

Routing

Julius Michaelis, Cornelius Diekmann

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Abstract

This entry contains definitions for routing with routing tables/longest prefix matching.

A routing table entry is modelled as a record of a prefix match, a metric, an output port, and an optional next hop. A routing table is a list of entries, sorted by prefix length and metric. Additionally, a parser and serializer for the output of the ip-route command, a function to create a relation from output port to corresponding destination IP space, and a model of a linux style router are included.

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Sorting a list by two keys

```
theory Linorder-Helper
imports Main
begin
```

Sorting is fun...

The problem is that Isabelle does not have anything like `sortBy`, only *sort-key*. This means that there is no way to sort something based on two properties, with one being infinitely more important.

Enter this:

```
datatype ('a,'b) linord-helper = LinordHelper 'a 'b

instantiation linord-helper :: (linorder, linorder) linorder
begin
  definition linord-helper-less-eq1 a b  $\equiv$  (case a of LinordHelper a1 a2  $\Rightarrow$  case b
of LinordHelper b1 b2  $\Rightarrow$  a1 < b1  $\vee$  a1 = b1  $\wedge$  a2  $\leq$  b2)
  definition a  $\leq$  b  $\longleftrightarrow$  linord-helper-less-eq1 a b
  definition a < b  $\longleftrightarrow$  (a  $\neq$  b  $\wedge$  linord-helper-less-eq1 a b)
instance
  <proof>
end
lemmas linord-helper-less = less-linord-helper-def linord-helper-less-eq1-def
lemmas linord-helper-le = less-eq-linord-helper-def linord-helper-less-eq1-def
```

Now, it is possible to use *sort-key* f , with f constructing a *LinordHelper* containing the two desired properties for sorting.

end

1 Routing Table

```
theory Routing-Table
imports IP-Addresses.Prefix-Match
         IP-Addresses.IPv4 IP-Addresses.IPv6
         Linorder-Helper
         IP-Addresses.Prefix-Match-toString
         Pure-ex.Guess
begin
```

This section makes the necessary definitions to work with a routing table using longest prefix matching.

1.1 Definition

```
record(overloaded) 'i routing-action =
  output-iface :: string
  next-hop :: 'i word option

record(overloaded) 'i routing-rule =
  routing-match :: ('i::len) prefix-match
  metric :: nat
  routing-action :: 'i routing-action
```

This definition is engineered to model routing tables on packet forwarding devices. It eludes, e.g., the source address hint, which is only relevant for packets originating from the device itself.

context
begin

definition *default-metric* = 0

type-synonym 'i *prefix-routing* = ('i *routing-rule*) list

abbreviation *routing-oiface* a \equiv *output-iface* (*routing-action* a)

abbreviation *routing-prefix* r \equiv *pfxm-length* (*routing-match* r)

definition *valid-prefixes* **where**

valid-prefixes r = *foldr* *conj* (*map* ($\lambda rr.$ *valid-prefix* (*routing-match* rr)) r) *True*

lemma *valid-prefixes-split*: *valid-prefixes* (r#rs) \implies *valid-prefix* (*routing-match* r) \wedge *valid-prefixes* rs
<proof>

lemma *foldr-True-set*: *foldr* ($\lambda x. (\wedge) (f\ x)$) l *True* = ($\forall x \in \text{set } l. f\ x$)
<proof>

lemma *valid-prefixes-alt-def*: *valid-prefixes* r = ($\forall e \in \text{set } r. \text{valid-prefix } (\text{routing-match } e)$)
<proof>

fun *has-default-route* :: ('i::len) *prefix-routing* \Rightarrow *bool* **where**
has-default-route (r#rs) = (((*pfxm-length* (*routing-match* r)) = 0) \vee *has-default-route* rs) |
has-default-route Nil = *False*

lemma *has-default-route-alt*: *has-default-route* rt \longleftrightarrow ($\exists r \in \text{set } rt. \text{pfxm-length } (\text{routing-match } r) = 0$) *<proof>*

1.2 Single Packet Semantics

fun *routing-table-semantics* :: ('i::len) *prefix-routing* \Rightarrow 'i *word* \Rightarrow 'i *routing-action*
where

routing-table-semantics [] - = *routing-action* (*undefined*::'i *routing-rule*) |

routing-table-semantics (r#rs) p = (if *prefix-match-semantics* (*routing-match* r) p then *routing-action* r else *routing-table-semantics* rs p)

lemma *routing-table-semantics-ports-from-table*: *valid-prefixes* rtbl \implies *has-default-route* rtbl \implies

routing-table-semantics rtbl packet = r \implies r \in *routing-action* ' set rtbl
<proof>

1.3 Longest Prefix Match

We can abuse *LinordHelper* to sort.

definition *routing-rule-sort-key* $\equiv \lambda r. \text{LinordHelper } (0 - (\text{of-nat} :: \text{nat} \Rightarrow \text{int})) (\text{pfxm-length } (\text{routing-match } r)) (\text{metric } r)$

There is actually a slight design choice here. We can choose to sort based on $(?a \leq ?b) = (\text{if } \text{pfxm-length } ?a = \text{pfxm-length } ?b \text{ then } \text{pfxm-prefix } ?a \leq \text{pfxm-prefix } ?b \text{ else } \text{pfxm-length } ?b < \text{pfxm-length } ?a)$ (thus including the address) or only the prefix length (excluding it). Which is taken does not matter gravely, since the bits of the prefix can't matter. They're either equal or the rules don't overlap and the metric decides. (It does matter for the resulting list though.) Ignoring the prefix and taking only its length is slightly easier.

definition *rr-ctor* $m \ l \ a \ nh \ me \equiv (\text{routing-match} = \text{PrefixMatch } (\text{ipv4addr-of-dotdecimal } m) \ l, \text{metric} = me, \text{routing-action} = (\text{output-iface} = a, \text{next-hop} = (\text{map-option } \text{ipv4addr-of-dotdecimal } nh)))$

value *sort-key* *routing-rule-sort-key* [
rr-ctor $(0,0,0,1) \ 3 \ \text{None } 0,$
rr-ctor $(0,0,0,2) \ 8 \ \text{None } 0,$
rr-ctor $(0,0,0,3) \ 4 \ \text{None } 13,$
rr-ctor $(0,0,0,3) \ 4 \ \text{None } 42]$

definition *is-longest-prefix-routing* $\equiv \text{sorted} \circ \text{map } \text{routing-rule-sort-key}$

definition *correct-routing* $:: ('i::\text{len}) \text{prefix-routing} \Rightarrow \text{bool}$ **where**
correct-routing $r \equiv \text{is-longest-prefix-routing } r \wedge \text{valid-prefixes } r$

Many proofs and functions around routing require at least parts of *correct-routing* as an assumption. Obviously, *correct-routing* is not given for arbitrary routing tables. Therefore, *correct-routing* is made to be executable and should be checked for any routing table after parsing. Note: *correct-routing* used to also require *has-default-route*, but none of the proofs require it anymore and it is not given for any routing table.

lemma *is-longest-prefix-routing-rule-exclusion*:

assumes *is-longest-prefix-routing* $(r1 \ \# \ rn \ \# \ rss)$

shows *is-longest-prefix-routing* $(r1 \ \# \ rss)$

<proof>

lemma *int-of-nat-less*: *int-of-nat* $a < \text{int-of-nat } b \implies a < b$ *<proof>*

lemma *is-longest-prefix-routing-sorted-by-length*:

assumes *is-longest-prefix-routing* r

and $r = r1 \ \# \ rs \ @ \ r2 \ \# \ rss$

shows $(\text{pfxm-length } (\text{routing-match } r1)) \geq \text{pfxm-length } (\text{routing-match } r2))$

<proof>

definition *sort-rtbl* $:: ('i::\text{len}) \text{routing-rule list} \Rightarrow 'i \text{routing-rule list} \equiv \text{sort-key } \text{routing-rule-sort-key}$

lemma *is-longest-prefix-routing-sort*: *is-longest-prefix-routing* (sort-rtbl r) <proof>

definition *unambiguous-routing* rtbl $\equiv (\forall rt1 rt2 rr ra. rtbl = rt1 @ rr \# rt2 \longrightarrow ra \in \text{set } (rt1 @ rt2) \longrightarrow \text{routing-match } rr = \text{routing-match } ra \longrightarrow \text{routing-rule-sort-key } rr \neq \text{routing-rule-sort-key } ra)$

lemma *unambiguous-routing-Cons*: *unambiguous-routing* (r # rtbl) \implies *unambiguous-routing* rtbl

<proof>

lemma *unambiguous-routing* (rr # rtbl) \implies *is-longest-prefix-routing* (rr # rtbl) \implies $ra \in \text{set } rtbl \implies \text{routing-match } rr = \text{routing-match } ra \implies \text{routing-rule-sort-key } rr < \text{routing-rule-sort-key } ra$

<proof>

primrec *unambiguous-routing-code* **where**

unambiguous-routing-code [] = True |

unambiguous-routing-code (rr#rtbl) = (list-all ($\lambda ra. \text{routing-match } rr \neq \text{routing-match } ra \vee \text{routing-rule-sort-key } rr \neq \text{routing-rule-sort-key } ra$) rtbl \wedge *unambiguous-routing-code* rtbl)

lemma *unambiguous-routing-code*[code-unfold]: *unambiguous-routing* rtbl \longleftrightarrow *unambiguous-routing-code* rtbl

<proof>

lemma *unambiguous-prefix-routing-weak-mono*:

assumes *lpfx*: *is-longest-prefix-routing* (rr#rtbl)

assumes *e*: $rr' \in \text{set } rtbl$

shows $\text{routing-rule-sort-key } rr' \geq \text{routing-rule-sort-key } rr$

<proof>

lemma *unambiguous-prefix-routing-strong-mono*:

assumes *lpfx*: *is-longest-prefix-routing* (rr#rtbl)

assumes *uam*: *unambiguous-routing* (rr#rtbl)

assumes *e*: $rr' \in \text{set } rtbl$

assumes *ne*: $\text{routing-match } rr' = \text{routing-match } rr$

shows $\text{routing-rule-sort-key } rr' > \text{routing-rule-sort-key } rr$

<proof>

lemma *routing-rule-sort-key* (rr-ctor (0,0,0,0) 8 [] None 0) $>$ *routing-rule-sort-key* (rr-ctor (0,0,0,0) 24 [] None 0) <proof>

In case you don't like that formulation of *is-longest-prefix-routing* over sorting, this is your lemma.

theorem *existential-routing*: *valid-prefixes* rtbl \implies *is-longest-prefix-routing* rtbl \implies *has-default-route* rtbl \implies *unambiguous-routing* rtbl \implies

routing-table-semantics rtbl addr = act \longleftrightarrow ($\exists rr \in \text{set } rtbl. \text{prefix-match-semantics } (\text{routing-match } rr) \text{ addr} \wedge \text{routing-action } rr = \text{act} \wedge$

($\forall ra \in \text{set } rtbl. \text{routing-rule-sort-key } ra < \text{routing-rule-sort-key } rr \longrightarrow \neg \text{prefix-match-semantics } (\text{routing-match } ra) \text{ addr}$))

<proof>

1.4 Printing

definition *routing-rule-32-toString* (*rr::32 routing-rule*) \equiv
prefix-match-32-toString (*routing-match rr*)
 @ (case *next-hop* (*routing-action rr*) of Some *nh* \Rightarrow " via " @ *ipv4addr-toString*
nh | - \Rightarrow [])
 @ " dev " @ *routing-oiface rr*
 @ " metric " @ *string-of-nat* (*metric rr*)

definition *routing-rule-128-toString* (*rr::128 routing-rule*) \equiv
prefix-match-128-toString (*routing-match rr*)
 @ (case *next-hop* (*routing-action rr*) of Some *nh* \Rightarrow " via " @ *ipv6addr-toString*
nh | - \Rightarrow [])
 @ " dev " @ *routing-oiface rr*
 @ " metric " @ *string-of-nat* (*metric rr*)

lemma *map routing-rule-32-toString*
 [rr-ctor (42,0,0,0) 7 "eth0" None 808,
 rr-ctor (0,0,0,0) 0 "eth1" (Some (222,173,190,239)) 707] =
 ["42.0.0.0/7 dev eth0 metric 808",
 "0.0.0.0/0 via 222.173.190.239 dev eth1 metric 707"] <proof>

2 Routing table to Relation

Walking through a routing table splits the (remaining) IP space when traversing a routing table into a pair of sets: the pair contains the IPs concerned by the current rule and those left alone.

private definition *ipset-prefix-match* **where**
ipset-prefix-match pfx rg = (let *pfxrg* = *prefix-to-wordset pfx* in (*rg* \cap *pfxrg*, *rg* - *pfxrg*))
private lemma *ipset-prefix-match-m[simp]*: *fst* (*ipset-prefix-match pfx rg*) = *rg* \cap (*prefix-to-wordset pfx*) <proof> **lemma** *ipset-prefix-match-nm[simp]*: *snd* (*ipset-prefix-match pfx rg*) = *rg* - (*prefix-to-wordset pfx*) <proof> **lemma** *ipset-prefix-match-distinct*:
rpm = *ipset-prefix-match pfx rg* \implies
 (*fst rpm*) \cap (*snd rpm*) = {} <proof> **lemma** *ipset-prefix-match-complete*: *rpm* =
ipset-prefix-match pfx rg \implies
 (*fst rpm*) \cup (*snd rpm*) = *rg* <proof> **lemma** *rpm-m-dup-simp*: *rg* \cap *fst* (*ipset-prefix-match*
(routing-match r) rg) = *fst* (*ipset-prefix-match* (*routing-match r*) *rg*)
 <proof> **definition** *range-prefix-match* :: 'i::len *prefix-match* \Rightarrow 'i *wordinterval* \Rightarrow
 'i *wordinterval* \times 'i *wordinterval* **where**
range-prefix-match pfx rg \equiv (let *pfxrg* = *prefix-to-wordinterval pfx* in
 (*wordinterval-intersection rg pfxrg*, *wordinterval-setminus rg pfxrg*))
private lemma *range-prefix-match-set-eq*:
 ($\lambda(r1,r2).$ (*wordinterval-to-set r1*, *wordinterval-to-set r2*)) (*range-prefix-match*
pfx rg) =
ipset-prefix-match pfx (*wordinterval-to-set rg*)
 <proof> **lemma** *range-prefix-match-sm[simp]*: *wordinterval-to-set* (*fst* (*range-prefix-match*
pfx rg)) =

```

    fst (ipset-prefix-match pfx (wordinterval-to-set rg))
  ⟨proof⟩ lemma range-prefix-match-snm[simp]: wordinterval-to-set (snd (range-prefix-match
pfx rg)) =
    snd (ipset-prefix-match pfx (wordinterval-to-set rg))
  ⟨proof⟩

```

2.1 Wordintervals for Ports by Routing

This split, although rather trivial, can be used to construct the sets (or rather: the intervals) of IPs that are actually matched by an entry in a routing table.

```

private fun routing-port-ranges :: 'i prefix-routing ⇒ 'i wordinterval ⇒ (string ×
('i::len) wordinterval) list where
routing-port-ranges [] lo = (if wordinterval-empty lo then [] else [(routing-oiface
(undefined::'i routing-rule),lo)]) |
routing-port-ranges (a#as) lo = (
  let rpm = range-prefix-match (routing-match a) lo; m = fst rpm; nm = snd rpm
  in (
    (routing-oiface a,m) # routing-port-ranges as nm))

```

```

private lemma routing-port-ranges-subsets:
(a1, b1) ∈ set (routing-port-ranges tbl s) ⇒ wordinterval-to-set b1 ⊆ wordinter-
val-to-set s
  ⟨proof⟩ lemma routing-port-ranges-sound: e ∈ set (routing-port-ranges tbl s) ⇒
k ∈ wordinterval-to-set (snd e) ⇒ valid-prefixes tbl ⇒
  fst e = output-iface (routing-table-semantics tbl k)
  ⟨proof⟩ lemma routing-port-ranges-disjoined:
assumes vpfx: valid-prefixes tbl
  and ins: (a1, b1) ∈ set (routing-port-ranges tbl s) (a2, b2) ∈ set (routing-port-ranges
tbl s)
  and nemp: wordinterval-to-set b1 ≠ {}
shows b1 ≠ b2 ⇔ wordinterval-to-set b1 ∩ wordinterval-to-set b2 = {}
  ⟨proof⟩ lemma routing-port-rangesI:
valid-prefixes tbl ⇒
  output-iface (routing-table-semantics tbl k) = output-port ⇒
  k ∈ wordinterval-to-set wi ⇒
  (∃ ip-range. (output-port, ip-range) ∈ set (routing-port-ranges tbl wi) ∧ k ∈ wordinter-
val-to-set ip-range)
  ⟨proof⟩

```

2.2 Reduction

So far, one entry in the list would be generated for each routing table entry. This next step reduces it to one for each port. The resulting list will represent a function from port to IP wordinterval. (It can also be understood as a function from IP (interval) to port (where the intervals don't overlap).

definition reduce-range-destination $l \equiv$

$\text{let } ps = \text{remdups } (\text{map fst } l) \text{ in}$
 $\text{let } c = \lambda s. (\text{wordinterval-Union} \circ \text{map snd} \circ \text{filter } (((=) s) \circ \text{fst})) l \text{ in}$
 $[(p, c p). p \leftarrow ps]$

definition *routing-ipassmt-wi tbl* \equiv *reduce-range-destination* (*routing-port-ranges* *tbl* *wordinterval-UNIV*)

lemma *routing-ipassmt-wi-distinct*: *distinct* (*map fst* (*routing-ipassmt-wi tbl*))

<proof> **lemma** *routing-port-ranges-superseted*:

$(a1, b1) \in \text{set } (\text{routing-port-ranges } \text{tbl } \text{wordinterval-UNIV}) \implies$

$\exists b2. (a1, b2) \in \text{set } (\text{routing-ipassmt-wi } \text{tbl}) \wedge \text{wordinterval-to-set } b1 \subseteq \text{wordinterval-to-set } b2$

<proof> **lemma** *routing-ipassmt-wi-subsetted*:

$(a1, b1) \in \text{set } (\text{routing-ipassmt-wi } \text{tbl}) \implies$

$(a1, b2) \in \text{set } (\text{routing-port-ranges } \text{tbl } \text{wordinterval-UNIV}) \implies \text{wordinterval-to-set } b2 \subseteq \text{wordinterval-to-set } b1$

<proof>

This lemma should hold without the *valid-prefixes* assumption, but that would break the semantic argument and make the proof a lot harder.

lemma *routing-ipassmt-wi-disjoint*:

assumes *vpfx*: *valid-prefixes* (*tbl::('i::len) prefix-routing*)

and *dif*: $a1 \neq a2$

and *ins*: $(a1, b1) \in \text{set } (\text{routing-ipassmt-wi } \text{tbl}) \ (a2, b2) \in \text{set } (\text{routing-ipassmt-wi } \text{tbl})$

shows $\text{wordinterval-to-set } b1 \cap \text{wordinterval-to-set } b2 = \{\}$

<proof>

lemma *routing-ipassmt-wi-sound*:

assumes *vpfx*: *valid-prefixes* *tbl*

and *ins*: $(ea, eb) \in \text{set } (\text{routing-ipassmt-wi } \text{tbl})$

and *x*: $k \in \text{wordinterval-to-set } eb$

shows $ea = \text{output-iface } (\text{routing-table-semantics } \text{tbl } k)$

<proof>

theorem *routing-ipassmt-wi*:

assumes *vpfx*: *valid-prefixes* *tbl*

shows

$\text{output-iface } (\text{routing-table-semantics } \text{tbl } k) = \text{output-port} \longleftrightarrow$

$(\exists \text{ip-range}. k \in \text{wordinterval-to-set } \text{ip-range} \wedge (\text{output-port}, \text{ip-range}) \in \text{set } (\text{routing-ipassmt-wi } \text{tbl}))$

<proof>

lemma *routing-ipassmt-wi-has-all-interfaces*:

assumes *in-tbl*: $r \in \text{set } \text{tbl}$

shows $\exists s. (\text{routing-oiface } r, s) \in \text{set } (\text{routing-ipassmt-wi } \text{tbl})$

<proof>

end

end

3 Linux Router

theory *Linux-Router*

imports

Routing-Table

Simple-Firewall.SimpleFw-Semantics

Simple-Firewall.Simple-Packet

HOL-Library.Monad-Syntax

begin

definition *fromMaybe* $a\ m = (\text{case } m \text{ of } \text{Some } a \Rightarrow a \mid \text{None} \Rightarrow a)$

Here, we present a heavily simplified model of a linux router. (i.e., a linux-based device with `net.ipv4.ip_forward`) It covers the following steps in packet processing:

- Packet arrives (destination port is empty, destination mac address is own address).
- Destination address is extracted and used for a routing table lookup.
- Packet is updated with output interface of routing decision.
- The FORWARD chain of iptables is considered.
- Next hop is extracted from the routing decision, fallback to destination address if directly attached.
- MAC address of next hop is looked up (using the mac lookup function `mlf`)
- L2 destination address of packet is updated.

This is stripped down to model only the most important and widely used aspects of packet processing. Here are a few examples of what was abstracted away:

- No local traffic.
- Only the `filter` table of iptables is considered, `raw` and `nat` are not.
- Only one routing table is considered. (Linux can have other tables than the `default` one.)

- No source MAC modification.
- ...

```
record interface =
  iface-name :: string
  iface-mac :: 48 word
```

```
definition iface-packet-check :: interface list  $\Rightarrow$  ('i::len,'b) simple-packet-ext-scheme
 $\Rightarrow$  interface option
where iface-packet-check ifs p  $\equiv$  find ( $\lambda i$ . iface-name i = p-iface p  $\wedge$  iface-mac i
= p-l2dst p) ifs
term simple-fw
definition simple-linux-router ::
  'i routing-rule list  $\Rightarrow$  'i simple-rule list  $\Rightarrow$  (('i::len) word  $\Rightarrow$  48 word option)  $\Rightarrow$ 
  interface list  $\Rightarrow$  'i simple-packet-ext  $\Rightarrow$  'i simple-packet-ext option where
simple-linux-router rt fw mlf ifl p  $\equiv$  do {
  -  $\leftarrow$  iface-packet-check ifl p;
  let rd — (routing decision) = routing-table-semantics rt (p-dst p);
  let p = p(p-iface := output-iface rd);
  let fd — (firewall decision) = simple-fw fw p;
  -  $\leftarrow$  (case fd of Decision FinalAllow  $\Rightarrow$  Some () | Decision FinalDeny  $\Rightarrow$  None);
  let nh = fromMaybe (p-dst p) (next-hop rd);
  ma  $\leftarrow$  mlf nh;
  Some (p(p-l2dst := ma))
}
```

However, the above model is still too powerful for some use-cases. Especially, the next hop look-up cannot be done without either a pre-distributed table of all MAC addresses, or the usual mechanic of sending out an ARP request and caching the answer. Doing ARP requests in the restricted environment of, e.g., an OpenFlow ruleset seems impossible. Therefore, we present this model:

```
definition simple-linux-router-nol12 ::
  'i routing-rule list  $\Rightarrow$  'i simple-rule list  $\Rightarrow$  ('i,'a) simple-packet-scheme  $\Rightarrow$ 
  ('i::len,'a) simple-packet-scheme option where
simple-linux-router-nol12 rt fw p  $\equiv$  do {
  let rd = routing-table-semantics rt (p-dst p);
  let p = p(p-iface := output-iface rd);
  let fd = simple-fw fw p;
  -  $\leftarrow$  (case fd of Decision FinalAllow  $\Rightarrow$  Some () | Decision FinalDeny  $\Rightarrow$  None);
  Some p
}
```

The differences to *simple-linux-router* are illustrated by the lemmata below.

```
lemma rtr-nomac-e1:
  fixes pi
```

assumes *simple-linux-router* *rt fw mlf ifl pi = Some po*
assumes *simple-linux-router-nol12* *rt fw pi = Some po'*
shows $\exists x. po = po' \langle p\text{-}l2dst := x \rangle$
 $\langle proof \rangle$

lemma *rtr-nomac-e2*:
fixes *pi*
assumes *simple-linux-router* *rt fw mlf ifl pi = Some po*
shows $\exists po'. \text{simple-linux-router-nol12 } rt \text{ fw } pi = \text{Some } po'$
 $\langle proof \rangle$

lemma *rtr-nomac-e3*:
fixes *pi*
assumes *simple-linux-router-nol12* *rt fw pi = Some po*
assumes *iface-packet-check* *ifl pi = Some i* — don't care
assumes *mlf* (*fromMaybe* (*p-dst pi*) (*next-hop* (*routing-table-semantics* *rt* (*p-dst pi*)))) = *Some i2*
shows $\exists po'. \text{simple-linux-router } rt \text{ fw } mlf \text{ ifl } pi = \text{Some } po'$
 $\langle proof \rangle$

lemma *rtr-nomac-eq*:
fixes *pi*
assumes *iface-packet-check* *ifl pi \neq None*
assumes *mlf* (*fromMaybe* (*p-dst pi*) (*next-hop* (*routing-table-semantics* *rt* (*p-dst pi*)))) \neq *None*
shows $\exists x. \text{map-option } (\lambda p. p \langle p\text{-}l2dst := x \rangle) (\text{simple-linux-router-nol12 } rt \text{ fw } pi) = \text{simple-linux-router } rt \text{ fw } mlf \text{ ifl } pi$
 $\langle proof \rangle$

end

4 Parser

theory *IpRoute-Parser*
imports *Routing-Table*
IP-Addresses.IP-Address-Parser
keywords *parse-ip-route parse-ip-6-route* :: *thy-decl*
begin

This helps to read the output of the `ip route` command into a *32 routing-rule list*.

definition *empty-rr-hlp* :: (*'a::len*) *prefix-match* \Rightarrow *'a routing-rule* **where**
empty-rr-hlp *pm* = *routing-rule.make* *pm* *default-metric* (*routing-action.make* "" *None*)

lemma *empty-rr-hlp-alt*:
empty-rr-hlp *pm* = \langle *routing-match* = *pm*, *metric* = *0*, *routing-action* = \langle *output-iface* = \langle , *next-hop* = *None* \rangle \rangle
 $\langle proof \rangle$

definition *routing-action-next-hop-update* :: 'a word \Rightarrow 'a routing-rule \Rightarrow ('a::len) routing-rule
where
routing-action-next-hop-update h pk = pk(\lfloor routing-action := (routing-action pk) \rfloor
next-hop := Some h \rfloor)
lemma *routing-action-next-hop-update* h pk = routing-action-update (next-hop-update
(λ -. (Some h))) (pk::32 routing-rule)
 \langle proof \rangle

definition *routing-action-oiface-update* :: string \Rightarrow 'a routing-rule \Rightarrow ('a::len) routing-rule
where
routing-action-oiface-update h pk = routing-action-update (output-iface-update
(λ -. h)) (pk::'a routing-rule)
lemma *routing-action-oiface-update* h pk = pk(\lfloor routing-action := (routing-action
pk) \rfloor output-iface := h \rfloor)
 \langle proof \rangle

definition *default-prefix* = PrefixMatch 0 0
lemma *default-prefix-matchall*: prefix-match-semantics default-prefix ip
 \langle proof \rangle

definition *sanity-ip-route* (r::('a::len) prefix-routing) \equiv correct-routing r \wedge unambiguous-routing r \wedge list-all ((\neq) \circ routing-oiface) r

The parser ensures that *sanity-ip-route* holds for any ruleset that is imported.

\langle ML \rangle

parse-ip-route rtbl-parser-test1 = ip-route-ex
lemma *sanity-ip-route* rtbl-parser-test1 \langle proof \rangle

lemma *rtbl-parser-test1* =
[\lfloor routing-match = PrefixMatch 0xFFFFFFFF 32, metric = 0, routing-action =
 \lfloor output-iface = "tun0", next-hop = None \rfloor \rfloor ,
 \lfloor routing-match = PrefixMatch 0xA0D2AA0 28, metric = 303, routing-action =
 \lfloor output-iface = "ewlan", next-hop = None \rfloor \rfloor ,
 \lfloor routing-match = PrefixMatch 0xA0D2500 24, metric = 0, routing-action =
 \lfloor output-iface = "tun0", next-hop = Some 0xFFFFFFFF \rfloor \rfloor ,
 \lfloor routing-match = PrefixMatch 0xA0D2C00 24, metric = 0, routing-action =
 \lfloor output-iface = "tun0", next-hop = Some 0xFFFFFFFF \rfloor \rfloor ,
 \lfloor routing-match = PrefixMatch 0 0, metric = 303, routing-action = \lfloor output-iface
= "ewlan", next-hop = Some 0xA0D2AA1 \rfloor \rfloor]
 \langle proof \rangle

parse-ip-6-route rtbl-parser-test2 = ip-6-route-ex
value[code] rtbl-parser-test2

lemma *sanity-ip-route rtbl-parser-test2* $\langle proof \rangle$

end