

# No Faster-Than-Light Observers

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## Abstract

We provide a formal proof within First Order Relativity Theory that no observer can travel faster than the speed of light. Originally reported by Stannett and Némethi [1].

## Contents

```
theory SpaceTime  
imports Main  
begin
```

```
record 'a Vector =  
  tdir :: 'a  
  xdir :: 'a  
  ydir :: 'a  
  zdir :: 'a
```

```
record 'a Point =  
  tval :: 'a  
  xval :: 'a  
  yval :: 'a  
  zval :: 'a
```

```
record 'a Line =  
  basepoint :: 'a Point  
  direction :: 'a Vector
```

```

record 'a Plane =
  pbasepoint :: 'a Point
  direction1  :: 'a Vector
  direction2  :: 'a Vector

```

```

record 'a Cone =
  vertex :: 'a Point
  slope  :: 'a

```

```

class Quantities = linordered-field

```

```

class Vectors = Quantities
begin

```

```

abbreviation vecZero :: 'a Vector (0) where
  vecZero ≡ (| tdir = (0::'a), xdir = 0, ydir = 0, zdir = 0 |)

```

```

fun vecPlus :: 'a Vector ⇒ 'a Vector ⇒ 'a Vector (infixr ⊕ 100) where
  vecPlus u v = (| tdir = tdir u + tdir v, xdir = xdir u + xdir v,
    ydir = ydir u + ydir v, zdir = zdir u + zdir v |)

```

```

fun vecMinus :: 'a Vector ⇒ 'a Vector ⇒ 'a Vector (infixr ⊖ 100) where
  vecMinus u v = (| tdir = tdir u - tdir v, xdir = xdir u - xdir v,
    ydir = ydir u - ydir v, zdir = zdir u - zdir v |)

```

```

fun vecNegate :: 'a Vector ⇒ 'a Vector (~ -) where
  vecNegate u = (| tdir = uminus (tdir u), xdir = uminus (xdir u),
    ydir = uminus (ydir u), zdir = uminus (zdir u) |)

```

```

fun innerProd :: 'a Vector ⇒ 'a Vector ⇒ 'a (infix dot 50) where
  innerProd u v = (tdir u * tdir v) + (xdir u * xdir v) +
    (ydir u * ydir v) + (zdir u * zdir v)

```

```

fun sqrlen :: 'a Vector ⇒ 'a where sqrlen u = (u dot u)

```

```

fun minkowskiProd :: 'a Vector ⇒ 'a Vector ⇒ 'a (infix mdot 50) where
  minkowskiProd u v = (tdir u * tdir v)
    - ((xdir u * xdir v) + (ydir u * ydir v) + (zdir u * zdir v))

```

```

fun mSqrLen :: 'a Vector ⇒ 'a where mSqrLen u = (u mdot u)

```

```

fun vecScale :: 'a ⇒ 'a Vector ⇒ 'a Vector (infix ** 200) where

```

$vecScale\ k\ u = (\mid\ tdir = k * tdir\ u, xdir = k * xdir\ u, ydir = k * ydir\ u, zdir = k * zdir\ u\ \mid)$

**fun** *orthogonal* :: 'a Vector  $\Rightarrow$  'a Vector  $\Rightarrow$  bool (**infix**  $\perp$  150) **where**  
*orthogonal* *u v* = (*u dot v* = 0)

**lemma** *lemVecZeroMinus*:

**shows**  $0 \ominus u = \sim u$   
(*proof*)

**lemma** *lemVecSelfMinus*:

**shows**  $u \ominus u = 0$   
(*proof*)

**lemma** *lemVecPlusCommute*:

**shows**  $u \oplus v = v \oplus u$   
(*proof*)

**lemma** *lemVecPlusAssoc*:

**shows**  $u \oplus (v \oplus w) = (u \oplus v) \oplus w$   
(*proof*)

**lemma** *lemVecPlusMinus*:

**shows**  $u \oplus (\sim v) = u \ominus v$   
(*proof*)

**lemma** *lemDotCommute*:

**shows** (*u dot v*) = (*v dot u*)  
(*proof*)

**lemma** *lemMDotCommute*:

**shows** (*u mdot v*) = (*v mdot u*)  
(*proof*)

**lemma** *lemScaleScale*:

**shows**  $a**(b**u) = (a*b)**u$   
(*proof*)

**lemma** *lemScale1*:  
  **shows**  $1 ** u = u$   
  ⟨*proof*⟩

**lemma** *lemScale0*:  
  **shows**  $0 ** u = 0$   
  ⟨*proof*⟩

**lemma** *lemScaleNeg*:  
  **shows**  $(-k)**u = \sim (k**u)$   
  ⟨*proof*⟩

**lemma** *lemScaleOrigin*:  
  **shows**  $k**0 = 0$   
  ⟨*proof*⟩

**lemma** *lemScaleOverAdd*:  
  **shows**  $k**(u \oplus v) = k**u \oplus k**v$   
  ⟨*proof*⟩

**lemma** *lemAddOverScale*:  
  **shows**  $a**u \oplus b**u = (a+b)**u$   
  ⟨*proof*⟩

**lemma** *lemScaleInverse*:  
  **assumes**  $k \neq (0::'a)$   
  **and**  $v = k**u$   
  **shows**  $u = (\text{inverse } k)**v$   
  ⟨*proof*⟩

**lemma** *lemOrthoSym*:  
  **assumes**  $u \perp v$

**shows**  $v \perp u$   
*<proof>*

**end**

**class** *Points* = *Quantities* + *Vectors*  
**begin**

**abbreviation** *origin* :: 'a *Point* **where**  
*origin*  $\equiv$  ( $\mid$  *tval* = 0, *xval* = 0, *yval* = 0, *zval* = 0  $\mid$ )

**fun** *vectorJoining* :: 'a *Point*  $\Rightarrow$  'a *Point*  $\Rightarrow$  'a *Vector* (*from* - *to* -) **where**  
*vectorJoining* *p q*  
= ( $\mid$  *tdir* = *tval* *q* - *tval* *p*, *xdir* = *xval* *q* - *xval* *p*,  
*ydir* = *yval* *q* - *yval* *p*, *zdir* = *zval* *q* - *zval* *p*  $\mid$ )

**fun** *moveBy* :: 'a *Point*  $\Rightarrow$  'a *Vector*  $\Rightarrow$  'a *Point* (**infixl**  $\rightsquigarrow$  100) **where**  
*moveBy* *p u*  
= ( $\mid$  *tval* = *tval* *p* + *tval* *u*, *xval* = *xval* *p* + *xval* *u*,  
*yval* = *yval* *p* + *yval* *u*, *zval* = *zval* *p* + *zval* *u*  $\mid$ )

**fun** *positionVector* :: 'a *Point*  $\Rightarrow$  'a *Vector* **where**  
*positionVector* *p* = ( $\mid$  *tval* = *tval* *p*, *xdir* = *xval* *p*, *ydir* = *yval* *p*, *zdir* = *zval* *p*  $\mid$ )

**fun** *before* :: 'a *Point*  $\Rightarrow$  'a *Point*  $\Rightarrow$  *bool* (**infixr**  $\lesssim$  100) **where**  
*before* *p q* = (*tval* *p* < *tval* *q*)

**fun** *after* :: 'a *Point*  $\Rightarrow$  'a *Point*  $\Rightarrow$  *bool* (**infixr**  $\gtrsim$  100) **where**  
*after* *p q* = (*tval* *p* > *tval* *q*)

**fun** *sametime* :: 'a *Point*  $\Rightarrow$  'a *Point*  $\Rightarrow$  *bool* (**infixr**  $\approx$  100) **where**  
*sametime* *p q* = (*tval* *p* = *tval* *q*)

**lemma** *lemFromToTo*:

**shows** (*from* *p* *to* *q*)  $\oplus$  (*from* *q* *to* *r*) = (*from* *p* *to* *r*)  
*<proof>*

**lemma** *lemMoveByMove*:

**shows**  $p \rightsquigarrow u \rightsquigarrow v = p \rightsquigarrow (u \oplus v)$   
*<proof>*

**lemma** *lemScaleLinear*:

**shows**  $p \rightsquigarrow a**u \rightsquigarrow b**v = p \rightsquigarrow (a**u \oplus b**v)$   
*<proof>*

**end**

**class** *Lines* = *Quantities* + *Vectors* + *Points*  
**begin**

**fun** *onAxisT* :: 'a *Point*  $\Rightarrow$  *bool* **where**  
*onAxisT* u = ((*xval* u = 0)  $\wedge$  (*yval* u = 0)  $\wedge$  (*zval* u = 0))

**fun** *space2* :: ('a *Point*)  $\Rightarrow$  ('a *Point*)  $\Rightarrow$  'a **where**  
*space2* u v  
= (*xval* u - *xval* v)\*(*xval* u - *xval* v)  
+ (*yval* u - *yval* v)\*(*yval* u - *yval* v)  
+ (*zval* u - *zval* v)\*(*zval* u - *zval* v)

**fun** *time2* :: ('a *Point*)  $\Rightarrow$  ('a *Point*)  $\Rightarrow$  'a **where**  
*time2* u v = (*tval* u - *tval* v)\*(*tval* u - *tval* v)

**fun** *speed* :: ('a *Point*)  $\Rightarrow$  ('a *Point*)  $\Rightarrow$  'a **where**  
*speed* u v = (*space2* u v / *time2* u v)

**fun** *mkLine* :: 'a *Point*  $\Rightarrow$  'a *Vector*  $\Rightarrow$  'a *Line* **where**  
*mkLine* b d = ( $\lfloor$  *basepoint* = b, *direction* = d  $\rfloor$ )

**fun** *lineJoining* :: 'a *Point*  $\Rightarrow$  'a *Point*  $\Rightarrow$  'a *Line* (*line joining - to -*) **where**  
*lineJoining* p q = ( $\lfloor$  *basepoint* = p, *direction* = *from p to q*  $\rfloor$ )

**fun** *parallel* :: 'a *Line*  $\Rightarrow$  'a *Line*  $\Rightarrow$  *bool* (- || ) **where**  
*parallel* lineA lineB = ((*direction* lineA = *vecZero*)  $\vee$  (*direction* lineB = *vecZero*)  
 $\vee$  ( $\exists$  k.(k  $\neq$  (0::'a)  $\wedge$  *direction* lineB = k\*\**direction*  
lineA)))

**fun** *collinear* :: 'a *Point*  $\Rightarrow$  'a *Point*  $\Rightarrow$  'a *Point*  $\Rightarrow$  *bool* **where**  
*collinear* p q r = ( $\exists$   $\alpha$   $\beta$ . (  $\alpha$  +  $\beta$  = 1 )  $\wedge$   
*positionVector* p =  $\alpha$ \*\*(*positionVector* q)  $\oplus$   $\beta$ \*\*(*positionVector* r) ))

**fun** *inLine* :: 'a *Point*  $\Rightarrow$  'a *Line*  $\Rightarrow$  *bool* **where**  
*inLine* p l = *collinear* p (*basepoint* l) (*basepoint* l  $\rightsquigarrow$  *direction* l)

**fun** *meets* :: 'a *Line*  $\Rightarrow$  'a *Line*  $\Rightarrow$  *bool* **where**  
*meets* line1 line2 = ( $\exists$  p.(*inLine* p line1  $\wedge$  *inLine* p line2))

**lemma** *lemParallelReflexive*:  
**shows** *lineA* || *lineA*  
*<proof>*

**lemma** *lemParallelSym*:  
  **assumes**  $lineA \parallel lineB$   
  **shows**  $lineB \parallel lineA$   
  ⟨*proof*⟩

**lemma** *lemParallelTrans*:  
  **assumes**  $lineA \parallel lineB$   
  **and**  $lineB \parallel lineC$   
  **and**  $direction\ lineB \neq vecZero$   
  **shows**  $lineA \parallel lineC$   
  ⟨*proof*⟩

**lemma** (in  $-$ ) *lemLineIdentity*:  
  **assumes**  $lineA = (| basepoint = basepoint\ lineB, direction = direction\ lineB |)$   
  **shows**  $lineA = lineB$   
  ⟨*proof*⟩

**lemma** *lemDirectionJoining*:  
  **shows**  $vectorJoining\ p\ (p \rightsquigarrow v) = v$   
  ⟨*proof*⟩

**lemma** *lemDirectionFromTo*:  
  **shows**  $direction\ (line\ joining\ p\ to\ (p \rightsquigarrow dir)) = dir$   
  ⟨*proof*⟩

**lemma** *lemLineEndpoint*:  
  **shows**  $q = p \rightsquigarrow (from\ p\ to\ q)$   
  ⟨*proof*⟩

**lemma** *lemNullLine*:  
  **assumes**  $direction\ lineA = vecZero$   
  **and**  $inLine\ x\ lineA$   
  **shows**  $x = basepoint\ lineA$   
  ⟨*proof*⟩

**lemma** *lemLineContainsBasepoint*:  
  **shows**  $inLine\ p\ (line\ joining\ p\ to\ q)$   
  ⟨*proof*⟩

**lemma** *lemLineContainsEndpoint*:  
  **shows**  $inLine\ q\ (line\ joining\ p\ to\ q)$

*<proof>*

**lemma** *lemDirectionReverse:*

**shows** *from q to p = vecNegate (from p to q)*

*<proof>*

**lemma** *lemParallelJoin:*

**assumes** *line joining p to q || line joining q to r*

**shows** *line joining p to q || line joining p to r*

*<proof>*

**lemma** *lemDirectionCollinear:*

**shows** *collinear u v (v  $\rightsquigarrow$  d)  $\longleftrightarrow$  ( $\exists \beta.$ (*from u to v = (- $\beta$ )\*\*d*)*

*<proof>*

**lemma** *lemParallelNotMeet:*

**assumes** *lineA || lineB*

**and** *direction lineA  $\neq$  vecZero*

**and** *direction lineB  $\neq$  vecZero*

**and** *inLine x lineA*

**and**  *$\neg$ (inLine x lineB)*

**shows**  *$\neg$ (meets lineA lineB)*

*<proof>*

**lemma** *lemAxisIsLine:*

**assumes** *onAxisT x*

**and** *onAxisT y*

**and** *onAxisT z*

**and** *x  $\neq$  y*

**and** *y  $\neq$  z*

**and** *z  $\neq$  x*

**shows** *collinear x y z*

*<proof>*

**lemma** *lemSpace2Sym:*

**shows** *space2 x y = space2 y x*

*<proof>*

**lemma** *lemTime2Sym:*

**shows** *time2 x y = time2 y x*

*<proof>*

**end**



```

class Planes = Quantities + Lines
begin
  fun mkPlane :: 'a Point  $\Rightarrow$  'a Vector  $\Rightarrow$  'a Vector  $\Rightarrow$  'a Plane where
    mkPlane b d1 d2 = ( | pbasepoint = b, direction1 = d1, direction2 = d2 | )

  fun coplanar :: 'a Point  $\Rightarrow$  'a Point  $\Rightarrow$  'a Point  $\Rightarrow$  'a Point  $\Rightarrow$  bool where
    coplanar e x y z
      = ( $\exists \alpha \beta \gamma. (\alpha + \beta + \gamma = 1) \wedge$ 
        positionVector e
          = ( $\alpha ** (\text{positionVector } x) \oplus \beta ** (\text{positionVector } y) \oplus \gamma ** (\text{positionVector } z)$ 
            )))

  fun inPlane :: 'a Point  $\Rightarrow$  'a Plane  $\Rightarrow$  bool where
    inPlane e pl = coplanar e (pbasepoint pl) (pbasepoint pl  $\rightsquigarrow$  direction1 pl)
                  (pbasepoint pl  $\rightsquigarrow$  direction2 pl)

  fun samePlane :: 'a Plane  $\Rightarrow$  'a Plane  $\Rightarrow$  bool where
    samePlane pl pl' = (inPlane (pbasepoint pl) pl'  $\wedge$ 
                       inPlane (pbasepoint pl  $\rightsquigarrow$  direction1 pl) pl'  $\wedge$ 
                       inPlane (pbasepoint pl  $\rightsquigarrow$  direction2 pl) pl')

lemma lemPlaneContainsBasePoint:
  shows inPlane (pbasepoint pl) pl
  <proof>

end

class Cones = Quantities + Lines + Planes +
fixes

  tangentPlane :: 'a Point  $\Rightarrow$  'a Cone  $\Rightarrow$  'a Plane
assumes

  AxTangentBase: pbasepoint (tangentPlane e cone) = e
and

  AxTangentVertex: inPlane (vertex cone) (tangentPlane e cone)
and

  AxConeTangent: (onCone e cone)  $\longrightarrow$ 
    ((inPlane pt (tangentPlane e cone)  $\wedge$  onCone pt cone)
      $\longleftrightarrow$  collinear (vertex cone) e pt)
and

```

*AxParallelCones*: (*onCone e econe*  $\wedge$  *e*  $\neq$  *vertex econe*  $\wedge$  *onCone f fccone*  $\wedge$  *f*  $\neq$  *vertex fccone*  
 $\wedge$  *inPlane f (tangentPlane e econe)*)  
 $\longrightarrow$  (*samePlane (tangentPlane e econe) (tangentPlane f fccone)*  
 $\wedge$  (*lineJoining (vertex econe) e*  $\parallel$  *lineJoining (vertex fccone)*  
*f*)))  
**and**

*AxParallelConesE*: *outsideCone f cone*  
 $\longrightarrow$  ( $\exists e.$ (*onCone e cone*  $\wedge$  *e*  $\neq$  *vertex cone*  $\wedge$  *inPlane f (tangentPlane e cone)*))  
**and**

*AxSlopedLineInVerticalPlane*: [*onAxisT e*; *onAxisT f*; *e*  $\neq$  *f*;  $\neg$ (*onAxisT g*)]  
 $\implies$  ( $\forall s.$ ( $\exists p .$  (*collinear e g p*  $\wedge$  (*space2 p f = (s\*s)\*time2 p f*))))

**begin**

**fun** *onCone* :: '*a Point*  $\Rightarrow$  '*a Cone*  $\Rightarrow$  *bool* **where**  
*onCone p cone*  
 $=$  (*space2 (vertex cone) p = (slope cone \* slope cone) \* time2 (vertex cone)*  
*p*)

**fun** *insideCone* :: '*a Point*  $\Rightarrow$  '*a Cone*  $\Rightarrow$  *bool* **where**  
*insideCone p cone*  
 $=$  (*space2 (vertex cone) p < (slope cone \* slope cone) \* time2 (vertex cone)*  
*p*)

**fun** *outsideCone* :: '*a Point*  $\Rightarrow$  '*a Cone*  $\Rightarrow$  *bool* **where**  
*outsideCone p cone*  
 $=$  (*space2 (vertex cone) p > (slope cone \* slope cone) \* time2 (vertex cone)*  
*p*)

**fun** *mkCone* :: '*a Point*  $\Rightarrow$  '*a*  $\Rightarrow$  '*a Cone* **where**  
*mkCone v s =* ( $\lfloor$  *vertex = v, slope = s*  $\rfloor$ )

**lemma** *lemVertexOnCone*:  
**shows** *onCone (vertex cone) cone*  
 $\langle$ *proof* $\rangle$

**lemma** *lemOutsideNotOnCone*:  
**assumes** *outsideCone f cone*  
**shows**  $\neg$  (*onCone f cone*)  
 $\langle$ *proof* $\rangle$

**end**

**class** *SpaceTime* = *Quantities* + *Vectors* + *Points* + *Lines* + *Planes* + *Cones*

```

end

theory SomeFunc
  imports Main
begin

fun someFunc :: ('a ⇒ 'b ⇒ bool) ⇒ 'a ⇒ 'b where
  someFunc P x = (SOME y. (P x y))

lemma lemSomeFunc:
  assumes ∃ y . P x y
  and f = someFunc P
  shows P x (f x)
  ⟨proof⟩

end

theory Axioms
  imports SpaceTime SomeFunc
begin

record Body =
  Ph :: bool
  IOb :: bool

class WorldView = SpaceTime +
fixes

  W :: Body ⇒ Body ⇒ 'a Point ⇒ bool (- sees - at -)
and

  wvt :: Body ⇒ Body ⇒ 'a Point ⇒ 'a Point
assumes
  AxWVT: [ IOb m; IOb k ] ⇒ (W k b x ↔ W m b (wvt m k x))
and
  AxWVTSym: [ IOb m; IOb k ] ⇒ (y = wvt k m x ↔ x = wvt m k y)
begin
end

class AxiomPreds = WorldView

```

```

begin
  fun sqrtTest :: 'a ⇒ 'a ⇒ bool where
    sqrtTest x r = ((r ≥ 0) ∧ (r*r = x))

  fun cTest :: Body ⇒ 'a ⇒ bool where
    cTest m v = ( (v > 0) ∧ (∀ x y . (
      (∃ p. (Ph p ∧ W m p x ∧ W m p y)) ↔ (space2 x y = (v * v)*(time2
x y))
      )))
end

class AxEuclidean = AxiomPreds + Quantities +
assumes
  AxEuclidean: (x ≥ Groups.zero-class.zero) ⇒ (∃ r. sqrtTest x r)
begin

  abbreviation sqrt :: 'a ⇒ 'a where
    sqrt ≡ someFunc sqrtTest

  lemma lemSqrt:
    assumes x ≥ 0
    and r = sqrt x
    shows r ≥ 0 ∧ r*r = x
    ⟨proof⟩
end

class AxLight = WorldView +
assumes
  AxLight: ∃ m v. ( IOb m ∧ (v > (0::'a)) ∧ (∀ x y. (
    (∃ p.(Ph p ∧ W m p x ∧ W m p y)) ↔ (space2 x y = (v * v)*time2 x
y)
    )))
begin
end

class AxPh = WorldView + AxiomPreds +
assumes
  AxPh: IOb m ⇒ (∃ v. cTest m v)
begin

  abbreviation c :: Body ⇒ 'a where
    c ≡ someFunc cTest

```

**fun** *lightcone* :: *Body*  $\Rightarrow$  'a *Point*  $\Rightarrow$  'a *Cone* **where**  
*lightcone* *m* *v* = *mkCone* *v* (*c m*)

**lemma** *lemCProps*:  
**assumes** *IOb m*  
**and** *v = c m*  
**shows**  $(v > 0) \wedge (\forall x y. (\exists p. (Ph\ p \wedge W\ m\ p\ x \wedge W\ m\ p\ y)))$   
 $\longleftrightarrow (space2\ x\ y = (c\ m * c\ m) * time2\ x\ y))$   
*<proof>*

**lemma** *lemCCone*:  
**assumes** *IOb m*  
**and** *onCone y (lightcone m x)*  
**shows**  $\exists p. (Ph\ p \wedge W\ m\ p\ x \wedge W\ m\ p\ y)$   
*<proof>*

**lemma** *lemCPos*:  
**assumes** *IOb m*  
**shows** *c m > 0*  
*<proof>*

**lemma** *lemCPhoton*:  
**assumes** *IOb m*  
**shows**  $\forall x y. (\exists p. (Ph\ p \wedge W\ m\ p\ x \wedge W\ m\ p\ y)) \longleftrightarrow (space2\ x\ y = (c\ m * c\ m) * (time2\ x\ y))$   
*<proof>*

**end**

**class** *AxEv* = *WorldView* +  
**assumes**  
*AxEv*:  $\llbracket IOb\ m; IOb\ k \rrbracket \Longrightarrow (\exists y. (\forall b. (W\ m\ b\ x \longleftrightarrow W\ k\ b\ y)))$   
**begin**  
**end**

**class** *AxThExp* = *WorldView* + *AxPh* +  
**assumes**  
*AxThExp*: *IOb m*  $\Longrightarrow (\forall x y. ($

$(\exists k.(IOb\ k \wedge W\ m\ k\ x \wedge W\ m\ k\ y)) \longleftrightarrow (space2\ x\ y < (c\ m * c\ m) * time2$   
 $x\ y)$   
 $)$

**begin**  
**end**

**class** *AxSelf* = *WorldView* +  
**assumes**  
*AxSelf*:  $IOb\ m \implies (W\ m\ m\ x) \longrightarrow (onAxisT\ x)$   
**begin**  
**end**

**class** *AxC* = *WorldView* + *AxPh* +  
**assumes**  
*AxC*:  $IOb\ m \implies c\ m = 1$   
**begin**  
**end**

**class** *AxSym* = *WorldView* +  
**assumes**  
*AxSym*:  $\llbracket IOb\ m; IOb\ k \rrbracket \implies$   
 $(W\ m\ e\ x \wedge W\ m\ f\ y \wedge W\ k\ e\ x' \wedge W\ k\ f\ y' \wedge$   
 $tval\ x = tval\ y \wedge tval\ x' = tval\ y')$   
 $\longrightarrow (space2\ x\ y = space2\ x'\ y')$   
**begin**  
**end**

**class** *AxLines* = *WorldView* +  
**assumes**  
*AxLines*:  $\llbracket IOb\ m; IOb\ k; collinear\ x\ p\ q \rrbracket \implies$   
 $collinear\ (wvt\ k\ m\ x)\ (wvt\ k\ m\ p)\ (wvt\ k\ m\ q)$   
**begin**  
**end**

```

class AxPlanes = WorldView +
assumes
  AxPlanes:  $\llbracket \text{IOb } m; \text{IOb } k \rrbracket \implies$ 
    (coplanar e x y z  $\longrightarrow$  coplanar (wvt k m e) (wvt k m x) (wvt k m y) (wvt k m z))
begin
end

```

```

class AxCones = WorldView + AxPh +
assumes
  AxCones:  $\llbracket \text{IOb } m; \text{IOb } k \rrbracket \implies$ 
    (onCone x (lightCone m v)  $\longrightarrow$  onCone (wvt k m x) (lightcone k (wvt k m v)))
begin
end

```

```

class AxTime = WorldView +
assumes
  AxTime:  $\llbracket \text{IOb } m; \text{IOb } k \rrbracket$ 
     $\implies (x \lesssim y \longrightarrow \text{wvt } k \ m \ x \lesssim \text{wvt } k \ m \ y)$ 
begin
end

```

**end**

```

theory SpecRel
imports Axioms
begin

```

```

class SpecRel = WorldView + AxPh + AxEv + AxSelf + AxSym

```

```

  + AxEuclidean

```

```

  + AxLines + AxPlanes + AxCones

```

```

begin

```

**lemma** *lemZEG*:  
  **shows**  $z - e = g - e + (z - g)$   
   $\langle proof \rangle$

**lemma** *noFTLObserver*:  
  **assumes** *iobm*: *IOb m*  
  **and**    *iobk*: *IOb k*  
  **and**    *mke*: *m sees k at e*  
  **and**    *mkf*: *m sees k at f*  
  **and**    *enotf*:  $e \neq f$   
**shows**     $space2\ e\ f \leq (c\ m * c\ m) * time2\ e\ f$   
   $\langle proof \rangle$

**end**

**end**

## References

- [1] M. Stannett and I. Németi. Using Isabelle/HOL to verify first-order relativity theory. *Journal of Automated Reasoning*, 52(4):361–378, 2014.