

# Regression Test Selection over JVM

Susannah Mansky

December 14, 2021

## Abstract

This development provides a general definition for safe Regression Test Selection (RTS) algorithms. RTS algorithms select which tests to rerun on revised code, reducing the time required to check for newly introduced errors. An RTS algorithm is considered safe if and only if all deselected tests would have unchanged results.

This definition is instantiated with two class-collection-based RTS algorithms run over the JVM as modeled by JinjaDCI. This is achieved with a general definition for Collection Semantics, small-step semantics instrumented to collect information during execution. As the RTS definition mandates safety, these instantiations include proofs of safety.

This work is described in Mansky and Gunter’s LSFA 2020 paper [1] and Mansky’s doctoral thesis [2].

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# 1 Theory Dependencies

Figure 1 shows the dependencies between the Isabelle theories in the following sections.

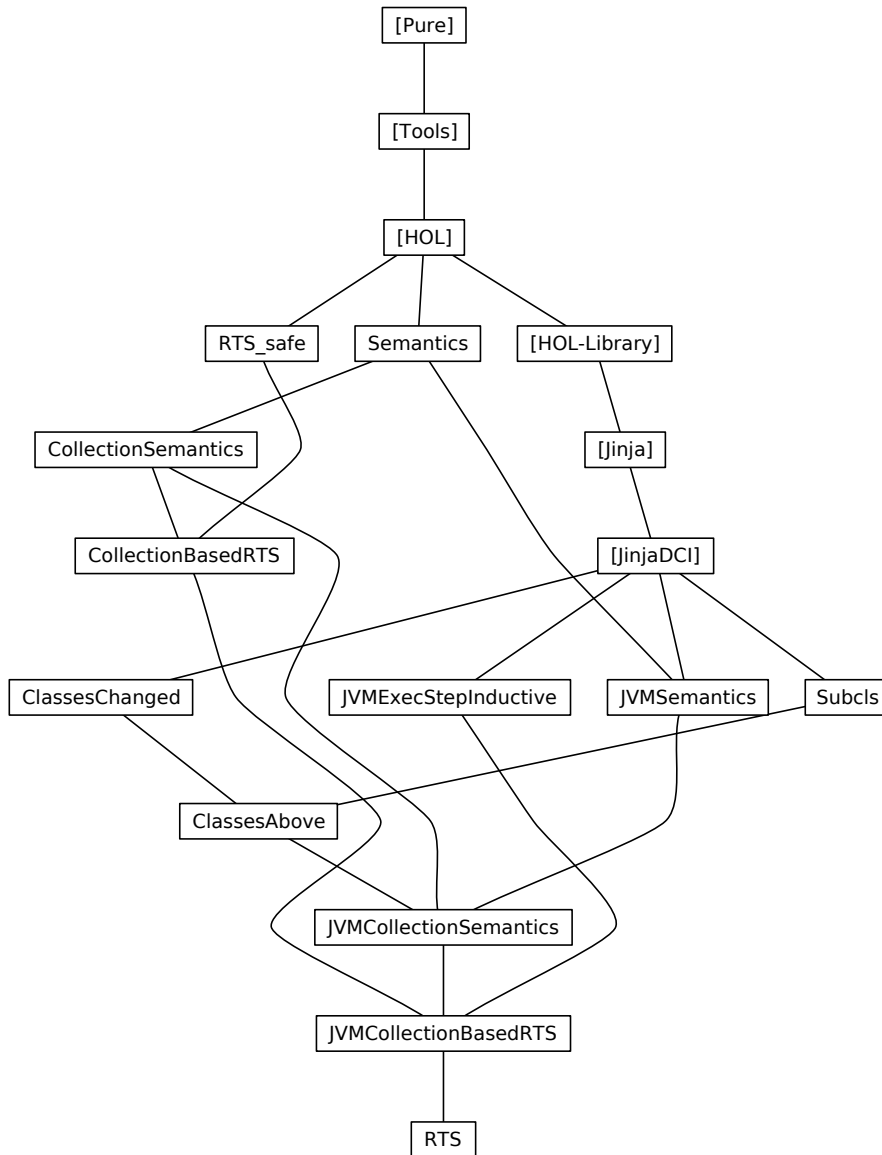


Figure 1: Theory Dependency Graph

## 2 Regression Test Selection algorithm model

```
theory RTS-safe
imports Main
begin
```

This describes an *existence safe* RTS algorithm: if a test is deselected based on an output, there is SOME equivalent output under the changed program.

```
locale RTS-safe =
```

```
fixes
```

```
  out :: 'prog  $\Rightarrow$  'test  $\Rightarrow$  'prog-out set and
  equiv-out :: 'prog-out  $\Rightarrow$  'prog-out  $\Rightarrow$  bool and
  deselect :: 'prog  $\Rightarrow$  'prog-out  $\Rightarrow$  'prog  $\Rightarrow$  bool and
  progs :: 'prog set and
  tests :: 'test set
```

```
assumes
```

```
  existence-safe:  $\llbracket P \in \text{progs}; P' \in \text{progs}; t \in \text{tests}; o1 \in \text{out } P \ t; \text{deselect } P \ o1 \ P' \rrbracket$ 
 $\implies (\exists o2 \in \text{out } P' \ t. \text{equiv-out } o1 \ o2)$  and
  equiv-out-equiv: equiv UNIV  $\{(x,y). \text{equiv-out } x \ y\}$  and
  equiv-out-deselect:  $\llbracket \text{equiv-out } o1 \ o2; \text{deselect } P \ o1 \ P' \rrbracket \implies \text{deselect } P \ o2 \ P'$ 
```

```
context RTS-safe begin
```

```
lemma equiv-out-refl: equiv-out a a
```

```
using equiv-class-eq-iff equiv-out-equiv by fastforce
```

```
lemma equiv-out-trans:  $\llbracket \text{equiv-out } a \ b; \text{equiv-out } b \ c \rrbracket \implies \text{equiv-out } a \ c$ 
```

```
using equiv-class-eq-iff equiv-out-equiv by fastforce
```

This shows that it is safe to continue deselecting a test based on its output under a previous program, to an arbitrary number of program changes, as long as the test is continually deselected. This is useful because it means changed programs don't need to generate new outputs for deselected tests to ensure safety of future deselections.

```
lemma existence-safe-trans:
```

```
assumes Pst-in:  $P_s \neq []$  set  $P_s \subseteq \text{progs}$   $t \in \text{tests}$  and
```

```
  o0:  $o_0 \in \text{out } (P_s!0) \ t$  and
```

```
  des:  $\forall n < (\text{length } P_s) - 1. \text{deselect } (P_s!n) \ o_0 \ (P_s!(\text{Suc } n))$ 
```

```
shows  $\exists o_n \in \text{out } (\text{last } P_s) \ t. \text{equiv-out } o_0 \ o_n$ 
```

```
using assms proof(induct length  $P_s$  arbitrary:  $P_s$ )
```

```
  case 0 with Pst-in show ?case by simp
```

```
next
```

```
  case (Suc x) then show ?case
```

```
  proof(induct x)
```

```
  case z: 0
```

```
    from z.premis(2,3) have  $P_s ! (\text{length } P_s - 2) = \text{last } P_s$ 
```

```
    by (simp add: last-conv-nth numeral-2-eq-2)
```

```
    with equiv-out-refl z.premis(2,6) show ?case by auto
```

```
  next
```

```

case  $Suc'$ :( $Suc\ x'$ )
let  $?Ps = take\ (Suc\ x')\ Ps$ 
have  $len'$ :  $Suc\ x' = length\ (take\ (Suc\ x')\ Ps)$  using  $Suc'.prems(2)$  by auto
moreover have  $nmt'$ :  $take\ (Suc\ x')\ Ps \neq []$  using  $len'$  by auto
moreover have  $sub'$ :  $set\ (take\ (Suc\ x')\ Ps) \subseteq progs$  using  $Suc'.prems(2)$ 
  by (meson order-trans set-take-subset)
moreover have  $t \in tests$  using  $Pst-in(3)$  by simp
moreover have  $o_0 \in out\ (take\ (Suc\ x')\ Ps\ !\ 0)\ t$  using  $Suc'.prems(4)$  by simp
moreover have  $\forall n < length\ (take\ (Suc\ x')\ Ps) - 1.$ 
   $deselect\ (take\ (Suc\ x')\ Ps\ !\ n)\ o_0\ (take\ (Suc\ x')\ Ps\ !\ (Suc\ n))$ 
  using  $Suc'.prems(5)\ len'$  by simp
ultimately have  $\exists o' \in out\ (last\ ?Ps)\ t.$  equiv-out  $o_0\ o'$  by(rule  $Suc'.prems(1)[of\ ?Ps]$ )
  then obtain  $o'$  where  $o'$ :  $o' \in out\ (last\ ?Ps)\ t$  and  $eo$ : equiv-out  $o_0\ o'$  by
clarify
  from  $Suc'.prems(1)\ Suc'.prems(2)\ len'\ nmt'$ 
  have  $last\ (take\ (Suc\ x')\ Ps) = Ps!\ x'\ last\ Ps = Ps!\ (Suc\ x')$ 
  by (metis diff-Suc-1 last-conv-nth lessI nth-take)+
  moreover have  $x' < length\ Ps - 1$  using  $Suc'.prems(2)$  by linarith
  ultimately have  $des'$ :  $deselect\ (last\ (take\ (Suc\ x')\ Ps))\ o_0\ (last\ Ps)$ 
  using  $Suc'.prems(5)$  by simp
  from  $Suc'.prems(1,2)\ sub'\ nmt'\ last-in-set$ 
  have  $Ps-in$ :  $last\ (take\ (Suc\ x')\ Ps) \in progs\ last\ Ps \in progs$  by blast+
  have  $\exists o_n \in out\ (last\ Ps)\ t.$  equiv-out  $o'\ o_n$ 
  by(rule existence-safe[where  $P=last\ (take\ (Suc\ x')\ Ps)$  and  $P'=last\ Ps$  and
 $t=t$ ,
     $OF\ Ps-in\ Pst-in(3)\ o'\ equiv-out-deselect[OF\ eo\ des']$ ])
  then obtain  $o_n$  where  $o_n$ :  $o_n \in out\ (last\ Ps)\ t$  and  $eo'$ : equiv-out  $o'\ o_n$  by
clarify
  moreover from  $eo\ eo'$  have equiv-out  $o_0\ o_n$  by(rule equiv-out-trans)
  ultimately show ?case by auto
qed
qed

end — RTS-safe

end

```

### 3 Semantics model

```

theory Semantics
imports Main
begin

```

General model for small-step semantics:

```

locale Semantics =
  fixes
   $small :: 'prog \Rightarrow 'state \Rightarrow 'state\ set$  and
   $endset :: 'state\ set$ 

```

**assumes**  
*endset-final*:  $\sigma \in \text{endset} \implies \forall P. \text{small } P \sigma = \{\}$

**context** *Semantics* **begin**

### 3.1 Extending *small* to multiple steps

**primrec** *small-nstep* :: '*prog*  $\Rightarrow$  '*state*  $\Rightarrow$  *nat*  $\Rightarrow$  '*state set* **where**

*small-nstep-base*:

*small-nstep* *P*  $\sigma$  0 =  $\{\sigma\}$  |

*small-nstep-Rec*:

*small-nstep* *P*  $\sigma$  (*Suc* *n*) =  
 $\{ \sigma 2. \exists \sigma 1. \sigma 1 \in \text{small-nstep } P \sigma n \wedge \sigma 2 \in \text{small } P \sigma 1 \}$

**lemma** *small-nstep-Rec2*:

*small-nstep* *P*  $\sigma$  (*Suc* *n*) =  
 $\{ \sigma 2. \exists \sigma 1. \sigma 1 \in \text{small } P \sigma \wedge \sigma 2 \in \text{small-nstep } P \sigma 1 n \}$

**proof**(*induct n arbitrary:  $\sigma$* )

**case** (*Suc* *n*)

**have** *right*:  $\bigwedge \sigma'. \sigma' \in \text{small-nstep } P \sigma (\text{Suc}(\text{Suc } n))$

$\implies \exists \sigma 1. \sigma 1 \in \text{small } P \sigma \wedge \sigma' \in \text{small-nstep } P \sigma 1 (\text{Suc } n)$

**proof** –

**fix**  $\sigma'$

**assume**  $\sigma' \in \text{small-nstep } P \sigma (\text{Suc}(\text{Suc } n))$

**then obtain**  $\sigma 1$  **where** *Sucnstep*:  $\sigma 1 \in \text{small-nstep } P \sigma (\text{Suc } n)$   $\sigma' \in \text{small } P$   
 $\sigma 1$  **by** *fastforce*

**obtain**  $\sigma 12$  **where** *nstep*:  $\sigma 12 \in \text{small } P \sigma \wedge \sigma 1 \in \text{small-nstep } P \sigma 12 n$

**using** *Suc Sucnstep(1)* **by** *fastforce*

**then show**  $\exists \sigma 1. \sigma 1 \in \text{small } P \sigma \wedge \sigma' \in \text{small-nstep } P \sigma 1 (\text{Suc } n)$

**using** *Sucnstep* **by** *fastforce*

**qed**

**have** *left*:  $\bigwedge \sigma'. \exists \sigma 1. \sigma 1 \in \text{small } P \sigma \wedge \sigma' \in \text{small-nstep } P \sigma 1 (\text{Suc } n)$

$\implies \sigma' \in \text{small-nstep } P \sigma (\text{Suc}(\text{Suc } n))$

**proof** –

**fix**  $\sigma'$

**assume**  $\exists \sigma 1. \sigma 1 \in \text{small } P \sigma \wedge \sigma' \in \text{small-nstep } P \sigma 1 (\text{Suc } n)$

**then obtain**  $\sigma 1$  **where** *Sucnstep*:  $\sigma 1 \in \text{small } P \sigma$   $\sigma' \in \text{small-nstep } P \sigma 1 (\text{Suc } n)$

**by** *fastforce*

**obtain**  $\sigma 12$  **where** *nstep*:  $\sigma 12 \in \text{small-nstep } P \sigma 1 n \wedge \sigma' \in \text{small } P \sigma 12$

**using** *Sucnstep(2)* **by** *auto*

**then show**  $\sigma' \in \text{small-nstep } P \sigma (\text{Suc}(\text{Suc } n))$  **using** *Suc Sucnstep* **by** *fastforce*

**qed**

**show** *?case* **using** *right left* **by** *fast*

**qed**(*simp*)

**lemma** *small-nstep-SucD*:

**assumes**  $\sigma' \in \text{small-nstep } P \sigma (\text{Suc } n)$

**shows**  $\exists \sigma 1. \sigma 1 \in \text{small } P \sigma \wedge \sigma' \in \text{small-nstep } P \sigma 1 n$

using *small-nstep-Rec2 case-prodD assms* by *fastforce*

**lemma** *small-nstep-Suc-nend*:  $\sigma' \in \text{small-nstep } P \sigma (\text{Suc } n1) \implies \sigma \notin \text{endset}$   
 using *endset-final small-nstep-SucD* by *fastforce*

### 3.2 Extending *small* to a big-step semantics

**definition** *big* :: *'prog*  $\Rightarrow$  *'state*  $\Rightarrow$  *'state set* **where**  
*big* *P*  $\sigma \equiv \{ \sigma'. \exists n. \sigma' \in \text{small-nstep } P \sigma n \wedge \sigma' \in \text{endset} \}$

**lemma** *bigI*:  
 $\llbracket \sigma' \in \text{small-nstep } P \sigma n; \sigma' \in \text{endset} \rrbracket \implies \sigma' \in \text{big } P \sigma$   
 by (*fastforce simp add: big-def*)

**lemma** *bigD*:  
 $\llbracket \sigma' \in \text{big } P \sigma \rrbracket \implies \exists n. \sigma' \in \text{small-nstep } P \sigma n \wedge \sigma' \in \text{endset}$   
 by (*simp add: big-def*)

**lemma** *big-def2*:  
 $\sigma' \in \text{big } P \sigma \iff (\exists n. \sigma' \in \text{small-nstep } P \sigma n \wedge \sigma' \in \text{endset})$   
**proof**(*rule iffI*)  
**assume**  $\sigma' \in \text{big } P \sigma$   
**then show**  $\exists n. \sigma' \in \text{small-nstep } P \sigma n \wedge \sigma' \in \text{endset}$  **using** *bigD big-def* **by**  
*auto*  
**next**  
**assume**  $\exists n. \sigma' \in \text{small-nstep } P \sigma n \wedge \sigma' \in \text{endset}$   
**then show**  $\sigma' \in \text{big } P \sigma$  **using** *big-def big-def* **by** *auto*  
**qed**

**lemma** *big-stepD*:  
**assumes** *big*:  $\sigma' \in \text{big } P \sigma$  **and** *nend*:  $\sigma \notin \text{endset}$   
**shows**  $\exists \sigma 1. \sigma 1 \in \text{small } P \sigma \wedge \sigma' \in \text{big } P \sigma 1$   
**proof** –  
**obtain** *n* **where**  $n: \sigma' \in \text{small-nstep } P \sigma n \wedge \sigma' \in \text{endset}$   
**using** *big-def2 big* **by** *auto*  
**then show** *?thesis* **using** *small-nstep-SucD nend big-def2* **by**(*cases n, simp*) *blast*  
**qed**

**lemma** *small-nstep-det-last-eq*:  
**assumes** *det*:  $\forall \sigma. \text{small } P \sigma = \{ \} \vee (\exists \sigma'. \text{small } P \sigma = \{ \sigma' \})$   
**shows**  $\llbracket \sigma' \in \text{big } P \sigma; \sigma' \in \text{small-nstep } P \sigma n; \sigma' \in \text{small-nstep } P \sigma n' \rrbracket \implies n = n'$   
**proof**(*induct n arbitrary: n'  $\sigma$   $\sigma'$* )  
**case** 0  
**have**  $\sigma' = \sigma$  **using** 0.premis(2) *small-nstep-base* **by** *blast*  
**then have**  $\sigma \in \text{endset}$  **using** 0.premis(1) *bigD* **by** *blast*  
**show** *?case*

```

proof(cases n')
  case Suc then show ?thesis using 0.premis(3) small-nstep-SucD endset-final[OF
endset] by blast
  qed(simp)
next
  case (Suc n1)
  then have endset:  $\sigma' \in \text{endset}$  using Suc.premis(1) bigD by blast
  have nend:  $\sigma \notin \text{endset}$  using small-nstep-Suc-nend[OF Suc.premis(2)] by simp
  then have neq:  $\sigma' \neq \sigma$  using endset by auto
  obtain  $\sigma 1$  where  $\sigma 1$ :  $\sigma 1 \in \text{small } P \ \sigma \ \sigma' \in \text{small-nstep } P \ \sigma 1 \ n1$ 
    using small-nstep-SucD[OF Suc.premis(2)] by clarsimp
  then have big:  $\sigma' \in \text{big } P \ \sigma 1$  using endset by(auto simp: big-def)
  show ?case
  proof(cases n')
    case 0 then show ?thesis using neq Suc.premis(3) using small-nstep-base by
blast
  next
    case Suc': (Suc n1')
    then obtain  $\sigma 1'$  where  $\sigma 1'$ :  $\sigma 1' \in \text{small } P \ \sigma \ \sigma' \in \text{small-nstep } P \ \sigma 1' \ n1'$ 
      using small-nstep-SucD[where  $\sigma = \sigma$  and  $\sigma' = \sigma'$  and  $n = n1'$ ] Suc.premis(3)
by blast
    then have  $\sigma 1 = \sigma 1'$  using  $\sigma 1(1)$  det by auto
    then show ?thesis using Suc.hyps(1)[OF big  $\sigma 1(2)$ ]  $\sigma 1'(2)$  Suc' by blast
  qed
qed

end — Semantics

```

**end**

## 4 Collection Semantics

```

theory CollectionSemantics
imports Semantics
begin

```

General model for small step semantics instrumented with an information collection mechanism:

```

locale CollectionSemantics = Semantics +
constrains

```

```

  small :: 'prog  $\Rightarrow$  'state  $\Rightarrow$  'state set and
  endset :: 'state set

```

```

fixes

```

```

  collect :: 'prog  $\Rightarrow$  'state  $\Rightarrow$  'state  $\Rightarrow$  'coll and
  combine :: 'coll  $\Rightarrow$  'coll  $\Rightarrow$  'coll and
  collect-id :: 'coll

```

```

assumes

```

```

  combine-assoc: combine (combine c1 c2) c3 = combine c1 (combine c2 c3) and

```



*collect-idl*[simp]: combine collect-id  $c = c$  and  
*collect-idr*[simp]: combine  $c$  collect-id  $= c$

**context** *CollectionSemantics* begin

## 4.1 Small-Step Collection Semantics

**definition** *csmall* :: 'prog  $\Rightarrow$  'state  $\Rightarrow$  ('state  $\times$  'coll) set **where**  
*csmall*  $P \sigma \equiv \{ (\sigma', coll). \sigma' \in small P \sigma \wedge collect P \sigma \sigma' = coll \}$

**lemma** *small-det-csmall-det*:

**assumes**  $\forall \sigma. small P \sigma = \{ \}$   $\vee (\exists \sigma'. small P \sigma = \{ \sigma' \})$

**shows**  $\forall \sigma. csmall P \sigma = \{ \}$   $\vee (\exists o'. csmall P \sigma = \{ o' \})$

**using** *assms* **by**(*fastforce simp: csmall-def*)

## 4.2 Extending *csmall* to multiple steps

**primrec** *csmall-nstep* :: 'prog  $\Rightarrow$  'state  $\Rightarrow$  nat  $\Rightarrow$  ('state  $\times$  'coll) set **where**  
*csmall-nstep-base*:

*csmall-nstep*  $P \sigma 0 = \{ (\sigma, collect-id) \}$  |

*csmall-nstep-Rec*:

*csmall-nstep*  $P \sigma (Suc n) =$

$\{ (\sigma 2, coll). \exists \sigma 1 coll 1 coll 2. (\sigma 1, coll 1) \in csmall-nstep P \sigma n$   
 $\wedge (\sigma 2, coll 2) \in csmall P \sigma 1 \wedge combine coll 1 coll 2 = coll \}$

**lemma** *small-nstep-csmall-nstep-equiv*:

*small-nstep*  $P \sigma n$

$= \{ \sigma'. \exists coll. (\sigma', coll) \in csmall-nstep P \sigma n \}$

**proof** (*induct n*) **qed**(*simp-all add: csmall-def*)

**lemma** *csmall-nstep-exists*:

$\sigma' \in big P \sigma \implies \exists n coll. (\sigma', coll) \in csmall-nstep P \sigma n \wedge \sigma' \in endset$

**proof**(*drule bigD*) **qed**(*clarsimp simp: small-nstep-csmall-nstep-equiv*)

**lemma** *csmall-det-csmall-nstep-det*:

**assumes**  $\forall \sigma. csmall P \sigma = \{ \}$   $\vee (\exists o'. csmall P \sigma = \{ o' \})$

**shows**  $\forall \sigma. csmall-nstep P \sigma n = \{ \}$   $\vee (\exists o'. csmall-nstep P \sigma n = \{ o' \})$

**using** *assms*

**proof**(*induct n*)

**case** (*Suc n*) **then show** ?*case* **by** *fastforce*

**qed**(*simp*)

**lemma** *csmall-nstep-Rec2*:

*csmall-nstep*  $P \sigma (Suc n) =$

$\{ (\sigma 2, coll). \exists \sigma 1 coll 1 coll 2. (\sigma 1, coll 1) \in csmall P \sigma$   
 $\wedge (\sigma 2, coll 2) \in csmall-nstep P \sigma 1 n \wedge combine coll 1 coll 2 = coll$

$\}$

**proof**(*induct n arbitrary:  $\sigma$* )

**case** (*Suc n*)

**have** *right*:  $\bigwedge \sigma' coll'. (\sigma', coll') \in csmall-nstep P \sigma (Suc(Suc n))$

$\implies \exists \sigma 1 \text{ coll1 coll2}. (\sigma 1, \text{coll1}) \in \text{csmall } P \sigma$   
 $\quad \wedge (\sigma', \text{coll2}) \in \text{csmall-nstep } P \sigma 1 (\text{Suc } n) \wedge \text{combine coll1 coll2}$   
 $= \text{coll}'$   
**proof** –  
**fix**  $\sigma' \text{ coll}'$   
**assume**  $(\sigma', \text{coll}') \in \text{csmall-nstep } P \sigma (\text{Suc}(\text{Suc } n))$   
**then obtain**  $\sigma 1 \text{ coll1 coll2}$  **where**  $\text{Sucnstep}: (\sigma 1, \text{coll1}) \in \text{csmall-nstep } P \sigma$   
 $(\text{Suc } n)$   
 $(\sigma', \text{coll2}) \in \text{csmall } P \sigma 1 \text{ combine coll1 coll2} = \text{coll}'$  **by fastforce**  
**obtain**  $\sigma 12 \text{ coll12 coll22}$  **where**  $\text{nstep}: (\sigma 12, \text{coll12}) \in \text{csmall } P \sigma$   
 $\quad \wedge (\sigma 1, \text{coll22}) \in \text{csmall-nstep } P \sigma 12 n \wedge \text{combine coll12 coll22}$   
 $= \text{coll1}$   
**using**  $\text{Suc Sucnstep}(1)$  **by fastforce**  
**then show**  $\exists \sigma 1 \text{ coll1 coll2}. (\sigma 1, \text{coll1}) \in \text{csmall } P \sigma$   
 $\quad \wedge (\sigma', \text{coll2}) \in \text{csmall-nstep } P \sigma 1 (\text{Suc } n) \wedge \text{combine coll1 coll2}$   
 $= \text{coll}'$   
**using**  $\text{combine-assoc}[\text{of coll12 coll22 coll2}] \text{Sucnstep}$  **by fastforce**  
**qed**  
**have left:**  $\bigwedge \sigma' \text{ coll}'. \exists \sigma 1 \text{ coll1 coll2}. (\sigma 1, \text{coll1}) \in \text{csmall } P \sigma$   
 $\quad \wedge (\sigma', \text{coll2}) \in \text{csmall-nstep } P \sigma 1 (\text{Suc } n) \wedge \text{combine coll1 coll2}$   
 $= \text{coll}'$   
 $\implies (\sigma', \text{coll}') \in \text{csmall-nstep } P \sigma (\text{Suc}(\text{Suc } n))$   
**proof** –  
**fix**  $\sigma' \text{ coll}'$   
**assume**  $\exists \sigma 1 \text{ coll1 coll2}. (\sigma 1, \text{coll1}) \in \text{csmall } P \sigma$   
 $\quad \wedge (\sigma', \text{coll2}) \in \text{csmall-nstep } P \sigma 1 (\text{Suc } n) \wedge \text{combine coll1 coll2}$   
 $= \text{coll}'$   
**then obtain**  $\sigma 1 \text{ coll1 coll2}$  **where**  $\text{Sucnstep}: (\sigma 1, \text{coll1}) \in \text{csmall } P \sigma$   
 $(\sigma', \text{coll2}) \in \text{csmall-nstep } P \sigma 1 (\text{Suc } n) \text{ combine coll1 coll2} = \text{coll}'$   
**by fastforce**  
**obtain**  $\sigma 12 \text{ coll12 coll22}$  **where**  $\text{nstep}: (\sigma 12, \text{coll12}) \in \text{csmall-nstep } P \sigma 1 n$   
 $\quad \wedge (\sigma', \text{coll22}) \in \text{csmall } P \sigma 12 n \wedge \text{combine coll12 coll22} = \text{coll2}$   
**using**  $\text{Sucnstep}(2)$  **by auto**  
**then show**  $(\sigma', \text{coll}') \in \text{csmall-nstep } P \sigma (\text{Suc}(\text{Suc } n))$   
**using**  $\text{combine-assoc}[\text{of coll1 coll12 coll22}] \text{Suc Sucnstep}$  **by fastforce**  
**qed**  
**show** *?case using right left by fast*  
**qed(simp)**

**lemma** *csmall-nstep-SucD:*

**assumes**  $(\sigma', \text{coll}') \in \text{csmall-nstep } P \sigma (\text{Suc } n)$

**shows**  $\exists \sigma 1 \text{ coll1}. (\sigma 1, \text{coll1}) \in \text{csmall } P \sigma$

$\quad \wedge (\exists \text{coll}. \text{coll}' = \text{combine coll1 coll} \wedge (\sigma', \text{coll}) \in \text{csmall-nstep } P \sigma 1 n)$

**using** *csmall-nstep-Rec2 CollectionSemantics-axioms case-prodD asms* **by fastforce**

**lemma** *csmall-nstep-Suc-nend:*  $\sigma' \in \text{csmall-nstep } P \sigma (\text{Suc } n1) \implies \sigma \notin \text{endset}$

**using** *endset-final csmall-nstep-SucD CollectionSemantics.csmall-def CollectionSemantics-axioms*

by fastforce

**lemma** *small-to-csmall-nstep-pres*:

**assumes**  $Qpres: \bigwedge P \sigma \sigma'. Q P \sigma \implies \sigma' \in \text{small } P \sigma \implies Q P \sigma'$

**shows**  $Q P \sigma \implies (\sigma', \text{coll}) \in \text{csmall-nstep } P \sigma n \implies Q P \sigma'$

**proof**(*induct n arbitrary:  $\sigma \sigma' \text{ coll}$* )

case (*Suc n*)

**then obtain**  $\sigma 1 \text{ coll1 coll2}$  **where**  $nstep: (\sigma 1, \text{coll1}) \in \text{csmall-nstep } P \sigma n$   
 $\wedge (\sigma', \text{coll2}) \in \text{csmall } P \sigma 1 \wedge \text{combine coll1 coll2} = \text{coll}$  **by**

*clarsimp*

**then show** *?case using Suc Qpres[where  $P=P$  and  $\sigma=\sigma 1$  and  $\sigma'=\sigma'$ ]* **by**(*auto simp: csmall-def*)

**qed**(*simp*)

**lemma** *csmall-to-csmall-nstep-prop*:

**assumes** *cond*:  $\bigwedge P \sigma \sigma' \text{ coll}. (\sigma', \text{coll}) \in \text{csmall } P \sigma \implies R P \text{ coll} \implies Q P \sigma \implies R' P \sigma \sigma' \text{ coll}$

**and** *Rcomb*:  $\bigwedge P \text{ coll1 coll2}. R P (\text{combine coll1 coll2}) = (R P \text{ coll1} \wedge R P \text{ coll2})$

**and** *Qpres*:  $\bigwedge P \sigma \sigma'. Q P \sigma \implies \sigma' \in \text{small } P \sigma \implies Q P \sigma'$

**and** *Rtrans'*:  $\bigwedge P \sigma \sigma 1 \sigma' \text{ coll1 coll2}.$

$R' P \sigma \sigma 1 \text{ coll1} \wedge R' P \sigma 1 \sigma' \text{ coll2} \implies R' P \sigma \sigma' (\text{combine}$

$\text{coll1 coll2})$

**and** *base*:  $\bigwedge \sigma. R' P \sigma \sigma \text{ collect-id}$

**shows**  $(\sigma', \text{coll}) \in \text{csmall-nstep } P \sigma n \implies R P \text{ coll} \implies Q P \sigma \implies R' P \sigma \sigma' \text{ coll}$

**proof**(*induct n arbitrary:  $\sigma \sigma' \text{ coll}$* )

case (*Suc n*)

**then obtain**  $\sigma 1 \text{ coll1 coll2}$  **where**  $nstep: (\sigma 1, \text{coll1}) \in \text{csmall-nstep } P \sigma n$

$\wedge (\sigma', \text{coll2}) \in \text{csmall } P \sigma 1 \wedge \text{combine coll1 coll2} = \text{coll}$  **by**

*clarsimp*

**then have**  $Q P \sigma 1$  **using** *small-to-csmall-nstep-pres[where  $Q=Q$ ]* *Qpres Suc* **by** *blast*

**then show** *?case using nstep assms Suc* **by** *auto blast*

**qed**(*simp add: base*)

**lemma** *csmall-to-csmall-nstep-prop2*:

**assumes** *cond*:  $\bigwedge P P' \sigma \sigma' \text{ coll}. (\sigma', \text{coll}) \in \text{csmall } P \sigma$

$\implies R P P' \text{ coll} \implies Q \sigma \implies (\sigma', \text{coll}) \in \text{csmall } P' \sigma$

**and** *Rcomb*:  $\bigwedge P P' \text{ coll1 coll2}. R P P' (\text{combine coll1 coll2}) = (R P P' \text{ coll1} \wedge R P P' \text{ coll2})$

**and** *Qpres*:  $\bigwedge P \sigma \sigma'. Q \sigma \implies \sigma' \in \text{small } P \sigma \implies Q \sigma'$

**shows**  $(\sigma', \text{coll}) \in \text{csmall-nstep } P \sigma n \implies R P P' \text{ coll} \implies Q \sigma \implies (\sigma', \text{coll}) \in \text{csmall-nstep } P' \sigma n$

**proof**(*induct n arbitrary:  $\sigma \sigma' \text{ coll}$* )

case (*Suc n*)

**then obtain**  $\sigma 1 \text{ coll1 coll2}$  **where**  $nstep: (\sigma 1, \text{coll1}) \in \text{csmall-nstep } P \sigma n$

$\wedge (\sigma', \text{coll2}) \in \text{csmall } P \sigma 1 \wedge \text{combine coll1 coll2} = \text{coll}$  **by**

*clarsimp*

**then have**  $Q \sigma 1$  **using** *small-to-csmall-nstep-pres[where  $Q=\lambda P. Q$ ]* *Qpres Suc* **by** *blast*

**then show** *?case using nstep assms Suc by auto blast*  
**qed**(*simp*)

### 4.3 Extending *csmall* to a big-step semantics

**definition** *cbig* :: '*prog*  $\Rightarrow$  '*state*  $\Rightarrow$  ('*state*  $\times$  '*coll*) set **where**

*cbig* *P*  $\sigma \equiv$   
 $\{ (\sigma', coll). \exists n. (\sigma', coll) \in csmall\text{-}nstep\ P\ \sigma\ n \wedge \sigma' \in endset \}$

**lemma** *cbigD*:

$\llbracket (\sigma', coll') \in cbig\ P\ \sigma \rrbracket \Longrightarrow \exists n. (\sigma', coll') \in csmall\text{-}nstep\ P\ \sigma\ n \wedge \sigma' \in endset$   
**by**(*simp add: cbig-def*)

**lemma** *cbigD'*:

$\llbracket \sigma' \in cbig\ P\ \sigma \rrbracket \Longrightarrow \exists n. \sigma' \in csmall\text{-}nstep\ P\ \sigma\ n \wedge fst\ \sigma' \in endset$   
**by**(*cases o', simp add: cbig-def*)

**lemma** *cbig-def2*:

$(\sigma', coll) \in cbig\ P\ \sigma \longleftrightarrow (\exists n. (\sigma', coll) \in csmall\text{-}nstep\ P\ \sigma\ n \wedge \sigma' \in endset)$

**proof**(*rule iffI*)

**assume**  $(\sigma', coll) \in cbig\ P\ \sigma$

**then show**  $\exists n. (\sigma', coll) \in csmall\text{-}nstep\ P\ \sigma\ n \wedge \sigma' \in endset$  **using** *bigD cbig-def*

**by** *auto*

**next**

**assume**  $\exists n. (\sigma', coll) \in csmall\text{-}nstep\ P\ \sigma\ n \wedge \sigma' \in endset$

**then show**  $(\sigma', coll) \in cbig\ P\ \sigma$  **using** *big-def cbig-def small-nstep-csmall-nstep-equiv*

**by** *auto*

**qed**

**lemma** *cbig-big-equiv*:

$(\exists coll. (\sigma', coll) \in cbig\ P\ \sigma) \longleftrightarrow \sigma' \in big\ P\ \sigma$

**proof**(*rule iffI*)

**assume**  $\exists coll. (\sigma', coll) \in cbig\ P\ \sigma$

**then show**  $\sigma' \in big\ P\ \sigma$  **by** (*auto simp: big-def cbig-def small-nstep-csmall-nstep-equiv*)

**next**

**assume**  $\sigma' \in big\ P\ \sigma$

**then show**  $\exists coll. (\sigma', coll) \in cbig\ P\ \sigma$  **by** (*fastforce simp: cbig-def dest: csmall-nstep-exists*)

**qed**

**lemma** *cbig-big-implies*:

$(\sigma', coll) \in cbig\ P\ \sigma \Longrightarrow \sigma' \in big\ P\ \sigma$

**using** *cbig-big-equiv by blast*

**lemma** *csmall-to-cbig-prop*:

**assumes**  $\bigwedge P\ \sigma\ \sigma'\ coll. (\sigma', coll) \in csmall\ P\ \sigma \Longrightarrow R\ P\ coll \Longrightarrow Q\ P\ \sigma \Longrightarrow R'\ P\ \sigma\ \sigma'\ coll$

**and**  $\bigwedge P\ coll1\ coll2. R\ P\ (combine\ coll1\ coll2) = (R\ P\ coll1 \wedge R\ P\ coll2)$

**and**  $\bigwedge P\ \sigma\ \sigma'. Q\ P\ \sigma \Longrightarrow \sigma' \in small\ P\ \sigma \Longrightarrow Q\ P\ \sigma'$

**and**  $\bigwedge P \sigma \sigma 1 \sigma' coll1 coll2.$   
 $R' P \sigma \sigma 1 coll1 \wedge R' P \sigma 1 \sigma' coll2 \implies R' P \sigma \sigma' (combine$   
 $coll1 coll2)$   
**and**  $\bigwedge \sigma. R' P \sigma \sigma collect-id$   
**shows**  $(\sigma', coll) \in cbig P \sigma \implies R P coll \implies Q P \sigma \implies R' P \sigma \sigma' coll$   
**using** *assms csmall-to-csmall-nstep-prop*[**where**  $R=R$  **and**  $Q=Q$  **and**  $R'=R'$  **and**  
 $\sigma=\sigma$ ]  
**by**(*auto simp: cbig-def2*)

**lemma** *csmall-to-cbig-prop2*:  
**assumes**  $\bigwedge P P' \sigma \sigma' coll. (\sigma', coll) \in csmall P \sigma \implies R P P' coll \implies Q \sigma \implies$   
 $(\sigma', coll) \in csmall P' \sigma$   
**and**  $\bigwedge P P' coll1 coll2. R P P' (combine coll1 coll2) = (R P P' coll1 \wedge R P P'$   
 $coll2)$   
**and**  $Qpres: \bigwedge P \sigma \sigma'. Q \sigma \implies \sigma' \in small P \sigma \implies Q \sigma'$   
**shows**  $(\sigma', coll) \in cbig P \sigma \implies R P P' coll \implies Q \sigma \implies (\sigma', coll) \in cbig P' \sigma$   
**using** *assms csmall-to-csmall-nstep-prop2*[**where**  $R=R$  **and**  $Q=Q$ ] **by**(*auto simp:*  
*cbig-def2*) *blast*

**lemma** *cbig-stepD*:  
**assumes** *cbig*:  $(\sigma', coll') \in cbig P \sigma$  **and** *nend*:  $\sigma \notin endset$   
**shows**  $\exists \sigma 1 coll1. (\sigma 1, coll1) \in csmall P \sigma$   
 $\wedge (\exists coll. coll' = combine coll1 coll \wedge (\sigma', coll') \in cbig P \sigma 1)$   
**proof** –  
**obtain**  $n$  **where**  $n: (\sigma', coll') \in csmall-nstep P \sigma n \sigma' \in endset$   
**using** *cbig-def2 cbig by auto*  
**then show** *?thesis using csmall-nstep-SucD nend cbig-def2 by*(*cases n, simp*)  
*blast*  
**qed**

**lemma** *csmall-nstep-det-last-eq*:  
**assumes** *det*:  $\forall \sigma. small P \sigma = \{\} \vee (\exists \sigma'. small P \sigma = \{\sigma'\})$   
**shows**  $\llbracket (\sigma', coll') \in cbig P \sigma; (\sigma', coll') \in csmall-nstep P \sigma n; (\sigma', coll'') \in cs-$   
 $mall-nstep P \sigma n' \rrbracket$   
 $\implies n = n'$   
**proof**(*induct n arbitrary: n' \sigma \sigma' coll' coll''*)  
**case** 0  
**have**  $\sigma' = \sigma$  **using** 0.prem(2) *csmall-nstep-base by blast*  
**then have** *endset*:  $\sigma \in endset$  **using** 0.prem(1) *cbigD by blast*  
**show** *?case*  
**proof**(*cases n'*)  
**case** *Suc* **then show** *?thesis using 0.prem(3) csmall-nstep-Suc-nend endset*  
**by** *blast*  
**qed**(*simp*)  
**next**  
**case** (*Suc n1*)  
**then have** *endset*:  $\sigma' \in endset$  **using** *Suc.prem(1) cbigD by blast*

```

have nend:  $\sigma \notin \text{endset}$  using csmall-nstep-Suc-nend[OF Suc.prems(2)] by simp
then have neq:  $\sigma' \neq \sigma$  using endset by auto
obtain  $\sigma 1 \text{ coll } \text{coll1}$  where  $\sigma 1$ :  $(\sigma 1, \text{coll1}) \in \text{csmall } P \sigma \text{ coll}' = \text{combine } \text{coll1}$ 
coll
   $(\sigma', \text{coll}) \in \text{csmall-nstep } P \sigma 1 n1$ 
  using csmall-nstep-SucD[OF Suc.prems(2)] by clarsimp
then have cbig:  $(\sigma', \text{coll}) \in \text{cbig } P \sigma 1$  using endset by(auto simp: cbig-def)
show ?case
proof(cases n^)
  case 0 then show ?thesis using neq Suc.prems(3) using csmall-nstep-base by
simp
  next
    case Suc': (Suc n1')
      then obtain  $\sigma 1' \text{ coll2 } \text{coll1}'$  where  $\sigma 1'$ :  $(\sigma 1', \text{coll1}') \in \text{csmall } P \sigma \text{ coll}'' =$ 
combine coll1' coll2
         $(\sigma', \text{coll2}) \in \text{csmall-nstep } P \sigma 1' n1'$ 
        using csmall-nstep-SucD[where  $\sigma = \sigma$  and  $\sigma' = \sigma'$  and  $\text{coll}' = \text{coll}''$  and  $n = n1'$ ]
Suc.prems(3) by blast
        then have  $\sigma 1 = \sigma 1'$  using  $\sigma 1 \text{ det csmall-def}$  by auto
        then show ?thesis using Suc.hyps(1)[OF cbig  $\sigma 1(3)$ ]  $\sigma 1'(3)$  Suc' by blast
      qed
    qed
end — CollectionSemantics
end

```

## 5 Collection-based RTS

```

theory CollectionBasedRTS
imports RTS-safe CollectionSemantics
begin

```

```

locale CollectionBasedRTS-base = RTS-safe + CollectionSemantics

```

General model for Regression Test Selection based on *CollectionSemantics*:

```

locale CollectionBasedRTS = CollectionBasedRTS-base where

```

```

  small = small :: 'prog  $\Rightarrow$  'state  $\Rightarrow$  'state set and
  collect = collect :: 'prog  $\Rightarrow$  'state  $\Rightarrow$  'state  $\Rightarrow$  'coll and
  out = out :: 'prog  $\Rightarrow$  'test  $\Rightarrow$  ('state  $\times$  'coll) set
for small collect out

```

+

**fixes**

```

  make-test-prog :: 'prog  $\Rightarrow$  'test  $\Rightarrow$  'prog and
  collect-start :: 'prog  $\Rightarrow$  'coll

```

**assumes**

*out-cbig*:

```

   $\exists i. \text{out } P t = \{(\sigma', \text{coll}'). \exists \text{coll}. (\sigma', \text{coll}) \in \text{cbig } (\text{make-test-prog } P t) i$ 
     $\wedge \text{coll}' = \text{combine } \text{coll } (\text{collect-start } P) \}$ 

```

**context** *CollectionBasedRTS* **begin**

**end** — *CollectionBasedRTS*

**end**

## 6 Instantiating *Semantics* with Jinja JVM

**theory** *JVMSemantics*

**imports** *../Common/Semantics JinjaDCI.JVMExec*

**begin**

**fun** *JVMsmall* :: *jvm-prog*  $\Rightarrow$  *jvm-state*  $\Rightarrow$  *jvm-state set* **where**  
*JVMsmall* *P*  $\sigma$  = {  $\sigma'$ . *exec* (*P*,  $\sigma$ ) = *Some*  $\sigma'$  }

**lemma** *JVMsmall-prealloc-pres*:

**assumes** *pre*: *preallocated* (*fst*(*snd*  $\sigma$ ))

**and**  $\sigma' \in$  *JVMsmall* *P*  $\sigma$

**shows** *preallocated* (*fst*(*snd*  $\sigma'$ ))

**using** *exec-prealloc-pres*[*OF pre*] *assms* **by**(*cases*  $\sigma$ , *cases*  $\sigma'$ , *auto*)

**lemma** *JVMsmall-det*: *JVMsmall* *P*  $\sigma$  = {}  $\vee$  ( $\exists \sigma'$ . *JVMsmall* *P*  $\sigma$  = { $\sigma'$ })

**by** *auto*

**definition** *JVMendset* :: *jvm-state set* **where**

*JVMendset*  $\equiv$  { (*xp*,*h*,*frs*,*sh*). *frs* = []  $\vee$  ( $\exists x$ . *xp* = *Some* *x*) }

**lemma** *JVMendset-final*:  $\sigma \in$  *JVMendset*  $\implies \forall P$ . *JVMsmall* *P*  $\sigma$  = {}

**by**(*auto simp: JVMendset-def*)

**lemma** *start-state-nend*:

*start-state* *P*  $\notin$  *JVMendset*

**by**(*simp add: start-state-def JVMendset-def*)

**interpretation** *JVMSemantics*: *Semantics* *JVMsmall* *JVMendset*

**by** *unfold-locales* (*auto dest: JVMendset-final*)

**end**

## 7 *classes-changed* theory

**theory** *ClassesChanged*

**imports** *JinjaDCI.Decl*

**begin**

A class is considered changed if it exists only in one program or the other,  
or exists in both but is different.

**definition** *classes-changed* :: 'm prog ⇒ 'm prog ⇒ cname set **where**  
*classes-changed* P1 P2 = {cn. class P1 cn ≠ class P2 cn}

**definition** *class-changed* :: 'm prog ⇒ 'm prog ⇒ cname ⇒ bool **where**  
*class-changed* P1 P2 cn = (class P1 cn ≠ class P2 cn)

**lemma** *classes-changed-class-changed*[simp]: cn ∈ *classes-changed* P1 P2 = *class-changed* P1 P2 cn  
**by** (*simp add: classes-changed-def class-changed-def*)

**lemma** *classes-changed-self*[simp]: *classes-changed* P P = {}  
**by** (*auto simp: class-changed-def*)

**lemma** *classes-changed-sym*: *classes-changed* P P' = *classes-changed* P' P  
**by** (*auto simp: class-changed-def*)

**lemma** *classes-changed-class*: [ cn ∉ *classes-changed* P P' ] ⇒ class P cn = class P' cn  
**by** (*clarsimp simp: class-changed-def*)

**lemma** *classes-changed-class-set*: [ S ∩ *classes-changed* P P' = {} ]  
⇒ ∀ C ∈ S. class P C = class P' C  
**by** (*fastforce simp: disjoint-iff-not-equal dest: classes-changed-class*)

We now relate *classes-changed* over two programs to those over programs with an added class (such as a test class).

**lemma** *classes-changed-cons-eq*:  
*classes-changed* (t # P) P' = (*classes-changed* P P' - {fst t})  
∪ (if *class-changed* [t] P' (fst t) then {fst t} else {})  
**by** (*auto simp: classes-changed-def class-changed-def class-def*)

**lemma** *class-changed-cons*:  
fst t ∉ *classes-changed* (t#P) (t#P')  
**by** (*simp add: class-changed-def class-def*)

**lemma** *classes-changed-cons*:  
*classes-changed* (t # P) (t # P') = *classes-changed* P P' - {fst t}  
**proof**(cases fst t ∈ *classes-changed* P P')  
**case** True  
**then show** ?thesis **using** *class-changed-cons*[**where** t=t **and** P=P **and** P'=P']  
*classes-changed-cons-eq*[**where** t=t] **by** (*auto simp: class-changed-def class-cons*)  
**next**  
**case** False  
**then show** ?thesis **using** *class-changed-cons*[**where** t=t **and** P=P **and** P'=P']  
**by** (*auto simp: class-changed-def*) (*metis (no-types, lifting) class-cons*)  
**qed**

**lemma** *classes-changed-int-Cons*:  
**assumes** coll ∩ *classes-changed* P P' = {}



```

shows  $coll \cap \text{classes-changed } (t \# P) (t \# P') = \{\}$ 
proof(cases fst t ∈ classes-changed P P')
  case True
    then have  $\text{classes-changed } P P' = \text{classes-changed } (t \# P) (t \# P') \cup \{\text{fst } t\}$ 
      using classes-changed-cons[where t=t and P=P and P'=P'] by fastforce
    then show ?thesis using assms by simp
  next
    case False
    then have  $\text{classes-changed } P P' = \text{classes-changed } (t \# P) (t \# P')$ 
      using classes-changed-cons[where t=t and P=P and P'=P'] by fastforce
    then show ?thesis using assms by simp
qed
end

```

## 8 subcls theory

```

theory Subcls
imports JinjaDCI.TypeRel
begin

```

```

lemma subcls-class-ex:  $\llbracket P \vdash C \preceq^* C'; C \neq C' \rrbracket$ 
   $\implies \exists D' \text{ fs } ms. \text{class } P C = \llbracket (D', \text{fs}, ms) \rrbracket$ 
proof(induct rule: converse-rtrancl-induct)
  case (step y z) then show ?case by(auto dest: subcls1D)
qed(simp)

```

```

lemma class-subcls1:
 $\llbracket \text{class } P y = \text{class } P' y; P \vdash y \prec^1 z \rrbracket \implies P' \vdash y \prec^1 z$ 
by(auto dest!: subcls1D intro!: subcls1I intro: sym)

```

```

lemma subcls1-single-valued: single-valued (subcls1 P)
by (clarsimp simp: single-valued-def subcls1.simps)

```

```

lemma subcls-confluent:
 $\llbracket P \vdash C \preceq^* C'; P \vdash C \preceq^* C'' \rrbracket \implies P \vdash C' \preceq^* C'' \vee P \vdash C'' \preceq^* C'$ 
by (simp add: single-valued-confluent subcls1-single-valued)

```

```

lemma subcls1-confluent:  $\llbracket P \vdash a \prec^1 b; P \vdash a \preceq^* c; a \neq c \rrbracket \implies P \vdash b \preceq^* c$ 
using subcls1-single-valued
by (auto elim!: converse-rtranclE[where x=a] simp: single-valued-def)

```

```

lemma subcls-self-superclass:  $\llbracket P \vdash C \prec^1 C; P \vdash C \preceq^* D \rrbracket \implies D = C$ 
using subcls1-single-valued
by (auto elim!: rtrancl-induct[where b=D] simp: single-valued-def)

```

```

lemma subcls-of-Obj-acyclic:

```

```

[[ P ⊢ C ≼* Object; C ≠ D ]] ⇒ ¬(P ⊢ C ≼* D ∧ P ⊢ D ≼* C)
proof(induct arbitrary: D rule: converse-rtrancl-induct)
  case base then show ?case by simp
next
  case (step y z) show ?case
  proof(cases y=z)
    case True with step show ?thesis by simp
  next
  case False show ?thesis
  proof(cases z = D)
    case True with False step.hyps(3)[of y] show ?thesis by clarsimp
  next
  case neg: False
  with step.hyps(3) have ¬(P ⊢ z ≼* D ∧ P ⊢ D ≼* z) by simp
  moreover from step.hyps(1)
  have P ⊢ D ≼* y ⇒ P ⊢ D ≼* z by(simp add: rtrancl-into-rtrancl)
  moreover from step.hyps(1) step.prem(1)
  have P ⊢ y ≼* D ⇒ P ⊢ z ≼* D by(simp add: subcls1-confluent)
  ultimately show ?thesis by clarsimp
  qed
qed
qed

```

```

lemma subcls-of-Obj: [[ P ⊢ C ≼* Object; P ⊢ C ≼* D ]] ⇒ P ⊢ D ≼* Object
by(auto dest: subcls-confluent)

```

**end**

## 9 classes-above theory

This section contains theory around the classes above (superclasses of) a class in the class structure, in particular noting that if their contents have not changed, then much of what that class sees (methods, fields) stays the same.

**theory** *ClassesAbove*

**imports** *ClassesChanged Subcls JinjaDCI.Exceptions*

**begin**

**abbreviation** *classes-above* :: 'm prog ⇒ cname ⇒ cname set **where**  
*classes-above* P c ≡ { cn. P ⊢ c ≼\* cn }

**abbreviation** *classes-between* :: 'm prog ⇒ cname ⇒ cname ⇒ cname set **where**  
*classes-between* P c d ≡ { cn. (P ⊢ c ≼\* cn ∧ P ⊢ cn ≼\* d) }

**abbreviation** *classes-above-xcpts* :: 'm prog ⇒ cname set **where**  
*classes-above-xcpts* P ≡ ⋃<sub>x∈sys-xcpts.</sub> *classes-above* P x

**lemma** *classes-above-def2*:

$P \vdash C \prec^1 D \implies \text{classes-above } P \ C = \{C\} \cup \text{classes-above } P \ D$   
**using** *subcls1-confluent* **by** *auto*

**lemma** *classes-above-class*:

$\llbracket \text{classes-above } P \ C \cap \text{classes-changed } P \ P' = \{\}; P \vdash C \preceq^* C' \rrbracket$   
 $\implies \text{class } P \ C' = \text{class } P' \ C'$   
**by** (*drule classes-changed-class-set, simp*)

**lemma** *classes-above-subset*:

**assumes**  $\text{classes-above } P \ C \cap \text{classes-changed } P \ P' = \{\}$   
**shows**  $\text{classes-above } P \ C \subseteq \text{classes-above } P' \ C$

**proof** –

**have** *ind*:  $\bigwedge x. P \vdash C \preceq^* x \implies P' \vdash C \preceq^* x$

**proof** –

**fix**  $x$  **assume** *sub*:  $P \vdash C \preceq^* x$

**then show**  $P' \vdash C \preceq^* x$

**proof**(*induct rule: rtrancl-induct*)

**case** *base* **then show** *?case* **by** *simp*

**next**

**case** (*step*  $y \ z$ )

**have**  $P' \vdash y \prec^1 z$  **by**(*rule class-subcls1[OF classes-above-class[OF assms step(1)] step(2)]*)

**then show** *?case* **using** *step(3)* **by** *simp*

**qed**

**qed**

**with** *classes-changed-class-set[OF assms]* **show** *?thesis* **by** *clarsimp*

**qed**

**lemma** *classes-above-subcls*:

$\llbracket \text{classes-above } P \ C \cap \text{classes-changed } P \ P' = \{\}; P \vdash C \preceq^* C' \rrbracket$   
 $\implies P' \vdash C \preceq^* C'$

**by** (*fastforce dest: classes-above-subset*)

**lemma** *classes-above-subset2*:

**assumes**  $\text{classes-above } P \ C \cap \text{classes-changed } P \ P' = \{\}$   
**shows**  $\text{classes-above } P' \ C \subseteq \text{classes-above } P \ C$

**proof** –

**have** *ind*:  $\bigwedge x. P' \vdash C \preceq^* x \implies P \vdash C \preceq^* x$

**proof** –

**fix**  $x$  **assume** *sub*:  $P' \vdash C \preceq^* x$

**then show**  $P \vdash C \preceq^* x$

**proof**(*induct rule: rtrancl-induct*)

**case** *base* **then show** *?case* **by** *simp*

**next**

**case** (*step*  $y \ z$ )

**with** *class-subcls1 classes-above-class[OF assms]* *rtrancl-into-rtrancl* **show** *?case* **by** *metis*

**qed**  
**qed**  
**with** *classes-changed-class-set*[*OF assms*] **show** *?thesis* **by** *clarsimp*  
**qed**

**lemma** *classes-above-subcls2*:  
 $\llbracket \text{classes-above } P \ C \cap \text{classes-changed } P \ P' = \{\}; P' \vdash C \preceq^* C' \rrbracket$   
 $\implies P \vdash C \preceq^* C'$   
**by** (*fastforce dest: classes-above-subset2*)

**lemma** *classes-above-set*:  
 $\llbracket \text{classes-above } P \ C \cap \text{classes-changed } P \ P' = \{\} \rrbracket$   
 $\implies \text{classes-above } P \ C = \text{classes-above } P' \ C$   
**by**(*fastforce dest: classes-above-subset classes-above-subset2*)

**lemma** *classes-above-classes-changed-sym*:  
**assumes** *classes-above*  $P \ C \cap \text{classes-changed } P \ P' = \{\}$   
**shows** *classes-above*  $P' \ C \cap \text{classes-changed } P' \ P = \{\}$   
**proof** –  
**have** *classes-above*  $P \ C = \text{classes-above } P' \ C$  **by**(*rule classes-above-set*[*OF assms*])  
**with** *classes-changed-sym*[**where**  $P=P$ ] *assms* **show** *?thesis* **by** *simp*  
**qed**

**lemma** *classes-above-sub-classes-between-eq*:  
 $P \vdash C \preceq^* D \implies \text{classes-above } P \ C = (\text{classes-between } P \ C \ D - \{D\}) \cup \text{classes-above } P \ D$   
**using** *subcls-confluent* **by** *auto*

**lemma** *classes-above-subcls-subset*:  
 $\llbracket P \vdash C \preceq^* C' \rrbracket \implies \text{classes-above } P \ C' \subseteq \text{classes-above } P \ C$   
**by** *auto*

## 9.1 Methods

**lemma** *classes-above-sees-methods*:  
**assumes** *int*: *classes-above*  $P \ C \cap \text{classes-changed } P \ P' = \{\}$  **and** *ms*:  $P \vdash C$   
*sees-methods*  $Mm$   
**shows**  $P' \vdash C$  *sees-methods*  $Mm$   
**proof** –  
**have** *cls*:  $\forall C' \in \text{classes-above } P \ C. \text{class } P \ C' = \text{class } P' \ C'$   
**by**(*rule classes-changed-class-set*[*OF int*])  
  
**have**  $\bigwedge C \ Mm. P \vdash C$  *sees-methods*  $Mm \implies$   
 $\forall C' \in \text{classes-above } P \ C. \text{class } P \ C' = \text{class } P' \ C' \implies P' \vdash C$   
*sees-methods*  $Mm$   
**proof** –  
**fix**  $C \ Mm$  **assume**  $P \vdash C$  *sees-methods*  $Mm$  **and**  $\forall C' \in \text{classes-above } P \ C. \text{class } P \ C' = \text{class } P' \ C'$   
**then show**  $P' \vdash C$  *sees-methods*  $Mm$

```

proof(induct rule: Methods.induct)
  case Obj: (sees-methods-Object  $D$   $fs$   $ms$   $Mm$ )
  with cls have class  $P' \text{ Object} = [(D, fs, ms)]$  by simp
  with Obj show ?case by(auto intro!: sees-methods-Object)
next
  case rec: (sees-methods-rec  $C$   $D$   $fs$   $ms$   $Mm$   $Mm'$ )
  then have  $P \vdash C \preceq^* D$  by (simp add: r-into-rtrancl[OF subcls1I])
  with converse-rtrancl-into-rtrancl have  $\bigwedge x. P \vdash D \preceq^* x \implies P \vdash C \preceq^* x$ 
by simp
  with rec.prem(1) have  $\forall C' \in \text{classes-above } P \ D. \text{class } P \ C' = \text{class } P' \ C'$  by
simp
  with rec show ?case by(auto intro: sees-methods-rec)
  qed
  qed
  with ms cls show ?thesis by simp
qed

```

**lemma** *classes-above-sees-method*:

```

[[ classes-above  $P$   $C \cap$  classes-changed  $P$   $P' = \{\}$ ;
   $P \vdash C \text{ sees } M, b: Ts \rightarrow T = m$  in  $C'$  ]]
 $\implies P' \vdash C \text{ sees } M, b: Ts \rightarrow T = m$  in  $C'$ 
by (auto dest: classes-above-sees-methods simp: Method-def)

```

**lemma** *classes-above-sees-method2*:

```

[[ classes-above  $P$   $C \cap$  classes-changed  $P$   $P' = \{\}$ ;
   $P' \vdash C \text{ sees } M, b: Ts \rightarrow T = m$  in  $C'$  ]]
 $\implies P \vdash C \text{ sees } M, b: Ts \rightarrow T = m$  in  $C'$ 
by (auto dest: classes-above-classes-changed-sym intro: classes-above-sees-method)

```

**lemma** *classes-above-method*:

```

assumes classes-above  $P$   $C \cap$  classes-changed  $P$   $P' = \{\}$ 
shows method  $P$   $C$   $M =$  method  $P'$   $C$   $M$ 
proof(cases  $\exists Ts \ T \ m \ D \ b. P \vdash C \text{ sees } M, b : Ts \rightarrow T = m$  in  $D$ )
  case True
  with assms show ?thesis by (auto dest: classes-above-sees-method)
next
  case False
  with assms have  $\neg(\exists Ts \ T \ m \ D \ b. P' \vdash C \text{ sees } M, b : Ts \rightarrow T = m$  in  $D$ )
  by (auto dest: classes-above-sees-method2)
  with False show ?thesis by(simp add: method-def)
qed

```

## 9.2 Fields

**lemma** *classes-above-has-fields*:

```

assumes int: classes-above  $P$   $C \cap$  classes-changed  $P$   $P' = \{\}$  and fs: P  $\vdash$   $C$ 
has-fields  $FDTs$ 
shows  $P' \vdash C$  has-fields  $FDTs$ 
proof –

```

**have**  $cls: \forall C' \in \text{classes-above } P \ C. \text{ class } P \ C' = \text{ class } P' \ C'$   
**by**(rule *classes-changed-class-set*[*OF int*])

**have**  $\bigwedge C \ Mm. P \vdash C \text{ has-fields } FDTs \implies$   
 $\forall C' \in \text{classes-above } P \ C. \text{ class } P \ C' = \text{ class } P' \ C' \implies P' \vdash C \text{ has-fields}$   
*FDTs*

**proof** –

**fix**  $C \ Mm$  **assume**  $P \vdash C \text{ has-fields } FDTs$  **and**  $\forall C' \in \text{classes-above } P \ C. \text{ class } P \ C' = \text{ class } P' \ C'$

**then show**  $P' \vdash C \text{ has-fields } FDTs$

**proof**(*induct rule: Fields.induct*)

**case** *Obj*: (*has-fields-Object*  $D \ fs \ ms \ FDTs$ )

**with**  $cls$  **have**  $\text{class } P' \ \text{Object} = \llbracket (D, fs, ms) \rrbracket$  **by** *simp*

**with** *Obj* **show** *?case* **by**(*auto intro!*: *has-fields-Object*)

**next**

**case** *rec*: (*has-fields-rec*  $C \ D \ fs \ ms \ FDTs \ FDTs'$ )

**then have**  $P \vdash C \preceq^* D$  **by** (*simp add: r-into-rtrancl*[*OF subcls1I*])

**with** *converse-rtrancl-into-rtrancl* **have**  $\bigwedge x. P \vdash D \preceq^* x \implies P \vdash C \preceq^* x$

**by** *simp*

**with** *rec.prem*(1) **have**  $\forall x. P \vdash D \preceq^* x \longrightarrow \text{class } P \ x = \text{class } P' \ x$  **by** *simp*

**with** *rec* **show** *?case* **by**(*auto intro: has-fields-rec*)

**qed**

**qed**

**with**  $fs \ cls$  **show** *?thesis* **by** *simp*

**qed**

**lemma** *classes-above-has-fields-dne*:

**assumes**  $\text{classes-above } P \ C \cap \text{classes-changed } P \ P' = \{\}$

**shows**  $(\forall FDTs. \neg P \vdash C \text{ has-fields } FDTs) = (\forall FDTs. \neg P' \vdash C \text{ has-fields } FDTs)$

**proof**(*rule iffI*)

**assume**  $asm: \forall FDTs. \neg P \vdash C \text{ has-fields } FDTs$

**from** *assms* *classes-changed-sym*[**where**  $P=P$ ] *classes-above-set*[*OF assms*]

**have** *int'*:  $\text{classes-above } P' \ C \cap \text{classes-changed } P' \ P = \{\}$  **by** *simp*

**from** *asm* *classes-above-has-fields*[*OF int'*] **show**  $\forall FDTs. \neg P' \vdash C \text{ has-fields}$   
*FDTs* **by** *auto*

**next**

**assume**  $\forall FDTs. \neg P' \vdash C \text{ has-fields } FDTs$

**with** *assms* **show**  $\forall FDTs. \neg P \vdash C \text{ has-fields } FDTs$  **by**(*auto dest: classes-above-has-fields*)

**qed**

**lemma** *classes-above-has-field*:

$\llbracket \text{classes-above } P \ C \cap \text{classes-changed } P \ P' = \{\};$   
 $P \vdash C \text{ has } F, b; t \text{ in } C' \rrbracket$   
 $\implies P' \vdash C \text{ has } F, b; t \text{ in } C'$

**by**(*auto dest: classes-above-has-fields simp: has-field-def*)

**lemma** *classes-above-has-field2*:

$\llbracket \text{classes-above } P \ C \cap \text{classes-changed } P \ P' = \{\};$   
 $P' \vdash C \text{ has } F, b; t \text{ in } C' \rrbracket$

$\Rightarrow P \vdash C \text{ has } F, b : t \text{ in } C'$   
**by**(*auto intro: classes-above-has-field dest: classes-above-classes-changed-sym*)

**lemma** *classes-above-sees-field*:  
 $\llbracket \text{classes-above } P \ C \cap \text{classes-changed } P \ P' = \{\} ;$   
 $P \vdash C \text{ sees } F, b : t \text{ in } C' \rrbracket$   
 $\Rightarrow P' \vdash C \text{ sees } F, b : t \text{ in } C'$   
**by**(*auto dest: classes-above-has-fields simp: sees-field-def*)

**lemma** *classes-above-sees-field2*:  
 $\llbracket \text{classes-above } P \ C \cap \text{classes-changed } P \ P' = \{\} ;$   
 $P' \vdash C \text{ sees } F, b : t \text{ in } C' \rrbracket$   
 $\Rightarrow P \vdash C \text{ sees } F, b : t \text{ in } C'$   
**by** (*auto intro: classes-above-sees-field dest: classes-above-classes-changed-sym*)

**lemma** *classes-above-field*:  
**assumes** *classes-above*  $P \ C \cap \text{classes-changed } P \ P' = \{\}$   
**shows** *field*  $P \ C \ F = \text{field } P' \ C \ F$   
**proof**(*cases*  $\exists T \ D \ b. P \vdash C \text{ sees } F, b : T \text{ in } D$ )  
  **case** *True*  
    **with** *assms show ?thesis by* (*auto dest: classes-above-sees-field*)  
  **next**  
    **case** *False*  
    **with** *assms have*  $\neg(\exists T \ D \ b. P' \vdash C \text{ sees } F, b : T \text{ in } D)$   
      **by** (*auto dest: classes-above-sees-field2*)  
    **with** *False show ?thesis by*(*simp add: field-def*)  
**qed**

**lemma** *classes-above-fields*:  
**assumes** *classes-above*  $P \ C \cap \text{classes-changed } P \ P' = \{\}$   
**shows** *fields*  $P \ C = \text{fields } P' \ C$   
**proof**(*cases*  $\exists \text{FDTs}. P \vdash C \text{ has-fields } \text{FDTs}$ )  
  **case** *True*  
    **with** *assms show ?thesis by*(*auto dest: classes-above-has-fields*)  
  **next**  
    **case** *False*  
    **with** *assms show ?thesis by* (*auto dest: classes-above-has-fields-dne simp: fields-def*)  
**qed**

**lemma** *classes-above-ifields*:  
 $\llbracket \text{classes-above } P \ C \cap \text{classes-changed } P \ P' = \{\} \rrbracket$   
 $\Rightarrow$   
 $\text{ifields } P \ C = \text{ifields } P' \ C$   
**by** (*simp add: ifields-def classes-above-fields*)

**lemma** *classes-above-blank*:  
 $\llbracket \text{classes-above } P \ C \cap \text{classes-changed } P \ P' = \{\} \rrbracket$   
 $\Rightarrow$

$blank\ P\ C = blank\ P'\ C$   
**by** (*simp add: blank-def classes-above-ifields*)

**lemma** *classes-above-isfields*:  
 $\llbracket\ classes-above\ P\ C \cap\ classes-changed\ P\ P' = \{\}\ \rrbracket$   
 $\implies$   
 $isfields\ P\ C = isfields\ P'\ C$   
**by** (*simp add: isfields-def classes-above-fields*)

**lemma** *classes-above-sblank*:  
 $\llbracket\ classes-above\ P\ C \cap\ classes-changed\ P\ P' = \{\}\ \rrbracket$   
 $\implies$   
 $sblank\ P\ C = sblank\ P'\ C$   
**by** (*simp add: sblank-def classes-above-isfields*)

### 9.3 Other

**lemma** *classes-above-start-heap*:  
**assumes** *classes-above-xcpts*  $P \cap classes-changed\ P\ P' = \{\}$   
**shows** *start-heap*  $P = start-heap\ P'$   
**proof** –  
**from** *assms have*  $\forall C \in sys-xcpts. blank\ P\ C = blank\ P'\ C$  **by** (*auto intro: classes-above-blank*)  
**then show** *?thesis* **by** (*simp add: start-heap-def*)  
**qed**  
**end**

## 10 Instantiating *CollectionSemantics* with Jinja JVM

**theory** *JVMCollectionSemantics*  
**imports** *../Common/CollectionSemantics JVMSemantics ../JinjaSuppl/ClassesAbove*  
**begin**

**abbreviation** *JVMcombine*  $::\ cname\ set \Rightarrow\ cname\ set \Rightarrow\ cname\ set$  **where**  
*JVMcombine*  $C\ C' \equiv C \cup C'$

**abbreviation** *JVMcollect-id*  $::\ cname\ set$  **where**  
*JVMcollect-id*  $\equiv \{\}$

### 10.1 JVM-specific *classes-above* theory

**fun** *classes-above-frames*  $::\ 'm\ prog \Rightarrow\ frame\ list \Rightarrow\ cname\ set$  **where**  
*classes-above-frames*  $P\ ((stk,loc,C,M,pc,ics)\#frs) = classes-above\ P\ C \cup classes-above-frames\ P\ frs$  |  
*classes-above-frames*  $P\ [] = \{\}$

**lemma** *classes-above-start-state*:



**assumes** *above-xcpts*:  $classes\text{-}above\text{-}xcpts\ P \cap classes\text{-}changed\ P\ P' = \{\}$   
**shows**  $start\text{-}state\ P = start\text{-}state\ P'$   
**using** *assms classes-above-start-heap* **by**(*simp add: start-state-def*)

**lemma** *classes-above-matches-ex-entry*:  
 $classes\text{-}above\ P\ C \cap classes\text{-}changed\ P\ P' = \{\}$   
 $\implies matches\text{-}ex\text{-}entry\ P\ C\ pc\ xcp = matches\text{-}ex\text{-}entry\ P'\ C\ pc\ xcp$   
**using** *classes-above-subcls classes-above-subcls2*  
**by**(*auto simp: matches-ex-entry-def*)

**lemma** *classes-above-match-ex-table*:  
**assumes**  $classes\text{-}above\ P\ C \cap classes\text{-}changed\ P\ P' = \{\}$   
**shows**  $match\text{-}ex\text{-}table\ P\ C\ pc\ es = match\text{-}ex\text{-}table\ P'\ C\ pc\ es$   
**using** *classes-above-matches-ex-entry*[*OF assms*] **proof**(*induct es*) **qed**(*auto*)

**lemma** *classes-above-find-handler*:  
**assumes**  $classes\text{-}above\ P\ (cname\text{-}of\ h\ a) \cap classes\text{-}changed\ P\ P' = \{\}$   
**shows**  $classes\text{-}above\text{-}frames\ P\ frs \cap classes\text{-}changed\ P\ P' = \{\}$   
 $\implies find\text{-}handler\ P\ a\ h\ frs\ sh = find\text{-}handler\ P'\ a\ h\ frs\ sh$   
**proof**(*induct frs*)  
**case** (*Cons fr' frs'*)  
**obtain** *stk loc C M pc ics* **where**  $fr': fr' = (stk, loc, C, M, pc, ics)$  **by**(*cases fr'*)  
**with** *Cons* **have**  
 $intC: classes\text{-}above\ P\ C \cap classes\text{-}changed\ P\ P' = \{\}$   
**and**  $int: classes\text{-}above\text{-}frames\ P\ frs' \cap classes\text{-}changed\ P\ P' = \{\}$  **by** *auto*  
**show** *?case* **using** *Cons fr' int classes-above-method*[*OF intC*]  
 $classes\text{-}above\text{-}match\text{-}ex\text{-}table$ [*OF assms(1)*] **by**(*auto split: bool.splits*)  
**qed**(*simp*)

**lemma** *find-handler-classes-above-frames*:  
 $find\text{-}handler\ P\ a\ h\ frs\ sh = (xp', h', frs', sh')$   
 $\implies classes\text{-}above\text{-}frames\ P\ frs' \subseteq classes\text{-}above\text{-}frames\ P\ frs$   
**proof**(*induct frs*)  
**case** (*Cons f1 frs1*)  
**then obtain** *stk loc C M pc ics* **where**  $f1: f1 = (stk, loc, C, M, pc, ics)$  **by**(*cases f1*)  
**show** *?case*  
**proof**(*cases match-ex-table P (cname-of h a) pc (ex-table-of P C M)*)  
**case** *None* **then show** *?thesis* **using** *f1 None Cons*  
**by**(*auto split: bool.splits list.splits init-call-status.splits*)  
**next**  
**case** (*Some a*) **then show** *?thesis* **using** *f1 Some Cons* **by** *auto*  
**qed**  
**qed**(*simp*)

**lemma** *find-handler-pieces*:  
 $find\text{-}handler\ P\ a\ h\ frs\ sh = (xp', h', frs', sh')$   
 $\implies h = h' \wedge sh = sh' \wedge classes\text{-}above\text{-}frames\ P\ frs' \subseteq classes\text{-}above\text{-}frames\ P\ frs$   
**using** *find-handler-classes-above-frames* **by**(*auto dest: find-handler-heap find-handler-sheap*)

## 10.2 Naive RTS algorithm

**fun** *JVMinstr-ncollect* ::

```
[jvm-prog, instr, heap, val list] ⇒ cname set where
JVMinstr-ncollect P (New C) h stk = classes-above P C |
JVMinstr-ncollect P (Getfield F C) h stk =
  (if (hd stk) = Null then {}
   else classes-above P (cname-of h (the-Addr (hd stk)))) |
JVMinstr-ncollect P (Getstatic C F D) h stk = classes-above P C |
JVMinstr-ncollect P (Putfield F C) h stk =
  (if (hd (tl stk)) = Null then {}
   else classes-above P (cname-of h (the-Addr (hd (tl stk)))) |
JVMinstr-ncollect P (Putstatic C F D) h stk = classes-above P C |
JVMinstr-ncollect P (Checkcast C) h stk =
  (if (hd stk) = Null then {}
   else classes-above P (cname-of h (the-Addr (hd stk)))) |
JVMinstr-ncollect P (Invoke M n) h stk =
  (if (stk ! n) = Null then {}
   else classes-above P (cname-of h (the-Addr (stk ! n)))) |
JVMinstr-ncollect P (Invokestatic C M n) h stk = classes-above P C |
JVMinstr-ncollect P Throw h stk =
  (if (hd stk) = Null then {}
   else classes-above P (cname-of h (the-Addr (hd stk)))) |
JVMinstr-ncollect P - h stk = {}
```

**fun** *JVMstep-ncollect* ::

```
[jvm-prog, heap, val list, cname, mname, pc, init-call-status] ⇒ cname set where
JVMstep-ncollect P h stk C M pc (Calling C' Cs) = classes-above P C' |
JVMstep-ncollect P h stk C M pc (Called (C'#Cs))
= classes-above P C' ∪ classes-above P (fst(method P C' clinit)) |
JVMstep-ncollect P h stk C M pc (Throwing Cs a) = classes-above P (cname-of h
a) |
JVMstep-ncollect P h stk C M pc ics = JVMinstr-ncollect P (instrs-of P C M !
pc) h stk
```

— naive collection function

**fun** *JVMexec-ncollect* :: *jvm-prog* ⇒ *jvm-state* ⇒ *cname set* **where**

```
JVMexec-ncollect P (None, h, (stk,loc,C,M,pc,ics)#frs, sh) =
  (JVMinstr-ncollect P h stk C M pc ics
   ∪ classes-above P C ∪ classes-above-frames P frs ∪ classes-above-xcpts P
  )
| JVMexec-ncollect P - = {}
```

**fun** *JVMNaiveCollect* :: *jvm-prog* ⇒ *jvm-state* ⇒ *jvm-state* ⇒ *cname set* **where**

```
JVMNaiveCollect P σ σ' = JVMexec-ncollect P σ
```

**interpretation** *JVMNaiveCollectionSemantics*:

*CollectionSemantics JVMsmall JVMendset JVMNaiveCollect JVMcombine JVM-*

*collect-id*  
**by** *unfold-locales auto*

### 10.3 Smarter RTS algorithm

**fun** *JVMinstr-scollect* ::  
 [*jvm-prog, instr*]  $\Rightarrow$  *cname set* **where**  
*JVMinstr-scollect* *P* (*Getstatic C F D*)  
 = (*if*  $\neg(\exists t. P \vdash C \text{ has } F, \text{Static}:t \text{ in } D)$  *then classes-above* *P C*  
   *else classes-between* *P C D - {D}*) |  
*JVMinstr-scollect* *P* (*Putstatic C F D*)  
 = (*if*  $\neg(\exists t. P \vdash C \text{ has } F, \text{Static}:t \text{ in } D)$  *then classes-above* *P C*  
   *else classes-between* *P C D - {D}*) |  
*JVMinstr-scollect* *P* (*Invokestatic C M n*)  
 = (*if*  $\neg(\exists Ts T m D. P \vdash C \text{ sees } M, \text{Static}:Ts \rightarrow T = m \text{ in } D)$  *then classes-above*  
*P C*  
   *else classes-between* *P C (fst(method P C M)) - {fst(method P C M)}*) |  
*JVMinstr-scollect* *P - = {}*

**fun** *JVMstep-scollect* ::  
 [*jvm-prog, instr, init-call-status*]  $\Rightarrow$  *cname set* **where**  
*JVMstep-scollect* *P i* (*Calling C' Cs*) = {*C'*} |  
*JVMstep-scollect* *P i* (*Called (C'#Cs)*) = {} |  
*JVMstep-scollect* *P i* (*Throwing Cs a*) = {} |  
*JVMstep-scollect* *P i ics* = *JVMinstr-scollect P i*

— smarter collection function

**fun** *JVMexec-scollect* :: *jvm-prog*  $\Rightarrow$  *jvm-state*  $\Rightarrow$  *cname set* **where**  
*JVMexec-scollect* *P* (*None, h, (stk,loc,C,M,pc,ics)#frs, sh*) =  
   *JVMstep-scollect P (instrs-of P C M ! pc) ics*  
 | *JVMexec-scollect P - = {}*

**fun** *JVMSmartCollect* :: *jvm-prog*  $\Rightarrow$  *jvm-state*  $\Rightarrow$  *jvm-state*  $\Rightarrow$  *cname set* **where**  
*JVMSmartCollect* *P*  $\sigma$   $\sigma'$  = *JVMexec-scollect P*  $\sigma$

**interpretation** *JVMSmartCollectionSemantics*:

*CollectionSemantics JVMsmall JVMendset JVMSmartCollect JVMcombine JVM-*  
*collect-id*  
**by** *unfold-locales*

### 10.4 A few lemmas using the instantiations

**lemma** *JVMnaive-csmallD*:

( $\sigma', cset$ )  $\in$  *JVMNaiveCollectionSemantics.csmall P*  $\sigma$   
 $\Rightarrow$  *JVMexec-ncollect P*  $\sigma$  = *cset*  $\wedge$   $\sigma' \in$  *JVMsmall P*  $\sigma$   
**by**(*simp add: JVMNaiveCollectionSemantics.csmall-def*)

**lemma** *JVMsmart-csmallD*:

$(\sigma', cset) \in \text{JVMSmartCollectionSemantics.csmall } P \sigma$   
 $\implies \text{JVExec-scollect } P \sigma = cset \wedge \sigma' \in \text{JVMSmall } P \sigma$   
**by**(*simp add: JVMSmartCollectionSemantics.csmall-def*)

**lemma** *jvm-naive-to-smart-csmall-nstep-last-eq*:

$\llbracket (\sigma', cset_n) \in \text{JVMSmartCollectionSemantics.cbig } P \sigma;$   
 $(\sigma', cset_n) \in \text{JVMSmartCollectionSemantics.csmall-nstep } P \sigma n;$   
 $(\sigma', cset_s) \in \text{JVMSmartCollectionSemantics.csmall-nstep } P \sigma n' \rrbracket$   
 $\implies n = n'$

**proof**(*induct n arbitrary: n'  $\sigma$   $\sigma'$   $cset_n$   $cset_s$* )

**case** 0

**have**  $\sigma' = \sigma$  **using** 0.premis(2) *JVMSmartCollectionSemantics.csmall-nstep-base*

**by** *blast*

**then have** *endset*:  $\sigma \in \text{JVMSendset}$  **using** 0.premis(1) *JVMSmartCollectionSemantics.cbigD* **by** *blast*

**show** ?*case*

**proof**(*cases n'*)

**case** *Suc* **then show** ?*thesis* **using** 0.premis(3) *JVMSmartCollectionSemantics.csmall-nstep-Suc-nend*

*endset* **by** *blast*

**qed**(*simp*)

**next**

**case** (*Suc n1*)

**then have** *endset*:  $\sigma' \in \text{JVMSendset}$  **using** *Suc.premis(1)* *JVMSmartCollectionSemantics.cbigD* **by** *blast*

**have** *nend*:  $\sigma \notin \text{JVMSendset}$

**using** *JVMSmartCollectionSemantics.csmall-nstep-Suc-nend[OF Suc.premis(2)]*

**by** *simp*

**then have** *neq*:  $\sigma' \neq \sigma$  **using** *endset* **by** *auto*

**obtain**  $\sigma 1$  *cset cset1* **where**  $\sigma 1$ :  $(\sigma 1, cset1) \in \text{JVMSmartCollectionSemantics.csmall } P \sigma$

$cset_n = cset1 \cup cset$   $(\sigma', cset) \in \text{JVMSmartCollectionSemantics.csmall-nstep } P \sigma 1 n1$

**using** *JVMSmartCollectionSemantics.csmall-nstep-SucD[OF Suc.premis(2)]* **by** *clarsimp*

**then have** *cbig*:  $(\sigma', cset) \in \text{JVMSmartCollectionSemantics.cbig } P \sigma 1$

**using** *endset* **by**(*auto simp: JVMSmartCollectionSemantics.cbig-def*)

**show** ?*case*

**proof**(*cases n'*)

**case** 0 **then show** ?*thesis*

**using** *neq Suc.premis(3)* *JVMSmartCollectionSemantics.csmall-nstep-base* **by**

*blast*

**next**

**case** *Suc'*: (*Suc n1'*)

**then obtain**  $\sigma 1'$  *cset2 cset1'* **where**  $\sigma 1'$ :  $(\sigma 1', cset1') \in \text{JVMSmartCollectionSemantics.csmall } P \sigma$

$cset_s = cset1' \cup cset2$   $(\sigma', cset2) \in \text{JVMSmartCollectionSemantics.csmall-nstep } P \sigma 1' n1'$

```

    using JVMSmartCollectionSemantics.csmall-nstep-SucD[where  $\sigma=\sigma$  and
 $\sigma'=\sigma'$  and  $coll'=cset_s$ 
      and  $n=n1'$ ] Suc.premis( $\mathcal{P}$ ) by blast
    then have  $\sigma1=\sigma1'$  using  $\sigma1$  JVMNaiveCollectionSemantics.csmall-def
      JVMSmartCollectionSemantics.csmall-def by auto
    then show ?thesis using Suc.hyps(1)[OF cbig  $\sigma1(\mathcal{P})$ ]  $\sigma1'(\mathcal{P})$  Suc' by blast
  qed
qed
end

```

## 11 Inductive JVM execution

```

theory JVMEExecStepInductive
imports JinjaDCI.JVMEExec
begin

```

```

datatype step-input = StepI instr |
                    StepC cname cname list | StepC2 cname cname list |
                    StepT cname list addr

```

```

inductive exec-step-ind :: [step-input, jvm-prog, heap, val list, val list,
                           cname, mname, pc, init-call-status, frame list, sheap.jvm-state]  $\Rightarrow$ 

```

```
bool
```

```
where
```

```
  exec-step-ind-Load:
```

```

exec-step-ind (StepI (Load n)) P h stk loc C0 M0 pc ics frs sh
  (None, h, ((loc ! n) # stk, loc, C0, M0, Suc pc, ics)#frs, sh)

```

```
| exec-step-ind-Store:
```

```

exec-step-ind (StepI (Store n)) P h stk loc C0 M0 pc ics frs sh
  (None, h, (tl stk, loc[n:=hd stk], C0, M0, Suc pc, ics)#frs, sh)

```

```
| exec-step-ind-Push:
```

```

exec-step-ind (StepI (Push v)) P h stk loc C0 M0 pc ics frs sh
  (None, h, (v # stk, loc, C0, M0, Suc pc, ics)#frs, sh)

```

```
| exec-step-ind-NewOOM-Called:
```

```

new-Addr h = None
 $\Rightarrow$  exec-step-ind (StepI (New C)) P h stk loc C0 M0 pc (Called Cs) frs sh
  ([addr-of-sys-xcpt OutOfMemory], h, (stk, loc, C0, M0, pc, No-ics)#frs, sh)

```

```
| exec-step-ind-NewOOM-Done:
```

```

[[ sh C = Some(obj, Done); new-Addr h = None;  $\forall$  Cs. ics  $\neq$  Called Cs ]]
 $\Rightarrow$  exec-step-ind (StepI (New C)) P h stk loc C0 M0 pc ics frs sh
  ([addr-of-sys-xcpt OutOfMemory], h, (stk, loc, C0, M0, pc, ics)#frs, sh)

```

```
| exec-step-ind-New-Called:
```

$new-Addr\ h = Some\ a$   
 $\implies exec-step-ind\ (StepI\ (New\ C))\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ (Called\ Cs)\ frs\ sh$   
 $(None, h(a \rightarrow blank\ P\ C), (Addr\ a \# stk, loc, C_0, M_0, Suc\ pc, No-ics) \# frs, sh)$

$| exec-step-ind-New-Done:$   
 $\llbracket sh\ C = Some(obj, Done); new-Addr\ h = Some\ a; \forall Cs. ics \neq Called\ Cs \rrbracket$   
 $\implies exec-step-ind\ (StepI\ (New\ C))\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ ics\ frs\ sh$   
 $(None, h(a \rightarrow blank\ P\ C), (Addr\ a \# stk, loc, C_0, M_0, Suc\ pc, ics) \# frs, sh)$

$| exec-step-ind-New-Init:$   
 $\llbracket \forall obj. sh\ C \neq Some(obj, Done); \forall Cs. ics \neq Called\ Cs \rrbracket$   
 $\implies exec-step-ind\ (StepI\ (New\ C))\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ ics\ frs\ sh$   
 $(None, h, (stk, loc, C_0, M_0, pc, Calling\ C\ []) \# frs, sh)$

$| exec-step-ind-Getfield-Null:$   
 $hd\ stk = Null$   
 $\implies exec-step-ind\ (StepI\ (Getfield\ F\ C))\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ ics\ frs\ sh$   
 $([addr-of-sys-xcpt\ NullPointer], h, (stk, loc, C_0, M_0, pc, ics) \# frs, sh)$

$| exec-step-ind-Getfield-NoField:$   
 $\llbracket v = hd\ stk; (D, fs) = the(h(the-Addr\ v)); v \neq Null; \neg(\exists t\ b. P \vdash D\ has\ F, b:t\ in\ C) \rrbracket$   
 $\implies exec-step-ind\ (StepI\ (Getfield\ F\ C))\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ ics\ frs\ sh$   
 $([addr-of-sys-xcpt\ NoSuchFieldError], h, (stk, loc, C_0, M_0, pc, ics) \# frs, sh)$

$| exec-step-ind-Getfield-Static:$   
 $\llbracket v = hd\ stk; (D, fs) = the(h(the-Addr\ v)); v \neq Null; P \vdash D\ has\ F, Static:t\ in\ C \rrbracket$   
 $\implies exec-step-ind\ (StepI\ (Getfield\ F\ C))\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ ics\ frs\ sh$   
 $([addr-of-sys-xcpt\ IncompatibleClassChangeError], h, (stk, loc, C_0, M_0, pc, ics) \# frs, sh)$

$| exec-step-ind-Getfield:$   
 $\llbracket v = hd\ stk; (D, fs) = the(h(the-Addr\ v)); (D', b, t) = field\ P\ C\ F; v \neq Null; P \vdash D\ has\ F, NonStatic:t\ in\ C \rrbracket$   
 $\implies exec-step-ind\ (StepI\ (Getfield\ F\ C))\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ ics\ frs\ sh$   
 $(None, h, (the(fs(F, C)) \# (tl\ stk), loc, C_0, M_0, pc+1, ics) \# frs, sh)$

$| exec-step-ind-Getstatic-NoField:$   
 $\neg(\exists t\ b. P \vdash C\ has\ F, b:t\ in\ D)$   
 $\implies exec-step-ind\ (StepI\ (Getstatic\ C\ F\ D))\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ ics\ frs\ sh$   
 $([addr-of-sys-xcpt\ NoSuchFieldError], h, (stk, loc, C_0, M_0, pc, ics) \# frs, sh)$

$| exec-step-ind-Getstatic-NonStatic:$   
 $P \vdash C\ has\ F, NonStatic:t\ in\ D$   
 $\implies exec-step-ind\ (StepI\ (Getstatic\ C\ F\ D))\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ ics\ frs\ sh$   
 $([addr-of-sys-xcpt\ IncompatibleClassChangeError], h, (stk, loc, C_0, M_0, pc, ics) \# frs, sh)$

$| exec-step-ind-Getstatic-Called:$

$\llbracket (D',b,t) = \text{field } P D F; P \vdash C \text{ has } F, \text{Static}:t \text{ in } D;$   
 $v = \text{the } ((\text{fst}(\text{the}(sh D')))) F \rrbracket$   
 $\implies \text{exec-step-ind } (\text{StepI } (\text{Getstatic } C F D)) P h \text{ stk loc } C_0 M_0 pc \text{ (Called } Cs)$   
 $\text{frs sh}$   
 $(None, h, (v\#stk, loc, C_0, M_0, Suc pc, No-ics)\#frs, sh)$

$| \text{exec-step-ind-Getstatic-Done:}$   
 $\llbracket (D',b,t) = \text{field } P D F; P \vdash C \text{ has } F, \text{Static}:t \text{ in } D;$   
 $\forall Cs. ics \neq \text{Called } Cs; sh D' = \text{Some}(sfs, Done);$   
 $v = \text{the } (sfs F) \rrbracket$   
 $\implies \text{exec-step-ind } (\text{StepI } (\text{Getstatic } C F D)) P h \text{ stk loc } C_0 M_0 pc ics \text{ frs sh}$   
 $(None, h, (v\#stk, loc, C_0, M_0, Suc pc, ics)\#frs, sh)$

$| \text{exec-step-ind-Getstatic-Init:}$   
 $\llbracket (D',b,t) = \text{field } P D F; P \vdash C \text{ has } F, \text{Static}:t \text{ in } D;$   
 $\forall sfs. sh D' \neq \text{Some}(sfs, Done); \forall Cs. ics \neq \text{Called } Cs \rrbracket$   
 $\implies \text{exec-step-ind } (\text{StepI } (\text{Getstatic } C F D)) P h \text{ stk loc } C_0 M_0 pc ics \text{ frs sh}$   
 $(None, h, (stk, loc, C_0, M_0, pc, Calling D' \square)\#frs, sh)$

$| \text{exec-step-ind-Putfield-Null:}$   
 $hd(tl \text{ stk}) = \text{Null}$   
 $\implies \text{exec-step-ind } (\text{StepI } (\text{Putfield } F C)) P h \text{ stk loc } C_0 M_0 pc ics \text{ frs sh}$   
 $(\lfloor \text{addr-of-sys-xcpt } \text{NullPointer} \rfloor, h, (stk, loc, C_0, M_0, pc, ics)\#frs, sh)$

$| \text{exec-step-ind-Putfield-NoField:}$   
 $\llbracket r = hd(tl \text{ stk}); a = \text{the-Addr } r; (D,fs) = \text{the } (h a); r \neq \text{Null}; \neg(\exists t b. P \vdash D \text{ has } F, b:t \text{ in } C) \rrbracket$   
 $\implies \text{exec-step-ind } (\text{StepI } (\text{Putfield } F C)) P h \text{ stk loc } C_0 M_0 pc ics \text{ frs sh}$   
 $(\lfloor \text{addr-of-sys-xcpt } \text{NoSuchFieldError} \rfloor, h, (stk, loc, C_0, M_0, pc, ics)\#frs, sh)$

$| \text{exec-step-ind-Putfield-Static:}$   
 $\llbracket r = hd(tl \text{ stk}); a = \text{the-Addr } r; (D,fs) = \text{the } (h a); r \neq \text{Null}; P \vdash D \text{ has } F, \text{Static}:t$   
 $\text{in } C \rrbracket$   
 $\implies \text{exec-step-ind } (\text{StepI } (\text{Putfield } F C)) P h \text{ stk loc } C_0 M_0 pc ics \text{ frs sh}$   
 $(\lfloor \text{addr-of-sys-xcpt } \text{IncompatibleClassChangeError} \rfloor, h, (stk, loc, C_0, M_0, pc, ics)\#frs,$   
 $sh)$

$| \text{exec-step-ind-Putfield:}$   
 $\llbracket v = hd \text{ stk}; r = hd(tl \text{ stk}); a = \text{the-Addr } r; (D,fs) = \text{the } (h a); (D',b,t) = \text{field } P$   
 $C F;$   
 $r \neq \text{Null}; P \vdash D \text{ has } F, \text{NonStatic}:t \text{ in } C \rrbracket$   
 $\implies \text{exec-step-ind } (\text{StepI } (\text{Putfield } F C)) P h \text{ stk loc } C_0 M_0 pc ics \text{ frs sh}$   
 $(None, h(a \mapsto (D, fs((F, C) \mapsto v))), (tl (tl \text{ stk}), loc, C_0, M_0, pc+1, ics)\#frs, sh)$

$| \text{exec-step-ind-Putstatic-NoField:}$   
 $\neg(\exists t b. P \vdash C \text{ has } F, b:t \text{ in } D)$   
 $\implies \text{exec-step-ind } (\text{StepI } (\text{Putstatic } C F D)) P h \text{ stk loc } C_0 M_0 pc ics \text{ frs sh}$   
 $(\lfloor \text{addr-of-sys-xcpt } \text{NoSuchFieldError} \rfloor, h, (stk, loc, C_0, M_0, pc, ics)\#frs, sh)$

| *exec-step-ind-Putstatic-NonStatic*:

$P \vdash C \text{ has } F, \text{NonStatic}:t \text{ in } D$

$\implies \text{exec-step-ind} (\text{StepI} (\text{Putstatic } C F D)) P h \text{ stk loc } C_0 M_0 \text{ pc ics frs sh}$   
 $(\lfloor \text{addr-of-sys-xcpt IncompatibleClassChangeError} \rfloor, h, (\text{stk}, \text{loc}, C_0, M_0, \text{pc}, \text{ics})\#\text{frs}, \text{sh})$

| *exec-step-ind-Putstatic-Called*:

$\llbracket (D', b, t) = \text{field } P D F; P \vdash C \text{ has } F, \text{Static}:t \text{ in } D; \text{the}(\text{sh } D') = (sfs, i) \rrbracket$

$\implies \text{exec-step-ind} (\text{StepI} (\text{Putstatic } C F D)) P h \text{ stk loc } C_0 M_0 \text{ pc} (\text{Called } Cs)$   
 $\text{frs sh}$   
 $(\text{None}, h, (\text{tl } \text{stk}, \text{loc}, C_0, M_0, \text{Suc } \text{pc}, \text{No-ics})\#\text{frs}, \text{sh}(D' := \text{Some} ((sfs(F \mapsto \text{hd } \text{stk}), i))))$

| *exec-step-ind-Putstatic-Done*:

$\llbracket (D', b, t) = \text{field } P D F; P \vdash C \text{ has } F, \text{Static}:t \text{ in } D;$

$\forall Cs. \text{ics} \neq \text{Called } Cs; \text{sh } D' = \text{Some} (sfs, \text{Done}) \rrbracket$

$\implies \text{exec-step-ind} (\text{StepI} (\text{Putstatic } C F D)) P h \text{ stk loc } C_0 M_0 \text{ pc ics frs sh}$   
 $(\text{None}, h, (\text{tl } \text{stk}, \text{loc}, C_0, M_0, \text{Suc } \text{pc}, \text{ics})\#\text{frs}, \text{sh}(D' := \text{Some} ((sfs(F \mapsto \text{hd } \text{stk}), \text{Done}))))$

| *exec-step-ind-Putstatic-Init*:

$\llbracket (D', b, t) = \text{field } P D F; P \vdash C \text{ has } F, \text{Static}:t \text{ in } D;$

$\forall sfs. \text{sh } D' \neq \text{Some} (sfs, \text{Done}); \forall Cs. \text{ics} \neq \text{Called } Cs \rrbracket$

$\implies \text{exec-step-ind} (\text{StepI} (\text{Putstatic } C F D)) P h \text{ stk loc } C_0 M_0 \text{ pc ics frs sh}$   
 $(\text{None}, h, (\text{stk}, \text{loc}, C_0, M_0, \text{pc}, \text{Calling } D' \square)\#\text{frs}, \text{sh})$

| *exec-step-ind-Checkcast*:

$\text{cast-ok } P C h (\text{hd } \text{stk})$

$\implies \text{exec-step-ind} (\text{StepI} (\text{Checkcast } C)) P h \text{ stk loc } C_0 M_0 \text{ pc ics frs sh}$   
 $(\text{None}, h, (\text{stk}, \text{loc}, C_0, M_0, \text{Suc } \text{pc}, \text{ics})\#\text{frs}, \text{sh})$

| *exec-step-ind-Checkcast-Error*:

$\neg \text{cast-ok } P C h (\text{hd } \text{stk})$

$\implies \text{exec-step-ind} (\text{StepI} (\text{Checkcast } C)) P h \text{ stk loc } C_0 M_0 \text{ pc ics frs sh}$   
 $(\lfloor \text{addr-of-sys-xcpt ClassCast} \rfloor, h, (\text{stk}, \text{loc}, C_0, M_0, \text{pc}, \text{ics})\#\text{frs}, \text{sh})$

| *exec-step-ind-Invoke-Null*:

$\text{stk!n} = \text{Null}$

$\implies \text{exec-step-ind} (\text{StepI} (\text{Invoke } M n)) P h \text{ stk loc } C_0 M_0 \text{ pc ics frs sh}$   
 $(\lfloor \text{addr-of-sys-xcpt NullPointerException} \rfloor, h, (\text{stk}, \text{loc}, C_0, M_0, \text{pc}, \text{ics})\#\text{frs}, \text{sh})$

| *exec-step-ind-Invoke-NoMethod*:

$\llbracket r = \text{stk!n}; C = \text{fst}(\text{the}(\text{h}(\text{the-Addr } r))); r \neq \text{Null};$

$\neg(\exists Ts T m D b. P \vdash C \text{ sees } M, b:Ts \rightarrow T = m \text{ in } D) \rrbracket$

$\implies \text{exec-step-ind} (\text{StepI} (\text{Invoke } M n)) P h \text{ stk loc } C_0 M_0 \text{ pc ics frs sh}$   
 $(\lfloor \text{addr-of-sys-xcpt NoSuchMethodError} \rfloor, h, (\text{stk}, \text{loc}, C_0, M_0, \text{pc}, \text{ics})\#\text{frs}, \text{sh})$

| *exec-step-ind-Invoke-Static*:

$\llbracket r = \text{stk!n}; C = \text{fst}(\text{the}(\text{h}(\text{the-Addr } r)));$



$(D, b, Ts, T, mxs, mxl_0, ins, xt) = \text{method } P \ C \ M; r \neq \text{Null};$   
 $P \vdash C \text{ sees } M, \text{Static}: Ts \rightarrow T = m \text{ in } D$  ]  
 $\Rightarrow \text{exec-step-ind } (\text{StepI } (\text{Invoke } M \ n)) \ P \ h \ stk \ loc \ C_0 \ M_0 \ pc \ ics \ frs \ sh$   
 $(\lfloor \text{addr-of-sys-xcpt } \text{IncompatibleClassChangeError} \rfloor, h, (stk, loc, C_0, M_0, pc, ics) \# frs, sh)$

$| \text{exec-step-ind-Invoke:}$   
 $\llbracket ps = \text{take } n \ stk; r = \text{stk}!n; C = \text{fst}(\text{the}(h(\text{the-Addr } r)));$   
 $(D, b, Ts, T, mxs, mxl_0, ins, xt) = \text{method } P \ C \ M; r \neq \text{Null};$   
 $P \vdash C \text{ sees } M, \text{NonStatic}: Ts \rightarrow T = m \text{ in } D;$   
 $f' = (\llbracket, [r] \text{@}(\text{rev } ps) \text{@}(\text{replicate } mxl_0 \ \text{undefined}), D, M, 0, \text{No-ics}) \rrbracket$   
 $\Rightarrow \text{exec-step-ind } (\text{StepI } (\text{Invoke } M \ n)) \ P \ h \ stk \ loc \ C_0 \ M_0 \ pc \ ics \ frs \ sh$   
 $(\text{None}, h, f' \# (stk, loc, C_0, M_0, pc, ics) \# frs, sh)$

$| \text{exec-step-ind-Invokestatic-NoMethod:}$   
 $\llbracket (D, b, Ts, T, mxs, mxl_0, ins, xt) = \text{method } P \ C \ M; \neg(\exists Ts \ T \ m \ D \ b. P \vdash C \text{ sees } M, b: Ts$   
 $\rightarrow T = m \text{ in } D) \rrbracket$   
 $\Rightarrow \text{exec-step-ind } (\text{StepI } (\text{Invokestatic } C \ M \ n)) \ P \ h \ stk \ loc \ C_0 \ M_0 \ pc \ ics \ frs \ sh$   
 $(\lfloor \text{addr-of-sys-xcpt } \text{NoSuchMethodError} \rfloor, h, (stk, loc, C_0, M_0, pc, ics) \# frs, sh)$

$| \text{exec-step-ind-Invokestatic-NonStatic:}$   
 $\llbracket (D, b, Ts, T, mxs, mxl_0, ins, xt) = \text{method } P \ C \ M; P \vdash C \text{ sees } M, \text{NonStatic}: Ts \rightarrow T$   
 $= m \text{ in } D \rrbracket$   
 $\Rightarrow \text{exec-step-ind } (\text{StepI } (\text{Invokestatic } C \ M \ n)) \ P \ h \ stk \ loc \ C_0 \ M_0 \ pc \ ics \ frs \ sh$   
 $(\lfloor \text{addr-of-sys-xcpt } \text{IncompatibleClassChangeError} \rfloor, h, (stk, loc, C_0, M_0, pc, ics) \# frs, sh)$

$| \text{exec-step-ind-Invokestatic-Called:}$   
 $\llbracket ps = \text{take } n \ stk; (D, b, Ts, T, mxs, mxl_0, ins, xt) = \text{method } P \ C \ M;$   
 $P \vdash C \text{ sees } M, \text{Static}: Ts \rightarrow T = m \text{ in } D;$   
 $f' = (\llbracket, (\text{rev } ps) \text{@}(\text{replicate } mxl_0 \ \text{undefined}), D, M, 0, \text{No-ics}) \rrbracket$   
 $\Rightarrow \text{exec-step-ind } (\text{StepI } (\text{Invokestatic } C \ M \ n)) \ P \ h \ stk \ loc \ C_0 \ M_0 \ pc \ (\text{Called } Cs)$   
 $frs \ sh$   
 $(\text{None}, h, f' \# (stk, loc, C_0, M_0, pc, \text{No-ics}) \# frs, sh)$

$| \text{exec-step-ind-Invokestatic-Done:}$   
 $\llbracket ps = \text{take } n \ stk; (D, b, Ts, T, mxs, mxl_0, ins, xt) = \text{method } P \ C \ M;$   
 $P \vdash C \text{ sees } M, \text{Static}: Ts \rightarrow T = m \text{ in } D;$   
 $\forall Cs. ics \neq \text{Called } Cs; sh \ D = \text{Some } (sfs, \text{Done});$   
 $f' = (\llbracket, (\text{rev } ps) \text{@}(\text{replicate } mxl_0 \ \text{undefined}), D, M, 0, \text{No-ics}) \rrbracket$   
 $\Rightarrow \text{exec-step-ind } (\text{StepI } (\text{Invokestatic } C \ M \ n)) \ P \ h \ stk \ loc \ C_0 \ M_0 \ pc \ ics \ frs \ sh$   
 $(\text{None}, h, f' \# (stk, loc, C_0, M_0, pc, ics) \# frs, sh)$

$| \text{exec-step-ind-Invokestatic-Init:}$   
 $\llbracket (D, b, Ts, T, mxs, mxl_0, ins, xt) = \text{method } P \ C \ M;$   
 $P \vdash C \text{ sees } M, \text{Static}: Ts \rightarrow T = m \text{ in } D;$   
 $\forall sfs. sh \ D \neq \text{Some } (sfs, \text{Done}); \forall Cs. ics \neq \text{Called } Cs \rrbracket$   
 $\Rightarrow \text{exec-step-ind } (\text{StepI } (\text{Invokestatic } C \ M \ n)) \ P \ h \ stk \ loc \ C_0 \ M_0 \ pc \ ics \ frs \ sh$   
 $(\text{None}, h, (stk, loc, C_0, M_0, pc, \text{Calling } D \ \llbracket) \# frs, sh)$

| *exec-step-ind-Return-Last-Init*:  
*exec-step-ind* (*StepI Return*) *P h stk<sub>0</sub> loc<sub>0</sub> C<sub>0</sub> clinit pc ics [] sh*  
*(None, h, [], sh(C<sub>0</sub>:=Some(fst(the(sh C<sub>0</sub>))), Done))*)

| *exec-step-ind-Return-Last*:  
*M<sub>0</sub> ≠ clinit*  
 $\implies$  *exec-step-ind* (*StepI Return*) *P h stk<sub>0</sub> loc<sub>0</sub> C<sub>0</sub> M<sub>0</sub> pc ics [] sh (None, h, [], sh)*

| *exec-step-ind-Return-Init*:  
 $\llbracket (D, b, Ts, T, m) = \text{method } P \ C_0 \ \text{clinit} \rrbracket$   
 $\implies$  *exec-step-ind* (*StepI Return*) *P h stk<sub>0</sub> loc<sub>0</sub> C<sub>0</sub> clinit pc ics ((stk', loc', C', m', pc', ics')#frs')*  
*sh*  
*(None, h, (stk', loc', C', m', pc', ics')#frs', sh(C<sub>0</sub>:=Some(fst(the(sh C<sub>0</sub>))), Done))*)

| *exec-step-ind-Return-NonStatic*:  
 $\llbracket (D, \text{NonStatic}, Ts, T, m) = \text{method } P \ C_0 \ M_0; M_0 \neq \text{clinit} \rrbracket$   
 $\implies$  *exec-step-ind* (*StepI Return*) *P h stk<sub>0</sub> loc<sub>0</sub> C<sub>0</sub> M<sub>0</sub> pc ics ((stk', loc', C', m', pc', ics')#frs')*  
*sh*  
*(None, h, ((hd stk<sub>0</sub>)#(drop (length Ts + 1) stk'), loc', C', m', Suc pc', ics')#frs', sh)*

| *exec-step-ind-Return-Static*:  
 $\llbracket (D, \text{Static}, Ts, T, m) = \text{method } P \ C_0 \ M_0; M_0 \neq \text{clinit} \rrbracket$   
 $\implies$  *exec-step-ind* (*StepI Return*) *P h stk<sub>0</sub> loc<sub>0</sub> C<sub>0</sub> M<sub>0</sub> pc ics ((stk', loc', C', m', pc', ics')#frs')*  
*sh*  
*(None, h, ((hd stk<sub>0</sub>)#(drop (length Ts) stk'), loc', C', m', Suc pc', ics')#frs', sh)*

| *exec-step-ind-Pop*:  
*exec-step-ind* (*StepI Pop*) *P h stk loc C<sub>0</sub> M<sub>0</sub> pc ics frs sh*  
*(None, h, (tl stk, loc, C<sub>0</sub>, M<sub>0</sub>, Suc pc, ics)#frs, sh)*

| *exec-step-ind-IAdd*:  
*exec-step-ind* (*StepI IAdd*) *P h stk loc C<sub>0</sub> M<sub>0</sub> pc ics frs sh*  
*(None, h, (Intg (the-Intg (hd (tl stk)) + the-Intg (hd stk))#(tl (tl stk)), loc, C<sub>0</sub>, M<sub>0</sub>, Suc pc, ics)#frs, sh)*

| *exec-step-ind-IfFalse-False*:  
*hd stk = Bool False*  
 $\implies$  *exec-step-ind* (*StepI (IfFalse i)*) *P h stk loc C<sub>0</sub> M<sub>0</sub> pc ics frs sh*  
*(None, h, (tl stk, loc, C<sub>0</sub>, M<sub>0</sub>, nat(int pc+i), ics)#frs, sh)*

| *exec-step-ind-IfFalse-nFalse*:  
*hd stk ≠ Bool False*  
 $\implies$  *exec-step-ind* (*StepI (IfFalse i)*) *P h stk loc C<sub>0</sub> M<sub>0</sub> pc ics frs sh*  
*(None, h, (tl stk, loc, C<sub>0</sub>, M<sub>0</sub>, Suc pc, ics)#frs, sh)*

| *exec-step-ind-CmpEq*:

*exec-step-ind (StepI CmpEq) P h stk loc C<sub>0</sub> M<sub>0</sub> pc ics frs sh*  
*(None, h, (Bool (hd (tl stk) = hd stk) # tl (tl stk), loc, C<sub>0</sub>, M<sub>0</sub>, Suc pc, ics)#frs,*  
*sh)*

| *exec-step-ind-Goto:*  
*exec-step-ind (StepI (Goto i)) P h stk loc C<sub>0</sub> M<sub>0</sub> pc ics frs sh*  
*(None, h, (stk, loc, C<sub>0</sub>, M<sub>0</sub>, nat(int pc+i), ics)#frs, sh)*

| *exec-step-ind-Throw:*  
*hd stk ≠ Null*  
 $\implies$  *exec-step-ind (StepI Throw) P h stk loc C<sub>0</sub> M<sub>0</sub> pc ics frs sh*  
*([the-Addr (hd stk)], h, (stk, loc, C<sub>0</sub>, M<sub>0</sub>, pc, ics)#frs, sh)*

| *exec-step-ind-Throw-Null:*  
*hd stk = Null*  
 $\implies$  *exec-step-ind (StepI Throw) P h stk loc C<sub>0</sub> M<sub>0</sub> pc ics frs sh*  
*([addr-of-sys-xcpt NullPointer], h, (stk, loc, C<sub>0</sub>, M<sub>0</sub>, pc, ics)#frs, sh)*

| *exec-step-ind-Init-None-Called:*  
 $\llbracket$  *sh C = None*  $\rrbracket$   
 $\implies$  *exec-step-ind (StepC C Cs) P h stk loc C<sub>0</sub> M<sub>0</sub> pc ics frs sh*  
*(None, h, (stk, loc, C<sub>0</sub>, M<sub>0</sub>, pc, Calling C Cs)#frs, sh(C := Some (sblank P*  
*C, Prepared)))*

| *exec-step-ind-Init-Done:*  
*sh C = Some (sfs, Done)*  
 $\implies$  *exec-step-ind (StepC C Cs) P h stk loc C<sub>0</sub> M<sub>0</sub> pc ics frs sh*  
*(None, h, (stk, loc, C<sub>0</sub>, M<sub>0</sub>, pc, Called Cs)#frs, sh)*

| *exec-step-ind-Init-Processing:*  
*sh C = Some (sfs, Processing)*  
 $\implies$  *exec-step-ind (StepC C Cs) P h stk loc C<sub>0</sub> M<sub>0</sub> pc ics frs sh*  
*(None, h, (stk, loc, C<sub>0</sub>, M<sub>0</sub>, pc, Called Cs)#frs, sh)*

| *exec-step-ind-Init-Error:*  
 $\llbracket$  *sh C = Some (sfs, Error)*  $\rrbracket$   
 $\implies$  *exec-step-ind (StepC C Cs) P h stk loc C<sub>0</sub> M<sub>0</sub> pc ics frs sh*  
*(None, h, (stk, loc, C<sub>0</sub>, M<sub>0</sub>, pc, Throwing Cs (addr-of-sys-xcpt NoClassDef-*  
*FoundError))#frs, sh)*

| *exec-step-ind-Init-Prepared-Object:*  
 $\llbracket$  *sh C = Some (sfs, Prepared);*  
*sh' = sh(C:=Some(fst(the(sh C)), Processing));*  
*C = Object*  $\rrbracket$   
 $\implies$  *exec-step-ind (StepC C Cs) P h stk loc C<sub>0</sub> M<sub>0</sub> pc ics frs sh*  
*(None, h, (stk, loc, C<sub>0</sub>, M<sub>0</sub>, pc, Called (C#Cs))#frs, sh')*

| *exec-step-ind-Init-Prepared-nObject:*  
 $\llbracket$  *sh C = Some (sfs, Prepared);*

$sh' = sh(C := \text{Some}(\text{fst}(\text{the}(sh\ C))), \text{Processing});$   
 $C \neq \text{Object}; D = \text{fst}(\text{the}(\text{class } P\ C)) \ ]$   
 $\implies \text{exec-step-ind } (\text{StepC } C\ Cs) P\ h\ stk\ loc\ C_0\ M_0\ pc\ ics\ frs\ sh$   
 $(\text{None}, h, (stk, loc, C_0, M_0, pc, \text{Calling } D\ (C\ \#Cs))\ \#frs, sh')$

| *exec-step-ind-Init*:

$\text{exec-step-ind } (\text{StepC2 } C\ Cs) P\ h\ stk\ loc\ C_0\ M_0\ pc\ ics\ frs\ sh$   
 $(\text{None}, h, \text{create-init-frame } P\ C\ \#(stk, loc, C_0, M_0, pc, \text{Called } Cs)\ \#frs, sh)$

| *exec-step-ind-InitThrow*:

$\text{exec-step-ind } (\text{StepT } (C\ \#Cs)\ a) P\ h\ stk\ loc\ C_0\ M_0\ pc\ ics\ frs\ sh$   
 $(\text{None}, h, (stk, loc, C_0, M_0, pc, \text{Throwing } Cs\ a)\ \#frs, (sh(C \mapsto (\text{fst}(\text{the}(sh\ C))),$   
 $\text{Error}))))$

| *exec-step-ind-InitThrow-End*:

$\text{exec-step-ind } (\text{StepT } []\ a) P\ h\ stk\ loc\ C_0\ M_0\ pc\ ics\ frs\ sh$   
 $([a], h, (stk, loc, C_0, M_0, pc, \text{No-ics})\ \#frs, sh)$

**inductive-cases** *exec-step-ind-cases* [cases set]:

$\text{exec-step-ind } (\text{StepI } (\text{Load } n)) P\ h\ stk\ loc\ C\ M\ pc\ ics\ frs\ sh\ \sigma$   
 $\text{exec-step-ind } (\text{StepI } (\text{Store } n)) P\ h\ stk\ loc\ C\ M\ pc\ ics\ frs\ sh\ \sigma$   
 $\text{exec-step-ind } (\text{StepI } (\text{Push } v)) P\ h\ stk\ loc\ C\ M\ pc\ ics\ frs\ sh\ \sigma$   
 $\text{exec-step-ind } (\text{StepI } (\text{New } C)) P\ h\ stk\ loc\ C\ M\ pc\ ics\ frs\ sh\ \sigma$   
 $\text{exec-step-ind } (\text{StepI } (\text{Getfield } F\ C)) P\ h\ stk\ loc\ C\ M\ pc\ ics\ frs\ sh\ \sigma$   
 $\text{exec-step-ind } (\text{StepI } (\text{Getstatic } C\ F\ D)) P\ h\ stk\ loc\ C\ M\ pc\ ics\ frs\ sh\ \sigma$   
 $\text{exec-step-ind } (\text{StepI } (\text{Putfield } F\ C)) P\ h\ stk\ loc\ C\ M\ pc\ ics\ frs\ sh\ \sigma$   
 $\text{exec-step-ind } (\text{StepI } (\text{Putstatic } C\ F\ D)) P\ h\ stk\ loc\ C\ M\ pc\ ics\ frs\ sh\ \sigma$   
 $\text{exec-step-ind } (\text{StepI } (\text{Checkcast } C)) P\ h\ stk\ loc\ C\ M\ pc\ ics\ frs\ sh\ \sigma$   
 $\text{exec-step-ind } (\text{StepI } (\text{Invoke } M\ n)) P\ h\ stk\ loc\ C\ M\ pc\ ics\ frs\ sh\ \sigma$   
 $\text{exec-step-ind } (\text{StepI } (\text{Invokestatic } C\ M\ n)) P\ h\ stk\ loc\ C\ M\ pc\ ics\ frs\ sh\ \sigma$   
 $\text{exec-step-ind } (\text{StepI } \text{Return}) P\ h\ stk\ loc\ C\ M\ pc\ ics\ frs\ sh\ \sigma$   
 $\text{exec-step-ind } (\text{StepI } \text{Pop}) P\ h\ stk\ loc\ C\ M\ pc\ ics\ frs\ sh\ \sigma$   
 $\text{exec-step-ind } (\text{StepI } \text{IAdd}) P\ h\ stk\ loc\ C\ M\ pc\ ics\ frs\ sh\ \sigma$   
 $\text{exec-step-ind } (\text{StepI } (\text{IfFalse } i)) P\ h\ stk\ loc\ C\ M\ pc\ ics\ frs\ sh\ \sigma$   
 $\text{exec-step-ind } (\text{StepI } \text{CmpEq}) P\ h\ stk\ loc\ C\ M\ pc\ ics\ frs\ sh\ \sigma$   
 $\text{exec-step-ind } (\text{StepI } (\text{Goto } i)) P\ h\ stk\ loc\ C\ M\ pc\ ics\ frs\ sh\ \sigma$   
 $\text{exec-step-ind } (\text{StepI } \text{Throw}) P\ h\ stk\ loc\ C\ M\ pc\ ics\ frs\ sh\ \sigma$   
 $\text{exec-step-ind } (\text{StepC } C'\ Cs) P\ h\ stk\ loc\ C\ M\ pc\ ics\ frs\ sh\ \sigma$   
 $\text{exec-step-ind } (\text{StepC2 } C'\ Cs) P\ h\ stk\ loc\ C\ M\ pc\ ics\ frs\ sh\ \sigma$   
 $\text{exec-step-ind } (\text{StepT } Cs\ a) P\ h\ stk\ loc\ C\ M\ pc\ ics\ frs\ sh\ \sigma$

— Deriving *step-input* for *exec-step-ind* from *exec-step* arguments

**fun** *exec-step-input* :: [jvm-prog, cname, mname, pc, init-call-status]  $\Rightarrow$  *step-input*

**where**

$\text{exec-step-input } P\ C\ M\ pc\ (\text{Calling } C'\ Cs) = \text{StepC } C'\ Cs \ |$   
 $\text{exec-step-input } P\ C\ M\ pc\ (\text{Called } (C'\ \#Cs)) = \text{StepC2 } C'\ Cs \ |$

$exec\text{-}step\text{-}input\ P\ C\ M\ pc\ (Throwing\ Cs\ a) = StepT\ Cs\ a \mid$   
 $exec\text{-}step\text{-}input\ P\ C\ M\ pc\ ics = StepI\ (instrs\text{-}of\ P\ C\ M\ !\ pc)$

**lemma** *exec-step-input-StepTD[simp]*:  
**assumes**  $exec\text{-}step\text{-}input\ P\ C\ M\ pc\ ics = StepT\ Cs\ a$  **shows**  $ics = Throwing\ Cs\ a$   
**using** *assms* **proof**(cases *ics*)  
  **case** (Called *Cs*) **with** *assms* **show** ?thesis **by**(cases *Cs*; *simp*)  
**qed**(*auto*)

**lemma** *exec-step-input-StepCD[simp]*:  
**assumes**  $exec\text{-}step\text{-}input\ P\ C\ M\ pc\ ics = StepC\ C'\ Cs$  **shows**  $ics = Calling\ C'\ Cs$   
**using** *assms* **proof**(cases *ics*)  
  **case** (Called *Cs*) **with** *assms* **show** ?thesis **by**(cases *Cs*; *simp*)  
**qed**(*auto*)

**lemma** *exec-step-input-StepC2D[simp]*:  
**assumes**  $exec\text{-}step\text{-}input\ P\ C\ M\ pc\ ics = StepC2\ C'\ Cs$  **shows**  $ics = Called\ (C'\#Cs)$   
**using** *assms* **proof**(cases *ics*)  
  **case** (Called *Cs*) **with** *assms* **show** ?thesis **by**(cases *Cs*; *simp*)  
**qed**(*auto*)

**lemma** *exec-step-input-StepID*:  
**assumes**  $exec\text{-}step\text{-}input\ P\ C\ M\ pc\ ics = StepI\ i$   
**shows**  $(ics = Called\ [] \vee ics = No\text{-}ics) \wedge instrs\text{-}of\ P\ C\ M\ !\ pc = i$   
**using** *assms* **proof**(cases *ics*)  
  **case** (Called *Cs*) **with** *assms* **show** ?thesis **by**(cases *Cs*; *simp*)  
**qed**(*auto*)

## 11.1 Equivalence of *exec-step* and *exec-step-input*

**lemma** *exec-step-imp-exec-step-ind*:  
**assumes**  $es: exec\text{-}step\ P\ h\ stk\ loc\ C\ M\ pc\ ics\ frs\ sh = (xp', h', frs', sh')$   
**shows**  $exec\text{-}step\text{-}ind\ (exec\text{-}step\text{-}input\ P\ C\ M\ pc\ ics)\ P\ h\ stk\ loc\ C\ M\ pc\ ics\ frs\ sh$   
 $(xp', h', frs', sh')$   
**proof**(cases *exec-step-input P C M pc ics*)  
  **case** (StepT *Cs a*)  
  **then have**  $ics = Throwing\ Cs\ a$  **by** *simp*  
  **then show** ?thesis **using** *exec-step-ind-InitThrow exec-step-ind-InitThrow-End StepT es*  
  **by**(cases *Cs*, *auto*)  
**next**  
  **case** (StepC *C1 Cs*)  
  **then have**  $ics: ics = Calling\ C1\ Cs$  **by** *simp*  
  **obtain**  $D\ b\ Ts\ T\ m$  **where** *lets: method P C1 clinit = (D,b,Ts,T,m)* **by**(cases *method P C1 clinit*)  
  **then obtain**  $mxs\ mxl_0\ ins\ xt$  **where**  $m: m = (mxs, mxl_0, ins, xt)$  **by**(cases *m*)  
  **show** ?thesis  
  **proof**(cases *sh C1*)

```

case None then show ?thesis
  using exec-step-ind-Init-None-Called ics assms by auto
next
case (Some a)
then obtain sfs i where sfsi: a = (sfs,i) by(cases a)
then show ?thesis using exec-step-ind-Init-Done exec-step-ind-Init-Processing
  exec-step-ind-Init-Error m lets Some ics assms
proof(cases i)
  case Prepared
  show ?thesis
using exec-step-ind-Init-Prepared-Object[where P=P] exec-step-ind-Init-Prepared-nObject
  sfsi m lets Prepared Some ics assms by(auto split: if-split-asm)
qed(auto)
qed
next
case (StepC2 C1 Cs)
then have ics: ics = Called (C1#Cs) by simp
then show ?thesis using exec-step-ind-Init assms by auto
next
case (StepI i)
then have
  ics: ics = Called [] ∨ ics = No-ics and
  exec-instr: exec-instr i P h stk loc C M pc ics frs sh = (xp', h', frs', sh')
using assms by(auto dest!: exec-step-input-StepID)
show ?thesis
proof(cases i)
  case (Load x1) then show ?thesis using exec-instr exec-step-ind-Load StepI
by auto
  next
  case (Store x2) then show ?thesis using exec-instr exec-step-ind-Store StepI
by auto
  next
  case (Push x3) then show ?thesis using exec-instr exec-step-ind-Push StepI
by auto
  next
  case (New C1)
then obtain sfs i where sfsi: the(sh C1) = (sfs,i) by(cases the(sh C1))
then show ?thesis using exec-step-ind-New-Called exec-step-ind-NewOOM-Called
  exec-step-ind-New-Done exec-step-ind-NewOOM-Done
  exec-step-ind-New-Init sfsi New StepI exec-instr ics by(auto split: init-state.splits)
next
case (Getfield F1 C1)
then obtain D fs D' b t where lets: the(h(the-Addr (hd stk))) = (D,fs)
  field P C1 F1 = (D',b,t) by(cases the(h(the-Addr (hd stk))), cases field P C1
F1)
then have  $\bigwedge b' t'. P \vdash D \text{ has } F1, b':t' \text{ in } C1 \implies (D', b, t) = (C1, b', t')$ 
using field-def2 has-field-idemp has-field-sees by fastforce
then show ?thesis using exec-step-ind-Getfield-Null exec-step-ind-Getfield-NoField
  exec-step-ind-Getfield-Static exec-step-ind-Getfield lets Getfield StepI exec-instr

```

```

    by(auto split: if-split-asm staticb.splits) metis+
next
case (Getstatic C1 F1 D1)
then obtain D' b t where lets: field P D1 F1 = (D',b,t) by(cases field P D1
F1)
then have field:  $\bigwedge b' t'. P \vdash C1 \text{ has } F1, b':t' \text{ in } D1 \implies (D', b, t) = (D1, b', t')$ 
using field-def2 has-field-idemp has-field-sees by fastforce
show ?thesis
proof(cases b)
case NonStatic then show ?thesis
using exec-step-ind-Getstatic-NoField exec-step-ind-Getstatic-NonStatic
field lets Getstatic exec-instr StepI by(auto split: if-split-asm) fastforce
next
case Static show ?thesis
proof(cases ics = Called [])
case True then show ?thesis using exec-step-ind-Getstatic-NoField
exec-step-ind-Getstatic-Called exec-step-ind-Getstatic-Init
Static field lets Getstatic exec-instr StepI ics
by(auto simp: split-beta split: if-split-asm) metis
next
case False
then have nCalled:  $\forall Cs. ics \neq Called Cs$  using ics by simp
show ?thesis
proof(cases sh D1)
case None
then have nDone:  $\forall sfs. sh D1 \neq Some(sfs, Done)$  by simp
then show ?thesis using exec-step-ind-Getstatic-NoField
exec-step-ind-Getstatic-Init[where sh=sh, OF - - nDone nCalled]
field lets None False Static Getstatic exec-instr StepI ics
by(auto split: if-split-asm) metis
next
case (Some a)
then obtain sfs i where sfsi:  $a=(sfs,i)$  by(cases a)
show ?thesis using exec-step-ind-Getstatic-NoField
exec-step-ind-Getstatic-Init sfsi False Static Some field lets Getstatic
exec-instr
proof(cases i = Done)
case True then show ?thesis using exec-step-ind-Getstatic-NoField
exec-step-ind-Getstatic-Done[OF - - nCalled] exec-step-ind-Getstatic-Init
sfsi False Static Some field lets Getstatic exec-instr StepI ics
by(auto split: if-split-asm) metis
next
case nD: False
then have nDone:  $\forall sfs. sh D1 \neq Some(sfs, Done)$  using sfsi Some by
simp
show ?thesis using nD
proof(cases i)
case Processing then show ?thesis using exec-step-ind-Getstatic-NoField
exec-step-ind-Getstatic-Init[where sh=sh, OF - - nDone nCalled]

```

```

      sfsi False Static Some field lets Getstatic exec-instr StepI ics
    by(auto split: if-split-asm) metis
  next
  case Prepared then show ?thesis using exec-step-ind-Getstatic-NoField
    exec-step-ind-Getstatic-Init[where sh=sh, OF - - nDone nCalled]
    sfsi False Static Some field lets Getstatic exec-instr StepI ics
    by(auto split: if-split-asm) metis
  next
  case Error then show ?thesis using exec-step-ind-Getstatic-NoField
    exec-step-ind-Getstatic-Init[where sh=sh, OF - - nDone nCalled]
    sfsi False Static Some field lets Getstatic exec-instr StepI ics
    by(auto split: if-split-asm) metis
  qed(simp)
  qed
  qed
  qed
  next
  case (Putfield F1 C1)
  then obtain D fs D' b t where lets: the(h(the-Addr (hd(tl stk)))) = (D,fs)
    field P C1 F1 = (D',b,t) by(cases the(h(the-Addr (hd(tl stk))))), cases field P
  C1 F1)
  then have  $\bigwedge b' t'. P \vdash D \text{ has } F1, b':t' \text{ in } C1 \implies (D', b, t) = (C1, b', t')$ 
    using field-def2 has-field-idemp has-field-sees by fastforce
  then show ?thesis using exec-step-ind-Putfield-Null exec-step-ind-Putfield-NoField
    exec-step-ind-Putfield-Static exec-step-ind-Putfield lets Putfield exec-instr StepI
    by(auto split: if-split-asm staticb.splits) metis+
  next
  case (Putstatic C1 F1 D1)
  then obtain D' b t where lets: field P D1 F1 = (D',b,t) by(cases field P D1
  F1)
  then have field:  $\bigwedge b' t'. P \vdash C1 \text{ has } F1, b':t' \text{ in } D1 \implies (D', b, t) = (D1, b', t')$ 
    using field-def2 has-field-idemp has-field-sees by fastforce
  show ?thesis
  proof(cases b)
  case NonStatic then show ?thesis
    using exec-step-ind-Putstatic-NoField exec-step-ind-Putstatic-NonStatic
    field lets Putstatic exec-instr StepI by(auto split: if-split-asm) fastforce
  next
  case Static show ?thesis
  proof(cases ics = Called [])
  case True then show ?thesis using exec-step-ind-Putstatic-NoField
    exec-step-ind-Putstatic-Called exec-step-ind-Putstatic-Init
    Static field lets Putstatic exec-instr StepI ics
    by(cases the(sh D1), auto split: if-split-asm) metis
  next
  case False
  then have nCalled:  $\forall Cs. ics \neq \text{Called } Cs$  using ics by simp
  show ?thesis

```



```

proof(cases sh D1)
  case None
  then have nDone:  $\forall$  sfs. sh D1  $\neq$  Some(sfs, Done) by simp
  then show ?thesis using exec-step-ind-Putstatic-NoField
    exec-step-ind-Putstatic-Init[where sh=sh, OF - - nDone nCalled]
    field lets None False Static Putstatic exec-instr StepI ics
  by(auto split: if-split-asm) metis
next
  case (Some a)
  then obtain sfs i where sfsi: a=(sfs,i) by(cases a)
  show ?thesis using exec-step-ind-Putstatic-NoField
    exec-step-ind-Putstatic-Init sfsi False Static Some field lets Putstatic
exec-instr
  proof(cases i = Done)
  case True then show ?thesis using exec-step-ind-Putstatic-NoField
    exec-step-ind-Putstatic-Done[OF - - nCalled] exec-step-ind-Putstatic-Init
    sfsi False Static Some field lets Putstatic exec-instr StepI ics
  by(auto split: if-split-asm) metis
next
  case nD: False
  then have nDone:  $\forall$  sfs. sh D1  $\neq$  Some(sfs, Done) using sfsi Some by
simp
  show ?thesis using nD
  proof(cases i)
  case Processing then show ?thesis using exec-step-ind-Putstatic-NoField
    exec-step-ind-Putstatic-Init[where sh=sh, OF - - nDone nCalled]
    sfsi False Static Some field lets Putstatic exec-instr StepI ics
  by(auto split: if-split-asm) metis
  next
  case Prepared then show ?thesis using exec-step-ind-Putstatic-NoField
    exec-step-ind-Putstatic-Init[where sh=sh, OF - - nDone nCalled]
    sfsi False Static Some field lets Putstatic exec-instr StepI ics
  by(auto split: if-split-asm) metis
  next
  case Error then show ?thesis using exec-step-ind-Putstatic-NoField
    exec-step-ind-Putstatic-Init[where sh=sh, OF - - nDone nCalled]
    sfsi False Static Some field lets Putstatic exec-instr StepI ics
  by(auto split: if-split-asm) metis
  qed(simp)
  qed
  qed
  qed
  qed
next
  case Checkcast then show ?thesis
  using exec-step-ind-Checkcast exec-step-ind-Checkcast-Error exec-instr StepI
  by(auto split: if-split-asm)
next
  case (Invoke M1 n) show ?thesis

```

```

proof(cases stk!n = Null)
case True then show ?thesis using exec-step-ind-Invoke-Null Invoke exec-instr
StepI
  by clarsimp
next
  case False
  let ?C = cname-of h (the-Addr (stk ! n))
  obtain D b Ts T m where method: method P ?C M1 = (D,b,Ts,T,m) by(cases
method P ?C M1)
  then obtain mxs mxl0 ins xt where m = (mxs,mxl0,ins,xt) by(cases m)
  then show ?thesis using exec-step-ind-Invoke-NoMethod
  exec-step-ind-Invoke-Static exec-step-ind-Invoke method False Invoke exec-instr
StepI
  by(auto split: if-split-asm staticb.splits)
qed
next
  case (Invokestatic C1 M1 n)
  obtain D b Ts T m where lets: method P C1 M1 = (D,b,Ts,T,m) by(cases
method P C1 M1)
  then obtain mxs mxl0 ins xt where m: m = (mxs,mxl0,ins,xt) by(cases m)
  have method:  $\bigwedge b' Ts' t' m' D'. P \vdash C1 \text{ sees } M1, b': Ts' \rightarrow t' = m' \text{ in } D'$ 
 $\implies (D,b,Ts,T,m) = (D',b',Ts',t',m')$  using lets by auto
  show ?thesis
  proof(cases b)
    case NonStatic then show ?thesis
    using exec-step-ind-Invokestatic-NoMethod exec-step-ind-Invokestatic-NonStatic
    m method lets Invokestatic exec-instr StepI by(auto split: if-split-asm)
  next
    case Static show ?thesis
    proof(cases ics = Called [])
      case True then show ?thesis using exec-step-ind-Invokestatic-NoMethod
      exec-step-ind-Invokestatic-Called exec-step-ind-Invokestatic-Init
      Static m method lets Invokestatic exec-instr StepI ics
      by(auto split: if-split-asm)
    next
      case False
      then have nCalled:  $\forall Cs. ics \neq Called Cs$  using ics by simp
      show ?thesis
      proof(cases sh D)
        case None
        then have nDone:  $\forall sfs. sh D \neq Some(sfs, Done)$  by simp
        show ?thesis using exec-step-ind-Invokestatic-NoMethod
        exec-step-ind-Invokestatic-Init[where sh=sh, OF - - nDone nCalled]
        method m lets None False Static Invokestatic exec-instr StepI ics
        by(auto split: if-split-asm)
      next
        case (Some a)
        then obtain sfs i where sfsi: a=(sfs,i) by(cases a)
        show ?thesis using exec-step-ind-Invokestatic-NoMethod

```

```

exec-step-ind-Invokestatic-Init sfsi False Static Some method lets Invokestatic
exec-instr
  proof(cases i = Done)
    case True then show ?thesis using exec-step-ind-Invokestatic-NoMethod
exec-step-ind-Invokestatic-Done[OF - - nCalled] exec-step-ind-Invokestatic-Init
      sfsi False Static Some m method lets Invokestatic exec-instr StepI ics
      by(auto split: if-split-asm)
    next
      case nD: False
        then have nDone:  $\forall$  sfs. sh D  $\neq$  Some(sfs, Done) using sfsi Some by
simp
          show ?thesis using nD
          proof(cases i)
            case Processing then show ?thesis using exec-step-ind-Invokestatic-NoMethod
exec-step-ind-Invokestatic-Init[where sh=sh, OF - - nDone nCalled]
              sfsi False Static Some m method lets Invokestatic exec-instr StepI ics
              by(auto split: if-split-asm)
            next
              case Prepared then show ?thesis using exec-step-ind-Invokestatic-NoMethod
exec-step-ind-Invokestatic-Init[where sh=sh, OF - - nDone nCalled]
                sfsi False Static Some m method lets Invokestatic exec-instr StepI ics
                by(auto split: if-split-asm)
              next
                case Error then show ?thesis using exec-step-ind-Invokestatic-NoMethod
exec-step-ind-Invokestatic-Init[where sh=sh, OF - - nDone nCalled]
                  sfsi False Static Some m method lets Invokestatic exec-instr StepI ics
                  by(auto split: if-split-asm)
                qed(simp)
              qed
            qed
          qed
        next
          case Return
            obtain D b Ts T m where method: method P C M = (D,b,Ts,T,m) by(cases
method P C M)
              then obtain mxs mxl0 ins xt where m = (mxs,mxl0,ins,xt) by(cases m)
              then show ?thesis using exec-step-ind-Return-Last-Init exec-step-ind-Return-Last
exec-step-ind-Return-Init exec-step-ind-Return-NonStatic exec-step-ind-Return-Static
                method Return exec-instr StepI ics
                by(auto split: if-split-asm staticb.splits bool.splits list.splits)
              next
                case Pop then show ?thesis using exec-instr StepI exec-step-ind-Pop by auto
                next
                  case IAdd then show ?thesis using exec-instr StepI exec-step-ind-IAdd by
auto
                  next
                    case Goto then show ?thesis using exec-instr StepI exec-step-ind-Goto by
auto

```

```

next
  case CmpEq then show ?thesis using exec-instr StepI exec-step-ind-CmpEq
by auto
next
  case (IfFalse x17) then show ?thesis
  using exec-instr StepI exec-step-ind-IfFalse-nFalse exec-step-ind-IfFalse-False
  exec-instr StepI by(auto split: val.splits staticb.splits)
next
  case Throw then show ?thesis
  using exec-instr StepI exec-step-ind-Throw exec-step-ind-Throw-Null
  by(auto split: val.splits)
qed
qed

```

lemma *exec-step-ind-imp-exec-step*:

assumes *esi*: *exec-step-ind si P h stk loc C M pc ics frs sh (xp', h', frs', sh')*

and *si*: *exec-step-input P C M pc ics = si*

shows *exec-step P h stk loc C M pc ics frs sh = (xp', h', frs', sh')*

proof –

have *StepI*:

$\bigwedge P C M pc Cs i . \text{exec-step-input } P C M pc (\text{Called } Cs) = \text{StepI } i$   
 $\implies \text{instrs-of } P C M ! pc = i \wedge Cs = []$

proof –

fix *P C M pc Cs i* show *exec-step-input P C M pc (Called Cs) = StepI i*  
 $\implies \text{instrs-of } P C M ! pc = i \wedge Cs = []$  by(*cases Cs; simp*)

qed

have *StepC*:

$\bigwedge P C M pc ics C' Cs . \text{exec-step-input } P C M pc ics = \text{StepC } C' Cs \implies ics =$   
*Calling C' Cs*

by *simp*

have *StepT*:

$\bigwedge P C M pc ics Cs a . \text{exec-step-input } P C M pc ics = \text{StepT } Cs a \implies ics =$   
*Throwing Cs a*

by *simp*

show ?thesis using *assms*

proof(*induct rule: exec-step-ind.induct*)

case (*exec-step-ind-NewOOM-Done sh C obj h ics P stk loc C<sub>0</sub> M<sub>0</sub> pc frs*)

then show ?case by(*cases ics, auto*)

next

case (*exec-step-ind-New-Done sh C obj h a ics P stk loc C<sub>0</sub> M<sub>0</sub> pc frs*)

then show ?case by(*cases ics, auto*)

next

case (*exec-step-ind-New-Init sh C ics P h stk loc C<sub>0</sub> M<sub>0</sub> pc frs*)

then show ?case by(*cases ics, auto split: init-state.splits*)

next

case (*exec-step-ind-Getfield-NoField v stk D fs h P F C loc C<sub>0</sub> M<sub>0</sub> pc ics frs*  
*sh*)

then show ?case by(*cases the (h (the-Addr (hd stk))), cases ics, auto dest!:*  
*StepI*)

```

next
  case (exec-step-ind-Getfield-Static v stk D fs h P F t C loc C0 M0 pc ics frs sh)
  then show ?case
    by(cases the (h (the-Addr (hd stk))), cases fst(snd(field P C F)),
      cases ics, auto simp: split-beta dest: has-field-sees[OF has-field-idemp] dest!:
StepI)
next
  case (exec-step-ind-Getfield v stk D fs h D' b t P C F loc C0 M0 pc ics frs sh)
  then show ?case
    by(cases the (h (the-Addr (hd stk))),
      cases ics; fastforce simp: split-beta dest: has-field-sees[OF has-field-idemp]
dest!: StepI)
next
  case (exec-step-ind-Getstatic-NonStatic P C F t D h stk loc C0 M0 pc ics frs
sh)
  then show ?case
    by(cases ics; fastforce simp: split-beta split: staticb.splits
      dest: has-field-sees[OF has-field-idemp] dest!: StepI)
next
  case exec-step-ind-Getstatic-Called
  then show ?case by(fastforce simp: split-beta split: staticb.splits dest!: StepI
    dest: has-field-sees[OF has-field-idemp])
next
  case (exec-step-ind-Getstatic-Done D' b t P D F C ics sh sfs v h stk loc C0 M0
pc frs)
  then show ?case by(cases ics, auto simp: split-beta split: staticb.splits
    dest: has-field-sees[OF has-field-idemp])
next
  case (exec-step-ind-Getstatic-Init D' b t P D F C sh ics h stk loc C0 M0 pc frs)
  then show ?case
    by(cases ics, auto simp: split-beta split: init-state.splits staticb.splits
    dest: has-field-sees[OF has-field-idemp])
next
  case (exec-step-ind-Putfield-NoField r stk a D fs h P F C loc C0 M0 pc ics frs
sh)
  then show ?case by(cases the (h (the-Addr (hd(tl stk)))), cases ics, auto dest!:
StepI)
next
  case (exec-step-ind-Putfield-Static r stk a D fs h P F t C loc C0 M0 pc ics frs
sh)
  then show ?case
    by(cases the (h (the-Addr (hd(tl stk)))), cases fst(snd(field P C F)),
      cases ics, auto simp: split-beta dest: has-field-sees[OF has-field-idemp] dest!:
StepI)
next
  case (exec-step-ind-Putfield v stk r a D fs h D' b t P C F loc C0 M0 pc ics frs
sh)
  then show ?case
    by(cases the (h (the-Addr (hd(tl stk)))),

```

```

      cases ics; fastforce simp: split-beta dest: has-field-sees[OF has-field-idemp]
dest!: StepI)
next
  case (exec-step-ind-Putstatic-NonStatic P C F t D h stk loc C0 M0 pc ics frs
sh)
  then show ?case
  by(cases ics; fastforce simp: split-beta split: staticb.splits
      dest: has-field-sees[OF has-field-idemp] dest!: StepI)
next
  case exec-step-ind-Putstatic-Called
  then show ?case by(fastforce simp: split-beta split: staticb.splits dest!: StepI
      dest: has-field-sees[OF has-field-idemp])
next
  case (exec-step-ind-Putstatic-Done D' b t P D F C ics sh sfs h stk loc C0 M0
pc frs)
  then show ?case by(cases ics, auto simp: split-beta split: staticb.splits
      dest: has-field-sees[OF has-field-idemp])
next
  case (exec-step-ind-Putstatic-Init D' b t P D F C sh ics h stk loc C0 M0 pc frs)
  then show ?case
  by(cases ics, auto simp: split-beta split: staticb.splits init-state.splits
      dest: has-field-sees[OF has-field-idemp])
next
  case (exec-step-ind-Invoke ps n stk r C h D b Ts T mxs mxl0 ins xt P M m f'
loc C0 M0 pc ics frs sh)
  then show ?case by(cases ics; fastforce dest!: StepI)
next
  case (exec-step-ind-Invokestatic-Called ps n stk D b Ts T mxs mxl0 ins xt P C
M m ics ics' sh)
  then show ?case by(cases ics; fastforce dest!: StepI)
next
  case (exec-step-ind-Invokestatic-Done ps n stk D b Ts T mxs mxl0 ins xt P C
M m ics sh sfs f')
  then show ?case by(cases ics; fastforce)
next
  case (exec-step-ind-Invokestatic-Init D b Ts T mxs mxl0 ins xt P C M m sh ics
n h stk loc C0 M0 pc frs)
  then show ?case by(cases ics; fastforce split: init-state.splits)
next
  case (exec-step-ind-Return-NonStatic D Ts T m P C0 M0 h stk0 loc0 pc ics stk'
loc' C' m' pc' ics' frs' sh)
  then show ?case by(cases method P C0 M0, cases ics, auto dest!: StepI)
next
  case (exec-step-ind-Return-Static D Ts T m P C0 M0 h stk0 loc0 pc ics stk'
loc' C' m' pc' ics' frs' sh)
  then show ?case by(cases method P C0 M0, cases ics, auto dest!: StepI)
next
  case (exec-step-ind-IfFalse-nFalse stk i P h loc C0 M0 pc ics frs sh)
  then show ?case by(cases hd stk; cases ics, auto dest!: StepI)

```

```

next
  case (exec-step-ind-Throw-Null stk P h loc C0 M0 pc ics frs sh)
  then show ?case by(cases hd stk; cases ics, auto dest!: StepI)
next
  case (exec-step-ind-Init C Cs P h stk loc C0 M0 pc ics frs sh)
  then have ics = Called (C#Cs) by simp
  then show ?case by auto

next
  case (exec-step-ind-Load n P h stk loc C0 M0 pc ics frs sh)
  then show ?case by(cases ics, auto dest!: StepI)
next
  case (exec-step-ind-Store n P h stk loc C0 M0 pc ics frs sh)
  then show ?case by(cases ics, auto dest!: StepI)
next
  case (exec-step-ind-Push v P h stk loc C0 M0 pc ics frs sh)
  then show ?case by(cases ics, auto dest!: StepI)
next
  case (exec-step-ind-NewOOM-Called h C P stk loc C0 M0 pc frs sh ics')
  then show ?case by(auto dest!: StepI)
next
  case (exec-step-ind-New-Called h a C P stk loc C0 M0 pc frs sh ics')
  then show ?case by(auto dest!: StepI)
next
  case (exec-step-ind-Getfield-Null stk F C P h loc C0 M0 pc ics frs sh)
  then show ?case by(cases ics, auto dest!: StepI)
next
  case (exec-step-ind-Getstatic-NoField P C F D h stk loc C0 M0 pc ics frs sh)
  then show ?case by(cases ics, auto dest!: StepI)
next
  case (exec-step-ind-Putfield-Null stk F C P h loc C0 M0 pc ics frs sh)
  then show ?case by(cases ics, auto dest!: StepI)
next
  case (exec-step-ind-Putstatic-NoField P C F D h stk loc C0 M0 pc ics frs sh)
  then show ?case by(cases ics, auto dest!: StepI)
next
  case (exec-step-ind-Checkcast P C h stk loc C0 M0 pc ics frs sh)
  then show ?case by(cases ics, auto dest!: StepI)
next
  case (exec-step-ind-Checkcast-Error P C h stk loc C0 M0 pc ics frs sh)
  then show ?case by(cases ics, auto dest!: StepI)
next
  case (exec-step-ind-Invoke-Null stk n M P h loc C0 M0 pc ics frs sh)
  then show ?case by(cases ics, auto dest!: StepI)
next
  case (exec-step-ind-Invoke-NoMethod r stk n C h P M loc C0 M0 pc ics frs sh)
  then show ?case by(cases ics, auto dest!: StepI)
next
  case (exec-step-ind-Invoke-Static r stk n C h D b Ts T mxs mxl0 ins xt P M m

```

```

loc C0 M0 pc ics)
  then show ?case by(cases ics, auto dest!: StepI)
next
  case (exec-step-ind-Invokestatic-NoMethod D b Ts T mxs mxl0 ins xt P C M n
h stk loc C0 M0 pc ics)
  then show ?case by(cases ics, auto dest!: StepI)
next
  case (exec-step-ind-Invokestatic-NonStatic D b Ts T mxs mxl0 ins xt P C M m
n h stk loc C0 M0 pc ics)
  then show ?case by(cases ics, auto dest!: StepI)
next
  case (exec-step-ind-Return-Last-Init P h stk0 loc0 C0 pc ics sh)
  then show ?case by(cases ics, auto dest!: StepI)
next
  case (exec-step-ind-Return-Last M0 P h stk0 loc0 C0 pc ics sh)
  then show ?case by(cases ics, auto dest!: StepI)
next
  case (exec-step-ind-Return-Init D b Ts T m P C0 h stk0 loc0 pc ics stk' loc' C'
m' pc' ics')
  then show ?case by(cases ics, auto dest!: StepI)
next
  case (exec-step-ind-Pop P h stk loc C0 M0 pc ics frs sh)
  then show ?case by(cases ics, auto dest!: StepI)
next
  case (exec-step-ind-IAdd P h stk loc C0 M0 pc ics frs sh)
  then show ?case by(cases ics, auto dest!: StepI)
next
  case (exec-step-ind-IfFalse-False stk i P h loc C0 M0 pc ics frs sh)
  then show ?case by(cases ics, auto dest!: StepI)
next
  case (exec-step-ind-CmpEq P h stk loc C0 M0 pc ics frs sh)
  then show ?case by(cases ics, auto dest!: StepI)
next
  case (exec-step-ind-Goto i P h stk loc C0 M0 pc ics frs sh)
  then show ?case by(cases ics, auto dest!: StepI)
next
  case (exec-step-ind-Throw stk P h loc C0 M0 pc ics frs sh)
  then show ?case by(cases ics, auto dest!: StepI)
next
  case (exec-step-ind-Init-None-Called sh C Cs P h stk loc C0 M0 pc ics frs)
  then show ?case by(auto dest!: StepC)
next
  case (exec-step-ind-Init-Done sh C sfs Cs P h stk loc C0 M0 pc ics frs)
  then show ?case by(auto dest!: StepC)
next
  case (exec-step-ind-Init-Processing sh C sfs Cs P h stk loc C0 M0 pc ics frs)
  then show ?case by(auto dest!: StepC)
next
  case (exec-step-ind-Init-Error sh C sfs Cs P h stk loc C0 M0 pc ics frs)

```



```

    then show ?case by(auto dest!: StepC)
  next
    case (exec-step-ind-Init-Prepared-Object sh C sfs sh' Cs P h stk loc C0 M0 pc
ics frs)
    then show ?case by(auto dest!: StepC)
  next
    case (exec-step-ind-Init-Prepared-nObject sh C sfs sh' D P Cs h stk loc C0 M0
pc ics frs)
    then show ?case by(auto dest!: StepC)
  next
    case (exec-step-ind-InitThrow C Cs a P h stk loc C0 M0 pc ics frs sh)
    then show ?case by(auto dest!: StepT)
  next
    case (exec-step-ind-InitThrow-End a P h stk loc C0 M0 pc ics frs sh)
    then show ?case by(auto dest!: StepT)
  qed
qed

```

— *exec-step* and *exec-step-ind* reach the same result given equivalent input

**lemma** *exec-step-ind-equiv*:

```

exec-step P h stk loc C M pc ics frs sh = (xp', h', frs', sh')
= exec-step-ind (exec-step-input P C M pc ics) P h stk loc C M pc ics frs sh (xp',
h', frs', sh')
using exec-step-imp-exec-step-ind exec-step-ind-imp-exec-step by auto

```

end

## 12 Instantiating *CollectionBasedRTS* with Jinja JVM

**theory** *JVMCollectionBasedRTS*

**imports** *../Common/CollectionBasedRTS JVMCollectionSemantics*

*JinjaDCI.BVSpecTypeSafe ../JinjaSuppl/JVMExecStepInductive*

**begin**

**lemma** *eq-equiv[simp]*: *equiv UNIV {(x, y). x = y}*  
**by**(*simp add: equiv-def refl-on-def sym-def trans-def*)

### 12.1 Some *classes-above* lemmas

**lemma** *start-prog-classes-above-Start*:

```

classes-above (start-prog P C M) Start = {Object, Start}
using start-prog-Start-super[of C M P] subcls1-confluent by auto

```

**lemma** *class-add-classes-above*:

```

assumes ns: ¬ is-class P C and ¬P ⊢ D ≼* C
shows classes-above (class-add P (C, cdec)) D = classes-above P D
using assms by(auto intro: class-add-subcls class-add-subcls-rev)

```

**lemma** *class-add-classes-above-xcpts*:  
**assumes** *ns*:  $\neg$  *is-class* *P C*  
**and** *ncp*:  $\bigwedge D. D \in \text{sys-xcpts} \implies \neg P \vdash D \preceq^* C$   
**shows** *classes-above-xcpts* (*class-add* *P* (*C*, *cdec*)) = *classes-above-xcpts* *P*  
**using** *assms class-add-classes-above* **by** *simp*

## 12.2 JVM next-step lemmas for initialization calling

**lemma** *JVM-New-next-step*:  
**assumes** *step*:  $\sigma' \in \text{JVMsmall}$  *P*  $\sigma$   
**and** *nend*:  $\sigma \notin \text{JVMendset}$   
**and** *curr*: *curr-instr* *P* (*hd*(*frames-of*  $\sigma$ )) = *New* *C*  
**and** *nDone*:  $\neg(\exists \text{sfs } i. \text{sheap } \sigma \ C = \text{Some}(\text{sfs}, i) \wedge i = \text{Done})$   
**and** *ics*: *ics-of*(*hd*(*frames-of*  $\sigma$ )) = *No-ics*  
**shows** *ics-of* (*hd*(*frames-of*  $\sigma'$ )) = *Calling* *C*  $\square$   $\wedge$  *sheap*  $\sigma = \text{sheap } \sigma' \wedge \sigma' \notin \text{JVMendset}$   
**proof** –  
**obtain** *xp h frs sh* **where**  $\sigma = (xp, h, frs, sh)$  **by**(*cases*  $\sigma$ )  
**then obtain** *f1 frs1* **where** *frs*: *frs*=*f1*#*frs1* **using** *nend* **by**(*cases* *frs*, *simp-all* *add*: *JVMendset-def*)  
**then obtain** *stk loc C' M' pc ics* **where** *f1*:*f1*=(*stk, loc, C', M', pc, ics*) **by**(*cases* *f1*)  
**have** *xp*: *xp* = *None* **using**  $\sigma$  *nend* **by**(*simp* *add*: *JVMendset-def*)  
**obtain** *xp' h' frs' sh'* **where**  $\sigma' = (xp', h', frs', sh')$  **by**(*cases*  $\sigma'$ )  
**have** *ics-of* (*hd* *frs'*) = *Calling* *C*  $\square$   $\wedge$  *sh* = *sh'*  $\wedge$  *frs'*  $\neq \square$   $\wedge$  *xp'* = *None*  
**proof**(*cases* *sh C*)  
**case** *None* **then show** *?thesis* **using**  $\sigma'$  *xp f1 frs*  $\sigma$  *assms* **by** *auto*  
**next**  
**case** (*Some a*)  
**then obtain** *sfs i* **where** *a*: *a*=(*sfs, i*) **by**(*cases* *a*)  
**then have** *nDone'*: *i*  $\neq$  *Done* **using** *nDone* *Some f1 frs*  $\sigma$  **by** *simp*  
**show** *?thesis* **using** *a* *Some*  $\sigma'$  *xp f1 frs*  $\sigma$  *assms* **by**(*auto split*: *init-state.splits*)  
**qed**  
**then show** *?thesis* **using** *ics*  $\sigma$   $\sigma'$  **by**(*cases* *frs'*, *auto simp*: *JVMendset-def*)  
**qed**

**lemma** *JVM-Getstatic-next-step*:  
**assumes** *step*:  $\sigma' \in \text{JVMsmall}$  *P*  $\sigma$   
**and** *nend*:  $\sigma \notin \text{JVMendset}$   
**and** *curr*: *curr-instr* *P* (*hd*(*frames-of*  $\sigma$ )) = *Getstatic* *C F D*  
**and** *fC*:  $P \vdash C$  *has* *F, Static:t* *in D*  
**and** *nDone*:  $\neg(\exists \text{sfs } i. \text{sheap } \sigma \ D = \text{Some}(\text{sfs}, i) \wedge i = \text{Done})$   
**and** *ics*: *ics-of*(*hd*(*frames-of*  $\sigma$ )) = *No-ics*  
**shows** *ics-of* (*hd*(*frames-of*  $\sigma'$ )) = *Calling* *D*  $\square$   $\wedge$  *sheap*  $\sigma = \text{sheap } \sigma' \wedge \sigma' \notin \text{JVMendset}$   
**proof** –  
**obtain** *xp h frs sh* **where**  $\sigma = (xp, h, frs, sh)$  **by**(*cases*  $\sigma$ )  
**then obtain** *f1 frs1* **where** *frs*: *frs*=*f1*#*frs1* **using** *nend* **by**(*cases* *frs*, *simp-all* *add*: *JVMendset-def*)

**then obtain**  $stk\ loc\ C'\ M'\ pc\ ics$  **where**  $f1:f1=(stk,loc,C',M',pc,ics)$  **by**(cases  $f1$ )  
**have**  $xp: xp = None$  **using**  $\sigma\ nend$  **by**(simp add: JVMendset-def)  
**obtain**  $xp'\ h'\ frs'\ sh'$  **where**  $\sigma': \sigma'=(xp',h',frs',sh')$  **by**(cases  $\sigma'$ )  
**have**  $ex': \exists t\ b. P \vdash C\ has\ F,b:t\ in\ D$  **using**  $fC$  **by** auto  
**have**  $field: \exists t. field\ P\ D\ F = (D,Static,t)$   
**using**  $fC\ field-def2\ has-field-idemp\ has-field-sees$  **by** blast  
**have**  $nCalled': \forall Cs. ics \neq Called\ Cs$  **using**  $ics\ f1\ frs\ \sigma$  **by** simp  
**have**  $ics-of\ (hd\ frs') = Calling\ D\ [] \wedge sh = sh' \wedge frs' \neq [] \wedge xp' = None$   
**proof**(cases  $sh\ D$ )  
**case**  $None$  **then show**  $?thesis$  **using**  $field\ ex'\ \sigma'\ xp\ f1\ frs\ \sigma\ assms$  **by** auto  
**next**  
**case** (Some  $a$ )  
**then obtain**  $sfs\ i$  **where**  $a: a=(sfs,i)$  **by**(cases  $a$ )  
**show**  $?thesis$  **using**  $field\ ex'\ a\ Some\ \sigma'\ xp\ f1\ frs\ \sigma\ assms$  **by**(auto split:  
*init-state.splits*)  
**qed**  
**then show**  $?thesis$  **using**  $ics\ \sigma\ \sigma'$  **by**(auto simp: JVMendset-def)  
**qed**

**lemma** *JVM-Putstatic-next-step:*

**assumes**  $step: \sigma' \in JVMsmall\ P\ \sigma$

**and**  $nend: \sigma \notin JVMendset$

**and**  $curr: curr-instr\ P\ (hd(frames-of\ \sigma)) = Putstatic\ C\ F\ D$

**and**  $fC: P \vdash C\ has\ F,Static:t\ in\ D$

**and**  $nDone: \neg(\exists sfs\ i. sheap\ \sigma\ D = Some(sfs,i) \wedge i = Done)$

**and**  $ics: ics-of(hd(frames-of\ \sigma)) = No-ics$

**shows**  $ics-of\ (hd(frames-of\ \sigma')) = Calling\ D\ [] \wedge sheap\ \sigma = sheap\ \sigma' \wedge \sigma' \notin JVMendset$

**proof** –

**obtain**  $xp\ h\ frs\ sh$  **where**  $\sigma: \sigma=(xp,h,frs,sh)$  **by**(cases  $\sigma$ )

**then obtain**  $f1\ frs1$  **where**  $frs: frs=f1\#\#frs1$  **using**  $nend$  **by**(cases  $frs$ , simp-all add: JVMendset-def)

**then obtain**  $stk\ loc\ C'\ M'\ pc\ ics$  **where**  $f1:f1=(stk,loc,C',M',pc,ics)$  **by**(cases  $f1$ )

**have**  $xp: xp = None$  **using**  $\sigma\ nend$  **by**(simp add: JVMendset-def)

**obtain**  $xp'\ h'\ frs'\ sh'$  **where**  $\sigma': \sigma'=(xp',h',frs',sh')$  **by**(cases  $\sigma'$ )

**have**  $ex': \exists t\ b. P \vdash C\ has\ F,b:t\ in\ D$  **using**  $fC$  **by** auto

**have**  $field: field\ P\ D\ F = (D,Static,t)$

**using**  $fC\ field-def2\ has-field-idemp\ has-field-sees$  **by** blast

**have**  $ics': ics-of\ (hd\ frs') = Calling\ D\ [] \wedge sh = sh' \wedge frs' \neq [] \wedge xp' = None$

**proof**(cases  $sh\ D$ )

**case**  $None$  **then show**  $?thesis$  **using**  $field\ ex'\ \sigma'\ xp\ f1\ frs\ \sigma\ assms$  **by** auto

**next**

**case** (Some  $a$ )

**then obtain**  $sfs\ i$  **where**  $a: a=(sfs,i)$  **by**(cases  $a$ )

**show**  $?thesis$  **using**  $field\ ex'\ a\ Some\ \sigma'\ xp\ f1\ frs\ \sigma\ assms$  **by**(auto split:  
*init-state.splits*)

**qed**

**then show** *?thesis* **using** *ics*  $\sigma$   $\sigma'$  **by**(*auto simp: JVMendset-def*)  
**qed**

**lemma** *JVM-Invokestatic-next-step*:

**assumes** *step*:  $\sigma' \in \text{JVMsmall } P \ \sigma$

**and** *nend*:  $\sigma \notin \text{JVMendset}$

**and** *curr*: *curr-instr*  $P$  (*hd*(*frames-of*  $\sigma$ )) = *Invokestatic*  $C \ M \ n$

**and** *mC*:  $P \vdash C$  *sees*  $M, \text{Static}: Ts \rightarrow T = m$  *in*  $D$

**and** *nDone*:  $\neg(\exists \text{sfs } i. \text{sheap } \sigma \ D = \text{Some}(\text{sfs}, i) \wedge i = \text{Done})$

**and** *ics*: *ics-of*(*hd*(*frames-of*  $\sigma$ )) = *No-ics*

**shows** *ics-of*(*hd*(*frames-of*  $\sigma'$ )) = *Calling*  $D \ [] \wedge \text{sheap } \sigma = \text{sheap } \sigma' \wedge \sigma' \notin \text{JVMendset}$

**proof** –

**obtain**  $xp \ h \ \text{frs} \ \text{sh}$  **where**  $\sigma = (xp, h, \text{frs}, \text{sh})$  **by**(*cases*  $\sigma$ )

**then obtain**  $f1 \ \text{frs1}$  **where**  $\text{frs} = f1 \# \text{frs1}$  **using** *nend* **by**(*cases frs, simp-all add: JVMendset-def*)

**then obtain**  $stk \ \text{loc} \ C' \ M' \ \text{pc} \ \text{ics}$  **where**  $f1: f1 = (stk, \text{loc}, C', M', \text{pc}, \text{ics})$  **by**(*cases f1*)

**have**  $xp$ :  $xp = \text{None}$  **using**  $\sigma$  *nend* **by**(*simp add: JVMendset-def*)

**obtain**  $xp' \ h' \ \text{frs}' \ \text{sh}'$  **where**  $\sigma' = (xp', h', \text{frs}', \text{sh}')$  **by**(*cases*  $\sigma'$ )

**have**  $ex'$ :  $\exists Ts \ T \ m \ D \ b. P \vdash C$  *sees*  $M, b: Ts \rightarrow T = m$  *in*  $D$  **using** *mC* **by** *fastforce*

**have** *method*:  $\exists m. \text{method } P \ C \ M = (D, \text{Static}, m)$  **using** *mC* **by**(*cases m, auto*)

**have**  $ics'$ : *ics-of*(*hd*  $\text{frs}'$ ) = *Calling*  $D \ [] \wedge \text{sh} = \text{sh}' \wedge \text{frs}' \neq [] \wedge xp' = \text{None}$

**proof**(*cases sh D*)

**case** *None* **then show** *?thesis* **using** *method ex'  $\sigma'$  xp f1 frs  $\sigma$  assms* **by** *auto next*

**case** (*Some a*)

**then obtain**  $\text{sfs} \ i$  **where**  $a = (\text{sfs}, i)$  **by**(*cases a*)

**then have** *nDone'*:  $i \neq \text{Done}$  **using** *nDone Some f1 frs  $\sigma$  by simp*

**show** *?thesis* **using** *method ex' a Some  $\sigma'$  xp f1 frs  $\sigma$  assms* **by**(*auto split: init-state.splits*)

**qed**

**then show** *?thesis* **using** *ics*  $\sigma$   $\sigma'$  **by**(*auto simp: JVMendset-def*)

**qed**

### 12.3 Definitions

**definition** *main* :: *string* **where** *main* = "main"

**definition** *Test* :: *string* **where** *Test* = "Test"

**definition** *test-oracle* :: *string* **where** *test-oracle* = "oracle"

**type-synonym** *jvm-class* = *jvm-method cdecl*

**type-synonym** *jvm-prog-out* = *jvm-state*  $\times$  *cname set*

A deselection algorithm based on classes that have changed from  $P1$  to  $P2$ :

**primrec** *jvm-deselect* :: *jvm-prog*  $\Rightarrow$  *jvm-prog-out*  $\Rightarrow$  *jvm-prog*  $\Rightarrow$  *bool* **where**  
*jvm-deselect*  $P1 \ (\sigma, \text{cset}) \ P2 = (\text{cset} \cap (\text{classes-changed } P1 \ P2)) = \{\}$

**definition** *jvm-progs* :: *jvm-prog* set **where**  
*jvm-progs*  $\equiv \{P. \text{wf-jvm-prog } P \wedge \neg \text{is-class } P \text{ Start} \wedge \neg \text{is-class } P \text{ Test}$   
 $\wedge (\forall b' Ts' T' m' D'. P \vdash \text{Object sees start-}m, b' : Ts' \rightarrow T' = m' \text{ in } D'$   
 $\rightarrow b' = \text{Static} \wedge Ts' = [] \wedge T' = \text{Void}) \}$

**definition** *jvm-tests* :: *jvm-class* set **where**  
*jvm-tests* =  $\{t. \text{fst } t = \text{Test}$   
 $\wedge (\forall P \in \text{jvm-progs}. \text{wf-jvm-prog } (t\#P) \wedge (\exists m. t\#P \vdash \text{Test sees main,Static: } []$   
 $\rightarrow \text{Void} = m \text{ in Test})) \}$

**abbreviation** *jvm-make-test-prog* :: *jvm-prog*  $\Rightarrow$  *jvm-class*  $\Rightarrow$  *jvm-prog* **where**  
*jvm-make-test-prog*  $P \ t \equiv \text{start-prog } (t\#P) \ (\text{fst } t) \ \text{main}$

**declare** *jvm-progs-def* [*simp*]  
**declare** *jvm-tests-def* [*simp*]

## 12.4 Definition lemmas

**lemma** *jvm-progs-tests-nStart*:  
**assumes**  $P: P \in \text{jvm-progs}$  **and**  $t: t \in \text{jvm-tests}$   
**shows**  $\neg \text{is-class } (t\#P) \ \text{Start}$   
**using** *assms* **by**(*simp add: is-class-def class-def Start-def Test-def*)

**lemma** *jvm-make-test-prog-classes-above-xcpts*:  
**assumes**  $P: P \in \text{jvm-progs}$  **and**  $t: t \in \text{jvm-tests}$   
**shows**  $\text{classes-above-xcpts } (\text{jvm-make-test-prog } P \ t) = \text{classes-above-xcpts } P$   
**proof** –

**have**  $nS: \neg \text{is-class } (t\#P) \ \text{Start}$  **by**(*rule jvm-progs-tests-nStart[OF P t]*)  
**from**  $P$  **have**  $nT: \neg \text{is-class } P \ \text{Test}$  **by** *simp*  
**from**  $P \ t$  **have**  $\text{wf-syscls } (t\#P) \wedge \text{wf-syscls } P$   
**by**(*simp add: wf-jvm-prog-def wf-jvm-prog-phi-def wf-prog-def*)  
**then have** [*simp*]:  $\bigwedge D. D \in \text{sys-xcpts} \implies \text{is-class } (t\#P) \ D \wedge \text{is-class } P \ D$   
**by**(*cases t, auto simp: wf-syscls-def is-class-def class-def dest!: weak-map-of-SomeI*)  
**from**  $\text{wf-nclass-nsub}[OF \ - \ nS] \ P \ t$  **have**  $nspS: \bigwedge D. D \in \text{sys-xcpts} \implies \neg(t\#P)$   
 $\vdash D \preceq^* \text{Start}$   
**by**(*auto simp: wf-jvm-prog-def wf-jvm-prog-phi-def*)  
**from**  $\text{wf-nclass-nsub}[OF \ - \ nT] \ P$  **have**  $nspT: \bigwedge D. D \in \text{sys-xcpts} \implies \neg P \vdash D$   
 $\preceq^* \text{Test}$   
**by**(*auto simp: wf-jvm-prog-def wf-jvm-prog-phi-def*)

**from** *class-add-classes-above-xcpts*[**where**  $P=t\#P$  **and**  $C=\text{Start}$ , *OF nS nspS*]  
**have**  $\text{classes-above-xcpts } (\text{jvm-make-test-prog } P \ t) = \text{classes-above-xcpts } (t\#P)$   
**by** *simp*  
**also from** *class-add-classes-above-xcpts*[**where**  $P=P$  **and**  $C=\text{Test}$ , *OF nT nspT*]  
 $t$   
**have**  $\dots = \text{classes-above-xcpts } P$  **by**(*cases t, simp*)  
**finally show** *?thesis* **by** *simp*  
**qed**

**lemma** *jvm-make-test-prog-sees-Test-main*:  
**assumes**  $P: P \in \text{jvm-progs}$  **and**  $t: t \in \text{jvm-tests}$   
**shows**  $\exists m. \text{jvm-make-test-prog } P \ t \vdash \text{Test sees main, Static} : [] \rightarrow \text{Void} = m \text{ in Test}$   
**proof** –  
**let**  $?P = \text{jvm-make-test-prog } P \ t$   
**from**  $P \ t$  **obtain**  $m$  **where**  
 $\text{meth}: t \# P \vdash \text{Test sees main, Static}: [] \rightarrow \text{Void} = m \text{ in Test}$  **and**  
 $\text{nstart}: \neg \text{is-class } (t \# P) \ \text{Start}$   
**by** (*auto simp: is-class-def class-def Start-def Test-def*)  
**from** *class-add-sees-method*[*OF meth nstart*] **show** *?thesis* **by** *fastforce*  
**qed**

## 12.5 Naive RTS algorithm

### 12.5.1 Definitions

**fun** *jvm-naive-out* :: *jvm-prog*  $\Rightarrow$  *jvm-class*  $\Rightarrow$  *jvm-prog-out set* **where**  
*jvm-naive-out*  $P \ t = \text{JVMNaiveCollectionSemantics.cbig } (\text{jvm-make-test-prog } P \ t)$   
(*start-state* ( $t \# P$ ))

**abbreviation** *jvm-naive-collect-start* :: *jvm-prog*  $\Rightarrow$  *cname set* **where**  
*jvm-naive-collect-start*  $P \equiv \{\}$

**lemma** *jvm-naive-out-xcpts-collected*:  
**assumes**  $o1 \in \text{jvm-naive-out } P \ t$   
**shows** *classes-above-xcpts* (*start-prog* ( $t \# P$ ) (*fst*  $t$ ) *main*)  $\subseteq \text{snd } o1$   
**using** *assms*  
**proof** –  
**obtain**  $\sigma' \ \text{coll}'$  **where**  $o1 = (\sigma', \text{coll}')$  **and**  
 $\text{cbig}: (\sigma', \text{coll}') \in \text{JVMNaiveCollectionSemantics.cbig } (\text{start-prog } (t \# P) \ (\text{fst } t)$   
*main*) (*start-state* ( $t \# P$ ))  
**using** *assms* **by** (*cases o1, simp*)  
**with** *JVMNaiveCollectionSemantics.cbig-stepD*[*OF cbig start-state-nend*]  
**show** *?thesis* **by** (*auto simp: JVMNaiveCollectionSemantics.csmall-def start-state-def*)  
**qed**

### 12.5.2 Naive algorithm correctness

We start with correctness over *exec-instr*, then all the functions/pieces that are used by naive *csmall* (that is, pieces used by *exec* - such as which frames are used based on *ics* - and all functions used by the collection function). We then prove that *csmall* is existence safe, extend this result to *cbig*, and finally prove the *existence-safe* statement over the locale pieces.

**lemma** *ncollect-exec-instr*:  
**assumes** *JVMinstr-ncollect*  $P \ i \ h \ \text{stk} \cap \text{classes-changed } P \ P' = \{\}$   
**and** *above-C*: *classes-above*  $P \ C \cap \text{classes-changed } P \ P' = \{\}$   
**and** *ics*:  $\text{ics} = \text{Called } [] \vee \text{ics} = \text{No-ics}$

```

and  $i: i = \text{instrs-of } P \ C \ M \ ! \ pc$ 
shows  $\text{exec-instr } i \ P \ h \ stk \ loc \ C \ M \ pc \ ics \ frs \ sh = \text{exec-instr } i \ P' \ h \ stk \ loc \ C \ M \ pc \ ics \ frs \ sh$ 
using  $assms$  proof( $cases \ i$ )
  case ( $New \ C1$ ) then show  $?thesis$  using  $assms \ classes\text{-above}\text{-blank}[of \ C1 \ P \ P']$ 
    by( $auto \ split: \ init\text{-state.splits} \ option.splits$ )
next
  case ( $Getfield \ F1 \ C1$ ) show  $?thesis$ 
  proof( $cases \ hd \ stk = \text{Null}$ )
    case  $True$  then show  $?thesis$  using  $Getfield \ assms$  by  $simp$ 
  next
    case  $False$ 
    let  $?D = (cname\text{-of } h \ (the\text{-Addr} \ (hd \ stk)))$ 
    have  $D: \text{classes}\text{-above } P \ ?D \cap \text{classes}\text{-changed } P \ P' = \{\}$ 
      using  $False \ Getfield \ assms$  by  $simp$ 
    show  $?thesis$ 
    proof( $cases \ \exists \ b \ t. \ P \vdash \ ?D \ \text{has } F1, b:t \ \text{in } C1$ )
      case  $True$ 
      then obtain  $b1 \ t1$  where  $P \vdash \ ?D \ \text{has } F1, b1:t1 \ \text{in } C1$  by  $auto$ 
      then have  $has: P' \vdash \ ?D \ \text{has } F1, b1:t1 \ \text{in } C1$ 
        using  $Getfield \ assms \ classes\text{-above}\text{-has}\text{-field}[OF \ D]$  by  $auto$ 
      have  $P \vdash \ ?D \preceq^* C1$  using  $has\text{-field}\text{-decl}\text{-above} \ True$  by  $auto$ 
      then have  $\text{classes}\text{-above } P \ C1 \subseteq \text{classes}\text{-above } P \ ?D$  by( $rule \ \text{classes}\text{-above}\text{-subcls}\text{-subset}$ )
      then have  $C1: \text{classes}\text{-above } P \ C1 \cap \text{classes}\text{-changed } P \ P' = \{\}$  using  $D$  by
 $auto$ 
      then show  $?thesis$  using  $has \ True \ Getfield \ assms \ classes\text{-above}\text{-field}[of \ C1 \ P \ P' \ F1]$ 
        by( $cases \ field \ P' \ C1 \ F1, \ cases \ the \ (h \ (the\text{-Addr} \ (hd \ stk))), \ auto$ )
      next
      case  $nex: False$ 
      then have  $\nexists \ b \ t. \ P' \vdash \ ?D \ \text{has } F1, b:t \ \text{in } C1$ 
        using  $False \ Getfield \ assms$ 
         $\text{classes}\text{-above}\text{-has}\text{-field}2[\text{where } C=?D \ \text{and } P=P \ \text{and } P'=P' \ \text{and } F=F1$ 
and } C'=C1]
        by  $auto$ 
      then show  $?thesis$  using  $nex \ Getfield \ assms \ classes\text{-above}\text{-field}[of \ C1 \ P \ P' \ F1]$ 
        by( $cases \ field \ P' \ C1 \ F1, \ cases \ the \ (h \ (the\text{-Addr} \ (hd \ stk))), \ auto$ )
      qed
    qed
  next
  case ( $Getstatic \ C1 \ F1 \ D1$ )
  then have  $C1: \text{classes}\text{-above } P \ C1 \cap \text{classes}\text{-changed } P \ P' = \{\}$  using  $assms$  by
 $auto$ 
  show  $?thesis$ 
  proof( $cases \ \exists \ b \ t. \ P \vdash \ C1 \ \text{has } F1, b:t \ \text{in } D1$ )
    case  $True$ 
    then obtain  $b \ t$  where  $meth: P \vdash \ C1 \ \text{has } F1, b:t \ \text{in } D1$  by  $auto$ 
    then have  $P \vdash \ C1 \preceq^* D1$  by( $rule \ has\text{-field}\text{-decl}\text{-above}$ )

```

```

then have D1: classes-above P D1  $\cap$  classes-changed P P' = {}
  using C1 rtranc-trans by fastforce
have P'  $\vdash$  C1 has F1,b:t in D1
  using meth Getstatic assms classes-above-has-field[OF C1] by auto
then show ?thesis using True D1 Getstatic assms classes-above-field[of D1 P
P' F1]
  by(cases field P' D1 F1, auto)
next
case False
then have  $\nexists$  b t. P'  $\vdash$  C1 has F1,b:t in D1
  using Getstatic assms
  classes-above-has-field2[where C=C1 and P=P and P'=P' and F=F1 and
C'=D1]
  by auto
then show ?thesis using False Getstatic assms
  by(cases field P' D1 F1, auto)
qed
next
case (Putfield F1 C1) show ?thesis
proof(cases hd(tl stk) = Null)
  case True then show ?thesis using Putfield assms by simp
next
case False
let ?D = (cname-of h (the-Addr (hd (tl stk))))
have D: classes-above P ?D  $\cap$  classes-changed P P' = {} using False Putfield
assms by simp
show ?thesis
proof(cases  $\exists$  b t. P  $\vdash$  ?D has F1,b:t in C1)
  case True
  then obtain b1 t1 where P  $\vdash$  ?D has F1,b1:t1 in C1 by auto
  then have has: P'  $\vdash$  ?D has F1,b1:t1 in C1
    using Putfield assms classes-above-has-field[OF D] by auto
  have P  $\vdash$  ?D  $\preceq^*$  C1 using has-field-decl-above True by auto
  then have classes-above P C1  $\subseteq$  classes-above P ?D by(rule classes-above-subcls-subset)
  then have C1: classes-above P C1  $\cap$  classes-changed P P' = {} using D by
auto
  then show ?thesis using has True Putfield assms classes-above-field[of C1 P
P' F1]
    by(cases field P' C1 F1, cases the (h (the-Addr (hd (tl stk))))), auto)
next
case nex: False
then have  $\nexists$  b t. P'  $\vdash$  ?D has F1,b:t in C1
  using False Putfield assms
  classes-above-has-field2[where C=?D and P=P and P'=P' and F=F1
and C'=C1]
  by auto
then show ?thesis using nex Putfield assms classes-above-field[of C1 P P'
F1]
  by(cases field P' C1 F1, cases the (h (the-Addr (hd (tl stk))))), auto)

```



```

    qed
  qed
next
  case (Putstatic C1 F1 D1)
  then have C1: classes-above P C1  $\cap$  classes-changed P P' = {} using Putstatic
  assms by auto
  show ?thesis
  proof(cases  $\exists b t. P \vdash C1 \text{ has } F1, b:t \text{ in } D1$ )
    case True
    then obtain b t where meth: P  $\vdash$  C1 has F1, b:t in D1 by auto
    then have P  $\vdash$  C1  $\preceq^*$  D1 by(rule has-field-decl-above)
    then have D1: classes-above P D1  $\cap$  classes-changed P P' = {}
      using C1 rtrancl-trans by fastforce
    then have P'  $\vdash$  C1 has F1, b:t in D1
    using meth Putstatic assms classes-above-has-field[OF C1] by auto
    then show ?thesis using True D1 Putstatic assms classes-above-field[of D1 P
P' F1]
      by(cases field P' D1 F1, auto)
    next
    case False
    then have  $\nexists b t. P' \vdash C1 \text{ has } F1, b:t \text{ in } D1$ 
      using Putstatic assms classes-above-has-field2[where C=C1 and P=P and
P'=P' and F=F1 and C'=D1]
      by auto
    then show ?thesis using False Putstatic assms
      by(cases field P' D1 F1, auto)
  qed
next
  case (Checkcast C1)
  then show ?thesis using assms
  proof(cases hd stk = Null)
    case False then show ?thesis
      using Checkcast assms classes-above-subcls classes-above-subcls2
      by(simp add: cast-ok-def) blast
  qed(simp add: cast-ok-def)
next
  case (Invoke M n)
  let ?C = cname-of h (the-Addr (stk ! n))
  show ?thesis
  proof(cases stk ! n = Null)
    case True then show ?thesis using Invoke assms by simp
  next
  case False
  then have above: classes-above P ?C  $\cap$  classes-changed P P' = {}
    using Invoke assms by simp
  obtain D b Ts T mxs mxl ins xt where meth: method P' ?C M = (D, b, Ts, T, mxs, mxl, ins, xt)
    by(cases method P' ?C M, clarsimp)
  have meq: method P ?C M = method P' ?C M
    using classes-above-method[OF above] by simp

```

```

then show ?thesis
proof(cases  $\exists Ts T m D b. P \vdash ?C \text{ sees } M, b:Ts \rightarrow T = m \text{ in } D$ )
  case nex: False
    then have  $\neg(\exists Ts T m D b. P' \vdash ?C \text{ sees } M, b:Ts \rightarrow T = m \text{ in } D)$ 
      using classes-above-sees-method2[OF above, of M] by clarsimp
    then show ?thesis using nex False Invoke by simp
  next
    case True
      then have  $\exists Ts T m D b. P' \vdash ?C \text{ sees } M, b:Ts \rightarrow T = m \text{ in } D$ 
        by(fastforce dest!: classes-above-sees-method[OF above, of M])
      then show ?thesis using meq meth True Invoke by simp
    qed
  qed
next
  case (Invokestatic C1 M n)
    then have above: classes-above P C1  $\cap$  classes-changed P P' = {}
      using assms by simp
    obtain D b Ts T mxs mxl ins xt where meth: method P' C1 M = (D, b, Ts, T, mxs, mxl, ins, xt)
      by(cases method P' C1 M, clarsimp)
    have meq: method P C1 M = method P' C1 M
      using classes-above-method[OF above] by simp
    show ?thesis
    proof(cases  $\exists Ts T m D b. P \vdash C1 \text{ sees } M, b:Ts \rightarrow T = m \text{ in } D$ )
      case False
        then have  $\neg(\exists Ts T m D b. P' \vdash C1 \text{ sees } M, b:Ts \rightarrow T = m \text{ in } D)$ 
          using classes-above-sees-method2[OF above, of M] by clarsimp
        then show ?thesis using False Invokestatic by simp
      next
        case True
          then have  $\exists Ts T m D b. P' \vdash C1 \text{ sees } M, b:Ts \rightarrow T = m \text{ in } D$ 
            by(fastforce dest!: classes-above-sees-method[OF above, of M])
          then show ?thesis using meq meth True Invokestatic by simp
        qed
      next
        case Return then show ?thesis using assms classes-above-method[OF above-C]
          by(cases frs, auto)
      next
        case (Load x1) then show ?thesis using assms by auto
      next
        case (Store x2) then show ?thesis using assms by auto
      next
        case (Push x3) then show ?thesis using assms by auto
      next
        case (Goto x15) then show ?thesis using assms by auto
      next
        case (IfFalse x17) then show ?thesis using assms by auto
    qed(auto)

```

— if collected classes unchanged, instruction collection unchanged

**lemma** *ncollect-JVMinstr-ncollect*:

**assumes**  $JVMinstr-ncollect\ P\ i\ h\ stk \cap\ classes-changed\ P\ P' = \{\}$

**shows**  $JVMinstr-ncollect\ P\ i\ h\ stk = JVMinstr-ncollect\ P'\ i\ h\ stk$

**proof**(*cases i*)

**case** (*New C1*)

**then show** *?thesis using assms classes-above-set[of C1 P P'] by auto*

**next**

**case** (*Getfield F C1*) **show** *?thesis*

**proof**(*cases hd stk = Null*)

**case** *True* **then show** *?thesis using Getfield assms by simp*

**next**

**case** *False*

**let**  $?D = cname-of\ h\ (the-Addr\ (hd\ stk))$

**have**  $classes-above\ P\ ?D \cap\ classes-changed\ P\ P' = \{\}$  **using** *False Getfield*

*assms by auto*

**then have**  $classes-above\ P\ ?D = classes-above\ P'\ ?D$

**using** *classes-above-set* **by** *blast*

**then show** *?thesis using assms Getfield by auto*

**qed**

**next**

**case** (*Getstatic C1 P1 D1*)

**then have**  $classes-above\ P\ C1 \cap\ classes-changed\ P\ P' = \{\}$  **using** *assms by auto*

**then have**  $classes-above\ P\ C1 = classes-above\ P'\ C1$

**using** *classes-above-set assms Getstatic* **by** *blast*

**then show** *?thesis using assms Getstatic by auto*

**next**

**case** (*Putfield F C1*) **show** *?thesis*

**proof**(*cases hd(tl stk) = Null*)

**case** *True* **then show** *?thesis using Putfield assms by simp*

**next**

**case** *False*

**let**  $?D = cname-of\ h\ (the-Addr\ (hd\ (tl\ stk)))$

**have**  $classes-above\ P\ ?D \cap\ classes-changed\ P\ P' = \{\}$  **using** *False Putfield*

*assms by auto*

**then have**  $classes-above\ P\ ?D = classes-above\ P'\ ?D$

**using** *classes-above-set* **by** *blast*

**then show** *?thesis using assms Putfield by auto*

**qed**

**next**

**case** (*Putstatic C1 F D1*)

**then have**  $classes-above\ P\ C1 \cap\ classes-changed\ P\ P' = \{\}$  **using** *assms by auto*

**then have**  $classes-above\ P\ C1 = classes-above\ P'\ C1$

**using** *classes-above-set assms Putstatic* **by** *blast*

**then show** *?thesis using assms Putstatic by auto*

**next**

**case** (*Checkcast C1*)

**then show** *?thesis using assms*

*classes-above-set[of cname-of h (the-Addr (hd stk)) P P'] by auto*

```

next
  case (Invoke M n)
  then show ?thesis using assms
    classes-above-set[of cname-of h (the-Addr (stk ! n)) P P'] by auto
next
  case (Invokestatic C1 M n)
  then show ?thesis using assms classes-above-set[of C1 P P'] by auto
next
  case Return
  then show ?thesis using assms classes-above-set[of - P P'] by auto
next
  case Throw
  then show ?thesis using assms
    classes-above-set[of cname-of h (the-Addr (hd stk)) P P'] by auto
qed(auto)

```

— if collected classes unchanged, *exec-step* unchanged

**lemma** *ncollect-exec-step*:

```

assumes JVMstep-ncollect P h stk C M pc ics  $\cap$  classes-changed P P' = {}
  and above-C: classes-above P C  $\cap$  classes-changed P P' = {}
shows exec-step P h stk loc C M pc ics frs sh = exec-step P' h stk loc C M pc ics
frs sh
proof(cases ics)
  case No-ics then show ?thesis
  using ncollect-exec-instr assms classes-above-method[OF above-C, THEN sym]
  by simp
next
  case (Calling C1 Cs)
  then have above-C1: classes-above P C1  $\cap$  classes-changed P P' = {}
  using assms(1) by auto
  show ?thesis
  proof(cases sh C1)
    case None
    then show ?thesis using Calling assms classes-above-sblank[OF above-C1] by
simp
  next
    case (Some a)
    then obtain sfs i where sfsi: a = (sfs, i) by(cases a)
    then show ?thesis using Calling Some assms
    proof(cases i)
      case Prepared then show ?thesis
      using above-C1 sfsi Calling Some assms classes-above-method[OF above-C1]
      by(simp add: split-beta classes-above-class classes-changed-class[where
cn=C1])
    next
      case Error then show ?thesis
      using above-C1 sfsi Calling Some assms classes-above-method[of C1 P P']
      by(simp add: split-beta classes-above-class classes-changed-class[where
cn=C1])
    end
  end

```

```

    qed(auto)
  qed
next
  case (Called Cs) show ?thesis
  proof(cases Cs)
    case Nil then show ?thesis
    using ncollect-exec-instr assms classes-above-method[OF above-C, THEN sym]
Called
    by simp
  next
    case (Cons C1 Cs1)
    then have above-C': classes-above P C1  $\cap$  classes-changed P P' = {} using
assms Called by auto
    show ?thesis using assms classes-above-method[OF above-C'] Cons Called by
simp
  qed
next
  case (Throwing Cs a) then show ?thesis using assms by(cases Cs; simp)
qed

```

— if collected classes unchanged, *exec-step* collection unchanged

**lemma** *ncollect-JVMstep-ncollect*:

**assumes** *JVMstep-ncollect P h stk C M pc ics*  $\cap$  *classes-changed P P' = {}*

**and** *above-C*: *classes-above P C*  $\cap$  *classes-changed P P' = {}*

**shows** *JVMstep-ncollect P h stk C M pc ics = JVMstep-ncollect P' h stk C M pc ics*

**proof**(*cases ics*)

case *No-ics* then show ?*thesis*

using *assms ncollect-JVMinstr-ncollect classes-above-method[OF above-C]*

by *simp*

**next**

case (*Calling C1 Cs*)

then have *above-C*: *classes-above P C1*  $\cap$  *classes-changed P P' = {}*

using *assms(1)* by *auto*

let ?*C* = *fst(method P C1 clinit)*

show ?*thesis* using *Calling assms classes-above-method[OF above-C]*

*classes-above-set[OF above-C]* by *auto*

**next**

case (*Called Cs*) show ?*thesis*

**proof**(*cases Cs*)

case *Nil* then show ?*thesis*

using *assms ncollect-JVMinstr-ncollect classes-above-method[OF above-C]* *Called*

by *simp*

**next**

case (*Cons C1 Cs1*)

then have *above-C1*: *classes-above P C1*  $\cap$  *classes-changed P P' = {}*

**and** *above-C'*: *classes-above P (fst (method P C1 clinit))*  $\cap$  *classes-changed P P' = {}*

using *assms Called* by *auto*

```

show ?thesis using assms Cons Called classes-above-set[OF above-C1]
  classes-above-set[OF above-C'] classes-above-method[OF above-C1]
  by simp
qed
next
  case (Throwing Cs a) then show ?thesis
  using assms classes-above-set[of cname-of h a P P'] by simp
qed

```

— if collected classes unchanged, *classes-above-frames* unchanged

**lemma** *ncollect-classes-above-frames*:

```

JVMexec-ncollect P (None, h, (stk,loc,C,M,pc,ics)#frs, sh) ∩ classes-changed P
P' = {}

```

```

 $\implies$  classes-above-frames P frs = classes-above-frames P' frs

```

**proof**(*induct frs*)

**case** (*Cons f frs'*)

**then obtain** *stk loc C M pc ics* **where** *f*: *f = (stk,loc,C,M,pc,ics)* **by**(*cases f*)

**then have** *above-C*: *classes-above P C ∩ classes-changed P P' = {}* **using** *Cons*

**by** *auto*

**show** ?*case using f Cons classes-above-subcls*[OF *above-C*]

*classes-above-subcls2*[OF *above-C*] **by** *auto*

**qed**(*auto*)

— if collected classes unchanged, *classes-above-xcpts* unchanged

**lemma** *ncollect-classes-above-xcpts*:

```

assumes JVMexec-ncollect P (None, h, (stk,loc,C,M,pc,ics)#frs, sh) ∩ classes-changed
P P' = {}

```

**shows** *classes-above-xcpts P = classes-above-xcpts P'*

**proof** —

```

have left:  $\bigwedge x x'. x' \in \text{sys-xcpts} \implies P \vdash x' \preceq^* x \implies \exists xa \in \text{sys-xcpts}. P' \vdash xa \preceq^*$ 
x

```

**proof** —

**fix** *x x'*

**assume** *x'*: *x' ∈ sys-xcpts* **and** *above*: *P ⊢ x' ≼\* x*

**then show**  $\exists xa \in \text{sys-xcpts}. P' \vdash xa \preceq^* x$  **using** *assms classes-above-subcls*[OF  
- *above*]

**by**(*rule-tac x=x' in bexI*) *auto*

**qed**

```

have right:  $\bigwedge x x'. x' \in \text{sys-xcpts} \implies P' \vdash x' \preceq^* x \implies \exists xa \in \text{sys-xcpts}. P \vdash xa$ 
 $\preceq^* x$ 

```

**proof** —

**fix** *x x'*

**assume** *x'*: *x' ∈ sys-xcpts* **and** *above*: *P' ⊢ x' ≼\* x*

**then show**  $\exists xa \in \text{sys-xcpts}. P \vdash xa \preceq^* x$  **using** *assms classes-above-subcls2*[OF  
- *above*]

**by**(*rule-tac x=x' in bexI*) *auto*

**qed**

**show** ?*thesis using left right by auto*

**qed**

— if collected classes unchanged, *exec* collection unchanged

**lemma** *ncollect-JVMexec-ncollect*:

**assumes** *JVMexec-ncollect*  $P \sigma \cap \text{classes-changed } P P' = \{\}$

**shows** *JVMexec-ncollect*  $P \sigma = \text{JVMexec-ncollect } P' \sigma$

**proof** –

**obtain**  $xp \ h \ frs \ sh$  **where**  $\sigma: \sigma = (xp, h, frs, sh)$  **by** (*cases*  $\sigma$ )

**then show** *?thesis* **using** *assms*

**proof**(*cases*  $\exists x. xp = \text{Some } x \vee frs = []$ )

**case** *False*

**then obtain**  $stk \ loc \ C \ M \ pc \ ics \ frs'$  **where**  $frs: frs = (stk, loc, C, M, pc, ics) \# frs'$

**by**(*cases*  $frs, \text{auto}$ )

**have** *step*: *JVMstep-ncollect*  $P \ h \ stk \ C \ M \ pc \ ics \ \cap \ \text{classes-changed } P P' = \{\}$

**using** *False*  $\sigma \ frs \ \text{assms}$  **by**(*cases*  $ics, \text{auto}$  *simp*: *JVMNaiveCollectionSemantics.csmall-def*)

**have** *above-C*: *classes-above*  $P \ C \ \cap \ \text{classes-changed } P P' = \{\}$

**using** *False*  $\sigma \ frs \ \text{assms}$  **by**(*auto* *simp*: *JVMNaiveCollectionSemantics.csmall-def*)

**have** *frames*: *classes-above-frames*  $P \ frs' = \text{classes-above-frames } P' \ frs'$

**using** *ncollect-classes-above-frames*  $frs \ \sigma \ \text{False} \ \text{assms}$  **by** *simp*

**have** *xcpts*: *classes-above-xcpts*  $P = \text{classes-above-xcpts } P'$

**using** *ncollect-classes-above-xcpts*  $frs \ \sigma \ \text{False} \ \text{assms}$  **by** *simp*

**show** *?thesis* **using** *False* *xcpts* *frames*  $frs \ \sigma \ \text{ncollect-JVMstep-ncollect}[OF \ \text{step} \ \text{above-C}]$

*classes-above-subcls*[*OF* *above-C*] *classes-above-subcls2*[*OF* *above-C*]

**by** *auto*

**qed**(*auto*)

**qed**

— if collected classes unchanged, classes above an exception returned by *exec-instr* unchanged

**lemma** *ncollect-exec-instr-xcpts*:

**assumes** *collect*: *JVMinstr-ncollect*  $P \ i \ h \ stk \ \cap \ \text{classes-changed } P P' = \{\}$

**and** *xpcollect*: *classes-above-xcpts*  $P \ \cap \ \text{classes-changed } P P' = \{\}$

**and** *prealloc*: *preallocated*  $h$

**and**  $\sigma': \sigma' = \text{exec-instr } i \ P \ h \ stk \ loc \ C \ M \ pc \ ics' \ frs \ sh$

**and**  $xp: \text{fst } \sigma' = \text{Some } a$

**and**  $i: i = \text{instrs-of } P \ C \ M \ ! \ pc$

**shows** *classes-above*  $P \ (\text{cname-of } h \ a) \ \cap \ \text{classes-changed } P P' = \{\}$

**using** *assms* *exec-instr-xcpts*[*OF*  $\sigma' \ xp$ ]

**proof**(*cases*  $i$ )

**case** *Throw* **then show** *?thesis* **using** *assms* **by**(*cases*  $hd \ stk, \text{fastforce+}$ )

**qed**(*fastforce+*)

— if collected classes unchanged, classes above an exception returned by *exec-step* unchanged

**lemma** *ncollect-exec-step-xcpts*:

**assumes** *collect*: *JVMstep-ncollect*  $P \ h \ stk \ C \ M \ pc \ ics \ \cap \ \text{classes-changed } P P' = \{\}$

**and** *xpcollect*: *classes-above-xcpts*  $P \ \cap \ \text{classes-changed } P P' = \{\}$

```

and prealloc: preallocated h
and  $\sigma'$ :  $\sigma' = \text{exec-step } P \ h \ \text{stk} \ \text{loc} \ C \ M \ \text{pc} \ \text{ics} \ \text{frs} \ \text{sh}$ 
and xp:  $\text{fst } \sigma' = \text{Some } a$ 
shows classes-above  $P \ (\text{cname-of } h \ a) \cap \text{classes-changed } P \ P' = \{\}$ 
proof(cases ics)
  case No-ics then show ?thesis using assms ncollect-exec-instr-xcpts by simp
next
  case (Calling x21 x22)
    then show ?thesis using assms by(clarsimp split: option.splits init-state.splits if-split-asm)
  next
  case (Called Cs) then show ?thesis using assms ncollect-exec-instr-xcpts by(cases Cs; simp)
next
  case (Throwing Cs a) then show ?thesis using assms by(cases Cs; simp)
qed

```

— if collected classes unchanged, if *csmall* returned a result under  $P$ ,  $P'$  returns the same

```

lemma ncollect-JVMsmall:
assumes collect:  $(\sigma', \text{cset}) \in \text{JVMNaiveCollectionSemantics.csmall } P \ \sigma$ 
and intersect:  $\text{cset} \cap \text{classes-changed } P \ P' = \{\}$ 
and prealloc: preallocated (fst(snd  $\sigma$ ))
shows  $(\sigma', \text{cset}) \in \text{JVMNaiveCollectionSemantics.csmall } P' \ \sigma$ 
proof –
  obtain xp h frs sh where  $\sigma: \sigma = (xp, h, frs, sh)$  by(cases  $\sigma$ )
  then have prealloc': preallocated h using prealloc by simp
  show ?thesis using  $\sigma$  assms
  proof(cases  $\exists x. xp = \text{Some } x \vee frs = []$ )
    case False
      then obtain stk loc C M pc ics frs' where  $\text{frs}: \text{frs} = (\text{stk}, \text{loc}, C, M, \text{pc}, \text{ics}) \# \text{frs}'$ 
        by(cases frs, auto)
      have step:  $\text{JVMstep-ncollect } P \ h \ \text{stk} \ C \ M \ \text{pc} \ \text{ics} \cap \text{classes-changed } P \ P' = \{\}$ 
        using False  $\sigma$  frs assms by(cases ics, auto simp: JVMNaiveCollectionSemantics.csmall-def)
      have above-C:  $\text{classes-above } P \ C \cap \text{classes-changed } P \ P' = \{\}$ 
        using False  $\sigma$  frs assms by(auto simp: JVMNaiveCollectionSemantics.csmall-def)
      obtain xp1 h1 frs1 sh1 where  $\text{exec}: \text{exec-step } P' \ h \ \text{stk} \ \text{loc} \ C \ M \ \text{pc} \ \text{ics} \ \text{frs}' \ \text{sh} =$ 
        (xp1, h1, frs1, sh1)
        by(cases exec-step P' h stk loc C M pc ics frs' sh)
      have collect:  $\text{JVMexec-ncollect } P \ \sigma = \text{JVMexec-ncollect } P' \ \sigma$ 
        using assms ncollect-JVMexec-ncollect by(simp add: JVMNaiveCollectionSemantics.csmall-def)
      have exec-instr:  $\text{exec-step } P \ h \ \text{stk} \ \text{loc} \ C \ M \ \text{pc} \ \text{ics} \ \text{frs}' \ \text{sh}$ 
        =  $\text{exec-step } P' \ h \ \text{stk} \ \text{loc} \ C \ M \ \text{pc} \ \text{ics} \ \text{frs}' \ \text{sh}$ 
        using ncollect-exec-step[OF step above-C]  $\sigma$  frs False by simp
      show ?thesis
    proof(cases xp1)
      case None then show ?thesis

```



```

using None  $\sigma$  frs step False assms ncollect-exec-step[OF step above-C] collect
exec
  by(auto simp: JVMNaiveCollectionSemantics.csmall-def)
next
  case (Some a)
  then show ?thesis using  $\sigma$  assms
  proof(cases xp)
    case None
    have frames: classes-above-frames P (frames-of  $\sigma$ )  $\cap$  classes-changed P P'
= {}
    using None Some frs  $\sigma$  assms by(auto simp: JVMNaiveCollectionSemantics.csmall-def)
    have frsi: classes-above-frames P frs  $\cap$  classes-changed P P' = {} using  $\sigma$ 
frames by simp
    have xpcollect: classes-above-xcpts P  $\cap$  classes-changed P P' = {}
    using None Some frs  $\sigma$  assms by(auto simp: JVMNaiveCollectionSemantics.csmall-def)
    have xp: classes-above P (cname-of h a)  $\cap$  classes-changed P P' = {}
    using ncollect-exec-step-xcpts[OF step xpcollect prealloc',
      where  $\sigma' = (xp1, h1, frs1, sh1)$  and frs=frs' and loc=loc and a=a and
sh=sh]
    exec exec-instr Some assms by auto
    show ?thesis using Some exec  $\sigma$  frs False assms exec-instr collect
      classes-above-find-handler[where h=h and sh=sh, OF xp frsi]
    by(auto simp: JVMNaiveCollectionSemantics.csmall-def)
  qed(auto simp: JVMNaiveCollectionSemantics.csmall-def)
qed
qed(auto simp: JVMNaiveCollectionSemantics.csmall-def)
qed

```

— if collected classes unchanged, if *cbig* returned a result under  $P$ ,  $P'$  returns the same

**lemma** ncollect-JVMbig:

```

assumes collect: ( $\sigma'$ , cset)  $\in$  JVMNaiveCollectionSemantics.cbig P  $\sigma$ 
and intersect: cset  $\cap$  classes-changed P P' = {}
and prealloc: preallocated (fst(snd  $\sigma$ ))
shows ( $\sigma'$ , cset)  $\in$  JVMNaiveCollectionSemantics.cbig P'  $\sigma$ 
using JVMNaiveCollectionSemantics.csmall-to-cbig-prop2[where R= $\lambda$ P P' cset.
cset  $\cap$  classes-changed P P' = {}]
and Q= $\lambda$  $\sigma$ . preallocated (fst(snd  $\sigma$ )) and P=P and P'=P' and  $\sigma$ = $\sigma$  and  $\sigma'$ = $\sigma'$ 
and coll=cset]
  ncollect-JVMsmall JVMsmall-prealloc-pres assms by auto

```

— and finally, RTS algorithm based on *ncollect* is existence safe

**theorem** jvm-naive-existence-safe:

```

assumes p: P  $\in$  jvm-progs and P'  $\in$  jvm-progs and t: t  $\in$  jvm-tests
and out: o1  $\in$  jvm-naive-out P t and jvm-deselect P o1 P'
shows  $\exists$  o2  $\in$  jvm-naive-out P' t. o1 = o2
using assms

```

**proof** –

```

let ?P = start-prog (t#P) (fst t) main
let ?P' = start-prog (t#P') (fst t) main
obtain wf-md where wf': wf-prog wf-md (t#P) using p t
  by(auto simp: wf-jvm-prog-def wf-jvm-prog-phi-def)
have ns: ¬is-class (t#P) Start using p t
  by(clarsimp simp: is-class-def class-def Start-def Test-def)
obtain σ1 coll1 where o1: o1 = (σ1, coll1) by(cases o1)
then have cbig: (σ1, coll1) ∈ JVMNaiveCollectionSemantics.cbig ?P (start-state
(t # P))
  using assms by simp
have coll1 ∩ classes-changed P P' = {} using cbig o1 assms by auto
then have changed: coll1 ∩ classes-changed (t#P) (t#P') = {} by(rule classes-changed-int-Cons)
then have changed': coll1 ∩ classes-changed ?P ?P' = {} by(rule classes-changed-int-Cons)
have classes-above-xcpts ?P = classes-above-xcpts (t#P)
  using class-add-classes-above[OF ns wf-sys-xcpt-nsub-Start[OF wf' ns]] by simp
then have classes-above-xcpts (t#P) ∩ classes-changed (t#P) (t#P') = {}
  using jvm-naive-out-xcpts-collected[OF out] o1 changed by auto
then have ss-eq: start-state (t#P) = start-state (t#P')
  using classes-above-start-state by simp
show ?thesis using ncollect-JVMbig[OF cbig changed']
  preallocated-start-state changed' ss-eq o1 assms by auto
qed

```

— ...thus *JVMNaiveCollection* is an instance of *CollectionBasedRTS*

**interpretation** *JVMNaiveCollectionRTS* :

*CollectionBasedRTS* (=) *jvm-deselect jvm-progs jvm-tests*

*JVMendset JVMcombine JVMcollect-id JVMsmall JVMNaiveCollect jvm-naive-out*

*jvm-make-test-prog jvm-naive-collect-start*

**by** *unfold-locales (rule jvm-naive-existence-safe, auto simp: start-state-def)*

## 12.6 Smarter RTS algorithm

### 12.6.1 Definitions and helper lemmas

**fun** *jvm-smart-out* :: *jvm-prog* ⇒ *jvm-class* ⇒ *jvm-prog-out set* **where**

*jvm-smart-out* P t

= {(σ', coll'). ∃ coll. (σ', coll) ∈ JVMSmartCollectionSemantics.cbig  
(jvm-make-test-prog P t) (start-state (t#P))  
∧ coll' = coll ∪ classes-above-xcpts P ∪ {Object, Start}}

**abbreviation** *jvm-smart-collect-start* :: *jvm-prog* ⇒ *cname set* **where**

*jvm-smart-collect-start* P ≡ classes-above-xcpts P ∪ {Object, Start}

**lemma** *jvm-naive-iff-smart*:

(∃ cset<sub>n</sub>. (σ', cset<sub>n</sub>) ∈ *jvm-naive-out* P t) ⇔ (∃ cset<sub>s</sub>. (σ', cset<sub>s</sub>) ∈ *jvm-smart-out* P t)

**by**(auto simp: JVMNaiveCollectionSemantics.cbig-big-equiv  
JVMSmartCollectionSemantics.cbig-big-equiv)

**lemma** *jvm-smart-out-classes-above-xcpts*:  
**assumes**  $s: (\sigma', cset_s) \in \text{jvm-smart-out } P \ t$  **and**  $P: P \in \text{jvm-progs}$  **and**  $t: t \in \text{jvm-tests}$   
**shows**  $\text{classes-above-xcpts } (\text{jvm-make-test-prog } P \ t) \subseteq cset_s$   
**using** *jvm-make-test-prog-classes-above-xcpts*[*OF P t*] *s* **by** *clarsimp*

**lemma** *jvm-smart-collect-start-make-test-prog*:  
 $\llbracket P \in \text{jvm-progs}; t \in \text{jvm-tests} \rrbracket$   
 $\implies \text{jvm-smart-collect-start } (\text{jvm-make-test-prog } P \ t) = \text{jvm-smart-collect-start } P$   
**using** *jvm-make-test-prog-classes-above-xcpts* **by** *simp*

**lemma** *jvm-smart-out-classes-above-start-heap*:  
**assumes**  $s: (\sigma', cset_s) \in \text{jvm-smart-out } P \ t$  **and**  $h: \text{start-heap } (t\#P) \ a = \text{Some}(C, fs)$   
**and**  $P: P \in \text{jvm-progs}$  **and**  $t: t \in \text{jvm-tests}$   
**shows**  $\text{classes-above } (\text{jvm-make-test-prog } P \ t) \ C \subseteq cset_s$   
**using** *start-heap-classes*[*OF h*] *jvm-smart-out-classes-above-xcpts*[*OF s P t*] **by** *auto*

**lemma** *jvm-smart-out-classes-above-start-sheap*:  
**assumes**  $(\sigma', cset_s) \in \text{jvm-smart-out } P \ t$  **and**  $\text{start-sheap } C = \text{Some}(sfs, i)$   
**shows**  $\text{classes-above } (\text{jvm-make-test-prog } P \ t) \ C \subseteq cset_s$   
**using** *assms start-prog-classes-above-Start* **by**(*clarsimp split: if-split-asm*)

**lemma** *jvm-smart-out-classes-above-frames*:  
 $(\sigma', cset_s) \in \text{jvm-smart-out } P \ t$   
 $\implies \text{classes-above-frames } (\text{jvm-make-test-prog } P \ t) \ (\text{frames-of } (\text{start-state } (t\#P)))$   
 $\subseteq cset_s$   
**using** *start-prog-classes-above-Start* **by**(*clarsimp split: if-split-asm simp: start-state-def*)

## 12.6.2 Additional well-formedness conditions

**fun** *coll-init-class* ::  $'m \text{ prog} \Rightarrow \text{instr} \Rightarrow \text{cname option}$  **where**  
*coll-init-class*  $P \ (\text{New } C) = \text{Some } C \mid$   
*coll-init-class*  $P \ (\text{Getstatic } C \ F \ D) = (\text{if } \exists t. P \vdash C \ \text{has } F, \text{Static}:t \ \text{in } D$   
 $\text{then } \text{Some } D \ \text{else } \text{None}) \mid$   
*coll-init-class*  $P \ (\text{Putstatic } C \ F \ D) = (\text{if } \exists t. P \vdash C \ \text{has } F, \text{Static}:t \ \text{in } D$   
 $\text{then } \text{Some } D \ \text{else } \text{None}) \mid$   
*coll-init-class*  $P \ (\text{Invokestatic } C \ M \ n) = \text{seeing-class } P \ C \ M \mid$   
*coll-init-class*  $- \ - = \text{None}$

— checks whether the given value is a pointer; if it's an address, checks whether it points to an object in the given heap

**fun** *is-ptr* ::  $\text{heap} \Rightarrow \text{val} \Rightarrow \text{bool}$  **where**  
*is-ptr*  $h \ \text{Null} = \text{True} \mid$   
*is-ptr*  $h \ (\text{Addr } a) = (\exists Cfs. h \ a = \text{Some } Cfs) \mid$   
*is-ptr*  $h \ - = \text{False}$

**lemma** *is-ptrD*:  $is\text{-ptr } h \ v \Longrightarrow v = \text{Null} \vee (\exists a. v = \text{Addr } a \wedge (\exists Cfs. h \ a = \text{Some } Cfs))$

**by**(*cases v, auto*)

— shorthand for: given stack has entries required by given instr, including pointer where necessary

**fun** *stack-safe* ::  $instr \Rightarrow heap \Rightarrow val \ list \Rightarrow bool$  **where**  
*stack-safe* (*Getfield F C*)  $h \ stk = (length \ stk > 0 \wedge is\text{-ptr } h \ (hd \ stk)) \mid$   
*stack-safe* (*Putfield F C*)  $h \ stk = (length \ stk > 1 \wedge is\text{-ptr } h \ (hd \ (tl \ stk))) \mid$   
*stack-safe* (*Checkcast C*)  $h \ stk = (length \ stk > 0 \wedge is\text{-ptr } h \ (hd \ stk)) \mid$   
*stack-safe* (*Invoke M n*)  $h \ stk = (length \ stk > n \wedge is\text{-ptr } h \ (stk \ ! \ n)) \mid$   
*stack-safe* *JVMInstructions.Throw*  $h \ stk = (length \ stk > 0 \wedge is\text{-ptr } h \ (hd \ stk)) \mid$   
*stack-safe* *i*  $h \ stk = \text{True}$

**lemma** *well-formed-stack-safe*:

**assumes** *wtp*:  $wf\text{-jvm-prog}_{\Phi} \ P$  **and** *correct*:  $P, \Phi \vdash (xp, h, (stk, loc, C, M, pc, ics) \# frs, sh) \checkmark$

**shows** *stack-safe* (*instrs-of P C M ! pc*)  $h \ stk$

**proof** –

**from** *correct* **obtain**  $b \ Ts \ T \ mxs \ mxl_0 \ ins \ xt$  **where**

$mC: P \vdash C \ sees \ M, b: Ts \rightarrow T = (mxs, mxl_0, ins, xt)$  **in** *C* **and**

$pc: pc < length \ ins$  **by** *clarsimp*

**from** *sees-wf-mdecl[OF - mC]* *wtp* **have** *wt-method P C b Ts T mxs mxl\_0 ins xt* ( $\Phi \ C \ M$ )

**by**(*auto simp: wf-jvm-prog-phi-def wf-mdecl-def*)

**with** *pc* **have**  $wt: P, T, mxs, length \ ins, xt \vdash ins \ ! \ pc, pc :: \Phi \ C \ M$  **by**(*simp add: wt-method-def*)

**from** *mC correct* **obtain**  $ST \ LT$  **where**

$\Phi: \Phi \ C \ M \ ! \ pc = \text{Some } (ST, LT)$  **and**

$stk: P, h \vdash stk \ [:\leq] \ ST$  **by** *fastforce*

**show** *?thesis*

**proof**(*cases instrs-of P C M ! pc*)

**case** (*Getfield F D*)

**with** *mC*  $\Phi \ wt \ stk$  **obtain**  $oT \ ST'$  **where**

$oT: P \vdash oT \leq \text{Class } D$  **and**

$ST: ST = oT \ \# \ ST'$  **by** *fastforce*

**with** *stk* **obtain**  $ref \ stk'$  **where**

$stk': stk = ref \ \# \ stk'$  **and**

$ref: P, h \vdash ref \ :\leq \ oT$  **by** *auto*

**with** *ref oT* **have**  $ref = \text{Null} \vee (ref \neq \text{Null} \wedge P, h \vdash ref \ :\leq \ \text{Class } D)$  **by** *auto*

**with** *Getfield mC* **have**  $is\text{-ptr } h \ ref$  **by**(*fastforce dest: non-npD*)

**with** *stk' Getfield* **show** *?thesis* **by** *auto*

**next**

**case** (*Putfield F D*)

**with** *mC*  $\Phi \ wt \ stk$  **obtain**  $vT \ oT \ ST'$  **where**

$oT: P \vdash oT \leq \text{Class } D$  **and**

$ST: ST = vT \ \# \ oT \ \# \ ST'$  **by** *fastforce*

**with** *stk* **obtain**  $v \ ref \ stk'$  **where**

$stk': stk = v \ \# \ ref \ \# \ stk'$  **and**

$ref: P, h \vdash ref \ :\leq \ oT$  **by** *auto*

```

with  $ref\ oT$  have  $ref = Null \vee (ref \neq Null \wedge P, h \vdash ref : \leq Class\ D)$  by auto
with  $Putfield\ mC$  have  $is\_ptr\ h\ ref$  by(fastforce dest: non-npD)
with  $stk'\ Putfield$  show ?thesis by auto
next
case (Checkcast D)
with  $mC\ \Phi\ wt\ stk$  obtain  $oT\ ST'$  where
   $oT: is\_refT\ oT$  and
   $ST: ST = oT \# ST'$  by fastforce
with  $stk$  obtain  $ref\ stk'$  where
   $stk': stk = ref \# stk'$  and
   $ref: P, h \vdash ref : \leq oT$  by auto
with  $ref\ oT$  have  $ref = Null \vee (ref \neq Null \wedge (\exists D'. P, h \vdash ref : \leq Class\ D'))$ 
  by(auto simp: is-refT-def)
with Checkcast mC have  $is\_ptr\ h\ ref$  by(fastforce dest: non-npD)
with  $stk'\ Checkcast$  show ?thesis by auto
next
case (Invoke M1 n)
with  $mC\ \Phi\ wt\ stk$  have
   $ST: n < size\ ST$  and
   $oT: ST!n = NT \vee (\exists D. ST!n = Class\ D)$  by auto
with  $stk$  have  $stk': n < size\ stk$  by (auto simp: list-all2-lengthD)
with  $stk\ ST\ oT$  list-all2-nthD2
have  $stk!n = Null \vee (stk!n \neq Null \wedge (\exists D. P, h \vdash stk!n : \leq Class\ D))$  by fastforce
with Invoke mC have  $is\_ptr\ h\ (stk!n)$  by(fastforce dest: non-npD)
with  $stk'\ Invoke$  show ?thesis by auto
next
case Throw
with  $mC\ \Phi\ wt\ stk$  obtain  $oT\ ST'$  where
   $oT: is\_refT\ oT$  and
   $ST: ST = oT \# ST'$  by fastforce
with  $stk$  obtain  $ref\ stk'$  where
   $stk': stk = ref \# stk'$  and
   $ref: P, h \vdash ref : \leq oT$  by auto
with  $ref\ oT$  have  $ref = Null \vee (ref \neq Null \wedge (\exists D'. P, h \vdash ref : \leq Class\ D'))$ 
  by(auto simp: is-refT-def)
with Throw mC have  $is\_ptr\ h\ ref$  by(fastforce dest: non-npD)
with  $stk'\ Throw$  show ?thesis by auto
qed(simp-all)
qed

```

### 12.6.3 Proving naive $\subseteq$ smart

We prove that, given well-formedness of the program and state, and "promises" about what has or will be collected in previous or future steps, *jvm-smart* collects everything *jvm-naive* does. We prove that promises about previously-collected classes ("backward promises") are maintained by execution, and promises about to-be-collected classes ("forward promises") are met by the end of execution. We then show that the required initial conditions (well-

formedness and backward promises) are met by the defined start states, and thus that a run test will collect at least those classes collected by the naive algorithm.

If backward promises have been kept, a single step preserves this property; i.e., any classes that have been added to this set (new heap objects, newly prepared sheap classes, new frames) are collected by the smart collection algorithm in that step or by forward promises:

**lemma** *backward-coll-promises-kept*:

**assumes**

— well-formedness

$wtp: wf-jvm-prog_{\Phi} P$

**and** *correct*:  $P, \Phi \vdash (xp, h, frs, sh) \surd$

— defs

**and** *f'*:  $hd\ frs = (stk, loc, C', M', pc, ics)$

— backward promises - will be collected prior

**and** *heap*:  $\bigwedge C\ fs. \exists a. h\ a = Some(C, fs) \implies classes\ above\ P\ C \subseteq cset$

**and** *sheap*:  $\bigwedge C\ sfs\ i. sh\ C = Some(sfs, i) \implies classes\ above\ P\ C \subseteq cset$

**and** *xcpts*:  $classes\ above\ xcpts\ P \subseteq cset$

**and** *frames*:  $classes\ above\ frames\ P\ frs \subseteq cset$

— forward promises - will be collected after if not already

**and** *init-class-prom*:  $\bigwedge C. ics = Called\ [] \vee ics = No\ ics$

$\implies coll\ init\ class\ P\ (instrs\ of\ P\ C'\ M'\ !\ pc) = Some\ C \implies classes\ above\ P\ C \subseteq cset$

**and** *Calling-prom*:  $\bigwedge C'\ Cs'. ics = Calling\ C'\ Cs' \implies classes\ above\ P\ C' \subseteq cset$

— collection and step

**and** *smart*:  $JVMexec\ scollect\ P\ (xp, h, frs, sh) \subseteq cset$

**and** *small*:  $(xp', h', frs', sh') \in JVMsmall\ P\ (xp, h, frs, sh)$

**shows**  $(h'\ a = Some(C, fs) \longrightarrow classes\ above\ P\ C \subseteq cset)$

$\wedge (sh'\ C = Some(sfs', i') \longrightarrow classes\ above\ P\ C \subseteq cset)$

$\wedge (classes\ above\ frames\ P\ frs' \subseteq cset)$

**using** *assms*

**proof**(*cases frs*)

**case** (*Cons f1 frs1*)

**then have** *cr'*:  $P, \Phi \vdash (xp, h, (stk, loc, C', M', pc, ics) \# frs1, sh) \surd$  **using** *correct f'* **by** *simp*

**let** *?i* = *instrs-of P C' M' ! pc*

**from** *well-formed-stack-safe*[*OF wtp cr'*] *correct-state-Throwing-ex*[*OF cr'*] **obtain**

*stack-safe*: *stack-safe ?i h stk* **and**

*Throwing-ex*:  $\bigwedge Cs\ a. ics = Throwing\ Cs\ a \implies \exists obj. h\ a = Some\ obj$  **by** *simp*

**have** *confc*: *conf-clinit P sh frs* **using** *correct Cons* **by** *simp*

**have** *Called-prom*:  $\bigwedge C'\ Cs'. ics = Called\ (C' \# Cs')$

$\implies classes\ above\ P\ C' \subseteq cset \wedge classes\ above\ P\ (fst(method\ P\ C'\ clinit))$

$\subseteq cset$

**proof** —

**fix** *C' Cs'* **assume** [*simp*]:  $ics = Called\ (C' \# Cs')$

**then have**  $C' \in set(clinit\ classes\ frs)$  **using** *f' Cons* **by** *simp*

```

then obtain sfs where shC': sh C' = Some(sfs, Processing) and is-class P C'
using confc by(auto simp: conf-clinit-def)
then have C'eq: C' = fst(method P C' clinit) using wf-sees-clinit wtp
by(fastforce simp: is-class-def wf-jvm-prog-phi-def)
then show classes-above P C' ⊆ cset ∧ classes-above P (fst(method P C'
clinit)) ⊆ cset
using sheap shC' by auto
qed
have Called-prom2:  $\bigwedge Cs. ics = Called Cs \implies \exists C1\ subj. Called\text{-context } P\ C1\ ?i$ 
 $\wedge sh\ C1 = Some\ subj$ 
using cr' by(auto simp: conf-f-def2)
have Throwing-prom:  $\bigwedge C'\ Cs\ a. ics = Throwing\ (C'\#Cs)\ a \implies \exists sfs. sh\ C' =$ 
 $Some(sfs, Processing)$ 
proof –
fix C' Cs a assume [simp]: ics = Throwing (C'#Cs) a
then have C' ∈ set(clinit-classes frs) using f' Cons by simp
then show  $\exists sfs. sh\ C' = Some(sfs, Processing)$  using confc by(clarsimp simp:
conf-clinit-def)
qed

show ?thesis using Cons assms
proof(cases xp)
case None
then have exec: exec (P, None, h, (stk,loc,C',M',pc,ics)#frs1, sh) = Some
(xp',h',frs',sh')
using small f' Cons by auto
obtain si where si: exec-step-input P C' M' pc ics = si by simp
obtain xp0 h0 frs0 sh0 where
exec-step: exec-step P h stk loc C' M' pc ics frs1 sh = (xp0, h0, frs0, sh0)
by(cases exec-step P h stk loc C' M' pc ics frs1 sh)
then have ind: exec-step-ind si P h stk loc C' M' pc ics frs1 sh
(xp0, h0, frs0, sh0) using exec-step-ind-equiv si by auto
then show ?thesis using heap sheap frames exec exec-step f' Cons
si init-class-prom stack-safe Calling-prom Called-prom Called-prom2 Throw-
ing-prom
proof(induct rule: exec-step-ind.induct)
case exec-step-ind-Load show ?case using exec-step-ind-Load.prem(1–7)
by auto
next
case exec-step-ind-Store show ?case using exec-step-ind-Store.prem(1–7)
by auto
next
case exec-step-ind-Push show ?case using exec-step-ind-Push.prem(1–7)
by auto
next
case exec-step-ind-NewOOM-Called show ?case using exec-step-ind-NewOOM-Called.prem(1–7)
by(auto simp del: find-handler.simps dest!: find-handler-pieces) blast+
next
case exec-step-ind-NewOOM-Done show ?case using exec-step-ind-NewOOM-Done.prem(1–7)

```

```

    by(auto simp del: find-handler.simps dest!: find-handler-pieces) blast+
next
case exec-step-ind-New-Called show ?case
using exec-step-ind-New-Called.hyps exec-step-ind-New-Called.prem1(1-9)
by(auto split: if-split-asm simp: blank-def dest!: exec-step-input-StepID)
blast+
next
case exec-step-ind-New-Done show ?case
using exec-step-ind-New-Done.hyps exec-step-ind-New-Done.prem1(1-9)
by(auto split: if-split-asm simp: blank-def dest!: exec-step-input-StepID)
blast+
next
case exec-step-ind-New-Init show ?case
using exec-step-ind-New-Init.prem1(1-7) by auto
next
case exec-step-ind-Getfield-Null show ?case using exec-step-ind-Getfield-Null.prem1(1-7)
by(auto simp del: find-handler.simps dest!: find-handler-pieces) blast+
next
case exec-step-ind-Getfield-NoField show ?case
using exec-step-ind-Getfield-NoField.prem1(1-7)
by(auto simp del: find-handler.simps dest!: find-handler-pieces) blast+
next
case exec-step-ind-Getfield-Static show ?case
using exec-step-ind-Getfield-Static.prem1(1-7)
by(auto simp del: find-handler.simps dest!: find-handler-pieces) blast+
next
case exec-step-ind-Getfield show ?case
using exec-step-ind-Getfield.prem1(1-7) by auto
next
case exec-step-ind-Getstatic-NoField show ?case
using exec-step-ind-Getstatic-NoField.prem1(1-7)
by(auto simp del: find-handler.simps dest!: find-handler-pieces) blast+
next
case exec-step-ind-Getstatic-NonStatic show ?case
using exec-step-ind-Getstatic-NonStatic.prem1(1-7)
by(auto simp del: find-handler.simps dest!: find-handler-pieces) blast+
next
case exec-step-ind-Getstatic-Called show ?case
using exec-step-ind-Getstatic-Called.prem1(1-7) by auto
next
case exec-step-ind-Getstatic-Done show ?case
using exec-step-ind-Getstatic-Done.prem1(1-7) by auto
next
case exec-step-ind-Getstatic-Init show ?case
using exec-step-ind-Getstatic-Init.prem1(1-7) by auto
next
case exec-step-ind-Putfield-Null show ?case
using exec-step-ind-Putfield-Null.prem1(1-7)
by(auto simp del: find-handler.simps dest!: find-handler-pieces) blast+

```



```

next
  case exec-step-ind-Putfield-NoField show ?case
    using exec-step-ind-Putfield-NoField.prems(1-7)
    by(auto simp del: find-handler.simps dest!: find-handler-pieces) blast+
next
  case exec-step-ind-Putfield-Static show ?case
    using exec-step-ind-Putfield-Static.prems(1-7)
    by(auto simp del: find-handler.simps dest!: find-handler-pieces) blast+
next
  case (exec-step-ind-Putfield v stk r a D fs h D' b t P C F loc C0 M0 pc ics frs
sh)
    obtain a C1 fs where addr: hd (tl stk) = Null  $\vee$  (hd (tl stk) = Addr a  $\wedge$  h
a = Some(C1,fs))
    using exec-step-ind-Putfield.prems(8,10) by(auto dest!: exec-step-input-StepID
is-ptrD)
    then have  $\bigwedge a. \text{hd}(tl\ stk) = \text{Addr } a \implies \text{classes-above } P\ C1 \subseteq \text{cset}$ 
    using exec-step-ind-Putfield.prems(1) addr by auto
    then show ?case using exec-step-ind-Putfield.hyps exec-step-ind-Putfield.prems(1-7)
addr
    by(auto split: if-split-asm) blast+
next
  case exec-step-ind-Putstatic-NoField show ?case
    using exec-step-ind-Putstatic-NoField.prems(1-7)
    by(auto simp del: find-handler.simps dest!: find-handler-pieces) blast+
next
  case exec-step-ind-Putstatic-NonStatic show ?case
    using exec-step-ind-Putstatic-NonStatic.prems(1-7)
    by(auto simp del: find-handler.simps dest!: find-handler-pieces) blast+
next
  case (exec-step-ind-Putstatic-Called D' b t P D F C sh sfs i h stk loc C0 M0
pc Cs frs)
    then have  $P \vdash D \text{ sees } F, \text{Static:t in } D$  by(simp add: has-field-sees[OF
has-field-idemp])
    then have  $D' \text{eq: } D' = D$  using exec-step-ind-Putstatic-Called.hyps(1) by
simp
    obtain sobj where sh D = Some sobj
    using exec-step-ind-Putstatic-Called.hyps(2) exec-step-ind-Putstatic-Called.prems(8,13)
    by(fastforce dest!: exec-step-input-StepID)
    then show ?case using exec-step-ind-Putstatic-Called.hyps
exec-step-ind-Putstatic-Called.prems(1-7)  $D' \text{eq}$ 
    by(auto split: if-split-asm) blast+
next
  case exec-step-ind-Putstatic-Done show ?case
    using exec-step-ind-Putstatic-Done.hyps exec-step-ind-Putstatic-Done.prems(1-7)
    by(auto split: if-split-asm) blast+
next
  case exec-step-ind-Putstatic-Init show ?case
    using exec-step-ind-Putstatic-Init.hyps exec-step-ind-Putstatic-Init.prems(1-7)
    by(auto split: if-split-asm) blast+

```

```

next
  case exec-step-ind-Checkcast show ?case
    using exec-step-ind-Checkcast.prems(1–7) by auto
next
case exec-step-ind-Checkcast-Error show ?case using exec-step-ind-Checkcast-Error.prems(1–7)
  by(auto simp del: find-handler.simps dest!: find-handler-pieces) blast+
next
case exec-step-ind-Invoke-Null show ?case using exec-step-ind-Invoke-Null.prems(1–7)
  by(auto simp del: find-handler.simps dest!: find-handler-pieces) blast+
next
case exec-step-ind-Invoke-NoMethod show ?case using exec-step-ind-Invoke-NoMethod.prems(1–7)
  by(auto simp del: find-handler.simps dest!: find-handler-pieces) blast+
next
case exec-step-ind-Invoke-Static show ?case using exec-step-ind-Invoke-Static.prems(1–7)
  by(auto simp del: find-handler.simps dest!: find-handler-pieces) blast+
next
case (exec-step-ind-Invoke ps n stk r C h D b Ts T mxs mxl0 ins xt P)
  have classes-above P D  $\subseteq$  cset
  using exec-step-ind-Invoke.hyps(2,3,5) exec-step-ind-Invoke.prems(1,8,10,13)
    rtrancl-trans[OF sees-method-decl-above[OF exec-step-ind-Invoke.hyps(6)]]
  by(auto dest!: exec-step-input-StepID is-ptrD) blast+
  then show ?case
  using exec-step-ind-Invoke.hyps(7) exec-step-ind-Invoke.prems(1–7) by auto
next
case exec-step-ind-Invokestatic-NoMethod
  show ?case using exec-step-ind-Invokestatic-NoMethod.prems(1–7)
  by(auto simp del: find-handler.simps dest!: find-handler-pieces) blast+
next
case exec-step-ind-Invokestatic-NonStatic
  show ?case using exec-step-ind-Invokestatic-NonStatic.prems(1–7)
  by(auto simp del: find-handler.simps dest!: find-handler-pieces) blast+
next
case (exec-step-ind-Invokestatic-Called ps n stk D b Ts T mxs mxl0 ins xt P
C M)
  have seeing-class P C M = Some D using exec-step-ind-Invokestatic-Called.hyps(2,3)
    by(fastforce simp: seeing-class-def)
  then have classes-above P D  $\subseteq$  cset using exec-step-ind-Invokestatic-Called.prems(8–9)
    by(fastforce dest!: exec-step-input-StepID)
  then show ?case
  using exec-step-ind-Invokestatic-Called.hyps exec-step-ind-Invokestatic-Called.prems(1–7)
    by(auto simp: seeing-class-def)
next
case (exec-step-ind-Invokestatic-Done ps n stk D b Ts T mxs mxl0 ins xt P C
M)
  have seeing-class P C M = Some D using exec-step-ind-Invokestatic-Done.hyps(2,3)
    by(fastforce simp: seeing-class-def)
  then have classes-above P D  $\subseteq$  cset using exec-step-ind-Invokestatic-Done.prems(8–9)
    by(fastforce dest!: exec-step-input-StepID)
  then show ?case

```

```

using exec-step-ind-Invokestatic-Done.hyps exec-step-ind-Invokestatic-Done.prems(1-7)
  by auto blast+
next
  case exec-step-ind-Invokestatic-Init show ?case
using exec-step-ind-Invokestatic-Init.hyps exec-step-ind-Invokestatic-Init.prems(1-7)
  by auto blast+
next
  case exec-step-ind-Return-Last-Init show ?case
  using exec-step-ind-Return-Last-Init.prems(1-7) by(auto split: if-split-asm)
blast+
next
  case exec-step-ind-Return-Last show ?case
  using exec-step-ind-Return-Last.prems(1-7) by auto
next
  case exec-step-ind-Return-Init show ?case
using exec-step-ind-Return-Init.prems(1-7) by(auto split: if-split-asm) blast+
next
  case exec-step-ind-Return-NonStatic show ?case
  using exec-step-ind-Return-NonStatic.prems(1-7) by auto
next
  case exec-step-ind-Return-Static show ?case
  using exec-step-ind-Return-Static.prems(1-7) by auto
next
  case exec-step-ind-Pop show ?case using exec-step-ind-Pop.prems(1-7) by
auto
next
  case exec-step-ind-IAdd show ?case using exec-step-ind-IAdd.prems(1-7)by
auto
next
  case exec-step-ind-IfFalse-False show ?case
  using exec-step-ind-IfFalse-False.prems(1-7) by auto
next
  case exec-step-ind-IfFalse-nFalse show ?case
  using exec-step-ind-IfFalse-nFalse.prems(1-7) by auto
next
  case exec-step-ind-CmpEq show ?case using exec-step-ind-CmpEq.prems(1-7)
by auto
next
  case exec-step-ind-Goto show ?case using exec-step-ind-Goto.prems(1-7)
by auto
next
  case exec-step-ind-Throw show ?case using exec-step-ind-Throw.prems(1-7)
  by(auto simp del: find-handler.simps dest!: find-handler-pieces) blast+
next
case exec-step-ind-Throw-Null show ?case using exec-step-ind-Throw-Null.prems(1-7)
  by(auto simp del: find-handler.simps dest!: find-handler-pieces) blast+
next
  case (exec-step-ind-Init-None-Called sh C Cs P)
have classes-above P C ⊆ cset using exec-step-ind-Init-None-Called.prems(8,11)

```

```

    by(auto dest!: exec-step-input-StepCD)
  then show ?case using exec-step-ind-Init-None-Called.prem(1-7)
    by(auto split: if-split-asm) blast+
next
  case exec-step-ind-Init-Done show ?case
    using exec-step-ind-Init-Done.prem(1-7) by auto
next
  case exec-step-ind-Init-Processing show ?case
    using exec-step-ind-Init-Processing.prem(1-7) by auto
next
  case exec-step-ind-Init-Error show ?case
    using exec-step-ind-Init-Error.prem(1-7) by auto
next
  case exec-step-ind-Init-Prepared-Object show ?case
    using exec-step-ind-Init-Prepared-Object.hyps
      exec-step-ind-Init-Prepared-Object.prem(1-7,10)
    by(auto split: if-split-asm dest!: exec-step-input-StepCD) blast+
next
  case exec-step-ind-Init-Prepared-nObject show ?case
    using exec-step-ind-Init-Prepared-nObject.hyps exec-step-ind-Init-Prepared-nObject.prem(1-7)
    by(auto split: if-split-asm) blast+
next
  case exec-step-ind-Init show ?case
    using exec-step-ind-Init.prem(1-7,8,12)
    by(auto simp: split-beta dest!: exec-step-input-StepC2D)
next
  case (exec-step-ind-InitThrow C Cs a P h stk loc C0 M0 pc ics frs sh)
    obtain sfs where sh C = Some(sfs,Processing)
    using exec-step-ind-InitThrow.prem(8,14) by(fastforce dest!: exec-step-input-StepTD)
    then show ?case using exec-step-ind-InitThrow.prem(1-7)
    by(auto split: if-split-asm) blast+
next
  case exec-step-ind-InitThrow-End show ?case using exec-step-ind-InitThrow-End.prem(1-7)
    by(auto simp del: find-handler.simps dest!: find-handler-pieces) blast+
qed
qed(simp)
qed(simp)

```

— Forward promises (classes that will be collected by the end of execution)  
 — - Classes that the current instruction will check initialization for will be collected  
 — - Class whose initialization is actively being called by the current frame will be collected

We prove that an *ics* of *Calling C Cs* (meaning *C*'s initialization procedure is actively being called) means that classes above *C* will be collected by *cbig* (i.e., by the end of execution) using proof by induction, proving the base and IH separately.

**lemma** *Calling-collects-base*:

**assumes** *big*:  $(\sigma', cset') \in JVMSmartCollectionSemantics.cbig P \sigma$

**and**  $nend: \sigma \notin JVMendset$   
**and**  $ics: ics\text{-of} (hd(frames\text{-of} \sigma)) = Calling\ Object\ Cs$   
**shows**  $classes\text{-above} P\ Object \subseteq cset \cup cset'$   
**proof**( $cases\ frames\text{-of} \sigma$ )  
  **case**  $Nil$  **then show**  $?thesis$  **using**  $nend$  **by**( $clarsimp\ simp: JVMendset\text{-def}$ )  
**next**  
  **case** ( $Cons\ f1\ frs1$ )  
  **then obtain**  $stk\ loc\ C\ M\ pc\ ics$  **where**  $f1 = (stk,loc,C,M,pc,ics)$  **by**( $cases\ f1$ )  
  **then show**  $?thesis$   
  **using**  $JVMSmartCollectionSemantics.cbig\text{-stepD}[OF\ big\ nend]$   $nend\ ics\ Cons$   
  **by**( $clarsimp\ simp: JVMSmartCollectionSemantics.csmall\text{-def}\ JVMendset\text{-def}$ )  
**qed**

— IH case where  $C$  has not been prepared yet

**lemma** *Calling-None-next-state:*

**assumes**  $ics: ics\text{-of} (hd(frames\text{-of} \sigma)) = Calling\ C\ Cs$

**and**  $none: sheap\ \sigma\ C = None$

**and**  $set: \forall C'. P \vdash C \preceq^* C' \longrightarrow (\exists sfs\ i. sheap\ \sigma\ C' = Some(sfs,i))$   
 $\longrightarrow classes\text{-above} P\ C' \subseteq cset$

**and**  $\sigma': (\sigma', cset') \in JVMSmartCollectionSemantics.csmall\ P\ \sigma$

**shows**  $\sigma' \notin JVMendset \wedge ics\text{-of} (hd(frames\text{-of} \sigma')) = Calling\ C\ Cs$

$\wedge (\exists sfs. sheap\ \sigma'\ C = Some(sfs,Prepared))$

$\wedge (\forall C'. P \vdash C \preceq^* C' \longrightarrow C \neq C')$

$\longrightarrow (\exists sfs\ i. sheap\ \sigma'\ C' = Some(sfs,i)) \longrightarrow classes\text{-above} P\ C' \subseteq cset$

**proof**( $cases\ frames\text{-of} \sigma = [] \vee (\exists x. fst\ \sigma = Some\ x)$ )

**case**  $True$  **then show**  $?thesis$  **using**  $assms$

**by**( $cases\ \sigma, auto\ simp: JVMSmartCollectionSemantics.csmall\text{-def}$ )

**next**

**case**  $False$

**then obtain**  $f1\ frs1$  **where**  $frs: frames\text{-of} \sigma = f1\#\ frs1$  **by**( $cases\ frames\text{-of} \sigma, auto$ )

**obtain**  $stk\ loc\ C'\ M\ pc\ ics$  **where**  $f1: f1 = (stk,loc,C',M,pc,ics)$  **by**( $cases\ f1$ )

**show**  $?thesis$  **using**  $f1\ frs\ False\ assms$

**by**( $cases\ \sigma, cases\ method\ P\ C\ clinit$ )

( $clarsimp\ simp: split\text{-beta}\ JVMSmartCollectionSemantics.csmall\text{-def}\ JVMendset\text{-def}$ )

**qed**

— IH case where  $C$  has been prepared (and has a direct superclass - i.e., is not *Object*)

**lemma** *Calling-Prepared-next-state:*

**assumes**  $sub: P \vdash C \prec^1 D$

**and**  $obj: P \vdash D \preceq^* Object$

**and**  $ics: ics\text{-of} (hd(frames\text{-of} \sigma)) = Calling\ C\ Cs$

**and**  $prep: sheap\ \sigma\ C = Some(sfs,Prepared)$

**and**  $set: \forall C'. P \vdash C \preceq^* C' \longrightarrow C \neq C' \longrightarrow (\exists sfs\ i. sheap\ \sigma\ C' = Some(sfs,i))$   
 $\longrightarrow classes\text{-above} P\ C' \subseteq cset$

**and**  $\sigma': (\sigma', cset') \in JVMSmartCollectionSemantics.csmall\ P\ \sigma$

**shows**  $\sigma' \notin JVMendset \wedge ics\text{-of} (hd(frames\text{-of} \sigma')) = Calling\ D\ (C\#\ Cs)$

$\wedge (\forall C'. P \vdash D \preceq^* C' \longrightarrow (\exists sfs\ i.\ sheap\ \sigma'\ C' = Some(sfs,i))$   
 $\longrightarrow classes\text{-}above\ P\ C' \subseteq cset)$

**using** *sub*  
**proof**(*cases C=Object*)  
**case** *nobj:False* **show** *?thesis*  
**proof**(*cases frames-of*  $\sigma = [] \vee (\exists x.\ fst\ \sigma = Some\ x)$ )  
**case** *True* **then show** *?thesis* **using** *assms*  
**by**(*cases*  $\sigma$ , *auto simp: JVMSmartCollectionSemantics.csmall-def*)  
**next**  
**case** *False*  
**then obtain** *f1 frs1* **where** *frs: frames-of*  $\sigma = f1\#\ frs1$  **by**(*cases frames-of*  $\sigma$ ,  
*auto*)  
**obtain** *stk loc C' M pc ics* **where** *f1: f1 = (stk,loc,C',M,pc,ics)* **by**(*cases f1*)  
**have**  $C \neq D$  **using** *sub obj subcls-self-superclass* **by** *auto*  
**then have** *dimp:  $\forall C'. P \vdash D \preceq^* C' \longrightarrow P \vdash C \preceq^* C' \wedge C \neq C'$*   
**using** *sub subcls-of-Obj-acyclic[OF obj]* **by** *fastforce*  
**have**  $\forall C'. P \vdash C \preceq^* C' \longrightarrow C \neq C' \longrightarrow (\exists sfs\ i.\ sheap\ \sigma'\ C' = Some(sfs,i))$   
 $\longrightarrow classes\text{-}above\ P\ C' \subseteq cset$   
**using** *f1 frs False nobj assms*  
**by**(*cases*  $\sigma$ , *cases method P C clinit*)  
*(auto simp: JVMSmartCollectionSemantics.csmall-def JVMendset-def)*  
**then have**  $\forall C'. P \vdash D \preceq^* C' \longrightarrow (\exists sfs\ i.\ sheap\ \sigma'\ C' = Some(sfs,i))$   
 $\longrightarrow classes\text{-}above\ P\ C' \subseteq cset$  **using** *sub dimp* **by** *auto*  
**then show** *?thesis* **using** *f1 frs False nobj assms*  
**by**(*cases*  $\sigma$ , *cases method P C clinit*)  
*(auto dest:subcls1D simp: JVMSmartCollectionSemantics.csmall-def JV-*  
*Mendset-def)*  
**qed**  
**qed**(*simp*)

— completed IH case: non-Object (pulls together above IH cases)

**lemma** *Calling-collects-IH:*

**assumes** *sub:  $P \vdash C \prec^1 D$*

**and** *obj:  $P \vdash D \preceq^* Object$*

**and** *step:  $\bigwedge \sigma\ cset'\ Cs. (\sigma', cset') \in JVMSmartCollectionSemantics.cbig\ P\ \sigma \implies \sigma \notin JVMendset$*

$\implies ics\text{-}of\ (hd(frames\text{-}of\ \sigma)) = Calling\ D\ Cs$

$\implies \forall C'. P \vdash D \preceq^* C' \longrightarrow (\exists sfs\ i.\ sheap\ \sigma\ C' = Some(sfs,i))$

$\longrightarrow classes\text{-}above\ P\ C' \subseteq cset$

$\implies classes\text{-}above\ P\ D \subseteq cset \cup cset'$

**and** *big:  $(\sigma', cset') \in JVMSmartCollectionSemantics.cbig\ P\ \sigma$*

**and** *nend:  $\sigma \notin JVMendset$*

**and** *curr:  $ics\text{-}of\ (hd(frames\text{-}of\ \sigma)) = Calling\ C\ Cs$*

**and** *set:  $\forall C'. P \vdash C \preceq^* C' \longrightarrow (\exists sfs\ i.\ sheap\ \sigma\ C' = Some(sfs,i))$*

$\longrightarrow classes\text{-}above\ P\ C' \subseteq cset$

**shows**  $classes\text{-}above\ P\ C \subseteq cset \cup cset'$

**proof**(*cases frames-of*  $\sigma$ )

**case** *Nil* **then show** *?thesis* **using** *nend* **by**(*clarsimp simp: JVMendset-def*)

**next**

```

case (Cons f1 frs1)
show ?thesis using sub
proof(cases  $\exists$  sfs i. sheap  $\sigma$   $C = \text{Some}(sfs, i)$ )
  case True then show ?thesis using set by auto
next
  case False
  obtain stk loc C' M pc ics where f1: f1 = (stk, loc, C', M, pc, ics) by(cases f1)
  then obtain  $\sigma 1$  coll1 coll where  $\sigma 1: (\sigma 1, coll1) \in \text{JVMSmartCollectionSemantics.csmall } P \sigma$ 
  cset' = coll1  $\cup$  coll ( $\sigma'$ , coll)  $\in$  JVMSmartCollectionSemantics.cbig }  $P \sigma 1$ 
  using JVMSmartCollectionSemantics.cbig-stepD[OF big nend] by clarsimp
  show ?thesis
  proof(cases  $\exists$  sfs. sheap  $\sigma$   $C = \text{Some}(sfs, Prepared)$ )
    case True
    then obtain sfs where sfs: sheap  $\sigma$   $C = \text{Some}(sfs, Prepared)$  by clarsimp
    have set':  $\forall C'. P \vdash C \preceq^* C' \longrightarrow C \neq C' \longrightarrow (\exists sfs i. sheap \sigma C' = \text{Some}(sfs, i))$ 
       $\longrightarrow$  classes-above  $P C' \subseteq cset$  using set by auto
    then have  $\sigma 1 \notin \text{JVMSendset} \wedge \text{ics-of} (\text{hd} (\text{frames-of } \sigma 1)) = \text{Calling } D (C \# Cs)$ 
       $\forall C'. P \vdash D \preceq^* C' \longrightarrow (\exists sfs i. sheap \sigma 1 C' = \text{Some}(sfs, i))$ 
       $\longrightarrow$  classes-above  $P C' \subseteq cset$ 
      using Calling-Prepared-next-state[OF sub obj curr sfs set'  $\sigma 1(1)$ ]
      by(auto simp: JVMSmartCollectionSemantics.csmall-def)
      then show ?thesis using step[of coll  $\sigma 1$ ] classes-above-def2[OF sub]  $\sigma 1 f1$ 
      Cons nend curr
      by(clarsimp simp: JVMSmartCollectionSemantics.csmall-def JVMSendset-def)
    next
    case none: False — Calling C Cs is the next ics, but after that is Calling D
    ( $C \# Cs$ )
    then have sNone: sheap  $\sigma$   $C = \text{None}$  using False by(cases sheap  $\sigma C$ , auto)
    then have nend1:  $\sigma 1 \notin \text{JVMSendset}$  and curr1: ics-of (hd (frames-of  $\sigma 1$ )) =
      Calling C Cs
      and prep:  $\exists sfs. sheap \sigma 1 C = \lfloor (sfs, Prepared) \rfloor$ 
      and set1:  $\forall C'. P \vdash C \preceq^* C' \longrightarrow C \neq C' \longrightarrow (\exists sfs i. sheap \sigma 1 C' = \lfloor (sfs,$ 
      i) \rfloor)
       $\longrightarrow$  classes-above  $P C' \subseteq cset$ 
      using Calling-None-next-state[OF curr sNone set  $\sigma 1(1)$ ] by simp+
    then obtain f2 frs2 where frs2: frames-of  $\sigma 1 = f2 \# frs2$ 
      by(cases  $\sigma 1$ , cases frames-of  $\sigma 1$ , clarsimp simp: JVMSendset-def)
    obtain sfs1 where sfs1: sheap  $\sigma 1$   $C = \text{Some}(sfs1, Prepared)$  using prep by
    clarsimp
    obtain stk2 loc2 C2 M2 pc2 ics2 where f2: f2 = (stk2, loc2, C2, M2, pc2, ics2)
by(cases f2)
    then obtain  $\sigma 2$  coll2 coll' where  $\sigma 2: (\sigma 2, coll2) \in \text{JVMSmartCollectionSemantics.csmall } P \sigma 1$ 
      coll = coll2  $\cup$  coll' ( $\sigma'$ , coll')  $\in$  JVMSmartCollectionSemantics.cbig }  $P \sigma 2$ 
      using JVMSmartCollectionSemantics.cbig-stepD[OF  $\sigma 1(3)$  nend1] by clar-
      simp
    then have  $\sigma 2 \notin \text{JVMSendset} \wedge \text{ics-of} (\text{hd} (\text{frames-of } \sigma 2)) = \text{Calling } D$ 
    ( $C \# Cs$ )

```

$\forall C'. P \vdash D \preceq^* C' \longrightarrow (\exists sfs\ i.\ sheap\ \sigma\ 2\ C' = Some(sfs,i))$   
 $\longrightarrow classes\text{-}above\ P\ C' \subseteq cset$   
**using** *Calling-Prepared-next-state*[*OF sub obj curr1 sfs1 set1*  $\sigma\ 2(1)$ ]  
**by**(*auto simp: JVMSmartCollectionSemantics.csmall-def*)  
**then show** *?thesis using step*[*of coll'  $\sigma\ 2$* ] *classes-above-def2*[*OF sub*]  $\sigma\ 2\ \sigma\ 1$   
*f2 frs2 f1 Cons*  
*nend1 nend curr1 curr*  
**by**(*clarsimp simp: JVMSmartCollectionSemantics.csmall-def JVMendset-def*)  
**qed**  
**qed**  
**qed**

— pulls together above base and IH cases

**lemma** *Calling-collects*:

**assumes** *sub*:  $P \vdash C \preceq^* Object$

**and**  $(\sigma', cset') \in JVMSmartCollectionSemantics.cbig\ P\ \sigma$

**and**  $\sigma \notin JVMendset$

**and** *ics-of* (*hd(frames-of  $\sigma$ )*) = *Calling C Cs*

**and**  $\forall C'. P \vdash C \preceq^* C' \longrightarrow (\exists sfs\ i.\ sheap\ \sigma\ C' = Some(sfs,i))$   
 $\longrightarrow classes\text{-}above\ P\ C' \subseteq cset$

**and**  $cset' \subseteq cset$

**shows** *classes-above P C*  $\subseteq cset$

**proof** —

**have** *base*:  $\forall \sigma\ cset'\ Cs.$

$(\sigma', cset') \in JVMSmartCollectionSemantics.cbig\ P\ \sigma \longrightarrow \sigma \notin JVMendset$

$\longrightarrow ics\text{-}of\ (hd\ (frames\text{-}of\ \sigma)) = Calling\ Object\ Cs$

$\longrightarrow (\forall C'. P \vdash Object \preceq^* C' \longrightarrow (\exists sfs\ i.\ sheap\ \sigma\ C' = [(sfs, i)])$

$\longrightarrow classes\text{-}above\ P\ C' \subseteq cset)$

$\longrightarrow classes\text{-}above\ P\ Object \subseteq JVMcombine\ cset\ cset'$  **using** *Calling-collects-base*

**by** *simp*

**have** *IH*:  $\bigwedge y\ z. P \vdash y \prec^1 z \implies$

$P \vdash z \preceq^* Object \implies$

$\forall \sigma\ cset'\ Cs. (\sigma', cset') \in JVMSmartCollectionSemantics.cbig\ P\ \sigma \longrightarrow \sigma \notin JVMendset$

$\longrightarrow ics\text{-}of\ (hd(frames\text{-}of\ \sigma)) = Calling\ z\ Cs$

$\longrightarrow (\forall C'. P \vdash z \preceq^* C' \longrightarrow (\exists sfs\ i.\ sheap\ \sigma\ C' = Some(sfs,i))$

$\longrightarrow classes\text{-}above\ P\ C' \subseteq cset)$

$\longrightarrow classes\text{-}above\ P\ z \subseteq cset \cup cset' \implies$

$\forall \sigma\ cset'\ Cs. (\sigma', cset') \in JVMSmartCollectionSemantics.cbig\ P\ \sigma \longrightarrow \sigma \notin JVMendset$

$\longrightarrow ics\text{-}of\ (hd(frames\text{-}of\ \sigma)) = Calling\ y\ Cs$

$\longrightarrow (\forall C'. P \vdash y \preceq^* C' \longrightarrow (\exists sfs\ i.\ sheap\ \sigma\ C' = Some(sfs,i))$

$\longrightarrow classes\text{-}above\ P\ C' \subseteq cset)$

$\longrightarrow classes\text{-}above\ P\ y \subseteq cset \cup cset'$

**using** *Calling-collects-IH by blast*

**have** *result*:  $\forall \sigma\ cset'\ Cs.$

$(\sigma', cset') \in JVMSmartCollectionSemantics.cbig\ P\ \sigma \longrightarrow \sigma \notin JVMendset$

$\longrightarrow ics\text{-}of\ (hd(frames\text{-}of\ \sigma)) = Calling\ C\ Cs$

$\longrightarrow (\forall C'. P \vdash C \preceq^* C' \longrightarrow (\exists sfs\ i.\ sheap\ \sigma\ C' = Some(sfs,i))$



$\longrightarrow$  *classes-above*  $P C' \subseteq cset$   
 $\longrightarrow$  *classes-above*  $P C \subseteq cset \cup cset'$   
**using** *converse-rtrancl-induct*[*OF sub*,  
**where**  $P = \lambda C. \forall \sigma cset' Cs. (\sigma', cset') \in JVMSmartCollectionSemantics.cbig$   
 $P \sigma \longrightarrow \sigma \notin JVMendset$   
 $\longrightarrow$  *ics-of* ( $hd(frames-of \sigma)$ ) = *Calling*  $C Cs$   
 $\longrightarrow (\forall C'. P \vdash C \preceq^* C' \longrightarrow (\exists sfs i. sheap \sigma C' = Some(sfs, i))$   
 $\longrightarrow$  *classes-above*  $P C' \subseteq cset$   
 $\longrightarrow$  *classes-above*  $P C \subseteq cset \cup cset'$ ]  
**using** *base IH by blast*  
**then show** *?thesis using assms by blast*  
**qed**

Instructions that call the initialization procedure will collect classes above the class initialized by the end of execution (using the above *Calling-collects*).

**lemma** *New-collects*:

**assumes** *sub*:  $P \vdash C \preceq^* Object$   
**and** *cbig*:  $(\sigma', cset') \in JVMSmartCollectionSemantics.cbig P \sigma$   
**and** *nend*:  $\sigma \notin JVMendset$   
**and** *curr*: *curr-instr*  $P (hd(frames-of \sigma)) = New C$   
**and** *ics*: *ics-of* ( $hd(frames-of \sigma)$ ) = *No-ics*  
**and** *sheap*:  $\forall C'. P \vdash C \preceq^* C' \longrightarrow (\exists sfs i. sheap \sigma C' = Some(sfs, i))$   
 $\longrightarrow$  *classes-above*  $P C' \subseteq cset$   
**and** *smart*:  $cset' \subseteq cset$   
**shows** *classes-above*  $P C \subseteq cset$   
**proof**(*cases* ( $\exists sfs i. sheap \sigma C = Some(sfs, i) \wedge i = Done$ ))  
**case** *True* **then show** *?thesis using sheap by auto*  
**next**  
**case** *False*  
**obtain** *n* **where** *nstep*:  $(\sigma', cset') \in JVMSmartCollectionSemantics.csmall-nstep$   
 $P \sigma n$   
**and**  $n \neq 0$  **using** *nend cbig JVMSmartCollectionSemantics.cbig-def2*  
*JVMSmartCollectionSemantics.csmall-nstep-base* **by** (*metis empty-iff insert-iff*)  
**then show** *?thesis*  
**proof**(*cases n*)  
**case** (*Suc n1*)  
**then obtain**  $\sigma 1 cset0 cset1$  **where**  $\sigma 1: (\sigma 1, cset1) \in JVMSmartCollectionSemantics.csmall P \sigma$   
 $cset' = cset1 \cup cset0$   $(\sigma', cset0) \in JVMSmartCollectionSemantics.csmall-nstep$   
 $P \sigma 1 n1$   
**using** *JVMSmartCollectionSemantics.csmall-nstep-SucD nstep by blast*  
**obtain**  $xp h frs sh$  **where**  $\sigma = (xp, h, frs, sh)$  **by** (*cases*  $\sigma$ )  
**then have** *ics1*: *ics-of* ( $hd(frames-of \sigma 1)$ ) = *Calling*  $C []$   
**and** *sheap'*: *sheap*  $\sigma = sheap \sigma 1$  **and** *nend1*:  $\sigma 1 \notin JVMendset$   
**using** *JVM-New-next-step*[*OF - nend curr*]  $\sigma 1(1)$  *False ics*  
**by** (*simp add: JVMSmartCollectionSemantics.csmall-def*) +  
**have**  $\sigma' \in JVMendset$  **using** *cbig JVMSmartCollectionSemantics.cbig-def2 by blast*  
**then have** *cbig1*:  $(\sigma', cset0) \in JVMSmartCollectionSemantics.cbig P \sigma 1$

```

    using JVMSmartCollectionSemantics.cbig-def2  $\sigma 1(3)$  by blast
  have sheap1:  $\forall C'. P \vdash C \preceq^* C' \longrightarrow (\exists sfs\ i. sheap\ \sigma 1\ C' = [(sfs, i)])$ 
     $\longrightarrow$  classes-above  $P\ C' \subseteq cset$  using sheap' sheap by simp
  have cset0  $\subseteq cset$  using  $\sigma 1(2)$  smart by blast
  then have classes-above  $P\ C \subseteq cset$ 
    using Calling-collects[OF sub cbig1 nend1 ics1 sheap1] by simp
  then show ?thesis using  $\sigma 1(2)$  smart by auto
qed(simp)
qed

```

lemma Getstatic-collects:

```

assumes sub:  $P \vdash D \preceq^* Object$ 
and cbig:  $(\sigma', cset') \in JVMSmartCollectionSemantics.cbig\ P\ \sigma$ 
and nend:  $\sigma \notin JVMendset$ 
and curr:  $curr\text{-instr}\ P\ (hd(frames\text{-of}\ \sigma)) = Getstatic\ C\ F\ D$ 
and ics:  $ics\text{-of}\ (hd(frames\text{-of}\ \sigma)) = No\text{-ics}$ 
and fC:  $P \vdash C\ has\ F, Static:t\ in\ D$ 
and sheap:  $\forall C'. P \vdash D \preceq^* C' \longrightarrow (\exists sfs\ i. sheap\ \sigma\ C' = Some(sfs, i))$ 
     $\longrightarrow$  classes-above  $P\ C' \subseteq cset$ 
and smart:  $cset' \subseteq cset$ 
shows classes-above  $P\ D \subseteq cset$ 
proof(cases  $(\exists sfs\ i. sheap\ \sigma\ D = Some(sfs, i) \wedge i = Done)$ 
   $\vee (ics\text{-of}(hd(frames\text{-of}\ \sigma)) = Called\ [])$ )
  case True then show ?thesis
  proof(cases  $\exists sfs\ i. sheap\ \sigma\ D = Some(sfs, i) \wedge i = Done$ )
    case True then show ?thesis using sheap by auto
  next
    case False
    then have  $ics\text{-of}(hd(frames\text{-of}\ \sigma)) = Called\ []$  using True by clarsimp
    then show ?thesis using ics by auto
  qed
next
  case False
  obtain n where nstep:  $(\sigma', cset') \in JVMSmartCollectionSemantics.csmall\text{-nstep}\ P\ \sigma\ n$ 
    and  $n \neq 0$  using nend cbig JVMSmartCollectionSemantics.cbig-def2
    JVMSmartCollectionSemantics.csmall-nstep-base by (metis empty-iff insert-iff)
  then show ?thesis
  proof(cases n)
    case (Suc n1)
    then obtain  $\sigma 1\ cset0\ cset1$  where  $\sigma 1$ :  $(\sigma 1, cset1) \in JVMSmartCollectionSemantics.csmall\ P\ \sigma$ 
       $cset' = cset1 \cup cset0$   $(\sigma', cset0) \in JVMSmartCollectionSemantics.csmall\text{-nstep}\ P\ \sigma 1\ n1$ 
    using JVMSmartCollectionSemantics.csmall-nstep-SucD nstep by blast
    obtain  $xp\ h\ frs\ sh$  where  $\sigma = (xp, h, frs, sh)$  by (cases  $\sigma$ )
    then have curr1:  $ics\text{-of}\ (hd(frames\text{-of}\ \sigma 1)) = Calling\ D\ []$ 
      and sheap':  $sheap\ \sigma = sheap\ \sigma 1$  and nend1:  $\sigma 1 \notin JVMendset$ 
      using JVM-Getstatic-next-step[OF - nend curr fC]  $\sigma 1(1)$  False ics

```

```

    by(simp add: JVMSmartCollectionSemantics.csmall-def) +
    have  $\sigma' \in \text{JVMEndset}$  using cbig JVMSmartCollectionSemantics.cbig-def2 by
    blast
    then have cbig1:  $(\sigma', \text{cset0}) \in \text{JVMSmartCollectionSemantics.cbig } P \sigma 1$ 
    using JVMSmartCollectionSemantics.cbig-def2  $\sigma 1(3)$  by blast
    have sheap1:  $\forall C'. P \vdash D \preceq^* C' \longrightarrow (\exists \text{sfs } i. \text{sheap } \sigma 1 C' = \llbracket (\text{sfs}, i) \rrbracket)$ 
     $\longrightarrow \text{classes-above } P C' \subseteq \text{cset}$  using sheap' sheap by simp
    have  $\text{cset0} \subseteq \text{cset}$  using  $\sigma 1(2)$  smart by blast
    then have classes-above  $P D \subseteq \text{cset}$ 
    using Calling-collects[OF sub cbig1 nend1 curr1 sheap1] by simp
    then show ?thesis using  $\sigma 1(2)$  smart by auto
  qed(simp)
qed

```

lemma *Putstatic-collects*:

```

assumes sub:  $P \vdash D \preceq^* \text{Object}$ 
and cbig:  $(\sigma', \text{cset}') \in \text{JVMSmartCollectionSemantics.cbig } P \sigma$ 
and nend:  $\sigma \notin \text{JVMEndset}$ 
and curr: curr-instr  $P (\text{hd}(\text{frames-of } \sigma)) = \text{Putstatic } C F D$ 
and ics: ics-of  $(\text{hd}(\text{frames-of } \sigma)) = \text{No-ics}$ 
and fC:  $P \vdash C \text{ has } F, \text{Static:}t \text{ in } D$ 
and sheap:  $\forall C'. P \vdash D \preceq^* C' \longrightarrow (\exists \text{sfs } i. \text{sheap } \sigma C' = \text{Some}(\text{sfs}, i))$ 
 $\longrightarrow \text{classes-above } P C' \subseteq \text{cset}$ 
and smart:  $\text{cset}' \subseteq \text{cset}$ 
shows classes-above  $P D \subseteq \text{cset}$ 
proof(cases  $(\exists \text{sfs } i. \text{sheap } \sigma D = \text{Some}(\text{sfs}, i) \wedge i = \text{Done})$ 
 $\vee (\text{ics-of}(\text{hd}(\text{frames-of } \sigma)) = \text{Called } \llbracket \rrbracket)$ )
  case True then show ?thesis
  proof(cases  $\exists \text{sfs } i. \text{sheap } \sigma D = \text{Some}(\text{sfs}, i) \wedge i = \text{Done}$ )
    case True then show ?thesis using sheap by auto
  next
  case False
  then have ics-of  $(\text{hd}(\text{frames-of } \sigma)) = \text{Called } \llbracket \rrbracket$  using True by clarsimp
  then show ?thesis using ics by auto
  qed
next
  case False
  obtain n where nstep:  $(\sigma', \text{cset}') \in \text{JVMSmartCollectionSemantics.csmall-nstep}$ 
   $P \sigma n$ 
  and  $n \neq 0$  using nend cbig JVMSmartCollectionSemantics.cbig-def2
  JVMSmartCollectionSemantics.csmall-nstep-base by (metis empty-iff insert-iff)
  then show ?thesis
  proof(cases n)
    case (Suc n1)
    then obtain  $\sigma 1 \text{cset0 } \text{cset1}$  where  $\sigma 1$ :  $(\sigma 1, \text{cset1}) \in \text{JVMSmartCollectionSe-}$ 
    mantics.csmall } P \sigma
     $\text{cset}' = \text{cset1} \cup \text{cset0}$   $(\sigma', \text{cset0}) \in \text{JVMSmartCollectionSemantics.csmall-nstep}$ 
     $P \sigma 1 n1$ 
    using JVMSmartCollectionSemantics.csmall-nstep-SucD nstep by blast

```

**obtain**  $xp\ h\ frs\ sh$  **where**  $\sigma = (xp, h, frs, sh)$  **by** (cases  $\sigma$ )  
**then have**  $curr1: ics\text{-of}\ (hd(\text{frames-of}\ \sigma 1)) = \text{Calling}\ D\ []$   
**and**  $sheap'$ :  $sheap\ \sigma = sheap\ \sigma 1$  **and**  $nend1: \sigma 1 \notin JVMendset$   
**using**  $JVM\text{-}Putstatic\text{-}next\text{-}step[OF\ \text{-}\ nend\ curr\ fC]\ \sigma 1(1)\ False\ ics$   
**by** ( $simp\ add: JVM\text{SmartCollectionSemantics.csmall-def}$ ) +  
**have**  $\sigma' \in JVMendset$  **using**  $cbig\ JVM\text{SmartCollectionSemantics.cbig-def2}$  **by**  
 $blast$   
**then have**  $cbig1: (\sigma', cset0) \in JVM\text{SmartCollectionSemantics.cbig}\ P\ \sigma 1$   
**using**  $JVM\text{SmartCollectionSemantics.cbig-def2}\ \sigma 1(3)$  **by**  $blast$   
**have**  $sheap1: \forall C'. P \vdash D \preceq^* C' \longrightarrow (\exists sfs\ i. sheap\ \sigma 1\ C' = [(sfs, i)])$   
 $\longrightarrow classes\text{-}above\ P\ C' \subseteq cset$  **using**  $sheap'\ sheap$  **by**  $simp$   
**have**  $cset0 \subseteq cset$  **using**  $\sigma 1(2)$   $smart$  **by**  $blast$   
**then have**  $classes\text{-}above\ P\ D \subseteq cset$   
**using**  $Calling\text{-}collects[OF\ sub\ cbig1\ nend1\ curr1\ sheap1]$  **by**  $simp$   
**then show**  $?thesis$  **using**  $\sigma 1(2)$   $smart$  **by**  $auto$   
**qed** ( $simp$ )  
**qed**

**lemma**  $Invokestatic\text{-}collects$ :  
**assumes**  $sub: P \vdash D \preceq^* Object$   
**and**  $cbig: (\sigma', cset') \in JVM\text{SmartCollectionSemantics.cbig}\ P\ \sigma$   
**and**  $smart: cset' \subseteq cset$   
**and**  $nend: \sigma \notin JVMendset$   
**and**  $curr: curr\text{-}instr\ P\ (hd(\text{frames-of}\ \sigma)) = Invokestatic\ C\ M\ n$   
**and**  $ics: ics\text{-of}\ (hd(\text{frames-of}\ \sigma)) = No\text{-}ics$   
**and**  $mC: P \vdash C\ sees\ M, Static: Ts \rightarrow T = m\ in\ D$   
**and**  $sheap: \forall C'. P \vdash D \preceq^* C' \longrightarrow (\exists sfs\ i. sheap\ \sigma\ C' = Some(sfs, i))$   
 $\longrightarrow classes\text{-}above\ P\ C' \subseteq cset$   
**shows**  $classes\text{-}above\ P\ D \subseteq cset$   
**proof** (cases  $(\exists sfs\ i. sheap\ \sigma\ D = Some(sfs, i) \wedge i = Done)$   
 $\vee (ics\text{-of}(hd(\text{frames-of}\ \sigma)) = Called\ [])$ )  
**case**  $True$  **then show**  $?thesis$   
**proof** (cases  $\exists sfs\ i. sheap\ \sigma\ D = Some(sfs, i) \wedge i = Done$ )  
**case**  $True$  **then show**  $?thesis$  **using**  $sheap$  **by**  $auto$   
**next**  
**case**  $False$   
**then have**  $ics\text{-of}(hd(\text{frames-of}\ \sigma)) = Called\ []$  **using**  $True$  **by**  $clarsimp$   
**then show**  $?thesis$  **using**  $ics$  **by**  $auto$   
**qed**  
**next**  
**case**  $False$   
**obtain**  $n$  **where**  $nstep: (\sigma', cset') \in JVM\text{SmartCollectionSemantics.csmall-nstep}$   
 $P\ \sigma\ n$   
**and**  $n \neq 0$  **using**  $nend\ cbig\ JVM\text{SmartCollectionSemantics.cbig-def2}$   
 $JVM\text{SmartCollectionSemantics.csmall-nstep-base}$  **by** ( $metis\ empty\text{-}iff\ insert\text{-}iff$ )  
**then show**  $?thesis$   
**proof** (cases  $n$ )  
**case** ( $Suc\ n1$ )  
**then obtain**  $\sigma 1\ cset0\ cset1$  **where**  $\sigma 1: (\sigma 1, cset1) \in JVM\text{SmartCollectionSe-}$

*mantics.csmall*  $P \sigma$   
 $cset' = cset1 \cup cset0$  ( $\sigma', cset0$ )  $\in$  *JVMSmartCollectionSemantics.csmall-nstep*  
 $P \sigma 1 n1$   
**using** *JVMSmartCollectionSemantics.csmall-nstep-SucD nstep* **by** *blast*  
**obtain**  $xp \ h \ frs \ sh$  **where**  $\sigma = (xp, h, frs, sh)$  **by** (*cases*  $\sigma$ )  
**then have**  $curr1$ : *ics-of* ( $hd(frames-of \ \sigma 1)$ ) = *Calling D*  $\square$   
**and**  $sheap'$ : *sheap*  $\sigma = sheap \ \sigma 1$  **and**  $nend1$ :  $\sigma 1 \notin JVMendset$   
**using** *JVM-Invokestatic-next-step[OF - nend curr mC]*  $\sigma 1(1)$  *False ics*  
**by** (*simp add: JVMSmartCollectionSemantics.csmall-def*)  
**have**  $\sigma' \in JVMendset$  **using** *cbig JVMSmartCollectionSemantics.cbig-def2* **by**  
*blast*  
**then have**  $cbig1$ : ( $\sigma', cset0$ )  $\in$  *JVMSmartCollectionSemantics.cbig*  $P \sigma 1$   
**using** *JVMSmartCollectionSemantics.cbig-def2*  $\sigma 1(3)$  **by** *blast*  
**have**  $sheap1$ :  $\forall C'. P \vdash D \preceq^* C' \longrightarrow (\exists sfs \ i. sheap \ \sigma 1 \ C' = [(sfs, i)])$   
 $\longrightarrow$  *classes-above*  $P \ C' \subseteq cset$  **using**  $sheap' \ sheap$  **by** *simp*  
**have**  $cset0 \subseteq cset$  **using**  $\sigma 1(2)$  *smart* **by** *blast*  
**then have** *classes-above*  $P \ D \subseteq cset$   
**using** *Calling-collects[OF sub cbig1 nend1 curr1 sheap1]* **by** *simp*  
**then show** *?thesis* **using**  $\sigma 1(2)$  *smart* **by** *auto*  
**qed**(*simp*)  
**qed**

The *smart-out* execution function keeps the promise to collect above the initial class (*Test*):

**lemma** *jvm-smart-out-classes-above-Test*:  
**assumes**  $s$ : ( $\sigma', cset_s$ )  $\in$  *jvm-smart-out*  $P \ t$  **and**  $P$ :  $P \in$  *jvm-progs* **and**  $t$ :  $t \in$   
*jvm-tests*  
**shows** *classes-above* (*jvm-make-test-prog*  $P \ t$ )  $Test \subseteq cset_s$   
(is *classes-above*  $?P \ ?D \subseteq ?cset$ )  
**proof** –  
**let**  $?\sigma = start-state$  ( $t\#P$ ) **and**  $?M = main$   
**let**  $?ics = ics-of$  ( $hd(frames-of \ ?\sigma)$ )  
**have**  $called$ :  $?ics = Called \ \square \implies$  *classes-above*  $?P \ ?D \subseteq ?cset$   
**by** (*simp add: start-state-def*)  
**then show** *?thesis*  
**proof**(*cases*  $?ics = Called \ \square$ )  
**case** *True* **then show** *?thesis* **using**  $called$  **by** *simp*  
**next**  
**case** *False*  
**from**  $P \ t$  **obtain**  $wf-md$  **where**  $wf$ : *wf-prog wf-md* ( $t\#P$ )  
**by** (*auto simp: wf-jvm-prog-phi-def wf-jvm-prog-def*)  
**from** *jvm-make-test-prog-sees-Test-main[OF P t]* **obtain**  $m$  **where**  
 $mC$ :  $?P \vdash ?D$  *sees*  $?M, Static$ :  $\square \rightarrow Void = m$  **in**  $?D$  **by** *fast*  
  
**then have**  $?P \vdash ?D \preceq^* Object$  **by** (*rule sees-method-sub-Obj*)  
**moreover from**  $s$  **obtain**  $cset'$  **where**  
 $cbig$ : ( $\sigma', cset'$ )  $\in$  *JVMSmartCollectionSemantics.cbig*  $?P \ ?\sigma$  **and**  $cset' \subseteq$   
 $?cset$  **by** *clarsimp*  
**moreover have**  $nend$ :  $?\sigma \notin JVMendset$  **by** (*rule start-state-nend*)

```

moreover from start-prog-start-m-instrs[OF wf] t
have curr: curr-instr ?P (hd(frames-of ?σ)) = Invokestatic ?D ?M 0
  by(simp add: start-state-def)
moreover have ics: ?ics = No-ics
  by(simp add: start-state-def)
moreover note mC
moreover from jvm-smart-out-classes-above-start-sheap[OF s]
have sheap: ∀ C'. ?P ⊢ ?D ≼* C' ⟶ (∃ sfs i. sheap ?σ C' = Some(sfs,i))
  ⟶ classes-above ?P C' ⊆ ?cset by(simp add: start-state-def)
ultimately show ?thesis by(rule Invokestatic-collects)
qed
qed

```

Using lemmas proving preservation of backward promises and keeping of forward promises, we prove that the smart algorithm collects at least the classes as the naive algorithm does.

**lemma** *jvm-naive-to-smart-exec-collect*:

**assumes**

— well-formedness

*wtp*: wf-jvm-prog<sub>Φ</sub> P

**and** *correct*: P, Φ ⊢ (xp, h, frs, sh)√

— defs

**and** *f'*: hd frs = (stk, loc, C', M', pc, ics)

— backward promises - will be collected prior

**and** *heap*:  $\bigwedge C fs. \exists a. h a = \text{Some}(C, fs) \implies \text{classes-above } P C \subseteq \text{cset}$

**and** *sheap*:  $\bigwedge C sfs i. sh C = \text{Some}(sfs, i) \implies \text{classes-above } P C \subseteq \text{cset}$

**and** *xcpts*: *classes-above-xcpts* P ⊆ cset

**and** *frames*: *classes-above-frames* P frs ⊆ cset

— forward promises - will be collected after if not already

**and** *init-class-prom*:  $\bigwedge C. ics = \text{Called } [] \vee ics = \text{No-ics}$

$\implies \text{coll-init-class } P (\text{instrs-of } P C' M' ! pc) = \text{Some } C \implies \text{classes-above } P$

$C \subseteq \text{cset}$

**and** *Calling-prom*:  $\bigwedge C' Cs'. ics = \text{Calling } C' Cs' \implies \text{classes-above } P C' \subseteq \text{cset}$

— collection

**and** *smart*: JVMexec-scollect P (xp, h, frs, sh) ⊆ cset

**shows** JVMexec-ncollect P (xp, h, frs, sh) ⊆ cset

**using** *assms*

**proof**(cases frs)

**case** (Cons f' frs')

**then have** [simp]: *classes-above* P C' ⊆ cset **using** frames f' **by** simp

**let** ?i = instrs-of P C' M' ! pc

**have** cr': P, Φ ⊢ (xp, h, (stk, loc, C', M', pc, ics)#frs', sh)√ **using** correct f' Cons **by** simp

**from** well-formed-stack-safe[OF wtp cr'] correct-state-Throwing-ex[OF cr'] **obtain**

*stack-safe*: stack-safe ?i h stk **and**

*Throwing-ex*:  $\bigwedge Cs a. ics = \text{Throwing } Cs a \implies \exists \text{obj}. h a = \text{Some obj}$  **by** simp

**have** confc: conf-clinit P sh frs **using** correct Cons **by** simp

**have** Called-prom:  $\bigwedge C' Cs'. ics = \text{Called } (C' \# Cs')$

```

    ⇒ classes-above P C' ⊆ cset ∧ classes-above P (fst(method P C' clinit))
⊆ cset
proof –
  fix C' Cs' assume [simp]: ics = Called (C'#Cs')
  then have C' ∈ set(clinit-classes frs) using f' Cons by simp
  then obtain sfs where shC': sh C' = Some(sfs, Processing) and is-class P C'
  using confc by(auto simp: conf-clinit-def)
  then have C'eq: C' = fst(method P C' clinit) using wf-sees-clinit wtp
  by(fastforce simp: is-class-def wf-jvm-prog-phi-def)
  then show classes-above P C' ⊆ cset ∧ classes-above P (fst(method P C'
clinit)) ⊆ cset
  using sheap shC' by auto
qed
show ?thesis using Cons assms
proof(cases xp)
  case None
  { assume ics: ics = Called [] ∨ ics = No-ics
    then have [simp]: JVMexec-ncollect P (xp,h,frs,sh)
      = JVMinstr-ncollect P ?i h stk ∪ classes-above P C'
      ∪ classes-above-frames P frs ∪ classes-above-xcpts P
    and [simp]: JVMexec-scollect P (xp,h,frs,sh) = JVMinstr-scollect P ?i
    using f' None Cons by auto
    have ?thesis using assms
    proof(cases ?i)
      case (New C)
      then have classes-above P C ⊆ cset using ics New assms by simp
      then show ?thesis using New xcpts frames smart f' by auto
    next
      case (Getfield F C) show ?thesis
      proof(cases hd stk = Null)
        case True then show ?thesis using Getfield assms by simp
      next
        case False
        let ?C = cname-of h (the-Addr (hd stk))
        have above-stk: classes-above P ?C ⊆ cset
        using stack-safe heap f' False Cons Getfield by(auto dest!: is-ptrD) blast
        then show ?thesis using Getfield assms by auto
      qed
    next
      case (Getstatic C F D)
      show ?thesis
      proof(cases ∃ t. P ⊢ C has F,Static:t in D)
        case True
        then have above-D: classes-above P D ⊆ cset using ics init-class-prom
        Getstatic by simp
        have sub: P ⊢ C ≼* D using has-field-decl-above True by blast
        then have above-C: classes-between P C D – {D} ⊆ cset
        using True Getstatic above-D smart f' by simp
        then have classes-above P C ⊆ cset

```

```

        using classes-above-sub-classes-between-eq[OF sub] above-D above-C by
auto
    then show ?thesis using Getstatic assms by auto
next
    case False then show ?thesis using Getstatic assms by auto
qed
next
case (Putfield F C) show ?thesis
proof(cases hd(tl stk) = Null)
    case True then show ?thesis using Putfield assms by simp
next
    case False
    let ?C = cname-of h (the-Addr (hd (tl stk)))
    have above-stk: classes-above P ?C  $\subseteq$  cset
        using stack-safe heap f' False Cons Putfield by(auto dest!: is-ptrD) blast
    then show ?thesis using Putfield assms by auto
qed
next
case (Putstatic C F D)
show ?thesis
proof(cases  $\exists t. P \vdash C \text{ has } F, \text{Static}:t \text{ in } D$ )
    case True
        then have above-D: classes-above P D  $\subseteq$  cset using ics init-class-prom
Putstatic by simp
        have sub:  $P \vdash C \preceq^* D$  using has-field-decl-above True by blast
        then have above-C: classes-between P C D - {D}  $\subseteq$  cset
            using True Putstatic above-D smart f' by simp
        then have classes-above P C  $\subseteq$  cset
            using classes-above-sub-classes-between-eq[OF sub] above-D above-C by
auto
        then show ?thesis using Putstatic assms by auto
    next
        case False then show ?thesis using Putstatic assms by auto
qed
next
case (Checkcast C) show ?thesis
proof(cases hd stk = Null)
    case True then show ?thesis using Checkcast assms by simp
next
    case False
    let ?C = cname-of h (the-Addr (hd stk))
    have above-stk: classes-above P ?C  $\subseteq$  cset
        using stack-safe heap False Cons f' Checkcast by(auto dest!: is-ptrD)
blast
    then show ?thesis using above-stk Checkcast assms by(cases hd stk =
Null, auto)
qed
next
case (Invoke M n) show ?thesis

```



```

proof(cases stk ! n = Null)
  case True then show ?thesis using Invoke assms by simp
next
  case False
  let ?C = cname-of h (the-Addr (stk ! n))
  have above-stk: classes-above P ?C  $\subseteq$  cset using stack-safe heap False
  Cons f' Invoke
    by(auto dest!: is-ptrD) blast
    then show ?thesis using Invoke assms by auto
  qed
next
  case (Invokestatic C M n)
  let ?D = fst (method P C M)
  show ?thesis
  proof(cases  $\exists Ts T m D. P \vdash C \text{ sees } M, \text{Static}: Ts \rightarrow T = m \text{ in } D$ )
    case True
    then have above-D: classes-above P ?D  $\subseteq$  cset using ics init-class-prom
    Invokestatic
      by(simp add: seeing-class-def)
      have sub:  $P \vdash C \preceq^* ?D$  using method-def2 sees-method-decl-above True
    by auto
    then show ?thesis
    proof(cases C = ?D)
      case True then show ?thesis
      using Invokestatic above-D xcpts frames smart f' by auto
    next
      case False
      then have above-C: classes-between P C ?D - {?D}  $\subseteq$  cset
      using True Invokestatic above-D smart f' by simp
      then have classes-above P C  $\subseteq$  cset
      using classes-above-sub-classes-between-eq[OF sub] above-D above-C by
      auto
      then show ?thesis using Invokestatic assms by auto
    qed
  next
    case False then show ?thesis using Invokestatic assms by auto
  qed
next
  case Throw show ?thesis
  proof(cases hd stk = Null)
    case True then show ?thesis using Throw assms by simp
  next
    case False
    let ?C = cname-of h (the-Addr (hd stk))
    have above-stk: classes-above P ?C  $\subseteq$  cset
    using stack-safe heap False Cons f' Throw by(auto dest!: is-ptrD) blast
    then show ?thesis using above-stk Throw assms by auto
  qed
next

```

```

    case Load then show ?thesis using assms by auto
next
    case Store then show ?thesis using assms by auto
next
    case Push then show ?thesis using assms by auto
next
    case Goto then show ?thesis using assms by auto
next
    case IfFalse then show ?thesis using assms by auto
qed(auto)
}
moreover
{ fix C1 Cs1 assume ics: ics = Called (C1#Cs1)
  then have ?thesis using None Cons Called-prom[OF ics] xcpts frames f' by
simp
}
moreover
{ fix Cs1 a assume ics: ics = Throwing Cs1 a
  then obtain C fs where h a = Some(C,fs) using Throwing-ex by fastforce
  then have above-stk: classes-above P (cname-of h a) ⊆ cset using heap by
auto
  then have ?thesis using ics None Cons xcpts frames f' by simp
}
moreover
{ fix C1 Cs1 assume ics: ics = Calling C1 Cs1
  then have ?thesis using None Cons Calling-prom[OF ics] xcpts frames f' by
simp
}
ultimately show ?thesis by (metis ics-classes.cases list.exhaust)
qed(simp)
qed(simp)

```

— ... which is the same as *csmall*

**lemma** *jvm-naive-to-smart-csmall:*

**assumes**

— well-formedness

*wtp: wf-jvm-prog<sub>Φ</sub> P*

**and** *correct: P, Φ ⊢ (xp, h, frs, sh)√*

— defs

**and** *f': hd frs = (stk, loc, C', M', pc, ics)*

— backward promises - will be collected prior

**and** *heap: ∧ C fs. ∃ a. h a = Some(C, fs) ⇒ classes-above P C ⊆ cset*

**and** *sheap: ∧ C sfs i. sh C = Some(sfs, i) ⇒ classes-above P C ⊆ cset*

**and** *xcpts: classes-above-xcpts P ⊆ cset*

**and** *frames: classes-above-frames P frs ⊆ cset*

— forward promises - will be collected after if not already

**and** *init-class-prom: ∧ C. ics = Called [] ∨ ics = No-ics*

*⇒ coll-init-class P (instrs-of P C' M' ! pc) = Some C ⇒ classes-above P C*

*⊆ cset*

**and** *Calling-prom*:  $\bigwedge C' Cs'. ics = \text{Calling } C' Cs' \implies \text{classes-above } P C' \subseteq cset$   
 — collections  
**and** *smart-coll*:  $(\sigma', cset_s) \in \text{JVMSmartCollectionSemantics.csmall } P (xp, h, frs, sh)$   
**and** *naive-coll*:  $(\sigma', cset_n) \in \text{JVMNaiveCollectionSemantics.csmall } P (xp, h, frs, sh)$   
**and** *smart*:  $cset_s \subseteq cset$   
**shows**  $cset_n \subseteq cset$   
**using** *jvm-naive-to-smart-exec-collect*[**where**  $h=h$  **and**  $sh=sh$ , *OF* *assms(1–9)*]  
     *smart smart-coll naive-coll*  
**by**(*fastforce simp*: *JVMNaiveCollectionSemantics.csmall-def*  
     *JVMSmartCollectionSemantics.csmall-def*)

— ...which means over *csmall-nstep*, stepping from the end state (the point by which future promises will have been fulfilled) (uses backward and forward promise lemmas)

**lemma** *jvm-naive-to-smart-csmall-nstep*:

$\llbracket$  *wf-jvm-prog* $\Phi$   $P$ ;  
 $P, \Phi \vdash (xp, h, frs, sh) \checkmark$ ;  
 $hd \text{ frs} = (stk, loc, C', M', pc, ics)$ ;  
 $\bigwedge C \text{ fs}. \exists a. h a = \text{Some}(C, fs) \implies \text{classes-above } P C \subseteq cset$ ;  
 $\bigwedge C \text{ sfs } i. sh C = \text{Some}(sfs, i) \implies \text{classes-above } P C \subseteq cset$ ;  
 $\text{classes-above-xcpts } P \subseteq cset$ ;  
 $\text{classes-above-frames } P \text{ frs} \subseteq cset$ ;  
 $\bigwedge C. ics = \text{Called } \square \vee ics = \text{No-ics}$   
 $\implies \text{coll-init-class } P (\text{instrs-of } P C' M' ! pc) = \text{Some } C \implies \text{classes-above } P$   
 $C \subseteq cset$ ;  
 $\bigwedge C' Cs'. ics = \text{Calling } C' Cs' \implies \text{classes-above } P C' \subseteq cset$ ;  
 $(\sigma', cset_n) \in \text{JVMNaiveCollectionSemantics.csmall-nstep } P (xp, h, frs, sh) n$ ;  
 $(\sigma', cset_s) \in \text{JVMSmartCollectionSemantics.csmall-nstep } P (xp, h, frs, sh) n$ ;  
 $cset_s \subseteq cset$ ;  
 $\sigma' \in \text{JVMendset } \square$   
 $\implies cset_n \subseteq cset$

**proof**(*induct*  $n$  *arbitrary*:  $xp \ h \ frs \ sh \ stk \ loc \ C' \ M' \ pc \ ics \ \sigma' \ cset_n \ cset_s \ cset$ )

**case**  $0$  **then show** *?case*

**using** *JVMNaiveCollectionSemantics.csmall-nstep-base subsetI old.prod.inject singletonD*

**by** (*metis (no-types, lifting) equals0D*)

**next**

**case** (*Suc*  $n1$ )

**let**  $? \sigma = (xp, h, frs, sh)$

**obtain**  $\sigma 1 \ cset 1 \ cset'$  **where**  $\sigma 1$ :  $(\sigma 1, cset 1) \in \text{JVMNaiveCollectionSemantics.csmall } P ? \sigma$

$cset_n = cset 1 \cup cset'$   $(\sigma', cset') \in \text{JVMNaiveCollectionSemantics.csmall-nstep } P \ \sigma 1 \ n1$

**using** *JVMNaiveCollectionSemantics.csmall-nstep-SucD[OF Suc.prem(10)]* **by** *clarsimp+*

**obtain**  $\sigma 1' \ cset 1' \ cset''$  **where**  $\sigma 1'$ :  $(\sigma 1', cset 1') \in \text{JVMSmartCollectionSemantics.csmall } P ? \sigma$

$cset_s = cset 1' \cup cset''$   $(\sigma', cset'') \in \text{JVMSmartCollectionSemantics.csmall-nstep } P \ \sigma 1' \ n1$

```

using JVMSmartCollectionSemantics.csmall-nstep-SucD[OF Suc.prem(11)] by
clarsimp+
have  $\sigma$ -eq:  $\sigma 1 = \sigma 1'$  using  $\sigma 1(1)$   $\sigma 1'(1)$  by(simp add: JVMNaiveCollectionSe-
mantics.csmall-def
                                JVMSmartCollectionSemantics.csmall-def)
have sub1': cset1'  $\subseteq$  cset and sub'': cset''  $\subseteq$  cset using Suc.prem(12)  $\sigma 1'(2)$ 
by auto
then have sub1: cset1  $\subseteq$  cset
using jvm-naive-to-smart-csmall[where  $h=h$  and  $sh=sh$  and  $\sigma'=\sigma 1$ , OF
Suc.prem(1-9) - - sub1']
Suc.prem(11,12)  $\sigma 1(1)$   $\sigma 1'(1)$   $\sigma$ -eq by fastforce
show ?case
proof(cases n1)
  case 0 then show ?thesis using  $\sigma 1(2,3)$  sub1 by auto
next
  case Suc2: (Suc n2)
  then have nend1:  $\sigma 1 \notin$  JVMendset
  using JVMNaiveCollectionSemantics.csmall-nstep-Suc-nend  $\sigma 1(3)$  by blast
  obtain xp1 h1 frs1 sh1 where  $\sigma 1$ -case [simp]:  $\sigma 1 = (xp1, h1, frs1, sh1)$  by(cases
 $\sigma 1$ )
  obtain stk1 loc1 C1' M1' pc1 ics1 where f1':  $hd$  frs1 = (stk1, loc1, C1', M1', pc1, ics1)
  by(cases hd frs1)
  then obtain frs1' where [simp]: frs1 = (stk1, loc1, C1', M1', pc1, ics1)#frs1'
  using JVMendset-def nend1 by(cases frs1, auto)
  have cbig1: ( $\sigma'$ , cset')  $\in$  JVMNaiveCollectionSemantics.cbig P  $\sigma 1$ 
  ( $\sigma'$ , cset'')  $\in$  JVMSmartCollectionSemantics.cbig P  $\sigma 1$  using  $\sigma 1(3)$   $\sigma 1'(3)$ 
Suc.prem(13)  $\sigma$ -eq
  using JVMNaiveCollectionSemantics.cbig-def2
  JVMSmartCollectionSemantics.cbig-def2 by blast+
  obtain  $\sigma 2'$  cset2' cset2'' where  $\sigma 2'$ : ( $\sigma 2'$ , cset2')  $\in$  JVMSmartCollectionSe-
mantics.csmall P  $\sigma 1$ 
  cset'' = cset2'  $\cup$  cset2'' ( $\sigma'$ , cset2'')  $\in$  JVMSmartCollectionSemantics.csmall-nstep
P  $\sigma 2'$  n2
  using JVMSmartCollectionSemantics.csmall-nstep-SucD  $\sigma 1'(3)$  Suc2  $\sigma$ -eq by
blast

have wtp: wf-jvm-prog $\Phi$  P by fact
let ?i1 = instrs-of P C1' M1' ! pc1
let ?ics1 = ics-of (hd (frames-of  $\sigma 1$ ))
have step: P  $\vdash$  (xp, h, frs, sh) -jvm $\rightarrow$  (xp1, h1, frs1, sh1)
proof -
  have exec (P, ? $\sigma$ ) = [ $\sigma 1'$ ] using JVMsmart-csmallD[OF  $\sigma 1'(1)$ ] by simp
  then have P  $\vdash$  ? $\sigma$  -jvm $\rightarrow$   $\sigma 1'$  using jvm-one-step1[OF exec-1.exec-1I] by
simp
  then show ?thesis using Suc.prem(12)  $\sigma$ -eq by fastforce
qed
have correct1: P,  $\Phi$   $\vdash$  (xp1, h1, frs1, sh1) $\surd$  by(rule BV-correct[OF wtp step
Suc.prem(2)])

```

**have**  $vics1: P, h1, sh1 \vdash_i (C1', M1', pc1, ics1)$   
**using**  $correct1\ Suc.premis(7)$  **by**  $(auto\ simp: conf-f-def2)$   
**from**  $correct1$  **obtain**  $b\ Ts\ T\ mxs\ mxl_0\ ins\ xt\ ST\ LT$  **where**  
 $meth1: P \vdash C1' \text{ sees } M1', b: Ts \rightarrow T = (mxs, mxl_0, ins, xt)$  **in**  $C1'$  **and**  
 $pc1: pc1 < length\ ins$  **and**  
 $\Phi\text{-}pc1: \Phi\ C1'\ M1' \!^! pc1 = Some\ (ST, LT)$  **by**  $(auto\ dest: sees-method-fun)$   
**then** **have**  $wt1: P, T, mxs, size\ ins, xt \vdash ins \!^! pc1, pc1 :: \Phi\ C1'\ M1'$   
**using**  $wt-jvm-prog-impl-wt-instr[OF\ wtp\ meth1]$  **by**  $fast$

**have**  $\bigwedge a\ C\ fs\ sfs'\ i'. (h1\ a = \lfloor (C, fs) \rfloor \longrightarrow classes\text{-}above\ P\ C \subseteq cset) \wedge$   
 $(sh1\ C = \lfloor (sfs', i') \rfloor \longrightarrow classes\text{-}above\ P\ C \subseteq cset) \wedge$   
 $classes\text{-}above\text{-}frames\ P\ frs1 \subseteq cset$   
**proof** –  
**fix**  $a\ C\ fs\ sfs'\ i'$   
**show**  $(h1\ a = \lfloor (C, fs) \rfloor \longrightarrow classes\text{-}above\ P\ C \subseteq cset) \wedge$   
 $(sh1\ C = \lfloor (sfs', i') \rfloor \longrightarrow classes\text{-}above\ P\ C \subseteq cset) \wedge$   
 $(classes\text{-}above\text{-}frames\ P\ frs1 \subseteq cset)$   
**using**  $Suc.premis(11-12)\ \sigma 1'\ \sigma\text{-}eq[THEN\ sym]\ JVMsmart\text{-}csmallD[OF\ \sigma 1'(1)]$   
 $backward\text{-}coll\text{-}promises\text{-}kept[\text{where } h=h\ \text{and } xp=xp\ \text{and } sh=sh\ \text{and } frs=frs$   
**and**  $frs'=frs1$   
**and**  $xp'=xp1\ \text{and } h'=h1\ \text{and } sh'=sh1, OF\ Suc.premis(1-9)]$  **by**  $auto$   
**qed**

**then** **have**  $heap1: \bigwedge C\ fs. \exists a. h1\ a = Some(C, fs) \implies classes\text{-}above\ P\ C \subseteq cset$   
**and**  $sheap1: \bigwedge C\ sfs\ i. sh1\ C = Some(sfs, i) \implies classes\text{-}above\ P\ C \subseteq cset$   
**and**  $frames1: classes\text{-}above\text{-}frames\ P\ frs1 \subseteq cset$  **by**  $blast+$   
**have**  $xcpts1: classes\text{-}above\text{-}xcpts\ P \subseteq cset$  **using**  $Suc.premis(6)$  **by**  $auto$   
–  $init\text{-}class$   $promise$

**have**  $sheap2: \bigwedge C. coll\text{-}init\text{-}class\ P\ ?i1 = Some\ C$   
 $\implies \forall C'. P \vdash C \preceq^* C' \longrightarrow (\exists sfs\ i. sheap\ \sigma 1\ C' = \lfloor (sfs, i) \rfloor)$   
 $\longrightarrow classes\text{-}above\ P\ C' \subseteq cset$  **using**  $sheap1$  **by**  $auto$   
**have**  $called: \bigwedge C. coll\text{-}init\text{-}class\ P\ ?i1 = Some\ C$   
 $\implies ics\text{-}of\ (hd\ (frames\text{-}of\ \sigma 1)) = Called\ [] \implies classes\text{-}above\ P\ C \subseteq cset$   
**proof** –  
**fix**  $C$  **assume**  $cic: coll\text{-}init\text{-}class\ P\ ?i1 = Some\ C$  **and**  
 $ics: ics\text{-}of\ (hd\ (frames\text{-}of\ \sigma 1)) = Called\ []$   
**then** **obtain**  $sobj$  **where**  $sh1\ C = Some\ sobj$  **using**  $vics1\ f1'$   
**by**  $(cases\ ?i1, auto\ simp: seeing\text{-}class\text{-}def\ split: if\text{-}split\text{-}asm)$   
**then** **show**  $classes\text{-}above\ P\ C \subseteq cset$  **using**  $sheap1$  **by**  $(cases\ sobj, simp)$   
**qed**

**have**  $init\text{-}class\text{-}prom1: \bigwedge C. ics1 = Called\ [] \vee ics1 = No\text{-}ics$   
 $\implies coll\text{-}init\text{-}class\ P\ ?i1 = Some\ C \implies classes\text{-}above\ P\ C \subseteq cset$   
**proof** –  
**fix**  $C$  **assume**  $ics1 = Called\ [] \vee ics1 = No\text{-}ics$  **and**  $cic: coll\text{-}init\text{-}class\ P\ ?i1$   
 $= Some\ C$   
**then** **have**  $ics: ?ics1 = Called\ [] \vee ?ics1 = No\text{-}ics$  **using**  $f1'$  **by**  $simp$   
**then** **show**  $classes\text{-}above\ P\ C \subseteq cset$  **using**  $cic$   
**proof**  $(cases\ ?ics1 = Called\ [])$   
**case**  $True$  **then** **show**  $?thesis$  **using**  $cic\ called$  **by**  $simp$   
**next**

```

case False
then have ics': ?ics1 = No-ics using ics by simp
then show ?thesis using cic
proof(cases ?i1)
  case (New C1)
    then have is-class P C1 using  $\Phi$ -pc1 wt1 meth1 by auto
    then have  $P \vdash C1 \preceq^* \text{Object}$  using wtp is-class-is-subcls
      by(auto simp: wf-jvm-prog-phi-def)
    then show ?thesis using New-collects[OF - cbig1(2) nend1 - ics' sheap2
sub'']
      f1' ics cic New by auto
  next
    case (Getstatic C1 F1 D1)
      then obtain t where mC1: P \vdash C1 has F1,Static:t in D1 and eq: C =
D1
        using cic by (metis coll-init-class.simps(2) option.inject option.simps(3))
        then have is-class P C using has-field-is-class'[OF mC1] by simp
        then have  $P \vdash C \preceq^* \text{Object}$  using wtp is-class-is-subcls
          by(auto simp: wf-jvm-prog-phi-def)
        then show ?thesis using Getstatic-collects[OF - cbig1(2) nend1 - ics' -
sheap2 sub'']
          eq f1' Getstatic ics cic by fastforce
      next
        case (Putstatic C1 F1 D1)
          then obtain t where mC1: P \vdash C1 has F1,Static:t in D1 and eq: C =
D1
            using cic by (metis coll-init-class.simps(3) option.inject option.simps(3))
            then have is-class P C using has-field-is-class'[OF mC1] by simp
            then have  $P \vdash C \preceq^* \text{Object}$  using wtp is-class-is-subcls
              by(auto simp: wf-jvm-prog-phi-def)
            then show ?thesis using Putstatic-collects[OF - cbig1(2) nend1 - ics' -
sheap2 sub'']
              eq f1' Putstatic ics cic by fastforce
          next
            case (Invokestatic C1 M1 n')
              then obtain Ts T m where mC: P \vdash C1 sees M1, Static : Ts \to T = m
in C
                using cic by(fastforce simp: seeing-class-def split: if-split-asm)
                then have is-class P C by(rule sees-method-is-class')
                then have Obj: P \vdash C \preceq^* Object using wtp is-class-is-subcls
                  by(auto simp: wf-jvm-prog-phi-def)
                show ?thesis using Invokestatic-collects[OF - cbig1(2) sub'' nend1 - ics'
mC sheap2]
                  Obj mC f1' Invokestatic ics cic by auto
            qed(simp+)
          qed
        qed
  — Calling promise
  have Calling-prom1: \bigwedge C' Cs'. ics1 = Calling C' Cs' \implies classes-above P C' \subseteq

```

*cset*

**proof** –

**fix**  $C' Cs'$  **assume**  $ics: ics1 = Calling\ C'\ Cs'$

**then have**  $is-class\ P\ C'$  **using**  $vics1$  **by**  $simp$

**then have**  $obj: P \vdash C' \preceq^* Object$  **using**  $wtp\ is-class-is-subcls$

**by**  $(auto\ simp: wf-jvm-prog-phi-def)$

**have**  $sheap3: \forall C1. P \vdash C' \preceq^* C1 \longrightarrow (\exists sfs\ i. sheap\ \sigma1\ C1 = [(sfs, i)])$

$\longrightarrow classes-above\ P\ C1 \subseteq cset$  **using**  $sheap1$  **by**  $auto$

**show**  $classes-above\ P\ C' \subseteq cset$

**using**  $Calling-collects[OF\ obj\ cbig1(2)\ nend1 - sheap3\ sub']$   $ics\ f1'$  **by**  $simp$

**qed**

**have**  $in-naive: (\sigma', cset') \in JVMNaiveCollectionSemantics.csmall-nstep\ P\ (xp1, h1, frs1, sh1)\ n1$

**and**  $in-smart: (\sigma', cset'') \in JVMSmartCollectionSemantics.csmall-nstep\ P\ (xp1, h1, frs1, sh1)\ n1$

**using**  $\sigma1(3)\ \sigma1'(3)\ \sigma-eq[THEN\ sym]$  **by**  $simp+$

**have**  $sub2: cset' \subseteq cset$

**by**  $(rule\ Suc.hyps[OF\ wtp\ correct1\ f1'\ heap1\ sheap1\ xcpts1\ frames1\ init-class-prom1\ Calling-prom1\ in-naive\ in-smart\ sub''\ Suc.prem1(13)])\ simp-all$

**then show**  $?thesis$  **using**  $\sigma1(2)\ \sigma1'(2)\ sub1\ sub2$  **by**  $fastforce$

**qed**

**qed**

— ...which means over  $cbig$

**lemma**  $jvm-naive-to-smart-cbig$ :

**assumes**

— well-formedness

$wtp: wf-jvm-prog_{\Phi}\ P$

**and**  $correct: P, \Phi \vdash (xp, h, frs, sh) \checkmark$

— defs

**and**  $f': hd\ frs = (stk, loc, C', M', pc, ics)$

— backward promises - will be collected/maintained prior

**and**  $heap: \bigwedge C\ fs. \exists a. h\ a = Some(C, fs) \implies classes-above\ P\ C \subseteq cset$

**and**  $sheap: \bigwedge C\ sfs\ i. sh\ C = Some(sfs, i) \implies classes-above\ P\ C \subseteq cset$

**and**  $xcpts: classes-above-xcpts\ P \subseteq cset$

**and**  $frames: classes-above-frames\ P\ frs \subseteq cset$

— forward promises - will be collected after if not already

**and**  $init-class-prom: \bigwedge C. ics = Called\ [] \vee ics = No-ics$

$\implies coll-init-class\ P\ (instrs-of\ P\ C'\ M'\ !\ pc) = Some\ C \implies classes-above\ P\ C$

$\subseteq cset$

**and**  $Calling-prom: \bigwedge C'\ Cs'. ics = Calling\ C'\ Cs' \implies classes-above\ P\ C' \subseteq cset$

— collections

**and**  $n: (\sigma', cset_n) \in JVMNaiveCollectionSemantics.cbig\ P\ (xp, h, frs, sh)$

**and**  $s: (\sigma', cset_s) \in JVMSmartCollectionSemantics.cbig\ P\ (xp, h, frs, sh)$

**and**  $smart: cset_s \subseteq cset$

**shows**  $cset_n \subseteq cset$

**proof** –

**let**  $?s = (xp, h, frs, sh)$

**have**  $nend: \sigma' \in JVMendset$  **using**  $n$  **by**  $(simp\ add: JVMNaiveCollectionSemant-$

```

tics.cbig-def)
  obtain n where n': ( $\sigma'$ , csetn) ∈ JVMNaiveCollectionSemantics.csmall-nstep P
  ? $\sigma$  n  $\sigma'$  ∈ JVMendset
  using JVMNaiveCollectionSemantics.cbig-def2 n by auto
  obtain s where s': ( $\sigma'$ , csets) ∈ JVMSmartCollectionSemantics.csmall-nstep P
  ? $\sigma$  s  $\sigma'$  ∈ JVMendset
  using JVMSmartCollectionSemantics.cbig-def2 s by auto
  have n=s using jvm-naive-to-smart-csmall-nstep-last-eq[OF n n'(1) s'(1)] by
  simp
  then have sn: ( $\sigma'$ , csets) ∈ JVMSmartCollectionSemantics.csmall-nstep P ? $\sigma$  n
  using s'(1) by simp
  then show ?thesis
  using jvm-naive-to-smart-csmall-nstep[OF assms(1-9) n'(1) sn assms(12) nend]
  by fast
qed

```

— ...thus naive  $\subseteq$  smart over the out function, since all conditions will be met - and promises kept - by the defined starting point

**lemma** *jvm-naive-to-smart-collection*:

**assumes** naive: ( $\sigma'$ , cset<sub>n</sub>) ∈ *jvm-naive-out* P t **and** smart: ( $\sigma'$ , cset<sub>s</sub>) ∈ *jvm-smart-out* P t

**and** P: P ∈ *jvm-progs* **and** t: t ∈ *jvm-tests*

**shows** cset<sub>n</sub>  $\subseteq$  cset<sub>s</sub>

**proof** –

**let** ?P = *jvm-make-test-prog* P t

**let** ? $\sigma$  = *start-state* (t#P)

**let** ?i = *instrs-of* ?P *Start start-m ! 0* **and** ?ics = *No-ics*

**obtain** xp h frs sh **where**

[*simp*]: ? $\sigma$  = (*xp, h, frs, sh*) **and**

[*simp*]: h = *start-heap* (t#P) **and**

[*simp*]: frs = [( $\square$ ), ( $\square$ ), *Start, start-m, 0, No-ics*] **and**

[*simp*]: sh = *start-sheap*

**by**(*clarsimp simp: start-state-def*)

**from** P t **have** nS:  $\neg$  *is-class* (t # P) *Start*

**by**(*simp add: is-class-def class-def Start-def Test-def*)

**from** P **have** nT:  $\neg$  *is-class* P *Test* **by** *simp*

**from** P t **obtain** m **where** tPm: t # P  $\vdash$  (*fst t*) *sees main, Static* :  $\square \rightarrow \text{Void} =$  m in (*fst t*)

**by** *auto*

**have** nclinit: *main*  $\neq$  *clinit* **by**(*simp add: main-def clinit-def*)

**have** Objp:  $\bigwedge b' Ts' T' m' D'$ .

$t\#P \vdash$  *Object sees start-m*,  $b' : Ts' \rightarrow T' = m'$  in  $D' \implies b' = \text{Static} \wedge Ts' =$

$\square \wedge T' = \text{Void}$

**proof** –

**fix** b' Ts' T' m' D'

**assume** mObj: t#P  $\vdash$  *Object sees start-m*,  $b' : Ts' \rightarrow T' = m'$  in  $D'$

**from** P **have** ot\_nsub:  $\neg$  P  $\vdash$  *Object*  $\preceq^*$  *Test*

**by**(*clarsimp simp: wf-jvm-prog-def wf-jvm-prog-phi-def*)



**from** *class-add-sees-method-rev*[*OF* - *ot-nsub*] *mObj* *t*  
**have**  $P \vdash \text{Object sees start-}m, b' : Ts' \rightarrow T' = m' \text{ in } D'$  **by** (*cases* *t*, *auto*)  
**with**  $P \text{ jvm-progs-def show } b' = \text{Static} \wedge Ts' = [] \wedge T' = \text{Void}$  **by** *blast*  
**qed**  
**from**  $P \text{ t obtain } \Phi \text{ where } wtp0: wf\text{-jvm-prog}_\Phi (t\#P)$  **by** (*auto simp: wf-jvm-prog-def*)  
**let**  $?\Phi' = \lambda C f. \text{if } C = \text{Start} \wedge (f = \text{start-}m \vee f = \text{clinit}) \text{ then } \text{start-}\varphi_m \text{ else } \Phi$   
*C f*  
**from**  $wtp0$  **have**  $wtp: wf\text{-jvm-prog}_{?\Phi'} ?P$   
**proof** –  
**note**  $wtp0 \ nS \ tPm \ nclinit$   
**moreover obtain**  $\bigwedge C. C \neq \text{Start} \implies ?\Phi' \ C = \Phi \ C \ ?\Phi' \ \text{Start} \ \text{start-}m =$   
*start-}\varphi\_m*  
 $?\Phi' \ \text{Start} \ \text{clinit} = \text{start-}\varphi_m$  **by** *simp*  
**moreover note** *Objp*  
**ultimately show** *?thesis* **by** (*rule start-prog-wf-jvm-prog-phi*)  
**qed**  
**have** *cic: coll-init-class ?P ?i = Some Test*  
**proof** –  
**from**  $wtp0$  **obtain** *wf-md* **where**  $wf: wf\text{-prog } wf\text{-md} (t\#P)$   
**by** (*clarsimp dest!: wt-jvm-progD*)  
**with** *start-prog-start-m-instrs t* **have**  $i: ?i = \text{Invokestatic Test main } 0$  **by** *simp*  
**from** *jvm-make-test-prog-sees-Test-main*[*OF*  $P \ t$ ] **obtain**  $m$  **where**  
 $?P \vdash \text{Test sees main, Static} : [] \rightarrow \text{Void} = m \text{ in Test}$  **by** *fast*  
**with**  $t$  **have** *seeing-class (jvm-make-test-prog P t) Test main = [Test]*  
**by** (*cases m, fastforce simp: seeing-class-def*)  
**with**  $i$  **show** *?thesis* **by** *simp*  
**qed**  
– well-formedness  
**note** *wtp*  
**moreover have** *correct: ?P, ?\Phi' \vdash (xp, h, frs, sh) \checkmark*  
**proof** –  
**note**  $wtp0 \ nS \ tPm \ nclinit$   
**moreover have**  $?\Phi' \ \text{Start} \ \text{start-}m = \text{start-}\varphi_m$  **by** *simp*  
**ultimately have**  $?P, ?\Phi' \vdash ?\sigma \checkmark$  **by** (*rule BV-correct-initial*)  
**then show** *?thesis* **by** *simp*  
**qed**  
– defs  
**moreover have**  $hd \ frs = ([], [], \text{Start}, \text{start-}m, 0, \text{No-ics})$  **by** *simp*  
– backward promises  
**moreover from** *jvm-smart-out-classes-above-start-heap*[*OF* *smart - P t*]  
**have** *heap: \bigwedge C fs. \exists a. h a = Some(C, fs) \implies classes-above ?P C \subseteq cset\_s* **by**  
*auto*  
**moreover from** *jvm-smart-out-classes-above-start-sheap*[*OF* *smart*]  
**have** *sheap: \bigwedge C sfs i. sh C = Some(sfs, i) \implies classes-above ?P C \subseteq cset\_s* **by**  
*simp*  
**moreover from** *jvm-smart-out-classes-above-xcpts*[*OF* *smart P t*]  
**have** *xcpts: classes-above-xcpts ?P \subseteq cset\_s* **by** *simp*  
**moreover from** *jvm-smart-out-classes-above-frames*[*OF* *smart*]  
**have** *frames: classes-above-frames ?P frs \subseteq cset\_s* **by** *simp*

— forward promises - will be collected after if not already  
**moreover from** *jvm-smart-out-classes-above-Test*[*OF smart P t*] *cic*  
**have** *init-class-prom*:  $\bigwedge C. ?ics = \text{Called } [] \vee ?ics = \text{No-ics}$   
 $\implies \text{coll-init-class } ?P ?i = \text{Some } C \implies \text{classes-above } ?P C \subseteq \text{cset}_s$  **by** *simp*  
**moreover have**  $\bigwedge C' Cs'. ?ics = \text{Calling } C' Cs' \implies \text{classes-above } ?P C' \subseteq \text{cset}_s$   
**by** *simp*  
— collections  
**moreover from** *naive*  
**have**  $n: (\sigma', \text{cset}_n) \in \text{JVMNaiveCollectionSemantics.cbig } ?P (xp, h, frs, sh)$  **by**  
*simp*  
**moreover from** *smart* **obtain**  $\text{cset}_s'$  **where**  
 $s: (\sigma', \text{cset}_s') \in \text{JVMSmartCollectionSemantics.cbig } ?P (xp, h, frs, sh)$  **and**  
 $\text{cset}_s' \subseteq \text{cset}_s$   
**by** *clarsimp*  
**ultimately show**  $\text{cset}_n \subseteq \text{cset}_s$  **by**(*rule jvm-naive-to-smart-cbig; simp*)  
**qed**

#### 12.6.4 Proving $\text{smart} \subseteq \text{naive}$

We prove that *jvm-naive* collects everything *jvm-smart* does. Combined with the other direction, this shows that the naive and smart algorithms collect the same set of classes.

**lemma** *jvm-smart-to-naive-exec-collect*:

*JVMexec-scollect P σ ⊆ JVMexec-ncollect P σ*

**proof** —

**obtain**  $xp\ h\ frs\ sh$  **where**  $\sigma: \sigma = (xp, h, frs, sh)$  **by**(*cases σ*)

**then show** *?thesis*

**proof**(*cases*  $\exists x. xp = \text{Some } x \vee frs = []$ )

**case** *False*

**then obtain**  $stk\ loc\ C\ M\ pc\ ics\ frs'$

**where** *none*:  $xp = \text{None}$  **and** *frs*:  $frs = (stk, loc, C, M, pc, ics) \# frs'$

**by**(*cases xp, auto, cases frs, auto*)

**have** *instr-case*:  $ics = \text{Called } [] \vee ics = \text{No-ics} \implies ?thesis$

**proof** —

**assume** *ics*:  $ics = \text{Called } [] \vee ics = \text{No-ics}$

**then show** *?thesis* **using**  $\sigma\ none\ frs$

**proof**(*cases curr-instr P (stk, loc, C, M, pc, ics)*) **qed**(*auto split: if-split-asm*)

**qed**

**then show** *?thesis* **using**  $\sigma\ none\ frs$

**proof**(*cases ics*)

**case**(*Called Cs*) **then show** *?thesis* **using** *instr-case σ none frs* **by**(*cases Cs, auto*)

**qed**(*auto*)

**qed**(*auto*)

**qed**

**lemma** *jvm-smart-to-naive-csmall*:

**assumes**  $(\sigma', \text{cset}_n) \in \text{JVMNaiveCollectionSemantics.csmall } P\ \sigma$

**and**  $(\sigma', \text{cset}_s) \in \text{JVMSmartCollectionSemantics.csmall } P\ \sigma$

**shows**  $cset_s \subseteq cset_n$   
**using** *jvm-smart-to-naive-exec-collect assms*  
**by** (*auto simp: JVMNaiveCollectionSemantics.csmall-def*  
*JVMSmartCollectionSemantics.csmall-def*)

**lemma** *jvm-smart-to-naive-csmall-nstep*:

$\llbracket (\sigma', cset_n) \in JVMNaiveCollectionSemantics.csmall-nstep\ P\ \sigma\ n;$   
 $(\sigma', cset_s) \in JVMSmartCollectionSemantics.csmall-nstep\ P\ \sigma\ n \rrbracket$   
 $\implies cset_s \subseteq cset_n$

**proof** (*induct n arbitrary:  $\sigma\ \sigma'\ cset_n\ cset_s$* )

**case** (*Suc n'*)

**obtain**  $\sigma 1\ cset 1\ cset'$  **where**  $\sigma 1$ :  $(\sigma 1, cset 1) \in JVMNaiveCollectionSemantics.csmall\ P\ \sigma$

$cset_n = cset 1 \cup cset'$   $(\sigma', cset') \in JVMNaiveCollectionSemantics.csmall-nstep\ P\ \sigma 1\ n'$

**using** *JVMNaiveCollectionSemantics.csmall-nstep-SucD [OF Suc.prem(1)]* **by** *clarsimp+*

**obtain**  $\sigma 1'\ cset 1'\ cset''$  **where**  $\sigma 1'$ :  $(\sigma 1', cset 1') \in JVMSmartCollectionSemantics.csmall\ P\ \sigma$

$cset_s = cset 1' \cup cset''$   $(\sigma', cset'') \in JVMSmartCollectionSemantics.csmall-nstep\ P\ \sigma 1'\ n'$

**using** *JVMSmartCollectionSemantics.csmall-nstep-SucD [OF Suc.prem(2)]* **by** *clarsimp+*

**have**  $\sigma\text{-eq}$ :  $\sigma 1 = \sigma 1'$  **using**  $\sigma 1(1)\ \sigma 1'(1)$  **by** (*simp add: JVMNaiveCollectionSemantics.csmall-def*

*JVMSmartCollectionSemantics.csmall-def*)

**then have**  $sub 1$ :  $cset 1' \subseteq cset 1$  **using**  $\sigma 1(1)\ \sigma 1'(1)$  *jvm-smart-to-naive-csmall* **by** *blast*

**have**  $sub 2$ :  $cset'' \subseteq cset'$  **using**  $\sigma 1(3)\ \sigma 1'(3)\ \sigma\text{-eq}\ Suc.hyps$  **by** *blast*

**then show** *?case* **using**  $\sigma 1(2)\ \sigma 1'(2)\ sub 1\ sub 2$  **by** *blast*

**qed** (*simp*)

**lemma** *jvm-smart-to-naive-cbig*:

**assumes**  $n$ :  $(\sigma', cset_n) \in JVMNaiveCollectionSemantics.cbig\ P\ \sigma$

**and**  $s$ :  $(\sigma', cset_s) \in JVMSmartCollectionSemantics.cbig\ P\ \sigma$

**shows**  $cset_s \subseteq cset_n$

**proof** –

**obtain**  $n$  **where**  $n'$ :  $(\sigma', cset_n) \in JVMNaiveCollectionSemantics.csmall-nstep\ P\ \sigma\ n\ \sigma' \in JVMendset$

**using** *JVMNaiveCollectionSemantics.cbig-def2 n* **by** *auto*

**obtain**  $s$  **where**  $s'$ :  $(\sigma', cset_s) \in JVMSmartCollectionSemantics.csmall-nstep\ P\ \sigma\ s\ \sigma' \in JVMendset$

**using** *JVMSmartCollectionSemantics.cbig-def2 s* **by** *auto*

**have**  $n=s$  **using** *jvm-naive-to-smart-csmall-nstep-last-eq [OF n n'(1) s'(1)]* **by** *simp*

**then show** *?thesis* **using** *jvm-smart-to-naive-csmall-nstep n'(1) s'(1)* **by** *blast*

**qed**

**lemma** *jvm-smart-to-naive-collection*:

**assumes** *naive*:  $(\sigma', cset_n) \in \text{jvm-naive-out } P t$  **and** *smart*:  $(\sigma', cset_s) \in \text{jvm-smart-out } P t$   
**and**  $P \in \text{jvm-progs}$  **and**  $t \in \text{jvm-tests}$   
**shows**  $cset_s \subseteq cset_n$   
**proof** –  
**have** *nend*:  $\text{start-state } (t\#P) \notin \text{JVMendset}$  **by** (*simp add*: *JVMendset-def start-state-def*)  
**from** *naive* **obtain** *n* **where**  
 $nstep$ :  $(\sigma', cset_n) \in \text{JVMNaiveCollectionSemantics.csmall-nstep}$   
 $(\text{jvm-make-test-prog } P t) (\text{start-state } (t\#P)) n$   
**by** (*auto dest!*: *JVMNaiveCollectionSemantics.cbigD*)  
**with** *nend naive* **obtain** *n'* **where**  $n' = \text{Suc } n'$   
**by** (*cases n*; *simp add*: *JVMNaiveCollectionSemantics.cbig-def*)  
**from** *start-prog-classes-above-Start*  
**have** *classes-above-frames*  $(\text{jvm-make-test-prog } P t) (\text{frames-of } (\text{start-state } (t\#P)))$   
 $= \{\text{Object}, \text{Start}\}$   
**by** (*simp add*: *start-state-def*)  
**with** *nstep n'*  
**have** *jvm-smart-collect-start*  $(\text{jvm-make-test-prog } P t) \subseteq cset_n$   
**by** (*auto simp*: *start-state-def JVMNaiveCollectionSemantics.csmall-def*  
 $\text{dest!}: \text{JVMNaiveCollectionSemantics.csmall-nstep-SucD}$   
 $\text{simp del}: \text{JVMNaiveCollectionSemantics.csmall-nstep-Rec}$ )  
**with** *jvm-smart-to-naive-cbig* [**where**  $P = \text{jvm-make-test-prog } P t$  **and**  $\sigma = \text{start-state } (t\#P)$  **and**  $\sigma' = \sigma$ ]  
 $\text{jvm-smart-collect-start-make-test-prog assms}$  **show** *?thesis* **by** *auto*  
**qed**

### 12.6.5 Safety of the smart algorithm

Having proved containment in both directions, we get *naive* = *smart*:

**lemma** *jvm-naive-eq-smart-collection*:

**assumes** *naive*:  $(\sigma', cset_n) \in \text{jvm-naive-out } P t$  **and** *smart*:  $(\sigma', cset_s) \in \text{jvm-smart-out } P t$

**and**  $P \in \text{jvm-progs}$  **and**  $t \in \text{jvm-tests}$

**shows**  $cset_n = cset_s$

**using** *jvm-naive-to-smart-collection* [*OF assms*] *jvm-smart-to-naive-collection* [*OF assms*] **by** *simp*

Thus, since the RTS algorithm based on *ncollect* is existence safe, the algorithm based on *scollect* is as well.

**theorem** *jvm-smart-existence-safe*:

**assumes**  $P: P \in \text{jvm-progs}$  **and**  $P': P' \in \text{jvm-progs}$  **and**  $t: t \in \text{jvm-tests}$

**and** *out*:  $o1 \in \text{jvm-smart-out } P t$  **and** *dss*:  $\text{jvm-deselect } P o1 P'$

**shows**  $\exists o2 \in \text{jvm-smart-out } P' t. o1 = o2$

**proof** –

**obtain**  $\sigma' cset_s$  **where**  $o1[\text{simp}]$ :  $o1 = (\sigma', cset_s)$  **by** (*cases o1*)

**with** *jvm-naive-iff-smart out* **obtain**  $cset_n$  **where**  $n$ :  $(\sigma', cset_n) \in \text{jvm-naive-out } P t$  **by** *blast*

```

from jvm-naive-eq-smart-collection[OF n - P t] out have eq: csetn = csets by
simp
with jvm-naive-existence-safe[OF P P' t n] dss have n': (σ', csetn) ∈ jvm-naive-out
P' t by simp
with jvm-naive-iff-smart obtain csets' where s': (σ', csets') ∈ jvm-smart-out
P' t by blast

from jvm-naive-eq-smart-collection[OF n' s' P' t] eq have csets = csets' by simp
then show ?thesis using s' by simp
qed

```

...thus *JVMSmartCollection* is an instance of *CollectionBasedRTS*:

```

interpretation JVMSmartCollectionRTS :
  CollectionBasedRTS (=) jvm-deselect jvm-progs jvm-tests
  JVMendset JVMcombine JVMcollect-id JVMsmall JVMSmartCollect jvm-smart-out
  jvm-make-test-prog jvm-smart-collect-start
by unfold-locales (rule jvm-smart-existence-safe, auto simp: start-state-def)

```

```

end
theory RTS
imports
  JVM-RTS/JVMCollectionBasedRTS
begin

end

```

## References

- [1] S. Mansky and E. L. Gunter. Safety of a smart classes-used regression test selection algorithm. *Electronic Notes in Theoretical Computer Science*, 351:51–73, 2020. Proceedings of LSFA 2020, the 15th International Workshop on Logical and Semantic Frameworks, with Applications (LSFA 2020).
- [2] S. E. Mansky. *Verified collection-based regression test selection via an extended Jinja semantics*. PhD thesis, University of Illinois at Urbana-Champaign, 2020.