the state of the CPU registers, user registers, memory, cache, and delayed write pool respectively. Additionally, a boolean indicates whether the state is undefined or not.

record (**overloaded**) ('*a*) *sparc-state* =
cpu-reg:: *cpu-context*
user-reg:: ('*a*) *user-context*
sys-reg:: *sys-context*
mem:: *mem-context*
mmu:: *MMU-state*
cache:: *cpu-cache*
dwrite:: *delayed-write-pool*
state-var:: *sparc-state-var*
traps:: *Trap set*
undef:: *bool*

22 functions for state member access

definition *cpu-reg-val*:: *CPU-register* ⇒ ('*a*) *sparc-state* ⇒ *reg-type*
where
cpu-reg-val reg state ≡ (*cpu-reg state*) *reg*

definition *cpu-reg-mod* :: *word32* ⇒ *CPU-register* ⇒ ('*a*) *sparc-state* ⇒
('*a*) *sparc-state*
where *cpu-reg-mod data-w32 cpu state* ≡
state(|*cpu-reg* := ((*cpu-reg state*)(*cpu* := *data-w32*))|)

r[0] = 0. Otherwise read the actual value.

definition *user-reg-val*:: ('*a*) *window-size* ⇒ *user-reg-type* ⇒ ('*a*) *sparc-state* ⇒
reg-type
where
user-reg-val window ur state ≡
if ur = *0 then 0*
else (*user-reg state*) *window ur*

Write a global register. win should be initialised as NWINDOWS.

fun (*sequential*) *global-reg-mod* :: *word32* ⇒ *nat* ⇒ *user-reg-type* ⇒
('*a*::*len*) *sparc-state* ⇒ ('*a*) *sparc-state*
where
global-reg-mod data-w32 0 ur state = *state*
|
global-reg-mod data-w32 win ur state = (
let win-word = *word-of-int* (*int* (*win*−*1*));
ns = *state*(|*user-reg* :=
(*user-reg state*)(*win-word* := ((*user-reg state*) *win-word*)(*ur* := *data-w32*))|)

in
global-reg-mod data-w32 (*win*−*1*) *ur ns*
)

Compute the next window.

definition *next-window* :: ('*a*::*len*) *window-size* ⇒ ('*a*) *window-size*
where
next-window win ≡
if (*uint win*) < (*NWINDOWS* − *1*) *then* (*win* + *1*)
else 0

Compute the previous window.

definition *pre-window* :: ('*a*::*len*) *window-size* ⇒ ('*a*::*len*) *window-size*
where
pre-window win ≡
if (*uint win*) > *0 then* (*win* − *1*)
else (*word-of-int* (*NWINDOWS* − *1*))

write an output register. Also write ur+16 of the previous window.

definition *out-reg-mod* :: *word32* ⇒ ('*a*::*len*) *window-size* ⇒ *user-reg-type* ⇒
('*a*) *sparc-state* ⇒ ('*a*) *sparc-state*
where
out-reg-mod data-w32 win ur state ≡
let state' = *state*(|*user-reg* :=
(*user-reg state*)(*win* := ((*user-reg state*) *win*)(*ur* := *data-w32*))|);
win' = *pre-window win*;
ur' = *ur* + *16*
in
state'(|*user-reg* :=
(*user-reg state'*)(*win'* := ((*user-reg state'*) *win'*)(*ur'* := *data-w32*))|)

Write a input register. Also write ur-16 of the next window.

definition *in-reg-mod* :: *word32* ⇒ ('*a*::*len*) *window-size* ⇒ *user-reg-type* ⇒
('*a*) *sparc-state* ⇒ ('*a*) *sparc-state*
where
in-reg-mod data-w32 win ur state ≡
let state' = *state*(|*user-reg* :=
(*user-reg state*)(*win* := ((*user-reg state*) *win*)(*ur* := *data-w32*))|);
win' = *next-window win*;
ur' = *ur* − *16*
in
state'(|*user-reg* :=
(*user-reg state'*)(*win'* := ((*user-reg state'*) *win'*)(*ur'* := *data-w32*))|)

Do not modify r[0].

definition *user-reg-mod* :: *word32* ⇒ ('*a*::*len*) *window-size* ⇒ *user-reg-type* ⇒
('*a*) *sparc-state* ⇒ ('*a*) *sparc-state*
where
user-reg-mod data-w32 win ur state ≡
if ur = *0 then state*
else if 0 < *ur* ∧ *ur* < *8 then*
global-reg-mod data-w32 (*nat NWINDOWS*) *ur state*
else if 7 < *ur* ∧ *ur* < *16 then*
out-reg-mod data-w32 win ur state
else if 15 < *ur* ∧ *ur* < *24 then*
state(|*user-reg* :=
(*user-reg state*)(*win* := ((*user-reg state*) *win*)(*ur* := *data-w32*))|)
else if 23 ≤ ur ∧ ur < 32 then
in-reg-mod data-w32 win ur state
else state

definition *sys-reg-val* :: *sys-reg* ⇒ ('*a*) *sparc-state* ⇒ *reg-type*
where
sys-reg-val reg state ≡ (*sys-reg state*) *reg*

definition *sys-reg-mod* :: *word32* ⇒ *sys-reg* ⇒
($'a$) *sparc-state* ⇒ ($'a$) *sparc-state*
where
sys-reg-mod data-w32 sys state ≡ *state*(|*sys-reg* := (*sys-reg state*)(*sys* := *data-w32*)|)

The following fucntions deal with physical memory. N.B. Physical memory address in SPARCv8 is 36-bit.

LEON3 doesn't distinguish ASI 8 and 9; 10 and 11 for read access for both user and supervisor. We recently discovered that the compiled machine code by the sparc-elf compiler often reads asi = 10 (user data) when the actual content is store in asi = 8 (user instruction). For testing purposes, we don't distinguish asi = 8,9,10,11 for reading access.

definition *mem-val*:: *asi-type* ⇒ *phys-address* ⇒
($'a$) *sparc-state* ⇒ *mem-val-type option*
where
mem-val asi add state ≡
let asi8 = *word-of-int 8*;
asi9 = *word-of-int 9*;
asi10 = *word-of-int 10*;
asi11 = *word-of-int 11*;
r1 = (*mem state*) *asi8 add*
in
if r1 = *None then*
let r2 = (*mem state*) *asi9 add in*
if r2 = *None then*
let r3 = (*mem state*) *asi10 add in*
if r3 = *None then*
(*mem state*) *asi11 add*
else r3
else r2
else r1

An alternative way to read values from memory. Some implementations may use this definition.

definition *mem-val-alt*:: *asi-type* ⇒ *phys-address* ⇒
($'a$) *sparc-state* ⇒ *mem-val-type option*
where
mem-val-alt asi add state ≡
let r1 = (*mem state*) *asi add*;
asi8 = *word-of-int 8*;
asi9 = *word-of-int 9*;
asi10 = *word-of-int 10*;
asi11 = *word-of-int 11*
in
if r1 = *None* ∧ (*uint asi*) = *8 then*

```
    let r2 = (mem state) asi9 add in
    r2
  else if r1 = None ∧ (uint asi) = 9 then
    let r2 = (mem state) asi8 add in
    r2
  else if r1 = None ∧ (uint asi) = 10 then
    let r2 = (mem state) asi11 add in
    if r2 = None then
      let r3 = (mem state) asi8 add in
      if r3 = None then
        (mem state) asi9 add
      else r3
    else r2
  else if r1 = None ∧ (uint asi) = 11 then
    let r2 = (mem state) asi10 add in
    if r2 = None then
      let r3 = (mem state) asi8 add in
      if r3 = None then
        (mem state) asi9 add
      else r3
    else r2
  else r1

definition mem-mod :: asi-type ⇒ phys-address ⇒ mem-val-type ⇒
                      ('a) sparc-state ⇒ ('a) sparc-state
where
mem-mod asi addr val state ≡
  let state1 = state(|mem := (mem state)
    (asi := ((mem state) asi)(addr := Some val))|)
  in — Only allow one of asi 8 and 9 (10 and 11) to have value.
  if (uint asi) = 8 ∨ (uint asi) = 10 then
    let asi2 = word-of-int ((uint asi) + 1) in
    state1(|mem := (mem state1)
      (asi2 := ((mem state1) asi2)(addr := None))|)
  else if (uint asi) = 9 ∨ (uint asi) = 11 then
    let asi2 = word-of-int ((uint asi) − 1) in
    state1(|mem := (mem state1)(asi2 := ((mem state1) asi2)(addr := None))|)
  else state1
```

An alternative way to write memory. This method insists that for each address, it can only hold a value in one of ASI = 8,9,10,11.

```
definition mem-mod-alt :: asi-type ⇒ phys-address ⇒ mem-val-type ⇒
                          ('a) sparc-state ⇒ ('a) sparc-state
where
mem-mod-alt asi addr val state ≡
  let state1 = state(|mem := (mem state)
    (asi := ((mem state) asi)(addr := Some val))|);
    asi8 = word-of-int 8;
```

```
  asi9 = word-of-int 9;
  asi10 = word-of-int 10;
  asi11 = word-of-int 11
in
— Only allow one of asi 8, 9, 10, 11 to have value.
if (uint asi) = 8 then
  let state2 = state1(|mem := (mem state1)
    (asi9 := ((mem state1) asi9)(addr := None))|);
   state3 = state2(|mem := (mem state2)
    (asi10 := ((mem state2) asi10)(addr := None))|);
   state4 = state3(|mem := (mem state3)
    (asi11 := ((mem state3) asi11)(addr := None))|)
  in
  state4
else if (uint asi) = 9 then
  let state2 = state1(|mem := (mem state1)
    (asi8 := ((mem state1) asi8)(addr := None))|);
   state3 = state2(|mem := (mem state2)
    (asi10 := ((mem state2) asi10)(addr := None))|);
   state4 = state3(|mem := (mem state3)
    (asi11 := ((mem state3) asi11)(addr := None))|)
  in
  state4
else if (uint asi) = 10 then
  let state2 = state1(|mem := (mem state1)
    (asi9 := ((mem state1) asi9)(addr := None))|);
   state3 = state2(|mem := (mem state2)
    (asi8 := ((mem state2) asi8)(addr := None))|);
   state4 = state3(|mem := (mem state3)
    (asi11 := ((mem state3) asi11)(addr := None))|)
  in
  state4
else if (uint asi) = 11 then
  let state2 = state1(|mem := (mem state1)
    (asi9 := ((mem state1) asi9)(addr := None))|);
   state3 = state2(|mem := (mem state2)
    (asi10 := ((mem state2) asi10)(addr := None))|);
   state4 = state3(|mem := (mem state3)
    (asi8 := ((mem state3) asi8)(addr := None))|)
  in
  state4
else state1
```

context
  includes *bit-operations-syntax*
begin

Given an ASI (word8), an address (word32) addr, read the 32bit value from

the memory addresses starting from address addr' where addr' = addr exception that the last two bits are 0's. That is, read the data from addr', addr'+1, addr'+2, addr'+3.

```
definition mem-val-w32 :: asi-type ⇒ phys-address ⇒
                          ('a) sparc-state ⇒ word32 option
where
mem-val-w32 asi addr state ≡
  let addr' = (AND) addr 0b11111111111111111111111111111100;
      addr0 = addr';
      addr1 = addr' + 1;
      addr2 = addr' + 2;
      addr3 = addr' + 3;
      r0 = mem-val-alt asi addr0 state;
      r1 = mem-val-alt asi addr1 state;
      r2 = mem-val-alt asi addr2 state;
      r3 = mem-val-alt asi addr3 state
  in
  if r0 = None ∨ r1 = None ∨ r2 = None ∨ r3 = None then
    None
  else
    let byte0 = case r0 of Some v ⇒ v;
        byte1 = case r1 of Some v ⇒ v;
        byte2 = case r2 of Some v ⇒ v;
        byte3 = case r3 of Some v ⇒ v
    in
    Some ((OR) ((OR) ((OR) ((ucast(byte0)) << 24)
                           ((ucast(byte1)) << 16))
                    ((ucast(byte2)) << 8))
               (ucast(byte3)))
```

Let *addr'* be *addr* with last two bits set to 0's. Write the 32bit data in the memory address *addr'* (and the following 3 addresses). *byte-mask* decides which byte of the 32bits are written.

```
definition mem-mod-w32 :: asi-type ⇒ phys-address ⇒ word4 ⇒ word32 ⇒
                          ('a) sparc-state ⇒ ('a) sparc-state
where
mem-mod-w32 asi addr byte-mask data-w32 state ≡
  let addr' = (AND) addr 0b11111111111111111111111111111100;
      addr0 = (OR) addr' 0b00000000000000000000000000000000;
      addr1 = (OR) addr' 0b00000000000000000000000000000001;
      addr2 = (OR) addr' 0b00000000000000000000000000000010;
      addr3 = (OR) addr' 0b00000000000000000000000000000011;
      byte0 = (ucast (data-w32 >> 24))::mem-val-type;
      byte1 = (ucast (data-w32 >> 16))::mem-val-type;
      byte2 = (ucast (data-w32 >> 8))::mem-val-type;
      byte3 = (ucast data-w32)::mem-val-type;
      s0 = if (((AND) byte-mask (0b1000::word4)) >> 3) = 1 then
```

```
        mem-mod asi addr0 byte0 state
      else state;
  s1 = if (((AND) byte-mask (0b0100::word4)) >> 2) = 1 then
        mem-mod asi addr1 byte1 s0
      else s0;
  s2 = if (((AND) byte-mask (0b0010::word4)) >> 1) = 1 then
        mem-mod asi addr2 byte2 s1
      else s1;
  s3 = if ((AND) byte-mask (0b0001::word4)) = 1 then
        mem-mod asi addr3 byte3 s2
      else s2
in
s3
```

The following functions deal with virtual addresses. These are based on functions written by David Sanan.

```
definition load-word-mem :: ('a) sparc-state ⇒ virtua-address ⇒ asi-type ⇒
                            machine-word option
where load-word-mem state va asi ≡
let pair = (virt-to-phys va (mmu state) (mem state)) in
case pair of
  Some pair ⇒ (
    if mmu-readable (get-acc-flag (snd pair)) asi then
      (mem-val-w32 asi (fst pair) state)
    else None)
  | None ⇒ None

definition store-word-mem ::('a) sparc-state ⇒ virtua-address ⇒ machine-word
⇒
                            word4 ⇒ asi-type ⇒ ('a) sparc-state option
where store-word-mem state va wd byte-mask asi ≡
let pair = (virt-to-phys va (mmu state) (mem state)) in
case pair of
  Some pair ⇒ (
    if mmu-writable (get-acc-flag (snd pair)) asi then
      Some (mem-mod-w32 asi (fst pair) byte-mask wd state)
    else None)
  | None ⇒ None

definition icache-val:: cache-type ⇒ ('a) sparc-state ⇒ mem-val-type option
where icache-val c state ≡ icache (cache state) c

definition dcache-val:: cache-type ⇒ ('a) sparc-state ⇒ mem-val-type option
where dcache-val c state ≡ dcache (cache state) c

definition icache-mod :: cache-type ⇒ mem-val-type ⇒
                         ('a) sparc-state ⇒ ('a) sparc-state
where icache-mod c val state ≡
```

```
  state(|cache := ((cache state)
    (|icache := (icache (cache state))(c := Some val)|))|)
```

```
definition dcache-mod :: cache-type ⇒ mem-val-type ⇒
                         ('a) sparc-state ⇒ ('a) sparc-state
where dcache-mod c val state ≡
  state(|cache := ((cache state)
    (|dcache := (dcache (cache state))(c := Some val)|))|)
```

Check if the memory address is in the cache or not.

```
definition icache-miss :: virtua-address ⇒ ('a) sparc-state ⇒ bool
where
icache-miss addr state ≡
  let line-len = 12;
      tag = (ucast (addr >> line-len))::cache-tag;
      line = (ucast (0b0::word1))::cache-line-size
  in
  if (icache-val (tag,line) state) = None then True
  else False
```

Check if the memory address is in the cache or not.

```
definition dcache-miss :: virtua-address ⇒ ('a) sparc-state ⇒ bool
where
dcache-miss addr state ≡
  let line-len = 12;
      tag = (ucast (addr >> line-len))::cache-tag;
      line = (ucast (0b0::word1))::cache-line-size
  in
  if (dcache-val (tag,line) state) = None then True
  else False
```

```
definition read-data-cache:: ('a) sparc-state ⇒ virtua-address ⇒ machine-word
option
where read-data-cache state va ≡
  let tag = (ucast (va >> 12))::word20;
      offset0 = (AND) ((ucast va)::word12) 0b111111111100;
      offset1 = (OR) offset0 0b000000000001;
      offset2 = (OR) offset0 0b000000000010;
      offset3 = (OR) offset0 0b000000000011;
      r0 = dcache-val (tag,offset0) state;
      r1 = dcache-val (tag,offset1) state;
      r2 = dcache-val (tag,offset2) state;
      r3 = dcache-val (tag,offset3) state
  in
  if r0 = None ∨ r1 = None ∨ r2 = None ∨ r3 = None then
```

```
    None
  else
    let byte0 = case r0 of Some v ⇒ v;
        byte1 = case r1 of Some v ⇒ v;
        byte2 = case r2 of Some v ⇒ v;
        byte3 = case r3 of Some v ⇒ v
    in
    Some ((OR) ((OR) ((OR) ((ucast(byte0)) << 24)
                           ((ucast(byte1)) << 16))
                      ((ucast(byte2)) << 8))
                (ucast(byte3)))
```

definition *read-instr-cache*:: ('*a*) *sparc-state* ⇒ *virtua-address* ⇒ *machine-word option*
where *read-instr-cache state va* ≡

```
  let tag = (ucast (va >> 12))::word20;
      offset0 = (AND) ((ucast va)::word12) 0b111111111100;
      offset1 = (OR) offset0 0b000000000001;
      offset2 = (OR) offset0 0b000000000010;
      offset3 = (OR) offset0 0b000000000011;
      r0 = icache-val (tag,offset0) state;
      r1 = icache-val (tag,offset1) state;
      r2 = icache-val (tag,offset2) state;
      r3 = icache-val (tag,offset3) state
  in
  if r0 = None ∨ r1 = None ∨ r2 = None ∨ r3 = None then
    None
  else
    let byte0 = case r0 of Some v ⇒ v;
        byte1 = case r1 of Some v ⇒ v;
        byte2 = case r2 of Some v ⇒ v;
        byte3 = case r3 of Some v ⇒ v
    in
    Some ((OR) ((OR) ((OR) ((ucast(byte0)) << 24)
                           ((ucast(byte1)) << 16))
                      ((ucast(byte2)) << 8))
                (ucast(byte3)))
```

definition *add-data-cache* :: ('*a*) *sparc-state* ⇒ *virtua-address* ⇒ *machine-word* ⇒

word4 ⇒ ('*a*) *sparc-state*
where

```
 add-data-cache state va word byte-mask ≡
   let tag = (ucast (va >> 12))::word20;
       offset0 = (AND) ((ucast va)::word12) 0b111111111100;
       offset1 = (OR) offset0 0b000000000001;
       offset2 = (OR) offset0 0b000000000010;
```

```
    offset3 = (OR) offset0 0b000000000011;
    byte0 = (ucast (word >> 24))::mem-val-type;
    byte1 = (ucast (word >> 16))::mem-val-type;
    byte2 = (ucast (word >> 8))::mem-val-type;
    byte3 = (ucast word)::mem-val-type;
    s0 = if (((AND) byte-mask (0b1000::word4)) >> 3) = 1 then
           dcache-mod (tag,offset0) byte0 state
         else state;
    s1 = if (((AND) byte-mask (0b0100::word4)) >> 2) = 1 then
           dcache-mod (tag,offset1) byte1 s0
         else s0;
    s2 = if (((AND) byte-mask (0b0010::word4)) >> 1) = 1 then
           dcache-mod (tag,offset2) byte2 s1
         else s1;
    s3 = if ((AND) byte-mask (0b0001::word4)) = 1 then
           dcache-mod (tag,offset3) byte3 s2
         else s2
  in s3
```

```
definition add-instr-cache :: ('a) sparc-state ⇒ virtua-address ⇒ machine-word
⇒
  word4 ⇒ ('a) sparc-state
where
 add-instr-cache state va word byte-mask ≡
   let tag = (ucast (va >> 12))::word20;
    offset0 = (AND) ((ucast va)::word12) 0b111111111100;
    offset1 = (OR) offset0 0b000000000001;
    offset2 = (OR) offset0 0b000000000010;
    offset3 = (OR) offset0 0b000000000011;
    byte0 = (ucast (word >> 24))::mem-val-type;
    byte1 = (ucast (word >> 16))::mem-val-type;
    byte2 = (ucast (word >> 8))::mem-val-type;
    byte3 = (ucast word)::mem-val-type;
    s0 = if (((AND) byte-mask (0b1000::word4)) >> 3) = 1 then
           icache-mod (tag,offset0) byte0 state
         else state;
    s1 = if (((AND) byte-mask (0b0100::word4)) >> 2) = 1 then
           icache-mod (tag,offset1) byte1 s0
         else s0;
    s2 = if (((AND) byte-mask (0b0010::word4)) >> 1) = 1 then
           icache-mod (tag,offset2) byte2 s1
         else s1;
    s3 = if ((AND) byte-mask (0b0001::word4)) = 1 then
           icache-mod (tag,offset3) byte3 s2
         else s2
   in s3
```

definition *empty-cache* ::*cache-context* **where** *empty-cache c* ≡ *None*

definition *flush-data-cache*:: ('*a*) *sparc-state* ⇒ ('*a*) *sparc-state* **where**
flush-data-cache state ≡ *state*(|*cache* := ((*cache state*)(|*dcache* := *empty-cache*|))|)

definition *flush-instr-cache*:: ('*a*) *sparc-state* ⇒ ('*a*) *sparc-state* **where**
flush-instr-cache state ≡ *state*(|*cache* := ((*cache state*)(|*icache* := *empty-cache*|))|)

definition *flush-cache-all*:: ('*a*) *sparc-state* ⇒ ('*a*) *sparc-state* **where**
flush-cache-all state ≡ *state*(|*cache* := ((*cache state*)(|
icache := *empty-cache*, *dcache* := *empty-cache*|))|)

Check if the FI or FD bit of CCR is 1. If FI is 1 then flush instruction cache. If FD is 1 then flush data cache.

definition *ccr-flush* :: ('*a*) *sparc-state* ⇒ ('*a*) *sparc-state*
where
ccr-flush state ≡
let ccr-val = *sys-reg-val CCR state*;
— *FI* is bit 21 of *CCR*
fi-val = ((*AND*) *ccr-val* (*0b00000000001000000000000000000000*)) >> *21*;
fd-val = ((*AND*) *ccr-val* (*0b00000000010000000000000000000000*)) >> *22*;
state1 = (*if fi-val* = *1 then flush-instr-cache state else state*)
in
if fd-val = *1 then flush-data-cache state1 else state1*

definition *get-delayed-pool* :: ('*a*) *sparc-state* ⇒ *delayed-write-pool*
where *get-delayed-pool state* ≡ *dwrite state*

definition *exe-pool* :: (*int* × *reg-type* × *CPU-register*) ⇒ (*int* × *reg-type* × *CPU-register*)
where *exe-pool w* ≡ *case w of* (*n*,*v*,*c*) ⇒ ((*n*−*1*),*v*,*c*)

Minus 1 to the delayed count for all the members in the set. Assuming all members have delay > 0.

primrec *delayed-pool-minus* :: *delayed-write-pool* ⇒ *delayed-write-pool*
where
delayed-pool-minus [] = []
|
delayed-pool-minus (*x*#*xs*) = (*exe-pool x*)#(*delayed-pool-minus xs*)

Add a delayed-write to the pool.

definition *delayed-pool-add* :: (*int* × *reg-type* × *CPU-register*) ⇒
('*a*) *sparc-state* ⇒ ('*a*) *sparc-state*
where
delayed-pool-add dw s ≡
let (*i*,*v*,*cr*) = *dw in*
if i = *0 then* — Write the value to the register immediately.
cpu-reg-mod v cr s
else — Add to delayed write pool.

```
  let curr-pool = get-delayed-pool s in
  s(|dwrite := curr-pool@[dw]|)
```

Remove a delayed-write from the pool. Assume that the delayed-write to be removed has delay 0. i.e., it has been executed.

```
definition delayed-pool-rm :: (int × reg-type × CPU-register) ⇒
                              ('a) sparc-state ⇒ ('a) sparc-state
where
delayed-pool-rm dw s ≡
  let curr-pool = get-delayed-pool s in
  case dw of (n,v,cr) ⇒
    (if n = 0 then
      s(|dwrite := List.remove1 dw curr-pool|)
     else s)
```

Remove all the entries with delay = 0, i.e., those that are written.

```
primrec delayed-pool-rm-written :: delayed-write-pool ⇒ delayed-write-pool
where
delayed-pool-rm-written [] = []
|
delayed-pool-rm-written (x#xs) =
  (if fst x = 0 then delayed-pool-rm-written xs else x#(delayed-pool-rm-written xs))
```

```
definition annul-val :: ('a) sparc-state ⇒ bool
where annul-val state ≡ get-annul (state-var state)

definition annul-mod :: bool ⇒ ('a) sparc-state ⇒ ('a) sparc-state
where annul-mod b s ≡ s(|state-var := write-annul b (state-var s)|)

definition reset-trap-val :: ('a) sparc-state ⇒ bool
where reset-trap-val state ≡ get-reset-trap (state-var state)

definition reset-trap-mod :: bool ⇒ ('a) sparc-state ⇒ ('a) sparc-state
where reset-trap-mod b s ≡ s(|state-var := write-reset-trap b (state-var s)|)

definition exe-mode-val :: ('a) sparc-state ⇒ bool
where exe-mode-val state ≡ get-exe-mode (state-var state)

definition exe-mode-mod :: bool ⇒ ('a) sparc-state ⇒ ('a) sparc-state
where exe-mode-mod b s ≡ s(|state-var := write-exe-mode b (state-var s)|)

definition reset-mode-val :: ('a) sparc-state ⇒ bool
where reset-mode-val state ≡ get-reset-mode (state-var state)

definition reset-mode-mod :: bool ⇒ ('a) sparc-state ⇒ ('a) sparc-state
where reset-mode-mod b s ≡ s(|state-var := write-reset-mode b (state-var s)|)
```

definition *err-mode-val* :: ('*a*) *sparc-state* ⇒ *bool*
where *err-mode-val state* ≡ *get-err-mode* (*state-var state*)

definition *err-mode-mod* :: *bool* ⇒ ('*a*) *sparc-state* ⇒ ('*a*) *sparc-state*
where *err-mode-mod b s* ≡ *s*(|*state-var* := *write-err-mode b* (*state-var s*)|)

definition *ticc-trap-type-val* :: ('*a*) *sparc-state* ⇒ *word7*
where *ticc-trap-type-val state* ≡ *get-ticc-trap-type* (*state-var state*)

definition *ticc-trap-type-mod* :: *word7* ⇒ ('*a*) *sparc-state* ⇒ ('*a*) *sparc-state*
where *ticc-trap-type-mod w s* ≡ *s*(|*state-var* := *write-ticc-trap-type w* (*state-var s*)|)

definition *interrupt-level-val* :: ('*a*) *sparc-state* ⇒ *word3*
where *interrupt-level-val state* ≡ *get-interrupt-level* (*state-var state*)

definition *interrupt-level-mod* :: *word3* ⇒ ('*a*) *sparc-state* ⇒ ('*a*) *sparc-state*
where *interrupt-level-mod w s* ≡ *s*(|*state-var* := *write-interrupt-level w* (*state-var s*)|)

definition *store-barrier-pending-val* :: ('*a*) *sparc-state* ⇒ *bool*
where *store-barrier-pending-val state* ≡
get-store-barrier-pending (*state-var state*)

definition *store-barrier-pending-mod* :: *bool* ⇒
('*a*) *sparc-state* ⇒ ('*a*) *sparc-state*
where *store-barrier-pending-mod w s* ≡
s(|*state-var* := *write-store-barrier-pending w* (*state-var s*)|)

definition *pb-block-ldst-byte-val* :: *virtua-address* ⇒ ('*a*) *sparc-state*
⇒ *bool*
where *pb-block-ldst-byte-val add state* ≡
(*atm-ldst-byte* (*state-var state*)) *add*

definition *pb-block-ldst-byte-mod* :: *virtua-address* ⇒ *bool* ⇒
('*a*) *sparc-state* ⇒ ('*a*) *sparc-state*
where *pb-block-ldst-byte-mod add b s* ≡
s(|*state-var* := ((*state-var s*)
(|*atm-ldst-byte* := (*atm-ldst-byte* (*state-var s*))(*add* := *b*)|))|)

We only read the address such that add mod 4 = 0. add mod 4 represents the current word.

definition *pb-block-ldst-word-val* :: *virtua-address* ⇒ ('*a*) *sparc-state*
⇒ *bool*
where *pb-block-ldst-word-val add state* ≡
let add0 = ((*AND*) *add* (*0b11111111111111111111111111111100*::*word32*)) *in*
(*atm-ldst-word* (*state-var state*)) *add0*

We only write the address such that add mod 4 = 0. add mod 4 represents

the current word.

```
definition pb-block-ldst-word-mod :: virtua-address ⇒ bool ⇒
  ('a) sparc-state ⇒ ('a) sparc-state
where pb-block-ldst-word-mod add b s ≡
  let add0 = ((AND) add (0b11111111111111111111111111111100::word32)) in
  s(|state-var := ((state-var s)
    (|atm-ldst-word := (atm-ldst-word (state-var s))(add0 := b)|))|)

definition get-trap-set :: ('a) sparc-state ⇒ Trap set
where get-trap-set state ≡ (traps state)

definition add-trap-set :: Trap ⇒ ('a) sparc-state ⇒ ('a) sparc-state
where add-trap-set t s ≡ s(|traps := (traps s) ∪ {t}|)

definition emp-trap-set :: ('a) sparc-state ⇒ ('a) sparc-state
where emp-trap-set s ≡ s(|traps := {}|)

definition state-undef:: ('a) sparc-state ⇒ bool
where state-undef state ≡ (undef state)
```

The *memory-read* interface that conforms with the SPARCv8 manual.

```
definition memory-read :: asi-type ⇒ virtua-address ⇒
                          ('a) sparc-state ⇒
                          ((word32 option) × ('a) sparc-state)
where memory-read asi addr state ≡
  let asi-int = uint asi in — See Page 25 and 35 for ASI usage in LEON 3FT.
  if asi-int = 1 then — Forced cache miss.
    — Directly read from memory.
    let r1 = load-word-mem state addr (word-of-int 8) in
    if r1 = None then
      let r2 = load-word-mem state addr (word-of-int 10) in
      if r2 = None then
        (None,state)
      else (r2,state)
    else (r1,state)
  else if asi-int = 2 then — System registers.
    — See Table 19, Page 34 for System Register address map in LEON 3FT.
    if uint addr = 0 then — Cache control register.
      ((Some (sys-reg-val CCR state)), state)
    else if uint addr = 8 then — Instruction cache configuration register.
      ((Some (sys-reg-val ICCR state)), state)
    else if uint addr = 12 then — Data cache configuration register.
      ((Some (sys-reg-val DCCR state)), state)
    else — Invalid address.
      (None, state)
  else if asi-int ∈ {8,9} then — Access instruction memory.
    let ccr-val = (sys-reg state) CCR in
    if ccr-val AND 1 ≠ 0 then — Cache is enabled. Update cache.
    — We don't go through the tradition, i.e., read from cache first,
```

```
  — if the address is not cached, then read from memory,
  — because performance is not an issue here.
  — Thus we directly read from memory and update the cache.
    let data = load-word-mem state addr asi in
    case data of
    Some w ⇒ (Some w,(add-instr-cache state addr w (0b1111::word4)))
    |None ⇒ (None, state)
  else — Cache is disabled. Just read from memory.
    ((load-word-mem state addr asi),state)
else if asi-int ∈ {10,11} then — Access data memory.
  let ccr-val = (sys-reg state) CCR in
  if ccr-val AND 1 ≠ 0 then — Cache is enabled. Update cache.
  — We don't go through the tradition, i.e., read from cache first,
  — if the address is not cached, then read from memory,
  — because performance is not an issue here.
  — Thus we directly read from memory and update the cache.
    let data = load-word-mem state addr asi in
    case data of
    Some w ⇒ (Some w,(add-data-cache state addr w (0b1111::word4)))
    |None ⇒ (None, state)
  else — Cache is disabled. Just read from memory.
    ((load-word-mem state addr asi),state)
— We don't access instruction cache tag. i.e., asi = 12.
else if asi-int = 13 then — Read instruction cache data.
  let cache-result = read-instr-cache state addr in
  case cache-result of
  Some w ⇒ (Some w, state)
  |None ⇒ (None, state)
— We don't access data cache tag. i.e., asi = 14.
else if asi-int = 15 then — Read data cache data.
  let cache-result = read-data-cache state addr in
  case cache-result of
  Some w ⇒ (Some w, state)
  |None ⇒ (None, state)
else if asi-int ∈ {16,17} then — Flush entire instruction/data cache.
  (None, state) — Has no effect for memory read.
else if asi-int ∈ {20,21} then — MMU diagnostic cache access.
  (None, state) — Not considered in this model.
else if asi-int = 24 then — Flush cache and TLB in LEON3.
  — But is not used for memory read.
  (None, state)
else if asi-int = 25 then — MMU registers.
  — Treat MMU registers as memory addresses that are not in the main memory.
  ((mmu-reg-val (mmu state) addr), state)
else if asi-int = 28 then — MMU bypass.
  — Directly use addr as a physical address.
  — Append 0000 in the front of addr.
  — In this case, (ucast addr) suffices.
  ((mem-val-w32 asi (ucast addr) state), state)
```

else if asi-int = 29 then — MMU diagnostic access.
(*None*, *state*) — Not considered in this model.
else — Not considered in this model.
(*None*, *state*)

Get the value of a memory address and an ASI.

definition *mem-val-asi*:: *asi-type* ⇒ *phys-address* ⇒
('*a*) *sparc-state* ⇒ *mem-val-type option*
where *mem-val-asi asi add state* ≡ (*mem state*) *asi add*

Check if an address is used in ASI 9 or 11.

definition *sup-addr* :: *phys-address* ⇒ ('*a*) *sparc-state* ⇒ *bool*
where
sup-addr addr state ≡
let addr' = (*AND*) *addr 0b11111111111111111111111111111100*;
addr0 = (*OR*) *addr'* *0b00000000000000000000000000000000*;
addr1 = (*OR*) *addr'* *0b00000000000000000000000000000001*;
addr2 = (*OR*) *addr'* *0b00000000000000000000000000000010*;
addr3 = (*OR*) *addr'* *0b00000000000000000000000000000011*;
r0 = *mem-val-asi 9 addr0 state*;
r1 = *mem-val-asi 9 addr1 state*;
r2 = *mem-val-asi 9 addr2 state*;
r3 = *mem-val-asi 9 addr3 state*;
r4 = *mem-val-asi 11 addr0 state*;
r5 = *mem-val-asi 11 addr1 state*;
r6 = *mem-val-asi 11 addr2 state*;
r7 = *mem-val-asi 11 addr3 state*
in
if r0 = *None* ∧ *r1* = *None* ∧ *r2* = *None* ∧ *r3* = *None* ∧
r4 = *None* ∧ *r5* = *None* ∧ *r6* = *None* ∧ *r7* = *None*
then False
else True

The *memory-write* interface that conforms with SPARCv8 manual.

LEON3 forbids user to write an address in ASI 9 and 11.

definition *memory-write-asi* :: *asi-type* ⇒ *virtua-address* ⇒ *word4* ⇒ *word32* ⇒
('*a*) *sparc-state* ⇒
('*a*) *sparc-state option*
where
memory-write-asi asi addr byte-mask data-w32 state ≡
let asi-int = *uint asi*; — See Page 25 and 35 for ASI usage in LEON 3FT.
psr-val = *cpu-reg-val PSR state*;
s-val = *get-S psr-val*
in
if asi-int = *1 then* — Forced cache miss.
— Directly write to memory.

— Assuming writing into *asi = 10*.
store-word-mem state addr data-w32 byte-mask (*word-of-int 10*)
else if asi-int = 2 then — System registers.
— See Table 19, Page 34 for System Register address map in LEON 3FT.
if uint addr = 0 then — Cache control register.
let s1 = (*sys-reg-mod data-w32 CCR state*) *in*
— Flush the instruction cache if FI of CCR is 1;
— flush the data cache if FD of CCR is 1.
Some (*ccr-flush s1*)
else if uint addr = 8 then — Instruction cache configuration register.
Some (*sys-reg-mod data-w32 ICCR state*)
else if uint addr = 12 then — Data cache configuration register.
Some (*sys-reg-mod data-w32 DCCR state*)
else — Invalid address.
None
else if asi-int ∈ {*8*,*9*} *then* — Access instruction memory.
— Write to memory. LEON3 does write-through. Both cache and the memory are updated.
let ns = add-instr-cache state addr data-w32 byte-mask in
store-word-mem ns addr data-w32 byte-mask asi
else if asi-int ∈ {*10*,*11*} *then* — Access data memory.
— Write to memory. LEON3 does write-through. Both cache and the memory are updated.
let ns = add-data-cache state addr data-w32 byte-mask in
store-word-mem ns addr data-w32 byte-mask asi
— We don't access instruction cache tag. i.e., *asi = 12*.
else if asi-int = 13 then — Write instruction cache data.
Some (*add-instr-cache state addr data-w32* (*0b1111*::*word4*))
— We don't access data cache tag. i.e., asi = 14.
else if asi-int = 15 then — Write data cache data.
Some (*add-data-cache state addr data-w32* (*0b1111*::*word4*))
else if asi-int = 16 then — Flush instruction cache.
Some (*flush-instr-cache state*)
else if asi-int = 17 then — Flush data cache.
Some (*flush-data-cache state*)
else if asi-int ∈ {*20*,*21*} *then* — MMU diagnostic cache access.
None — Not considered in this model.
else if asi-int = 24 then — Flush TLB and cache in LEON3.
— We don't consider TLB here.
Some (*flush-cache-all state*)
else if asi-int = 25 then — MMU registers.
— Treat MMU registers as memory addresses that are not in the main memory.
let mmu-state′ = mmu-reg-mod (*mmu state*) *addr data-w32 in*
case mmu-state′ of
Some mmus ⇒ *Some* (*state*(|*mmu* := *mmus*|))
|*None* ⇒ *None*
else if asi-int = 28 then — MMU bypass.
— Write to virtual address as physical address.
— Append 0000 in front of addr.

```
    Some (mem-mod-w32 asi (ucast addr) byte-mask data-w32 state)
  else if asi-int = 29 then — MMU diagnostic access.
    None — Not considered in this model.
  else — Not considered in this model.
    None


definition memory-write :: asi-type ⇒ virtua-address ⇒ word4 ⇒ word32 ⇒
                           ('a) sparc-state ⇒
                           ('a) sparc-state option
where
memory-write asi addr byte-mask data-w32 state ≡
  let result = memory-write-asi asi addr byte-mask data-w32 state in
  case result of
  None ⇒ None
  | Some s1 ⇒ Some (store-barrier-pending-mod False s1)
```

monad for sequential operations over the register representation

```
type-synonym ('a,'e) sparc-state-monad = (('a) sparc-state,'e) det-monad
```

Given a word32 value, a cpu register, write the value in the cpu register.

```
definition write-cpu :: word32 ⇒ CPU-register ⇒ ('a,unit) sparc-state-monad
where write-cpu w cr ≡
  do
    modify (λs. (cpu-reg-mod w cr s));
    return ()
  od

definition write-cpu-tt :: word8 ⇒ ('a,unit) sparc-state-monad
where write-cpu-tt w ≡
  do
    tbr-val ← gets (λs. (cpu-reg-val TBR s));
    new-tbr-val ← gets (λs. (write-tt w tbr-val));
    write-cpu new-tbr-val TBR;
    return ()
  od
```

Given a word32 value, a word4 window, a user register, write the value in the user register. N.B. CWP is a 5 bit value, but we only use the last 4 bits, since there are only 16 windows.

```
definition write-reg :: word32 ⇒ ('a::len) word ⇒ user-reg-type ⇒
  ('a,unit) sparc-state-monad
where write-reg w win ur ≡
  do
    modify (λs.(user-reg-mod w win ur s));
    return ()
  od
```

definition *set-annul* :: *bool* ⇒ (*'a*,*unit*) *sparc-state-monad*
where *set-annul b* ≡
  do
    modify (λ*s*. (*annul-mod b s*));
    return ()
  od

definition *set-reset-trap* :: *bool* ⇒ (*'a*,*unit*) *sparc-state-monad*
where *set-reset-trap b* ≡
  do
    modify (λ*s*. (*reset-trap-mod b s*));
    return ()
  od

definition *set-exe-mode* :: *bool* ⇒ (*'a*,*unit*) *sparc-state-monad*
where *set-exe-mode b* ≡
  do
    modify (λ*s*. (*exe-mode-mod b s*));
    return ()
  od

definition *set-reset-mode* :: *bool* ⇒ (*'a*,*unit*) *sparc-state-monad*
where *set-reset-mode b* ≡
  do
    modify (λ*s*. (*reset-mode-mod b s*));
    return ()
  od

definition *set-err-mode* :: *bool* ⇒ (*'a*,*unit*) *sparc-state-monad*
where *set-err-mode b* ≡
  do
    modify (λ*s*. (*err-mode-mod b s*));
    return ()
  od

fun *get-delayed-0* :: (*int* × *reg-type* × *CPU-register*) *list* ⇒
  (*int* × *reg-type* × *CPU-register*) *list*
where
get-delayed-0 [] = []
|
get-delayed-0 (*x* # *xs*) =
  (*if fst x* = *0 then x* # (*get-delayed-0 xs*)
  else get-delayed-0 xs)

Get a list of delayed-writes with delay 0.

definition *get-delayed-write* :: *delayed-write-pool* ⇒ (*int* × *reg-type* × *CPU-register*)
list
where
get-delayed-write dwp ≡ *get-delayed-0 dwp*

definition *delayed-write* :: (*int* × *reg-type* × *CPU-register*) ⇒ ('*a*) *sparc-state* ⇒
('*a*) *sparc-state*
where *delayed-write dw s* ≡
let (*n*,*v*,*r*) = *dw in*
if n = *0 then*
cpu-reg-mod v r s
else s

primrec *delayed-write-all* :: (*int* × *reg-type* × *CPU-register*) *list* ⇒
('*a*) *sparc-state* ⇒ ('*a*) *sparc-state*
where *delayed-write-all* [] *s* = *s*
|*delayed-write-all* (*x* # *xs*) *s* =
delayed-write-all xs (*delayed-write x s*)

primrec *delayed-pool-rm-list* :: (*int* × *reg-type* × *CPU-register*) *list*⇒
('*a*) *sparc-state* ⇒ ('*a*) *sparc-state*
where *delayed-pool-rm-list* [] *s* = *s*
|*delayed-pool-rm-list* (*x* # *xs*) *s* =
delayed-pool-rm-list xs (*delayed-pool-rm x s*)

definition *delayed-pool-write* :: ('*a*) *sparc-state* ⇒ ('*a*) *sparc-state*
where *delayed-pool-write s* ≡
let dwp0 = *get-delayed-pool s*;
dwp1 = *delayed-pool-minus dwp0*;
wl = *get-delayed-write dwp1*;
s1 = *delayed-write-all wl s*;
s2 = *delayed-pool-rm-list wl s1*
in s2

definition *raise-trap* :: *Trap* ⇒ ('*a*,*unit*) *sparc-state-monad*
where *raise-trap t* ≡
do
modify (λ*s*. (*add-trap-set t s*));
return ()
od

end

end

23 SPARC instruction model

theory *Sparc-Instruction*
imports *Main Sparc-Types Sparc-State HOL−Eisbach.Eisbach-Tools*
begin

This theory provides a formal model for assembly instruction to be executed in the model.

An instruction is defined as a tuple composed of a *sparc-operation* element, defining the operation the instruction carries out, and a list of operands *inst-operand*. *inst-operand* can be a user register *user-reg* or a memory address *mem-add-type*.

datatype *inst-operand* =
W5 word5
|*W30 word30*
|*W22 word22*
|*Cond word4*
|*Flag word1*
|*Asi asi-type*
|*Simm13 word13*
|*Opf word9*
|*Imm7 word7*

primrec *get-operand-w5*::*inst-operand* ⇒ *word5*
where *get-operand-w5* (*W5 r*) = *r*

primrec *get-operand-w30*::*inst-operand* ⇒ *word30*
where *get-operand-w30* (*W30 r*) = *r*

primrec *get-operand-w22*::*inst-operand* ⇒ *word22*
where *get-operand-w22* (*W22 r*) = *r*

primrec *get-operand-cond*::*inst-operand* ⇒ *word4*
where *get-operand-cond* (*Cond r*) = *r*

primrec *get-operand-flag*::*inst-operand* ⇒ *word1*
where *get-operand-flag* (*Flag r*) = *r*

primrec *get-operand-asi*::*inst-operand* ⇒ *asi-type*
where *get-operand-asi* (*Asi r*) = *r*

primrec *get-operand-simm13*::*inst-operand* ⇒ *word13*
where *get-operand-simm13* (*Simm13 r*) = *r*

primrec *get-operand-opf*::*inst-operand* ⇒ *word9*
where *get-operand-opf* (*Opf r*) = *r*

primrec *get-operand-imm7*:: *inst-operand* ⇒ *word7*
where *get-operand-imm7* (*Imm7 r*) = *r*

context
includes *bit-operations-syntax*
begin

type-synonym *instruction* = (*sparc-operation* × *inst-operand list*)

definition *get-op*::*word32* ⇒ *int*

where *get-op w* ≡ *uint* (*w* >> *30*)

definition *get-op2*::*word32* ⇒ *int*
where *get-op2 w* ≡
  let mask-op2 = *0b00000001111000000000000000000000 in*
  uint (((*AND*) *mask-op2 w*) >> *22*)

definition *get-op3*::*word32* ⇒ *int*
where *get-op3 w* ≡
  let mask-op3 = *0b00000001111111000000000000000000 in*
  uint (((*AND*) *mask-op3 w*) >> *19*)

definition *get-disp30*::*word32* ⇒ *int*
where *get-disp30 w* ≡
  let mask-disp30 = *0b00111111111111111111111111111111 in*
  uint ((*AND*) *mask-disp30 w*)

definition *get-a*::*word32* ⇒ *int*
where *get-a w* ≡
  let mask-a = *0b00100000000000000000000000000000 in*
  uint (((*AND*) *mask-a w*) >> *29*)

definition *get-cond*::*word32* ⇒ *int*
where *get-cond w* ≡
  let mask-cond = *0b00011110000000000000000000000000 in*
  uint (((*AND*) *mask-cond w*) >> *25*)

definition *get-disp-imm22*::*word32* ⇒ *int*
where *get-disp-imm22 w* ≡
  let mask-disp-imm22 = *0b00000000001111111111111111111111 in*
  uint ((*AND*) *mask-disp-imm22 w*)

definition *get-rd*::*word32* ⇒ *int*
where *get-rd w* ≡
  let mask-rd = *0b00111110000000000000000000000000 in*
  uint (((*AND*) *mask-rd w*) >> *25*)

definition *get-rs1*::*word32* ⇒ *int*
where *get-rs1 w* ≡
  let mask-rs1 = *0b00000000000001111100000000000000 in*
  uint (((*AND*) *mask-rs1 w*) >> *14*)

definition *get-i*::*word32* ⇒ *int*
where *get-i w* ≡
  let mask-i = *0b00000000000000000010000000000000 in*
  uint (((*AND*) *mask-i w*) >> *13*)

definition *get-opf*::*word32* ⇒ *int*
where *get-opf w* ≡

```
  let mask-opf = 0b00000000000000000011111111100000 in
  uint (((AND) mask-opf w) >> 5)

definition get-rs2::word32 ⇒ int
where get-rs2 w ≡
  let mask-rs2 = 0b00000000000000000000000000011111 in
  uint ((AND) mask-rs2 w)

definition get-simm13::word32 ⇒ int
where get-simm13 w ≡
  let mask-simm13 = 0b00000000000000000001111111111111 in
  uint ((AND) mask-simm13 w)

definition get-asi::word32 ⇒ int
where get-asi w ≡
  let mask-asi = 0b00000000000000000001111111100000 in
  uint (((AND) mask-asi w) >> 5)

definition get-trap-cond:: word32 ⇒ int
where get-trap-cond w ≡
  let mask-cond = 0b00011110000000000000000000000000 in
  uint (((AND) mask-cond w) >> 25)

definition get-trap-imm7:: word32 ⇒ int
where get-trap-imm7 w ≡
  let mask-imm7 = 0b00000000000000000000000001111111 in
  uint ((AND) mask-imm7 w)

definition parse-instr-f1::word32 ⇒
  (Exception list + instruction)
where — CALL, with a single operand disp30+00
parse-instr-f1 w ≡
  Inr (call-type CALL,[W30 (word-of-int (get-disp30 w))])

definition parse-instr-f2::word32 ⇒
  (Exception list + instruction)
where parse-instr-f2 w ≡
  let op2 = get-op2 w in
  if op2 = uint(0b100::word3) then — SETHI or NOP
    let rd = get-rd w in
    let imm22 = get-disp-imm22 w in
    if rd = 0 ∧ imm22 = 0 then — NOP
      Inr (nop-type NOP,[])
    else — SETHI, with operands [imm22,rd]
      Inr (sethi-type SETHI,[(W22 (word-of-int imm22)),
                  (W5 (word-of-int rd))])
  else if op2 = uint(0b010::word3) then — Bicc, with operands [a,disp22]
    let cond = get-cond w in
    let flaga = Flag (word-of-int (get-a w)) in
```

```
  let disp22 = W22 (word-of-int (get-disp-imm22 w)) in
  if cond = uint(0b0001::word4) then — BE
    Inr (bicc-type BE,[flaga,disp22])
  else if cond = uint(0b1001::word4) then — BNE
    Inr (bicc-type BNE,[flaga,disp22])
  else if cond = uint(0b1100::word4) then — BGU
    Inr (bicc-type BGU,[flaga,disp22])
  else if cond = uint(0b0010::word4) then — BLE
    Inr (bicc-type BLE,[flaga,disp22])
  else if cond = uint(0b0011::word4) then — BL
    Inr (bicc-type BL,[flaga,disp22])
  else if cond = uint(0b1011::word4) then — BGE
    Inr (bicc-type BGE,[flaga,disp22])
  else if cond = uint(0b0110::word4) then — BNEG
    Inr (bicc-type BNEG,[flaga,disp22])
  else if cond = uint(0b1010::word4) then — BG
    Inr (bicc-type BG,[flaga,disp22])
  else if cond = uint(0b0101::word4) then — BCS
    Inr (bicc-type BCS,[flaga,disp22])
  else if cond = uint(0b0100::word4) then — BLEU
    Inr (bicc-type BLEU,[flaga,disp22])
  else if cond = uint(0b1101::word4) then — BCC
    Inr (bicc-type BCC,[flaga,disp22])
  else if cond = uint(0b1000::word4) then — BA
    Inr (bicc-type BA,[flaga,disp22])
  else if cond = uint(0b0000::word4) then — BN
    Inr (bicc-type BN,[flaga,disp22])
  else if cond = uint(0b1110::word4) then — BPOS
    Inr (bicc-type BPOS,[flaga,disp22])
  else if cond = uint(0b1111::word4) then — BVC
    Inr (bicc-type BVC,[flaga,disp22])
  else if cond = uint(0b0111::word4) then — BVS
    Inr (bicc-type BVS,[flaga,disp22])
  else Inl [invalid-cond-f2]
else Inl [invalid-op2-f2]
```

We don't consider floating-point operations, so we don't consider the third type of format 3.

```
definition parse-instr-f3::word32 ⇒ (Exception list + instruction)
where parse-instr-f3 w ≡
  let this-op = get-op w in
  let rd = get-rd w in
  let op3 = get-op3 w in
  let rs1 = get-rs1 w in
  let flagi = get-i w in
  let asi = get-asi w in
  let rs2 = get-rs2 w in
  let simm13 = get-simm13 w in
```

```
if this-op = uint(0b11::word2) then — Load and Store
— If an instruction accesses alternative space but flagi = 1,
— may need to throw a trap.
  if op3 = uint(0b001001::word6) then — LDSB
    if flagi = 1 then — Operant list is [i,rs1,simm13,rd]
      Inr (load-store-type LDSB,[(Flag (word-of-int flagi)),
                                 (W5 (word-of-int rs1)),
                                 (Simm13 (word-of-int simm13)),
                                 (W5 (word-of-int rd))])
    else — Operant list is [i,rs1,rs2,rd]
      Inr (load-store-type LDSB,[(Flag (word-of-int flagi)),
                                 (W5 (word-of-int rs1)),
                                 (W5 (word-of-int rs2)),
                                 (W5 (word-of-int rd))])
  else if op3 = uint(0b011001::word6) then — LDSBA
    Inr (load-store-type LDSBA,[(Flag (word-of-int flagi)),
                                (W5 (word-of-int rs1)),
                                (W5 (word-of-int rs2)),
                                (Asi (word-of-int asi)),
                                (W5 (word-of-int rd))])
  else if op3 = uint(0b001010::word6) then — LDSH
    if flagi = 1 then — Operant list is [i,rs1,simm13,rd]
      Inr (load-store-type LDSH,[(Flag (word-of-int flagi)),
                                 (W5 (word-of-int rs1)),
                                 (Simm13 (word-of-int simm13)),
                                 (W5 (word-of-int rd))])
    else — Operant list is [i,rs1,rs2,rd]
      Inr (load-store-type LDSH,[(Flag (word-of-int flagi)),
                                 (W5 (word-of-int rs1)),
                                 (W5 (word-of-int rs2)),
                                 (W5 (word-of-int rd))])
  else if op3 = uint(0b011010::word6) then — LDSHA
    Inr (load-store-type LDSHA,[(Flag (word-of-int flagi)),
                                (W5 (word-of-int rs1)),
                                (W5 (word-of-int rs2)),
                                (Asi (word-of-int asi)),
                                (W5 (word-of-int rd))])
  else if op3 = uint(0b000001::word6) then — LDUB
    if flagi = 1 then — Operant list is [i,rs1,simm13,rd]
      Inr (load-store-type LDUB,[(Flag (word-of-int flagi)),
                                 (W5 (word-of-int rs1)),
                                 (Simm13 (word-of-int simm13)),
                                 (W5 (word-of-int rd))])
    else — Operant list is [i,rs1,rs2,rd]
      Inr (load-store-type LDUB,[(Flag (word-of-int flagi)),
                                 (W5 (word-of-int rs1)),
                                 (W5 (word-of-int rs2)),
                                 (W5 (word-of-int rd))])
  else if op3 = uint(0b010001::word6) then — LDUBA
```

```
   Inr (load-store-type LDUBA,[(Flag (word-of-int flagi)),
                               (W5 (word-of-int rs1)),
                               (W5 (word-of-int rs2)),
                               (Asi (word-of-int asi)),
                               (W5 (word-of-int rd))])
else if op3 = uint(0b000010::word6) then — LDUH
  if flagi = 1 then — Operant list is [i,rs1,simm13,rd]
     Inr (load-store-type LDUH,[(Flag (word-of-int flagi)),
                               (W5 (word-of-int rs1)),
                               (Simm13 (word-of-int simm13)),
                               (W5 (word-of-int rd))])
  else — Operant list is [i,rs1,rs2,rd]
     Inr (load-store-type LDUH,[(Flag (word-of-int flagi)),
                               (W5 (word-of-int rs1)),
                               (W5 (word-of-int rs2)),
                               (W5 (word-of-int rd))])
else if op3 = uint(0b010010::word6) then — LDUHA
   Inr (load-store-type LDUHA,[(Flag (word-of-int flagi)),
                               (W5 (word-of-int rs1)),
                               (W5 (word-of-int rs2)),
                               (Asi (word-of-int asi)),
                               (W5 (word-of-int rd))])
else if op3 = uint(0b000000::word6) then — LD
  if flagi = 1 then — Operant list is [i,rs1,simm13,rd]
     Inr (load-store-type LD,[(Flag (word-of-int flagi)),
                              (W5 (word-of-int rs1)),
                              (Simm13 (word-of-int simm13)),
                              (W5 (word-of-int rd))])
  else — Operant list is [i,rs1,rs2,rd]
     Inr (load-store-type LD,[(Flag (word-of-int flagi)),
                              (W5 (word-of-int rs1)),
                              (W5 (word-of-int rs2)),
                              (W5 (word-of-int rd))])
else if op3 = uint(0b010000::word6) then — LDA
   Inr (load-store-type LDA,[(Flag (word-of-int flagi)),
                              (W5 (word-of-int rs1)),
                              (W5 (word-of-int rs2)),
                              (Asi (word-of-int asi)),
                              (W5 (word-of-int rd))])
else if op3 = uint(0b000011::word6) then — LDD
  if flagi = 1 then — Operant list is [i,rs1,simm13,rd]
     Inr (load-store-type LDD,[(Flag (word-of-int flagi)),
                              (W5 (word-of-int rs1)),
                              (Simm13 (word-of-int simm13)),
                              (W5 (word-of-int rd))])
  else — Operant list is [i,rs1,rs2,rd]
     Inr (load-store-type LDD,[(Flag (word-of-int flagi)),
                              (W5 (word-of-int rs1)),
                              (W5 (word-of-int rs2)),
```

```
                (W5 (word-of-int rd))])
else if op3 = uint(0b010011::word6) then — LDDA
  Inr (load-store-type LDDA,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (W5 (word-of-int rs2)),
                (Asi (word-of-int asi)),
                (W5 (word-of-int rd))])
else if op3 = uint(0b001101::word6) then — LDSTUB
  if flagi = 1 then — Operant list is [i,rs1,simm13,rd]
    Inr (load-store-type LDSTUB,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (Simm13 (word-of-int simm13)),
                (W5 (word-of-int rd))])
  else — Operant list is [i,rs1,rs2,rd]
    Inr (load-store-type LDSTUB,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (W5 (word-of-int rs2)),
                (W5 (word-of-int rd))])
else if op3 = uint(0b011101::word6) then — LDSTUBA
  Inr (load-store-type LDSTUBA,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (W5 (word-of-int rs2)),
                (Asi (word-of-int asi)),
                (W5 (word-of-int rd))])
else if op3 = uint(0b000101::word6) then — STB
  if flagi = 1 then — Operant list is [i,rs1,simm13,rd]
    Inr (load-store-type STB,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (Simm13 (word-of-int simm13)),
                (W5 (word-of-int rd))])
  else — Operant list is [i,rs1,rs2,rd]
    Inr (load-store-type STB,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (W5 (word-of-int rs2)),
                (W5 (word-of-int rd))])
else if op3 = uint(0b010101::word6) then — STBA
  Inr (load-store-type STBA,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (W5 (word-of-int rs2)),
                (Asi (word-of-int asi)),
                (W5 (word-of-int rd))])
else if op3 = uint(0b000110::word6) then — STH
  if flagi = 1 then — Operant list is [i,rs1,simm13,rd]
    Inr (load-store-type STH,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (Simm13 (word-of-int simm13)),
                (W5 (word-of-int rd))])
  else — Operant list is [i,rs1,rs2,rd]
    Inr (load-store-type STH,[(Flag (word-of-int flagi)),
```

```
                    (W5 (word-of-int rs1)),
                    (W5 (word-of-int rs2)),
                    (W5 (word-of-int rd))])
else if op3 = uint(0b010110::word6) then — STHA
  Inr (load-store-type STHA,[(Flag (word-of-int flagi)),
                    (W5 (word-of-int rs1)),
                    (W5 (word-of-int rs2)),
                    (Asi (word-of-int asi)),
                    (W5 (word-of-int rd))])
else if op3 = uint(0b000100::word6) then — ST
  if flagi = 1 then — Operant list is [i,rs1,simm13,rd]
    Inr (load-store-type ST,[(Flag (word-of-int flagi)),
                    (W5 (word-of-int rs1)),
                    (Simm13 (word-of-int simm13)),
                    (W5 (word-of-int rd))])
  else — Operant list is [i,rs1,rs2,rd]
    Inr (load-store-type ST,[(Flag (word-of-int flagi)),
                    (W5 (word-of-int rs1)),
                    (W5 (word-of-int rs2)),
                    (W5 (word-of-int rd))])
else if op3 = uint(0b010100::word6) then — STA
  Inr (load-store-type STA,[(Flag (word-of-int flagi)),
                    (W5 (word-of-int rs1)),
                    (W5 (word-of-int rs2)),
                    (Asi (word-of-int asi)),
                    (W5 (word-of-int rd))])
else if op3 = uint(0b000111::word6) then — STD
  if flagi = 1 then — Operant list is [i,rs1,simm13,rd]
    Inr (load-store-type STD,[(Flag (word-of-int flagi)),
                    (W5 (word-of-int rs1)),
                    (Simm13 (word-of-int simm13)),
                    (W5 (word-of-int rd))])
  else — Operant list is [i,rs1,rs2,rd]
    Inr (load-store-type STD,[(Flag (word-of-int flagi)),
                    (W5 (word-of-int rs1)),
                    (W5 (word-of-int rs2)),
                    (W5 (word-of-int rd))])
else if op3 = uint(0b010111::word6) then — STDA
  Inr (load-store-type STDA,[(Flag (word-of-int flagi)),
                    (W5 (word-of-int rs1)),
                    (W5 (word-of-int rs2)),
                    (Asi (word-of-int asi)),
                    (W5 (word-of-int rd))])
else if op3 = uint(0b001111::word6) then — SWAP
  if flagi = 1 then — Operant list is [i,rs1,simm13,rd]
    Inr (load-store-type SWAP,[(Flag (word-of-int flagi)),
                    (W5 (word-of-int rs1)),
                    (Simm13 (word-of-int simm13)),
                    (W5 (word-of-int rd))])
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    else — Operant list is [i,rs1,rs2,rd]
      Inr (load-store-type SWAP,[(Flag (word-of-int flagi)),
                                 (W5 (word-of-int rs1)),
                                 (W5 (word-of-int rs2)),
                                 (W5 (word-of-int rd))])
  else if op3 = uint(0b011111::word6) then — SWAPA
    Inr (load-store-type SWAPA,[(Flag (word-of-int flagi)),
                                (W5 (word-of-int rs1)),
                                (W5 (word-of-int rs2)),
                                (Asi (word-of-int asi)),
                                (W5 (word-of-int rd))])
  else Inl [invalid-op3-f3-op11]
else if this-op = uint(0b10::word2) then — Others
  if op3 = uint(0b111000::word6) then — JMPL
    if flagi = 0 then — return [i,rs1,rs2,rd]
      Inr (ctrl-type JMPL,[(Flag (word-of-int flagi)),
                           (W5 (word-of-int rs1)),
                           (W5 (word-of-int rs2)),
                           (W5 (word-of-int rd))])
    else — return [i,rs1,simm13,rd]
      Inr (ctrl-type JMPL,[(Flag (word-of-int flagi)),
                           (W5 (word-of-int rs1)),
                           (Simm13 (word-of-int simm13)),
                           (W5 (word-of-int rd))])
  else if op3 = uint(0b111001::word6) then — RETT
    if flagi = 0 then — return [i,rs1,rs2]
      Inr (ctrl-type RETT,[(Flag (word-of-int flagi)),
                           (W5 (word-of-int rs1)),
                           (W5 (word-of-int rs2))])
    else — return [i,rs1,simm13]
      Inr (ctrl-type RETT,[(Flag (word-of-int flagi)),
                           (W5 (word-of-int rs1)),
                           (Simm13 (word-of-int simm13))])
  — The following are Read and Write instructions,
  — only return [rs1,rd] as operand.
  else if op3 = uint(0b101000::word6) ∧ rs1 ≠ 0 then — RDASR
    if rs1 = uint(0b01111::word6) ∧ rd = 0 then — STBAR is a special case of
RDASR
      Inr (load-store-type STBAR,[])
    else Inr (sreg-type RDASR,[(W5 (word-of-int rs1)),
                               (W5 (word-of-int rd))])
  else if op3 = uint(0b101000::word6) ∧ rs1 = 0 then — RDY
    Inr (sreg-type RDY,[(W5 (word-of-int rs1)),
                        (W5 (word-of-int rd))])
  else if op3 = uint(0b101001::word6) then — RDPSR
    Inr (sreg-type RDPSR,[(W5 (word-of-int rs1)),
                          (W5 (word-of-int rd))])
  else if op3 = uint(0b101010::word6) then — RDWIM
    Inr (sreg-type RDWIM,[(W5 (word-of-int rs1)),
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                (W5 (word-of-int rd))])
else if op3 = uint(0b101011::word6) then — RDTBR
  Inr (sreg-type RDTBR,[(W5 (word-of-int rs1)),
                (W5 (word-of-int rd))])
else if op3 = uint(0b110000::word6) ∧ rd ≠ 0 then — WRASR
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (sreg-type WRASR,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (W5 (word-of-int rs2)),
                (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (sreg-type WRASR,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (Simm13 (word-of-int simm13)),
                (W5 (word-of-int rd))])
else if op3 = uint(0b110000::word6) ∧ rd = 0 then — WRY
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (sreg-type WRY,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (W5 (word-of-int rs2)),
                (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (sreg-type WRY,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (Simm13 (word-of-int simm13)),
                (W5 (word-of-int rd))])
else if op3 = uint(0b110001::word6) then — WRPSR
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (sreg-type WRPSR,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (W5 (word-of-int rs2)),
                (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (sreg-type WRPSR,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (Simm13 (word-of-int simm13)),
                (W5 (word-of-int rd))])
else if op3 = uint(0b110010::word6) then — WRWIM
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (sreg-type WRWIM,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (W5 (word-of-int rs2)),
                (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (sreg-type WRWIM,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (Simm13 (word-of-int simm13)),
                (W5 (word-of-int rd))])
else if op3 = uint(0b110011::word6) then — WRTBR
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  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (sreg-type WRTBR,[(Flag (word-of-int flagi)),
                          (W5 (word-of-int rs1)),
                          (W5 (word-of-int rs2)),
                          (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (sreg-type WRTBR,[(Flag (word-of-int flagi)),
                          (W5 (word-of-int rs1)),
                          (Simm13 (word-of-int simm13)),
                          (W5 (word-of-int rd))])
— FLUSH instruction
else if op3 = uint(0b111011::word6) then — FLUSH
  if flagi = 0 then — return [1,rs1,rs2]
    Inr (load-store-type FLUSH,[(Flag (word-of-int flagi)),
                          (W5 (word-of-int rs1)),
                          (W5 (word-of-int rs2))])
  else — return [i,rs1,simm13]
    Inr (load-store-type FLUSH,[(Flag (word-of-int flagi)),
                          (W5 (word-of-int rs1)),
                          (Simm13 (word-of-int simm13))])
— The following are arithmetic instructions.
else if op3 = uint(0b000001::word6) then — AND
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (logic-type ANDs,[(Flag (word-of-int flagi)),
                          (W5 (word-of-int rs1)),
                          (W5 (word-of-int rs2)),
                          (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (logic-type ANDs,[(Flag (word-of-int flagi)),
                          (W5 (word-of-int rs1)),
                          (Simm13 (word-of-int simm13)),
                          (W5 (word-of-int rd))])
else if op3 = uint(0b010001::word6) then — ANDcc
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (logic-type ANDcc,[(Flag (word-of-int flagi)),
                          (W5 (word-of-int rs1)),
                          (W5 (word-of-int rs2)),
                          (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (logic-type ANDcc,[(Flag (word-of-int flagi)),
                          (W5 (word-of-int rs1)),
                          (Simm13 (word-of-int simm13)),
                          (W5 (word-of-int rd))])
else if op3 = uint(0b000101::word6) then — ANDN
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (logic-type ANDN,[(Flag (word-of-int flagi)),
                          (W5 (word-of-int rs1)),
                          (W5 (word-of-int rs2)),
                          (W5 (word-of-int rd))])
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  else — return [i,rs1,simm13,rd]
    Inr (logic-type ANDN,[(Flag (word-of-int flagi)),
                          (W5 (word-of-int rs1)),
                          (Simm13 (word-of-int simm13)),
                          (W5 (word-of-int rd))])
else if op3 = uint(0b010101::word6) then — ANDNcc
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (logic-type ANDNcc,[(Flag (word-of-int flagi)),
                            (W5 (word-of-int rs1)),
                            (W5 (word-of-int rs2)),
                            (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (logic-type ANDNcc,[(Flag (word-of-int flagi)),
                            (W5 (word-of-int rs1)),
                            (Simm13 (word-of-int simm13)),
                            (W5 (word-of-int rd))])
else if op3 = uint(0b000010::word6) then — OR
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (logic-type ORs,[(Flag (word-of-int flagi)),
                         (W5 (word-of-int rs1)),
                         (W5 (word-of-int rs2)),
                         (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (logic-type ORs,[(Flag (word-of-int flagi)),
                         (W5 (word-of-int rs1)),
                         (Simm13 (word-of-int simm13)),
                         (W5 (word-of-int rd))])
else if op3 = uint(0b010010::word6) then — ORcc
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (logic-type ORcc,[(Flag (word-of-int flagi)),
                          (W5 (word-of-int rs1)),
                          (W5 (word-of-int rs2)),
                          (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (logic-type ORcc,[(Flag (word-of-int flagi)),
                          (W5 (word-of-int rs1)),
                          (Simm13 (word-of-int simm13)),
                          (W5 (word-of-int rd))])
else if op3 = uint(0b000110::word6) then — ORN
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (logic-type ORN,[(Flag (word-of-int flagi)),
                         (W5 (word-of-int rs1)),
                         (W5 (word-of-int rs2)),
                         (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (logic-type ORN,[(Flag (word-of-int flagi)),
                         (W5 (word-of-int rs1)),
                         (Simm13 (word-of-int simm13)),
                         (W5 (word-of-int rd))])
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else if op3 = uint(0b010110::word6) then — ORNcc
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (logic-type ORNcc,[(Flag (word-of-int flagi)),
                           (W5 (word-of-int rs1)),
                           (W5 (word-of-int rs2)),
                           (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (logic-type ORNcc,[(Flag (word-of-int flagi)),
                           (W5 (word-of-int rs1)),
                           (Simm13 (word-of-int simm13)),
                           (W5 (word-of-int rd))])
else if op3 = uint(0b000011::word6) then — XORs
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (logic-type XORs,[(Flag (word-of-int flagi)),
                          (W5 (word-of-int rs1)),
                          (W5 (word-of-int rs2)),
                          (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (logic-type XORs,[(Flag (word-of-int flagi)),
                          (W5 (word-of-int rs1)),
                          (Simm13 (word-of-int simm13)),
                          (W5 (word-of-int rd))])
else if op3 = uint(0b010011::word6) then — XORcc
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (logic-type XORcc,[(Flag (word-of-int flagi)),
                           (W5 (word-of-int rs1)),
                           (W5 (word-of-int rs2)),
                           (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (logic-type XORcc,[(Flag (word-of-int flagi)),
                           (W5 (word-of-int rs1)),
                           (Simm13 (word-of-int simm13)),
                           (W5 (word-of-int rd))])
else if op3 = uint(0b000111::word6) then — XNOR
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (logic-type XNOR,[(Flag (word-of-int flagi)),
                          (W5 (word-of-int rs1)),
                          (W5 (word-of-int rs2)),
                          (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (logic-type XNOR,[(Flag (word-of-int flagi)),
                          (W5 (word-of-int rs1)),
                          (Simm13 (word-of-int simm13)),
                          (W5 (word-of-int rd))])
else if op3 = uint(0b010111::word6) then — XNORcc
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (logic-type XNORcc,[(Flag (word-of-int flagi)),
                           (W5 (word-of-int rs1)),
                           (W5 (word-of-int rs2)),
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                (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (logic-type XNORcc,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (Simm13 (word-of-int simm13)),
                (W5 (word-of-int rd))])
else if op3 = uint(0b100101::word6) then — SLL
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (shift-type SLL,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (W5 (word-of-int rs2)),
                (W5 (word-of-int rd))])
  else — return [i,rs1,shcnt,rd]
    let shcnt = rs2 in
    Inr (shift-type SLL,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (W5 (word-of-int shcnt)),
                (W5 (word-of-int rd))])
else if op3 = uint (0b100110::word6) then — SRL
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (shift-type SRL,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (W5 (word-of-int rs2)),
                (W5 (word-of-int rd))])
  else — return [i,rs1,shcnt,rd]
    let shcnt = rs2 in
    Inr (shift-type SRL,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (W5 (word-of-int shcnt)),
                (W5 (word-of-int rd))])
else if op3 = uint(0b100111::word6) then — SRA
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (shift-type SRA,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (W5 (word-of-int rs2)),
                (W5 (word-of-int rd))])
  else — return [i,rs1,shcnt,rd]
    let shcnt = rs2 in
    Inr (shift-type SRA,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (W5 (word-of-int shcnt)),
                (W5 (word-of-int rd))])
else if op3 = uint(0b000000::word6) then — ADD
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (arith-type ADD,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (W5 (word-of-int rs2)),
                (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
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    Inr (arith-type ADD,[(Flag (word-of-int flagi)),
                         (W5 (word-of-int rs1)),
                         (Simm13 (word-of-int simm13)),
                         (W5 (word-of-int rd))])
else if op3 = uint(0b010000::word6) then — ADDcc
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (arith-type ADDcc,[(Flag (word-of-int flagi)),
                           (W5 (word-of-int rs1)),
                           (W5 (word-of-int rs2)),
                           (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (arith-type ADDcc,[(Flag (word-of-int flagi)),
                           (W5 (word-of-int rs1)),
                           (Simm13 (word-of-int simm13)),
                           (W5 (word-of-int rd))])
else if op3 = uint(0b001000::word6) then — ADDX
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (arith-type ADDX,[(Flag (word-of-int flagi)),
                          (W5 (word-of-int rs1)),
                          (W5 (word-of-int rs2)),
                          (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (arith-type ADDX,[(Flag (word-of-int flagi)),
                          (W5 (word-of-int rs1)),
                          (Simm13 (word-of-int simm13)),
                          (W5 (word-of-int rd))])
else if op3 = uint(0b011000::word6) then — ADDXcc
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (arith-type ADDXcc,[(Flag (word-of-int flagi)),
                            (W5 (word-of-int rs1)),
                            (W5 (word-of-int rs2)),
                            (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (arith-type ADDXcc,[(Flag (word-of-int flagi)),
                            (W5 (word-of-int rs1)),
                            (Simm13 (word-of-int simm13)),
                            (W5 (word-of-int rd))])
else if op3 = uint(0b100000::word6) then — TADDcc
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (arith-type TADDcc,[(Flag (word-of-int flagi)),
                            (W5 (word-of-int rs1)),
                            (W5 (word-of-int rs2)),
                            (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (arith-type TADDcc,[(Flag (word-of-int flagi)),
                            (W5 (word-of-int rs1)),
                            (Simm13 (word-of-int simm13)),
                            (W5 (word-of-int rd))])
else if op3 = uint(0b100010::word6) then — TADDccTV
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  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (arith-type TADDccTV,[(Flag (word-of-int flagi)),
                  (W5 (word-of-int rs1)),
                  (W5 (word-of-int rs2)),
                  (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (arith-type TADDccTV,[(Flag (word-of-int flagi)),
                  (W5 (word-of-int rs1)),
                  (Simm13 (word-of-int simm13)),
                  (W5 (word-of-int rd))])
else if op3 = uint(0b000100::word6) then — SUB
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (arith-type SUB,[(Flag (word-of-int flagi)),
                  (W5 (word-of-int rs1)),
                  (W5 (word-of-int rs2)),
                  (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (arith-type SUB,[(Flag (word-of-int flagi)),
                  (W5 (word-of-int rs1)),
                  (Simm13 (word-of-int simm13)),
                  (W5 (word-of-int rd))])
else if op3 = uint(0b010100::word6) then — SUBcc
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (arith-type SUBcc,[(Flag (word-of-int flagi)),
                  (W5 (word-of-int rs1)),
                  (W5 (word-of-int rs2)),
                  (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (arith-type SUBcc,[(Flag (word-of-int flagi)),
                  (W5 (word-of-int rs1)),
                  (Simm13 (word-of-int simm13)),
                  (W5 (word-of-int rd))])
else if op3 = uint(0b001100::word6) then — SUBX
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (arith-type SUBX,[(Flag (word-of-int flagi)),
                  (W5 (word-of-int rs1)),
                  (W5 (word-of-int rs2)),
                  (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (arith-type SUBX,[(Flag (word-of-int flagi)),
                  (W5 (word-of-int rs1)),
                  (Simm13 (word-of-int simm13)),
                  (W5 (word-of-int rd))])
else if op3 = uint(0b011100::word6) then — SUBXcc
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (arith-type SUBXcc,[(Flag (word-of-int flagi)),
                  (W5 (word-of-int rs1)),
                  (W5 (word-of-int rs2)),
                  (W5 (word-of-int rd))])
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  else — return [i,rs1,simm13,rd]
    Inr (arith-type SUBXcc,[(Flag (word-of-int flagi)),
                 (W5 (word-of-int rs1)),
                 (Simm13 (word-of-int simm13)),
                 (W5 (word-of-int rd))])
else if op3 = uint(0b100001::word6) then — TSUBcc
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (arith-type TSUBcc,[(Flag (word-of-int flagi)),
                 (W5 (word-of-int rs1)),
                 (W5 (word-of-int rs2)),
                 (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (arith-type TSUBcc,[(Flag (word-of-int flagi)),
                 (W5 (word-of-int rs1)),
                 (Simm13 (word-of-int simm13)),
                 (W5 (word-of-int rd))])
else if op3 = uint(0b100011::word6) then — TSUBccTV
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (arith-type TSUBccTV,[(Flag (word-of-int flagi)),
                 (W5 (word-of-int rs1)),
                 (W5 (word-of-int rs2)),
                 (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (arith-type TSUBccTV,[(Flag (word-of-int flagi)),
                 (W5 (word-of-int rs1)),
                 (Simm13 (word-of-int simm13)),
                 (W5 (word-of-int rd))])
else if op3 = uint(0b100100::word6) then — MULScc
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (arith-type MULScc,[(Flag (word-of-int flagi)),
                 (W5 (word-of-int rs1)),
                 (W5 (word-of-int rs2)),
                 (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (arith-type MULScc,[(Flag (word-of-int flagi)),
                 (W5 (word-of-int rs1)),
                 (Simm13 (word-of-int simm13)),
                 (W5 (word-of-int rd))])
else if op3 = uint(0b001010::word6) then — UMUL
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (arith-type UMUL,[(Flag (word-of-int flagi)),
                 (W5 (word-of-int rs1)),
                 (W5 (word-of-int rs2)),
                 (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (arith-type UMUL,[(Flag (word-of-int flagi)),
                 (W5 (word-of-int rs1)),
                 (Simm13 (word-of-int simm13)),
                 (W5 (word-of-int rd))])
```

```
else if op3 = uint(0b011010::word6) then — UMULcc
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (arith-type UMULcc,[(Flag (word-of-int flagi)),
                  (W5 (word-of-int rs1)),
                  (W5 (word-of-int rs2)),
                  (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (arith-type UMULcc,[(Flag (word-of-int flagi)),
                  (W5 (word-of-int rs1)),
                  (Simm13 (word-of-int simm13)),
                  (W5 (word-of-int rd))])
else if op3 = uint(0b001011::word6) then — SMUL
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (arith-type SMUL,[(Flag (word-of-int flagi)),
                  (W5 (word-of-int rs1)),
                  (W5 (word-of-int rs2)),
                  (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (arith-type SMUL,[(Flag (word-of-int flagi)),
                  (W5 (word-of-int rs1)),
                  (Simm13 (word-of-int simm13)),
                  (W5 (word-of-int rd))])
else if op3 = uint(0b011011::word6) then — SMULcc
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (arith-type SMULcc,[(Flag (word-of-int flagi)),
                   (W5 (word-of-int rs1)),
                   (W5 (word-of-int rs2)),
                   (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (arith-type SMULcc,[(Flag (word-of-int flagi)),
                   (W5 (word-of-int rs1)),
                   (Simm13 (word-of-int simm13)),
                   (W5 (word-of-int rd))])
else if op3 = uint(0b001110::word6) then — UDIV
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (arith-type UDIV,[(Flag (word-of-int flagi)),
                  (W5 (word-of-int rs1)),
                  (W5 (word-of-int rs2)),
                  (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (arith-type UDIV,[(Flag (word-of-int flagi)),
                  (W5 (word-of-int rs1)),
                  (Simm13 (word-of-int simm13)),
                  (W5 (word-of-int rd))])
else if op3 = uint(0b011110::word6) then — UDIVcc
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (arith-type UDIVcc,[(Flag (word-of-int flagi)),
                   (W5 (word-of-int rs1)),
                   (W5 (word-of-int rs2)),
```

```
                    (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (arith-type UDIVcc,[(Flag (word-of-int flagi)),
                    (W5 (word-of-int rs1)),
                    (Simm13 (word-of-int simm13)),
                    (W5 (word-of-int rd))])
else if op3 = uint(0b001111::word6) then — SDIV
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (arith-type SDIV,[(Flag (word-of-int flagi)),
                    (W5 (word-of-int rs1)),
                    (W5 (word-of-int rs2)),
                    (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (arith-type SDIV,[(Flag (word-of-int flagi)),
                    (W5 (word-of-int rs1)),
                    (Simm13 (word-of-int simm13)),
                    (W5 (word-of-int rd))])
else if op3 = uint(0b011111::word6) then — SDIVcc
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (arith-type SDIVcc,[(Flag (word-of-int flagi)),
                    (W5 (word-of-int rs1)),
                    (W5 (word-of-int rs2)),
                    (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (arith-type SDIVcc,[(Flag (word-of-int flagi)),
                    (W5 (word-of-int rs1)),
                    (Simm13 (word-of-int simm13)),
                    (W5 (word-of-int rd))])
else if op3 = uint(0b111100::word6) then — SAVE
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (ctrl-type SAVE,[(Flag (word-of-int flagi)),
                    (W5 (word-of-int rs1)),
                    (W5 (word-of-int rs2)),
                    (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (ctrl-type SAVE,[(Flag (word-of-int flagi)),
                    (W5 (word-of-int rs1)),
                    (Simm13 (word-of-int simm13)),
                    (W5 (word-of-int rd))])
else if op3 = uint(0b111101::word6) then — RESTORE
  if flagi = 0 then — return [i,rs1,rs2,rd]
    Inr (ctrl-type RESTORE,[(Flag (word-of-int flagi)),
                    (W5 (word-of-int rs1)),
                    (W5 (word-of-int rs2)),
                    (W5 (word-of-int rd))])
  else — return [i,rs1,simm13,rd]
    Inr (ctrl-type RESTORE,[(Flag (word-of-int flagi)),
                    (W5 (word-of-int rs1)),
                    (Simm13 (word-of-int simm13)),
```

```
                    (W5 (word-of-int rd))])
else if op3 = uint(0b111010::word6) then — Ticc
  let trap-cond = get-trap-cond w in
  let trap-imm7 = get-trap-imm7 w in
  if trap-cond = uint(0b1000::word4) then — TA
    if flagi = 0 then — return [i,rs1,rs2]
      Inr (ticc-type TA,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (W5 (word-of-int rs2))])
    else — return [i,rs1,trap-imm7]
      Inr (ticc-type TA,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (Imm7 (word-of-int trap-imm7))])
  else if trap-cond = uint(0b0000::word4) then — TN
    if flagi = 0 then — return [i,rs1,rs2]
      Inr (ticc-type TN,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (W5 (word-of-int rs2))])
    else — return [i,rs1,trap-imm7]
      Inr (ticc-type TN,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (Imm7 (word-of-int trap-imm7))])
  else if trap-cond = uint(0b1001::word4) then — TNE
    if flagi = 0 then — return [i,rs1,rs2]
      Inr (ticc-type TNE,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (W5 (word-of-int rs2))])
    else — return [i,rs1,trap-imm7]
      Inr (ticc-type TNE,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (Imm7 (word-of-int trap-imm7))])
  else if trap-cond = uint(0b0001::word4) then — TE
    if flagi = 0 then — return [i,rs1,rs2]
      Inr (ticc-type TE,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (W5 (word-of-int rs2))])
    else — return [i,rs1,trap-imm7]
      Inr (ticc-type TE,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (Imm7 (word-of-int trap-imm7))])
  else if trap-cond = uint(0b1010::word4) then — TG
    if flagi = 0 then — return [i,rs1,rs2]
      Inr (ticc-type TG,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (W5 (word-of-int rs2))])
    else — return [i,rs1,trap-imm7]
      Inr (ticc-type TG,[(Flag (word-of-int flagi)),
                (W5 (word-of-int rs1)),
                (Imm7 (word-of-int trap-imm7))])
```

```
else if trap-cond = uint(0b0010::word4) then — TLE
  if flagi = 0 then — return [i,rs1,rs2]
    Inr (ticc-type TLE,[(Flag (word-of-int flagi)),
                        (W5 (word-of-int rs1)),
                        (W5 (word-of-int rs2))])
  else — return [i,rs1,trap-imm7]
    Inr (ticc-type TLE,[(Flag (word-of-int flagi)),
                        (W5 (word-of-int rs1)),
                        (Imm7 (word-of-int trap-imm7))])
else if trap-cond = uint(0b1011::word4) then — TGE
  if flagi = 0 then — return [i,rs1,rs2]
    Inr (ticc-type TGE,[(Flag (word-of-int flagi)),
                        (W5 (word-of-int rs1)),
                        (W5 (word-of-int rs2))])
  else — return [i,rs1,trap-imm7]
    Inr (ticc-type TGE,[(Flag (word-of-int flagi)),
                        (W5 (word-of-int rs1)),
                        (Imm7 (word-of-int trap-imm7))])
else if trap-cond = uint(0b0011::word4) then — TL
  if flagi = 0 then — return [i,rs1,rs2]
    Inr (ticc-type TL,[(Flag (word-of-int flagi)),
                       (W5 (word-of-int rs1)),
                       (W5 (word-of-int rs2))])
  else — return [i,rs1,trap-imm7]
    Inr (ticc-type TL,[(Flag (word-of-int flagi)),
                       (W5 (word-of-int rs1)),
                       (Imm7 (word-of-int trap-imm7))])
else if trap-cond = uint(0b1100::word4) then — TGU
  if flagi = 0 then — return [i,rs1,rs2]
    Inr (ticc-type TGU,[(Flag (word-of-int flagi)),
                        (W5 (word-of-int rs1)),
                        (W5 (word-of-int rs2))])
  else — return [i,rs1,trap-imm7]
    Inr (ticc-type TGU,[(Flag (word-of-int flagi)),
                        (W5 (word-of-int rs1)),
                        (Imm7 (word-of-int trap-imm7))])
else if trap-cond = uint(0b0100::word4) then — TLEU
  if flagi = 0 then — return [i,rs1,rs2]
    Inr (ticc-type TLEU,[(Flag (word-of-int flagi)),
                         (W5 (word-of-int rs1)),
                         (W5 (word-of-int rs2))])
  else — return [i,rs1,trap-imm7]
    Inr (ticc-type TLEU,[(Flag (word-of-int flagi)),
                         (W5 (word-of-int rs1)),
                         (Imm7 (word-of-int trap-imm7))])
else if trap-cond = uint(0b1101::word4) then — TCC
  if flagi = 0 then — return [i,rs1,rs2]
    Inr (ticc-type TCC,[(Flag (word-of-int flagi)),
                        (W5 (word-of-int rs1)),
```

```
            (W5 (word-of-int rs2))])
  else — return [i,rs1,trap-imm7]
    Inr (ticc-type TCC,[(Flag (word-of-int flagi)),
            (W5 (word-of-int rs1)),
            (Imm7 (word-of-int trap-imm7))])
else if trap-cond = uint(0b0101::word4) then — TCS
  if flagi = 0 then — return [i,rs1,rs2]
    Inr (ticc-type TCS,[(Flag (word-of-int flagi)),
            (W5 (word-of-int rs1)),
            (W5 (word-of-int rs2))])
  else — return [i,rs1,trap-imm7]
    Inr (ticc-type TCS,[(Flag (word-of-int flagi)),
            (W5 (word-of-int rs1)),
            (Imm7 (word-of-int trap-imm7))])
else if trap-cond = uint(0b1110::word4) then — TPOS
  if flagi = 0 then — return [i,rs1,rs2]
    Inr (ticc-type TPOS,[(Flag (word-of-int flagi)),
            (W5 (word-of-int rs1)),
            (W5 (word-of-int rs2))])
  else — return [i,rs1,trap-imm7]
    Inr (ticc-type TPOS,[(Flag (word-of-int flagi)),
            (W5 (word-of-int rs1)),
            (Imm7 (word-of-int trap-imm7))])
else if trap-cond = uint(0b0110::word4) then — TNEG
  if flagi = 0 then — return [i,rs1,rs2]
    Inr (ticc-type TNEG,[(Flag (word-of-int flagi)),
            (W5 (word-of-int rs1)),
            (W5 (word-of-int rs2))])
  else — return [i,rs1,trap-imm7]
    Inr (ticc-type TNEG,[(Flag (word-of-int flagi)),
            (W5 (word-of-int rs1)),
            (Imm7 (word-of-int trap-imm7))])
else if trap-cond = uint(0b1111::word4) then — TVC
  if flagi = 0 then — return [i,rs1,rs2]
    Inr (ticc-type TVC,[(Flag (word-of-int flagi)),
            (W5 (word-of-int rs1)),
            (W5 (word-of-int rs2))])
  else — return [i,rs1,trap-imm7]
    Inr (ticc-type TVC,[(Flag (word-of-int flagi)),
            (W5 (word-of-int rs1)),
            (Imm7 (word-of-int trap-imm7))])
else if trap-cond = uint(0b0111::word4) then — TVS
  if flagi = 0 then — return [i,rs1,rs2]
    Inr (ticc-type TVS,[(Flag (word-of-int flagi)),
            (W5 (word-of-int rs1)),
            (W5 (word-of-int rs2))])
  else — return [i,rs1,trap-imm7]
    Inr (ticc-type TVS,[(Flag (word-of-int flagi)),
            (W5 (word-of-int rs1)),
```

```
                (Imm7 (word-of-int trap-imm7))])
    else Inl [invalid-trap-cond]
  else Inl [invalid-op3-f3-op10]
else Inl [invalid-op-f3]
```

Read the word32 value from the Program Counter in the current state. Find the instruction in the memory address of the word32 value. Return a word32 value of the insturction.

```
definition fetch-instruction::('a) sparc-state ⇒
  (Exception list + word32)
where fetch-instruction s ≡
  — pc-val is the 32-bit memory address of the instruction.
  let pc-val = cpu-reg-val PC s;
      psr-val = cpu-reg-val PSR s;
      s-val = get-S psr-val;
      asi = if s-val = 0 then word-of-int 8 else word-of-int 9
  in
  — Check if pc-val is aligned to 4-byte (32-bit) boundary.
  — That is, check if the least significant two bits of
  — pc-val are 0s.
    if uint((AND) (0b00000000000000000000000000000011) pc-val) = 0 then
    — Get the 32-bit value from the address of pc-val
    — to the address of pc-val+3
      let (mem-result,n-s) = memory-read asi pc-val s in
      case mem-result of
       None ⇒ Inl [fetch-instruction-error]
      |Some v ⇒ Inr v
    else Inl [fetch-instruction-error]
```

Decode the word32 value of an instruction into the name of the instruction and its operands.

```
definition decode-instruction::word32 ⇒
  Exception list + instruction
where decode-instruction w ≡
  let this-op = get-op w in
  if this-op = uint(0b01::word2) then — Instruction format 1
    parse-instr-f1 w
  else if this-op = uint(0b00::word2) then — Instruction format 2
    parse-instr-f2 w
  else — op = 11 0r 10, instruction format 3
    parse-instr-f3 w
```

Get the current window from the PSR

```
definition get-curr-win::unit ⇒ ('a,('a::len window-size)) sparc-state-monad
where get-curr-win - ≡
```

```
  do
    curr-win ← gets (λs. (ucast (get-CWP (cpu-reg-val PSR s))));
    return curr-win
  od
```

Operational semantics for CALL

```
definition call-instr::instruction ⇒ ('a::len,unit) sparc-state-monad
where call-instr instr ≡
  let op-list = snd instr;
      mem-addr = ((ucast (get-operand-w30 (op-list!0)))::word32) << 2
  in
  do
    curr-win ← get-curr-win();
    pc-val ← gets (λs. (cpu-reg-val PC s));
    npc-val ← gets (λs. (cpu-reg-val nPC s));
    write-reg pc-val curr-win (word-of-int 15);
    write-cpu npc-val PC;
    write-cpu (pc-val + mem-addr) nPC;
    return ()
  od
```

Evaluate icc based on the bits N, Z, V, C in PSR and the type of branching instruction. See Sparcv8 manual Page 178.

```
definition eval-icc::sparc-operation ⇒ word1 ⇒ word1 ⇒ word1 ⇒ word1 ⇒ int
where
eval-icc instr-name n-val z-val v-val c-val ≡
    if instr-name = bicc-type BNE then
      if z-val = 0 then 1 else 0
    else if instr-name = bicc-type BE then
      if z-val = 1 then 1 else 0
    else if instr-name = bicc-type BG then
      if ((OR) z-val (n-val XOR v-val)) = 0 then 1 else 0
    else if instr-name = bicc-type BLE then
      if ((OR) z-val (n-val XOR v-val)) = 1 then 1 else 0
    else if instr-name = bicc-type BGE then
      if (n-val XOR v-val) = 0 then 1 else 0
    else if instr-name = bicc-type BL then
      if (n-val XOR v-val) = 1 then 1 else 0
    else if instr-name = bicc-type BGU then
      if (c-val = 0 ∧ z-val = 0) then 1 else 0
    else if instr-name = bicc-type BLEU then
      if (c-val = 1 ∨ z-val = 1) then 1 else 0
    else if instr-name = bicc-type BCC then
      if c-val = 0 then 1 else 0
    else if instr-name = bicc-type BCS then
      if c-val = 1 then 1 else 0
    else if instr-name = bicc-type BNEG then
      if n-val = 1 then 1 else 0
    else if instr-name = bicc-type BA then 1
```

```
  else if instr-name = bicc-type BN then 0
  else if instr-name = bicc-type BPOS then
    if n-val = 0 then 1 else 0
  else if instr-name = bicc-type BVC then
    if v-val = 0 then 1 else 0
  else if instr-name = bicc-type BVS then
    if v-val = 1 then 1 else 0
  else −1
```

```
definition branch-instr-sub1:: sparc-operation ⇒ ('a) sparc-state ⇒ int
where branch-instr-sub1 instr-name s ≡
  let n-val = get-icc-N ((cpu-reg s) PSR);
      z-val = get-icc-Z ((cpu-reg s) PSR);
      v-val = get-icc-V ((cpu-reg s) PSR);
      c-val = get-icc-C ((cpu-reg s) PSR)
  in
  eval-icc instr-name n-val z-val v-val c-val
```

Operational semantics for Branching insturctions. Return exception or a bool value for annulment. If the bool value is 1, then the delay instruciton is not executed, otherwise the delay instruction is executed.

```
definition branch-instr::instruction ⇒ ('a,unit) sparc-state-monad
where branch-instr instr ≡
  let instr-name = fst instr;
      op-list = snd instr;
      disp22 = get-operand-w22 (op-list!1);
      flaga = get-operand-flag (op-list!0)
  in
  do
    icc-val ← gets( λs. (branch-instr-sub1 instr-name s));
    npc-val ← gets (λs. (cpu-reg-val nPC s));
    pc-val ← gets (λs. (cpu-reg-val PC s));
    write-cpu npc-val PC;
    if icc-val = 1 then
      do
        write-cpu (pc-val + (sign-ext24 (((ucast(disp22))::word24) << 2))) nPC;
        if (instr-name = bicc-type BA) ∧ (flaga = 1) then
          do
            set-annul True;
            return ()
          od
        else
          return ()
      od
    else — icc-val = 0
      do
        write-cpu (npc-val + 4) nPC;
        if flaga = 1 then
```

```
        do
          set-annul True;
          return ()
        od
      else return ()
    od
  od
```

Operational semantics for NOP

definition *nop-instr*::*instruction* ⇒ ('*a*,*unit*) *sparc-state-monad*
where *nop-instr instr* ≡ *return* ()

Operational semantics for SETHI

definition *sethi-instr*::*instruction* ⇒ ('*a*::*len*,*unit*) *sparc-state-monad*
where *sethi-instr instr* ≡
let op-list = *snd instr*;
imm22 = *get-operand-w22* (*op-list*!*0*);
rd = *get-operand-w5* (*op-list*!*1*)
in
if rd ≠ *0 then*
do
curr-win ← *get-curr-win*();
write-reg (((*ucast*(*imm22*))::*word32*) << *10*) *curr-win rd*;
return ()
od
else return ()

Get *operand2* based on the flag *i*, *rs1*, *rs2*, and *simm13*. If *i* = *0* then *operand2* = *r*[*rs2*], else *operand2* = *sign-ext13*(*simm13*). *op-list* should be [*i*,*rs1*,*rs2*,...] or [*i*,*rs1*,*simm13*,...].

definition *get-operand2*::*inst-operand list* ⇒ ('*a*::*len*) *sparc-state*
⇒ *virtua-address*
where *get-operand2 op-list s* ≡
let flagi = *get-operand-flag* (*op-list*!*0*);
curr-win = *ucast* (*get-CWP* (*cpu-reg-val PSR s*))
in
if flagi = *0 then*
let rs2 = *get-operand-w5* (*op-list*!*2*);
rs2-val = *user-reg-val curr-win rs2 s*
in rs2-val
else
let ext-simm13 = *sign-ext13* (*get-operand-simm13* (*op-list*!*2*)) *in*
ext-simm13

Get *operand2-val* based on the flag *i*, *rs1*, *rs2*, and *simm13*. If *i* = *0* then *operand2-val* = *uint r*[*rs2*], else *operand2-val* = *sint sign-ext13*(*simm13*). *op-list* should be [*i*,*rs1*,*rs2*,...] or [*i*,*rs1*,*simm13*,...].

```
definition get-operand2-val::inst-operand list ⇒ ('a::len) sparc-state ⇒ int
where get-operand2-val op-list s ≡
  let flagi = get-operand-flag (op-list!0);
      curr-win = ucast (get-CWP (cpu-reg-val PSR s))
  in
  if flagi = 0 then
    let rs2 = get-operand-w5 (op-list!2);
        rs2-val = user-reg-val curr-win rs2 s
    in sint rs2-val
  else
    let ext-simm13 = sign-ext13 (get-operand-simm13 (op-list!2)) in
    sint ext-simm13
```

Get the address based on the flag *i*, *rs1*, *rs2*, and *simm13*. If $i = 0$ then $addr = r[rs1] + r[rs2]$, else $addr = r[rs1] + sign\text{-}ext13(simm13)$. *op-list* should be [*i*,*rs1*,*rs2*,...] or [*i*,*rs1*,*simm13*,...].

```
definition get-addr::inst-operand list ⇒ ('a::len) sparc-state ⇒ virtua-address
where get-addr op-list s ≡
  let rs1 = get-operand-w5 (op-list!1);
      curr-win = ucast (get-CWP (cpu-reg-val PSR s));
      rs1-val = user-reg-val curr-win rs1 s;
      op2 = get-operand2 op-list s
  in
  (rs1-val + op2)
```

Operational semantics for JMPL

```
definition jmpl-instr::instruction ⇒ ('a::len,unit) sparc-state-monad
where jmpl-instr instr ≡
  let op-list = snd instr;
      rd = get-operand-w5 (op-list!3)
  in
  do
    curr-win ← get-curr-win();
    jmp-addr ← gets (λs. (get-addr op-list s));
    if ((AND) jmp-addr 0b00000000000000000000000000000011) ≠ 0 then
      do
        raise-trap mem-address-not-aligned;
        return ()
      od
    else
      do
        rd-next-val ← gets (λs. (if rd ≠ 0 then
                                   (cpu-reg-val PC s)
                                 else
                                   user-reg-val curr-win rd s));
        write-reg rd-next-val curr-win rd;
        npc-val ← gets (λs. (cpu-reg-val nPC s));
```

```
      write-cpu npc-val PC;
      write-cpu jmp-addr nPC;
      return ()
    od
  od
```

Operational semantics for RETT

```
definition rett-instr :: instruction ⇒ ('a::len,unit) sparc-state-monad
where rett-instr instr ≡
  let op-list = snd instr in
  do
    psr-val ← gets (λs. (cpu-reg-val PSR s));
    curr-win ← gets (λs. (get-CWP (cpu-reg-val PSR s)));
    new-cwp ← gets (λs. (word-of-int (((uint curr-win) + 1) mod NWINDOWS)));
    new-cwp-int ← gets (λs. ((uint curr-win) + 1) mod NWINDOWS);
    addr ← gets (λs. (get-addr op-list s));
    et-val ← gets (λs. ((ucast (get-ET psr-val))::word1));
    s-val ← gets (λs. ((ucast (get-S psr-val))::word1));
    ps-val ← gets (λs. ((ucast (get-PS psr-val))::word1));
    wim-val ← gets (λs. (cpu-reg-val WIM s));
    npc-val ← gets (λs. (cpu-reg-val nPC s));
    if et-val = 1 then
      if s-val = 0 then
        do
          raise-trap privileged-instruction;
          return ()
        od
      else
        do
          raise-trap illegal-instruction;
          return ()
        od
    else if s-val = 0 then
      do
        write-cpu-tt (0b00000011::word8);
        set-exe-mode False;
        set-err-mode True;
        raise-trap privileged-instruction;
        fail ()
      od
    else if (get-WIM-bit (nat new-cwp-int) wim-val) ≠ 0 then
      do
        write-cpu-tt (0b00000110::word8);
        set-exe-mode False;
        set-err-mode True;
        raise-trap window-underflow;
        fail ()
      od
     else if ((AND) addr (0b00000000000000000000000000000011::word32)) ≠ 0
```

```
then
      do
        write-cpu-tt (0b00000111::word8);
        set-exe-mode False;
        set-err-mode True;
        raise-trap mem-address-not-aligned;
        fail ()
      od
    else
      do
        write-cpu npc-val PC;
        write-cpu addr nPC;
        new-psr-val ← gets (λs. (update-PSR-rett new-cwp 1 ps-val psr-val));
        write-cpu new-psr-val PSR;
        return ()
      od
  od
```

```
definition save-retore-sub1 :: word32 ⇒ word5 ⇒ word5 ⇒ ('a::len,unit) sparc-state-monad
where save-retore-sub1 result new-cwp rd ≡
  do
  psr-val ← gets (λs. (cpu-reg-val PSR s));
  new-psr-val ← gets (λs. (update-CWP new-cwp psr-val));
  write-cpu new-psr-val PSR; — Change CWP to the new window value.
  write-reg result (ucast new-cwp) rd; — Write result in rd of the new window.
  return ()
  od
```

Operational semantics for SAVE and RESTORE.

```
definition save-restore-instr :: instruction ⇒ ('a::len,unit) sparc-state-monad
where save-restore-instr instr ≡
  let instr-name = fst instr;
      op-list = snd instr;
      rd = get-operand-w5 (op-list!3)
  in
  do
    psr-val ← gets (λs. (cpu-reg-val PSR s));
    curr-win ← get-curr-win();
    wim-val ← gets (λs. (cpu-reg-val WIM s));
    if instr-name = ctrl-type SAVE then
      do
          new-cwp ← gets (λs. ((word-of-int (((uint curr-win) − 1) mod NWIN-
DOWS)))::word5);
        if (get-WIM-bit (unat new-cwp) wim-val) ≠ 0 then
          do
            raise-trap window-overflow;
            return ()
          od
        else
```

```
        do
            result ← gets (λs. (get-addr op-list s)); — operands are from the old
window.
          save-retore-sub1 result new-cwp rd
        od
      od
    else — instr-name = RESTORE
      do
          new-cwp ← gets (λs. ((word-of-int (((uint curr-win) + 1) mod NWIN-
DOWS)))::word5);
        if (get-WIM-bit (unat new-cwp) wim-val) ≠ 0 then
          do
            raise-trap window-underflow;
            return ()
          od
        else
          do
            result ← gets (λs. (get-addr op-list s)); — operands are from the old
window.
            save-retore-sub1 result new-cwp rd
          od
      od
  od
```

definition *flush-cache-line* :: *word32* ⇒ (*'a*,*unit*) *sparc-state-monad*
where *flush-cache-line* ≡ *undefined*

definition *flush-Ibuf-and-pipeline* :: *word32* ⇒ (*'a*,*unit*) *sparc-state-monad*
where *flush-Ibuf-and-pipeline* ≡ *undefined*

Operational semantics for FLUSH. Flush the all the caches.

```
definition flush-instr :: instruction ⇒ ('a::len,unit) sparc-state-monad
where flush-instr instr ≡
  let op-list = snd instr in
  do
    addr ← gets (λs. (get-addr op-list s));
    modify (λs. (flush-cache-all s));
    flush-cache-line(addr);
    flush-Ibuf-and-pipeline(addr);
    return ()
  od
```

Operational semantics for read state register instructions. We do not consider RDASR here.

```
definition read-state-reg-instr :: instruction ⇒ ('a::len,unit) sparc-state-monad
where read-state-reg-instr instr ≡
  let instr-name = fst instr;
      op-list = snd instr;
      rs1 = get-operand-w5 (op-list!0);
```

```
    rd = get-operand-w5 (op-list!1)
in
do
  curr-win ← get-curr-win();
  psr-val ← gets (λs. (cpu-reg-val PSR s));
  s-val ← gets (λs. (get-S psr-val));
  if (instr-name ∈ {sreg-type RDPSR,sreg-type RDWIM,sreg-type RDTBR} ∨
      (instr-name = sreg-type RDASR ∧ privileged-ASR rs1))
     ∧ ((ucast s-val)::word1) = 0 then
    do
      raise-trap privileged-instruction;
      return ()
    od
  else if illegal-instruction-ASR rs1 then
    do
      raise-trap illegal-instruction;
      return ()
    od
  else if rd ≠ 0 then
    if instr-name = sreg-type RDY then
      do
        y-val ← gets (λs. (cpu-reg-val Y s));
        write-reg y-val curr-win rd;
        return ()
      od
    else if instr-name = sreg-type RDASR then
      do
        asr-val ← gets (λs. (cpu-reg-val (ASR rs1) s));
        write-reg asr-val curr-win rd;
        return ()
      od
    else if instr-name = sreg-type RDPSR then
      do
        write-reg psr-val curr-win rd;
        return ()
      od
    else if instr-name = sreg-type RDWIM then
      do
        wim-val ← gets (λs. (cpu-reg-val WIM s));
        write-reg wim-val curr-win rd;
        return ()
      od
    else — Must be RDTBR.
      do
        tbr-val ← gets (λs. (cpu-reg-val TBR s));
        write-reg tbr-val curr-win rd;
        return ()
      od
  else return ()
```

```
  od
```

Operational semantics for write state register instructions. We do not consider WRASR here.

```
definition write-state-reg-instr :: instruction ⇒ ('a::len,unit) sparc-state-monad
where write-state-reg-instr instr ≡
  let instr-name = fst instr;
      op-list = snd instr;
      rs1 = get-operand-w5 (op-list!1);
      rd = get-operand-w5 (op-list!3)
  in
  do
    curr-win ← get-curr-win();
    psr-val ← gets (λs. (cpu-reg-val PSR s));
    s-val ← gets (λs. (get-S psr-val));
    op2 ← gets (λs. (get-operand2 op-list s));
    rs1-val ← gets (λs. (user-reg-val curr-win rs1 s));
    result ← gets (λs. ((XOR) rs1-val op2));
    if instr-name = sreg-type WRY then
      do
        modify (λs. (delayed-pool-add (DELAYNUM, result, Y) s));
        return ()
      od
    else if instr-name = sreg-type WRASR then
        if privileged-ASR rd ∧ s-val = 0 then
          do
            raise-trap privileged-instruction;
            return ()
          od
        else if illegal-instruction-ASR rd then
          do
            raise-trap illegal-instruction;
            return ()
          od
        else
          do
            modify (λs. (delayed-pool-add (DELAYNUM, result, (ASR rd)) s));
            return ()
          od
    else if instr-name = sreg-type WRPSR then
      if s-val = 0 then
        do
          raise-trap privileged-instruction;
          return ()
        od
      else if (uint ((ucast result)::word5)) ≥ NWINDOWS then
        do
          raise-trap illegal-instruction;
          return ()
```

```
      od
    else
      do — ET and PIL appear to be written IMMEDIATELY w.r.t. interrupts.
        pil-val ← gets (λs. (get-PIL result));
        et-val ← gets (λs. (get-ET result));
        new-psr-val ← gets (λs. (update-PSR-et-pil et-val pil-val psr-val));
        write-cpu new-psr-val PSR;
        modify (λs. (delayed-pool-add (DELAYNUM, result, PSR) s));
        return ()
      od
  else if instr-name = sreg-type WRWIM then
    if s-val = 0 then
      do
        raise-trap privileged-instruction;
        return ()
      od
    else
      do — Don't write bits corresponding to non-existent windows.
       result-f ← gets (λs. ((result << nat (32 − NWINDOWS)) >> nat (32 −
NWINDOWS)));
        modify (λs. (delayed-pool-add (DELAYNUM, result-f, WIM) s));
        return ()
      od
  else — Must be WRTBR
    if s-val = 0 then
      do
        raise-trap privileged-instruction;
        return ()
      od
    else
      do — Only write the bits <31:12> of the result to TBR.
        tbr-val ← gets (λs. (cpu-reg-val TBR s));
      tbr-val-11-0 ← gets (λs. ((AND) tbr-val 0b00000000000000000000111111111111));
      result-tmp ← gets (λs. ((AND) result 0b11111111111111111111000000000000));
        result-f ← gets (λs. ((OR) tbr-val-11-0 result-tmp));
        modify (λs. (delayed-pool-add (DELAYNUM, result-f, TBR) s));
        return ()
      od
od

definition logical-result :: sparc-operation ⇒ word32 ⇒ word32 ⇒ word32
where logical-result instr-name rs1-val operand2 ≡
  if (instr-name = logic-type ANDs) ∨
                    (instr-name = logic-type ANDcc) then
                  (AND) rs1-val operand2
                 else if (instr-name = logic-type ANDN) ∨
                         (instr-name = logic-type ANDNcc) then
                  (AND) rs1-val (NOT operand2)
                 else if (instr-name = logic-type ORs) ∨
```

```
        (instr-name = logic-type ORcc) then
   (OR) rs1-val operand2
  else if instr-name ∈ {logic-type ORN,logic-type ORNcc} then
   (OR) rs1-val (NOT operand2)
  else if instr-name ∈ {logic-type XORs,logic-type XORcc} then
   (XOR) rs1-val operand2
  else — Must be XNOR or XNORcc
   (XOR) rs1-val (NOT operand2)
```

```
definition logical-new-psr-val :: word32 ⇒ ('a) sparc-state ⇒ word32
where logical-new-psr-val result s ≡
  let psr-val = cpu-reg-val PSR s;
      n-val = (ucast (result >> 31))::word1;
      z-val = if (result = 0) then 1 else 0;
      v-val = 0;
      c-val = 0
  in
  update-PSR-icc n-val z-val v-val c-val psr-val
```

```
definition logical-instr-sub1 :: sparc-operation ⇒ word32 ⇒
  ('a::len,unit) sparc-state-monad
where
logical-instr-sub1 instr-name result ≡
  if instr-name ∈ {logic-type ANDcc,logic-type ANDNcc,logic-type ORcc,
     logic-type ORNcc,logic-type XORcc,logic-type XNORcc} then
     do
       new-psr-val ← gets (λs. (logical-new-psr-val result s));
       write-cpu new-psr-val PSR;
       return ()
     od
   else return ()
```

Operational semantics for logical instructions.

```
definition logical-instr :: instruction ⇒ ('a::len,unit) sparc-state-monad
where logical-instr instr ≡
  let instr-name = fst instr;
      op-list = snd instr;
      rs1 = get-operand-w5 (op-list!1);
      rd = get-operand-w5 (op-list!3)
  in
  do
    operand2 ← gets (λs. (get-operand2 op-list s));
    curr-win ← get-curr-win();
    rs1-val ← gets (λs. (user-reg-val curr-win rs1 s));
    rd-val ← gets (λs. (user-reg-val curr-win rd s));
    result ← gets (λs. (logical-result instr-name rs1-val operand2));
```

```
    new-rd-val ← gets (λs. (if rd ≠ 0 then result else rd-val));
    write-reg new-rd-val curr-win rd;
    logical-instr-sub1 instr-name result
  od
```

Operational semantics for shift instructions.

```
definition shift-instr :: instruction ⇒ ('a::len,unit) sparc-state-monad
where shift-instr instr ≡
  let instr-name = fst instr;
      op-list = snd instr;
      flagi = get-operand-flag (op-list!0);
      rs1 = get-operand-w5 (op-list!1);
      rs2-shcnt = get-operand-w5 (op-list!2);
      rd = get-operand-w5 (op-list!3)
  in
  do
    curr-win ← get-curr-win();
    shift-count ← gets (λs. (if flagi = 0 then
                               ucast (user-reg-val curr-win rs2-shcnt s)
                             else rs2-shcnt));
    rs1-val ← gets (λs. (user-reg-val curr-win rs1 s));
    if (instr-name = shift-type SLL) ∧ (rd ≠ 0) then
      do
        rd-val ← gets (λs. (rs1-val << (unat shift-count)));
        write-reg rd-val curr-win rd;
        return ()
      od
    else if (instr-name = shift-type SRL) ∧ (rd ≠ 0) then
      do
        rd-val ← gets (λs. (rs1-val >> (unat shift-count)));
        write-reg rd-val curr-win rd;
        return ()
      od
    else if (instr-name = shift-type SRA) ∧ (rd ≠ 0) then
      do
        rd-val ← gets (λs. (rs1-val >>> (unat shift-count)));
        write-reg rd-val curr-win rd;
        return ()
      od
    else return ()
  od

definition add-instr-sub1 :: sparc-operation ⇒ word32 ⇒ word32 ⇒ word32
  ⇒ ('a::len,unit) sparc-state-monad
where add-instr-sub1 instr-name result rs1-val operand2 ≡
  if instr-name ∈ {arith-type ADDcc,arith-type ADDXcc} then
      do
        psr-val ← gets (λs. (cpu-reg-val PSR s));
        result-31 ← gets (λs. ((ucast (result >> 31))::word1));
```

```
    rs1-val-31 ← gets (λs. ((ucast (rs1-val >> 31))::word1));
    operand2-31 ← gets (λs. ((ucast (operand2 >> 31))::word1));
    new-n-val ← gets (λs. (result-31));
    new-z-val ← gets (λs. (if result = 0 then 1::word1 else 0::word1));
    new-v-val ← gets (λs. ((OR) ((AND) rs1-val-31
                                       ((AND) operand2-31
                                              (NOT result-31)))
                                ((AND) (NOT rs1-val-31)
                                       ((AND) (NOT operand2-31)
                                              result-31))));
    new-c-val ← gets (λs. ((OR) ((AND) rs1-val-31
                                       operand2-31)
                                ((AND) (NOT result-31)
                                       ((OR) rs1-val-31
                                             operand2-31))));
    new-psr-val ← gets (λs. (update-PSR-icc new-n-val
                                            new-z-val
                                            new-v-val
                                            new-c-val psr-val));
    write-cpu new-psr-val PSR;
    return ()
   od
  else return ()
```

Operational semantics for add instructions. These include ADD, ADDcc, ADDX.

```
definition add-instr :: instruction ⇒ ('a::len,unit) sparc-state-monad
where add-instr instr ≡
  let instr-name = fst instr;
      op-list = snd instr;
      rs1 = get-operand-w5 (op-list!1);
      rd = get-operand-w5 (op-list!3)
  in
  do
    operand2 ← gets (λs. (get-operand2 op-list s));
    curr-win ← get-curr-win();
    rs1-val ← gets (λs. (user-reg-val curr-win rs1 s));
    psr-val ← gets (λs. (cpu-reg-val PSR s));
    c-val ← gets (λs. (get-icc-C psr-val));
    result ← gets (λs. (if (instr-name = arith-type ADD) ∨
                           (instr-name = arith-type ADDcc) then
                        rs1-val + operand2
                       else — Must be ADDX or ADDXcc
                        rs1-val + operand2 + (ucast c-val)));
    rd-val ← gets (λs. (user-reg-val curr-win rd s));
    new-rd-val ← gets (λs. (if rd ≠ 0 then result else rd-val));
    write-reg new-rd-val curr-win rd;
    add-instr-sub1 instr-name result rs1-val operand2
```

```
  od

definition sub-instr-sub1 :: sparc-operation ⇒ word32 ⇒ word32 ⇒ word32
  ⇒ ('a::len,unit) sparc-state-monad
where sub-instr-sub1 instr-name result rs1-val operand2 ≡
  if instr-name ∈ {arith-type SUBcc,arith-type SUBXcc} then
      do
        psr-val ← gets (λs. (cpu-reg-val PSR s));
        result-31 ← gets (λs. ((ucast (result >> 31))::word1));
        rs1-val-31 ← gets (λs. ((ucast (rs1-val >> 31))::word1));
        operand2-31 ← gets (λs. ((ucast (operand2 >> 31))::word1));
        new-n-val ← gets (λs. (result-31));
        new-z-val ← gets (λs. (if result = 0 then 1::word1 else 0::word1));
        new-v-val ← gets (λs. ((OR) ((AND) rs1-val-31
                                           ((AND) (NOT operand2-31)
                                                  (NOT result-31)))
                                    ((AND) (NOT rs1-val-31)
                                           ((AND) operand2-31
                                                  result-31))));
        new-c-val ← gets (λs. ((OR) ((AND) (NOT rs1-val-31)
                                           operand2-31)
                                    ((AND) result-31
                                           ((OR) (NOT rs1-val-31)
                                                 operand2-31))));
        new-psr-val ← gets (λs. (update-PSR-icc new-n-val
                                                new-z-val
                                                new-v-val
                                                new-c-val psr-val));
        write-cpu new-psr-val PSR;
        return ()
      od
    else return ()
```

Operational semantics for subtract instructions. These include SUB, SUBcc, SUBX.

```
definition sub-instr :: instruction ⇒ ('a::len,unit) sparc-state-monad
where sub-instr instr ≡
  let instr-name = fst instr;
      op-list = snd instr;
      rs1 = get-operand-w5 (op-list!1);
      rd = get-operand-w5 (op-list!3)
  in
  do
    operand2 ← gets (λs. (get-operand2 op-list s));
    curr-win ← get-curr-win();
    rs1-val ← gets (λs. (user-reg-val curr-win rs1 s));
    psr-val ← gets (λs. (cpu-reg-val PSR s));
    c-val ← gets (λs. (get-icc-C psr-val));
```

```
    result ← gets (λs. (if (instr-name = arith-type SUB) ∨
                           (instr-name = arith-type SUBcc) then
                         rs1-val − operand2
                        else — Must be SUBX or SUBXcc
                         rs1-val − operand2 − (ucast c-val)));
    rd-val ← gets (λs. (user-reg-val curr-win rd s));
    new-rd-val ← gets (λs. (if rd ≠ 0 then result else rd-val));
    write-reg new-rd-val curr-win rd;
    sub-instr-sub1 instr-name result rs1-val operand2
  od

definition mul-instr-sub1 :: sparc-operation ⇒ word32 ⇒
  ('a::len,unit) sparc-state-monad
where mul-instr-sub1 instr-name result ≡
  if instr-name ∈ {arith-type SMULcc,arith-type UMULcc} then
      do
        psr-val ← gets (λs. (cpu-reg-val PSR s));
        new-n-val ← gets (λs. ((ucast (result >> 31))::word1));
        new-z-val ← gets (λs. (if result = 0 then 1 else 0));
        new-v-val ← gets (λs. 0);
        new-c-val ← gets (λs. 0);
        new-psr-val ← gets (λs. (update-PSR-icc new-n-val
                                                new-z-val
                                                new-v-val
                                                new-c-val psr-val));
        write-cpu new-psr-val PSR;
        return ()
      od
    else return ()
```

Operational semantics for multiply instructions.

```
definition mul-instr :: instruction ⇒ ('a::len,unit) sparc-state-monad
where mul-instr instr ≡
  let instr-name = fst instr;
      op-list = snd instr;
      rs1 = get-operand-w5 (op-list!1);
      rd = get-operand-w5 (op-list!3)
  in
  do
    operand2 ← gets (λs. (get-operand2 op-list s));
    curr-win ← get-curr-win();
    rs1-val ← gets (λs. (user-reg-val curr-win rs1 s));
    psr-val ← gets (λs. (cpu-reg-val PSR s));
     result0 ← gets (λs. (if instr-name ∈ {arith-type UMUL,arith-type UMULcc}
then
                           (word-of-int ((uint rs1-val) *
                                         (uint operand2)))::word64
                         else — Must be SMUL or SMULcc
```

```
                    (word-of-int ((sint rs1-val) *
                                  (sint operand2)))::word64));
    — whether to use ucast or scast does not matter below.
    y-val ← gets (λs. ((ucast (result0 >> 32))::word32));
    write-cpu y-val Y;
    result ← gets (λs. ((ucast result0)::word32));
    rd-val ← gets (λs. (user-reg-val curr-win rd s));
    new-rd-val ← gets (λs. (if rd ≠ 0 then result else rd-val));
    write-reg new-rd-val curr-win rd;
    mul-instr-sub1 instr-name result
  od

definition div-comp-temp-64bit :: instruction ⇒ word64 ⇒
  virtua-address ⇒ word64
where div-comp-temp-64bit i y-rs1 operand2 ≡
  if ((fst i) = arith-type UDIV) ∨ ((fst i) = arith-type UDIVcc) then
    (word-of-int ((uint y-rs1) div (uint operand2)))::word64
  else — Must be SDIV or SDIVcc.
    — Due to Isabelle's rounding method is not nearest to zero,
    — we have to implement division in a different way.
    let sop1 = sint y-rs1;
        sop2 = sint operand2;
        pop1 = abs sop1;
        pop2 = abs sop2
    in
    if sop1 > 0 ∧ sop2 > 0 then
      (word-of-int (sop1 div sop2))
    else if sop1 > 0 ∧ sop2 < 0 then
      (word-of-int (− (sop1 div pop2)))
    else if sop1 < 0 ∧ sop2 > 0 then
      (word-of-int (− (pop1 div sop2)))
    else — sop1 < 0 ∧ sop2 < 0
      (word-of-int (pop1 div pop2))

definition div-comp-temp-V :: instruction ⇒ word32 ⇒ word33 ⇒ word1
where div-comp-temp-V i w32 w33 ≡
  if ((fst i) = arith-type UDIV) ∨ ((fst i) = arith-type UDIVcc) then
    if w32 = 0 then 0 else 1
  else — Must be SDIV or SDIVcc.
    if (w33 = 0) ∨ (w33 = (0b111111111111111111111111111111111::word33))
    then 0 else 1

definition div-comp-result :: instruction ⇒ word1 ⇒ word64 ⇒ word32
where div-comp-result i temp-V temp-64bit ≡
  if temp-V = 1 then
    if ((fst i) = arith-type UDIV) ∨ ((fst i) = arith-type UDIVcc) then
      (0b11111111111111111111111111111111::word32)
    else if (fst i) ∈ {arith-type SDIV,arith-type SDIVcc} then
      if temp-64bit > 0 then
```

```
      (0b01111111111111111111111111111111::word32)
    else ((word-of-int (0 - (uint (0b10000000000000000000000000000000::word32))))::word32)
    else ((ucast temp-64bit)::word32)
  else ((ucast temp-64bit)::word32)

definition div-write-new-val :: instruction ⇒ word32 ⇒ word1 ⇒
  ('a::len,unit) sparc-state-monad
where div-write-new-val i result temp-V ≡
  if (fst i) ∈ {arith-type UDIVcc,arith-type SDIVcc} then
  do
    psr-val ← gets (λs. (cpu-reg-val PSR s));
    new-n-val ← gets (λs. ((ucast (result >> 31))::word1));
    new-z-val ← gets (λs. (if result = 0 then 1 else 0));
    new-v-val ← gets (λs. temp-V);
    new-c-val ← gets (λs. 0);
    new-psr-val ← gets (λs. (update-PSR-icc new-n-val
                                            new-z-val
                                            new-v-val
                                            new-c-val psr-val));
    write-cpu new-psr-val PSR;
    return ()
  od
  else return ()

definition div-comp :: instruction ⇒ word5 ⇒ word5 ⇒ virtua-address ⇒
  ('a::len,unit) sparc-state-monad
where div-comp instr rs1 rd operand2 ≡
  do
    curr-win ← get-curr-win();
    rs1-val ← gets (λs. (user-reg-val curr-win rs1 s));

    y-val ← gets (λs. (cpu-reg-val Y s));
    y-rs1 ← gets (λs. ((word-cat y-val rs1-val)::word64));
    temp-64bit ← gets (λs. (div-comp-temp-64bit instr y-rs1 operand2));
    result ← gets (λs. ((ucast temp-64bit)::word32));
    temp-high32 ← gets (λs. ((ucast (temp-64bit >> 32))::word32));
    temp-high33 ← gets (λs. ((ucast (temp-64bit >> 31))::word33));
    temp-V ← gets (λs. (div-comp-temp-V instr temp-high32 temp-high33));
    result ← gets (λs. (div-comp-result instr temp-V temp-64bit));
    rd-val ← gets (λs. (user-reg-val curr-win rd s));
    new-rd-val ← gets (λs. (if rd ≠ 0 then result else rd-val));
    write-reg new-rd-val curr-win rd;
    div-write-new-val instr result temp-V
  od
```

Operational semantics for divide instructions.

```
definition div-instr :: instruction ⇒ ('a::len,unit) sparc-state-monad
where div-instr instr ≡
  let instr-name = fst instr;
```

```
    op-list = snd instr;
    rs1 = get-operand-w5 (op-list!1);
    rd = get-operand-w5 (op-list!3)
  in
  do
    operand2 ← gets (λs. (get-operand2 op-list s));
    if (uint operand2) = 0 then
      do
        raise-trap division-by-zero;
        return ()
      od
    else
      div-comp instr rs1 rd operand2
  od

definition ld-word0 :: instruction ⇒ word32 ⇒ virtua-address ⇒  word32
where ld-word0 instr data-word address ≡
  if (fst instr) ∈ {load-store-type LDSB,load-store-type LDUB,
   load-store-type LDUBA,load-store-type LDSBA} then
    let byte = if (uint ((ucast address)::word2)) = 0 then
                 (ucast (data-word >> 24))::word8
               else if (uint ((ucast address)::word2)) = 1 then
                 (ucast (data-word >> 16))::word8
               else if (uint ((ucast address)::word2)) = 2 then
                 (ucast (data-word >> 8))::word8
               else — Must be 3.
                 (ucast data-word)::word8
    in
     if (fst instr) = load-store-type LDSB ∨ (fst instr) = load-store-type LDSBA
then
      sign-ext8 byte
    else
      zero-ext8 byte
  else if (fst instr) = load-store-type LDUH ∨ (fst instr) = load-store-type LDSH
∨
        (fst instr) = load-store-type LDSHA ∨ (fst instr) = load-store-type LDUHA
       then
    let halfword = if (uint ((ucast address)::word2)) = 0 then
                     (ucast (data-word >> 16))::word16
                   else — Must be 2.
                     (ucast data-word)::word16
    in
     if (fst instr) = load-store-type LDSH ∨ (fst instr) = load-store-type LDSHA
then
      sign-ext16 halfword
    else
      zero-ext16 halfword
  else — Must be LDD
    data-word
```

```
definition ld-asi :: instruction ⇒ word1 ⇒ asi-type
where ld-asi instr s-val ≡
  if (fst instr) ∈ {load-store-type LDD,load-store-type LD,load-store-type LDUH,
    load-store-type LDSB,load-store-type LDUB,load-store-type LDSH} then
    if s-val = 0 then (word-of-int 10)::asi-type
    else (word-of-int 11)::asi-type
  else — Must be LDA, LDUBA, LDSBA, LDSHA, LDUHA, or LDDA.
    get-operand-asi ((snd instr)!3)


definition load-sub2 :: virtua-address ⇒ asi-type ⇒ word5 ⇒
  ('a::len) window-size ⇒ word32 ⇒ ('a,unit) sparc-state-monad
where load-sub2 address asi rd curr-win word0 ≡
  do
    write-reg word0 curr-win ((AND) rd 0b11110);
    (result1,new-state1) ← gets (λs. (memory-read asi (address + 4) s));
    if result1 = None then
    do
      raise-trap data-access-exception;
      return ()
    od
    else
    do
      word1 ← gets (λs. (case result1 of Some v ⇒ v));
      modify (λs. (new-state1));
      write-reg word1 curr-win ((OR) rd 1);
      return ()
    od
  od

definition load-sub3 :: instruction ⇒ ('a::len) window-size ⇒
  word5 ⇒ asi-type ⇒ virtua-address ⇒
  ('a::len,unit) sparc-state-monad
where load-sub3 instr curr-win rd asi address ≡
  do
    (result,new-state) ← gets (λs. (memory-read asi address s));
    if result = None then
    do
      raise-trap data-access-exception;
      return ()
    od
    else
    do
      data-word ← gets (λs. (case result of Some v ⇒ v));
      modify (λs. (new-state));
      word0 ← gets (λs. (ld-word0 instr data-word address));
      if rd ≠ 0 ∧ (fst instr) ∈ {load-store-type LD,load-store-type LDA,
```

```
      load-store-type LDUH,load-store-type LDSB,load-store-type LDUB,
      load-store-type LDUBA,load-store-type LDSH,load-store-type LDSHA,
      load-store-type LDUHA,load-store-type LDSBA} then
    do
      write-reg word0 curr-win rd;
      return ()
    od
    else — Must be LDD or LDDA
      load-sub2 address asi rd curr-win word0
  od
od

definition load-sub1 :: instruction ⇒ word5 ⇒ word1 ⇒
  ('a::len,unit) sparc-state-monad
where load-sub1 instr rd s-val ≡
  do
    curr-win ← get-curr-win();
    address ← gets (λs. (get-addr (snd instr) s));
    asi ← gets (λs. (ld-asi instr s-val));
    if (((fst instr) = load-store-type LDD ∨ (fst instr) = load-store-type LDDA)
       ∧ ((ucast address)::word3) ≠ 0)
        ∨ ((fst instr) ∈ {load-store-type LD,load-store-type LDA}
           ∧ ((ucast address)::word2) ≠ 0)
           ∨ (((fst instr) = load-store-type LDUH ∨ (fst instr) = load-store-type
LDUHA
               ∨ (fst instr) = load-store-type LDSH ∨ (fst instr) = load-store-type
LDSHA)
           ∧ ((ucast address)::word1) ≠ 0)
    then
    do
      raise-trap mem-address-not-aligned;
      return ()
    od
    else
      load-sub3 instr curr-win rd asi address
  od
```

Operational semantics for Load instructions.

```
definition load-instr :: instruction ⇒ ('a::len,unit) sparc-state-monad
where load-instr instr ≡
  let instr-name = fst instr;
      op-list = snd instr;
      flagi = get-operand-flag (op-list!0);
      rd = if instr-name ∈ {load-store-type LDUBA,load-store-type LDA,
        load-store-type LDSBA,load-store-type LDSHA,
        load-store-type LDSHA,load-store-type LDDA} then — rd is member 4
            get-operand-w5 (op-list!4)
          else — rd is member 3
            get-operand-w5 (op-list!3)
```

```
  in
  do
    psr-val ← gets (λs. (cpu-reg-val PSR s));
    s-val ← gets (λs. (get-S psr-val));
    if instr-name ∈ {load-store-type LDA,load-store-type LDUBA,
      load-store-type LDSBA,load-store-type LDSHA,
      load-store-type LDUHA,load-store-type LDDA} ∧ s-val = 0 then
      do
        raise-trap privileged-instruction;
        return ()
      od
    else if instr-name ∈ {load-store-type LDA,load-store-type LDUBA,
      load-store-type LDSBA,load-store-type LDSHA,load-store-type LDUHA,
      load-store-type LDDA} ∧ flagi = 1 then
      do
        raise-trap illegal-instruction;
        return ()
      od
    else
      load-sub1 instr rd s-val
  od

definition st-asi :: instruction ⇒ word1 ⇒ asi-type
where st-asi instr s-val ≡
  if (fst instr) ∈ {load-store-type STD,load-store-type ST,
    load-store-type STH,load-store-type STB} then
    if s-val = 0 then (word-of-int 10)::asi-type
    else (word-of-int 11)::asi-type
  else — Must be STA, STBA, STHA, STDA.
    get-operand-asi ((snd instr)!3)


definition st-byte-mask :: instruction ⇒ virtua-address ⇒ word4
where st-byte-mask instr address ≡
  if (fst instr) ∈ {load-store-type STD,load-store-type ST,
    load-store-type STA,load-store-type STDA} then
    (0b1111::word4)
  else if (fst instr) ∈ {load-store-type STH,load-store-type STHA} then
    if ((ucast address)::word2) = 0 then
      (0b1100::word4)
    else — Must be 2.
      (0b0011::word4)
  else — Must be STB or STBA.
    if ((ucast address)::word2) = 0 then
      (0b1000::word4)
    else if ((ucast address)::word2) = 1 then
      (0b0100::word4)
    else if ((ucast address)::word2) = 2 then
      (0b0010::word4)
```

```
    else — Must be 3.
      (0b0001::word4)


definition st-data0 :: instruction ⇒ ('a::len) window-size ⇒
  word5 ⇒ virtua-address ⇒ ('a) sparc-state ⇒ reg-type
where st-data0 instr curr-win rd address s ≡
  if (fst instr) ∈ {load-store-type STD,load-store-type STDA} then
    user-reg-val curr-win ((AND) rd 0b11110) s
  else if (fst instr) ∈ {load-store-type ST,load-store-type STA} then
    user-reg-val curr-win rd s
  else if (fst instr) ∈ {load-store-type STH,load-store-type STHA} then
    if ((ucast address)::word2) = 0 then
      (user-reg-val curr-win rd s) << 16
    else — Must be 2.
      user-reg-val curr-win rd s
  else — Must be STB or STBA.
    if ((ucast address)::word2) = 0 then
      (user-reg-val curr-win rd s) << 24
    else if ((ucast address)::word2) = 1 then
      (user-reg-val curr-win rd s) << 16
    else if ((ucast address)::word2) = 2 then
       (user-reg-val curr-win rd s) << 8
    else — Must be 3.
       user-reg-val curr-win rd s


definition store-sub2 :: instruction ⇒ ('a::len) window-size ⇒
  word5 ⇒ asi-type ⇒ virtua-address ⇒
  ('a::len,unit) sparc-state-monad
where store-sub2 instr curr-win rd asi address ≡
  do
    byte-mask ← gets (λs. (st-byte-mask instr address));
    data0 ← gets (λs. (st-data0 instr curr-win rd address s));
    result0 ← gets (λs. (memory-write asi address byte-mask data0 s));
    if result0 = None then
    do
      raise-trap data-access-exception;
      return ()
    od
    else
    do
      new-state ← gets (λs. (case result0 of Some v ⇒ v));
      modify (λs. (new-state));
      if (fst instr) ∈ {load-store-type STD,load-store-type STDA} then
      do
        data1 ← gets (λs. (user-reg-val curr-win ((OR) rd 0b00001) s));
       result1 ← gets (λs. (memory-write asi (address + 4) (0b1111::word4) data1
s));
```

```
      if result1 = None then
      do
        raise-trap data-access-exception;
        return ()
      od
      else
        do
        new-state1 ← gets (λs. (case result1 of Some v ⇒ v));
        modify (λs. (new-state1));
        return ()
        od
    od
    else
      return ()
  od
od

definition store-sub1 :: instruction ⇒ word5 ⇒ word1 ⇒
  ('a::len,unit) sparc-state-monad
where store-sub1 instr rd s-val ≡
  do
    curr-win ← get-curr-win();
    address ← gets (λs. (get-addr (snd instr) s));
    asi ← gets (λs. (st-asi instr s-val));
    — The following code is intentionally long to match the definitions in SPARCv8.
    if ((fst instr) = load-store-type STH ∨ (fst instr) = load-store-type STHA)
       ∧ ((ucast address)::word1) ≠ 0 then
    do
      raise-trap mem-address-not-aligned;
      return ()
    od
    else if (fst instr) ∈ {load-store-type ST,load-store-type STA}
          ∧ ((ucast address)::word2) ≠ 0 then
    do
      raise-trap mem-address-not-aligned;
      return ()
    od
    else if (fst instr) ∈ {load-store-type STD,load-store-type STDA}
          ∧ ((ucast address)::word3) ≠ 0 then
    do
      raise-trap mem-address-not-aligned;
      return ()
    od
    else
      store-sub2 instr curr-win rd asi address
  od
```

Operational semantics for Store instructions.

```
definition store-instr :: instruction ⇒
```

```
  ('a::len,unit) sparc-state-monad
where store-instr instr ≡
  let instr-name = fst instr;
      op-list = snd instr;
      flagi = get-operand-flag (op-list!0);
      rd = if instr-name ∈ {load-store-type STA,load-store-type STBA,
            load-store-type STHA,load-store-type STDA} then — rd is member 4
            get-operand-w5 (op-list!4)
           else — rd is member 3
            get-operand-w5 (op-list!3)
  in
  do
    psr-val ← gets (λs. (cpu-reg-val PSR s));
    s-val ← gets (λs. (get-S psr-val));
    if instr-name ∈ {load-store-type STA,load-store-type STDA,
      load-store-type STHA,load-store-type STBA} ∧ s-val = 0 then
      do
        raise-trap privileged-instruction;
        return ()
      od
    else if instr-name ∈ {load-store-type STA,load-store-type STDA,
      load-store-type STHA,load-store-type STBA} ∧ flagi = 1 then
      do
        raise-trap illegal-instruction;
        return ()
      od
    else
      store-sub1 instr rd s-val
  od
```

The instructions below are not used by Xtratum and they are not tested.

```
definition ldst-asi :: instruction ⇒ word1 ⇒ asi-type
where ldst-asi instr s-val ≡
  if (fst instr) ∈ {load-store-type LDSTUB} then
    if s-val = 0 then (word-of-int 10)::asi-type
    else (word-of-int 11)::asi-type
  else — Must be LDSTUBA.
    get-operand-asi ((snd instr)!3)


definition ldst-word0 :: instruction ⇒ word32 ⇒ virtua-address ⇒  word32
where ldst-word0 instr data-word address ≡
  let byte = if (uint ((ucast address)::word2)) = 0 then
                (ucast (data-word >> 24))::word8
              else if (uint ((ucast address)::word2)) = 1 then
                (ucast (data-word >> 16))::word8
              else if (uint ((ucast address)::word2)) = 2 then
                (ucast (data-word >> 8))::word8
              else — Must be 3.
```

```
                (ucast data-word)::word8
  in
  zero-ext8 byte


definition ldst-byte-mask :: instruction ⇒ virtua-address ⇒ word4
where ldst-byte-mask instr address ≡
    if ((ucast address)::word2) = 0 then
      (0b1000::word4)
    else if ((ucast address)::word2) = 1 then
      (0b0100::word4)
    else if ((ucast address)::word2) = 2 then
      (0b0010::word4)
    else — Must be 3.
      (0b0001::word4)


definition load-store-sub1 :: instruction ⇒ word5 ⇒ word1 ⇒
  ('a::len,unit) sparc-state-monad
where load-store-sub1 instr rd s-val ≡
  do
    curr-win ← get-curr-win();
    address ← gets (λs. (get-addr (snd instr) s));
    asi ← gets (λs. (ldst-asi instr s-val));
    — wait for locks to be lifted.
   — an implementation actually need only block when another LDSTUB or SWAP
   — is pending on the same byte in memory as the one addressed by this LDSTUB
    — Should wait when block-type = 1 ∨ block-word = 1
    — until another processes write both to be 0.
    — We implement this as setting pc as npc when the instruction
    — is blocked. This way, in the next iteration, we will still execution
    — the current instruction.
    block-byte ← gets (λs. (pb-block-ldst-byte-val address s));
    block-word ← gets (λs. (pb-block-ldst-word-val address s));
    if block-byte ∨ block-word then
    do
      pc-val ← gets (λs. (cpu-reg-val PC s));
      write-cpu pc-val nPC;
      return ()
    od
    else
    do
      modify (λs. (pb-block-ldst-byte-mod address True s));
      (result,new-state) ← gets (λs. (memory-read asi address s));
      if result = None then
      do
        raise-trap data-access-exception;
        return ()
      od
```

```
      else
      do
        data-word ← gets (λs. (case result of Some v ⇒ v));
        modify (λs. (new-state));
        byte-mask ← gets (λs. (ldst-byte-mask instr address));
        data0 ← gets (λs. (0b11111111111111111111111111111111::word32));
        result0 ← gets (λs. (memory-write asi address byte-mask data0 s));
        modify (λs. (pb-block-ldst-byte-mod address False s));
        if result0 = None then
        do
          raise-trap data-access-exception;
          return ()
        od
        else
        do
          new-state1 ← gets (λs. (case result0 of Some v ⇒ v));
          modify (λs. (new-state1));
          word0 ← gets (λs. (ldst-word0 instr data-word address));
          if rd ≠ 0 then
          do
            write-reg word0 curr-win rd;
            return ()
          od
          else
            return ()
        od
      od
    od
  od
```

Operational semantics for atomic load-store.

```
definition load-store-instr :: instruction ⇒ ('a::len,unit) sparc-state-monad
where load-store-instr instr ≡
  let instr-name = fst instr;
      op-list = snd instr;
      flagi = get-operand-flag (op-list!0);
      rd = if instr-name ∈ {load-store-type LDSTUBA} then — rd is member 4
             get-operand-w5 (op-list!4)
           else — rd is member 3
             get-operand-w5 (op-list!3)
  in
  do
    psr-val ← gets (λs. (cpu-reg-val PSR s));
    s-val ← gets (λs. (get-S psr-val));
    if instr-name ∈ {load-store-type LDSTUBA} ∧ s-val = 0 then
      do
        raise-trap privileged-instruction;
        return ()
      od
```

```
    else if instr-name ∈ {load-store-type LDSTUBA} ∧ flagi = 1 then
      do
        raise-trap illegal-instruction;
        return ()
      od
    else
      load-store-sub1 instr rd s-val
  od

definition swap-sub1 :: instruction ⇒ word5 ⇒ word1 ⇒
  ('a::len,unit) sparc-state-monad
where swap-sub1 instr rd s-val ≡
  do
    curr-win ← get-curr-win();
    address ← gets (λs. (get-addr (snd instr) s));
    asi ← gets (λs. (ldst-asi instr s-val));
    temp ← gets (λs. (user-reg-val curr-win rd s));
    — wait for locks to be lifted.
   — an implementation actually need only block when another LDSTUB or SWAP
   — is pending on the same byte in memory as the one addressed by this LDSTUB
    — Should wait when block-type = 1 ∨ block-word = 1
    — until another processes write both to be 0.
    — We implement this as setting pc as npc when the instruction
    — is blocked. This way, in the next iteration, we will still execution
    — the current instruction.
    block-byte ← gets (λs. (pb-block-ldst-byte-val address s));
    block-word ← gets (λs. (pb-block-ldst-word-val address s));
    if block-byte ∨ block-word then
    do
      pc-val ← gets (λs. (cpu-reg-val PC s));
      write-cpu pc-val nPC;
      return ()
    od
    else
    do
      modify (λs. (pb-block-ldst-word-mod address True s));
      (result,new-state) ← gets (λs. (memory-read asi address s));
      if result = None then
      do
        raise-trap data-access-exception;
        return ()
      od
      else
      do
        word ← gets (λs. (case result of Some v ⇒ v));
        modify (λs. (new-state));
        byte-mask ← gets (λs. (0b1111::word4));
        result0 ← gets (λs. (memory-write asi address byte-mask temp s));
        modify (λs. (pb-block-ldst-word-mod address False s));
```

```
      if result0 = None then
      do
        raise-trap data-access-exception;
        return ()
      od
      else
      do
        new-state1 ← gets (λs. (case result0 of Some v ⇒ v));
        modify (λs. (new-state1));
        if rd ≠ 0 then
        do
          write-reg word curr-win rd;
          return ()
        od
        else
          return ()
      od
    od
  od
od
```

Operational semantics for swap.

```
definition swap-instr :: instruction ⇒ ('a::len,unit) sparc-state-monad
where swap-instr instr ≡
  let instr-name = fst instr;
      op-list = snd instr;
      flagi = get-operand-flag (op-list!0);
      rd = if instr-name ∈ {load-store-type SWAPA} then — rd is member 4
             get-operand-w5 (op-list!4)
           else — rd is member 3
             get-operand-w5 (op-list!3)
  in
  do
    psr-val ← gets (λs. (cpu-reg-val PSR s));
    s-val ← gets (λs. (get-S psr-val));
    if instr-name ∈ {load-store-type SWAPA} ∧ s-val = 0 then
      do
        raise-trap privileged-instruction;
        return ()
      od
    else if instr-name ∈ {load-store-type SWAPA} ∧ flagi = 1 then
      do
        raise-trap illegal-instruction;
        return ()
      od
    else
      swap-sub1 instr rd s-val
  od
```

definition *bit2-zero* :: *word2* ⇒ *word1*
where *bit2-zero w2* ≡ *if w2* ≠ *0 then 1 else 0*

Operational semantics for tagged add instructions.

```
definition tadd-instr :: instruction ⇒ ('a::len,unit) sparc-state-monad
where tadd-instr instr ≡
  let instr-name = fst instr;
      op-list = snd instr;
      rs1 = get-operand-w5 (op-list!1);
      rd = get-operand-w5 (op-list!3)
  in
  do
    operand2 ← gets (λs. (get-operand2 op-list s));
    curr-win ← get-curr-win();
    rs1-val ← gets (λs. (user-reg-val curr-win rs1 s));
    psr-val ← gets (λs. (cpu-reg-val PSR s));
    c-val ← gets (λs. (get-icc-C psr-val));
    result ← gets (λs. (rs1-val + operand2));
    result-31 ← gets (λs. ((ucast (result >> 31))::word1));
    rs1-val-31 ← gets (λs. ((ucast (rs1-val >> 31))::word1));
    operand2-31 ← gets (λs. ((ucast (operand2 >> 31))::word1));
    rs1-val-2 ← gets (λs. (bit2-zero ((ucast rs1-val)::word2)));
    operand2-2 ← gets (λs. (bit2-zero ((ucast operand2)::word2)));
    temp-V ← gets (λs. ((OR) ((OR) ((AND) rs1-val-31
                                          ((AND) operand2-31
                                                 (NOT result-31)))
                                    ((AND) (NOT rs1-val-31)
                                           ((AND) (NOT operand2-31)
                                                  result-31)))
                             ((OR) rs1-val-2 operand2-2)));
    if instr-name = arith-type TADDccTV ∧ temp-V = 1 then
    do
      raise-trap tag-overflow;
      return ()
    od
    else
    do
      rd-val ← gets (λs. (user-reg-val curr-win rd s));
      new-rd-val ← gets (λs. (if rd ≠ 0 then result else rd-val));
      write-reg new-rd-val curr-win rd;
      new-n-val ← gets (λs. (result-31));
      new-z-val ← gets (λs. (if result = 0 then 1::word1 else 0::word1));
      new-v-val ← gets (λs. temp-V);
      new-c-val ← gets (λs. ((OR) ((AND) rs1-val-31
                                         operand2-31)
                                  ((AND) (NOT result-31)
                                         ((OR) rs1-val-31
                                               operand2-31))));
      new-psr-val ← gets (λs. (update-PSR-icc new-n-val
```

```
                                        new-z-val
                                        new-v-val
                                        new-c-val psr-val));
    write-cpu new-psr-val PSR;
    rd-val ← gets (λs. (user-reg-val curr-win rd s));
    new-rd-val ← gets (λs. (if rd ≠ 0 then result else rd-val));
    write-reg new-rd-val curr-win rd;
    return ()
  od
od
```

Operational semantics for tagged add instructions.

```
definition tsub-instr :: instruction ⇒ ('a::len,unit) sparc-state-monad
where tsub-instr instr ≡
  let instr-name = fst instr;
      op-list = snd instr;
      rs1 = get-operand-w5 (op-list!1);
      rd = get-operand-w5 (op-list!3)
  in
  do
    operand2 ← gets (λs. (get-operand2 op-list s));
    curr-win ← get-curr-win();
    rs1-val ← gets (λs. (user-reg-val curr-win rs1 s));
    psr-val ← gets (λs. (cpu-reg-val PSR s));
    c-val ← gets (λs. (get-icc-C psr-val));
    result ← gets (λs. (rs1-val − operand2));
    result-31 ← gets (λs. ((ucast (result >> 31))::word1));
    rs1-val-31 ← gets (λs. ((ucast (rs1-val >> 31))::word1));
    operand2-31 ← gets (λs. ((ucast (operand2 >> 31))::word1));
    rs1-val-2 ← gets (λs. (bit2-zero ((ucast rs1-val)::word2)));
    operand2-2 ← gets (λs. (bit2-zero ((ucast operand2)::word2)));
    temp-V ← gets (λs. ((OR) ((OR) ((AND) rs1-val-31
                                          ((AND) operand2-31
                                                 (NOT result-31)))
                                   ((AND) (NOT rs1-val-31)
                                          ((AND) (NOT operand2-31)
                                                 result-31)))
                             ((OR) rs1-val-2 operand2-2)));
    if instr-name = arith-type TSUBccTV ∧ temp-V = 1 then
    do
      raise-trap tag-overflow;
      return ()
    od
    else
    do
      rd-val ← gets (λs. (user-reg-val curr-win rd s));
      new-rd-val ← gets (λs. (if rd ≠ 0 then result else rd-val));
      write-reg new-rd-val curr-win rd;
      new-n-val ← gets (λs. (result-31));
```

```
    new-z-val ← gets (λs. (if result = 0 then 1::word1 else 0::word1));
    new-v-val ← gets (λs. temp-V);
    new-c-val ← gets (λs. ((OR) ((AND) rs1-val-31
                                       operand2-31)
                                ((AND) (NOT result-31)
                                       ((OR) rs1-val-31
                                             operand2-31))));
    new-psr-val ← gets (λs. (update-PSR-icc new-n-val
                                            new-z-val
                                            new-v-val
                                            new-c-val psr-val));
    write-cpu new-psr-val PSR;
    rd-val ← gets (λs. (user-reg-val curr-win rd s));
    new-rd-val ← gets (λs. (if rd ≠ 0 then result else rd-val));
    write-reg new-rd-val curr-win rd;
    return ()
   od
  od
```

definition *muls-op2* :: *inst-operand list* ⇒ (*'a*::*len*) *sparc-state* ⇒ *word32*
where *muls-op2 op-list s* ≡
  let y-val = cpu-reg-val Y s in
  if ((*ucast y-val*)::*word1*) = *0 then 0*
  else get-operand2 op-list s

Operational semantics for multiply step instruction.

definition *muls-instr* :: *instruction* ⇒ (*'a*::*len*,*unit*) *sparc-state-monad*
where *muls-instr instr* ≡

```
  let instr-name = fst instr;
      op-list = snd instr;
      rs1 = get-operand-w5 (op-list!1);
      rd = get-operand-w5 (op-list!3)
  in
  do
    curr-win ← get-curr-win();
    rs1-val ← gets (λs. (user-reg-val curr-win rs1 s));
    psr-val ← gets (λs. (cpu-reg-val PSR s));
    n-val ← gets (λs. (get-icc-N psr-val));
    v-val ← gets (λs. (get-icc-V psr-val));
    c-val ← gets (λs. (get-icc-C psr-val));
    y-val ← gets (λs. (cpu-reg-val Y s));
    operand1 ← gets (λs. (word-cat ((XOR) n-val v-val)
                                   ((ucast (rs1-val >> 1))::word31)));
    operand2 ← gets (λs. (muls-op2 op-list s));
    result ← gets (λs. (operand1 + operand2));
     new-y-val ← gets (λs. (word-cat ((ucast rs1-val)::word1) ((ucast (y-val >>
1))::word31)));
    write-cpu new-y-val Y;
```

```
  rd-val ← gets (λs. (user-reg-val curr-win rd s));
  new-rd-val ← gets (λs. (if rd ≠ 0 then result else rd-val));
  write-reg new-rd-val curr-win rd;
  result-31 ← gets (λs. ((ucast (result >> 31))::word1));
  operand1-31 ← gets (λs. ((ucast (operand1 >> 31))::word1));
  operand2-31 ← gets (λs. ((ucast (operand2 >> 31))::word1));
  new-n-val ← gets (λs. (result-31));
  new-z-val ← gets (λs. (if result = 0 then 1::word1 else 0::word1));
  new-v-val ← gets (λs. ((OR) ((AND) operand1-31
                               ((AND) operand2-31
                                  (NOT result-31)))
                               ((AND) (NOT operand1-31)
                                  ((AND) (NOT operand2-31)
                                         result-31))));
  new-c-val ← gets (λs. ((OR) ((AND) operand1-31
                                     operand2-31)
                               ((AND) (NOT result-31)
                                      ((OR) operand1-31
                                            operand2-31))));
  new-psr-val ← gets (λs. (update-PSR-icc new-n-val
                                          new-z-val
                                          new-v-val
                                          new-c-val psr-val));
  write-cpu new-psr-val PSR;
  return ()
od
```

Evaluate icc based on the bits N, Z, V, C in PSR and the type of ticc instruction. See Sparcv8 manual Page 182.

```
definition trap-eval-icc::sparc-operation ⇒ word1 ⇒ word1 ⇒ word1 ⇒ word1
⇒ int
where trap-eval-icc instr-name n-val z-val v-val c-val ≡
  if instr-name = ticc-type TNE then
    if z-val = 0 then 1 else 0
  else if instr-name = ticc-type TE then
    if z-val = 1 then 1 else 0
  else if instr-name = ticc-type TG then
    if ((OR) z-val (n-val XOR v-val)) = 0 then 1 else 0
  else if instr-name = ticc-type TLE then
    if ((OR) z-val (n-val XOR v-val)) = 1 then 1 else 0
  else if instr-name = ticc-type TGE then
    if (n-val XOR v-val) = 0 then 1 else 0
  else if instr-name = ticc-type TL then
    if (n-val XOR v-val) = 1 then 1 else 0
  else if instr-name = ticc-type TGU then
    if (c-val = 0 ∧ z-val = 0) then 1 else 0
  else if instr-name = ticc-type TLEU then
    if (c-val = 1 ∨ z-val = 1) then 1 else 0
  else if instr-name = ticc-type TCC then
```

```
    if c-val = 0 then 1 else 0
  else if instr-name = ticc-type TCS then
    if c-val = 1 then 1 else 0
  else if instr-name = ticc-type TPOS then
    if n-val = 0 then 1 else 0
  else if instr-name = ticc-type TNEG then
    if n-val = 1 then 1 else 0
  else if instr-name = ticc-type TVC then
    if v-val = 0 then 1 else 0
  else if instr-name = ticc-type TVS then
    if v-val = 1 then 1 else 0
  else if instr-name = ticc-type TA then 1
  else if instr-name = ticc-type TN then 0
  else −1
```

Get *operand2* for *ticc* based on the flag *i*, *rs1*, *rs2*, and *trap-imm7*. If $i = 0$ then *operand2* = *r*[*rs2*], else *operand2* = *sign-ext7*(*trap-imm7*). *op-list* should be [*i*,*rs1*,*rs2*] or [*i*,*rs1*,*trap-imm7*].

```
definition get-trap-op2::inst-operand list ⇒ ('a::len) sparc-state
  ⇒ virtua-address
where get-trap-op2 op-list s ≡
  let flagi = get-operand-flag (op-list!0);
      curr-win = ucast (get-CWP (cpu-reg-val PSR s))
  in
  if flagi = 0 then
    let rs2 = get-operand-w5 (op-list!2);
        rs2-val = user-reg-val curr-win rs2 s
    in rs2-val
  else
    let ext-simm7 = sign-ext7 (get-operand-imm7 (op-list!2)) in
    ext-simm7
```

Operational semantics for Ticc insturctions.

```
definition ticc-instr::instruction ⇒
  ('a::len,unit) sparc-state-monad
where ticc-instr instr ≡
  let instr-name = fst instr;
      op-list = snd instr;
      rs1 = get-operand-w5 (op-list!1)
  in
  do
    n-val ← gets (λs. get-icc-N ((cpu-reg s) PSR));
    z-val ← gets (λs. get-icc-Z ((cpu-reg s) PSR));
    v-val ← gets (λs. get-icc-V ((cpu-reg s) PSR));
    c-val ← gets (λs. get-icc-C ((cpu-reg s) PSR));
    icc-val ← gets(λs. (trap-eval-icc instr-name n-val z-val v-val c-val));
    curr-win ← get-curr-win();
```

```
    rs1-val ← gets (λs. (user-reg-val curr-win rs1 s));
    trap-number ← gets (λs. (rs1-val + (get-trap-op2 op-list s)));
    npc-val ← gets (λs. (cpu-reg-val nPC s));
    pc-val ← gets (λs. (cpu-reg-val PC s));
    if icc-val = 1 then
      do
        raise-trap trap-instruction;
        trap-number7 ← gets (λs. ((ucast trap-number)::word7));
        modify (λs. (ticc-trap-type-mod trap-number7 s));
        return ()
      od
    else — icc-val = 0
      do
        write-cpu npc-val PC;
        write-cpu (npc-val + 4) nPC;
        return ()
      od
  od
```

Operational semantics for store barrier.

```
definition store-barrier-instr::instruction ⇒ ('a::len,unit) sparc-state-monad
where store-barrier-instr instr ≡
  do
    modify (λs. (store-barrier-pending-mod True s));
    return ()
  od

end

end


theory Sparc-Execution
imports Main Sparc-Instruction Sparc-State Sparc-Types
HOL−Eisbach.Eisbach-Tools
begin

primrec sum :: nat ⇒ nat where
sum 0 = 0 |
sum (Suc n) = Suc n + sum n

definition select-trap :: unit ⇒ ('a,unit) sparc-state-monad
where select-trap - ≡
  do
    traps ← gets (λs. (get-trap-set s));
    rt-val ← gets (λs. (reset-trap-val s));
    psr-val ← gets (λs. (cpu-reg-val PSR s));
    et-val ← gets (λs. (get-ET psr-val));
    modify (λs. (emp-trap-set s));
```

```
if rt-val = True then — ignore ET, and leave tt unchaged
  return ()
else if et-val = 0 then — go to error mode, machine needs reset
  do
    set-err-mode True;
    set-exe-mode False;
    fail ()
  od
— By the SPARCv8 manual only 1 of the following traps could be in traps.
else if data-store-error ∈ traps  then
  do
    write-cpu-tt (0b00101011::word8);
    return ()
  od
else if instruction-access-error ∈ traps then
  do
    write-cpu-tt (0b00100001::word8);
    return ()
  od
else if r-register-access-error ∈ traps then
  do
    write-cpu-tt (0b00100000::word8);
    return ()
  od
else if instruction-access-exception ∈ traps then
  do
    write-cpu-tt (0b00000001::word8);
    return ()
  od
else if privileged-instruction ∈ traps then
  do
    write-cpu-tt (0b00000011::word8);
    return ()
  od
else if illegal-instruction ∈ traps then
  do
    write-cpu-tt (0b00000010::word8);
    return ()
  od
else if fp-disabled ∈ traps then
  do
    write-cpu-tt (0b00000100::word8);
    return ()
  od
else if cp-disabled ∈ traps then
  do
    write-cpu-tt (0b00100100::word8);
    return ()
  od
```

```
else if unimplemented-FLUSH ∈ traps then
  do
    write-cpu-tt (0b00100101::word8);
    return ()
  od
else if window-overflow ∈ traps then
  do
    write-cpu-tt (0b00000101::word8);
    return ()
  od
else if window-underflow ∈ traps then
  do
    write-cpu-tt (0b00000110::word8);
    return ()
  od
else if mem-address-not-aligned ∈ traps then
  do
    write-cpu-tt (0b00000111::word8);
    return ()
  od
else if fp-exception ∈ traps then
  do
    write-cpu-tt (0b00001000::word8);
    return ()
  od
else if cp-exception ∈ traps then
  do
    write-cpu-tt (0b00101000::word8);
    return ()
  od
else if data-access-error ∈ traps then
  do
    write-cpu-tt (0b00101001::word8);
    return ()
  od
else if data-access-exception ∈ traps then
  do
    write-cpu-tt (0b00001001::word8);
    return ()
  od
else if tag-overflow ∈ traps then
  do
    write-cpu-tt (0b00001010::word8);
    return ()
  od
else if division-by-zero ∈ traps then
  do
    write-cpu-tt (0b00101010::word8);
    return ()
```

```
    od
  else if trap-instruction ∈ traps then
    do
      ticc-trap-type ← gets (λs. (ticc-trap-type-val s));
      write-cpu-tt (word-cat (1::word1) ticc-trap-type);
      return ()
    od
  else if interrupt-level > 0 then
  — We don't consider interrupt-level
  else return ()
od

definition exe-trap-st-pc :: unit ⇒ ('a::len,unit) sparc-state-monad
where exe-trap-st-pc - ≡
  do
    annul ← gets (λs. (annul-val s));
    pc-val ← gets (λs. (cpu-reg-val PC s));
    npc-val ← gets (λs. (cpu-reg-val nPC s));
    curr-win ← get-curr-win();
    if annul = False then
      do
        write-reg pc-val curr-win (word-of-int 17);
        write-reg npc-val curr-win (word-of-int 18);
        return ()
      od
    else — annul = True
      do
        write-reg npc-val curr-win (word-of-int 17);
        write-reg (npc-val + 4) curr-win (word-of-int 18);
        set-annul False;
        return ()
      od
  od

definition exe-trap-wr-pc :: unit ⇒ ('a::len,unit) sparc-state-monad
where exe-trap-wr-pc - ≡
  do
    psr-val ← gets (λs. (cpu-reg-val PSR s));
    new-psr-val ← gets (λs. (update-S (1::word1) psr-val));
    write-cpu new-psr-val PSR;
    reset-trap ← gets (λs. (reset-trap-val s));
    tbr-val ← gets (λs. (cpu-reg-val TBR s));
    if reset-trap = False then
      do
        write-cpu tbr-val PC;
        write-cpu (tbr-val + 4) nPC;
        return ()
      od
    else — reset-trap = True
```

```
    do
      write-cpu 0 PC;
      write-cpu 4 nPC;
      set-reset-trap False;
      return ()
    od
  od

definition execute-trap :: unit ⇒ ('a::len,unit) sparc-state-monad
where execute-trap - ≡
  do
    select-trap();
    err-mode ← gets (λs. (err-mode-val s));
    if err-mode = True then
      — The SparcV8 manual doesn't say what to do.
      return ()
    else
      do
        psr-val ← gets (λs. (cpu-reg-val PSR s));
        s-val ← gets (λs. ((ucast (get-S psr-val))::word1));
        curr-win ← get-curr-win();
          new-cwp ← gets (λs. ((word-of-int (((uint curr-win) − 1) mod NWIN-
DOWS)))::word5);
          new-psr-val ← gets (λs. (update-PSR-exe-trap new-cwp (0::word1) s-val
psr-val));
        write-cpu new-psr-val PSR;
        exe-trap-st-pc();
        exe-trap-wr-pc();
        return ()
      od
  od

definition dispatch-instruction :: instruction ⇒ ('a::len,unit) sparc-state-monad
where dispatch-instruction instr ≡
  let instr-name = fst instr in
  do
    traps ← gets (λs. (get-trap-set s));
    if traps = {} then
      if instr-name ∈ {load-store-type LDSB,load-store-type LDUB,
        load-store-type LDUBA,load-store-type LDUH,load-store-type LD,
        load-store-type LDA,load-store-type LDD} then
        load-instr instr
      else if instr-name ∈ {load-store-type STB,load-store-type STH,
        load-store-type ST,load-store-type STA,load-store-type STD} then
        store-instr instr
      else if instr-name ∈ {sethi-type SETHI} then
        sethi-instr instr
      else if instr-name ∈ {nop-type NOP} then
        nop-instr instr
```

```
      else if instr-name ∈ {logic-type ANDs,logic-type ANDcc,logic-type ANDN,
        logic-type ANDNcc,logic-type ORs,logic-type ORcc,logic-type ORN,
        logic-type XORs,logic-type XNOR} then
        logical-instr instr
      else if instr-name ∈ {shift-type SLL,shift-type SRL,shift-type SRA} then
        shift-instr instr
       else if instr-name ∈ {arith-type ADD,arith-type ADDcc,arith-type ADDX}
then
        add-instr instr
     else if instr-name ∈ {arith-type SUB,arith-type SUBcc,arith-type SUBX} then
        sub-instr instr
      else if instr-name ∈ {arith-type UMUL,arith-type SMUL,arith-type SMULcc}
then
        mul-instr instr
       else if instr-name ∈ {arith-type UDIV,arith-type UDIVcc,arith-type SDIV}
then
        div-instr instr
      else if instr-name ∈ {ctrl-type SAVE,ctrl-type RESTORE} then
        save-restore-instr instr
      else if instr-name ∈ {call-type CALL} then
        call-instr instr
      else if instr-name ∈ {ctrl-type JMPL} then
        jmpl-instr instr
      else if instr-name ∈ {ctrl-type RETT} then
        rett-instr instr
      else if instr-name ∈ {sreg-type RDY,sreg-type RDPSR,sreg-type RDWIM,
        sreg-type RDTBR} then
        read-state-reg-instr instr
      else if instr-name ∈ {sreg-type WRY,sreg-type WRPSR,sreg-type WRWIM,
        sreg-type WRTBR} then
        write-state-reg-instr instr
      else if instr-name ∈ {load-store-type FLUSH} then
        flush-instr instr
      else if instr-name ∈ {bicc-type BE,bicc-type BNE,bicc-type BGU,
        bicc-type BLE,bicc-type BL,bicc-type BGE,bicc-type BNEG,bicc-type BG,
          bicc-type BCS,bicc-type BLEU,bicc-type BCC,bicc-type BA,bicc-type BN}
then
        branch-instr instr
      else fail ()
    else return ()
  od

definition supported-instruction :: sparc-operation ⇒ bool
where supported-instruction instr ≡
  if instr ∈ {load-store-type LDSB,load-store-type LDUB,load-store-type LDUBA,
              load-store-type LDUH,load-store-type LD,load-store-type LDA,
              load-store-type LDD,
              load-store-type STB,load-store-type STH,load-store-type ST,
              load-store-type STA,load-store-type STD,
```

```
        sethi-type SETHI,
        nop-type NOP,
       logic-type ANDs,logic-type ANDcc,logic-type ANDN,logic-type ANDNcc,
        logic-type ORs,logic-type ORcc,logic-type ORN,logic-type XORs,
        logic-type XNOR,
        shift-type SLL,shift-type SRL,shift-type SRA,
        arith-type ADD,arith-type ADDcc,arith-type ADDX,
        arith-type SUB,arith-type SUBcc,arith-type SUBX,
        arith-type UMUL,arith-type SMUL,arith-type SMULcc,
        arith-type UDIV,arith-type UDIVcc,arith-type SDIV,
        ctrl-type SAVE,ctrl-type RESTORE,
        call-type CALL,
        ctrl-type JMPL,
        ctrl-type RETT,
        sreg-type RDY,sreg-type RDPSR,sreg-type RDWIM,sreg-type RDTBR,
       sreg-type WRY,sreg-type WRPSR,sreg-type WRWIM,sreg-type WRTBR,
        load-store-type FLUSH,
        bicc-type BE,bicc-type BNE,bicc-type BGU,bicc-type BLE,
        bicc-type BL,bicc-type BGE,bicc-type BNEG,bicc-type BG,
        bicc-type BCS,bicc-type BLEU,bicc-type BCC,bicc-type BA,
        bicc-type BN}
  then True
 else False


definition execute-instr-sub1 :: instruction ⇒ ('a::len,unit) sparc-state-monad
where execute-instr-sub1 instr ≡
 do
  instr-name ← gets (λs. (fst instr));
  traps2 ← gets (λs. (get-trap-set s));
  if traps2 = {} ∧ instr-name ∉ {call-type CALL,ctrl-type RETT,ctrl-type JMPL,
                                 bicc-type BE,bicc-type BNE,bicc-type BGU,
                                 bicc-type BLE,bicc-type BL,bicc-type BGE,
                                 bicc-type BNEG,bicc-type BG,
                                 bicc-type BCS,bicc-type BLEU,bicc-type BCC,
                                 bicc-type BA,bicc-type BN} then
  do
    npc-val ← gets (λs. (cpu-reg-val nPC s));
    write-cpu npc-val PC;
    write-cpu (npc-val + 4) nPC;
    return ()
  od
  else return ()
 od

definition execute-instruction :: unit ⇒ ('a::len,unit) sparc-state-monad
where execute-instruction - ≡
 do
  traps ← gets (λs. (get-trap-set s));
```

```
  if traps = {} then
  do
    exe-mode ← gets (λs. (exe-mode-val s));
    if exe-mode = True then
    do
      modify (λs. (delayed-pool-write s));
      fetch-result ← gets (λs. (fetch-instruction s));
      case fetch-result of
      Inl e1 ⇒ (do — Memory address in PC is not aligned.
                   — Actually, SparcV8 manual doens't check alignment here.
                 raise-trap instruction-access-exception;
                 return ()
               od)
      | Inr v1 ⇒ (do
        dec ← gets (λs. (decode-instruction v1));
        case dec of
          Inl e2 ⇒ (— Instruction is ill-formatted.
                     fail ()
                   )
        | Inr v2 ⇒ (do
          instr ← gets (λs. (v2));
          annul ← gets (λs. (annul-val s));
          if annul = False then
          do
            dispatch-instruction instr;
            execute-instr-sub1 instr;
            return ()
          od
          else — annul ≠ False
          do
            set-annul False;
            npc-val ← gets (λs. (cpu-reg-val nPC s));
            write-cpu npc-val PC;
            write-cpu (npc-val + 4) nPC;
            return ()
          od
        od)
      od)
    od
    else return () — Not in execute-mode.
  od
  else — traps is not empty, which means trap = 1.
  do
    execute-trap();
    return ()
  od
od

definition NEXT :: ('a::len)sparc-state ⇒ ('a)sparc-state option
```

```
where NEXT s ≡ case execute-instruction () s of (-,True) ⇒ None
| (s',False) ⇒ Some (snd s')

context
  includes bit-operations-syntax
begin

definition good-context :: ('a::len) sparc-state ⇒ bool
where good-context s ≡
  let traps = get-trap-set s;
      psr-val = cpu-reg-val PSR s;
      et-val = get-ET psr-val;
      rt-val = reset-trap-val s
  in
  if traps ≠ {} ∧ rt-val = False ∧ et-val = 0 then False — enter error-mode in
select-traps.
  else
    let s' = delayed-pool-write s in
    case fetch-instruction s' of
    — instruction-access-exception is handled in the next state.
    Inl - ⇒ True
    |Inr v ⇒ (
      case decode-instruction v of
      Inl - ⇒ False
      |Inr instr ⇒ (
        let annul = annul-val s' in
        if annul = True then True
        else — annul = False
          if supported-instruction (fst instr) then
            — The only instruction that could fail is RETT.
            if (fst instr) = ctrl-type RETT then
              let curr-win-r = (get-CWP (cpu-reg-val PSR s'));
                  new-cwp-int-r = (((uint curr-win-r) + 1) mod NWINDOWS);
                  wim-val-r = cpu-reg-val WIM s';
                  psr-val-r = cpu-reg-val PSR s';
                  et-val-r = get-ET psr-val-r;
                  s-val-r = (ucast (get-S psr-val-r))::word1;
                  op-list-r = snd instr;
                  addr-r = get-addr (snd instr) s'
              in
              if et-val-r = 1 then True
              else if s-val-r = 0 then False
              else if (get-WIM-bit (nat new-cwp-int-r) wim-val-r) ≠ 0 then False
           else if ((AND) addr-r (0b00000000000000000000000000000011::word32))
≠ 0 then False
              else True
            else True
          else False — Unsupported instruction.
      )
```

)

end

function (*sequential*) *seq-exec*:: *nat* ⇒ ('*a*::*len*,*unit*) *sparc-state-monad*
where *seq-exec 0* = *return* ()
|
seq-exec n = (*do execute-instruction*();
(*seq-exec* (*n*−*1*))
od)

⟨*proof*⟩
termination ⟨*proof*⟩

type-synonym *leon3-state* = (*word-length5*) *sparc-state*

type-synonym ('*e*) *leon3-state-monad* = (*leon3-state*, '*e*) *det-monad*

definition *execute-leon3-instruction*:: *unit* ⇒ (*unit*) *leon3-state-monad*
where *execute-leon3-instruction* ≡ *execute-instruction*

definition *seq-exec-leon3*:: *nat* ⇒ (*unit*) *leon3-state-monad*
where *seq-exec-leon3* ≡ *seq-exec*

end

theory *Sparc-Properties*

imports *Main Sparc-Execution*

begin

24 Single step theorem

The following shows that, if the pre-state satisfies certain conditions called *good-context*, there must be a defined post-state after a single step execution.

method *save-restore-proof* =
((*simp add*: *save-restore-instr-def*),
(*simp add*: *Let-def simpler-gets-def bind-def h1-def h2-def*),
(*simp add*: *case-prod-unfold*),
(*simp add*: *raise-trap-def simpler-modify-def*),
(*simp add*: *simpler-gets-def bind-def h1-def h2-def*),
(*simp add*: *save-retore-sub1-def*),
(*simp add*: *write-cpu-def simpler-modify-def*),
(*simp add*: *write-reg-def simpler-modify-def*),
(*simp add*: *get-curr-win-def*),

(*simp add*: *simpler-gets-def bind-def h1-def h2-def*))

method *select-trap-proof0* =
((*simp add*: *select-trap-def exec-gets return-def*),
(*simp add*: *DetMonad.bind-def h1-def h2-def simpler-modify-def*),
(*simp add*: *write-cpu-tt-def write-cpu-def*),
(*simp add*: *DetMonad.bind-def h1-def h2-def simpler-modify-def*),
(*simp add*: *return-def simpler-gets-def*))

method *select-trap-proof1* =
((*simp add*: *select-trap-def exec-gets return-def*),
(*simp add*: *DetMonad.bind-def h1-def h2-def simpler-modify-def*),
(*simp add*: *write-cpu-tt-def write-cpu-def*),
(*simp add*: *DetMonad.bind-def h1-def h2-def simpler-modify-def*),
(*simp add*: *return-def simpler-gets-def*),
(*simp add*: *emp-trap-set-def err-mode-val-def cpu-reg-mod-def*))

method *dispatch-instr-proof1* =
((*simp add*: *dispatch-instruction-def*),
(*simp add*: *simpler-gets-def bind-def h1-def h2-def*),
(*simp add*: *Let-def*))

method *exe-proof-to-decode* =
((*simp add*: *execute-instruction-def*),
(*simp add*: *exec-gets bind-def h1-def h2-def Let-def return-def*),
clarsimp,
(*simp add*: *simpler-gets-def bind-def h1-def h2-def Let-def simpler-modify-def*),
(*simp add*: *return-def*))

method *exe-proof-dispatch-rett* =
((*simp add*: *dispatch-instruction-def*),
(*simp add*: *simpler-gets-def bind-def h1-def h2-def Let-def*),
(*simp add*: *rett-instr-def*),
(*simp add*: *simpler-gets-def bind-def h1-def h2-def Let-def*))

lemma *write-cpu-result*: *snd* (*write-cpu w r s*) = *False*
⟨*proof*⟩

lemma *set-annul-result*: *snd* (*set-annul b s*) = *False*
⟨*proof*⟩

lemma *raise-trap-result* : *snd* (*raise-trap t s*) = *False*
⟨*proof*⟩

context
includes *bit-operations-syntax*
begin

lemma *rett-instr-result*: (*fst i*) = *ctrl-type RETT* ∧

(*get-ET* (*cpu-reg-val PSR s*) ≠ *1* ∧
(((*get-S* (*cpu-reg-val PSR s*)))::*word1*) ≠ *0* ∧
(*get-WIM-bit* (*nat* (((*uint* (*get-CWP* (*cpu-reg-val PSR s*))) + *1*) *mod NWINDOWS*))
(*cpu-reg-val WIM s*)) = *0* ∧
((*AND*) (*get-addr* (*snd i*) *s*) (*0b*00000000000000000000000000000011::*word32*)) = *0*) ⟹
snd (*rett-instr i s*) = *False*
⟨*proof*⟩

lemma *call-instr-result*: (*fst i*) = *call-type CALL* ⟹
snd (*call-instr i s*) = *False*
⟨*proof*⟩

lemma *branch-instr-result*: (*fst i*) ∈ {*bicc-type BE*,*bicc-type BNE*,*bicc-type BGU*,
bicc-type BLE,*bicc-type BL*,*bicc-type BGE*,*bicc-type BNEG*,*bicc-type BG*,
bicc-type BCS,*bicc-type BLEU*,*bicc-type BCC*,*bicc-type BA*,*bicc-type BN*} ⟹
snd (*branch-instr i s*) = *False*
⟨*proof*⟩

lemma *nop-instr-result*: (*fst i*) = *nop-type NOP* ⟹
snd (*nop-instr i s*) = *False*
⟨*proof*⟩

lemma *sethi-instr-result*: (*fst i*) = *sethi-type SETHI* ⟹
snd (*sethi-instr i s*) = *False*
⟨*proof*⟩

lemma *jmpl-instr-result*: (*fst i*) = *ctrl-type JMPL* ⟹
snd (*jmpl-instr i s*) = *False*
⟨*proof*⟩

lemma *save-restore-instr-result*: (*fst i*) ∈ {*ctrl-type SAVE*,*ctrl-type RESTORE*}
⟹
snd (*save-restore-instr i s*) = *False*
⟨*proof*⟩

lemma *flush-instr-result*: (*fst i*) = *load-store-type FLUSH* ⟹
snd (*flush-instr i s*) = *False*
⟨*proof*⟩

lemma *read-state-reg-instr-result*: (*fst i*) ∈ {*sreg-type RDY*,*sreg-type RDPSR*,
sreg-type RDWIM,*sreg-type RDTBR*} ⟹
snd (*read-state-reg-instr i s*) = *False*
⟨*proof*⟩

lemma *write-state-reg-instr-result*: (*fst i*) ∈ {*sreg-type WRY*,*sreg-type WRPSR*,
sreg-type WRWIM,*sreg-type WRTBR*} ⟹
snd (*write-state-reg-instr i s*) = *False*

⟨*proof*⟩

lemma *logical-instr-result*: (*fst i*) ∈ {*logic-type ANDs*,*logic-type ANDcc*,
logic-type ANDN,*logic-type ANDNcc*,*logic-type ORs*,*logic-type ORcc*,
logic-type ORN,*logic-type XORs*,*logic-type XNOR*} ⟹
snd (*logical-instr i s*) = *False*
⟨*proof*⟩

lemma *shift-instr-result*: (*fst i*) ∈ {*shift-type SLL*,*shift-type
SRL*,*shift-type SRA*} ⟹
snd (*shift-instr i s*) = *False*
⟨*proof*⟩

method *add-sub-instr-proof* =
((*simp add*: *Let-def*),
auto,
(*simp add*: *write-reg-def simpler-modify-def*),
(*simp add*: *simpler-gets-def bind-def*),
(*simp add*: *get-curr-win-def simpler-gets-def*),
(*simp add*: *write-reg-def write-cpu-def simpler-modify-def*),
(*simp add*: *bind-def*),
(*simp add*: *case-prod-unfold*),
(*simp add*: *simpler-gets-def*),
(*simp add*: *get-curr-win-def simpler-gets-def*),
(*simp add*: *write-reg-def simpler-modify-def*),
(*simp add*: *simpler-gets-def bind-def*),
(*simp add*: *get-curr-win-def simpler-gets-def*))

lemma *add-instr-result*: (*fst i*) ∈ {*arith-type ADD*,*arith-type
ADDcc*,*arith-type ADDX*} ⟹
snd (*add-instr i s*) = *False*
⟨*proof*⟩

lemma *sub-instr-result*: (*fst i*) ∈ {*arith-type SUB*,*arith-type SUBcc*,
arith-type SUBX} ⟹
snd (*sub-instr i s*) = *False*
⟨*proof*⟩

lemma *mul-instr-result*: (*fst i*) ∈ {*arith-type UMUL*,*arith-type SMUL*,
arith-type SMULcc} ⟹
snd (*mul-instr i s*) = *False*
⟨*proof*⟩

lemma *div-write-new-val-result*: *snd* (*div-write-new-val i result temp-V s*) = *False*
⟨*proof*⟩

lemma *div-result*: *snd* (*div-comp instr rs1 rd operand2 s*) = *False*
⟨*proof*⟩

lemma *div-instr-result*: (*fst i*) ∈ {*arith-type UDIV*,*arith-type UDIVcc*,
arith-type SDIV} ⟹
snd (*div-instr i s*) = *False*
⟨*proof*⟩

lemma *load-sub2-result*: *snd* (*load-sub2 address asi rd curr-win word0 s*) = *False*
⟨*proof*⟩

lemma *load-sub3-result*: *snd* (*load-sub3 instr curr-win rd asi address s*) = *False*
⟨*proof*⟩

lemma *load-sub1-result*: *snd* (*load-sub1 i rd s-val s*) = *False*
⟨*proof*⟩

lemma *load-instr-result*: (*fst i*) ∈ {*load-store-type LDSB*,*load-store-type LDUB*,
load-store-type LDUBA,*load-store-type LDUH*,*load-store-type LD*,
load-store-type LDA,*load-store-type LDD*} ⟹
snd (*load-instr i s*) = *False*
⟨*proof*⟩

lemma *store-sub2-result*: *snd* (*store-sub2 instr curr-win rd asi address s*) = *False*
⟨*proof*⟩

lemma *store-sub1-result*: *snd* (*store-sub1 instr rd s-val s*) = *False*
⟨*proof*⟩

lemma *store-instr-result*: (*fst i*) ∈ {*load-store-type STB*,*load-store-type STH*,
load-store-type ST,*load-store-type STA*,*load-store-type STD*} ⟹
snd (*store-instr i s*) = *False*
⟨*proof*⟩

lemma *supported-instr-set*: *supported-instruction i* = *True* ⟹
i ∈ {*load-store-type LDSB*,*load-store-type LDUB*,*load-store-type LDUBA*,
load-store-type LDUH,*load-store-type LD*,*load-store-type LDA*,
load-store-type LDD,
load-store-type STB,*load-store-type STH*,*load-store-type ST*,
load-store-type STA,*load-store-type STD*,
sethi-type SETHI,
nop-type NOP,
logic-type ANDs,*logic-type ANDcc*,*logic-type ANDN*,*logic-type ANDNcc*,
logic-type ORs,*logic-type ORcc*,*logic-type ORN*,*logic-type XORs*,
logic-type XNOR,
shift-type SLL,*shift-type SRL*,*shift-type SRA*,
arith-type ADD,*arith-type ADDcc*,*arith-type ADDX*,
arith-type SUB,*arith-type SUBcc*,*arith-type SUBX*,
arith-type UMUL,*arith-type SMUL*,*arith-type SMULcc*,
arith-type UDIV,*arith-type UDIVcc*,*arith-type SDIV*,
ctrl-type SAVE,*ctrl-type RESTORE*,
call-type CALL,

ctrl-type JMPL,
ctrl-type RETT,
sreg-type RDY,*sreg-type RDPSR*,*sreg-type RDWIM*,*sreg-type RDTBR*,
sreg-type WRY,*sreg-type WRPSR*,*sreg-type WRWIM*,*sreg-type WRTBR*,
load-store-type FLUSH,
bicc-type BE,*bicc-type BNE*,*bicc-type BGU*,*bicc-type BLE*,
bicc-type BL,*bicc-type BGE*,*bicc-type BNEG*,*bicc-type BG*,
bicc-type BCS,*bicc-type BLEU*,*bicc-type BCC*,*bicc-type BA*,
bicc-type BN}
⟨*proof*⟩

lemma *dispatch-instr-result*:
assumes *a1*: *supported-instruction* (*fst i*) = *True* ∧ (*fst i*) ≠ *ctrl-type RETT*
shows *snd* (*dispatch-instruction i s*) = *False*
⟨*proof*⟩

lemma *dispatch-instr-result-rett*:
assumes *a1*: (*fst i*) = *ctrl-type RETT* ∧ (*get-ET* (*cpu-reg-val PSR s*) ≠ *1* ∧
(((*get-S* (*cpu-reg-val PSR s*)))::*word1*) ≠ *0* ∧
(*get-WIM-bit* (*nat* (((*uint* (*get-CWP* (*cpu-reg-val PSR s*))) + *1*) *mod NWINDOWS*))
(*cpu-reg-val WIM s*)) = *0* ∧
((*AND*) (*get-addr* (*snd i*) *s*) (*0b*00000000000000000000000000000011::*word32*))
= *0*)
shows *snd* (*dispatch-instruction i s*) = *False*
⟨*proof*⟩

lemma *execute-instr-sub1-result*: *snd* (*execute-instr-sub1 i s*) = *False*
⟨*proof*⟩

lemma *next-match* : *snd* (*execute-instruction* () *s*) = *False* ⟹
NEXT s = *Some* (*snd* (*fst* (*execute-instruction* () *s*)))
⟨*proof*⟩

lemma *exec-ss1* : ∃ *s'*. (*execute-instruction* () *s* = (*s'*, *False*)) ⟹
∃ *s''*. (*execute-instruction*() *s* = (*s''*, *False*))
⟨*proof*⟩

lemma *exec-ss2* : *snd* (*execute-instruction*() *s*) = *False* ⟹
snd (*execute-instruction* () *s*) = *False*
⟨*proof*⟩

lemma *good-context-1* : *good-context s* ∧ *s'* = *s* ∧
(*get-trap-set s'*) ≠ {} ∧ (*reset-trap-val s'*) = *False* ∧ *get-ET* (*cpu-reg-val PSR s'*)
= *0*
⟹ *False*
⟨*proof*⟩

lemma *fetch-instr-result-1* : ¬ (∃ *e*. *fetch-instruction s'* = *Inl e*) ⟹

$(\exists\, v.\ \mathit{fetch\text{-}instruction}\ s' = \mathit{Inr}\ v)$
⟨*proof*⟩

lemma *fetch-instr-result-2* : $(\exists\, v.\ \mathit{fetch\text{-}instruction}\ s' = \mathit{Inr}\ v) \Longrightarrow$
$\neg\ (\exists\, e.\ \mathit{fetch\text{-}instruction}\ s' = \mathit{Inl}\ e)$
⟨*proof*⟩

lemma *fetch-instr-result-3* : $(\exists\, e.\ \mathit{fetch\text{-}instruction}\ s' = \mathit{Inl}\ e) \Longrightarrow$
$\neg\ (\exists\, v.\ \mathit{fetch\text{-}instruction}\ s' = \mathit{Inr}\ v)$
⟨*proof*⟩

lemma *decode-instr-result-1* :
$\neg(\exists\, v2.\ ((\mathit{decode\text{-}instruction}\ v1)::(\mathit{Exception\ list} + \mathit{instruction})) = \mathit{Inr}\ v2) \Longrightarrow$
$(\exists\, e.\ ((\mathit{decode\text{-}instruction}\ v1)::(\mathit{Exception\ list} + \mathit{instruction})) = \mathit{Inl}\ e)$
⟨*proof*⟩

lemma *decode-instr-result-2* :
$(\exists\, e.\ ((\mathit{decode\text{-}instruction}\ v1)::(\mathit{Exception\ list} + \mathit{instruction})) = \mathit{Inl}\ e) \Longrightarrow$
$\neg(\exists\, v2.\ ((\mathit{decode\text{-}instruction}\ v1)::(\mathit{Exception\ list} + \mathit{instruction})) = \mathit{Inr}\ v2)$
⟨*proof*⟩

lemma *decode-instr-result-3* : $x = \mathit{decode\text{-}instruction}\ v1 \wedge y = \mathit{decode\text{-}instruction}\ v2$
$\wedge\ v1 = v2 \Longrightarrow x = y$
⟨*proof*⟩

lemma *decode-instr-result-4* :
$\neg\ (\exists\, e.\ ((\mathit{decode\text{-}instruction}\ v1)::(\mathit{Exception\ list} + \mathit{instruction})) = \mathit{Inl}\ e) \Longrightarrow$
$(\exists\, v2.\ ((\mathit{decode\text{-}instruction}\ v1)::(\mathit{Exception\ list} + \mathit{instruction})) = \mathit{Inr}\ v2)$
⟨*proof*⟩

lemma *good-context-2* :
$\mathit{good\text{-}context}\ (s::((\mathit{'a}::\mathit{len})\ \mathit{sparc\text{-}state})) \wedge$
$\mathit{fetch\text{-}instruction}\ (\mathit{delayed\text{-}pool\text{-}write}\ s) = \mathit{Inr}\ v1\ \wedge$
$\neg(\exists\, v2.\ (\mathit{decode\text{-}instruction}\ v1::(\mathit{Exception\ list} + \mathit{instruction})) = \mathit{Inr}\ v2)$
$\Longrightarrow \mathit{False}$
⟨*proof*⟩

lemma *good-context-3* :
$\mathit{good\text{-}context}\ (s::((\mathit{'a}::\mathit{len})\ \mathit{sparc\text{-}state})) \wedge$
$s'' = \mathit{delayed\text{-}pool\text{-}write}\ s \wedge$
$\mathit{fetch\text{-}instruction}\ s'' = \mathit{Inr}\ v1\ \wedge$
$(\mathit{decode\text{-}instruction}\ v1::(\mathit{Exception\ list} + \mathit{instruction})) = \mathit{Inr}\ v2 \wedge$
$\mathit{annul\text{-}val}\ s'' = \mathit{False} \wedge \mathit{supported\text{-}instruction}\ (\mathit{fst}\ v2) = \mathit{False}$
$\Longrightarrow \mathit{False}$
⟨*proof*⟩

lemma *good-context-4* :
$\mathit{good\text{-}context}\ (s::((\mathit{'a}::\mathit{len})\ \mathit{sparc\text{-}state})) \wedge$

$s'' = delayed\text{-}pool\text{-}write\ s \wedge$
$fetch\text{-}instruction\ s'' = Inr\ v1 \wedge$
$((decode\text{-}instruction\ v1)::(Exception\ list + instruction)) = Inr\ v2 \wedge$
$annul\text{-}val\ s'' = False \wedge$
$supported\text{-}instruction\ (fst\ v2) = True \wedge$ — This line is redundant
$(fst\ v2) = ctrl\text{-}type\ RETT \wedge get\text{-}ET\ (cpu\text{-}reg\text{-}val\ PSR\ s'') \neq 1 \wedge$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
$\Longrightarrow False$
$\langle proof \rangle$

lemma *good-context-5* :
$good\text{-}context\ (s::(('a::len)\ sparc\text{-}state)) \wedge$
$s'' = delayed\text{-}pool\text{-}write\ s \wedge$
$fetch\text{-}instruction\ s'' = Inr\ v1 \wedge$
$((decode\text{-}instruction\ v1)::(Exception\ list + instruction)) = Inr\ v2 \wedge$
$annul\text{-}val\ s'' = False \wedge$
$supported\text{-}instruction\ (fst\ v2) = True \wedge$ — This line is redundant
$(fst\ v2) = ctrl\text{-}type\ RETT \wedge get\text{-}ET\ (cpu\text{-}reg\text{-}val\ PSR\ s'') \neq 1 \wedge$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) \neq 0 \wedge$
$(get\text{-}WIM\text{-}bit\ (nat\ (((uint\ (get\text{-}CWP\ (cpu\text{-}reg\text{-}val\ PSR\ s'')))\ +\ 1)\ mod\ NWINDOWS))$
$(cpu\text{-}reg\text{-}val\ WIM\ s'')) \neq 0$
$\Longrightarrow False$
$\langle proof \rangle$

lemma *good-context-6* :
$good\text{-}context\ (s::(('a::len)\ sparc\text{-}state)) \wedge$
$s'' = delayed\text{-}pool\text{-}write\ s \wedge$
$fetch\text{-}instruction\ s'' = Inr\ v1 \wedge$
$((decode\text{-}instruction\ v1)::(Exception\ list + instruction)) = Inr\ v2 \wedge$
$annul\text{-}val\ s'' = False \wedge$
$supported\text{-}instruction\ (fst\ v2) = True \wedge$ — This line is redundant
$(fst\ v2) = ctrl\text{-}type\ RETT \wedge get\text{-}ET\ (cpu\text{-}reg\text{-}val\ PSR\ s'') \neq 1 \wedge$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) \neq 0 \wedge$
$(get\text{-}WIM\text{-}bit\ (nat\ (((uint\ (get\text{-}CWP\ (cpu\text{-}reg\text{-}val\ PSR\ s'')))\ +\ 1)\ mod\ NWINDOWS))$
$(cpu\text{-}reg\text{-}val\ WIM\ s'')) = 0 \wedge$
$((AND)\ (get\text{-}addr\ (snd\ v2)\ s'')\ (0b00000000000000000000000000000011::word32)) \neq 0$
$\Longrightarrow False$
$\langle proof \rangle$

lemma *good-context-all* :
$good\text{-}context\ (s::(('a::len)\ sparc\text{-}state)) \wedge$
$s'' = delayed\text{-}pool\text{-}write\ s \Longrightarrow$
$(get\text{-}trap\text{-}set\ s = \{\} \vee (reset\text{-}trap\text{-}val\ s) \neq False \vee get\text{-}ET\ (cpu\text{-}reg\text{-}val\ PSR\ s) \neq 0) \wedge$
$((\exists e.\ fetch\text{-}instruction\ s'' = Inl\ e) \vee$
$(\exists v1\ v2.\ fetch\text{-}instruction\ s'' = Inr\ v1 \wedge$

$((decode\text{-}instruction\ v1)::(Exception\ list\ +\ instruction)) = Inr\ v2\ \wedge$
$(annul\text{-}val\ s'' = True\ \vee$
$(annul\text{-}val\ s'' = False\ \wedge$
$(\forall\ v1'\ v2'.\ fetch\text{-}instruction\ s'' = Inr\ v1'\ \wedge$
$((decode\text{-}instruction\ v1')::(Exception\ list\ +\ instruction)) = Inr\ v2' \longrightarrow$
$supported\text{-}instruction\ (fst\ v2') = True)\ \wedge$
$((fst\ v2) \neq ctrl\text{-}type\ RETT\ \vee$
$((fst\ v2) = ctrl\text{-}type\ RETT\ \wedge$
$(get\text{-}ET\ (cpu\text{-}reg\text{-}val\ PSR\ s'') = 1\ \vee$
$(get\text{-}ET\ (cpu\text{-}reg\text{-}val\ PSR\ s'') \neq 1\ \wedge$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s'')))::word1) \neq 0\ \wedge$
$(get\text{-}WIM\text{-}bit\ (nat\ (((uint\ (get\text{-}CWP\ (cpu\text{-}reg\text{-}val\ PSR\ s'')))\ +\ 1)\ mod$
$NWINDOWS))$
$(cpu\text{-}reg\text{-}val\ WIM\ s'')) = 0\ \wedge$
$((AND)\ (get\text{-}addr\ (snd\ v2)\ s'')\ (0b00000000000000000000000000000011::word32))$
$= 0))))))))$
⟨*proof*⟩

lemma *select-trap-result1* : $(reset\text{-}trap\text{-}val\ s) = True \Longrightarrow$
$snd\ (select\text{-}trap()\ s) = False$
⟨*proof*⟩

lemma *select-trap-result2* :
assumes *a1*: $\neg(reset\text{-}trap\text{-}val\ s = False \wedge get\text{-}ET\ (cpu\text{-}reg\text{-}val\ PSR\ s) = 0)$
shows $snd\ (select\text{-}trap()\ s) = False$
⟨*proof*⟩

lemma *emp-trap-set-err-mode* : $err\text{-}mode\text{-}val\ s = err\text{-}mode\text{-}val\ (emp\text{-}trap\text{-}set\ s)$
⟨*proof*⟩

lemma *write-cpu-tt-err-mode* : $err\text{-}mode\text{-}val\ s = err\text{-}mode\text{-}val\ (snd\ (fst\ (write\text{-}cpu\text{-}tt$
$w\ s)))$
⟨*proof*⟩

lemma *select-trap-monad* : $snd\ (select\text{-}trap()\ s) = False \Longrightarrow$
$err\text{-}mode\text{-}val\ s = err\text{-}mode\text{-}val\ (snd\ (fst\ (select\text{-}trap\ ()\ s)))$
⟨*proof*⟩

lemma *exe-trap-st-pc-result* : $snd\ (exe\text{-}trap\text{-}st\text{-}pc()\ s) = False$
⟨*proof*⟩

lemma *exe-trap-wr-pc-result* : $snd\ (exe\text{-}trap\text{-}wr\text{-}pc()\ s) = False$
⟨*proof*⟩

lemma *execute-trap-result* : $\neg(reset\text{-}trap\text{-}val\ s = False \wedge get\text{-}ET\ (cpu\text{-}reg\text{-}val\ PSR$
$s) = 0) \Longrightarrow$
$snd\ (execute\text{-}trap()\ s) = False$
⟨*proof*⟩

lemma *execute-trap-result2* : $\neg$(*reset-trap-val s = False* $\wedge$ *get-ET* (*cpu-reg-val PSR s*) = *0*) $\Longrightarrow$
  snd (*execute-trap*() *s*) = *False*
⟨*proof*⟩

lemma *exe-instr-all* :
good-context (*s*::((*'a*::*len*) *sparc-state*)) $\Longrightarrow$
  snd (*execute-instruction*() *s*) = *False*
⟨*proof*⟩

lemma *dispatch-fail*:
snd (*execute-instruction*() (*s*::((*'a*::*len*) *sparc-state*))) = *False* $\wedge$
  get-trap-set s = {} $\wedge$
  exe-mode-val s $\wedge$
  fetch-instruction (*delayed-pool-write s*) = *Inr v* $\wedge$
  ((*decode-instruction v*)::(*Exception list* + *instruction*)) = *Inl e*
 $\Longrightarrow$ *False*
⟨*proof*⟩

lemma *no-error* : *good-context s* $\Longrightarrow$ *snd* (*execute-instruction* () *s*) = *False*
⟨*proof*⟩

theorem *single-step* : *good-context s* $\Longrightarrow$ *NEXT s* = *Some* (*snd* (*fst* (*execute-instruction* () *s*)))
⟨*proof*⟩

25 Privilege safty

The following shows that, if the pre-state is under user mode, then after a singel step execution, the post-state is aslo under user mode.

lemma *write-cpu-pc-privilege*: *s'* = *snd* (*fst* (*write-cpu w PC s*)) $\wedge$
  (((*get-S* (*cpu-reg-val PSR s*)))::*word1*) = *0* $\Longrightarrow$
  (((*get-S* (*cpu-reg-val PSR s'*)))::*word1*) = *0*
⟨*proof*⟩

lemma *write-cpu-npc-privilege*: *s'* = *snd* (*fst* (*write-cpu w nPC s*)) $\wedge$
  (((*get-S* (*cpu-reg-val PSR s*)))::*word1*) = *0* $\Longrightarrow$
  (((*get-S* (*cpu-reg-val PSR s'*)))::*word1*) = *0*
⟨*proof*⟩

lemma *write-cpu-y-privilege*: *s'* = *snd* (*fst* (*write-cpu w Y s*)) $\wedge$
  (((*get-S* (*cpu-reg-val PSR s*)))::*word1*) = *0*
  $\Longrightarrow$ (((*get-S* (*cpu-reg-val PSR s'*)))::*word1*) = *0*
⟨*proof*⟩

lemma *cpu-reg-mod-y-privilege*: *s'* = *cpu-reg-mod w Y s* $\wedge$
  (((*get-S* (*cpu-reg-val PSR s*)))::*word1*) = *0*
  $\Longrightarrow$ (((*get-S* (*cpu-reg-val PSR s'*)))::*word1*) = *0*

⟨*proof*⟩

lemma *cpu-reg-mod-asr-privilege*: $s' = cpu\text{-}reg\text{-}mod\ w\ (ASR\ r)\ s \wedge$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
$\Longrightarrow (((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *global-reg-mod-privilege*: $s' = global\text{-}reg\text{-}mod\ w1\ n\ w2\ s \wedge$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0 \Longrightarrow$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *out-reg-mod-privilege*: $s' = out\text{-}reg\text{-}mod\ a\ w\ r\ s \wedge$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0 \Longrightarrow$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *in-reg-mod-privilege*: $s' = in\text{-}reg\text{-}mod\ a\ w\ r\ s \wedge$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0 \Longrightarrow$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *user-reg-mod-privilege*:
assumes *a1*: $s' = user\text{-}reg\text{-}mod\ d\ (w::(('a::len)\ window\text{-}size))\ r$
$(s::(('a::len)\ sparc\text{-}state)) \wedge$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
shows $(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *write-reg-privilege*: $s' = snd\ (fst\ (write\text{-}reg\ w1\ w2\ w3$
$(s::(('a::len)\ sparc\text{-}state)))) \wedge$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0 \Longrightarrow$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *set-annul-privilege*: $s' = snd\ (fst\ (set\text{-}annul\ b\ s)) \wedge$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0 \Longrightarrow$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *set-reset-trap-privilege*: $s' = snd\ (fst\ (set\text{-}reset\text{-}trap\ b\ s)) \wedge$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0 \Longrightarrow$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *empty-delayed-pool-write-privilege*: $get\text{-}delayed\text{-}pool\ s = [] \wedge$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0 \wedge$
$s' = delayed\text{-}pool\text{-}write\ s \Longrightarrow$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$

⟨*proof*⟩

lemma *raise-trap-privilege*:
(((*get-S* (*cpu-reg-val PSR s*)))::*word1*) = *0* ∧
s′ = *snd* (*fst* (*raise-trap t s*)) ⟹
(((*get-S* (*cpu-reg-val PSR s′*)))::*word1*) = *0*
⟨*proof*⟩

lemma *write-cpu-tt-privilege*: *s′* = *snd* (*fst* (*write-cpu-tt w s*)) ∧
(((*get-S* (*cpu-reg-val PSR s*)))::*word1*) = *0*
⟹ (((*get-S* (*cpu-reg-val PSR s′*)))::*word1*) = *0*
⟨*proof*⟩

lemma *emp-trap-set-privilege*: *s′* = *emp-trap-set s* ∧
(((*get-S* (*cpu-reg-val PSR s*)))::*word1*) = *0*
⟹ (((*get-S* (*cpu-reg-val PSR s′*)))::*word1*) = *0*
⟨*proof*⟩

lemma *sys-reg-mod-privilege*: *s′* = *sys-reg-mod w r s*
∧ (((*get-S* (*cpu-reg-val PSR s*)))::*word1*) = *0*
⟹ (((*get-S* (*cpu-reg-val PSR s′*)))::*word1*) = *0*
⟨*proof*⟩

lemma *mem-mod-privilege*:
assumes *a1*: *s′* = *mem-mod a1 a2 v s* ∧
(((*get-S* (*cpu-reg-val PSR s*)))::*word1*) = *0*
shows (((*get-S* (*cpu-reg-val PSR s′*)))::*word1*) = *0*
⟨*proof*⟩

lemma *mem-mod-w32-privilege*: *s′* = *mem-mod-w32 a1 a2 b d s* ∧
(((*get-S* (*cpu-reg-val PSR s*)))::*word1*) = *0*
⟹ (((*get-S* (*cpu-reg-val PSR s′*)))::*word1*) = *0*
⟨*proof*⟩

lemma *add-instr-cache-privilege*: *s′* = *add-instr-cache s addr y m* ⟹
(((*get-S* (*cpu-reg-val PSR s*)))::*word1*) = *0* ⟹
(((*get-S* (*cpu-reg-val PSR s′*)))::*word1*) = *0*
⟨*proof*⟩

lemma *add-data-cache-privilege*: *s′* = *add-data-cache s addr y m* ⟹
(((*get-S* (*cpu-reg-val PSR s*)))::*word1*) = *0* ⟹
(((*get-S* (*cpu-reg-val PSR s′*)))::*word1*) = *0*
⟨*proof*⟩

lemma *memory-read-privilege*:
assumes *a1*: *s′* = *snd* (*memory-read asi addr s*) ∧
(((*get-S* (*cpu-reg-val PSR s*)))::*word1*) = *0*
shows (((*get-S* (*cpu-reg-val PSR s′*)))::*word1*) = *0*
⟨*proof*⟩

lemma *get-curr-win-privilege*: $s' = snd\ (fst\ (get\text{-}curr\text{-}win()\ s)) \wedge$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
$\Longrightarrow (((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *load-sub2-privilege*:
assumes *a1*: $s' = snd\ (fst\ (load\text{-}sub2\ addr\ asi\ r\ win\ w\ s))$
$\wedge\ (((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
shows $(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *load-sub3-privilege*:
assumes *a1*: $s' = snd\ (fst\ (load\text{-}sub3\ instr\ curr\text{-}win\ rd\ asi\ address\ s))$
$\wedge\ (((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
shows $(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *load-sub1-privilege*:
assumes *a1*: $s' = snd\ (fst\ (load\text{-}sub1\ instr\ rd\ s\text{-}val\ s))$
$\wedge\ (((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
shows $(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *load-instr-privilege*: $s' = snd\ (fst\ (load\text{-}instr\ i\ s))$
$\wedge\ (((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
$\Longrightarrow (((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *store-barrier-pending-mod-privilege*: $s' = store\text{-}barrier\text{-}pending\text{-}mod\ b\ s$
$\wedge\ (((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
$\Longrightarrow (((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *store-word-mem-privilege*:
assumes *a1*: $store\text{-}word\text{-}mem\ s\ addr\ data\ byte\text{-}mask\ asi = Some\ s' \wedge$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
shows $(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *flush-instr-cache-privilege*: $(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0 \Longrightarrow$
$s' = flush\text{-}instr\text{-}cache\ s \Longrightarrow$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *flush-data-cache-privilege*: $(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0 \Longrightarrow$
$s' = flush\text{-}data\text{-}cache\ s \Longrightarrow$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *flush-cache-all-privilege*: $(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0 \Longrightarrow$
$s' = flush\text{-}cache\text{-}all\ s \Longrightarrow$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *memory-write-asi-privilege*:
assumes *a1*: $r = memory\text{-}write\text{-}asi\ asi\ addr\ byte\text{-}mask\ data\ s \wedge$
$r = Some\ s' \wedge$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
shows $(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *memory-write-privilege*:
assumes *a1*: $r = memory\text{-}write\ asi\ addr\ byte\text{-}mask\ data$
$(s::(('a::len)\ sparc\text{-}state)) \wedge$
$r = Some\ s' \wedge$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
shows $(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR$
$(s'::(('a::len)\ sparc\text{-}state)))))::word1) = 0$
⟨*proof*⟩

lemma *store-sub2-privilege*:
assumes *a1*: $s' = snd\ (fst\ (store\text{-}sub2\ instr\ curr\text{-}win\ rd\ asi\ address\ s))$
$\wedge\ (((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
shows $(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *store-sub1-privilege*:
assumes *a1*: $s' = snd\ (fst\ (store\text{-}sub1\ instr\ rd\ s\text{-}val$
$(s::(('a::len)\ sparc\text{-}state))))$
$\wedge\ (((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
shows $(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR$
$(s'::(('a::len)\ sparc\text{-}state)))))::word1) = 0$
⟨*proof*⟩

lemma *store-instr-privilege*:
assumes *a1*: $s' = snd\ (fst\ (store\text{-}instr\ instr$
$(s::(('a::len)\ sparc\text{-}state))))$
$\wedge\ (((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
shows $(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR$
$(s'::(('a::len)\ sparc\text{-}state)))))::word1) = 0$
⟨*proof*⟩

lemma *sethi-instr-privilege*:
assumes *a1*: $s' = snd\ (fst\ (sethi\text{-}instr\ instr$
$(s::(('a::len)\ sparc\text{-}state))))$
$\wedge\ (((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
shows $(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$

⟨*proof*⟩

lemma *nop-instr-privilege*:
assumes *a1*: $s' = snd\ (fst\ (nop\text{-}instr\ instr$
$(s::(('a::len)\ sparc\text{-}state))))$
$\wedge\ (((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
shows $(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *ucast-0*: $(((get\text{-}S\ w))::word1) = 0 \Longrightarrow get\text{-}S\ w = 0$
⟨*proof*⟩

lemma *ucast-02*: $get\text{-}S\ w = 0 \Longrightarrow (((get\text{-}S\ w))::word1) = 0$
⟨*proof*⟩

lemma *ucast-s*: $(((get\text{-}S\ w))::word1) = 0 \Longrightarrow$
$(AND)\ w\ (0b00000000000000000000000010000000::word32) = 0$
⟨*proof*⟩

lemma *ucast-s2*: $(AND)\ w\ 0b00000000000000000000000010000000 = 0$
$\Longrightarrow (((get\text{-}S\ w))::word1) = 0$
⟨*proof*⟩

lemma *update-PSR-icc-1*: $w' = (AND)\ w\ (0b11111111000011111111111111111111::word32)$
$\wedge\ (((get\text{-}S\ w))::word1) = 0$
$\Longrightarrow (((get\text{-}S\ w'))::word1) = 0$
⟨*proof*⟩

lemma *and-num-1048576-128*: $(AND)\ (0b00000000000010000000000000000000::word32)$
$(0b00000000000000000000000010000000::word32) = 0$
⟨*proof*⟩

lemma *and-num-2097152-128*: $(AND)\ (0b00000000000100000000000000000000::word32)$
$(0b00000000000000000000000010000000::word32) = 0$
⟨*proof*⟩

lemma *and-num-4194304-128*: $(AND)\ (0b00000000001000000000000000000000::word32)$
$(0b00000000000000000000000010000000::word32) = 0$
⟨*proof*⟩

lemma *and-num-8388608-128*: $(AND)\ (0b00000000010000000000000000000000::word32)$
$(0b00000000000000000000000010000000::word32) = 0$
⟨*proof*⟩

lemma *or-and-s*: $(AND)\ w1\ (0b00000000000000000000000010000000::word32) =$
0
$\wedge\ (AND)\ w2\ (0b00000000000000000000000010000000::word32) = 0$
$\Longrightarrow (AND)\ ((OR)\ w1\ w2)\ (0b00000000000000000000000010000000::word32) =$
0

⟨*proof*⟩

lemma *and-or-s*:
assumes (((*get-S w1*))::*word1*) = *0* ∧
(*AND*) *w2* (*0b00000000000000000000000010000000*::*word32*) = *0*
shows (((*get-S* ((*OR*) ((*AND*) *w1*
(*0b11111111000011111111111111111111*::*word32*)) *w2*)))::*word1*) = *0*
⟨*proof*⟩

lemma *and-or-or-s*:
assumes *a1*: (((*get-S w1*))::*word1*) = *0* ∧
(*AND*) *w2* (*0b00000000000000000000000010000000*::*word32*) = *0* ∧
(*AND*) *w3* (*0b00000000000000000000000010000000*::*word32*) = *0*
shows (((*get-S* ((*OR*) ((*OR*) ((*AND*) *w1*
(*0b11111111000011111111111111111111*::*word32*)) *w2*) *w3*)))::*word1*) = *0*
⟨*proof*⟩

lemma *and-or-or-or-s*:
assumes *a1*: (((*get-S w1*))::*word1*) = *0* ∧
(*AND*) *w2* (*0b00000000000000000000000010000000*::*word32*) = *0* ∧
(*AND*) *w3* (*0b00000000000000000000000010000000*::*word32*) = *0* ∧
(*AND*) *w4* (*0b00000000000000000000000010000000*::*word32*) = *0*
shows (((*get-S* ((*OR*) ((*OR*) ((*OR*) ((*AND*) *w1*
(*0b11111111000011111111111111111111*::*word32*)) *w2*) *w3*) *w4*)))::*word1*) = *0*
⟨*proof*⟩

lemma *and-or-or-or-or-s*:
assumes *a1*: (((*get-S w1*))::*word1*) = *0* ∧
(*AND*) *w2* (*0b00000000000000000000000010000000*::*word32*) = *0* ∧
(*AND*) *w3* (*0b00000000000000000000000010000000*::*word32*) = *0* ∧
(*AND*) *w4* (*0b00000000000000000000000010000000*::*word32*) = *0* ∧
(*AND*) *w5* (*0b00000000000000000000000010000000*::*word32*) = *0*
shows (((*get-S* ((*OR*) ((*OR*) ((*OR*) ((*OR*) ((*AND*) *w1*
(*0b11111111000011111111111111111111*::*word32*)) *w2*) *w3*) *w4*) *w5*)))::*word1*)
= *0*
⟨*proof*⟩

lemma *write-cpu-PSR-icc-privilege*:
assumes *a1*: *s′* = *snd* (*fst* (*write-cpu* (*update-PSR-icc n-val z-val v-val c-val*
(*cpu-reg-val PSR s*))
PSR
(*s*::((*′a*::*len*) *sparc-state*))))
∧ (((*get-S* (*cpu-reg-val PSR s*)))::*word1*) = *0*
shows (((*get-S* (*cpu-reg-val PSR s′*)))::*word1*) = *0*
⟨*proof*⟩

lemma *and-num-4294967167-128*: (*AND*) (*0b11111111111111111111111101111111*::*word32*)
(*0b00000000000000000000000010000000*::*word32*) = *0*
⟨*proof*⟩

lemma *s-0-word*: (((*get-S* ((*AND*) *w*
(*0b11111111111111111111111101111111*::*word32*))))::*word1*) = *0*
⟨*proof*⟩

lemma *update-PSR-CWP-1*: *w'* = (*AND*) *w* (*0b11111111111111111111111111100000*::*word32*)
∧ (((*get-S w*))::*word1*) = *0*
⟹ (((*get-S w'*))::*word1*) = *0*
⟨*proof*⟩

lemma *write-cpu-PSR-CWP-privilege*:
assumes *a1*: *s'* = *snd* (*fst* (*write-cpu* (*update-CWP cwp-val*
(*cpu-reg-val PSR s*))
PSR
(*s*::((*'a*::*len*) *sparc-state*))))
∧ (((*get-S* (*cpu-reg-val PSR s*)))::*word1*) = *0*
shows (((*get-S* (*cpu-reg-val PSR s'*)))::*word1*) = *0*
⟨*proof*⟩

lemma *logical-instr-sub1-privilege*:
assumes *a1*: *s'* = *snd* (*fst* (*logical-instr-sub1 instr-name result*
(*s*::((*'a*::*len*) *sparc-state*))))
∧ (((*get-S* (*cpu-reg-val PSR s*)))::*word1*) = *0*
shows (((*get-S* (*cpu-reg-val PSR s'*)))::*word1*) = *0*
⟨*proof*⟩

lemma *logical-instr-privilege*:
assumes *a1*: *s'* = *snd* (*fst* (*logical-instr instr*
(*s*::((*'a*::*len*) *sparc-state*))))
∧ (((*get-S* (*cpu-reg-val PSR s*)))::*word1*) = *0*
shows (((*get-S* (*cpu-reg-val PSR s'*)))::*word1*) = *0*
⟨*proof*⟩

method *shift-instr-privilege-proof* = (
(*simp add*: *shift-instr-def*),
(*simp add*: *Let-def*),
(*simp add*: *simpler-gets-def*),
(*simp add*: *bind-def h1-def h2-def Let-def case-prod-unfold*),
auto,
(*blast intro*: *get-curr-win-privilege write-reg-privilege*),
(*blast intro*: *get-curr-win-privilege write-reg-privilege*)
)

lemma *shift-instr-privilege*:
assumes *a1*: *s'* = *snd* (*fst* (*shift-instr instr*
(*s*::((*'a*::*len*) *sparc-state*))))
∧ (((*get-S* (*cpu-reg-val PSR s*)))::*word1*) = *0*
shows (((*get-S* (*cpu-reg-val PSR s'*)))::*word1*) = *0*
⟨*proof*⟩

lemma *add-instr-sub1-privilege*:
assumes *a1*: $s' = snd\ (fst\ (add\text{-}instr\text{-}sub1\ instr\text{-}name\ result\ rs1\text{-}val\ operand2$
$(s::(('a::len)\ sparc\text{-}state))))$
$\wedge\ (((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
shows $(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *add-instr-privilege*:
assumes *a1*: $s' = snd\ (fst\ (add\text{-}instr\ instr$
$(s::(('a::len)\ sparc\text{-}state))))$
$\wedge\ (((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
shows $(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *sub-instr-sub1-privilege*:
assumes *a1*: $s' = snd\ (fst\ (sub\text{-}instr\text{-}sub1\ instr\text{-}name\ result\ rs1\text{-}val\ operand2$
$(s::(('a::len)\ sparc\text{-}state))))$
$\wedge\ (((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
shows $(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *sub-instr-privilege*:
assumes *a1*: $s' = snd\ (fst\ (sub\text{-}instr\ instr$
$(s::(('a::len)\ sparc\text{-}state))))$
$\wedge\ (((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
shows $(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *mul-instr-sub1-privilege*:
assumes *a1*: $s' = snd\ (fst\ (mul\text{-}instr\text{-}sub1\ instr\text{-}name\ result$
$(s::(('a::len)\ sparc\text{-}state))))$
$\wedge\ (((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
shows $(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *mul-instr-privilege*:
assumes *a1*: $s' = snd\ (fst\ (mul\text{-}instr\ instr$
$(s::(('a::len)\ sparc\text{-}state))))$
$\wedge\ (((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
shows $(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *div-write-new-val-privilege*:
assumes *a1*: $s' = snd\ (fst\ (div\text{-}write\text{-}new\text{-}val\ i\ result\ temp\text{-}V$
$(s::(('a::len)\ sparc\text{-}state))))$
$\wedge\ (((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
shows $(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *div-comp-privilege*:
assumes *a1*: $s' = snd\ (fst\ (div\text{-}comp\ instr\ rs1\ rd\ operand2$
$(s::(('a::len)\ sparc\text{-}state))))$
$\wedge\ (((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
shows $(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *div-instr-privilege*:
assumes *a1*: $s' = snd\ (fst\ (div\text{-}instr\ instr$
$(s::(('a::len)\ sparc\text{-}state))))$
$\wedge\ (((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
shows $(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *save-retore-sub1-privilege*:
assumes *a1*: $s' = snd\ (fst\ (save\text{-}retore\text{-}sub1\ result\ new\text{-}cwp\ rd$
$(s::(('a::len)\ sparc\text{-}state))))$
$\wedge\ (((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
shows $(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

method *save-restore-instr-privilege-proof* = (
(*simp add*: *save-restore-instr-def*),
(*simp add*: *Let-def*),
(*simp add*: *simpler-gets-def bind-def h1-def h2-def Let-def*),
(*simp add*: *case-prod-unfold*),
auto,
(*blast intro*: *get-curr-win-privilege raise-trap-privilege*),
(*simp add*: *simpler-gets-def bind-def h1-def h2-def Let-def case-prod-unfold*),
(*blast intro*: *get-curr-win-privilege save-retore-sub1-privilege*)
)

lemma *save-restore-instr-privilege*:
assumes *a1*: $s' = snd\ (fst\ (save\text{-}restore\text{-}instr\ instr$
$(s::(('a::len)\ sparc\text{-}state))))$
$\wedge\ (((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
shows $(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *call-instr-privilege*:
assumes *a1*: $s' = snd\ (fst\ (call\text{-}instr\ instr$
$(s::(('a::len)\ sparc\text{-}state))))$
$\wedge\ (((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
shows $(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *jmpl-instr-privilege*:
assumes *a1*: $s' = snd\ (fst\ (jmpl\text{-}instr\ instr$

(*s*::(('*a*::*len*) *sparc-state*))))
∧ (((*get-S* (*cpu-reg-val PSR s*)))::*word1*) = *0*
shows (((*get-S* (*cpu-reg-val PSR s'*)))::*word1*) = *0*
⟨*proof*⟩

lemma *rett-instr-privilege*:
assumes *a1*: *snd* (*rett-instr i s*) = *False* ∧
s' = *snd* (*fst* (*rett-instr instr*
(*s*::(('*a*::*len*) *sparc-state*))))
∧ (((*get-S* (*cpu-reg-val PSR s*)))::*word1*) = *0*
shows (((*get-S* (*cpu-reg-val PSR s'*)))::*word1*) = *0*
⟨*proof*⟩

method *read-state-reg-instr-privilege-proof* = (
(*simp add*: *read-state-reg-instr-def*),
(*simp add*: *Let-def*),
(*simp add*: *simpler-gets-def bind-def h1-def h2-def Let-def*),
(*simp add*: *case-prod-unfold*)
)

lemma *read-state-reg-instr-privilege*:
assumes *a1*: *s'* = *snd* (*fst* (*read-state-reg-instr instr*
(*s*::(('*a*::*len*) *sparc-state*))))
∧ (((*get-S* (*cpu-reg-val PSR s*)))::*word1*) = *0*
shows (((*get-S* (*cpu-reg-val PSR s'*)))::*word1*) = *0*
⟨*proof*⟩

method *write-state-reg-instr-privilege-proof* = (
(*simp add*: *write-state-reg-instr-def*),
(*simp add*: *Let-def*),
(*simp add*: *simpler-gets-def bind-def h1-def h2-def Let-def*),
(*simp add*: *case-prod-unfold*)
)

lemma *write-state-reg-instr-privilege*:
assumes *a1*: *s'* = *snd* (*fst* (*write-state-reg-instr instr*
(*s*::(('*a*::*len*) *sparc-state*))))
∧ (((*get-S* (*cpu-reg-val PSR s*)))::*word1*) = *0*
shows (((*get-S* (*cpu-reg-val PSR s'*)))::*word1*) = *0*
⟨*proof*⟩

lemma *flush-instr-privilege*:
assumes *a1*: *s'* = *snd* (*fst* (*flush-instr instr*
(*s*::(('*a*::*len*) *sparc-state*))))
∧ (((*get-S* (*cpu-reg-val PSR s*)))::*word1*) = *0*
shows (((*get-S* (*cpu-reg-val PSR s'*)))::*word1*) = *0*
⟨*proof*⟩

lemma *branch-instr-privilege*:

assumes *a1*: $s' = snd\ (fst\ (branch\text{-}instr\ instr$
$(s::(('a::len)\ sparc\text{-}state))))$
$\wedge\ (((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
shows $(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

method *dispath-instr-privilege-proof* = (
(*simp add*: *dispatch-instruction-def*),
(*simp add*: *simpler-gets-def bind-def h1-def h2-def Let-def*),
(*simp add*: *Let-def*)
)

lemma *dispath-instr-privilege*:
assumes *a1*: $snd\ (dispatch\text{-}instruction\ instr\ s) = False \wedge$
$s' = snd\ (fst\ (dispatch\text{-}instruction\ instr\ s))$
$\wedge\ (((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
shows $(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

lemma *execute-instr-sub1-privilege*:
assumes *a1*: $snd\ (execute\text{-}instr\text{-}sub1\ i\ s) = False \wedge$
$s' = snd\ (fst\ (execute\text{-}instr\text{-}sub1\ i\ s))$
$\wedge\ (((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
shows $(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

Assume that there is no *delayed-write* and there is no traps to be executed. If an instruction is executed as a user, the privilege will not be changed to supervisor after the execution.

theorem *safe-privilege* :
assumes *a1*: $get\text{-}delayed\text{-}pool\ s = [] \wedge get\text{-}trap\text{-}set\ s = \{\} \wedge$
$snd\ (execute\text{-}instruction()\ s) = False \wedge$
$s' = snd\ (fst\ (execute\text{-}instruction()\ s)) \wedge$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s)))::word1) = 0$
shows $(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ s')))::word1) = 0$
⟨*proof*⟩

26 Single step non-interference property.

definition *user-accessible*:: $('a::len)\ sparc\text{-}state \Rightarrow phys\text{-}address \Rightarrow bool$ **where**
$user\text{-}accessible\ s\ pa \equiv \exists\, va\ p.\ (virt\text{-}to\text{-}phys\ va\ (mmu\ s)\ (mem\ s)) = Some\ p \wedge$
$mmu\text{-}readable\ (get\text{-}acc\text{-}flag\ (snd\ p))\ 10 \wedge$
$(fst\ p) = pa$ — Passing $asi = 8$ is the same.

lemma *user-accessible-8*:
assumes *a1*: $mmu\text{-}readable\ (get\text{-}acc\text{-}flag\ (snd\ p))\ 8$
shows $mmu\text{-}readable\ (get\text{-}acc\text{-}flag\ (snd\ p))\ 10$
⟨*proof*⟩

definition *mem-equal*:: ('*a*) *sparc-state* ⇒ ('*a*) *sparc-state* ⇒
phys-address ⇒ *bool* **where**
mem-equal s1 s2 pa ≡
(*mem s1*) *8* (*pa AND 68719476732*) = (*mem s2*) *8* (*pa AND 68719476732*) ∧
(*mem s1*) *8* ((*pa AND 68719476732*) + *1*) = (*mem s2*) *8* ((*pa AND 68719476732*)
+ *1*) ∧
(*mem s1*) *8* ((*pa AND 68719476732*) + *2*) = (*mem s2*) *8* ((*pa AND 68719476732*)
+ *2*) ∧
(*mem s1*) *8* ((*pa AND 68719476732*) + *3*) = (*mem s2*) *8* ((*pa AND 68719476732*)
+ *3*) ∧
(*mem s1*) *9* (*pa AND 68719476732*) = (*mem s2*) *9* (*pa AND 68719476732*) ∧
(*mem s1*) *9* ((*pa AND 68719476732*) + *1*) = (*mem s2*) *9* ((*pa AND 68719476732*)
+ *1*) ∧
(*mem s1*) *9* ((*pa AND 68719476732*) + *2*) = (*mem s2*) *9* ((*pa AND 68719476732*)
+ *2*) ∧
(*mem s1*) *9* ((*pa AND 68719476732*) + *3*) = (*mem s2*) *9* ((*pa AND 68719476732*)
+ *3*) ∧
(*mem s1*) *10* (*pa AND 68719476732*) = (*mem s2*) *10* (*pa AND 68719476732*) ∧
(*mem s1*) *10* ((*pa AND 68719476732*) + *1*) = (*mem s2*) *10* ((*pa AND 68719476732*)
+ *1*) ∧
(*mem s1*) *10* ((*pa AND 68719476732*) + *2*) = (*mem s2*) *10* ((*pa AND 68719476732*)
+ *2*) ∧
(*mem s1*) *10* ((*pa AND 68719476732*) + *3*) = (*mem s2*) *10* ((*pa AND 68719476732*)
+ *3*) ∧
(*mem s1*) *11* (*pa AND 68719476732*) = (*mem s2*) *11* (*pa AND 68719476732*) ∧
(*mem s1*) *11* ((*pa AND 68719476732*) + *1*) = (*mem s2*) *11* ((*pa AND 68719476732*)
+ *1*) ∧
(*mem s1*) *11* ((*pa AND 68719476732*) + *2*) = (*mem s2*) *11* ((*pa AND 68719476732*)
+ *2*) ∧
(*mem s1*) *11* ((*pa AND 68719476732*) + *3*) = (*mem s2*) *11* ((*pa AND 68719476732*)
+ *3*)

low-equal defines the equivalence relation over two sparc states that is an analogy to the $=_L$ relation over memory contexts in the traditional non-interference theorem.

definition *low-equal*:: ('*a*::*len*) *sparc-state* ⇒ ('*a*) *sparc-state* ⇒ *bool* **where**
low-equal s1 s2 ≡
(*cpu-reg s1*) = (*cpu-reg s2*) ∧
(*user-reg s1*) = (*user-reg s2*) ∧
(*sys-reg s1*) = (*sys-reg s2*) ∧
(∀ *va*. (*virt-to-phys va* (*mmu s1*) (*mem s1*)) = (*virt-to-phys va* (*mmu s2*) (*mem
s2*))) ∧
(∀ *pa*. (*user-accessible s1 pa*) ⟶ *mem-equal s1 s2 pa*) ∧
(*mmu s1*) = (*mmu s2*) ∧
(*state-var s1*) = (*state-var s2*) ∧
(*traps s1*) = (*traps s2*) ∧
(*undef s1*) = (*undef s2*)

lemma *low-equal-com*: *low-equal s1 s2* $\Longrightarrow$ *low-equal s2 s1*
⟨*proof*⟩

lemma *non-exe-mode-equal*: *exe-mode-val s* = *False* ∧
get-trap-set s = {} ∧
Some t = *NEXT s* $\Longrightarrow$
t = *s*
⟨*proof*⟩

lemma *exe-mode-low-equal*:
assumes *a1*: *low-equal s1 s2*
shows *exe-mode-val s1* = *exe-mode-val s2*
⟨*proof*⟩

lemma *mem-val-mod-state*: *mem-val-alt asi a s* = *mem-val-alt asi a*
(*s*(|*cpu-reg* := *new-cpu-reg*,
user-reg := *new-user-reg*,
dwrite := *new-dwrite*,
state-var := *new-state-var*,
traps := *new-traps*,
undef := *new-undef*|))
⟨*proof*⟩

lemma *mem-val-w32-mod-state*: *mem-val-w32 asi a s* = *mem-val-w32 asi a*
(*s*(|*cpu-reg* := *new-cpu-reg*,
user-reg := *new-user-reg*,
dwrite := *new-dwrite*,
state-var := *new-state-var*,
traps := *new-traps*,
undef := *new-undef*|))
⟨*proof*⟩

lemma *load-word-mem-mod-state*: *load-word-mem s addr asi* = *load-word-mem*
(*s*(|*cpu-reg* := *new-cpu-reg*,
user-reg := *new-user-reg*,
dwrite := *new-dwrite*,
state-var := *new-state-var*,
traps := *new-traps*,
undef := *new-undef*|)) *addr asi*
⟨*proof*⟩

lemma *load-word-mem2-mod-state*:
fst (*case load-word-mem s addr asi of None* ⇒ (*None*, *s*)
| *Some w* ⇒ (*Some w*, *add-data-cache s addr w 15*)) =
fst (*case load-word-mem* (*s*(|*cpu-reg* := *new-cpu-reg*,
user-reg := *new-user-reg*,
dwrite := *new-dwrite*,
state-var := *new-state-var*,

traps := *new-traps*,
undef := *new-undef*⦈) *addr asi of*
None ⇒ (*None*, (*s*⦇*cpu-reg* := *new-cpu-reg*,
user-reg := *new-user-reg*,
dwrite := *new-dwrite*,
state-var := *new-state-var*,
traps := *new-traps*,
undef := *new-undef*⦈)))
| *Some w* ⇒ (*Some w*, *add-data-cache* (*s*⦇*cpu-reg* := *new-cpu-reg*,
user-reg := *new-user-reg*,
dwrite := *new-dwrite*,
state-var := *new-state-var*,
traps := *new-traps*,
undef := *new-undef*⦈) *addr w 15*))
⟨*proof*⟩

lemma *load-word-mem3-mod-state*:
fst (*case load-word-mem s addr asi of None* ⇒ (*None*, *s*)
| *Some w* ⇒ (*Some w*, *add-instr-cache s addr w 15*)) =
fst (*case load-word-mem* (*s*⦇*cpu-reg* := *new-cpu-reg*,
user-reg := *new-user-reg*,
dwrite := *new-dwrite*,
state-var := *new-state-var*,
traps := *new-traps*,
undef := *new-undef*⦈) *addr asi of*
None ⇒ (*None*, (*s*⦇*cpu-reg* := *new-cpu-reg*,
user-reg := *new-user-reg*,
dwrite := *new-dwrite*,
state-var := *new-state-var*,
traps := *new-traps*,
undef := *new-undef*⦈)))
| *Some w* ⇒ (*Some w*, *add-instr-cache* (*s*⦇*cpu-reg* := *new-cpu-reg*,
user-reg := *new-user-reg*,
dwrite := *new-dwrite*,
state-var := *new-state-var*,
traps := *new-traps*,
undef := *new-undef*⦈) *addr w 15*))
⟨*proof*⟩

lemma *read-dcache-mod-state*: *read-data-cache s addr* = *read-data-cache*
(*s*⦇*cpu-reg* := *new-cpu-reg*,
user-reg := *new-user-reg*,
dwrite := *new-dwrite*,
state-var := *new-state-var*,
traps := *new-traps*,
undef := *new-undef*⦈) *addr*
⟨*proof*⟩

lemma *read-dcache2-mod-state*:

fst (*case read-data-cache s addr of None* ⇒ (*None*, *s*)
  | *Some w* ⇒ (*Some w*, *s*)) =
 fst (*case read-data-cache* (*s*(|*cpu-reg* := *new-cpu-reg*,
  user-reg := *new-user-reg*,
  dwrite := *new-dwrite*,
  state-var := *new-state-var*,
  traps := *new-traps*,
  undef := *new-undef*|)) *addr of*
    None ⇒ (*None*, (*s*(|*cpu-reg* := *new-cpu-reg*,
  user-reg := *new-user-reg*,
  dwrite := *new-dwrite*,
  state-var := *new-state-var*,
  traps := *new-traps*,
  undef := *new-undef*|)))
  | *Some w* ⇒ (*Some w*, (*s*(|*cpu-reg* := *new-cpu-reg*,
  user-reg := *new-user-reg*,
  dwrite := *new-dwrite*,
  state-var := *new-state-var*,
  traps := *new-traps*,
  undef := *new-undef*|))))
⟨*proof*⟩

lemma *read-icache-mod-state*: *read-instr-cache s addr* = *read-instr-cache*
(*s*(|*cpu-reg* := *new-cpu-reg*,
  user-reg := *new-user-reg*,
  dwrite := *new-dwrite*,
  state-var := *new-state-var*,
  traps := *new-traps*,
  undef := *new-undef*|)) *addr*
⟨*proof*⟩

lemma *read-icache2-mod-state*:
fst (*case read-instr-cache s addr of None* ⇒ (*None*, *s*)
  | *Some w* ⇒ (*Some w*, *s*)) =
 fst (*case read-instr-cache* (*s*(|*cpu-reg* := *new-cpu-reg*,
  user-reg := *new-user-reg*,
  dwrite := *new-dwrite*,
  state-var := *new-state-var*,
  traps := *new-traps*,
  undef := *new-undef*|)) *addr of*
    None ⇒ (*None*, (*s*(|*cpu-reg* := *new-cpu-reg*,
  user-reg := *new-user-reg*,
  dwrite := *new-dwrite*,
  state-var := *new-state-var*,
  traps := *new-traps*,
  undef := *new-undef*|)))
  | *Some w* ⇒ (*Some w*, (*s*(|*cpu-reg* := *new-cpu-reg*,
  user-reg := *new-user-reg*,
  dwrite := *new-dwrite*,

state-var := *new-state-var*,
traps := *new-traps*,
undef := *new-undef*⦈))))
⟨*proof*⟩

lemma *mem-read-mod-state*: *fst* (*memory-read asi addr s*) =
fst (*memory-read asi addr*
(*s*⦇*cpu-reg* := *new-cpu-reg*,
user-reg := *new-user-reg*,
dwrite := *new-dwrite*,
state-var := *new-state-var*,
traps := *new-traps*,
undef := *new-undef*⦈)))
⟨*proof*⟩

lemma *insert-trap-mem*: *fst* (*memory-read asi addr s*) =
fst (*memory-read asi addr* (*s*⦇*traps* := *new-traps*⦈))
⟨*proof*⟩

lemma *cpu-reg-mod-mem*: *fst* (*memory-read asi addr s*) =
fst (*memory-read asi addr* (*s*⦇*cpu-reg* := *new-cpu-reg*⦈))
⟨*proof*⟩

lemma *user-reg-mod-mem*: *fst* (*memory-read asi addr s*) =
fst (*memory-read asi addr* (*s*⦇*user-reg* := *new-user-reg*⦈))
⟨*proof*⟩

lemma *annul-mem*: *fst* (*memory-read asi addr s*) =
fst (*memory-read asi addr*
(*s*⦇*state-var* := *new-state-var*,
cpu-reg := *new-cpu-reg*⦈))
⟨*proof*⟩

lemma *state-var-mod-mem*: *fst* (*memory-read asi addr s*) =
fst (*memory-read asi addr* (*s*⦇*state-var* := *new-state-var*⦈))
⟨*proof*⟩

lemma *mod-state-low-equal*: *low-equal s1 s2* ∧
t1 = (*s1*⦇*cpu-reg* := *new-cpu-reg*,
user-reg := *new-user-reg*,
dwrite := *new-dwrite*,
state-var := *new-state-var*,
traps := *new-traps*,
undef := *new-undef*⦈) ∧
t2 = (*s2*⦇*cpu-reg* := *new-cpu-reg*,
user-reg := *new-user-reg*,
dwrite := *new-dwrite*,
state-var := *new-state-var*,
traps := *new-traps*,

undef := *new-undef*⦈) ⟹
low-equal t1 t2
⟨*proof*⟩

lemma *user-reg-state-mod-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = (*s1*⦇*user-reg* := *new-user-reg*⦈) ∧
t2 = (*s2*⦇*user-reg* := *new-user-reg*⦈)
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *mod-trap-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = (*s1*⦇*traps* := *new-traps*⦈) ∧
t2 = (*s2*⦇*traps* := *new-traps*⦈)
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *state-var-low-equal*: *low-equal s1 s2* ⟹
state-var s1 = *state-var s2*
⟨*proof*⟩

lemma *state-var2-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = (*s1*⦇*state-var* := *new-state-var*⦈) ∧
t2 = (*s2*⦇*state-var* := *new-state-var*⦈)
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *traps-low-equal*: *low-equal s1 s2* ⟹ *traps s1* = *traps s2*
⟨*proof*⟩

lemma *s-low-equal*: *low-equal s1 s2* ⟹
(*get-S* (*cpu-reg-val PSR s1*)) = (*get-S* (*cpu-reg-val PSR s2*))
⟨*proof*⟩

lemma *cpu-reg-val-low-equal*: *low-equal s1 s2* ⟹
(*cpu-reg-val cr s1*) = (*cpu-reg-val cr s2*)
⟨*proof*⟩

lemma *get-curr-win-low-equal*: *low-equal s1 s2* ⟹
(*fst* (*fst* (*get-curr-win* () *s1*))) = (*fst* (*fst* (*get-curr-win* () *s2*)))
⟨*proof*⟩

lemma *get-curr-win2-low-equal*: *low-equal s1 s2* ⟹
t1 = (*snd* (*fst* (*get-curr-win* () *s1*))) ⟹
t2 = (*snd* (*fst* (*get-curr-win* () *s2*))) ⟹
low-equal t1 t2
⟨*proof*⟩

lemma *get-curr-win3-low-equal*: *low-equal s1 s2* $\Longrightarrow$
(*traps* (*snd* (*fst* (*get-curr-win* () *s1*)))) =
(*traps* (*snd* (*fst* (*get-curr-win* () *s2*))))
⟨*proof*⟩

lemma *get-addr-low-equal*: *low-equal s1 s2* $\Longrightarrow$
((*ucast* (*get-addr* (*snd instr*) (*snd* (*fst* (*get-curr-win* () *s1*))))::*word3*) =
((*ucast* (*get-addr* (*snd instr*) (*snd* (*fst* (*get-curr-win* () *s2*))))::*word3*) ∧
((*ucast* (*get-addr* (*snd instr*) (*snd* (*fst* (*get-curr-win* () *s1*))))::*word2*) =
((*ucast* (*get-addr* (*snd instr*) (*snd* (*fst* (*get-curr-win* () *s2*))))::*word2*) ∧
((*ucast* (*get-addr* (*snd instr*) (*snd* (*fst* (*get-curr-win* () *s1*))))::*word1*) =
((*ucast* (*get-addr* (*snd instr*) (*snd* (*fst* (*get-curr-win* () *s2*))))::*word1*)
⟨*proof*⟩

lemma *get-addr2-low-equal*: *low-equal s1 s2* $\Longrightarrow$
get-addr (*snd instr*) (*snd* (*fst* (*get-curr-win* () *s1*))) =
get-addr (*snd instr*) (*snd* (*fst* (*get-curr-win* () *s2*)))
⟨*proof*⟩

lemma *sys-reg-low-equal*: *low-equal s1 s2* $\Longrightarrow$
sys-reg s1 = *sys-reg s2*
⟨*proof*⟩

lemma *user-reg-low-equal*: *low-equal s1 s2* $\Longrightarrow$
user-reg s1 = *user-reg s2*
⟨*proof*⟩

lemma *user-reg-val-low-equal*: *low-equal s1 s2* $\Longrightarrow$
user-reg-val win ur s1 = *user-reg-val win ur s2*
⟨*proof*⟩

lemma *get-operand2-low-equal*: *low-equal s1 s2* $\Longrightarrow$
get-operand2 op-list s1 = *get-operand2 op-list s2*
⟨*proof*⟩

lemma *mem-val-mod-cache*: *mem-val-alt asi a s* =
mem-val-alt asi a (*s*(|*cache* := *new-cache*|))
⟨*proof*⟩

lemma *mem-val-w32-mod-cache*: *mem-val-w32 asi a s* =
mem-val-w32 asi a (*s*(|*cache* := *new-cache*|))
⟨*proof*⟩

lemma *load-word-mem-mod-cache*:
load-word-mem s addr asi =
load-word-mem (*s*(|*cache* := *new-cache*|)) *addr asi*
⟨*proof*⟩

lemma *memory-read-8-mod-cache*:
fst (*memory-read 8 addr s*) = *fst* (*memory-read 8 addr* (*s*(|*cache* := *new-cache*|)))
⟨*proof*⟩

lemma *memory-read-10-mod-cache*:
fst (*memory-read 10 addr s*) = *fst* (*memory-read 10 addr* (*s*(|*cache* := *new-cache*|)))
⟨*proof*⟩

lemma *mem-equal-mod-cache*: *mem-equal s1 s2 pa* ⟹
mem-equal (*s1*(|*cache* := *new-cache1*|)) (*s2*(|*cache* := *new-cache2*|)) *pa*
⟨*proof*⟩

lemma *user-accessible-mod-cache*: *user-accessible* (*s*(|*cache* := *new-cache*|)) *pa* =
user-accessible s pa
⟨*proof*⟩

lemma *mem-equal-mod-user-reg*: *mem-equal s1 s2 pa* ⟹
mem-equal (*s1*(|*user-reg* := *new-user-reg1*|)) (*s2*(|*user-reg* := *user-reg2*|)) *pa*
⟨*proof*⟩

lemma *user-accessible-mod-user-reg*: *user-accessible* (*s*(|*user-reg* := *new-user-reg*|))
pa =
user-accessible s pa
⟨*proof*⟩

lemma *mem-equal-mod-cpu-reg*: *mem-equal s1 s2 pa* ⟹
mem-equal (*s1*(|*cpu-reg* := *new-cpu1*|)) (*s2*(|*cpu-reg* := *cpu-reg2*|)) *pa*
⟨*proof*⟩

lemma *user-accessible-mod-cpu-reg*: *user-accessible* (*s*(|*cpu-reg* := *new-cpu-reg*|)) *pa*
=
user-accessible s pa
⟨*proof*⟩

lemma *mem-equal-mod-trap*: *mem-equal s1 s2 pa* ⟹
mem-equal (*s1*(|*traps* := *new-traps1*|)) (*s2*(|*traps* := *traps2*|)) *pa*
⟨*proof*⟩

lemma *user-accessible-mod-trap*: *user-accessible* (*s*(|*traps* := *new-traps*|)) *pa* =
user-accessible s pa
⟨*proof*⟩

lemma *mem-equal-annul*: *mem-equal s1 s2 pa* ⟹
mem-equal (*s1*(|*state-var* := *new-state-var*,
cpu-reg := *new-cpu-reg*|)) (*s2*(|*state-var* := *new-state-var2*,
cpu-reg := *new-cpu-reg2*|)) *pa*
⟨*proof*⟩

lemma *user-accessible-annul*: *user-accessible* (*s*(|*state-var* := *new-state-var*,

cpu-reg := *new-cpu-reg*⦈) *pa* =
user-accessible s pa
⟨*proof*⟩

lemma *mem-val-alt-10-mem-equal-0*: *mem-equal s1 s2 pa* ⟹
mem-val-alt 10 (*pa AND 68719476732*) *s1* = *mem-val-alt 10* (*pa AND 68719476732*)
s2
⟨*proof*⟩

lemma *mem-val-alt-10-mem-equal-1*: *mem-equal s1 s2 pa* ⟹
mem-val-alt 10 ((*pa AND 68719476732*) + *1*) *s1* = *mem-val-alt 10* ((*pa AND*
68719476732) + *1*) *s2*
⟨*proof*⟩

lemma *mem-val-alt-10-mem-equal-2*: *mem-equal s1 s2 pa* ⟹
mem-val-alt 10 ((*pa AND 68719476732*) + *2*) *s1* = *mem-val-alt 10* ((*pa AND*
68719476732) + *2*) *s2*
⟨*proof*⟩

lemma *mem-val-alt-10-mem-equal-3*: *mem-equal s1 s2 pa* ⟹
mem-val-alt 10 ((*pa AND 68719476732*) + *3*) *s1* = *mem-val-alt 10* ((*pa AND*
68719476732) + *3*) *s2*
⟨*proof*⟩

lemma *mem-val-alt-10-mem-equal*:
assumes *a1*: *mem-equal s1 s2 pa*
shows *mem-val-alt 10* (*pa AND 68719476732*) *s1* = *mem-val-alt 10* (*pa AND*
68719476732) *s2* ∧
mem-val-alt 10 ((*pa AND 68719476732*) + *1*) *s1* = *mem-val-alt 10* ((*pa AND*
68719476732) + *1*) *s2* ∧
mem-val-alt 10 ((*pa AND 68719476732*) + *2*) *s1* = *mem-val-alt 10* ((*pa AND*
68719476732) + *2*) *s2* ∧
mem-val-alt 10 ((*pa AND 68719476732*) + *3*) *s1* = *mem-val-alt 10* ((*pa AND*
68719476732) + *3*) *s2*
⟨*proof*⟩

lemma *mem-val-w32-10-mem-equal*:
assumes *a1*: *mem-equal s1 s2 a*
shows *mem-val-w32 10 a s1* = *mem-val-w32 10 a s2*
⟨*proof*⟩

lemma *mem-val-alt-8-mem-equal-0*: *mem-equal s1 s2 pa* ⟹
mem-val-alt 8 (*pa AND 68719476732*) *s1* = *mem-val-alt 8* (*pa AND 68719476732*)
s2
⟨*proof*⟩

lemma *mem-val-alt-8-mem-equal-1*: *mem-equal s1 s2 pa* ⟹
mem-val-alt 8 ((*pa AND 68719476732*) + *1*) *s1* = *mem-val-alt 8* ((*pa AND 68719476732*)
+ *1*) *s2*

⟨*proof*⟩

lemma *mem-val-alt-8-mem-equal-2*: *mem-equal s1 s2 pa* ⟹
mem-val-alt 8 ((*pa AND 68719476732*) + *2*) *s1* = *mem-val-alt 8* ((*pa AND 68719476732*)
+ *2*) *s2*
⟨*proof*⟩

lemma *mem-val-alt-8-mem-equal-3*: *mem-equal s1 s2 pa* ⟹
mem-val-alt 8 ((*pa AND 68719476732*) + *3*) *s1* = *mem-val-alt 8* ((*pa AND 68719476732*)
+ *3*) *s2*
⟨*proof*⟩

lemma *mem-val-alt-8-mem-equal*:
assumes *a1*: *mem-equal s1 s2 pa*
shows *mem-val-alt 8* (*pa AND 68719476732*) *s1* = *mem-val-alt 8* (*pa AND 68719476732*)
s2 ∧
mem-val-alt 8 ((*pa AND 68719476732*) + *1*) *s1* = *mem-val-alt 8* ((*pa AND*
68719476732) + *1*) *s2* ∧
mem-val-alt 8 ((*pa AND 68719476732*) + *2*) *s1* = *mem-val-alt 8* ((*pa AND*
68719476732) + *2*) *s2* ∧
mem-val-alt 8 ((*pa AND 68719476732*) + *3*) *s1* = *mem-val-alt 8* ((*pa AND*
68719476732) + *3*) *s2*
⟨*proof*⟩

lemma *mem-val-w32-8-mem-equal*:
assumes *a1*: *mem-equal s1 s2 a*
shows *mem-val-w32 8 a s1* = *mem-val-w32 8 a s2*
⟨*proof*⟩

lemma *load-word-mem-10-low-equal*:
assumes *a1*: *low-equal s1 s2*
shows *load-word-mem s1 address 10* = *load-word-mem s2 address 10*
⟨*proof*⟩

lemma *load-word-mem-8-low-equal*:
assumes *a1*: *low-equal s1 s2*
shows *load-word-mem s1 address 8* = *load-word-mem s2 address 8*
⟨*proof*⟩

lemma *mem-read-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧ *asi* ∈ {*8*,*10*}
shows *fst* (*memory-read asi address s1*) = *fst* (*memory-read asi address s2*)
⟨*proof*⟩

lemma *read-mem-pc-low-equal*:
assumes *a1*: *low-equal s1 s2*
shows *fst* (*memory-read 8* (*cpu-reg-val PC s1*) *s1*) =
fst (*memory-read 8* (*cpu-reg-val PC s2*) *s2*)
⟨*proof*⟩

lemma *dcache-mod-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *dcache-mod c v s1* ∧
t2 = *dcache-mod c v s2*
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *add-data-cache-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *add-data-cache s1 address w bm* ∧
t2 = *add-data-cache s2 address w bm*
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *mem-read2-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *snd* (*memory-read* (*10*::*word8*) *address s1*) ∧
t2 = *snd* (*memory-read* (*10*::*word8*) *address s2*)
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *mem-read-delayed-write-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧ *get-delayed-pool s1* = [] ∧ *get-delayed-pool s2* = []
shows *fst* (*memory-read 8* (*cpu-reg-val PC* (*delayed-pool-write s1*)) (*delayed-pool-write s1*)) =
fst (*memory-read 8* (*cpu-reg-val PC* (*delayed-pool-write s2*)) (*delayed-pool-write s2*))
⟨*proof*⟩

lemma *global-reg-mod-low-equal*:
assumes *a1*: *low-equal s1 s2*∧
t1 = (*global-reg-mod w n rd s1*) ∧
t2 = (*global-reg-mod w n rd s2*)
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *out-reg-mod-low-equal*:
assumes *a1*: *low-equal s1 s2*∧
t1 = (*out-reg-mod w curr-win rd s1*) ∧
t2 = (*out-reg-mod w curr-win rd s2*)
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *in-reg-mod-low-equal*:
assumes *a1*: *low-equal s1 s2*∧
t1 = (*in-reg-mod w curr-win rd s1*) ∧
t2 = (*in-reg-mod w curr-win rd s2*)
shows *low-equal t1 t2*

⟨*proof*⟩

lemma *user-reg-mod-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *user-reg-mod w curr-win rd s1* ∧ *t2* = *user-reg-mod w curr-win rd s2*
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *virt-to-phys-low-equal*: *low-equal s1 s2* ⟹
virt-to-phys addr (*mmu s1*) (*mem s1*) = *virt-to-phys addr* (*mmu s2*) (*mem s2*)
⟨*proof*⟩

lemma *write-reg-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = (*snd* (*fst* (*write-reg w curr-win rd s1*))) ∧
t2 = (*snd* (*fst* (*write-reg w curr-win rd s2*)))
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *write-cpu-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *snd* (*fst* (*write-cpu w cr s1*)) ∧
t2 = (*snd* (*fst* (*write-cpu w cr s2*)))
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *cpu-reg-mod-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *cpu-reg-mod w cr s1* ∧
t2 = *cpu-reg-mod w cr s2*
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *load-sub2-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = (*snd* (*fst* (*load-sub2 address 10 rd curr-win w s1*))) ∧
t2 = (*snd* (*fst* (*load-sub2 address 10 rd curr-win w s2*)))
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *load-sub3-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *snd* (*fst* (*load-sub3 instr curr-win rd* (*10*::*word8*) *address s1*)) ∧
t2 = *snd* (*fst* (*load-sub3 instr curr-win rd* (*10*::*word8*) *address s2*))
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *ld-asi-user*:
(*fst instr* = *load-store-type LDSB* ∨

fst instr = load-store-type LDUB ∨
fst instr = load-store-type LDUH ∨
fst instr = load-store-type LD ∨
fst instr = load-store-type LDD) ⟹
ld-asi instr 0 = 10
⟨*proof*⟩

lemma *load-sub1-low-equal*:
assumes *a1*: *low-equal s1 s2 ∧*
(*fst instr = load-store-type LDSB ∨*
fst instr = load-store-type LDUB ∨
fst instr = load-store-type LDUH ∨
fst instr = load-store-type LD ∨
fst instr = load-store-type LDD) ∧
t1 = snd (fst (load-sub1 instr rd 0 s1)) ∧
t2 = snd (fst (load-sub1 instr rd 0 s2))
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *load-instr-low-equal*:
assumes *a1*: *low-equal s1 s2 ∧*
(*fst instr = load-store-type LDSB ∨*
fst instr = load-store-type LDUB ∨
fst instr = load-store-type LDUBA ∨
fst instr = load-store-type LDUH ∨
fst instr = load-store-type LD ∨
fst instr = load-store-type LDA ∨
fst instr = load-store-type LDD) ∧
(((get-S (cpu-reg-val PSR s1)))::word1) = 0 ∧
(((get-S (cpu-reg-val PSR s2)))::word1) = 0 ∧
t1 = snd (fst (load-instr instr s1)) ∧ t2 = snd (fst (load-instr instr s2))
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *st-data0-low-equal*: *low-equal s1 s2 ⟹*
st-data0 instr curr-win rd addr s1 = st-data0 instr curr-win rd addr s2
⟨*proof*⟩

lemma *store-word-mem-low-equal-none*: *low-equal s1 s2 ⟹*
store-word-mem (add-data-cache s1 addr data bm) addr data bm 10 = None ⟹
store-word-mem (add-data-cache s2 addr data bm) addr data bm 10 = None
⟨*proof*⟩

lemma *memory-write-asi-low-equal-none*: *low-equal s1 s2 ⟹*
memory-write-asi 10 addr bm data s1 = None ⟹
memory-write-asi 10 addr bm data s2 = None
⟨*proof*⟩

lemma *memory-write-low-equal-none*: *low-equal s1 s2 ⟹*

memory-write 10 addr bm data s1 = None $\Longrightarrow$
memory-write 10 addr bm data s2 = None
⟨*proof*⟩

lemma *memory-write-low-equal-none2*: *low-equal s1 s2* $\Longrightarrow$
memory-write 10 addr bm data s2 = None $\Longrightarrow$
memory-write 10 addr bm data s1 = None
⟨*proof*⟩

lemma *mem-context-val-9-unchanged*:
mem-context-val 9 addr1 (mem s1) =
mem-context-val 9 addr1
((*mem s1*)(*10* := *mem s1 10*(*addr* ↦ *val*), *11* := (*mem s1 11*)(*addr* := *None*)))
⟨*proof*⟩

lemma *mem-context-val-w32-9-unchanged*:
mem-context-val-w32 9 addr1 (mem s1) =
mem-context-val-w32 9 addr1
((*mem s1*)(*10* := *mem s1 10*(*addr* ↦ *val*), *11* := (*mem s1 11*)(*addr* := *None*)))
⟨*proof*⟩

lemma *ptd-lookup-unchanged-4*:
ptd-lookup va ptp (mem s1) 4 =
ptd-lookup va ptp ((*mem s1*)(*10* := *mem s1 10*(*addr* ↦ *val*),
11 := (*mem s1 11*)(*addr* := *None*))) *4*
⟨*proof*⟩

lemma *ptd-lookup-unchanged-3*:
ptd-lookup va ptp (mem s1) 3 =
ptd-lookup va ptp ((*mem s1*)(*10* := *mem s1 10*(*addr* ↦ *val*),
11 := (*mem s1 11*)(*addr* := *None*))) *3*
⟨*proof*⟩

lemma *ptd-lookup-unchanged-2*:
ptd-lookup va ptp (mem s1) 2 =
ptd-lookup va ptp ((*mem s1*)(*10* := *mem s1 10*(*addr* ↦ *val*),
11 := (*mem s1 11*)(*addr* := *None*))) *2*
⟨*proof*⟩

lemma *ptd-lookup-unchanged-1*:
ptd-lookup va ptp (mem s1) 1 =
ptd-lookup va ptp ((*mem s1*)(*10* := *mem s1 10*(*addr* ↦ *val*),
11 := (*mem s1 11*)(*addr* := *None*))) *1*
⟨*proof*⟩

lemma *virt-to-phys-unchanged-sub1*:
assumes *a1*: (*let context-table-entry* = (*v1* >> *11* << *11*) *OR* (*v2 AND 511* <<

2)
  in Let (*mem-context-val-w32* (*word-of-int 9*) (*ucast context-table-entry*) (*mem s1*))
   (*case-option None* (λ*lvl1-page-table. ptd-lookup va lvl1-page-table* (*mem s1*) *1*)))
=
 (*let context-table-entry* = (*v1* >> *11* << *11*) *OR* (*v2 AND 511* << *2*)
  in Let (*mem-context-val-w32* (*word-of-int 9*) (*ucast context-table-entry*) (*mem s2*))
   (*case-option None* (λ*lvl1-page-table. ptd-lookup va lvl1-page-table* (*mem s2*) *1*)))
shows (*let context-table-entry* = (*v1* >> *11* << *11*) *OR* (*v2 AND 511* << *2*)
  in Let (*mem-context-val-w32* (*word-of-int 9*) (*ucast context-table-entry*)
   ((*mem s1*)(*10* := *mem s1 10*(*addr* ↦ *val*), *11* := (*mem s1 11*)(*addr* := *None*))))
    (*case-option None* (λ*lvl1-page-table. ptd-lookup va lvl1-page-table*
      ((*mem s1*)(*10* := *mem s1 10*(*addr* ↦ *val*), *11* := (*mem s1 11*)(*addr* := *None*))) *1*))) =
 (*let context-table-entry* = (*v1* >> *11* << *11*) *OR* (*v2 AND 511* << *2*)
  in Let (*mem-context-val-w32* (*word-of-int 9*) (*ucast context-table-entry*)
   ((*mem s2*)(*10* := *mem s2 10*(*addr* ↦ *val*), *11* := (*mem s2 11*)(*addr* := *None*))))
    (*case-option None* (λ*lvl1-page-table. ptd-lookup va lvl1-page-table*
      ((*mem s2*)(*10* := *mem s2 10*(*addr* ↦ *val*), *11* := (*mem s2 11*)(*addr* := *None*))) *1*)))
⟨*proof*⟩

lemma *virt-to-phys-unchanged*:
assumes *a1*: (∀ *va. virt-to-phys va* (*mmu s2*) (*mem s1*) = *virt-to-phys va* (*mmu s2*) (*mem s2*))
shows (∀ *va. virt-to-phys va* (*mmu s2*) ((*mem s1*)(*10* := *mem s1 10*(*addr* ↦ *val*),
                                11 := (*mem s1 11*)(*addr* := *None*))) =
          virt-to-phys va (*mmu s2*) ((*mem s2*)(*10* := *mem s2 10*(*addr* ↦ *val*),
                                11 := (*mem s2 11*)(*addr* := *None*))))
⟨*proof*⟩

lemma *virt-to-phys-unchanged2-sub1*:
(*case mem-context-val-w32* (*word-of-int 9*)
  (*ucast* ((*v1* >> *11* << *11*) *OR* (*v2 AND 511* << *2*))) (*mem s2*) *of*
  None ⇒ *None* | *Some lvl1-page-table* ⇒ *ptd-lookup va lvl1-page-table* (*mem s2*) *1*) =
(*case mem-context-val-w32* (*word-of-int 9*)
  (*ucast* ((*v1* >> *11* << *11*) *OR* (*v2 AND 511* << *2*))) ((*mem s2*)
   (*10* := *mem s2 10*(*addr* ↦ *val*), *11* := (*mem s2 11*)(*addr* := *None*))) *of*
  None ⇒ *None* | *Some lvl1-page-table* ⇒ *ptd-lookup va lvl1-page-table* ((*mem s2*)
   (*10* := *mem s2 10*(*addr* ↦ *val*), *11* := (*mem s2 11*)(*addr* := *None*))) *1*)
⟨*proof*⟩

lemma *virt-to-phys-unchanged2*:
virt-to-phys va (*mmu s2*) (*mem s2*) =
virt-to-phys va (*mmu s2*) ((*mem s2*)(*10* := *mem s2 10*(*addr* ↦ *val*),
                                11 := (*mem s2 11*)(*addr* := *None*)))

⟨*proof*⟩

lemma *virt-to-phys-unchanged-low-equal*:
assumes *a1*: *low-equal s1 s2*
shows (∀ *va*. *virt-to-phys va* (*mmu s2*) ((*mem s1*)(*10* := *mem s1 10*(*addr* ↦ *val*),

11 := (*mem s1 11*)(*addr* := *None*))) =
virt-to-phys va (*mmu s2*) ((*mem s2*)(*10* := *mem s2 10*(*addr* ↦ *val*),
11 := (*mem s2 11*)(*addr* := *None*))))
⟨*proof*⟩

lemma *mmu-low-equal*: *low-equal s1 s2* ⟹ *mmu s1* = *mmu s2*
⟨*proof*⟩

lemma *mem-val-alt-8-unchanged0*:
assumes *a1*: *mem-equal s1 s2 pa*
shows *mem-val-alt 8* (*pa AND 68719476732*) (*s1*(|*mem* := (*mem s1*)(*10* := *mem s1 10*(*addr* ↦ *val*),
11 := (*mem s1 11*)(*addr* := *None*))|)) =
mem-val-alt 8 (*pa AND 68719476732*) (*s2*(|*mem* := (*mem s2*)(*10* := *mem s2 10*(*addr* ↦ *val*),
11 := (*mem s2 11*)(*addr* := *None*))|))
⟨*proof*⟩

lemma *mem-val-alt-8-unchanged1*:
assumes *a1*: *mem-equal s1 s2 pa*
shows *mem-val-alt 8* ((*pa AND 68719476732*) + *1*) (*s1*(|*mem* := (*mem s1*)(*10* := *mem s1 10*(*addr* ↦ *val*),
11 := (*mem s1 11*)(*addr* := *None*))|)) =
mem-val-alt 8 ((*pa AND 68719476732*) + *1*) (*s2*(|*mem* := (*mem s2*)(*10* := *mem s2 10*(*addr* ↦ *val*),
11 := (*mem s2 11*)(*addr* := *None*))|))
⟨*proof*⟩

lemma *mem-val-alt-8-unchanged2*:
assumes *a1*: *mem-equal s1 s2 pa*
shows *mem-val-alt 8* ((*pa AND 68719476732*) + *2*) (*s1*(|*mem* := (*mem s1*)(*10* := *mem s1 10*(*addr* ↦ *val*),
11 := (*mem s1 11*)(*addr* := *None*))|)) =
mem-val-alt 8 ((*pa AND 68719476732*) + *2*) (*s2*(|*mem* := (*mem s2*)(*10* := *mem s2 10*(*addr* ↦ *val*),
11 := (*mem s2 11*)(*addr* := *None*))|))
⟨*proof*⟩

lemma *mem-val-alt-8-unchanged3*:
assumes *a1*: *mem-equal s1 s2 pa*
shows *mem-val-alt 8* ((*pa AND 68719476732*) + *3*) (*s1*(|*mem* := (*mem s1*)(*10* := *mem s1 10*(*addr* ↦ *val*),
11 := (*mem s1 11*)(*addr* := *None*))|)) =

mem-val-alt 8 ((pa AND 68719476732) + 3) (s2⦇mem := (mem s2)(10 := mem s2 10(addr ↦ val),
11 := (mem s2 11)(addr := None))⦈)
⟨*proof*⟩

lemma *mem-val-alt-8-unchanged*:
assumes *a1*: *mem-equal s1 s2 pa*
shows *mem-val-alt 8 (pa AND 68719476732) (s1⦇mem := (mem s1)(10 := mem s1 10(addr ↦ val),*
11 := (mem s1 11)(addr := None))⦈) =
mem-val-alt 8 (pa AND 68719476732) (s2⦇mem := (mem s2)(10 := mem s2 10(addr ↦ val),
11 := (mem s2 11)(addr := None))⦈) ∧
mem-val-alt 8 ((pa AND 68719476732) + 1) (s1⦇mem := (mem s1)(10 := mem s1 10(addr ↦ val),
11 := (mem s1 11)(addr := None))⦈) =
mem-val-alt 8 ((pa AND 68719476732) + 1) (s2⦇mem := (mem s2)(10 := mem s2 10(addr ↦ val),
11 := (mem s2 11)(addr := None))⦈) ∧
mem-val-alt 8 ((pa AND 68719476732) + 2) (s1⦇mem := (mem s1)(10 := mem s1 10(addr ↦ val),
11 := (mem s1 11)(addr := None))⦈) =
mem-val-alt 8 ((pa AND 68719476732) + 2) (s2⦇mem := (mem s2)(10 := mem s2 10(addr ↦ val),
11 := (mem s2 11)(addr := None))⦈) ∧
mem-val-alt 8 ((pa AND 68719476732) + 3) (s1⦇mem := (mem s1)(10 := mem s1 10(addr ↦ val),
11 := (mem s1 11)(addr := None))⦈) =
mem-val-alt 8 ((pa AND 68719476732) + 3) (s2⦇mem := (mem s2)(10 := mem s2 10(addr ↦ val),
11 := (mem s2 11)(addr := None))⦈)
⟨*proof*⟩

lemma *mem-val-w32-8-unchanged*:
assumes *a1*: *mem-equal s1 s2 a*
shows *mem-val-w32 8 a (s1⦇mem := (mem s1)(10 := mem s1 10(addr ↦ val),*
11 := (mem s1 11)(addr := None))⦈) =
mem-val-w32 8 a (s2⦇mem := (mem s2)(10 := mem s2 10(addr ↦ val),
11 := (mem s2 11)(addr := None))⦈)
⟨*proof*⟩

lemma *load-word-mem-8-unchanged*:
assumes *a1*: *low-equal s1 s2 ∧*
load-word-mem s1 addra 8 = load-word-mem s2 addra 8
shows *load-word-mem (s1⦇mem := (mem s1)(10 := mem s1 10(addr ↦ val),*
11 := (mem s1 11)(addr := None))⦈) addra 8 =
load-word-mem (s2⦇mem := (mem s2)(10 := mem s2 10(addr ↦ val),
11 := (mem s2 11)(addr := None))⦈) addra 8
⟨*proof*⟩

lemma *load-word-mem-select-8*:
assumes *a1*: *fst* (*case load-word-mem s1 addra 8 of None* ⇒ (*None*, *s1*)
  | *Some w* ⇒ (*Some w*, *add-instr-cache s1 addra w 15*)) =
fst (*case load-word-mem s2 addra 8 of None* ⇒ (*None*, *s2*)
  | *Some w* ⇒ (*Some w*, *add-instr-cache s2 addra w 15*))
shows *load-word-mem s1 addra 8* = *load-word-mem s2 addra 8*
⟨*proof*⟩

lemma *memory-read-8-unchanged*:
assumes *a1*: *low-equal s1 s2* ∧
fst (*memory-read 8 addra s1*) = *fst* (*memory-read 8 addra s2*)
shows *fst* (*memory-read 8 addra*
        (*s1*(|*mem* := (*mem s1*)(*10* := *mem s1 10*(*addr* ↦ *val*),
          11 := (*mem s1 11*)(*addr* := *None*))|))) =
    fst (*memory-read 8 addra*
        (*s2*(|*mem* := (*mem s2*)(*10* := *mem s2 10*(*addr* ↦ *val*),
          11 := (*mem s2 11*)(*addr* := *None*))|)))
⟨*proof*⟩

lemma *mem-val-alt-mod*:
assumes *a1*: *addr1* ≠ *addr2*
shows *mem-val-alt 10 addr1 s* =
mem-val-alt 10 addr1 (*s*(|*mem* := (*mem s*)(*10* := *mem s 10*(*addr2* ↦ *val*),
  11 := (*mem s 11*)(*addr2* := *None*))|))
⟨*proof*⟩

lemma *mem-val-alt-mod2*:
mem-val-alt 10 addr (*s*(|*mem* := (*mem s*)(*10* := *mem s 10*(*addr* ↦ *val*),
  11 := (*mem s 11*)(*addr* := *None*))|)) = *Some val*
⟨*proof*⟩

lemma *mem-val-alt-10-unchanged0*:
assumes *a1*: *mem-equal s1 s2 pa*
shows *mem-val-alt 10* (*pa AND 68719476732*) (*s1*(|*mem* := (*mem s1*)(*10* := *mem s1 10*(*addr* ↦ *val*),
  11 := (*mem s1 11*)(*addr* := *None*))|)) =
  mem-val-alt 10 (*pa AND 68719476732*) (*s2*(|*mem* := (*mem s2*)(*10* := *mem s2 10*(*addr* ↦ *val*),
  11 := (*mem s2 11*)(*addr* := *None*))|))
⟨*proof*⟩

lemma *mem-val-alt-10-unchanged1*:
assumes *a1*: *mem-equal s1 s2 pa*
shows *mem-val-alt 10* ((*pa AND 68719476732*) + *1*) (*s1*(|*mem* := (*mem s1*)(*10* := *mem s1 10*(*addr* ↦ *val*),
  11 := (*mem s1 11*)(*addr* := *None*))|)) =
  mem-val-alt 10 ((*pa AND 68719476732*) + *1*) (*s2*(|*mem* := (*mem s2*)(*10* := *mem s2 10*(*addr* ↦ *val*),

11 := (*mem s2 11*)(*addr* := *None*))⦈)
⟨*proof*⟩

lemma *mem-val-alt-10-unchanged2*:
assumes *a1*: *mem-equal s1 s2 pa*
shows *mem-val-alt 10* ((*pa AND 68719476732*) + *2*) (*s1*⦇*mem* := (*mem s1*)(*10* := *mem s1 10*(*addr* ↦ *val*),
11 := (*mem s1 11*)(*addr* := *None*))⦈) =
mem-val-alt 10 ((*pa AND 68719476732*) + *2*) (*s2*⦇*mem* := (*mem s2*)(*10* := *mem s2 10*(*addr* ↦ *val*),
11 := (*mem s2 11*)(*addr* := *None*))⦈)
⟨*proof*⟩

lemma *mem-val-alt-10-unchanged3*:
assumes *a1*: *mem-equal s1 s2 pa*
shows *mem-val-alt 10* ((*pa AND 68719476732*) + *3*) (*s1*⦇*mem* := (*mem s1*)(*10* := *mem s1 10*(*addr* ↦ *val*),
11 := (*mem s1 11*)(*addr* := *None*))⦈) =
mem-val-alt 10 ((*pa AND 68719476732*) + *3*) (*s2*⦇*mem* := (*mem s2*)(*10* := *mem s2 10*(*addr* ↦ *val*),
11 := (*mem s2 11*)(*addr* := *None*))⦈)
⟨*proof*⟩

lemma *mem-val-alt-10-unchanged*:
assumes *a1*: *mem-equal s1 s2 pa*
shows *mem-val-alt 10* (*pa AND 68719476732*) (*s1*⦇*mem* := (*mem s1*)(*10* := *mem s1 10*(*addr* ↦ *val*),
11 := (*mem s1 11*)(*addr* := *None*))⦈) =
mem-val-alt 10 (*pa AND 68719476732*) (*s2*⦇*mem* := (*mem s2*)(*10* := *mem s2 10*(*addr* ↦ *val*),
11 := (*mem s2 11*)(*addr* := *None*))⦈) ∧
mem-val-alt 10 ((*pa AND 68719476732*) + *1*) (*s1*⦇*mem* := (*mem s1*)(*10* := *mem s1 10*(*addr* ↦ *val*),
11 := (*mem s1 11*)(*addr* := *None*))⦈) =
mem-val-alt 10 ((*pa AND 68719476732*) + *1*) (*s2*⦇*mem* := (*mem s2*)(*10* := *mem s2 10*(*addr* ↦ *val*),
11 := (*mem s2 11*)(*addr* := *None*))⦈) ∧
mem-val-alt 10 ((*pa AND 68719476732*) + *2*) (*s1*⦇*mem* := (*mem s1*)(*10* := *mem s1 10*(*addr* ↦ *val*),
11 := (*mem s1 11*)(*addr* := *None*))⦈) =
mem-val-alt 10 ((*pa AND 68719476732*) + *2*) (*s2*⦇*mem* := (*mem s2*)(*10* := *mem s2 10*(*addr* ↦ *val*),
11 := (*mem s2 11*)(*addr* := *None*))⦈) ∧
mem-val-alt 10 ((*pa AND 68719476732*) + *3*) (*s1*⦇*mem* := (*mem s1*)(*10* := *mem s1 10*(*addr* ↦ *val*),
11 := (*mem s1 11*)(*addr* := *None*))⦈) =
mem-val-alt 10 ((*pa AND 68719476732*) + *3*) (*s2*⦇*mem* := (*mem s2*)(*10* := *mem s2 10*(*addr* ↦ *val*),
11 := (*mem s2 11*)(*addr* := *None*))⦈)

⟨*proof*⟩

lemma *mem-val-w32-10-unchanged*:
assumes *a1*: *mem-equal s1 s2 a*
shows *mem-val-w32 10 a* (*s1*(|*mem* := (*mem s1*)(*10* := *mem s1 10*(*addr* ↦ *val*),
11 := (*mem s1 11*)(*addr* := *None*))|)) =
mem-val-w32 10 a (*s2*(|*mem* := (*mem s2*)(*10* := *mem s2 10*(*addr* ↦ *val*),
11 := (*mem s2 11*)(*addr* := *None*))|))
⟨*proof*⟩

lemma *is-accessible*: *low-equal s1 s2* ⟹
virt-to-phys addra (*mmu s1*) (*mem s1*) = *Some* (*a*, *b*) ⟹
virt-to-phys addra (*mmu s2*) (*mem s2*) = *Some* (*a*, *b*) ⟹
mmu-readable (*get-acc-flag b*) *10* ⟹
mem-equal s1 s2 a
⟨*proof*⟩

lemma *load-word-mem-10-unchanged*:
assumes *a1*: *low-equal s1 s2* ∧
load-word-mem s1 addra 10 = *load-word-mem s2 addra 10*
shows *load-word-mem* (*s1*(|*mem* := (*mem s1*)(*10* := *mem s1 10*(*addr* ↦ *val*),
11 := (*mem s1 11*)(*addr* := *None*))|)) *addra 10* =
load-word-mem (*s2*(|*mem* := (*mem s2*)(*10* := *mem s2 10*(*addr* ↦ *val*),
11 := (*mem s2 11*)(*addr* := *None*))|)) *addra 10*
⟨*proof*⟩

lemma *load-word-mem-select-10*:
assumes *a1*: *fst* (*case load-word-mem s1 addra 10 of None* ⇒ (*None*, *s1*)
| *Some w* ⇒ (*Some w*, *add-data-cache s1 addra w 15*)) =
fst (*case load-word-mem s2 addra 10 of None* ⇒ (*None*, *s2*)
| *Some w* ⇒ (*Some w*, *add-data-cache s2 addra w 15*))
shows *load-word-mem s1 addra 10* = *load-word-mem s2 addra 10*
⟨*proof*⟩

lemma *memory-read-10-unchanged*:
assumes *a1*: *low-equal s1 s2* ∧
fst (*memory-read 10 addra s1*) = *fst* (*memory-read 10 addra s2*)
shows *fst* (*memory-read 10 addra*
(*s1*(|*mem* := (*mem s1*)(*10* := *mem s1 10*(*addr* ↦ *val*),
11 := (*mem s1 11*)(*addr* := *None*))|))) =
fst (*memory-read 10 addra*
(*s2*(|*mem* := (*mem s2*)(*10* := *mem s2 10*(*addr* ↦ *val*),
11 := (*mem s2 11*)(*addr* := *None*))|)))
⟨*proof*⟩

lemma *state-mem-mod-1011-low-equal-sub1*:
assumes *a1*: (∀ *va*. *virt-to-phys va* (*mmu s2*) (*mem s1*) =
virt-to-phys va (*mmu s2*) (*mem s2*)) ∧
(∀ *pa*. (∃ *va b*. *virt-to-phys va* (*mmu s2*) (*mem s2*) = *Some* (*pa*, *b*) ∧

mmu-readable (*get-acc-flag b*) *10*) ⟶
mem-equal s1 s2 pa) ∧
mmu s1 = *mmu s2* ∧
virt-to-phys va (*mmu s2*)
((*mem s1*)(*10* := *mem s1 10*(*addr* ↦ *val*), *11* := (*mem s1 11*)(*addr* := *None*)))
=
Some (*pa*, *b*) ∧
mmu-readable (*get-acc-flag b*) *10*
shows *mem-equal s1 s2 pa*
⟨*proof*⟩

lemma *mem-equal-unchanged*:
assumes *a1*: *mem-equal s1 s2 pa*
shows *mem-equal* (*s1*(|*mem* := (*mem s1*)(*10* := *mem s1 10*(*addr* ↦ *val*),
11 := (*mem s1 11*)(*addr* := *None*))|))
(*s2*(|*mem* := (*mem s2*)(*10* := *mem s2 10*(*addr* ↦ *val*),
11 := (*mem s2 11*)(*addr* := *None*))|))
pa
⟨*proof*⟩

lemma *state-mem-mod-1011-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *s1*(|*mem* := (*mem s1*)(*10* := *mem s1 10*(*addr* ↦ *val*), *11* := (*mem s1 11*)(*addr* := *None*))|) ∧
t2 = *s2*(|*mem* := (*mem s2*)(*10* := *mem s2 10*(*addr* ↦ *val*), *11* := (*mem s2 11*)(*addr* := *None*))|)
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *mem-mod-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = (*mem-mod 10 addr val s1*) ∧
t2 = (*mem-mod 10 addr val s2*)
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *mem-mod-w32-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *mem-mod-w32 10 a bm data s1* ∧
t2 = *mem-mod-w32 10 a bm data s2*
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *store-word-mem-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
Some t1 = *store-word-mem s1 addr data bm 10* ∧
Some t2 = *store-word-mem s2 addr data bm 10*
shows *low-equal t1 t2* ⟨*proof*⟩

lemma *memory-write-asi-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
Some t1 = *memory-write-asi 10 addr bm data s1* ∧
Some t2 = *memory-write-asi 10 addr bm data s2*
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *store-barrier-pending-mod-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *store-barrier-pending-mod False s1* ∧
t2 = *store-barrier-pending-mod False s2*
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *memory-write-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
Some t1 = *memory-write 10 addr bm data s1* ∧
Some t2 = *memory-write 10 addr bm data s2*
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *memory-write-low-equal2*:
assumes *a1*: *low-equal s1 s2* ∧
Some t1 = *memory-write 10 addr bm data s1*
shows ∃ *t2*. *Some t2* = *memory-write 10 addr bm data s2*
⟨*proof*⟩

lemma *store-sub2-low-equal-sub1*:
assumes *a1*: *low-equal s1 s2* ∧
memory-write 10 addr (*st-byte-mask instr addr*)
(*st-data0 instr curr-win rd addr s2*) *s1* = *Some y* ∧
memory-write 10 addr (*st-byte-mask instr addr*)
(*st-data0 instr curr-win rd addr s2*) *s2* = *Some ya*
shows *low-equal* (*y*(|*traps* := *insert data-access-exception* (*traps y*)|))
(*ya*(|*traps* := *insert data-access-exception* (*traps ya*)|))
⟨*proof*⟩

lemma *store-sub2-low-equal-sub2*:
assumes *a1*: *low-equal s1 s2* ∧
memory-write 10 addr (*st-byte-mask instr addr*)
(*st-data0 instr curr-win rd addr s2*) *s1* = *Some y* ∧
memory-write 10 addr (*st-byte-mask instr addr*)
(*st-data0 instr curr-win rd addr s2*) *s2* = *Some ya* ∧
memory-write 10 (*addr* + *4*) *15* (*user-reg-val curr-win* (*rd OR 1*) *y*) *y* = *None* ∧
memory-write 10 (*addr* + *4*) *15* (*user-reg-val curr-win* (*rd OR 1*) *ya*) *ya* = *Some yb*
shows *False*
⟨*proof*⟩

lemma *store-sub2-low-equal-sub3*:
assumes *a1*: *low-equal s1 s2* ∧
memory-write 10 addr (*st-byte-mask instr addr*)
(*st-data0 instr curr-win rd addr s2*) *s1* = *Some y* ∧
memory-write 10 addr (*st-byte-mask instr addr*)
(*st-data0 instr curr-win rd addr s2*) *s2* = *Some ya* ∧
memory-write 10 (*addr* + *4*) *15* (*user-reg-val curr-win* (*rd OR 1*) *y*) *y* = *Some yb*
∧
memory-write 10 (*addr* + *4*) *15* (*user-reg-val curr-win* (*rd OR 1*) *ya*) *ya* = *None*
shows *False*
⟨*proof*⟩

lemma *store-sub2-low-equal-sub4*:
assumes *a1*: *low-equal s1 s2* ∧
memory-write 10 addr (*st-byte-mask instr addr*)
(*st-data0 instr curr-win rd addr s2*) *s1* = *Some y* ∧
memory-write 10 addr (*st-byte-mask instr addr*)
(*st-data0 instr curr-win rd addr s2*) *s2* = *Some ya* ∧
memory-write 10 (*addr* + *4*) *15* (*user-reg-val curr-win* (*rd OR 1*) *y*) *y* = *Some yb*
∧
memory-write 10 (*addr* + *4*) *15* (*user-reg-val curr-win* (*rd OR 1*) *ya*) *ya* = *Some*
yc
shows *low-equal yb yc*
⟨*proof*⟩

lemma *store-sub2-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *snd* (*fst* (*store-sub2 instr curr-win rd 10 addr s1*)) ∧
t2 = *snd* (*fst* (*store-sub2 instr curr-win rd 10 addr s2*))
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *store-sub1-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
(*fst instr* = *load-store-type STB* ∨
fst instr = *load-store-type STH* ∨
fst instr = *load-store-type ST* ∨
fst instr = *load-store-type STD*) ∧
t1 = *snd* (*fst* (*store-sub1 instr rd 0 s1*)) ∧
t2 = *snd* (*fst* (*store-sub1 instr rd 0 s2*))
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *store-instr-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
(*fst instr* = *load-store-type STB* ∨
fst instr = *load-store-type STH* ∨
fst instr = *load-store-type ST* ∨
fst instr = *load-store-type STA* ∨

fst instr = load-store-type STD) ∧
(((*get-S* (*cpu-reg-val PSR s1*)))::*word1*) = *0* ∧
(((*get-S* (*cpu-reg-val PSR s2*)))::*word1*) = *0* ∧
t1 = *snd* (*fst* (*store-instr instr s1*)) ∧ *t2* = *snd* (*fst* (*store-instr instr s2*))
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *sethi-low-equal*: *low-equal s1 s2* ∧
t1 = *snd* (*fst* (*sethi-instr instr s1*)) ∧ *t2* = *snd* (*fst* (*sethi-instr instr s2*)) ⟹
low-equal t1 t2
⟨*proof*⟩

lemma *nop-low-equal*: *low-equal s1 s2* ∧
t1 = *snd* (*fst* (*nop-instr instr s1*)) ∧ *t2* = *snd* (*fst* (*nop-instr instr s2*)) ⟹
low-equal t1 t2
⟨*proof*⟩

lemma *logical-instr-sub1-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *snd* (*fst* (*logical-instr-sub1 instr-name result s1*)) ∧
t2 = *snd* (*fst* (*logical-instr-sub1 instr-name result s2*))
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *logical-instr-low-equal*: *low-equal s1 s2* ∧
t1 = *snd* (*fst* (*logical-instr instr s1*)) ∧ *t2* = *snd* (*fst* (*logical-instr instr s2*)) ⟹
low-equal t1 t2
⟨*proof*⟩

lemma *shift-instr-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *snd* (*fst* (*shift-instr instr s1*)) ∧ *t2* = *snd* (*fst* (*shift-instr instr s2*))
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *add-instr-sub1-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *snd* (*fst* (*add-instr-sub1 instr-name result rs1-val operand2 s1*)) ∧
t2 = *snd* (*fst* (*add-instr-sub1 instr-name result rs1-val operand2 s2*))
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *add-instr-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *snd* (*fst* (*add-instr instr s1*)) ∧ *t2* = *snd* (*fst* (*add-instr instr s2*))
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *sub-instr-sub1-low-equal*:

assumes *a1*: *low-equal s1 s2* ∧
t1 = *snd* (*fst* (*sub-instr-sub1 instr-name result rs1-val operand2 s1*)) ∧
t2 = *snd* (*fst* (*sub-instr-sub1 instr-name result rs1-val operand2 s2*))
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *sub-instr-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *snd* (*fst* (*sub-instr instr s1*)) ∧ *t2* = *snd* (*fst* (*sub-instr instr s2*))
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *mul-instr-sub1-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *snd* (*fst* (*mul-instr-sub1 instr-name result s1*)) ∧
t2 = *snd* (*fst* (*mul-instr-sub1 instr-name result s2*))
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *mul-instr-low-equal*:
‹*low-equal t1 t2*›
if ‹*low-equal s1 s2* ∧ *t1* = *snd* (*fst* (*mul-instr instr s1*)) ∧ *t2* = *snd* (*fst* (*mul-instr instr s2*))›
⟨*proof*⟩

lemma *div-write-new-val-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *snd* (*fst* (*div-write-new-val i result temp-V s1*)) ∧
t2 = *snd* (*fst* (*div-write-new-val i result temp-V s2*))
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *div-comp-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *snd* (*fst* (*div-comp instr rs1 rd operand2 s1*)) ∧
t2 = *snd* (*fst* (*div-comp instr rs1 rd operand2 s2*))
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *div-instr-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *snd* (*fst* (*div-instr instr s1*)) ∧ *t2* = *snd* (*fst* (*div-instr instr s2*))
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *get-curr-win-traps-low-equal*:
assumes *a1*: *low-equal s1 s2*
shows *low-equal*
(*snd* (*fst* (*get-curr-win* () *s1*))

(|traps := insert some-trap (traps (snd (fst (get-curr-win () s1))))|)
(snd (fst (get-curr-win () s2))
(|traps := insert some-trap (traps (snd (fst (get-curr-win () s2))))|)
⟨proof⟩

lemma *save-restore-instr-sub1-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *snd* (*fst* (*save-retore-sub1 result new-cwp rd s1*)) ∧
t2 = *snd* (*fst* (*save-retore-sub1 result new-cwp rd s2*))
shows *low-equal t1 t2*
⟨proof⟩

lemma *get-WIM-bit-low-equal*:
‹*get-WIM-bit* (*nat* ((*uint* (*fst* (*fst* (*get-curr-win* () *s1*))) − *1*) *mod NWINDOWS*))
(*cpu-reg-val WIM* (*snd* (*fst* (*get-curr-win* () *s1*)))) =
get-WIM-bit (*nat* ((*uint* (*fst* (*fst* (*get-curr-win* () *s2*))) − *1*) *mod NWINDOWS*))
(*cpu-reg-val WIM* (*snd* (*fst* (*get-curr-win* () *s2*))))›
if ‹*low-equal s1 s2*›
⟨proof⟩

lemma *get-WIM-bit-low-equal2*:
‹*get-WIM-bit* (*nat* ((*uint* (*fst* (*fst* (*get-curr-win* () *s1*))) + *1*) *mod NWINDOWS*))
(*cpu-reg-val WIM* (*snd* (*fst* (*get-curr-win* () *s1*)))) =
get-WIM-bit (*nat* ((*uint* (*fst* (*fst* (*get-curr-win* () *s2*))) + *1*) *mod NWINDOWS*))
(*cpu-reg-val WIM* (*snd* (*fst* (*get-curr-win* () *s2*))))›
if ‹*low-equal s1 s2*›
⟨proof⟩

lemma *take-bit-5-mod-NWINDOWS-eq* [*simp*]:
‹*take-bit 5* (*k mod NWINDOWS*) = *k mod NWINDOWS*›
⟨proof⟩

lemma *save-restore-instr-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *snd* (*fst* (*save-restore-instr instr s1*)) ∧ *t2* = *snd* (*fst* (*save-restore-instr instr s2*))
shows *low-equal t1 t2*
⟨proof⟩

lemma *call-instr-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *snd* (*fst* (*call-instr instr s1*)) ∧ *t2* = *snd* (*fst* (*call-instr instr s2*))
shows *low-equal t1 t2*
⟨proof⟩

lemma *jmpl-instr-low-equal-sub1*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *snd* (*fst* (*write-cpu* (*get-addr* (*snd instr*) (*snd* (*fst* (*get-curr-win* () *s2*)))))
nPC

(*snd* (*fst* (*write-cpu* (*cpu-reg-val nPC*
(*snd* (*fst* (*write-reg* (*cpu-reg-val PC* (*snd* (*fst* (*get-curr-win* () *s1*))))
(*fst* (*fst* (*get-curr-win* () *s2*))) (*get-operand-w5* (*snd instr* ! *3*))
(*snd* (*fst* (*get-curr-win* () *s1*)))))))
PC (*snd* (*fst* (*write-reg* (*cpu-reg-val PC* (*snd* (*fst* (*get-curr-win* () *s1*))))
(*fst* (*fst* (*get-curr-win* () *s2*))) (*get-operand-w5* (*snd instr* ! *3*))
(*snd* (*fst* (*get-curr-win* () *s1*)))))))))) ∧
t2 = *snd* (*fst* (*write-cpu* (*get-addr* (*snd instr*) (*snd* (*fst* (*get-curr-win* () *s2*))))
nPC
(*snd* (*fst* (*write-cpu* (*cpu-reg-val nPC*
(*snd* (*fst* (*write-reg* (*cpu-reg-val PC* (*snd* (*fst* (*get-curr-win* () *s2*))))
(*fst* (*fst* (*get-curr-win* () *s2*))) (*get-operand-w5* (*snd instr* ! *3*))
(*snd* (*fst* (*get-curr-win* () *s2*)))))))
PC (*snd* (*fst* (*write-reg* (*cpu-reg-val PC* (*snd* (*fst* (*get-curr-win* () *s2*))))
(*fst* (*fst* (*get-curr-win* () *s2*))) (*get-operand-w5* (*snd instr* ! *3*))
(*snd* (*fst* (*get-curr-win* () *s2*)))))))))))
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *jmpl-instr-low-equal-sub2*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *snd* (*fst* (*write-cpu* (*get-addr* (*snd instr*) (*snd* (*fst* (*get-curr-win* () *s2*))))
nPC
(*snd* (*fst* (*write-cpu* (*cpu-reg-val nPC*
(*snd* (*fst* (*write-reg* (*user-reg-val* (*fst* (*fst* (*get-curr-win* () *s2*))) *0*
(*snd* (*fst* (*get-curr-win* () *s1*)))) (*fst* (*fst* (*get-curr-win* () *s2*))) *0*
(*snd* (*fst* (*get-curr-win* () *s1*)))))))) *PC* (*snd* (*fst* (*write-reg*
(*user-reg-val* (*fst* (*fst* (*get-curr-win* () *s2*))) *0*
(*snd* (*fst* (*get-curr-win* () *s1*)))) (*fst* (*fst* (*get-curr-win* () *s2*))) *0*
(*snd* (*fst* (*get-curr-win* () *s1*)))))))))))) ∧
t2 = *snd* (*fst* (*write-cpu* (*get-addr* (*snd instr*) (*snd* (*fst* (*get-curr-win* () *s2*))))
nPC
(*snd* (*fst* (*write-cpu* (*cpu-reg-val nPC* (*snd* (*fst* (*write-reg*
(*user-reg-val* (*fst* (*fst* (*get-curr-win* () *s2*))) *0*
(*snd* (*fst* (*get-curr-win* () *s2*)))) (*fst* (*fst* (*get-curr-win* () *s2*))) *0*
(*snd* (*fst* (*get-curr-win* () *s2*)))))))) *PC* (*snd* (*fst* (*write-reg*
(*user-reg-val* (*fst* (*fst* (*get-curr-win* () *s2*))) *0*
(*snd* (*fst* (*get-curr-win* () *s2*)))) (*fst* (*fst* (*get-curr-win* () *s2*))) *0*
(*snd* (*fst* (*get-curr-win* () *s2*)))))))))))
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *jmpl-instr-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *snd* (*fst* (*jmpl-instr instr s1*)) ∧ *t2* = *snd* (*fst* (*jmpl-instr instr s2*))
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *rett-instr-low-equal*:

assumes *a1*: *low-equal s1 s2* ∧
¬ *snd* (*rett-instr instr s1*) ∧
¬ *snd* (*rett-instr instr s2*) ∧
(((*get-S* (*cpu-reg-val PSR s1*)))::*word1*) = *0* ∧
(((*get-S* (*cpu-reg-val PSR s2*)))::*word1*) = *0* ∧
t1 = *snd* (*fst* (*rett-instr instr s1*)) ∧ *t2* = *snd* (*fst* (*rett-instr instr s2*))
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *read-state-reg-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
(((*get-S* (*cpu-reg-val PSR s1*)))::*word1*) = *0* ∧
(((*get-S* (*cpu-reg-val PSR s2*)))::*word1*) = *0* ∧
t1 = *snd* (*fst* (*read-state-reg-instr instr s1*)) ∧
t2 = *snd* (*fst* (*read-state-reg-instr instr s2*))
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *get-s-get-curr-win*:
assumes *a1*: *low-equal s1 s2*
shows *get-S* (*cpu-reg-val PSR* (*snd* (*fst* (*get-curr-win* () *s1*)))) =
get-S (*cpu-reg-val PSR* (*snd* (*fst* (*get-curr-win* () *s2*))))
⟨*proof*⟩

lemma *write-state-reg-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
(((*get-S* (*cpu-reg-val PSR s1*)))::*word1*) = *0* ∧
(((*get-S* (*cpu-reg-val PSR s2*)))::*word1*) = *0* ∧
t1 = *snd* (*fst* (*write-state-reg-instr instr s1*)) ∧
t2 = *snd* (*fst* (*write-state-reg-instr instr s2*))
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *flush-instr-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *snd* (*fst* (*flush-instr instr s1*)) ∧
t2 = *snd* (*fst* (*flush-instr instr s2*))
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *branch-instr-sub1-low-equal*:
assumes *a1*: *low-equal s1 s2*
shows *branch-instr-sub1 instr-name s1* = *branch-instr-sub1 instr-name s2*
⟨*proof*⟩

lemma *set-annul-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *snd* (*fst* (*set-annul True s1*)) ∧
t2 = *snd* (*fst* (*set-annul True s2*))

shows *low-equal t1 t2*
⟨*proof*⟩

lemma *branch-instr-low-equal-sub0*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *snd* (*fst* (*write-cpu* (*cpu-reg-val PC s2* +
sign-ext24 (*ucast* (*get-operand-w22* (*snd instr* ! *Suc 0*)) << *2*))
nPC (*snd* (*fst* (*write-cpu* (*cpu-reg-val nPC s2*) *PC s1*))))) ∧
t2 = *snd* (*fst* (*write-cpu* (*cpu-reg-val PC s2* +
sign-ext24 (*ucast* (*get-operand-w22* (*snd instr* ! *Suc 0*)) << *2*))
nPC (*snd* (*fst* (*write-cpu* (*cpu-reg-val nPC s2*) *PC s2*)))))
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *branch-instr-low-equal-sub1*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *snd* (*fst* (*set-annul True* (*snd* (*fst* (*write-cpu*
(*cpu-reg-val PC s2* + *sign-ext24*
(*ucast* (*get-operand-w22* (*snd instr* ! *Suc 0*)) << *2*))
nPC (*snd* (*fst* (*write-cpu* (*cpu-reg-val nPC s2*) *PC s1*)))))))) ∧
t2 = *snd* (*fst* (*set-annul True* (*snd* (*fst* (*write-cpu*
(*cpu-reg-val PC s2* + *sign-ext24*
(*ucast* (*get-operand-w22* (*snd instr* ! *Suc 0*)) << *2*))
nPC (*snd* (*fst* (*write-cpu* (*cpu-reg-val nPC s2*) *PC s2*)))))))
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *branch-instr-low-equal-sub2*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *snd* (*fst* (*set-annul True*
(*snd* (*fst* (*write-cpu* (*cpu-reg-val nPC s2* + *4*) *nPC*
(*snd* (*fst* (*write-cpu* (*cpu-reg-val nPC s2*) *PC s1*)))))))) ∧
t2 = *snd* (*fst* (*set-annul True*
(*snd* (*fst* (*write-cpu* (*cpu-reg-val nPC s2* + *4*) *nPC*
(*snd* (*fst* (*write-cpu* (*cpu-reg-val nPC s2*) *PC s2*)))))))
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *branch-instr-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
t1 = *snd* (*fst* (*branch-instr instr s1*)) ∧
t2 = *snd* (*fst* (*branch-instr instr s2*))
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *dispath-instr-low-equal*:
assumes *a1*: *low-equal s1 s2* ∧
(((*get-S* (*cpu-reg-val PSR s1*)))::*word1*) = *0* ∧
(((*get-S* (*cpu-reg-val PSR s2*)))::*word1*) = *0* ∧

$\neg$ *snd* (*dispatch-instruction instr s1*) $\wedge$
$\neg$ *snd* (*dispatch-instruction instr s2*) $\wedge$
t1 = (*snd* (*fst* (*dispatch-instruction instr s1*))) $\wedge$
t2 = (*snd* (*fst* (*dispatch-instruction instr s2*)))
shows *low-equal t1 t2*
⟨*proof*⟩

lemma *execute-instr-sub1-low-equal*:
assumes *a1*: *low-equal s1 s2* $\wedge$
$\neg$ *snd* (*execute-instr-sub1 instr s1*) $\wedge$
$\neg$ *snd* (*execute-instr-sub1 instr s2*) $\wedge$
t1 = (*snd* (*fst* (*execute-instr-sub1 instr s1*))) $\wedge$
t2 = (*snd* (*fst* (*execute-instr-sub1 instr s2*)))
shows *low-equal t1 t2*
⟨*proof*⟩

theorem *non-interference-step*:
assumes *a1*: (((*get-S* (*cpu-reg-val PSR s1*)))::*word1*) = *0* $\wedge$
good-context s1 $\wedge$
get-delayed-pool s1 = [] $\wedge$ *get-trap-set s1* = {} $\wedge$
(((*get-S* (*cpu-reg-val PSR s2*)))::*word1*) = *0* $\wedge$
get-delayed-pool s2 = [] $\wedge$ *get-trap-set s2* = {} $\wedge$
good-context s2 $\wedge$
low-equal s1 s2
shows $\exists$ *t1 t2*. *Some t1* = *NEXT s1* $\wedge$ *Some t2* = *NEXT s2* $\wedge$
(((*get-S* (*cpu-reg-val PSR t1*)))::*word1*) = *0* $\wedge$
(((*get-S* (*cpu-reg-val PSR t2*)))::*word1*) = *0* $\wedge$
low-equal t1 t2
⟨*proof*⟩

function (*sequential*) *SEQ*:: *nat* $\Rightarrow$ ('*a*::*len*) *sparc-state* $\Rightarrow$ ('*a*) *sparc-state option*
where *SEQ 0 s* = *Some s*
|*SEQ n s* = (
  case SEQ (*n*−*1*) *s of None* $\Rightarrow$ *None*
  | *Some t* $\Rightarrow$ *NEXT t*
)
⟨*proof*⟩
termination ⟨*proof*⟩

lemma *SEQ-suc*: *SEQ n s* = *Some t* $\Longrightarrow$ *SEQ* (*Suc n*) *s* = *NEXT t*
⟨*proof*⟩

definition *user-seq-exe*:: *nat* $\Rightarrow$ ('*a*::*len*) *sparc-state* $\Rightarrow$ *bool* **where**
user-seq-exe n s $\equiv$ $\forall$ *i t*. (*i* $\leq$ *n* $\wedge$ *SEQ i s* = *Some t*) $\longrightarrow$
  (*good-context t* $\wedge$ *get-delayed-pool t* = [] $\wedge$ *get-trap-set t* = {})

NIA is short for non-interference assumption.

definition *NIA t1 t2* $\equiv$
    (((*get-S* (*cpu-reg-val PSR t1*)))::*word1*) = *0* $\wedge$

$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ t2)))::word1) = 0\ \wedge$
$good\text{-}context\ t1\ \wedge\ get\text{-}delayed\text{-}pool\ t1 = []\ \wedge\ get\text{-}trap\text{-}set\ t1 = \{\}\ \wedge$
$good\text{-}context\ t2\ \wedge\ get\text{-}delayed\text{-}pool\ t2 = []\ \wedge\ get\text{-}trap\text{-}set\ t2 = \{\}\ \wedge$
$low\text{-}equal\ t1\ t2$

NIC is short for non-interference conclusion.

definition $NIC\ t1\ t2 \equiv (\exists\, u1\ u2.\ Some\ u1 = NEXT\ t1\ \wedge\ Some\ u2 = NEXT\ t2$
$\wedge$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ u1)))::word1) = 0\ \wedge$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ u2)))::word1) = 0\ \wedge$
$low\text{-}equal\ u1\ u2)$

lemma *NIS-short*: $\forall\, t1\ t2.\ NIA\ t1\ t2 \longrightarrow NIC\ t1\ t2$
⟨*proof*⟩

lemma *non-interference-induct-case-sub1*:
assumes *a1*: $(\exists\, t1.\ Some\ t1 = SEQ\ n\ s1\ \wedge$
$(\exists\, t2.\ Some\ t2 = SEQ\ n\ s2\ \wedge$
$NIA\ t1\ t2))$
shows $(\exists\, t1.\ Some\ t1 = SEQ\ n\ s1\ \wedge$
$(\exists\, t2.\ Some\ t2 = SEQ\ n\ s2\ \wedge$
$NIA\ t1\ t2\ \wedge$
$NIC\ t1\ t2))$
⟨*proof*⟩

lemma *non-interference-induct-case*:
assumes *a1*:
$((\forall\, i\ t.\ i \leq n\ \wedge\ SEQ\ i\ s1 = Some\ t \longrightarrow$
$good\text{-}context\ t\ \wedge\ get\text{-}delayed\text{-}pool\ t = []\ \wedge\ get\text{-}trap\text{-}set\ t = \{\})\ \wedge$
$(\forall\, i\ t.\ i \leq n\ \wedge\ SEQ\ i\ s2 = Some\ t \longrightarrow$
$good\text{-}context\ t\ \wedge\ get\text{-}delayed\text{-}pool\ t = []\ \wedge\ get\text{-}trap\text{-}set\ t = \{\}) \longrightarrow$
$(\exists\, t1.\ Some\ t1 = SEQ\ n\ s1\ \wedge$
$(\exists\, t2.\ Some\ t2 = SEQ\ n\ s2\ \wedge$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ t1)))::word1) = 0\ \wedge$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ t2)))::word1) = 0\ \wedge\ low\text{-}equal\ t1\ t2)))\ \wedge$
$(\forall\, i\ t.\ i \leq Suc\ n\ \wedge\ SEQ\ i\ s1 = Some\ t \longrightarrow$
$good\text{-}context\ t\ \wedge\ get\text{-}delayed\text{-}pool\ t = []\ \wedge\ get\text{-}trap\text{-}set\ t = \{\})\ \wedge$
$(\forall\, i\ t.\ i \leq Suc\ n\ \wedge\ SEQ\ i\ s2 = Some\ t \longrightarrow$
$good\text{-}context\ t\ \wedge\ get\text{-}delayed\text{-}pool\ t = []\ \wedge\ get\text{-}trap\text{-}set\ t = \{\})$
shows $\exists\, t1.\ Some\ t1 = (case\ SEQ\ n\ s1\ of\ None \Rightarrow None \mid Some\ x \Rightarrow NEXT\ x)\ \wedge$
$(\exists\, t2.\ Some\ t2 = (case\ SEQ\ n\ s2\ of\ None \Rightarrow None \mid Some\ x \Rightarrow NEXT$
$x)\ \wedge$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ t1)))::word1) = 0\ \wedge$
$(((get\text{-}S\ (cpu\text{-}reg\text{-}val\ PSR\ t2)))::word1) = 0\ \wedge\ low\text{-}equal\ t1\ t2)$
⟨*proof*⟩

lemma *non-interference-induct-case-sub2*:
assumes *a1*:
$(user\text{-}seq\text{-}exe\ n\ s1\ \wedge$

user-seq-exe n s2 $\longrightarrow$
($\exists$ *t1*. *Some t1* = *SEQ n s1* $\wedge$
($\exists$ *t2*. *Some t2* = *SEQ n s2* $\wedge$
(((*get-S* (*cpu-reg-val PSR t1*)))::*word1*) = *0* $\wedge$
(((*get-S* (*cpu-reg-val PSR t2*)))::*word1*) = *0* $\wedge$ *low-equal t1 t2*))) $\wedge$
user-seq-exe (*Suc n*) *s1* $\wedge$
user-seq-exe (*Suc n*) *s2*
shows $\exists$ *t1*. *Some t1* = (*case SEQ n s1 of None* $\Rightarrow$ *None* | *Some x* $\Rightarrow$ *NEXT x*) $\wedge$
($\exists$ *t2*. *Some t2* = (*case SEQ n s2 of None* $\Rightarrow$ *None* | *Some x* $\Rightarrow$ *NEXT x*) $\wedge$
(((*get-S* (*cpu-reg-val PSR t1*)))::*word1*) = *0* $\wedge$
(((*get-S* (*cpu-reg-val PSR t2*)))::*word1*) = *0* $\wedge$ *low-equal t1 t2*)
⟨*proof*⟩

theorem *non-interference*:
assumes *a1*:
(((*get-S* (*cpu-reg-val PSR s1*)))::*word1*) = *0* $\wedge$
good-context s1 $\wedge$
get-delayed-pool s1 = [] $\wedge$ *get-trap-set s1* = {} $\wedge$
(((*get-S* (*cpu-reg-val PSR s2*)))::*word1*) = *0* $\wedge$
get-delayed-pool s2 = [] $\wedge$ *get-trap-set s2* = {} $\wedge$
good-context s2 $\wedge$
user-seq-exe n s1 $\wedge$ *user-seq-exe n s2* $\wedge$
low-equal s1 s2
shows ($\exists$ *t1 t2*. *Some t1* = *SEQ n s1* $\wedge$ *Some t2* = *SEQ n s2* $\wedge$
(((*get-S* (*cpu-reg-val PSR t1*)))::*word1*) = *0* $\wedge$
(((*get-S* (*cpu-reg-val PSR t2*)))::*word1*) = *0* $\wedge$
low-equal t1 t2)
⟨*proof*⟩

end

end