

Promela Formalization

By René Neumann

May 26, 2024

Abstract

We present an executable formalization of the language Promela, the description language for models of the model checker SPIN. This formalization is part of the work for a completely verified model checker (CAVA), but also serves as a useful (and executable!) description of the semantics of the language itself, something that is currently missing. The formalization uses three steps: It takes an abstract syntax tree generated from an SML parser, removes syntactic sugar and enriches it with type information. This further gets translated into a transition system, on which the semantic engine (read: successor function) operates.

Contents

1	Introduction	3
2	Abstract Syntax Tree	4
3	Data structures as used in Promela	7
3.1	Abstract Syntax Tree <i>after</i> preprocessing	8
3.2	Preprocess the AST of the parser into our variant	10
3.3	The transition system	24
3.4	State	25
3.5	Printing	26
4	Invariants for Promela data structures	29
4.1	Bounds	29
4.2	Variables and similar	30
4.3	Invariants of a process	33
4.4	Invariants of the global state	35
4.5	Invariants of the program	38
5	Formalization of Promela semantics	39
5.1	Misc Helpers	39
5.2	Variable handling	41
5.3	Expressions	45
5.4	Variable declaration	48
5.5	Folding	53
5.6	Starting processes	55
5.7	AST to edges	62
5.7.1	Setup	66
5.8	Semantic Engine	70
5.8.1	Evaluation of Edges	70
5.8.2	Executable edges	78
5.8.3	Successor calculation	82
5.8.4	Handle non-termination	89
5.9	Finiteness of the state space	90
5.10	Traces	91
5.10.1	Printing of traces	92
5.11	Code export	93
6	LTL integration	93
6.1	LTL optimization	93
6.2	Language of a Promela program	97
6.3	Proposition types and conversion	100

1 Introduction

Promela [1] is a modeling language, mainly used in the model checker SPIN [2]. It offers a C-like syntax and allows to define processes to be run concurrently. Those processes can communicate via shared global variables or by message-passing via channels. Inside a process, constructs exist for non-deterministic choice, starting other processes and enforcing atomicity. It furthermore allows different means for specifying properties: LTL formulae, assertions in the code, never claims (i.e. an automata that explicitly specifies unwanted behavior) and others.

Some constructs found in Promela models, like `#include` and `#define`, are not part of the language Promela itself, but belong to the language of the C preprocessor. SPIN does not process those, but calls the C compiler internally to process them. We do not deal with them here, but also expect the sources to be preprocessed.

Observing the output of SPIN and examining the generated graphs often is the only way of determining the semantics of a certain construct. This is complicated further by SPIN unconditionally applying optimizations. For the current formalization we chose to copy the semantics of SPIN, including the aforementioned optimizations. For some constructs, we had to restrict the semantics, i.e. some models are accepted by SPIN, but not by this formalization. Those deviations are:

- `run` is a statement instead of an expression. SPIN here has a complicated set of restrictions unto where `run` can occur inside an expression. The sole use of it is to be able to get the ID of a spawned process. We omitted this feature to guarantee expressions to be free of side-effects.
- Variable declarations which got jumped over are seen as not existing. In SPIN, such constructs show surprising behavior:
`int i; goto L; i = 5; L: printf("%d", i)` yields 0, while
`goto L; int i = 5; L: printf("%d", i)` yields 5.
The latter is forbidden in our formalization (it will get rejected with “unknown variable i”), while the first behaves as in SPIN.
- Violating an `assert` does not abort, but instead sets the variable `__assert__` to true. This needs to be checked explicitly in the LTL formula. We plan on adding this check in an automatic manner.
- Types are bounded. Except for well-defined types like booleans, overflow is not allowed and will result in an error. The same holds for assigning a value that is outside the bounds. SPIN does not specify any explicit semantics here, but solely refers to the underlying C-compiler and its semantics. This might result in two models behaving differently on different systems when run with SPIN, while this formalization, due to the explicit bounds in the semantics, is not affected.

Additionally, some constructs are currently not supported, and the compilation will abort if they are encountered: `d_step`¹, `typedef`, remote references, bit-operations, `unsigned`, and property specifications except `ltl` and `assert`. Other constructs are accepted but ignored, because they do not change the behavior of a model: advanced variable scoping, `xr`, `xs`, `print*`, priorities, and visibility of variables.

Nonetheless, for models not using those unsupported constructs, we generate the very same number of states as SPIN does. An exception applies for large `goto` chains and when simultaneous termination of multiple processes is involved, as SPIN's semantics is too vague here.

2 Abstract Syntax Tree

```
theory PromelaAST
imports Main
begin
```

The abstract syntax tree is generated from the handwritten SML parser. This theory only mirrors the data structures from the SML level to make them available in Isabelle.

```
context
begin
```

```
local-setup <
  Local-Theory.map-background-naming (Name-Space.mandatory-path AST)
>
```

```
datatype binOp =
  | BinOpAdd
  | BinOpSub
  | BinOpMul
  | BinOpDiv
  | BinOpMod
  | BinOpBitAnd
  | BinOpBitXor
  | BinOpBitOr
  | BinOpGr
  | BinOpLe
  | BinOpGEq
  | BinOpLEq
  | BinOpEq
  | BinOpNEq
  | BinOpShiftL
  | BinOpShiftR
```

¹This can be safely replaced by `atomic`, though larger models will be produced then.

```

| BinOpAnd
| BinOpOr

datatype unOp =
  UnOpComp
| UnOpMinus
| UnOpNeg

datatype expr =
  ExprBinOp binOp expr expr
| ExprUnOp unOp expr
| ExprCond expr expr expr
| ExprLen varRef
| ExprPoll varRef recvArg list
| ExprRndPoll varRef recvArg list
| ExprVarRef varRef
| ExprConst integer
| ExprTimeOut
| ExprNP
| ExprEnabled expr
| ExprPC expr
| ExprRemoteRef String.literal
   expr option
   String.literal
| ExprGetPrio expr
| ExprSetPrio expr expr
| ExprFull varRef
| ExprEmpty varRef
| ExprNFull varRef
| ExprNEmpty varRef

and varRef = VarRef String.literal
   expr option
   varRef option

and recvArg = RecvArgVar varRef
| RecvArgEval expr
| RecvArgConst integer

datatype range =
  RangeFromTo varRef
   expr
   expr
| RangeIn varRef varRef

datatype varType =
  VarTypeBit
| VarTypeBool
| VarTypeByte

```

```

| VarTypePid
| VarTypeShort
| VarTypeInt
| VarTypeMType
| VarTypeChan
| VarTypeUnsigned
| VarTypeCustom String.literal

datatype varDecl =
  VarDeclNum String.literal
             integer option
             expr option
| VarDeclChan String.literal
             integer option
             (integer * varType list) option
| VarDeclUnsigned String.literal
             integer
             expr option
| VarDeclMType String.literal
             integer option
             String.literal option

datatype decl =
  Decl bool option
        varType
        varDecl list

datatype stmt =
  StmtIf (step list) list
| StmtDo (step list) list
| StmtFor range step list
| StmtAtomic step list
| StmtDStep step list
| StmtSelect range
| StmtSeq step list
| StmtSend varRef expr list
| StmtSortSend varRef expr list
| StmtRecv varRef recvArg list
| StmtRndRecv varRef recvArg list
| StmtRecvX varRef recvArg list
| StmtRndRecvX varRef recvArg list
| StmtAssign varRef expr
| StmtIncr varRef
| StmtDecr varRef
| StmtElse
| StmtBreak
| StmtGoTo String.literal

```

```

      | StmtntLabeled String.literal stmtnt
      | StmtntPrintf String.literal expr list
      | StmtntPrintM String.literal
      | StmtntRun String.literal
          expr list
          integer option
      | StmtntAssert expr
      | StmtntCond expr
      | StmtntCall String.literal varRef list

and step = StepStmtnt stmtnt stmtnt option
      | StepDecl decl
      | StepXR varRef list
      | StepXS varRef list

datatype module =
      ProcType (integer option) option
          String.literal
          decl list
          integer option
          expr option
          step list
      | DProcType (integer option) option
          String.literal
          decl list
          integer option
          expr option
          step list
      | Init integer option step list
      | Never step list
      | Trace step list
      | NoTrace step list
      | Inline String.literal String.literal list step list
      | TypeDef String.literal decl list
      | MType String.literal list
      | ModuDecl decl
      | Ltl String.literal String.literal

end
end

```

3 Data structures as used in Promela

```

theory PromelaDatastructures
imports
  CAVA-Base.CAVA-Base
  CAVA-Base.Lexord-List
  PromelaAST
  HOL-Library.IArray

```

Deriving.Compare-Instances
CAVA-Base.CAVA-Code-Target
begin

3.1 Abstract Syntax Tree *after* preprocessing

From the plain AST stemming from the parser, we'd like to have one containing more information while also removing duplicated constructs. This is achieved in the preprocessing step.

The additional information contains:

- variable type (including whether it represents a channel or not)
- global vs local variable

Also certain constructs are expanded (like for-loops) or different nodes in the AST are collapsed into one parametrized node (e.g. the different send-operations).

This preprocessing phase also tries to detect certain static errors and will bail out with an exception if such is encountered.

```
datatype binOp = BinOpAdd
    | BinOpSub
    | BinOpMul
    | BinOpDiv
    | BinOpMod
    | BinOpGr
    | BinOpLe
    | BinOpGEq
    | BinOpLEq
    | BinOpEq
    | BinOpNEq
    | BinOpAnd
    | BinOpOr

datatype unOp = UnOpMinus
    | UnOpNeg

datatype expr = ExprBinOp binOp expr expr
    | ExprUnOp unOp expr
    | ExprCond expr expr expr
    | ExprLen chanRef
    | ExprVarRef varRef
    | ExprConst integer
    | ExprMConst integer String.literal
    | ExprTimeOut
    | ExprFull chanRef
    | ExprEmpty chanRef
    | ExprPoll chanRef recvArg list bool
```



```

and varRef = VarRef bool
               String.literal
               expr option
and chanRef = ChanRef varRef — explicit type for channels
and recvArg = RecvArgVar varRef
               | RecvArgEval expr
               | RecvArgConst integer
               | RecvArgMConst integer String.literal

datatype varType = VTBounded integer integer
                  | VTChan

```

Variable declarations at the beginning of a proctype or at global level.

```

datatype varDecl = VarDeclNum integer integer
                   String.literal
                   integer option
                   expr option
                   | VarDeclChan String.literal
                   integer option
                   (integer * varType list) option

```

Variable declarations during a proctype.

```

datatype procVarDecl = ProcVarDeclNum integer integer
                      String.literal
                      integer option
                      expr option
                      | ProcVarDeclChan String.literal
                      integer option

```

```

datatype procArg = ProcArg varType String.literal

```

```

datatype stmnt = StmntIf (step list) list
                 | StmntDo (step list) list
                 | StmntAtomic step list
                 | StmntSeq step list
                 | StmntSend chanRef expr list bool
                 | StmntRecv chanRef recvArg list bool bool
                 | StmntAssign varRef expr
                 | StmntElse
                 | StmntBreak
                 | StmntSkip
                 | StmntGoTo String.literal
                 | StmntLabeled String.literal stmnt
                 | StmntRun String.literal
                   expr list
                 | StmntCond expr
                 | StmntAssert expr

```

```

and step = StepStmnt stmnt stmnt option
      | StepDecl proc VarDecl list
      | StepSkip

datatype proc = ProcType (integer option) option
                String.literal
                procArg list
                varDecl list
                step list
      | Init varDecl list step list

type-synonym ltl = — name: String.literal × — formula: String.literal
type-synonym promela = varDecl list × proc list × ltl list

```

3.2 Preprocess the AST of the parser into our variant

We setup some functionality for printing warning or even errors.

All those constants are logically *undefined*, but replaced by the parser for something meaningful.

consts

```
warn :: String.literal ⇒ unit
```

abbreviation *with-warn msg e* \equiv let - = warn msg in e

abbreviation *the-warn opt msg* \equiv case opt of None \Rightarrow () | - \Rightarrow warn msg

usc: "Unsupported Construct"

definition [code del]: *usc* (c :: String.literal) \equiv undefined

definition [code del]: *err* (e :: String.literal) = undefined

abbreviation *errv e v* \equiv err (e + v)

definition [simp, code del]: *abort* (msg :: String.literal) f = f ()

abbreviation *abortv msg v f* \equiv abort (msg + v) f

code-printing

```

code-module PromelaUtils  $\rightarrow$  (SML)  $\langle$ 
  structure PromelaUtils = struct
    exception UnsupportedConstruct of string
    exception StaticError of string
    exception RuntimeError of string
    fun warn msg = TextIO.print (Warning: ^msg ^\n)
    fun usc c = raise (UnsupportedConstruct c)
    fun err e = raise (StaticError e)
    fun abort msg - = raise (RuntimeError msg)
  end $\rangle$ 
| constant warn  $\rightarrow$  (SML) PromelaUtils.warn
| constant usc  $\rightarrow$  (SML) PromelaUtils.usc
| constant err  $\rightarrow$  (SML) PromelaUtils.err

```

| **constant** *abort* \rightarrow (*SML*) *PromelaUtils.abort*
code-reserved *SML PromelaUtils*

The preprocessing is done for each type on its own.

```
primrec ppBinOp :: AST.binOp  $\Rightarrow$  binOp
where
  ppBinOp AST.BinOpAdd = BinOpAdd
| ppBinOp AST.BinOpSub = BinOpSub
| ppBinOp AST.BinOpMul = BinOpMul
| ppBinOp AST.BinOpDiv = BinOpDiv
| ppBinOp AST.BinOpMod = BinOpMod
| ppBinOp AST.BinOpGr = BinOpGr
| ppBinOp AST.BinOpLe = BinOpLe
| ppBinOp AST.BinOpGEq = BinOpGEq
| ppBinOp AST.BinOpLEq = BinOpLEq
| ppBinOp AST.BinOpEq = BinOpEq
| ppBinOp AST.BinOpNEq = BinOpNEq
| ppBinOp AST.BinOpAnd = BinOpAnd
| ppBinOp AST.BinOpOr = BinOpOr
| ppBinOp AST.BinOpBitAnd = usc STR "BinOpBitAnd"
| ppBinOp AST.BinOpBitXor = usc STR "BinOpBitXor"
| ppBinOp AST.BinOpBitOr = usc STR "BinOpBitOr"
| ppBinOp AST.BinOpShiftL = usc STR "BinOpShiftL"
| ppBinOp AST.BinOpShiftR = usc STR "BinOpShiftR"
```

```
primrec ppUnOp :: AST.unOp  $\Rightarrow$  unOp
where
  ppUnOp AST.UnOpMinus = UnOpMinus
| ppUnOp AST.UnOpNeg = UnOpNeg
| ppUnOp AST.UnOpComp = usc STR "UnOpComp"
```

The data structure holding all information on variables we found so far.

```
type-synonym var-data =
  (String.literal, (integer option  $\times$  bool)) lm — channels
   $\times$  (String.literal, (integer option  $\times$  bool)) lm — variables
   $\times$  (String.literal, integer) lm — mtotypes
   $\times$  (String.literal, varRef) lm — aliases (used for inlines)
```

```
definition dealWithVar
  :: var-data  $\Rightarrow$  String.literal
   $\Rightarrow$  (String.literal  $\Rightarrow$  integer option  $\times$  bool  $\Rightarrow$  expr option  $\Rightarrow$  'a)
   $\Rightarrow$  (String.literal  $\Rightarrow$  integer option  $\times$  bool  $\Rightarrow$  expr option  $\Rightarrow$  'a)
   $\Rightarrow$  (integer  $\Rightarrow$  'a)  $\Rightarrow$  'a
where
  dealWithVar cvm n fC fV fM  $\equiv$  (
    let (c,v,m,a) = cvm in
    let (n, idx) = case lm.lookup n a of
```

```

        None ⇒ (n, None)
      | Some (VarRef - name idx) ⇒ (name, idx)
in
case lm.lookup n m of
  Some i ⇒ (case idx of None ⇒ fM i
                    | - ⇒ err STR "Array subscript used on MType (via alias).")
| None ⇒ (case lm.lookup n v of
          Some g ⇒ fV n g idx
          | None ⇒ (case lm.lookup n c of
                    Some g ⇒ fC n g idx
                    | None ⇒ err (STR "Unknown variable referenced: " + n))))

primrec enforceChan :: varRef + chanRef ⇒ chanRef where
  enforceChan (Inl -) = err STR "Channel expected. Got normal variable."
| enforceChan (Inr c) = c

fun liftChan :: varRef + chanRef ⇒ varRef where
  liftChan (Inl v) = v
| liftChan (Inr (ChanRef v)) = v

fun resolveIdx :: expr option ⇒ expr option ⇒ expr option
where
  resolveIdx None None = None
| resolveIdx idx None = idx
| resolveIdx None aliasIdx = aliasIdx
| resolveIdx - - = err STR "Array subscript used twice (via alias)."

fun ppExpr :: var-data ⇒ AST.expr ⇒ expr
and ppVarRef :: var-data ⇒ AST.varRef ⇒ varRef + chanRef
and ppRecvArg :: var-data ⇒ AST.recvArg ⇒ recvArg
where
  ppVarRef cvm (AST.VarRef name idx None) = dealWithVar cvm name
    (λname (-,g) aIdx. let idx = map-option (ppExpr cvm) idx in
      Inr (ChanRef (VarRef g name (resolveIdx idx aIdx))))
  (λname (-,g) aIdx. let idx = map-option (ppExpr cvm) idx in
    Inl (VarRef g name (resolveIdx idx aIdx)))
  (λ-. err STR "Variable expected. Got MType.")
| ppVarRef cvm (AST.VarRef - - (Some -)) =
  usc STR "next operation on variables"

| ppExpr cvm AST.ExprTimeOut = ExprTimeOut
| ppExpr cvm (AST.ExprConst c) = ExprConst c

| ppExpr cvm (AST.ExprBinOp bo l r) =
  ExprBinOp (ppBinOp bo) (ppExpr cvm l) (ppExpr cvm r)
| ppExpr cvm (AST.ExprUnOp uo e) =
  ExprUnOp (ppUnOp uo) (ppExpr cvm e)
| ppExpr cvm (AST.ExprCond c t f) =
  ExprCond (ppExpr cvm c) (ppExpr cvm t) (ppExpr cvm f)

```

```

| ppExpr cvm (AST.ExprLen v) =
  ExprLen (enforceChan (ppVarRef cvm v))
| ppExpr cvm (AST.ExprFull v) =
  ExprFull (enforceChan (ppVarRef cvm v))
| ppExpr cvm (AST.ExprEmpty v) =
  ExprEmpty (enforceChan (ppVarRef cvm v))

| ppExpr cvm (AST.ExprNFull v) =
  ExprUnOp UnOpNeg (ExprFull (enforceChan (ppVarRef cvm v)))
| ppExpr cvm (AST.ExprNEmpty v) =
  ExprUnOp UnOpNeg (ExprEmpty (enforceChan (ppVarRef cvm v)))

| ppExpr cvm (AST.ExprVarRef v) = (
  let to-exp = λ-. ExprVarRef (liftChan (ppVarRef cvm v)) in
  case v of
    AST.VarRef name None None ⇒
      dealWithVar cvm name
        (λ- - -. to-exp())
        (λ- - -. to-exp())
        (λi. ExprMConst i name)
    | - ⇒ to-exp())

| ppExpr cvm (AST.ExprPoll v es) =
  ExprPoll (enforceChan (ppVarRef cvm v)) (map (ppRecvArg cvm) es) False
| ppExpr cvm (AST.ExprRndPoll v es) =
  ExprPoll (enforceChan (ppVarRef cvm v)) (map (ppRecvArg cvm) es) True

| ppExpr cvm AST.ExprNP = usc STR "ExprNP"
| ppExpr cvm (AST.ExprEnabled -) = usc STR "ExprEnabled"
| ppExpr cvm (AST.ExprPC -) = usc STR "ExprPC"
| ppExpr cvm (AST.ExprRemoteRef - - -) = usc STR "ExprRemoteRef"
| ppExpr cvm (AST.ExprGetPrio -) = usc STR "ExprGetPrio"
| ppExpr cvm (AST.ExprSetPrio - -) = usc STR "ExprSetPrio"

| ppRecvArg cvm (AST.RecvArgVar v) = (
  let to-ra = λ-. RecvArgVar (liftChan (ppVarRef cvm v)) in
  case v of
    AST.VarRef name None None ⇒
      dealWithVar cvm name
        (λ- - -. to-ra())
        (λ- - -. to-ra())
        (λi. RecvArgMConst i name)
    | - ⇒ to-ra())
| ppRecvArg cvm (AST.RecvArgEval e) = RecvArgEval (ppExpr cvm e)
| ppRecvArg cvm (AST.RecvArgConst c) = RecvArgConst c

primrec ppVarType :: AST.varType ⇒ varType where
  ppVarType AST.VarTypeBit = VTBounded 0 1

```

```

| ppVarType AST.VarTypeBool = VTBounded 0 1
| ppVarType AST.VarTypeByte = VTBounded 0 255
| ppVarType AST.VarTypePid = VTBounded 0 255
| ppVarType AST.VarTypeShort = VTBounded  $-(2^{15})$   $((2^{15}) - 1)$ 
| ppVarType AST.VarTypeInt = VTBounded  $-(2^{31})$   $((2^{31}) - 1)$ 
| ppVarType AST.VarTypeMType = VTBounded 1 255
| ppVarType AST.VarTypeChan = VTChan
| ppVarType AST.VarTypeUnsigned = usc STR "VarTypeUnsigned"
| ppVarType (AST.VarTypeCustom _) = usc STR "VarTypeCustom"

fun ppVarDecl
  :: var-data  $\Rightarrow$  varType  $\Rightarrow$  bool  $\Rightarrow$  AST.varDecl  $\Rightarrow$  var-data  $\times$  varDecl
where
  ppVarDecl (c,v,m,a) (VTBounded l h) g
    (AST.VarDeclNum name size init) = (
      case lm.lookup name v of
        Some -  $\Rightarrow$  errv STR "Duplicate variable " name
      | -  $\Rightarrow$  (case lm.lookup name a of
        Some -  $\Rightarrow$  errv
          STR "Variable name clashes with alias: " name
        | -  $\Rightarrow$  ((c, lm.update name (size,g) v, m, a),
          VarDeclNum l h name size
            (map-option (ppExpr (c,v,m,a)) init))))
  | ppVarDecl - - g (AST.VarDeclNum name size init) =
    err STR "Assiging num to non-num"

  | ppVarDecl (c,v,m,a) VTChan g
    (AST.VarDeclChan name size cap) = (
      let cap' = map-option (apsnd (map ppVarType)) cap in
      case lm.lookup name c of
        Some -  $\Rightarrow$  errv STR "Duplicate variable " name
      | -  $\Rightarrow$  (case lm.lookup name a of
        Some -  $\Rightarrow$  errv
          STR "Variable name clashes with alias: " name
        | -  $\Rightarrow$  ((lm.update name (size, g) c, v, m, a),
          VarDeclChan name size cap'))
  | ppVarDecl - - g (AST.VarDeclChan name size init) =
    err STR "Assiging chan to non-chan"

  | ppVarDecl (c,v,m,a) (VTBounded l h) g
    (AST.VarDeclMType name size init) = (
      let init = map-option ( $\lambda$ mty.
        case lm.lookup mty m of
          None  $\Rightarrow$  errv STR "Unknown MType " mty
        | Some mval  $\Rightarrow$  ExprMConst mval mty) init in
      case lm.lookup name c of
        Some -  $\Rightarrow$  errv STR "Duplicate variable " name
      | -  $\Rightarrow$  (case lm.lookup name a of Some -
         $\Rightarrow$  errv STR "Variable name clashes with alias: " name

```

```

| - => ((c, lm.update name (sze,g) v, m, a),
        VarDeclNum l h name sze init)))

| ppVarDecl - - g (AST.VarDeclMType name sze init) =
  err STR "Assiging num to non-num"

| ppVarDecl - - - (AST.VarDeclUnsigned - - -) =
  usc STR "VarDeclUnsigned"

definition ppProcVarDecl
  :: var-data => varType => bool => AST.varDecl => var-data × procVarDecl
where
  ppProcVarDecl cvm ty g v = (case ppVarDecl cvm ty g v of
    (cvm, VarDeclNum l h name sze init) => (cvm, ProcVarDeclNum l h name
  sze init)
    | (cvm, VarDeclChan name sze None) => (cvm, ProcVarDeclChan name sze)
    | - => err STR "Channel initilizations only allowed at the beginning of proc-
  types.")

fun ppProcArg
  :: var-data => varType => bool => AST.varDecl => var-data × procArg
where
  ppProcArg (c,v,m,a) (VTBounded l h) g
    (AST.VarDeclNum name None None) = (
    case lm.lookup name v of
      Some - => errv STR "Duplicate variable " name
    | - => (case lm.lookup name a of
      Some - => errv
        STR "Variable name clashes with alias: " name
      | - => ((c, lm.update name (None, g) v, m, a),
        ProcArg (VTBounded l h) name)))
  | ppProcArg - - - (AST.VarDeclNum - - -) =
    err STR "Invalid proctype arguments"

  | ppProcArg (c,v,m,a) VTChan g
    (AST.VarDeclChan name None None) = (
    case lm.lookup name c of
      Some - => errv STR "Duplicate variable " name
    | - => (case lm.lookup name a of
      Some - => errv
        STR "Variable name clashes with alias: " name
      | - => ((lm.update name (None, g) c, v, m, a), ProcArg VTChan name)))
  | ppProcArg - - - (AST.VarDeclChan - - -) =
    err STR "Invalid proctype arguments"

  | ppProcArg (c,v,m,a) (VTBounded l h) g
    (AST.VarDeclMType name None None) = (
    case lm.lookup name v of
      Some - => errv STR "Duplicate variable " name

```

```

| - => (case lm.lookup name a of
  Some - => errv
    STR "Variable name clashes with alias: " name
  | - => ((c, lm.update name (None, g) v, m, a),
    ProcArg (VTBounded l h) name)))
| ppProcArg - - - (AST.VarDeclMType - - -) =
  err STR "Invalid proctype arguments"

| ppProcArg - - - (AST.VarDeclUnsigned - - -) = usc STR "VarDeclUnsigned"

```

Some preprocessing functions enrich the *var-data* argument and hence return a new updated one. When chaining multiple calls to such functions after another, we need to make sure, the *var-data* is passed accordingly. *cvm-fold* does exactly that for such a function *g* and a list of nodes *ss*.

definition *cvm-fold* **where**

```

cvm-fold g cvm ss = foldl (λ(cvm,ss) s. apsnd (λs'. ss@[s']) (g cvm s))
  (cvm, []) ss

```

lemma *cvm-fold-cong*[*fundef-cong*]:

```

assumes cvm = cvm'
and stepss = stepss'
and ∧x d. x ∈ set stepss ⇒ g d x = g' d x
shows cvm-fold g cvm stepss = cvm-fold g' cvm' stepss'

```

unfolding *cvm-fold-def*

using *assms*

by (*fastforce* *intro*: *foldl-cong* *split*: *prod.split*)

fun *liftDecl* **where**

```

liftDecl f g cvm (AST.Decl vis t decls) = (
  let - = the-warn vis STR "Visibility in declarations not supported. Ignored." in
  let t = ppVarType t in
  cvm-fold (λcvm. f cvm t g) cvm decls)

```

definition *ppDecl*

```

:: bool ⇒ var-data ⇒ AST.decl ⇒ var-data × varDecl list

```

where

```

ppDecl = liftDecl ppVarDecl

```

definition *ppDeclProc*

```

:: var-data ⇒ AST.decl ⇒ var-data × procVarDecl list

```

where

```

ppDeclProc = liftDecl ppProcVarDecl False

```

definition *ppDeclProcArg*

```

:: var-data ⇒ AST.decl ⇒ var-data × procArg list

```

where

```

ppDeclProcArg = liftDecl ppProcArg False

```


definition *incr* :: *varRef* \Rightarrow *stmtnt* **where**
incr *v* = *StmntAssign* *v* (*ExprBinOp* *BinOpAdd* (*ExprVarRef* *v*) (*ExprConst* 1))

definition *decr* :: *varRef* \Rightarrow *stmtnt* **where**
decr *v* = *StmntAssign* *v* (*ExprBinOp* *BinOpSub* (*ExprVarRef* *v*) (*ExprConst* 1))

Transforms **for** (*i* : *lb* .. *ub*) **steps** into

```
{
  i = lb;
  do
    :: i =< ub -> steps; i++
    :: else -> break
  od
}
```

definition *forFromTo* :: *varRef* \Rightarrow *expr* \Rightarrow *expr* \Rightarrow *step list* \Rightarrow *stmtnt* **where**
forFromTo *i* *lb* *ub* *steps* = (
 let
 — *i* = *lb*
loop-pre = *StepStmnt* (*StmntAssign* *i* *lb*) *None*;
 — *i* \leq *ub*
loop-cond = *StepStmnt* (*StmntCond*
 (*ExprBinOp* *BinOpLEq* (*ExprVarRef* *i*) *ub*))
None;
 — *i*++
loop-incr = *StepStmnt* (*incr* *i*) *None*;
 — *i* \leq *ub* \rightarrow ...; *i*++
loop-body = *loop-cond* # *steps* @ [*loop-incr*];
 — *else* \rightarrow *break*
loop-abort = [*StepStmnt* *StmntElse* *None*, *StepStmnt* *StmntBreak* *None*];
 — *do* :: *i* \leq *ub* \rightarrow ... :: *else* \rightarrow *break* *od*
loop = *StepStmnt* (*StmntDo* [*loop-body*, *loop-abort*]) *None*
 in
StmntSeq [*loop-pre*, *loop*])

Transforms (where *a* is an array with *N* entries) **for** (*i* in *a*) **steps** into

```
{
  i = 0;
  do
    :: i < N -> steps; i++
    :: else -> break
  od
}
```

definition *forInArray* :: *varRef* \Rightarrow *integer* \Rightarrow *step list* \Rightarrow *stmtnt* **where**

```

forInArray i N steps = (
  let
    — i = 0
    loop-pre = StepStmnt (StmntAssign i (ExprConst 0)) None;
    — i < N
    loop-cond = StepStmnt (StmntCond
                          (ExprBinOp BinOpLe (ExprVarRef i)
                          (ExprConst N)))
                          None;

    — i++
    loop-incr = StepStmnt (incr i) None;
    — i < N -> ...; i++
    loop-body = loop-cond # steps @ [loop-incr];
    — else -> break
    loop-abort = [StepStmnt StmntElse None, StepStmnt StmntBreak None];
    — do :: i < N -> ... :: else -> break od
    loop = StepStmnt (StmntDo [loop-body, loop-abort]) None
  in
    StmntSeq [loop-pre, loop])

```

Transforms (where *c* is a channel) **for (msg in c) steps** into

```

{
  byte :tmp: = 0;
  do
    :: :tmp: < len(c) ->
      c?msg; c!msg;
      steps;
      :tmp:++
    :: else -> break
  od
}

```

definition *forInChan* :: *varRef* ⇒ *chanRef* ⇒ *step list* ⇒ *stmnt* **where**

```

forInChan msg c steps = (
  let
    — byte :tmp: = 0
    tmpStr = STR ":tmp:";
    loop-pre = StepDecl
              [ProcVarDeclNum 0 255 tmpStr None (Some (ExprConst 0))];
    tmp = VarRef False tmpStr None;
    — :tmp: < len(c)
    loop-cond = StepStmnt (StmntCond
                          (ExprBinOp BinOpLe (ExprVarRef tmp)
                          (ExprLen c)))
                          None;

    — :tmp:++
    loop-incr = StepStmnt (incr tmp) None;

```

```

— c?msg
recv = StepStmnt (StmntRecv c [RecvArgVar msg] False True) None;
— c!msg
send = StepStmnt (StmntSend c [ExprVarRef msg] False) None;
— :tmp: < len(c) -> c?msg; c!msg; ...; :tmp:++
loop-body = [loop-cond, recv, send] @ steps @ [loop-incr];
— else -> break
loop-abort = [StepStmnt StmntElse None, StepStmnt StmntBreak None];
— do :: :tmp: < len(c) -> ... :: else -> break od
loop = StepStmnt (StmntDo [loop-body, loop-abort]) None
in
  StmntSeq [loop-pre, loop])

```

Transforms `select (i : lb .. ub)` into

```

{
  i = lb;
do
  :: i < ub -> i++
  :: break
od
}

```

definition `select :: varRef \Rightarrow expr \Rightarrow expr \Rightarrow stmnt` **where**

```

select i lb ub = (
  let
    — i = lb
    pre = StepStmnt (StmntAssign i lb) None;
    — i < ub
    cond = StepStmnt (StmntCond (ExprBinOp BinOpLe (ExprVarRef i) ub))
              None;
    — i++
    incr = StepStmnt (incr i) None;
    — i < ub -> i++
    loop-body = [cond, incr];
    — break
    loop-abort = [StepStmnt StmntBreak None];
    — do :: i < ub -> ... :: break od
    loop = StepStmnt (StmntDo [loop-body, loop-abort]) None
  in
    StmntSeq [pre, loop])

```

type-synonym `inlines =`

`(String.literal, String.literal list \times (var-data \Rightarrow var-data \times step list)) lm`

type-synonym `stmnt-data =`

`bool \times varDecl list \times inlines \times var-data`

fun `ppStep :: stmnt-data \Rightarrow AST.step \Rightarrow stmnt-data \times step`

and `ppStmnt :: stmnt-data \Rightarrow AST.stmnt \Rightarrow stmnt-data \times stmnt`

where

```

ppStep cvm (AST.StepStmnt s u) = (
  let (cvm', s') = ppStmnt cvm s in
  case u of None => (cvm', StepStmnt s' None)
    | Some u => let (cvm'', u') = ppStmnt cvm' u in
      (cvm'', StepStmnt s' (Some u'))
| ppStep (False, ps, i, cvm) (AST.StepDecl d) =
  map-prod (λcvm. (False, ps, i, cvm)) StepDecl (ppDeclProc cvm d)
| ppStep (True, ps, i, cvm) (AST.StepDecl d) = (
  let (cvm', ps') = ppDecl False cvm d
  in ((True, ps@ps', i, cvm'), StepSkip))
| ppStep (-, cvm) (AST.StepXR -) =
  with-warn STR "StepXR not supported. Ignored." ((False, cvm), StepSkip)
| ppStep (-, cvm) (AST.StepXS -) =
  with-warn STR "StepXS not supported. Ignored." ((False, cvm), StepSkip)

| ppStmnt (-, cvm) (AST.StmntBreak) = ((False, cvm), StmntBreak)
| ppStmnt (-, cvm) (AST.StmntElse) = ((False, cvm), StmntElse)
| ppStmnt (-, cvm) (AST.StmntGoTo l) = ((False, cvm), StmntGoTo l)
| ppStmnt (-, cvm) (AST.StmntLabeled l s) =
  apsnd (StmntLabeled l) (ppStmnt (False, cvm) s)
| ppStmnt (-, ps, i, cvm) (AST.StmntCond e) =
  ((False, ps, i, cvm), StmntCond (ppExpr cvm e))
| ppStmnt (-, ps, i, cvm) (AST.StmntAssert e) =
  ((False, ps, i, cvm), StmntAssert (ppExpr cvm e))
| ppStmnt (-, ps, i, cvm) (AST.StmntAssign v e) =
  ((False, ps, i, cvm), StmntAssign (liftChan (ppVarRef cvm v)) (ppExpr cvm e))
| ppStmnt (-, ps, i, cvm) (AST.StmntSend v es) =
  ((False, ps, i, cvm), StmntSend (enforceChan (ppVarRef cvm v))
    (map (ppExpr cvm) es) False)
| ppStmnt (-, ps, i, cvm) (AST.StmntSortSend v es) =
  ((False, ps, i, cvm), StmntSend (enforceChan (ppVarRef cvm v))
    (map (ppExpr cvm) es) True)
| ppStmnt (-, ps, i, cvm) (AST.StmntRecv v rs) =
  ((False, ps, i, cvm), StmntRecv (enforceChan (ppVarRef cvm v))
    (map (ppRecvArg cvm) rs) False True)
| ppStmnt (-, ps, i, cvm) (AST.StmntRecvX v rs) =
  ((False, ps, i, cvm), StmntRecv (enforceChan (ppVarRef cvm v))
    (map (ppRecvArg cvm) rs) False False)
| ppStmnt (-, ps, i, cvm) (AST.StmntRndRecv v rs) =
  ((False, ps, i, cvm), StmntRecv (enforceChan (ppVarRef cvm v))
    (map (ppRecvArg cvm) rs) True True)
| ppStmnt (-, ps, i, cvm) (AST.StmntRndRecvX v rs) =
  ((False, ps, i, cvm), StmntRecv (enforceChan (ppVarRef cvm v))
    (map (ppRecvArg cvm) rs) True False)
| ppStmnt (-, ps, i, cvm) (AST.StmntRun n es p) = (
  let - = the-warn p STR "Priorities for 'run' not supported. Ignored." in
  ((False, ps, i, cvm), StmntRun n (map (ppExpr cvm) es)))
| ppStmnt (-, cvm) (AST.StmntSeq ss) =

```

```

    apsnd StmtntSeq (cvm-fold ppStep (False,cvm) ss)
| ppStmtnt (-,cvm) (AST.StmntAtomic ss) =
    apsnd StmtntAtomic (cvm-fold ppStep (False,cvm) ss)
| ppStmtnt (-,cvm) (AST.StmntIf sss) =
    apsnd StmtntIf (cvm-fold (cvm-fold ppStep) (False,cvm) sss)
| ppStmtnt (-,cvm) (AST.StmntDo sss) =
    apsnd StmtntDo (cvm-fold (cvm-fold ppStep) (False,cvm) sss)

| ppStmtnt (-,ps,i,cvm) (AST.StmntIncr v) =
    ((False,ps,i,cvm), incr (liftChan (ppVarRef cvm v)))
| ppStmtnt (-,ps,i,cvm) (AST.StmntDecr v) =
    ((False,ps,i,cvm), decr (liftChan (ppVarRef cvm v)))

| ppStmtnt (-,cvm) (AST.StmntPrintF -) =
    with-warn STR "PrintF ignored" ((False,cvm), StmtntSkip)
| ppStmtnt (-,cvm) (AST.StmntPrintM -) =
    with-warn STR "PrintM ignored" ((False,cvm), StmtntSkip)

| ppStmtnt (-,ps,inl,cvm) (AST.StmntFor
    (AST.RangeFromTo i lb ub)
    steps) = (
    let
        i = liftChan (ppVarRef cvm i);
        (lb,ub) = (ppExpr cvm lb, ppExpr cvm ub)
    in
        apsnd (forFromTo i lb ub) (cvm-fold ppStep (False,ps,inl,cvm) steps))
| ppStmtnt (-,ps,inl,cvm) (AST.StmntFor
    (AST.RangeIn i v)
    steps) = (
    let
        i = liftChan (ppVarRef cvm i);
        (cvm',steps) = cvm-fold ppStep (False,ps,inl,cvm) steps
    in
        case ppVarRef cvm v of
            Inr c ⇒ (cvm', forInChan i c steps)
        | Inl (VarRef - (Some -)) ⇒ err STR "Iterating over array-member."
        | Inl (VarRef - name None) ⇒ (
            let (-,v,-) = cvm in
            case fst (the (lm.lookup name v)) of
                None ⇒ err STR "Iterating over non-array variable."
                | Some N ⇒ (cvm', forInArray i N steps)))

| ppStmtnt (-,ps,inl,cvm) (AST.StmntSelect
    (AST.RangeFromTo i lb ub)) = (
    let
        i = liftChan (ppVarRef cvm i);
        (lb, ub) = (ppExpr cvm lb, ppExpr cvm ub)
    in

```

```

      ((False,ps,inl,cvm), select i lb ub))
| ppStmnt (-,cvm) (AST.StmntSelect (AST.RangeIn - -)) =
  err STR "in not allowed in select"

| ppStmnt (-,ps,inl,cvm) (AST.StmntCall macro args) = (
  let
    args = map (liftChan ∘ ppVarRef cvm) args;
    (c,v,m,a) = cvm
  in
    case lm.lookup macro inl of
      None ⇒ errv STR "Calling unknown macro " macro
    | Some (names,sF) ⇒
      if length names ≠ length args then
        (err STR "Called macro with wrong number of arguments.")
      else
        let a' = foldl (λa (k,v). lm.update k v a) a (zip names args) in
        let ((c,v,m,-),steps) = sF (c,v,m,a') in
        ((False,ps,inl,c,v,m,a), StmntSeq steps))

| ppStmnt cvm (AST.StmntDStep -) = usc STR "StmntDStep"

fun ppModule
  :: var-data × inlines ⇒ AST.module
  ⇒ var-data × inlines × (varDecl list + proc + ltl)
where
  ppModule (cvm, inl) (AST.ProcType act name args prio prov steps) = (
    let
      - = the-warn prio STR "Priorities for procs not supported. Ignored.";
      - = the-warn prov STR "Prio (??) for procs not supported. Ignored.";
      (cvm', args) = cvm-fold ppDeclProcArg cvm args;
      ((-, vars, -, -), steps) = cvm-fold ppStep (True,[],inl,cvm') steps
    in
      (cvm, inl, Inr (Inl (ProcType act name (concat args) vars steps))))

| ppModule (cvm,inl) (AST.Init prio steps) = (
  let - = the-warn prio STR "Priorities for procs not supported. Ignored." in
  let ((-, vars, -, -), steps) = cvm-fold ppStep (True,[],inl,cvm) steps in
  (cvm, inl, Inr (Inl (Init vars steps))))

| ppModule (cvm,inl) (AST.Ltl name formula) =
  (cvm, inl, Inr (Inr (name, formula)))

| ppModule (cvm,inl) (AST.ModuDecl decl) =
  apsnd (λds. (inl,Inl ds)) (ppDecl True cvm decl)

| ppModule (cvm,inl) (AST.MType mtys) = (
  let (c,v,m,a) = cvm in
  let num = integer-of-nat (lm.size m) + 1 in
  let (m',-) = foldr (λmty (m,num).

```

```

      let m' = lm.update mty num m
      in (m', num+1)) mtys (m, num)
in
  ((c, v, m', a), inl, Inl [])

| ppModule (cvm, inl) (AST.Inline name args steps) = (
  let stepF = (λcvm. let ((-, -, cvm), steps) =
    cvm-fold ppStep (False, [], inl, cvm) steps
    in (cvm, steps))
  in let inl = lm.update name (args, stepF) inl
  in (cvm, inl, Inl []))

| ppModule cvm (AST.DProcType - - - -) = usc STR "DProcType"
| ppModule cvm (AST.Never -) = usc STR "Never"
| ppModule cvm (AST.Trace -) = usc STR "Trace"
| ppModule cvm (AST.NoTrace -) = usc STR "NoTrace"
| ppModule cvm (AST.TypeDef - -) = usc STR "TypeDef"

definition preprocess :: AST.module list ⇒ promela where
  preprocess ms = (
    let
      dflt-vars = [(STR "-pid", (None, False)),
        (STR "--assert--", (None, True)),
        (STR "--", (None, True))];
      cvm = (lm.empty(), lm.to-map dflt-vars, lm.empty(), lm.empty());
      (-, -, pr) = (foldl (λ(cvm, inl, vs, ps, ls) m.
        let (cvm', inl', m') = ppModule (cvm, inl) m in
        case m' of
          Inl v ⇒ (cvm', inl', vs@v, ps, ls)
        | Inr (Inl p) ⇒ (cvm', inl', vs, ps@[p], ls)
        | Inr (Inr l) ⇒ (cvm', inl', vs, ps, ls@[l]))
        (cvm, lm.empty(), [], [], []) ms)
    in
      pr)

fun extractLTL
  :: AST.module ⇒ ltl option
where
  extractLTL (AST.Ltl name formula) = Some (name, formula)
| extractLTL - = None

primrec extractLTLs
  :: AST.module list ⇒ (String.literal, String.literal) lm
where
  extractLTLs [] = lm.empty()
| extractLTLs (m#ms) = (case extractLTL m of
  None ⇒ extractLTLs ms
  | Some (n, f) ⇒ lm.update n f (extractLTLs ms))

```

definition *lookupLTL*

:: AST.module list \Rightarrow String.literal \Rightarrow String.literal option

where *lookupLTL ast k = lm.lookup k (extractLTLs ast)*

3.3 The transition system

The edges in our transition system consist of a condition (evaluated under the current environment) and an effect (modifying the current environment). Further they may be atomic, i. e. a whole row of such edges is taken before yielding a new state. Additionally, they carry a priority: the edges are checked from highest to lowest priority, and if one edge on a higher level can be taken, the lower levels are ignored.

The states of the system do not carry any information.

datatype *edgeCond = ECElse*

| *ECTrue*
 | *ECFalse*
 | *ECExpr expr*
 | *ECRun String.literal*
 | *ECSend chanRef*
 | *ECRecv chanRef recvArg list bool*

datatype *edgeEffect = EEEnd*

| *EEId*
 | *EEGoto*
 | *EEAssert expr*
 | *EEAssign varRef expr*
 | *EEDecl procVarDecl*
 | *EERun String.literal expr list*
 | *EESend chanRef expr list bool*
 | *EERecv chanRef recvArg list bool bool*

datatype *edgeIndex = Index nat | LabelJump String.literal nat option*

datatype *edgeAtomic = NonAtomic | Atomic | InAtomic*

record *edge =*

cond :: edgeCond
effect :: edgeEffect
target :: edgeIndex
prio :: integer
atomic :: edgeAtomic

definition *isAtomic :: edge \Rightarrow bool where*

isAtomic e = (case atomic e of Atomic \Rightarrow True | - \Rightarrow False)

definition *inAtomic :: edge \Rightarrow bool where*

inAtomic e = (case atomic e of NonAtomic \Rightarrow False | - \Rightarrow True)

3.4 State

datatype *variable* = *Var varType integer*
 | *VArray varType nat integer iarray*

datatype *channel* = *Channel integer varType list integer list list*
 | *HSChannel varType list*
 | *InvChannel*

type-synonym *var-dict* = (*String.literal, variable*) *lm*

type-synonym *labels* = (*String.literal, nat*) *lm*

type-synonym *lts* = (*String.literal, String.literal*) *lm*

type-synonym *states* = ($\text{— prio: integer} \times \text{edge list}$) *iarray*

type-synonym *channels* = *channel list*

type-synonym *process* =

nat — offset

\times *edgeIndex* — start

\times *procArg list* — args

\times *varDecl list* — top decls

record *program* =

processes :: *process iarray*

labels :: *labels iarray*

states :: *states iarray*

proc-names :: *String.literal iarray*

proc-data :: (*String.literal, nat*) *lm*

record *pState* = — State of a process

pid :: *nat* — Process identifier

vars :: *var-dict* — Dictionary of variables

pc :: *nat* — Program counter

channels :: *integer list* — List of channels created in the process. Used to close them on finalization.

idx :: *nat* — Offset into the arrays of *program*

hide-const (**open**) *idx*

record *gState* = — Global state

vars :: *var-dict* — Global variables

channels :: *channels* — Channels are by construction part of the global state, even when created in a process.

timeout :: *bool* — Set to True if no process can take a transition.

procs :: *pState list* — List of all running processes. A process is removed from it, when there is no running one with a higher index.

record *gState_I* = *gState* + — Additional internal infos

handshake :: *nat*

hsdata :: *integer list* — Data transferred via a handshake.

exclusive :: *nat* — Set to the PID of the process, which is in an exclusive (=

atomic) state.

else :: *bool* — Set to True for each process, if it can not take a transition.
Used before timeout.

3.5 Printing

primrec *printBinOp* :: *binOp* \Rightarrow *string* **where**

```

  printBinOp BinOpAdd = "+"
| printBinOp BinOpSub = "-"
| printBinOp BinOpMul = "*"
| printBinOp BinOpDiv = "/"
| printBinOp BinOpMod = "mod"
| printBinOp BinOpGr = ">"
| printBinOp BinOpLe = "<"
| printBinOp BinOpGEq = ">="
| printBinOp BinOpLEq = "<="
| printBinOp BinOpEq = "=="
| printBinOp BinOpNEq = "!="
| printBinOp BinOpAnd = "&&"
| printBinOp BinOpOr = "||"

```

primrec *printUnOp* :: *unOp* \Rightarrow *string* **where**

```

  printUnOp UnOpMinus = "-"
| printUnOp UnOpNeg = "!"

```

definition *printList* :: (*a* \Rightarrow *string*) \Rightarrow '*a* list \Rightarrow *string* \Rightarrow *string* \Rightarrow *string* \Rightarrow *string*

where

```

  printList f xs l r sep = (
    let f' = ( $\lambda$ str x. if str = [] then f x
                  else str @ sep @ f x)
    in l @ (foldl f' [] xs) @ r)

```

lemma *printList-cong* [*fundef-cong*]:

```

assumes xs = xs'
and l = l'
and r = r'
and sep = sep'
and  $\bigwedge x. x \in \text{set } xs \implies f x = f' x$ 
shows printList f xs l r sep = printList f' xs' l' r' sep'
unfolding printList-def
using assms
by (auto intro: foldl-cong)

```

fun *printExpr* :: (*integer* \Rightarrow *string*) \Rightarrow *expr* \Rightarrow *string*

and *printFun* :: (*integer* \Rightarrow *string*) \Rightarrow *string* \Rightarrow *chanRef* \Rightarrow *string*

and *printVarRef* :: (*integer* \Rightarrow *string*) \Rightarrow *varRef* \Rightarrow *string*

and *printChanRef* :: (*integer* \Rightarrow *string*) \Rightarrow *chanRef* \Rightarrow *string*

and *printRecvArg* :: (*integer* \Rightarrow *string*) \Rightarrow *recvArg* \Rightarrow *string* **where**

```

    printExpr f ExprTimeOut = "timeout"
| printExpr f (ExprBinOp binOp left right) =
    printExpr f left @ " " @ printBinOp binOp @ " " @ printExpr f right
| printExpr f (ExprUnOp unOp e) = printUnOp unOp @ printExpr f e
| printExpr f (ExprVarRef varRef) = printVarRef f varRef
| printExpr f (ExprConst i) = f i
| printExpr f (ExprMConst i m) = String.explode m
| printExpr f (ExprCond c l r) =
    "(" @ printExpr f c @ " " @ printExpr f l @ " " @ printExpr f r @ " " @ ")"
| printExpr f (ExprLen chan) = printFun f "len" chan
| printExpr f (ExprEmpty chan) = printFun f "empty" chan
| printExpr f (ExprFull chan) = printFun f "full" chan
| printExpr f (ExprPoll chan es srt) = (
    let p = if srt then "???" else "???" in
    printChanRef f chan @ p
    @ printList (printRecvArg f) es "[" "]" " ", ")")

| printVarRef - (VarRef - name None) = String.explode name
| printVarRef f (VarRef - name (Some indx)) =
    String.explode name @ "[" @ printExpr f indx @ "]"

| printChanRef f (ChanRef v) = printVarRef f v

| printFun f fun var = fun @ "(" @ printChanRef f var @ ")"

| printRecvArg f (RecvArgVar v) = printVarRef f v
| printRecvArg f (RecvArgConst c) = f c
| printRecvArg f (RecvArgMConst - m) = String.explode m
| printRecvArg f (RecvArgEval e) = "eval(" @ printExpr f e @ ")"

fun printVarDecl :: (integer ⇒ string) ⇒ procVarDecl ⇒ string where
    printVarDecl f (ProcVarDeclNum - - n None None) =
        String.explode n @ " = 0"
| printVarDecl f (ProcVarDeclNum - - n None (Some e)) =
    String.explode n @ " = " @ printExpr f e
| printVarDecl f (ProcVarDeclNum - - n (Some i) None) =
    String.explode n @ "[" @ f i @ "]" = 0
| printVarDecl f (ProcVarDeclNum - - n (Some i) (Some e)) =
    String.explode n @ "[" @ f i @ "]" = " @ printExpr f e
| printVarDecl f (ProcVarDeclChan n None) =
    "chan " @ String.explode n
| printVarDecl f (ProcVarDeclChan n (Some i)) =
    "chan " @ String.explode n @ "[" @ f i @ "]"

primrec printCond :: (integer ⇒ string) ⇒ edgeCond ⇒ string where
    printCond f ECElse = "else"
| printCond f ECTrue = "true"

```

```

| printCond f ECFalse = "false"
| printCond f (ECRun n) = "run " @ String.explode n @ "(...)"
| printCond f (ECEExpr e) = printExpr f e
| printCond f (ECSend c) = printChanRef f c @ "! ..."
| printCond f (ECRecv c -) = printChanRef f c @ "? ..."

```

primrec printEffect :: (integer \Rightarrow string) \Rightarrow edgeEffect \Rightarrow string **where**

```

  printEffect f EEEnd = "-- end --"
| printEffect f EEId = "ID"
| printEffect f EEGoto = "goto"
| printEffect f (EEAssert e) = "assert(" @ printExpr f e @ ")"
| printEffect f (EERun n -) = "run " @ String.explode n @ "(...)"
| printEffect f (EEAssign v expr) =
  printVarRef f v @ " = " @ printExpr f expr
| printEffect f (EEDecl d) = printVarDecl f d
| printEffect f (EESend v es srt) = (
  let s = if srt then "!" else "" in
  printChanRef f v @ s @ printList (printExpr f) es "(" " " " ", ")
| printEffect f (EERecv v rs srt rem) = (
  let p = if srt then "??" else "?" in
  let (l,r) = if rem then "(" " " else "<", ">" in
  printChanRef f v @ p @ printList (printRecvArg f) rs l r " ", ")

```

primrec printIndex :: (integer \Rightarrow string) \Rightarrow edgeIndex \Rightarrow string **where**

```

  printIndex f (Index pos) = f (integer-of-nat pos)
| printIndex - (LabelJump l -) = String.explode l

```

definition printEdge :: (integer \Rightarrow string) \Rightarrow nat \Rightarrow edge \Rightarrow string **where**

```

  printEdge f indx e = (
    let
      tStr = printIndex f (target e);
      pStr = if prio e < 0 then " Prio: " @ f (prio e) else [];
      atom = if isAtomic e then  $\lambda x. x @ "\{A\}"$  else id;
      pEff =  $\lambda -. atom (printEffect f (effect e))$ ;
      contStr = case (cond e) of
        ECTrue  $\Rightarrow$  pEff ()
      | ECFalse  $\Rightarrow$  pEff ()
      | ECSend -  $\Rightarrow$  pEff ()
      | ECRecv - -  $\Rightarrow$  pEff ()
      | -  $\Rightarrow atom ("(" @ printCond f (cond e) @ ")")$ 
    in
    f (integer-of-nat indx) @ " ----> " @ tStr @ " => " @ contStr @ pStr)

```

definition printEdges :: (integer \Rightarrow string) \Rightarrow states \Rightarrow string list **where**

```

  printEdges f es = concat (map ( $\lambda n. map (printEdge f n) (snd (es !! n))$ )
    (rev [0.. $IArray.length$  es]))

```

definition printLabels :: (integer \Rightarrow string) \Rightarrow labels \Rightarrow string list **where**

```

  printLabels f ls = lm.iterate ls ( $\lambda(k,l) res.$ 

```

```

      ("Label " @ String.explode k @ ": "
       @ f (integer-of-nat l)) # res) []

fun printProcesses :: (integer ⇒ string) ⇒ program ⇒ string list where
  printProcesses f prog = lm.iterate (proc-data prog)
    (λ(k,idx) res.
      let (-,start,-) = processes prog !! idx in
      [] # ("Process " @ String.explode k) # [] # printEdges f (states prog !!
idx)
      @ ["START ---> " @ printIndex f start, []]
      @ printLabels f (labels prog !! idx) @ res) []
end

```

4 Invariants for Promela data structures

```

theory PromelaInvariants
imports PromelaDatastructures
begin

```

The different data structures used in the Promela implementation require different invariants, which are specified in this file. As there is no (useful) way of specifying *correctness* of the implementation, those invariants are tailored towards proving the finiteness of the generated state-space.

4.1 Bounds

Finiteness requires that possible variable ranges are finite, as is the maximum number of processes. Currently, they are supplied here as constants. In a perfect world, they should be able to be set dynamically.

```

definition min-var-value :: integer where
  min-var-value = -(231)
definition max-var-value :: integer where
  max-var-value = (231) - 1

```

```

lemma min-max-var-value-simps [simp, intro!]:
  min-var-value < max-var-value
  min-var-value < 0
  min-var-value ≤ 0
  max-var-value > 0
  max-var-value ≥ 0
by (simp-all add: min-var-value-def max-var-value-def)

```

```

definition max-procs ≡ 255
definition max-channels ≡ 65535
definition max-array-size = 65535

```

4.2 Variables and similar

```

fun varType-inv :: varType  $\Rightarrow$  bool where
  varType-inv (VTBounded l h)
     $\longleftrightarrow l \geq \text{min-var-value} \wedge h \leq \text{max-var-value} \wedge l < h$ 
| varType-inv VTChan  $\longleftrightarrow$  True

fun variable-inv :: variable  $\Rightarrow$  bool where
  variable-inv (Var t val)
     $\longleftrightarrow \text{varType-inv } t \wedge \text{val} \in \{\text{min-var-value}..\text{max-var-value}\}$ 
| variable-inv (VArray t sz ar)
     $\longleftrightarrow \text{varType-inv } t$ 
       $\wedge \text{sz} \leq \text{max-array-size}$ 
       $\wedge \text{IArray.length } ar = \text{sz}$ 
       $\wedge \text{set } (\text{IArray.list-of } ar) \subseteq \{\text{min-var-value}..\text{max-var-value}\}$ 

fun channel-inv :: channel  $\Rightarrow$  bool where
  channel-inv (Channel cap ts q)
     $\longleftrightarrow \text{cap} \leq \text{max-array-size}$ 
       $\wedge \text{cap} \geq 0$ 
       $\wedge \text{set } ts \subseteq \text{Collect varType-inv}$ 
       $\wedge \text{length } ts \leq \text{max-array-size}$ 
       $\wedge \text{length } q \leq \text{max-array-size}$ 
       $\wedge (\forall x \in \text{set } q. \text{length } x = \text{length } ts$ 
       $\wedge \text{set } x \subseteq \{\text{min-var-value}..\text{max-var-value}\})$ 
| channel-inv (HChannel ts)
     $\longleftrightarrow \text{set } ts \subseteq \text{Collect varType-inv} \wedge \text{length } ts \leq \text{max-array-size}$ 
| channel-inv InvChannel  $\longleftrightarrow$  True

lemma varTypes-finite:
  finite (Collect varType-inv)
proof (rule finite-subset)
  show Collect (varType-inv)  $\subseteq$ 
    {VTChan}
     $\cup (\lambda(l,h). \text{VTBounded } l \ h)$ 
    ' ( $\{\text{min-var-value}..\text{max-var-value}\} \times \{\text{min-var-value}..\text{max-var-value}\}$ )
  apply (rule subsetI)
  apply (case-tac x)
  apply auto
  done

  show finite ... by auto
qed

lemma variables-finite:
  finite (Collect variable-inv)
proof (rule finite-subset)
  let ?mm =  $\{\text{min-var-value}..\text{max-var-value}\}$ 
  let ?V1 =  $(\lambda(t, \text{val}). \text{Var } t \ \text{val})$  ' ( $\{\text{vt}. \text{varType-inv } vt\} \times ?mm$ )
  let ?V2 =  $(\lambda(t, \text{sz}, ar). \text{VArray } t \ \text{sz} \ ar)$ 

```

```

    ' ({vt. varType-inv vt}
      × {0..max-array-size}
      × {ar. IArray.length ar ≤ max-array-size
        ∧ set (IArray.list-of ar) ⊆ ?mm})

{
  fix A :: 'a set
  let ?LS = {xs. set xs ⊆ A ∧ length xs ≤ max-array-size }
  let ?AS = {ar. IArray.length ar ≤ max-array-size
    ∧ set (IArray.list-of ar) ⊆ A}

  assume finite A
  hence finite ?LS by (simp add: finite-lists-length-le)
  moreover have ?AS ⊆ IArray ' ?LS
    apply (auto simp: image-def)
    apply (rule-tac x = IArray.list-of x in exI)
    apply auto
    apply (metis iarray.exhaust list-of.simps)
    done
  ultimately have finite ?AS by (auto simp add: finite-subset)
} note finite-arr = this

show Collect variable-inv ⊆ (?V1 ∪ ?V2)
  apply (rule subsetI)
  apply (case-tac x)
  apply (auto simp add: image-def)
  done

show finite ... by (blast intro: varTypes-finite finite-arr)
qed

lemma channels-finite:
  finite (Collect channel-inv)
proof (rule finite-subset)
  let ?C1 =
    (λ(cap,ts,q). Channel cap ts q)
    ' ({0..max-array-size}
      × {ts. set ts ⊆ Collect varType-inv ∧ length ts ≤ max-array-size}
      × {q. set q ⊆ {x. set x ⊆ {min-var-value..max-var-value}
        ∧ length x ≤ max-array-size}
        ∧ length q ≤ max-array-size})
  let ?C2 =
    HSChannel ' {ts. set ts ⊆ Collect varType-inv ∧ length ts ≤ max-array-size}
  let ?C3 = {InvChannel}

show (Collect channel-inv) ⊆ ?C1 ∪ ?C2 ∪ ?C3
  apply (rule subsetI)
  apply (case-tac x)
  apply (auto simp add: image-def)

```

done

show *finite ...* **by** (*blast intro: finite-lists-length-le varTypes-finite*)
qed

To give an upper bound of variable names, we need a way to calculate it.

primrec *procArgName* :: *procArg* \Rightarrow *String.literal* **where**
procArgName (*ProcArg* - *name*) = *name*

primrec *varDeclName* :: *varDecl* \Rightarrow *String.literal* **where**
varDeclName (*VarDeclNum* - - *name* - -) = *name*
| *varDeclName* (*VarDeclChan* *name* - -) = *name*

primrec *procVarDeclName* :: *procVarDecl* \Rightarrow *String.literal* **where**
procVarDeclName (*ProcVarDeclNum* - - *name* - -) = *name*
| *procVarDeclName* (*ProcVarDeclChan* *name* -) = *name*

definition *edgeDecls* :: *edge* \Rightarrow *procVarDecl set* **where**
edgeDecls *e* = (
 case effect e of
 EEDecl p \Rightarrow {*p*}
 | - \Rightarrow {})

lemma *edgeDecls-finite*:
 finite (edgeDecls e)
by (*simp add: edgeDecls-def split: edgeEffect.split*)

definition *edgeSet* :: *states* \Rightarrow *edge set* **where**
edgeSet *s* = *set (concat (map snd (IArray.list-of s)))*

lemma *edgeSet-finite*:
 finite (edgeSet s)
by (*simp add: edgeSet-def*)

definition *statesDecls* :: *states* \Rightarrow *procVarDecl set* **where**
statesDecls *s* = \bigcup (*edgeDecls* ‘ (*edgeSet* *s*))

definition *statesNames* :: *states* \Rightarrow *String.literal set* **where**
statesNames *s* = *procVarDeclName* ‘ *statesDecls* *s*

lemma *statesNames-finite*:
 finite (statesNames s)
by (*simp add: edgeSet-finite edgeDecls-finite statesNames-def statesDecls-def*)

fun *process-names* :: *states* \Rightarrow *process* \Rightarrow *String.literal set* **where**
process-names *ss* (-, -, *args*, *decls*) =
 statesNames *ss*
 \cup *procArgName* ‘ *set args*

$\cup \text{varDeclName } \text{' set decls}$
 $\cup \{ \text{STR } \text{'-'}, \text{STR } \text{'--assert--'}, \text{STR } \text{'-pid'} \}$

lemma *process-names-finite*:
 $\text{finite } (\text{process-names } ss \ p)$
by (*cases* p) (*simp add: statesNames-finite*)

definition *vardict-inv* :: $\text{states} \Rightarrow \text{process} \Rightarrow \text{var-dict} \Rightarrow \text{bool}$ **where**
 $\text{vardict-inv } ss \ p \ vs$
 $\longleftrightarrow \text{lm.ball } vs \ (\lambda(k,v). k \in \text{process-names } ss \ p \wedge \text{variable-inv } v)$

lemma *vardicts-finite*:
 $\text{finite } (\text{Collect } (\text{vardict-inv } ss \ p))$

proof –
have $\text{Assoc-List.set } \text{' Collect } (\text{vardict-inv } ss \ p) \subseteq$
 $\text{Pow } (\text{process-names } ss \ p \times \{v. \text{variable-inv } v\})$
by (*auto simp add: lm-ball-Assoc-List-set vardict-inv-def*)

moreover have *finite ...*
using *process-names-finite variables-finite*
by *simp*
ultimately show *?thesis* **by** (*metis finite-Assoc-List-set-image finite-subset*)
qed

lemma *lm-to-map-vardict-inv*:
assumes $\forall (k,v) \in \text{set } xs. k \in \text{process-names } ss \ \text{proc} \wedge \text{variable-inv } v$
shows $\text{vardict-inv } ss \ \text{proc} \ (\text{lm.to-map } xs)$
using *assms*
unfolding *vardict-inv-def*
by (*auto simp add: lm.correct dest: map-of-SomeD*)

4.3 Invariants of a process

definition *pState-inv* :: $\text{program} \Rightarrow \text{pState} \Rightarrow \text{bool}$ **where**
 $\text{pState-inv } \text{prog } p$
 $\longleftrightarrow \text{pid } p \leq \text{max-procs}$
 $\wedge \text{pState.idx } p < \text{IArray.length } (\text{states } \text{prog})$
 $\wedge \text{IArray.length } (\text{states } \text{prog}) = \text{IArray.length } (\text{processes } \text{prog})$
 $\wedge \text{pc } p < \text{IArray.length } ((\text{states } \text{prog}) \text{ !! } \text{pState.idx } p)$
 $\wedge \text{set } (\text{pState.channels } p) \subseteq \{-1..<\text{integer-of-nat } \text{max-channels}\}$
 $\wedge \text{length } (\text{pState.channels } p) \leq \text{max-channels}$
 $\wedge \text{vardict-inv } ((\text{states } \text{prog}) \text{ !! } \text{pState.idx } p)$
 $\quad ((\text{processes } \text{prog}) \text{ !! } \text{pState.idx } p)$
 $\quad (\text{pState.vars } p)$

lemma *pStates-finite*:
 $\text{finite } (\text{Collect } (\text{pState-inv } \text{prog}))$
proof –
let $?P1 = \{.. \text{max-procs} :: \text{nat}\}$

```

let ?P2 = {..IArray.length (states prog)}
let ?P3 = {..Max (IArray.length ‘(set (IArray.list-of (states prog))))}
let ?P4 = {cs.set cs  $\subseteq$  { $-1..<\text{integer-of-nat } \text{max-channels}$ }
            $\wedge$  length cs  $\leq$  max-channels}
let ?P5 =  $\bigcup_{x \in \{..IArray.length (states prog)\}}$ 
           Collect (vardict-inv (states prog !! x) (processes prog !! x))
let ?P = ?P1  $\times$  ?P2  $\times$  ?P3  $\times$  ?P4  $\times$  ?P5

have {p. pState-inv prog p}  $\subseteq$ 
  ( $\lambda(\text{pid}, \text{idx}, \text{pc}, \text{channels}, \text{vars}). \text{pState.make pid vars pc channels idx}$ ) ‘ ?P
unfolding pState-inv-def image-def [of - ?P]
apply (clarsimp simp add: pState.defs)
apply (tactic ‘Record.split-simp-tac @{context} [] (K ~ 1) 1’)
apply auto
apply (rule order-trans [OF less-imp-le])
apply (auto intro!: Max-ge)
done
moreover
have finite ?P4 by (fastforce intro: finite-lists-length-le)
hence finite ?P by (auto intro: finite-cartesian-product simp: vardicts-finite)

ultimately show ?thesis by (elim finite-subset) (rule finite-imageI)
qed

```

Throughout the calculation of the semantic engine, a modified process is not necessarily part of *procs g*. Hence we need to establish an additional constraint for the relation between a global and a process state.

definition *cl-inv* :: (*'a gState-scheme * pState*) \Rightarrow *bool* **where**
cl-inv gp = (*case gp of* (*g,p*) \Rightarrow
length (*pState.channels p*) \leq *length* (*gState.channels g*))

lemma *cl-inv-lengthD*:
cl-inv (*g,p*) \Longrightarrow *length* (*pState.channels p*) \leq *length* (*gState.channels g*)
unfolding *cl-inv-def*
by *auto*

lemma *cl-invI*:
length (*pState.channels p*) \leq *length* (*gState.channels g*) \Longrightarrow *cl-inv* (*g,p*)
unfolding *cl-inv-def* **by** *auto*

lemma *cl-inv-trans*:
length (*channels g*) \leq *length* (*channels g'*) \Longrightarrow *cl-inv* (*g,p*) \Longrightarrow *cl-inv* (*g',p*)
by (*simp add: cl-inv-def*)

lemma *cl-inv-vars-update[intro!]*:
cl-inv (*g,p*) \Longrightarrow *cl-inv* (*g, pState.vars-update vs p*)
cl-inv (*g,p*) \Longrightarrow *cl-inv* (*gState.vars-update vs g, p*)
by (*simp-all add: cl-inv-def*)

lemma *cl-inv-handshake-update*[intro!]:
 $cl_inv\ (g,p) \implies cl_inv\ (g[handsake := h],p)$
by (*simp add: cl-inv-def*)

lemma *cl-inv-hsdata-update*[intro!]:
 $cl_inv\ (g,p) \implies cl_inv\ (g[hsdata := h],p)$
by (*simp add: cl-inv-def*)

lemma *cl-inv-procs-update*[intro!]:
 $cl_inv\ (g,p) \implies cl_inv\ (g[procs := ps],p)$
by (*simp add: cl-inv-def*)

lemma *cl-inv-channels-update*:
assumes *cl-inv* (*g,p*)
shows *cl-inv* (*gState.channels-update* ($\lambda cs. cs[i:=c]$) *g*, *p*)
using *assms unfolding cl-inv-def*
by *simp*

4.4 Invariants of the global state

Note that *gState-inv* must be defined in a way to be applicable to both *gState* and *gState_I*.

definition *gState-inv* :: *program* \Rightarrow 'a *gState-scheme* \Rightarrow *bool* **where**
 $gState_inv\ prog\ g$
 $\iff length\ (procs\ g) \leq max_procs$
 $\wedge (\forall p \in set\ (procs\ g). pState_inv\ prog\ p \wedge cl_inv\ (g,p))$
 $\wedge length\ (channels\ g) \leq max_channels$
 $\wedge set\ (channels\ g) \subseteq Collect\ channel_inv$
 $\wedge lm.ball\ (vars\ g)\ (\lambda(k,v). variable_inv\ v)$

The set of global states adhering to the terms of *gState-inv* is not finite. But the set of all global states that can be constructed by the semantic engine from one starting state is. Thus we establish a progress relation, i.e. all successors of a state *g* relate to *g* under this specification.

definition *gState-progress-rel* :: *program* \Rightarrow ('a *gState-scheme*) *rel* **where**
 $gState_progress_rel\ p = \{(g,g').\ gState_inv\ p\ g \wedge gState_inv\ p\ g'$
 $\wedge length\ (channels\ g) \leq length\ (channels\ g')$
 $\wedge dom\ (lm.\alpha\ (vars\ g)) = dom\ (lm.\alpha\ (vars\ g'))\}$

lemma *gState-progress-rel-gState-invI1*[intro]:
 $(g,g') \in gState_progress_rel\ prog \implies gState_inv\ prog\ g$
by (*simp add: gState-progress-rel-def*)

lemma *gState-progress-rel-gState-invI2*[intro]:
 $(g,g') \in gState_progress_rel\ prog \implies gState_inv\ prog\ g'$
by (*simp add: gState-progress-rel-def*)

lemma *gState-progress-relI*:

```

assumes  $gState\text{-}inv\ prog\ g$ 
and  $gState\text{-}inv\ prog\ g'$ 
and  $length\ (channels\ g) \leq length\ (channels\ g')$ 
and  $dom\ (lm.\alpha\ (vars\ g)) = dom\ (lm.\alpha\ (vars\ g'))$ 
shows  $(g, g') \in gState\text{-}progress\text{-}rel\ prog$ 
unfolding  $gState\text{-}progress\text{-}rel\text{-}def$ 
using  $assms$ 
by  $auto$ 

```

```

lemma  $gState\text{-}progress\text{-}refl[simp, intro!]$ :
 $gState\text{-}inv\ prog\ g \implies (g, g) \in (gState\text{-}progress\text{-}rel\ prog)$ 
unfolding  $gState\text{-}progress\text{-}rel\text{-}def$ 
by  $auto$ 

```

```

lemma  $refl\text{-}on\text{-}gState\text{-}progress\text{-}rel$ :
 $refl\text{-}on\ (Collect\ (gState\text{-}inv\ prog))\ (gState\text{-}progress\text{-}rel\ prog)$ 
by  $(auto\ intro!\!: refl\text{-}onI)$ 

```

```

lemma  $trans\text{-}gState\text{-}progress\text{-}rel[simp]$ :
 $trans\ (gState\text{-}progress\text{-}rel\ prog)$ 
by  $(intro\ transI)\ (simp\ add!\: gState\text{-}progress\text{-}rel\text{-}def)$ 

```

```

lemmas  $gState\text{-}progress\text{-}rel\text{-}trans\ [trans] = trans\text{-}gState\text{-}progress\text{-}rel[THEN\ transD]$ 

```

```

lemma  $gState\text{-}progress\text{-}rel\text{-}trancl\text{-}id[simp]$ :
 $(gState\text{-}progress\text{-}rel\ prog)^+ = gState\text{-}progress\text{-}rel\ prog$ 
by  $simp$ 

```

```

lemma  $gState\text{-}progress\text{-}rel\text{-}rtrancl\text{-}absorb$ :
assumes  $gState\text{-}inv\ prog\ g$ 
shows  $(gState\text{-}progress\text{-}rel\ prog)^* \text{ `` } \{g\} = gState\text{-}progress\text{-}rel\ prog \text{ `` } \{g\}$ 
using  $assms\ refl\text{-}on\text{-}gState\text{-}progress\text{-}rel$ 
by  $(intro\ Image\text{-}absorb\text{-}rtrancl)\ auto$ 

```

The main theorem: The set of all global states reachable from an initial state, is finite.

```

lemma  $gStates\text{-}finite$ :
fixes  $g :: gState$ 
shows  $finite\ ((gState\text{-}progress\text{-}rel\ prog)^* \text{ `` } \{g\})$ 
proof  $(cases\ gState\text{-}inv\ prog\ g)$ 
case  $False$  hence  $(gState\text{-}progress\text{-}rel\ prog)^* \text{ `` } \{g\} = \{g\}$ 
by  $(intro\ Image\text{-}empty\text{-}rtrancl\text{-}Image\text{-}id)$ 
 $(auto\ simp\ add!\: gState\text{-}progress\text{-}rel\text{-}def)$ 
thus  $?thesis$  by  $simp$ 
next
case  $True$ 
let  $?G1 = \{m.\ dom\ (lm.\alpha\ m) = dom\ (lm.\alpha\ (vars\ g))$ 
 $\wedge\ ran\ (lm.\alpha\ m) \subseteq Collect\ variable\text{-}inv\ \}$ 
let  $?G2 = \{cs.\ set\ cs \subseteq Collect\ channel\text{-}inv\$ 

```

```

       $\wedge \text{length } cs \leq \text{max-channels}\}$ 
let ?G3 = {True, False}
let ?G4 = {ps. set ps  $\subseteq$  Collect (pState-inv prog)
       $\wedge \text{length } ps \leq \text{max-procs}\}$ 

let ?G = ?G1  $\times$  ?G2  $\times$  ?G3  $\times$  ?G4
let ?G' = ( $\lambda(\text{vars}, \text{chans}, t, ps)$ ). gState.make vars chans t ps) ‘ ?G

have G1: finite ?G1
proof (rule finite-subset)
  show ?G1  $\subseteq$  {v'. fst ‘ Assoc-List.set v' = fst ‘ Assoc-List.set (vars g)
     $\wedge$  snd ‘ Assoc-List.set v'  $\subseteq$  Collect variable-inv}
  by (simp add: dom-lm- $\alpha$ -Assoc-List-set ran-lm- $\alpha$ -Assoc-List-set)
show finite ... (is finite ?X)
proof (rule finite-Assoc-List-set-image, rule finite-subset)
  show Assoc-List.set ‘ ?X  $\subseteq$ 
    Pow (fst ‘ Assoc-List.set (vars g)  $\times$  Collect variable-inv)
  by auto
show finite ... by (auto simp add: variables-finite dom-lm- $\alpha$ -Assoc-List-set[symmetric])
qed
qed

have finite ((gState-progress-rel prog) “ {g})
proof (rule finite-subset)
  show (gState-progress-rel prog) “ {g}  $\subseteq$ 
    ( $\lambda(\text{vars}, \text{chans}, t, ps)$ ). gState.make vars chans t ps) ‘ ?G
  apply (clarsimp simp add: image-def gState-inv-def gState.defs gState-progress-rel-def)
  apply (rule-tac x = vars x in exI)
  apply (simp add: lm-ball-eq-ran)
  apply (rule-tac x = channels x in exI)
  apply (case-tac timeout x)
  apply clarsimp
  apply (rule-tac x=procs x in exI)
  apply auto
done
show finite ... using G1
  by (blast intro: finite-lists-length-le channels-finite pStates-finite)
qed
with gState-progress-rel-rtrancl-absorb[OF True] show ?thesis by simp
qed

lemma gState-progress-rel-channels-update:
  assumes gState-inv prog g
  and channel-inv c
  and i < length (channels g)
  shows (g, gState.channels-update ( $\lambda cs$ . cs[i:=c]) g)  $\in$  gState-progress-rel prog
using assms
by (auto intro!: gState-progress-relI
    simp add: gState-inv-def cl-inv-def

```

dest!: *subsetD*[*OF set-update-subset-insert*]

lemma *gState-progress-rel-channels-update-step*:

assumes *gState-inv prog g*

and *step*: $(g, g') \in \text{gState-progress-rel prog}$

and *channel-inv c*

and $i < \text{length}(\text{channels } g')$

shows $(g, \text{gState.channels-update } (\lambda cs. cs[i:=c]) \ g') \in \text{gState-progress-rel prog}$

proof –

note *step*

also hence *gState-inv prog g'* **by** *blast*

note *gState-progress-rel-channels-update*[*OF this assms*(3,4)]

finally show *?thesis* .

qed

4.5 Invariants of the program

Naturally, we need our program to also adhere to certain invariants. Else we can't show, that the generated states are correct according to the invariants above.

definition *program-inv where*

program-inv prog

$\longleftrightarrow \text{IArray.length}(\text{states prog}) > 0$

$\wedge \text{IArray.length}(\text{states prog}) = \text{IArray.length}(\text{processes prog})$

$\wedge (\forall s \in \text{set}(\text{IArray.list-of}(\text{states prog})). \text{IArray.length } s > 0)$

$\wedge \text{lm.ball}(\text{proc-data prog})$

$(\lambda(-, \text{sid}x).$

$\text{sid}x < \text{IArray.length}(\text{processes prog})$

$\wedge \text{fst}(\text{processes prog} !! \text{sid}x) = \text{sid}x)$

$\wedge (\forall (\text{sid}x, \text{start}, \text{procArgs}, \text{args}) \in \text{set}(\text{IArray.list-of}(\text{processes prog})).$

$(\exists s. \text{start} = \text{Index } s \wedge s < \text{IArray.length}(\text{states prog} !! \text{sid}x)))$

lemma *program-inv-length-states*:

assumes *program-inv prog*

and $n < \text{IArray.length}(\text{states prog})$

shows $\text{IArray.length}(\text{states prog} !! n) > 0$

using *assms* **by** (*simp add: program-inv-def*)

lemma *program-invI*:

assumes $0 < \text{IArray.length}(\text{states prog})$

and $\text{IArray.length}(\text{states prog}) = \text{IArray.length}(\text{processes prog})$

and $\bigwedge s. s \in \text{set}(\text{IArray.list-of}(\text{states prog}))$

$\implies 0 < \text{IArray.length } s$

and $\bigwedge \text{sid}x. \text{sid}x \in \text{ran}(\text{lm}.\alpha(\text{proc-data prog}))$

$\implies \text{sid}x < \text{IArray.length}(\text{processes prog})$

$\wedge \text{fst}(\text{processes prog} !! \text{sid}x) = \text{sid}x$

and $\bigwedge \text{sid}x \text{ start procArgs args.}$

$(\text{sid}x, \text{start}, \text{procArgs}, \text{args}) \in \text{set}(\text{IArray.list-of}(\text{processes prog}))$

$\implies \exists s. \text{start} = \text{Index } s \wedge s < \text{IArray.length}(\text{states prog} !! \text{sid}x)$

```

    shows program-inv prog
  unfolding program-inv-def
  using assms
  by (auto simp add: lm-ball-eq-ran)

end

```

5 Formalization of Promela semantics

```

theory Promela
imports
  PromelaDatastructures
  PromelaInvariants
  PromelaStatistics
begin

Auxiliary

lemma mod-integer-le:
   $\langle x \bmod (a + 1) \leq b \rangle$  if  $\langle a \leq b \rangle \langle 0 < a \rangle$  for  $a \ b \ x :: \text{integer}$ 
  using that including integer.lifting proof transfer
  fix  $a \ b \ x :: \text{int}$ 
  assume  $\langle 0 < a \rangle \langle a \leq b \rangle$ 
  have  $\langle x \bmod (a + 1) < a + 1 \rangle$ 
    by (rule pos-mod-bound) (use  $\langle 0 < a \rangle$  in simp)
  with  $\langle a \leq b \rangle$  show  $\langle x \bmod (a + 1) \leq b \rangle$ 
    by simp
qed

lemma mod-integer-ge:
   $\langle b \leq x \bmod (a + 1) \rangle$  if  $\langle b \leq 0 \rangle \langle 0 < a \rangle$  for  $a \ b \ x :: \text{integer}$ 
  using that including integer.lifting proof transfer
  fix  $a \ b \ x :: \text{int}$ 
  assume  $\langle b \leq 0 \rangle \langle 0 < a \rangle$ 
  then have  $\langle 0 \leq x \bmod (a + 1) \rangle$ 
    by simp
  with  $\langle b \leq 0 \rangle$  show  $\langle b \leq x \bmod (a + 1) \rangle$ 
    by simp
qed

```

After having defined the datastructures, we present in this theory how to construct the transition system and how to generate the successors of a state, i. e. the real semantics of a Promela program. For the first task, we take the enriched AST as input, the second one operates on the transition system.

5.1 Misc Helpers

```

definition add-label :: String.literal  $\Rightarrow$  labels  $\Rightarrow$  nat  $\Rightarrow$  labels where

```

```

add-label l lbls pos = (
  case lm.lookup l lbls of
    None => lm.update l pos lbls
  | Some - => abortv STR "Label given twice: " l (λ-. lbls))

```

definition *min-prio* :: *edge list* ⇒ *integer* ⇒ *integer* **where**
min-prio es start = Min ((prio ‘ set es) ∪ {start})

lemma *min-prio-code* [code]:

min-prio es start = fold (λe pri. if prio e < pri then prio e else pri) es start

proof –

from Min.set-eq-fold **have** Min (set (start # map prio es)) = fold min (map prio es) start **by** metis

also have ... = fold (min ∘ prio) es start **by** (simp add: fold-map)

also have ... = fold (λe pri. if prio e < pri then prio e else pri) es start **by**
(auto intro!: fold-cong)

finally show ?thesis **by** (simp add: min-prio-def)

qed

definition *for-all* :: ('a ⇒ bool) ⇒ 'a list ⇒ bool **where**

for-all f xs ⟷ (∀ x ∈ set xs. f x)

lemma *for-all-code*[code]:

for-all f xs ⟷ foldli xs id (λkv σ. f kv) True

by (simp add: for-all-def foldli-conj)

definition *find-remove* :: ('a ⇒ bool) ⇒ 'a list ⇒ 'a option × 'a list **where**

find-remove P xs = (case List.find P xs of None ⇒ (None, xs)

| Some x ⇒ (Some x, List.remove1 x xs))

lemma *find-remove-code* [code]:

find-remove P [] = (None, [])

find-remove P (x#xs) = (if P x then (Some x, xs)

else apsnd (Cons x) (find-remove P xs))

by (induct xs) (auto simp add: find-remove-def dest: find-SomeD split: option.split)

lemma *find-remove-subset*:

find-remove P xs = (res, xs') ⟹ set xs' ⊆ set xs

unfolding *find-remove-def*

using set-remove1-subset

by (force split: option.splits)

lemma *find-remove-length*:

find-remove P xs = (res, xs') ⟹ length xs' ≤ length xs

unfolding *find-remove-def*

by (induct xs arbitrary: res xs') (auto split: if-splits option.splits)

5.2 Variable handling

Handling variables, with their different scopes (global vs. local), and their different types (array vs channel vs bounded) is one of the main challenges of the implementation.

```

fun lookupVar :: variable  $\Rightarrow$  integer option  $\Rightarrow$  integer where
  lookupVar (Var - val) None = val
| lookupVar (Var - -) (Some -) = abort STR "Array used on var" ( $\lambda$ -. 0)
| lookupVar (VArray - - vals) None = vals !! 0
| lookupVar (VArray - siz vals) (Some idx) = vals !! nat-of-integer idx

primrec checkVarValue :: varType  $\Rightarrow$  integer  $\Rightarrow$  integer where
  checkVarValue (VTBounded lRange hRange) val = (
    if val  $\leq$  hRange  $\wedge$  val  $\geq$  lRange then val
    else — overflowing is well-defined and may actually be used (e.g. bool)
      if lRange = 0  $\wedge$  val > 0
      then val mod (hRange + 1)
      else — we do not want to implement C-semantics (ie type casts)
        abort STR "Value overflow" ( $\lambda$ -. lRange))
| checkVarValue VTChan val = (
  if val < min-var-value  $\vee$  val > max-var-value
  then abort STR "Value overflow" ( $\lambda$ -. 0)
  else val)

lemma [simp]:
  variable-inv (Var VTChan 0)
by simp

context
  fixes type :: varType
  assumes varType-inv type
begin

lemma checkVarValue-bounded:
  checkVarValue type val  $\in$  {min-var-value..max-var-value}
  using <varType-inv type>
  by (cases type) (auto intro: mod-integer-le mod-integer-ge)

lemma checkVarValue-bounds:
  min-var-value  $\leq$  checkVarValue type val
  checkVarValue type val  $\leq$  max-var-value
  using checkVarValue-bounded [of val] by simp-all

lemma checkVarValue-Var:
  variable-inv (Var type (checkVarValue type val))
  using <varType-inv type> by (simp add: checkVarValue-bounds)

end

```

```

fun editVar :: variable ⇒ integer option ⇒ integer ⇒ variable where
  editVar (Var type -) None val = Var type (checkVarValue type val)
| editVar (Var - -) (Some -) - = abort STR "Array used on var" (λ-. Var VTChan
0)
| editVar (VArray type siz vals) None val = (
  let lv = IArray.list-of vals in
  let v' = lv[0:=checkVarValue type val] in
  VArray type siz (IArray v'))
| editVar (VArray type siz vals) (Some idx) val = (
  let lv = IArray.list-of vals in
  let v' = lv[(nat-of-integer idx):=checkVarValue type val] in
  VArray type siz (IArray v'))

lemma editVar-variable-inv:
  assumes variable-inv v
  shows variable-inv (editVar v idx val)
proof (cases v)
  case (Var type val) with assms have varType-inv type by simp
  with Var show ?thesis
  by (cases idx)
    (auto intro!: checkVarValue-Var
      simp del: checkVarValue.simps variable-inv.simps)
next
  case (VArray type siz vals)
  with assms have [simp, intro!]: varType-inv type by simp

  show ?thesis
  proof (cases idx)
    case None with assms VArray show ?thesis
    by (cases IArray.list-of vals) (auto intro!: checkVarValue-bounds)
  next
    case (Some i)
    note upd-cases = in-set-upd-cases[where l=IArray.list-of vals and i=nat-of-integer
i]

    from Some VArray assms show ?thesis
    by (cases type)
      (auto elim!: upd-cases intro!: mod-integer-le mod-integer-ge simp add:
min-var-value-def)
    qed
  qed

definition getVar'
  :: bool ⇒ String.literal ⇒ integer option
  ⇒ 'a gState-scheme ⇒ pState
  ⇒ integer option
where
  getVar' gl v idx g p = (
    let vars = if gl then gState.vars g else pState.vars p in

```

$map-option (\lambda x. lookupVar\ x\ idx)\ (lm.lookup\ v\ vars))$

definition *setVar'*

$:: bool \Rightarrow String.literal \Rightarrow integer\ option$
 $\Rightarrow integer$
 $\Rightarrow 'a\ gState-scheme \Rightarrow pState$
 $\Rightarrow 'a\ gState-scheme * pState$

where

setVar' gl v idx val g p = (
if gl then
if v = STR "-" then (g,p) — "-" is a write-only scratch variable
else case lm.lookup v (gState.vars g) of
None $\Rightarrow abortv\ STR\ "Unknown\ global\ variable:\ " v (\lambda-. (g,p))$
| Some x $\Rightarrow (g (gState.vars := lm.update\ v\ (editVar\ x\ idx\ val)$
(gState.vars g))
, p)
else
case lm.lookup v (pState.vars p) of
None $\Rightarrow abortv\ STR\ "Unknown\ proc\ variable:\ " v (\lambda-. (g,p))$
| Some x $\Rightarrow (g, p (pState.vars := lm.update\ v\ (editVar\ x\ idx\ val)$
(pState.vars p)))

lemma *setVar'-gState-inv:*

assumes *gState-inv prog g*
shows *gState-inv prog (fst (setVar' gl v idx val g p))*
unfolding *setVar'-def using assms*
by (*auto simp add: gState-inv-def lm.correct*
intro: editVar-variable-inv
split: option.splits)

lemma *setVar'-gState-progress-rel:*

assumes *gState-inv prog g*
shows $(g, fst (setVar' gl v idx val g p)) \in gState-progress-rel\ prog$
apply (*intro gState-progress-relI*)
apply (*fact assms*)
apply (*fact setVar'-gState-inv[OF assms]*)
apply (*auto simp: setVar'-def lm.correct split: option.splits*)
done

lemma *vardict-inv-process-names:*

assumes *vardict-inv ss proc v*
and *lm.lookup k v = Some x*
shows $k \in process-names\ ss\ proc$
using *assms*
by (*auto simp add: lm.correct vardict-inv-def*)

lemma *vardict-inv-variable-inv:*

assumes *vardict-inv ss proc v*
and *lm.lookup k v = Some x*

shows *variable-inv* x
using *assms*
by (*auto simp add: lm.correct vardict-inv-def*)

lemma *vardict-inv-updateI*:
assumes *vardict-inv* ss *proc* vs
and $x \in \text{process-names } ss$ *proc*
and *variable-inv* v
shows *vardict-inv* ss *proc* (*lm.update* x v vs)
using *assms*
by (*auto simp add: lm.correct vardict-inv-def*)

lemma *update-vardict-inv*:
assumes *vardict-inv* ss *proc* v
and *lm.lookup* k $v = \text{Some } x$
and *variable-inv* x'
shows *vardict-inv* ss *proc* (*lm.update* k x' v)
using *assms*
by (*auto intro!: vardict-inv-updateI vardict-inv-process-names*)

lemma *setVar'-pState-inv*:
assumes *pState-inv* *prog* p
shows *pState-inv* *prog* (*snd* (*setVar'* gl v *idx* *val* g p))
unfolding *setVar'-def* **using** *assms*
by (*auto split: if-splits option.splits*
simp add: pState-inv-def
intro: update-vardict-inv editVar-variable-inv vardict-inv-variable-inv)

lemma *setVar'-cl-inv*:
assumes *cl-inv* (g, p)
shows *cl-inv* (*setVar'* gl v *idx* *val* g p)
unfolding *setVar'-def* **using** *assms*
by (*auto split: if-splits option.splits*)

definition *withVar'*
 $:: \text{bool} \Rightarrow \text{String.literal} \Rightarrow \text{integer option}$
 $\Rightarrow (\text{integer} \Rightarrow 'x)$
 $\Rightarrow 'a \text{ gState-scheme} \Rightarrow \text{pState}$
 $\Rightarrow 'x$

where
withVar' gl v *idx* f g $p = f$ (*the* (*getVar'* gl v *idx* g p))

definition *withChannel'*
 $:: \text{bool} \Rightarrow \text{String.literal} \Rightarrow \text{integer option}$
 $\Rightarrow (\text{nat} \Rightarrow \text{channel} \Rightarrow 'x)$
 $\Rightarrow 'a \text{ gState-scheme} \Rightarrow \text{pState}$
 $\Rightarrow 'x$

where
withChannel' gl v *idx* f g $p = ($

```

let error = λ-. abortv STR "Variable is not a channel: " v
              (λ-. f 0 InvChannel) in
let abort = λ-. abortv STR "Channel already closed / invalid: " v
              (λ-. f 0 InvChannel)
in withVar' gl v idx (λi. let i = nat-of-integer i in
  if i ≥ length (channels g) then error ()
  else let c = channels g ! i in
    case c of
      InvChannel ⇒ abort ()
    | - ⇒ f i c) g p)

```

5.3 Expressions

Expressions are free of side-effects.

This is in difference to SPIN, where *run* is an expression with side-effect.

We treat *run* as a statement.

abbreviation *trivCond* $x \equiv \text{if } x \text{ then } 1 \text{ else } 0$

```

fun exprArith :: 'a gState-scheme ⇒ pState ⇒ expr ⇒ integer
and pollCheck :: 'a gState-scheme ⇒ pState ⇒ channel ⇒ recvArg list ⇒ bool
              ⇒ bool
and recvArgsCheck :: 'a gState-scheme ⇒ pState ⇒ recvArg list ⇒ integer list
              ⇒ bool

```

where

```

  exprArith g p (ExprConst x) = x
| exprArith g p (ExprMConst x -) = x

| exprArith g p ExprTimeOut = trivCond (timeout g)

| exprArith g p (ExprLen (ChanRef (VarRef gl name None))) =
  withChannel' gl name None (
    λ- c. case c of
      Channel - - q ⇒ integer-of-nat (length q)
    | HSCchannel - ⇒ 0) g p

| exprArith g p (ExprLen (ChanRef (VarRef gl name (Some idx)))) =
  withChannel' gl name (Some (exprArith g p idx)) (
    λ- c. case c of
      Channel - - q ⇒ integer-of-nat (length q)
    | HSCchannel - ⇒ 0) g p

| exprArith g p (ExprEmpty (ChanRef (VarRef gl name None))) =
  trivCond (withChannel' gl name None (
    λ- c. case c of Channel - - q ⇒ (q = [])
    | HSCchannel - ⇒ True) g p)

| exprArith g p (ExprEmpty (ChanRef (VarRef gl name (Some idx)))) =
  trivCond (withChannel' gl name (Some (exprArith g p idx)) (

```

$$\begin{aligned}
& \lambda\text{- } c. \text{ case } c \text{ of } Channel - - q \Rightarrow (q = []) \\
& \quad | HSCchannel - \Rightarrow True) \ g \ p) \\
| \text{ exprArith } g \ p \ (ExprFull \ (ChanRef \ (VarRef \ gl \ name \ None))) = \\
& \quad \text{trivCond} \ (\text{withChannel}' \ gl \ name \ None \ (\\
& \quad \lambda\text{- } c. \text{ case } c \text{ of} \\
& \quad \quad Channel \ cap - q \Rightarrow \text{integer-of-nat} \ (\text{length } q) \geq \text{cap} \\
& \quad \quad | HSCchannel - \Rightarrow False) \ g \ p) \\
| \text{ exprArith } g \ p \ (ExprFull \ (ChanRef \ (VarRef \ gl \ name \ (Some \ idx)))) = \\
& \quad \text{trivCond} \ (\text{withChannel}' \ gl \ name \ (Some \ (\text{exprArith } g \ p \ idx)) \ (\\
& \quad \lambda\text{- } c. \text{ case } c \text{ of} \\
& \quad \quad Channel \ cap - q \Rightarrow \text{integer-of-nat} \ (\text{length } q) \geq \text{cap} \\
& \quad \quad | HSCchannel - \Rightarrow False) \ g \ p) \\
| \text{ exprArith } g \ p \ (ExprVarRef \ (VarRef \ gl \ name \ None)) = \\
& \quad \text{withVar}' \ gl \ name \ None \ id \ g \ p \\
| \text{ exprArith } g \ p \ (ExprVarRef \ (VarRef \ gl \ name \ (Some \ idx))) = \\
& \quad \text{withVar}' \ gl \ name \ (Some \ (\text{exprArith } g \ p \ idx)) \ id \ g \ p \\
| \text{ exprArith } g \ p \ (ExprUnOp \ UnOpMinus \ expr) = 0 - \text{exprArith } g \ p \ expr \\
| \text{ exprArith } g \ p \ (ExprUnOp \ UnOpNeg \ expr) = ((\text{exprArith } g \ p \ expr) + 1) \bmod 2 \\
| \text{ exprArith } g \ p \ (ExprBinOp \ BinOpAdd \ lexpr \ rexpr) = \\
& \quad (\text{exprArith } g \ p \ lexpr) + (\text{exprArith } g \ p \ rexpr) \\
| \text{ exprArith } g \ p \ (ExprBinOp \ BinOpSub \ lexpr \ rexpr) = \\
& \quad (\text{exprArith } g \ p \ lexpr) - (\text{exprArith } g \ p \ rexpr) \\
| \text{ exprArith } g \ p \ (ExprBinOp \ BinOpMul \ lexpr \ rexpr) = \\
& \quad (\text{exprArith } g \ p \ lexpr) * (\text{exprArith } g \ p \ rexpr) \\
| \text{ exprArith } g \ p \ (ExprBinOp \ BinOpDiv \ lexpr \ rexpr) = \\
& \quad (\text{exprArith } g \ p \ lexpr) \text{ div } (\text{exprArith } g \ p \ rexpr) \\
| \text{ exprArith } g \ p \ (ExprBinOp \ BinOpMod \ lexpr \ rexpr) = \\
& \quad (\text{exprArith } g \ p \ lexpr) \bmod (\text{exprArith } g \ p \ rexpr) \\
| \text{ exprArith } g \ p \ (ExprBinOp \ BinOpGr \ lexpr \ rexpr) = \\
& \quad \text{trivCond} \ (\text{exprArith } g \ p \ lexpr > \text{exprArith } g \ p \ rexpr) \\
| \text{ exprArith } g \ p \ (ExprBinOp \ BinOpLe \ lexpr \ rexpr) = \\
& \quad \text{trivCond} \ (\text{exprArith } g \ p \ lexpr < \text{exprArith } g \ p \ rexpr) \\
| \text{ exprArith } g \ p \ (ExprBinOp \ BinOpGEq \ lexpr \ rexpr) = \\
& \quad \text{trivCond} \ (\text{exprArith } g \ p \ lexpr \geq \text{exprArith } g \ p \ rexpr) \\
| \text{ exprArith } g \ p \ (ExprBinOp \ BinOpLEq \ lexpr \ rexpr) =
\end{aligned}$$

```

    trivCond (exprArith g p lexr ≤ exprArith g p rexr)

| exprArith g p (ExprBinOp BinOpEq lexr rexr) =
    trivCond (exprArith g p lexr = exprArith g p rexr)

| exprArith g p (ExprBinOp BinOpNEq lexr rexr) =
    trivCond (exprArith g p lexr ≠ exprArith g p rexr)

| exprArith g p (ExprBinOp BinOpAnd lexr rexr) =
    trivCond (exprArith g p lexr ≠ 0 ∧ exprArith g p rexr ≠ 0)

| exprArith g p (ExprBinOp BinOpOr lexr rexr) =
    trivCond (exprArith g p lexr ≠ 0 ∨ exprArith g p rexr ≠ 0)

| exprArith g p (ExprCond cexpr texpr fexpr) =
    (if exprArith g p cexpr ≠ 0 then exprArith g p texpr
     else exprArith g p fexpr)

| exprArith g p (ExprPoll (ChanRef (VarRef gl name None)) rs srt) =
    trivCond (withChannel' gl name None (
        λ- c. pollCheck g p c rs srt) g p)

| exprArith g p (ExprPoll (ChanRef (VarRef gl name (Some idx))) rs srt) =
    trivCond (withChannel' gl name (Some (exprArith g p idx)) (
        λ- c. pollCheck g p c rs srt) g p)

| pollCheck g p InvChannel - - =
    abort STR "Channel already closed / invalid." (λ-. False)
| pollCheck g p (HSCChannel -) - - = False
| pollCheck g p (Channel - - q) rs srt = (
    if q = [] then False
    else if ¬ srt then recvArgsCheck g p rs (hd q)
    else List.find (recvArgsCheck g p rs) q ≠ None)

| recvArgsCheck - - [] = True
| recvArgsCheck - - - [] =
    abort STR "Length mismatch on receiving." (λ-. False)
| recvArgsCheck - - [] - =
    abort STR "Length mismatch on receiving." (λ-. False)
| recvArgsCheck g p (r#rs) (v#vs) = ((
    case r of
        RecvArgConst c ⇒ c = v
    | RecvArgMConst c - ⇒ c = v
    | RecvArgVar var ⇒ True
    | RecvArgEval e ⇒ exprArith g p e = v ) ∧ recvArgsCheck g p rs vs)

```

getVar' etc. do operate on name, index, ... directly. Lift them to use *VarRef* instead.

fun *liftVar* **where**

$\text{liftVar } f \text{ (VarRef } gl \text{ } v \text{ } idx) \text{ argm } g \text{ } p =$
 $f \text{ } gl \text{ } v \text{ (map-option (exprArith } g \text{ } p) \text{ } idx) \text{ argm } g \text{ } p$

definition $\text{getVar } v = \text{liftVar } (\lambda gl \text{ } v \text{ } idx \text{ } arg. \text{getVar}' \text{ } gl \text{ } v \text{ } idx) \text{ } v \text{ } ()$

definition $\text{setVar} = \text{liftVar } \text{setVar}'$

definition $\text{withVar} = \text{liftVar } \text{withVar}'$

primrec withChannel

where $\text{withChannel } (\text{ChanRef } v) = \text{liftVar } \text{withChannel}' \text{ } v$

lemma $\text{setVar-gState-progress-rel}$:

assumes $g\text{State-inv } prog \text{ } g$

shows $(g, \text{fst } (\text{setVar } v \text{ } val \text{ } g \text{ } p)) \in g\text{State-progress-rel } prog$

unfolding setVar-def

by $(\text{cases } v) \text{ (simp add: setVar'-gState-progress-rel[OF assms])}$

lemmas $\text{setVar-gState-inv} =$

$\text{setVar-gState-progress-rel[THEN gState-progress-rel-gState-invI2]}$

lemma setVar-pState-inv :

assumes $p\text{State-inv } prog \text{ } p$

shows $p\text{State-inv } prog \text{ (snd (setVar } v \text{ } val \text{ } g \text{ } p))}$

unfolding setVar-def

by $(\text{cases } v) \text{ (auto simp add: setVar'-pState-inv assms)}$

lemma setVar-cl-inv :

assumes $cl\text{-inv } (g, p)$

shows $cl\text{-inv } (\text{setVar } v \text{ } val \text{ } g \text{ } p)$

unfolding setVar-def

by $(\text{cases } v) \text{ (auto simp add: setVar'-cl-inv assms)}$

5.4 Variable declaration

lemma $\text{channel-inv-code } [code]$:

$\text{channel-inv } (\text{Channel } cap \text{ } ts \text{ } q)$

$\longleftrightarrow cap \leq \text{max-array-size}$

$\wedge 0 \leq cap$

$\wedge \text{for-all } varType\text{-inv } ts$

$\wedge \text{length } ts \leq \text{max-array-size}$

$\wedge \text{length } q \leq \text{max-array-size}$

$\wedge \text{for-all } (\lambda x. \text{length } x = \text{length } ts$

$\wedge \text{for-all } (\lambda y. y \geq \text{min-var-value}$

$\wedge y \leq \text{max-var-value}) \text{ } x) \text{ } q$

$\text{channel-inv } (\text{HChannel } ts)$

$\longleftrightarrow \text{for-all } varType\text{-inv } ts \wedge \text{length } ts \leq \text{max-array-size}$

by $(\text{auto simp add: for-all-def}) \text{ force+}$

primrec toVariable

$:: 'a \text{ } g\text{State-scheme} \Rightarrow p\text{State} \Rightarrow \text{varDecl} \Rightarrow \text{String.literal} * \text{variable} * \text{channels}$

where

```

toVariable g p (VarDeclNum lb hb name siz init) = (
  let type = VTBounded lb hb in
  if ¬ varType-inv type then abortv STR "Invalid var def (varType-inv failed): "
  " name
                                     (λ-. (name, Var VTChan 0, []))
else
  let
    init = checkVarValue type (case init of
      None ⇒ 0
      | Some e ⇒ exprArith g p e);
    v = (case siz of
      None ⇒ Var type init
      | Some s ⇒ if nat-of-integer s ≤ max-array-size
        then VArray type (nat-of-integer s)
          (IArray.tabulate (s, λ-. init))
        else abortv STR "Invalid var def (array too large): " name
          (λ-. Var VTChan 0))
  in
    (name, v, []))

| toVariable g p (VarDeclChan name siz types) = (
  let
    size = (case siz of None ⇒ 1 | Some s ⇒ nat-of-integer s);
    chans = (case types of
      None ⇒ []
      | Some (cap, tys) ⇒
        let C = (if cap = 0 then HSCchannel tys
          else Channel cap tys []) in
        if ¬ channel-inv C
        then abortv STR "Invalid var def (channel-inv failed): "
          name (λ-. [])
        else replicate size C);
    cidx = (case types of
      None ⇒ 0
      | Some - ⇒ integer-of-nat (length (channels g)));
    v = (case siz of
      None ⇒ Var VTChan cidx
      | Some s ⇒ if nat-of-integer s ≤ max-array-size
        then VArray VTChan (nat-of-integer s)
          (IArray.tabulate (s,
            λi. if cidx = 0 then 0
            else i + cidx))
        else abortv STR "Invalid var def (array too large): "
          name (λ-. Var VTChan 0))
  in
    (name, v, chans))

```

lemma toVariable-variable-inv:

```

assumes  $gState\text{-}inv$  prog  $g$ 
shows  $variable\text{-}inv$  (fst (snd (toVariable  $g$   $p$   $v$ )))
using  $assms$ 
apply (cases  $v$ )
  apply (auto intro!: checkVarValue-Var
    simp del: variable-inv.simps checkVarValue.simps varType-inv.simps
    split: if-splits option.splits)
  apply (auto intro!: mod-integer-ge mod-integer-le simp add: min-var-value-def)
  apply (simp-all add: assms  $gState\text{-}inv\text{-}def$ 
    max-channels-def max-var-value-def min-var-value-def max-array-size-def)
  including integer.lifting
  apply (transfer', simp)+
done

```

```

lemma toVariable-channels-inv:
shows  $\forall x \in set$  (snd (snd (toVariable  $g$   $p$   $v$ ))). channel-inv  $x$ 
by (cases  $v$ ) auto

```

```

lemma toVariable-channels-inv':
shows toVariable  $g$   $p$   $v = (a,b,c) \implies \forall x \in set$   $c$ . channel-inv  $x$ 
using toVariable-channels-inv
by (metis snd-conv)

```

```

lemma toVariable-variable-inv':
shows  $gState\text{-}inv$  prog  $g \implies toVariable$   $g$   $p$   $v = (a,b,c) \implies variable\text{-}inv$   $b$ 
by (metis snd-conv fst-conv toVariable-variable-inv)

```

```

definition mkChannels
  :: 'a  $gState\text{-}scheme \Rightarrow pState \Rightarrow channels \Rightarrow 'a$   $gState\text{-}scheme * pState$ 
where
  mkChannels  $g$   $p$   $cs =$  (
    if  $cs = []$  then ( $g,p$ ) else
    let  $l = length$  (channels  $g$ ) in
    if  $l + length$   $cs > max\text{-}channels$ 
    then abort STR "Too much channels" ( $\lambda\cdot$ . ( $g,p$ ))
    else let
       $cs_p = map$  integer-of-nat [ $l..l + length$   $cs$ ];
       $g' = g$ (channels := channels  $g$  @  $cs$ );
       $p' = p$ ( $pState.channels := pState.channels$   $p$  @  $cs_p$ )
    in
    ( $g'$ ,  $p'$ ))

```

```

lemma mkChannels-gState-progress-rel:
   $gState\text{-}inv$  prog  $g$ 
   $\implies set$   $cs \subseteq Collect$  channel-inv
   $\implies (g, fst$  (mkChannels  $g$   $p$   $cs$ ))  $\in gState\text{-}progress\text{-}rel$  prog
unfolding mkChannels-def
by (intro  $gState\text{-}progress\text{-}relI$ )
  (auto simp add:  $gState\text{-}inv\text{-}def$   $gState.defs$  cl-inv-def)

```

```

lemmas mkChannels-gState-inv =
  mkChannels-gState-progress-rel[THEN gState-progress-rel-gState-invI2]

lemma mkChannels-pState-inv:
  pState-inv prog p
   $\implies$  cl-inv (g,p)
   $\implies$  pState-inv prog (snd (mkChannels g p cs))
unfolding mkChannels-def
including integer.lifting
apply (auto simp add: pState-inv-def pState.defs gState-inv-def dest!: cl-inv-lengthD)
  apply (transfer', simp)+
  done

lemma mkChannels-cl-inv:
  cl-inv (g,p)  $\implies$  cl-inv (mkChannels g p cs)
unfolding mkChannels-def
by (auto simp add: pState.defs dest: cl-inv-lengthD intro!: cl-invI)

definition mkVarChannel
  :: varDecl
   $\Rightarrow$  ((var-dict  $\Rightarrow$  var-dict)  $\Rightarrow$  'a gState-scheme * pState)
   $\Rightarrow$  'a gState-scheme * pState)
   $\Rightarrow$  'a gState-scheme  $\Rightarrow$  pState
   $\Rightarrow$  'a gState-scheme * pState
where
  mkVarChannel v upd g p = (
    let
      (k,v,cs) = toVariable g p v;
      (g',p') = upd (lm.update k v) (g,p)
    in
      mkChannels g' p' cs)

lemma mkVarChannel-gState-inv:
  assumes gState-inv prog g
  and  $\bigwedge k v' cs. \text{toVariable } g p v = (k,v',cs)$ 
   $\implies$  gState-inv prog (fst (upd (lm.update k v') (g,p)))
  shows gState-inv prog (fst (mkVarChannel v upd g p))
using assms unfolding mkVarChannel-def
by (force split: varDecl.split prod.split
  intro!: mkChannels-gState-inv
  dest: toVariable-channels-inv')

lemma mkVarChannel-gState-progress-rel:
  assumes gState-inv prog g
  and  $\bigwedge k v' cs. \text{toVariable } g p v = (k,v',cs)$ 
   $\implies$  (g, fst (upd (lm.update k v') (g,p)))  $\in$  gState-progress-rel prog
  shows (g, fst (mkVarChannel v upd g p))  $\in$  gState-progress-rel prog
proof -

```

obtain $k\ v'\ cs$ **where** $1: \text{toVariable } g\ p\ v = (k, v', cs)$ **by** $(\text{metis prod.exhaust})$
obtain $g'\ p'$ **where** $2: (g', p') = \text{upd } (lm.\text{update } k\ v')\ (g, p)$ **by** $(\text{metis prod.exhaust})$
with $1\ \text{assms}$ **have** $*$: $(g, g') \in g\text{State-progress-rel prog}$ **by** (metis fst-conv)

also from $1\ 2$ **have** $(g', \text{fst } (\text{mkChannels } g'\ p'\ cs)) \in g\text{State-progress-rel prog}$
by $(\text{force intro!}: \text{mkChannels-gState-progress-rel } g\text{State-progress-rel-gState-invI2}[OF\ *])$

 $\text{dest: toVariable-channels-inv'}$
finally have $(g, \text{fst } (\text{mkChannels } g'\ p'\ cs)) \in g\text{State-progress-rel prog}$.
thus ?thesis using $1\ 2$ **by** $(\text{auto simp add: mkVarChannel-def split: prod.split})$
qed

lemma $\text{mkVarChannel-pState-inv}$:
assumes $p\text{State-inv prog } p$
and $cl\text{-inv } (g, p)$
and $\bigwedge k\ v'\ cs. \text{toVariable } g\ p\ v = (k, v', cs)$
 $\implies cl\text{-inv } (\text{upd } (lm.\text{update } k\ v')\ (g, p))$
and $\bigwedge k\ v'\ cs. \text{toVariable } g\ p\ v = (k, v', cs)$
 $\implies p\text{State-inv prog } (\text{snd } (\text{upd } (lm.\text{update } k\ v')\ (g, p)))$
shows $p\text{State-inv prog } (\text{snd } (\text{mkVarChannel } v\ \text{upd } g\ p))$
using $\text{assms unfolding mkVarChannel-def}$
by $(\text{force split: varDecl.split prod.split intro!}: \text{mkChannels-pState-inv})$

lemma $\text{mkVarChannel-cl-inv}$:
assumes $cl\text{-inv } (g, p)$
and $\bigwedge k\ v'\ cs. \text{toVariable } g\ p\ v = (k, v', cs)$
 $\implies cl\text{-inv } (\text{upd } (lm.\text{update } k\ v')\ (g, p))$
shows $cl\text{-inv } (\text{mkVarChannel } v\ \text{upd } g\ p)$
using $\text{assms unfolding mkVarChannel-def}$
by $(\text{force split: varDecl.split prod.splits intro!}: \text{mkChannels-cl-inv})$

definition mkVarChannelProc
 $:: \text{procVarDecl} \Rightarrow 'a\ g\text{State-scheme} \Rightarrow p\text{State} \Rightarrow 'a\ g\text{State-scheme} * p\text{State}$
where
 $\text{mkVarChannelProc } v\ g\ p = ($
 let
 $v' = \text{case } v\ \text{of}$
 $\quad \text{ProcVarDeclNum } lb\ hb\ name\ siz\ init \Rightarrow$
 $\quad \quad \text{VarDeclNum } lb\ hb\ name\ siz\ init$
 $\quad | \text{ProcVarDeclChan } name\ siz \Rightarrow$
 $\quad \quad \text{VarDeclChan } name\ siz\ None;$
 $(k, v, cs) = \text{toVariable } g\ p\ v'$
 in
 $\text{mkVarChannel } v'\ (\text{apsnd } \circ\ p\text{State.vars-update})\ g\ p)$

lemma $\text{mkVarChannelProc-gState-progress-rel}$:
assumes $g\text{State-inv prog } g$

```

  shows (g, fst (mkVarChannelProc v g p)) ∈ gState-progress-rel prog
unfolding mkVarChannelProc-def
using assms
by (auto intro!: mkVarChannel-gState-progress-rel)

lemmas mkVarChannelProc-gState-inv =
  mkVarChannelProc-gState-progress-rel[THEN gState-progress-rel-gState-invI2]

lemma toVariable-name:
  toVariable g p (VarDeclNum lb hb name sz init) = (x,a,b) ⇒ x = name
  toVariable g p (VarDeclChan name sz t) = (x, a, b) ⇒ x = name
by (auto split: if-splits)

declare toVariable.simps[simp del]

lemma statesDecls-process-names:
  assumes v ∈ statesDecls (states prog !! (pState.idx p))
  shows procVarDeclName v ∈ process-names (states prog !! (pState.idx p))
    (processes prog !! (pState.idx p))
using assms
by (cases processes prog !! (pState.idx p)) (auto simp add: statesNames-def)

lemma mkVarChannelProc-pState-inv:
  assumes pState-inv prog p
  and gState-inv prog g
  and cl-inv (g, p)
  and decl: v ∈ statesDecls (states prog !! (pState.idx p))
  shows pState-inv prog (snd (mkVarChannelProc v g p))
unfolding mkVarChannelProc-def
using assms statesDecls-process-names[OF decl]
by (auto intro!: mkVarChannel-pState-inv)
  (auto dest: toVariable-name
    split: procVarDecl.splits
    intro: toVariable-variable-inv' vardict-inv-updateI
    simp add: pState-inv-def)

lemma mkVarChannelProc-cl-inv:
  assumes cl-inv (g,p)
  shows cl-inv (mkVarChannelProc v g p)
unfolding mkVarChannelProc-def using assms
by (auto intro!: mkVarChannel-cl-inv)

```

5.5 Folding

Fold over lists (and lists of lists) of *step/stmnt*. The folding functions are doing a bit more than that, e.g. ensuring the offset into the program array is correct.

definition *step-fold'* where
step-fold' g steps (lbls :: labels) pri pos

```

      (nxt :: edgeIndex) (onxt :: edgeIndex option) iB =
foldr (λstep (pos, nxt, lbls, es).
      let (e, enxt, lbls) = g step (lbls, pri, pos, nxt, onxt, iB)
      in (pos + length e, enxt, lbls, es@e)
) steps (pos, nxt, lbls, [])

```

definition *step-fold* **where**

```

step-fold g steps lbls pri pos nxt onxt iB = (
  let (-,nxt,lbls,s) = step-fold' g steps lbls pri pos nxt onxt iB
  in (s,nxt,lbls))

```

lemma *step-fold'-cong*:

```

assumes lbls = lbls'
and pri = pri'
and pos = pos'
and steps = steps'
and nxt = nxt'
and onxt = onxt'
and iB = iB'
and ∧x d. x ∈ set steps ⇒ g x d = g' x d
shows step-fold' g steps lbls pri pos nxt onxt iB =
      step-fold' g' steps' lbls' pri' pos' nxt' onxt' iB'

```

unfolding *step-fold'-def*

by (auto intro: foldr-cong simp add: assms)

lemma *step-fold-cong[fundef-cong]*:

```

assumes lbls = lbls'
and pri = pri'
and pos = pos'
and steps = steps'
and nxt = nxt'
and onxt = onxt'
and iB = iB'
and ∧x d. x ∈ set steps ⇒ g x d = g' x d
shows step-fold g steps lbls pri pos nxt onxt iB =
      step-fold g' steps' lbls' pri' pos' nxt' onxt' iB'

```

unfolding *step-fold-def*

by (auto simp: assms cong: step-fold'-cong)

fun *step-foldL-step* **where**

```

step-foldL-step - - - [] (pos, nxt, lbls, es, is) = (pos, nxt, lbls, es, is)
| step-foldL-step g pri onxt (s#steps) (pos, nxt, lbls, es, is) = (
  let (pos', nxt', lbls', ss') = step-fold' g steps lbls pri pos nxt onxt False in
  let (s', nxt'', lbls'') = g s (lbls', pri, pos', nxt', onxt, True) in
  let rs = butlast s'; s'' = last s' in
  (pos' + length rs, nxt, lbls'', es@ss'@rs, s''#is))

```

definition *step-foldL* **where**

```

step-foldL g stepss lbls pri pos nxt onxt =

```

foldr (step-foldL-step g pri onxt) stepss (pos,nxt,lbls,[],[])

lemma *step-foldL-step-cong*:

assumes *pri = pri'*
and *onxt = onxt'*
and *s = s'*
and *d = d'*
and $\bigwedge x d. x \in \text{set } s \implies g \ x \ d = g' \ x \ d$
shows *step-foldL-step g pri onxt s d = step-foldL-step g' pri' onxt' s' d'*
using *assms*
by (*cases d', cases s'*) (*simp-all cong: step-fold'-cong*)

lemma *step-foldL-cong[fundef-cong]*:

assumes *lbls = lbls'*
and *pri = pri'*
and *pos = pos'*
and *stepss = stepss'*
and *nxt = nxt'*
and *onxt = onxt'*
and $\bigwedge x x' d. x \in \text{set } \text{stepss} \implies x' \in \text{set } x \implies g \ x' \ d = g' \ x' \ d$
shows *step-foldL g stepss lbls pri pos nxt onxt =*
step-foldL g' stepss' lbls' pri' pos' nxt' onxt'
unfolding *step-foldL-def*
using *assms*
apply (*cases stepss'*)
apply *simp*
apply (*force intro!: foldr-cong step-foldL-step-cong*)
done

5.6 Starting processes

definition *modProcArg*

*:: (procArg * integer) \Rightarrow String.literal * variable*

where

modProcArg x = (
case x of
(ProcArg ty name, val) \Rightarrow if varType-inv ty
then let init = checkVarValue ty val
in (name, Var ty init)
else abortv STR "Invalid proc arg (varType-inv failed)"
name (λ -. (name, Var VTChan 0)))

definition *emptyProc :: pState*

— The empty process.

where

emptyProc = (pid = 0, vars = lm.empty (), pc = 0, channels = [], idx = 0)

lemma *vardict-inv-empty*:

vardict-inv ss proc (lm.empty())

```

unfolding vardict-inv-def
by (simp add: lm.correct)

lemma emptyProc-cl-inv[simp]:
  cl-inv (g, emptyProc)
by (simp add: cl-inv-def emptyProc-def)

lemma emptyProc-pState-inv:
  assumes program-inv prog
  shows pState-inv prog emptyProc
proof -
  from assms have IArray.length (states prog !! 0) > 0
  by (intro program-inv-length-states) (auto simp add: program-inv-def)
  with assms show ?thesis
  unfolding pState-inv-def program-inv-def emptyProc-def
  by (auto simp add: vardict-inv-empty)
qed

fun mkProc
  :: 'a gState-scheme  $\Rightarrow$  pState
   $\Rightarrow$  String.literal  $\Rightarrow$  expr list  $\Rightarrow$  process  $\Rightarrow$  nat
   $\Rightarrow$  'a gState-scheme * pState
where
  mkProc g p name args (sidx, start, argDecls, decls) pidN = (
    let start = case start of
      Index x  $\Rightarrow$  x
      | -  $\Rightarrow$  abortv STR "Process start is not index: " name ( $\lambda$ -. 0)
    in
    — sanity check
    if length args  $\neq$  length argDecls
    then abortv STR "Signature mismatch: " name ( $\lambda$ -. (g, emptyProc))
    else
    let
      — evaluate args (in the context of the calling process)
      eArgs = map (exprArith g p) args;

      — replace the init part of argDecls
      argVars = map modProcArg (zip argDecls eArgs);

      — add -pid to vars
      pidI = integer-of-nat pidN;
      argVars = (STR "-pid", Var (VTBounded 0 pidI) pidI) # argVars;
      argVars = lm.to-map argVars;

      — our new process
      p = ( $\lfloor$  pid = pidN, vars = argVars, pc = start, channels = [], idx = sidx  $\rfloor$ )
    in
    — apply the declarations
    foldl ( $\lambda$ (g,p) d. mkVarChannel d (apsnd  $\circ$  pState.vars-update) g p)

```


(g,p)
 $decls$

lemma *mkProc-gState-progress-rel*:

assumes *gState-inv prog g*

shows $(g, fst (mkProc\ g\ p\ name\ args\ (processes\ prog\ !!\ sid\ x)\ pidN)) \in gState\ progress\ rel\ prog$

proof –

obtain *sidx'* *start argDecls decls* **where**

p: *processes prog !! sidx* = (*sidx'*, *start*, *argDecls*, *decls*)

by (*metis prod.exhaust*)

from *assms* **have**

$\bigwedge p. (g, fst (foldl (\lambda(g,p)\ d.\ mkVarChannel\ d\ (apsnd\ \circ\ pState.vars\ update)\ g\ p)$

$(g,p)\ decls))$

$\in gState\ progress\ rel\ prog$

proof (*induction decls arbitrary: g p*)

case (*Cons d decls*)

obtain *g' p'* **where**

new: (*g',p'*) = (*mkVarChannel d (apsnd ∘ pState.vars-update) g p*)

by (*metis prod.exhaust*)

hence *g' = fst ...* **by** (*metis fst-conv*)

with *Cons.prem*s **have** *g-g'*: (*g,g'*) $\in gState\ progress\ rel\ prog$

by (*auto intro: mkVarChannel-gState-progress-rel*)

also note *Cons.IH*[*OF g-g'*[*THEN gState-progress-rel-gState-invI2*], *of p*]

finally show *?case* **by** (*auto simp add: o-def new*)

qed *simp*

thus *?thesis* **using** *assms p* **by** *auto*

qed

lemmas *mkProc-gState-inv =*

mkProc-gState-progress-rel[*THEN gState-progress-rel-gState-invI2*]

lemma *mkProc-pState-inv*:

assumes *program-inv prog*

and *gState-inv prog g*

and *pidN* $\leq max\ procs$ **and** *pidN* > 0

and *sidx* $< IArray.length\ (processes\ prog)$

and *fst (processes prog !! sidx)* = *sidx*

shows *pState-inv prog (snd (mkProc g p name args (processes prog !! sidx) pidN))*

proof –

obtain *sidx'* *start argDecls decls* **where**

processes prog !! sidx = (*sidx'*, *start*, *argDecls*, *decls*)

by (*metis prod.exhaust*)

with *assms* **have**

p-def: *processes prog !! sidx* = (*sidx*, *start*, *argDecls*, *decls*)

IArray.list-of (processes prog) ! sidx = (*sidx*, *start*, *argDecls*, *decls*)

by *simp-all*

```

with assms have (sidx,start,argDecls,decls) ∈ set (IArray.list-of (processes prog))

  by (force dest: nth-mem)

with assms obtain s where s: start = Index s s < IArray.length (states prog !! sidx)
  unfolding program-inv-def by auto

hence P-inv: pState-inv prog ()
  pid = pidN,
  vars = lm.to-map
    ((STR "-pid", Var (VTBounded 0 (integer-of-nat pidN))
      (integer-of-nat pidN))
    # map modProcArg (zip argDecls (map (exprArith g p) args))),
  pc = s, channels = [], idx = sidx)
unfolding pState-inv-def
using assms[unfolded program-inv-def]
including integer.lifting
apply (simp add: p-def)
apply (intro lm-to-map-vardict-inv)
apply auto
  apply (simp add: max-procs-def max-var-value-def)
  apply transfer'
  apply simp
  apply transfer'
  apply simp
  apply (simp add: min-var-value-def)
  apply transfer'
  apply simp
  apply (simp add: max-var-value-def max-procs-def)
  apply transfer'
  apply simp
  apply (drule set-zip-leftD)
  apply (force simp add: modProcArg-def
    split: procArg.splits if-splits
    intro!: image-eqI)
apply (clarsimp simp add: modProcArg-def
  split: procArg.splits if-splits
  simp del: variable-inv.simps)
apply (intro checkVarValue-Var)
apply assumption
done

from p-def have
  varDeclName ' set decls ⊆
  process-names (states prog !! sidx) (processes prog !! sidx)
  by auto
with ⟨gState-inv prog g⟩ have
  F-inv: ∧p. [ pState-inv prog p; sidx = pState.idx p; cl-inv (g,p) ]

```

$\implies pState\text{-}inv\ prog$
 $(snd\ (foldl\ (\lambda(g,p)\ d.\ mkVarChannel\ d\ (apsnd\ \circ\ pState.vars\text{-}update))$
 $g\ p)$
 $(g,p)\ decls))$
proof (*induction decls arbitrary: g p*)
case (*Cons d ds*) **hence**
 $decl: varDeclName\ d \in process\text{-}names\ (states\ prog\ !!\ pState.idx\ p)$
 $(processes\ prog\ !!\ pState.idx\ p)$
by *simp*

obtain $g'\ p'$ **where**
 $new: (g',p') = (mkVarChannel\ d\ (apsnd\ \circ\ pState.vars\text{-}update))\ g\ p)$
by (*metis prod.exhaust*)
hence $p': p' = snd\ \dots$ **and** $g': g' = fst\ \dots$
by (*metis snd-conv fst-conv*) $+$
with *Cons.prem*s **have** $pState\text{-}inv\ prog\ p'$
apply (*auto intro!: mkVarChannel-pState-inv*)
apply (*simp add: pState-inv-def*)
apply (*intro vardict-inv-updateI*)
apply *simp*
apply (*cases d*)
apply (*force dest!: toVariable-name*)
apply (*force dest!: toVariable-name*)
apply (*intro toVariable-variable-inv'*)
apply *assumption* $+$
done
moreover
from $p'\ Cons.prem$ s **have** $pState.idx\ p' = sid_x$
by (*auto simp add: mkVarChannel-def mkChannels-def split: prod.split*)
moreover
from $new\ Cons.prem$ s **have** $cl\text{-}inv\ (g',p')$
by (*auto intro!: mkVarChannel-cl-inv*)
moreover
from $g'\ Cons.prem$ s **have** $gState\text{-}inv\ prog\ g'$
by (*auto intro!: mkVarChannel-gState-inv*)
moreover
from *Cons.prem*s **have**
 $varDeclName\ 'set\ ds \subseteq$
 $process\text{-}names\ (states\ prog\ !!\ sid_x)\ (processes\ prog\ !!\ sid_x)$
by *simp*
ultimately
have
 $pState\text{-}inv\ prog$
 $(snd\ (foldl\ (\lambda(g,p)\ d.\ mkVarChannel\ d\ (apsnd\ \circ\ pState.vars\text{-}update))\ g\ p)$
 $(g',p')\ ds))$
using *Cons.IH*[*of p' g'*] **by** (*simp add: o-def*)
with *new show ?case* **by** (*simp add: o-def*)
qed *simp*

```

show ?thesis
  by (auto simp add: p-def s cl-inv-def
        intro: F-inv[OF P-inv])
      (blast intro: emptyProc-pState-inv assms)
qed

lemma mkProc-cl-inv:
  assumes cl-inv (g,p)
  shows cl-inv (mkProc g p name args (processes prog !! sidx) pidN)
proof -
  note IArray.sub-def [simp del]
  obtain sidx' start argDecls decls
    where [simp]: processes prog !! sidx = (sidx', start, argDecls, decls)
    by (metis prod.exhaust)

  have
    P-inv:  $\bigwedge s v. \text{cl-inv } (g, (\text{pid} = \text{pidN}, \text{vars} = v, \text{pc} = s, \text{channels} = [], \text{id} = \text{sidx'}))$ 
    by (simp add: cl-inv-def)

  have
     $\bigwedge p. \text{cl-inv}(g,p) \implies$ 
     $\text{cl-inv } (\text{foldl } (\lambda(g,p) d. \text{mkVarChannel } d (\text{apsnd} \circ \text{pState.vars-update}) g p) (g,p) \text{ decls})$ 
  proof (induction decls arbitrary: g p)
  case (Cons d ds)
  obtain g' p' where
    new: (g',p') = (mkVarChannel d (apsnd  $\circ$  pState.vars-update) g p)
    by (metis prod.exhaust)
  with Cons.premis have cl-inv (g',p')
    by (auto intro!: mkVarChannel-cl-inv)

  from Cons.IH[OF this] new show ?case by (simp add: o-def)
qed simp

from this[OF P-inv] show ?thesis by auto
qed

declare mkProc.simps[simp del]

definition runProc
  :: String.literal  $\Rightarrow$  expr list  $\Rightarrow$  program
   $\Rightarrow$  'a gState-scheme  $\Rightarrow$  pState
   $\Rightarrow$  'a gState-scheme * pState
where
  runProc name args prog g p = (
    if length (procs g)  $\geq$  max-procs
    then abort STR "Too many processes" ( $\lambda\_. (g,p)$ )
    else let pid = length (procs g) + 1 in

```

```

      case lm.lookup name (proc-data prog) of
        None  $\Rightarrow$  abortv STR "No such process: " name
              ( $\lambda\_. (g,p)$ )
      | Some proc-idx  $\Rightarrow$ 
        let (g', proc) = mkProc g p name args (processes prog !! proc-idx) pid
        in (g' [procs := procs g @ [proc]], p))

lemma runProc-gState-progress-rel:
  assumes program-inv prog
  and gState-inv prog g
  and pState-inv prog p
  and cl-inv (g,p)
  shows (g, fst (runProc name args prog g p))  $\in$  gState-progress-rel prog
proof (cases length (procs g) < max-procs)
  note IArray.sub-def [simp del]

  case True thus ?thesis
  proof (cases lm.lookup name (proc-data prog))
    case (Some proc-idx)
    hence *: proc-idx < IArray.length (processes prog)
           fst (processes prog !! proc-idx) = proc-idx
    using assms
    by (simp-all add: lm.correct program-inv-def)

    obtain g' p' where
      new: (g',p') = mkProc g p name args (processes prog !! proc-idx) (length (procs
g) + 1)
    by (metis prod.exhaust)
    hence g': g' = fst ... and p': p' = snd ...
    by (metis snd-conv fst-conv)+
    from assms g' have (g, g')  $\in$  gState-progress-rel prog
    by (auto intro!: mkProc-gState-progress-rel)

    moreover
    from * assms True p' have pState-inv prog p'
    by (auto intro!: mkProc-pState-inv)
    moreover
    from assms new have cl-inv (g',p')
    by (auto intro!: mkProc-cl-inv)
    ultimately show ?thesis
    using True Some new assms
    unfolding runProc-def gState-progress-rel-def
    by (clarsimp split: prod.split)
    (auto simp add: gState-inv-def cl-inv-def)
  next
    case None with assms show ?thesis by (auto simp add: runProc-def)
  qed
next
case False with assms show ?thesis by (auto simp add: runProc-def)

```

qed

lemmas *runProc-gState-inv* =
runProc-gState-progress-rel[THEN gState-progress-rel-gState-invI2]

lemma *runProc-pState-id*:
snd (runProc name args prog g p) = p
unfolding *runProc-def*
by (*auto split: if-splits split: option.split prod.split*)

lemma *runProc-pState-inv*:
assumes *pState-inv prog p*
shows *pState-inv prog (snd (runProc name args prog g p))*
by (*simp add: assms runProc-pState-id*)

lemma *runProc-cl-inv*:
assumes *program-inv prog*
and *gState-inv prog g*
and *pState-inv prog p*
and *cl-inv (g,p)*
shows *cl-inv (runProc name args prog g p)*
proof –
obtain *g' p'* **where** **: runProc name args prog g p = (g',p')*
by (*metis prod.exhaust*)
with *runProc-gState-progress-rel[OF assms, of name args]* **have**
length (channels g) ≤ length (channels g')
by (*simp add: gState-progress-rel-def*)
moreover from ** runProc-pState-id* **have** *p' = p* **by** (*metis snd-conv*)
ultimately show *?thesis* **by** (*metis <cl-inv (g,p)> * cl-inv-trans*)
qed

5.7 AST to edges

type-synonym *ast* = *AST.module list*

In this section, the AST is translated into the transition system.

Handling atomic blocks is non-trivial. Therefore, we do this in an extra pass: *lp* and *hp* are the positions of the start and the end of the atomic block. Every edge pointing into this range is therefore marked as *Atomic*. If they are pointing somewhere else, they are set to *InAtomic*, meaning: they start *in* an atomic block, but leave it afterwards.

definition *atomize* :: *nat ⇒ nat ⇒ edge list ⇒ edge list* **where**

atomize lp hp es = *fold (λe es.*

let e' = case target e of

LabelJump - *None* ⇒

- Labels are checked again later on, when they
- are going to be resolved. Hence it is safe to say
- *atomic* here, especially as the later algorithm

— relies on targets in atomic blocks to be marked as such.

```

    e⟦ atomic := InAtomic ⟧
  | LabelJump - (Some via) ⇒
    if lp ≤ via ∧ hp ≥ via then e⟦ atomic := Atomic ⟧
    else e⟦ atomic := InAtomic ⟧
  | Index p' ⇒
    if lp ≤ p' ∧ hp ≥ p' then e⟦ atomic := Atomic ⟧
    else e⟦ atomic := InAtomic ⟧
in e'#es) es []

```

```

fun skip — No-(edge)
where
  skip (lbls, pri, pos, nxt, -) =
    ([[cond = ECEExpr (ExprConst 1),
      effect = EEId, target = nxt, prio = pri,
      atomic = NonAtomic]], Index pos, lbls)

```

The AST is walked backwards. This allows to know the next state directly.

Parameters used:

lbls Map of Labels

pri Current priority

pos Current position in the array

nxt Next state

onxt Previous 'next state' (where to jump after a 'do')

inBlock Needed for certain constructs to calculate the layout of the array

```

fun stepToState
  :: step
  ⇒ (labels * integer * nat * edgeIndex * edgeIndex option * bool)
  ⇒ edge list list * edgeIndex * labels
and stmtnToState
  :: stmtnt
  ⇒ (labels * integer * nat * edgeIndex * edgeIndex option * bool)
  ⇒ edge list list * edgeIndex * labels
where
  stepToState (StepStmnt s None) data = stmtnToState s data
  | stepToState (StepStmnt s (Some u)) (lbls, pri, pos, nxt, onxt, -) = (
    let
      — the unless part
      (ues, -, lbls') = stmtnToState u (lbls, pri, pos, nxt, onxt, True);
      u = last ues; ues = butlast ues;
      pos' = pos + length ues;

      — find minimal current priority

```

$pri = \text{min-prio } u \text{ } pri;$
 — the guarded part –
 — priority is decreased, because there is now a new unless part with
 — higher prio
 $(ses, spos, lbls'') = \text{stmntToState } s \text{ } (lbls', pri - 1, pos', nxt, onxt, False);$
 — add an edge to the unless part for each generated state
 $ses = \text{map } (List.append \ u) \ ses$
in
 $(ues @ ses, spos, lbls'')$

$| \text{stepToState } (StepDecl \ decls) \ (lbls, pri, pos, nxt, onxt, -) = ($
 $\quad \text{let } edgeF = \lambda d \ (lbls, pri, pos, nxt, -).$
 $\quad \quad ([[(cond = ECTrue, effect = EEDecl \ d, target = nxt,$
 $\quad \quad \quad prio = pri, atomic = NonAtomic)]]], Index \ pos, lbls)$
in
 $\quad \text{step-fold } edgeF \ decls \ lbls \ pri \ pos \ nxt \ onxt \ False)$

$| \text{stepToState } StepSkip \ (lbls, -, -, nxt, -) = ([], nxt, lbls)$

$| \text{stmntToState } (StmntAtomic \ steps) \ (lbls, pri, pos, nxt, onxt, inBlock) = ($
 $\quad \text{let } (es, pos', lbls') = \text{step-fold } stepToState \ steps \ lbls \ pri \ pos \ nxt \ onxt \ inBlock \ \text{in}$
 $\quad \text{let } es' = \text{map } (atomize \ pos \ (pos + length \ es)) \ es \ \text{in}$
 $\quad (es', pos', lbls'))$

$| \text{stmntToState } (StmntLabeled \ l \ s) \ (lbls, pri, pos, d) = ($
 $\quad \text{let}$
 $\quad \quad (es, pos', lbls) = \text{stmntToState } s \ (lbls, pri, pos, d);$
 — We don't resolve goto-chains. If the labeled stmnt returns only a jump,
 — use this goto state.
 $\quad \quad lpos = \text{case } pos' \ \text{of } Index \ p \Rightarrow p \mid - \Rightarrow pos;$
 $\quad \quad lbls' = \text{add-label } l \ lbls \ lpos$
in
 $\quad (es, pos', lbls'))$

$| \text{stmntToState } (StmntDo \ stepss) \ (lbls, pri, pos, nxt, onxt, inBlock) = ($
 $\quad \text{let}$
 — construct the different branches
 — nxt in those branches points current pos (it is a loop after all)
 — $onxt$ then is the current nxt (needed for break, f.ex.)
 $\quad \quad (-, -, lbls, es, is) = \text{step-foldL } stepToState \ stepss \ lbls \ pri$
 $\quad \quad \quad (pos+1) \ (Index \ pos) \ (Some \ nxt);$
 — put the branch starting points (is) into the array
 $\quad \quad es' = \text{concat } is \ \# \ es$
in
 $\quad \text{if } inBlock \ \text{then}$

— inside another DO or IF or UNLESS
 — \longrightarrow append branches again, so they can be consumed
 ($es' @ [concat\ is], Index\ pos, lbls$)
 else
 ($es', Index\ pos, lbls$)
)

| $stmntToState\ (StmntIf\ stepss)\ (lbls, pri, pos, nxt, onxt, -) =$
 $\quad let\ (pos, -, lbls, es, is) = step-foldL\ stepToState\ stepss\ lbls\ pri\ pos\ nxt\ onxt$
 $\quad in\ (es @ [concat\ is], Index\ pos, lbls)$

| $stmntToState\ (StmntSeq\ steps)\ (lbls, pri, pos, nxt, onxt, inBlock) =$
 $\quad step-fold\ stepToState\ steps\ lbls\ pri\ pos\ nxt\ onxt\ inBlock$

| $stmntToState\ (StmntAssign\ v\ e)\ (lbls, pri, pos, nxt, -) =$
 $\quad ([[(cond = ECTrue, effect = EEAssign\ v\ e, target = nxt, prio = pri,$
 $\quad \quad atomic = NonAtomic)]]], Index\ pos, lbls)$

| $stmntToState\ (StmntAssert\ e)\ (lbls, pri, pos, nxt, -) =$
 $\quad ([[(cond = ECTrue, effect = EEAssert\ e, target = nxt, prio = pri,$
 $\quad \quad atomic = NonAtomic)]]], Index\ pos, lbls)$

| $stmntToState\ (StmntCond\ e)\ (lbls, pri, pos, nxt, -) =$
 $\quad ([[(cond = ECEExpr\ e, effect = EEId, target = nxt, prio = pri,$
 $\quad \quad atomic = NonAtomic)]]], Index\ pos, lbls)$

| $stmntToState\ StmntElse\ (lbls, pri, pos, nxt, -) =$
 $\quad ([[(cond = ECElse, effect = EEId, target = nxt, prio = pri,$
 $\quad \quad atomic = NonAtomic\ \emptyset)]]], Index\ pos, lbls)$

| $stmntToState\ StmntBreak\ (lbls, pri, -, -, Some\ onxt, -) =$
 $\quad ([[(cond = ECTrue, effect = EEGoto, target = onxt, prio = pri,$
 $\quad \quad atomic = NonAtomic\ \emptyset)]]], onxt, lbls)$

| $stmntToState\ StmntBreak\ (-, -, -, None, -) =$
 $\quad abort\ STR\ "Misplaced\ break"\ (\lambda-. ([], Index\ 0, lm.empty()))$

| $stmntToState\ (StmntRun\ n\ args)\ (lbls, pri, pos, nxt, onxt, -) =$
 $\quad ([[(cond = ECRun\ n, effect = EERun\ n\ args, target = nxt, prio = pri,$
 $\quad \quad atomic = NonAtomic\ \emptyset)]]], Index\ pos, lbls)$

| $stmntToState\ (StmntGoTo\ l)\ (lbls, pri, pos, -) =$
 $\quad ([[(cond = ECTrue, effect = EEGoto, target = LabelJump\ l\ None, prio = pri,$
 $\quad \quad atomic = NonAtomic\ \emptyset)]]], LabelJump\ l\ (Some\ pos), lbls)$

| $stmntToState\ (StmntSend\ v\ e\ srt)\ (lbls, pri, pos, nxt, -) =$
 $\quad ([[(cond = ECSend\ v, effect = EESend\ v\ e\ srt, target = nxt, prio = pri,$
 $\quad \quad atomic = NonAtomic\ \emptyset)]]], Index\ pos, lbls)$

```

| stmntToState (StmntRecv v r srt rem) (lbls, pri, pos, nxt, -) =
  ([[cond = ERecv v r srt, effect = ERecv v r srt rem, target = nxt, prio =
    pri,
    atomic = NonAtomic 0]], Index pos, lbls)

| stmntToState StmntSkip d = skip d

```

5.7.1 Setup

definition *endState* :: *edge list* **where**

— An extra state added to each process marking its end.

```

endState = [[cond = ECFalse, effect = EEEnd, target = Index 0, prio = 0,
  atomic = NonAtomic 0]]

```

definition *resolveLabel* :: *String.literal* \Rightarrow *labels* \Rightarrow *nat* **where**

```

resolveLabel l lbls = (
  case lm.lookup l lbls of
    None  $\Rightarrow$  abortv STR "Unresolved label: " l (\-. 0)
  | Some pos  $\Rightarrow$  pos)

```

primrec *resolveLabels* :: *edge list list* \Rightarrow *labels* \Rightarrow *edge list* \Rightarrow *edge list* **where**

```

resolveLabels - - [] = []
| resolveLabels edges lbls (e#es) = (
  let check-atomic =  $\lambda$ pos. fold ( $\lambda$ e a. a  $\wedge$  inAtomic e) (edges ! pos) True in
  case target e of
    Index -  $\Rightarrow$  e
  | LabelJump l None  $\Rightarrow$ 
    let pos = resolveLabel l lbls in
    e[target := Index pos,
      atomic := if inAtomic e then
        if check-atomic pos then Atomic
        else InAtomic
      else NonAtomic 0]
  | LabelJump l (Some via)  $\Rightarrow$ 
    let pos = resolveLabel l lbls in
    e[target := Index pos,
      — NB: isAtomic instead of inAtomic, cf atomize()
      atomic := if isAtomic e then
        if check-atomic pos  $\wedge$  check-atomic via then Atomic
        else InAtomic
      else atomic e 0]
    ) # (resolveLabels edges lbls es)

```

definition *calculatePrios* :: *edge list list* \Rightarrow (*integer* * *edge list*) *list* **where**

```

calculatePrios ess = map ( $\lambda$ es. (min-prio es 0, es)) ess

```

definition *toStates* :: *step list* \Rightarrow *states* * *edgeIndex* * *labels* **where**

```

toStates steps = (
  let

```

```

    (states,pos,lbls) = step-fold stepToState steps (lm.empty())
                                0 1 (Index 0) None False;

    pos = (case pos of
            Index - => pos
          | LabelJump l - => Index (resolveLabel l lbls));
    states = endState # states;
    states = map (resolveLabels states lbls) states;
    states = calculatePrios states
  in
    case pos of Index s =>
      if s < length states then (IArray states, pos, lbls)
      else abort STR "Start index out of bounds" (λ-. (IArray states, Index 0,
lbls)))

```

lemma *toStates-inv*:

```

  assumes toStates steps = (ss,start,lbls)
  shows ∃ s. start = Index s ∧ s < IArray.length ss
    and IArray.length ss > 0
  using assms
  unfolding toStates-def calculatePrios-def
  by (auto split: prod.splits edgeIndex.splits if-splits)

```

primrec *toProcess*

```

  :: nat => proc => states * nat * String.literal * (labels * process)
  where
    toProcess sidx (ProcType act name args decls steps) = (
      let
        (states, start, lbls) = toStates steps;
        act = (case act of
              None => 0
            | Some None => 1
            | Some (Some x) => nat-of-integer x)
      in
        (states, act, name, lbls, sidx, start, args, decls))
  | toProcess sidx (Init decls steps) = (
    let (states, start, lbls) = toStates steps in
    (states, 1, STR ":init:", lbls, sidx, start, [], decls))

```

lemma *toProcess-sidx*:

```

  toProcess sidx p = (ss,a,n,l,idx,r) ==> idx = sidx
  by (cases p) (simp-all split: prod.splits)

```

lemma *toProcess-states-nonempty*:

```

  toProcess sidx p = (ss,a,n,l,idx,r) ==> IArray.length ss > 0
  by (cases p) (force split: prod.splits dest: toStates-inv(2))+

```

lemma *toProcess-start*:

```

  toProcess sidx p = (ss,a,n,l,idx,start,r)

```

$\Rightarrow \exists s. \text{start} = \text{Index } s \wedge s < \text{IArray.length } ss$
by (cases p) (force split: prod.splits dest: toStates-inv(1))+

lemma *toProcess-startE*:

assumes *toProcess* $\text{sid } p = (ss, a, n, l, \text{idx}, \text{start}, r)$
obtains s **where** $\text{start} = \text{Index } s \wedge s < \text{IArray.length } ss$
using *toProcess-start* [OF *assms*]
by *blast*

The main construction function. Takes an AST and returns an initial state, and the program (= transition system).

definition *setUp* :: $\text{ast} \Rightarrow \text{program} \times \text{gState}$ **where**

setUp ast = (
 let
 (decls, procs, -) = preprocess ast;
 assertVar = Var (VTBounded 0 1) 0;

 pre-procs = map (case-prod toProcess) (List.enumerate 1 procs);

 procs = IArray ((0, Index 0, [], []) # map ($\lambda(-, -, -, -, p). p$) pre-procs);
 labels = IArray (lm.empty() # map ($\lambda(-, -, -, l, -). l$) pre-procs);
 states = IArray (IArray [(0, [])] # map ($\lambda(s, -). s$) pre-procs);
 names = IArray (STR "invalid" # map ($\lambda(-, -, n, -). n$) pre-procs);

 proc-data = lm.to-map (map ($\lambda(-, -, n, -, \text{idx}, -). (n, \text{idx})$) pre-procs);

 prog = [] processes = procs, labels = labels, states = states,
 proc-names = names, proc-data = proc-data [];

 g = [] vars = lm.sng (STR "--assert--") assertVar,
 channels = [InvChannel], timeout = False, procs = [] [];
 g' = foldl ($\lambda g d.$
 fst (mkVarChannel d (apfst \circ gState.vars-update) g emptyProc)
) g decls;
 g'' = foldl ($\lambda g (-, a, \text{name}, -).$
 foldl ($\lambda g \text{name}.$
 fst (runProc name [] prog g emptyProc)
) g (replicate a name)
) g' pre-procs
 in
 (prog, g''))

lemma *setUp-program-inv'*:

program-inv (fst (setUp ast))

proof (rule *program-invI*, goal-cases)

case 1 show ?case **by** (simp add: setUp-def split: prod.split)

next

case 2 show ?case **by** (simp add: setUp-def split: prod.split)

```

next
  case 3 thus ?case
    by (auto simp add: setUp-def o-def
        split: prod.splits
        dest!: toProcess-states-nonempty)
next
  case 4 thus ?case
    unfolding setUp-def
    by (auto simp add: lm.correct o-def in-set-enumerate-eq nth-enumerate-eq
        dest!: subsetD[OF Misc.ran-map-of] toProcess-sidx
        split: prod.splits)

next
  case 5 thus ?case
    apply (auto simp add: setUp-def o-def split: prod.splits)
    apply (frule toProcess-sidx)
    apply (frule toProcess-start)
    apply (auto simp: in-set-enumerate-eq nth-enumerate-eq)
    done
qed

lemma setUp-program-inv:
  assumes setUp ast = (prog, g)
  shows program-inv prog
using assms setUp-program-inv'
by (metis fst-conv)

lemma setUp-gState-inv:
  assumes setUp ast = (prog, g)
  shows gState-inv prog g
proof -
  from assms have p-INV: program-inv prog by (fact setUp-program-inv)

  {
    fix prog :: program
    assume *: program-inv prog
    let ?g = (| vars = lm.sng (STR "--assert--") (Var (VTBounded 0 1) 0),
              channels = [InvChannel], timeout = False, procs = [] |)

    have g1: gState-inv prog ?g
      by (simp add: gState-inv-def max-channels-def lm-correct max-var-value-def)
    {
      fix g decls
      assume gState-inv prog g
      hence gState-inv prog (foldl (λg d.
        fst (mkVarChannel d (apfst ∘ gState.vars-update) g emptyProc)
      ) g decls)
      apply (rule foldl-rule)
      apply (intro mkVarChannel-gState-inv)
    }
  }

```

```

    apply simp
  apply (frule-tac g= $\sigma$  in toVariable-variable-inv')
    apply assumption
  apply (auto simp add: gState-inv-def lm.correct)
done
}
note g2 = this[OF g1]

{
  fix g :: 'a gState-scheme and pre-procs
  assume gState-inv prog g
  hence gState-inv prog (foldl ( $\lambda g$  (-,a,name,-).
    foldl ( $\lambda g$  name.
      fst (runProc name [] prog g emptyProc)
    ) g (replicate a name)
  ) g pre-procs)
  apply (rule foldl-rule)
  apply (clarsimp split: prod.splits)
  apply (rule foldl-rule)
  apply (auto intro!: runProc-gState-inv emptyProc-pState-inv *)
done
}
note this[OF g2]
} note g-INV = this

from assms p-INV show ?thesis
  unfolding setUp-def
  by (auto split: prod.splits intro!: g-INV)
qed

```

5.8 Semantic Engine

After constructing the transition system, we are missing the final part: The successor function on this system. We use SPIN-nomenclature and call it *semantic engine*.

definition $\text{assertVar} \equiv \text{VarRef True (STR "--assert--")} \text{ None}$

5.8.1 Evaluation of Edges

```

fun evalRecvArgs
  :: recvArg list  $\Rightarrow$  integer list  $\Rightarrow$  gStateI  $\Rightarrow$  pState  $\Rightarrow$  gStateI * pState
where
  evalRecvArgs [] [] g l = (g,l)
| evalRecvArgs - [] g l =
  abort STR "Length mismatch on receiving." ( $\lambda$ -. (g,l))
| evalRecvArgs [] - g l =
  abort STR "Length mismatch on receiving." ( $\lambda$ -. (g,l))
| evalRecvArgs (r#rs) (v#vs) g l = (
  let (g,l) =

```

```

    case r of
      RecvArgVar var  $\Rightarrow$  setVar var v g l
    | -  $\Rightarrow$  (g,l)
  in evalRecvArgs rs vs g l)

primrec evalCond
  :: edgeCond  $\Rightarrow$  gStateI  $\Rightarrow$  pState  $\Rightarrow$  bool
where
  evalCond ECTrue - -  $\longleftrightarrow$  True
| evalCond ECFalse - -  $\longleftrightarrow$  False
| evalCond (ECEExpr e) g l  $\longleftrightarrow$  exprArith g l e  $\neq$  0
| evalCond (ECRun -) g l  $\longleftrightarrow$  length (procs g) < 255
| evalCond ECElse g l  $\longleftrightarrow$  gStateI.else g
| evalCond (ECSend v) g l  $\longleftrightarrow$ 
  withChannel v ( $\lambda$ - c.
    case c of
      Channel cap - q  $\Rightarrow$  integer-of-nat (length q) < cap
    | HSChannel -  $\Rightarrow$  True) g l
| evalCond (ECRecv v rs srt) g l  $\longleftrightarrow$ 
  withChannel v ( $\lambda$ i c.
    case c of
      HSChannel -  $\Rightarrow$  handshake g  $\neq$  0  $\wedge$  recvArgsCheck g l rs (hsdata g)
    | -  $\Rightarrow$  pollCheck g l c rs srt) g l

fun evalHandshake
  :: edgeCond  $\Rightarrow$  nat  $\Rightarrow$  gStateI  $\Rightarrow$  pState  $\Rightarrow$  bool
where
  evalHandshake (ECRecv v - -) h g l
     $\longleftrightarrow$  h = 0
     $\vee$  withChannel v ( $\lambda$ i c. case c of
      HSChannel -  $\Rightarrow$  i = h
    | Channel - - -  $\Rightarrow$  False) g l
| evalHandshake - h - -  $\longleftrightarrow$  h = 0

primrec evalEffect
  :: edgeEffect  $\Rightarrow$  program  $\Rightarrow$  gStateI  $\Rightarrow$  pState  $\Rightarrow$  gStateI * pState
where
  evalEffect EEEnd - g l = (g,l)
| evalEffect EEId - g l = (g,l)
| evalEffect EEGoto - g l = (g,l)
| evalEffect EEAssign v e - g l = setVar v (exprArith g l e) g l
| evalEffect EEDecl d - g l = mkVarChannelProc d g l
| evalEffect EERun name args prog g l = runProc name args prog g l
| evalEffect EEAssert e - g l = (
  if exprArith g l e = 0
  then setVar assertVar 1 g l
  else (g,l))
| evalEffect (EESend v es srt) - g l = withChannel v ( $\lambda$ i c.
  let

```

```

    ab = λ-. abort STR "Length mismatch on sending." (λ-. (g,l));
    es = map (exprArith g l) es
  in
    if ¬ for-all (λx. x ≥ min-var-value ∧ x ≤ max-var-value) es
    then abort STR "Invalid Channel" (λ-. (g,l))
    else
      case c of
        Channel cap ts q ⇒
          if length ts ≠ length es ∨ ¬ (length q < max-array-size)
          then ab()
          else let
            q' = if ¬ srt then q@[es]
            else let
              q = map lexic q;
              q' = insort (lexic es) q
            in map unlex q';
            g = gState.channels-update (λcs.
              cs[ i := Channel cap ts q' ]) g
          in (g,l)
        | HSChannel ts ⇒
          if length ts ≠ length es then ab()
          else (g[hsdata := es, handshake := i], l)
        | InvChannel ⇒ abort STR "Trying to send on invalid channel" (λ-. (g,l))
      ) g l
  | evalEffect (EERecv v rs srt rem) - g l = withChannel v (λi c.
    case c of
      Channel cap ts qs ⇒
        if qs = [] then abort STR "Recv from empty channel" (λ-. (g,l))
        else
          let
            (q', qs') = if ¬ srt then (hd qs, tl qs)
            else apfst the (find-remove (recvArgsCheck g l rs) qs);
            (g,l) = evalRecvArgs rs q' g l;
            g = if rem
              then gState.channels-update (λcs. cs[ i := Channel cap ts qs' ]) g
              else g
            — messages are not removed – so no need to update anything
          in (g,l)
        | HSChannel - ⇒
          let (g,l) = evalRecvArgs rs (hsdata g) g l in
          let g = g[ handshake := 0, hsdata := [] ]
          in (g,l)
        | InvChannel ⇒ abort STR "Receiving on invalid channel" (λ-. (g,l))
      ) g l

```

lemma *statesDecls-effect*:
assumes $ef \in \text{effect}$ ‘ edgeSet ss
and $ef = \text{EEDecl } d$
shows $d \in \text{statesDecls ss}$


```

proof –
  from assms obtain e where  $e \in \text{edgeSet } ss \text{ ef} = \text{effect } e$  by auto
  thus ?thesis using assms
    unfolding statesDecls-def
    by (auto simp add: edgeDecls-def
      intro!: bezI[where x = e]
      split: edgeEffect.split)
qed

lemma evalRecvArgs-pState-inv:
  assumes pState-inv prog p
  shows pState-inv prog (snd (evalRecvArgs rargs xs g p))
  using assms
proof (induction rargs xs arbitrary: p g rule: list-induct2')
  case ( $\lambda r \text{ rs } x \text{ xs}$ ) thus ?case
  proof (cases r)
    case (RecvArgVar v)
      obtain  $g' \text{ } p'$  where  $\text{new: setVar } v \text{ } x \text{ } g \text{ } p = (g', p')$  by (metis prod.exhaust)
      hence  $p' = \text{snd (setVar } v \text{ } x \text{ } g \text{ } p)$  by simp
      with  $\lambda$  have pState-inv prog p' by (auto intro!: setVar-pState-inv)
      from  $\lambda$ .IH[OF this] RecvArgVar new show ?thesis by simp
    qed simp-all
  qed simp-all

lemma evalRecvArgs-pState-inv':
  assumes evalRecvArgs rargs xs g p = (g', p')
  and pState-inv prog p
  shows pState-inv prog p'
using assms evalRecvArgs-pState-inv
by (metis snd-conv)

lemma evalRecvArgs-gState-progress-rel:
  assumes gState-inv prog g
  shows  $(g, \text{fst (evalRecvArgs rargs xs g p)}) \in \text{gState-progress-rel prog}$ 
  using assms
proof (induction rargs xs arbitrary: p g rule: list-induct2')
  case ( $\lambda r \text{ rs } x \text{ xs}$ ) thus ?case
  proof (cases r)
    case (RecvArgVar v)
      obtain  $g' \text{ } p'$  where  $\text{new: setVar } v \text{ } x \text{ } g \text{ } p = (g', p')$  by (metis prod.exhaust)
      hence  $g' = \text{fst (setVar } v \text{ } x \text{ } g \text{ } p)$  by simp
      with  $\lambda$  have  $(g, g') \in \text{gState-progress-rel prog}$ 
      by (auto intro!: setVar-gState-progress-rel)
      also hence gState-inv prog g'
      by (rule gState-progress-rel-gState-invI2)
      note  $\lambda$ .IH[OF this, of p']
      finally show ?thesis using RecvArgVar new by simp
    qed simp-all
  qed simp-all

```

```

lemmas evalRecvArgs-gState-inv =
  evalRecvArgs-gState-progress-rel[THEN gState-progress-rel-gState-invI2]

lemma evalRecvArgs-cl-inv:
  assumes cl-inv (g,p)
  shows cl-inv (evalRecvArgs rargs xs g p)
  using assms
proof (induction rargs xs arbitrary: p g rule: list-induct2')
  case (4 r rs x xs) thus ?case
  proof (cases r)
    case (RecvArg Var v)
      with 4 have cl-inv (setVar v x g p)
      by (auto intro!: setVar-cl-inv)
      with 4.IH RecvArg Var show ?thesis
      by (auto split: prod.splits)
    qed simp-all
  qed simp-all

lemma evalEffect-pState-inv:
  assumes pState-inv prog p
  and gState-inv prog g
  and cl-inv (g,p)
  and e ∈ effect ' edgeSet (states prog !! pState.idx p)
  shows pState-inv prog (snd (evalEffect e prog g p))
  using assms
proof (cases e)
  case (EEDecl d)
    with assms have d ∈ statesDecls (states prog !! pState.idx p)
    using statesDecls-effect
    by simp
    with assms EEDecl show ?thesis
    by (auto simp del: IArray.sub-def
      intro!: mkVarChannelProc-pState-inv)
  next
    case (EESend c es srt)
    then obtain v where ChanRef v = c by (cases c) simp
    with EESend assms show ?thesis
    by (cases v) (auto simp add: withChannel'-def withVar'-def split: channel.split)
  next
    case (EERecv c es srt)
    then obtain v where ChanRef v = c by (cases c) simp
    with EERecv assms show ?thesis
    by (cases v)
      (auto simp: withChannel'-def withVar'-def
        split: prod.splits channel.split
        dest: evalRecvArgs-pState-inv')
  qed (clarsimp-all intro!: setVar-pState-inv runProc-pState-inv)

```

```

lemma evalEffect-gState-progress-rel:
  assumes program-inv prog
  and gState-inv prog g
  and pState-inv prog p
  and cl-inv (g,p)
  shows (g, fst (evalEffect e prog g p)) ∈ gState-progress-rel prog
using assms
proof (cases e)
  case EEAssert
  with assms show ?thesis
    by (auto intro: setVar-gState-progress-rel)
next
  case EEAssign
  with assms show ?thesis
    by (auto intro: setVar-gState-progress-rel)
next
  case EEDecl
  with assms show ?thesis
    by (auto intro: mkVarChannelProc-gState-progress-rel)
next
  case EERun
  with assms show ?thesis
    by (auto intro: runProc-gState-progress-rel)
next
  case (EESend c es srt)
  then obtain v where v: c = ChanRef v by (metis chanRef.exhaust)
  obtain idx where idx: nat-of-integer (the (getVar v g p)) = idx by blast
  note idx' = idx[symmetric, unfolded getVar-def]
  show ?thesis
  proof (cases idx < length (gState.channels g))
    case True
    note DEF = True EESend v idx' assms

    show ?thesis
    proof (cases gState.channels g ! idx)
      case (Channel cap ts q)
      with True have Channel cap ts q ∈ set (gState.channels g)
        by (metis nth-mem)
      with assms have channel-inv (Channel cap ts q)
        by (auto simp add: gState-inv-def simp del: channel-inv.simps)
      with Channel DEF show ?thesis
        by (cases v)
          (auto simp add: withChannel'-def withVar'-def for-all-def
            split: channel.split
            intro!: gState-progress-rel-channels-update)
    next
    case HSCchannel with DEF show ?thesis
      by (cases v)
        (auto simp: withChannel'-def withVar'-def gState-progress-rel-def

```

```

      gState-inv-def
      split: channel.split)
next
  case InvChannel with DEF show ?thesis
    by (cases v) (auto simp add: withChannel'-def withVar'-def)
qed
next
  case False with EESend idx' v assms show ?thesis
    by (cases v) (auto simp add: withChannel'-def withVar'-def)
qed
next
  case (EERcv c rs srt rem)
  then obtain v where v: c = ChanRef v by (metis chanRef.exhaust)
  obtain idx where idx: nat-of-integer (the (getVar v g p)) = idx by blast
  note idx' = idx[symmetric, unfolded getVar-def]
  show ?thesis
  proof (cases idx < length (gState.channels g))
    case True
    note DEF = True EERcv v idx' assms
    show ?thesis
    proof (cases gState.channels g ! idx)
      note channel-inv.simps[simp del]
      case (Channel cap ts q)
      with True have Channel cap ts q ∈ set (gState.channels g)
        by (metis nth-mem)
      with assms have c-inv: channel-inv (Channel cap ts q)
        by (auto simp add: gState-inv-def simp del: channel-inv.simps)
      moreover obtain res q' where
        apfst the (find-remove (recvArgsCheck g p rs) q) = (res, q')
        by (metis prod.exhaust)
      moreover hence q' = snd (find-remove (recvArgsCheck g p rs) q)
        by (simp add: apfst-def map-prod-def split: prod.splits)
      with find-remove-subset find-remove-length have
        set q' ⊆ set q length q' ≤ length q
        by (metis surjective-pairing)+
      with c-inv have channel-inv (Channel cap ts q')
        by (auto simp add: channel-inv.simps)
      moreover {
        assume q ≠ []
        hence set (tl q) ⊆ set q using tl-subset by auto
        with c-inv have channel-inv (Channel cap ts (tl q))
          by (auto simp add: channel-inv.simps)
      }
    }
    moreover {
      fix res g' p'
      assume evalRecvArgs rs res g p = (g', p')
      with evalRecvArgs-gState-progress-rel assms have
        (g, g') ∈ gState-progress-rel prog
        by (metis fst-conv)
    }
  }

```

```

    hence length (channels g) ≤ length (channels g')
    by (simp add: gState-progress-rel-def)
  }
ultimately show ?thesis using Channel DEF
  apply (cases v)
  apply (auto simp add: withChannel'-def withVar'-def for-all-def
    split: channel.split prod.split
    elim: fstE
    intro!: evalRecvArgs-gState-progress-rel
    gState-progress-rel-channels-update-step)
    apply force+
  done
next
case HSCchannel
obtain g' p' where *: evalRecvArgs rs (hsdata g) g p = (g', p')
  by (metis prod.exhaust)
with assms have (g, g') ∈ gState-progress-rel prog
  by (auto elim!: fstE intro!: evalRecvArgs-gState-progress-rel)
also hence gState-inv prog g' by blast
hence (g', g'(|handshake := 0, hsdata := [])) ∈ gState-progress-rel prog
  by (auto simp add: gState-progress-rel-def gState-inv-def)
finally
have (g, g'(|handshake := 0, hsdata := [])) ∈ gState-progress-rel prog .
with DEF HSCchannel * show ?thesis
  by (cases v) (auto simp add: withChannel'-def withVar'-def for-all-def
    split: channel.split prod.split)
next
case InvChannel with DEF show ?thesis
  by (cases v) (auto simp add: withChannel'-def withVar'-def)
qed
next
case False with EERecv idx' v assms show ?thesis
  by (cases v) (auto simp add: withChannel'-def withVar'-def)
qed
qed simp-all

lemma evalEffect-cl-inv:
  assumes cl-inv (g, p)
  and program-inv prog
  and gState-inv prog g
  and pState-inv prog p
  shows cl-inv (evalEffect e prog g p)
  using assms
proof (cases e)
  case EERun with assms show ?thesis by (force intro!: runProc-cl-inv)
next
case (EESend c es srt) then obtain v where ChanRef v = c by (cases c) simp
with EESend assms show ?thesis
  by (cases v)

```

```

      (auto simp add: withChannel'-def withVar'-def split: channel.split
        intro!: cl-inv-channels-update)
next
case (EERecv c es srt) then obtain v where ChanRef v = c by (cases c) simp
with EERecv assms show ?thesis
  apply (cases v)
  apply (auto simp add: withChannel'-def withVar'-def split: channel.split prod.split
    intro!: cl-inv-channels-update)
  apply (metis evalRecvArgs-cl-inv)+
  done
qed (simp-all add: setVar-cl-inv mkVarChannelProc-cl-inv)

```

5.8.2 Executable edges

To find a successor global state, we first need to find all those edges which are executable (i. e. the condition evaluates to true).

type-synonym *choices* = (edge * pState) list

— A choice is an executable edge and the process it belongs to.

definition *getChoices* :: gState_I ⇒ pState ⇒ edge list ⇒ choices **where**
getChoices g p = foldl (λE e.
 if evalHandshake (cond e) (handshake g) g p ∧ evalCond (cond e) g p
 then (e,p)#E
 else E) []

lemma *getChoices-sub-edges-fst*:
 fst ‘ set (getChoices g p es) ⊆ set es
unfolding *getChoices-def*
by (rule foldl-rule-aux) auto

lemma *getChoices-sub-edges*:
 (a,b) ∈ set (getChoices g p es) ⇒ a ∈ set es
using *getChoices-sub-edges-fst*
by force

lemma *getChoices-p-snd*:
 snd ‘ set (getChoices g p es) ⊆ {p}
unfolding *getChoices-def*
by (rule foldl-rule-aux) auto

lemma *getChoices-p*:
 (a,b) ∈ set (getChoices g p es) ⇒ b = p
using *getChoices-p-snd*
by force

definition *sort-by-pri* **where**
sort-by-pri min-pri edges = foldl (λes e.
 let idx = nat-of-integer (abs (prio e))
 in if idx > min-pri

```

    then abort STR "Invalid priority" (λ-. es)
    else let ep = e # (es ! idx) in es[idx := ep]
  ) (replicate (min-pri + 1) []) edges

```

```

lemma sort-by-pri-edges':
  assumes set edges ⊆ A
  shows set (sort-by-pri min-pri edges) ⊆ {xs. set xs ⊆ A}
using assms
unfolding sort-by-pri-def
apply (rule-tac I=λσ -. (∀x ∈ set σ. set x ⊆ A) ∧ length σ = min-pri + 1 in
  foldl-rule-aux-P)
  apply simp
  apply (force dest!: subsetD[OF set-update-subset-insert]
    split: if-splits)
apply force
done

```

```

lemma sort-by-pri-edges:
  assumes set edges ⊆ A
  and es ∈ set (sort-by-pri min-pri edges)
  shows set es ⊆ A
using sort-by-pri-edges'[OF assms(1)] assms
by auto

```

```

lemma sort-by-pri-length:
  length (sort-by-pri min-pri edges) = min-pri + 1
unfolding sort-by-pri-def
by (rule foldl-rule-aux-P [where I=λσ -. length σ = min-pri + 1])
  simp-all

```

```

definition executable
  :: states iarray ⇒ gState_I ⇒ choices nres
  — Find all executable edges
where
  executable ss g = (
    let procs = procs g in
    nfoldli procs (λ-. True) (λp E.
      if (exclusive g = 0 ∨ exclusive g = pid p) then do {
        let (min-pri, edges) = (ss !! pState.idx p) !! pc p;
        ASSERT(set edges ⊆ edgeSet (ss !! pState.idx p));

        (E', -, -) ←
          if min-pri = 0 then do {
            WHILE_T (λ(E, brk, -). E = [] ∧ brk = 0) (λ (-, -, ELSE). do {
              let g = g[gState_I.else := ELSE];
              E = getChoices g p edges
            })
          in
            if E = [] then (
              if ¬ ELSE then RETURN (E, 0::nat, True)

```

```

      else RETURN (E, 1, False))
      else RETURN (E, 1, ELSE) }) ([], 0::nat, False)
    } else do {
      let min-pri = nat-of-integer (abs min-pri);
      let pri-edges = sort-by-pri min-pri edges;
      ASSERT (∀ es ∈ set pri-edges.
        set es ⊆ edgeSet (ss !! pState.idx p));
      let pri-edges = IArray pri-edges;

      WHILE_T (λ(E,pri,-). E = [] ∧ pri ≤ min-pri) (λ(-, pri, ELSE). do
{
  let es = pri-edges !! pri;
  let g = g(gState_I.else := ELSE);
  let E = getChoices g p es;
  if E = [] then (
    if ¬ ELSE then RETURN (E,pri,True)
    else RETURN (E, pri + 1, False))
  else RETURN (E, pri, ELSE) }) ([], 0, False)
    };
    RETURN (E'@E)
  } else RETURN E
) []
)

```

definition

```

while-rel1 =
  measure (λx. if x = [] then 1 else 0)
<*lex*> measure (λx. if x = 0 then 1 else 0)
<*lex*> measure (λx. if ¬ x then 1 else 0)

```

lemma wf-while-rel1:

wf while-rel1

unfolding while-rel1-def

by auto

definition

```

while-rel2 mp =
  measure (λx. if x = [] then 1 else 0)
<*lex*> measure (λx. (mp + 1) - x)
<*lex*> measure (λx. if ¬ x then 1 else 0)

```

lemma wf-while-rel2:

wf (while-rel2 mp)

unfolding while-rel2-def

by auto

lemma executable-edgeSet:

assumes gState-inv prog g

and program-inv prog


```

and  $ss = \text{states } prog$ 
shows  $\text{executable } ss \ g$ 
 $\leq SPEC \ (\lambda cs. \forall (e,p) \in \text{set } cs.$ 
 $\quad e \in \text{edgeSet } (ss \ !! \ pState.idx \ p)$ 
 $\quad \wedge \ pState\text{-inv } prog \ p$ 
 $\quad \wedge \ cl\text{-inv } (g,p))$ 
unfolding  $\text{executable-def}$ 
using  $assms$ 
apply  $(\text{refine-rcg } \text{refine-vcg } \text{nfoldli-rule}[\text{where}$ 
 $\quad I = \lambda - \text{ } cs. \forall (e,p) \in \text{set } cs.$ 
 $\quad \quad e \in \text{edgeSet } (ss \ !! \ pState.idx \ p)$ 
 $\quad \quad \wedge \ pState\text{-inv } prog \ p$ 
 $\quad \quad \wedge \ cl\text{-inv } (g,p))]$ 
 $\quad \text{apply } \text{simp}$ 
 $\quad \text{apply } (\text{rename-tac } p \ l1 \ l2 \ \sigma \ e \ p')$ 
 $\quad \text{apply } (\text{subgoal-tac } pState\text{-inv } prog \ p)$ 
 $\quad \text{apply } (\text{clarsimp } \text{simp } \text{add: } \text{edgeSet-def } pState\text{-inv-def})[]$ 
 $\quad \text{apply } (\text{rule-tac } x = IArray.\text{list-of}$ 
 $\quad \quad (IArray.\text{list-of } (\text{states } prog) \ ! \ pState.idx \ p) \ ! \ pc \ p \ \text{in } \text{be } I)$ 
 $\quad \text{apply } \text{simp}$ 
 $\quad \text{apply } (\text{force } \text{intro!} : \text{nth-mem})$ 
 $\quad \text{apply } (\text{force } \text{simp } \text{add: } gState\text{-inv-def})$ 
apply  $(\text{refine-rcg } \text{refine-vcg } \text{wf-while-rel1 } \text{WHILET-rule}[\text{where}$ 
 $\quad I = \lambda (cs, -, -). \forall (e,p) \in \text{set } cs.$ 
 $\quad \quad e \in \text{edgeSet } (ss \ !! \ pState.idx \ p)$ 
 $\quad \quad \wedge \ pState\text{-inv } prog \ p$ 
 $\quad \quad \wedge \ cl\text{-inv } (g,p)$ 
 $\quad \text{and } R = \text{while-rel1}])$ 
 $\quad \text{apply } (\text{vc-solve } \text{solve: } \text{asm-rl } \text{simp } \text{add: } \text{while-rel1-def})$ 
 $\quad \text{apply } (\text{frule } \text{getChoices-p})$ 
 $\quad \text{using } \text{getChoices-sub-edges } \text{apply } (\text{force } \text{simp } \text{add: } gState\text{-inv-def})$ 
 $\quad \text{using } \text{sort-by-pri-edges } \text{apply } \text{fastforce}$ 
apply  $(\text{rename-tac } \text{min-pri } \text{pri-edges})$ 
apply  $(\text{rule-tac } I = \lambda (cs, -, -). \forall (e,p) \in \text{set } cs.$ 
 $\quad \quad e \in \text{edgeSet } (ss \ !! \ pState.idx \ p)$ 
 $\quad \quad \wedge \ pState\text{-inv } prog \ p$ 
 $\quad \quad \wedge \ cl\text{-inv } (g,p)$ 
 $\quad \text{and } R = \text{while-rel2 } (\text{nat-of-integer } (\text{abs } \text{min-pri}))$ 
 $\quad \text{in } \text{WHILET-rule})$ 
 $\quad \text{apply } (\text{simp } \text{add: } \text{wf-while-rel2})$ 
 $\quad \text{apply } \text{simp}$ 
apply  $(\text{refine-rcg } \text{refine-vcg})$ 
 $\quad \text{apply } (\text{clarsimp-all } \text{split: } \text{prod.split } \text{simp } \text{add: } \text{while-rel2-def})$ 
 $\quad \text{apply } (\text{metis } \text{diff-less-mono } \text{lessI})$ 
 $\quad \text{apply } (\text{rename-tac } idx' \ \text{else } e \ p')$ 
 $\quad \text{apply } (\text{frule } \text{getChoices-p})$ 
 $\quad \text{apply } (\text{frule } \text{getChoices-sub-edges})$ 
 $\quad \text{apply } (\text{subgoal-tac}$ 
 $\quad \quad \text{sort-by-pri } (\text{nat-of-integer } |\text{min-pri}|) \ \text{pri-edges } \ ! \ idx')$ 

```

```

      ∈ set (sort-by-pri (nat-of-integer |min-pri|) pri-edges))
    apply (auto simp add: assms gState-inv-def)[]
    apply (force simp add: sort-by-pri-length)
    apply (auto simp add: assms)
  done

```

```

lemma executable-edgeSet':
  assumes gState-inv prog g
  and program-inv prog
  shows executable (states prog) g
  ≤ SPEC (λcs. ∀ (e,p) ∈ set cs.
    e ∈ edgeSet ((states prog) !! pState.idx p)
    ∧ pState-inv prog p
    ∧ cl-inv(g,p))
using executable-edgeSet[where ss = states prog] assms
by simp

```

```

schematic-goal executable-refine:
  RETURN (?ex s g) ≤ executable s g
unfolding executable-def
by (refine-transfer)

```

concrete-definition executable-impl **for** s g **uses** executable-refine

5.8.3 Successor calculation

```

function toI where
  toI (| gState.vars = v, channels = ch, timeout = t, procs = p |)
    = (| gState.vars = v, channels = ch, timeout = False, procs = p,
      handshake = 0, hsdata = [], exclusive = 0, gStateI.else = False |)
by (metis gState.cases) (metis gState.ext-inject)
termination by lexicographic-order

```

```

function fromI where
  fromI (| gState.vars = v, channels = ch, timeout = t, procs = p, ... = m |)
    = (| gState.vars = v, channels = ch, timeout = t, procs = p |)
by (metis gState.surjective) (metis gState.ext-inject)
termination by lexicographic-order

```

```

function resetI where
  resetI (| gState.vars = v, channels = ch, timeout = t, procs = p,
    handshake = hs, hsdata = hsd, exclusive = -, gStateI.else = - |)
    = (| gState.vars = v, channels = ch, timeout = False, procs = p,
      handshake = 0, hsdata = if hs ≠ 0 then hsd else [], exclusive = 0,
      gStateI.else = False |)
by (metis (full-types) gStateI.surjective unit.exhaust)
  (metis gState.select-convs gStateI.select-convs)
termination by lexicographic-order

```

lemma *gState-inv-to_I*:
gState-inv prog g = gState-inv prog (to_I g)
by (*cases g, simp add: gState-inv-def cl-inv-def*)

lemma *gState-inv-from_I*:
gState-inv prog g = gState-inv prog (from_I g)
by (*cases g, simp add: gState-inv-def cl-inv-def*)

lemma *gState-inv-reset_I*:
gState-inv prog g = gState-inv prog (reset_I g)
by (*cases g, simp add: gState-inv-def cl-inv-def*)

lemmas *gState-inv-I-simps =*
gState-inv-to_I gState-inv-from_I gState-inv-reset_I

definition *removeProcs*

— Remove ended processes, if there is no running one with a higher pid.

where

*removeProcs ps = foldr (λp (dead,sd,ps,dc).
 if dead ∧ pc p = 0 then (True, True, ps, pState.channels p @ dcs)
 else (False, sd, p#ps, dcs)) ps (True, False, [], [])*

lemma *removeProcs-subset'*:
set (fst (snd (snd (removeProcs ps)))) ⊆ set ps
unfolding *removeProcs-def*
apply (*subst foldr-conv-foldl*)
apply (*rule foldl-rule-aux-P[where I=λ(-,ps',-) . set ps' ⊆ set ps]*)
apply *simp*
apply (*clarsimp split: if-splits*)
apply *force*
apply (*rename-tac p*)
apply (*case-tac p = x*)
apply (*subst set-rev[symmetric]*)
apply *simp*
apply *force*
apply *force*
done

lemma *removeProcs-length'*:
length (fst (snd (snd (removeProcs ps)))) ≤ length ps
unfolding *removeProcs-def*
apply (*subst foldr-conv-foldl*)
apply (*rule foldl-rule-aux-P[where*
I=λ(-,ps',-) ps''. length ps' + length ps'' ≤ length ps])
apply (*auto split: if-splits*)
done

lemma *removeProcs-subset*:

$\text{removeProcs } ps = (\text{dead}, \text{sd}, \text{ps}', \text{dcs}) \implies \text{set } \text{ps}' \subseteq \text{set } ps$
using $\text{removeProcs-subset}'$
by ($\text{metis fst-conv snd-conv}$)

lemma $\text{removeProcs-length}$:
 $\text{removeProcs } ps = (\text{dead}, \text{sd}, \text{ps}', \text{dcs}) \implies \text{length } \text{ps}' \leq \text{length } ps$
using $\text{removeProcs-length}'$
by ($\text{metis fst-conv snd-conv}$)

definition $\text{cleanChans} :: \text{integer list} \Rightarrow \text{channels} \Rightarrow \text{channels}$
 — Mark channels of closed processes as invalid.
where
 $\text{cleanChans dchans cs} = \text{snd } (\text{foldl } (\lambda(i, cs) c.$
 if $\text{List.member dchans } i$ *then* $(i + 1, \text{cs@[InvChannel]})$
 else $(i + 1, \text{cs@[c]})$) $(0, [])$ cs)

lemma $\text{cleanChans-channel-inv}$:
assumes $\text{set } cs \subseteq \text{Collect channel-inv}$
shows $\text{set } (\text{cleanChans dchans cs}) \subseteq \text{Collect channel-inv}$
using assms
unfolding cleanChans-def
apply ($\text{rule-tac foldl-rule-aux}$)
apply ($\text{force split: if-splits}$) +
done

lemma cleanChans-length :
 $\text{length } (\text{cleanChans dchans cs}) = \text{length } cs$
unfolding cleanChans-def
by ($\text{rule foldl-rule-aux-P}$ [**where**
 $I = \lambda(-, cs') cs''. \text{length } cs' + \text{length } cs'' = \text{length } cs$])
 ($\text{force split: if-splits}$) +

definition $\text{checkDeadProcs} :: 'a \text{ gState-scheme} \Rightarrow 'a \text{ gState-scheme}$ **where**
 $\text{checkDeadProcs } g =$
 let $(-, \text{soDied}, \text{procs}, \text{dchans}) = \text{removeProcs } (\text{procs } g)$ *in*
 if soDied *then*
 $g \setminus \text{procs} := \text{procs}, \text{channels} := \text{cleanChans dchans } (\text{channels } g)$
 else g)

lemma $\text{checkDeadProcs-gState-progress-rel}$:
assumes $g \text{ gState-inv prog } g$
shows $(g, \text{checkDeadProcs } g) \in g \text{ gState-progress-rel prog}$
using $\text{assms cleanChans-length}$ [**where** $cs = \text{channels } g$]
 $\text{cleanChans-channel-inv}$ [**where** $cs = \text{channels } g$]
unfolding $\text{checkDeadProcs-def}$
apply ($\text{intro gState-progress-relI}$)
 apply assumption
 apply ($\text{clarsimp split: if-splits prod.splits}$)
 apply ($\text{frule removeProcs-length}$)

```

    apply (frule removeProcs-subset)
    apply (auto simp add: gState-inv-def cl-inv-def)[]
  apply (clarsimp split: if-splits prod.splits)+
done

```

lemma *gState-progress-rel-exclusive*:

```

  (g, g') ∈ gState-progress-rel prog
  ⇒ (g, g'⟦exclusive := p⟧) ∈ gState-progress-rel prog
by (simp add: gState-progress-rel-def gState-inv-def cl-inv-def)

```

definition *applyEdge*

```

  :: program ⇒ edge ⇒ pState ⇒ gStateI ⇒ gStateI nres

```

where

```

  applyEdge prog e p g = do {

```

```

    let (g',p') = evalEffect (effect e) prog g p;
    ASSERT ((g,g') ∈ gState-progress-rel prog);
    ASSERT (pState-inv prog p');
    ASSERT (cl-inv (g',p'));

    let p'' = (case target e of Index t ⇒
      if t < IArray.length (states prog !! pState.idx p') then p'⟦pc := t⟧
      else abort STR "Edge target out of bounds" (λ-. p')
    | - ⇒ abort STR "Edge target not Index" (λ-. p'));
    ASSERT (pState-inv prog p'');

    let g'' = g'⟦procs := list-update (procs g') (pid p'' - 1) p''⟧;
    ASSERT ((g',g'') ∈ gState-progress-rel prog);

    let g''' = (if isAtomic e ∧ handshake g'' = 0
      then g''⟦exclusive := pid p''⟧
      else g'');
    ASSERT ((g',g''') ∈ gState-progress-rel prog);

    let gf = (if pc p'' = 0 then checkDeadProcs g''' else g''');
    ASSERT ((g''',gf) ∈ gState-progress-rel prog);
    RETURN gf }

```

lemma *applyEdge-gState-progress-rel*:

```

  assumes program-inv prog
  and gState-inv prog g
  and pState-inv prog p
  and cl-inv (g,p)
  and e ∈ edgeSet (states prog !! pState.idx p)
  shows applyEdge prog e p g ≤ SPEC (λg'. (g,g') ∈ gState-progress-rel prog)
using assms
unfolding applyEdge-def
apply (intro refine-vcg)

```

```

apply (force elim: fstE intro!: evalEffect-gState-progress-rel)

apply (force elim: sndE intro!: evalEffect-pState-inv)

apply (drule subst)
apply (rule evalEffect-cl-inv)
apply assumption+

apply (cases target e)
apply (clarsimp simp add: pState-inv-def)
apply simp

apply (frule gState-progress-rel-gState-invI2)
apply (cases target e)
  apply (clarsimp split: if-splits)
  apply (intro gState-progress-relI)
    apply assumption
    apply (auto simp add: gState-inv-def cl-inv-def
      dest!: subsetD[OF set-update-subset-insert])[]
  apply (simp add: cl-inv-def)
  apply simp
apply (intro gState-progress-relI)
  apply assumption
  apply (auto simp add: gState-inv-def cl-inv-def
    dest!: subsetD[OF set-update-subset-insert])[]
  apply (simp add: cl-inv-def)
  apply simp
apply (clarsimp split: if-splits)
apply (intro gState-progress-relI)
  apply assumption
  apply (auto simp add: gState-inv-def cl-inv-def
    dest!: subsetD[OF set-update-subset-insert])[]
  apply (simp add: cl-inv-def)
  apply simp

apply (auto split: if-splits intro!: gState-progress-rel-exclusive)[]

apply (force intro!: checkDeadProcs-gState-progress-rel)

apply (blast intro!: gState-progress-rel-trans)
done

schematic-goal applyEdge-refine:
  RETURN (?ae prog e p g) ≤ applyEdge prog e p g
unfolding applyEdge-def
by (refine-transfer)

concrete-definition applyEdge-impl for e p g uses applyEdge-refine

```

definition *nexts*

$:: \text{program} \Rightarrow g\text{State} \Rightarrow g\text{State} \text{ ls nres}$

— The successor function

where

nexts prog g = (
let
f = from_I;
g = to_I g
in

REC ($\lambda D \ g. \text{ do } \{$
E $\leftarrow \text{executable } (\text{states prog}) \ g;$
if *E* $= []$ *then*
if *handshake g* $\neq 0$ *then*
— HS not possible – remove current step
RETURN (ls.empty())
else if *exclusive g* $\neq 0$ *then*
— Atomic blocks – just return current state
RETURN (ls.sng (f g))
else if $\neg \text{timeout } g$ *then*
— Set timeout
D (g(|timeout := True|))
else
— If all else fails: stutter
RETURN (ls.sng (f (reset_I g)))
else
— Setting the internal variables (exclusive, handshake, ...) to 0
— is safe – they are either set by the edges, or not thought
— to be used outside executable.
let *g = reset_I g in*
nfoldli E ($\lambda -. \text{True}$) ($\lambda (e,p) \ G.$
applyEdge prog e p g $\gg= (\lambda \ g'.$
if *handshake g'* $\neq 0 \vee \text{isAtomic } e$ *then do* $\{$
G_R $\leftarrow D \ g';$
if *ls.isEmpty G_R \wedge handshake g' = 0 then*
— this only happens if the next step is a handshake, which fails
— hence we stay at the current state
RETURN (ls.ins (f g') G)
else
RETURN (ls.union G_R G)
} else RETURN (ls.ins (f g') G))) (ls.empty())
}) g
$\gg= (\lambda G. \text{ if } \text{ls.isEmpty } G \text{ then RETURN (ls.sng (f g)) else RETURN } G)$
)

lemma *gState-progress-rel-intros:*

$(\text{to}_I \ g, \ gI') \in g\text{State-progress-rel prog}$

$\implies (g, \text{from}_I \ gI') \in g\text{State-progress-rel prog}$

$(gI, \ gI') \in g\text{State-progress-rel prog}$

$\implies (gI, \text{reset}_I \ gI') \in \text{gState-progress-rel prog}$
 $(\text{to}_I \ g, gI') \in \text{gState-progress-rel prog}$
 $\implies (\text{to}_I \ g, gI' \backslash \text{timeout} := t) \in \text{gState-progress-rel prog}$
unfolding *gState-progress-rel-def*
by (cases *g*, cases *gI'*, cases *gI*, force simp add: *gState-inv-def cl-inv-def*)+

lemma *gState-progress-rel-step-intros*:
 $(\text{to}_I \ g, g') \in \text{gState-progress-rel prog}$
 $\implies (\text{reset}_I \ g', g'') \in \text{gState-progress-rel prog}$
 $\implies (g, \text{from}_I \ g'') \in \text{gState-progress-rel prog}$
 $(\text{to}_I \ g, g') \in \text{gState-progress-rel prog}$
 $\implies (\text{reset}_I \ g', g'') \in \text{gState-progress-rel prog}$
 $\implies (\text{to}_I \ g, g'') \in \text{gState-progress-rel prog}$
unfolding *gState-progress-rel-def*
by (cases *g*, cases *g'*, cases *g''*, force simp add: *gState-inv-def cl-inv-def*)+

lemma *cl-inv-reset_I*:
 $\text{cl-inv}(g, p) \implies \text{cl-inv}(\text{reset}_I \ g, p)$
by (cases *g*) (simp add: *cl-inv-def*)

lemmas *refine-helpers* =
gState-progress-rel-intros gState-progress-rel-step-intros cl-inv-reset_I

lemma *nexts-SPEC*:
assumes *gState-inv prog g*
and *program-inv prog*
shows *nexts prog g* \leq *SPEC* ($\lambda g s. \forall g' \in \text{ls}. \alpha \ g s. (g, g') \in \text{gState-progress-rel prog}$)
using *assms*
unfolding *nexts-def*
apply (*refine-rcg refine-vcg REC-rule*[**where**
 $\text{pre} = \lambda g'. (\text{to}_I \ g, g') \in \text{gState-progress-rel prog}$])
apply (*simp add: gState-inv-to_I*)
apply (*rule order-trans*[*OF executable-edgeSet'*])
apply (*drule gState-progress-rel-gState-invI2*)
apply *assumption*
apply *assumption*
apply (*refine-rcg refine-vcg nfoldli-rule*[**where**
 $I = \lambda - . \sigma. \forall g' \in \text{ls}. \alpha \ \sigma. (g, g') \in \text{gState-progress-rel prog}$
 $\text{order-trans}[\text{OF applyEdge-gState-progress-rel}]$])
apply (*vc-solve intro: refine-helpers solve: asm-rl simp add: ls.correct*)
apply (*rule order-trans*)
apply (*rprems*)
apply (*vc-solve intro: refine-helpers solve: asm-rl simp add: ls.correct*)
apply (*rule order-trans*)
apply (*rprems*)
apply (*vc-solve intro: refine-helpers solve: asm-rl simp add: ls.correct*)
done

lemma *RETURN-dRETURN*:

$RETURN\ f \leq f' \implies nres\text{-}of\ (dRETURN\ f) \leq f'$

unfolding *nres-of-def*

by *simp*

lemma *executable-dRETURN*:

$nres\text{-}of\ (dRETURN\ (executable\text{-}impl\ prog\ g)) \leq executable\ prog\ g$

using *executable-impl.refine*

by (*simp add: RETURN-dRETURN*)

lemma *applyEdge-dRETURN*:

$nres\text{-}of\ (dRETURN\ (applyEdge\text{-}impl\ prog\ e\ p\ g)) \leq applyEdge\ prog\ e\ p\ g$

using *applyEdge-impl.refine*

by (*simp add: RETURN-dRETURN*)

schematic-goal *nexts-code-aux*:

$nres\text{-}of\ (?nexts\ prog\ g) \leq nexts\ prog\ g$

unfolding *nexts-def*

by (*refine-transfer the-resI executable-dRETURN applyEdge-dRETURN*)

concrete-definition *nexts-code-aux* **for** *prog g* **uses** *nexts-code-aux*

prepare-code-thms *nexts-code-aux-def*

5.8.4 Handle non-termination

A Promela model may include non-terminating parts. Therefore we cannot guarantee, that *nexts* will actually terminate. To avoid having to deal with this in the model checker, we fail in case of non-termination.

definition *SUCCEED-abort* **where**

$SUCCEED\text{-}abort\ msg\ dm\ m = ($

$\text{case } m \text{ of}$

$RES\ X \Rightarrow \text{if } X = \{\} \text{ then } Code.abort\ msg\ (\lambda\cdot. dm) \text{ else } RES\ X$

$| \cdot \Rightarrow m)$

definition *dSUCCEED-abort* **where**

$dSUCCEED\text{-}abort\ msg\ dm\ m = ($

$\text{case } m \text{ of}$

$dSUCCEEDi \Rightarrow Code.abort\ msg\ (\lambda\cdot. dm)$

$| \cdot \Rightarrow m)$

definition *ref-succeed* **where**

$ref\text{-}succeed\ m\ m' \longleftrightarrow m \leq m' \wedge (m = SUCCEED \longrightarrow m' = SUCCEED)$

lemma *dSUCCEED-abort-SUCCEED-abort*:

$\llbracket RETURN\ dm' \leq dm; ref\text{-}succeed\ (nres\text{-}of\ m')\ m \rrbracket$

$\implies nres\text{-}of\ (dSUCCEED\text{-}abort\ msg\ (dRETURN\ dm')\ (m'))$

$\leq SUCCEED\text{-}abort\ msg\ dm\ m$

unfolding *dSUCCEED-abort-def SUCCEED-abort-def ref-succeed-def*

by (auto split: dres.splits nres.splits)

The final successor function now incorporates:

1. *nexts*
2. handling of non-termination

definition *nexts-code* **where**

```

nexts-code prog g =
  the-res (dSUCCEED-abort (STR "The Universe is broken!"))
    (dRETURN (ls.sng g))
    (nexts-code-aux prog g)

```

lemma *nexts-code-SPEC*:

```

assumes gState-inv prog g
and program-inv prog
shows g' ∈ ls.α (nexts-code prog g)
      ⇒ (g,g') ∈ gState-progress-rel prog
unfolding nexts-code-def
unfolding dSUCCEED-abort-def
using assms
using order-trans[OF nexts-code-aux.refine nexts-SPEC[OF assms(1,2)]]
by (auto split: dres.splits simp: ls.correct)

```

5.9 Finiteness of the state space

inductive-set *reachable-states*

```

for P :: program
and gs :: gState — start state
where

```

```

gs ∈ reachable-states P gs |
g ∈ reachable-states P gs ⇒ x ∈ ls.α (nexts-code P g)
      ⇒ x ∈ reachable-states P gs

```

lemmas *reachable-states-induct*[case-names *init step*] =
reachable-states.induct[split-format (complete)]

lemma *reachable-states-finite*:

```

assumes program-inv prog
and gState-inv prog g
shows finite (reachable-states prog g)
proof (rule finite-subset[OF - gStates-finite[of - g]])
  define INV where INV g' ⇔ g' ∈ (gState-progress-rel prog)* “{g} ∧ gState-inv
prog g' for g'

```

```

{
  fix g'
  have g' ∈ reachable-states prog g ⇒ INV g'
  proof (induct rule: reachable-states-induct)

```

```

    case init with assms show ?case by (simp add: INV-def)
next
case (step g g')
from step(2,3) have
  (g, g') ∈ gState-progress-rel prog
  using nexts-code-SPEC[OF - ⟨program-inv prog⟩]
  unfolding INV-def by auto
thus ?case using step(2) unfolding INV-def by auto
qed
}

thus reachable-states prog g ⊆
  (gState-progress-rel prog)* “{g}”
  unfolding INV-def by auto
qed

```

5.10 Traces

When trying to generate a lasso, we have a problem: We only have a list of global states. But what are the transitions to come from one to the other?

This problem shall be tackled by *replay*: Given two states, it generates a list of transitions that was taken.

definition *replay* :: *program* \Rightarrow *gState* \Rightarrow *gState* \Rightarrow *choices nres* **where**
replay prog g₁ g₂ = (
 let
 g₁ = to_I g₁;
 check = λg. from_I g = g₂
 in
 REC_T (λD g. do {
 E ← executable (states prog) g;
 if E = [] then
 if check g then RETURN []
 else if ¬ timeout g then D (g (timeout := True))
 else abort STR "Stuttering should not occur on replay"
 (λ-. RETURN [])
 else
 let g = reset_I g in
 nfoldli E (λE. E = []) (λ(e,p) -.
 applyEdge prog e p g \gg (λg'.
 if handshake g' ≠ 0 ∨ isAtomic e then do {
 E_R ← D g';
 if E_R = [] then
 if check g' then RETURN [(e,p)] else RETURN []
 else
 RETURN ((e,p) # E_R)
 } else if check g' then RETURN [(e,p)] else RETURN [])
) []
 }) g₁

)

lemma *abort-refine*[*refine-transfer*]:

$nres\text{-}of\ (f\ ()) \leq F\ () \implies nres\text{-}of\ (abort\ s\ f) \leq abort\ s\ F$

$f() \neq dSUCCEED \implies abort\ s\ f \neq dSUCCEED$

by *auto*

schematic-goal *replay-code-aux*:

$RETURN\ (?replay\ prog\ g_1\ g_2) \leq replay\ prog\ g_1\ g_2$

unfolding *replay-def applyEdge-def*

by (*refine-transfer the-resI executable-dRETURN*)

concrete-definition *replay-code* **for** *prog g₁ g₂* **uses** *replay-code-aux*
prepare-code-thms *replay-code-def*

5.10.1 Printing of traces

definition *procDescr*

$:: (integer \Rightarrow string) \Rightarrow program \Rightarrow pState \Rightarrow string$

where

procDescr f prog p = (

let

name = String.explode (proc-names prog !! pState.idx p);

id = f (integer-of-nat (pid p))

in

name @ " (" @ id @ " "))

definition *printInitial*

$:: (integer \Rightarrow string) \Rightarrow program \Rightarrow gState \Rightarrow string$

where

printInitial f prog g₀ = (

let psS = printList (procDescr f prog) (procs g₀) [] [] " " in

"Initially running: " @ psS)

abbreviation *lf* $\equiv CHR\ 0x0A$

fun *printConfig*

$:: (integer \Rightarrow string) \Rightarrow program \Rightarrow gState\ option \Rightarrow gState \Rightarrow string$

where

printConfig f prog None g₀ = printInitial f prog g₀

| *printConfig f prog (Some g₀) g₁ = (*

let eps = replay-code prog g₀ g₁ in

let print = ($\lambda(e,p).$ procDescr f prog p @ ": " @ printEdge f (pc p) e)

in if eps = [] \wedge g₁ = g₀ then " -- stutter --"

else printList print eps [] [] (lf# " "))

definition *printConfigFromAST f* $\equiv printConfig\ f\ o\ fst\ o\ setUp$

5.11 Code export

code-identifier

code-module *PromelaInvariants* \rightarrow (SML) *Promela*
code-module *PromelaDatastructures* \rightarrow (SML) *Promela*

definition *executable-triv* *prog g* = *executable-impl* (*snd prog*) *g*

definition *apply-triv* *prog g ep* = *applyEdge-impl* *prog* (*fst ep*) (*snd ep*) (*reset_I* *g*)

export-code

setUp printProcesses printConfigFromAST nexts-code executable-triv apply-triv
extractLTLs lookupLTL
checking *SML*

export-code

setUp printProcesses printConfigFromAST nexts-code executable-triv apply-triv
extractLTLs lookupLTL
in *SML*
file \langle *Promela.sml* \rangle

end

6 LTL integration

theory *PromelaLTL*

imports

Promela
LTL.LTL

begin

We have a semantic engine for Promela. But we need to have an integration with LTL – more specifically, we must know when a proposition is true in a global state. This is achieved in this theory.

6.1 LTL optimization

For efficiency reasons, we do not store the whole *expr* on the labels of a system automaton, but *nat* instead. This index then is used to look up the corresponding *expr*.

type-synonym *APs* = *expr iarray*

primrec *ltlc-aps-list'* :: '*a* *ltlc* \Rightarrow '*a* *list* \Rightarrow '*a* *list*

where

ltlc-aps-list' *True-ltlc l* = *l*
| *ltlc-aps-list'* *False-ltlc l* = *l*
| *ltlc-aps-list'* (*Prop-ltlc p*) *l* = (if *List.member l p* then *l* else *p#l*)
| *ltlc-aps-list'* (*Not-ltlc x*) *l* = *ltlc-aps-list' x l*
| *ltlc-aps-list'* (*Next-ltlc x*) *l* = *ltlc-aps-list' x l*

```

| ltlc-aps-list' (Final-ltlc x) l = ltlc-aps-list' x l
| ltlc-aps-list' (Global-ltlc x) l = ltlc-aps-list' x l
| ltlc-aps-list' (And-ltlc x y) l = ltlc-aps-list' y (ltlc-aps-list' x l)
| ltlc-aps-list' (Or-ltlc x y) l = ltlc-aps-list' y (ltlc-aps-list' x l)
| ltlc-aps-list' (Implies-ltlc x y) l = ltlc-aps-list' y (ltlc-aps-list' x l)
| ltlc-aps-list' (Until-ltlc x y) l = ltlc-aps-list' y (ltlc-aps-list' x l)
| ltlc-aps-list' (Release-ltlc x y) l = ltlc-aps-list' y (ltlc-aps-list' x l)
| ltlc-aps-list' (WeakUntil-ltlc x y) l = ltlc-aps-list' y (ltlc-aps-list' x l)
| ltlc-aps-list' (StrongRelease-ltlc x y) l = ltlc-aps-list' y (ltlc-aps-list' x l)

```

lemma *ltlc-aps-list'-correct*:

```

set (ltlc-aps-list'  $\varphi$  l) = atoms-ltlc  $\varphi \cup set\ l$ 
by (induct  $\varphi$  arbitrary: l) (auto simp add: in-set-member)

```

lemma *ltlc-aps-list'-distinct*:

```

distinct l  $\implies distinct (ltlc-aps-list' \varphi l)$ 
by (induct  $\varphi$  arbitrary: l) (auto simp add: in-set-member)

```

definition *ltlc-aps-list* :: '*a* *ltlc* \Rightarrow '*a* *list*

where

```

ltlc-aps-list  $\varphi$  = ltlc-aps-list'  $\varphi []$ 

```

lemma *ltlc-aps-list-correct*:

```

set (ltlc-aps-list  $\varphi$ ) = atoms-ltlc  $\varphi$ 
unfolding ltlc-aps-list-def
by (force simp: ltlc-aps-list'-correct)

```

lemma *ltlc-aps-list-distinct*:

```

distinct (ltlc-aps-list  $\varphi$ )
unfolding ltlc-aps-list-def
by (auto intro: ltlc-aps-list'-distinct)

```

primrec *idx'* :: *nat* \Rightarrow '*a* *list* \Rightarrow '*a* \Rightarrow *nat option* **where**

```

idx' - [] = None
| idx' ctr (x#xs) y = (if x = y then Some ctr else idx' (ctr+1) xs y)

```

definition *idx* = *idx' 0*

lemma *idx'-correct*:

```

assumes distinct xs
shows idx' ctr xs y = Some n  $\longleftrightarrow n \geq ctr \wedge n < length\ xs + ctr \wedge xs ! (n-ctr)$ 
= y
using assms
proof (induction xs arbitrary: n ctr)
case (Cons x xs)
show ?case
proof (cases x=y)
case True with Cons.prems have *: y  $\notin set\ xs$  by auto
{

```

```

    assume A: (y#xs)!(n-ctr) = y
    and less: ctr ≤ n
    and length: n < length (y#xs) + ctr
    have n = ctr
    proof (rule ccontr)
      assume n ≠ ctr with less have n-ctr > 0 by auto
      moreover from ⟨n≠ctr⟩ length have n-ctr < length(y#xs) by auto
      ultimately have (y#xs)!(n-ctr) ∈ set xs by simp
      with A * show False by auto
    qed
  }
  with True Cons show ?thesis by auto
next
case False
from Cons have distinct xs by simp
with Cons.IH False have idx' (Suc ctr) xs y = Some n ⟷ Suc ctr ≤ n ∧ n
< length xs + Suc ctr ∧ xs ! (n - Suc ctr) = y
  by simp
with False show ?thesis
  apply -
  apply (rule iffI)
  apply clarsimp-all
  done
qed
qed simp

lemma idx-correct:
  assumes distinct xs
  shows idx xs y = Some n ⟷ n < length xs ∧ xs ! n = y
  using idx'-correct[OF assms]
  by (simp add: idx-def)

lemma idx-dom:
  assumes distinct xs
  shows dom (idx xs) = set xs
  by (auto simp add: idx-correct assms in-set-conv-nth)

lemma idx-image-self:
  assumes distinct xs
  shows (the ∘ idx xs) ' set xs = {..

```

assume $n < \text{length } xs$
moreover with nth-mem **have** $xs ! n \in \text{set } xs$ **by** *simp*
then obtain x **where** $xs ! n = x$ $x \in \text{set } xs$ **by** *simp-all*
ultimately have $\text{idx } xs \ x = \text{Some } n$ **by** (*simp add: idx-correct*[*OF assms*])
hence the $(\text{idx } xs \ x) = n$ **by** *simp*
thus $n \in (\text{the} \circ \text{idx } xs)$ ‘*set xs*
using $\langle x \in \text{set } xs \rangle$
by *auto*
qed

lemma *idx-ran*:
assumes *distinct xs*
shows $\text{ran } (\text{idx } xs) = \{..<\text{length } xs\}$
using *ran-is-image*[**where** $M = \text{idx } xs$]
using *idx-image-self*[*OF assms*] *idx-dom*[*OF assms*]
by *simp*

lemma *idx-inj-on-dom*:
assumes *distinct xs*
shows *inj-on* $(\text{idx } xs)$ $(\text{dom } (\text{idx } xs))$
by (*fastforce simp add: idx-dom assms in-set-conv-nth idx-correct*
intro!: inj-onI)

definition *ltl-convert* :: *expr ltlc* \Rightarrow *APs* \times *nat ltlc* **where**
ltl-convert $\varphi =$ (
let *APs* = *ltlc-aps-list* φ ;
 $\varphi_i = \text{map-ltlc } (\text{the} \circ \text{idx } \text{APs}) \ \varphi$
in (*IArray APs, φ_i*)

lemma *ltl-convert-correct*:
assumes *ltl-convert* $\varphi = (\text{APs}, \varphi_i)$
shows *atoms-ltlc* $\varphi = \text{set } (\text{IArray.list-of } \text{APs})$ (**is** ?*P1*)
and *atoms-ltlc* $\varphi_i = \{..<\text{IArray.length } \text{APs}\}$ (**is** ?*P2*)
and $\varphi_i = \text{map-ltlc } (\text{the} \circ \text{idx } (\text{IArray.list-of } \text{APs})) \ \varphi$ (**is** ?*P3*)
and *distinct* $(\text{IArray.list-of } \text{APs})$

proof –
let ?*APs* = *IArray.list-of* *APs*

from *assms* **have** *APs-def*: ?*APs* = *ltlc-aps-list* φ
unfolding *ltl-convert-def* **by** *auto*

with *ltlc-aps-list-correct* **show** *APs-set*: ?*P1* **by** *metis*

from *assms* **show** ?*P3*
unfolding *ltl-convert-def*
by *auto*

from *assms* **have** *atoms-ltlc* $\varphi_i = (\text{the} \circ \text{idx } ?\text{APs})$ ‘*atoms-ltlc* φ
unfolding *ltl-convert-def*

by (auto simp add: ltlc.set-map)
 moreover from APs-def ltlc-aps-list-distinct show distinct ?APs by simp
 note idx-image-self[OF this]
 moreover note APs-set
 ultimately show ?P2 by simp
 qed

definition prepare

$:: - \times (\text{program} \Rightarrow \text{unit}) \Rightarrow \text{ast} \Rightarrow \text{expr ltlc} \Rightarrow (\text{program} \times \text{APs} \times \text{gState}) \times \text{nat}$
 ltlc

where

prepare cfg ast $\varphi \equiv$

let

(prog, g₀) = Promela.setUp ast;

(APs, φ_i) = PromelaLTL.ltl-convert φ

in

((prog, APs, g₀), φ_i)

lemma prepare-instrument[code]:

prepare cfg ast $\varphi \equiv$

let

(-, printF) = cfg;

- = PromelaStatistics.start ();

(prog, g₀) = Promela.setUp ast;

- = printF prog;

(APs, φ_i) = PromelaLTL.ltl-convert φ ;

- = PromelaStatistics.stop-timer ();

in

((prog, APs, g₀), φ_i)

by (simp add: prepare-def)

export-code prepare checking SML

6.2 Language of a Promela program

definition propValid :: APs \Rightarrow gState \Rightarrow nat \Rightarrow bool where

propValid APs g $i \longleftrightarrow i < \text{IArray.length APs} \wedge \text{exprArith g emptyProc (APs!!i)}$
 $\neq 0$

definition promela-E :: program \Rightarrow (gState \times gState) set

— Transition relation of a promela program

where

promela-E prog $\equiv \{(g, g'). g' \in \text{ls.}\alpha (\text{nexts-code prog } g)\}$

definition promela-E-ltl :: program \times APs \Rightarrow (gState \times gState) set where

promela-E-ltl = promela-E \circ fst

definition promela-is-run' :: program \times gState \Rightarrow gState word \Rightarrow bool

— Predicate defining runs of promela programs

where

$promela-is-run' \text{ prog } r \equiv$
 $\text{let } (prog, g_0) = prog \text{ in}$
 $r \ 0 = g_0$
 $\wedge (\forall i. r \ (Suc \ i) \in ls.\alpha \ (nexts-code \ prog \ (r \ i)))$

abbreviation $promela-is-run \equiv promela-is-run' \circ setUp$

definition $promela-is-run-ltl :: program \times APs \times gState \Rightarrow gState \text{ word} \Rightarrow bool$

where

$promela-is-run-ltl \text{ promg } r \equiv \text{let } (prog, APs, g) = \text{promg} \text{ in } promela-is-run' \ (prog, g)$
 r

definition $promela-props :: gState \Rightarrow \text{expr set}$

where

$promela-props \ g = \{e. \text{exprArith } g \ \text{emptyProc } e \neq 0\}$

definition $promela-props-ltl :: APs \Rightarrow gState \Rightarrow \text{nat set}$

where

$promela-props-ltl \ APs \ g \equiv \text{Collect } (\text{propValid } APs \ g)$

definition $promela-language :: ast \Rightarrow \text{expr set word set}$ **where**

$promela-language \ ast \equiv \{promela-props \circ r \mid r. \text{promela-is-run } ast \ r\}$

definition $promela-language-ltl :: program \times APs \times gState \Rightarrow \text{nat set word set}$

where

$promela-language-ltl \text{ promg} \equiv \text{let } (prog, APs, g) = \text{promg} \text{ in}$
 $\{promela-props-ltl \ APs \circ r \mid r. \text{promela-is-run-ltl } \text{promg } r\}$

lemma $promela-props-ltl-map-aprops:$

assumes $ltl-convert \ \varphi = (APs, \varphi_i)$

shows $promela-props-ltl \ APs =$

$\text{map-props } (idx \ (IArray.list-of \ APs)) \circ promela-props$

proof –

let $?APs = IArray.list-of \ APs$

let $?idx = idx \ ?APs$

from $ltl-convert-correct \ \text{assms}$ **have** $D: \text{distinct } ?APs$ **by** simp

show $?thesis$

proof $(intro \ ext \ set-eqI \ iffI)$

fix $g \ i$

assume $i \in promela-props-ltl \ APs \ g$

hence $\text{propValid } APs \ g \ i$ **by** $(\text{simp add: } promela-props-ltl-def)$

hence $l: i < IArray.length \ APs \ \text{exprArith } g \ \text{emptyProc } (APs!!i) \neq 0$

by $(\text{simp-all add: } \text{propValid-def})$

hence $APs!!i \in promela-props \ g$ **by** $(\text{simp add: } promela-props-def)$

moreover from $idx-correct \ l \ D$ **have** $?idx \ (APs!!i) = \text{Some } i$ **by** fastforce

ultimately show $i \in (\text{map-props } ?idx \circ promela-props) \ g$

```

    unfolding o-def map-props-def
    by blast
next
fix g i
assume i ∈ (map-props ?idx ∘ promela-props) g
then obtain p where p-def: p ∈ promela-props g ?idx p = Some i
    unfolding map-props-def o-def
    by auto
hence expr: exprArith g emptyProc p ≠ 0 by (simp add: promela-props-def)

from D p-def have i < IArray.length APs APs !! i = p
    using idx-correct by fastforce+
with expr have propValid APs g i by (simp add: propValid-def)
thus i ∈ promela-props-ltl APs g
    by (simp add: promela-props-ltl-def)
qed
qed

```

lemma *promela-run-in-language-iff*:
assumes *conv*: *ltl-convert* $\varphi = (APs, \varphi_i)$
shows *promela-props* $\circ \xi \in \text{language-ltlc } \varphi$
 $\longleftrightarrow \text{promela-props-ltl } APs \circ \xi \in \text{language-ltlc } \varphi_i$ (**is** $?L \longleftrightarrow ?R$)

proof –

let $?APs = IArray.\text{list-of } APs$

from *conv* **have** *D*: *distinct* $?APs$
 by (simp add: ltl-convert-correct)
with *conv* **have** *APs*: *atoms-ltlc* $\varphi \subseteq \text{dom } (idx ?APs)$
 by (simp add: idx-dom ltl-convert-correct)

note *map- semantics* = *map- semantics-ltlc*[*OF* *idx-inj-on-dom*[*OF* *D*] *APs*]
promela-props-ltl-map-aprops[*OF* *conv*]
ltl-convert-correct[*OF* *conv*]

have $?L \longleftrightarrow (\text{promela-props} \circ \xi) \models_c \varphi$ **by** (simp add: language-ltlc-def)
also have $\dots \longleftrightarrow (\text{promela-props-ltl } APs \circ \xi) \models_c \varphi_i$
 using *map- semantics*
 by (simp add: o-assoc)
also have $\dots \longleftrightarrow ?R$
 by (simp add: language-ltlc-def)
finally show *?thesis* .

qed

lemma *promela-language-sub-iff*:
assumes *conv*: *ltl-convert* $\varphi = (APs, \varphi_i)$
and *setUp*: *setUp* *ast* = (*prog*, *g*)
shows *promela-language-ltl* (*prog*, *APs*, *g*) $\subseteq \text{language-ltlc } \varphi_i \longleftrightarrow \text{promela-language}$
ast $\subseteq \text{language-ltlc } \varphi$
 using *promela-run-in-language-iff*[*OF* *conv*] *setUp*

```
by (auto simp add: promela-language-ltl-def promela-language-def promela-is-run-ltl-def)
```

```
hide-const (open) abort abortv
           err errv
           usc
           warn the-warn with-warn
```

```
hide-const (open) idx idx'
end
theory PromelaLTLConv
imports
  Promela
  LTL.LTL
begin
```

6.3 Proposition types and conversion

LTL formulae and propositions are also generated by an SML parser. Hence we have the same setup as for Promela itself: Mirror the data structures and (sometimes) map them to new ones.

This theory is intended purely to be used by frontend code to convert from *propc* to *expr*. The other theories work on *expr* directly.

While we could of course convert directly, that would introduce yet a semantic level.

```
datatype binOp = Eq | Le | LEq | Gr | GEq
```

```
datatype ident = Ident String.literal integer option
```

```
datatype propc = CProp ident
              | BProp binOp ident ident
              | BExpProp binOp ident integer
```

```
fun identConv :: ident ⇒ varRef where
  identConv (Ident name None) = VarRef True name None
| identConv (Ident name (Some i)) = VarRef True name (Some (ExprConst i))
```

```
definition ident2expr :: ident ⇒ expr where
  ident2expr = ExprVarRef ∘ identConv
```

```
primrec binOpConv :: binOp ⇒ PromelaDatastructures.binOp where
  binOpConv Eq = BinOpEq
| binOpConv Le = BinOpLe
| binOpConv LEq = BinOpLEq
| binOpConv Gr = BinOpGr
| binOpConv GEq = BinOpGEq
```

```

primrec propc2expr :: propc  $\Rightarrow$  expr where
  propc2expr (CProp ident) =
    ExprBinOp BinOpEq (ident2expr ident) (ExprConst 1)
| propc2expr (BProp bop il ir) =
    ExprBinOp (binOpConv bop) (ident2expr il) (ident2expr ir)
| propc2expr (BExpProp bop il ir) =
    ExprBinOp (binOpConv bop) (ident2expr il) (ExprConst ir)

```

```

definition ltl_conv :: propc ltlc  $\Rightarrow$  expr ltlc where
  ltl_conv = map_ltlc propc2expr

```

```

definition printPropc
  :: (integer  $\Rightarrow$  char list)  $\Rightarrow$  propc  $\Rightarrow$  char list
where
  printPropc f p = printExpr f (propc2expr p)

```

The semantics of a *propc* is given just for reference.

```

definition evalPropc :: gState  $\Rightarrow$  propc  $\Rightarrow$  bool where
  evalPropc g p  $\longleftrightarrow$  exprArith g emptyProc (propc2expr p)  $\neq$  0

```

end

References

- [1] Promela manual pages. <http://spinroot.com/spin/Man/promela.html>. Accessed: 2013-02-07.
- [2] G. J. Holzmann. *The Spin Model Checker — Primer and Reference Manual*. Addison-Wesley, 2003.