Network Security Policy Verification

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April 20, 2020

Abstract. We present a unified theory for verifying network security policies. A security policy is represented as directed graph. To check high-level security goals, security invariants over the policy are expressed. We cover monotonic security invariants, i.e. prohibiting more does not harm security. We provide the following contributions for the security invariant theory. (*i*) Secure auto-completion of scenario-specific knowledge, which eases usability. (*ii*) Security violations can be repaired by tightening the policy iff the security invariants hold for the deny-all policy. (*iii*) An algorithm to compute a security policy. (*iv*) A formalization of stateful connection semantics in network security mechanisms. (*v*) An algorithm to compute a secure stateful implementation of a policy. (*vi*) An executable implementation of all the theory. (*vii*) Examples, ranging from an aircraft cabin data network to the analysis of a large real-world firewall.

For a detailed description, see [2, 3, 1].

Acknowledgements. This entry contains contributions by Lars Hupel and would not have made it into the AFP without him. I want to thank the Isabelle group Munich for always providing valuable help. I would like to express my deep gratitude to my supervisor, Georg Carle, for supporting this topic and facilitating further research possibilities in this field.

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theory *TopoS-Vertices*
imports *Main*
HOL−Library.Char-ord
HOL−Library.List-Lexorder
begin

1 A type for vertices

This theory makes extensive use of graphs. We define a typeclass *vertex* for the vertices we will use in our theory. The vertices will correspond to network or policy entities.

Later, we will conduct some proves by providing counterexamples. Therefore, we say that the type of a vertex has at least three pairwise distinct members.

For example, the types *string*, *nat*, *bool* × *bool* and many other fulfill this assumption. The type *bool* alone does not fulfill this assumption, because it only has two elements.

This is only a constraint over the type, of course, a policy with less than three entities can also be verified.

TL;DR: We define *'a vertex*, which is as good as *'a*.

— We need at least some vertices available for a graph ...
class *vertex* =
  fixes *vertex-1* :: *'a*
  fixes *vertex-2* :: *'a*
  fixes *vertex-3* :: *'a*
  assumes *distinct-vertices*: *distinct* [*vertex-1*, *vertex-2*, *vertex-3*]
begin
  lemma *distinct-vertices12*[*simp*]: *vertex-1* ≠ *vertex-2* **using** *distinct-vertices* **by**(*simp*)
  lemma *distinct-vertices13*[*simp*]: *vertex-1* ≠ *vertex-3* **using** *distinct-vertices* **by**(*simp*)
  lemma *distinct-vertices23*[*simp*]: *vertex-2* ≠ *vertex-3* **using** *distinct-vertices* **by**(*simp*)

  lemmas *distinct-vertices-sym* = *distinct-vertices12*[*symmetric*] *distinct-vertices13*[*symmetric*]
      distinct-vertices23[*symmetric*]
  declare *distinct-vertices-sym*[*simp*]
end

Numbers, chars and strings are good candidates for vertices.

instantiation *nat*::*vertex*
begin
  definition *vertex-1-nat* ::*nat* **where** *vertex-1* ≡ (*1*::*nat*)
  definition *vertex-2-nat* ::*nat* **where** *vertex-2* ≡ (*2*::*nat*)
  definition *vertex-3-nat* ::*nat* **where** *vertex-3* ≡ (*3*::*nat*)
instance proof qed(*simp add*: *vertex-1-nat-def vertex-2-nat-def vertex-3-nat-def*)
end
value *vertex-1*::*nat*

instantiation *int*::*vertex*
begin
  definition *vertex-1-int* ::*int* **where** *vertex-1* ≡ (*1*::*int*)
  definition *vertex-2-int* ::*int* **where** *vertex-2* ≡ (*2*::*int*)
  definition *vertex-3-int* ::*int* **where** *vertex-3* ≡ (*3*::*int*)
instance proof qed(*simp add*: *vertex-1-int-def vertex-2-int-def vertex-3-int-def*)
end

instantiation *char*::*vertex*
begin
  definition *vertex-1-char* ::*char* **where** *vertex-1* ≡ *CHR ''A''*
  definition *vertex-2-char* ::*char* **where** *vertex-2* ≡ *CHR ''B''*
  definition *vertex-3-char* ::*char* **where** *vertex-3* ≡ *CHR ''C''*
instance proof(*intro-classes*) **qed**(*simp add*: *vertex-1-char-def vertex-2-char-def vertex-3-char-def*)
end
value *vertex-1*::*char*

instantiation *list* :: (*vertex*) *vertex*
begin
  definition *vertex-1-list* **where** *vertex-1* ≡ []
  definition *vertex-2-list* **where** *vertex-2* ≡ [*vertex-1*]
  definition *vertex-3-list* **where** *vertex-3* ≡ [*vertex-1*, *vertex-1*]
instance proof qed(*simp add*: *vertex-1-list-def vertex-2-list-def vertex-3-list-def*)
end

— for the ML graphviz visualizer
ML ‹
fun tune-string-vertex-format (*t*: *term*) (*s*: *string*) : *string* =
    if fastype-of t = @{*typ string*} *then*
      if String.isPrefix '' s then
        String.substring (*s*, (*size ''*), (*size s* − (*size ''''*)))
      else let val - = *writeln* (*no tune-string-vertex-format for* \ ^*s*^\) *in s end*
    else s
    handle Subscript => *let val* - = *writeln* (*tune-string-vertex-format Subscript excpetion*) *in s end*;
›

end
theory *TopoS-Interface*
imports *Main Lib*/*FiniteGraph TopoS-Vertices Lib*/*TopoS-Util*
begin

2 Security Invariants

A good documentation of this formalization is available in [3].

We define security invariants over a graph. The graph corresponds to the network's access control structure.

— *'v* is the type of the nodes in the graph (hosts in the network). *'a* is the type of the host attributes.
record (*'v*::*vertex*, *'a*) *TopoS-Params* =
  node-properties :: *'v*::*vertex* ⇒ *'a option*

A Security Invariant is defined as locale.

We successively define more and more locales with more and more assumptions. This clearly depicts which assumptions are necessary to use certain features of a Security Invariant. In addition, it makes instance proofs of Security Invariants easier, since the lemmas obtained by an (easy, few assumptions) instance proof can be used for the complicated (more assumptions) instance proofs.

A security Invariant consists of one function: *sinvar*. Essentially, it is a predicate over the policy (depicted as graph G and a host attribute mapping (nP)).

A Security Invariant where the offending flows (flows that invalidate the policy) can be defined and calculated. No assumptions are necessary for this step.

locale *SecurityInvariant-withOffendingFlows* =
fixes *sinvar*::('*v*::*vertex*) *graph* ⇒ ('*v*::*vertex* ⇒ '*a*) ⇒ *bool* — policy ⇒ host attribute mapping ⇒ bool
begin
— Offending Flows definitions:
definition *is-offending-flows*::('*v* × '*v*) *set* ⇒ '*v graph* ⇒ ('*v* ⇒ '*a*) ⇒ *bool* **where**
is-offending-flows f G nP ≡ ¬ *sinvar G nP* ∧ *sinvar* (*delete-edges G f*) *nP*

— Above definition is not minimal:
definition *is-offending-flows-min-set*::('*v* × '*v*) *set* ⇒ '*v graph* ⇒ ('*v* ⇒ '*a*) ⇒ *bool* **where**
is-offending-flows-min-set f G nP ≡ *is-offending-flows f G nP* ∧
(∀ (*e1*, *e2*) ∈ *f*. ¬ *sinvar* (*add-edge e1 e2* (*delete-edges G f*)) *nP*)

— The set of all offending flows.
definition *set-offending-flows*::'*v graph* ⇒ ('*v* ⇒ '*a*) ⇒ ('*v* × '*v*) *set set* **where**
set-offending-flows G nP = {*F*. *F* ⊆ (*edges G*) ∧ *is-offending-flows-min-set F G nP*}

Some of the *set-offending-flows* definition

lemma *offending-not-empty*: ⟦ *F* ∈ *set-offending-flows G nP* ⟧ ⟹ *F* ≠ {}
by(*auto simp add*: *set-offending-flows-def is-offending-flows-def is-offending-flows-min-set-def*)
lemma *empty-offending-contra*:
⟦ *F* ∈ *set-offending-flows G nP*; *F* = {}⟧ ⟹ *False*
by(*simp add*: *set-offending-flows-def is-offending-flows-def is-offending-flows-min-set-def*)
lemma *offending-notevalD*: *F* ∈ *set-offending-flows G nP* ⟹ ¬ *sinvar G nP*
by(*simp add*: *set-offending-flows-def is-offending-flows-def is-offending-flows-min-set-def*)
lemma *sinvar-no-offending*: *sinvar G nP* ⟹ *set-offending-flows G nP* = {}
by(*simp add*: *set-offending-flows-def is-offending-flows-def is-offending-flows-min-set-def*)
theorem *removing-offending-flows-makes-invariant-hold*:
∀ *F* ∈ *set-offending-flows G nP*. *sinvar* (*delete-edges G F*) *nP*
proof(*cases sinvar G nP*)
case *True*
hence *no-offending*: *set-offending-flows G nP* = {} **using** *sinvar-no-offending* **by** *simp*
thus ∀ *F*∈*set-offending-flows G nP*. *sinvar* (*delete-edges G F*) *nP* **using** *empty-iff* **by** *simp*
next
case *False* **thus** ∀ *F*∈*set-offending-flows G nP*. *sinvar* (*delete-edges G F*) *nP*
by(*simp add*: *set-offending-flows-def is-offending-flows-def is-offending-flows-min-set-def graph-ops*)
qed
corollary *valid-without-offending-flows*:
⟦ *F* ∈ *set-offending-flows G nP* ⟧ ⟹ *sinvar* (*delete-edges G F*) *nP*
by(*simp add*: *removing-offending-flows-makes-invariant-hold*)

lemma *set-offending-flows-simp*:
⟦ *wf-graph G* ⟧ ⟹
set-offending-flows G nP = {*F*. *F* ⊆ *edges G* ∧
(¬ *sinvar G nP* ∧ *sinvar* (⦇*nodes* = *nodes G*, *edges* = *edges G* − *F*⦈) *nP*) ∧
(∀ (*e1*, *e2*)∈*F*. ¬ *sinvar* (⦇*nodes* = *nodes G*, *edges* = {(*e1*, *e2*)} ∪ (*edges G* − *F*)⦈) *nP*)}
apply(*simp only*: *set-offending-flows-def is-offending-flows-min-set-def*
is-offending-flows-def delete-edges-simp2 add-edge-def graph.select-convs)
apply(*subgoal-tac* ⋀*F e1 e2*. *F* ⊆ *edges G* ⟹ (*e1*, *e2*) ∈ *F* ⟹ *nodes G* ∪ {*e1*, *e2*} = *nodes G*)

apply *fastforce*
apply(*simp add*: *wf-graph-def*)
by (*metis fst-conv imageI in-mono insert-absorb snd-conv*)

end

print-locale! *SecurityInvariant-withOffendingFlows*

The locale *SecurityInvariant-withOffendingFlows* has no assumptions about the security invariant *sinvar*. Undesirable things may happen: The offending flows can be empty, even for a violated invariant.

We provide an example, the security invariant λ- -. *False*. As host attributes, we simply use the identity function *id*.

lemma *SecurityInvariant-withOffendingFlows.set-offending-flows* (λ- -. *False*) (| *nodes* = {''*v1*''}, *edges*={} |) *id* = {}
lemma *SecurityInvariant-withOffendingFlows.set-offending-flows* (λ- -. *False*)
(| *nodes* = {''*v1*'', ''*v2*''}, *edges* = {(''*v1*'', ''*v2*'')} |) *id* = {}

In general, there exists a *sinvar* such that the invariant does not hold and no offending flows exits.

lemma ∃ *sinvar*. ¬ *sinvar G nP* ∧ *SecurityInvariant-withOffendingFlows.set-offending-flows sinvar G nP* = {}
apply(*simp add*: *SecurityInvariant-withOffendingFlows.set-offending-flows-def*
SecurityInvariant-withOffendingFlows.is-offending-flows-min-set-def SecurityInvariant-withOffendingFlows.is-offendi
apply(*rule-tac x*=(λ- -. *False*) **in** *exI*)
apply(*simp*)
done

Thus, we introduce usefulness properties that prohibits such useless invariants.

We summarize them in an invariant. It requires the following:

1. The offending flows are always defined.
2. The invariant is monotonic, i.e. prohibiting more is more secure.
3. And, the (non-minimal) offending flows are monotonic, i.e. prohibiting more solves more security issues.

Later, we will show that is suffices to show that the invariant is monotonic. The other two properties can be derived.

locale *SecurityInvariant-preliminaries* = *SecurityInvariant-withOffendingFlows sinvar*
for *sinvar*
+
assumes
defined-offending:
⟦ *wf-graph G*; ¬ *sinvar G nP* ⟧ ⟹ *set-offending-flows G nP* ≠ {}
and
mono-sinvar:
⟦ *wf-graph* (| *nodes* = *N*, *edges* = *E* |); *E'* ⊆ *E*; *sinvar* (| *nodes* = *N*, *edges* = *E* |) *nP* ⟧ ⟹

sinvar (| *nodes* = *N*, *edges* = *E'* |) *nP*
and *mono-offending*:
⟦ *wf-graph G*; *is-offending-flows ff G nP* ⟧ ⟹ *is-offending-flows* (*ff* ∪ *f'*) *G nP*
begin

To instantiate a *SecurityInvariant-preliminaries*, here are some hints: Have a look at the *TopoS-withOffendingFlows.thy* file. There is a definition of *sinvar-mono*. It impplies *mono-sinvar* and *mono-offending apply*(*fact SecurityInvariant-withOffendingFlows.sinvar-mono-imp-sinvar-mono*[*OF sinvar-mono*]) *apply*(*fact SecurityInvariant-wi sinvar-mono*])
In addition, *SecurityInvariant-withOffendingFlows.mono-imp-set-offending-flows-not-empty*[*OF sinvar-mono*] gives a nice proof rule for *defined-offending*

Basically, *sinvar-mono*. implies almost all assumptions here and is equal to *mono-sinvar*.

end

2.1 Security Invariants with secure auto-completion of host attribute mappings

We will now add a new artifact to the Security Invariant. It is a secure default host attribute, we will use the symbol ⊥.

The newly introduced Boolean *receiver-violation* tells whether a security violation happens at the sender's or the receiver's side.

The details can be looked up in [3].

— Some notes about the notation: *fst* ' *F* means to apply the function *fst* to the set *F* element-wise. Example: If *F* is a set of directed edges, *F* ⊆ *edges G*, then *fst* ' *F* is the set of senders and *snd* ' *f* the set of receivers.

locale *SecurityInvariant* = *SecurityInvariant-preliminaries sinvar*
for *sinvar*::('*v*::*vertex*) *graph* ⇒ ('*v*::*vertex* ⇒ '*a*) ⇒ *bool*
+
fixes *default-node-properties* :: '*a* (⊥)
and *receiver-violation* :: *bool*
assumes
— default value can never fix a security violation.
— Idea: Assume there is a violation, then there is some offending flow. *receiver-violation* defines whether the violation happens at the sender's or the receiver's side. We call the place of the violation the *offending host*. We replace the host attribute of the offending host with the default attribute. Giving an offending host, a *secure* default attribute does not change whether the invariant holds. I.e. this reconfiguration does not remove information, thus preserves all security critical information. Thought experiment preliminaries: Can a default configuration ever solve an existing security violation? NO! Thought experiment 1: admin forgot to configure host, hence it is handled by default configuration value ... Thought experiment 2: new node (attacker) is added to the network. What is its default configuration value ...
default-secure:
⟦ *wf-graph G*; ¬ *sinvar G nP*; *F* ∈ *set-offending-flows G nP* ⟧ ⟹
(¬ *receiver-violation* ⟶ *i* ∈ *fst* ' *F* ⟶ ¬ *sinvar G* (*nP*(*i* := ⊥))) ∧
(*receiver-violation* ⟶ *i* ∈ *snd* ' *F* ⟶ ¬ *sinvar G* (*nP*(*i* := ⊥)))
and
default-unique:
otherbot ≠ ⊥ ⟹
∃ (*G*::('*v*::*vertex*) *graph*) *nP i F*. *wf-graph G* ∧ ¬ *sinvar G nP* ∧ *F* ∈ *set-offending-flows G nP*
∧
sinvar (*delete-edges G F*) *nP* ∧

```
    (¬ receiver-violation ⟶ i ∈ fst ‘ F ∧ sinvar G (nP(i := otherbot))) ∧
    (receiver-violation ⟶ i ∈ snd ‘ F ∧ sinvar G (nP(i := otherbot)))
  begin
  — Removes option type, replaces with default host attribute
  fun node-props :: ('v, 'a) TopoS-Params ⇒ ('v ⇒ 'a) where
  node-props P = (λ i. (case (node-properties P) i of Some property ⇒ property | None ⇒ ⊥))

  definition node-props-formaldef :: ('v, 'a) TopoS-Params ⇒ ('v ⇒ 'a) where
  node-props-formaldef P ≡
  (λ i. (if i ∈ dom (node-properties P) then the (node-properties P i) else ⊥))

  lemma node-props-eq-node-props-formaldef: node-props-formaldef = node-props
   by(simp add: fun-eq-iff node-props-formaldef-def option.case-eq-if domIff)
```

Checking whether a security invariant holds.

1. check that the policy G is syntactically valid
2. check the security invariant *sinvar*

```
  definition eval::'v graph ⇒ ('v, 'a) TopoS-Params ⇒ bool where
  eval G P ≡ wf-graph G ∧ sinvar G (node-props P)


  lemma unique-common-math-notation:
  assumes ∀ G nP i F. wf-graph (G::('v::vertex) graph) ∧ ¬ sinvar G nP ∧ F ∈ set-offending-flows
G nP ∧
      sinvar (delete-edges G F) nP ∧
      (¬ receiver-violation ⟶ i ∈ fst ‘ F ⟶ ¬ sinvar G (nP(i := otherbot))) ∧
      (receiver-violation ⟶ i ∈ snd ‘ F ⟶ ¬ sinvar G (nP(i := otherbot)))
  shows otherbot = ⊥
  apply(rule ccontr)
  apply(drule default-unique)
  using assms by blast
 end

print-locale! SecurityInvariant
```

2.2 Information Flow Security and Access Control

receiver-violation defines the offending host. Thus, it defines when the violation happens. We found that this coincides with the invariant's security strategy.

ACS If the violation happens at the sender, we have an access control strategy (*ACS*). I.e. the sender does not have the appropriate rights to initiate the connection.

IFS If the violation happens at the receiver, we have an information flow security strategy (*IFS*) I.e. the receiver lacks the appropriate security level to retrieve the (confidential) information. The violations happens only when the receiver reads the data.

We refine our *SecurityInvariant* locale.

2.3 Information Flow Security Strategy (IFS)

locale *SecurityInvariant-IFS* = *SecurityInvariant-preliminaries sinvar*
for *sinvar*::('*v*::*vertex*) *graph* ⇒ ('*v*::*vertex* ⇒ '*a*) ⇒ *bool*
+
fixes *default-node-properties* :: '*a* (⊥)
assumes *default-secure-IFS*:
⟦ *wf-graph G*; *f* ∈ *set-offending-flows G nP* ⟧ ⟹
∀ *i* ∈ *snd*‘ *f*. ¬ *sinvar G* (*nP*(*i* := ⊥))
and
— If some otherbot fulfills *default-secure*, it must be ⊥ Hence, ⊥ is uniquely defined
default-unique-IFS:
(∀ *G f nP i*. *wf-graph G* ∧ *f* ∈ *set-offending-flows G nP* ∧ *i* ∈ *snd*‘ *f*
⟶ ¬ *sinvar G* (*nP*(*i* := *otherbot*))) ⟹ *otherbot* = ⊥
begin
lemma *default-unique-EX-notation*: *otherbot* ≠ ⊥ ⟹
∃ *G nP i f*. *wf-graph G* ∧ ¬ *sinvar G nP* ∧ *f* ∈ *set-offending-flows G nP* ∧
sinvar (*delete-edges G f*) *nP* ∧
(*i* ∈ *snd*‘ *f* ∧ *sinvar G* (*nP*(*i* := *otherbot*)))
apply(*erule contrapos-pp*)
apply(*simp*)
using *default-unique-IFS SecurityInvariant-withOffendingFlows.valid-without-offending-flows offending-notevalD*
by *metis*
end

sublocale *SecurityInvariant-IFS* ⊆ *SecurityInvariant* **where** *receiver-violation*=*True*
apply(*unfold-locales*)
apply(*simp add*: *default-secure-IFS*)
apply(*simp only*: *HOL.simp-thms*)
apply(*drule default-unique-EX-notation*)
apply(*assumption*)
done

locale *SecurityInvariant-IFS-otherDirectrion* = *SecurityInvariant* **where** *receiver-violation*=*True*
sublocale *SecurityInvariant-IFS-otherDirectrion* ⊆ *SecurityInvariant-IFS*
apply(*unfold-locales*)
apply (*metis default-secure offending-notevalD*)
apply(*erule contrapos-pp*)
apply(*simp*)
apply(*drule default-unique*)
apply(*simp*)
apply(*blast*)
done

lemma *default-uniqueness-by-counterexample-IFS*:
assumes (∀ *G F nP i*. *wf-graph G* ∧ *F* ∈ *SecurityInvariant-withOffendingFlows.set-offending-flows sinvar G nP* ∧ *i* ∈ *snd*‘ *F*
⟶ ¬ *sinvar G* (*nP*(*i* := *otherbot*)))
and *otherbot* ≠ *default-value* ⟹
∃ *G nP i F*. *wf-graph G* ∧ ¬ *sinvar G nP* ∧ *F* ∈ (*SecurityInvariant-withOffendingFlows.set-offending-flows sinvar G nP*) ∧

sinvar (*delete-edges G F*) *nP* ∧
i ∈ *snd* ‘ *F* ∧ *sinvar G* (*nP*(*i* := *otherbot*))
shows *otherbot* = *default-value*
using *assms* **by** *blast*

2.4 Access Control Strategy (ACS)

locale *SecurityInvariant-ACS* = *SecurityInvariant-preliminaries sinvar*
for *sinvar*::(′*v*::*vertex*) *graph* ⇒ (′*v*::*vertex* ⇒ ′*a*) ⇒ *bool*
+
fixes *default-node-properties* :: ′*a* (⊥)
assumes *default-secure-ACS*:
⟦ *wf-graph G*; *f* ∈ *set-offending-flows G nP* ⟧ ⟹
∀ *i* ∈ *fst*‘ *f*. ¬ *sinvar G* (*nP*(*i* := ⊥))
and
default-unique-ACS:
(∀ *G f nP i*. *wf-graph G* ∧ *f* ∈ *set-offending-flows G nP* ∧ *i* ∈ *fst*‘ *f*
⟶ ¬ *sinvar G* (*nP*(*i* := *otherbot*))) ⟹ *otherbot* = ⊥
begin
lemma *default-unique-EX-notation*: *otherbot* ≠ ⊥ ⟹
∃ *G nP i f*. *wf-graph G* ∧ ¬ *sinvar G nP* ∧ *f* ∈ *set-offending-flows G nP* ∧
sinvar (*delete-edges G f*) *nP* ∧
(*i* ∈ *fst*‘ *f* ∧ *sinvar G* (*nP*(*i* := *otherbot*)))
apply(*erule contrapos-pp*)
apply(*simp*)
using *default-unique-ACS SecurityInvariant-withOffendingFlows.valid-without-offending-flows offending-notevalD*
by *metis*
end

sublocale *SecurityInvariant-ACS* ⊆ *SecurityInvariant* **where** *receiver-violation=False*
apply(*unfold-locales*)
apply(*simp add*: *default-secure-ACS*)
apply(*simp only*: *HOL.simp-thms*)
apply(*drule default-unique-EX-notation*)
apply(*assumption*)
done

locale *SecurityInvariant-ACS-otherDirectrion* = *SecurityInvariant* **where** *receiver-violation=False*
sublocale *SecurityInvariant-ACS-otherDirectrion* ⊆ *SecurityInvariant-ACS*
apply(*unfold-locales*)
apply (*metis default-secure offending-notevalD*)
apply(*erule contrapos-pp*)
apply(*simp*)
apply(*drule default-unique*)
apply(*simp*)
apply(*blast*)
done

lemma *default-uniqueness-by-counterexample-ACS*:
assumes (∀ *G F nP i*. *wf-graph G* ∧ *F* ∈ *SecurityInvariant-withOffendingFlows.set-offending-flows*

sinvar G nP ∧ *i* ∈ *fst* ‘ *F*
⟶ ¬ *sinvar G* (*nP*(*i* := *otherbot*)))
and *otherbot* ≠ *default-value* ⟹
∃ *G nP i F*. *wf-graph G* ∧ ¬ *sinvar G nP* ∧ *F* ∈ (*SecurityInvariant-withOffendingFlows.set-offending-flows sinvar G nP*) ∧
sinvar (*delete-edges G F*) *nP* ∧
i ∈ *fst* ‘ *F* ∧ *sinvar G* (*nP*(*i* := *otherbot*))
shows *otherbot* = *default-value*
using *assms* **by** *blast*

The sublocale relationships tell that the simplified *SecurityInvariant-ACS* and *SecurityInvariant-IFS* assumptions suffice to do tho generic SecurityInvariant assumptions.

end
theory *TopoS-withOffendingFlows*
imports *TopoS-Interface*
begin

3 *SecurityInvariant* Instantiation Helpers

The security invariant locales are set up hierarchically to ease instantiation proofs. The first locale, *SecurityInvariant-withOffendingFlows* has no assumptions, thus instantiations is for free. The first step focuses on monotonicity,

context *SecurityInvariant-withOffendingFlows*
begin

We define the monotonicity of *sinvar*:
⋀*nP N E′ E*. ⟦*wf-graph* (|*nodes* = *N*, *edges* = *E*|); *E′* ⊆ *E*; *sinvar* (|*nodes* = *N*, *edges* = *E*|) *nP*⟧ ⟹ *sinvar* (|*nodes* = *N*, *edges* = *E′*|) *nP*
Having a valid invariant, removing edges retains the validity. I.e. prohibiting more, is more or equally secure.

definition *sinvar-mono* :: *bool* **where**
sinvar-mono ⟷ (∀ *nP N E′ E*.
wf-graph (| *nodes* = *N*, *edges* = *E* |) ∧
E′ ⊆ *E* ∧
sinvar (| *nodes* = *N*, *edges* = *E* |) *nP* ⟶ *sinvar* (| *nodes* = *N*, *edges* = *E′* |) *nP*)

If one can show *sinvar-mono*, then the instantiation of the *SecurityInvariant-preliminaries* locale is tremendously simplified.

lemma *sinvar-mono-I-proofrule-simple*:
⟦ (∀ *G nP*. *sinvar G nP* = (∀ (*e1*, *e2*) ∈ *edges G*. *P e1 e2 nP*)) ⟧ ⟹ *sinvar-mono*
apply(*simp add*: *sinvar-mono-def*)
apply(*clarify*)
apply(*fast*)
done

lemma *sinvar-mono-I-proofrule*:
⟦ (∀ *nP* (*G*:: *'v graph*). *sinvar G nP* = (∀ (*e1*, *e2*) ∈ *edges G*. *P e1 e2 nP G*));
(∀ *nP e1 e2 N E′ E*.
wf-graph (| *nodes* = *N*, *edges* = *E* |) ∧
(*e1*,*e2*) ∈ *E* ∧
E′ ⊆ *E* ∧

P *e1 e2 nP* (|*nodes* = *N*, *edges* = *E*|) ⟶ *P e1 e2 nP* (|*nodes* = *N*, *edges* = *E′*|) ⟧ ⟹ *sinvar-mono*
unfolding *sinvar-mono-def*
proof(*clarify*)
fix *nP N E′ E*
assume *AllForm*: (∀ *nP* (*G*:: *′v graph*). *sinvar G nP* = (∀ (*e1*, *e2*) ∈ *edges G*. *P e1 e2 nP G*))
and *Pmono*: ∀ *nP e1 e2 N E′ E*. *wf-graph* (| *nodes* = *N*, *edges* = *E* |) ∧ (*e1*,*e2*) ∈ *E* ∧ *E′* ⊆ *E* ∧ *P e1 e2 nP* (|*nodes* = *N*, *edges* = *E*|) ⟶ *P e1 e2 nP* (|*nodes* = *N*, *edges* = *E′*|)
and *wfG*: *wf-graph* (|*nodes* = *N*, *edges* = *E*|)
and *E′subset*: *E′* ⊆ *E*
and *evalE*: *sinvar* (|*nodes* = *N*, *edges* = *E*|) *nP*

from *Pmono* **have** *Pmono1*:
⋀*nP N E′ E*. *wf-graph* (| *nodes* = *N*, *edges* = *E* |) ⟹ *E′* ⊆ *E* ⟹ (∀(*e1*,*e2*) ∈ *E*. *P e1 e2 nP* (|*nodes* = *N*, *edges* = *E*|) ⟶ *P e1 e2 nP* (|*nodes* = *N*, *edges* = *E′*|))
by *blast*

from *AllForm* **have** *sinvar* (|*nodes* = *N*, *edges* = *E*|) *nP* = (∀ (*e1*, *e2*) ∈ *E*. *P e1 e2 nP* (|*nodes* = *N*, *edges* = *E*|)) **by** *force*
from *this evalE* **have** (∀ (*e1*, *e2*) ∈ *E*. *P e1 e2 nP* (|*nodes* = *N*, *edges* = *E*|)) **by** *simp*
from *Pmono1*[*OF wfG E′subset*, *of nP*] *this* **have** ∀(*e1*, *e2*) ∈ *E*. *P e1 e2 nP* (|*nodes* = *N*, *edges* = *E′*|) **by** *fast*
from *this E′subset* **have** ∀(*e1*, *e2*) ∈ *E′*. *P e1 e2 nP* (|*nodes* = *N*, *edges* = *E′*|) **by** *fast*
from *this* **have** ∀(*e1*, *e2*) ∈ (*edges* (|*nodes* = *N*, *edges* = *E′*|)). *P e1 e2 nP* (|*nodes* = *N*, *edges* = *E′*|) **by** *simp*
from *this AllForm* **show** *sinvar* (|*nodes* = *N*, *edges* = *E′*|) *nP* **by** *presburger*
qed

Invariant violations do not disappear if we add more flows.

lemma *sinvar-mono-imp-negative-mono*:
sinvar-mono ⟹ *wf-graph* (| *nodes* = *N*, *edges* = *E* |) ⟹ *E′* ⊆ *E* ⟹
¬ *sinvar* (| *nodes* = *N*, *edges* = *E′* |) *nP* ⟹ ¬ *sinvar* (| *nodes* = *N*, *edges* = *E* |) *nP*
unfolding *sinvar-mono-def* **by**(*blast*)

corollary *sinvar-mono-imp-negative-delete-edge-mono*:
sinvar-mono ⟹ *wf-graph G* ⟹ *X* ⊆ *Y* ⟹ ¬ *sinvar* (*delete-edges G Y*) *nP* ⟹ ¬ *sinvar* (*delete-edges G X*) *nP*
proof −
assume *sinvar-mono*
and *wf-graph G* **and** *X* ⊆ *Y* **and** ¬ *sinvar* (*delete-edges G Y*) *nP*
from *delete-edges-wf*[*OF* ‹*wf-graph G*›] **have** *valid-G-delete*: *wf-graph* (|*nodes* = *nodes G*, *edges* = *edges G* − *X*|) **by**(*simp add*: *delete-edges-simp2*)
from ‹*X* ⊆ *Y*› **have** *edges G* − *Y* ⊆ *edges G* − *X* **by** *blast*
with ‹*sinvar-mono*› *sinvar-mono-def valid-G-delete* **have**
sinvar (|*nodes* = *nodes G*, *edges* = *edges G* − *X*|) *nP* ⟹ *sinvar* (|*nodes* = *nodes G*, *edges* = *edges G* − *Y*|) *nP* **by** *blast*
hence *sinvar* (*delete-edges G X*) *nP* ⟹ *sinvar* (*delete-edges G Y*) *nP* **by**(*simp add*: *delete-edges-simp2*)
with ‹¬ *sinvar* (*delete-edges G Y*) *nP*› **show** *?thesis* **by** *blast*
qed

lemma *sinvar-mono-imp-is-offending-flows-mono*:
assumes *mono*: *sinvar-mono*
and *wfG*: *wf-graph G*
shows *is-offending-flows FF G nP* $\Longrightarrow$ *is-offending-flows* $(FF \cup F)$ *G nP*
proof −
from *wfG* **have** *wfG'*: *wf-graph* $(\!|nodes = nodes\ G,\ edges = \{(e1,\ e2).\ (e1,\ e2) \in edges\ G \wedge (e1,\ e2) \notin FF\}|\!)$
by (*metis delete-edges-def delete-edges-wf*)
from *mono* **have** *sinvarE*: $(\bigwedge nP\ N\ E'\ E.$ *wf-graph* $(\!|\ nodes = N,\ edges = E\ |\!) \Longrightarrow E' \subseteq E \Longrightarrow$ *sinvar* $(\!|\ nodes = N,\ edges = E\ |\!)\ nP \Longrightarrow$ *sinvar* $(\!|\ nodes = N,\ edges = E'\ |\!)\ nP\ )$
unfolding *sinvar-mono-def*
by *metis*
have $\bigwedge\ G\ FF\ F.\ \{(e1,\ e2).\ (e1,\ e2) \in edges\ G \wedge (e1,\ e2) \notin FF \wedge (e1,\ e2) \notin F\} \subseteq \{(e1,\ e2).\ (e1,\ e2) \in edges\ G \wedge (e1,\ e2) \notin FF\}$
by(*rule Collect-mono*) (*simp*)
from *sinvarE*[*OF wfG' this*]
show *is-offending-flows FF G nP* $\Longrightarrow$ *is-offending-flows* $(FF \cup F)$ *G nP*
by(*simp add*: *is-offending-flows-def delete-edges-def*)
qed

lemma *sinvar-mono-imp-sinvar-mono*:
sinvar-mono $\Longrightarrow$ *wf-graph* $(\!|\ nodes = N,\ edges = E\ |\!) \Longrightarrow E' \subseteq E \Longrightarrow$ *sinvar* $(\!|\ nodes = N,\ edges = E\ |\!)\ nP \Longrightarrow$
sinvar $(\!|\ nodes = N,\ edges = E'\ |\!)\ nP$
apply(*simp add*: *sinvar-mono-def*)
by *blast*

end

3.1 Offending Flows Not Empty Helper Lemmata

context *SecurityInvariant-withOffendingFlows*
begin

Give an over-approximation of offending flows (e.g. all edges) and get back a minimal set

fun *minimalize-offending-overapprox* :: $('v \times 'v)$ *list* $\Rightarrow$ $('v \times 'v)$ *list* $\Rightarrow$
$'v$ *graph* $\Rightarrow$ $('v \Rightarrow 'a) \Rightarrow ('v \times 'v)$ *list* **where**
minimalize-offending-overapprox [] *keep - - = keep* |
minimalize-offending-overapprox $(f\#fs)$ *keep G nP* = (*if sinvar* (*delete-edges-list G* (*fs@keep*)) *nP then*
minimalize-offending-overapprox fs keep G nP
else
minimalize-offending-overapprox fs $(f\#keep)$ *G nP*
)

The graph we check in *minimalize-offending-overapprox*, *G* $(-)$ $(fs \cup keep)$ is the graph from the *offending-flows-min-set* condition. We add f and remove it.

lemma *minimalize-offending-overapprox-subset*:
set (*minimalize-offending-overapprox ff keeps G nP*) $\subseteq$ *set ff* $\cup$ *set keeps*
proof(*induction ff arbitrary*: *keeps*)
case *Nil*
thus *?case* **by** *simp*
next

case (*Cons a ff*)
from *Cons* **have** *case1*: (*sinvar* (*delete-edges-list G* (*ff* @ *keeps*)) *nP* ⟹
set (*minimalize-offending-overapprox ff keeps G nP*) ⊆ *insert a* (*set ff* ∪ *set keeps*))
by *blast*
from *Cons* **have** *case2*: (¬ *sinvar* (*delete-edges-list G* (*ff* @ *keeps*)) *nP* ⟹
set (*minimalize-offending-overapprox ff* (*a* # *keeps*) *G nP*) ⊆ *insert a* (*set ff* ∪ *set keeps*))
by *fastforce*
from *case1 case2* **show** *?case* **by** *simp*
qed

lemma *not-model-mono-imp-addedge-mono*:
assumes *mono*: *sinvar-mono*
and *vG*: *wf-graph G* **and** *ain*: (*a1*,*a2*) ∈ *edges G* **and** *xy*: *X* ⊆ *Y* **and** *ns*: ¬ *sinvar* (*add-edge a1 a2* (*delete-edges G* (*Y*))) *nP*
shows ¬ *sinvar* (*add-edge a1 a2* (*delete-edges G X*)) *nP*
proof −
have *wf-graph-add-delete-edge-simp*:
⋀*Y*. *add-edge a1 a2* (*delete-edges G Y*) = (*delete-edges G* (*Y* − {(*a1*,*a2*)}))
apply(*simp add*: *delete-edges-simp2 add-edge-def*)
apply(*rule conjI*)
using *ain* **apply** (*metis insert-absorb vG wf-graph.E-wfD*(*1*) *wf-graph.E-wfD*(*2*))
apply(*auto simp add*: *ain*)
done
from *this ns* **have** *1*: ¬ *sinvar* (*delete-edges G* (*Y* − {(*a1*, *a2*)})) *nP* **by** *simp*
have *2*: *X* − {(*a1*, *a2*)} ⊆ *Y* − {(*a1*, *a2*)} **by** (*metis Diff-mono subset-refl xy*)
from *sinvar-mono-imp-negative-delete-edge-mono*[*OF mono*] *vG* **have**
⋀*X Y*. *X* ⊆ *Y* ⟹ ¬ *sinvar* (*delete-edges G Y*) *nP* ⟹ ¬ *sinvar* (*delete-edges G X*) *nP* **by** *blast*
from *this*[*OF 2 1*] **have** ¬ *sinvar* (*delete-edges G* (*X* − {(*a1*, *a2*)})) *nP* **by** *simp*
from *this wf-graph-add-delete-edge-simp*[*symmetric*] **show** *?thesis* **by** *simp*
qed

theorem *is-offending-flows-min-set-minimalize-offending-overapprox*:
assumes *mono*: *sinvar-mono*
and *vG*: *wf-graph G* **and** *iO*: *is-offending-flows* (*set ff*) *G nP* **and** *sF*: *set ff* ⊆ *edges G* **and** *dF*: *distinct ff*
shows *is-offending-flows-min-set* (*set* (*minimalize-offending-overapprox ff* [] *G nP*)) *G nP*
(**is** *is-offending-flows-min-set ?minset G nP*)
proof −
from *iO* **have** *sinvar* (*delete-edges G* (*set ff*)) *nP* **by** (*metis is-offending-flows-def*)

— *sinvar* holds if we delete *ff*. With the following generalized statement, we show that it also holds if we delete *minimalize-offending-overapprox ff* []
{
fix *keeps*
— Generalized for arbitrary *keeps*
have *sinvar* (*delete-edges G* (*set ff* ∪ *set keeps*)) *nP* ⟹
sinvar (*delete-edges G* (*set* (*minimalize-offending-overapprox ff keeps G nP*))) *nP*
apply(*induction ff arbitrary*: *keeps*)
apply(*simp*)
apply(*simp*)

apply(*rule impI*)
apply(*simp add*:*delete-edges-list-union*)
done
}
— *keeps* = []
note *minimalize-offending-overapprox-maintains-evalmodel*=*this*[*of* []]

from ‹*sinvar* (*delete-edges G* (*set ff*)) *nP*› *minimalize-offending-overapprox-maintains-evalmodel*
have
sinvar (*delete-edges G ?minset*) *nP* **by** *simp*
hence *1*: *is-offending-flows ?minset G nP* **by** (*metis iO is-offending-flows-def*)

We need to show minimality of *minimalize-offending-overapprox ff* []. Minimality means ∀(*e1*, *e2*)∈*set* (*minimalize-offending-overapprox ff* [] *G nP*). ¬ *sinvar* (*add-edge e1 e2* (*delete-edges G* (*set* (*minimalize-offending-overap*
ff [] *G nP*)))) *nP*. We show the following generalized fact.

{
fix *ff keeps*
have ∀ *x* ∈ *set ff*. *x* ∉ *set keeps* ⟹
∀ *x* ∈ *set ff*. *x* ∈ *edges G* ⟹
distinct ff ⟹
∀(*e1*, *e2*)∈ *set keeps*.
¬ *sinvar* (*add-edge e1 e2* (*delete-edges G* (*set* (*minimalize-offending-overapprox ff keeps G*
nP)))) *nP* ⟹
∀(*e1*, *e2*)∈*set* (*minimalize-offending-overapprox ff keeps G nP*).
¬ *sinvar* (*add-edge e1 e2* (*delete-edges G* (*set* (*minimalize-offending-overapprox ff keeps G*
nP)))) *nP*
proof(*induction ff arbitrary*: *keeps*)
case *Nil*
from *Nil* **show** *?case* **by**(*simp*)
next
case (*Cons a ff*)
assume *not-in-keeps*: ∀*x*∈*set* (*a* # *ff*). *x* ∉ *set keeps*
hence *a-not-in-keeps*: *a* ∉ *set keeps* **by** *simp*
assume *in-edges*: ∀*x*∈*set* (*a* # *ff*). *x* ∈ *edges G*
hence *ff-in-edges*: ∀*x*∈*set ff*. *x* ∈ *edges G* **and** *a-in-edges*: *a* ∈ *edges G* **by** *simp-all*
assume *distinct*: *distinct* (*a* # *ff*)
hence *ff-distinct*: *distinct ff* **and** *a-not-in-ff*: *a* ∉ *set ff* **by** *simp-all*
assume *minimal*: ∀(*e1*, *e2*)∈*set keeps*.
¬ *sinvar* (*add-edge e1 e2* (*delete-edges G* (*set* (*minimalize-offending-overapprox* (*a* # *ff*) *keeps*
G nP)))) *nP*

have *delete-edges-list-union-insert*: ⋀ *f fs keep*. *delete-edges-list G* (*f*#*fs*@*keep*) = *delete-edges G* ({*f*} ∪ *set fs* ∪ *set keep*)
by(*simp add*: *graph-ops delete-edges-list-set*)

let *?goal*=*?case* — we show this by case distinction
show *?case*
proof(*cases sinvar* (*delete-edges-list G* (*ff*@*keeps*)) *nP*)
case *True*
from *True* **have** *sinvar* (*delete-edges-list G* (*ff*@*keeps*)) *nP* **.**
from *this Cons* **show** *?goal* **using** *delete-edges-list-union* **by** *simp*
next

case *False*

{ — a lemma we only need once here
fix *a ff keeps*
assume *mono*: *sinvar-mono* **and** *ankeeps*: $a \notin$ *set keeps*
and *anff*: $a \notin$ *set ff* **and** *aE*: $a \in$ *edges G*
and *nsinvar*: $\neg$ *sinvar* (*delete-edges-list G* (*ff* @ *keeps*)) *nP*
have $\neg$ *sinvar* (*add-edge* (*fst a*) (*snd a*) (*delete-edges G* (*set* (*minimalize-offending-overapprox* (*a # ff*) *keeps G nP*)))) *nP*
proof −
{ fix *F Fs keep*
from *vG* **have** $F \in$ *edges G* $\Longrightarrow$ $F \notin$ *set Fs* $\Longrightarrow$ $F \notin$ *set keep* $\Longrightarrow$
(*add-edge* (*fst F*) (*snd F*) (*delete-edges-list G* (*F#Fs@keep*))) = (*delete-edges-list G* (*Fs@keep*))
apply(*simp add*:*delete-edges-list-union delete-edges-list-union-insert*)
apply(*simp add*: *graph-ops*)
apply(*rule conjI*)
apply(*simp add*: *wf-graph-def*)
apply *blast*
apply(*simp add*: *wf-graph-def*)
by *fastforce*
} note *delete-edges-list-add-add-iff=this*
from *aE* **have** (*fst a*, *snd a*) $\in$ *edges G* **by** *simp*
from *delete-edges-list-add-add-iff*[*of a ff keeps*] **have**
delete-edges-list G (*ff* @ *keeps*) = *add-edge* (*fst a*) (*snd a*) (*delete-edges-list G* (*a # ff* @ *keeps*))
by (*metis aE anff ankeeps*)
from *this nsinvar* **have** $\neg$ *sinvar* (*add-edge* (*fst a*) (*snd a*) (*delete-edges-list G* (*a # ff* @ *keeps*))) *nP* **by** *simp*
from *this delete-edges-list-union-insert* **have** *1*:
$\neg$ *sinvar* (*add-edge* (*fst a*) (*snd a*) (*delete-edges G* (*insert a* (*set ff* $\cup$ *set keeps*)))) *nP* **by** (*metis insert-is-Un sup-assoc*)

from *minimalize-offending-overapprox-subset*[*of ff a#keeps G nP*] **have**
set (*minimalize-offending-overapprox ff* (*a # keeps*) *G nP*) $\subseteq$ *insert a* (*set ff* $\cup$ *set keeps*) **by** *simp*

from *not-model-mono-imp-addedge-mono*[*OF mono vG* ‹(*fst a*, *snd a*) $\in$ *edges G*› *this 1*] **show** *?thesis*
by (*metis minimalize-offending-overapprox.simps*(*2*) *nsinvar*)
qed
} note *not-model-mono-imp-addedge-mono-minimalize-offending-overapprox=this*

from *not-model-mono-imp-addedge-mono-minimalize-offending-overapprox*[*OF mono a-not-in-keeps a-not-in-ff a-in-edges False*] **have** *a-minimal*:
$\neg$ *sinvar* (*add-edge* (*fst a*) (*snd a*) (*delete-edges G* (*set* (*minimalize-offending-overapprox* (*a # ff*) *keeps G nP*)))) *nP*
by *simp*
from *minimal a-minimal*
have *a-keeps-minimal*: $\forall$(*e1*, *e2*)$\in$*set* (*a # keeps*).
$\neg$ *sinvar* (*add-edge e1 e2* (*delete-edges G* (*set* (*minimalize-offending-overapprox ff* (*a # keeps*) *G nP*)))) *nP*
using *False* **by** *fastforce*
from *Cons.prems* **have** *a-not-in-keeps*: $\forall x \in$*set ff*. $x \notin$ *set* (*a#keeps*) **by** *auto*

from *Cons.IH*[*OF a-not-in-keeps ff-in-edges ff-distinct a-keeps-minimal*] **have** *IH*:
$\forall (e1,\ e2) \in set\ (minimalize\text{-}offending\text{-}overapprox\ ff\ (a\ \#\ keeps)\ G\ nP).$
$\neg\ sinvar\ (add\text{-}edge\ e1\ e2\ (delete\text{-}edges\ G\ (set\ (minimalize\text{-}offending\text{-}overapprox\ ff\ (a\ \#\ keeps)\ G\ nP))))\ nP$ **.**

from *False* **have** $\neg\ sinvar\ (delete\text{-}edges\ G\ (set\ ff \cup set\ keeps))\ nP$ **using** *delete-edges-list-union* **by** *metis*
from *this* **have** $set\ (minimalize\text{-}offending\text{-}overapprox\ (a\ \#\ ff)\ keeps\ G\ nP) =$
$set\ (minimalize\text{-}offending\text{-}overapprox\ ff\ (a\#keeps)\ G\ nP)$
by(*simp add*: *delete-edges-list-union*)
from *this IH* **have** *?goal* **by** *presburger*
thus *?goal* **.**
qed
qed
} **note** *mono-imp-minimalize-offending-overapprox-minimal=this*[*of ff* []]

from *mono-imp-minimalize-offending-overapprox-minimal*[*OF - - dF*] *sF* **have** *2*:
$\forall (e1,\ e2) \in ?minset.\ \neg\ sinvar\ (add\text{-}edge\ e1\ e2\ (delete\text{-}edges\ G\ ?minset))\ nP$
by *auto*
from *1 2* **show** *?thesis*
by(*simp add*: *is-offending-flows-def is-offending-flows-min-set-def*)
qed

corollary *mono-imp-set-offending-flows-not-empty*:
assumes *mono-sinvar*: *sinvar-mono*
and *vG*: *wf-graph G* **and** *iO*: *is-offending-flows* (*set ff*) *G nP* **and** *sS*: *set ff* $\subseteq$ *edges G* **and** *dF*: *distinct ff*
shows
set-offending-flows G nP $\neq \{\}$
proof −
from *iO SecurityInvariant-withOffendingFlows.is-offending-flows-def* **have** *nS*: $\neg\ sinvar\ G\ nP$ **by** *metis*
from *sinvar-mono-imp-negative-delete-edge-mono*[*OF mono-sinvar*] **have** *negative-delete-edge-mono*:

$\forall\ G\ nP\ X\ Y.\ wf\text{-}graph\ G \wedge X \subseteq Y \wedge \neg\ sinvar\ (delete\text{-}edges\ G\ (Y))\ nP \longrightarrow \neg\ sinvar\ (delete\text{-}edges\ G\ X)\ nP$ **by** *blast*

from *is-offending-flows-min-set-minimalize-offending-overapprox*[*OF mono-sinvar vG iO sS dF*]
have *is-offending-flows-min-set* (*set* (*minimalize-offending-overapprox ff* [] *G nP*)) *G nP* **by** *simp*
from *this set-offending-flows-def sS* **have**
$(set\ (minimalize\text{-}offending\text{-}overapprox\ ff\ []\ G\ nP)) \in set\text{-}offending\text{-}flows\ G\ nP$
using *minimalize-offending-overapprox-subset*[**where** *keeps*=[]] **by** *fastforce*
thus *?thesis* **by** *blast*
qed

To show that *set-offending-flows* is not empty, the previous corollary $[\![$*sinvar-mono*; *wf-graph ?G*; *is-offending-flows* (*set ?ff*) *?G ?nP*; *set ?ff* $\subseteq$ *edges ?G*; *distinct ?ff*$]\!] \Longrightarrow$ *set-offending-flows ?G ?nP* $\neq \{\}$ is very useful. Just select *set ff* = *edges G*.

If there exists a security violations, there a means to fix it if and only if the network in which nobody communicates with anyone fulfills the security requirement

theorem *valid-empty-edges-iff-exists-offending-flows*:
assumes *mono*: *sinvar-mono* **and** *wfG*: *wf-graph G* **and** *noteval*: $\neg\ sinvar\ G\ nP$
shows $sinvar\ (\!|\ nodes = nodes\ G,\ edges = \{\}\ |\!)\ nP \longleftrightarrow set\text{-}offending\text{-}flows\ G\ nP \neq \{\}$

proof
assume *a*: *sinvar* (| *nodes* = *nodes G*, *edges* = {} |) *nP*

from *finite-distinct-list*[*OF wf-graph.finiteE*] *wfG*
obtain *list-edges* **where** *list-edges-props*: *set list-edges* = *edges G* ∧ *distinct list-edges* **by** *blast*
hence *listedges-subseteq-edges*: *set list-edges* ⊆ *edges G* **by** *blast*

have *empty-edge-graph-simp*: (*delete-edges G* (*edges G*)) = (| *nodes* = *nodes G*, *edges* = {} |) **by**(*auto simp add*: *graph-ops*)
from *a is-offending-flows-def noteval list-edges-props empty-edge-graph-simp*
have *overapprox*: *is-offending-flows* (*set list-edges*) *G nP* **by** *auto*

from *mono-imp-set-offending-flows-not-empty*[*OF mono wfG overapprox listedges-subseteq-edges*] *list-edges-props*
show *set-offending-flows G nP* ≠ {} **by** *simp*
next
assume *a*: *set-offending-flows G nP* ≠ {}

from *a* **obtain** *f* **where** *f-props*: *f* ⊆ *edges G* ∧ *is-offending-flows-min-set f G nP* **using** *set-offending-flows-def* **by** *fastforce*

from *f-props* **have** *sinvar* (*delete-edges G f*) *nP* **using** *is-offending-flows-min-set-def is-offending-flows-def* **by** *simp*
hence *evalGf*: *sinvar* (|*nodes* = *nodes G*, *edges* = {(*e1*, *e2*). (*e1*, *e2*) ∈ *edges G* ∧ (*e1*, *e2*) ∉ *f*}|) *nP* **by**(*simp add*: *delete-edges-def*)
from *delete-edges-wf*[*OF wfG*, *unfolded delete-edges-def*]
have *wfGf*: *wf-graph* (|*nodes* = *nodes G*, *edges* = {(*e1*, *e2*). (*e1*, *e2*) ∈ *edges G* ∧ (*e1*, *e2*) ∉ *f*}|) **by** *simp*
have *emptyseqGf*: {} ⊆ {(*e1*, *e2*). (*e1*, *e2*) ∈ *edges G* ∧ (*e1*, *e2*) ∉ *f*} **by** *simp*

from *mono*[*unfolded sinvar-mono-def*] *evalGf wfGf emptyseqGf* **have** *sinvar* (|*nodes* = *nodes G*, *edges* = {}|) *nP* **by** *blast*
thus *sinvar* (|*nodes* = *nodes G*, *edges* = {}|) *nP* .
qed

minimalize-offending-overapprox not only computes a set where *is-offending-flows-min-set* holds, but it also returns a subset of the input.

lemma *minimalize-offending-overapprox-keeps-keeps*: (*set keeps*) ⊆ *set* (*minimalize-offending-overapprox ff keeps G nP*)
proof(*induction ff keeps G nP rule*: *minimalize-offending-overapprox.induct*)
qed(*simp-all*)

lemma *minimalize-offending-overapprox-subseteq-input*: *set* (*minimalize-offending-overapprox ff keeps G nP*) ⊆ (*set ff*) ∪ (*set keeps*)
proof(*induction ff keeps G nP rule*: *minimalize-offending-overapprox.induct*)
case *1* **thus** *?case* **by** *simp*
next
case *2* **thus** *?case* **by**(*simp add*: *delete-edges-list-set delete-edges-simp2*) *blast*
qed

end

context *SecurityInvariant-preliminaries*
begin

sinvar-mono naturally holds in *SecurityInvariant-preliminaries*

lemma *sinvar-monoI*: *sinvar-mono*
unfolding *sinvar-mono-def* **using** *mono-sinvar* **by** *blast*

Note: due to monotonicity, the minimality also holds for arbitrary subsets

lemma assumes *wf-graph G* **and** *is-offending-flows-min-set F G nP* **and** $F \subseteq$ *edges G* **and** $E \subseteq F$ **and** $E \neq \{\}$
shows $\neg$ *sinvar* (| *nodes* = *nodes G*, *edges* = ((*edges G*) − *F*) ∪ *E* |) *nP*
proof −
from *sinvar-mono-imp-negative-delete-edge-mono*[*OF sinvar-monoI* ‹*wf-graph G*›] **have** *negative-delete-edge-mono*:

$\bigwedge X\ Y.\ X \subseteq Y \Longrightarrow \neg$ *sinvar* (| *nodes* = *nodes G*, *edges* = (*edges G*) − *Y* |) *nP* $\Longrightarrow \neg$ *sinvar* (| *nodes* = *nodes G*, *edges* = *edges G* − *X* |) *nP*
using *delete-edges-simp2* **by** *metis*
from *assms*(*2*) **have** (∀(*e1*, *e2*)∈*F*. $\neg$ *sinvar* (*add-edge e1 e2* (*delete-edges G F*)) *nP*)
unfolding *is-offending-flows-min-set-def* **by** *simp*
with ‹*wf-graph G*› **have** *min*: (∀(*e1*, *e2*)∈*F*. $\neg$ *sinvar* (| *nodes* = *nodes G*, *edges* = ((*edges G*) − *F*) ∪ {(*e1*,*e2*)} |) *nP*)
apply(*simp add*: *delete-edges-simp2 add-edge-def*)
apply(*rule*, *rename-tac x*, *case-tac x*, *rename-tac e1 e2*, *simp*)
apply(*erule-tac x*=(*e1*, *e2*) **in** *ballE*)
apply(*simp-all*)
apply(*subgoal-tac insert e1* (*insert e2* (*nodes G*)) = *nodes G*)
apply(*simp*)
by (*metis assms*(*3*) *insert-absorb rev-subsetD wf-graph.E-wfD*(*1*) *wf-graph.E-wfD*(*2*))
from ‹*E* ≠ {}› **obtain** *e* **where** *e* ∈ *E* **by** *blast*
with *min* ‹*E* ⊆ *F*› **have** *mine*: $\neg$ *sinvar* (| *nodes* = *nodes G*, *edges* = ((*edges G*) − *F*) ∪ {*e*} |) *nP* **by** *fast*
have *e1*: *edges G* − (*F* − {*e*}) = *insert e* (*edges G* − *F*) **using** *DiffD2* ‹*e* ∈ *E*› *assms*(*3*) *assms*(*4*) **by** *auto*
have *e2*: *edges G* − (*F* − *E*) = ((*edges G*) − *F*) ∪ *E* **using** *assms*(*3*) *assms*(*4*) **by** *auto*
from *negative-delete-edge-mono*[**where** *Y*=*F* − {*e*} **and** *X*=*F* − *E*] ‹*e* ∈ *E*› **have**
$\neg$ *sinvar* (|*nodes* = *nodes G*, *edges* = *edges G* − (*F* − {*e*})|) *nP* $\Longrightarrow \neg$ *sinvar* (|*nodes* = *nodes G*, *edges* = *edges G* − (*F* − *E*)|) *nP* **by** *blast*
with *mine e1 e2* **show** *?thesis* **by** *simp*
qed

The algorithm *minimalize-offending-overapprox* is correct

lemma *minimalize-offending-overapprox-sound*:
⟦ *wf-graph G*; *is-offending-flows* (*set ff*) *G nP*; *set ff* ⊆ *edges G*; *distinct ff* ⟧
$\Longrightarrow$ *is-offending-flows-min-set* (*set* (*minimalize-offending-overapprox ff* [] *G nP*)) *G nP*
using *is-offending-flows-min-set-minimalize-offending-overapprox sinvar-monoI* **by** *blast*

If $\neg$ *sinvar G nP* Given a list ff, (ff is distinct and a subset of G's edges) such that *sinvar* (*V*, *E* − *ff*) *nP* *minimalize-offending-overapprox* minimizes ff such that we get an offending flows Note: choosing ff = edges G is a good choice!

theorem *minimalize-offending-overapprox-gives-back-an-offending-flow*:
⟦ *wf-graph G*; *is-offending-flows* (*set ff*) *G nP*; *set ff* ⊆ *edges G*; *distinct ff* ⟧
$\Longrightarrow$

(*set* (*minimalize-offending-overapprox ff* [] *G nP*)) ∈ *set-offending-flows G nP*
apply(*frule*(*3*) *minimalize-offending-overapprox-sound*)
apply(*simp add*: *set-offending-flows-def*)
using *minimalize-offending-overapprox-subseteq-input*[**where** *keeps*=[], *simplified*] **by** *blast*

end

A version which acts on configured security invariants. I.e. there is no type *'a* for the host attributes in it.

fun *minimalize-offending-overapprox* :: ('*v graph* ⇒ *bool*) ⇒ ('*v* × '*v*) *list* ⇒ ('*v* × '*v*) *list* ⇒
'*v graph* ⇒('*v* × '*v*) *list* **where**
minimalize-offending-overapprox - [] keep - = keep |
minimalize-offending-overapprox m (*f*#*fs*) *keep G* = (*if m* (*delete-edges-list G* (*fs*@*keep*)) *then*
minimalize-offending-overapprox m fs keep G
else
minimalize-offending-overapprox m fs (*f*#*keep*) *G*
)

lemma *minimalize-offending-overapprox-boundnP*:
shows *minimalize-offending-overapprox* (λ*G*. *m G nP*) *fs keeps G* =
SecurityInvariant-withOffendingFlows.minimalize-offending-overapprox m fs keeps G nP
apply(*induction fs arbitrary*: *keeps*)
apply(*simp add*: *SecurityInvariant-withOffendingFlows.minimalize-offending-overapprox.simps*; *fail*)
apply(*simp add*: *SecurityInvariant-withOffendingFlows.minimalize-offending-overapprox.simps*)
done

context *SecurityInvariant-withOffendingFlows*
begin

If there is a violation and there are no offending flows, there does not exist a possibility to fix the violation by tightening the policy. ⟦*sinvar-mono*; *wf-graph ?G*; ¬ *sinvar ?G ?nP*⟧ ⟹ *sinvar* (⦇*nodes* = *nodes ?G*, *edges* = {}⦈) *?nP* = (*set-offending-flows ?G ?nP* ≠ {}) already hints this.

lemma *mono-imp-emptyoffending-eq-nevervalid*:
⟦ *sinvar-mono*; *wf-graph G*; ¬ *sinvar G nP*; *set-offending-flows G nP* = {}⟧ ⟹
¬ (∃ *F* ⊆ *edges G*. *sinvar* (*delete-edges G F*) *nP*)
proof −
assume *mono*: *sinvar-mono*
and *wfG*: *wf-graph G*
and *a1*: ¬ *sinvar G nP*
and *a2*: *set-offending-flows G nP* = {}

from *wfG* **have** *wfG'*: *wf-graph* (⦇*nodes* = *nodes G*, *edges* = *edges G*⦈) **by**(*simp add*:*wf-graph-def*)

from *a2 set-offending-flows-def* **have** ∀ *f* ⊆ *edges G*. ¬ *is-offending-flows-min-set f G nP* **by** *simp*
from *this is-offending-flows-min-set-def is-offending-flows-def a1* **have** *notdeleteconj*:
∀ *f* ⊆ *edges G*.

$\neg$ *sinvar* (*delete-edges G f*) *nP* $\vee$
$\neg$ (($\forall$(*e1*, *e2*)$\in$*f*. $\neg$ *sinvar* (*add-edge e1 e2* (*delete-edges G f*)) *nP*))
by *simp*
have $\forall f \subseteq$*edges G*. $\neg$ *sinvar* (*delete-edges G f*) *nP*
proof (*rule allI*, *rule impI*)
fix *f*
assume *f* $\subseteq$ *edges G*
from *this notdeleteconj* **have**
$\neg$ *sinvar* (*delete-edges G f*) *nP* $\vee$
$\neg$ (($\forall$(*e1*, *e2*)$\in$*f*. $\neg$ *sinvar* (*add-edge e1 e2* (*delete-edges G f*)) *nP*)) **by** *simp*
from *this* **show** $\neg$ *sinvar* (*delete-edges G f*) *nP*
proof
assume $\neg$ *sinvar* (*delete-edges G f*) *nP* **thus** $\neg$ *sinvar* (*delete-edges G f*) *nP* **.**
next
assume $\neg$ ($\forall$(*e1*, *e2*)$\in$*f*. $\neg$ *sinvar* (*add-edge e1 e2* (*delete-edges G f*)) *nP*)
hence $\exists$(*e1*,*e2*)$\in$*f*. *sinvar* (*add-edge e1 e2* (*delete-edges G f*)) *nP* **by**(*auto*)
from *this* **obtain** *e1 e2* **where** *e1e2cond*: (*e1*,*e2*)$\in$*f* $\wedge$ *sinvar* (*add-edge e1 e2* (*delete-edges G f*)) *nP* **by** *blast*

from ‹*f* $\subseteq$ *edges G*› *wfG* **have** *finite f* **apply**(*simp add*: *wf-graph-def*) **by** (*metis rev-finite-subset*)
from *this* **obtain** *listf* **where** *listf*: *set listf* = *f* $\wedge$ *distinct listf* **by** (*metis finite-distinct-list*)

from *e1e2cond* ‹*f* $\subseteq$ *edges G*› **have** *Geq*:
(*add-edge e1 e2* (*delete-edges G f*)) = (| *nodes* = *nodes G*, *edges* = *edges G* $-$ *f* $\cup$ {(*e1*,*e2*)}|)
apply(*simp add*: *graph-ops wfG'*)
apply(*clarify*)
using *wfG*[*unfolded wf-graph-def*] **by** *force*

from *this*[*symmetric*] *add-edge-wf*[*OF delete-edges-wf*[*OF wfG*]] **have**
wf-graph (|*nodes* = *nodes G*, *edges* = *edges G* $-$ *f* $\cup$ {(*e1*, *e2*)}|) **by** *simp*
from *mono this* **have** *mono''*:
$\bigwedge$ *E'*. *E'* $\subseteq$ *edges G* $-$ *f* $\cup$ {(*e1*, *e2*)} $\Longrightarrow$
sinvar (|*nodes* = *nodes G*, *edges* = *edges G* $-$ *f* $\cup$ {(*e1*, *e2*)}|) *nP* $\Longrightarrow$
sinvar (|*nodes* = *nodes G*, *edges* = *E'*|) *nP* **unfolding** *sinvar-mono-def* **by** *blast*

from *e1e2cond Geq* **have** *sinvar* (| *nodes* = *nodes G*, *edges* = *edges G* $-$ *f* $\cup$ {(*e1*,*e2*)}|) *nP* **by** *simp*
from *this mono''* **have** *sinvar* (| *nodes* = *nodes G*, *edges* = *edges G* $-$ *f*|) *nP* **by** *auto*
hence *overapprox*: *sinvar* (*delete-edges G f*) *nP* **by** (*simp add*: *delete-edges-simp2*)

from *a1 overapprox* **have** *is-offending-flows f G nP* **by**(*simp add*: *is-offending-flows-def*)
from *this listf* **have** *c1*: *is-offending-flows* (*set listf*) *G nP* **by**(*simp add*: *is-offending-flows-def*)
from *listf* ‹*f* $\subseteq$ *edges G*› **have** *c2*: *set listf* $\subseteq$ *edges G* **by** *simp*

from *mono-imp-set-offending-flows-not-empty*[*OF mono wfG c1 c2 conjunct2*[*OF listf*]] **have**

set-offending-flows G nP $\neq$ {} **.**
from *this a2* **have** *False* **by** *simp*

thus $\neg$ *sinvar* (*delete-edges G f*) *nP* **by** *simp*

```
    qed
  qed
  thus ?thesis by simp
 qed
end
```

3.2 Monotonicity of offending flows

```
context SecurityInvariant-preliminaries
begin
```

If there is some F' in the offending flows of a small graph and you have a bigger graph, you can extend F' by some *Fadd* and minimality in F is preserved

```
 lemma minimality-offending-flows-mono-edges-graph-extend:
 ⟦ wf-graph (| nodes = V, edges = E |); E' ⊆ E; Fadd ∩ E' = {}; F' ∈ set-offending-flows (|nodes
= V, edges = E'|) nP ⟧ ⟹
        (∀(e1, e2)∈F'. ¬ sinvar (add-edge e1 e2 (delete-edges (|nodes = V, edges = E |) (F' ∪
Fadd))) nP)
 proof −
  assume a1: wf-graph (| nodes = V, edges = E |)
  and    a2: E' ⊆ E
  and    a3: Fadd ∩ E' = {}
  and    a4: F' ∈ set-offending-flows (|nodes = V, edges = E'|) nP

  from a4 have F' ⊆ E' by(simp add: set-offending-flows-def)

  obtain Eadd where Eadd-prop: E' ∪ Eadd = E and E' ∩ Eadd = {} using a2 by blast

  have Fadd-notinE': ⋀Fadd. Fadd ∩ E' = {} ⟹ E' − (F' ∪ Fadd) = E' − F' by blast
  from ‹F' ⊆ E'› a1[simplified wf-graph-def] a2 have FinV1: fst ‘ F' ⊆ V and FinV2: snd ‘ F'
⊆ V
  proof −
   from a1 have fst ‘ E ⊆ V by(simp add: wf-graph-def)
   with ‹F' ⊆ E'› a2 show fst ‘ F' ⊆ V by fast
   from a1 have snd ‘ E ⊆ V by(simp add: wf-graph-def)
   with ‹F' ⊆ E'› a2 show snd ‘ F' ⊆ V by fast
  qed
  hence insert-e1-e2-V: ∀ (e1, e2) ∈ F'. insert e1 (insert e2 V) = V by auto
  hence add-edge-F: ∀ (e1, e2) ∈ F'. add-edge e1 e2 (|nodes = V, edges = E' − F' |) = (|nodes
= V, edges = (E' − F') ∪ {(e1, e2)}|)
   by(simp add: add-edge-def)

  have Fadd-notinE': ⋀Fadd. Fadd ∩ E' = {} ⟹ E' − (F' ∪ Fadd) = E' − F' by blast
  from ‹F' ⊆ E'› this have Fadd-notinF: ⋀Fadd. Fadd ∩ E' = {} ⟹ F' ∩ Fadd = {} by blast

  have Fadd-subseteq-Eadd: ⋀Fadd. (Fadd ∩ E' = {} ∧ Fadd ⊆ E) = (Fadd ⊆ Eadd)
   proof(rule iffI, goal-cases)
   case 1 thus ?case using Eadd-prop a2 by blast
   next
   case 2 thus ?case using Eadd-prop a2 ‹E' ∩ Eadd = {}› by blast
   qed

  from a4 have (∀(e1, e2)∈F'. ¬ sinvar (add-edge e1 e2 (|nodes = V, edges = E' − F'|)) nP)
   by(simp add: set-offending-flows-def is-offending-flows-min-set-def delete-edges-simp2)
```

with *add-edge-F* **have** *noteval-F*: $\forall (e1,\ e2){\in}F'.\ \neg\ sinvar\ (\!|nodes = V,\ edges = (E' - F') \cup \{(e1,\ e2)\}|\!)\ nP$
by *fastforce*

have *tupleBallI*: $\bigwedge A\ P.\ (\bigwedge e1\ e2.\ (e1,\ e2){\in}A \Longrightarrow P\ (e1,\ e2)) \Longrightarrow ALL\ (e1,\ e2){:}A.\ P\ (e1,\ e2)$ **by** *force*
have $\forall (e1,\ e2){\in}F'.\ \neg\ sinvar\ (\!|nodes = V,\ edges = (E - (F' \cup Fadd)) \cup \{(e1,\ e2)\}|\!)\ nP$
proof(*rule tupleBallI*)
fix *e1 e2*
assume *f2*: $(e1,\ e2) \in F'$
with *a3* **have** *gFadd1*: $\neg\ sinvar\ (\!|nodes = V,\ edges = (E' - (F' \cup Fadd)) \cup \{(e1,\ e2)\}|\!)\ nP$
using *Fadd-notinE' noteval-F* **by** *fastforce*

from *a1 FinV1 FinV2 a3 f2* **have** *gFadd2*:
wf-graph $(\!|nodes = V,\ edges = (E - (F' \cup Fadd)) \cup \{(e1,\ e2)\}|\!)$
by(*auto simp add*: *wf-graph-def*)
from *a2 a3 f2* **have** *gFadd3*:
$(E' - (F' \cup Fadd)) \cup \{(e1,\ e2)\} \subseteq (E - (F' \cup Fadd)) \cup \{(e1,\ e2)\}$ **by** *blast*

from *mono-sinvar*[*OF gFadd2 gFadd3*] *gFadd1*
show $\neg\ sinvar\ (\!|nodes = V,\ edges = (E - (F' \cup Fadd)) \cup \{(e1,\ e2)\}|\!)\ nP$ **by** *blast*
qed
thus *?thesis*
apply(*simp add*: *delete-edges-simp2 Fadd-notinE' add-edge-def*)
apply(*clarify*)
using *insert-e1-e2-V* **by** *fastforce*
qed

The minimality condition of the offending flows also holds if we increase the graph.

corollary *minimality-offending-flows-mono-edges-graph*:
$[\![$ *wf-graph* $(\!|$ *nodes* $= V,$ *edges* $= E\ |\!);$
$E' \subseteq E;$
$F \in$ *set-offending-flows* $(\!|nodes = V,\ edges = E'|\!)\ nP\ ]\!] \Longrightarrow$
$\forall (e1,\ e2){\in}F.\ \neg\ sinvar\ (add\text{-}edge\ e1\ e2\ (delete\text{-}edges\ (\!|nodes = V,\ edges = E\ |\!)\ F))\ nP$
using *minimality-offending-flows-mono-edges-graph-extend*[**where** *Fadd*={}, *simplified*] **by** *presburger*

all sets in the set of offending flows are monotonic, hence, for a larger graph, they can be extended to match the smaller graph. I.e. everything is monotonic.

theorem *mono-extend-set-offending-flows*: $[\![$ *wf-graph* $(\!|$ *nodes* $= V,$ *edges* $= E\ |\!);\ E' \subseteq E;\ F' \in$ *set-offending-flows* $(\!|$ *nodes* $= V,$ *edges* $= E'\ |\!)\ nP\ ]\!] \Longrightarrow$
$\exists\ F \in$ *set-offending-flows* $(\!|$ *nodes* $= V,$ *edges* $= E\ |\!)\ nP.\ F' \subseteq F$
proof −
fix *F'V E E'*
assume *a1*: *wf-graph* $(\!|$ *nodes* $= V,$ *edges* $= E\ |\!)$
and *a2*: $E' \subseteq E$
and *a4*: $F' \in$ *set-offending-flows* $(\!|nodes = V,\ edges = E'|\!)\ nP$

— Idea: $F = F' \cup$ *minimize* $(E - E')$

have $\bigwedge f.$ *wf-graph* $(delete\text{-}edges\ (\!|nodes = V,\ edges = E|\!)\ f)$
using *delete-edges-wf*[*OF a1*] **by** *fast*
hence *wf1*: $\bigwedge f.$ *wf-graph* $(\!|nodes = V,\ edges = E - f|\!)$

by(*simp add*: *delete-edges-simp2*)

obtain *Eadd* **where** *Eadd-prop*: $E' \cup Eadd = E$ **and** $E' \cap Eadd = \{\}$ **using** *a2* **by** *blast*

from *a4* **have** $F' \subseteq E'$ **by**(*simp add*: *set-offending-flows-def*)

from *wf1* **have** *wf2*: *wf-graph* (|*nodes* = *V*, *edges* = $E' - F' \cup Eadd$|)
apply(*subgoal-tac* $E' - F' \cup Eadd = E - F'$)
apply *fastforce*
using *Eadd-prop* ‹$E' \cap Eadd = \{\}$› ‹$F' \subseteq E'$› **by** *fast*

from *a4* **have** *offending-F*: ¬ *sinvar* (|*nodes* = *V*, *edges* = E'|) *nP*
by(*simp add*: *set-offending-flows-def is-offending-flows-min-set-def is-offending-flows-def*)
from *this mono-sinvar*[*OF a1 a2*] **have**
goal-noteval: ¬ *sinvar* (|*nodes* = *V*, *edges* = *E*|) *nP* **by** *blast*

from *a4* **have** *eval-E-minus-FEadd-simp*: *sinvar* (|*nodes* = *V*, *edges* = $E' - F'$|) *nP*
by(*simp add*: *set-offending-flows-def is-offending-flows-min-set-def is-offending-flows-def delete-edges-simp2*)

show ∃ *F* ∈ *set-offending-flows* (| *nodes* = *V*, *edges* = *E* |) *nP*. $F' \subseteq F$
proof(*cases* ¬ *sinvar* (|*nodes* = *V*, *edges* = $E' - F' \cup Eadd$|) *nP*)
assume *assumption-new-violation*: ¬ *sinvar* (|*nodes* = *V*, *edges* = $E' - F' \cup Eadd$|) *nP*
from *a1* **have** *finite Eadd*
apply(*simp add*: *wf-graph-def*)
using *Eadd-prop wf-graph.finiteE* **by** *blast*
from *this* **obtain** *Eadd-list* **where** *Eadd-list-prop*: *set Eadd-list* = *Eadd* **and** *distinct Eadd-list* **by** (*metis finite-distinct-list*)
from *a1* **have** *finite* E'
apply(*simp add*: *wf-graph-def*)
using *Eadd-prop* **by** *blast*
from *this* **obtain** E'*-list* **where** E'*-list-prop*: *set* E'*-list* = E' **and** *distinct* E'*-list* **by** (*metis finite-distinct-list*)
from ‹*finite* E'› ‹$F' \subseteq E'$› **obtain** F'*-list* **where** *set* F'*-list* = F' **and** *distinct* F'*-list* **by** (*metis finite-distinct-list rev-finite-subset*)

have $E' - F' \cup Eadd - Eadd = E' - F'$ **using** *Eadd-prop* ‹$E' \cap Eadd = \{\}$› ‹$F' \subseteq E'$› **by** *blast*
with *assumption-new-violation eval-E-minus-FEadd-simp* **have**
is-offending-flows (*set* (*Eadd-list*)) (|*nodes* = *V*, *edges* = ($E' - F'$) ∪ *Eadd*|) *nP*
by (*simp add*: *Eadd-list-prop delete-edges-simp2 is-offending-flows-def*)
from *minimalize-offending-overapprox-sound*[*OF wf2 this* - ‹*distinct Eadd-list*›] **have**
is-offending-flows-min-set
(*set* (*minimalize-offending-overapprox Eadd-list* []
(|*nodes* = *V*, *edges* = $E' - F' \cup Eadd$|) *nP*)) (|*nodes* = *V*, *edges* = $E' - F' \cup Eadd$|) *nP*
by(*simp add*: *Eadd-list-prop*)
with *minimalize-offending-overapprox-subseteq-input*[*of Eadd-list* [] (|*nodes* = *V*, *edges* = $E' - F' \cup Eadd$|) *nP*, *simplified Eadd-list-prop*]
obtain *Fadd* **where** *Fadd-prop*: *is-offending-flows-min-set Fadd* (|*nodes* = *V*, *edges* = $E' - F' \cup Eadd$|) *nP* **and** *Fadd* ⊆ *Eadd* **by** *auto*

have *graph-edges-simp-helper*: $E' - F' \cup Eadd - Fadd = E - (F' \cup Fadd)$
using ‹$E' \cap Eadd = \{\}$› *Eadd-prop* ‹$F' \subseteq E'$› **by** *blast*

from *Fadd-prop graph-edges-simp-helper* **have**
goal-eval-Fadd: *sinvar* (*delete-edges* (|*nodes* = *V*, *edges* = *E*|) (*F'* ∪ *Fadd*)) *nP* **and**
pre-goal-minimal-Fadd: (∀(*e1*, *e2*)∈*Fadd*. ¬ *sinvar* (*add-edge e1 e2* (*delete-edges* (|*nodes* = *V*, *edges* = *E* |) (*F'* ∪ *Fadd*))) *nP*)
by(*simp add*: *is-offending-flows-min-set-def is-offending-flows-def delete-edges-simp2*)+

from ‹*E'* ∩ *Eadd* = {}› ‹*Fadd* ⊆ *Eadd*› **have** *Fadd* ∩ *E'* = {} **by** *blast*
from *minimality-offending-flows-mono-edges-graph-extend*[*OF a1* ‹*E'* ⊆ *E*› ‹*Fadd* ∩ *E'* = {}› *a4*]
have *mono-delete-edges-minimal*: (∀(*e1*, *e2*)∈*F'*. ¬ *sinvar* (*add-edge e1 e2* (*delete-edges* (|*nodes* = *V*, *edges* = *E* |) (*F'* ∪ *Fadd*))) *nP*) **.**

from *mono-delete-edges-minimal pre-goal-minimal-Fadd* **have** *goal-minimal*:
∀(*e1*, *e2*)∈*F'* ∪ *Fadd*. ¬ *sinvar* (*add-edge e1 e2* (*delete-edges* (|*nodes* = *V*, *edges* = *E*|) (*F'* ∪ *Fadd*))) *nP* **by** *fastforce*

from *Eadd-prop* ‹*Fadd* ⊆ *Eadd*› ‹*F'* ⊆ *E'*› **have** *goal-subset*: *F'* ⊆ *E* ∧ *Fadd* ⊆ *E* **by** *blast*

show ∃ *F* ∈ *set-offending-flows* (| *nodes* = *V*, *edges* = *E* |) *nP*. *F'* ⊆ *F*
apply(*simp add*: *set-offending-flows-def is-offending-flows-min-set-def is-offending-flows-def*)
apply(*rule-tac x*=*F'* ∪ *Fadd* **in** *exI*)
apply(*simp add*: *goal-noteval goal-eval-Fadd goal-minimal goal-subset*)
done
next
assume ¬ ¬ *sinvar* (|*nodes* = *V*, *edges* = *E'* − *F'* ∪ *Eadd*|) *nP*
hence *assumption-no-new-violation*: *sinvar* (|*nodes* = *V*, *edges* = *E'* − *F'* ∪ *Eadd*|) *nP* **by** *simp*
from *this* ‹*F'* ⊆ *E'*› ‹*E'* ∩ *Eadd* = {}› **have** *sinvar* (|*nodes* = *V*, *edges* = *E* − *F'*|) *nP*
proof(*subst Eadd-prop*[*symmetric*])
assume *a1*: *F'* ⊆ *E'*
assume *a2*: *E'* ∩ *Eadd* = {}
assume *a3*: *sinvar* (|*nodes* = *V*, *edges* = *E'* − *F'* ∪ *Eadd*|) *nP*
have ⋀*x*₁. *x*₁ ∩ *E'* − *Eadd* = *x*₁ ∩ *E'*
using *a2 Un-Diff-Int* **by** *auto*
hence *F'* − *Eadd* = *F'*
using *a1* **by** *auto*
hence {} ∪ (*Eadd* − *F'*) = *Eadd*
using *Int-Diff Un-Diff-Int sup-commute* **by** *auto*
thus *sinvar* (|*nodes* = *V*, *edges* = *E'* ∪ *Eadd* − *F'*|) *nP*
using *a3* **by** (*metis Un-Diff sup-bot.left-neutral*)
qed
from *this* **have** *goal-eval*: *sinvar* (*delete-edges* (|*nodes* = *V*, *edges* = *E*|) *F'*) *nP*
by(*simp add*: *delete-edges-simp2*)

from *Eadd-prop* ‹*F'* ⊆ *E'*› **have** *goal-subset*: *F'* ⊆ *E* **by**(*blast*)

from *minimality-offending-flows-mono-edges-graph*[*OF a1 a2 a4*]
have *goal-minimal*: (∀(*e1*, *e2*)∈*F'*. ¬ *sinvar* (*add-edge e1 e2* (*delete-edges* (|*nodes* = *V*, *edges* = *E* |) *F'*)) *nP*) **.**

show ∃ *F* ∈ *set-offending-flows* (| *nodes* = *V*, *edges* = *E* |) *nP*. *F'* ⊆ *F*
apply(*simp add*: *set-offending-flows-def is-offending-flows-min-set-def is-offending-flows-def*)
apply(*rule-tac x*=*F'* **in** *exI*)

apply(*simp add*: *goal-noteval goal-subset goal-minimal goal-eval*)
done
qed
qed

The offending flows are monotonic.

corollary *offending-flows-union-mono*: ⟦ *wf-graph* (| *nodes* = *V*, *edges* = *E* |); *E'* ⊆ *E* ⟧ ⟹
⋃ (*set-offending-flows* (| *nodes* = *V*, *edges* = *E'* |) *nP*) ⊆ ⋃ (*set-offending-flows* (| *nodes* = *V*, *edges* = *E* |) *nP*)
apply(*clarify*)
apply(*drule*(2) *mono-extend-set-offending-flows*)
by *blast*

lemma *set-offending-flows-insert-contains-new*:
⟦ *wf-graph* (| *nodes* = *V*, *edges* = *insert e E* |); *set-offending-flows* (|*nodes* = *V*, *edges* = *E*|) *nP* = {}; *set-offending-flows* (|*nodes* = *V*, *edges* = *insert e E*|) *nP* ≠ {} ⟧ ⟹
{*e*} ∈ *set-offending-flows* (|*nodes* = *V*, *edges* = *insert e E*|) *nP*
proof −
assume *wfG*: *wf-graph* (| *nodes* = *V*, *edges* = *insert e E* |)
and *a1*: *set-offending-flows* (|*nodes* = *V*, *edges* = *E*|) *nP* = {}
and *a2*: *set-offending-flows* (|*nodes* = *V*, *edges* = *insert e E*|) *nP* ≠ {}

from *a1 a2* **have** *e* ∉ *E* **by** (*metis insert-absorb*)

from *a1* **have** *a1'*: ∀ *F* ⊆ *E*. ¬ *is-offending-flows-min-set F* (|*nodes* = *V*, *edges* = *E*|) *nP*
by(*simp add*: *set-offending-flows-def*)
from *a2* **have** *a2'*: ∃ *F* ⊆ *insert e E*. *is-offending-flows-min-set F* (|*nodes* = *V*, *edges* = *insert e E*|) *nP*
by(*simp add*: *set-offending-flows-def*)

from *wfG* **have** *wfG'*: *wf-graph* (| *nodes* = *V*, *edges* = *E* |) **by**(*simp add*:*wf-graph-def*)

from *a1 defined-offending*[*OF wfG'*] **have** *evalG*: *sinvar* (|*nodes* = *V*, *edges* = *E* |) *nP* **by** *blast*
from *sinvar-monoI*[*unfolded sinvar-mono-def*] *wfG' this*
have *goal-eval*: *sinvar* (|*nodes* = *V*, *edges* = *E* − {*e*}|) *nP* **by** (*metis Diff-subset*)

from *sinvar-no-offending a2* **have** *goal-not-eval*: ¬ *sinvar* (|*nodes* = *V*, *edges* = *insert e E*|) *nP* **by** *blast*

obtain *a b* **where** *e*: *e* = (*a*,*b*) **by** (*cases e*) *blast*
with *wfG* **have** *insert-e-V*: *insert a* (*insert b V*) = *V* **by**(*auto simp add*: *wf-graph-def*)

from *a1' a2'* **have** *min-set-e*: *is-offending-flows-min-set* {*e*} (|*nodes* = *V*, *edges* = *insert e E*|) *nP*
apply(*simp add*: *is-offending-flows-min-set-def is-offending-flows-def add-edge-def delete-edges-simp2 goal-not-eval goal-eval*)
using *goal-not-eval* **by**(*simp add*: *e insert-e-V*)

thus {*e*} ∈ *set-offending-flows* (|*nodes* = *V*, *edges* = *insert e E*|) *nP*
by(*simp add*: *set-offending-flows-def*)
qed

end

value *Pow* {*1*::*int*, *2*, *3*} ∪ {{*8*}, {*9*}}
value ⋃ *x*∈*Pow* {*1*::*int*, *2*, *3*}. ⋃ *y* ∈ {{*8*::*int*}, {*9*}}. {*x* ∪ *y*}

— combines powerset of A with B
definition *pow-combine* :: *'x set* ⇒ *'x set set* ⇒ *'x set set* **where**
pow-combine A B ≡ (⋃ *X* ∈ *Pow A*. ⋃ *Y* ∈ *B*. {*X* ∪ *Y*}) ∪ *Pow A*

value *pow-combine* {*1*::*int*,*2*} {{*5*::*int*, *6*}, {*8*}}
value *pow-combine* {*1*::*int*,*2*} {}

lemma *pow-combine-mono*:
fixes *S* :: *'a set set*
and *X* :: *'a set*
and *Y* :: *'a set*
assumes *a1*: ∀ *F* ∈ *S*. *F* ⊆ *X*
shows ∀ *F* ∈ *pow-combine Y S*. *F* ⊆ *Y* ∪ *X*
apply(*simp add*: *pow-combine-def*)
apply(*rule*)
apply(*simp*)
by (*metis Pow-iff assms sup.coboundedI1 sup.orderE sup.orderI sup-assoc*)

lemma *S* ⊆ *pow-combine X S* **by**(*auto simp add*: *pow-combine-def*)
lemma *Pow X* ⊆ *pow-combine X S* **by**(*auto simp add*: *pow-combine-def*)

lemma *rule-pow-combine-fixfst*: *B* ⊆ *C* ⟹ *pow-combine A B* ⊆ *pow-combine A C*
by(*auto simp add*: *pow-combine-def*)

value *pow-combine* {*1*::*int*,*2*} {{*5*::*int*, *6*}, {*1*}} ⊆ *pow-combine* {*1*::*int*,*2*} {{*5*::*int*, *6*}, {*8*}}

lemma *rule-pow-combine-fixfst-Union*: ⋃ *B* ⊆ ⋃ *C* ⟹ ⋃ (*pow-combine A B*) ⊆ ⋃ (*pow-combine A C*)
apply(*rule*)
apply(*fastforce simp*: *pow-combine-def*)
done

context *SecurityInvariant-preliminaries*
begin

lemma *offending-partition-subset-empty*:
assumes *a1*:∀ *F* ∈ (*set-offending-flows* (|*nodes* = *V*, *edges* = *E* ∪ *X*|) *nP*). *F* ⊆ *X*
and *wfGEX*: *wf-graph* (|*nodes* = *V*, *edges* = *E* ∪ *X*|)
and *disj*: *E* ∩ *X* = {}
shows (*set-offending-flows* (|*nodes* = *V*, *edges* = *E*|) *nP*) = {}
proof(*rule ccontr*)
assume *c*: *set-offending-flows* (|*nodes* = *V*, *edges* = *E*|) *nP* ≠ {}
from *this* **obtain** *F'* **where** *F'-prop*: *F'* ∈ *set-offending-flows* (|*nodes* = *V*, *edges* = *E*|) *nP* **by**

blast
from *F′-prop* **have** $F' \subseteq E$ **using** *set-offending-flows-def* **by** *simp*
from *mono-extend-set-offending-flows*[*OF wfGEX - F′-prop*] **have**
$\exists F \in$*set-offending-flows* $(\!|nodes = V, edges = E \cup X|\!)$ *nP*. $F' \subseteq F$ **by** *blast*
from *this a1* **have** $F' \subseteq X$ **by** *fast*
from *F′-prop* **have** $\{\} \neq F'$ **by** (*metis empty-offending-contra*)
from ‹$F' \subseteq X$› ‹$F' \subseteq E$› *disj* ‹$\{\} \neq F'$›
show *False* **by** *blast*
qed

corollary *partitioned-offending-subseteq-pow-combine*:
assumes *wfGEX*: *wf-graph* $(\!|nodes = V, edges = E \cup X|\!)$
and *disj*: $E \cap X = \{\}$
and *partitioned-offending*: $\forall\ F \in$ (*set-offending-flows* $(\!|nodes = V, edges = E \cup X|\!)$ *nP*). $F \subseteq X$
shows (*set-offending-flows* $(\!|nodes = V, edges = E \cup X|\!)$ *nP*) $\subseteq$ *pow-combine X* (*set-offending-flows* $(\!|nodes = V, edges = E|\!)$ *nP*)
apply(*subst offending-partition-subset-empty*[*OF partitioned-offending wfGEX disj*])
apply(*simp add*: *pow-combine-def*)
apply(*rule*)
apply(*simp*)
using *partitioned-offending* **by** *simp*
end

context *SecurityInvariant-preliminaries*
begin

Knowing that the $\bigcup$ *offending is* $\subseteq X$, removing something from the graphs's edges, it also disappears from the offending flows.

lemma *Un-set-offending-flows-bound-minus*:
assumes *wfG*: *wf-graph* $(\!|\ nodes = V, edges = E\ |\!)$
and *Foffending*: $\bigcup$(*set-offending-flows* $(\!|nodes = V, edges = E|\!)$ *nP*) $\subseteq X$
shows $\bigcup$(*set-offending-flows* $(\!|nodes = V, edges = E - \{f\}|\!)$ *nP*) $\subseteq X - \{f\}$
proof −
from *wfG* **have** *wfG′*: *wf-graph* $(\!|\ nodes = V, edges = E - \{f\}\ |\!)$
by(*auto simp add*: *wf-graph-def finite-subset*)

from *offending-flows-union-mono*[*OF wfG*, **where** E'=$E - \{f\}$] **have**
$\bigcup$(*set-offending-flows* $(\!|nodes = V, edges = E - \{f\}|\!)$ *nP*) $- \{f\} \subseteq \bigcup$(*set-offending-flows* $(\!|nodes = V, edges = E|\!)$ *nP*) $- \{f\}$ **by** *blast*
also have
$\bigcup$(*set-offending-flows* $(\!|nodes = V, edges = E - \{f\}|\!)$ *nP*) $\subseteq \bigcup$(*set-offending-flows* $(\!|nodes = V, edges = E - \{f\}|\!)$ *nP*) $- \{f\}$
apply(*simp add*: *set-offending-flows-simp*[*OF wfG′*]) **by** *blast*
ultimately have *Un-set-offending-flows-minus*:
$\bigcup$(*set-offending-flows* $(\!|nodes = V, edges = E - \{f\}|\!)$ *nP*) $\subseteq \bigcup$(*set-offending-flows* $(\!|nodes = V, edges = E\ |\!)$ *nP*) $- \{f\}$
by *blast*

from *Foffending Un-set-offending-flows-minus*
show *?thesis* **by** *blast*
qed

If the offending flows are bound by some X, the we can remove all finite $E'$from the graph's

edges and the offending flows from the smaller graph are bound by $X - E'$.

lemma *Un-set-offending-flows-bound-minus-subseteq*:
assumes *wfG*: *wf-graph* (| *nodes* = *V*, *edges* = *E* |)
and *Foffending*: $\bigcup$ (*set-offending-flows* (|*nodes* = *V*, *edges* = *E*|) *nP*) $\subseteq$ *X*
shows $\bigcup$ (*set-offending-flows* (|*nodes* = *V*, *edges* = *E* − *E'*|) *nP*) $\subseteq$ *X* − *E'*
proof −
from *wfG* **have** *wfG'*: *wf-graph* (| *nodes* = *V*, *edges* = *E* − *E'* |)
by(*auto simp add*: *wf-graph-def finite-subset*)

from *offending-flows-union-mono*[*OF wfG*, **where** *E'*=*E* − *E'*] **have**
$\bigcup$(*set-offending-flows* (|*nodes* = *V*, *edges* = *E* − *E'*|) *nP*) − *E'* $\subseteq$ $\bigcup$(*set-offending-flows* (|*nodes* = *V*, *edges* = *E*|) *nP*) − *E'* **by** *blast*
also have
$\bigcup$(*set-offending-flows* (|*nodes* = *V*, *edges* = *E* − *E'*|) *nP*) $\subseteq$ $\bigcup$(*set-offending-flows* (|*nodes* = *V*, *edges* = *E* − *E'*|) *nP*) − *E'*
apply(*simp add*: *set-offending-flows-simp*[*OF wfG'*]) **by** *blast*
ultimately have *Un-set-offending-flows-minus*:
$\bigcup$ (*set-offending-flows* (|*nodes* = *V*, *edges* = *E* − *E'*|) *nP*) $\subseteq$ $\bigcup$ (*set-offending-flows* (|*nodes* = *V*, *edges* = *E* |) *nP*) − *E'*
by *blast*

from *Foffending Un-set-offending-flows-minus*
show *?thesis* **by** *blast*
qed

corollary *Un-set-offending-flows-bound-minus-subseteq'*:
⟦ *wf-graph* (| *nodes* = *V*, *edges* = *E* |);
$\bigcup$ (*set-offending-flows* (| *nodes* = *V*, *edges* = *E* |) *nP*) $\subseteq$ *X* ⟧ ⟹
$\bigcup$ (*set-offending-flows* (| *nodes* = *V*, *edges* = *E* − *E'* |) *nP*) $\subseteq$ *X* − *E'*
apply(*drule*(*1*) *Un-set-offending-flows-bound-minus-subseteq*) **by** *blast*

end

end
theory *TopoS-ENF*
imports *Main TopoS-Interface Lib/TopoS-Util TopoS-withOffendingFlows*
begin

4 Special Structures of Security Invariants

Security Invariants may have a common structure: If the function *sinvar* is predicate which starts with ∀ (v_1, v_2) ∈ *edges G*. ..., we call this the all edges normal form (ENF). We found that this form has some nice properties. Also, locale instantiation is easier in ENF with the help of the following lemmata.

4.1 Simple Edges Normal Form (ENF)

context *SecurityInvariant-withOffendingFlows*
begin

definition *sinvar-all-edges-normal-form* :: ('*a* ⇒ '*a* ⇒ *bool*) ⇒ *bool* **where**

sinvar-all-edges-normal-form P ≡ ∀ *G nP*. *sinvar G nP* = (∀ (*e1*, *e2*)∈ *edges G*. *P* (*nP e1*) (*nP e2*))

reflexivity is needed for convenience. If a security invariant is not reflexive, that means that all nodes with the default parameter ⊥ are not allowed to communicate with each other. Non-reflexivity is possible, but requires more work.

definition *ENF-refl* :: ('*a* ⇒ '*a* ⇒ *bool*) ⇒ *bool* **where**
ENF-refl P ≡ *sinvar-all-edges-normal-form P* ∧ (∀ *p1*. *P p1 p1*)

lemma *monotonicity-sinvar-mono*: *sinvar-all-edges-normal-form P* ⟹ *sinvar-mono*
unfolding *sinvar-all-edges-normal-form-def sinvar-mono-def*
by *auto*

end

4.1.1 Offending Flows

context *SecurityInvariant-withOffendingFlows*
begin

The insight: for all edges in the members of the offending flows, ¬ *P* holds.

lemma *ENF-offending-imp-not-P*:
assumes *sinvar-all-edges-normal-form P F* ∈ *set-offending-flows G nP* (*e1*, *e2*) ∈ *F*
shows ¬ *P* (*nP e1*) (*nP e2*)
using *assms*
unfolding *sinvar-all-edges-normal-form-def set-offending-flows-def is-offending-flows-min-set-def is-offending-flows-def*
by (*fastforce simp*: *graph-ops*)

Hence, the members of *set-offending-flows* must look as follows.

lemma *ENF-offending-set-P-representation*:
assumes *sinvar-all-edges-normal-form P F* ∈ *set-offending-flows G nP*
shows *F* = {(*e1*,*e2*). (*e1*, *e2*) ∈ *edges G* ∧ ¬ *P* (*nP e1*) (*nP e2*)} (**is** *F* = *?E*)
proof −
{ fix *a b*
assume (*a*, *b*) ∈ *F*
hence (*a*, *b*) ∈ *?E*
using *assms*
by (*auto simp*: *set-offending-flows-def ENF-offending-imp-not-P*)
}
moreover
{ fix *x*
assume *x* ∈ *?E*
hence *x* ∈ *F*
using *assms*
unfolding *sinvar-all-edges-normal-form-def set-offending-flows-def is-offending-flows-min-set-def*
by (*fastforce simp*: *is-offending-flows-def graph-ops*)
}
ultimately show *?thesis*
by *blast*
qed

We can show left to right of the desired representation of *set-offending-flows*

lemma *ENF-offending-subseteq-lhs*:

assumes *sinvar-all-edges-normal-form P*
shows *set-offending-flows G nP* $\subseteq$ { {(*e1*,*e2*). (*e1*, *e2*) $\in$ *edges G* $\wedge$ $\neg$ *P* (*nP e1*) (*nP e2*)} }
using *assms*
by (*force simp*: *ENF-offending-set-P-representation*)

if *set-offending-flows* is not empty, we have the other direction.

lemma *ENF-offending-not-empty-imp-ENF-offending-subseteq-rhs*:
assumes *sinvar-all-edges-normal-form P set-offending-flows G nP* $\neq$ {}
shows { {(*e1*,*e2*) $\in$ *edges G*. $\neg$ *P* (*nP e1*) (*nP e2*)} } $\subseteq$ *set-offending-flows G nP*
using *assms ENF-offending-set-P-representation*
by *blast*

lemma *ENF-notevalmodel-imp-offending-not-empty*:
sinvar-all-edges-normal-form P $\Longrightarrow$ $\neg$ *sinvar G nP* $\Longrightarrow$ *set-offending-flows G nP* $\neq$ {}

proof −
assume *enf*: *sinvar-all-edges-normal-form P*
and *ns*: $\neg$ *sinvar G nP*

{
let *?F'*={(*e1*,*e2*) $\in$ (*edges G*). $\neg$*P* (*nP e1*) (*nP e2*)}
— select {(*e1*, *e2*). (*e1*, *e2*) $\in$ *edges G* $\wedge$ $\neg$ *P* (*nP e1*) (*nP e2*)} as the list of all edges which violate *P*

from *enf* **have** *ENF-notevalmodel-offending-imp-ex-offending-min*:
$\bigwedge$*F*. *is-offending-flows F G nP* $\Longrightarrow$ *F* $\subseteq$ *edges G* $\Longrightarrow$
$\exists$ *F'*. *F'* $\subseteq$ *edges G* $\wedge$ *is-offending-flows-min-set F' G nP*
unfolding *sinvar-all-edges-normal-form-def is-offending-flows-min-set-def is-offending-flows-def*
by (−) (*rule exI*[**where** *x*=*?F'*], *fastforce simp*: *graph-ops*)

from *enf ns* **have** $\exists$ *F*. *F* $\subseteq$ (*edges G*) $\wedge$ *is-offending-flows F G nP*
unfolding *sinvar-all-edges-normal-form-def is-offending-flows-def*
by (−) (*rule exI*[**where** *x*=*?F'*], *fastforce simp*: *graph-ops*)

from *enf ns this ENF-notevalmodel-offending-imp-ex-offending-min* **have** *ENF-notevalmodel-imp-ex-offending-min*:
$\exists$ *F*. *F* $\subseteq$ *edges G* $\wedge$ *is-offending-flows-min-set F G nP* **by** *blast*
} note *ENF-notevalmodel-imp-ex-offending-min*=*this*

from *ENF-notevalmodel-imp-ex-offending-min* **show** *set-offending-flows G nP* $\neq$ {} **using** *set-offending-flows-def* **by** *simp*
qed

lemma *ENF-offending-case1*:
⟦ *sinvar-all-edges-normal-form P*; $\neg$ *sinvar G nP* ⟧ $\Longrightarrow$
{ {(*e1*,*e2*). (*e1*, *e2*) $\in$ (*edges G*) $\wedge$ $\neg$ *P* (*nP e1*) (*nP e2*)} } = *set-offending-flows G nP*
apply(*rule*)
apply(*frule ENF-notevalmodel-imp-offending-not-empty*, *simp*)
apply(*rule ENF-offending-not-empty-imp-ENF-offending-subseteq-rhs*, *simp*)
apply *simp*
apply(*rule ENF-offending-subseteq-lhs*)
apply *simp*
done

lemma *ENF-offending-case2*:

⟦ *sinvar-all-edges-normal-form P*; *sinvar G nP* ⟧ ⟹
{} = *set-offending-flows G nP*
apply(*drule sinvar-no-offending*[*of G nP*])
apply *simp*
done

theorem *ENF-offending-set*:
⟦ *sinvar-all-edges-normal-form P* ⟧ ⟹
set-offending-flows G nP = (*if sinvar G nP then*
{}
else
{ {(*e1*,*e2*). (*e1*, *e2*) ∈ *edges G* ∧ ¬ *P* (*nP e1*) (*nP e2*)} })
by(*simp add*: *ENF-offending-case1 ENF-offending-case2*)
end

4.1.2 Lemmata

lemma (**in** *SecurityInvariant-withOffendingFlows*) *ENF-offending-members*:
⟦ ¬ *sinvar G nP*; *sinvar-all-edges-normal-form P*; *f* ∈ *set-offending-flows G nP*⟧ ⟹
f ⊆ (*edges G*) ∧ (∀ (*e1*, *e2*)∈ *f*. ¬ *P* (*nP e1*) (*nP e2*))
by(*auto simp add*: *ENF-offending-set*)

4.1.3 Instance Helper

lemma (**in** *SecurityInvariant-withOffendingFlows*) *ENF-refl-not-offedning*:
⟦ ¬ *sinvar G nP*; *f* ∈ *set-offending-flows G nP*;
ENF-refl P⟧ ⟹
∀(*e1*,*e2*) ∈ *f*. *e1* ≠ *e2*
proof −
assume *a-not-eval*: ¬ *sinvar G nP*
and *a-enf-refl*: *ENF-refl P*
and *a-offedning*: *f* ∈ *set-offending-flows G nP*

from *a-enf-refl* **have** *a-enf*: *sinvar-all-edges-normal-form P* **using** *ENF-refl-def* **by** *simp*
hence *a-ENF*: ⋀ *G nP*. *sinvar G nP* = (∀ (*e1*, *e2*) ∈ *edges G*. *P* (*nP e1*) (*nP e2*)) **using** *sinvar-all-edges-normal-form-def* **by** *simp*

from *a-enf-refl ENF-refl-def* **have** *a-refl*: ∀ (*e1*,*e1*) ∈ *f*. *P* (*nP e1*) (*nP e1*) **by** *simp*
from *ENF-offending-members*[*OF a-not-eval a-enf a-offedning*] **have** ∀ (*e1*, *e2*) ∈ *f*. ¬ *P* (*nP e1*) (*nP e2*) **by** *fast*
from *this a-refl* **show** ∀(*e1*,*e2*) ∈ *f*. *e1* ≠ *e2* **by** *fast*
qed

lemma (**in** *SecurityInvariant-withOffendingFlows*) *ENF-default-update-fst*:
fixes *default-node-properties* :: *'a* (⊥)
assumes *modelInv*: ¬ *sinvar G nP*
and *ENFdef*: *sinvar-all-edges-normal-form P*
and *secdef*: ∀ (*nP*::*'v* ⇒ *'a*) *e1 e2*. ¬ (*P* (*nP e1*) (*nP e2*)) ⟶ ¬ (*P* ⊥ (*nP e2*))
shows
¬ (∀ (*e1*, *e2*) ∈ *edges G*. *P* ((*nP*(*i* := ⊥)) *e1*) (*nP e2*))
proof −
from *ENFdef* **have** *ENF*: ⋀ *G nP*. *sinvar G nP* = (∀ (*e1*, *e2*)∈ *edges G*. *P* (*nP e1*) (*nP e2*))
using *sinvar-all-edges-normal-form-def* **by** *simp*

from *modelInv ENF* **have** *modelInv'*: $\neg$ ($\forall$ (*e1*, *e2*)$\in$ *edges G*. *P* (*nP e1*) (*nP e2*)) **by** *simp*
from *this secdef* **have** *modelInv''*: $\neg$ ($\forall$ (*e1*, *e2*)$\in$ *edges G*. *P* $\bot$ (*nP e2*)) **by** *blast*
have *simpUpdateI*: $\bigwedge$ *e1 e2*. $\neg$ *P* (*nP e1*) (*nP e2*) $\Longrightarrow$ $\neg$ *P* $\bot$ (*nP e2*) $\Longrightarrow$ $\neg$ *P* ((*nP*(*i* := $\bot$)) *e1*) (*nP e2*) **by** *simp*
hence $\bigwedge$ *X*. $\exists$(*e1*,*e2*) $\in$ *X*. $\neg$ *P* (*nP e1*) (*nP e2*) $\Longrightarrow$ $\exists$(*e1*,*e2*) $\in$ *X*.$\neg$ *P* $\bot$ (*nP e2*) $\Longrightarrow$ $\exists$(*e1*,*e2*) $\in$ *X*.$\neg$ *P* ((*nP*(*i* := $\bot$)) *e1*) (*nP e2*)
using *secdef* **by** *blast*
from *this modelInv' modelInv''* **show** $\neg$ ($\forall$ (*e1*, *e2*) $\in$ *edges G*. *P* ((*nP*(*i* := $\bot$)) *e1*) (*nP e2*)) **by** *blast*
qed

lemma (**in** *SecurityInvariant-withOffendingFlows*)
fixes *default-node-properties* :: *'a* ($\bot$)
shows $\neg$ *sinvar G nP* $\Longrightarrow$ *sinvar-all-edges-normal-form P* $\Longrightarrow$
($\forall$ (*nP*::*'v* $\Rightarrow$ *'a*) *e1 e2*. $\neg$ (*P* (*nP e1*) (*nP e2*)) $\longrightarrow$ $\neg$ (*P* $\bot$ (*nP e2*))) $\Longrightarrow$
($\forall$ (*nP*::*'v* $\Rightarrow$ *'a*) *e1 e2*. $\neg$ (*P* (*nP e1*) (*nP e2*)) $\longrightarrow$ $\neg$ (*P* (*nP e1*) $\bot$)) $\Longrightarrow$
($\forall$ (*nP*::*'v* $\Rightarrow$ *'a*) *e1 e2*. $\neg$ *P* $\bot$ $\bot$)
$\Longrightarrow$ $\neg$ *sinvar G* (*nP*(*i* := $\bot$))
proof $-$
assume *a1*: $\neg$ *sinvar G nP*
and *a2d*: *sinvar-all-edges-normal-form P*
and *a3*: $\forall$ (*nP*::*'v* $\Rightarrow$ *'a*) *e1 e2*. $\neg$ (*P* (*nP e1*) (*nP e2*)) $\longrightarrow$ $\neg$ (*P* $\bot$ (*nP e2*))
and *a4*: $\forall$ (*nP*::*'v* $\Rightarrow$ *'a*) *e1 e2*. $\neg$ (*P* (*nP e1*) (*nP e2*)) $\longrightarrow$ $\neg$ (*P* (*nP e1*) $\bot$)
and *a5*: $\forall$ (*nP*::*'v* $\Rightarrow$ *'a*) *e1 e2*. $\neg$ *P* $\bot$ $\bot$

from *a2d* **have** *a2*: $\bigwedge$ *G nP*. *sinvar G nP* = ($\forall$ (*e1*, *e2*) $\in$ *edges G*. *P* (*nP e1*) (*nP e2*))
using *sinvar-all-edges-normal-form-def* **by** *simp*

from *ENF-default-update-fst*[*OF a1 a2d*] *a3* **have** *subgoal1*: $\neg$ ($\forall$ (*e1*, *e2*) $\in$ *edges G*. *P* ((*nP*(*i* := $\bot$)) *e1*) (*nP e2*)) **by** *blast*

let *?nP'* = (*nP*(*i* := $\bot$))

from *subgoal1* **have** $\exists$ (*e1*, *e2*) $\in$ *edges G*. $\neg$ *P* (*?nP' e1*) (*nP e2*) **by** *blast*
from *this* **obtain** *e11 e21* **where** *s1cond*: (*e11*, *e21*) $\in$ *edges G* $\wedge$ $\neg$ *P* (*?nP' e11*) (*nP e21*) **by** *blast*

from *s1cond* **have** *i* $\neq$ *e11* $\Longrightarrow$ $\neg$ *P* (*nP e11*) (*nP e21*) **by** *simp*
from *s1cond* **have** *e11* $\neq$ *e21* $\Longrightarrow$ $\neg$ *P* (*?nP' e11*) (*?nP' e21*)
apply *simp*
apply(*rule conjI*)
apply *blast*
apply(*insert a4*)
by *force*
from *s1cond a4 fun-upd-apply* **have** *ex1*: *e11* $\neq$ *e21* $\Longrightarrow$ $\neg$ *P* (*?nP' e11*) (*?nP' e21*) **by** *metis*
from *s1cond a5* **have** *ex2*: *e11* = *e21* $\Longrightarrow$ $\neg$ *P* (*?nP' e11*) (*?nP' e21*) **by** *auto*

from *ex1 ex2 s1cond* **have** $\exists$ (*e1*, *e2*) $\in$ *edges G*. $\neg$ *P* (*?nP' e1*) (*?nP' e2*) **by** *blast*
hence $\neg$ ($\forall$ (*e1*, *e2*) $\in$ *edges G*. *P* ((*nP*(*i* := $\bot$)) *e1*) ((*nP*(*i* := $\bot$)) *e2*)) **by** *fast*
from *this a2* **show** $\neg$ *sinvar G* (*nP*(*i* := $\bot$)) **by** *presburger*
qed

lemma (**in** *SecurityInvariant-withOffendingFlows*) *ENF-fsts-refl-instance*:
fixes *default-node-properties* :: *'a* (⊥)
assumes *a-enf-refl*: *ENF-refl P*
and *a3*: ∀ (*nP*::*'v* ⇒ *'a*) *e1 e2*. ¬ (*P* (*nP e1*) (*nP e2*)) ⟶ ¬ (*P* ⊥ (*nP e2*))
and *a-offending*: *f* ∈ *set-offending-flows G nP*
and *a-i-fsts*: *i* ∈ *fst* ‘ *f*
shows
¬ *sinvar G* (*nP*(*i* := ⊥))
proof −
from *a-offending* **have** *a-not-eval*: ¬ *sinvar G nP* **by** (*metis equals0D sinvar-no-offending*)
from *valid-without-offending-flows*[*OF a-offending*] **have** *a-offending-rm*: *sinvar* (*delete-edges G f*) *nP* **.**

from *a-enf-refl* **have** *a-enf*: *sinvar-all-edges-normal-form P* **using** *ENF-refl-def* **by** *simp*
hence *a2*: ⋀ *G nP*. *sinvar G nP* = (∀ (*e1*, *e2*) ∈ *edges G*. *P* (*nP e1*) (*nP e2*)) **using** *sinvar-all-edges-normal-form-def* **by** *simp*

from *ENF-offending-members*[*OF a-not-eval a-enf a-offending*] **have** *a-f-3-in-f*: ⋀ *e1 e2*. (*e1*, *e2*) ∈ *f* ⟹ ¬ *P* (*nP e1*) (*nP e2*) **by** *fast*

let *?nP'* = (*nP*(*i* := ⊥))

from *offending-not-empty*[*OF a-offending*] *ENF-offending-members*[*OF a-not-eval a-enf a-offending*] *a-i-fsts hd-in-set*
obtain *e1 e2* **where** *e1e2cond*: (*e1*, *e2*) ∈ *f* ∧ *e1* = *i* **by** *force*

from *conjunct1*[*OF e1e2cond*] *a-f-3-in-f* **have** *e1e2notP*: ¬ *P* (*nP e1*) (*nP e2*) **by** *simp*
from *this a3* **have** ¬ *P* ⊥ (*nP e2*) **by** *simp*
from *this e1e2notP* **have** *e1e2subgoal1*: ¬ *P* (*?nP' e1*) (*nP e2*) **by** *simp*

from *ENF-refl-not-offedning*[*OF a-not-eval a-offending a-enf-refl*] *conjunct1*[*OF e1e2cond*] **have** *ENF-refl*: *e1* ≠ *e2* **by** *fast*

from *e1e2subgoal1* **have** *e1* ≠ *e2* ⟹ ¬ *P* (*?nP' e1*) (*?nP' e2*)
apply *simp*
apply(*rule conjI*)
apply *blast*
apply(*insert conjunct2*[*OF e1e2cond*])
by *simp*

from *this ENF-refl ENF-offending-members*[*OF a-not-eval a-enf a-offending*] *conjunct1*[*OF e1e2cond*] **have**
∃ (*e1*, *e2*) ∈ *edges G*. ¬ *P* (*?nP' e1*) (*?nP' e2*) **by** *blast*
hence ¬ (∀ (*e1*, *e2*) ∈ *edges G*. *P* ((*nP*(*i* := ⊥)) *e1*) ((*nP*(*i* := ⊥)) *e2*)) **by** *fast*
from *this a2* **show** ¬ *sinvar G* (*nP*(*i* := ⊥)) **by** *presburger*
qed

lemma (**in** *SecurityInvariant-withOffendingFlows*) *ENF-snds-refl-instance*:
fixes *default-node-properties* :: *'a* (⊥)
assumes *a-enf-refl*: *ENF-refl P*
and *a3*: ∀ (*nP*::*'v* ⇒ *'a*) *e1 e2*. ¬ (*P* (*nP e1*) (*nP e2*)) ⟶ ¬ (*P* (*nP e1*) ⊥)
and *a-offending*: *f* ∈ *set-offending-flows G nP*

and *a-i-snds*: $i \in snd\ `\ f$
shows
$\neg$ *sinvar G* (*nP*($i := \bot$))
proof −
from *a-offending* **have** *a-not-eval*: $\neg$ *sinvar G nP* **by** (*metis equals0D sinvar-no-offending*)
from *valid-without-offending-flows*[*OF a-offending*] **have** *a-offending-rm*: *sinvar* (*delete-edges G f*) *nP* **.**
from *a-enf-refl* **have** *a-enf*: *sinvar-all-edges-normal-form P* **using** *ENF-refl-def* **by** *simp*
hence *a2*: $\bigwedge$ *G nP*. *sinvar G nP* = ($\forall$ (*e1*, *e2*) $\in$ *edges G*. *P* (*nP e1*) (*nP e2*)) **using** *sinvar-all-edges-normal-form-def* **by** *simp*

from *ENF-offending-members*[*OF a-not-eval a-enf a-offending*] **have** *a-f-3-in-f*: $\bigwedge$ *e1 e2*. (*e1*, *e2*) $\in f \Longrightarrow \neg$ *P* (*nP e1*) (*nP e2*) **by** *fast*

let *?nP′* = (*nP*($i := \bot$))

from *offending-not-empty*[*OF a-offending*] *ENF-offending-members*[*OF a-not-eval a-enf a-offending*] *a-i-snds hd-in-set*
obtain *e1 e2* **where** *e1e2cond*: (*e1*, *e2*) $\in f \wedge$ *e2* = *i* **by** *force*

from *conjunct1*[*OF e1e2cond*] *a-f-3-in-f* **have** *e1e2notP*: $\neg$ *P* (*nP e1*) (*nP e2*) **by** *simp*
from *this a3* **have** $\neg$ *P* (*nP e1*) $\bot$ **by** *auto*
from *this e1e2notP* **have** *e1e2subgoal1*: $\neg$ *P* (*nP e1*) (*?nP′ e2*) **by** *simp*

from *ENF-refl-not-offedning*[*OF a-not-eval a-offending a-enf-refl*] *e1e2cond* **have** *ENF-refl*: *e1* $\neq$ *e2* **by** *fast*

from *e1e2subgoal1* **have** *e1* $\neq$ *e2* $\Longrightarrow \neg$ *P* (*?nP′ e1*) (*?nP′ e2*)
apply *simp*
apply(*rule conjI*)
apply(*insert conjunct2*[*OF e1e2cond*])
by *simp-all*

from *this ENF-refl e1e2cond ENF-offending-members*[*OF a-not-eval a-enf a-offending*] *conjunct1*[*OF e1e2cond*] **have**
$\exists$ (*e1*, *e2*) $\in$ *edges G*. $\neg$ *P* (*?nP′ e1*) (*?nP′ e2*) **by** *blast*
hence $\neg$ ($\forall$ (*e1*, *e2*) $\in$ *edges G*. *P* ((*nP*($i := \bot$)) *e1*) ((*nP*($i := \bot$)) *e2*)) **by** *fast*
from *this a2* **show** $\neg$ *sinvar G* (*nP*($i := \bot$)) **by** *presburger*
qed

4.2 edges normal form ENF with sender and receiver names

definition (**in** *SecurityInvariant-withOffendingFlows*) *sinvar-all-edges-normal-form-sr* :: ('*a* $\Rightarrow$ '*v* $\Rightarrow$ '*a* $\Rightarrow$ '*v* $\Rightarrow$ *bool*) $\Rightarrow$ *bool* **where**
sinvar-all-edges-normal-form-sr P $\equiv \forall$ *G nP*. *sinvar G nP* = ($\forall$ (*s*, *r*)$\in$ *edges G*. *P* (*nP s*) *s* (*nP r*) *r*)

lemma (**in** *SecurityInvariant-withOffendingFlows*) *ENFsr-monotonicity-sinvar-mono*: $[\![$ *sinvar-all-edges-normal-form-sr P* $]\!] \Longrightarrow$
sinvar-mono
apply(*simp add*: *sinvar-all-edges-normal-form-sr-def sinvar-mono-def*)
by *blast*

4.2.1 Offending Flows:

theorem (**in** *SecurityInvariant-withOffendingFlows*) *ENFsr-offending-set*:
assumes *ENFsr*: *sinvar-all-edges-normal-form-sr P*
shows *set-offending-flows G nP* = (*if sinvar G nP then*
{}
else
{ {(*s*,*r*). (*s*, *r*) ∈ *edges G* ∧ ¬ *P* (*nP s*) *s* (*nP r*) *r*} }) (**is** *?A* = *?B*)
proof(*cases sinvar G nP*)
case *True* **thus** *?A* = *?B*
by(*simp add*: *set-offending-flows-def is-offending-flows-min-set-def is-offending-flows-def*)
next
case *False*
from *ENFsr* **have** *ENFsr-offending-imp-not-P*: ⋀ *F s r*. *F* ∈ *set-offending-flows G nP* ⟹ (*s*, *r*) ∈ *F* ⟹ ¬ *P* (*nP s*) *s* (*nP r*) *r*
unfolding *sinvar-all-edges-normal-form-sr-def*
apply(*simp add*: *set-offending-flows-def is-offending-flows-def is-offending-flows-min-set-def graph-ops*)
apply *clarify*
by *fastforce*
from *ENFsr* **have** *ENFsr-offending-set-P-representation*:
⋀ *F*. *F* ∈ *set-offending-flows G nP* ⟹ *F* = {(*s*,*r*). (*s*, *r*) ∈ *edges G* ∧ ¬ *P* (*nP s*) *s* (*nP r*) *r*}
apply −
apply *rule*
apply *rule*
apply *clarify*
apply(*rename-tac a b*)
apply *rule*
apply(*auto simp add*:*set-offending-flows-def*)[*1*]
apply(*simp add*: *ENFsr-offending-imp-not-P*)
unfolding *sinvar-all-edges-normal-form-sr-def*
apply(*simp add*:*set-offending-flows-def is-offending-flows-def is-offending-flows-min-set-def graph-ops*)
apply *clarify*
apply(*rename-tac a b a1 b1*)
apply(*blast*)
done

from *ENFsr False* **have** *ENFsr-offending-flows-exist*: *set-offending-flows G nP* ≠ {}
apply(*simp add*: *set-offending-flows-def is-offending-flows-min-set-def is-offending-flows-def sinvar-all-edges-normal-*
delete-edges-def add-edge-def)
apply(*clarify*)
apply(*rename-tac s r*)
apply(*rule-tac x*={(*s*,*r*). (*s*,*r*) ∈ (*edges G*) ∧ ¬*P* (*nP s*) *s* (*nP r*) *r*} **in** *exI*)
apply(*simp*)
by *blast*

from *ENFsr* **have** *ENFsr-offenindg-not-empty-imp-ENF-offending-subseteq-rhs*:
set-offending-flows G nP ≠ {} ⟹
{ {(*s*,*r*). (*s*, *r*) ∈ *edges G* ∧ ¬ *P* (*nP s*) *s* (*nP r*) *r*} } ⊆ *set-offending-flows G nP*
apply −
apply *rule*
using *ENFsr-offending-set-P-representation*
by *blast*

from *ENFsr* **have** *ENFsr-offending-subseteq-lhs*:
(*set-offending-flows G nP*) ⊆ { {(*s*,*r*). (*s*, *r*) ∈ *edges G* ∧ ¬ *P* (*nP s*) *s* (*nP r*) *r*} }
apply −
apply *rule*
by(*simp add*: *ENFsr-offending-set-P-representation*)

from *False ENFsr-offenindg-not-empty-imp-ENF-offending-subseteq-rhs*[*OF ENFsr-offending-flows-exist*] *ENFsr-offending-subseteq-lhs* **show** *?A* = *?B*
by *force*
qed

4.3 edges normal form not refl ENFnrSR

definition (**in** *SecurityInvariant-withOffendingFlows*) *sinvar-all-edges-normal-form-not-refl-SR* :: ('*a* ⇒ '*v* ⇒ '*a* ⇒ '*v* ⇒ *bool*) ⇒ *bool* **where**
sinvar-all-edges-normal-form-not-refl-SR P ≡
∀ *G nP*. *sinvar G nP* = (∀ (*s*, *r*) ∈ *edges G*. *s* ≠ *r* ⟶ *P* (*nP s*) *s* (*nP r*) *r*)

we derive everything from the ENFnrSR form

lemma (**in** *SecurityInvariant-withOffendingFlows*) *ENFnrSR-to-ENFsr*:
sinvar-all-edges-normal-form-not-refl-SR P ⟹ *sinvar-all-edges-normal-form-sr* (λ *p1 v1 p2 v2*. *v1* ≠ *v2* ⟶ *P p1 v1 p2 v2*)
by(*simp add*: *sinvar-all-edges-normal-form-sr-def sinvar-all-edges-normal-form-not-refl-SR-def*)

4.3.1 Offending Flows

theorem (**in** *SecurityInvariant-withOffendingFlows*) *ENFnrSR-offending-set*:
⟦ *sinvar-all-edges-normal-form-not-refl-SR P* ⟧ ⟹
set-offending-flows G nP = (*if sinvar G nP then*
{}
else
{ {(*e1*,*e2*). (*e1*, *e2*) ∈ *edges G* ∧ *e1* ≠ *e2* ∧ ¬ *P* (*nP e1*) *e1* (*nP e2*) *e2*} })
by(*auto dest*: *ENFnrSR-to-ENFsr simp*: *ENFsr-offending-set*)

4.3.2 Instance helper

lemma (**in** *SecurityInvariant-withOffendingFlows*) *ENFnrSR-fsts-weakrefl-instance*:
fixes *default-node-properties* :: '*a* (⊥)
assumes *a-enf*: *sinvar-all-edges-normal-form-not-refl-SR P*
and *a-weakrefl*: ∀ *s r*. *P* ⊥ *s* ⊥ *r*
and *a-botdefault*: ∀ *s r*. (*nP r*) ≠ ⊥ ⟶ ¬ *P* (*nP s*) *s* (*nP r*) *r* ⟶ ¬ *P* ⊥ *s* (*nP r*) *r*
and *a-alltobot*: ∀ *s r*. *P* (*nP s*) *s* ⊥ *r*
and *a-offending*: *f* ∈ *set-offending-flows G nP*
and *a-i-fsts*: *i* ∈ *fst*‘ *f*
shows
¬ *sinvar G* (*nP*(*i* := ⊥))
proof −
from *a-offending* **have** *a-not-eval*: ¬ *sinvar G nP* **by** (*metis ex-in-conv sinvar-no-offending*)
from *valid-without-offending-flows*[*OF a-offending*] **have** *a-offending-rm*: *sinvar* (*delete-edges G f*) *nP* **.**
from *a-enf* **have** *a-enf'*: ⋀ *G nP*. *sinvar G nP* = (∀ (*e1*, *e2*)∈ (*edges G*). *e1* ≠ *e2* ⟶ *P* (*nP e1*) *e1* (*nP e2*) *e2*)
using *sinvar-all-edges-normal-form-not-refl-SR-def* **by** *simp*

from *ENFnrSR-offending-set*[*OF a-enf*] *a-not-eval a-offending* **have** *a-f-3-in-f*: $\bigwedge$ *e1 e2*. (*e1*, *e2*)∈*f* $\Longrightarrow$ ¬ *P* (*nP e1*) *e1* (*nP e2*) *e2* **by**(*simp*)
from *ENFnrSR-offending-set*[*OF a-enf*] *a-not-eval a-offending* **have** *a-f-3-neq*: $\bigwedge$ *e1 e2*. (*e1*, *e2*)∈*f* $\Longrightarrow$ *e1* ≠ *e2* **by** *simp*

let *?nP'* = (*nP*(*i* := ⊥))

from *ENFnrSR-offending-set*[*OF a-enf*] *a-not-eval a-offending a-i-fsts*
obtain *e1 e2* **where** *e1e2cond*: (*e1*, *e2*) ∈ *f* ∧ *e1* = *i* **by** *fastforce*

from *conjunct1*[*OF e1e2cond*] *a-offending* **have** (*e1*, *e2*) ∈ *edges G*
by (*metis* (*lifting*, *no-types*) *SecurityInvariant-withOffendingFlows.set-offending-flows-def mem-Collect-eq rev-subsetD*)

from *conjunct1*[*OF e1e2cond*] *a-f-3-in-f* **have** *e1e2notP*: ¬ *P* (*nP e1*) *e1* (*nP e2*) *e2* **by** *simp*
from *e1e2notP a-weakrefl* **have** *e1ore2neqbot*: (*nP e1*) ≠ ⊥ ∨ (*nP e2*) ≠ ⊥ **by** *fastforce*
from *e1e2notP a-alltobot* **have** (*nP e2*) ≠ ⊥ **by** *fastforce*
from *this e1e2notP a-botdefault* **have** ¬ *P* ⊥ *e1* (*nP e2*) *e2* **by** *simp*
from *this e1e2notP* **have** *e1e2subgoal1*: ¬ *P* (*?nP' e1*) *e1* (*nP e2*) *e2* **by** *auto*

from *a-f-3-neq e1e2cond* **have** *e2* ≠ *e1* **by** *blast*

from *e1e2subgoal1* **have** *e1* ≠ *e2* $\Longrightarrow$ ¬ *P* (*?nP' e1*) *e1* (*?nP' e2*) *e2*
apply *simp*
apply(*rule conjI*)
apply *blast*
apply(*insert e1e2cond*)
by *simp*
from *this* ‹*e2* ≠ *e1*› **have** ¬ *P* (*?nP' e1*) *e1* (*?nP' e2*) *e2* **by** *simp*

from *this* ‹*e2* ≠ *e1*› *ENFnrSR-offending-set*[*OF a-enf*] *a-offending* ‹(*e1*, *e2*) ∈ *edges G*› **have**
∃ (*e1*, *e2*)∈(*edges G*). *e2* ≠ *e1* ∧ ¬ *P* (*?nP' e1*) *e1* (*?nP' e2*) *e2* **by** *blast*
hence ¬ (∀ (*e1*, *e2*)∈(*edges G*). *e2* ≠ *e1* ⟶ *P* ((*nP*(*i* := ⊥)) *e1*) *e1* ((*nP*(*i* := ⊥)) *e2*) *e2*) **by** *fast*
from *this a-enf'* **show** ¬ *sinvar G* (*nP*(*i* := ⊥)) **by** *fast*
qed

lemma (**in** *SecurityInvariant-withOffendingFlows*) *ENFnrSR-snds-weakrefl-instance*:
fixes *default-node-properties* :: *'a* (⊥)
assumes *a-enf*: *sinvar-all-edges-normal-form-not-refl-SR P*
and *a-weakrefl*: ∀ *s r*. *P* ⊥ *s* ⊥ *r*
and *a-botdefault*: ∀ *s r*. (*nP s*) ≠ ⊥ ⟶ ¬ *P* (*nP s*) *s* (*nP r*) *r* ⟶ ¬ *P* (*nP s*) *s* ⊥ *r*
and *a-bottoall*: ∀ *s r*. *P* ⊥ *s* (*nP r*) *r*
and *a-offending*: *f* ∈ *set-offending-flows G nP*
and *a-i-snds*: *i* ∈ *snd‘ f*
shows
¬ *sinvar G* (*nP*(*i* := ⊥))
proof −
from *a-offending* **have** *a-not-eval*: ¬ *sinvar G nP* **by** (*metis equals0D sinvar-no-offending*)
from *valid-without-offending-flows*[*OF a-offending*] **have** *a-offending-rm*: *sinvar* (*delete-edges G f*) *nP* **.**

from *a-enf* **have** *a-enf'*: $\bigwedge$ *G nP*. *sinvar G nP* = (∀ (*e1*, *e2*)∈(*edges G*). *e1* ≠ *e2* ⟶ *P* (*nP e1*) *e1* (*nP e2*) *e2*)
using *sinvar-all-edges-normal-form-not-refl-SR-def* **by** *simp*

from *ENFnrSR-offending-set*[*OF a-enf*] *a-not-eval a-offending* **have** *a-f-3-in-f*: $\bigwedge$ *s r*. (*s*, *r*)∈*f* ⟹ ¬ *P* (*nP s*) *s* (*nP r*) *r* **by** *simp*
from *ENFnrSR-offending-set*[*OF a-enf*] *a-not-eval a-offending* **have** *a-f-3-neq*: $\bigwedge$ *s r*. (*s*, *r*)∈*f* ⟹ *s* ≠ *r* **by** *simp*

let *?nP'* = (*nP*(*i* := ⊥))

from *ENFnrSR-offending-set*[*OF a-enf*] *a-not-eval a-offending a-i-snds*
obtain *e1 e2* **where** *e1e2cond*: (*e1*, *e2*)∈*f* ∧ *e2* = *i* **by** *fastforce*

from *conjunct1*[*OF e1e2cond*] *a-offending* **have** (*e1*, *e2*) ∈ *edges G*
by (*metis* (*lifting*, *no-types*) *SecurityInvariant-withOffendingFlows.set-offending-flows-def mem-Collect-eq rev-subsetD*)

from *conjunct1*[*OF e1e2cond*] *a-f-3-in-f* **have** *e1e2notP*: ¬ *P* (*nP e1*) *e1* (*nP e2*) *e2* **by** *simp*
from *e1e2notP a-weakrefl* **have** *e1ore2neqbot*: (*nP e1*) ≠ ⊥ ∨ (*nP e2*) ≠ ⊥ **by** *fastforce*
from *e1e2notP a-bottoall* **have** *x1*: (*nP e1*) ≠ ⊥ **by** *fastforce*
from *this e1e2notP a-botdefault* **have** *x2*: ¬ *P* (*nP e1*) *e1* ⊥ *e2* **by** *fast*
from *this e1e2notP* **have** *e1e2subgoal1*: ¬ *P* (*nP e1*) *e1* (*?nP'* *e2*) *e2* **by** *auto*

from *a-f-3-neq e1e2cond* **have** *e2* ≠ *e1* **by** *blast*

from *e1e2subgoal1* **have** *e1* ≠ *e2* ⟹ ¬ *P* (*?nP'* *e1*) *e1* (*?nP'* *e2*) *e2* **by**(*simp add*: *e1e2cond*)

from *this* ‹*e2* ≠ *e1*› *ENFnrSR-offending-set*[*OF a-enf*] *a-offending* ‹(*e1*, *e2*) ∈ *edges G*› **have**
∃ (*e1*, *e2*)∈(*edges G*). *e2* ≠ *e1* ∧ ¬ *P* (*?nP'* *e1*) *e1* (*?nP'* *e2*) *e2* **by** *fastforce*
hence ¬ (∀ (*e1*, *e2*)∈(*edges G*). *e2* ≠ *e1* ⟶ *P* ((*nP*(*i* := ⊥)) *e1*) *e1* ((*nP*(*i* := ⊥)) *e2*) *e2*) **by** *fast*
from *this a-enf'* **show** ¬ *sinvar G* (*nP*(*i* := ⊥)) **by** *fast*
qed

4.4 edges normal form not refl ENFnr

definition (**in** *SecurityInvariant-withOffendingFlows*) *sinvar-all-edges-normal-form-not-refl* :: ('*a* ⇒ '*a* ⇒ *bool*) ⇒ *bool* **where**
sinvar-all-edges-normal-form-not-refl P ≡ ∀ *G nP*. *sinvar G nP* = (∀ (*e1*, *e2*) ∈ *edges G*. *e1* ≠ *e2* ⟶ *P* (*nP e1*) (*nP e2*))

we derive everything from the ENFnrSR form

lemma (**in** *SecurityInvariant-withOffendingFlows*) *ENFnr-to-ENFnrSR*:
sinvar-all-edges-normal-form-not-refl P ⟹ *sinvar-all-edges-normal-form-not-refl-SR* (λ *v1 - v2 -*. *P v1 v2*)
by(*simp add*: *sinvar-all-edges-normal-form-not-refl-def sinvar-all-edges-normal-form-not-refl-SR-def*)

4.4.1 Offending Flows

theorem (**in** *SecurityInvariant-withOffendingFlows*) *ENFnr-offending-set*:
⟦ *sinvar-all-edges-normal-form-not-refl P* ⟧ ⟹
set-offending-flows G nP = (*if sinvar G nP then*
{}

else
{ {(*e1*,*e2*). (*e1*, *e2*) ∈ *edges G* ∧ *e1* ≠ *e2* ∧ ¬ *P* (*nP e1*) (*nP e2*)} })
apply(*drule ENFnr-to-ENFnrSR*)
by(*drule*(*1*) *ENFnrSR-offending-set*)

4.4.2 Instance helper

lemma (**in** *SecurityInvariant-withOffendingFlows*) *ENFnr-fsts-weakrefl-instance*:
fixes *default-node-properties* :: 'a (⊥)
assumes *a-enf*: *sinvar-all-edges-normal-form-not-refl P*
and *a-botdefault*: ∀ *e1 e2*. *e2* ≠ ⊥ ⟶ ¬ *P e1 e2* ⟶ ¬ *P* ⊥ *e2*
and *a-alltobot*: ∀ *e1*. *P e1* ⊥
and *a-offending*: *f* ∈ *set-offending-flows G nP*
and *a-i-fsts*: *i* ∈ *fst' f*
shows
¬ *sinvar G* (*nP*(*i* := ⊥))
proof −
from *assms* **show** *?thesis*
apply −
apply(*drule ENFnr-to-ENFnrSR*)
apply(*drule ENFnrSR-fsts-weakrefl-instance*)
by *auto*
qed

lemma (**in** *SecurityInvariant-withOffendingFlows*) *ENFnr-snds-weakrefl-instance*:
fixes *default-node-properties* :: 'a (⊥)
assumes *a-enf*: *sinvar-all-edges-normal-form-not-refl P*
and *a-botdefault*: ∀ *e1 e2*. ¬ *P e1 e2* ⟶ ¬ *P e1* ⊥
and *a-bottoall*: ∀ *e2*. *P* ⊥ *e2*
and *a-offending*: *f* ∈ *set-offending-flows G nP*
and *a-i-snds*: *i* ∈ *snd' f*
shows
¬ *sinvar G* (*nP*(*i* := ⊥))
proof −
from *assms* **show** *?thesis*
apply −
apply(*drule ENFnr-to-ENFnrSR*)
apply(*drule ENFnrSR-snds-weakrefl-instance*)
by *auto*
qed

lemma (**in** *SecurityInvariant-withOffendingFlows*) *ENF-weakrefl-instance-FALSE*:
fixes *default-node-properties* :: 'a (⊥)
assumes *a-wfG*: *wf-graph G*
and *a-not-eval*: ¬ *sinvar G nP*
and *a-enf*: *sinvar-all-edges-normal-form P*
and *a-weakrefl*: *P* ⊥ ⊥
and *a-botisolated*: ⋀ *e2*. *e2* ≠ ⊥ ⟹ ¬ *P* ⊥ *e2*
and *a-botdefault*: ⋀ *e1 e2*. *e1* ≠ ⊥ ⟹ ¬ *P e1 e2* ⟹ ¬ *P e1* ⊥

and *a-offending*: $f \in$ *set-offending-flows G nP*
and *a-offending-rm*: *sinvar* (*delete-edges G f*) *nP*
and *a-i-fsts*: $i \in$ *snd*‘ *f*
and *a-not-eval-upd*: $\neg$ *sinvar G* (*nP*(*i* := $\bot$))
shows *False*
oops

end
theory *vertex-example-simps*
imports *Lib/FiniteGraph TopoS-Vertices*
beginend
theory *TopoS-Helper*
imports *Main TopoS-Interface*
TopoS-ENF
vertex-example-simps
begin

lemma (**in** *SecurityInvariant-preliminaries*) *sinvar-valid-remove-flattened-offending-flows*:
assumes *wf-graph* (|*nodes* = *nodesG*, *edges* = *edgesG*|)
shows *sinvar* (|*nodes* = *nodesG*, *edges* = *edgesG* − $\bigcup$ (*set-offending-flows* (|*nodes* = *nodesG*, *edges* = *edgesG*|) *nP*) |) *nP*
proof −
{ fix *f*
assume ∗: *f*∈*set-offending-flows* (|*nodes* = *nodesG*, *edges* = *edgesG*|) *nP*

from ∗ **have** *1*: *sinvar* (|*nodes* = *nodesG*, *edges* = *edgesG* − *f* |) *nP*
by (*metis* (*hide-lams*, *mono-tags*) *SecurityInvariant-withOffendingFlows.valid-without-offending-flows delete-edges-simp2 graph.select-convs*(*1*) *graph.select-convs*(*2*))
from ∗ **have** *2*: *edgesG* − $\bigcup$ (*set-offending-flows* (|*nodes* = *nodesG*, *edges* = *edgesG*|) *nP*) ⊆ *edgesG* − *f*
by *blast*
note *1 2*
}
with *assms* **show** *?thesis*
by (*metis* (*hide-lams*, *no-types*) *Diff-empty Union-empty defined-offending equals0I mono-sinvar wf-graph-remove-edges*)
qed

lemma (**in** *SecurityInvariant-preliminaries*) *sinvar-valid-remove-SOME-offending-flows*:
assumes *set-offending-flows* (|*nodes* = *nodesG*, *edges* = *edgesG*|) *nP* ≠ {}
shows *sinvar* (|*nodes* = *nodesG*, *edges* = *edgesG* − (*SOME F*. *F* ∈ *set-offending-flows* (|*nodes* = *nodesG*, *edges* = *edgesG*|) *nP*) |) *nP*
proof −
{ fix *f*
assume ∗: *f*∈*set-offending-flows* (|*nodes* = *nodesG*, *edges* = *edgesG*|) *nP*

from ∗ **have** *1*: *sinvar* (|*nodes* = *nodesG*, *edges* = *edgesG* − *f* |) *nP*
by (*metis* (*hide-lams*, *mono-tags*) *SecurityInvariant-withOffendingFlows.valid-without-offending-flows delete-edges-simp2 graph.select-convs*(*1*) *graph.select-convs*(*2*))
from ∗ **have** *2*: *edgesG* − $\bigcup$ (*set-offending-flows* (|*nodes* = *nodesG*, *edges* = *edgesG*|) *nP*) ⊆ *edgesG* − *f*

```
    by blast
  note 1 2
 }
 with assms show ?thesis by (simp add: some-in-eq)
qed


lemma (in SecurityInvariant-preliminaries) sinvar-valid-remove-minimalize-offending-overapprox:
 assumes wf-graph (|nodes = nodesG, edges = edgesG|)
    and ¬ sinvar (|nodes = nodesG, edges = edgesG|) nP
    and set Es = edgesG and distinct Es
 shows sinvar (|nodes = nodesG, edges = edgesG −
          set (minimalize-offending-overapprox Es [] (|nodes = nodesG, edges = edgesG|) nP) |) nP
proof −
 from assms have off-Es: is-offending-flows (set Es) (|nodes = nodesG, edges = edgesG|) nP
  by (metis (no-types, lifting) Diff-cancel
    SecurityInvariant-withOffendingFlows.valid-empty-edges-iff-exists-offending-flows defined-offending
      delete-edges-simp2 graph.select-convs(2) is-offending-flows-def sinvar-monoI)
 from minimalize-offending-overapprox-gives-back-an-offending-flow[OF assms(1) off-Es - assms(4)]
have
   in-offending: set (minimalize-offending-overapprox Es [] (|nodes = nodesG, edges = edgesG|) nP)
    ∈ set-offending-flows (|nodes = nodesG, edges = edgesG|) nP
    using assms(3) by simp

 { fix f
   assume ∗: f∈set-offending-flows (|nodes = nodesG, edges = edgesG|) nP
   from ∗ have 1: sinvar (|nodes = nodesG, edges = edgesG − f |) nP
   by (metis (hide-lams, mono-tags) SecurityInvariant-withOffendingFlows.valid-without-offending-flows
delete-edges-simp2 graph.select-convs(1) graph.select-convs(2))
   note 1
 }
 with in-offending show ?thesis by (simp add: some-in-eq)
qed

end
theory SINVAR-Subnets2
imports../TopoS-Helper
begin
```

4.5 SecurityInvariant Subnets2

Warning, This is just a test. Please look at `SINVAR_Subnets.thy`. This security invariant has the following changes, compared to `SINVAR_Subnets.thy`: A new BorderRouter' is introduced which can send to the members of its subnet. A new InboundRouter is accessible by anyone. It can access all other routers and the outside.

```
datatype subnets = Subnet nat | BorderRouter nat | BorderRouter' nat | InboundRouter | Unassigned

definition default-node-properties :: subnets
 where default-node-properties ≡ Unassigned

fun allowed-subnet-flow :: subnets ⇒ subnets ⇒ bool where
 allowed-subnet-flow (Subnet s1) (Subnet s2) = (s1 = s2) |
 allowed-subnet-flow (Subnet s1) (BorderRouter s2) = (s1 = s2) |
```

allowed-subnet-flow (*Subnet s1*) (*BorderRouter′ s2*) = (*s1* = *s2*) |
allowed-subnet-flow (*Subnet* -) - = *True* |
allowed-subnet-flow (*BorderRouter* -) (*Subnet* -) = *False* |
allowed-subnet-flow (*BorderRouter* -) - = *True* |
allowed-subnet-flow (*BorderRouter′ s1*) (*Subnet s2*) = (*s1* = *s2*) |
allowed-subnet-flow (*BorderRouter′* -) - = *True* |
allowed-subnet-flow InboundRouter (*Subnet* -) = *False* |
allowed-subnet-flow InboundRouter - = *True* |
allowed-subnet-flow Unassigned Unassigned = *True* |
allowed-subnet-flow Unassigned InboundRouter = *True* |
allowed-subnet-flow Unassigned - = *False*

fun *sinvar* :: *′v graph* ⇒ (*′v* ⇒ *subnets*) ⇒ *bool* **where**
sinvar G nP = (∀ (*e1*,*e2*) ∈ *edges G*. *allowed-subnet-flow* (*nP e1*) (*nP e2*))

definition *receiver-violation* :: *bool* **where** *receiver-violation* = *False*

Only members of the same subnet or their *BorderRouter′* can access them.

lemma *allowed-subnet-flow a* (*Subnet s1*) ⟹ *a* = (*BorderRouter′ s1*) ∨ *a* = (*Subnet s1*)
apply(*cases a*)
apply(*simp-all*)
done

4.5.1 Preliminaries

lemma *sinvar-mono*: *SecurityInvariant-withOffendingFlows.sinvar-mono sinvar*
apply(*simp only*: *SecurityInvariant-withOffendingFlows.sinvar-mono-def*)
apply(*clarify*)
by *auto*

interpretation *SecurityInvariant-preliminaries*
where *sinvar* = *sinvar*
apply *unfold-locales*
apply(*frule-tac finite-distinct-list*[*OF wf-graph.finiteE*])
apply(*erule-tac exE*)
apply(*rename-tac list-edges*)
apply(*rule-tac ff*=*list-edges* **in** *SecurityInvariant-withOffendingFlows.mono-imp-set-offending-flows-not-empty*[*OF sinvar-mono*])
apply(*auto*)[*6*]
apply(*auto simp add*: *SecurityInvariant-withOffendingFlows.is-offending-flows-def graph-ops*)[*1*]
apply(*fact SecurityInvariant-withOffendingFlows.sinvar-mono-imp-is-offending-flows-mono*[*OF sinvar-mono*])
done

4.5.2 ENF

lemma *All-to-Unassigned*: ∀ *e1*. *allowed-subnet-flow e1 Unassigned*
by (*rule allI*, *case-tac e1*, *simp-all*)
lemma *Unassigned-default-candidate*: ∀ *nP e1 e2*. ¬ *allowed-subnet-flow* (*nP e1*) (*nP e2*) ⟶ ¬ *allowed-subnet-flow Unassigned* (*nP e2*)
apply(*intro allI*)
apply(*case-tac nP e2*)
apply *simp-all*
apply(*case-tac nP e1*)

```
      apply simp-all
   by(simp add: All-to-Unassigned)
 lemma allowed-subnet-flow-refl: ∀ e. allowed-subnet-flow e e
   by(rule allI, case-tac e, simp-all)
 lemma Subnets-ENF: SecurityInvariant-withOffendingFlows.sinvar-all-edges-normal-form sinvar allowed-subnet-flow
   unfolding SecurityInvariant-withOffendingFlows.sinvar-all-edges-normal-form-def
   by simp
 lemma Subnets-ENF-refl: SecurityInvariant-withOffendingFlows.ENF-refl sinvar allowed-subnet-flow
   unfolding SecurityInvariant-withOffendingFlows.ENF-refl-def
   apply(rule conjI)
    apply(simp add: Subnets-ENF)
   apply(simp add: allowed-subnet-flow-refl)
 done


 definition Subnets-offending-set:: 'v graph ⇒ ('v ⇒ subnets) ⇒ ('v × 'v) set set where
 Subnets-offending-set G nP = (if sinvar G nP then
    {}
   else
    { {e ∈ edges G. case e of (e1,e2) ⇒ ¬ allowed-subnet-flow (nP e1) (nP e2)} })
 lemma Subnets-offending-set:
 SecurityInvariant-withOffendingFlows.set-offending-flows sinvar = Subnets-offending-set
   apply(simp only: fun-eq-iff ENF-offending-set[OF Subnets-ENF] Subnets-offending-set-def)
   apply(rule allI)+
   apply(rename-tac G nP)
   apply(auto)
 done


interpretation Subnets: SecurityInvariant-ACS
where default-node-properties = SINVAR-Subnets2.default-node-properties
and sinvar = SINVAR-Subnets2.sinvar
rewrites SecurityInvariant-withOffendingFlows.set-offending-flows sinvar = Subnets-offending-set
 unfolding SINVAR-Subnets2.default-node-properties-def
 apply unfold-locales
  apply(rule ballI)
 apply (rule SecurityInvariant-withOffendingFlows.ENF-fsts-refl-instance[OF Subnets-ENF-refl Unassigned-default-ca
   apply(simp-all)[2]
 apply(erule default-uniqueness-by-counterexample-ACS)
 apply (simp add: SecurityInvariant-withOffendingFlows.set-offending-flows-def
    SecurityInvariant-withOffendingFlows.is-offending-flows-min-set-def
    SecurityInvariant-withOffendingFlows.is-offending-flows-def)
 apply (simp add:graph-ops)
 apply (simp split: prod.split-asm prod.split)
 apply(rule-tac x=⦇ nodes={vertex-1,vertex-2}, edges = {(vertex-1,vertex-2)} ⦈ in exI, simp)
 apply(rule conjI)
  apply(simp add: wf-graph-def)
 apply(case-tac otherbot, simp-all)
   apply(rename-tac mysubnetcase)
   apply(rule-tac x=(λ x. Unassigned)(vertex-1 := Unassigned, vertex-2 := BorderRouter mysub-
netcase) in exI, simp)
   apply(rule-tac x=vertex-1 in exI, simp)
   apply(rule-tac x={(vertex-1,vertex-2)} in exI, simp)
  apply(rule-tac x=(λ x. Unassigned)(vertex-1 := Unassigned, vertex-2 := BorderRouter whatever)
```

in *exI*, *simp*)
apply(*rule-tac x=vertex-1* **in** *exI*, *simp*)
apply(*rule-tac x={(vertex-1,vertex-2)}* **in** *exI*, *simp*)
apply(*rule-tac x=(λ x. Unassigned)(vertex-1 := Unassigned, vertex-2 := BorderRouter whatever)*
in *exI*, *simp*)
apply(*rule-tac x=vertex-1* **in** *exI*, *simp*)
apply(*rule-tac x={(vertex-1,vertex-2)}* **in** *exI*, *simp*)
apply(*rule-tac x=(λ x. Unassigned)(vertex-1 := Unassigned, vertex-2 := BorderRouter whatever)*
in *exI*, *simp*)
apply(*rule-tac x=vertex-1* **in** *exI*, *simp*)
apply(*rule-tac x={(vertex-1,vertex-2)}* **in** *exI*, *simp*)
apply(*fact Subnets-offending-set*)
done

lemma *TopoS-Subnets2*: *SecurityInvariant sinvar default-node-properties receiver-violation*
unfolding *receiver-violation-def* **by** *unfold-locales*

hide-fact (**open**) *sinvar-mono*
hide-const (**open**) *sinvar receiver-violation default-node-properties*

end
theory *SINVAR-BLPstrict*
imports *../TopoS-Helper*
begin

4.6 Stricter Bell LaPadula SecurityInvariant

All unclassified data sources must be labeled, default assumption: all is secret.

Warning: This is considered here an access control strategy. By default, everything is secret and one explicitly prohibits sending to non-secret hosts.

datatype *security-level* = *Unclassified* | *Confidential* | *Secret*

instantiation *security-level* :: *linorder*
begin
fun *less-eq-security-level* :: *security-level* ⇒ *security-level* ⇒ *bool* **where**
(*Unclassified* ≤ *Unclassified*) = *True* |
(*Confidential* ≤ *Confidential*) = *True* |
(*Secret* ≤ *Secret*) = *True* |
(*Unclassified* ≤ *Confidential*) = *True* |
(*Confidential* ≤ *Secret*) = *True* |
(*Unclassified* ≤ *Secret*) = *True* |
(*Secret* ≤ *Confidential*) = *False* |
(*Confidential* ≤ *Unclassified*) = *False* |
(*Secret* ≤ *Unclassified*) = *False*

fun *less-security-level* :: *security-level* ⇒ *security-level* ⇒ *bool* **where**
(*Unclassified* < *Unclassified*) = *False* |
(*Confidential* < *Confidential*) = *False* |
(*Secret* < *Secret*) = *False* |
(*Unclassified* < *Confidential*) = *True* |

```
    (Confidential < Secret) = True  |
    (Unclassified < Secret) = True  |
    (Secret < Confidential) = False  |
    (Confidential < Unclassified) = False  |
    (Secret < Unclassified) = False
  instance
    apply(intro-classes)
    apply(case-tac [!] x)
    apply(simp-all)
    apply(case-tac  [!] y)
    apply(simp-all)
    apply(case-tac  [!] z)
    apply(simp-all)
    done
  end



definition default-node-properties :: security-level
  where  default-node-properties ≡ Secret



fun sinvar :: 'v graph ⇒ ('v ⇒ security-level) ⇒ bool where
  sinvar G nP = (∀ (e1,e2) ∈ edges G. (nP e1) ≤ (nP e2))

definition receiver-violation :: bool where receiver-violation ≡ False


lemma sinvar-mono: SecurityInvariant-withOffendingFlows.sinvar-mono sinvar
  apply(simp only: SecurityInvariant-withOffendingFlows.sinvar-mono-def)
  apply(clarify)
  by auto


interpretation SecurityInvariant-preliminaries
where sinvar = sinvar
  apply unfold-locales
    apply(frule-tac finite-distinct-list[OF wf-graph.finiteE])
    apply(erule-tac exE)
    apply(rename-tac list-edges)
   apply(rule-tac ff=list-edges in SecurityInvariant-withOffendingFlows.mono-imp-set-offending-flows-not-empty[OF
sinvar-mono])
       apply(auto)[6]
   apply(auto simp add: SecurityInvariant-withOffendingFlows.is-offending-flows-def graph-ops)[1]
 apply(fact SecurityInvariant-withOffendingFlows.sinvar-mono-imp-is-offending-flows-mono[OF sinvar-mono])
 done
```

4.7 ENF

```
  lemma secret-default-candidate: ⋀ (nP::('v ⇒ security-level)) e1 e2. ¬ (nP e1) ≤ (nP e2) ⟹ ¬
Secret ≤ (nP e2)
    apply(case-tac nP e1)
    apply(simp-all)
```

apply(*case-tac* [!] *nP e2*)
apply(*simp-all*)
done
lemma *BLP-ENF*: *SecurityInvariant-withOffendingFlows.sinvar-all-edges-normal-form sinvar* (≤)
unfolding *SecurityInvariant-withOffendingFlows.sinvar-all-edges-normal-form-def*
by *simp*
lemma *BLP-ENF-refl*: *SecurityInvariant-withOffendingFlows.ENF-refl sinvar* (≤)
unfolding *SecurityInvariant-withOffendingFlows.ENF-refl-def*
apply(*rule conjI*)
apply(*simp add*: *BLP-ENF*)
apply(*simp*)
done

definition *BLP-offending-set*:: *'v graph* ⇒ (*'v* ⇒ *security-level*) ⇒ (*'v* × *'v*) *set set* **where**
BLP-offending-set G nP = (*if sinvar G nP then*
{}
else
{ {*e* ∈ *edges G*. *case e of* (*e1*,*e2*) ⇒ (*nP e1*) > (*nP e2*)} })
lemma *BLP-offending-set*: *SecurityInvariant-withOffendingFlows.set-offending-flows sinvar* = *BLP-offending-set*
apply(*simp only*: *fun-eq-iff SecurityInvariant-withOffendingFlows.ENF-offending-set*[*OF BLP-ENF*]
BLP-offending-set-def)
apply(*rule allI*)+
apply(*rename-tac G nP*)
apply(*auto*)
done

interpretation *BLPstrict*: *SecurityInvariant-ACS sinvar default-node-properties*

rewrites *SecurityInvariant-withOffendingFlows.set-offending-flows sinvar* = *BLP-offending-set*
unfolding *default-node-properties-def*
apply(*unfold-locales*)
apply(*rule ballI*)
apply(*rule SecurityInvariant-withOffendingFlows.ENF-fsts-refl-instance*[*OF BLP-ENF-refl*])
apply(*simp-all add*: *BLP-ENF BLP-ENF-refl*)[*3*]
apply(*simp add*: *secret-default-candidate*; *fail*)
apply(*erule default-uniqueness-by-counterexample-ACS*)
apply(*rule-tac x*=⦇ *nodes*=*set* [*vertex-1*,*vertex-2*], *edges* = *set* [(*vertex-1*,*vertex-2*)] ⦈ **in** *exI*,
simp)
apply(*simp add*: *BLP-offending-set graph-ops wf-graph-def*)
apply(*rule-tac x*=(λ *x*. *Secret*)(*vertex-1* := *Secret*, *vertex-2* := *Confidential*) **in** *exI*, *simp*)
apply(*rule-tac x*=*vertex-1* **in** *exI*, *simp*)
apply(*rule-tac x*=*set* [(*vertex-1*,*vertex-2*)] **in** *exI*, *simp*)
apply(*simp add*: *BLP-offending-set-def*)
apply(*rule conjI*)
apply *fastforce*
apply (*case-tac otherbot*, *simp-all*)
apply(*fact BLP-offending-set*)
done

lemma *TopoS-BLPstrict*: *SecurityInvariant sinvar default-node-properties receiver-violation*
unfolding *receiver-violation-def* **by** *unfold-locales*

hide-fact (**open**) *sinvar-mono*

hide-const (**open**) *sinvar receiver-violation default-node-properties*

end
theory *SINVAR-Tainting*
imports *../TopoS-Helper*
begin

4.8 SecurityInvariant Tainting for IFS

context
begin

qualified type-synonym *taints* = *string set*

Warning: an infinite set has cardinality 0

lemma *card* (*UNIV*::*taints*) = *0* **by** (*simp add*: *infinite-UNIV-listI*)

qualified definition *default-node-properties* :: *taints*
where *default-node-properties* ≡ {}

For all nodes n in the graph, for all nodes r which are reachable from n, node n needs the appropriate tainting fields which are set by r

definition *sinvar-tainting* :: *'v graph* ⇒ (*'v* ⇒ *taints*) ⇒ *bool* **where**
sinvar-tainting G nP ≡ ∀ *n* ∈ (*nodes G*). ∀ *r* ∈ (*succ-tran G n*). *nP n* ⊆ *nP r*

private lemma *sinvar-tainting-edges-def*: *wf-graph G* ⟹
sinvar-tainting G nP ⟷ (∀ (*v1*,*v2*) ∈ *edges G*. ∀ *r* ∈ (*succ-tran G v1*). *nP v1* ⊆ *nP r*)
unfolding *sinvar-tainting-def*
proof
assume *a1*: *wf-graph G*
and *a2*: ∀ *n*∈*nodes G*. ∀ *r*∈*succ-tran G n*. *nP n* ⊆ *nP r*
from *a1*[*simplified wf-graph-def*] **have** *f1*: *fst ' edges G* ⊆ *nodes G* **by** *simp*
from *f1 a2* **have** ∀ *v* ∈ (*fst ' edges G*). ∀ *r*∈*succ-tran G v*. *nP v* ⊆ *nP r* **by** *auto*
thus ∀ (*v1*, -)∈*edges G*. ∀ *r*∈*succ-tran G v1*. *nP v1* ⊆ *nP r* **by** *fastforce*
next
assume *a1*: *wf-graph G*
and *a2*: ∀ (*v1*, *v2*)∈*edges G*. ∀ *r*∈*succ-tran G v1*. *nP v1* ⊆ *nP r*
from *a2* **have** *g1*: ∀ *v* ∈ (*fst ' edges G*). ∀ *r*∈*succ-tran G v*. *nP v* ⊆ *nP r* **by** *fastforce*
from *FiniteGraph.succ-tran-empty*[*OF a1*]
have *g2*: ∀ *v*. *v* ∉ (*fst ' edges G*) ⟶ (∀ *r*∈*succ-tran G v*. *nP v* ⊆ *nP r*) **by** *blast*
from *g1 g2* **show** ∀ *n*∈*nodes G*. ∀ *r*∈*succ-tran G n*. *nP n* ⊆ *nP r* **by** *metis*
qed

Alternative definition of the *sinvar-tainting*

qualified definition *sinvar* :: *'v graph* ⇒ (*'v* ⇒ *taints*) ⇒ *bool* **where**
sinvar G nP ≡ ∀ (*v1*,*v2*) ∈ *edges G*. *nP v1* ⊆ *nP v2*

qualified lemma *sinvar-preferred-def*:
wf-graph G ⟹ *sinvar-tainting G nP* = *sinvar G nP*
proof(*unfold sinvar-tainting-edges-def sinvar-def*, *rule iffI*, *goal-cases*)

case *2*
have $(v, v') \in (edges\ G)^+ \Longrightarrow nP\ v \subseteq nP\ v'$ **for** $v\ v'$
proof(*induction rule*: *trancl-induct*)
case *base* **thus** *?case* **using** *2*(*2*) **by** *fastforce*
next
case *step* **thus** *?case* **using** *2*(*2*) **by** *fastforce*
qed
thus *?case*
by(*simp add*: *succ-tran-def*)
next
case *1*
from *1*(*1*)[*simplified wf-graph-def*] **have** *f1*: *fst ' edges G* $\subseteq$ *nodes G* **by** *simp*
from *f1 1*(*2*) **have** $\forall v \in (fst\ `\ edges\ G).\ \forall v' \in succ\text{-}tran\ G\ v.\ nP\ v \subseteq nP\ v'$ **by** *fastforce*
thus *?case* **unfolding** *succ-tran-def* **by** *fastforce*
qed

Information Flow Security

qualified definition *receiver-violation* :: *bool* **where** *receiver-violation* $\equiv$ *True*

private lemma *sinvar-mono*: *SecurityInvariant-withOffendingFlows.sinvar-mono sinvar*
apply(*simp add*: *SecurityInvariant-withOffendingFlows.sinvar-mono-def sinvar-def*)
apply(*clarify*)
by *blast*
interpretation *SecurityInvariant-preliminaries*
where *sinvar* = *sinvar*
proof(*unfold-locales*, *goal-cases*)
case (*1 G nP*)
from *1* **show** *?case*
apply(*frule-tac finite-distinct-list*[*OF wf-graph.finiteE*])
apply(*erule-tac exE*)
apply(*rename-tac list-edges*)
apply(*rule-tac ff*=*list-edges* **in** *SecurityInvariant-withOffendingFlows.mono-imp-set-offending-flows-not-empty*[*OF sinvar-mono*])
apply(*auto simp add*: *sinvar-def*)
apply(*auto simp add*: *sinvar-def SecurityInvariant-withOffendingFlows.is-offending-flows-def graph-ops*)[*1*]
done
next
case (*2 N E E' nP*) **thus** *?case* **by**(*simp add*: *sinvar-def*) *blast*
next
case *3* **thus** *?case* **by**(*fact SecurityInvariant-withOffendingFlows.sinvar-mono-imp-is-offending-flows-mono*[*OF sinvar-mono*])
qed

private lemma *Taints-def-unique*: *otherbot* $\neq \{\} \Longrightarrow$
$\exists\ G\ p\ i\ f.$ *wf-graph G* $\wedge \neg$ *sinvar G p* $\wedge\ f \in$ (*SecurityInvariant-withOffendingFlows.set-offending-flows sinvar G p*) $\wedge$
sinvar (*delete-edges G f*) *p* $\wedge$
i $\in$ *snd ' f* $\wedge$ *sinvar G* (*p*(*i* := *otherbot*))
apply(*simp*)
apply (*simp add*: *SecurityInvariant-withOffendingFlows.set-offending-flows-def*

SecurityInvariant-withOffendingFlows.is-offending-flows-min-set-def
SecurityInvariant-withOffendingFlows.is-offending-flows-def)
apply (*simp add*:*graph-ops*)
apply (*simp split*: *prod.split-asm prod.split*)
apply(*rule-tac x*=(| *nodes*=*set* [*vertex-1*,*vertex-2*], *edges* = *set* [(*vertex-1*,*vertex-2*)] |) **in** *exI*, *simp*)
apply(*rule conjI*)
apply(*simp add*: *wf-graph-def*; *fail*)
apply(*subgoal-tac* ∃*foo*. *foo* ∈ *otherbot*)
prefer *2*
subgoal by *fastforce*
apply(*erule exE*, *rename-tac foo*)
apply(*rule-tac x*=(λ *x*. {})(*vertex-1* := {*foo*}, *vertex-2* := {}) **in** *exI*)
apply(*rule conjI*)
apply(*simp add*: *sinvar-def*; *fail*)
apply(*rule-tac x*=*vertex-2* **in** *exI*)
apply(*rule-tac x*=*set* [(*vertex-1*,*vertex-2*)] **in** *exI*, *simp*)
apply(*simp add*: *sinvar-def*)
done

4.8.1 ENF

private lemma *Taints-ENF*: *SecurityInvariant-withOffendingFlows.sinvar-all-edges-normal-form sinvar* (⊆)
unfolding *SecurityInvariant-withOffendingFlows.sinvar-all-edges-normal-form-def sinvar-def*
by *simp*
private lemma *Taints-ENF-refl*: *SecurityInvariant-withOffendingFlows.ENF-refl sinvar* (⊆)
unfolding *SecurityInvariant-withOffendingFlows.ENF-refl-def*
by(*auto simp add*: *Taints-ENF*)

qualified definition *Taints-offending-set*:: *'v graph* ⇒ (*'v* ⇒ *taints*) ⇒ (*'v* × *'v*) *set set* **where**
Taints-offending-set G nP = (*if sinvar G nP then*
{}
else
{ {*e* ∈ *edges G*. *case e of* (*e1*,*e2*) ⇒ ¬ (*nP e1*) ⊆ (*nP e2*)} })
lemma *Taints-offending-set*: *SecurityInvariant-withOffendingFlows.set-offending-flows sinvar* = *Taints-offending-set*
by(*auto simp add*: *fun-eq-iff*
SecurityInvariant-withOffendingFlows.ENF-offending-set[*OF Taints-ENF*]
Taints-offending-set-def)

interpretation *Taints*: *SecurityInvariant-IFS sinvar default-node-properties*
rewrites *SecurityInvariant-withOffendingFlows.set-offending-flows sinvar* = *Taints-offending-set*
unfolding *receiver-violation-def*
unfolding *default-node-properties-def*
proof(*unfold-locales*, *goal-cases*)
case *1*
from *1*(*2*) **show** *?case*
apply(*intro ballI*)
apply(*rule SecurityInvariant-withOffendingFlows.ENF-snds-refl-instance*[*OF Taints-ENF-refl*])
apply(*simp-all add*: *Taints-ENF Taints-ENF-refl*)
by *blast*
next
case *2* **thus** *?case*

proof(*elim default-uniqueness-by-counterexample-IFS*)
qed(*fact Taints-def-unique*)
next
case *3* **show** *set-offending-flows = Taints-offending-set* **by**(*fact Taints-offending-set*)
qed

lemma *TopoS-Tainting*: *SecurityInvariant sinvar default-node-properties receiver-violation*
unfolding *receiver-violation-def* **by** *unfold-locales*

end

end
theory *SINVAR-BLPbasic*
imports *../TopoS-Helper*
begin

4.9 SecurityInvariant Basic Bell LaPadula

type-synonym *security-level = nat*

definition *default-node-properties* :: *security-level*
where *default-node-properties ≡ 0*

fun *sinvar* :: *'v graph ⇒ ('v ⇒ security-level) ⇒ bool* **where**
sinvar G nP = (∀ (e1,e2) ∈ edges G. (nP e1) ≤ (nP e2))

What we call a *security-level* is also referred to as security label (or security clearance of subjects and classification of objects) in the literature. The lowest security level is *0*, which can be understood as unclassified. Consequently, 1 = confidential, 2 = secret, 3 = topSecret, ... The total order of the security levels corresponds to the total order of the natural numbers ≤. It is important that there is smallest security level (i.e. *default-node-properties*), otherwise, a unique and secure default parameter could not exist. Hence, it is not possible to extend the security levels to *int* to model unlimited "un-confidentialness".

definition *receiver-violation* :: *bool* **where** *receiver-violation ≡ True*

lemma *sinvar-mono*: *SecurityInvariant-withOffendingFlows.sinvar-mono sinvar*
apply(*simp only*: *SecurityInvariant-withOffendingFlows.sinvar-mono-def*)
apply(*clarify*)
by *auto*

interpretation *SecurityInvariant-preliminaries*
where *sinvar = sinvar*
apply *unfold-locales*
apply(*frule-tac finite-distinct-list*[*OF wf-graph.finiteE*])
apply(*erule-tac exE*)
apply(*rename-tac list-edges*)
apply(*rule-tac ff=list-edges* **in** *SecurityInvariant-withOffendingFlows.mono-imp-set-offending-flows-not-empty*[*OF sinvar-mono*])
apply(*auto*)[*6*]
apply(*auto simp add*: *SecurityInvariant-withOffendingFlows.is-offending-flows-def graph-ops*)[*1*]

apply(*fact SecurityInvariant-withOffendingFlows.sinvar-mono-imp-is-offending-flows-mono*[*OF sinvar-mono*])
done

lemma *BLP-def-unique*: *otherbot* ≠ *0* ⟹
∃ *G p i f*. *wf-graph G* ∧ ¬ *sinvar G p* ∧ *f* ∈ (*SecurityInvariant-withOffendingFlows.set-offending-flows sinvar G p*) ∧
sinvar (*delete-edges G f*) *p* ∧
i ∈ *snd* ‘ *f* ∧ *sinvar G* (*p*(*i* := *otherbot*))
apply(*simp*)
apply (*simp add*: *SecurityInvariant-withOffendingFlows.set-offending-flows-def SecurityInvariant-withOffendingFlows.is-offending-flows-min-set-def SecurityInvariant-withOffendingFlows.is-offending-flows-def*)
apply (*simp add*:*graph-ops*)
apply (*simp split*: *prod.split-asm prod.split*)
apply(*rule-tac x*=(| *nodes*=*set* [*vertex-1*,*vertex-2*], *edges* = *set* [(*vertex-1*,*vertex-2*)] |) **in** *exI*, *simp*)
apply(*rule conjI*)
apply(*simp add*: *wf-graph-def*)
apply(*rule-tac x*=(λ *x*. *0*)(*vertex-1* := *1*, *vertex-2* := *0*) **in** *exI*, *simp*)
apply(*rule-tac x*=*vertex-2* **in** *exI*, *simp*)
apply(*rule-tac x*=*set* [(*vertex-1*,*vertex-2*)] **in** *exI*, *simp*)
done

4.9.1 ENF

lemma *zero-default-candidate*: ⋀ *nP e1 e2*. ¬ ((≤)::*security-level* ⇒ *security-level* ⇒ *bool*) (*nP e1*) (*nP e2*) ⟹ ¬ (≤) (*nP e1*) *0*
by *simp-all*
lemma *zero-default-candidate-rule*: ⋀ (*nP*::(*'v* ⇒ *security-level*)) *e1 e2*. ¬ (*nP e1*) ≤ (*nP e2*) ⟹ ¬ (*nP e1*) ≤ *0*
by *simp-all*
lemma *privacylevel-refl*: ((≤)::*security-level* ⇒ *security-level* ⇒ *bool*) *e e*
by(*simp-all*)
lemma *BLP-ENF*: *SecurityInvariant-withOffendingFlows.sinvar-all-edges-normal-form sinvar* (≤)
unfolding *SecurityInvariant-withOffendingFlows.sinvar-all-edges-normal-form-def*
by *simp*
lemma *BLP-ENF-refl*: *SecurityInvariant-withOffendingFlows.ENF-refl sinvar* (≤)
unfolding *SecurityInvariant-withOffendingFlows.ENF-refl-def*
apply(*rule conjI*)
apply(*simp add*: *BLP-ENF*)
apply(*simp add*: *privacylevel-refl*)
done

definition *BLP-offending-set*:: *'v graph* ⇒ (*'v* ⇒ *security-level*) ⇒ (*'v* × *'v*) *set set* **where**
BLP-offending-set G nP = (*if sinvar G nP then*
{}
else
{ {*e* ∈ *edges G*. *case e of* (*e1*,*e2*) ⇒ (*nP e1*) > (*nP e2*)} })
lemma *BLP-offending-set*: *SecurityInvariant-withOffendingFlows.set-offending-flows sinvar* = *BLP-offending-set*
apply(*simp only*: *fun-eq-iff SecurityInvariant-withOffendingFlows.ENF-offending-set*[*OF BLP-ENF*] *BLP-offending-set-def*)
apply(*rule allI*)+
apply(*rename-tac G nP*)
apply(*auto*)

done

interpretation *BLPbasic*: *SecurityInvariant-IFS sinvar default-node-properties*
rewrites *SecurityInvariant-withOffendingFlows.set-offending-flows sinvar = BLP-offending-set*
unfolding *receiver-violation-def*
unfolding *default-node-properties-def*
apply(*unfold-locales*)
apply(*rule ballI*)
apply(*rule SecurityInvariant-withOffendingFlows.ENF-snds-refl-instance*[*OF BLP-ENF-refl*])
apply(*simp-all add*: *BLP-ENF BLP-ENF-refl*)[*3*]
apply(*erule default-uniqueness-by-counterexample-IFS*)
apply(*fact BLP-def-unique*)
apply(*fact BLP-offending-set*)
done

lemma *TopoS-BLPBasic*: *SecurityInvariant sinvar default-node-properties receiver-violation*
unfolding *receiver-violation-def* **by** *unfold-locales*

Alternate definition of the *sinvar*: For all reachable nodes, the security level is higher

lemma *sinvar-BLPbasic-tancl*:
wf-graph G $\Longrightarrow$ *sinvar G nP* = ($\forall$ *v* $\in$ *nodes G*. $\forall$ *v'* $\in$ *succ-tran G v*. (*nP v*) $\leq$ (*nP v'*))
proof(*unfold sinvar.simps*, *rule iffI*, *goal-cases*)
case *1*
have (*v*, *v'*) $\in$ (*edges G*)$^+$ $\Longrightarrow$ *nP v* $\leq$ *nP v'* **for** *v v'*
proof(*induction rule*: *trancl-induct*)
case *base* **thus** *?case* **using** *1*(*2*) **by** *fastforce*
next
case *step* **thus** *?case* **using** *1*(*2*) **by** *fastforce*
qed
thus *?case*
by(*simp add*: *succ-tran-def*)
next
case *2*
from *2*(*1*)[*simplified wf-graph-def*] **have** *f1*: *fst ' edges G* $\subseteq$ *nodes G* **by** *simp*
from *f1 2*(*2*) **have** $\forall$ *v* $\in$ (*fst ' edges G*). $\forall$ *v'*$\in$*succ-tran G v*. *nP v* $\leq$ *nP v'* **by** *auto*
thus *?case* **unfolding** *succ-tran-def* **by** *fastforce*
qed

hide-fact (**open**) *sinvar-mono*
hide-fact *BLP-def-unique zero-default-candidate zero-default-candidate-rule privacylevel-refl BLP-ENF BLP-ENF-refl*

hide-const (**open**) *sinvar receiver-violation default-node-properties*

end
theory *SINVAR-TaintingTrusted*
imports *../TopoS-Helper*
begin

4.10 SecurityInvariant Tainting with Untainting-Feature for IFS

context
begin
qualified datatype *taints-raw* = *TaintsUntaints-Raw* (*taints-raw*: *string set*) (*untaints-raw*: *string set*)

The *untaints-raw* set must be a subset of *taints-raw*. Otherwise, there can be entries in the untaints set, which do not affect anything. This is certainly undesirable. In addition, a unique default parameter cannot exist if we allow such dead entries.

qualified typedef *taints* = {*ts*::*taints-raw*. *untaints-raw ts* ⊆ *taints-raw ts*}
morphisms *raw-of-taints Abs-taints*
proof
show *TaintsUntaints-Raw* {} {} ∈ {*ts*. *untaints-raw ts* ⊆ *taints-raw ts*} **by** *simp*
qed

setup-lifting *type-definition-taints*

lemma *taints-eq-iff*:
tsx = *tsy* ⟷ *raw-of-taints tsx* = *raw-of-taints tsy*
by (*simp add*: *raw-of-taints-inject*)

definition *taints* :: *taints* ⇒ *string set* **where**
taints ts ≡ *taints-raw* (*raw-of-taints ts*)
definition *untaints* :: *taints* ⇒ *string set* **where**
untaints ts ≡ *untaints-raw* (*raw-of-taints ts*)

lemma *taints-wellformedness*: *untaints ts* ⊆ *taints ts*
using *raw-of-taints taints-def untaints-def* **by** *auto*

Constructor for *taints*:

definition *TaintsUntaints* :: *string set* ⇒ *string set* ⇒ *taints* **where**
TaintsUntaints ts uts = *Abs-taints* (*TaintsUntaints-Raw* (*ts* ∪ *uts*) *uts*)

lemma *raw-of-taints-TaintsUntaints*:
raw-of-taints (*TaintsUntaints ts uts*) = (*TaintsUntaints-Raw* (*ts* ∪ *uts*) *uts*)
by (*simp add*: *TaintsUntaints-def Abs-taints-inverse*)

lemma *taints-TaintsUntaints*[*code*]: *taints* (*TaintsUntaints ts uts*) = *ts* ∪ *uts*
by(*simp add*: *taints-def raw-of-taints-TaintsUntaints*)
lemma *untaints-TaintsUntaints*[*code*]: *untaints* (*TaintsUntaints ts uts*) = *uts*
by(*simp add*: *untaints-def raw-of-taints-TaintsUntaints*)

The things in the first set are tainted, those in the second set are untainted. For example, a machine produces *''foo''*: *TaintsUntaints* {*''foo''*} {}
For example, a machine consumes *''foo''* and *''bar''*, combines them in a way that they are no longer critical and outputs *''baz''*: *TaintsUntaints* {*''foo''*, *''bar''*, *''baz''*} {*''foo''*, *''bar''*} abbreviated: *TaintsUntaints* {*''baz''*} {*''foo''*, *''bar''*}

lemma *TaintsUntaints* {*''foo''*, *''bar''*, *''baz''*} {*''foo''*, *''bar''*} =
TaintsUntaints {*''baz''*} {*''foo''*, *''bar''*}
apply(*simp add*: *taints-eq-iff raw-of-taints-TaintsUntaints*)
by *blast*

qualified definition *default-node-properties* :: *taints*
where *default-node-properties* ≡ *TaintsUntaints* {} {}

qualified definition *sinvar* :: *'v graph* ⇒ (*'v* ⇒ *taints*) ⇒ *bool* **where**
sinvar G nP ≡ ∀ (*v1*,*v2*) ∈ *edges G*.
taints (*nP v1*) − *untaints* (*nP v1*) ⊆ *taints* (*nP v2*)

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qualified definition *receiver-violation* :: *bool* **where** *receiver-violation* ≡ *True*

private lemma *sinvar-mono*: *SecurityInvariant-withOffendingFlows.sinvar-mono sinvar*
apply(*simp add*: *SecurityInvariant-withOffendingFlows.sinvar-mono-def sinvar-def*)
apply(*clarify*)
by *blast*
interpretation *SecurityInvariant-preliminaries*
where *sinvar* = *sinvar*
proof(*unfold-locales*, *goal-cases*)
case (*1 G nP*)
from *1* **show** *?case*
apply(*frule-tac finite-distinct-list*[*OF wf-graph.finiteE*])
apply(*erule-tac exE*)
apply(*rename-tac list-edges*)
apply(*rule-tac ff*=*list-edges* **in** *SecurityInvariant-withOffendingFlows.mono-imp-set-offending-flows-not-empty*[*OF sinvar-mono*])
apply(*auto simp add*: *sinvar-def*)
apply(*auto simp add*: *sinvar-def SecurityInvariant-withOffendingFlows.is-offending-flows-def graph-ops*)
done
next
case (*2 N E E' nP*) **thus** *?case* **by**(*simp add*: *sinvar-def*) *blast*
next
case *3* **thus** *?case* **by**(*fact SecurityInvariant-withOffendingFlows.sinvar-mono-imp-is-offending-flows-mono*[*OF sinvar-mono*])
qed

Needs the well-formedness condition that *untaints otherbot* ⊆ *taints otherbot*

private lemma *Taints-def-unique*: *otherbot* ≠ *default-node-properties* ⟹
∃ *G p i f*. *wf-graph G* ∧ ¬ *sinvar G p* ∧ *f* ∈ (*SecurityInvariant-withOffendingFlows.set-offending-flows sinvar G p*) ∧
sinvar (*delete-edges G f*) *p* ∧
i ∈ *snd* ‘ *f* ∧ *sinvar G* (*p*(*i* := *otherbot*))
apply(*subgoal-tac untaints otherbot* ⊆ *taints otherbot*)
prefer *2*
subgoal using *taints-wellformedness* **by** *simp*
apply(*simp add*: *default-node-properties-def*)
apply (*simp add*: *SecurityInvariant-withOffendingFlows.set-offending-flows-def*
SecurityInvariant-withOffendingFlows.is-offending-flows-min-set-def
SecurityInvariant-withOffendingFlows.is-offending-flows-def)
apply (*simp add*:*graph-ops*)
apply (*simp split*: *prod.split-asm prod.split*)
apply(*rule-tac x*=(| *nodes*=*set* [*vertex-1*,*vertex-2*], *edges* = *set* [(*vertex-1*,*vertex-2*)] |) **in** *exI*, *simp*)
apply(*rule conjI*)

apply(*simp add*: *wf-graph-def*; *fail*)
apply(*subgoal-tac* ∃*foo*. *foo* ∈ *taints otherbot*)
prefer *2*
subgoal
apply(*case-tac otherbot*, *rename-tac tsraw*)
apply(*simp*)
apply(*subgoal-tac taints-raw tsraw* ≠ {})
prefer *2* **subgoal for** *tsraw*
apply(*case-tac tsraw*)
apply(*simp add*: *TaintsUntaints-def*)
by *fastforce*
by (*simp add*: *Abs-taints-inverse ex-in-conv taints-def*)
apply(*elim exE*, *rename-tac foo*)
apply(*rule-tac x*=(λ *x*. *default-node-properties*)
(*vertex-1* := *TaintsUntaints* {*foo*} {}, *vertex-2* := *default-node-properties*) **in** *exI*)
apply(*simp add*: *default-node-properties-def*)
apply(*rule conjI*)
apply(*simp add*: *sinvar-def taints-TaintsUntaints untaints-TaintsUntaints*; *fail*)
apply(*rule-tac x*=*vertex-2* **in** *exI*)
apply(*rule-tac x*=*set* [(*vertex-1*,*vertex-2*)] **in** *exI*, *simp*)
apply(*simp add*: *sinvar-def taints-TaintsUntaints untaints-TaintsUntaints*; *fail*)
done

4.10.1 ENF

private lemma *Taints-ENF*: *SecurityInvariant-withOffendingFlows.sinvar-all-edges-normal-form*
sinvar (λ*c1 c2*. *taints c1* − *untaints c1* ⊆ *taints c2*)
unfolding *SecurityInvariant-withOffendingFlows.sinvar-all-edges-normal-form-def sinvar-def*
by *blast*
private lemma *Taints-ENF-refl*: *SecurityInvariant-withOffendingFlows.ENF-refl*
sinvar (λ*c1 c2*. *taints c1* − *untaints c1* ⊆ *taints c2*)
unfolding *SecurityInvariant-withOffendingFlows.ENF-refl-def*
apply(*intro conjI*)
subgoal using *Taints-ENF* **by** *simp*
by *auto*

qualified definition *Taints-offending-set*:: *'v graph* ⇒ (*'v* ⇒ *taints*) ⇒ (*'v* × *'v*) *set set* **where**
Taints-offending-set G nP = (*if sinvar G nP then*
{}
else
{ {*e* ∈ *edges G*. *case e of* (*e1*,*e2*) ⇒ ¬ *taints* (*nP e1*) − *untaints* (*nP e1*) ⊆ *taints* (*nP e2*)} })
lemma *Taints-offending-set*: *SecurityInvariant-withOffendingFlows.set-offending-flows sinvar* = *Taints-offending-set*
by(*auto simp add*: *fun-eq-iff*
SecurityInvariant-withOffendingFlows.ENF-offending-set[*OF Taints-ENF*]
Taints-offending-set-def)

interpretation *Taints*: *SecurityInvariant-IFS sinvar default-node-properties*
rewrites *SecurityInvariant-withOffendingFlows.set-offending-flows sinvar* = *Taints-offending-set*
unfolding *receiver-violation-def*
unfolding *default-node-properties-def*
proof(*unfold-locales*, *goal-cases*)

```
  case (1 G f nP)
    from 1(2) show ?case
    apply(intro ballI)
   apply(rule SecurityInvariant-withOffendingFlows.ENF-snds-refl-instance[OF Taints-ENF-refl])
     apply(simp add: sinvar-def taints-TaintsUntaints untaints-TaintsUntaints, blast)
    by(simp)+
  next
  case 2 thus ?case
    apply(elim default-uniqueness-by-counterexample-IFS)
    apply(rule Taints-def-unique)
    apply(simp-all add: default-node-properties-def)
    done
  next
  case 3 show set-offending-flows = Taints-offending-set by(fact Taints-offending-set)
 qed


 lemma TopoS-TaintingTrusted: SecurityInvariant sinvar default-node-properties receiver-violation
 unfolding receiver-violation-def by unfold-locales

end


code-datatype TaintsUntaints

value[code] TaintsUntaints {''foo''} {''bar''}

value[code] taints (TaintsUntaints {''foo''} {''bar''})

end
theory SINVAR-BLPtrusted
imports ../TopoS-Helper
begin
```

4.11 SecurityInvariant Basic Bell LaPadula with trusted entities

```
type-synonym security-level = nat

record node-config =
    security-level::security-level
    trusted::bool

definition default-node-properties :: node-config
  where  default-node-properties ≡ (| security-level = 0, trusted = False |)

fun sinvar :: 'v graph ⇒ ('v ⇒ node-config) ⇒  bool where
  sinvar G nP = (∀ (e1,e2) ∈ edges G. (if trusted (nP e2) then True else security-level (nP e1) ≤
security-level (nP e2) ))
```

A simplified version of the Bell LaPadula model was presented in `SINVAR_BLPbasic.thy`. In this theory, we extend this template with a notion of trust by adding a Boolean flag *trusted* to the host attributes. This is a refinement to represent real-world scenarios more accurately and analogously happened to the original Bell LaPadula model (see publication "Looking Back at the Bell-La Padula Model" A trusted host can receive information of any security level and

may declassify it, i.e. distribute the information with its own security level. For example, a *trusted sc = True* host is allowed to receive any information and with the *0* level, it is allowed to reveal it to anyone.

definition *receiver-violation* :: *bool* **where** *receiver-violation* ≡ *True*

lemma *sinvar-mono*: *SecurityInvariant-withOffendingFlows.sinvar-mono sinvar*
apply(*simp only*: *SecurityInvariant-withOffendingFlows.sinvar-mono-def*)
apply(*clarify*)
apply(*simp split*: *prod.split prod.split-asm*)
by *auto*

interpretation *SecurityInvariant-preliminaries*
where *sinvar* = *sinvar*
apply *unfold-locales*
apply(*frule-tac finite-distinct-list*[*OF wf-graph.finiteE*])
apply(*erule-tac exE*)
apply(*rename-tac list-edges*)
apply(*rule-tac ff=list-edges* **in** *SecurityInvariant-withOffendingFlows.mono-imp-set-offending-flows-not-empty*[*OF sinvar-mono*])
apply(*auto split*: *prod.split prod.split-asm*)[*6*]
apply(*simp add*: *SecurityInvariant-withOffendingFlows.is-offending-flows-def graph-ops split*: *prod.split prod.split-asm*)[*1*]
apply (*metis prod.inject*)
apply(*fact SecurityInvariant-withOffendingFlows.sinvar-mono-imp-is-offending-flows-mono*[*OF sinvar-mono*])
done

lemma *a* ≠ *b* ⟹ ((∃ *x*. *y x*)) ⟹ ((∀ *x*. ¬ *y x*) ⟹ *a* = *b*) **by** *simp*

lemma *BLP-def-unique*: *otherbot* ≠ *default-node-properties* ⟹
∃ *G p i f*. *wf-graph G* ∧ ¬ *sinvar G p* ∧ *f* ∈ (*SecurityInvariant-withOffendingFlows.set-offending-flows sinvar G p*) ∧
sinvar (*delete-edges G f*) *p* ∧
i ∈ *snd* ‘ *f* ∧ *sinvar G* (*p*(*i* := *otherbot*))
apply(*simp add*:*default-node-properties-def*)
apply (*simp add*: *SecurityInvariant-withOffendingFlows.set-offending-flows-def*
SecurityInvariant-withOffendingFlows.is-offending-flows-min-set-def
SecurityInvariant-withOffendingFlows.is-offending-flows-def)
apply (*simp add*:*graph-ops*)
apply (*simp split*: *prod.split-asm prod.split*)
apply(*rule-tac x*=⦇ *nodes*={*vertex-1*, *vertex-2*}, *edges* = {(*vertex-1*,*vertex-2*)} ⦈ **in** *exI*, *simp*)
apply(*rule conjI*)
apply(*simp add*: *wf-graph-def*)
apply(*rule-tac x*=(λ *x*. *default-node-properties*)(*vertex-1* := ⦇*security-level* = *1*, *trusted* = *False* ⦈, *vertex-2* := ⦇*security-level* = *0*, *trusted* = *False* ⦈) **in** *exI*, *simp add*:*default-node-properties-def*)
apply(*rule-tac x*=*vertex-2* **in** *exI*, *simp*)
apply(*rule-tac x*={(*vertex-1*,*vertex-2*)} **in** *exI*, *simp*)
apply(*case-tac otherbot*)
apply *simp*
apply(*erule disjE*)
apply *force*

apply *fast*
done

4.11.1 ENF

definition *BLP-P* **where** *BLP-P* ≡ (λ*n1 n2*.(*if trusted n2 then True else security-level n1* ≤ *security-level n2*))
lemma *zero-default-candidate*: ∀ *nP e1 e2*. ¬ *BLP-P* (*nP e1*) (*nP e2*) ⟶ ¬ *BLP-P* (*nP e1*) *default-node-properties*
apply(*rule allI*)+
apply(*case-tac nP e1*)
apply(*case-tac nP e2*)
apply(*rename-tac privacy2 trusted2 more2*)
apply (*simp add*: *BLP-P-def default-node-properties-def*)
done
lemma *privacylevel-refl*: *BLP-P e e*
by(*simp-all add*: *BLP-P-def*)
lemma *BLP-ENF*: *SecurityInvariant-withOffendingFlows.sinvar-all-edges-normal-form sinvar BLP-P*
unfolding *SecurityInvariant-withOffendingFlows.sinvar-all-edges-normal-form-def*
unfolding *BLP-P-def*
by *simp*
lemma *BLP-ENF-refl*: *SecurityInvariant-withOffendingFlows.ENF-refl sinvar BLP-P*
unfolding *SecurityInvariant-withOffendingFlows.ENF-refl-def*
apply(*rule conjI*)
apply(*simp add*: *BLP-ENF*)
apply(*simp add*: *privacylevel-refl*)
done

definition *BLP-offending-set*:: *'v graph* ⇒ (*'v* ⇒ *node-config*) ⇒ (*'v* × *'v*) *set set* **where**
BLP-offending-set G nP = (*if sinvar G nP then*
{}
else
{ {*e* ∈ *edges G*. *case e of* (*e1*,*e2*) ⇒ ¬ *BLP-P* (*nP e1*) (*nP e2*)} })
lemma *BLP-offending-set*: *SecurityInvariant-withOffendingFlows.set-offending-flows sinvar* = *BLP-offending-set*
apply(*simp only*: *fun-eq-iff SecurityInvariant-withOffendingFlows.ENF-offending-set*[*OF BLP-ENF*] *BLP-offending-set-def*)
apply(*rule allI*)+
apply(*rename-tac G nP*)
apply(*auto*)
done

interpretation *BLPtrusted*: *SecurityInvariant-IFS*
where *default-node-properties* = *default-node-properties*
and *sinvar* = *sinvar*
rewrites *SecurityInvariant-withOffendingFlows.set-offending-flows sinvar* = *BLP-offending-set*
apply *unfold-locales*
apply(*rule ballI*)
apply (*rule-tac f*=*f* **in** *SecurityInvariant-withOffendingFlows.ENF-snds-refl-instance*[*OF BLP-ENF-refl zero-default-candidate*])
apply(*simp*)
apply(*simp*)
apply(*erule default-uniqueness-by-counterexample-IFS*)

apply(*fact BLP-def-unique*)
apply(*fact BLP-offending-set*)
done

lemma *TopoS-BLPtrusted*: *SecurityInvariant sinvar default-node-properties receiver-violation*
unfolding *receiver-violation-def* **by** *unfold-locales*

hide-type (**open**) *node-config*
hide-const (**open**) *sinvar-mono*

hide-const (**open**) *BLP-P*
hide-fact *BLP-def-unique zero-default-candidate privacylevel-refl BLP-ENF BLP-ENF-refl*

hide-const (**open**) *sinvar receiver-violation default-node-properties*

end
theory *Analysis-Tainting*
imports *SINVAR-Tainting SINVAR-BLPbasic*
SINVAR-TaintingTrusted SINVAR-BLPtrusted
begin

term *SINVAR-Tainting.sinvar*
term *SINVAR-BLPbasic.sinvar*

lemma *tainting-imp-blp-cutcard*: $\forall$ *ts v*. *nP v* = *ts* $\longrightarrow$ *finite ts* $\Longrightarrow$
SINVAR-Tainting.sinvar G nP $\Longrightarrow$ *SINVAR-BLPbasic.sinvar G* (λ*ts*. *card* (*ts* $\cap$ *X*)) $\circ$ *nP*)
apply(*simp add*: *SINVAR-Tainting.sinvar-def*)
apply(*clarify*, *rename-tac a b*)
apply(*erule-tac x*=(*a*,*b*) **in** *ballE*)
apply(*simp-all*)
apply(*subgoal-tac finite* (*nP a* $\cap$ *X*))
prefer *2* **subgoal using** *finite-Int* **by** *blast*
apply(*subgoal-tac finite* (*nP b* $\cap$ *X*))
prefer *2* **subgoal using** *finite-Int* **by** *blast*
using *card-mono* **by** (*metis Int-subset-iff order-refl subset-antisym*)

lemma *tainting-imp-blp-cutcard2*: *finite X* $\Longrightarrow$
SINVAR-Tainting.sinvar G nP $\Longrightarrow$ *SINVAR-BLPbasic.sinvar G* (λ*ts*. *card* (*ts* $\cap$ *X*)) $\circ$ *nP*)
apply(*simp add*: *SINVAR-Tainting.sinvar-def*)
apply(*clarify*, *rename-tac a b*)
apply(*erule-tac x*=(*a*,*b*) **in** *ballE*)
apply(*simp-all*)
apply(*subgoal-tac finite* (*nP a* $\cap$ *X*))
prefer *2* **subgoal using** *finite-Int* **by** *blast*
apply(*subgoal-tac finite* (*nP b* $\cap$ *X*))
prefer *2* **subgoal using** *finite-Int* **by** *blast*
using *card-mono* **by** (*metis Int-subset-iff order-refl subset-antisym*)

lemma $\forall$ *ts v*. *nP v* = *ts* $\longrightarrow$ *finite ts* $\Longrightarrow$
SINVAR-Tainting.sinvar G nP $\Longrightarrow$ *SINVAR-BLPbasic.sinvar G* (*card* $\circ$ *nP*)
apply(*drule*(*1*) *tainting-imp-blp-cutcard*[**where** *X*=*UNIV*])

by(*simp*)

lemma $\forall\, b \in snd\ `\ edges\ G.\ finite\ (nP\ b) \Longrightarrow$
  SINVAR-Tainting.sinvar G nP $\Longrightarrow$ *SINVAR-BLPbasic.sinvar G* (*card* $\circ$ *nP*)
apply(*simp add*: *SINVAR-Tainting.sinvar-def*)
apply(*clarify*, *rename-tac a b*)
apply(*erule-tac x*=(*a*,*b*) **in** *ballE*)
 apply(*simp-all*)
apply(*case-tac finite* (*nP a*))
 apply(*case-tac* [!] *finite* (*nP b*))
  using *card-mono* **apply** *blast*
 apply(*simp-all*)
done

One tainting invariant is equal to many BLP invariants. The BLP invariants are the projection of the tainting mapping for exactly one label

lemma *tainting-iff-blp*:
  defines *extract* $\equiv \lambda a\ ts.$ *if a* $\in$ *ts then 1*::*security-level else 0*::*security-level*
  shows *SINVAR-Tainting.sinvar G nP* $\longleftrightarrow$ ($\forall\, a.$ *SINVAR-BLPbasic.sinvar G* (*extract a* $\circ$ *nP*))
proof
  show*SINVAR-Tainting.sinvar G nP* $\Longrightarrow$ $\forall\, a.$ *SINVAR-BLPbasic.sinvar G* (*extract a* $\circ$ *nP*)
    apply(*simp add*: *extract-def*)
    apply(*safe*)
     apply *simp*
    apply(*simp add*: *SINVAR-Tainting.sinvar-def*)
    by *fast*
  next
    assume *blp*: $\forall\, a.$ *SINVAR-BLPbasic.sinvar G* (*extract a* $\circ$ *nP*)
    { fix *v1 v2*
      assume $*$: (*v1*,*v2*)$\in$*edges G*
      { fix *a*
        from *blp* $*$ **have** (*if a* $\in$ *nP v1 then 1*::*security-level else 0*) $\leq$ (*if a* $\in$ *nP v2 then 1 else 0*)
          unfolding *extract-def*
          apply(*simp*)
          apply(*erule-tac x*=*a* **in** *allE*)
          apply(*erule-tac x*=(*v1*, *v2*) **in** *ballE*)
           apply(*simp-all*)
          apply(*simp split*: *if-split-asm*)
          done
        hence *a* $\in$ *nP v1* $\Longrightarrow$ *a* $\in$ *nP v2* **by**(*simp split*: *if-split-asm*)
      }
      from *this* **have** *nP v1* $\subseteq$ *nP v2* **by** *auto*
    }
    thus *SINVAR-Tainting.sinvar G nP* **unfolding** *SINVAR-Tainting.sinvar-def* **by** *blast*
qed

If the labels are finite, the above can be generalized to arbitrary subsets of tainting labels.

lemma *tainting-iff-blp-extended*:
  defines *extract* $\equiv \lambda A\ ts.$ *card* (*A* $\cap$ *ts*)
  assumes *finite*: $\forall\, ts\ v.$ *nP v* = *ts* $\longrightarrow$ *finite ts*
  shows *SINVAR-Tainting.sinvar G nP* $\longleftrightarrow$ ($\forall\, A.$ *SINVAR-BLPbasic.sinvar G* (*extract A* $\circ$ *nP*))
proof

```
  show SINVAR-Tainting.sinvar G nP ⟹ ∀ A. SINVAR-BLPbasic.sinvar G (extract A ∘ nP)
    apply(simp add: extract-def)
    apply(safe)
    apply(simp add: SINVAR-Tainting.sinvar-def)
    apply(rename-tac A a b)
    apply(subgoal-tac finite (A ∩ nP a))
     prefer 2 subgoal using finite by blast
    apply(rule card-mono)
     apply(simp add: finite; fail)
    by blast
  next
    assume blp: ∀ A. SINVAR-BLPbasic.sinvar G (extract A ∘ nP)
    { fix v1 v2
      assume ∗: (v1,v2)∈edges G
      { fix A
        from blp ∗ have card (A ∩ nP v1) ≤ card (A ∩ nP v2)
          unfolding extract-def
          apply(clarsimp)
          apply(erule-tac x=A in allE)
          apply(erule-tac x=(v1, v2) in ballE)
           by(simp-all)
      }
      from this finite card-seteq have nP v1 ⊆ nP v2 by (metis Int-absorb Int-lower1 inf.orderI)
    }
    thus SINVAR-Tainting.sinvar G nP unfolding SINVAR-Tainting.sinvar-def by blast
qed
```

Translated to the Bell LaPadula model with trust: security level is the number of tainted minus the untainted things We set the Trusted flag if a machine untaints things.

```
lemma ∀ ts v. nP v = ts ⟶ finite (taints ts) ⟹
  SINVAR-TaintingTrusted.sinvar G nP ⟹
    SINVAR-BLPtrusted.sinvar G ((λ ts. (|security-level = card (taints ts − untaints ts), trusted =
(untaints ts ≠ {})|) ) ∘ nP)
apply(simp add: SINVAR-TaintingTrusted.sinvar-def)
apply(clarify, rename-tac a b)
apply(erule-tac x=(a,b) in ballE)
 apply(simp-all)
apply(subgoal-tac finite (taints (nP a) − untaints (nP a)))
 prefer 2 subgoal by blast
apply(rule card-mono)
 by blast+

lemma tainting-iff-blp-trusted:
  defines project ≡ λa ts. (|
      security-level =
        if
          a ∈ (taints ts − untaints ts)
        then
          1::security-level
        else
          0::security-level
      , trusted = a ∈ untaints ts|)
  shows SINVAR-TaintingTrusted.sinvar G nP ⟷ (∀ a. SINVAR-BLPtrusted.sinvar G (project a ∘
nP))
```

unfolding *project-def*
apply(*rule iffI*)
 subgoal
 apply(*simp add*: *SINVAR-TaintingTrusted.sinvar-def*)
 apply(*clarify*, *rename-tac a b*)
 apply(*erule-tac x*=(*a*,*b*) **in** *ballE*)
  apply(*simp-all*)
 by *blast*
apply(*simp*)
apply(*simp add*: *SINVAR-TaintingTrusted.sinvar-def*)
apply(*clarify*, *rename-tac a b taintlabel*)
apply(*erule-tac x*=*taintlabel* **in** *allE*)
apply(*erule-tac x*=(*a*,*b*) **in** *ballE*)
 apply(*simp-all*)
apply(*simp split*: *if-split-asm*)
using *taints-wellformedness* **by** *blast*

If the labels are finite, the above can be generalized to arbitrary subsets of tainting labels.

lemma *tainting-iff-blp-trusted-extended*:
  defines *project* ≡ λ*A ts*.
      (|
         security-level = *card* (*A* ∩ (*taints ts* − *untaints ts*))
       , *trusted* = (*A* ∩ *untaints ts*) ≠ {}
      |)
  assumes *finite*: ∀ *ts v*. *nP v* = *ts* ⟶ *finite* (*taints ts*)
  shows *SINVAR-TaintingTrusted.sinvar G nP* ⟷ (∀ *A*. *SINVAR-BLPtrusted.sinvar G* (*project A* ∘ *nP*))
unfolding *project-def*
apply(*rule iffI*)
 subgoal
 apply(*simp add*: *SINVAR-TaintingTrusted.sinvar-def*)
 apply(*clarify*, *rename-tac a b*)
 apply(*erule-tac x*=(*a*,*b*) **in** *ballE*)
  apply(*simp-all*)
 apply(*rule card-mono*)
  using *finite* **apply** *blast*
 by *blast*
apply(*simp*)
apply(*simp add*: *SINVAR-TaintingTrusted.sinvar-def*)
apply(*clarify*, *rename-tac a b taintlabel*)
apply(*erule-tac x*={*taintlabel*} **in** *allE*)
apply(*erule-tac x*=(*a*,*b*) **in** *ballE*)
 apply(*simp-all*)
apply(*simp split*: *if-split-asm*)
 using *taints-wellformedness* **apply** *blast*
using *Diff-insert-absorb* **by** *fastforce*

end
theory *TopoS-Interface-impl*
imports *Lib/FiniteGraph Lib/FiniteListGraph TopoS-Interface TopoS-Helper*
begin

5 Executable Implementation with Lists

Correspondence List Implementation and set Specification

5.1 Abstraction from list implementation to set specification

Nomenclature: *-spec* is the specification, *-impl* the corresponding implementation.

-spec and *-impl* only need to comply for *wf-graph*s. We will always require the stricter *wf-list-graph*, which implies *wf-graph*.

lemma *wf-list-graph G* $\Longrightarrow$ *wf-graph* (*list-graph-to-graph G*)

locale *TopoS-List-Impl* =
fixes *default-node-properties* :: *'a* ($\bot$)
and *sinvar-spec*::(*'v*::*vertex*) *graph* $\Rightarrow$ (*'v*::*vertex* $\Rightarrow$ *'a*) $\Rightarrow$ *bool*
and *sinvar-impl*::(*'v*::*vertex*) *list-graph* $\Rightarrow$ (*'v*::*vertex* $\Rightarrow$ *'a*) $\Rightarrow$ *bool*
and *receiver-violation* :: *bool*
and *offending-flows-impl*::(*'v*::*vertex*) *list-graph* $\Rightarrow$ (*'v* $\Rightarrow$ *'a*) $\Rightarrow$ (*'v* $\times$ *'v*) *list list*
and *node-props-impl*::(*'v*::*vertex*, *'a*) *TopoS-Params* $\Rightarrow$ (*'v* $\Rightarrow$ *'a*)
and *eval-impl*::(*'v*::*vertex*) *list-graph* $\Rightarrow$ (*'v*, *'a*) *TopoS-Params* $\Rightarrow$ *bool*
assumes
spec: *SecurityInvariant sinvar-spec default-node-properties receiver-violation* — specification is valid
and
sinvar-spec-impl: *wf-list-graph G* $\Longrightarrow$
(*sinvar-spec* (*list-graph-to-graph G*) *nP*) = (*sinvar-impl G nP*)
and
offending-flows-spec-impl: *wf-list-graph G* $\Longrightarrow$
(*SecurityInvariant-withOffendingFlows.set-offending-flows sinvar-spec* (*list-graph-to-graph G*) *nP*) =
set'set (*offending-flows-impl G nP*)
and
node-props-spec-impl:
SecurityInvariant.node-props-formaldef default-node-properties P = *node-props-impl P*
and
eval-spec-impl:
(*distinct* (*nodesL G*) $\wedge$ *distinct* (*edgesL G*) $\wedge$
SecurityInvariant.eval sinvar-spec default-node-properties (*list-graph-to-graph G*) *P*) =
(*eval-impl G P*)

5.2 Security Invariants Packed

We pack all necessary functions and properties of a security invariant in a struct-like data structure.

record (*'v*::*vertex*, *'a*) *TopoS-packed* =
nm-name :: *string*
nm-receiver-violation :: *bool*
nm-default :: *'a*
nm-sinvar::(*'v*::*vertex*) *list-graph* $\Rightarrow$ (*'v* $\Rightarrow$ *'a*) $\Rightarrow$ *bool*
nm-offending-flows::(*'v*::*vertex*) *list-graph* $\Rightarrow$ (*'v* $\Rightarrow$ *'a*) $\Rightarrow$ (*'v* $\times$ *'v*) *list list*
nm-node-props::(*'v*::*vertex*, *'a*) *TopoS-Params* $\Rightarrow$ (*'v* $\Rightarrow$ *'a*)
nm-eval::(*'v*::*vertex*) *list-graph* $\Rightarrow$ (*'v*, *'a*)*TopoS-Params* $\Rightarrow$ *bool*

The packed list implementation must comply with the formal definition.

locale *TopoS-modelLibrary* =
fixes m :: ($'v$::*vertex*, $'a$) *TopoS-packed* — concrete model implementation
and *sinvar-spec*::($'v$::*vertex*) *graph* ⇒ ($'v$::*vertex* ⇒ $'a$) ⇒ *bool* — specification
assumes
name-not-empty: *length* (*nm-name m*) > *0*
and
impl-spec: *TopoS-List-Impl*
(*nm-default m*)
sinvar-spec
(*nm-sinvar m*)
(*nm-receiver-violation m*)
(*nm-offending-flows m*)
(*nm-node-props m*)
(*nm-eval m*)

5.3 Helpful Lemmata

show that *sinvar* complies

lemma *TopoS-eval-impl-proofrule*:
assumes *inst*: *SecurityInvariant sinvar-spec default-node-properties receiver-violation*
assumes *ev*: ⋀*nP*. *wf-list-graph G* ⟹ *sinvar-spec* (*list-graph-to-graph G*) *nP* = *sinvar-impl G nP*
shows
(*distinct* (*nodesL G*) ∧ *distinct* (*edgesL G*) ∧
SecurityInvariant.eval sinvar-spec default-node-properties (*list-graph-to-graph G*) *P*) =
(*wf-list-graph G* ∧ *sinvar-impl G* (*SecurityInvariant.node-props default-node-properties P*))
proof (*cases wf-list-graph G*)
case *True*
hence *sinvar-spec* (*list-graph-to-graph G*) (*SecurityInvariant.node-props default-node-properties P*)
=
sinvar-impl G (*SecurityInvariant.node-props default-node-properties P*)
using *ev* **by** *blast*

with *inst* **show** *?thesis*
unfolding *wf-list-graph-def*
by (*simp add*: *wf-list-graph-iff-wf-graph SecurityInvariant.eval-def*)
next
case *False*
hence (*distinct* (*nodesL G*) ∧ *distinct* (*edgesL G*) ∧ *wf-list-graph-axioms G*) = *False*
unfolding *wf-list-graph-def* **by** *blast*
with *False* **show** *?thesis*
unfolding *SecurityInvariant.eval-def*[*OF inst*]
by (*fastforce simp*: *wf-list-graph-iff-wf-graph*)
qed

5.4 Helper lemmata

Provide *sinvar* function and get back a function that computes the list of offending flows
Exponential time!

definition *Generic-offending-list*:: ($'v$ *list-graph* ⇒ ($'v$ ⇒ $'a$) ⇒ *bool*)⇒ $'v$ *list-graph* ⇒ ($'v$ ⇒ $'a$)
⇒ ($'v$ × $'v$) *list list* **where**
Generic-offending-list sinvar G nP = [*f* ← (*subseqs* (*edgesL G*)).

$(\neg\ sinvar\ G\ nP \wedge sinvar\ (FiniteListGraph.delete\text{-}edges\ G\ f)\ nP) \wedge$
$(\forall (e1,\ e2) \in set\ f.\ \neg\ sinvar\ (add\text{-}edge\ e1\ e2\ (FiniteListGraph.delete\text{-}edges\ G\ f))\ nP)]$

proof rule: if *sinvar* complies, *Generic-offending-list* complies

lemma *Generic-offending-list-correct*:
assumes *valid*: *wf-list-graph G*
assumes *spec-impl*: $\bigwedge G\ nP.$ *wf-list-graph G* $\Longrightarrow$ *sinvar-spec* (*list-graph-to-graph G*) *nP* = *sinvar-impl G nP*
shows *SecurityInvariant-withOffendingFlows.set-offending-flows sinvar-spec* (*list-graph-to-graph G*) *nP* =
set‘set(*Generic-offending-list sinvar-impl G nP*)
proof −
have $\bigwedge$ *P G. set ‘* $\{x \in set\ (subseqs\ (edgesL\ G)).\ P\ G\ (set\ x)\}$ = $\{x \in set\ ‘\ set\ (subseqs\ (edgesL\ G)).\ P\ G\ (x)\}$
by *fastforce*
hence *subset-subseqs-filter*: $\bigwedge$ *G P.* $\{f.\ f \subseteq edges\ (list\text{-}graph\text{-}to\text{-}graph\ G) \wedge P\ G\ f\}$
= *set ‘ set* [*f*←*subseqs* (*edgesL G*) . *P G* (*set f*)]
unfolding *list-graph-to-graph-def*
by (*auto simp*: *subseqs-powset*)

from *valid delete-edges-wf* **have** $\forall f.$ *wf-list-graph*(*FiniteListGraph.delete-edges G f*) **by** *fast*
with *spec-impl*[*symmetric*] *FiniteListGraph.delete-edges-correct*[*of G*] **have** *impl-spec-delete*:
$\forall f.$ *sinvar-impl* (*FiniteListGraph.delete-edges G f*) *nP* =
sinvar-spec (*FiniteGraph.delete-edges* (*list-graph-to-graph G*) (*set f*)) *nP* **by** *simp*

from *spec-impl*[*OF valid*, *symmetric*] **have** *impl-spec-not*:
(¬ *sinvar-impl G nP*) = (¬ *sinvar-spec* (*list-graph-to-graph G*) *nP*) **by** *auto*

from *spec-impl*[*symmetric*, *OF FiniteListGraph.add-edge-wf*[*OF FiniteListGraph.delete-edges-wf*[*OF valid*]]] **have** *impl-spec-allE*:
∀ *e1 e2 E. sinvar-impl* (*FiniteListGraph.add-edge e1 e2* (*FiniteListGraph.delete-edges G E*)) *nP* =
sinvar-spec (*list-graph-to-graph* (*FiniteListGraph.add-edge e1 e2* (*FiniteListGraph.delete-edges G E*))) *nP* **by** *simp*

have *list-graph*: $\bigwedge$ *e1 e2 G f.* (*list-graph-to-graph* (*FiniteListGraph.add-edge e1 e2* (*FiniteListGraph.delete-edges G f*))) =
(*FiniteGraph.add-edge e1 e2* (*FiniteGraph.delete-edges* (*list-graph-to-graph G*) (*set f*)))
by(*simp add*: *FiniteListGraph.add-edge-correct FiniteListGraph.delete-edges-correct*)

show *?thesis*
unfolding *SecurityInvariant-withOffendingFlows.set-offending-flows-def*
SecurityInvariant-withOffendingFlows.is-offending-flows-min-set-def
SecurityInvariant-withOffendingFlows.is-offending-flows-def
Generic-offending-list-def
apply(*subst impl-spec-delete*)
apply(*subst impl-spec-not*)
apply(*subst impl-spec-allE*)
apply(*subst list-graph*)
apply(*rule subset-subseqs-filter*)
done
qed

lemma *all-edges-list-I*: *P* (*list-graph-to-graph G*) = *Pl G* $\Longrightarrow$

```
  (∀(e1, e2)∈ (edges (list-graph-to-graph G)). P (list-graph-to-graph G) e1 e2) = (∀(e1, e2)∈set
(edgesL G). Pl G e1 e2)
  unfolding list-graph-to-graph-def
  by simp

  lemma all-nodes-list-I: P (list-graph-to-graph G) = Pl G ⟹
    (∀ n ∈ (nodes (list-graph-to-graph G)). P (list-graph-to-graph G) n) = (∀  n ∈set (nodesL G). Pl
G n)
  unfolding list-graph-to-graph-def
  by simp
```

```
  fun minimalize-offending-overapprox :: ('v list-graph ⇒ bool) ⇒
    ('v × 'v) list ⇒ ('v × 'v) list ⇒ 'v list-graph ⇒ ('v × 'v) list where
  minimalize-offending-overapprox - [] keep - = keep |
  minimalize-offending-overapprox m (f#fs) keep G = (if m (delete-edges G (fs@keep)) then
      minimalize-offending-overapprox m fs keep G
    else
      minimalize-offending-overapprox m fs (f#keep) G
    )

  thm minimalize-offending-overapprox-boundnP
  lemma minimalize-offending-overapprox-spec-impl:
    assumes valid: wf-list-graph (G::'v::vertex list-graph)
        and spec-impl: ⋀G nP::('v ⇒ 'a). wf-list-graph G ⟹ sinvar-spec (list-graph-to-graph G) nP
= sinvar-impl G nP
    shows minimalize-offending-overapprox (λG. sinvar-impl G nP) fs keeps G =
        TopoS-withOffendingFlows.minimalize-offending-overapprox (λG. sinvar-spec G nP) fs keeps
(list-graph-to-graph G)
    apply(subst minimalize-offending-overapprox-boundnP)
    using valid spec-impl apply(induction fs arbitrary: keeps)
      apply(simp add: SecurityInvariant-withOffendingFlows.minimalize-offending-overapprox.simps;
fail)
    apply(simp add: SecurityInvariant-withOffendingFlows.minimalize-offending-overapprox.simps)
    apply (metis FiniteListGraph.delete-edges-wf delete-edges-list-set list-graph-correct(5))
    done
```

With *TopoS-Interface-impl.minimalize-offending-overapprox*, we can get one offending flow

```
  lemma minimalize-offending-overapprox-gives-some-offending-flow:
    assumes wf: wf-list-graph G
        and NetModelLib: TopoS-modelLibrary m sinvar-spec
        and violation: ¬ (nm-sinvar m) G nP
    shows set (minimalize-offending-overapprox (λG. (nm-sinvar m) G nP) (edgesL G) [] G) ∈
          SecurityInvariant-withOffendingFlows.set-offending-flows sinvar-spec (list-graph-to-graph G)
nP
    proof −
      from wf have wfG: wf-graph (list-graph-to-graph G)
        by (simp add: wf-list-graph-def wf-list-graph-iff-wf-graph)
      from wf have dist-edges: distinct (edgesL G) by (simp add: wf-list-graph-def)
```

let *?spec-algo=TopoS-withOffendingFlows.minimalize-offending-overapprox*
(λG. sinvar-spec G nP) (edgesL G) [] (list-graph-to-graph G)

note *spec=TopoS-List-Impl.spec[OF TopoS-modelLibrary.impl-spec[OF NetModelLib]]*

from *spec* **have** *spec-prelim*: *SecurityInvariant-preliminaries sinvar-spec*
by(*simp add*: *SecurityInvariant-def*)
from *spec-prelim SecurityInvariant-preliminaries.sinvar-monoI* **have** *mono*:
SecurityInvariant-withOffendingFlows.sinvar-mono sinvar-spec **by** *blast*

from *spec-prelim* **have** *empty-edges*: *sinvar-spec (|nodes = set (nodesL G), edges = {}|) nP*
using *SecurityInvariant-preliminaries.defined-offending*
SecurityInvariant-withOffendingFlows.sinvar-mono-imp-sinvar-mono
SecurityInvariant-withOffendingFlows.valid-empty-edges-iff-exists-offending-flows
mono empty-subsetI graph.simps(1)
list-graph-to-graph-def local.wf wf-list-graph-def wf-list-graph-iff-wf-graph
by (*metis*)

have *spec-impl*: *wf-list-graph G ⟹ sinvar-spec (list-graph-to-graph G) nP = (nm-sinvar m) G nP* **for** *G nP*
using *NetModelLib TopoS-List-Impl.sinvar-spec-impl TopoS-modelLibrary.impl-spec* **by** *fastforce*

from *minimalize-offending-overapprox-spec-impl[OF wf] spec-impl* **have** *alog-spec*:
minimalize-offending-overapprox (λG. (nm-sinvar m) G nP) fs keeps G =
TopoS-withOffendingFlows.minimalize-offending-overapprox (λG. sinvar-spec G nP) fs keeps (list-graph-to-graph G)
for *fs keeps* **by** *blast*

from *spec-impl violation* **have**
SecurityInvariant-withOffendingFlows.is-offending-flows sinvar-spec (set (edgesL G)) (list-graph-to-graph G) nP
apply(*simp add*: *SecurityInvariant-withOffendingFlows.is-offending-flows-def*)
apply(*intro conjI*)
apply (*simp add*: *local.wf*; *fail*)
apply(*simp add*: *FiniteGraph.delete-edges-simp2 list-graph-to-graph-def*)
apply(*simp add*: *empty-edges*)
done
hence *goal*: *SecurityInvariant-withOffendingFlows.is-offending-flows-min-set sinvar-spec*
(*set ?spec-algo*) (*list-graph-to-graph G*) *nP*
apply(*subst minimalize-offending-overapprox-boundnP*)
apply(*rule SecurityInvariant-withOffendingFlows.is-offending-flows-min-set-minimalize-offending-overapprox[OF mono wfG - - dist-edges]*)
apply(*simp add*: *list-graph-to-graph-def*)+
done

from *SecurityInvariant-withOffendingFlows.minimalize-offending-overapprox-subseteq-input[of sinvar-spec (edgesL G) []]* **have** *subset-edges*:
set ?spec-algo ⊆ edges (list-graph-to-graph G)
apply(*subst minimalize-offending-overapprox-boundnP*)
by(*simp add*: *list-graph-to-graph-def*)

from *goal* **show** *?thesis*

by(*simp add*: *SecurityInvariant-withOffendingFlows.set-offending-flows-def alog-spec subset-edges*)
qed

6 Security Invariant Library

end
theory *SINVAR-BLPbasic-impl*
imports *SINVAR-BLPbasic ../TopoS-Interface-impl*
begin

6.0.1 SecurityInvariant BLPbasic List Implementation

code-identifier code-module *SINVAR-BLPbasic-impl* => (*Scala*) *SINVAR-BLPbasic*

fun *sinvar* :: *'v list-graph* ⇒ (*'v* ⇒ *security-level*) ⇒ *bool* **where**
sinvar G nP = (∀ (*e1*,*e2*) ∈ *set* (*edgesL G*). (*nP e1*) ≤ (*nP e2*))

definition *BLP-offending-list*:: *'v list-graph* ⇒ (*'v* ⇒ *security-level*) ⇒ (*'v* × *'v*) *list list* **where**
BLP-offending-list G nP = (*if sinvar G nP then*
[]
else
[[*e* ← *edgesL G*. *case e of* (*e1*,*e2*) ⇒ (*nP e1*) > (*nP e2*)]])

definition *NetModel-node-props P* = (λ *i*. (*case* (*node-properties P*) *i of Some property* ⇒ *property*
| *None* ⇒ *SINVAR-BLPbasic.default-node-properties*))
lemma[*code-unfold*]: *SecurityInvariant.node-props SINVAR-BLPbasic.default-node-properties P* = *NetModel-node-props P*
apply(*simp add*: *NetModel-node-props-def*)
done

definition *BLP-eval G P* = (*wf-list-graph G* ∧
sinvar G (*SecurityInvariant.node-props SINVAR-BLPbasic.default-node-properties P*))

interpretation *BLPbasic-impl*:*TopoS-List-Impl*
where *default-node-properties*=*SINVAR-BLPbasic.default-node-properties*
and *sinvar-spec*=*SINVAR-BLPbasic.sinvar*
and *sinvar-impl*=*sinvar*
and *receiver-violation*=*SINVAR-BLPbasic.receiver-violation*
and *offending-flows-impl*=*BLP-offending-list*
and *node-props-impl*=*NetModel-node-props*
and *eval-impl*=*BLP-eval*
apply(*unfold TopoS-List-Impl-def*)
apply(*rule conjI*)
apply(*simp add*: *TopoS-BLPBasic*)
apply(*simp add*: *list-graph-to-graph-def*; *fail*)
apply(*rule conjI*)
apply(*simp add*: *list-graph-to-graph-def*)
apply(*simp add*: *list-graph-to-graph-def BLP-offending-set BLP-offending-set-def BLP-offending-list-def*; *fail*)
apply(*rule conjI*)
apply(*simp only*: *NetModel-node-props-def*)

apply(*metis BLPbasic.node-props.simps BLPbasic.node-props-eq-node-props-formaldef*)
apply(*simp only*: *BLP-eval-def*)
apply(*simp add*: *TopoS-eval-impl-proofrule*[*OF TopoS-BLPBasic*])
apply(*simp add*: *list-graph-to-graph-def*)
done

6.0.2 BLPbasic packing

definition *SINVAR-LIB-BLPbasic* :: ('*v*::*vertex*, *security-level*) *TopoS-packed* **where**
SINVAR-LIB-BLPbasic ≡
(| *nm-name* = ''*BLPbasic*'',
nm-receiver-violation = *SINVAR-BLPbasic.receiver-violation*,
nm-default = *SINVAR-BLPbasic.default-node-properties*,
nm-sinvar = *sinvar*,
nm-offending-flows = *BLP-offending-list*,
nm-node-props = *NetModel-node-props*,
nm-eval = *BLP-eval*
|)
interpretation *SINVAR-LIB-BLPbasic-interpretation*: *TopoS-modelLibrary SINVAR-LIB-BLPbasic*

SINVAR-BLPbasic.sinvar
apply(*unfold TopoS-modelLibrary-def SINVAR-LIB-BLPbasic-def*)
apply(*rule conjI*)
apply(*simp*)
apply(*simp*)
by(*unfold-locales*)

6.0.3 Example

definition *fabNet* :: *string list-graph* **where**
fabNet ≡ (| *nodesL* = [''*Statistics*'', ''*SensorSink*'', ''*PresenceSensor*'', ''*Webcam*'', ''*TempSensor*'', ''*FireSesnsor*'',
''*MissionControl1*'', ''*MissionControl2*'', ''*Watchdog*'', ''*Bot1*'', ''*Bot2*''],
edgesL =[(''*PresenceSensor*'', ''*SensorSink*''), (''*Webcam*'', ''*SensorSink*''),
(''*TempSensor*'', ''*SensorSink*''), (''*FireSesnsor*'', ''*SensorSink*''),
(''*SensorSink*'', ''*Statistics*''),
(''*MissionControl1*'', ''*Bot1*''), (''*MissionControl1*'', ''*Bot2*''),
(''*MissionControl2*'', ''*Bot2*''),
(''*Watchdog*'', ''*Bot1*''), (''*Watchdog*'', ''*Bot2*'')] |)
value *wf-list-graph fabNet*

definition *sensorProps-try1* :: *string* ⇒ *security-level* **where**
sensorProps-try1 ≡ (λ *n*. *SINVAR-BLPbasic.default-node-properties*)(''*PresenceSensor*'' := *2*, ''*Webcam*'' := *3*)
value *BLP-offending-list fabNet sensorProps-try1*
value *sinvar fabNet sensorProps-try1*

definition *sensorProps-try2* :: *string* ⇒ *security-level* **where**
sensorProps-try2 ≡ (λ *n*. *SINVAR-BLPbasic.default-node-properties*)(''*PresenceSensor*'' := *2*, ''*Webcam*'' := *3*,
''*SensorSink*'' := *3*)
value *BLP-offending-list fabNet sensorProps-try2*
value *sinvar fabNet sensorProps-try2*

definition *sensorProps-try3* :: *string* ⇒ *security-level* **where**
sensorProps-try3 ≡ (λ *n*. *SINVAR-BLPbasic.default-node-properties*)(*''PresenceSensor''* := *2*, *''Webcam''* := *3*,
''SensorSink'' := *3*, *''Statistics''* := *3*)
value *BLP-offending-list fabNet sensorProps-try3*
value *sinvar fabNet sensorProps-try3*

Another parameter set for confidential controlling information

definition *sensorProps-conf* :: *string* ⇒ *security-level* **where**
sensorProps-conf ≡ (λ *n*. *SINVAR-BLPbasic.default-node-properties*)(*''MissionControl1''* := *1*, *''MissionControl2''* := *2*,
''Bot1'' := *1*, *''Bot2''* := *2*)
value *BLP-offending-list fabNet sensorProps-conf*
value *sinvar fabNet sensorProps-conf*

Complete example:

definition *sensorProps-NMParams-try3* :: (*string*, *nat*) *TopoS-Params* **where**
sensorProps-NMParams-try3 ≡ (| *node-properties* = [*''PresenceSensor''* ↦ *2*,
''Webcam'' ↦ *3*,
''SensorSink'' ↦ *3*,
''Statistics'' ↦ *3*] |)
value *BLP-eval fabNet sensorProps-NMParams-try3*

export-code *SINVAR-LIB-BLPbasic* **checking** *Scala*

hide-const (**open**) *NetModel-node-props BLP-offending-list BLP-eval*

hide-const (**open**) *sinvar*

end
theory *SINVAR-Subnets*
imports../*TopoS-Helper*
begin

6.1 SecurityInvariant Subnets

If unsure, maybe you should look at `SINVAR_SubnetsInGW.thy`

datatype *subnets* = *Subnet nat* | *BorderRouter nat* | *Unassigned*

definition *default-node-properties* :: *subnets*
where *default-node-properties* ≡ *Unassigned*

fun *allowed-subnet-flow* :: *subnets* ⇒ *subnets* ⇒ *bool* **where**
allowed-subnet-flow (*Subnet s1*) (*Subnet s2*) = (*s1* = *s2*) |
allowed-subnet-flow (*Subnet s1*) (*BorderRouter s2*) = (*s1* = *s2*) |
allowed-subnet-flow (*Subnet s1*) *Unassigned* = *True* |
allowed-subnet-flow (*BorderRouter s1*) (*Subnet s2*) = *False* |
allowed-subnet-flow (*BorderRouter s1*) *Unassigned* = *True* |
allowed-subnet-flow (*BorderRouter s1*) (*BorderRouter s2*) = *True* |
allowed-subnet-flow Unassigned Unassigned = *True* |
allowed-subnet-flow Unassigned - = *False*

fun *sinvar* :: *'v graph* ⇒ (*'v* ⇒ *subnets*) ⇒ *bool* **where**
sinvar G nP = (∀ (*e1*,*e2*) ∈ *edges G*. *allowed-subnet-flow* (*nP e1*) (*nP e2*))

definition *receiver-violation* :: *bool* **where** *receiver-violation* = *False*

6.1.1 Preliminaries

lemma *sinvar-mono*: *SecurityInvariant-withOffendingFlows.sinvar-mono sinvar*
apply(*simp only*: *SecurityInvariant-withOffendingFlows.sinvar-mono-def*)
apply(*clarify*)
by *auto*

interpretation *SecurityInvariant-preliminaries*
where *sinvar* = *sinvar*
apply *unfold-locales*
apply(*frule-tac finite-distinct-list*[*OF wf-graph.finiteE*])
apply(*erule-tac exE*)
apply(*rename-tac list-edges*)
apply(*rule-tac ff*=*list-edges* **in** *SecurityInvariant-withOffendingFlows.mono-imp-set-offending-flows-not-empty*[*OF sinvar-mono*])
apply(*auto*)[*6*]
apply(*auto simp add*: *SecurityInvariant-withOffendingFlows.is-offending-flows-def graph-ops*)[*1*]
apply(*fact SecurityInvariant-withOffendingFlows.sinvar-mono-imp-is-offending-flows-mono*[*OF sinvar-mono*])
done

6.1.2 ENF

lemma *Unassigned-only-to-Unassigned*: *allowed-subnet-flow Unassigned e2* ⟷ *e2* = *Unassigned*
by(*case-tac e2*, *simp-all*)
lemma *All-to-Unassigned*: ∀ *e1*. *allowed-subnet-flow e1 Unassigned*
by (*rule allI*, *case-tac e1*, *simp-all*)
lemma *Unassigned-default-candidate*: ∀ *nP e1 e2*. ¬ *allowed-subnet-flow* (*nP e1*) (*nP e2*) ⟶ ¬ *allowed-subnet-flow Unassigned* (*nP e2*)
apply(*rule allI*)+
apply(*case-tac nP e2*)
apply *simp*
apply *simp*
by(*simp add*: *All-to-Unassigned*)
lemma *allowed-subnet-flow-refl*: ∀ *e*. *allowed-subnet-flow e e*
by(*rule allI*, *case-tac e*, *simp-all*)
lemma *Subnets-ENF*: *SecurityInvariant-withOffendingFlows.sinvar-all-edges-normal-form sinvar allowed-subnet-flow*
unfolding *SecurityInvariant-withOffendingFlows.sinvar-all-edges-normal-form-def*
by *simp*
lemma *Subnets-ENF-refl*: *SecurityInvariant-withOffendingFlows.ENF-refl sinvar allowed-subnet-flow*
unfolding *SecurityInvariant-withOffendingFlows.ENF-refl-def*
apply(*rule conjI*)
apply(*simp add*: *Subnets-ENF*)
apply(*simp add*: *allowed-subnet-flow-refl*)
done

definition *Subnets-offending-set*:: *'v graph* ⇒ (*'v* ⇒ *subnets*) ⇒ (*'v* × *'v*) *set set* **where**

Subnets-offending-set G nP = (*if sinvar G nP then*
{}
else
{ {*e* ∈ *edges G*. *case e of* (*e1*,*e2*) ⇒ ¬ *allowed-subnet-flow* (*nP e1*) (*nP e2*)} })
lemma *Subnets-offending-set*:
SecurityInvariant-withOffendingFlows.set-offending-flows sinvar = *Subnets-offending-set*
apply(*simp only*: *fun-eq-iff ENF-offending-set*[*OF Subnets-ENF*] *Subnets-offending-set-def*)
apply(*rule allI*)+
apply(*rename-tac G nP*)
apply(*auto*)
done

interpretation *Subnets*: *SecurityInvariant-ACS*
where *default-node-properties* = *SINVAR-Subnets.default-node-properties*
and *sinvar* = *SINVAR-Subnets.sinvar*
rewrites *SecurityInvariant-withOffendingFlows.set-offending-flows sinvar* = *Subnets-offending-set*
unfolding *SINVAR-Subnets.default-node-properties-def*
apply *unfold-locales*
apply(*rule ballI*)
apply (*rule SecurityInvariant-withOffendingFlows.ENF-fsts-refl-instance*[*OF Subnets-ENF-refl Unassigned-default-ca*
apply(*simp-all*)[*2*]
apply(*erule default-uniqueness-by-counterexample-ACS*)
apply (*simp add*: *SecurityInvariant-withOffendingFlows.set-offending-flows-def*
SecurityInvariant-withOffendingFlows.is-offending-flows-min-set-def
SecurityInvariant-withOffendingFlows.is-offending-flows-def)
apply (*simp add*:*graph-ops*)
apply (*simp split*: *prod.split-asm prod.split*)
apply(*rule-tac x*=(| *nodes*={*vertex-1*,*vertex-2*}, *edges* = {(*vertex-1*,*vertex-2*)} |) **in** *exI*, *simp*)
apply(*rule conjI*)
apply(*simp add*: *wf-graph-def*)
apply(*case-tac otherbot*, *simp-all*)
apply(*rename-tac mysubnetcase*)
apply(*rule-tac x*=(λ *x*. *Unassigned*)(*vertex-1* := *Unassigned*, *vertex-2* := *BorderRouter mysubnet-case*) **in** *exI*, *simp*)
apply(*rule-tac x*=*vertex-1* **in** *exI*, *simp*)
apply(*rule-tac x*={(*vertex-1*,*vertex-2*)} **in** *exI*, *simp*)
apply(*rule-tac x*=(λ *x*. *Unassigned*)(*vertex-1* := *Unassigned*, *vertex-2* := *BorderRouter whatever*) **in** *exI*, *simp*)
apply(*rule-tac x*=*vertex-1* **in** *exI*, *simp*)
apply(*rule-tac x*={(*vertex-1*,*vertex-2*)} **in** *exI*, *simp*)
apply(*fact Subnets-offending-set*)
done

lemma *TopoS-Subnets*: *SecurityInvariant sinvar default-node-properties receiver-violation*
unfolding *receiver-violation-def* **by** *unfold-locales*

6.1.3 Analysis

lemma *violating-configurations*: ¬ *sinvar G nP* ⟹
∃ (*e1*, *e2*) ∈ *edges G*. *nP e1* = *Unassigned* ∨ (∃ *s1*. *nP e1* = *Subnet s1*) ∨ (∃ *s1*. *nP e1* = *BorderRouter s1*)
apply *simp*

```
  apply clarify
  apply(rename-tac a b)
  apply(case-tac nP b, simp-all)
    apply(case-tac nP a, simp-all)
      apply blast
     apply blast
    apply blast
   apply(case-tac nP a, simp-all)
    apply blast
   apply blast
  apply(simp add: All-to-Unassigned)
done
```

All cases where the model can become invalid:

```
theorem violating-configurations-exhaust: ¬ sinvar G nP ⟷
   (∃ (e1, e2) ∈ (edges G).
      nP e1 = Unassigned ∧ nP e2 ≠ Unassigned ∨
     (∃ s1 s2. nP e1 = Subnet s1 ∧ s1 ≠ s2 ∧ (nP e2 = Subnet s2 ∨ nP e2 = BorderRouter s2)) ∨
     (∃ s1 s2. nP e1 = BorderRouter s1 ∧ nP e2 = Subnet s2)
   ) (is ?l ⟷ ?r)
 proof
  assume ?l
    have violating-configurations-exhaust-Unassigned:
     (n1, n2) ∈ (edges G) ⟹ nP n1 = Unassigned ⟹ ¬ allowed-subnet-flow (nP n1) (nP n2) ⟹
       ∃ (e1, e2) ∈ (edges G).  nP e1 = Unassigned ∧ nP e2 ≠ Unassigned  for n1 n2
     by(cases nP n2, simp-all) force+

    have violating-configurations-exhaust-Subnet:
     (n1, n2) ∈ (edges G) ⟹ nP n1 = Subnet s1' ⟹ ¬ allowed-subnet-flow (nP n1) (nP n2) ⟹
     ∃ (e1, e2) ∈ (edges G). ∃ s1 s2. nP e1 = Subnet s1 ∧ s1 ≠ s2 ∧ (nP e2 = Subnet s2 ∨ nP e2
= BorderRouter s2)
     for n1 n2 s1' by(cases nP n2, simp-all) blast+

    have violating-configurations-exhaust-BorderRouter:
   (n1, n2) ∈ (edges G) ⟹ nP n1 = BorderRouter s1' ⟹ ¬ allowed-subnet-flow (nP n1) (nP n2)
⟹
     ∃ (e1, e2) ∈ (edges G). ∃ s1 s2. nP e1 = BorderRouter s1 ∧ nP e2 = Subnet s2 for n1 n2 s1'
     by(cases nP n2, simp-all) blast+

    from ‹?l› show ?r
    apply simp
    apply clarify
    apply(rename-tac n1 n2)
    apply(case-tac nP n1, simp-all)
     apply(rename-tac s1)
     apply(drule-tac s1'=s1 in violating-configurations-exhaust-Subnet, simp-all)
     apply blast
    apply(rename-tac s1)
    apply(drule-tac s1'=s1 in violating-configurations-exhaust-BorderRouter, simp-all)
    apply blast
   apply(drule-tac violating-configurations-exhaust-Unassigned, simp-all)
   apply blast
   done
  next
```

```
 assume ?r thus ?l
  apply simp
  apply(clarify)
  apply(safe)
    apply(rule-tac x=(a,b) in bexI)
     apply (simp add: Unassigned-only-to-Unassigned; fail)
    apply(simp; fail)
   apply(rule-tac x=(a,b) in bexI)
    apply(simp; fail)
   apply(simp; fail)
  apply(rule-tac x=(a,b) in bexI)
   apply(simp; fail)
  apply(simp; fail)
 apply(rule-tac x=(a,b) in bexI)
  apply(simp; fail)
 apply(simp; fail)
 done
qed


hide-fact (open) sinvar-mono
hide-const (open) sinvar receiver-violation default-node-properties

end
theory SINVAR-Subnets-impl
imports SINVAR-Subnets ../TopoS-Interface-impl
begin
```

6.1.4 SecurityInvariant Subnets List Implementation

```
code-identifier code-module SINVAR-Subnets-impl => (Scala) SINVAR-Subnets

fun sinvar :: 'v list-graph ⇒ ('v ⇒ subnets) ⇒ bool where
  sinvar G nP = (∀ (e1,e2) ∈ set (edgesL G). allowed-subnet-flow (nP e1) (nP e2))


definition Subnets-offending-list:: 'v list-graph ⇒ ('v ⇒ subnets) ⇒ ('v × 'v) list list where
  Subnets-offending-list G nP = (if sinvar G nP then
    []
   else
    [ [e ← edgesL G. case e of (e1,e2) ⇒ ¬ allowed-subnet-flow (nP e1) (nP e2)] ])



definition NetModel-node-props P = (λ i. (case (node-properties P) i of Some property ⇒ property
| None ⇒ SINVAR-Subnets.default-node-properties))
lemma[code-unfold]: SecurityInvariant.node-props SINVAR-Subnets.default-node-properties P = NetModel-node-props
P
apply(simp add: NetModel-node-props-def)
done

definition Subnets-eval G P = (wf-list-graph G ∧
  sinvar G (SecurityInvariant.node-props SINVAR-Subnets.default-node-properties P))
```

interpretation *Subnets-impl*:*TopoS-List-Impl*
where *default-node-properties=SINVAR-Subnets.default-node-properties*
and *sinvar-spec=SINVAR-Subnets.sinvar*
and *sinvar-impl=sinvar*
and *receiver-violation=SINVAR-Subnets.receiver-violation*
and *offending-flows-impl=Subnets-offending-list*
and *node-props-impl=NetModel-node-props*
and *eval-impl=Subnets-eval*
apply(*unfold TopoS-List-Impl-def*)
apply(*rule conjI*)
apply(*simp add*: *TopoS-Subnets list-graph-to-graph-def*)
apply(*rule conjI*)
apply(*simp add*: *list-graph-to-graph-def Subnets-offending-set Subnets-offending-set-def Subnets-offending-list-def*)
apply(*rule conjI*)
apply(*simp only*: *NetModel-node-props-def*)
apply(*metis Subnets.node-props.simps Subnets.node-props-eq-node-props-formaldef*)
apply(*simp only*: *Subnets-eval-def*)
apply(*simp add*: *TopoS-eval-impl-proofrule*[*OF TopoS-Subnets*])
apply(*simp-all add*: *list-graph-to-graph-def*)
done

6.1.5 Subnets packing

definition *SINVAR-LIB-Subnets* :: (*'v*::*vertex*, *SINVAR-Subnets.subnets*) *TopoS-packed* **where**
SINVAR-LIB-Subnets ≡
(‖ *nm-name* = *''Subnets''*,
nm-receiver-violation = *SINVAR-Subnets.receiver-violation*,
nm-default = *SINVAR-Subnets.default-node-properties*,
nm-sinvar = *sinvar*,
nm-offending-flows = *Subnets-offending-list*,
nm-node-props = *NetModel-node-props*,
nm-eval = *Subnets-eval*
‖)
interpretation *SINVAR-LIB-Subnets-interpretation*: *TopoS-modelLibrary SINVAR-LIB-Subnets*
SINVAR-Subnets.sinvar
apply(*unfold TopoS-modelLibrary-def SINVAR-LIB-Subnets-def*)
apply(*rule conjI*)
apply(*simp*)
apply(*simp*)
by(*unfold-locales*)

Examples

definition *example-net-sub* :: *nat list-graph* **where**
example-net-sub ≡ (‖ *nodesL* = [*1*::*nat*,*2*,*3*,*4*, *8*,*9*, *11*,*12*, *42*],
edgesL = [(*1*,*2*),(*1*,*3*),(*1*,*4*),(*2*,*1*),(*2*,*3*),(*2*,*4*),(*3*,*1*),(*3*,*2*),(*3*,*4*),(*4*,*1*),(*4*,*2*),(*4*,*3*),
(*4*,*11*),(*1*,*11*),
(*8*,*9*),(*9*,*8*),
(*8*,*12*),
(*11*,*12*),
(*11*,*42*), (*12*,*42*), (*3*,*42*)] ‖)
value *wf-list-graph example-net-sub*

definition *example-conf-sub* **where**

example-conf-sub ≡ ((λe. *SINVAR-Subnets.default-node-properties*)
(*1* := *Subnet 1*, *2*:= *Subnet 1*, *3*:= *Subnet 1*, *4*:=*Subnet 1*,
11:=*BorderRouter 1*,
8:=*Subnet 2*, *9*:=*Subnet 2*,
12:=*BorderRouter 2*,
42 := *Unassigned*))

value *sinvar example-net-sub example-conf-sub*

definition *example-net-sub-invalid* **where**
example-net-sub-invalid ≡ *example-net-sub*(|*edgesL* := (*42*,*4*)#(*3*,*8*)#(*11*,*8*)#(*edgesL example-net-sub*)|)

value *sinvar example-net-sub-invalid example-conf-sub*
value *Subnets-offending-list example-net-sub-invalid example-conf-sub*

value *sinvar*
(| *nodesL* = [*1*::*nat*,*2*,*3*,*4*], *edgesL* = [(*1*,*2*), (*2*,*3*), (*3*,*4*), (*8*,*9*),(*9*,*8*)] |)
(λe. *SINVAR-Subnets.default-node-properties*)
value *sinvar*
(| *nodesL* = [*1*::*nat*,*2*,*3*,*4*,*8*,*9*,*11*,*12*], *edgesL* = [(*1*,*2*),(*2*,*3*),(*3*,*4*), (*4*,*11*),(*1*,*11*), (*8*,*9*),(*9*,*8*),(*8*,*12*), (*11*,*12*)] |)
((λe. *SINVAR-Subnets.default-node-properties*)(*1* := *Subnet 1*, *2*:= *Subnet 1*, *3*:= *Subnet 1*, *4*:=*Subnet 1*, *11*:=*BorderRouter 1*,
8:=*Subnet 2*, *9*:=*Subnet 2*, *12*:=*BorderRouter 2*))
value *sinvar*
(| *nodesL* = [*1*::*nat*,*2*,*3*,*4*,*8*,*9*,*11*,*12*], *edgesL* = [(*1*,*2*),(*2*,*3*),(*3*,*4*), (*4*,*11*),(*1*,*11*), (*8*,*9*),(*9*,*8*),(*8*,*12*), (*11*,*12*)] |)
((λe. *SINVAR-Subnets.default-node-properties*)(*1* := *Subnet 1*, *2*:= *Subnet 1*, *3*:= *Subnet 1*, *4*:=*Subnet 1*, *11*:=*BorderRouter 1*))
value *sinvar*
(| *nodesL* = [*1*::*nat*,*2*,*3*,*4*,*8*,*9*,*10*], *edgesL* = [(*1*,*2*), (*2*,*3*), (*3*,*4*), (*8*,*9*),(*9*,*8*)] |)
((λe. *SINVAR-Subnets.default-node-properties*)(*8*:=*Subnet 8*, *9*:=*Subnet 8*))

hide-const (**open**) *NetModel-node-props*
hide-const (**open**) *sinvar*

end
theory *SINVAR-DomainHierarchyNG*
imports *../TopoS-Helper*
HOL−Lattice.CompleteLattice
begin

6.2 SecurityInvariant DomainHierarchyNG

6.2.1 Datatype Domain Hierarchy

A fully qualified domain name for an entity in a tree-like hierarchy

datatype *domainNameDept* = *Dept string domainNameDept* (**infixr** −− *65*) |
Leaf — leaf of the tree, end of all domainNames

Example: the CoffeeMachine of I8

value *''i8''−−''CoffeeMachine''−−Leaf*

A tree strucuture to represent the general hierarchy, i.e. possible domainNameDepts

datatype *domainTree* = *Department*
string — division
domainTree list — sub divisions

one step in tree to find matching department

fun *hierarchy-next* :: *domainTree list* ⇒ *domainNameDept* ⇒ *domainTree option* **where**
hierarchy-next [] - = *None* |
hierarchy-next (*s*#*ss*) *Leaf* = *None* |
hierarchy-next ((*Department d ds*)#*ss*) (*Dept n ns*) = (*if d*=*n then Some* (*Department d ds*) *else hierarchy-next ss* (*Dept n ns*))

Examples:

lemma *hierarchy-next* [*Department ''i20''* [], *Department ''i8''* [*Department ''CoffeeMachine''* [], *Department ''TeaMachine''* []]]
(*''i8''−−Leaf*)
=
Some (*Department ''i8''* [*Department ''CoffeeMachine''* [], *Department ''TeaMachine''* []]) **by** *eval*
lemma *hierarchy-next* [*Department ''i20''* [], *Department ''i8''* [*Department ''CoffeeMachine''* [], *Department ''TeaMachine''* []]]
(*''i8''−−''whatsoever''−−Leaf*)
=
Some (*Department ''i8''* [*Department ''CoffeeMachine''* [], *Department ''TeaMachine''* []]) **by** *eval*
lemma *hierarchy-next* [*Department ''i20''* [], *Department ''i8''* [*Department ''CoffeeMachine''* [], *Department ''TeaMachine''* []]]
Leaf
= *None* **by** *eval*
lemma *hierarchy-next* [*Department ''i20''* [], *Department ''i8''* [*Department ''CoffeeMachine''* [], *Department ''TeaMachine''* []]]
(*''i0''−−Leaf*)
= *None* **by** *eval*

Does a given *domainNameDept* match the specified tree structure?

fun *valid-hierarchy-pos* :: *domainTree* ⇒ *domainNameDept* ⇒ *bool* **where**
valid-hierarchy-pos (*Department d ds*) *Leaf* = *True* |
valid-hierarchy-pos (*Department d ds*) (*Dept n Leaf*) = (*d*=*n*) |
valid-hierarchy-pos (*Department d ds*) (*Dept n ns*) = (*n*=*d* ∧
(*case hierarchy-next ds ns of*
None ⇒ *False* |
Some t ⇒ *valid-hierarchy-pos t ns*))

Examples:

lemma *valid-hierarchy-pos* (*Department ''TUM''* []) *Leaf* **by** *eval*
lemma *valid-hierarchy-pos* (*Department ''TUM''* []) *Leaf* **by** *eval*
lemma *valid-hierarchy-pos* (*Department ''TUM''* []) (*''TUM''−−Leaf*) **by** *eval*
lemma *valid-hierarchy-pos* (*Department ''TUM''* []) (*''TUM''−−''facilityManagement''−−Leaf*) = *False* **by** *eval*

lemma *valid-hierarchy-pos* (*Department ''TUM''* []) (*''LMU''−−Leaf*) = *False* **by** *eval*
lemma *valid-hierarchy-pos* (*Department ''TUM''* [*Department ''i8''* [], (*Department ''i20''* [])])
(*''TUM''−−Leaf*) **by** *eval*
lemma *valid-hierarchy-pos* (*Department ''TUM''* [*Department ''i8''* [], *Department ''i20''* []])
(*''TUM''−−''i8''−−Leaf*) **by** *eval*
lemma *valid-hierarchy-pos*
(*Department ''TUM''* [
Department ''i8'' [
Department ''CoffeeMachine'' [],
Department ''TeaMachine'' []
],
Department ''i20'' []
])
(*''TUM''−−''i8''−−''CoffeeMachine''−−Leaf*) **by** *eval*
lemma *valid-hierarchy-pos* (*Department ''TUM''* [*Department ''i8''* [*Department ''CoffeeMachine''*
[], *Department ''TeaMachine''* []], *Department ''i20''* []])
(*''TUM''−−''i8''−−''CleanKitchen''−−Leaf*) = *False* **by** *eval*

instantiation *domainNameDept* :: *order*
begin
print-context

fun *less-eq-domainNameDept* :: *domainNameDept* ⇒ *domainNameDept* ⇒ *bool* **where**
Leaf ≤ (*Dept* - -) = *False* |
(*Dept* - -) ≤ *Leaf* = *True* |
Leaf ≤ *Leaf* = *True* |
(*Dept n1 n1s*) ≤ (*Dept n2 n2s*) = (*n1*=*n2* ∧ *n1s* ≤ *n2s*)

fun *less-domainNameDept* :: *domainNameDept* ⇒ *domainNameDept* ⇒ *bool* **where**
Leaf < *Leaf* = *False* |
Leaf < (*Dept* - -) = *False* |
(*Dept* - -) < *Leaf* = *True* |
(*Dept n1 n1s*) < (*Dept n2 n2s*) = (*n1*=*n2* ∧ *n1s* < *n2s*)

lemma *Leaf-Top*: *a* ≤ *Leaf*
apply(*case-tac a*)
by(*simp-all*)

lemma *Leaf-Top-Unique*: *Leaf* ≤ *a* = (*a* = *Leaf*)
apply(*case-tac a*)
by(*simp-all*)

lemma *no-Bot*: *n1* ≠ *n2* ⟹ *z* ≤ *n1* −− *n1s* ⟹ *z* ≤ *n2* −− *n2s* ⟹ *False*
apply(*case-tac z*)
by(*simp-all*)

lemma *uncomparable-sup-is-Top*: *n1* ≠ *n2* ⟹ *n1* −− *x* ≤ *z* ⟹ *n2* −− *y* ≤ *z* ⟹ *z* = *Leaf*
apply(*case-tac z*)
by(*simp-all*)

lemma *common-inf-imp-comparable*: (*z*::*domainNameDept*) ≤ *a* ⟹ *z* ≤ *b* ⟹ *a* ≤ *b* ∨ *b* ≤ *a*

```
      apply(induction z arbitrary: a b)
       apply(rename-tac zn zdpt a b)
       apply(simp-all add: Leaf-Top-Unique)
      apply(case-tac a)
       apply(rename-tac an adpt)
       apply(simp-all add: Leaf-Top)
      apply(case-tac b)
       apply(rename-tac bn bdpt)
       apply(simp-all add: Leaf-Top)
      done

    lemma prepend-domain: a ≤ b ⟹ x−−a ≤ x−−b
      by(simp)
    lemma unfold-dmain-leq: y ≤ zn −− zns ⟹ ∃ yns. y = zn −− yns ∧ yns ≤ zns
      proof −
        assume a1: y ≤ zn −− zns
       obtain sk30 :: domainNameDept ⇒ char list and sk31 :: domainNameDept ⇒ domainNameDept
where ∀ x0. sk30 x0 −− sk31 x0 = x0 ∨ Leaf = x0
          by (metis domainNameDept.exhaust)
        thus ∃ yns. y = zn −− yns ∧ yns ≤ zns
          using a1 by (metis less-eq-domainNameDept.simps(1) less-eq-domainNameDept.simps(4))
      qed

    lemma less-eq-refl:
      fixes x :: domainNameDept
      shows x ≤ y ⟹ y ≤ z ⟹ x ≤ z
      proof −
      have x ≤ y ⟶ y ≤ z ⟶ x ≤ z
        proof(induction z arbitrary:x y)
        case Leaf
          have x ≤ Leaf using Leaf-Top by simp
          thus ?case by simp
        next
        case (Dept zn zns)
          show ?case proof(clarify)
            assume a1: x ≤ y and a2: y ≤ zn−−zns
             from unfold-dmain-leq[OF a2] obtain yns where y1: y = zn−−yns and y2: yns ≤ zns
by auto
            from unfold-dmain-leq this a1 obtain xns where x1: x = zn −− xns and x2: xns ≤ yns
by blast
            from Dept y2 x2 have xns ≤ zns by simp
            from this x1 show x ≤ zn−−zns by simp
          qed
        qed
      thus x ≤ y ⟹ y ≤ z ⟹ x ≤ z by simp
      qed

    instance
      proof
        fix x y ::domainNameDept
        show (x < y) = (x ≤ y ∧ ¬ y ≤ x)
          apply(induction rule: less-domainNameDept.induct)
           apply(simp-all)
          by blast
```

```
    next
      fix x::domainNameDept
      show x ≤ x
        using[[show-types]] apply(induction x)
        by simp-all
    next
      fix x y z :: domainNameDept
      show x ≤ y ⟹ y ≤ z ⟹ x ≤ z  apply (rule less-eq-refl) by simp-all
    next
      fix x y ::domainNameDept
      show x ≤ y ⟹ y ≤ x ⟹ x = y
        apply(induction rule: less-domainNameDept.induct)
          by(simp-all)
  qed
 end

 instantiation domainNameDept :: Orderings.top
 begin
  definition top-domainNameDept where Orderings.top ≡ Leaf
  instance
    by intro-classes
 end

  lemma (''TUM''--''BLUBB''--Leaf) ≤ (''TUM''--Leaf) by eval

  lemma (''TUM''--''i8''--Leaf) ≤ (''TUM''--Leaf) by eval
  lemma ¬ (''TUM''--Leaf) ≤ (''TUM''--''i8''--Leaf)  by eval
   lemma valid-hierarchy-pos (Department ''TUM'' [Department ''i8'' [], Department ''i20'' []])
(''TUM''--''i8''--Leaf) by eval

  lemma (''TUM''--Leaf) ≤ Leaf by eval
   lemma valid-hierarchy-pos (Department ''TUM'' [Department ''i8'' [], Department ''i20'' []])
(Leaf) by eval

  lemma ¬ Leaf ≤ (''TUM''--Leaf) by eval
   lemma valid-hierarchy-pos (Department ''TUM'' [Department ''i8'' [], Department ''i20'' []])
(''TUM''--Leaf) by eval

  lemma ¬ (''TUM''--''BLUBB''--Leaf) ≤ (''X''--''TUM''--''BLUBB''--Leaf) by eval

  lemma (''TUM''--''i8''--''CoffeeMachine''--Leaf) ≤ (''TUM''--''i8''--Leaf) by eval
  lemma (''TUM''--''i8''--Leaf) ≤ (''TUM''--''i8''--Leaf) by eval
  lemma (''TUM''--''i8''--''CoffeeMachine''--Leaf) ≤ (''TUM''--Leaf) by eval
  lemma (''TUM''--''i8''--''CoffeeMachine''--Leaf) ≤ (Leaf) by eval
  lemma ¬ (''TUM''--''i8''--Leaf) ≤ (''TUM''--''i20''--Leaf) by eval
  lemma ¬ (''TUM''--''i20''--Leaf) ≤ (''TUM''--''i8''--Leaf) by eval
```

6.2.2 Adding Chop

by putting entities higher in the hierarchy.

```
  fun domainNameDeptChopOne :: domainNameDept ⇒ domainNameDept where
    domainNameDeptChopOne Leaf = Leaf |
    domainNameDeptChopOne (name--Leaf) = Leaf |
    domainNameDeptChopOne (name--dpt) = name--(domainNameDeptChopOne dpt)
```

lemma *domainNameDeptChopOne (''i8''--''CoffeeMachine''--Leaf) = ''i8'' -- Leaf* **by** *eval*
lemma *domainNameDeptChopOne (''i8''--''CoffeeMachine''--''CoffeeSlave''--Leaf) = ''i8'' -- ''CoffeeMachine'' -- Leaf* **by** *eval*
lemma *domainNameDeptChopOne Leaf = Leaf* **by**(*fact domainNameDeptChopOne.simps(1)*)

theorem *chopOne-not-decrease*: *dn ≤ domainNameDeptChopOne dn*
apply(*induction dn*)
apply(*rename-tac name dpt*)
apply(*drule-tac x=name* **in** *prepend-domain*)
apply(*case-tac dpt*)
apply *simp-all*
done

lemma *chopOneContinue*: *dpt ≠ Leaf ⟹ domainNameDeptChopOne (name -- dpt) = name -- domainNameDeptChopOne (dpt)*
apply(*case-tac dpt*)
by *simp-all*

fun *domainNameChop* :: *domainNameDept ⇒ nat ⇒ domainNameDept* **where**
domainNameChop Leaf - = Leaf |
domainNameChop namedpt 0 = namedpt |
domainNameChop namedpt (Suc n) = domainNameChop (domainNameDeptChopOne namedpt) n

lemma *domainNameChop (''i8''--''CoffeeMachine''--Leaf) 2 = Leaf* **by** *eval*
lemma *domainNameChop (''i8''--''CoffeeMachine''--''CoffeeSlave''--Leaf) 2 = ''i8''--Leaf* **by** *eval*
lemma *domainNameChop (''i8''--Leaf) 0 = ''i8''--Leaf* **by** *eval*
lemma *domainNameChop (Leaf) 8 = Leaf* **by** *eval*

lemma *chop0*[*simp*]: *domainNameChop dn 0 = dn*
apply(*case-tac dn*)
by *simp-all*

lemma (*domainNameDeptChopOne^^2*) (*''d1''--''d2''--''d3''--Leaf*) = *''d1''--Leaf* **by** *eval*

domainNameChop is equal to applying n times chop one

lemma *domainNameChopFunApply*: *domainNameChop dn n = (domainNameDeptChopOne^^n) dn*
apply(*induction dn n rule*: *domainNameChop.induct*)
apply (*simp-all*)
apply(*rename-tac nat,induct-tac nat, simp-all*)
apply(*rename-tac n*)
by (*metis funpow-swap1*)

lemma *domainNameChopRotateSuc*: *domainNameChop dn (Suc n) = domainNameDeptChopOne (domainNameChop dn n)*
by(*simp add*: *domainNameChopFunApply*)

lemma *domainNameChopRotate*: *domainNameChop (domainNameDeptChopOne dn) n = domain-*

NameDeptChopOne (*domainNameChop dn n*)
apply(*subgoal-tac domainNameChop* (*domainNameDeptChopOne dn*) *n* = *domainNameChop dn* (*Suc n*))
apply *simp*
apply(*simp add*: *domainNameChopFunApply*)
apply(*case-tac dn*)
by(*simp-all*)

theorem *chop-not-decrease-hierarchy*: *dn* ≤ *domainNameChop dn n*
apply(*induction n*)
apply(*simp*)
apply(*case-tac dn*)
apply(*rename-tac name dpt*)
apply (*simp*)
apply(*simp add*:*domainNameChopRotate*)
apply (*metis chopOne-not-decrease less-eq-refl*)
apply *simp*
done

corollary *dn* ≤ *domainNameDeptChopOne* ((*domainNameDeptChopOne* ^^ *n*) (*dn*))
by (*metis chop-not-decrease-hierarchy domainNameChopFunApply domainNameChopRotateSuc*)

compute maximum common level of both inputs

fun *chop-sup* :: *domainNameDept* ⇒ *domainNameDept* ⇒ *domainNameDept* **where**
chop-sup Leaf - = *Leaf* |
chop-sup - *Leaf* = *Leaf* |
chop-sup (*a*−−*as*) (*b*−−*bs*) = (*if a* ≠ *b then Leaf else a*−−(*chop-sup as bs*))

lemma *chop-sup* (''*a*''−−''*b*''−−''*c*''−−*Leaf*) (''*a*''−−''*b*''−−''*d*''−−*Leaf*) = ''*a*'' −− ''*b*'' −− *Leaf* **by** *eval*
lemma *chop-sup* (''*a*''−−''*b*''−−''*c*''−−*Leaf*) (''*a*''−−''*x*''−−''*d*''−−*Leaf*) = ''*a*'' −− *Leaf* **by** *eval*
lemma *chop-sup* (''*a*''−−''*b*''−−''*c*''−−*Leaf*) (''*x*''−−''*x*''−−''*d*''−−*Leaf*) = *Leaf* **by** *eval*

lemma *chop-sup-commute*: *chop-sup a b* = *chop-sup b a*
apply(*induction a b rule*: *chop-sup.induct*)
apply(*rename-tac a*)
apply(*simp-all*)
apply(*case-tac a*, *simp-all*)
done
lemma *chop-sup-max1*: *a* ≤ *chop-sup a b*
apply(*induction a b rule*: *chop-sup.induct*)
by(*simp-all*)
lemma *chop-sup-max2*: *b* ≤ *chop-sup a b*
apply(*subst chop-sup-commute*)
by(*simp add*: *chop-sup-max1*)

lemma *chop-sup-is-sup*: ∀ *z*. *a* ≤ *z* ∧ *b* ≤ *z* ⟶ *chop-sup a b* ≤ *z*
apply(*clarify*)
apply(*induction a b rule*: *chop-sup.induct*)
apply(*simp-all*)
apply(*rule conjI*)

apply(*clarify*)
apply(*subgoal-tac z=Leaf*)
apply(*simp*)
apply(*simp add*: *uncomparable-sup-is-Top*)
apply(*clarify*)
apply(*case-tac z*)
by(*simp-all*)

datatype *domainName* = *DN domainNameDept* | *Unassigned*

6.2.3 Makeing it a complete Lattice

instantiation *domainName* :: *partial-order*
begin

fun *leq-domainName* :: *domainName* ⇒ *domainName* ⇒ *bool* **where**
leq-domainName Unassigned - = True |
leq-domainName - Unassigned = False |
leq-domainName (*DN dnA*) (*DN dnB*) = (*dnA* ≤ *dnB*)
instance
apply(*intro-classes*)

apply(*case-tac x*)
apply(*simp-all*)

apply(*case-tac x*, *rename-tac dnX*)
apply(*case-tac y*, *rename-tac dnY*)
apply(*case-tac z*, *rename-tac dnZ*)
apply(*simp-all*)

apply(*case-tac x*, *rename-tac dnX*)
apply(*case-tac y*, *rename-tac dnY*)
apply(*simp-all*)
apply(*metis domainName.exhaust leq-domainName.simps*(*2*))
done
end

lemma *is-Inf* {*Unassigned*, *DN Leaf*} *Unassigned*
by(*simp add*: *is-Inf-def*)

The infinum of two elements:

fun *DN-inf* :: *domainName* ⇒ *domainName* ⇒ *domainName* **where**
DN-inf Unassigned - = Unassigned |
DN-inf - Unassigned = Unassigned |
DN-inf (*DN a*) (*DN b*) = (*if a* ≤ *b then DN a else if b* ≤ *a then DN b else Unassigned*)

lemma *DN-inf* (*DN* (*''TUM''−−''i8''−−Leaf*)) (*DN* (*''TUM''−−''i20''−−Leaf*)) = *Unassigned* **by** *eval*
lemma *DN-inf* (*DN* (*''TUM''−−''i8''−−Leaf*)) (*DN* (*''TUM''−−Leaf*)) = *DN* (*''TUM''* −− *''i8''* −− *Leaf*) **by** *eval*

```
lemma DN-inf-commute: DN-inf x y = DN-inf y x
  apply(induction x y rule: DN-inf.induct)
    apply(rename-tac x)
    apply(case-tac x)
     by (simp-all)

lemma DN-inf-is-inf: is-inf x y (DN-inf x y)
  apply(induction x y rule: DN-inf.induct)
    apply(simp add: is-inf-def)
   apply(simp add: is-inf-def)
  apply(simp add: is-inf-def)
  apply(clarify)
  apply(rename-tac z)
  apply(case-tac z)
   apply(simp)
   apply(rename-tac zn)
   apply(simp-all)
  using common-inf-imp-comparable by blast


fun DN-sup :: domainName ⇒ domainName ⇒ domainName where
  DN-sup Unassigned a = a |
  DN-sup a Unassigned = a |
  DN-sup (DN a) (DN b) = DN (chop-sup a b)

lemma DN-sup-commute: DN-sup x y = DN-sup y x
  apply(induction x y rule: DN-sup.induct)
    apply(rename-tac x)
    apply(case-tac x)
     by(simp-all add: chop-sup-commute)

lemma DN-sup-is-sup: is-sup x y (DN-sup x y)
  apply(induction x y rule: DN-inf.induct)
    apply(simp add: is-sup-def leq-refl)
   apply(simp add: is-sup-def)
  apply(simp add: is-sup-def chop-sup-max1 chop-sup-max2)
  apply(clarify)
  apply(rename-tac z)
  apply(case-tac z)
   apply(simp)
   apply(rename-tac zn)
   apply(simp-all)
  apply(clarify)
  apply(simp add: chop-sup-is-sup)
  done
```

domainName is a Lattice:

```
instantiation domainName :: lattice
  begin
  instance
    apply intro-classes
     apply(rule-tac x=DN-inf x y in exI)
     apply(fact DN-inf-is-inf)
    apply(rule-tac x=DN-sup x y in exI)
```

```
    apply(rule DN-sup-is-sup)
   done
 end
```

```
datatype domainNameTrust = DN (domainNameDept × nat) | Unassigned
```

```
 fun leq-domainNameTrust :: domainNameTrust ⇒ domainNameTrust ⇒ bool (infixr ⊑trust 65)
where
   leq-domainNameTrust Unassigned - = True |
   leq-domainNameTrust - Unassigned = False |
    leq-domainNameTrust (DN (dnA, trustA)) (DN (dnB, trustB)) = (dnA ≤ (domainNameChop
dnB trustB))

 lemma leq-domainNameTrust-refl: x ⊑trust x
  apply(case-tac x)
   apply(rename-tac prod)
   apply(case-tac prod)
   apply(simp add: chop-not-decrease-hierarchy)
  by(simp)

 lemma leq-domainNameTrust-NOT-trans: ∃ x y z. x ⊑trust y ∧ y ⊑trust z ∧ ¬ x ⊑trust z
  apply(rule-tac x=DN (''TUM''--Leaf, 0) in exI)
  apply(rule-tac x=DN (''TUM''--''i8''--Leaf, 1) in exI)
  apply(rule-tac x=DN (''TUM''--''i8''--Leaf, 0) in exI)
  apply(simp)
  done

 lemma leq-domainNameTrust-NOT-antisym: ∃ x y. x ⊑trust y ∧ y ⊑trust x ∧ x ≠ y
  apply(rule-tac x=DN (Leaf, 3) in exI)
  apply(rule-tac x=DN (Leaf, 4) in exI)
  apply(simp)
  done
```

6.2.4 The network security invariant

```
definition default-node-properties :: domainNameTrust
  where default-node-properties = Unassigned
```

The sender is, noticing its trust level, on the same or higher hierarchy level as the receiver.

```
fun sinvar :: 'v graph ⇒ ('v ⇒ domainNameTrust) ⇒ bool where
  sinvar G nP = (∀ (s, r) ∈ edges G. (nP r) ⊑trust (nP s))
```

a domain name must be in the supplied tree

```
fun verify-globals :: 'v graph ⇒ ('v ⇒ domainNameTrust) ⇒ domainTree ⇒ bool where
  verify-globals G nP tree = (∀ v ∈ nodes G.
    case (nP v) of Unassigned ⇒ True | DN (level, trust) ⇒ valid-hierarchy-pos tree level
   )
```

lemma *verify-globals* (| *nodes=set* [*1*,*2*,*3*], *edges=set* [] |) (λ*n*. *default-node-properties*) (*Department* ''*TUM*'' [])
by (*simp add*: *default-node-properties-def*)

definition *receiver-violation* :: *bool* **where** *receiver-violation* = *False*

thm *SecurityInvariant-withOffendingFlows.sinvar-mono-def*
lemma *sinvar-mono*: *SecurityInvariant-withOffendingFlows.sinvar-mono sinvar*
apply(*rule-tac SecurityInvariant-withOffendingFlows.sinvar-mono-I-proofrule*)
apply(*auto*)
apply(*rename-tac nP e1 e2 N E' e1' e2' E*)
apply(*blast*)
done

interpretation *SecurityInvariant-preliminaries*
where *sinvar* = *sinvar*
apply *unfold-locales*
apply(*frule-tac finite-distinct-list*[*OF wf-graph.finiteE*])
apply(*erule-tac exE*)
apply(*rename-tac list-edges*)
apply(*rule-tac ff=list-edges* **in** *SecurityInvariant-withOffendingFlows.mono-imp-set-offending-flows-not-empty*[*OF sinvar-mono*])
apply(*auto*)[*4*]
apply(*auto simp add*: *SecurityInvariant-withOffendingFlows.is-offending-flows-def graph-ops*)[*1*]
apply(*fact SecurityInvariant-withOffendingFlows.sinvar-mono-imp-sinvar-mono*[*OF sinvar-mono*])
apply(*fact SecurityInvariant-withOffendingFlows.sinvar-mono-imp-is-offending-flows-mono*[*OF sinvar-mono*])
done

6.2.5 ENF

lemma *DomainHierarchyNG-ENF*: *SecurityInvariant-withOffendingFlows.sinvar-all-edges-normal-form sinvar* (λ *s r*. *r* $\sqsubseteq_{trust}$ *s*)
unfolding *SecurityInvariant-withOffendingFlows.sinvar-all-edges-normal-form-def*
by *simp*
lemma *DomainHierarchyNG-ENF-refl*: *SecurityInvariant-withOffendingFlows.ENF-refl sinvar* (λ *s r*. *r* $\sqsubseteq_{trust}$ *s*)
unfolding *SecurityInvariant-withOffendingFlows.ENF-refl-def*
apply(*rule conjI*)
apply(*simp add*: *DomainHierarchyNG-ENF*)
apply(*simp add*: *leq-domainNameTrust-refl*)
done
lemma *unassigned-default-candidate*: ∀ *nP s r*. ¬ (*nP r*) $\sqsubseteq_{trust}$ (*nP s*) ⟶ ¬ (*nP r*) $\sqsubseteq_{trust}$ *default-node-properties*
apply(*clarify*)
apply (*simp add*: *default-node-properties-def*)
by (*metis leq-domainNameTrust.elims*(*3*) *leq-domainNameTrust.simps*(*2*))

definition *DomainHierarchyNG-offending-set*:: *'v graph* ⇒ (*'v* ⇒ *domainNameTrust*) ⇒ (*'v* × *'v*) *set set* **where**
DomainHierarchyNG-offending-set G nP = (*if sinvar G nP then*
{}
else
{ {*e* ∈ *edges G*. *case e of* (*e1*,*e2*) ⇒ ¬ (*nP e2*) ⊑$_{trust}$ (*nP e1*)} })
lemma *DomainHierarchyNG-offending-set*: *SecurityInvariant-withOffendingFlows.set-offending-flows sinvar* = *DomainHierarchyNG-offending-set*
apply(*simp only*: *fun-eq-iff SecurityInvariant-withOffendingFlows.ENF-offending-set*[*OF DomainHierarchyNG-ENF*] *DomainHierarchyNG-offending-set-def*)
apply(*rule allI*)+
apply(*rename-tac G nP*)
apply(*auto split*:*prod.split-asm prod.split simp add*: *Let-def*)
done

lemma *Unassigned-unique-default*: *otherbot* ≠ *default-node-properties* ⟹
∃ *G nP gP i f*.
wf-graph G ∧
¬ *sinvar G nP* ∧
f ∈ *SecurityInvariant-withOffendingFlows.set-offending-flows sinvar G nP* ∧
sinvar (*delete-edges G f*) *nP* ∧
(*i* ∈ *fst* ‘ *f* ∧ *sinvar G* (*nP*(*i* := *otherbot*)))
apply (*simp add*: *SecurityInvariant-withOffendingFlows.set-offending-flows-def*
SecurityInvariant-withOffendingFlows.is-offending-flows-min-set-def
SecurityInvariant-withOffendingFlows.is-offending-flows-def)
apply (*simp add*:*graph-ops*)
apply (*simp split*: *prod.split-asm prod.split domainNameTrust.split*)
apply(*rule-tac x*=(| *nodes*={*vertex-1*,*vertex-2*}, *edges* = {(*vertex-1*,*vertex-2*)} |) **in** *exI*, *simp*)
apply(*rule conjI*)
apply(*simp add*: *wf-graph-def*)
apply(*case-tac otherbot*)
apply(*rename-tac prod*)
apply(*case-tac prod*)
apply(*rename-tac dn trustlevel*)
apply(*clarify*)

apply(*case-tac dn*)

apply(*rename-tac name dpt*)
apply(*simp*)
apply(*rule-tac x*=(λ *x*. *default-node-properties*)(*vertex-1* := *Unassigned*, *vertex-2* := *DN* (*name*−−*dpt*, *0*)) **in** *exI*, *simp*)
apply(*rule-tac x*=*vertex-1* **in** *exI*, *simp*)
apply(*rule-tac x*={(*vertex-1*,*vertex-2*)} **in** *exI*, *simp*)
apply(*simp add*:*chop-not-decrease-hierarchy*)

apply(*simp*)
apply(*rule-tac x*=(λ *x*. *default-node-properties*)(*vertex-1* := *Unassigned*, *vertex-2* := *DN* (*Leaf*, *0*)) **in** *exI*, *simp*)
apply(*rule-tac x*=*vertex-1* **in** *exI*, *simp*)
apply(*rule-tac x*={(*vertex-1*,*vertex-2*)} **in** *exI*, *simp*)

apply(*simp add*: *default-node-properties-def*)
done

interpretation *DomainHierarchyNG*: *SecurityInvariant-ACS*
where *default-node-properties* = *default-node-properties*
and *sinvar* = *sinvar*
rewrites *SecurityInvariant-withOffendingFlows.set-offending-flows sinvar* = *DomainHierarchyNG-offending-set*
apply *unfold-locales*
apply(*rule ballI*)
apply(*drule SecurityInvariant-withOffendingFlows.ENF-fsts-refl-instance*[*OF DomainHierarchyNG-ENF-refl unassigned-default-candidate*], *simp-all*)[*1*]
apply(*erule default-uniqueness-by-counterexample-ACS*)
apply(*drule Unassigned-unique-default*)
apply(*simp*)
apply(*fact DomainHierarchyNG-offending-set*)
done

lemma *TopoS-DomainHierarchyNG*: *SecurityInvariant sinvar default-node-properties receiver-violation*
unfolding *receiver-violation-def* **by**(*unfold-locales*)

hide-const (**open**) *sinvar receiver-violation*

end
theory *SINVAR-DomainHierarchyNG-impl*
imports *SINVAR-DomainHierarchyNG ../TopoS-Interface-impl*
begin

6.2.6 SecurityInvariant DomainHierarchy List Implementation

code-identifier code-module *SINVAR-DomainHierarchyNG-impl* => (*Scala*) *SINVAR-DomainHierarchyNG*

fun *sinvar* :: *'v list-graph* ⇒ (*'v* ⇒ *domainNameTrust*) ⇒ *bool* **where**
sinvar G nP = (∀ (*s*, *r*) ∈ *set* (*edgesL G*). (*nP r*) $\sqsubseteq_{trust}$ (*nP s*))

definition *DomainHierarchyNG-sanity-check-config* :: *domainNameTrust list* ⇒ *domainTree* ⇒ *bool*
where
DomainHierarchyNG-sanity-check-config host-attributes tree = (∀ *c* ∈ *set host-attributes*.
case c of Unassigned ⇒ *True*
| *DN* (*level*, *trust*) ⇒ *valid-hierarchy-pos tree level*
)

fun *verify-globals* :: *'v list-graph* ⇒ (*'v* ⇒ *domainNameTrust*) ⇒ *domainTree* ⇒ *bool* **where**
verify-globals G nP tree = (∀ *v* ∈ *set* (*nodesL G*).
case (*nP v*) *of Unassigned* ⇒ *True* | *DN* (*level*, *trust*) ⇒ *valid-hierarchy-pos tree level*
)

lemma *DomainHierarchyNG-sanity-check-config c tree* $\Longrightarrow$
$\{x.\ \exists\, v.\ nP\ v = x\} = set\ c \Longrightarrow$
verify-globals G nP tree
apply(*simp add*: *DomainHierarchyNG-sanity-check-config-def split*: *if-split-asm*)
apply(*clarify*)
apply(*case-tac nP v*)
apply(*simp-all*)
apply(*clarify*)
by *force*

definition *DomainHierarchyNG-offending-list*:: *'v list-graph* $\Rightarrow$ (*'v* $\Rightarrow$ *domainNameTrust*) $\Rightarrow$ (*'v* $\times$ *'v*) *list list* **where**
DomainHierarchyNG-offending-list G nP = (*if sinvar G nP then*
[]
else
[[*e* $\leftarrow$ *edgesL G. case e of* (*s*,*r*) $\Rightarrow$ $\neg$ (*nP r*) $\sqsubseteq_{trust}$ (*nP s*)]])

lemma *DomainHierarchyNG.node-props P* =
(*λi. case node-properties P i of None* $\Rightarrow$ *SINVAR-DomainHierarchyNG.default-node-properties* | *Some property* $\Rightarrow$ *property*)
by(*fact SecurityInvariant.node-props.simps*[*OF TopoS-DomainHierarchyNG*, *of P*])

definition *NetModel-node-props P* = (*λ i.* (*case* (*node-properties P*) *i of Some property* $\Rightarrow$ *property* | *None* $\Rightarrow$ *SINVAR-DomainHierarchyNG.default-node-properties*))

lemma[*code-unfold*]: *DomainHierarchyNG.node-props P* = *NetModel-node-props P*
by(*simp add*: *NetModel-node-props-def*)

definition *DomainHierarchyNG-eval G P* = (*wf-list-graph G* $\wedge$
sinvar G (*SecurityInvariant.node-props SINVAR-DomainHierarchyNG.default-node-properties P*))

interpretation *DomainHierarchyNG-impl*:*TopoS-List-Impl*
where *default-node-properties*=*SINVAR-DomainHierarchyNG.default-node-properties*
and *sinvar-spec*=*SINVAR-DomainHierarchyNG.sinvar*
and *sinvar-impl*=*sinvar*
and *receiver-violation*=*SINVAR-DomainHierarchyNG.receiver-violation*
and *offending-flows-impl*=*DomainHierarchyNG-offending-list*
and *node-props-impl*=*NetModel-node-props*
and *eval-impl*=*DomainHierarchyNG-eval*
apply(*unfold TopoS-List-Impl-def*)
apply(*rule conjI*)
apply(*simp add*: *TopoS-DomainHierarchyNG list-graph-to-graph-def*; *fail*)
apply(*rule conjI*)
apply(*simp add*: *list-graph-to-graph-def DomainHierarchyNG-offending-set*
DomainHierarchyNG-offending-set-def DomainHierarchyNG-offending-list-def; *fail*)
apply(*rule conjI*)

apply(*simp only*: *NetModel-node-props-def*)
apply(*metis DomainHierarchyNG.node-props.simps DomainHierarchyNG.node-props-eq-node-props-formaldef*)
apply(*simp only*: *DomainHierarchyNG-eval-def*)
apply(*intro allI*)
apply(*rule TopoS-eval-impl-proofrule*[*OF TopoS-DomainHierarchyNG*])
apply(*simp add*: *list-graph-to-graph-def*)
done

6.2.7 DomainHierarchyNG packing

definition *SINVAR-LIB-DomainHierarchyNG* :: ('*v*::*vertex*, *domainNameTrust*) *TopoS-packed* **where**
SINVAR-LIB-DomainHierarchyNG ≡
(| *nm-name* = ''*DomainHierarchyNG*'',
nm-receiver-violation = *SINVAR-DomainHierarchyNG.receiver-violation*,
nm-default = *SINVAR-DomainHierarchyNG.default-node-properties*,
nm-sinvar = *sinvar*,
nm-offending-flows = *DomainHierarchyNG-offending-list*,
nm-node-props = *NetModel-node-props*,
nm-eval = *DomainHierarchyNG-eval*
|)
interpretation *SINVAR-LIB-DomainHierarchyNG-interpretation*: *TopoS-modelLibrary SINVAR-LIB-DomainHierarc*

SINVAR-DomainHierarchyNG.sinvar
apply(*unfold TopoS-modelLibrary-def SINVAR-LIB-DomainHierarchyNG-def*)
apply(*rule conjI*)
apply(*simp*)
apply(*simp*)
by(*unfold-locales*)

Examples:

definition *example-TUM-net* :: *string list-graph* **where**
example-TUM-net ≡ (| *nodesL*=[''*Gateway*'', ''*LowerSVR*'', ''*UpperSRV*''],
edgesL=[
(''*Gateway*'',''*LowerSVR*''), (''*Gateway*'',''*UpperSRV*''),
(''*LowerSVR*'', ''*Gateway*''),
(''*UpperSRV*'', ''*Gateway*'')
] |)
value *wf-list-graph example-TUM-net*

definition *example-TUM-config* :: *string* ⇒ *domainNameTrust* **where**
example-TUM-config ≡ ((λ *e*. *default-node-properties*)
(''*Gateway*'':= *DN* (''*ACD*''−−''*AISD*''−−*Leaf*, *1*),
''*LowerSVR*'':= *DN* (''*ACD*''−−''*AISD*''−−*Leaf*, *0*),
''*UpperSRV*'':= *DN* (''*ACD*''−−*Leaf*, *0*)
))

definition *example-TUM-hierarchy* :: *domainTree* **where**
example-TUM-hierarchy ≡ (*Department* ''*ACD*'' [
Department ''*AISD*'' []
])

value *verify-globals example-TUM-net example-TUM-config example-TUM-hierarchy*
value *sinvar* *example-TUM-net example-TUM-config*

definition *example-TUM-net-invalid* **where**
example-TUM-net-invalid ≡ *example-TUM-net*(|*edgesL* :=
("*LowerSRV*", "*UpperSRV*")#(*edgesL example-TUM-net*)|)

value *verify-globals example-TUM-net-invalid example-TUM-config example-TUM-hierarchy*
value *sinvar* *example-TUM-net-invalid example-TUM-config*
value *DomainHierarchyNG-offending-list example-TUM-net-invalid example-TUM-config*

hide-const (**open**) *NetModel-node-props*

hide-const (**open**) *sinvar*

end
theory *SINVAR-BLPtrusted-impl*
imports *SINVAR-BLPtrusted ../TopoS-Interface-impl*
begin

6.2.8 SecurityInvariant List Implementation

code-identifier code-module *SINVAR-BLPtrusted-impl* => (*Scala*) *SINVAR-BLPtrusted*

fun *sinvar* :: *'v list-graph* ⇒ (*'v* ⇒ *SINVAR-BLPtrusted.node-config*) ⇒ *bool* **where**
sinvar G nP = (∀ (*e1*,*e2*) ∈ *set* (*edgesL G*). (*if trusted* (*nP e2*) *then True else security-level* (*nP e1*) ≤ *security-level* (*nP e2*)))

definition *BLP-offending-list*:: *'v list-graph* ⇒ (*'v* ⇒ *SINVAR-BLPtrusted.node-config*) ⇒ (*'v* × *'v*) *list list* **where**
BLP-offending-list G nP = (*if sinvar G nP then*
[]
else
[[*e* ← *edgesL G*. *case e of* (*e1*,*e2*) ⇒ ¬ *SINVAR-BLPtrusted.BLP-P* (*nP e1*) (*nP e2*)]])

definition *NetModel-node-props P* = (λ *i*. (*case* (*node-properties P*) *i of Some property* ⇒ *property* | *None* ⇒ *SINVAR-BLPtrusted.default-node-properties*))
lemma[*code-unfold*]: *SecurityInvariant.node-props SINVAR-BLPtrusted.default-node-properties P* = *NetModel-node-props P*
apply(*simp add*: *NetModel-node-props-def*)
done

definition *BLP-eval G P* = (*wf-list-graph G* ∧
sinvar G (*SecurityInvariant.node-props SINVAR-BLPtrusted.default-node-properties P*))

interpretation *BLPtrusted-impl*:*TopoS-List-Impl*
where *default-node-properties*=*SINVAR-BLPtrusted.default-node-properties*
and *sinvar-spec*=*SINVAR-BLPtrusted.sinvar*
and *sinvar-impl*=*sinvar*
and *receiver-violation*=*SINVAR-BLPtrusted.receiver-violation*
and *offending-flows-impl*=*BLP-offending-list*
and *node-props-impl*=*NetModel-node-props*

and *eval-impl=BLP-eval*
apply(*unfold TopoS-List-Impl-def*)
apply(*rule conjI*)
apply(*simp add*: *TopoS-BLPtrusted list-graph-to-graph-def*; *fail*)
apply(*rule conjI*)
apply(*simp add*: *list-graph-to-graph-def BLP-offending-set BLP-offending-set-def BLP-offending-list-def*)
apply(*rule conjI*)
apply(*simp only*: *NetModel-node-props-def*)
apply(*metis BLPtrusted.node-props.simps BLPtrusted.node-props-eq-node-props-formaldef*)
apply(*simp only*: *BLP-eval-def*)
apply(*intro allI*)
apply(*rule TopoS-eval-impl-proofrule*[*OF TopoS-BLPtrusted*])
apply(*simp-all add*: *list-graph-to-graph-def*)
done

6.2.9 BLPtrusted packing

definition *SINVAR-LIB-BLPtrusted* :: ('*v*::*vertex*, *SINVAR-BLPtrusted.node-config*) *TopoS-packed*
where
SINVAR-LIB-BLPtrusted ≡
(| *nm-name* = ''*BLPtrusted*'',
nm-receiver-violation = *SINVAR-BLPtrusted.receiver-violation*,
nm-default = *SINVAR-BLPtrusted.default-node-properties*,
nm-sinvar = *sinvar*,
nm-offending-flows = *BLP-offending-list*,
nm-node-props = *NetModel-node-props*,
nm-eval = *BLP-eval*
|)
interpretation *SINVAR-LIB-BLPtrusted-interpretation*: *TopoS-modelLibrary SINVAR-LIB-BLPtrusted*

SINVAR-BLPtrusted.sinvar
apply(*unfold TopoS-modelLibrary-def SINVAR-LIB-BLPtrusted-def*)
apply(*rule conjI*)
apply(*simp*)
apply(*simp*)
by(*unfold-locales*)

6.2.10 Example

export-code *SINVAR-LIB-BLPtrusted* **checking** *Scala*

hide-const (**open**) *NetModel-node-props BLP-offending-list BLP-eval*

hide-const (**open**) *sinvar*

end
theory *SINVAR-SecGwExt*
imports *../TopoS-Helper*
begin

6.3 SecurityInvariant PolEnforcePointExtended

A PolEnforcePoint is an application-level central policy enforcement point. Legacy note: The old verions called it a SecurityGateway.

Hosts may belong to a certain domain. Sometimes, a pattern where intra-domain communication between domain members must be approved by a central instance is required.

We call such a central instance PolEnforcePoint and present a template for this architecture. Five host roles are distinguished:. A PolEnforcePoint, aPolEnforcePointIN which accessible from the outside, a DomainMember, a less-restricted AccessibleMember which is accessible from the outside world, and a default value Unassigned that reflects none of these roles.

datatype *secgw-member* = *PolEnforcePoint* | *PolEnforcePointIN* | *DomainMember* | *AccessibleMember* | *Unassigned*

definition *default-node-properties* :: *secgw-member*
where *default-node-properties* ≡ *Unassigned*

fun *allowed-secgw-flow* :: *secgw-member* ⇒ *secgw-member* ⇒ *bool* **where**
allowed-secgw-flow PolEnforcePoint - = True |
allowed-secgw-flow PolEnforcePointIN - = True |
allowed-secgw-flow DomainMember DomainMember = False |
allowed-secgw-flow DomainMember - = True |
allowed-secgw-flow AccessibleMember DomainMember = False |
allowed-secgw-flow AccessibleMember - = True |
allowed-secgw-flow Unassigned Unassigned = True |
allowed-secgw-flow Unassigned PolEnforcePointIN = True |
allowed-secgw-flow Unassigned AccessibleMember = True |
allowed-secgw-flow Unassigned - = False

fun *sinvar* :: *'v graph* ⇒ (*'v* ⇒ *secgw-member*) ⇒ *bool* **where**
sinvar G nP = (∀ (*e1*,*e2*) ∈ *edges G*. *e1* ≠ *e2* ⟶ *allowed-secgw-flow* (*nP e1*) (*nP e2*))

definition *receiver-violation* :: *bool* **where** *receiver-violation = False*

6.3.1 Preliminaries

lemma *sinvar-mono*: *SecurityInvariant-withOffendingFlows.sinvar-mono sinvar*
apply(*simp only*: *SecurityInvariant-withOffendingFlows.sinvar-mono-def*)
apply(*clarify*)
by *auto*

interpretation *SecurityInvariant-preliminaries*
where *sinvar = sinvar*
apply *unfold-locales*
apply(*frule-tac finite-distinct-list*[*OF wf-graph.finiteE*])
apply(*erule-tac exE*)
apply(*rename-tac list-edges*)
apply(*rule-tac ff=list-edges* **in** *SecurityInvariant-withOffendingFlows.mono-imp-set-offending-flows-not-empty*[*OF sinvar-mono*])
apply(*auto*)[*6*]
apply(*auto simp add*: *SecurityInvariant-withOffendingFlows.is-offending-flows-def graph-ops*)[*1*]
apply(*fact SecurityInvariant-withOffendingFlows.sinvar-mono-imp-is-offending-flows-mono*[*OF sinvar-mono*])

done

6.3.2 ENF

lemma *PolEnforcePoint-ENFnr*: *SecurityInvariant-withOffendingFlows.sinvar-all-edges-normal-form-not-refl sinvar allowed-secgw-flow*
by(*simp add*: *SecurityInvariant-withOffendingFlows.sinvar-all-edges-normal-form-not-refl-def*)
lemma *Unassigned-botdefault*: $\forall$ *e1 e2*. *e2* $\neq$ *Unassigned* $\longrightarrow$ $\neg$ *allowed-secgw-flow e1 e2* $\longrightarrow$ $\neg$ *allowed-secgw-flow Unassigned e2*
apply(*rule allI*)+
apply(*case-tac e2*)
apply(*simp-all*)
apply(*case-tac e1*)
apply(*simp-all*)
apply(*case-tac e1*)
apply(*simp-all*)
done
lemma *Unassigned-not-to-Member*: $\neg$ *allowed-secgw-flow Unassigned DomainMember*
by(*simp*)
lemma *All-to-Unassigned*: $\forall$ *e1*. *allowed-secgw-flow e1 Unassigned*
by (*rule allI*, *case-tac e1*, *simp-all*)

definition *PolEnforcePointExtended-offending-set*:: *'v graph* $\Rightarrow$ (*'v* $\Rightarrow$ *secgw-member*) $\Rightarrow$ (*'v* $\times$ *'v*) *set set* **where**
PolEnforcePointExtended-offending-set G nP = (*if sinvar G nP then*
{}
else
{ {*e* $\in$ *edges G*. *case e of* (*e1*,*e2*) $\Rightarrow$ *e1* $\neq$ *e2* $\wedge$ $\neg$ *allowed-secgw-flow* (*nP e1*) (*nP e2*)} })
lemma *PolEnforcePointExtended-offending-set*: *SecurityInvariant-withOffendingFlows.set-offending-flows sinvar* = *PolEnforcePointExtended-offending-set*
apply(*simp only*: *fun-eq-iff ENFnr-offending-set*[*OF PolEnforcePoint-ENFnr*] *PolEnforcePointExtended-offending-set*
apply(*rule allI*)+
apply(*rename-tac G nP*)
apply(*auto*)
done

interpretation *PolEnforcePointExtended*: *SecurityInvariant-ACS*
where *default-node-properties* = *default-node-properties*
and *sinvar* = *sinvar*
rewrites *SecurityInvariant-withOffendingFlows.set-offending-flows sinvar* = *PolEnforcePointExtended-offending-set*
unfolding *default-node-properties-def*
apply *unfold-locales*
apply(*rule ballI*)
apply (*rule SecurityInvariant-withOffendingFlows.ENFnr-fsts-weakrefl-instance*[*OF PolEnforcePoint-ENFnr Unassigned-botdefault All-to-Unassigned*])[*1*]
apply(*simp*)
apply(*simp*)
apply(*erule default-uniqueness-by-counterexample-ACS*)
apply (*simp add*: *SecurityInvariant-withOffendingFlows.set-offending-flows-def*
SecurityInvariant-withOffendingFlows.is-offending-flows-min-set-def
SecurityInvariant-withOffendingFlows.is-offending-flows-def)
apply (*simp add*:*graph-ops*)
apply (*simp split*: *prod.split-asm prod.split*)
apply(*rule-tac x*=(| *nodes*={*vertex-1*,*vertex-2*}, *edges* = {(*vertex-1*,*vertex-2*)} |) **in** *exI*, *simp*)

apply(*rule conjI*)
apply(*simp add*: *wf-graph-def*)
apply(*case-tac otherbot*, *simp-all*)
apply(*rename-tac secgwcase*)
apply(*rule-tac x*=(λ *x*. *Unassigned*)(*vertex-1* := *Unassigned*, *vertex-2* := *DomainMember*) **in** *exI*, *simp*)
apply(*rule-tac x*={(*vertex-1*,*vertex-2*)} **in** *exI*, *simp*)
apply(*rename-tac secgwINcase*)
apply(*rule-tac x*=(λ *x*. *Unassigned*)(*vertex-1* := *Unassigned*, *vertex-2* := *DomainMember*) **in** *exI*, *simp*)
apply(*rule-tac x*=*vertex-1* **in** *exI*, *simp*)
apply(*rule-tac x*={(*vertex-1*,*vertex-2*)} **in** *exI*, *simp*)
apply(*rename-tac membercase*)
apply(*rule-tac x*=(λ *x*. *Unassigned*)(*vertex-1* := *Unassigned*, *vertex-2* := *PolEnforcePoint*) **in** *exI*, *simp*)
apply(*rule-tac x*=*vertex-1* **in** *exI*, *simp*)
apply(*rule-tac x*={(*vertex-1*,*vertex-2*)} **in** *exI*, *simp*)
apply(*rule-tac x*=(λ *x*. *Unassigned*)(*vertex-1* := *Unassigned*, *vertex-2* := *PolEnforcePoint*) **in** *exI*, *simp*)
apply(*rule-tac x*=*vertex-1* **in** *exI*, *simp*)
apply(*rule-tac x*={(*vertex-1*,*vertex-2*)} **in** *exI*, *simp*)

apply(*fact PolEnforcePointExtended-offending-set*)
done

lemma *TopoS-PolEnforcePointExtended*: *SecurityInvariant sinvar default-node-properties receiver-violation*
unfolding *receiver-violation-def* **by** *unfold-locales*

hide-const (**open**) *sinvar receiver-violation*

end
theory *SINVAR-SecGwExt-impl*
imports *SINVAR-SecGwExt ../TopoS-Interface-impl*
begin

code-identifier code-module *SINVAR-SecGwExt-impl* => (*Scala*) *SINVAR-SecGwExt*

6.3.3 SecurityInvariant PolEnforcePointExtended List Implementation

fun *sinvar* :: *'v list-graph* $\Rightarrow$ (*'v* $\Rightarrow$ *SINVAR-SecGwExt.secgw-member*) $\Rightarrow$ *bool* **where**
sinvar G nP = ($\forall$ (*e1*,*e2*) $\in$ *set* (*edgesL G*). *e1* $\neq$ *e2* $\longrightarrow$ *SINVAR-SecGwExt.allowed-secgw-flow* (*nP e1*) (*nP e2*))

definition *PolEnforcePointExtended-offending-list*:: *'v list-graph* $\Rightarrow$ (*'v* $\Rightarrow$ *secgw-member*) $\Rightarrow$ (*'v* $\times$ *'v*) *list list* **where**
PolEnforcePointExtended-offending-list G nP = (*if sinvar G nP then*
[]
else
[[*e* $\leftarrow$ *edgesL G*. *case e of* (*e1*,*e2*) $\Rightarrow$ *e1* $\neq$ *e2* $\wedge$ $\neg$ *allowed-secgw-flow* (*nP e1*) (*nP e2*)]])

definition *NetModel-node-props P* = (λ *i*. (*case* (*node-properties P*) *i of Some property* ⇒ *property*
| *None* ⇒ *SINVAR-SecGwExt.default-node-properties*))
lemma[*code-unfold*]: *SecurityInvariant.node-props SINVAR-SecGwExt.default-node-properties P* = *NetModel-node-prop*
P
apply(*simp add*: *NetModel-node-props-def*)
done

definition *PolEnforcePoint-eval G P* = (*wf-list-graph G* ∧
sinvar G (*SecurityInvariant.node-props SINVAR-SecGwExt.default-node-properties P*))

interpretation *PolEnforcePoint-impl*:*TopoS-List-Impl*
where *default-node-properties*=*SINVAR-SecGwExt.default-node-properties*
and *sinvar-spec*=*SINVAR-SecGwExt.sinvar*
and *sinvar-impl*=*sinvar*
and *receiver-violation*=*SINVAR-SecGwExt.receiver-violation*
and *offending-flows-impl*=*PolEnforcePointExtended-offending-list*
and *node-props-impl*=*NetModel-node-props*
and *eval-impl*=*PolEnforcePoint-eval*
apply(*unfold TopoS-List-Impl-def*)
apply(*rule conjI*)
apply(*simp add*: *TopoS-PolEnforcePointExtended list-graph-to-graph-def*)
apply(*rule conjI*)
apply(*simp add*: *list-graph-to-graph-def PolEnforcePointExtended-offending-set PolEnforcePointExtended-offending-set*
PolEnforcePointExtended-offending-list-def)
apply(*rule conjI*)
apply(*simp only*: *NetModel-node-props-def*)
apply(*metis PolEnforcePointExtended.node-props.simps PolEnforcePointExtended.node-props-eq-node-props-formaldef*
apply(*simp only*: *PolEnforcePoint-eval-def*)
apply(*simp add*: *TopoS-eval-impl-proofrule*[*OF TopoS-PolEnforcePointExtended*])
apply(*simp-all add*: *list-graph-to-graph-def*)
done

6.3.4 PolEnforcePoint packing

definition *SINVAR-LIB-PolEnforcePointExtended* :: ('*v*::*vertex*, *secgw-member*) *TopoS-packed* **where**
SINVAR-LIB-PolEnforcePointExtended ≡
(| *nm-name* = ''*PolEnforcePointExtended*'',
nm-receiver-violation = *SINVAR-SecGwExt.receiver-violation*,
nm-default = *SINVAR-SecGwExt.default-node-properties*,
nm-sinvar = *sinvar*,
nm-offending-flows = *PolEnforcePointExtended-offending-list*,
nm-node-props = *NetModel-node-props*,
nm-eval = *PolEnforcePoint-eval*
|)
interpretation *SINVAR-LIB-PolEnforcePointExtended-interpretation*: *TopoS-modelLibrary SINVAR-LIB-PolEnforce*

SINVAR-SecGwExt.sinvar
apply(*unfold TopoS-modelLibrary-def SINVAR-LIB-PolEnforcePointExtended-def*)
apply(*rule conjI*)
apply(*simp*)
apply(*simp*)
by(*unfold-locales*)

Examples

definition *example-net-secgw* :: *nat list-graph* **where**
example-net-secgw ≡ (| *nodesL* = [*1*::*nat*,*2*, *3*, *8*,*9*, *11*,*12*],
edgesL = [(*3*,*8*),(*8*,*3*),(*2*,*8*),(*8*,*1*),(*1*,*9*),(*9*,*2*),(*2*,*9*),(*9*,*1*), (*1*,*3*), (*8*,*11*),(*8*,*12*), (*11*,*9*), (*11*,*3*), (*11*,*12*)] |)
value *wf-list-graph example-net-secgw*

definition *example-conf-secgw* **where**
example-conf-secgw ≡ ((λ*e*. *SINVAR-SecGwExt.default-node-properties*)
(*1* := *DomainMember*, *2*:= *DomainMember*, *3*:= *AccessibleMember*,
8:= *PolEnforcePoint*, *9*:= *PolEnforcePointIN*))

export-code *sinvar* **checking** *SML*
definition *test* = *sinvar* (| *nodesL*=[*1*::*nat*], *edgesL*=[] |) (λ-. *SINVAR-SecGwExt.default-node-properties*)
export-code *test* **checking** *SML*
value *sinvar* (| *nodesL*=[*1*::*nat*], *edgesL*=[] |) (λ-. *SINVAR-SecGwExt.default-node-properties*)

value *sinvar example-net-secgw example-conf-secgw*
value *PolEnforcePoint-offending-list example-net-secgw example-conf-secgw*

definition *example-net-secgw-invalid* **where**
example-net-secgw-invalid ≡ *example-net-secgw*(|*edgesL* := (*3*,*1*)#(*11*,*1*)#(*11*,*8*)#(*1*,*2*)#(*edgesL example-net-secgw*)|)

value *sinvar example-net-secgw-invalid example-conf-secgw*
value *PolEnforcePoint-offending-list example-net-secgw-invalid example-conf-secgw*

hide-const (**open**) *NetModel-node-props*
hide-const (**open**) *sinvar*

end
theory *SINVAR-Sink*
imports *../TopoS-Helper*
begin

6.4 SecurityInvariant Sink (IFS)

datatype *node-config* = *Sink* | *SinkPool* | *Unassigned*

definition *default-node-properties* :: *node-config*
where *default-node-properties* = *Unassigned*

fun *allowed-sink-flow* :: *node-config* ⇒ *node-config* ⇒ *bool* **where**
allowed-sink-flow Sink - = *False* |
allowed-sink-flow SinkPool SinkPool = *True* |
allowed-sink-flow SinkPool Sink = *True* |
allowed-sink-flow SinkPool - = *False* |
allowed-sink-flow Unassigned - = *True*

fun *sinvar* :: *'v graph* ⇒ (*'v* ⇒ *node-config*) ⇒ *bool* **where**
sinvar G nP = (∀ (*e1*,*e2*) ∈ *edges G*. *e1* ≠ *e2* ⟶ *allowed-sink-flow* (*nP e1*) (*nP e2*))

definition *receiver-violation* :: *bool* **where** *receiver-violation* = *True*

6.4.1 Preliminaries

lemma *sinvar-mono*: *SecurityInvariant-withOffendingFlows.sinvar-mono sinvar*
apply(*simp only*: *SecurityInvariant-withOffendingFlows.sinvar-mono-def*)
apply(*clarify*)
by *auto*

interpretation *SecurityInvariant-preliminaries*
where *sinvar* = *sinvar*
apply *unfold-locales*
apply(*frule-tac finite-distinct-list*[*OF wf-graph.finiteE*])
apply(*erule-tac exE*)
apply(*rename-tac list-edges*)
apply(*rule-tac ff*=*list-edges* **in** *SecurityInvariant-withOffendingFlows.mono-imp-set-offending-flows-not-empty*[*OF sinvar-mono*])
apply(*auto*)[*6*]
apply(*auto simp add*: *SecurityInvariant-withOffendingFlows.is-offending-flows-def graph-ops*)[*1*]
apply(*fact SecurityInvariant-withOffendingFlows.sinvar-mono-imp-is-offending-flows-mono*[*OF sinvar-mono*])
done

6.4.2 ENF

lemma *Sink-ENFnr*: *SecurityInvariant-withOffendingFlows.sinvar-all-edges-normal-form-not-refl sinvar allowed-sink-flow*
by(*simp add*: *SecurityInvariant-withOffendingFlows.sinvar-all-edges-normal-form-not-refl-def*)
lemma *Unassigned-to-All*: ∀ *e2*. *allowed-sink-flow Unassigned e2*
by (*rule allI*, *case-tac e2*, *simp-all*)
lemma *Unassigned-default-candidate*: ∀ *e1 e2*. ¬ *allowed-sink-flow e1 e2* ⟶ ¬ *allowed-sink-flow e1 Unassigned*
apply(*rule allI*)+
apply(*case-tac e2*)
apply *simp-all*
apply(*case-tac e1*)
apply *simp-all*
apply(*case-tac e1*)
apply *simp-all*
done

definition *Sink-offending-set*:: *'v graph* ⇒ (*'v* ⇒ *node-config*) ⇒ (*'v* × *'v*) *set set* **where**
Sink-offending-set G nP = (*if sinvar G nP then*
{}
else
{ {*e* ∈ *edges G*. *case e of* (*e1*,*e2*) ⇒ *e1* ≠ *e2* ∧ ¬ *allowed-sink-flow* (*nP e1*) (*nP e2*)} })
lemma *Sink-offending-set*:
SecurityInvariant-withOffendingFlows.set-offending-flows sinvar = *Sink-offending-set*
apply(*simp only*: *fun-eq-iff ENFnr-offending-set*[*OF Sink-ENFnr*] *Sink-offending-set-def*)
apply(*rule allI*)+
apply(*rename-tac G nP*)
apply(*auto*)
done

interpretation *Sink*: *SecurityInvariant-IFS*
where *default-node-properties = default-node-properties*
and *sinvar = sinvar*
rewrites *SecurityInvariant-withOffendingFlows.set-offending-flows sinvar = Sink-offending-set*
unfolding *default-node-properties-def*
apply *unfold-locales*
apply(*rule ballI*)
apply (*rule SecurityInvariant-withOffendingFlows.ENFnr-snds-weakrefl-instance*[*OF Sink-ENFnr Unassigned-default-candidate Unassigned-to-All*])
apply(*simp-all*)[*2*]

apply(*erule default-uniqueness-by-counterexample-IFS*)
apply (*simp add*: *SecurityInvariant-withOffendingFlows.set-offending-flows-def*
SecurityInvariant-withOffendingFlows.is-offending-flows-min-set-def
SecurityInvariant-withOffendingFlows.is-offending-flows-def)
apply (*simp add*:*graph-ops*)
apply (*simp split*: *prod.split-asm prod.split*)
apply(*rule-tac x=*⦇ *nodes=*{*vertex-1*,*vertex-2*}, *edges* = {(*vertex-1*,*vertex-2*)} ⦈ **in** *exI*, *simp*)
apply(*rule conjI*)
apply(*simp add*: *wf-graph-def*)
apply(*case-tac otherbot*, *simp-all*)
apply(*rule-tac x=*(λ *x*. *Unassigned*)(*vertex-1* := *SinkPool*, *vertex-2* := *Unassigned*) **in** *exI*, *simp*)
apply(*rule-tac x=vertex-2* **in** *exI*, *simp*)
apply(*rule-tac x=*{(*vertex-1*, *vertex-2*)} **in** *exI*, *simp*)
apply(*rule-tac x=*(λ *x*. *Unassigned*)(*vertex-1* := *SinkPool*, *vertex-2* := *Unassigned*) **in** *exI*, *simp*)
apply(*rule-tac x=vertex-2* **in** *exI*, *simp*)
apply(*rule-tac x=*{(*vertex-1*, *vertex-2*)} **in** *exI*, *simp*)

apply(*fact Sink-offending-set*)
done

lemma *TopoS-Sink*: *SecurityInvariant sinvar default-node-properties receiver-violation*
unfolding *receiver-violation-def* **by** *unfold-locales*

hide-fact (**open**) *sinvar-mono*
hide-const (**open**) *sinvar receiver-violation default-node-properties*

end
theory *SINVAR-Sink-impl*
imports *SINVAR-Sink ../TopoS-Interface-impl*
begin

code-identifier code-module *SINVAR-Sink-impl* => (*Scala*) *SINVAR-Sink*

6.4.3 SecurityInvariant Sink (IFS) List Implementation

fun *sinvar* :: *'v list-graph* ⇒ (*'v* ⇒ *node-config*) ⇒ *bool* **where**
sinvar G nP = (∀ (*e1*,*e2*) ∈ *set* (*edgesL G*). *e1* ≠ *e2* ⟶ *SINVAR-Sink.allowed-sink-flow* (*nP e1*) (*nP e2*))

definition *Sink-offending-list*:: *'v list-graph* ⇒ (*'v* ⇒ *SINVAR-Sink.node-config*) ⇒ (*'v* × *'v*) *list list*
where

```
  Sink-offending-list G nP = (if sinvar G nP then
    []
   else
    [ [e ← edgesL G. case e of (e1,e2) ⇒ e1 ≠ e2 ∧ ¬ allowed-sink-flow (nP e1) (nP e2)] ])


definition NetModel-node-props P = (λ i. (case (node-properties P) i of Some property ⇒ property
| None ⇒ SINVAR-Sink.default-node-properties))
lemma[code-unfold]: SecurityInvariant.node-props SINVAR-Sink.default-node-properties P = NetModel-node-props
P
apply(simp add: NetModel-node-props-def)
done

definition Sink-eval G P = (wf-list-graph G ∧
  sinvar G (SecurityInvariant.node-props SINVAR-Sink.default-node-properties P))


interpretation Sink-impl:TopoS-List-Impl
  where default-node-properties=SINVAR-Sink.default-node-properties
  and sinvar-spec=SINVAR-Sink.sinvar
  and sinvar-impl=sinvar
  and receiver-violation=SINVAR-Sink.receiver-violation
  and offending-flows-impl=Sink-offending-list
  and node-props-impl=NetModel-node-props
  and eval-impl=Sink-eval
 apply(unfold TopoS-List-Impl-def)
 apply(rule conjI)
  apply(simp add: TopoS-Sink list-graph-to-graph-def)
 apply(rule conjI)
 apply(simp add: list-graph-to-graph-def Sink-offending-set Sink-offending-set-def Sink-offending-list-def)
 apply(rule conjI)
  apply(simp only: NetModel-node-props-def)
  apply(metis Sink.node-props.simps Sink.node-props-eq-node-props-formaldef)
 apply(simp only: Sink-eval-def)
 apply(intro allI)
 apply(rule TopoS-eval-impl-proofrule[OF TopoS-Sink])
 apply(simp-all add: list-graph-to-graph-def)
done
```

6.4.4 Sink packing

```
  definition SINVAR-LIB-Sink :: ('v::vertex, node-config) TopoS-packed where
    SINVAR-LIB-Sink ≡
    (| nm-name = ''Sink'',
      nm-receiver-violation = SINVAR-Sink.receiver-violation,
      nm-default = SINVAR-Sink.default-node-properties,
      nm-sinvar = sinvar,
      nm-offending-flows = Sink-offending-list,
      nm-node-props = NetModel-node-props,
      nm-eval = Sink-eval
      |)
  interpretation SINVAR-LIB-Sink-interpretation: TopoS-modelLibrary SINVAR-LIB-Sink
      SINVAR-Sink.sinvar
```

apply(*unfold TopoS-modelLibrary-def SINVAR-LIB-Sink-def*)
apply(*rule conjI*)
apply(*simp*)
apply(*simp*)
by(*unfold-locales*)

Examples

definition *example-net-sink* :: *nat list-graph* **where**
example-net-sink ≡ (| *nodesL* = [*1*::*nat*,*2*,*3*, *8*, *11*,*12*],
edgesL = [(*1*,*8*),(*1*,*2*), (*2*,*8*),(*3*,*8*),(*4*,*8*), (*2*,*3*),(*3*,*2*), (*11*,*8*),(*12*,*8*), (*11*,*12*), (*1*,*12*)] |)
value *wf-list-graph example-net-sink*

definition *example-conf-sink* **where**
example-conf-sink ≡ (λ*e*. *SINVAR-Sink.default-node-properties*)(*8*:= *Sink*, *2*:= *SinkPool*, *3*:= *SinkPool*, *4*:= *SinkPool*)

value *sinvar example-net-sink example-conf-sink*
value *Sink-offending-list example-net-sink example-conf-sink*

definition *example-net-sink-invalid* **where**
example-net-sink-invalid ≡ *example-net-sink*(|*edgesL* := (*2*,*1*)#(*8*,*11*)#(*8*,*2*)#(*edgesL example-net-sink*)|)

value *sinvar example-net-sink-invalid example-conf-sink*
value *Sink-offending-list example-net-sink-invalid example-conf-sink*

hide-const (**open**) *NetModel-node-props*
hide-const (**open**) *sinvar*

end
theory *SINVAR-SubnetsInGW*
imports../*TopoS-Helper*
begin

6.5 SecurityInvariant SubnetsInGW

datatype *subnets* = *Member* | *InboundGateway* | *Unassigned*

definition *default-node-properties* :: *subnets*
where *default-node-properties* ≡ *Unassigned*

fun *allowed-subnet-flow* :: *subnets* ⇒ *subnets* ⇒ *bool* **where**
allowed-subnet-flow Member - = True |
allowed-subnet-flow InboundGateway - = True |
allowed-subnet-flow Unassigned Unassigned = True |
allowed-subnet-flow Unassigned InboundGateway = True|
allowed-subnet-flow Unassigned Member = False

fun *sinvar* :: *'v graph* ⇒ (*'v* ⇒ *subnets*) ⇒ *bool* **where**
sinvar G nP = (∀ (*e1*,*e2*) ∈ *edges G*. *allowed-subnet-flow* (*nP e1*) (*nP e2*))

definition *receiver-violation* :: *bool* **where** *receiver-violation* = *False*

6.5.1 Preliminaries

lemma *sinvar-mono*: *SecurityInvariant-withOffendingFlows.sinvar-mono sinvar*
apply(*simp only*: *SecurityInvariant-withOffendingFlows.sinvar-mono-def*)
apply(*clarify*)
by *auto*

interpretation *SecurityInvariant-preliminaries*
where *sinvar = sinvar*
apply *unfold-locales*
apply(*frule-tac finite-distinct-list*[*OF wf-graph.finiteE*])
apply(*erule-tac exE*)
apply(*rename-tac list-edges*)
apply(*rule-tac ff=list-edges* **in** *SecurityInvariant-withOffendingFlows.mono-imp-set-offending-flows-not-empty*[*OF sinvar-mono*])
apply(*auto*)[*6*]
apply(*auto simp add*: *SecurityInvariant-withOffendingFlows.is-offending-flows-def graph-ops*)[*1*]
apply(*fact SecurityInvariant-withOffendingFlows.sinvar-mono-imp-is-offending-flows-mono*[*OF sinvar-mono*])
done

6.5.2 ENF

lemma *Unassigned-not-to-Member*: ¬ *allowed-subnet-flow Unassigned Member*
by(*simp*)
lemma *All-to-Unassigned*: *allowed-subnet-flow e1 Unassigned*
by (*case-tac e1*, *simp-all*)
lemma *Member-to-All*: *allowed-subnet-flow Member e2*
by (*case-tac e2*, *simp-all*)
lemma *Unassigned-default-candidate*: ∀ *nP e1 e2*. ¬ *allowed-subnet-flow* (*nP e1*) (*nP e2*) ⟶ ¬ *allowed-subnet-flow Unassigned* (*nP e2*)
apply(*rule allI*)+
apply(*case-tac nP e2*)
apply *simp*
apply(*case-tac nP e1*)
apply(*simp-all*)[*3*]
by(*simp add*: *All-to-Unassigned*)
lemma *allowed-subnet-flow-refl*: *allowed-subnet-flow e e*
by(*case-tac e*, *simp-all*)
lemma *SubnetsInGW-ENF*: *SecurityInvariant-withOffendingFlows.sinvar-all-edges-normal-form sinvar allowed-subnet-flow*
unfolding *SecurityInvariant-withOffendingFlows.sinvar-all-edges-normal-form-def*
by *simp*
lemma *SubnetsInGW-ENF-refl*: *SecurityInvariant-withOffendingFlows.ENF-refl sinvar allowed-subnet-flow*
unfolding *SecurityInvariant-withOffendingFlows.ENF-refl-def*
apply(*rule conjI*)
apply(*simp add*: *SubnetsInGW-ENF*)
apply(*simp add*: *allowed-subnet-flow-refl*)
done

definition *SubnetsInGW-offending-set*:: *'v graph* ⇒ (*'v* ⇒ *subnets*) ⇒ (*'v* × *'v*) *set set* **where**
SubnetsInGW-offending-set G nP = (*if sinvar G nP then*
{}
else
{ {*e* ∈ *edges G*. *case e of* (*e1*,*e2*) ⇒ ¬ *allowed-subnet-flow* (*nP e1*) (*nP e2*)} })
lemma *SubnetsInGW-offending-set*:

SecurityInvariant-withOffendingFlows.set-offending-flows sinvar = SubnetsInGW-offending-set
apply(*simp only*: *fun-eq-iff ENF-offending-set*[*OF SubnetsInGW-ENF*] *SubnetsInGW-offending-set-def*)
apply(*rule allI*)+
apply(*rename-tac G nP*)
apply(*auto*)
done

interpretation *SubnetsInGW*: *SecurityInvariant-ACS*
where *default-node-properties = SINVAR-SubnetsInGW.default-node-properties*
and *sinvar = SINVAR-SubnetsInGW.sinvar*
rewrites *SecurityInvariant-withOffendingFlows.set-offending-flows sinvar = SubnetsInGW-offending-set*
unfolding *SINVAR-SubnetsInGW.default-node-properties-def*
apply *unfold-locales*

apply(*rule ballI*)
thm *SecurityInvariant-withOffendingFlows.ENF-fsts-refl-instance*[*OF SubnetsInGW-ENF-refl Unassigned-default-can*
apply(*rule SecurityInvariant-withOffendingFlows.ENF-fsts-refl-instance*[*OF SubnetsInGW-ENF-refl*
Unassigned-default-candidate])
apply(*simp-all*)[*2*]

apply(*erule default-uniqueness-by-counterexample-ACS*)
apply (*simp add*: *SecurityInvariant-withOffendingFlows.set-offending-flows-def*
SecurityInvariant-withOffendingFlows.is-offending-flows-min-set-def
SecurityInvariant-withOffendingFlows.is-offending-flows-def)
apply (*simp add*:*graph-ops*)
apply (*simp split*: *prod.split-asm prod.split*)
apply(*rule-tac x=*⦇ *nodes=*{*vertex-1*,*vertex-2*}, *edges* = {(*vertex-1*,*vertex-2*)} ⦈ **in** *exI*, *simp*)
apply(*rule conjI*)
apply(*simp add*: *wf-graph-def*)
apply(*case-tac otherbot*, *simp-all*)
apply(*rule-tac x=*(λ *x*. *Unassigned*)(*vertex-1* := *Unassigned*, *vertex-2* := *Member*) **in** *exI*, *simp*)
apply(*rule-tac x=*{(*vertex-1*,*vertex-2*)} **in** *exI*, *simp*)
apply(*rule-tac x=*(λ *x*. *Unassigned*)(*vertex-1* := *Unassigned*, *vertex-2* := *Member*) **in** *exI*, *simp*)
apply(*rule-tac x=vertex-1* **in** *exI*, *simp*)
apply(*rule-tac x=*{(*vertex-1*,*vertex-2*)} **in** *exI*, *simp*)

apply(*fact SubnetsInGW-offending-set*)
done

lemma *TopoS-SubnetsInGW*: *SecurityInvariant sinvar default-node-properties receiver-violation*
unfolding *receiver-violation-def* **by** *unfold-locales*

hide-fact (**open**) *sinvar-mono*
hide-const (**open**) *sinvar receiver-violation default-node-properties*

end
theory *SINVAR-SubnetsInGW-impl*
imports *SINVAR-SubnetsInGW ../TopoS-Interface-impl*
begin

code-identifier code-module *SINVAR-SubnetsInGW-impl => (Scala) SINVAR-SubnetsInGW*

6.5.3 SecurityInvariant SubnetsInGw List Implementation

fun *sinvar* :: *'v list-graph* ⇒ (*'v* ⇒ *subnets*) ⇒ *bool* **where**
sinvar G nP = (∀ (*e1*,*e2*) ∈ *set* (*edgesL G*). *SINVAR-SubnetsInGW.allowed-subnet-flow* (*nP e1*) (*nP e2*))

definition *SubnetsInGW-offending-list*:: *'v list-graph* ⇒ (*'v* ⇒ *subnets*) ⇒ (*'v* × *'v*) *list list* **where**
SubnetsInGW-offending-list G nP = (*if sinvar G nP then*
[]
else
[[*e* ← *edgesL G*. *case e of* (*e1*,*e2*) ⇒ ¬ *allowed-subnet-flow* (*nP e1*) (*nP e2*)]])

definition *NetModel-node-props P* = (λ *i*. (*case* (*node-properties P*) *i of Some property* ⇒ *property* | *None* ⇒ *SINVAR-SubnetsInGW.default-node-properties*))
lemma[*code-unfold*]: *SecurityInvariant.node-props SINVAR-SubnetsInGW.default-node-properties P* = *NetModel-node-props P*
apply(*simp add*: *NetModel-node-props-def*)
done

definition *SubnetsInGW-eval G P* = (*wf-list-graph G* ∧
sinvar G (*SecurityInvariant.node-props SINVAR-SubnetsInGW.default-node-properties P*))

interpretation *SubnetsInGW-impl*:*TopoS-List-Impl*
where *default-node-properties*=*SINVAR-SubnetsInGW.default-node-properties*
and *sinvar-spec*=*SINVAR-SubnetsInGW.sinvar*
and *sinvar-impl*=*sinvar*
and *receiver-violation*=*SINVAR-SubnetsInGW.receiver-violation*
and *offending-flows-impl*=*SubnetsInGW-offending-list*
and *node-props-impl*=*NetModel-node-props*
and *eval-impl*=*SubnetsInGW-eval*
apply(*unfold TopoS-List-Impl-def*)
apply(*rule conjI*)
apply(*simp add*: *TopoS-SubnetsInGW list-graph-to-graph-def*)
apply(*rule conjI*)
apply(*simp add*: *list-graph-to-graph-def SubnetsInGW-offending-set SubnetsInGW-offending-set-def SubnetsInGW-offending-list-def*)
apply(*rule conjI*)
apply(*simp only*: *NetModel-node-props-def*)
apply(*metis SubnetsInGW.node-props.simps SubnetsInGW.node-props-eq-node-props-formaldef*)
apply(*simp only*: *SubnetsInGW-eval-def*)
apply(*simp add*: *TopoS-eval-impl-proofrule*[*OF TopoS-SubnetsInGW*])
apply(*simp-all add*: *list-graph-to-graph-def*)
done

6.5.4 SubnetsInGW packing

definition *SINVAR-LIB-SubnetsInGW* :: (*'v*::*vertex*, *subnets*) *TopoS-packed* **where**
SINVAR-LIB-SubnetsInGW ≡
(| *nm-name* = *''SubnetsInGW''*,
nm-receiver-violation = *SINVAR-SubnetsInGW.receiver-violation*,
nm-default = *SINVAR-SubnetsInGW.default-node-properties*,

nm-sinvar = *sinvar*,
nm-offending-flows = *SubnetsInGW-offending-list*,
nm-node-props = *NetModel-node-props*,
nm-eval = *SubnetsInGW-eval*
⦈
interpretation *SINVAR-LIB-SubnetsInGW-interpretation*: *TopoS-modelLibrary SINVAR-LIB-SubnetsInGW SINVAR-SubnetsInGW.sinvar*
apply(*unfold TopoS-modelLibrary-def SINVAR-LIB-SubnetsInGW-def*)
apply(*rule conjI*)
apply(*simp*)
apply(*simp*)
by(*unfold-locales*)

Examples

definition *example-net-sub* :: *nat list-graph* **where**
example-net-sub ≡ ⦇ *nodesL* = [*1*::*nat*,*2*,*3*,*4*, *8*, *11*,*12*,*42*],
edgesL = [(*1*,*2*),(*1*,*3*),(*1*,*4*),(*2*,*1*),(*2*,*3*),(*2*,*4*),(*3*,*1*),(*3*,*2*),(*3*,*4*),(*4*,*1*),(*4*,*2*),(*4*,*3*),
(*8*,*1*),(*8*,*2*),
(*8*,*11*),
(*11*,*8*), (*12*,*8*),
(*11*,*42*), (*12*,*42*), (*8*,*42*)] ⦈
value *wf-list-graph example-net-sub*

definition *example-conf-sub* **where**
example-conf-sub ≡ ((λ*e*. *SINVAR-SubnetsInGW.default-node-properties*)
(*1* := *Member*, *2*:= *Member*, *3*:= *Member*, *4*:=*Member*,
8:=*InboundGateway*))

value *sinvar example-net-sub example-conf-sub*

definition *example-net-sub-invalid* **where**
example-net-sub-invalid ≡ *example-net-sub*(⦇*edgesL* := (*42*,*4*)#(*edgesL example-net-sub*)⦈

value *sinvar example-net-sub-invalid example-conf-sub*
value *SubnetsInGW-offending-list example-net-sub-invalid example-conf-sub*

hide-const (**open**) *NetModel-node-props*
hide-const (**open**) *sinvar*

end
theory *SINVAR-CommunicationPartners*
imports *../TopoS-Helper*
begin

6.6 SecurityInvariant CommunicationPartners

Idea of this securityinvariant: Only some nodes can communicate with Master nodes. It constrains who may access master nodes, Master nodes can access the world (except other prohibited master nodes). A node configured as Master has a list of nodes that can access it. Also, in order to be able to access a Master node, the sender must be denoted as a node we

Care about. By default, all nodes are set to DontCare, thus they cannot access Master nodes. But they can access all other DontCare nodes and Care nodes.

TL;DR: An access control list determines who can access a master node.

datatype *'v node-config = DontCare | Care | Master 'v list*

definition *default-node-properties :: 'v node-config*
where *default-node-properties = DontCare*

Unrestricted accesses among DontCare nodes!

fun *allowed-flow :: 'v node-config ⇒ 'v ⇒ 'v node-config ⇒ 'v ⇒ bool* **where**
allowed-flow DontCare - DontCare - = True |
allowed-flow DontCare - Care - = True |
allowed-flow DontCare - (Master -) - = False |
allowed-flow Care - Care - = True |
allowed-flow Care - DontCare - = True |
allowed-flow Care s (Master M) r = (s ∈ set M) |
allowed-flow (Master -) s (Master M) r = (s ∈ set M) |
allowed-flow (Master -) - Care - = True |
allowed-flow (Master -) - DontCare - = True

fun *sinvar :: 'v graph ⇒ ('v ⇒ 'v node-config) ⇒ bool* **where**
sinvar G nP = (∀ (s,r) ∈ edges G. s ≠ r ⟶ allowed-flow (nP s) s (nP r) r)

definition *receiver-violation :: bool* **where** *receiver-violation = False*

6.6.1 Preliminaries

lemma *sinvar-mono*: *SecurityInvariant-withOffendingFlows.sinvar-mono sinvar*
apply(*simp only*: *SecurityInvariant-withOffendingFlows.sinvar-mono-def*)
apply(*clarify*)
by *auto*

interpretation *SecurityInvariant-preliminaries*
where *sinvar = sinvar*
apply *unfold-locales*
apply(*frule-tac finite-distinct-list*[*OF wf-graph.finiteE*])
apply(*erule-tac exE*)
apply(*rename-tac list-edges*)
apply(*rule-tac ff=list-edges* **in** *SecurityInvariant-withOffendingFlows.mono-imp-set-offending-flows-not-empty*[*OF sinvar-mono*])
apply(*auto*)[*6*]
apply(*auto simp add*: *SecurityInvariant-withOffendingFlows.is-offending-flows-def graph-ops*)[*1*]
apply(*fact SecurityInvariant-withOffendingFlows.sinvar-mono-imp-is-offending-flows-mono*[*OF sinvar-mono*])
done

6.6.2 ENRnr

lemma *CommunicationPartners-ENRnrSR*: *SecurityInvariant-withOffendingFlows.sinvar-all-edges-normal-form-not-r sinvar allowed-flow*
by(*simp add*: *SecurityInvariant-withOffendingFlows.sinvar-all-edges-normal-form-not-refl-SR-def*)
lemma *Unassigned-weakrefl*: *∀ s r. allowed-flow DontCare s DontCare r*
by(*simp*)

lemma *Unassigned-botdefault*: $\forall\ s\ r.\ (nP\ r) \neq DontCare \longrightarrow \neg\ allowed\text{-}flow\ (nP\ s)\ s\ (nP\ r)\ r \longrightarrow \neg\ allowed\text{-}flow\ DontCare\ s\ (nP\ r)\ r$
apply(*rule allI*)+
apply(*case-tac nP r*)
apply(*simp-all*)
apply(*case-tac nP s*)
apply(*simp-all*)
done
lemma $\neg\ allowed\text{-}flow\ DontCare\ s\ (Master\ M)\ r$ **by**(*simp*)
lemma $\neg\ allowed\text{-}flow\ any\ s\ (Master\ [])\ r$ **by**(*cases any*, *simp-all*)

lemma *All-to-Unassigned*: $\forall\ s\ r.\ allowed\text{-}flow\ (nP\ s)\ s\ DontCare\ r$
by (*rule allI*, *rule allI*, *case-tac nP s*, *simp-all*)
lemma *Unassigned-default-candidate*: $\forall\ s\ r.\ \neg\ allowed\text{-}flow\ (nP\ s)\ s\ (nP\ r)\ r \longrightarrow \neg\ allowed\text{-}flow\ DontCare\ s\ (nP\ r)\ r$
apply(*intro allI*, *rename-tac s r*)+
apply(*case-tac nP s*)
apply(*simp-all*)
apply(*case-tac nP r*)
apply(*simp-all*)
apply(*case-tac nP r*)
apply(*simp-all*)
done

definition *CommunicationPartners-offending-set*:: *'v graph* $\Rightarrow$ (*'v* $\Rightarrow$ *'v node-config*) $\Rightarrow$ (*'v* $\times$ *'v*) *set set* **where**
CommunicationPartners-offending-set G nP = (*if sinvar G nP then*
{}
else
{ {*e* $\in$ *edges G*. *case e of* (*e1*,*e2*) $\Rightarrow$ *e1* $\neq$ *e2* $\wedge$ $\neg$ *allowed-flow* (*nP e1*) *e1* (*nP e2*) *e2*} })
lemma *CommunicationPartners-offending-set*:
SecurityInvariant-withOffendingFlows.set-offending-flows sinvar = *CommunicationPartners-offending-set*
apply(*simp only*: *fun-eq-iff ENFnrSR-offending-set*[*OF CommunicationPartners-ENRnrSR*] *CommunicationPartners*
apply(*rule allI*)+
apply(*rename-tac G nP*)
apply(*auto*)
done

interpretation *CommunicationPartners*: *SecurityInvariant-ACS*
where *default-node-properties* = *default-node-properties*
and *sinvar* = *sinvar*
rewrites *SecurityInvariant-withOffendingFlows.set-offending-flows sinvar* = *CommunicationPartners-offending-set*
unfolding *receiver-violation-def*
unfolding *default-node-properties-def*
apply *unfold-locales*
apply(*rule ballI*)
apply (*rule-tac f*=*f* **in** *SecurityInvariant-withOffendingFlows.ENFnrSR-fsts-weakrefl-instance*[*OF CommunicationPartners-ENRnrSR Unassigned-weakrefl Unassigned-botdefault All-to-Unassigned*])
apply(*simp*)
apply(*simp*)
apply(*erule default-uniqueness-by-counterexample-ACS*)
apply(*rule-tac x*=(| *nodes*={*vertex-1*,*vertex-2*}, *edges* = {(*vertex-1*,*vertex-2*)} |) **in** *exI*, *simp*)
apply(*rule conjI*)

apply(*simp add*: *wf-graph-def*)
apply(*simp add*: *CommunicationPartners-offending-set CommunicationPartners-offending-set-def delete-edges-simp2*)
apply(*case-tac otherbot*, *simp-all*)
apply(*rule-tac x*=(λ *x*. *DontCare*)(*vertex-1* := *DontCare*, *vertex-2* := *Master* [*vertex-1*]) **in** *exI*, *simp*)
apply(*rule-tac x*=*vertex-1* **in** *exI*, *simp*)
apply(*simp split*: *prod.split*)
apply(*clarify*)
apply *force*
apply(*rename-tac M*)
apply(*rule-tac x*=(λ *x*. *DontCare*)(*vertex-1* := *DontCare*, *vertex-2* := (*Master* (*vertex-1*#*M'*))) **in** *exI*, *simp*)
apply(*simp split*: *prod.split*)
apply(*clarify*)
apply *force*
apply(*fact CommunicationPartners-offending-set*)
done

lemma *TopoS-SubnetsInGW*: *SecurityInvariant sinvar default-node-properties receiver-violation*
unfolding *receiver-violation-def* **by** *unfold-locales*

Example:

lemma *sinvar* (|*nodes* = {''*db1*'', ''*db2*'', ''*h1*'', ''*h2*'', ''*foo*'', ''*bar*''},
edges = {(''*h1*'', ''*db1*''), (''*h2*'', ''*db1*''), (''*h1*'', ''*h2*''),
(''*db1*'', ''*h1*''), (''*db1*'', ''*foo*''), (''*db1*'', ''*db2*''), (''*db1*'', ''*db1*''),
(''*h1*'', ''*foo*''), (''*foo*'', ''*h1*''), (''*foo*'', ''*bar*'')}|)
(((((λ*h*. *default-node-properties*)(''*h1*'' := *Care*))(''*h2*'' := *Care*))
(''*db1*'' := *Master* [''*h1*'', ''*h2*'']))(''*db2*'' := *Master* [''*db1*'']))) **by** *eval*

hide-fact (**open**) *sinvar-mono*
hide-const (**open**) *sinvar receiver-violation default-node-properties*

end
theory *SINVAR-CommunicationPartners-impl*
imports *SINVAR-CommunicationPartners ../TopoS-Interface-impl*
begin

code-identifier code-module *SINVAR-CommunicationPartners-impl* => (*Scala*) *SINVAR-CommunicationPartners*

6.6.3 SecurityInvariant CommunicationPartners List Implementation

fun *sinvar* :: ′*v list-graph* ⇒ (′*v* ⇒ ′*v node-config*) ⇒ *bool* **where**
sinvar G nP = (∀ (*s*,*r*) ∈ *set* (*edgesL G*). *s* ≠ *r* ⟶ *SINVAR-CommunicationPartners.allowed-flow* (*nP s*) *s* (*nP r*) *r*)

definition *CommunicationPartners-offending-list*:: ′*v list-graph* ⇒ (′*v* ⇒ ′*v node-config*) ⇒ (′*v* × ′*v*) *list list* **where**
CommunicationPartners-offending-list G nP = (*if sinvar G nP then*
[]

else
[[*e* ← *edgesL G*. *case e of* (*e1*,*e2*) ⇒ *e1* ≠ *e2* ∧ ¬ *allowed-flow* (*nP e1*) *e1* (*nP e2*) *e2*]])

thm *SINVAR-CommunicationPartners.CommunicationPartners.node-props.simps*
definition *NetModel-node-props* (*P*::('*v*::*vertex*, '*v node-config*) *TopoS-Params*) =
(λ *i*. (*case* (*node-properties P*) *i of Some property* ⇒ *property* | *None* ⇒ *SINVAR-CommunicationPartners.default-node-*
lemma[*code-unfold*]: *SecurityInvariant.node-props SINVAR-CommunicationPartners.default-node-properties*
P = *NetModel-node-props P*
apply(*simp add*: *NetModel-node-props-def*)
done

definition *CommunicationPartners-eval G P* = (*wf-list-graph G* ∧
sinvar G (*SecurityInvariant.node-props SINVAR-CommunicationPartners.default-node-properties P*))

interpretation *CommunicationPartners-impl*:*TopoS-List-Impl*
where *default-node-properties*=*SINVAR-CommunicationPartners.default-node-properties*
and *sinvar-spec*=*SINVAR-CommunicationPartners.sinvar*
and *sinvar-impl*=*sinvar*
and *receiver-violation*=*SINVAR-CommunicationPartners.receiver-violation*
and *offending-flows-impl*=*CommunicationPartners-offending-list*
and *node-props-impl*=*NetModel-node-props*
and *eval-impl*=*CommunicationPartners-eval*
apply(*unfold TopoS-List-Impl-def*)
apply(*rule conjI*)
apply(*simp add*: *TopoS-SubnetsInGW list-graph-to-graph-def*; *fail*)
apply(*rule conjI*)
apply(*simp add*: *list-graph-to-graph-def CommunicationPartners-offending-set CommunicationPartners-offending-set-d*
CommunicationPartners-offending-list-def)
apply(*rule conjI*)
apply(*simp only*: *NetModel-node-props-def*)
apply(*metis CommunicationPartners.node-props.simps CommunicationPartners.node-props-eq-node-props-formaldef*)
apply(*simp only*: *CommunicationPartners-eval-def*)
apply(*simp add*: *TopoS-eval-impl-proofrule*[*OF TopoS-SubnetsInGW*])
apply(*simp-all add*: *list-graph-to-graph-def*)
done

6.6.4 CommunicationPartners packing

definition *SINVAR-LIB-CommunicationPartners* :: ('*v*::*vertex*, '*v SINVAR-CommunicationPartners.node-config*)
TopoS-packed **where**
SINVAR-LIB-CommunicationPartners ≡
(| *nm-name* = ''*CommunicationPartners*'',
nm-receiver-violation = *SINVAR-CommunicationPartners.receiver-violation*,
nm-default = *SINVAR-CommunicationPartners.default-node-properties*,
nm-sinvar = *sinvar*,
nm-offending-flows = *CommunicationPartners-offending-list*,
nm-node-props = *NetModel-node-props*,
nm-eval = *CommunicationPartners-eval*
|)
interpretation *SINVAR-LIB-CommunicationPartners-interpretation*: *TopoS-modelLibrary SINVAR-LIB-Communica*
SINVAR-CommunicationPartners.sinvar
apply(*unfold TopoS-modelLibrary-def SINVAR-LIB-CommunicationPartners-def*)

apply(*rule conjI*)
apply(*simp*)
apply(*simp*)
by(*unfold-locales*)

Examples

hide-const (**open**) *NetModel-node-props*
hide-const (**open**) *sinvar*

end
theory *SINVAR-NoRefl*
imports *../TopoS-Helper*
begin

6.7 SecurityInvariant NoRefl

Hosts are not allowed to communicate with themselves.

This can be used to effectively lift hosts to roles. Just list all roles that are allowed to communicate with themselves. Otherwise, communication between hosts of the same role (group) is prohibited. Useful in conjunction with the security gateway.

datatype *node-config* = *NoRefl* | *Refl*

definition *default-node-properties* :: *node-config*
where *default-node-properties* = *NoRefl*

fun *sinvar* :: *'v graph* ⇒ (*'v* ⇒ *node-config*) ⇒ *bool* **where**
sinvar G nP = (∀ (*s*, *r*) ∈ *edges G*. *s* = *r* ⟶ *nP s* = *Refl*)

definition *receiver-violation* :: *bool* **where** *receiver-violation* = *False*

6.7.1 Preliminaries

lemma *sinvar-mono*: *SecurityInvariant-withOffendingFlows.sinvar-mono sinvar*
apply(*simp only*: *SecurityInvariant-withOffendingFlows.sinvar-mono-def*)
apply(*clarify*)
by *auto*

interpretation *SecurityInvariant-preliminaries*
where *sinvar* = *sinvar*
apply *unfold-locales*
apply(*frule-tac finite-distinct-list*[*OF wf-graph.finiteE*])
apply(*erule-tac exE*)
apply(*rename-tac list-edges*)
apply(*rule-tac ff*=*list-edges* **in** *SecurityInvariant-withOffendingFlows.mono-imp-set-offending-flows-not-empty*[*OF sinvar-mono*])
apply(*auto*)[*6*]
apply(*auto simp add*: *SecurityInvariant-withOffendingFlows.is-offending-flows-def graph-ops*)[*1*]
apply(*fact SecurityInvariant-withOffendingFlows.sinvar-mono-imp-is-offending-flows-mono*[*OF sinvar-mono*])
done

lemma *NoRfl-ENRsr*: *SecurityInvariant-withOffendingFlows.sinvar-all-edges-normal-form-sr sinvar* (λ nP_s *s* nP_r *r*. *s* = *r* ⟶ nP_s = *Refl*)
by(*simp add*: *SecurityInvariant-withOffendingFlows.sinvar-all-edges-normal-form-sr-def*)

definition *NoRefl-offending-set*:: *'v graph* ⇒ (*'v* ⇒ *node-config*) ⇒ (*'v* × *'v*) *set set* **where**
NoRefl-offending-set G nP = (*if sinvar G nP then*
{}
else
{ {*e* ∈ *edges G*. *case e of* (*e1*,*e2*) ⇒ *e1* = *e2* ∧ *nP e1* = *NoRefl*} })

thm *SecurityInvariant-withOffendingFlows.ENFsr-offending-set*[*OF NoRfl-ENRsr*]

lemma *NoRefl-offending-set*: *SecurityInvariant-withOffendingFlows.set-offending-flows sinvar* = *NoRefl-offending-set*
apply(*simp only*: *fun-eq-iff NoRefl-offending-set-def*)
apply(*intro allI*, *rename-tac G nP*)
apply(*simp only*: *SecurityInvariant-withOffendingFlows.ENFsr-offending-set*[*OF NoRfl-ENRsr*])
apply(*case-tac sinvar G nP*)
apply(*simp*; *fail*)
apply(*simp*)
apply(*rule*)
apply(*rule*)
apply(*clarsimp*)
using *node-config.exhaust* **apply** *blast*
apply(*rule*)
apply(*rule*)
apply(*clarsimp*)
done

lemma *NoRefl-unique-default*:
∀ *G f nP i*. *wf-graph G* ∧ *f* ∈ *set-offending-flows G nP* ∧ *i* ∈ *fst* ‘ *f* ⟶ ¬ *sinvar G* (*nP*(*i* := *otherbot*)) ⟹
otherbot = *NoRefl*
apply(*erule default-uniqueness-by-counterexample-ACS*)
apply (*simp add*: *SecurityInvariant-withOffendingFlows.set-offending-flows-def*
SecurityInvariant-withOffendingFlows.is-offending-flows-min-set-def
SecurityInvariant-withOffendingFlows.is-offending-flows-def)
apply (*simp add*:*graph-ops*)
apply (*simp split*: *prod.split-asm prod.split*)
apply(*rule-tac x*=(| *nodes*={*vertex-1*}, *edges* = {(*vertex-1*,*vertex-1*)} |) **in** *exI*, *simp*)
apply(*rule conjI*)
apply(*simp add*: *wf-graph-def*)
apply(*case-tac otherbot*, *simp-all*)
apply(*rule-tac x*=(λ *x*. *NoRefl*)(*vertex-1* := *NoRefl*, *vertex-2* := *NoRefl*) **in** *exI*, *simp*)
apply(*rule-tac x*={(*vertex-1*,*vertex-1*)} **in** *exI*, *simp*)
done

interpretation *NoRefl*: *SecurityInvariant-ACS*
where *default-node-properties* = *default-node-properties*
and *sinvar* = *sinvar*
rewrites *SecurityInvariant-withOffendingFlows.set-offending-flows sinvar* = *NoRefl-offending-set*
unfolding *default-node-properties-def*
apply *unfold-locales*
apply(*rule ballI*)

apply(*frule SINVAR-NoRefl.offending-notevalD*)
apply(*simp only*: *SecurityInvariant-withOffendingFlows.ENFsr-offending-set*[*OF NoRfl-ENRsr*])
apply *fastforce*
apply(*fact NoRefl-unique-default*)
apply(*fact NoRefl-offending-set*)
done

It can also be interpreted as IFS

lemma *NoRefl-SecurityInvariant-IFS*: *SecurityInvariant-IFS sinvar default-node-properties*
unfolding *default-node-properties-def*
apply *unfold-locales*
apply(*rule ballI*)
apply(*frule SINVAR-NoRefl.offending-notevalD*)
apply(*simp only*: *SecurityInvariant-withOffendingFlows.ENFsr-offending-set*[*OF NoRfl-ENRsr*])
apply *fastforce*
apply(*erule default-uniqueness-by-counterexample-IFS*)
apply (*simp add*: *SecurityInvariant-withOffendingFlows.set-offending-flows-def*
SecurityInvariant-withOffendingFlows.is-offending-flows-min-set-def
SecurityInvariant-withOffendingFlows.is-offending-flows-def)
apply (*simp add*:*graph-ops*)
apply (*simp split*: *prod.split-asm prod.split*)
apply(*rule-tac x*=(| *nodes*={*vertex-1*}, *edges* = {(*vertex-1*,*vertex-1*)} |) **in** *exI*, *simp*)
apply(*rule conjI*)
apply(*simp add*: *wf-graph-def*)
apply(*case-tac otherbot*, *simp-all*)
apply(*rule-tac x*=(λ *x*. *NoRefl*)(*vertex-1* := *NoRefl*, *vertex-2* := *NoRefl*) **in** *exI*, *simp*)
apply(*rule-tac x*={(*vertex-1*,*vertex-1*)} **in** *exI*, *simp*)
done

lemma *TopoS-NoRefl*: *SecurityInvariant sinvar default-node-properties receiver-violation*
unfolding *receiver-violation-def* **by** *unfold-locales*

hide-fact (**open**) *sinvar-mono*
hide-const (**open**) *sinvar receiver-violation default-node-properties*

end
theory *SINVAR-NoRefl-impl*
imports *SINVAR-NoRefl ../TopoS-Interface-impl*
begin

code-identifier code-module *SINVAR-NoRefl-impl* => (*Scala*) *SINVAR-NoRefl*

6.7.2 SecurityInvariant NoRefl List Implementation

fun *sinvar* :: *'v list-graph* ⇒ (*'v* ⇒ *node-config*) ⇒ *bool* **where**
sinvar G nP = (∀ (*s*,*r*) ∈ *set* (*edgesL G*). *s* = *r* ⟶ *nP s* = *Refl*)

definition *NoRefl-offending-list*:: *'v list-graph* ⇒ (*'v* ⇒ *node-config*) ⇒ (*'v* × *'v*) *list list* **where**
NoRefl-offending-list G nP = (*if sinvar G nP then*
[]
else
[[*e* ← *edgesL G*. *case e of* (*e1*,*e2*) ⇒ *e1* = *e2* ∧ *nP e1* = *NoRefl*]])

definition *NetModel-node-props P* = (λ *i*. (*case* (*node-properties P*) *i of Some property* ⇒ *property*
| *None* ⇒ *SINVAR-NoRefl.default-node-properties*))
lemma[*code-unfold*]: *SecurityInvariant.node-props SINVAR-NoRefl.default-node-properties P* = *NetModel-node-props P*
apply(*simp add*: *NetModel-node-props-def*)
done

definition *NoRefl-eval G P* = (*wf-list-graph G* ∧
sinvar G (*SecurityInvariant.node-props SINVAR-NoRefl.default-node-properties P*))

interpretation *NoRefl-impl*:*TopoS-List-Impl*
where *default-node-properties*=*SINVAR-NoRefl.default-node-properties*
and *sinvar-spec*=*SINVAR-NoRefl.sinvar*
and *sinvar-impl*=*sinvar*
and *receiver-violation*=*SINVAR-NoRefl.receiver-violation*
and *offending-flows-impl*=*NoRefl-offending-list*
and *node-props-impl*=*NetModel-node-props*
and *eval-impl*=*NoRefl-eval*
apply(*unfold TopoS-List-Impl-def*)
apply(*rule conjI*)
apply(*simp add*: *TopoS-NoRefl list-graph-to-graph-def*)
apply(*rule conjI*)
apply(*simp add*: *list-graph-to-graph-def NoRefl-offending-set NoRefl-offending-set-def NoRefl-offending-list-def*)
apply(*rule conjI*)
apply(*simp only*: *NetModel-node-props-def*)
apply(*metis NoRefl.node-props.simps NoRefl.node-props-eq-node-props-formaldef*)
apply(*simp only*: *NoRefl-eval-def*)
apply(*simp add*: *TopoS-eval-impl-proofrule*[*OF TopoS-NoRefl*])
apply(*simp add*: *list-graph-to-graph-def*)
done

6.7.3 PolEnforcePoint packing

definition *SINVAR-LIB-NoRefl* :: ('*v*::*vertex*, *node-config*) *TopoS-packed* **where**
SINVAR-LIB-NoRefl ≡
(| *nm-name* = ''*NoRefl*'',
nm-receiver-violation = *SINVAR-NoRefl.receiver-violation*,
nm-default = *SINVAR-NoRefl.default-node-properties*,
nm-sinvar = *sinvar*,
nm-offending-flows = *NoRefl-offending-list*,
nm-node-props = *NetModel-node-props*,
nm-eval = *NoRefl-eval*
|)
interpretation *SINVAR-LIB-NoRefl-interpretation*: *TopoS-modelLibrary SINVAR-LIB-NoRefl SINVAR-NoRefl.sinvar*
apply(*unfold TopoS-modelLibrary-def SINVAR-LIB-NoRefl-def*)
apply(*rule conjI*)
apply(*simp*)
apply(*simp*)
by(*unfold-locales*)

Examples

definition *example-net* :: *nat list-graph* **where**
example-net ≡ (| *nodesL* = [*1*::*nat*,*2*,*3*],
edgesL = [(*1*,*2*),(*2*,*2*),(*2*,*1*),(*1*,*3*)] |)
lemma *wf-list-graph example-net* **by** *eval*

definition *example-conf* **where**
example-conf ≡ ((λ*e*. *SINVAR-NoRefl.default-node-properties*)(*2*:= *Refl*))

lemma *sinvar example-net example-conf* **by** *eval*
lemma *NoRefl-offending-list example-net* (λ*e*. *SINVAR-NoRefl.default-node-properties*) = [[(*2*, *2*)]]
by *eval*

hide-const (**open**) *NetModel-node-props*
hide-const (**open**) *sinvar*

end
theory *SINVAR-Tainting-impl*
imports *SINVAR-Tainting ../TopoS-Interface-impl*
begin

6.7.4 SecurityInvariant Tainting List Implementation

code-identifier code-module *SINVAR-Tainting-impl* => (*Scala*) *SINVAR-Tainting*

fun *sinvar* :: *'v list-graph* ⇒ (*'v* ⇒ *SINVAR-Tainting.taints*) ⇒ *bool* **where**
sinvar G nP = (∀ (*e1*,*e2*) ∈ *set* (*edgesL G*). (*nP e1*) ⊆ (*nP e2*))

definition *Tainting-offending-list*:: *'v list-graph* ⇒ (*'v* ⇒ *SINVAR-Tainting.taints*) ⇒ (*'v* × *'v*) *list list* **where**
Tainting-offending-list G nP = (*if sinvar G nP then*
[]
else
[[*e* ← *edgesL G*. *case e of* (*e1*,*e2*) ⇒ ¬(*nP e1*) ⊆ (*nP e2*)]])

definition *NetModel-node-props P* =
(λ *i*. (*case* (*node-properties P*) *i of*
Some property ⇒ *property*
| *None* ⇒ *SINVAR-Tainting.default-node-properties*))
lemma[*code-unfold*]: *SecurityInvariant.node-props SINVAR-Tainting.default-node-properties P* = *NetModel-node-props P*
by(*simp add*: *NetModel-node-props-def SecurityInvariant.node-props.simps*[*OF TopoS-Tainting*])

definition *Tainting-eval G P* = (*wf-list-graph G* ∧
sinvar G (*SecurityInvariant.node-props SINVAR-Tainting.default-node-properties P*))

interpretation *Tainting-impl*:*TopoS-List-Impl*
where *default-node-properties*=*SINVAR-Tainting.default-node-properties*
and *sinvar-spec*=*SINVAR-Tainting.sinvar*

and *sinvar-impl=sinvar*
and *receiver-violation=SINVAR-Tainting.receiver-violation*
and *offending-flows-impl=Tainting-offending-list*
and *node-props-impl=NetModel-node-props*
and *eval-impl=Tainting-eval*
apply(*unfold TopoS-List-Impl-def*)
apply(*rule conjI*)
apply(*simp add*: *TopoS-Tainting*)
apply(*simp add*: *list-graph-to-graph-def SINVAR-Tainting.sinvar-def*; *fail*)
apply(*rule conjI*)
apply(*simp add*: *list-graph-to-graph-def*)
apply(*simp add*: *list-graph-to-graph-def SINVAR-Tainting.sinvar-def Taints-offending-set SINVAR-Tainting.Taints-offending-set-def Tainting-offending-list-def*; *fail*)
apply(*rule conjI*)
apply(*simp only*: *NetModel-node-props-def*)

apply (*metis SecurityInvariant.node-props.simps SecurityInvariant.node-props-eq-node-props-formaldef TopoS-Tainting*)
apply(*simp only*: *Tainting-eval-def*)
apply(*simp add*: *TopoS-eval-impl-proofrule*[*OF TopoS-Tainting*])
apply(*simp add*: *list-graph-to-graph-def SINVAR-Tainting.sinvar-def*)
done

6.7.5 Tainting packing

definition *SINVAR-LIB-Tainting* :: (*'v*::*vertex*, *SINVAR-Tainting.taints*) *TopoS-packed* **where**
SINVAR-LIB-Tainting ≡
(| *nm-name* = *''Tainting''*,
nm-receiver-violation = *SINVAR-Tainting.receiver-violation*,
nm-default = *SINVAR-Tainting.default-node-properties*,
nm-sinvar = *sinvar*,
nm-offending-flows = *Tainting-offending-list*,
nm-node-props = *NetModel-node-props*,
nm-eval = *Tainting-eval*
|)
interpretation *SINVAR-LIB-BLPbasic-interpretation*: *TopoS-modelLibrary SINVAR-LIB-Tainting SINVAR-Tainting.sinvar*
apply(*unfold TopoS-modelLibrary-def SINVAR-LIB-Tainting-def*)
apply(*rule conjI*)
apply(*simp*)
apply(*simp*)
by(*unfold-locales*)

6.7.6 Example

context
begin
private definition *tainting-example* :: *string list-graph* **where**
tainting-example ≡ (| *nodesL* = [*''produce 1''*,
''produce 2'',
''produce 3'',
''read 1 2'',
''read 3'',
''consume 1 2 3'',

```
                    ''consume 3''],
        edgesL =[(''produce 1'', ''read 1 2''),
                 (''produce 2'', ''read 1 2''),
                 (''produce 3'', ''read 3''),
                 (''read 3'', ''read 1 2''),
                 (''read 1 2'', ''consume 1 2 3''),
                 (''read 3'', ''consume 3'')] ⦈
lemma wf-list-graph tainting-example by eval


private definition tainting-example-props :: string ⇒ SINVAR-Tainting.taints where
  tainting-example-props ≡ (λ n. SINVAR-Tainting.default-node-properties)
                    (''produce 1'' := {''1''},
                     ''produce 2'' := {''2''},
                     ''produce 3'' := {''3''},
                     ''read 1 2'' := {''1'',''2'', ''3''},
                     ''read 3'' := {''3''},
                     ''consume 1 2 3'' := {''1'',''2'',''3''},
                     ''consume 3'' := {''3''})
private lemma sinvar tainting-example tainting-example-props by eval
end

export-code SINVAR-LIB-Tainting checking Scala

hide-const (open) NetModel-node-props Tainting-offending-list Tainting-eval

hide-const (open) sinvar

end
theory SINVAR-TaintingTrusted-impl
imports SINVAR-TaintingTrusted ../TopoS-Interface-impl
begin
```

6.7.7 SecurityInvariant Tainting with Trust List Implementation

```
code-identifier code-module SINVAR-Tainting-impl => (Scala) SINVAR-Tainting


lemma A − B ⊆ C ⟷ (∀ a∈A. a ∈ C ∨ a ∈ B) by blast
lemma ¬(A − B ⊆ C) ⟷ (∃ a ∈ A. a ∉ C ∧ a ∉ B) by blast

fun sinvar :: 'v list-graph ⇒ ('v ⇒ SINVAR-TaintingTrusted.taints) ⇒ bool where
  sinvar G nP = (∀ (v1,v2) ∈ set (edgesL G). taints (nP v1) − untaints (nP v1) ⊆ taints (nP v2))



export-code sinvar checking SML
value[code] sinvar (⦇ nodesL = [], edgesL =[] ⦈) (λ-. SINVAR-TaintingTrusted.default-node-properties)
lemma sinvar (⦇ nodesL = [], edgesL =[] ⦈) (λ-. SINVAR-TaintingTrusted.default-node-properties) by
eval


definition TaintingTrusted-offending-list
  :: 'v list-graph ⇒ ('v ⇒ SINVAR-TaintingTrusted.taints) ⇒ ('v × 'v) list list where
```

TaintingTrusted-offending-list G nP = (if sinvar G nP then
[]
else
[[e ← edgesL G. case e of (v1,v2) ⇒ ¬(taints (nP v1) − untaints (nP v1) ⊆ taints (nP v2))]])

export-code *TaintingTrusted-offending-list* **checking** *SML*

definition *NetModel-node-props P =*
(λ i. (case (node-properties P) i of
Some property ⇒ property
| None ⇒ SINVAR-TaintingTrusted.default-node-properties))
lemma[*code-unfold*]: *SecurityInvariant.node-props SINVAR-TaintingTrusted.default-node-properties P = NetModel-node-props P*
by(*simp add*: *NetModel-node-props-def SecurityInvariant.node-props.simps*[*OF TopoS-TaintingTrusted*])

definition *TaintingTrusted-eval G P = (wf-list-graph G ∧*
sinvar G (SecurityInvariant.node-props SINVAR-TaintingTrusted.default-node-properties P))

interpretation *TaintingTrusted-impl*:*TopoS-List-Impl*
where *default-node-properties=SINVAR-TaintingTrusted.default-node-properties*
and *sinvar-spec=SINVAR-TaintingTrusted.sinvar*
and *sinvar-impl=sinvar*
and *receiver-violation=SINVAR-TaintingTrusted.receiver-violation*
and *offending-flows-impl=TaintingTrusted-offending-list*
and *node-props-impl=NetModel-node-props*
and *eval-impl=TaintingTrusted-eval*
apply(*unfold TopoS-List-Impl-def*)
apply(*rule conjI*)
apply(*simp add*: *TopoS-TaintingTrusted*)
apply(*simp add*: *list-graph-to-graph-def SINVAR-TaintingTrusted.sinvar-def*; *fail*)
apply(*rule conjI*)
apply(*simp add*: *list-graph-to-graph-def*)
apply(*simp add*: *list-graph-to-graph-def SINVAR-TaintingTrusted.sinvar-def Taints-offending-set SINVAR-TaintingTrusted.Taints-offending-set-def TaintingTrusted-offending-list-def*; *fail*)
apply(*rule conjI*)
apply(*simp only*: *NetModel-node-props-def*)

apply (*metis SecurityInvariant.node-props.simps SecurityInvariant.node-props-eq-node-props-formaldef TopoS-TaintingTrusted*)
apply(*simp only*: *TaintingTrusted-eval-def*)
apply(*simp add*: *TopoS-eval-impl-proofrule*[*OF TopoS-TaintingTrusted*])
apply(*simp add*: *list-graph-to-graph-def SINVAR-TaintingTrusted.sinvar-def*; *fail*)
done

6.7.8 TaintingTrusted packing

definition *SINVAR-LIB-TaintingTrusted* :: (*'v*::*vertex*, *SINVAR-TaintingTrusted.taints*) *TopoS-packed*
where
SINVAR-LIB-TaintingTrusted ≡

(| *nm-name* = *''TaintingTrusted''*,
nm-receiver-violation = *SINVAR-TaintingTrusted.receiver-violation*,
nm-default = *SINVAR-TaintingTrusted.default-node-properties*,
nm-sinvar = *sinvar*,
nm-offending-flows = *TaintingTrusted-offending-list*,
nm-node-props = *NetModel-node-props*,
nm-eval = *TaintingTrusted-eval*
|)
interpretation *SINVAR-LIB-BLPbasic-interpretation*: *TopoS-modelLibrary SINVAR-LIB-TaintingTrusted SINVAR-TaintingTrusted.sinvar*
apply(*unfold TopoS-modelLibrary-def SINVAR-LIB-TaintingTrusted-def*)
apply(*rule conjI*)
apply(*simp*)
apply(*simp*)
by(*unfold-locales*)

6.7.9 Example

context
begin
private definition *tainting-example* :: *string list-graph* **where**
tainting-example ≡ (| *nodesL* = [*''produce 1''*,
''produce 2'',
''produce 3'',
''read 1 2'',
''read 3'',
''consume 1 2 3'',
''consume 3''],
edgesL =[(*''produce 1''*, *''read 1 2''*),
(*''produce 2''*, *''read 1 2''*),
(*''produce 3''*, *''read 3''*),
(*''read 3''*, *''read 1 2''*),
(*''read 1 2''*, *''consume 1 2 3''*),
(*''read 3''*, *''consume 3''*)] |)
lemma *wf-list-graph tainting-example* **by** *eval*

private definition *tainting-example-props* :: *string* ⇒ *SINVAR-TaintingTrusted.taints* **where**
tainting-example-props ≡ (λ *n*. *SINVAR-TaintingTrusted.default-node-properties*)
(*''produce 1''* := *TaintsUntaints* {*''1''*} {},
''produce 2'' := *TaintsUntaints* {*''2''*} {},
''produce 3'' := *TaintsUntaints* {*''3''*} {},
''read 1 2'' := *TaintsUntaints* {*''3''*,*''foo''*} {*''1''*,*''2''*},
''read 3'' := *TaintsUntaints* {*''3''*} {},
''consume 1 2 3'' := *TaintsUntaints* {*''foo''*,*''3''*} {},
''consume 3'' := *TaintsUntaints* {*''3''*} {})

value *tainting-example-props* (*''consume 1 2 3''*)
value[*code*] *TaintingTrusted-offending-list tainting-example tainting-example-props*
private lemma *sinvar tainting-example tainting-example-props* **by** *eval*
end

export-code *SINVAR-LIB-TaintingTrusted* **checking** *Scala*

export-code *SINVAR-LIB-TaintingTrusted* **checking** *SML*

hide-const (**open**) *NetModel-node-props TaintingTrusted-offending-list TaintingTrusted-eval*

hide-const (**open**) *sinvar*

end
theory *SINVAR-Dependability*
imports *../TopoS-Helper*
begin

6.8 SecurityInvariant Dependability

type-synonym *dependability-level* = *nat*

definition *default-node-properties* :: *dependability-level*
where *default-node-properties* ≡ *0*

Less-equal other nodes depend on the output of a node than its dependability level.

fun *sinvar* :: *'v graph* ⇒ (*'v* ⇒ *dependability-level*) ⇒ *bool* **where**
sinvar G nP = (∀ (*e1*,*e2*) ∈ *edges G*. (*num-reachable G e1*) ≤ (*nP e1*))

definition *receiver-violation* :: *bool* **where**
receiver-violation ≡ *False*

It does not matter whether we iterate over all edges or all nodes. We chose all edges because it is in line with the other models.

fun *sinvar-nodes* :: *'v graph* ⇒ (*'v* ⇒ *dependability-level*) ⇒ *bool* **where**
sinvar-nodes G nP = (∀ *v* ∈ *nodes G*. (*num-reachable G v*) ≤ (*nP v*))

theorem *sinvar-edges-nodes-iff*: *wf-graph G* ⟹
sinvar-nodes G nP = *sinvar G nP*
proof(*unfold sinvar-nodes.simps sinvar.simps*, *rule iffI*)
assume *a1*: *wf-graph G*
and *a2*: ∀ *v*∈*nodes G*. *num-reachable G v* ≤ *nP v*

from *a1*[*simplified wf-graph-def*] **have** *f1*: *fst* ‘ *edges G* ⊆ *nodes G* **by** *simp*
from *f1 a2* **have** ∀ *v* ∈ (*fst* ‘ *edges G*). *num-reachable G v* ≤ *nP v* **by** *auto*

thus ∀ (*e1*, -) ∈ *edges G*. *num-reachable G e1* ≤ *nP e1* **by** *auto*
next
assume *a1*: *wf-graph G*
and *a2*: ∀ (*e1*, -)∈*edges G*. *num-reachable G e1* ≤ *nP e1*

from *a2* **have** *g1*: ∀ *v* ∈ (*fst* ‘ *edges G*). *num-reachable G v* ≤ *nP v* **by** *auto*

from *FiniteGraph.succ-tran-empty*[*OF a1*] *num-reachable-zero-iff*[*OF a1*, *symmetric*]
have *g2*: ∀ *v*. *v* ∉ (*fst* ‘ *edges G*) ⟶ *num-reachable G v* ≤ *nP v* **by** (*metis le0*)

from *g1 g2* **show** ∀ *v*∈*nodes G*. *num-reachable G v* ≤ *nP v* **by** *metis*
qed

lemma *num-reachable-le-nodes*: ⟦ *wf-graph G* ⟧ ⟹ *num-reachable G v* ≤ *card* (*nodes G*)
unfolding *num-reachable-def*
using *succ-tran-subseteq-nodes card-seteq nat-le-linear wf-graph.finiteV* **by** *metis*

nP is valid if all dependability level are greater equal the total number of nodes in the graph

lemma ⟦ *wf-graph G*; ∀ *v* ∈ *nodes G*. *nP v* ≥ *card* (*nodes G*) ⟧ ⟹ *sinvar G nP*
apply(*subst sinvar-edges-nodes-iff*[*symmetric*], *simp*)
apply(*simp add*:)
using *num-reachable-le-nodes* **by** (*metis le-trans*)

Generate a valid configuration to start from:

Takes arbitrary configuration, returns a valid one

fun *dependability-fix-nP* :: *'v graph* ⇒ (*'v* ⇒ *dependability-level*) ⇒ (*'v* ⇒ *dependability-level*)
where
dependability-fix-nP G nP = (λ*v*. *if num-reachable G v* ≤ (*nP v*) *then* (*nP v*) *else num-reachable G v*)

dependability-fix-nP always gives you a valid solution

lemma *dependability-fix-nP-valid*: ⟦ *wf-graph G* ⟧ ⟹ *sinvar G* (*dependability-fix-nP G nP*)
by(*subst sinvar-edges-nodes-iff*[*symmetric*], *simp-all*)

furthermore, it gives you a minimal solution, i.e. if someone supplies a configuration with a value lower than calculated by *dependability-fix-nP*, this is invalid!

lemma *dependability-fix-nP-minimal-solution*: ⟦ *wf-graph G*; ∃ *v* ∈ *nodes G*. (*nP v*) < (*dependability-fix-nP G* (λ-. *0*)) *v* ⟧ ⟹ ¬ *sinvar G nP*
apply(*subst sinvar-edges-nodes-iff*[*symmetric*], *simp*)
apply(*simp*)
apply(*clarify*)
apply(*rule-tac x*=*v* **in** *bexI*)
apply(*simp-all*)
done

lemma *sinvar-mono*: *SecurityInvariant-withOffendingFlows.sinvar-mono sinvar*
apply(*rule-tac SecurityInvariant-withOffendingFlows.sinvar-mono-I-proofrule*)
apply(*auto*)
apply(*rename-tac nP e1 e2 N E' e1' e2' E*)
apply(*drule-tac E'*=*E'* **and** *v*=*e1'* **in** *num-reachable-mono*)
apply *simp*
apply(*subgoal-tac* (*e1'*, *e2'*) ∈ *E*)
apply(*force*)
apply(*blast*)
done

interpretation *SecurityInvariant-preliminaries*
where *sinvar* = *sinvar*
apply *unfold-locales*
apply(*frule-tac finite-distinct-list*[*OF wf-graph.finiteE*])

apply(*erule-tac exE*)
apply(*rename-tac list-edges*)
apply(*rule-tac ff=list-edges* **in** *SecurityInvariant-withOffendingFlows.mono-imp-set-offending-flows-not-empty*[*OF sinvar-mono*])
apply(*auto*)[*4*]
apply(*auto simp add*: *SecurityInvariant-withOffendingFlows.is-offending-flows-def graph-ops*)[*1*]
apply(*fact SecurityInvariant-withOffendingFlows.sinvar-mono-imp-sinvar-mono*[*OF sinvar-mono*])
apply(*fact SecurityInvariant-withOffendingFlows.sinvar-mono-imp-is-offending-flows-mono*[*OF sinvar-mono*])
done

interpretation *Dependability*: *SecurityInvariant-ACS*
where *default-node-properties = SINVAR-Dependability.default-node-properties*
and *sinvar = SINVAR-Dependability.sinvar*
unfolding *SINVAR-Dependability.default-node-properties-def*
proof
fix *G*::*'a graph* **and** *f nP*
assume *wf-graph G* **and** *f* ∈ *set-offending-flows G nP*
thus ∀ *i*∈*fst* ‘ *f*. ¬ *sinvar G* (*nP*(*i* := *0*))
apply (*simp add*: *SecurityInvariant-withOffendingFlows.set-offending-flows-def*
SecurityInvariant-withOffendingFlows.is-offending-flows-min-set-def
SecurityInvariant-withOffendingFlows.is-offending-flows-def)
apply (*simp split*: *prod.split-asm prod.split*)
apply (*simp add*:*graph-ops*)
apply(*clarify*)
apply (*metis gr0I le0*)
done
next
fix *otherbot*
assume *assm*: ∀ *G f nP i*. *wf-graph G* ∧ *f* ∈ *set-offending-flows G nP* ∧ *i* ∈ *fst* ‘ *f* ⟶ ¬ *sinvar G* (*nP*(*i* := *otherbot*))
have *unique-default-example-succ-tran*:
succ-tran (⦇*nodes* = {*vertex-1*, *vertex-2*}, *edges* = {(*vertex-1*, *vertex-2*)}⦈) *vertex-1* = {*vertex-2*}
using *unique-default-example1* **by** *blast*
from *assm* **show** *otherbot = 0*
apply −
apply(*elim default-uniqueness-by-counterexample-ACS*)
apply(*simp*)
apply (*simp add*: *SecurityInvariant-withOffendingFlows.set-offending-flows-def*
SecurityInvariant-withOffendingFlows.is-offending-flows-min-set-def
SecurityInvariant-withOffendingFlows.is-offending-flows-def)
apply (*simp add*:*graph-ops*)
apply (*simp split*: *prod.split-asm prod.split*)
apply(*rule-tac x=*⦇ *nodes*={*vertex-1*,*vertex-2*}, *edges* = {(*vertex-1*,*vertex-2*)} ⦈ **in** *exI*, *simp*)
apply(*rule conjI*)
apply(*simp add*: *wf-graph-def*)
apply(*rule-tac x=*(λ *x*. *0*)(*vertex-1* := *0*, *vertex-2* := *0*) **in** *exI*, *simp*)
apply(*rule conjI*)
apply(*simp add*: *unique-default-example-succ-tran num-reachable-def*)
apply(*rule-tac x=vertex-1* **in** *exI*, *simp*)
apply(*rule-tac x=*{(*vertex-1*,*vertex-2*)} **in** *exI*, *simp*)
apply(*simp add*: *unique-default-example-succ-tran num-reachable-def*)
apply(*simp add*: *succ-tran-def unique-default-example-simp1 unique-default-example-simp2*)
done

qed

lemma *TopoS-Dependability*: *SecurityInvariant sinvar default-node-properties receiver-violation*
unfolding *receiver-violation-def* **by** *unfold-locales*

hide-const (**open**) *sinvar receiver-violation default-node-properties*

end
theory *SINVAR-Dependability-impl*
imports *SINVAR-Dependability ../TopoS-Interface-impl*
begin

code-identifier code-module *SINVAR-Dependability-impl* => (*Scala*) *SINVAR-Dependability*

6.8.1 SecurityInvariant Dependability List Implementation

Less-equal other nodes depend on the output of a node than its dependability level.

fun *sinvar* :: *'v list-graph* ⇒ (*'v* ⇒ *dependability-level*) ⇒ *bool* **where**
sinvar G nP = (∀ (*e1*,*e2*) ∈ *set* (*edgesL G*). (*num-reachable G e1*) ≤ (*nP e1*))

value *sinvar*
(⦇ *nodesL* = [*1*::*nat*,*2*,*3*,*4*], *edgesL* = [(*1*,*2*), (*2*,*3*), (*3*,*4*), (*8*,*9*),(*9*,*8*)] ⦈)
(λ*e*. *3*)
value *sinvar*
(⦇ *nodesL* = [*1*::*nat*,*2*,*3*,*4*,*8*,*9*,*10*], *edgesL* = [(*1*,*2*), (*2*,*3*), (*3*,*4*), (*8*,*9*),(*9*,*8*)] ⦈)
(λ*e*. *2*)

Generate a valid configuration to start from:

fun *dependability-fix-nP* :: *'v list-graph* ⇒ (*'v* ⇒ *dependability-level*) ⇒ (*'v* ⇒ *dependability-level*)
where
dependability-fix-nP G nP = (λ*v*. *let nr* = *num-reachable G v in* (*if nr* ≤ (*nP v*) *then* (*nP v*) *else nr*))

theorem *dependability-fix-nP-impl-correct*: *wf-list-graph G* ⟹ *dependability-fix-nP G nP* = *SINVAR-Dependability.d*
(*list-graph-to-graph G*) *nP*
by(*simp add*: *num-reachable-correct fun-eq-iff*)

value *let G* = (⦇ *nodesL* = [*1*::*nat*,*2*,*3*,*4*], *edgesL* = [(*1*,*1*), (*2*,*1*), (*3*,*1*), (*4*,*1*), (*1*,*2*), (*1*,*3*)] ⦈ *in*
(*let nP* = *dependability-fix-nP G* (λ*e*. *0*) *in map* (λ*v*. *nP v*) (*nodesL G*))

value *let G* = (⦇ *nodesL* = [*1*::*nat*,*2*,*3*,*4*], *edgesL* = [(*1*,*1*)] ⦈ *in* (*let nP* = *dependability-fix-nP G*
(λ*e*. *0*) *in map* (λ*v*. *nP v*) (*nodesL G*))

definition *Dependability-offending-list*:: *'v list-graph* ⇒ (*'v* ⇒ *dependability-level*) ⇒ (*'v* × *'v*) *list list*
where
Dependability-offending-list = *Generic-offending-list sinvar*

definition *NetModel-node-props P* = (*λ i.* (*case* (*node-properties P*) *i of Some property ⇒ property* | *None ⇒ SINVAR-Dependability.default-node-properties*))
lemma[*code-unfold*]: *SecurityInvariant.node-props SINVAR-Dependability.default-node-properties P* = *NetModel-node-props P*
apply(*simp add*: *NetModel-node-props-def*)
done

definition *Dependability-eval G P* = (*wf-list-graph G* ∧
sinvar G (*SecurityInvariant.node-props SINVAR-Dependability.default-node-properties P*))

lemma *sinvar-correct*: *wf-list-graph G ⟹ SINVAR-Dependability.sinvar* (*list-graph-to-graph G*) *nP* = *sinvar G nP*
apply(*simp*)
apply(*rule all-edges-list-I*)
apply(*simp add*: *fun-eq-iff*)
apply(*clarify*)
apply(*rename-tac x*)
apply(*drule-tac v*=*x* **in** *num-reachable-correct*)
apply *presburger*
done

interpretation *Dependability-impl*:*TopoS-List-Impl*
where *default-node-properties*=*SINVAR-Dependability.default-node-properties*
and *sinvar-spec*=*SINVAR-Dependability.sinvar*
and *sinvar-impl*=*sinvar*
and *receiver-violation*=*SINVAR-Dependability.receiver-violation*
and *offending-flows-impl*=*Dependability-offending-list*
and *node-props-impl*=*NetModel-node-props*
and *eval-impl*=*Dependability-eval*
apply(*unfold TopoS-List-Impl-def*)
apply(*rule conjI*)
apply(*rule conjI*)
apply(*simp add*: *TopoS-Dependability*; *fail*)
apply(*intro allI impI*)
apply(*fact sinvar-correct*)
apply(*rule conjI*)
apply(*unfold Dependability-offending-list-def*)
apply(*intro allI impI*)
apply(*rule Generic-offending-list-correct*)
apply(*assumption*)
apply(*simp only*: *sinvar-correct*)
apply(*rule conjI*)
apply(*intro allI*)
apply(*simp only*: *NetModel-node-props-def*)
apply(*metis Dependability.node-props.simps Dependability.node-props-eq-node-props-formaldef*)
apply(*simp only*: *Dependability-eval-def*)
apply(*intro allI impI*)
apply(*rule TopoS-eval-impl-proofrule*[*OF TopoS-Dependability*])
apply(*simp only*: *sinvar-correct*)

done

6.8.2 Dependability packing

definition *SINVAR-LIB-Dependability* :: (*'v*::*vertex*, *SINVAR-Dependability.dependability-level*) *TopoS-packed*
where
SINVAR-LIB-Dependability ≡
(| *nm-name* = *''Dependability''*,
nm-receiver-violation = *SINVAR-Dependability.receiver-violation*,
nm-default = *SINVAR-Dependability.default-node-properties*,
nm-sinvar = *sinvar*,
nm-offending-flows = *Dependability-offending-list*,
nm-node-props = *NetModel-node-props*,
nm-eval = *Dependability-eval*
|)
interpretation *SINVAR-LIB-Dependability-interpretation*: *TopoS-modelLibrary SINVAR-LIB-Dependability SINVAR-Dependability.sinvar*
apply(*unfold TopoS-modelLibrary-def SINVAR-LIB-Dependability-def*)
apply(*rule conjI*)
apply(*simp*)
apply(*simp*)
by(*unfold-locales*)

Example:

value *let G* = (| *nodesL* = [*1*::*nat*,*2*,*3*,*4*,*8*,*9*,*10*], *edgesL* = [(*1*,*2*), (*2*,*3*), (*3*,*4*), (*8*,*9*),(*9*,*8*)] |)
in sinvar G ((λ *n*. *SINVAR-Dependability.default-node-properties*)(*1*:=*3*, *2*:=*2*, *3*:=*1*, *4*:=*0*, *8*:=*2*, *9*:=*2*, *10*:=*0*))

value *let G* = (| *nodesL* = [*1*::*nat*,*2*,*3*,*4*,*8*,*9*,*10*], *edgesL* = [(*1*,*2*), (*2*,*3*), (*3*,*4*), (*8*,*9*),(*9*,*8*)] |)
in sinvar G ((λ *n*. *SINVAR-Dependability.default-node-properties*)(*1*:=*10*, *2*:=*10*, *3*:=*10*, *4*:=*10*, *8*:=*10*, *9*:=*10*, *10*:=*10*))

value *let G* = (| *nodesL* = [*1*::*nat*,*2*,*3*,*4*,*8*,*9*,*10*], *edgesL* = [(*1*,*2*), (*2*,*3*), (*3*,*4*), (*8*,*9*),(*9*,*8*)] |)
in sinvar G ((λ *n*. *2*))

value *let G* = (| *nodesL* = [*1*::*nat*,*2*,*3*,*4*,*8*,*9*,*10*], *edgesL* = [(*1*,*2*), (*2*,*3*), (*3*,*4*), (*8*,*9*),(*9*,*8*)] |)
in Dependability-eval G (|*node-properties*=[*1*↦*3*, *2*↦*2*, *3*↦*1*, *4*↦*0*, *8*↦*2*, *9*↦*2*, *10*↦*0*] |)

value *Dependability-offending-list* (| *nodesL* = [*1*::*nat*,*2*,*3*,*4*,*8*,*9*,*10*], *edgesL* = [(*1*,*2*), (*2*,*3*), (*3*,*4*), (*8*,*9*),(*9*,*8*)] |) (λ *n*. *2*)

hide-fact (**open**) *sinvar-correct*
hide-const (**open**) *sinvar NetModel-node-props*

end
theory *SINVAR-NonInterference*
imports *../TopoS-Helper*
begin

6.9 SecurityInvariant NonInterference

datatype *node-config* = *Interfering* | *Unrelated*

definition *default-node-properties* :: *node-config*

where *default-node-properties = Interfering*

definition *undirected-reachable* :: *'v graph ⇒ 'v => 'v set* **where**
undirected-reachable G v = (succ-tran (undirected G) v) − {v}

lemma *undirected-reachable-mono*:
E' ⊆ E ⟹ undirected-reachable (|nodes = N, edges = E'|) n ⊆ undirected-reachable (|nodes = N, edges = E|) n
unfolding *undirected-reachable-def undirected-def succ-tran-def*
by (*fastforce intro*: *trancl-mono*)

fun *sinvar* :: *'v graph ⇒ ('v ⇒ node-config) ⇒ bool* **where**
sinvar G nP = (∀ n ∈ (nodes G). (nP n) = Interfering ⟶ (nP ‘ (undirected-reachable G n)) ⊆ {Unrelated})

lemma *sinvar G nP ⟷*
(∀ n ∈ {v' ∈ (nodes G). (nP v') = Interfering}. {nP v' | v'. v' ∈ undirected-reachable G n} ⊆ {Unrelated})
by *auto*

definition *receiver-violation* :: *bool* **where**
receiver-violation = True

simplifications for sets we need in the uniqueness proof

lemma *tmp1*: {(*b*, *a*). *a = vertex-1 ∧ b = vertex-2*} = {(*vertex-2*, *vertex-1*)} **by** *auto*
lemma *tmp6*: {(*vertex-1*, *vertex-2*), (*vertex-2*, *vertex-1*)}$^+$ =
{(*vertex-1*, *vertex-1*), (*vertex-2*, *vertex-2*), (*vertex-1*, *vertex-2*), (*vertex-2*, *vertex-1*)}
apply(*rule*)
apply(*rule*)
apply(*case-tac x*, *simp*)
apply(*erule-tac r*={(*vertex-1*, *vertex-2*), (*vertex-2*, *vertex-1*)} **in** *trancl-induct*)
apply(*auto*)
apply (*metis* (*mono-tags*) *insertCI r-r-into-trancl*)+
done
lemma *tmp2*: (*insert* (*vertex-1*, *vertex-2*) {(*b*, *a*). *a = vertex-1 ∧ b = vertex-2*})$^+$ =
{(*vertex-1*, *vertex-1*), (*vertex-2*, *vertex-2*), (*vertex-1*, *vertex-2*), (*vertex-2*, *vertex-1*)}
apply(*subst tmp1*)
apply(*fact tmp6*)
done
lemma *tmp4*: {(*e1*, *e2*). *e1 = vertex-1 ∧ e2 = vertex-2 ∧ (e1 = vertex-1 ⟶ e2 ≠ vertex-2)*} = {} **by** *blast*
lemma *tmp5*: {(*b*, *a*). *a = vertex-1 ∧ b = vertex-2 ∨ a = vertex-1 ∧ b = vertex-2 ∧ (a = vertex-1 ⟶ b ≠ vertex-2)*} =
{(*vertex-2*, *vertex-1*)} **by** *fastforce*
lemma *unique-default-example*: *undirected-reachable (|nodes = {vertex-1, vertex-2}, edges = {(vertex-1, vertex-2)}|) vertex-1 = {vertex-2}*
by(*auto simp add*: *undirected-def undirected-reachable-def succ-tran-def tmp2*)
lemma *unique-default-example-hlp1*: *delete-edges (|nodes = {vertex-1, vertex-2}, edges = {(vertex-1, vertex-2)}|) {(vertex-1, vertex-2)} =*
(|nodes = {vertex-1, vertex-2}, edges = {}|)
by(*simp add*: *delete-edges-def*)
lemma *unique-default-example-2*:
undirected-reachable (*delete-edges* (|*nodes* = {*vertex-1*, *vertex-2*}, *edges* = {(*vertex-1*, *vertex-2*)}|)

{(*vertex-1*,*vertex-2*)}) *vertex-1* = {}
by(*simp add*: *undirected-def undirected-reachable-def succ-tran-def unique-default-example-hlp1*)
lemma *unique-default-example-3*:
undirected-reachable (*delete-edges* (⦇*nodes* = {*vertex-1*, *vertex-2*}, *edges* = {(*vertex-1*, *vertex-2*)}⦈
{(*vertex-1*,*vertex-2*)}) *vertex-2* = {}
by(*simp add*: *undirected-def undirected-reachable-def succ-tran-def unique-default-example-hlp1*)
lemma *unique-default-example-4*:
(*undirected-reachable* (*add-edge vertex-1 vertex-2* (*delete-edges* (⦇*nodes* = {*vertex-1*, *vertex-2*},
edges = {(*vertex-1*, *vertex-2*)}⦈) {(*vertex-1*, *vertex-2*)})) *vertex-1*) = {*vertex-2*}
apply(*simp add*: *delete-edges-def add-edge-def undirected-def undirected-reachable-def succ-tran-def*)
apply(*subst tmp4*)
apply(*subst tmp5*)
apply(*simp*)
apply(*subst tmp6*)
by *force*
lemma *unique-default-example-5*:
(*undirected-reachable* (*add-edge vertex-1 vertex-2* (*delete-edges* (⦇*nodes* = {*vertex-1*, *vertex-2*},
edges = {(*vertex-1*, *vertex-2*)}⦈) {(*vertex-1*, *vertex-2*)})) *vertex-2*) = {*vertex-1*}
apply(*simp add*: *delete-edges-def add-edge-def undirected-def undirected-reachable-def succ-tran-def*)
apply(*subst tmp4*)
apply(*subst tmp5*)
apply(*simp*)
apply(*subst tmp6*)
by *force*

lemma *empty-undirected-reachable-false*: *xb* ∈ *undirected-reachable* (*delete-edges G* (*edges G*)) *na* ⟷ *False*
by(*simp add*: *undirected-reachable-def succ-tran-def undirected-def delete-edges-edges-empty*)

6.9.1 monotonic and preliminaries

lemma *sinvar-mono*: *SecurityInvariant-withOffendingFlows.sinvar-mono sinvar*
unfolding *SecurityInvariant-withOffendingFlows.sinvar-mono-def*
apply(*clarsimp*)
apply(*rename-tac nP N E' n E xa*)
apply(*erule-tac x*=*n* **in** *ballE*)
prefer *2*
apply *simp*
apply(*simp*)
apply(*drule-tac N*=*N* **and** *n*=*n* **in** *undirected-reachable-mono*)
apply(*blast*)
done

interpretation *SecurityInvariant-preliminaries*
where *sinvar* = *sinvar*
apply *unfold-locales*
apply(*frule-tac finite-distinct-list*[*OF wf-graph.finiteE*])
apply(*erule-tac exE*)
apply(*rename-tac list-edges*)
apply(*rule-tac ff*=*list-edges* **in** *SecurityInvariant-withOffendingFlows.mono-imp-set-offending-flows-not-empty*[*OF*

sinvar-mono])
apply(*auto*)[*4*]
apply(*auto simp add*: *SecurityInvariant-withOffendingFlows.is-offending-flows-def empty-undirected-reachable-false*)
apply(*fact SecurityInvariant-withOffendingFlows.sinvar-mono-imp-sinvar-mono*[*OF sinvar-mono*])
apply(*fact SecurityInvariant-withOffendingFlows.sinvar-mono-imp-is-offending-flows-mono*[*OF sinvar-mono*])
done

interpretation *NonInterference*: *SecurityInvariant-IFS*
where *default-node-properties* = *SINVAR-NonInterference.default-node-properties*
and *sinvar* = *SINVAR-NonInterference.sinvar*
unfolding *SINVAR-NonInterference.default-node-properties-def*
apply *unfold-locales*
apply(*rule ballI*)
apply(*drule SINVAR-NonInterference.offending-notevalD*)
apply(*simp*)
apply *clarify*
apply(*rename-tac xa*)
apply(*case-tac nP xa*)

apply *simp*
apply(*erule-tac x*=*n* **and** *A*=*nodes G* **in** *ballE*)
prefer *2*
apply *fast*
apply(*simp*)
apply(*thin-tac wf-graph G*)
apply(*thin-tac* (*a*,*b*) ∈ *f*)
apply(*thin-tac n* ∈ *nodes G*)
apply(*thin-tac nP n* = *Interfering*)
apply(*erule disjE*)
apply *fastforce*
apply *fastforce*

apply *simp*

apply(*erule default-uniqueness-by-counterexample-IFS*)
apply (*simp add*: *SecurityInvariant-withOffendingFlows.set-offending-flows-def*
SecurityInvariant-withOffendingFlows.is-offending-flows-min-set-def
SecurityInvariant-withOffendingFlows.is-offending-flows-def)
apply (*simp add*:*delete-edges-set-nodes*)
apply (*simp split*: *prod.split-asm prod.split*)
apply(*rule-tac x*=(| *nodes*={*vertex-1*,*vertex-2*}, *edges* = {(*vertex-1*,*vertex-2*)} |) **in** *exI*, *simp*)
apply(*rule conjI*)
apply(*simp add*: *wf-graph-def*)
apply(*rule-tac x*=(λ *x*. *default-node-properties*)(*vertex-1* := *Interfering*, *vertex-2* := *Interfering*) **in** *exI*, *simp*)
apply(*rule conjI*)
apply(*simp add*: *unique-default-example*)
apply(*rule-tac x*=*vertex-2* **in** *exI*, *simp*)
apply(*rule-tac x*={(*vertex-1*,*vertex-2*)} **in** *exI*, *simp*)
apply(*simp add*: *unique-default-example*)
apply(*simp add*: *unique-default-example-2*)
apply(*simp add*: *unique-default-example-3*)
apply(*simp add*: *unique-default-example-4*)

apply(*simp add*: *unique-default-example-5*)
apply(*case-tac otherbot*)
apply *simp*
apply(*simp add*:*graph-ops*)
done

lemma *TopoS-NonInterference*: *SecurityInvariant sinvar default-node-properties receiver-violation*
unfolding *receiver-violation-def* **by** *unfold-locales*

hide-const (**open**) *sinvar receiver-violation default-node-properties*

— Hide all the helper lemmas.
hide-fact *tmp1 tmp2 tmp4 tmp5 tmp6 unique-default-example*
unique-default-example-2 unique-default-example-3 unique-default-example-4
unique-default-example-5 empty-undirected-reachable-false

end
theory *SINVAR-NonInterference-impl*
imports *SINVAR-NonInterference ../TopoS-Interface-impl*
begin

code-identifier code-module *SINVAR-NonInterference-impl* => (*Scala*) *SINVAR-NonInterference*

6.9.2 SecurityInvariant NonInterference List Implementation

definition *undirected-reachable* :: *'v list-graph* ⇒ *'v* => *'v list* **where**
undirected-reachable G v = *removeAll v* (*succ-tran* (*undirected G*) *v*)

lemma *undirected-reachable-set*: *set* (*undirected-reachable G v*) = {*e2*. (*v*,*e2*) ∈ (*set* (*edgesL* (*undirected G*)))$^+$} − {*v*}
by(*simp add*: *undirected-succ-tran-set undirected-nodes-set undirected-reachable-def*)

fun *sinvar-set* :: *'v list-graph* ⇒ (*'v* ⇒ *node-config*) ⇒ *bool* **where**
sinvar-set G nP = (∀ *n* ∈ *set* (*nodesL G*). (*nP n*) = *Interfering* ⟶ *set* (*map nP* (*undirected-reachable G n*)) ⊆ {*Unrelated*})

fun *sinvar* :: *'v list-graph* ⇒ (*'v* ⇒ *node-config*) ⇒ *bool* **where**
sinvar G nP = (∀ *n* ∈ *set* (*nodesL G*). (*nP n*) = *Interfering* ⟶ (*let result* = *remdups* (*map nP* (*undirected-reachable G n*)) *in result* = [] ∨ *result* = [*Unrelated*]))

lemma *P* = *Q* ⟹ (∀ *x*. *P x*) = (∀ *x*. *Q x*)
by(*erule arg-cong*)

lemma *sinvar-eq-help1*: *nP* ‘ *set* (*undirected-reachable G n*) = *set* (*map nP* (*undirected-reachable G n*))
by *auto*
lemma *sinvar-eq-help2*: *set l* = {*Unrelated*} ⟹ *remdups l* = [*Unrelated*]
apply(*induction l*)
apply *simp*

apply(*simp*)
apply (*metis empty-iff insertI1 set-empty2 subset-singletonD*)
done
lemma *sinvar-eq-help3*: (*let result* = *remdups* (*map nP* (*undirected-reachable G n*)) *in result* = [] ∨ *result* = [*Unrelated*]) = (*set* (*map nP* (*undirected-reachable G n*)) ⊆ {*Unrelated*})
apply *simp*
apply(*rule iffI*)
apply(*erule disjE*)
apply *simp*
apply(*simp only*: *set-map*[*symmetric*])
apply(*subst set-remdups*[*symmetric*])
apply *simp*
apply(*case-tac nP ' set* (*undirected-reachable G n*) = {})
apply *fast*
apply(*case-tac nP ' set* (*undirected-reachable G n*) = {*Unrelated*})
defer
apply(*subgoal-tac nP ' set* (*undirected-reachable G n*) ⊆ {*Unrelated*} ⟹
nP ' set (*undirected-reachable G n*) ≠ {} ⟹
nP ' set (*undirected-reachable G n*) ≠ {*Unrelated*} ⟹ *False*)
apply *fast*
apply (*metis subset-singletonD*)
apply *simp*
apply(*rule disjI2*)
apply(*simp only*: *sinvar-eq-help1*)
apply(*simp add*:*sinvar-eq-help2*)
done

lemma *sinvar-list-eq-set*: *sinvar* = *sinvar-set*
apply(*insert sinvar-eq-help3*)
apply(*simp add*: *fun-eq-iff*)
apply(*rule allI*)+
apply *fastforce*
done

value *sinvar*
(| *nodesL* = [*1*::*nat*,*2*,*3*,*4*], *edgesL* = [(*1*,*2*), (*2*,*3*), (*3*,*4*), (*8*,*9*),(*9*,*8*)] |)
(λ*e*. *SINVAR-NonInterference.default-node-properties*)
value *sinvar*
(| *nodesL* = [*1*::*nat*,*2*,*3*,*4*,*8*,*9*,*10*], *edgesL* = [(*1*,*2*), (*2*,*3*), (*3*,*4*)] |)
((λ*e*. *SINVAR-NonInterference.default-node-properties*)(*1*:= *Interfering*, *2*:= *Unrelated*, *3*:= *Unrelated*, *4*:= *Unrelated*))
value *sinvar*
(| *nodesL* = [*1*::*nat*,*2*,*3*,*4*,*5*, *8*,*9*,*10*], *edgesL* = [(*1*,*2*), (*2*,*3*), (*3*,*4*), (*5*,*4*), (*8*,*9*),(*9*,*8*)] |)
((λ*e*. *SINVAR-NonInterference.default-node-properties*)(*1*:= *Interfering*, *2*:= *Unrelated*, *3*:= *Unrelated*, *4*:= *Unrelated*))
value *sinvar*
(| *nodesL* = [*1*::*nat*], *edgesL* = [(*1*,*1*)] |)
((λ*e*. *SINVAR-NonInterference.default-node-properties*)(*1*:= *Interfering*))

value (*undirected-reachable* (| *nodesL* = [*1*::*nat*], *edgesL* = [(*1*,*1*)] |) *1*) = []

definition *NonInterference-offending-list*:: *'v list-graph* ⇒ (*'v* ⇒ *node-config*) ⇒ (*'v* × *'v*) *list list*
where
NonInterference-offending-list = *Generic-offending-list sinvar*

definition *NetModel-node-props P* = (λ *i*. (*case* (*node-properties P*) *i of Some property* ⇒ *property*
| *None* ⇒ *SINVAR-NonInterference.default-node-properties*))
lemma[*code-unfold*]: *SecurityInvariant.node-props SINVAR-NonInterference.default-node-properties P*
= *NetModel-node-props P*
apply(*simp add*: *NetModel-node-props-def*)
done

definition *NonInterference-eval G P* = (*wf-list-graph G* ∧
sinvar G (*SecurityInvariant.node-props SINVAR-NonInterference.default-node-properties P*))

lemma *sinvar-correct*: *wf-list-graph G* ⟹ *SINVAR-NonInterference.sinvar* (*list-graph-to-graph G*)
nP = *sinvar G nP*
apply(*simp add*: *sinvar-list-eq-set*)
apply(*rule all-nodes-list-I*)
by (*simp add*: *SINVAR-NonInterference.undirected-reachable-def succ-tran-correct undirected-correct*
undirected-reachable-def)

interpretation *NonInterference-impl*:*TopoS-List-Impl*
where *default-node-properties*=*SINVAR-NonInterference.default-node-properties*
and *sinvar-spec*=*SINVAR-NonInterference.sinvar*
and *sinvar-impl*=*sinvar*
and *receiver-violation*=*SINVAR-NonInterference.receiver-violation*
and *offending-flows-impl*=*NonInterference-offending-list*
and *node-props-impl*=*NetModel-node-props*
and *eval-impl*=*NonInterference-eval*
apply(*unfold TopoS-List-Impl-def*)
apply(*rule conjI*)
apply(*rule conjI*)
apply(*simp add*: *TopoS-NonInterference*; *fail*)
apply(*intro allI impI*)
apply(*fact sinvar-correct*)
apply(*rule conjI*)
apply(*unfold NonInterference-offending-list-def*)
apply(*intro allI impI*)
apply(*rule Generic-offending-list-correct*)
apply(*assumption*)
apply(*simp only*: *sinvar-correct*)
apply(*rule conjI*)
apply(*intro allI*)
apply(*simp only*: *NetModel-node-props-def*)

apply(*metis NonInterference.node-props.simps NonInterference.node-props-eq-node-props-formaldef*)
apply(*simp only*: *NonInterference-eval-def*)
apply(*intro allI impI*)
apply(*rule TopoS-eval-impl-proofrule*[*OF TopoS-NonInterference*])
apply(*simp only*: *sinvar-correct*)
done

6.9.3 NonInterference packing

definition *SINVAR-LIB-NonInterference* :: (*'v*::*vertex*, *node-config*) *TopoS-packed* **where**
SINVAR-LIB-NonInterference ≡
(| *nm-name* = *''NonInterference''*,
nm-receiver-violation = *SINVAR-NonInterference.receiver-violation*,
nm-default = *SINVAR-NonInterference.default-node-properties*,
nm-sinvar = *sinvar*,
nm-offending-flows = *NonInterference-offending-list*,
nm-node-props = *NetModel-node-props*,
nm-eval = *NonInterference-eval*
|)
interpretation *SINVAR-LIB-NonInterference-interpretation*: *TopoS-modelLibrary SINVAR-LIB-NonInterference SINVAR-NonInterference.sinvar*
apply(*unfold TopoS-modelLibrary-def SINVAR-LIB-NonInterference-def*)
apply(*rule conjI*)
apply(*simp*)
apply(*simp*)
by(*unfold-locales*)

Example:

context begin
private definition *example-graph* = (| *nodesL* = [*1*::*nat*,*2*,*3*,*4*,*5*, *8*,*9*,*10*], *edgesL* = [(*1*,*2*), (*2*,*3*), (*3*,*4*), (*5*,*4*), (*8*,*9*), (*9*,*8*)] |)
private definition*example-conf* = ((λ*e*. *SINVAR-NonInterference.default-node-properties*) (*1*:= *Interfering*, *2*:= *Unrelated*, *3*:= *Unrelated*, *4*:= *Unrelated*, *8*:= *Unrelated*, *9*:= *Unrelated*))

private lemma ¬ *sinvar example-graph example-conf* **by** *eval*
private lemma *NonInterference-offending-list example-graph example-conf* =
[[(*1*, *2*)], [(*2*, *3*)], [(*3*, *4*)], [(*5*, *4*)]] **by** *eval*
end

hide-const (**open**) *NetModel-node-props*
hide-const (**open**) *sinvar*

end
theory *SINVAR-ACLcommunicateWith*
imports *../TopoS-Helper*
begin

6.10 SecurityInvariant ACLcommunicateWith

An access control list strategy that says that hosts must only transitively access each other if allowed

Warning: this transitive model has exponential computational complexity

definition *default-node-properties* :: *'v list*
where *default-node-properties* ≡ []

fun *sinvar* :: *'v graph* ⇒ (*'v* ⇒ *'v list*) ⇒ *bool* **where**
sinvar G nP = (∀ *v* ∈ *nodes G*. (∀ *a* ∈ (*succ-tran G v*). *a* ∈ *set* (*nP v*)))

definition *receiver-violation* :: *bool* **where**
receiver-violation ≡ *False*

lemma *ACLcommunicateWith-sinvar-alternative*:
wf-graph G ⟹ *sinvar G nP* = (∀ (*e1*,*e2*) ∈ (*edges G*)⁺. *e2* ∈ *set* (*nP e1*))
proof(*unfold sinvar.simps*, *rule iffI*, *goal-cases*)
case *1*
from *1*(*1*) **have** *e1-nodes*: (*e1*, *e2*) ∈ *edges G* ⟹ *e1* ∈ *nodes G* **for** *e1 e2*
by (*simp add*: *wf-graph.E-wfD*(*1*))
from *1*(*2*) **have** ∀ *v*∈*nodes G*. ∀ *a*. (*v*, *a*) ∈ (*edges G*)⁺ ⟶ *a* ∈ *set* (*nP v*)
by(*simp add*: *succ-tran-def*)
with *e1-nodes* **have** (*e1*, *e2*)∈(*edges G*)⁺ ⟹ *e2* ∈ *set* (*nP e1*) **for** *e1 e2*
by (*meson tranclD*)
thus *?case* **by** *blast*
next
case *2*
from *2*(*1*) **have** *e1-nodes*: (*v*, *a*) ∈ *edges G* ⟹ *v* ∈ *nodes G* **for** *v a*
by (*simp add*: *wf-graph.E-wfD*(*1*))
with *2*(*2*) **show** *?case* **by**(*auto simp add*: *succ-tran-def*)
qed

lemma *sinvar-mono*: *SecurityInvariant-withOffendingFlows.sinvar-mono sinvar*
unfolding *SecurityInvariant-withOffendingFlows.sinvar-mono-def*
proof(*clarify*)
fix *nP*::(*'v* ⇒ *'v list*) **and** *N E' E*
assume *a1*: *wf-graph* (|*nodes* = *N*, *edges* = *E*|)
and *a2*: *E'* ⊆ *E*
and *a3*: *sinvar* (|*nodes* = *N*, *edges* = *E*|) *nP*

from *a3* **have** *v* ∈ *N* ⟹ ∀ *a*∈(*succ-tran* (|*nodes* = *N*, *edges* = *E*|) *v*). *a* ∈ *set* (*nP v*) **for** *v* **by** *fastforce*
with *a2* **have** *g2*: *v* ∈ *N* ⟹ (∀ *a* ∈ (*succ-tran* (|*nodes* = *N*, *edges* = *E'*|) *v*). *a* ∈ *set* (*nP v*)) **for** *v*
using *succ-tran-mono*[*OF a1*] **by** *blast*
thus *sinvar* (|*nodes* = *N*, *edges* = *E'*|) *nP* **by** *simp*
qed

interpretation *SecurityInvariant-preliminaries*
where *sinvar* = *sinvar*
apply *unfold-locales*
apply(*frule-tac finite-distinct-list*[*OF wf-graph.finiteE*])
apply(*erule-tac exE*)
apply(*rename-tac list-edges*)
apply(*rule-tac ff*=*list-edges* **in** *SecurityInvariant-withOffendingFlows.mono-imp-set-offending-flows-not-empty*[*OF*

sinvar-mono])
apply(*auto*)[*4*]
apply(*auto simp add*: *SecurityInvariant-withOffendingFlows.is-offending-flows-def graph-ops False-set succ-tran-empty*)[*1*]
apply(*fact SecurityInvariant-withOffendingFlows.sinvar-mono-imp-sinvar-mono*[*OF sinvar-mono*])
apply(*fact SecurityInvariant-withOffendingFlows.sinvar-mono-imp-is-offending-flows-mono*[*OF sinvar-mono*])
done

lemma *unique-default-example*: *succ-tran* (⦇*nodes* = {*vertex-1*, *vertex-2*}, *edges* = {(*vertex-1*, *vertex-2*)}⦈ *vertex-2* = {}
apply (*simp add*: *succ-tran-def*)
by (*metis Domain.DomainI Domain-empty Domain-insert distinct-vertices12 singleton-iff trancl-domain*)

interpretation *ACLcommunicateWith*: *SecurityInvariant-ACS*
where *default-node-properties* = *SINVAR-ACLcommunicateWith.default-node-properties*
and *sinvar* = *SINVAR-ACLcommunicateWith.sinvar*
unfolding *SINVAR-ACLcommunicateWith.default-node-properties-def*
apply *unfold-locales*

apply *simp*
apply(*subst*(*asm*) *SecurityInvariant-withOffendingFlows.set-offending-flows-simp*, *simp*)
apply(*clarsimp*)
apply (*metis*)

apply(*erule default-uniqueness-by-counterexample-ACS*)
apply(*simp*)
apply (*simp add*: *SecurityInvariant-withOffendingFlows.set-offending-flows-def*
SecurityInvariant-withOffendingFlows.is-offending-flows-min-set-def
SecurityInvariant-withOffendingFlows.is-offending-flows-def)
apply (*simp add*:*graph-ops*)
apply (*simp split*: *prod.split-asm prod.split*)
apply(*simp add*: *List.neq-Nil-conv*)
apply(*erule exE*)
apply(*rename-tac canAccessThis*)
apply(*case-tac canAccessThis* = *vertex-1*)
apply(*rule-tac x*=⦇ *nodes*={*canAccessThis*,*vertex-2*}, *edges* = {(*vertex-2*,*canAccessThis*)} ⦈ **in** *exI*, *simp*)
apply(*rule conjI*)
apply(*simp add*: *wf-graph-def*)
apply(*rule-tac x*=(λ *x*. [])(*vertex-1* := [], *vertex-2* := []) **in** *exI*, *simp*)
apply(*simp add*: *example-simps*)
apply(*rule-tac x*={(*vertex-2*,*vertex-1*)} **in** *exI*, *simp*)
apply(*simp add*: *example-simps*)
apply(*fastforce*)

apply(*rule-tac x*=⦇ *nodes*={*vertex-1*,*canAccessThis*}, *edges* = {(*vertex-1*,*canAccessThis*)} ⦈ **in** *exI*, *simp*)
apply(*rule conjI*)
apply(*simp add*: *wf-graph-def*)
apply(*rule-tac x*=(λ *x*. [])(*vertex-1* := [], *canAccessThis* := []) **in** *exI*, *simp*)
apply(*simp add*: *example-simps*)
apply(*rule-tac x*={(*vertex-1*,*canAccessThis*)} **in** *exI*, *simp*)

apply(*simp add*: *example-simps*)
apply(*fastforce*)
done

lemma *TopoS-ACLcommunicateWith*: *SecurityInvariant sinvar default-node-properties receiver-violation*
unfolding *receiver-violation-def* **by** *unfold-locales*

hide-const (**open**) *sinvar receiver-violation default-node-properties*

end
theory *SINVAR-ACLnotCommunicateWith*
imports *../TopoS-Helper SINVAR-ACLcommunicateWith*
begin

6.11 SecurityInvariant ACLnotCommunicateWith

An access control list strategy that says that hosts must not transitively access each other.

node properties: a set of hosts this host must not access

definition *default-node-properties* :: *'v set*
where *default-node-properties* ≡ *UNIV*

fun *sinvar* :: *'v graph* ⇒ (*'v* ⇒ *'v set*) ⇒ *bool* **where**
sinvar G nP = (∀ *v* ∈ *nodes G*. ∀ *a* ∈ (*succ-tran G v*). *a* ∉ (*nP v*))

definition *receiver-violation* :: *bool* **where**
receiver-violation ≡ *False*

It is the inverse of *SINVAR-ACLcommunicateWith.sinvar*

lemma *ACLcommunicateNotWith-inverse-ACLcommunicateWith*:
∀ *v*. *UNIV* − *nP' v* = *set* (*nP v*) ⟹ *SINVAR-ACLcommunicateWith.sinvar G nP* ⟷ *sinvar G nP'*
by *auto*

lemma *sinvar-mono*: *SecurityInvariant-withOffendingFlows.sinvar-mono sinvar*
unfolding *SecurityInvariant-withOffendingFlows.sinvar-mono-def*
proof(*clarify*)
fix *nP*::(*'v* ⇒ *'v set*) **and** *N E' E*
assume *a1*: *wf-graph* (|*nodes* = *N*, *edges* = *E*|)
and *a2*: *E'* ⊆ *E*
and *a3*: *sinvar* (|*nodes* = *N*, *edges* = *E*|) *nP*

from *a3* **have** ⋀*v a*. *v* ∈ *N* ⟹ *a* ∈ (*succ-tran* (|*nodes* = *N*, *edges* = *E*|) *v*) ⟹ *a* ∉ (*nP v*) **by** *fastforce*
from *this a2* **have** *g1*: ⋀*v a*. *v* ∈ *N* ⟹ *a* ∈ (*succ-tran* (|*nodes* = *N*, *edges* = *E'*|) *v*) ⟹ *a* ∉ (*nP v*)
using *succ-tran-mono*[*OF a1*] **by** *blast*

thus *sinvar* (|*nodes* = *N*, *edges* = *E'*|) *nP*
by(*clarsimp*)
qed

lemma *succ-tran-empty*: (*succ-tran* (|*nodes* = *nodes G*, *edges* = {}|) *v*) = {}
by(*simp add*: *succ-tran-def*)

interpretation *SecurityInvariant-preliminaries*
where *sinvar* = *sinvar*
apply *unfold-locales*
apply(*frule-tac finite-distinct-list*[*OF wf-graph.finiteE*])
apply(*erule-tac exE*)
apply(*rename-tac list-edges*)
apply(*rule-tac ff*=*list-edges* **in** *SecurityInvariant-withOffendingFlows.mono-imp-set-offending-flows-not-empty*[*OF sinvar-mono*])
apply(*auto*)[*4*]
apply(*auto simp add*: *SecurityInvariant-withOffendingFlows.is-offending-flows-def graph-ops succ-tran-empty*)[*1*]
apply(*fact SecurityInvariant-withOffendingFlows.sinvar-mono-imp-sinvar-mono*[*OF sinvar-mono*])
apply(*fact SecurityInvariant-withOffendingFlows.sinvar-mono-imp-is-offending-flows-mono*[*OF sinvar-mono*])
done

lemma *unique-default-example*: *succ-tran* (|*nodes* = {*vertex-1*, *vertex-2*}, *edges* = {(*vertex-1*, *vertex-2*)}|) *vertex-2* = {}
apply (*simp add*: *succ-tran-def*)
by (*metis Domain.DomainI Domain-empty Domain-insert distinct-vertices12 singleton-iff trancl-domain*)

interpretation *ACLnotCommunicateWith*: *SecurityInvariant-ACS*
where *default-node-properties* = *SINVAR-ACLnotCommunicateWith.default-node-properties*
and *sinvar* = *SINVAR-ACLnotCommunicateWith.sinvar*
unfolding *SINVAR-ACLnotCommunicateWith.default-node-properties-def*
apply *unfold-locales*

apply *simp*
apply(*subst*(*asm*) *SecurityInvariant-withOffendingFlows.set-offending-flows-simp*, *simp*)
apply(*clarsimp*)
apply (*metis*)

apply(*erule default-uniqueness-by-counterexample-ACS*)
apply(*simp*)
apply (*simp add*: *SecurityInvariant-withOffendingFlows.set-offending-flows-def*
SecurityInvariant-withOffendingFlows.is-offending-flows-min-set-def
SecurityInvariant-withOffendingFlows.is-offending-flows-def)
apply (*simp add*:*graph-ops*)
apply (*simp split*: *prod.split-asm prod.split*)
apply(*case-tac otherbot* = {})

apply(*rule-tac x*=(| *nodes*={*vertex-1*,*vertex-2*}, *edges* = {(*vertex-1*,*vertex-2*)} |) **in** *exI*, *simp*)
apply(*rule conjI*)
apply(*simp add*: *wf-graph-def*)
apply(*rule-tac x*=(λ *x*. *UNIV*)(*vertex-1* := {*vertex-2*}, *vertex-2* := {}) **in** *exI*, *simp*)
apply(*simp add*: *example-simps*)
apply(*rule-tac x*={(*vertex-1*,*vertex-2*)} **in** *exI*, *simp*)
apply(*simp add*: *example-simps*)

apply(*subgoal-tac* ∃ *canAccess*. *canAccess* ∈ *UNIV* ∧ *canAccess* ∉ *otherbot*)

prefer *2*
apply *blast*
apply(*erule exE*)
apply(*rename-tac canAccessThis*)
apply(*case-tac vertex-1* $\neq$ *canAccessThis*)

apply(*rule-tac x*=(| *nodes*={*vertex-1*,*canAccessThis*}, *edges* = {(*vertex-1*,*canAccessThis*)} |) **in** *exI*, *simp*)
apply(*rule conjI*)
apply(*simp add*: *wf-graph-def*)
apply(*rule-tac x*=(λ *x*. *UNIV*)(*vertex-1* := *UNIV*, *canAccessThis* := {}) **in** *exI*, *simp*)
apply(*simp add*: *example-simps*)
apply(*rule-tac x*={(*vertex-1*,*canAccessThis*)} **in** *exI*, *simp*)
apply(*simp add*: *example-simps*)

apply(*rule-tac x*=(| *nodes*={*canAccessThis*,*vertex-2*}, *edges* = {(*vertex-2*,*canAccessThis*)} |) **in** *exI*, *simp*)
apply(*rule conjI*)
apply(*simp add*: *wf-graph-def*)
apply(*rule-tac x*=(λ *x*. *UNIV*)(*vertex-2* := *UNIV*, *canAccessThis* := {}) **in** *exI*, *simp*)
apply(*simp add*: *example-simps*)
apply(*rule-tac x*={(*vertex-2*,*canAccessThis*)} **in** *exI*, *simp*)
apply(*simp add*: *example-simps*)
done

lemma *TopoS-ACLnotCommunicateWith*: *SecurityInvariant sinvar default-node-properties receiver-violation*
unfolding *receiver-violation-def* **by** *unfold-locales*

hide-const (**open**) *sinvar receiver-violation default-node-properties*

end
theory *SINVAR-ACLnotCommunicateWith-impl*
imports *SINVAR-ACLnotCommunicateWith ../TopoS-Interface-impl*
begin

code-identifier code-module *SINVAR-ACLnotCommunicateWith-impl* => (*Scala*) *SINVAR-ACLnotCommunicateW*

6.11.1 SecurityInvariant ACLnotCommunicateWith List Implementation

fun *sinvar* :: *'v list-graph* $\Rightarrow$ (*'v* $\Rightarrow$ *'v set*) $\Rightarrow$ *bool* **where**
sinvar G nP = ($\forall$ *v* $\in$ *set* (*nodesL G*). $\forall$ *a* $\in$ *set* (*succ-tran G v*). *a* $\notin$ (*nP v*))

definition *NetModel-node-props* (*P*::(*'v*::*vertex*, *'v set*) *TopoS-Params*) =
(λ *i*. (*case* (*node-properties P*) *i of Some property* $\Rightarrow$ *property* | *None* $\Rightarrow$ *SINVAR-ACLnotCommunicateWith.default-no*
lemma[*code-unfold*]: *SecurityInvariant.node-props SINVAR-ACLnotCommunicateWith.default-node-properties P* = *NetModel-node-props P*
apply(*simp add*: *NetModel-node-props-def*)
done

definition *ACLnotCommunicateWith-offending-list* = *Generic-offending-list sinvar*

definition *ACLnotCommunicateWith-eval G P* = (*wf-list-graph G* $\wedge$
sinvar G (*SecurityInvariant.node-props SINVAR-ACLnotCommunicateWith.default-node-properties*

P))

lemma *sinvar-correct*: *wf-list-graph G* ⟹ *SINVAR-ACLnotCommunicateWith.sinvar* (*list-graph-to-graph G*) *nP* = *sinvar G nP*
by (*metis SINVAR-ACLnotCommunicateWith.sinvar.simps SINVAR-ACLnotCommunicateWith-impl.sinvar.simps graph.select-convs*(*1*) *list-graph-to-graph-def succ-tran-correct*)

interpretation *ACLnotCommunicateWith-impl*:*TopoS-List-Impl*
where *default-node-properties*=*SINVAR-ACLnotCommunicateWith.default-node-properties*
and *sinvar-spec*=*SINVAR-ACLnotCommunicateWith.sinvar*
and *sinvar-impl*=*sinvar*
and *receiver-violation*=*SINVAR-ACLnotCommunicateWith.receiver-violation*
and *offending-flows-impl*=*ACLnotCommunicateWith-offending-list*
and *node-props-impl*=*NetModel-node-props*
and *eval-impl*=*ACLnotCommunicateWith-eval*
apply(*unfold TopoS-List-Impl-def*)
apply(*rule conjI*)
apply(*rule conjI*)
apply(*simp add*: *TopoS-ACLnotCommunicateWith*; *fail*)
apply(*intro allI impI*)
apply(*fact sinvar-correct*)
apply(*rule conjI*)
apply(*unfold ACLnotCommunicateWith-offending-list-def*)
apply(*intro allI impI*)
apply(*rule Generic-offending-list-correct*)
apply(*assumption*)
apply(*simp only*: *sinvar-correct*; *fail*)
apply(*rule conjI*)
apply(*intro allI*)
apply(*simp only*: *NetModel-node-props-def*)
apply(*metis ACLnotCommunicateWith.node-props.simps ACLnotCommunicateWith.node-props-eq-node-props-formal*
apply(*simp only*: *ACLnotCommunicateWith-eval-def*)
apply(*intro allI impI*)
apply(*rule TopoS-eval-impl-proofrule*[*OF TopoS-ACLnotCommunicateWith*])
apply(*simp only*: *sinvar-correct*)
done

6.11.2 packing

definition *SINVAR-LIB-ACLnotCommunicateWith*:: (*'v*::*vertex*, *'v set*) *TopoS-packed* **where**
SINVAR-LIB-ACLnotCommunicateWith ≡
(| *nm-name* = *''ACLnotCommunicateWith''*,
nm-receiver-violation = *SINVAR-ACLnotCommunicateWith.receiver-violation*,
nm-default = *SINVAR-ACLnotCommunicateWith.default-node-properties*,
nm-sinvar = *sinvar*,
nm-offending-flows = *ACLnotCommunicateWith-offending-list*,
nm-node-props = *NetModel-node-props*,
nm-eval = *ACLnotCommunicateWith-eval*
|)
interpretation *SINVAR-LIB-ACLnotCommunicateWith-interpretation*: *TopoS-modelLibrary SINVAR-LIB-ACLnotC*
SINVAR-ACLnotCommunicateWith.sinvar
apply(*unfold TopoS-modelLibrary-def SINVAR-LIB-ACLnotCommunicateWith-def*)

apply(*rule conjI*)
apply(*simp*)
apply(*simp*)
by(*unfold-locales*)

Examples

hide-const (**open**) *NetModel-node-props*
hide-const (**open**) *sinvar*

end
theory *SINVAR-ACLcommunicateWith-impl*
imports *SINVAR-ACLcommunicateWith ../TopoS-Interface-impl*
begin

code-identifier code-module *SINVAR-ACLcommunicateWith-impl* => (*Scala*) *SINVAR-ACLcommunicateWith*

6.11.3 List Implementation

fun *sinvar* :: *'v list-graph* ⇒ (*'v* ⇒ *'v list*) ⇒ *bool* **where**
sinvar G nP = (∀ *v* ∈ *set* (*nodesL G*). ∀ *a* ∈ (*set* (*succ-tran G v*)). *a* ∈ *set* (*nP v*))

definition *NetModel-node-props* (*P*::(*'v*::*vertex*, *'v list*) *TopoS-Params*) =
(λ *i*. (*case* (*node-properties P*) *i of Some property* ⇒ *property* | *None* ⇒ *SINVAR-ACLcommunicateWith.default-node-p*
lemma[*code-unfold*]: *SecurityInvariant.node-props SINVAR-ACLcommunicateWith.default-node-properties*
P = *NetModel-node-props P*
by(*simp add*: *NetModel-node-props-def*)

definition *ACLcommunicateWith-offending-list* = *Generic-offending-list sinvar*

definition *ACLcommunicateWith-eval G P* = (*wf-list-graph G* ∧
sinvar G (*SecurityInvariant.node-props SINVAR-ACLcommunicateWith.default-node-properties P*))

lemma *sinvar-correct*: *wf-list-graph G* ⟹ *SINVAR-ACLcommunicateWith.sinvar* (*list-graph-to-graph*
G) *nP* = *sinvar G nP*
by (*metis SINVAR-ACLcommunicateWith.sinvar.simps SINVAR-ACLcommunicateWith-impl.sinvar.simps*
graph.select-convs(*1*) *list-graph-to-graph-def succ-tran-correct*)

interpretation *SINVAR-ACLcommunicateWith-impl*:*TopoS-List-Impl*
where *default-node-properties*=*SINVAR-ACLcommunicateWith.default-node-properties*
and *sinvar-spec*=*SINVAR-ACLcommunicateWith.sinvar*
and *sinvar-impl*=*sinvar*
and *receiver-violation*=*SINVAR-ACLcommunicateWith.receiver-violation*
and *offending-flows-impl*=*ACLcommunicateWith-offending-list*
and *node-props-impl*=*NetModel-node-props*
and *eval-impl*=*ACLcommunicateWith-eval*
apply(*unfold TopoS-List-Impl-def*)
apply(*rule conjI*)
apply(*rule conjI*)
apply(*simp add*: *TopoS-ACLcommunicateWith*; *fail*)
apply(*intro allI impI*)
apply(*fact sinvar-correct*)

apply(*rule conjI*)
apply(*unfold ACLcommunicateWith-offending-list-def*)
apply(*intro allI impI*)
apply(*rule Generic-offending-list-correct*)
apply(*assumption*)
apply(*simp only*: *sinvar-correct*; *fail*)
apply(*rule conjI*)
apply(*intro allI*)
apply(*simp only*: *NetModel-node-props-def*)
apply(*metis ACLcommunicateWith.node-props.simps ACLcommunicateWith.node-props-eq-node-props-formaldef*)
apply(*simp only*: *ACLcommunicateWith-eval-def*)
apply(*intro allI impI*)
apply(*rule TopoS-eval-impl-proofrule*[*OF TopoS-ACLcommunicateWith*])
apply(*simp only*: *sinvar-correct*; *fail*)
done

6.11.4 packing

definition *SINVAR-LIB-ACLcommunicateWith*:: (*'v*::*vertex*, *'v list*) *TopoS-packed* **where**
SINVAR-LIB-ACLcommunicateWith ≡
(| *nm-name* = *''ACLcommunicateWith''*,
nm-receiver-violation = *SINVAR-ACLcommunicateWith.receiver-violation*,
nm-default = *SINVAR-ACLcommunicateWith.default-node-properties*,
nm-sinvar = *sinvar*,
nm-offending-flows = *ACLcommunicateWith-offending-list*,
nm-node-props = *NetModel-node-props*,
nm-eval = *ACLcommunicateWith-eval*
|)
interpretation *SINVAR-LIB-ACLcommunicateWith-interpretation*: *TopoS-modelLibrary SINVAR-LIB-ACLcommunic*
SINVAR-ACLcommunicateWith.sinvar
apply(*unfold TopoS-modelLibrary-def SINVAR-LIB-ACLcommunicateWith-def*)
apply(*rule conjI*)
apply(*simp*)
apply(*simp*)
by(*unfold-locales*)

Examples

context begin

1 can access 2 and 3 2 can access 3

private lemma *sinvar*
(| *nodesL* = [*1*::*nat*, *2*, *3*],
edgesL = [(*1*,*2*), (*2*,*3*)]|)
(((λ*v*. *SINVAR-ACLcommunicateWith.default-node-properties*)
(*1* := [*2*,*3*]))
(*2* := [*3*])) **by** *eval*

Everyone can access everyone, except for 1: 1 must not access 4. The offending flows may be any edge on the path from 1 to 4

lemma *ACLcommunicateWith-offending-list*
(| *nodesL* = [*1*::*nat*, *2*, *3*, *4*],
edgesL = [(*1*,*2*), (*2*,*3*), (*3*, *4*)]|)
(((((λ*v*. *SINVAR-ACLcommunicateWith.default-node-properties*)

(1 := [1,2,3]))
(2 := [1,2,3,4]))
(3 := [1,2,3,4]))
(4 := [1,2,3,4])) =
[[(1, 2)], [(2, 3)], [(3, 4)]] **by** *eval*

If we add the additional edge from 1 to 3, then the offending flows are either

(3.4) , because this disconnects 4 from the graph completely

- any pair of edges which disconnects 1 from 3

lemma *ACLcommunicateWith-offending-list*
(| *nodesL* = [1::*nat*, 2, 3, 4],
edgesL = [(1,2), (1,3), (2,3), (3, 4)]|)
(((((λ*v*. *SINVAR-ACLcommunicateWith.default-node-properties*)
(1 := [1,2,3]))
(2 := [1,2,3,4]))
(3 := [1,2,3,4]))
(4 := [1,2,3,4])) =
[[(1, 2), (1, 3)], [(1, 3), (2, 3)], [(3, 4)]] **by** *eval*
end

hide-const (**open**) *NetModel-node-props*
hide-const (**open**) *sinvar*

end
theory *SINVAR-Dependability-norefl*
imports *../TopoS-Helper*
begin

6.12 SecurityInvariant *Dependability-norefl*

A version of the Dependability model but if a node reaches itself, it is ignored

type-synonym *dependability-level* = *nat*

definition *default-node-properties* :: *dependability-level*
where *default-node-properties* ≡ *0*

Less-equal other nodes depend on the output of a node than its dependability level.

fun *sinvar* :: *'v graph* ⇒ (*'v* ⇒ *dependability-level*) ⇒ *bool* **where**
sinvar G nP = (∀ (*e1*,*e2*) ∈ *edges G*. (*num-reachable-norefl G e1*) ≤ (*nP e1*))

definition *receiver-violation* :: *bool* **where**
receiver-violation ≡ *False*

lemma *sinvar-mono*: *SecurityInvariant-withOffendingFlows.sinvar-mono sinvar*

```
 apply(rule-tac SecurityInvariant-withOffendingFlows.sinvar-mono-I-proofrule)
  apply(auto)
 apply(rename-tac nP e1 e2 N E' e1' e2' E)
 apply(drule-tac E'=E' and v=e1' in num-reachable-norefl-mono)
  apply simp
 apply(subgoal-tac (e1', e2') ∈ E)
  apply(force)
 apply(blast)
done


interpretation SecurityInvariant-preliminaries
where sinvar = sinvar
 apply unfold-locales
   apply(frule-tac finite-distinct-list[OF wf-graph.finiteE])
   apply(erule-tac exE)
   apply(rename-tac list-edges)
  apply(rule-tac ff=list-edges in SecurityInvariant-withOffendingFlows.mono-imp-set-offending-flows-not-empty[OF
sinvar-mono])
       apply(auto)[4]
   apply(auto simp add: SecurityInvariant-withOffendingFlows.is-offending-flows-def graph-ops)[1]
  apply(fact SecurityInvariant-withOffendingFlows.sinvar-mono-imp-sinvar-mono[OF sinvar-mono])
 apply(fact SecurityInvariant-withOffendingFlows.sinvar-mono-imp-is-offending-flows-mono[OF sinvar-mono])
done



interpretation Dependability: SecurityInvariant-ACS
where default-node-properties = SINVAR-Dependability-norefl.default-node-properties
and sinvar = SINVAR-Dependability-norefl.sinvar
 unfolding SINVAR-Dependability-norefl.default-node-properties-def
 proof
   fix G::'a graph and f nP
   assume wf-graph G and f ∈ set-offending-flows G nP
   thus ∀ i∈fst ‘ f. ¬ sinvar G (nP(i := 0))
    apply (simp add: SecurityInvariant-withOffendingFlows.set-offending-flows-def
     SecurityInvariant-withOffendingFlows.is-offending-flows-min-set-def
     SecurityInvariant-withOffendingFlows.is-offending-flows-def)
    apply (simp split: prod.split-asm prod.split)
    apply (simp add:graph-ops)
    apply(clarify)
    apply (metis gr0I le0)
    done
 next
  fix otherbot
  assume assm: ∀ G f nP i. wf-graph G ∧ f ∈ set-offending-flows G nP ∧ i ∈ fst ‘ f ⟶ ¬ sinvar
G (nP(i := otherbot))
  have unique-default-example-succ-tran:
    succ-tran (⦇nodes = {vertex-1, vertex-2}, edges = {(vertex-1, vertex-2)}⦈) vertex-1 = {vertex-2}
   using unique-default-example1 by blast
  from assm show otherbot = 0
   apply −
   apply(elim default-uniqueness-by-counterexample-ACS)
   apply(simp)
```

apply (*simp add*: *SecurityInvariant-withOffendingFlows.set-offending-flows-def*
SecurityInvariant-withOffendingFlows.is-offending-flows-min-set-def
SecurityInvariant-withOffendingFlows.is-offending-flows-def)
apply (*simp add*:*graph-ops*)
apply (*simp split*: *prod.split-asm prod.split*)
apply(*rule-tac x*=⦇ *nodes*={*vertex-1*,*vertex-2*}, *edges* = {(*vertex-1*,*vertex-2*)} ⦈ **in** *exI*, *simp*)
apply(*rule conjI*)
apply(*simp add*: *wf-graph-def*)
apply(*rule-tac x*=(λ *x*. *0*)(*vertex-1* := *0*, *vertex-2* := *0*) **in** *exI*, *simp*)
apply(*rule conjI*)
apply(*simp add*: *unique-default-example-succ-tran num-reachable-noref-def*; *fail*)
apply(*rule-tac x*=*vertex-1* **in** *exI*, *simp*)
apply(*rule-tac x*={(*vertex-1*,*vertex-2*)} **in** *exI*, *simp*)
apply(*simp add*: *unique-default-example-succ-tran num-reachable-noref-def*)
apply(*simp add*: *succ-tran-def unique-default-example-simp1 unique-default-example-simp2*)
done
qed

lemma *TopoS-Dependability-noref*: *SecurityInvariant sinvar default-node-properties receiver-violation*
unfolding *receiver-violation-def* **by** *unfold-locales*

hide-const (**open**) *sinvar receiver-violation default-node-properties*

end
theory *SINVAR-Dependability-noref-impl*
imports *SINVAR-Dependability-noref ../TopoS-Interface-impl*
begin

code-identifier code-module *SINVAR-Dependability-noref-impl* => (*Scala*) *SINVAR-Dependability-noref*

6.12.1 SecurityInvariant Dependability noref List Implementation

fun *sinvar* :: *'v list-graph* ⇒ (*'v* ⇒ *dependability-level*) ⇒ *bool* **where**
sinvar G nP = (∀ (*e1*,*e2*) ∈ *set* (*edgesL G*). (*num-reachable-noref G e1*) ≤ (*nP e1*))

value *sinvar*
⦇ *nodesL* = [*1*::*nat*,*2*,*3*,*4*], *edgesL* = [(*1*,*2*), (*2*,*3*), (*3*,*4*), (*8*,*9*),(*9*,*8*)] ⦈
(λ*e*. *3*)
value *sinvar*
⦇ *nodesL* = [*1*::*nat*,*2*,*3*,*4*,*8*,*9*,*10*], *edgesL* = [(*1*,*2*), (*2*,*3*), (*3*,*4*), (*8*,*9*),(*9*,*8*)] ⦈
(λ*e*. *2*)

definition *Dependability-noref-offending-list*:: *'v list-graph* ⇒ (*'v* ⇒ *dependability-level*) ⇒ (*'v* × *'v*) *list list* **where**
Dependability-noref-offending-list = *Generic-offending-list sinvar*

definition *NetModel-node-props P* = (λ *i*. (*case* (*node-properties P*) *i of Some property* ⇒ *property*

| *None* ⇒ *SINVAR-Dependability-noreﬂ.default-node-properties*))
lemma[*code-unfold*]: *SecurityInvariant.node-props SINVAR-Dependability-noreﬂ.default-node-properties P* = *NetModel-node-props P*
apply(*simp add*: *NetModel-node-props-def*)
done

definition *Dependability-noreﬂ-eval G P* = (*wf-list-graph G* ∧
sinvar G (*SecurityInvariant.node-props SINVAR-Dependability-noreﬂ.default-node-properties P*))

lemma *sinvar-correct*: *wf-list-graph G* ⟹ *SINVAR-Dependability-noreﬂ.sinvar* (*list-graph-to-graph G*) *nP* = *sinvar G nP*
apply(*simp*)
apply(*rule all-edges-list-I*)
apply(*simp add*: *fun-eq-iff*)
apply(*clarify*)
apply(*rename-tac x*)
apply(*drule-tac v*=*x* **in** *num-reachable-noreﬂ-correct*)
apply *presburger*
done

interpretation *Dependability-noreﬂ-impl*:*TopoS-List-Impl*
where *default-node-properties*=*SINVAR-Dependability-noreﬂ.default-node-properties*
and *sinvar-spec*=*SINVAR-Dependability-noreﬂ.sinvar*
and *sinvar-impl*=*sinvar*
and *receiver-violation*=*SINVAR-Dependability-noreﬂ.receiver-violation*
and *offending-flows-impl*=*Dependability-noreﬂ-offending-list*
and *node-props-impl*=*NetModel-node-props*
and *eval-impl*=*Dependability-noreﬂ-eval*
apply(*unfold TopoS-List-Impl-def*)
apply(*rule conjI*)
apply(*rule conjI*)
apply(*simp add*: *TopoS-Dependability-noreﬂ*; *fail*)
apply(*intro allI impI*)
apply(*fact sinvar-correct*)
apply(*rule conjI*)
apply(*unfold Dependability-noreﬂ-offending-list-def*)
apply(*intro allI impI*)
apply(*rule Generic-offending-list-correct*)
apply(*assumption*)
apply(*simp only*: *sinvar-correct*)
apply(*rule conjI*)
apply(*intro allI*)
apply(*simp only*: *NetModel-node-props-def*)
apply(*metis Dependability.node-props.simps Dependability.node-props-eq-node-props-formaldef*)
apply(*simp only*: *Dependability-noreﬂ-eval-def*)
apply(*intro allI impI*)
apply(*rule TopoS-eval-impl-proofrule*[*OF TopoS-Dependability-noreﬂ*])
apply(*simp only*: *sinvar-correct*)
done

6.12.2 packing

definition *SINVAR-LIB-Dependability-noreﬂ* :: (*'v*::*vertex*, *SINVAR-Dependability-noreﬂ.dependability-level*) *TopoS-packed* **where**
SINVAR-LIB-Dependability-noreﬂ ≡
(| *nm-name* = *''Dependability-noreﬂ''*,
nm-receiver-violation = *SINVAR-Dependability-noreﬂ.receiver-violation*,
nm-default = *SINVAR-Dependability-noreﬂ.default-node-properties*,
nm-sinvar = *sinvar*,
nm-offending-flows = *Dependability-noreﬂ-offending-list*,
nm-node-props = *NetModel-node-props*,
nm-eval = *Dependability-noreﬂ-eval*
|)
interpretation *SINVAR-LIB-Dependability-noreﬂ-interpretation*: *TopoS-modelLibrary SINVAR-LIB-Dependability-no*
SINVAR-Dependability-noreﬂ.sinvar
apply(*unfold TopoS-modelLibrary-def SINVAR-LIB-Dependability-noreﬂ-def*)
apply(*rule conjI*)
apply(*simp*)
apply(*simp*)
by(*unfold-locales*)

hide-fact (**open**) *sinvar-correct*
hide-const (**open**) *sinvar NetModel-node-props*

end
theory *TopoS-Library*
imports
Lib/FiniteListGraph-Impl
Security-Invariants/SINVAR-BLPbasic-impl
Security-Invariants/SINVAR-Subnets-impl
Security-Invariants/SINVAR-DomainHierarchyNG-impl
Security-Invariants/SINVAR-BLPtrusted-impl
Security-Invariants/SINVAR-SecGwExt-impl
Security-Invariants/SINVAR-Sink-impl
Security-Invariants/SINVAR-SubnetsInGW-impl
Security-Invariants/SINVAR-CommunicationPartners-impl
Security-Invariants/SINVAR-NoRefl-impl
Security-Invariants/SINVAR-Tainting-impl
Security-Invariants/SINVAR-TaintingTrusted-impl

Security-Invariants/SINVAR-Dependability-impl
Security-Invariants/SINVAR-NonInterference-impl
Security-Invariants/SINVAR-ACLnotCommunicateWith-impl
Security-Invariants/SINVAR-ACLcommunicateWith-impl
Security-Invariants/SINVAR-Dependability-noreﬂ-impl
Lib/Efficient-Distinct
HOL−Library.Code-Target-Nat
begin

end
theory *TopoS-Composition-Theory*
imports *TopoS-Interface TopoS-Helper*

begin

7 Composition Theory

Several invariants may apply to one policy.

The security invariants are all collected in a list. The list corresponds to the security requirements. The list should have the type $('v\ graph \Rightarrow bool)\ list$, i.e. a list of predicates over the policy. We need in instantiated security invariant, i.e. get rid of $'a$ and $'b$

— An instance (configured) a security invariant I.e. a concrete security requirement, in different terminology.
record $('v)$ *SecurityInvariant-configured* =
c-sinvar::$('v)$ *graph* $\Rightarrow$ *bool*
c-offending-flows::$('v)$ *graph* $\Rightarrow$ $('v \times 'v)$ *set set*
c-isIFS::*bool*

— parameters 1-3 are the *SecurityInvariant*: *sinvar* $\perp$ *receiver-violation*
Fourth parameter is the host attribute mapping *nP*
TODO: probably check *wf-graph* here and optionally some host-attribute sanity checker as in DomainHierachy.
fun *new-configured-SecurityInvariant* ::
$((('v$::*vertex*$)\ graph \Rightarrow ('v \Rightarrow 'a) \Rightarrow bool) \times 'a \times bool \times ('v \Rightarrow 'a)) \Rightarrow ('v$ *SecurityInvariant-configured*$)$ *option* **where**
new-configured-SecurityInvariant (*sinvar*, *defbot*, *receiver-violation*, *nP*) =
(
if SecurityInvariant sinvar defbot receiver-violation then
Some (|
c-sinvar = (λG. *sinvar G nP*),
c-offending-flows = (λG. *SecurityInvariant-withOffendingFlows.set-offending-flows sinvar G nP*),
c-isIFS = *receiver-violation*
|)
else None
)

declare *new-configured-SecurityInvariant.simps*[*simp del*]

lemma *new-configured-TopoS-sinvar-correct*:
SecurityInvariant sinvar defbot receiver-violation $\Longrightarrow$
c-sinvar (*the* (*new-configured-SecurityInvariant* (*sinvar*, *defbot*, *receiver-violation*, *nP*))) = (λG. *sinvar G nP*)
by(*simp add*: *Let-def new-configured-SecurityInvariant.simps*)

lemma *new-configured-TopoS-offending-flows-correct*:
SecurityInvariant sinvar defbot receiver-violation $\Longrightarrow$
c-offending-flows (*the* (*new-configured-SecurityInvariant* (*sinvar*, *defbot*, *receiver-violation*, *nP*))) =

(λG. *SecurityInvariant-withOffendingFlows.set-offending-flows sinvar G nP*)
by(*simp add*: *Let-def new-configured-SecurityInvariant.simps*)

We now collect all the core properties of a security invariant, but without the $'a$ $'b$ types, so it is instantiated with a concrete configuration.

locale *configured-SecurityInvariant* =

fixes $m :: ('v::vertex)$ *SecurityInvariant-configured*
assumes
— As in SecurityInvariant definition
valid-c-offending-flows:
c-offending-flows $m\ G = \{F.\ F \subseteq (edges\ G) \wedge \neg\ c\text{-}sinvar\ m\ G \wedge c\text{-}sinvar\ m\ (delete\text{-}edges\ G\ F) \wedge$
$(\forall\ (e1,\ e2) \in F.\ \neg\ c\text{-}sinvar\ m\ (add\text{-}edge\ e1\ e2\ (delete\text{-}edges\ G\ F)))\}$
and
— A empty network can have no security violations
defined-offending:
$[\![\ wf\text{-}graph\ (\!|\ nodes = N,\ edges = \{\}\ |\!)\ ]\!] \Longrightarrow c\text{-}sinvar\ m\ (\!|\ nodes = N,\ edges = \{\}|\!)$
and
— prohibiting more does not decrease security
mono-sinvar:
$[\![\ wf\text{-}graph\ (\!|\ nodes = N,\ edges = E\ |\!);\ E' \subseteq E;\ c\text{-}sinvar\ m\ (\!|\ nodes = N,\ edges = E\ |\!)\ ]\!] \Longrightarrow$
$c\text{-}sinvar\ m\ (\!|\ nodes = N,\ edges = E'\ |\!)$
begin

lemma *sinvar-monoI*:
SecurityInvariant-withOffendingFlows.sinvar-mono $(\lambda\ (G::('v::vertex)\ graph)\ (nP::'v \Rightarrow 'a).\ c\text{-}sinvar\ m\ G)$
apply(*simp add*: *SecurityInvariant-withOffendingFlows.sinvar-mono-def*, *clarify*)
by(*fact mono-sinvar*)

if the network where nobody communicates with anyone fulfilles its security requirement, the offending flows are always defined.

lemma *defined-offending'*:
$[\![\ wf\text{-}graph\ G;\ \neg\ c\text{-}sinvar\ m\ G\ ]\!] \Longrightarrow c\text{-}offending\text{-}flows\ m\ G \neq \{\}$
proof −
assume *a1*: *wf-graph G*
and *a2*: ¬ *c-sinvar m G*
have *subst-set-offending-flows*:
$\bigwedge nP.\ SecurityInvariant\text{-}withOffendingFlows.set\text{-}offending\text{-}flows\ (\lambda G\ nP.\ c\text{-}sinvar\ m\ G)\ G\ nP = c\text{-}offending\text{-}flows\ m\ G$
by(*simp add*: *valid-c-offending-flows fun-eq-iff*
SecurityInvariant-withOffendingFlows.set-offending-flows-def
SecurityInvariant-withOffendingFlows.is-offending-flows-min-set-def
SecurityInvariant-withOffendingFlows.is-offending-flows-def)

from *a1* **have** *wfG-empty*: *wf-graph* (|*nodes = nodes G*, *edges* = {}|) **by**(*simp add*:*wf-graph-def*)

from *a1* **have** $\bigwedge nP.\ \neg\ c\text{-}sinvar\ m\ G \Longrightarrow SecurityInvariant\text{-}withOffendingFlows.set\text{-}offending\text{-}flows\ (\lambda G\ nP.\ c\text{-}sinvar\ m\ G)\ G\ nP \neq \{\}$
apply(*frule-tac finite-distinct-list*[*OF wf-graph.finiteE*])
apply(*erule-tac exE*)
apply(*rename-tac list-edges*)
apply(*rule-tac ff*=*list-edges* **in** *SecurityInvariant-withOffendingFlows.mono-imp-set-offending-flows-not-empty*[*OF sinvar-monoI*])
by(*auto simp add*: *SecurityInvariant-withOffendingFlows.is-offending-flows-def delete-edges-simp2 defined-offending*[*OF wfG-empty*])

thus *?thesis* **by**(*simp add*: *a2 subst-set-offending-flows*)
qed

lemma *subst-offending-flows*: $\bigwedge$ *nP. SecurityInvariant-withOffendingFlows.set-offending-flows* (λ*G nP. c-sinvar m G*) *G nP* = *c-offending-flows m G*
apply (*unfold SecurityInvariant-withOffendingFlows.set-offending-flows-def*
SecurityInvariant-withOffendingFlows.is-offending-flows-min-set-def
SecurityInvariant-withOffendingFlows.is-offending-flows-def)
by(*simp add*: *valid-c-offending-flows*)

all the *SecurityInvariant-preliminaries* stuff must hold, for an arbitrary *nP*

lemma *SecurityInvariant-preliminariesD*:
SecurityInvariant-preliminaries (λ (*G*::('*v*::*vertex*) *graph*) (*nP*::'*v* $\Rightarrow$ '*a*). *c-sinvar m G*)
proof(*unfold-locales*, *goal-cases*)
case *1* **thus** *?case* **using** *defined-offending'* **by**(*simp add*: *subst-offending-flows*)
next case *2* **thus** *?case* **by**(*fact mono-sinvar*)
next case *3* **thus** *?case* **by**(*fact SecurityInvariant-withOffendingFlows.sinvar-mono-imp-is-offending-flows-mono*[*OF sinvar-monoI*])
qed

lemma *negative-mono*:
$\bigwedge$ *N E' E. wf-graph* (| *nodes* = *N*, *edges* = *E* |) $\Longrightarrow$
E' $\subseteq$ *E* $\Longrightarrow$ $\neg$ *c-sinvar m* (| *nodes* = *N*, *edges* = *E'* |) $\Longrightarrow$ $\neg$ *c-sinvar m* (| *nodes* = *N*, *edges* = *E* |)
by(*blast dest*: *mono-sinvar*)

7.1 Reusing Lemmata

lemmas *mono-extend-set-offending-flows* =
SecurityInvariant-preliminaries.mono-extend-set-offending-flows[*OF SecurityInvariant-preliminariesD*, *simplified subst-offending-flows*]

⟦*wf-graph* (|*nodes* = *V*, *edges* = *E*|); *E'* $\subseteq$ *E*; *F'* $\in$ *c-offending-flows m* (|*nodes* = *V*, *edges* = *E'*|)⟧ $\Longrightarrow$ $\exists$ *F*$\in$*c-offending-flows m* (|*nodes* = *V*, *edges* = *E*|). *F'* $\subseteq$ *F*

lemmas *offending-flows-union-mono* =
SecurityInvariant-preliminaries.offending-flows-union-mono[*OF SecurityInvariant-preliminariesD*, *simplified subst-offending-flows*]

⟦*wf-graph* (|*nodes* = *V*, *edges* = *E*|); *E'* $\subseteq$ *E*⟧ $\Longrightarrow$ $\bigcup$ (*c-offending-flows m* (|*nodes* = *V*, *edges* = *E'*|)) $\subseteq$ $\bigcup$ (*c-offending-flows m* (|*nodes* = *V*, *edges* = *E*|))

lemmas *sinvar-valid-remove-flattened-offending-flows* =
SecurityInvariant-preliminaries.sinvar-valid-remove-flattened-offending-flows[*OF SecurityInvariant-preliminariesD*, *simplified subst-offending-flows*]

wf-graph (|*nodes* = *nodesG*, *edges* = *edgesG*|) $\Longrightarrow$ *c-sinvar m* (|*nodes* = *nodesG*, *edges* = *edgesG* $-$ $\bigcup$ (*c-offending-flows m* (|*nodes* = *nodesG*, *edges* = *edgesG*|))|)

lemmas *sinvar-valid-remove-SOME-offending-flows* =
SecurityInvariant-preliminaries.sinvar-valid-remove-SOME-offending-flows[*OF SecurityInvariant-preliminariesD*, *simplified subst-offending-flows*]

c-offending-flows m (|*nodes* = *nodesG*, *edges* = *edgesG*|) $\neq$ {} $\Longrightarrow$ *c-sinvar m* (|*nodes* = *nodesG*, *edges* = *edgesG* $-$ (*SOME F*. *F* $\in$ *c-offending-flows m* (|*nodes* = *nodesG*, *edges* = *edgesG*|))|)

lemmas *sinvar-valid-remove-minimalize-offending-overapprox* =
SecurityInvariant-preliminaries.sinvar-valid-remove-minimalize-offending-overapprox[*OF SecurityInvariant-prelimin*
simplified subst-offending-flows]

⟦*wf-graph* (|*nodes* = *nodesG*, *edges* = *edgesG*|); ¬ *c-sinvar m* (|*nodes* = *nodesG*, *edges* = *edgesG*|); *set Es* = *edgesG*; *distinct Es*⟧ ⟹ *c-sinvar m* (|*nodes* = *nodesG*, *edges* = *edgesG* − *set* (*SecurityInvariant-withOffendingFlows.minimalize-offending-overapprox* (λ*G nP*. *c-sinvar m G*) *Es* [] (|*nodes* = *nodesG*, *edges* = *edgesG*|) *nP*)|)

lemmas *empty-offending-contra* = *SecurityInvariant-withOffendingFlows.empty-offending-contra*[**where** *sinvar*=(λ*G nP*. *c-sinvar m G*), *simplified subst-offending-flows*]

⟦*F* ∈ *c-offending-flows m G*; *F* = {}⟧ ⟹ *False*

lemmas *Un-set-offending-flows-bound-minus-subseteq* = *SecurityInvariant-preliminaries.Un-set-offending-flows-bound-minus-subseteq*[*OF SecurityInvariant-preliminariesD*, *simplified subst-offending-flows*]

⟦*wf-graph* (|*nodes* = *V*, *edges* = *E*|); ⋃ (*c-offending-flows m* (|*nodes* = *V*, *edges* = *E*|)) ⊆ *X*⟧ ⟹ ⋃ (*c-offending-flows m* (|*nodes* = *V*, *edges* = *E* − *E*′|)) ⊆ *X* − *E*′

lemmas *Un-set-offending-flows-bound-minus-subseteq*′ = *SecurityInvariant-preliminaries.Un-set-offending-flows-bound-minus-subseteq*′[*OF SecurityInvariant-preliminariesD*, *simplified subst-offending-flows*]

⟦*wf-graph* (|*nodes* = *V*, *edges* = *E*|); ⋃ (*c-offending-flows m* (|*nodes* = *V*, *edges* = *E*|)) ⊆ *X*⟧ ⟹ ⋃ (*c-offending-flows m* (|*nodes* = *V*, *edges* = *E* − *E*′|)) ⊆ *X* − *E*′

end

thm *configured-SecurityInvariant-def*

configured-SecurityInvariant m ≡ (∀ *G*. *c-offending-flows m G* = {*F*. *F* ⊆ *edges G* ∧ ¬ *c-sinvar m G* ∧ *c-sinvar m* (*delete-edges G F*) ∧ (∀ (*e1*, *e2*)∈*F*. ¬ *c-sinvar m* (*add-edge e1 e2* (*delete-edges G F*)))}) ∧ (∀ *N*. *wf-graph* (|*nodes* = *N*, *edges* = {}|) ⟶ *c-sinvar m* (|*nodes* = *N*, *edges* = {}|)) ∧ (∀ *N E E*′. *wf-graph* (|*nodes* = *N*, *edges* = *E*|) ⟶ *E*′ ⊆ *E* ⟶ *c-sinvar m* (|*nodes* = *N*, *edges* = *E*|) ⟶ *c-sinvar m* (|*nodes* = *N*, *edges* = *E*′|))

thm *configured-SecurityInvariant.mono-sinvar*

⟦*configured-SecurityInvariant m*; *wf-graph* (|*nodes* = *N*, *edges* = *E*|); *E*′ ⊆ *E*; *c-sinvar m* (|*nodes* = *N*, *edges* = *E*|)⟧ ⟹ *c-sinvar m* (|*nodes* = *N*, *edges* = *E*′|)

Naming convention: m :: network security requirement M :: network security requirement list

The function *new-configured-SecurityInvariant* takes some tuple and if it returns a result, the locale assumptions are automatically fulfilled.

theorem *new-configured-SecurityInvariant-sound*:
⟦ *new-configured-SecurityInvariant* (*sinvar*, *defbot*, *receiver-violation*, *nP*) = *Some m* ⟧ ⟹
configured-SecurityInvariant m
proof −
assume *a*: *new-configured-SecurityInvariant* (*sinvar*, *defbot*, *receiver-violation*, *nP*) = *Some m*
hence *NetModel*: *SecurityInvariant sinvar defbot receiver-violation*
by(*simp add*: *new-configured-SecurityInvariant.simps split*: *if-split-asm*)
hence *NetModel-p*: *SecurityInvariant-preliminaries sinvar* **by**(*simp add*: *SecurityInvariant-def*)

from *a* **have** *c-eval*: *c-sinvar m* = (λ*G*. *sinvar G nP*)
and *c-offending*: *c-offending-flows m* = (λ*G*. *SecurityInvariant-withOffendingFlows.set-offending-flows sinvar G nP*)

and *c-isIFS m = receiver-violation*
by(*auto simp add*: *new-configured-SecurityInvariant.simps NetModel split*: *if-split-asm*)

have *monoI*: *SecurityInvariant-withOffendingFlows.sinvar-mono sinvar*
apply(*simp add*: *SecurityInvariant-withOffendingFlows.sinvar-mono-def*, *clarify*)
by(*fact SecurityInvariant-preliminaries.mono-sinvar*[*OF NetModel-p*])
from *SecurityInvariant-withOffendingFlows.valid-empty-edges-iff-exists-offending-flows*[*OF monoI*, *symmetric*]
SecurityInvariant-preliminaries.defined-offending[*OF NetModel-p*]
have *eval-empty-graph*: ⋀ *N nP*. *wf-graph* (|*nodes* = *N*, *edges* = {}|) ⟹ *sinvar* (|*nodes* = *N*, *edges* = {}|) *nP*
by *fastforce*

show *?thesis*
apply(*unfold-locales*)
apply(*simp add*: *c-eval c-offending SecurityInvariant-withOffendingFlows.set-offending-flows-def SecurityInvariant-withOffendingFlows.is-offending-flows-min-set-def SecurityInvariant-withOffendingFlows.is-offending-*
apply(*simp add*: *c-eval eval-empty-graph*)
apply(*simp add*: *c-eval*,*drule*(*3*) *SecurityInvariant-preliminaries.mono-sinvar*[*OF NetModel-p*])
done
qed

All security invariants are valid according to the definition

definition *valid-reqs* :: (*'v*::*vertex*) *SecurityInvariant-configured list* ⇒ *bool* **where**
valid-reqs M ≡ ∀ *m* ∈ *set M*. *configured-SecurityInvariant m*

7.2 Algorithms

A (generic) security invariant corresponds to a type of security requirements (type: *'v graph* ⇒ (*'v* ⇒ *'a*) ⇒ *bool*). A configured security invariant is a security requirement in a scenario specific setting (type: *'v graph* ⇒ *bool*). I.e., it is a security requirement as listed in the requirements document. All security requirements are fulfilled for a fixed policy G if all security requirements are fulfilled for G.

Get all possible offending flows from all security requirements

definition *get-offending-flows* :: *'v SecurityInvariant-configured list* ⇒ *'v graph* ⇒ ((*'v* × *'v*) *set set*) **where**
get-offending-flows M G = (⋃ *m*∈*set M*. *c-offending-flows m G*)

definition *all-security-requirements-fulfilled* :: (*'v*::*vertex*) *SecurityInvariant-configured list* ⇒ *'v graph* ⇒ *bool* **where**
all-security-requirements-fulfilled M G ≡ ∀ *m* ∈ *set M*. (*c-sinvar m*) *G*

Generate a valid topology from the security requirements

fun *generate-valid-topology* :: *'v SecurityInvariant-configured list* ⇒ *'v graph* ⇒ *'v graph* **where**
generate-valid-topology [] *G* = *G* |
generate-valid-topology (*m*#*Ms*) *G* = *delete-edges* (*generate-valid-topology Ms G*) (⋃ (*c-offending-flows m G*))

— return all Access Control Strategy models from a list of models
definition *get-ACS* :: (*'v*::*vertex*) *SecurityInvariant-configured list* ⇒ *'v SecurityInvariant-configured list* **where**

get-ACS M ≡ [*m* ← *M*. ¬ *c-isIFS m*]
— return all Information Flows Strategy models from a list of models
definition *get-IFS* :: ('*v*::*vertex*) *SecurityInvariant-configured list* ⇒ '*v SecurityInvariant-configured list* **where**
get-IFS M ≡ [*m* ← *M*. *c-isIFS m*]
lemma *get-ACS-union-get-IFS*: *set* (*get-ACS M*) ∪ *set* (*get-IFS M*) = *set M*
by(*auto simp add*: *get-ACS-def get-IFS-def*)

7.3 Lemmata

lemma *valid-reqs1*: *valid-reqs* (*m* # *M*) ⟹ *configured-SecurityInvariant m*
by(*simp add*: *valid-reqs-def*)
lemma *valid-reqs2*: *valid-reqs* (*m* # *M*) ⟹ *valid-reqs M*
by(*simp add*: *valid-reqs-def*)
lemma *get-offending-flows-alt1*: *get-offending-flows M G* = ⋃ {*c-offending-flows m G* | *m*. *m* ∈ *set M*}
apply(*simp add*: *get-offending-flows-def*)
by *fastforce*
lemma *get-offending-flows-un*: ⋃(*get-offending-flows M G*) = (⋃*m*∈*set M*. ⋃(*c-offending-flows m G*))
apply(*simp add*: *get-offending-flows-def*)
by *blast*

lemma *all-security-requirements-fulfilled-mono*:
⟦ *valid-reqs M*; *E'* ⊆ *E*; *wf-graph* (| *nodes* = *V*, *edges* = *E* |) ⟧ ⟹
all-security-requirements-fulfilled M (| *nodes* = *V*, *edges* = *E* |) ⟹
all-security-requirements-fulfilled M (| *nodes* = *V*, *edges* = *E'* |)
apply(*induction M arbitrary*: *E' E*)
apply(*simp-all add*: *all-security-requirements-fulfilled-def*)
apply(*rename-tac m M E' E*)
apply(*rule conjI*)
apply(*erule*(*2*) *configured-SecurityInvariant.mono-sinvar*[*OF valid-reqs1*])
apply(*simp-all*)
apply(*drule valid-reqs2*)
apply *blast*
done

7.4 generate valid topology

lemma *generate-valid-topology-nodes*:
nodes (*generate-valid-topology M G*) = (*nodes G*)
apply(*induction M arbitrary*: *G*)
by(*simp-all add*: *graph-ops*)

lemma *generate-valid-topology-def-alt*:
generate-valid-topology M G = *delete-edges G* (⋃ (*get-offending-flows M G*))
proof(*induction M arbitrary*: *G*)
case *Nil*
thus *?case* **by**(*simp add*: *get-offending-flows-def*)
next
case (*Cons m M*)
from *Cons*[*simplified delete-edges-simp2 get-offending-flows-def*]
have *edges* (*generate-valid-topology M G*) = *edges G* − ⋃(⋃*m*∈*set M*. *c-offending-flows m*

G)
by (*metis graph.select-convs*(*2*))
thus *?case*
apply(*simp add*: *get-offending-flows-def delete-edges-simp2*)
apply(*rule*)
apply(*simp add*: *generate-valid-topology-nodes*)
by *blast*
qed

lemma *wf-graph-generate-valid-topology*: *wf-graph G* ⟹ *wf-graph* (*generate-valid-topology M G*)
proof(*induction M arbitrary*: *G*)
qed(*simp-all*)

lemma *generate-valid-topology-mono-models*:
edges (*generate-valid-topology* (*m*#*M*) (| *nodes* = *V*, *edges* = *E* |)) ⊆ *edges* (*generate-valid-topology M* (| *nodes* = *V*, *edges* = *E* |))
proof(*induction M arbitrary*: *E m*)
case *Nil* **thus** *?case* **by**(*simp add*: *delete-edges-simp2*)
case *Cons* **thus** *?case* **by**(*simp add*: *delete-edges-simp2*)
qed

lemma *generate-valid-topology-subseteq-edges*:
edges (*generate-valid-topology M G*) ⊆ (*edges G*)
proof(*induction M arbitrary*: *G*)
case *Cons* **thus** *?case* **by** (*simp add*: *delete-edges-simp2*) *blast*
qed(*simp*)

generate-valid-topology generates a valid topology (Policy)!

theorem *generate-valid-topology-sound*:
⟦ *valid-reqs M*; *wf-graph* (|*nodes* = *V*, *edges* = *E*|) ⟧ ⟹
all-security-requirements-fulfilled M (*generate-valid-topology M* (|*nodes* = *V*, *edges* = *E*|))
proof(*induction M arbitrary*: *V E*)
case *Nil*
thus *?case* **by**(*simp add*: *all-security-requirements-fulfilled-def*)
next
case (*Cons m M*)
from *valid-reqs1*[*OF Cons*(*2*)] **have** *validReq*: *configured-SecurityInvariant m* **.**

from *Cons*(*3*) **have** *valid-rmUnOff*: *wf-graph* (|*nodes* = *V*, *edges* = *E* − ⋃(*c-offending-flows m* (|*nodes* = *V*, *edges* = *E*|)) |)
by(*simp add*: *wf-graph-remove-edges*)

from *configured-SecurityInvariant.sinvar-valid-remove-flattened-offending-flows*[*OF validReq Cons*(*3*)]
have *valid-eval-rmUnOff*: *c-sinvar m* (|*nodes* = *V*, *edges* = *E* − ⋃(*c-offending-flows m* (|*nodes* = *V*, *edges* = *E*|)) |) **.**

from *generate-valid-topology-subseteq-edges* **have** *edges-gentopo-subseteq*:
edges (*generate-valid-topology M* (|*nodes* = *V*, *edges* = *E*|)) − ⋃(*c-offending-flows m* (|*nodes* = *V*, *edges* = *E*|))
⊆
E − ⋃(*c-offending-flows m* (|*nodes* = *V*, *edges* = *E*|)) **by** *fastforce*

from *configured-SecurityInvariant.mono-sinvar*[*OF validReq valid-rmUnOff edges-gentopo-subseteq*

valid-eval-rmUnOff]
have *c-sinvar m* (⦇*nodes* = *V*, *edges* = (*edges* (*generate-valid-topology M* (⦇*nodes* = *V*, *edges* = *E*⦈)) − ⋃(*c-offending-flows m* (⦇*nodes* = *V*, *edges* = *E*⦈)) ⦈ **.**
from *this* **have** *goal1*:
c-sinvar m (*delete-edges* (*generate-valid-topology M* (⦇*nodes* = *V*, *edges* = *E*⦈)) (⋃(*c-offending-flows m* (⦇*nodes* = *V*, *edges* = *E*⦈))))
by(*simp add*: *delete-edges-simp2 generate-valid-topology-nodes*)

from *valid-reqs2*[*OF Cons*(*2*)] **have** *valid-reqs M* **.**
from *Cons.IH*[*OF* ‹*valid-reqs M*› *Cons*(*3*)] **have** *IH*:
all-security-requirements-fulfilled M (*generate-valid-topology M* (⦇*nodes* = *V*, *edges* = *E*⦈)) **.**

have *generate-valid-topology-EX-graph-record*:
∃ *hypE*. (*generate-valid-topology M* (⦇*nodes* = *V*, *edges* = *E*⦈)) = (⦇*nodes* = *V*, *edges* = *hypE*⦈)
apply(*induction M arbitrary*: *V E*)
by(*simp-all add*: *delete-edges-simp2 generate-valid-topology-nodes*)

from *generate-valid-topology-EX-graph-record* **obtain** *E-IH* **where** *E-IH-prop*:
(*generate-valid-topology M* (⦇*nodes* = *V*, *edges* = *E*⦈)) = (⦇*nodes* = *V*, *edges* = *E-IH*⦈) **by** *blast*

from *wf-graph-generate-valid-topology*[*OF Cons*(*3*)] *E-IH-prop*
have *valid-G-E-IH*: *wf-graph* (⦇*nodes* = *V*, *edges* = *E-IH*⦈) **by** *metis*

— *all-security-requirements-fulfilled M* (⦇*nodes* = *V*, *edges* = *E-IH*⦈)
— *?E′* ⊆ *E-IH* ⟹ *all-security-requirements-fulfilled M* (⦇*nodes* = *V*, *edges* = *?E′*⦈)

from *all-security-requirements-fulfilled-mono*[*OF* ‹*valid-reqs M*› - *valid-G-E-IH IH*[*simplified E-IH-prop*]] **have** *mono-rule*:
⋀ *E′*. *E′* ⊆ *E-IH* ⟹ *all-security-requirements-fulfilled M* (⦇*nodes* = *V*, *edges* = *E′*⦈) **.**

have *all-security-requirements-fulfilled M*
(*delete-edges* (*generate-valid-topology M* (⦇*nodes* = *V*, *edges* = *E*⦈)) (⋃(*c-offending-flows m* (⦇*nodes* = *V*, *edges* = *E*⦈))))
apply(*subst E-IH-prop*)
apply(*simp add*: *delete-edges-simp2*)
apply(*rule mono-rule*)
by *fast*

from *this* **have** *goal2*:
(∀ *ma*∈*set M*.
c-sinvar ma (*delete-edges* (*generate-valid-topology M* (⦇*nodes* = *V*, *edges* = *E*⦈)) (⋃(*c-offending-flows m* (⦇*nodes* = *V*, *edges* = *E*⦈)))))
by(*simp add*: *all-security-requirements-fulfilled-def*)

from *goal1 goal2*
show *all-security-requirements-fulfilled* (*m* # *M*) (*generate-valid-topology* (*m* # *M*) (⦇*nodes* = *V*, *edges* = *E*⦈)
by (*simp add*: *all-security-requirements-fulfilled-def*)
qed

lemma *generate-valid-topology-as-set*:

generate-valid-topology M G = delete-edges G ($\bigcup m \in$ *set M*. ($\bigcup$ (*c-offending-flows m G*)))
apply(*induction M arbitrary*: *G*)
apply(*simp-all add*: *delete-edges-simp2 generate-valid-topology-nodes*) **by** *fastforce*

lemma *c-offending-flows-subseteq-edges*: *configured-SecurityInvariant m* $\Longrightarrow \bigcup$(*c-offending-flows m G*) $\subseteq$ *edges G*
apply(*clarify*)
apply(*simp only*: *configured-SecurityInvariant.valid-c-offending-flows*)
apply(*thin-tac configured-SecurityInvariant x* **for** *x*)
by *auto*

Does it also generate a maximum topology? It does, if the security invariants are in ENF-form. That means, if all security invariants can be expressed as a predicate over the edges, $\exists P.\ \forall G.$ *c-sinvar m G* = ($\forall$ (*v1*, *v2*)$\in$*edges G*. *P* (*v1*, *v2*))

definition *max-topo* :: ('*v*::*vertex*) *SecurityInvariant-configured list* $\Rightarrow$ '*v graph* $\Rightarrow$ *bool* **where**
max-topo M G $\equiv$ *all-security-requirements-fulfilled M G* $\wedge$ (
$\forall$ (*v1*, *v2*) $\in$ (*nodes G* $\times$ *nodes G*) $-$ (*edges G*). $\neg$ *all-security-requirements-fulfilled M* (*add-edge v1 v2 G*))

lemma *unique-offending-obtain*:
assumes *m*: *configured-SecurityInvariant m* **and** *unique*: *c-offending-flows m G* = {*F*}
obtains *P* **where** *F* = {(*v1*, *v2*) $\in$ *edges G*. $\neg$ *P* (*v1*, *v2*)} **and** *c-sinvar m G* = ($\forall$(*v1*,*v2*) $\in$ *edges G*. *P* (*v1*, *v2*)) **and**
($\forall$(*v1*,*v2*) $\in$ *edges G* $-$ *F*. *P* (*v1*, *v2*))
proof $-$
assume *EX*: ($\bigwedge$*P*. *F* = {(*v1*, *v2*). (*v1*, *v2*) $\in$ *edges G* $\wedge$ $\neg$ *P* (*v1*, *v2*)} $\Longrightarrow$ *c-sinvar m G* = ($\forall$(*v1*, *v2*)$\in$*edges G*. *P* (*v1*, *v2*)) $\Longrightarrow$ $\forall$(*v1*, *v2*)$\in$*edges G* $-$ *F*. *P* (*v1*, *v2*) $\Longrightarrow$ *thesis*)

from *unique c-offending-flows-subseteq-edges*[*OF m*] **have** *F* $\subseteq$ *edges G* **by** *force*
from *this* **obtain** *P* **where** *F* = {*e* $\in$ *edges G*. $\neg$ *P e*} **by** (*metis double-diff set-diff-eq subset-refl*)
hence *1*: *F* = {(*v1*, *v2*) $\in$ *edges G*. $\neg$ *P* (*v1*, *v2*)} **by** *auto*

from *configured-SecurityInvariant.valid-c-offending-flows*[*OF m*] **have** *c-offending-flows m G* =
{*F*. *F* $\subseteq$ *edges G* $\wedge$ $\neg$ *c-sinvar m G* $\wedge$ *c-sinvar m* (*delete-edges G F*) $\wedge$
($\forall$(*e1*, *e2*)$\in$*F*. $\neg$ *c-sinvar m* (*add-edge e1 e2* (*delete-edges G F*)))} **.**

from *this unique* **have** $\neg$ *c-sinvar m G* **and** *2*: *c-sinvar m* (*delete-edges G F*) **and**
3: ($\forall$(*e1*, *e2*)$\in$*F*. $\neg$ *c-sinvar m* (*add-edge e1 e2* (*delete-edges G F*))) **by** *auto*

from *this* ‹*F* = {*e* $\in$ *edges G*. $\neg$ *P e*}› **have** *x3*: $\forall$ *e* $\in$ *edges G* $-$ *F*. *P e* **by** (*metis* (*lifting*) *mem-Collect-eq set-diff-eq*)
hence *4*: $\forall$(*v1*,*v2*) $\in$ *edges G* $-$ *F*. *P* (*v1*, *v2*) **by** *blast*

have *F* $\neq$ {} **by** (*metis assms*(*1*) *assms*(*2*) *configured-SecurityInvariant.empty-offending-contra insertCI*)
from *this* ‹*F* = {*e* $\in$ *edges G*. $\neg$ *P e*}› ‹$\neg$ *c-sinvar m G*› **have** *5*: *c-sinvar m G* = ($\forall$(*v1*,*v2*) $\in$ *edges G*. *P* (*v1*, *v2*))
apply(*simp add*: *graph-ops*)
by(*blast*)

from *EX*[*of P*] *unique 1 x3 5* **show** *?thesis* **by** *fast*
qed

lemma *enf-offending-flows*:

assumes *vm*: *configured-SecurityInvariant m* **and** *enf*: $\forall\, G.$ *c-sinvar m G* = $(\forall\, e \in$ *edges G*. *P e*)
shows $\forall\, G.$ *c-offending-flows m G* = (*if c-sinvar m G then* {} *else* {{*e* ∈ *edges G*. ¬ *P e*}})
proof −
{ fix *G*
from *vm configured-SecurityInvariant.valid-c-offending-flows* **have** *offending-formaldef*:
c-offending-flows m G =
{*F*. *F* ⊆ *edges G* ∧ ¬ *c-sinvar m G* ∧ *c-sinvar m* (*delete-edges G F*) ∧
(∀ (*e1*, *e2*)∈*F*. ¬ *c-sinvar m* (*add-edge e1 e2* (*delete-edges G F*)))} **by** *auto*
have *c-offending-flows m G* = (*if c-sinvar m G then* {} *else* {{*e* ∈ *edges G*. ¬ *P e*}})
proof(*cases c-sinvar m G*)
case *True* **thus** *?thesis* — {}
by(*simp add*: *offending-formaldef*)
next
case *False* **thus** *?thesis* **by**(*auto simp add*: *offending-formaldef graph-ops enf*)
qed
} thus *?thesis* **by** *simp*
qed

lemma *enf-not-fulfilled-if-in-offending*:
assumes *validRs*: *valid-reqs M*
and *wfG*: *wf-graph G*
and *enf*: ∀ *m* ∈ *set M*. ∃ *P*. ∀ *G*. *c-sinvar m G* = (∀ *e* ∈ *edges G*. *P e*)
shows ∀ *x* ∈ (⋃ *m*∈*set M*. ⋃(*c-offending-flows m* (*fully-connected G*))).
¬ *all-security-requirements-fulfilled M* (| *nodes* = *V*, *edges* = *insert x E*|)
unfolding *all-security-requirements-fulfilled-def*
proof(*simp*, *clarify*, *rename-tac m F a b*)
let *?G*=(*fully-connected G*)
fix *m F v1 v2*
assume *m* ∈ *set M* **and** *F* ∈ *c-offending-flows m ?G* **and** (*v1*, *v2*) ∈ *F*

from *validRs* **have** *valid-mD*:⋀*m*. *m* ∈ *set M* ⟹ *configured-SecurityInvariant m*
by(*simp add*: *valid-reqs-def*)

from ‹*m* ∈ *set M*› *valid-mD* **have** *configured-SecurityInvariant m* **by** *simp*

from *enf* ‹*m* ∈ *set M*› **obtain** *P* **where** *enf-m*: ∀ *G*. *c-sinvar m G* = (∀ *e*∈*edges G*. *P e*) **by** *blast*

from ‹(*v1*, *v2*) ∈ *F*› **have** *F* ≠ {} **by** *auto*

from *enf-offending-flows*[*OF* ‹*configured-SecurityInvariant m*› ‹∀ *G*. *c-sinvar m G* = (∀ *e*∈*edges G*. *P e*)›] **have**
offending: ⋀*G*. *c-offending-flows m G* = (*if c-sinvar m G then* {} *else* {{*e* ∈ *edges G*. ¬ *P e*}})
by *simp*
from ‹*F* ∈ *c-offending-flows m ?G*› ‹*F* ≠ {}› **have** *F* = {*e* ∈ *edges ?G*. ¬ *P e*}
by(*simp split*: *if-split-asm add*: *offending*)
from *this* ‹(*v1*, *v2*) ∈ *F*› **have** ¬ *P* (*v1*, *v2*) **by** *simp*

from *this enf-m* **have** ¬ *c-sinvar m* (|*nodes* = *V*, *edges* = *insert* (*v1*, *v2*) *E*|) **by**(*simp*)
thus ∃ *m*∈*set M*. ¬ *c-sinvar m* (|*nodes* = *V*, *edges* = *insert* (*v1*, *v2*) *E*|) **using** ‹*m* ∈ *set M*›
apply(*rule-tac x*=*m* **in** *bexI*)
by *simp-all*
qed

theorem *generate-valid-topology-max-topo*: ⟦ *valid-reqs M*; *wf-graph G*;
∀ *m* ∈ *set M*. ∃ *P*. ∀ *G*. *c-sinvar m G* = (∀ *e* ∈ *edges G*. *P e*)⟧ ⟹
max-topo M (*generate-valid-topology M* (*fully-connected G*))
proof −
let *?G*=(*fully-connected G*)
assume *validRs*: *valid-reqs M*
and *wfG*: *wf-graph G*
and *enf*: ∀ *m* ∈ *set M*. ∃ *P*. ∀ *G*. *c-sinvar m G* = (∀ *e* ∈ *edges G*. *P e*)

obtain *V E* **where** *VE-prop*: (| *nodes* = *V*, *edges* = *E* |) = *generate-valid-topology M ?G* **by** (*metis graph.cases*)
hence *VE-prop-asset*:
(| *nodes* = *V*, *edges* = *E* |) = (| *nodes* = *V*, *edges* = *V* × *V* − (⋃ *m*∈*set M*. ⋃ (*c-offending-flows m ?G*))|)
by(*simp add*: *fully-connected-def generate-valid-topology-as-set delete-edges-simp2*)

from *VE-prop-asset* **have** *E-prop*: *E* = *V* × *V* − (⋃ *m*∈*set M*. ⋃ (*c-offending-flows m ?G*)) **by** *fast*
from *VE-prop* **have** *V-prop*: *nodes G* = *V*
by (*simp add*: *fully-connected-def delete-edges-simp2 generate-valid-topology-def-alt*)
from *VE-prop* **have** *V-full-prop*: *nodes* (*generate-valid-topology M ?G*) = *V* **by** (*metis graph.select-convs*(*1*))
from *VE-prop* **have** *E-full-prop*: *edges* (*generate-valid-topology M ?G*) = *E* **by** (*metis graph.select-convs*(*2*))

from *VE-prop wf-graph-generate-valid-topology*[*OF fully-connected-wf*[*OF wfG*]]
have *wfG-VE*: *wf-graph* (| *nodes* = *V*, *edges* = *E* |) **by** *force*

from *generate-valid-topology-sound*[*OF validRs wfG-VE*] *fully-connected-wf*[*OF wfG*] **have** *VE-all-valid*:

all-security-requirements-fulfilled M (| *nodes* = *V*, *edges* = *V* × *V* − (⋃ *m*∈*set M*. ⋃ (*c-offending-flows m ?G*))|)
by (*metis VE-prop VE-prop-asset fully-connected-def generate-valid-topology-sound validRs*)
hence *goal1*: *all-security-requirements-fulfilled M* (*generate-valid-topology M* (*fully-connected G*)) **by** (*metis VE-prop VE-prop-asset*)

from *validRs* **have** *valid-mD*:⋀*m*. *m* ∈ *set M* ⟹ *configured-SecurityInvariant m*
by(*simp add*: *valid-reqs-def*)

from *c-offending-flows-subseteq-edges*[**where** *G*=*?G*] *validRs* **have** *hlp1*: (⋃ *m*∈*set M*. ⋃ (*c-offending-flows m ?G*)) ⊆ *V* × *V*
apply(*simp add*: *fully-connected-def V-prop*)
using *valid-reqs-def* **by** *blast*
have ⋀*A B*. *A* − (*A* − *B*) = *B* ∩ *A* **by** *fast*
from *E-prop hlp1* **have** *V* × *V* − *E* = (⋃ *m*∈*set M*. ⋃ (*c-offending-flows m ?G*)) **by** *force*

from *enf-not-fulfilled-if-in-offending*[*OF validRs wfG enf*]
have ∀ (*v1*, *v2*) ∈ (⋃ *m*∈*set M*. ⋃ (*c-offending-flows m ?G*)).
¬ *all-security-requirements-fulfilled M* (| *nodes* = *V*, *edges* = *E* ∪ {(*v1*, *v2*)}|) **by** *simp*

from *this* ‹*V* × *V* − *E* = (⋃ *m*∈*set M*. ⋃ (*c-offending-flows m ?G*))› **have** ∀ (*v1*, *v2*) ∈ *V* × *V* − *E*.
¬ *all-security-requirements-fulfilled M* (| *nodes* = *V*, *edges* = *E* ∪ {(*v1*, *v2*)}|) **by** *simp*
hence *goal2*: (∀ (*v1*, *v2*)∈*nodes* (*generate-valid-topology M ?G*) × *nodes* (*generate-valid-topology*

M ?G) −
edges (*generate-valid-topology M ?G*).
¬ *all-security-requirements-fulfilled M* (*add-edge v1 v2* (*generate-valid-topology M ?G*)))
proof(*unfold V-full-prop E-full-prop graph-ops*)
assume *a*: ∀(*v1*, *v2*)∈*V* × *V* − *E*. ¬ *all-security-requirements-fulfilled M* (|*nodes* = *V*, *edges* = *E* ∪ {(*v1*, *v2*)}|)
have ∀(*v1*, *v2*)∈*V* × *V* − *E*. *V* ∪ {*v1*, *v2*} = *V* **by** *blast*
hence ∀(*v1*, *v2*)∈*V* × *V* − *E*. (|*nodes* = *V* ∪ {*v1*, *v2*}, *edges* = {(*v1*, *v2*)} ∪ *E*|) = (|*nodes* = *V*, *edges* = *E* ∪ {(*v1*, *v2*)}|) **by** *blast*
from *this a* **show** ∀(*v1*, *v2*)∈*V* × *V* − *E*. ¬ *all-security-requirements-fulfilled M* (|*nodes* = *V* ∪ {*v1*, *v2*}, *edges* = {(*v1*, *v2*)} ∪ *E*|)
— TODO: this should be trivial ...
apply(*simp*)
apply(*rule ballI*)
apply(*erule-tac x*=*x* **and** *A*=*V* × *V* − *E* **in** *ballE*)
prefer *2* **apply**(*simp*; *fail*)
apply(*erule-tac x*=*x* **and** *A*=*V* × *V* − *E* **in** *ballE*)
prefer *2* **apply**(*simp*; *fail*)
apply(*clarify*)
by *presburger*
qed

from *goal1 goal2* **show** *?thesis*
unfolding *max-topo-def* **by** *presburger*
qed

lemma *enf-all-valid-policy-subset-of-max*:
assumes *validRs*: *valid-reqs M*
and *wfG*: *wf-graph G*
and *enf*: ∀ *m* ∈ *set M*. ∃ *P*. ∀ *G*. *c-sinvar m G* = (∀ *e* ∈ *edges G*. *P e*)
and *nodesG'*: *nodes G* = *nodes G'*
shows ⟦ *wf-graph G'*;
all-security-requirements-fulfilled M G'⟧ ⟹
edges G' ⊆ *edges* (*generate-valid-topology M* (*fully-connected G*))
using *nodesG'* **apply**(*cases generate-valid-topology M* (*fully-connected G*), *rename-tac V E*, *simp*)
apply(*cases G'*, *rename-tac V' E'*, *simp*)
apply(*subgoal-tac nodes G* = *V*)
prefer *2*
apply (*metis fully-connected-def generate-valid-topology-nodes graph.select-convs*(*1*))
apply(*simp*)
proof(*rule ccontr*)
fix *V E V' E'*
assume *a5*: *all-security-requirements-fulfilled M* (|*nodes* = *V*, *edges* = *E'*|) **and**
a6: *generate-valid-topology M* (*fully-connected G*) = (|*nodes* = *V*, *edges* = *E*|) **and**
a10: *wf-graph* (|*nodes* = *V*, *edges* = *E'*|) **and**
contr: ¬ *E'* ⊆ *E*

from *wfG a6* **have** *wf-graph* (|*nodes* = *V*, *edges* = *E*|)
by (*metis fully-connected-wf wf-graph-generate-valid-topology*)
with *a10* **have** *EE'subsets*: *fst* ‘ *E* ⊆ *V* ∧ *snd* ‘ *E* ⊆ *V* ∧ *fst* ‘ *E'* ⊆ *V* ∧ *snd* ‘ *E'* ⊆ *V*
by(*simp add*: *wf-graph-def*)
hence *EE'subsets'*: *E* ⊆ *V* × *V* ∧ *E'* ⊆ *V* × *V* **by** *auto*

from *generate-valid-topology-max-topo*[*OF validRs wfG enf*]

```
    have m1: all-security-requirements-fulfilled M (|nodes = V, edges = E|) and
        m2: (∀ x∈V × V − E. case x of (v1, v2) ⇒ ¬ all-security-requirements-fulfilled M (add-edge
v1 v2 (|nodes = V, edges = E|)))
    by(simp add: max-topo-def a6)+

  from m2 have m2': ∀ x∈V × V − E. ¬ all-security-requirements-fulfilled M (|nodes = V, edges
= insert x E|)
    apply(simp add: add-edge-def)
    apply(rule ballI, rename-tac x)
    apply(erule-tac x=x in ballE, simp-all)
    apply(case-tac x, simp)
    by (simp add: insert-absorb)

   show False
    proof(cases V = {})
     case True
     with EE'subsets a10 have E = {} and E' = {}
      by(simp add: wf-graph-def)+
     with True contr show ?thesis by simp
    next
     case False
     with EE'subsets' contr obtain x where x: x ∈ E' ∧ x ∉ E ∧ x ∈ V × V
      by blast
     from m2' x have ¬ all-security-requirements-fulfilled M (|nodes = V, edges = insert x E|)
      by (simp)

     from a6 x have x-offedning: x ∈ (⋃ m∈set M. ⋃(c-offending-flows m (fully-connected G)))
      apply(simp add: generate-valid-topology-as-set delete-edges-simp2 fully-connected-def)
      by blast

     from enf-not-fulfilled-if-in-offending[OF validRs wfG enf] x-offedning have
       1: ¬ all-security-requirements-fulfilled M (|nodes = V, edges = insert x myE|) for myE by
blast

     from x have insertxE': insert x E' = E' by blast
     with a5 have
      all-security-requirements-fulfilled M (|nodes = V, edges = insert x E'|) by simp
     with insertxE' all-security-requirements-fulfilled-mono[OF validRs - a10 a5] have
      2: all-security-requirements-fulfilled M (|nodes = V, edges = insert x {}|) by blast
     from 1 2 show ?thesis by blast
    qed
  qed
```

7.5 More Lemmata

```
   lemma (in configured-SecurityInvariant) c-sinvar-valid-imp-no-offending-flows:
    c-sinvar m G ⟹ c-offending-flows m G = {}
     by(simp add: valid-c-offending-flows)

   lemma all-security-requirements-fulfilled-imp-no-offending-flows:
      valid-reqs M ⟹ all-security-requirements-fulfilled M G ⟹ (⋃ m∈set M. ⋃(c-offending-flows
m G)) = {}
     proof(induction M)
     case Cons thus ?case
```

unfolding *all-security-requirements-fulfilled-def*
apply(*simp*)
by(*blast dest*: *valid-reqs2 valid-reqs1 configured-SecurityInvariant.c-sinvar-valid-imp-no-offending-flows*)
qed(*simp*)

corollary *all-security-requirements-fulfilled-imp-get-offending-empty*:
valid-reqs M ⟹ *all-security-requirements-fulfilled M G* ⟹ *get-offending-flows M G* = {}
apply(*frule*(*1*) *all-security-requirements-fulfilled-imp-no-offending-flows*)
apply(*simp add*: *get-offending-flows-def*)
apply(*thin-tac all-security-requirements-fulfilled M G*)
apply(*simp add*: *valid-reqs-def*)
apply(*clarify*)
using *configured-SecurityInvariant.empty-offending-contra* **by** *fastforce*

corollary *generate-valid-topology-does-nothing-if-valid*:
⟦ *valid-reqs M*; *all-security-requirements-fulfilled M G*⟧ ⟹
generate-valid-topology M G = *G*
by(*simp add*: *generate-valid-topology-as-set graph-ops all-security-requirements-fulfilled-imp-no-offending-flows*)

lemma *mono-extend-get-offending-flows*: ⟦ *valid-reqs M*;
wf-graph (|*nodes* = *V*, *edges* = *E*|);
E' ⊆ *E*;
F' ∈ *get-offending-flows M* (|*nodes* = *V*, *edges* = *E'*|) ⟧ ⟹
∃ *F*∈*get-offending-flows M* (|*nodes* = *V*, *edges* = *E*|). *F'* ⊆ *F*
proof(*induction M*)
case *Nil* **thus** *?case* **by**(*simp add*: *get-offending-flows-def*)
next
case (*Cons m M*)
from *Cons.prems* **have** *configured-SecurityInvariant m*
and *valid-reqs M* **using** *valid-reqs2 valid-reqs1* **by** *blast+*
from *Cons.prems*(*4*) **have**
F' ∈ *c-offending-flows m* (|*nodes* = *V*, *edges* = *E'*|) ∨
(*F'* ∈ *get-offending-flows M* (|*nodes* = *V*, *edges* = *E'*|))
by(*simp add*: *get-offending-flows-def*)
from *this* **show** *?case*
proof(*elim disjE*, *goal-cases*)
case *1*
with ‹*configured-SecurityInvariant m*› *Cons.prems*(*2*,*3*,*4*) **obtain** *F* **where**
F∈*c-offending-flows m* (|*nodes* = *V*, *edges* = *E*|) **and** *F'* ⊆ *F*
by(*blast dest*: *configured-SecurityInvariant.mono-extend-set-offending-flows*)
hence *F*∈*get-offending-flows* (*m* # *M*) (|*nodes* = *V*, *edges* = *E*|)
by (*simp add*: *get-offending-flows-def*)
with ‹*F'* ⊆ *F*› **show** *?case* **by** *blast*
next
case *2* **with** *Cons* ‹*valid-reqs M*› **show** *?case* **by**(*simp add*: *get-offending-flows-def*) *blast*
qed
qed

lemma *get-offending-flows-subseteq-edges*: *valid-reqs M* ⟹ *F* ∈ *get-offending-flows M* (|*nodes* = *V*, *edges* = *E*|) ⟹ *F* ⊆ *E*
apply(*induction M*)
apply(*simp add*: *get-offending-flows-def*)
apply(*simp add*: *get-offending-flows-def*)

apply(*frule valid-reqs2*, *drule valid-reqs1*)
apply(*simp add*: *configured-SecurityInvariant.valid-c-offending-flows*)
by *blast*

thm *configured-SecurityInvariant.offending-flows-union-mono*
lemma *get-offending-flows-union-mono*: ⟦*valid-reqs M*;
wf-graph (|*nodes* = *V*, *edges* = *E*|); *E'* ⊆ *E* ⟧ ⟹
⋃(*get-offending-flows M* (|*nodes* = *V*, *edges* = *E'*|)) ⊆ ⋃(*get-offending-flows M* (|*nodes* = *V*, *edges* = *E*|))
apply(*induction M*)
apply(*simp add*: *get-offending-flows-def*)
apply(*frule valid-reqs2*, *drule valid-reqs1*)
apply(*drule*(2) *configured-SecurityInvariant.offending-flows-union-mono*)
apply(*simp add*: *get-offending-flows-def*)
by *auto*

thm *configured-SecurityInvariant.Un-set-offending-flows-bound-minus-subseteq'*
lemma *Un-set-offending-flows-bound-minus-subseteq'*:⟦*valid-reqs M*;
wf-graph (|*nodes* = *V*, *edges* = *E*|); *E'* ⊆ *E*;
⋃(*get-offending-flows M* (|*nodes* = *V*, *edges* = *E*|)) ⊆ *X* ⟧ ⟹ ⋃(*get-offending-flows M* (|*nodes* = *V*, *edges* = *E* − *E'*|)) ⊆ *X* − *E'*
proof(*induction M*)
case *Nil* **thus** *?case* **by** (*simp add*: *get-offending-flows-def*)
next
case (*Cons m M*)
from *Cons.prems*(1) *valid-reqs2* **have** *valid-reqs M* **by** *force*
from *Cons.prems*(1) *valid-reqs1* **have** *configured-SecurityInvariant m* **by** *force*
from *Cons.prems*(4) **have** ⋃(*get-offending-flows M* (|*nodes* = *V*, *edges* = *E*|)) ⊆ *X* **by**(*simp add*: *get-offending-flows-def*)
from *Cons.IH*[*OF* ‹*valid-reqs M*› *Cons.prems*(2) *Cons.prems*(3) ‹⋃(*get-offending-flows M* (|*nodes* = *V*, *edges* = *E*|)) ⊆ *X*›] **have** *IH*: ⋃(*get-offending-flows M* (|*nodes* = *V*, *edges* = *E* − *E'*|)) ⊆ *X* − *E'* **.**
from *Cons.prems*(4) **have** ⋃(*c-offending-flows m* (|*nodes* = *V*, *edges* = *E*|)) ⊆ *X* **by**(*simp add*: *get-offending-flows-def*)
from *configured-SecurityInvariant.Un-set-offending-flows-bound-minus-subseteq'*[*OF* ‹*configured-SecurityInvariant m*› *Cons.prems*(2) ‹⋃(*c-offending-flows m* (|*nodes* = *V*, *edges* = *E*|)) ⊆ *X*›] **have** ⋃(*c-offending-flows m* (|*nodes* = *V*, *edges* = *E* − *E'*|)) ⊆ *X* − *E'* **.**
from *this IH* **show** *?case* **by**(*simp add*: *get-offending-flows-def*)
qed

lemma *ENF-uniquely-defined-offedning*: *valid-reqs M* ⟹ *wf-graph G* ⟹
∀ *m* ∈ *set M*. ∃ *P*. ∀ *G*. *c-sinvar m G* = (∀ *e* ∈ *edges G*. *P e*) ⟹
∀ *m* ∈ *set M*. ∀ *G*. ¬ *c-sinvar m G* ⟶ (∃ *OFF*. *c-offending-flows m G* = {*OFF*})
apply −
apply(*induction M*)
apply(*simp*; *fail*)
apply(*rename-tac m M*)
apply(*frule valid-reqs1*)
apply(*drule valid-reqs2*)
apply(*simp*)

apply(*elim conjE*)
apply(*erule-tac x=m* **in** *ballE*)
apply(*simp-all*; *fail*)
apply(*erule exE*, *rename-tac P*)
apply(*drule-tac P=P* **in** *enf-offending-flows*)
apply(*simp*; *fail*)
apply(*simp*; *fail*)
done

lemma assumes *configured-SecurityInvariant m*
and $\forall$ *G*. $\neg$ *c-sinvar m G* $\longrightarrow$ ($\exists$ *OFF*. *c-offending-flows m G* = {*OFF*})
shows $\exists$ *OFF-P*. $\forall$ *G*. *c-offending-flows m G* = (*if c-sinvar m G then* {} *else* {*OFF-P G*})
proof −
from *assms* **have** $\exists$ *OFF-P*.
c-offending-flows m G = (*if c-sinvar m G then* {} *else* {*OFF-P G*}) **for** *G*
apply(*erule-tac x=G* **in** *allE*)
apply(*cases c-sinvar m G*)
apply(*drule configured-SecurityInvariant.c-sinvar-valid-imp-no-offending-flows*, *simp*)
apply(*simp*; *fail*)
apply(*simp*)
by *meson*
with *assms* **show** *?thesis* **by** *metis*
qed

Hilber's eps operator example

lemma (*SOME x*. *x* : {*1*::*nat*, *2*, *3*}) = *x* $\Longrightarrow$ *x* = *1* $\vee$ *x* = *2* $\vee$ *x* = *3*
proof −
have (*SOME x*. *x* $\in$ {*1*::*nat*, *2*, *3*}) $\in$ {*1*::*nat*, *2*, *3*} **unfolding** *some-in-eq* **by** *simp*
thus (*SOME x*. *x* : {*1*::*nat*, *2*, *3*}) = *x* $\Longrightarrow$ *x* = *1* $\vee$ *x* = *2* $\vee$ *x* = *3* **by** *fast*
qed

Only removing one offending flow should be enough

fun *generate-valid-topology-SOME* :: *'v SecurityInvariant-configured list* $\Rightarrow$ *'v graph* $\Rightarrow$ *'v graph*
where
generate-valid-topology-SOME [] *G* = *G* |
generate-valid-topology-SOME (*m*#*Ms*) *G* = (*if c-sinvar m G*
then generate-valid-topology-SOME Ms G
else delete-edges (*generate-valid-topology-SOME Ms G*) (*SOME F*. *F* $\in$ *c-offending-flows m G*)
)

lemma *generate-valid-topology-SOME-nodes*: *nodes* (*generate-valid-topology-SOME M* (|*nodes* = *V*, *edges* = *E*|)) = *V*
proof(*induction M*)
qed(*simp-all add*: *delete-edges-simp2*)

theorem *generate-valid-topology-SOME-sound*:
⟦ *valid-reqs M*; *wf-graph* (|*nodes* = *V*, *edges* = *E*|) ⟧ $\Longrightarrow$
all-security-requirements-fulfilled M (*generate-valid-topology-SOME M* (|*nodes* = *V*, *edges* = *E*|))
proof(*induction M*)
case *Nil*
thus *?case* **by**(*simp add*: *all-security-requirements-fulfilled-def*)
next
case (*Cons m M*)
from *valid-reqs1*[*OF Cons*(*2*)] **have** *validReq*: *configured-SecurityInvariant m* **.**

from *configured-SecurityInvariant.sinvar-valid-remove-SOME-offending-flows*[*OF validReq*] **have**
c-offending-flows m (|*nodes* = *V*, *edges* = *E*|) ≠ {} ⟹
c-sinvar m (|*nodes* = *V*, *edges* = *E* − (*SOME F*. *F* ∈ *c-offending-flows m* (|*nodes* = *V*, *edges* = *E*|))|) .

have *generate-valid-topology-SOME-edges*: *edges* (*generate-valid-topology-SOME M* (|*nodes* = *V*, *edges* = *E*|)) ⊆ *E*
for *M*::'*a SecurityInvariant-configured list* **and** *V E*
proof(*induction M*)
qed(*auto simp add*: *delete-edges-simp2*)

from *configured-SecurityInvariant.mono-sinvar*[*OF validReq Cons.prems*(*2*),
of edges (*generate-valid-topology-SOME M* (|*nodes* = *V*, *edges* = *E*|))]
generate-valid-topology-SOME-edges
have *c-sinvar m* (|*nodes* = *V*, *edges* = *E*|) ⟹
c-sinvar m (|*nodes* = *V*, *edges* = *edges* (*generate-valid-topology-SOME M* (|*nodes* = *V*, *edges* = *E*|))|)
by *simp*
moreover from *configured-SecurityInvariant.defined-offending'*[*OF validReq Cons.prems*(*2*)] **have** *not-sinvar-off*:
¬ *c-sinvar m* (|*nodes* = *V*, *edges* = *E*|) ⟹ *c-offending-flows m* (|*nodes* = *V*, *edges* = *E*|) ≠ {} **by** *blast*
ultimately have *goal-sinvar-m*:
c-offending-flows m (|*nodes* = *V*, *edges* = *E*|) = {} ⟹
c-sinvar m (*generate-valid-topology-SOME M* (|*nodes* = *V*, *edges* = *E*|))
using *generate-valid-topology-SOME-nodes*
by (*metis graph.select-convs*(*1*) *graph.select-convs*(*2*) *graph-eq-intro*)

from *valid-reqs2*[*OF Cons*(*2*)] **have** *valid-reqs M* .
from *Cons.IH*[*OF* ‹*valid-reqs M*› *Cons*(*3*)] **have** *IH*:
all-security-requirements-fulfilled M (*generate-valid-topology-SOME M* (|*nodes* = *V*, *edges* = *E*|)) .

have *goal-rm-SOME-m*: *c-offending-flows m* (|*nodes* = *V*, *edges* = *E*|) ≠ {} ⟹
c-sinvar m (*delete-edges* (*generate-valid-topology-SOME M* (|*nodes* = *V*, *edges* = *E*|))
(*SOME F*. *F* ∈ *c-offending-flows m* (|*nodes* = *V*, *edges* = *E*|)))
proof −
assume *a1*: *c-offending-flows m* (|*nodes* = *V*, *edges* = *E*|) ≠ {}
have *f2*: (∀ *r ra p*. ¬ *r* ⊆ *ra* ∨ (*p*::'*a* × '*a*) ∉ *r* ∨ *p* ∈ *ra*) = (∀ *r ra p*. ¬ *r* ⊆ *ra* ∨ (*p*::'*a* × '*a*) ∉ *r* ∨ *p* ∈ *ra*)
by *meson*
have *f3*: *wf-graph* (|*nodes* = *V*, *edges* = *E* − (*SOME r*. *r* ∈ *c-offending-flows m* (|*nodes* = *V*, *edges* = *E*|))|)
by (*simp add*: *Cons.prems*(*2*) *wf-graph-remove-edges*)
have *edges* (*generate-valid-topology-SOME M* (|*nodes* = *V*, *edges* = *E*|)) − (*SOME r*. *r* ∈ *c-offending-flows m* (|*nodes* = *V*, *edges* = *E*|)) ⊆ *E* − (*SOME r*. *r* ∈ *c-offending-flows m* (|*nodes* = *V*, *edges* = *E*|))
using *f2 generate-valid-topology-SOME-edges*[*of M V E*] **by** *blast*
then have *c-sinvar m* (|*nodes* = *V*, *edges* = *edges* (*generate-valid-topology-SOME M* (|*nodes* = *V*, *edges* = *E*|)) − (*SOME r*. *r* ∈ *c-offending-flows m* (|*nodes* = *V*, *edges* = *E*|))|)
using *f3 a1* ‹*c-offending-flows m* (|*nodes* = *V*, *edges* = *E*|) ≠ {} ⟹ *c-sinvar m* (|*nodes* = *V*, *edges* = *E* − (*SOME F*. *F* ∈ *c-offending-flows m* (|*nodes* = *V*, *edges* = *E*|))|)› *configured-SecurityInvariant.negative-mono validReq* **by** *blast*

then show *c-sinvar m* (*delete-edges* (*generate-valid-topology-SOME M* (|*nodes* = *V*, *edges* = *E*|)) (*SOME r*. *r* ∈ *c-offending-flows m* (|*nodes* = *V*, *edges* = *E*|)))
by (*simp add*: *generate-valid-topology-SOME-nodes graph-ops*(*5*))
qed

have *wf-graph-generate-valid-topology-SOME*: *wf-graph G* ⟹ *wf-graph* (*generate-valid-topology-SOME M G*)
for *G*
apply(*cases G*)
apply(*simp add*: *wf-graph-def generate-valid-topology-SOME-nodes*)
using *generate-valid-topology-SOME-edges* **by** (*meson dual-order.trans image-mono rev-finite-subset*)

{ assume *notempty*: *c-offending-flows m* (|*nodes* = *V*, *edges* = *E*|) ≠ {}
hence ∃ *hypE*. (*generate-valid-topology-SOME M* (|*nodes* = *V*, *edges* = *E*|)) = (|*nodes* = *V*, *edges* = *hypE*|)
proof(*induction M arbitrary*: *V E*)
qed(*simp-all add*: *delete-edges-simp2 generate-valid-topology-SOME-nodes*)
from *this* **obtain** *E-IH* **where** *E-IH-prop*:
(*generate-valid-topology-SOME M* (|*nodes* = *V*, *edges* = *E*|)) = (|*nodes* = *V*, *edges* = *E-IH*|)
by *blast*

from *wf-graph-generate-valid-topology-SOME*[*OF Cons*(*3*)] *E-IH-prop*
have *valid-G-E-IH*: *wf-graph* (|*nodes* = *V*, *edges* = *E-IH*|) **by** *simp*

from *all-security-requirements-fulfilled-mono*[*OF* ‹*valid-reqs M*› *- valid-G-E-IH*] *IH E-IH-prop*
have *mono-rule*: *E'* ⊆ *E-IH* ⟹ *all-security-requirements-fulfilled M* (|*nodes* = *V*, *edges* = *E'*|) **for** *E'* **by** *simp*

have *all-security-requirements-fulfilled M*
(*delete-edges* (*generate-valid-topology-SOME M* (|*nodes* = *V*, *edges* = *E*|))
(*SOME F*. *F* ∈ *c-offending-flows m* (|*nodes* = *V*, *edges* = *E*|)))
unfolding *E-IH-prop* **by**(*auto simp add*: *delete-edges-simp2 intro*:*mono-rule*)
} note *goal-fulfilled-M*=*this*

have *no-offending*: *c-sinvar m* (|*nodes* = *V*, *edges* = *E*|) ⟹ *c-offending-flows m* (|*nodes* = *V*, *edges* = *E*|) = {}
by (*simp add*: *configured-SecurityInvariant.c-sinvar-valid-imp-no-offending-flows validReq*)

show *all-security-requirements-fulfilled* (*m* # *M*) (*generate-valid-topology-SOME* (*m* # *M*) (|*nodes* = *V*, *edges* = *E*|))
apply(*simp add*: *all-security-requirements-fulfilled-def*)
apply(*intro conjI impI*)
subgoal using *goal-sinvar-m no-offending* **by** *blast*
subgoal using *IH* **by**(*simp add*: *all-security-requirements-fulfilled-def*; *fail*)
subgoal using *goal-rm-SOME-m not-sinvar-off* **by** *blast*
subgoal using *goal-fulfilled-M not-sinvar-off* **by**(*simp add*: *all-security-requirements-fulfilled-def*)
done
qed

lemma *generate-valid-topology-SOME-def-alt*:
generate-valid-topology-SOME M G = *delete-edges G* (⋃ *m* ∈ *set M*. *if c-sinvar m G then* {} *else* (*SOME F*. *F* ∈ *c-offending-flows m G*))

```
    proof(induction M arbitrary: G)
      case Nil
        thus ?case by(simp add: get-offending-flows-def)
      next
      case (Cons m M)
        from Cons[simplified delete-edges-simp2 get-offending-flows-def]
        have IH :edges (generate-valid-topology-SOME M G) =
              edges G − (⋃m∈set M. if c-sinvar m G then {} else SOME F. F ∈ c-offending-flows
m G)
          by simp
        hence ¬ c-sinvar m G ⟹
                edges (generate-valid-topology-SOME (m # M) G) =
                    (edges G) − (⋃m∈set (m#M). if c-sinvar m G then {} else SOME F. F ∈
c-offending-flows m G)
          apply(simp add: get-offending-flows-def delete-edges-simp2)
          by blast
        with Cons.IH show ?case by(simp add: get-offending-flows-def delete-edges-simp2)
    qed

  lemma generate-valid-topology-SOME-superset:
    ⟦ valid-reqs M; wf-graph G ⟧ ⟹
    edges (generate-valid-topology M G) ⊆ edges (generate-valid-topology-SOME M G)
  proof −
    have isabelle2016-1-helper:
      x ∈ (⋃m∈set M. if c-sinvar m G then {} else SOME F. F ∈ c-offending-flows m G) ⟷
       (∃m∈set M. ¬ c-sinvar m G ∧ (c-sinvar m G ∨ x ∈ (SOME F. F ∈ c-offending-flows m G)))
      for x by auto
    have 1: m∈set M ⟹ ¬ c-sinvar m G ∧ (c-sinvar m G ∨ x ∈ (SOME F. F ∈ c-offending-flows
m G)) ⟹
          c-offending-flows m G ≠ {} ⟹
          x ∈ ⋃(⋃m∈set M. c-offending-flows m G)
    for x m
    apply(simp)
    apply(rule-tac x=m in bexI)
    apply(simp-all)
     using some-in-eq by blast

    show valid-reqs M ⟹ wf-graph G ⟹ ?thesis
    unfolding generate-valid-topology-SOME-def-alt generate-valid-topology-def-alt
    apply (rule delete-edges-edges-mono)
    apply (auto simp add: delete-edges-simp2 get-offending-flows-def valid-reqs-def)
    apply (metis (full-types) configured-SecurityInvariant.defined-offending' some-in-eq)
    done
  qed
```

Notation: *generate-valid-topology-SOME*: non-deterministic choice *generate-valid-topology-some*: executable which selects always the same

```
  fun generate-valid-topology-some :: 'v SecurityInvariant-configured list ⇒ ('v×'v) list ⇒ 'v graph ⇒
'v graph where
    generate-valid-topology-some [] - G = G |
    generate-valid-topology-some (m#Ms) Es G = (if c-sinvar m G
      then generate-valid-topology-some Ms Es G
       else delete-edges (generate-valid-topology-some Ms Es G) (set (minimalize-offending-overapprox
(c-sinvar m) Es [] G))
```

```
  )
 theorem generate-valid-topology-some-sound:
  ⟦ valid-reqs M; wf-graph (|nodes = V, edges = E|); set Es = E; distinct Es ⟧ ⟹
  all-security-requirements-fulfilled M (generate-valid-topology-some M Es (|nodes = V, edges = E|))
    proof(induction M)
      case Nil
      thus ?case by(simp add: all-security-requirements-fulfilled-def)
    next
      case (Cons m M)
      from valid-reqs1[OF Cons(2)] have validReq: configured-SecurityInvariant m .

      from configured-SecurityInvariant.sinvar-valid-remove-minimalize-offending-overapprox[OF
        validReq Cons.prems(2) -  Cons.prems(3) Cons.prems(4)] have rm-off-valid:
       ¬ c-sinvar m (|nodes = V, edges = E|)⟹
          c-sinvar m (|nodes = V, edges = E − (set (minimalize-offending-overapprox (c-sinvar m)
Es [] (|nodes = V, edges = E|)))|)
      apply(subst(asm) minimalize-offending-overapprox-boundnP[symmetric])
      by blast

      have generate-valid-topology-some-nodes: nodes (generate-valid-topology-some M Es (|nodes =
V, edges = E|)) = V
        for M::'a SecurityInvariant-configured list and V E
        proof(induction M)
        qed(simp-all add: delete-edges-simp2)


      have generate-valid-topology-some-edges: edges (generate-valid-topology-some M Es (|nodes =
V, edges = E|)) ⊆ E
        for M::'a SecurityInvariant-configured list and V E
        proof(induction M)
        qed(auto simp add: delete-edges-simp2)

      from configured-SecurityInvariant.mono-sinvar[OF validReq Cons.prems(2),
            of edges (generate-valid-topology-some M Es (|nodes = V, edges = E|))]
          generate-valid-topology-some-edges
        have c-sinvar m (|nodes = V, edges = E|) ⟹
          c-sinvar m (|nodes = V, edges = edges (generate-valid-topology-some M Es (|nodes = V,
edges = E|))|)
        by simp
       moreover from configured-SecurityInvariant.defined-offending'[OF validReq Cons.prems(2)]
have not-sinvar-off:
        ¬ c-sinvar m (|nodes = V, edges = E|) ⟹ c-offending-flows m (|nodes = V, edges = E|) ≠
{} by blast
      ultimately have goal-sinvar-m:
        c-offending-flows m (|nodes = V, edges = E|) = {} ⟹
          c-sinvar m (generate-valid-topology-some M Es (|nodes = V, edges = E|))
         using generate-valid-topology-some-nodes
         by (metis graph.select-convs(1) graph.select-convs(2) graph-eq-intro)


      from valid-reqs2[OF Cons(2)] have valid-reqs M .
      from Cons.IH[OF ‹valid-reqs M› Cons(3)] Cons.prems have IH:
       all-security-requirements-fulfilled M (generate-valid-topology-some M Es (|nodes = V, edges =
E|)) by simp
```

have *wf-graph-generate-valid-topology-some*: *wf-graph G* $\Longrightarrow$ *wf-graph* (*generate-valid-topology-some M Es G*)
for *G*
apply(*cases G*)
apply(*simp add*: *wf-graph-def generate-valid-topology-some-nodes*)
using *generate-valid-topology-some-edges* **by** (*meson dual-order.trans image-mono rev-finite-subset*)

{ assume *notempty*: *c-offending-flows m* (|*nodes* = *V*, *edges* = *E*|) ≠ {}
hence ∃ *hypE*. (*generate-valid-topology-some M Es* (|*nodes* = *V*, *edges* = *E*|)) = (|*nodes* = *V*, *edges* = *hypE*|)
proof(*induction M arbitrary*: *V E*)
qed(*simp-all add*: *delete-edges-simp2 generate-valid-topology-some-nodes*)
from *this* **obtain** *E-IH* **where** *E-IH-prop*:
(*generate-valid-topology-some M Es* (|*nodes* = *V*, *edges* = *E*|)) = (|*nodes* = *V*, *edges* = *E-IH*|) **by** *blast*

from *wf-graph-generate-valid-topology-some*[*OF Cons*(*3*)] *E-IH-prop*
have *valid-G-E-IH*: *wf-graph* (|*nodes* = *V*, *edges* = *E-IH*|) **by** *simp*

from *all-security-requirements-fulfilled-mono*[*OF* ‹*valid-reqs M*› *- valid-G-E-IH*] *IH E-IH-prop*
have *mono-rule*: *E'* ⊆ *E-IH* $\Longrightarrow$ *all-security-requirements-fulfilled M* (|*nodes* = *V*, *edges* = *E'*|) **for** *E'* **by** *simp*

have *all-security-requirements-fulfilled M*
(*delete-edges* (*generate-valid-topology-some M Es* (|*nodes* = *V*, *edges* = *E*|))
(*set* (*minimalize-offending-overapprox* (*c-sinvar m*) *Es* [] (|*nodes* = *V*, *edges* = *E*|))))
unfolding *E-IH-prop* **by**(*auto simp add*: *delete-edges-simp2 intro*:*mono-rule*)
} note *goal-fulfilled-M=this*

have *no-offending*: *c-sinvar m* (|*nodes* = *V*, *edges* = *E*|) $\Longrightarrow$ *c-offending-flows m* (|*nodes* = *V*, *edges* = *E*|) = {}
by (*simp add*: *configured-SecurityInvariant.c-sinvar-valid-imp-no-offending-flows validReq*)

show *all-security-requirements-fulfilled* (*m* # *M*) (*generate-valid-topology-some* (*m* # *M*) *Es* (|*nodes* = *V*, *edges* = *E*|))
apply(*simp add*: *all-security-requirements-fulfilled-def*)
apply(*intro conjI impI*)
subgoal using *goal-sinvar-m no-offending* **by** *blast*
subgoal using *IH* **by**(*simp add*: *all-security-requirements-fulfilled-def*; *fail*)
subgoal using *rm-off-valid* **by** (*metis* (*no-types*, *lifting*) *Cons.prems*(*2*) *Diff-mono*
configured-SecurityInvariant.mono-sinvar delete-edges-simp2 generate-valid-topology-some-edges
generate-valid-topology-some-nodes order-refl validReq wf-graph-remove-edges)
subgoal using *goal-fulfilled-M not-sinvar-off* **by**(*simp add*: *all-security-requirements-fulfilled-def*)
done
qed

end
theory *TopoS-Stateful-Policy*
imports *TopoS-Composition-Theory*
begin

8 Stateful Policy

Details described in [1].

Algorithm

term *TopoS-Composition-Theory.generate-valid-topology*

generates a valid high-level topology. Now we discuss how to turn this into a stateful policy.

Example: SensorNode produces data and has no security level. SensorSink has high security level SensorNode -¿ SensorSink, but not the other way round. Implementation: UDP in one direction

Alice is in internal protected subnet. Google can not arbitrarily access Alice. Alice sends requests to google. It is desirable that Alice gets the response back Implementation: TCP and stateful packet filter that allows, once Alice establishes a connection, to get a response back via this connection.

Result: IFS violations undesirable. ACS violations may be okay under certain conditions.

term *all-security-requirements-fulfilled*

$G = (V, E_{fix}, E_{state})$

record *'v stateful-policy* =
  hosts :: *'v set* — nodes, vertices
  flows-fix :: *('v × 'v) set* — edges in high-level policy
  flows-state :: *('v × 'v) set* — edges that can have stateful flows, i.e. backflows

All the possible ways packets can travel in a *'v stateful-policy*. They can either choose the fixed links; Or use a stateful link, i.e. establish state. Once state is established, packets can flow back via the established link.

definition *all-flows* :: *'v stateful-policy* ⇒ *('v × 'v) set* **where**
  all-flows $\mathcal{T}$ ≡ *flows-fix* $\mathcal{T}$ ∪ *flows-state* $\mathcal{T}$ ∪ *backflows* (*flows-state* $\mathcal{T}$)

definition *stateful-policy-to-network-graph* :: *'v stateful-policy* ⇒ *'v graph* **where**
  stateful-policy-to-network-graph $\mathcal{T}$ = ⦇ *nodes* = *hosts* $\mathcal{T}$, *edges* = *all-flows* $\mathcal{T}$ ⦈

'v stateful-policy syntactically well-formed

locale *wf-stateful-policy* =
  fixes $\mathcal{T}$ :: *'v stateful-policy*
  assumes *E-wf*: *fst* ‘ (*flows-fix* $\mathcal{T}$) ⊆ (*hosts* $\mathcal{T}$)
    snd ‘ (*flows-fix* $\mathcal{T}$) ⊆ (*hosts* $\mathcal{T}$)
  and *E-state-fix*: *flows-state* $\mathcal{T}$ ⊆ *flows-fix* $\mathcal{T}$
  and *finite-Hosts*: *finite* (*hosts* $\mathcal{T}$)
begin

lemma *E-wfD*: **assumes** $(v,v') \in$ *flows-fix* $\mathcal{T}$
shows $v \in$ *hosts* $\mathcal{T}$ $v' \in$ *hosts* $\mathcal{T}$
apply −
apply (*rule subsetD*[*OF E-wf*(*1*)])
using *assms* **apply** *force*
apply (*rule subsetD*[*OF E-wf*(*2*)])
using *assms* **apply** *force*
done

lemma *E-state-valid*: *fst* ‘ (*flows-state* $\mathcal{T}$) $\subseteq$ (*hosts* $\mathcal{T}$)
snd ‘ (*flows-state* $\mathcal{T}$) $\subseteq$ (*hosts* $\mathcal{T}$)
apply −
using *E-wf*(*1*) *E-state-fix* **apply**(*blast*)
using *E-wf*(*2*) *E-state-fix* **apply**(*blast*)
done

lemma *E-state-validD*: **assumes** $(v,v') \in$ *flows-state* $\mathcal{T}$
shows $v \in$ *hosts* $\mathcal{T}$ $v' \in$ *hosts* $\mathcal{T}$
apply −
apply (*rule subsetD*[*OF E-state-valid*(*1*)])
using *assms* **apply** *force*
apply (*rule subsetD*[*OF E-state-valid*(*2*)])
using *assms* **apply** *force*
done

lemma *finite-fix*: *finite* (*flows-fix* $\mathcal{T}$)
proof −
from *finite-subset*[*OF E-wf*(*1*) *finite-Hosts*] **have** *1*: *finite* (*fst* ‘ *flows-fix* $\mathcal{T}$) **.**
from *finite-subset*[*OF E-wf*(*2*) *finite-Hosts*] **have** *2*: *finite* (*snd* ‘ *flows-fix* $\mathcal{T}$) **.**
have *s*: *flows-fix* $\mathcal{T} \subseteq$ (*fst* ‘ *flows-fix* $\mathcal{T}$ $\times$ *snd* ‘ *flows-fix* $\mathcal{T}$) **by** *force*
from *finite-cartesian-product*[*OF 1 2*] **have** *finite* (*fst* ‘ *flows-fix* $\mathcal{T}$ $\times$ *snd* ‘ *flows-fix* $\mathcal{T}$) **.**
from *finite-subset*[*OF s this*] **show** *?thesis* **.**
qed

lemma *finite-state*: *finite* (*flows-state* $\mathcal{T}$)
using *finite-subset*[*OF E-state-fix finite-fix*] **by** *assumption*

lemma *finite-backflows-state*: *finite* (*backflows* (*flows-state* $\mathcal{T}$))
using [[*simproc add*: *finite-Collect*]] **by**(*simp add*: *backflows-def finite-state*)

lemma *E-state-backflows-wf*: *fst* ‘ *backflows* (*flows-state* $\mathcal{T}$) $\subseteq$ (*hosts* $\mathcal{T}$)
snd ‘ *backflows* (*flows-state* $\mathcal{T}$) $\subseteq$ (*hosts* $\mathcal{T}$)
by(*auto simp add*: *backflows-def E-state-valid E-state-validD*)

end

Minimizing stateful flows such that only newly added backflows remain

definition *filternew-flows-state* :: *'v stateful-policy* $\Rightarrow$ (*'v* $\times$ *'v*) *set* **where**
filternew-flows-state $\mathcal{T} \equiv \{(s,\ r) \in$ *flows-state* $\mathcal{T}$. $(r,\ s) \notin$ *flows-fix* $\mathcal{T}\}$

lemma *filternew-subseteq-flows-state*: *filternew-flows-state* $\mathcal{T} \subseteq$ *flows-state* $\mathcal{T}$
by(*auto simp add*: *filternew-flows-state-def*)

— alternative definitions, all are equal
lemma *filternew-flows-state-alt*: *filternew-flows-state* $\mathcal{T}$ = *flows-state* $\mathcal{T}$ − (*backflows* (*flows-fix* $\mathcal{T}$))
apply(*simp add*: *backflows-def filternew-flows-state-def*)
apply(*rule*)
apply *blast*+
done
lemma *filternew-flows-state-alt2*: *filternew-flows-state* $\mathcal{T}$ = {$e \in$ *flows-state* $\mathcal{T}$. $e \notin$ *backflows* (*flows-fix* $\mathcal{T}$)}
apply(*simp add*: *backflows-def filternew-flows-state-def*)
apply(*rule*)
apply *blast*+
done
lemma *backflows-filternew-flows-state*: *backflows* (*filternew-flows-state* $\mathcal{T}$) = (*backflows* (*flows-state* $\mathcal{T}$)) − (*flows-fix* $\mathcal{T}$)
by(*simp add*: *filternew-flows-state-alt backflows-minus-backflows*)

lemma *stateful-policy-to-network-graph-filternew*: ⟦ *wf-stateful-policy* $\mathcal{T}$ ⟧ ⟹
stateful-policy-to-network-graph $\mathcal{T}$ =
stateful-policy-to-network-graph (|*hosts* = *hosts* $\mathcal{T}$, *flows-fix* = *flows-fix* $\mathcal{T}$, *flows-state* = *filternew-flows-state* $\mathcal{T}$ |)
apply(*drule wf-stateful-policy.E-state-fix*)
apply(*simp add*: *stateful-policy-to-network-graph-def all-flows-def*)
apply(*rule Set.equalityI*)
apply(*simp add*: *filternew-flows-state-def backflows-def*)
apply(*rule*, *blast*)+
apply(*simp add*: *filternew-flows-state-def backflows-def*)
apply *fastforce*
done

lemma *backflows-filternew-disjunct-flows-fix*:
∀ $b \in$ (*backflows* (*filternew-flows-state* $\mathcal{T}$)). $b \notin$ *flows-fix* $\mathcal{T}$
by(*simp add*: *filternew-flows-state-def backflows-def*)

Given a high-level policy, we can construct a pretty large syntactically valid low level policy. However, the stateful policy will almost certainly violate security requirements!

lemma *wf-graph* G ⟹ *wf-stateful-policy* (| *hosts* = *nodes* G, *flows-fix* = *nodes* G × *nodes* G, *flows-state* = *nodes* G × *nodes* G |)
by(*simp add*: *wf-stateful-policy-def wf-graph-def*)

wf-stateful-policy implies *wf-graph*

lemma *wf-stateful-policy-is-wf-graph*: *wf-stateful-policy* $\mathcal{T}$ ⟹ *wf-graph* (|*nodes* = *hosts* $\mathcal{T}$, *edges* = *all-flows* $\mathcal{T}$|)
apply(*frule wf-stateful-policy.E-state-backflows-wf*)
apply(*frule wf-stateful-policy.E-state-backflows-wf*(*2*))
apply(*frule wf-stateful-policy.E-state-valid*)
apply(*frule wf-stateful-policy.E-state-valid*(*2*))
apply(*frule wf-stateful-policy.E-wf*)
apply(*frule wf-stateful-policy.E-wf*(*2*))
apply(*simp add*: *all-flows-def wf-graph-def wf-stateful-policy-def*
wf-stateful-policy.finite-fix wf-stateful-policy.finite-state wf-stateful-policy.finite-backflows-state)
apply(*rule conjI*)
apply (*metis image-Un sup.bounded-iff*)+
done

lemma ($\forall F \in$ *get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* $\mathcal{T}$). $F \subseteq$ *backflows* (*filternew-flows-state* $\mathcal{T}$)) $\longleftrightarrow$
$\bigcup$(*get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* $\mathcal{T}$)) $\subseteq$ (*backflows* (*flows-state* $\mathcal{T}$)) $-$ (*flows-fix* $\mathcal{T}$)
by(*simp add*: *filternew-flows-state-alt backflows-minus-backflows*, *blast*)

When is a stateful policy $\mathcal{T}$ compliant with a high-level policy G and the security requirements M?

locale *stateful-policy-compliance* =
fixes $\mathcal{T}$ :: ('*v*::*vertex*) *stateful-policy*
fixes G :: '*v graph*
fixes M :: ('*v*) *SecurityInvariant-configured list*
assumes
— the graph must be syntactically valid
wfG: *wf-graph G*
and
— security requirements must be valid
validReqs: *valid-reqs M*
and
— the high-level policy must be valid
high-level-policy-valid: *all-security-requirements-fulfilled M G*
and
— the stateful policy must be syntactically valid
stateful-policy-wf:
wf-stateful-policy $\mathcal{T}$
and
— the stateful policy must talk about the same nodes as the high-level policy
hosts-nodes:
hosts $\mathcal{T}$ = *nodes G*
and
— only flows that are allowed in the high-level policy are allowed in the stateful policy
flows-edges:
flows-fix $\mathcal{T} \subseteq$ *edges G*
and
— the low level policy must comply with the high-level policy
— all information flow strategy requirements must be fulfilled, i.e. no leaks!
compliant-stateful-IFS:
all-security-requirements-fulfilled (*get-IFS M*) (*stateful-policy-to-network-graph* $\mathcal{T}$)
and
— No Access Control side effects must occur
compliant-stateful-ACS:
$\forall F \in$ *get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* $\mathcal{T}$). $F \subseteq$ *backflows* (*filternew-flows-state* $\mathcal{T}$)

begin
lemma *compliant-stateful-ACS-no-side-effects-filternew-helper*:
$\forall E \subseteq$ *backflows* (*filternew-flows-state* $\mathcal{T}$). $\forall F \in$ *get-offending-flows* (*get-ACS M*) (| *nodes* = *hosts* $\mathcal{T}$, *edges* = *flows-fix* $\mathcal{T} \cup E$ |). $F \subseteq E$
proof(*rule*, *rule*)
fix *E*
assume *a1*: $E \subseteq$ *backflows* (*filternew-flows-state* $\mathcal{T}$)
from *validReqs* **have** *valid-ReqsACS*: *valid-reqs* (*get-ACS M*) **by**(*simp add*: *get-ACS-def valid-reqs-def*)

from *compliant-stateful-ACS stateful-policy-to-network-graph-filternew*[*OF stateful-policy-wf*] **have** *compliant-stateful-ACS-only-state-violations-filternew*:
$\forall F \in$ *get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* (|*hosts* = *hosts* $\mathcal{T}$, *flows-fix* = *flows-fix* $\mathcal{T}$, *flows-state* = *filternew-flows-state* $\mathcal{T}$ |)). $F \subseteq$ *backflows* (*filternew-flows-state* $\mathcal{T}$) **by** *simp*

from *wf-stateful-policy-is-wf-graph*[*OF stateful-policy-wf*] **have** *wfGfilternew*:
wf-graph (|*nodes* = *hosts* $\mathcal{T}$, *edges* = *flows-fix* $\mathcal{T}$ $\cup$ *filternew-flows-state* $\mathcal{T}$ $\cup$ *backflows* (*filternew-flows-state* $\mathcal{T}$)|)
apply(*simp add*: *all-flows-def filternew-flows-state-alt backflows-minus-backflows*)
by(*auto simp add*: *wf-graph-def*)

from *wf-stateful-policy.E-state-fix*[*OF stateful-policy-wf*] *filternew-subseteq-flows-state* **have** *flows-fix-un-filternew-sim*
flows-fix $\mathcal{T}$ $\cup$ *filternew-flows-state* $\mathcal{T}$ = *flows-fix* $\mathcal{T}$ **by** *blast*

from *compliant-stateful-ACS-only-state-violations-filternew* **have**
$\bigwedge m.\ m \in$ *set* (*get-ACS M*) $\Longrightarrow$
$\bigcup$(*c-offending-flows m* (|*nodes* = *hosts* $\mathcal{T}$, *edges* = *flows-fix* $\mathcal{T}$ $\cup$ *filternew-flows-state* $\mathcal{T}$ $\cup$ *backflows* (*filternew-flows-state* $\mathcal{T}$)|)) $\subseteq$ *backflows* (*filternew-flows-state* $\mathcal{T}$)
by(*simp add*: *stateful-policy-to-network-graph-def all-flows-def get-offending-flows-def*, *blast*)

— idea: use $\forall F \in$*get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* $\mathcal{T}$). $F \subseteq$ *backflows* (*filternew-flows-state* $\mathcal{T}$) with the ⟦*configured-SecurityInvariant ?m*; *wf-graph* (|*nodes* = *?V*, *edges* = *?E*|); $\bigcup$ (*c-offending-flows ?m* (|*nodes* = *?V*, *edges* = *?E*|)) $\subseteq$ *?X*⟧ $\Longrightarrow$ $\bigcup$ (*c-offending-flows ?m* (|*nodes* = *?V*, *edges* = *?E* − *?E'*|)) $\subseteq$ *?X* − *?E'* lemma and substract *backflows* (*filternew-flows-state* $\mathcal{T}$) − *E*, on the right hand side *E* remains, as Graph's edges *flows-fix* $\mathcal{T}$ $\cup$ *E* remains

from *configured-SecurityInvariant.Un-set-offending-flows-bound-minus-subseteq*[**where** *X*=*backflows* (*filternew-flows-state* $\mathcal{T}$), *OF - wfGfilternew this*]
‹*valid-reqs* (*get-ACS M*)›
have
$\bigwedge$ *m E*. *m* $\in$ *set* (*get-ACS M*) $\Longrightarrow$
$\forall F \in$*c-offending-flows m* (|*nodes* = *hosts* $\mathcal{T}$, *edges* = *flows-fix* $\mathcal{T}$ $\cup$ *filternew-flows-state* $\mathcal{T}$ $\cup$ *backflows* (*filternew-flows-state* $\mathcal{T}$) − *E*|). $F \subseteq$ *backflows* (*filternew-flows-state* $\mathcal{T}$) − *E*
by(*auto simp add*: *all-flows-def valid-reqs-def*)
from *this flows-fix-un-filternew-simp* **have** *rule*:
$\bigwedge$ *m E*. *m* $\in$ *set* (*get-ACS M*) $\Longrightarrow$
$\forall F \in$*c-offending-flows m* (|*nodes* = *hosts* $\mathcal{T}$, *edges* = *flows-fix* $\mathcal{T}$ $\cup$ *backflows* (*filternew-flows-state* $\mathcal{T}$) − *E*|). $F \subseteq$ *backflows* (*filternew-flows-state* $\mathcal{T}$) − *E*
by *simp*

from *backflows-finite rev-finite-subset*[*OF wf-stateful-policy.finite-state*[*OF stateful-policy-wf*] *filternew-subseteq-flows-state*] **have**
finite (*backflows* (*filternew-flows-state* $\mathcal{T}$)) **by** *blast*
from *a1 this* **have** *finite E* **by** (*metis rev-finite-subset*)

from *a1* **obtain** *E'* **where** *E'-prop1*: *backflows* (*filternew-flows-state* $\mathcal{T}$) − *E'* = *E* **and** *E'-prop2*: *E'* = *backflows* (*filternew-flows-state* $\mathcal{T}$) − *E* **by** *blast*
from *E'-prop2* ‹*finite* (*backflows* (*filternew-flows-state* $\mathcal{T}$))› ‹*finite E*› **have** *finite E'* **by** *blast*

from *Set.double-diff*[**where** *B*=*backflows* (*filternew-flows-state* $\mathcal{T}$) **and** *C*=*backflows* (*filternew-flows-state* $\mathcal{T}$) **and** *A*=*E*, *OF a1*, *simplified*] **have** *Ebackflowssimp*:
backflows (*filternew-flows-state* $\mathcal{T}$) − (*backflows* (*filternew-flows-state* $\mathcal{T}$) − *E*) = *E* **.**

have *flows-fix* $\mathcal{T}$ ∪ *backflows* (*filternew-flows-state* $\mathcal{T}$) − (*backflows* (*filternew-flows-state* $\mathcal{T}$) − *E*) =
(*flows-fix* $\mathcal{T}$ − (*backflows* (*filternew-flows-state* $\mathcal{T}$))) ∪ *E*
apply(*simp add*: *Set.Un-Diff*)
apply(*simp add*: *Ebackflowssimp*)
by *blast*
also have (*flows-fix* $\mathcal{T}$ − (*backflows* (*filternew-flows-state* $\mathcal{T}$))) ∪ *E* = *flows-fix* $\mathcal{T}$ ∪ *E* **using** *backflows-filternew-disjunct-flows-fix* **by** *blast*
finally have *flows-E-simp*: *flows-fix* $\mathcal{T}$ ∪ *backflows* (*filternew-flows-state* $\mathcal{T}$) − (*backflows* (*filternew-flows-state* $\mathcal{T}$) − *E*) = *flows-fix* $\mathcal{T}$ ∪ *E* **.**

from *rule*[*simplified E'-prop1 E'-prop2*] **have**
⋀*m*. *m* ∈ *set* (*get-ACS M*) ⟹
∀ *F*∈*c-offending-flows m* (|*nodes* = *hosts* $\mathcal{T}$, *edges* = *flows-fix* $\mathcal{T}$ ∪ *backflows* (*filternew-flows-state* $\mathcal{T}$) − (*backflows* (*filternew-flows-state* $\mathcal{T}$) − *E*)|).
F ⊆ *backflows* (*filternew-flows-state* $\mathcal{T}$) − (*backflows* (*filternew-flows-state* $\mathcal{T}$) − *E*)
by(*simp*)
from *this Ebackflowssimp flows-E-simp* **have**
⋀*m*. *m* ∈ *set* (*get-ACS M*) ⟹
∀ *F*∈*c-offending-flows m* (|*nodes* = *hosts* $\mathcal{T}$, *edges* = *flows-fix* $\mathcal{T}$ ∪ *E*|). *F* ⊆ *E*
by *simp*
thus ∀ *F*∈*get-offending-flows* (*get-ACS M*) (|*nodes* = *hosts* $\mathcal{T}$, *edges* = *flows-fix* $\mathcal{T}$ ∪ *E*|). *F* ⊆ *E*
by(*simp add*: *get-offending-flows-def*)
qed

theorem *compliant-stateful-ACS-no-side-effects*:
∀ *E* ⊆ *backflows* (*flows-state* $\mathcal{T}$). ∀ *F* ∈ *get-offending-flows*(*get-ACS M*) (| *nodes* = *hosts* $\mathcal{T}$, *edges* = *flows-fix* $\mathcal{T}$ ∪ *E* |). *F* ⊆ *E*
proof −
from *compliant-stateful-ACS stateful-policy-to-network-graph-filternew*[*OF stateful-policy-wf*] **have** *a1*:
∀ *F* ∈ *get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* (|*hosts* = *hosts* $\mathcal{T}$, *flows-fix* = *flows-fix* $\mathcal{T}$, *flows-state* = *filternew-flows-state* $\mathcal{T}$ |)). *F* ⊆ *backflows* (*filternew-flows-state* $\mathcal{T}$) **by** *simp*

have *backflows-split*: *backflows* (*filternew-flows-state* $\mathcal{T}$) ∪ (*backflows* (*flows-state* $\mathcal{T}$) − *backflows* (*filternew-flows-state* $\mathcal{T}$)) = *backflows* (*flows-state* $\mathcal{T}$)
by (*metis Diff-subset Un-Diff-cancel Un-absorb1 backflows-minus-backflows filternew-flows-state-alt*)

have
∀ *E*⊆*backflows* (*filternew-flows-state* $\mathcal{T}$) ∪ (*backflows* (*flows-state* $\mathcal{T}$) − *backflows* (*filternew-flows-state* $\mathcal{T}$)).
∀ *F*∈*get-offending-flows* (*get-ACS M*) (|*nodes* = *hosts* $\mathcal{T}$, *edges* = *flows-fix* $\mathcal{T}$ ∪ *E*|). *F* ⊆ *E*
proof(*rule allI*, *rule impI*)
fix *E*
assume *h1*: *E* ⊆ *backflows* (*filternew-flows-state* $\mathcal{T}$) ∪ (*backflows* (*flows-state* $\mathcal{T}$) − *backflows* (*filternew-flows-state* $\mathcal{T}$))

have ∃ *E1 E2*. *E1* ⊆ *backflows* (*filternew-flows-state* $\mathcal{T}$) ∧ *E2* ⊆ (*backflows* (*flows-state* $\mathcal{T}$) − *backflows* (*filternew-flows-state* $\mathcal{T}$)) ∧ *E1* ∪ *E2* = *E* ∧ *E1* ∩ *E2* = {}
apply(*rule-tac x*={*e* ∈ *E*. *e* ∈ *backflows* (*filternew-flows-state* $\mathcal{T}$)} **in** *exI*)
apply(*rule-tac x*={*e* ∈ *E*. *e* ∈(*backflows* (*flows-state* $\mathcal{T}$) − *backflows* (*filternew-flows-state* $\mathcal{T}$))} **in** *exI*)
apply(*simp*)

apply(*rule*)
apply *blast*
apply(*rule*)
apply *blast*
apply(*rule*)
using *h1* **apply** *blast*
using *backflows-filternew-disjunct-flows-fix* **by** *blast*

from *this* **obtain** *E1 E2* **where** *E1-prop*: $E1 \subseteq$ *backflows* (*filternew-flows-state* $\mathcal{T}$) **and** *E2-prop*: $E2 \subseteq$ (*backflows* (*flows-state* $\mathcal{T}$) − *backflows* (*filternew-flows-state* $\mathcal{T}$)) **and** $E = E1 \cup E2$ **and** $E1 \cap E2 = \{\}$ **by** *blast*

— the stateful flows are ⊆ fix flows. If substracting the new stateful flows, onyly the existing fix flows remain
from *E2-prop filternew-flows-state-alt* **have** $E2 \subseteq$ *flows-fix* $\mathcal{T}$ **by** (*metis* (*hide-lams*, *no-types*) *Diff-subset-conv Un-Diff-cancel2 backflows-minus-backflows inf-sup-ord*(*3*) *order.trans*)
— hence, E2 disappears
from *Set.Un-absorb1*[*OF this*] **have** *E2-absorb*: *flows-fix* $\mathcal{T} \cup E2 =$ *flows-fix* $\mathcal{T}$ **by** *blast*

from ‹$E = E1 \cup E2$› **have** *E2E1eq*: $E2 \cup E1 = E$ **by** *blast*

from ‹$E = E1 \cup E2$› ‹$E1 \cap E2 = \{\}$› **have** $E1 \subseteq E$ **by** *simp*

from *compliant-stateful-ACS-no-side-effects-filternew-helper E1-prop* **have** $\forall F \in$ *get-offending-flows* (*get-ACS M*) (|*nodes* = *hosts* $\mathcal{T}$, *edges* = *flows-fix* $\mathcal{T} \cup E1$ |). $F \subseteq E1$ **by** *simp*
hence $\forall F \in$ *get-offending-flows* (*get-ACS M*) (|*nodes* = *hosts* $\mathcal{T}$, *edges* = *flows-fix* $\mathcal{T} \cup E2 \cup E1$ |). $F \subseteq E1$ **using** *E2-absorb*[*symmetric*] **by** *simp*
hence $\forall F \in$ *get-offending-flows* (*get-ACS M*) (|*nodes* = *hosts* $\mathcal{T}$, *edges* = *flows-fix* $\mathcal{T} \cup E$ |). $F \subseteq E1$ **using** *E2E1eq* **by** (*metis Un-assoc*)

from *this* ‹$E1 \subseteq E$› **show** $\forall F \in$ *get-offending-flows* (*get-ACS M*) (|*nodes* = *hosts* $\mathcal{T}$, *edges* = *flows-fix* $\mathcal{T} \cup E$|). $F \subseteq E$ **by** *blast*
qed

from *this backflows-split* **show** *?thesis* **by** *presburger*
qed

corollary *compliant-stateful-ACS-no-side-effects'*: ∀ $E \subseteq$ *backflows* (*flows-state* $\mathcal{T}$). ∀ $F \in$ *get-offending-flows*(*get-ACS M*) (| *nodes* = *hosts* $\mathcal{T}$, *edges* = *flows-fix* $\mathcal{T} \cup$ *flows-state* $\mathcal{T} \cup E$ |). $F \subseteq E$
using *compliant-stateful-ACS-no-side-effects wf-stateful-policy.E-state-fix*[*OF stateful-policy-wf*] **by** (*metis Un-absorb2*)

The high level graph generated from the low level policy is a valid graph

lemma *valid-stateful-policy*: *wf-graph* (|*nodes* = *hosts* $\mathcal{T}$, *edges* = *all-flows* $\mathcal{T}$|)
by(*rule wf-stateful-policy-is-wf-graph*,*fact stateful-policy-wf*)

The security requirements are definitely fulfilled if we consider only the fixed flows and the normal direction of the stateful flows (i.e. no backflows). I.e. considering no states, everything must be fulfilled

lemma *compliant-stateful-ACS-static-valid*: *all-security-requirements-fulfilled* (*get-ACS M*) (| *nodes* = *hosts* $\mathcal{T}$, *edges* = *flows-fix* $\mathcal{T}$ |)
proof −
from *validReqs* **have** *valid-ReqsACS*: *valid-reqs* (*get-ACS M*) **by**(*simp add*: *get-ACS-def valid-reqs-def*)

from *wfG hosts-nodes*[*symmetric*] **have** *wfG'*: *wf-graph* (| *nodes* = *hosts* $\mathcal{T}$, *edges* = *edges G* |) **by**(*case-tac G*, *simp*)
from *high-level-policy-valid* **have** *all-security-requirements-fulfilled* (*get-ACS M*) *G*
by(*simp add*: *get-ACS-def all-security-requirements-fulfilled-def*)
from *this hosts-nodes*[*symmetric*] **have** *all-security-requirements-fulfilled* (*get-ACS M*) (| *nodes* = *hosts* $\mathcal{T}$, *edges* = *edges G* |)
by(*case-tac G*, *simp*)
from *all-security-requirements-fulfilled-mono*[*OF valid-ReqsACS flows-edges wfG' this*] **show** *?thesis* **.**
qed
theorem *compliant-stateful-ACS-static-valid'*:
all-security-requirements-fulfilled M (| *nodes* = *hosts* $\mathcal{T}$, *edges* = *flows-fix* $\mathcal{T}$ ∪ *flows-state* $\mathcal{T}$ |)
proof −
from *validReqs* **have** *valid-ReqsIFS*: *valid-reqs* (*get-IFS M*) **by**(*simp add*: *get-IFS-def valid-reqs-def*)

— show that it holds for IFS, by monotonicity as it holds for more in IFS
from *all-security-requirements-fulfilled-mono*[*OF valid-ReqsIFS - valid-stateful-policy compliant-stateful-IFS*[*unfolde*
stateful-policy-to-network-graph-def]] **have**
goalIFS: *all-security-requirements-fulfilled* (*get-IFS M*) (| *nodes* = *hosts* $\mathcal{T}$, *edges* = *flows-fix* $\mathcal{T}$ ∪ *flows-state* $\mathcal{T}$ |) **by**(*simp add*: *all-flows-def*)

from *wf-stateful-policy.E-state-fix*[*OF stateful-policy-wf*] **have** *flows-fix* $\mathcal{T}$ ∪ *flows-state* $\mathcal{T}$ = *flows-fix* $\mathcal{T}$ **by** *blast*
from *this compliant-stateful-ACS-static-valid* **have** *goalACS*:
all-security-requirements-fulfilled (*get-ACS M*) (| *nodes* = *hosts* $\mathcal{T}$, *edges* = *flows-fix* $\mathcal{T}$ ∪ *flows-state* $\mathcal{T}$ |) **by** *simp*

— ACS and IFS together form M, we know it holds for ACS
from *goalACS goalIFS* **show** *?thesis*
apply(*simp add*: *all-security-requirements-fulfilled-def get-IFS-def get-ACS-def*)
by *fastforce*
qed

The flows with state are a subset of the flows allowed by the policy

theorem *flows-state-edges*: *flows-state* $\mathcal{T}$ ⊆ *edges G*
using *wf-stateful-policy.E-state-fix*[*OF stateful-policy-wf*] *flows-edges* **by** *simp*

All offending flows are subsets of the reveres stateful flows

lemma *compliant-stateful-ACS-only-state-violations*:
∀ *F* ∈ *get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* $\mathcal{T}$). *F* ⊆ *backflows* (*flows-state* $\mathcal{T}$)
proof −
have *backflows* (*filternew-flows-state* $\mathcal{T}$) ⊆ *backflows* (*flows-state* $\mathcal{T}$) **by** (*metis Diff-subset backflows-minus-backflows filternew-flows-state-alt*)
from *compliant-stateful-ACS this* **have**
∀ *F* ∈ *get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* $\mathcal{T}$). *F* ⊆ *backflows* (*flows-state* $\mathcal{T}$)
by (*metis subset-trans*)
thus *?thesis* **.**
qed
theorem *compliant-stateful-ACS-only-state-violations'*: ∀ *F* ∈ *get-offending-flows M* (*stateful-policy-to-network-graph* $\mathcal{T}$). *F* ⊆ *backflows* (*flows-state* $\mathcal{T}$)
proof −
from *validReqs* **have** *valid-ReqsIFS*: *valid-reqs* (*get-IFS M*) **by**(*simp add*: *get-IFS-def valid-reqs-def*)

have *offending-split*: $\bigwedge G$. *get-offending-flows M G* = (*get-offending-flows* (*get-IFS M*) *G* ∪ *get-offending-flows* (*get-ACS M*) *G*)
apply(*simp add*: *get-offending-flows-def get-IFS-def get-ACS-def*) **by** *blast*
show *?thesis*
apply(*subst offending-split*)
using *compliant-stateful-ACS-only-state-violations*
all-security-requirements-fulfilled-imp-get-offending-empty[*OF valid-ReqsIFS compliant-stateful-IFS*]
by *auto*
qed

All violations are backflows of valid flows

corollary *compliant-stateful-ACS-only-state-violations-union*: ⋃(*get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* $\mathcal{T}$)) ⊆ *backflows* (*flows-state* $\mathcal{T}$)
using *compliant-stateful-ACS-only-state-violations* **by** *fastforce*

corollary *compliant-stateful-ACS-only-state-violations-union'*: ⋃(*get-offending-flows M* (*stateful-policy-to-network-gr*
$\mathcal{T}$)) ⊆ *backflows* (*flows-state* $\mathcal{T}$)
using *compliant-stateful-ACS-only-state-violations'* **by** *fastforce*

All individual flows cause no side effects, i.e. each backflow causes at most itself as violation, no other side-effect violations are induced.

lemma *compliant-stateful-ACS-no-state-singleflow-side-effect*:
∀ (v_1, v_2) ∈ *backflows* (*flows-state* $\mathcal{T}$).
⋃(*get-offending-flows*(*get-ACS M*) (| *nodes* = *hosts* $\mathcal{T}$, *edges* = *flows-fix* $\mathcal{T}$ ∪ *flows-state* $\mathcal{T}$ ∪ {(v_1, v_2)} |)) ⊆ {(v_1, v_2)}
using *compliant-stateful-ACS-no-side-effects'* **by** *blast*
end

8.1 Summarizing the important theorems

No information flow security requirements are violated (including all added stateful flows)

thm *stateful-policy-compliance.compliant-stateful-IFS*

There are not access control side effects when allowing stateful backflows. I.e. for all possible subsets of the to-allow backflows, the violations they cause are only these backflows themselves

thm *stateful-policy-compliance.compliant-stateful-ACS-no-side-effects'*

Also, considering all backflows individually, they cause no side effect, i.e. the only violation added is the backflow itself

thm *stateful-policy-compliance.compliant-stateful-ACS-no-state-singleflow-side-effect*

In particular, all introduced offending flows for access control strategies are at most the stateful backflows

thm *stateful-policy-compliance.compliant-stateful-ACS-only-state-violations-union*

Which implies: all introduced offending flows are at most the stateful backflows

thm *stateful-policy-compliance.compliant-stateful-ACS-only-state-violations-union'*

Disregarding the backflows of stateful flows, all security requirements are fulfilled.

thm *stateful-policy-compliance.compliant-stateful-ACS-static-valid'*

end
theory *TopoS-Composition-Theory-impl*
imports *TopoS-Interface-impl TopoS-Composition-Theory*
begin

9 Composition Theory – List Implementation

Several invariants may apply to one policy.

term *X*::(*'v*::*vertex*, *'a*) *TopoS-packed*

9.1 Generating instantiated (configured) network security invariants

— a configured network security invariant in list implementaion

record (*'v*) *SecurityInvariant* =
implc-type :: *string*
implc-description :: *string*
implc-sinvar ::(*'v*) *list-graph* ⇒ *bool*
implc-offending-flows ::(*'v*) *list-graph* ⇒ (*'v* × *'v*) *list list*
implc-isIFS :: *bool*

Test if this definition is compliant with the formal definition on sets.

definition *SecurityInvariant-complies-formal-def* ::
(*'v*) *SecurityInvariant* ⇒ *'v TopoS-Composition-Theory.SecurityInvariant-configured* ⇒ *bool* **where**
SecurityInvariant-complies-formal-def impl spec ≡
(∀ *G*. *wf-list-graph G* ⟶ *implc-sinvar impl G* = *c-sinvar spec* (*list-graph-to-graph G*)) ∧
(∀ *G*. *wf-list-graph G* ⟶ *set'set* (*implc-offending-flows impl G*) = *c-offending-flows spec* (*list-graph-to-graph G*)) ∧
(*implc-isIFS impl* = *c-isIFS spec*)

fun *new-configured-list-SecurityInvariant* ::
(*'v*::*vertex*, *'a*) *TopoS-packed* ⇒ (*'v*::*vertex*, *'a*) *TopoS-Params* ⇒ *string* ⇒
(*'v SecurityInvariant*) **where**
new-configured-list-SecurityInvariant m C description =
(*let nP* = *nm-node-props m C in*
(⦇
implc-type = *nm-name m*,
implc-description = *description*,
implc-sinvar = (λ*G*. (*nm-sinvar m*) *G nP*),
implc-offending-flows = (λ*G*. (*nm-offending-flows m*) *G nP*),
implc-isIFS = *nm-receiver-violation m*
⦈))

the *new-configured-SecurityInvariant* must give a result if we have the SecurityInvariant modelLibrary

lemma *TopoS-modelLibrary-yields-new-configured-SecurityInvariant*:
assumes *NetModelLib*: *TopoS-modelLibrary m sinvar-spec*
and *nPdef*: *nP* = *nm-node-props m C*
and *formalSpec*: *Spec* = ⦇

```
                    c-sinvar = (λG. sinvar-spec G nP),
                  c-offending-flows = (λG. SecurityInvariant-withOffendingFlows.set-offending-flows
sinvar-spec G nP),
                    c-isIFS = nm-receiver-violation m
                  ⦆
  shows new-configured-SecurityInvariant (sinvar-spec, nm-default m, nm-receiver-violation m, nP)
= Some Spec
  proof −
  from NetModelLib have NetModel: SecurityInvariant sinvar-spec (nm-default m) (nm-receiver-violation
m)
    by(simp add: TopoS-modelLibrary-def TopoS-List-Impl-def)

    have Spec: (⦇c-sinvar = λG. sinvar-spec G nP,
      c-offending-flows = λG. SecurityInvariant-withOffendingFlows.set-offending-flows sinvar-spec
G nP,
        c-isIFS = nm-receiver-violation m⦈) = Spec
    by(simp add: formalSpec)
    show ?thesis
      unfolding new-configured-SecurityInvariant.simps
      by(simp add: NetModel Spec)
  qed
  thm TopoS-modelLibrary-yields-new-configured-SecurityInvariant[simplified]



 lemma new-configured-list-SecurityInvariant-complies:
  assumes NetModelLib: TopoS-modelLibrary m sinvar-spec
  and     nPdef:       nP = nm-node-props m C
 and formalSpec:     Spec = new-configured-SecurityInvariant (sinvar-spec, nm-default m, nm-receiver-violation
m, nP)
  and implSpec:       Impl = new-configured-list-SecurityInvariant m C description
  shows SecurityInvariant-complies-formal-def Impl (the Spec)
  proof −
    from TopoS-modelLibrary-yields-new-configured-SecurityInvariant[OF NetModelLib nPdef]
  have SpecUnfolded: new-configured-SecurityInvariant (sinvar-spec, nm-default m, nm-receiver-violation
m, nP) =
      Some (⦇c-sinvar = λG. sinvar-spec G nP,
        c-offending-flows = λG. SecurityInvariant-withOffendingFlows.set-offending-flows sinvar-spec
G nP,
        c-isIFS = nm-receiver-violation m⦈) by simp

    from NetModelLib show ?thesis
      apply(simp add: SpecUnfolded formalSpec implSpec Let-def)
      apply(simp add: SecurityInvariant-complies-formal-def-def)
      apply(simp add: TopoS-modelLibrary-def TopoS-List-Impl-def)
      apply(simp add: nPdef)
      done
  qed


 corollary new-configured-list-SecurityInvariant-complies′:
  ⟦ TopoS-modelLibrary m sinvar-spec ⟧ ⟹
  SecurityInvariant-complies-formal-def (new-configured-list-SecurityInvariant m C description)
      (the (new-configured-SecurityInvariant (sinvar-spec, nm-default m, nm-receiver-violation m,
```

nm-node-props m C)))
by(*blast dest*: *new-configured-list-SecurityInvariant-complies*)

— From
thm *new-configured-SecurityInvariant-sound*
— we get that *new-configured-list-SecurityInvariant* has all the necessary properties (modulo *SecurityInvariant-complies-*

9.2 About security invariants

specification and implementation comply.

type-synonym *'v security-models-spec-impl*=(*'v SecurityInvariant* × *'v TopoS-Composition-Theory.SecurityInvariant*
list

definition *get-spec* :: *'v security-models-spec-impl* ⇒ (*'v TopoS-Composition-Theory.SecurityInvariant-configured*)
list **where**
get-spec M ≡ [*snd m*. *m* ← *M*]
definition *get-impl* :: *'v security-models-spec-impl* ⇒ (*'v SecurityInvariant*) *list* **where**
get-impl M ≡ [*fst m*. *m* ← *M*]

9.3 Calculating offending flows

fun *implc-get-offending-flows* :: (*'v*) *SecurityInvariant list* ⇒ *'v list-graph* ⇒ ((*'v* × *'v*) *list list*)
where
implc-get-offending-flows [] *G* = [] |
implc-get-offending-flows (*m*#*Ms*) *G* = (*implc-offending-flows m G*)@(*implc-get-offending-flows*
Ms G)

lemma *implc-get-offending-flows-fold*:
implc-get-offending-flows M G = *fold* (λ*m accu*. *accu*@(*implc-offending-flows m G*)) *M* []
proof−
{ **fix** *accu*
have *accu*@(*implc-get-offending-flows M G*) = *fold* (λ*m accu*. *accu*@(*implc-offending-flows m*
G)) *M accu*
apply(*induction M arbitrary*: *accu*)
apply(*simp-all*)
by(*metis append-eq-appendI*) }
from *this*[**where** *accu2*=[]] **show** *?thesis* **by** *simp*
qed

lemma *implc-get-offending-flows-Un*: *set'set* (*implc-get-offending-flows M G*) = (⋃ *m*∈*set M*. *set'set*
(*implc-offending-flows m G*))
apply(*induction M*)
apply(*simp-all*)
by (*metis image-Un*)

lemma *implc-get-offending-flows-map-concat*: (*implc-get-offending-flows M G*) = *concat* [*implc-offending-flows*
m G. *m* ← *M*]
apply(*induction M*)
by(*simp-all*)

theorem *implc-get-offending-flows-complies*:

assumes *a1*: ∀ (*m-impl*, *m-spec*) ∈ *set M*. *SecurityInvariant-complies-formal-def m-impl m-spec*
and *a2*: *wf-list-graph G*
shows *set'set* (*implc-get-offending-flows* (*get-impl M*) *G*) = (*get-offending-flows* (*get-spec M*) (*list-graph-to-graph G*))
proof −
from *a1* **have** ∀ (*m-impl*, *m-spec*) ∈ *set M*. *set ' set* (*implc-offending-flows m-impl G*) = *c-offending-flows m-spec* (*list-graph-to-graph G*)
apply(*simp add*: *SecurityInvariant-complies-formal-def-def*)
using *a2* **by** *blast*
hence ∀ *m* ∈ *set M*. *set ' set* (*implc-offending-flows* (*fst m*) *G*) = *c-offending-flows* (*snd m*) (*list-graph-to-graph G*) **by** *fastforce*
thus *?thesis*
by(*simp add*: *get-impl-def get-spec-def implc-get-offending-flows-Un get-offending-flows-def*)
qed

9.4 Accessors

definition *get-IFS* :: *'v SecurityInvariant list* ⇒ *'v SecurityInvariant list* **where**
get-IFS M ≡ [*m* ← *M*. *implc-isIFS m*]
definition *get-ACS* :: *'v SecurityInvariant list* ⇒ *'v SecurityInvariant list* **where**
get-ACS M ≡ [*m* ← *M*. ¬ *implc-isIFS m*]

lemma *get-IFS-get-ACS-complies*:
assumes *a*: ∀ (*m-impl*, *m-spec*) ∈ *set M*. *SecurityInvariant-complies-formal-def m-impl m-spec*
shows ∀ (*m-impl*, *m-spec*) ∈ *set* (*zip* (*get-IFS* (*get-impl M*)) (*TopoS-Composition-Theory.get-IFS* (*get-spec M*))).
SecurityInvariant-complies-formal-def m-impl m-spec
and ∀ (*m-impl*, *m-spec*) ∈ *set* (*zip* (*get-ACS* (*get-impl M*)) (*TopoS-Composition-Theory.get-ACS* (*get-spec M*))).
SecurityInvariant-complies-formal-def m-impl m-spec
proof −
from *a* **have** ∀ (*m-impl*, *m-spec*) ∈ *set M*. *implc-isIFS m-impl* = *c-isIFS m-spec*
apply(*simp add*: *SecurityInvariant-complies-formal-def-def*) **by** *fastforce*
hence *set-zip-IFS*: *set* (*zip* (*filter implc-isIFS* (*get-impl M*)) (*filter c-isIFS* (*get-spec M*))) ⊆ *set M*
apply(*simp add*: *get-impl-def get-spec-def*)
apply(*induction M*)
apply(*simp-all*)
by *force*
from *set-zip-IFS a* **show** ∀ (*m-impl*, *m-spec*) ∈ *set* (*zip* (*get-IFS* (*get-impl M*)) (*TopoS-Composition-Theory.get-IFS* (*get-spec M*))).
SecurityInvariant-complies-formal-def m-impl m-spec
apply(*simp add*: *get-IFS-def get-ACS-def*
TopoS-Composition-Theory.get-IFS-def TopoS-Composition-Theory.get-ACS-def) **by** *blast*
next
from *a* **have** ∀ (*m-impl*, *m-spec*) ∈ *set M*. *implc-isIFS m-impl* = *c-isIFS m-spec*
apply(*simp add*: *SecurityInvariant-complies-formal-def-def*) **by** *fastforce*
hence *set-zip-ACS*: *set* (*zip* [*m*←*get-impl M* . ¬ *implc-isIFS m*] [*m*←*get-spec M* . ¬ *c-isIFS m*]) ⊆ *set M*
apply(*simp add*: *get-impl-def get-spec-def*)
apply(*induction M*)
apply(*simp-all*)
by *force*
from *this a* **show** ∀ (*m-impl*, *m-spec*) ∈ *set* (*zip* (*get-ACS* (*get-impl M*)) (*TopoS-Composition-Theory.get-ACS*

(*get-spec M*))).
SecurityInvariant-complies-formal-def m-impl m-spec
apply(*simp add*: *get-IFS-def get-ACS-def*
TopoS-Composition-Theory.get-IFS-def TopoS-Composition-Theory.get-ACS-def) **by** *fast*
qed

lemma *get-IFS-get-ACS-select-simps*:
assumes *a1*: ∀ (*m-impl*, *m-spec*) ∈ *set M*. *SecurityInvariant-complies-formal-def m-impl m-spec*
shows ∀ (*m-impl*, *m-spec*) ∈ *set* (*zip* (*get-IFS* (*get-impl M*)) (*TopoS-Composition-Theory.get-IFS* (*get-spec M*))). *SecurityInvariant-complies-formal-def m-impl m-spec* (**is** ∀ (*m-impl*, *m-spec*) ∈ *set* *?zippedIFS*. *SecurityInvariant-complies-formal-def m-impl m-spec*)
and (*get-impl* (*zip* (*TopoS-Composition-Theory-impl.get-IFS* (*get-impl M*)) (*TopoS-Composition-Theory.get-IFS* (*get-spec M*)))) = *TopoS-Composition-Theory-impl.get-IFS* (*get-impl M*)
and (*get-spec* (*zip* (*TopoS-Composition-Theory-impl.get-IFS* (*get-impl M*)) (*TopoS-Composition-Theory.get-IFS* (*get-spec M*)))) = *TopoS-Composition-Theory.get-IFS* (*get-spec M*)
and ∀ (*m-impl*, *m-spec*) ∈ *set* (*zip* (*get-ACS* (*get-impl M*)) (*TopoS-Composition-Theory.get-ACS* (*get-spec M*))). *SecurityInvariant-complies-formal-def m-impl m-spec* (**is** ∀ (*m-impl*, *m-spec*) ∈ *set* *?zippedACS*. *SecurityInvariant-complies-formal-def m-impl m-spec*)
and (*get-impl* (*zip* (*TopoS-Composition-Theory-impl.get-ACS* (*get-impl M*)) (*TopoS-Composition-Theory.get-ACS* (*get-spec M*)))) = *TopoS-Composition-Theory-impl.get-ACS* (*get-impl M*)
and (*get-spec* (*zip* (*TopoS-Composition-Theory-impl.get-ACS* (*get-impl M*)) (*TopoS-Composition-Theory.get-ACS* (*get-spec M*)))) = *TopoS-Composition-Theory.get-ACS* (*get-spec M*)
proof −
from *get-IFS-get-ACS-complies*(*1*)[*OF a1*]
show ∀ (*m-impl*, *m-spec*) ∈ *set* (*?zippedIFS*). *SecurityInvariant-complies-formal-def m-impl m-spec* **by** *simp*
next
from *a1* **show** (*get-impl ?zippedIFS*) = *TopoS-Composition-Theory-impl.get-IFS* (*get-impl M*)
apply(*simp add*: *TopoS-Composition-Theory-impl.get-IFS-def get-spec-def get-impl-def TopoS-Composition-Theory*
apply(*induction M*)
apply(*simp*)
apply(*simp*)
apply(*rule conjI*)
apply(*clarify*)
using *SecurityInvariant-complies-formal-def-def* **apply** (*auto*)[*1*]
apply(*clarify*)
using *SecurityInvariant-complies-formal-def-def* **apply** (*auto*)[*1*]
done
next
from *a1* **show** (*get-spec ?zippedIFS*) = *TopoS-Composition-Theory.get-IFS* (*get-spec M*)
apply(*simp add*: *TopoS-Composition-Theory-impl.get-IFS-def get-spec-def get-impl-def TopoS-Composition-Theory*
apply(*induction M*)
apply(*simp*)
apply(*simp*)
apply(*rule conjI*)
apply(*clarify*)
using *SecurityInvariant-complies-formal-def-def* **apply** (*auto*)[*1*]
apply(*clarify*)
using *SecurityInvariant-complies-formal-def-def* **apply** (*auto*)[*1*]
done
next
from *get-IFS-get-ACS-complies*(*2*)[*OF a1*]

show $\forall$ (*m-impl*, *m-spec*) $\in$ *set* (*?zippedACS*). *SecurityInvariant-complies-formal-def m-impl m-spec* **by** *simp*
next
from *a1* **show** (*get-impl ?zippedACS*) = *TopoS-Composition-Theory-impl.get-ACS* (*get-impl M*)
apply(*simp add*: *TopoS-Composition-Theory-impl.get-ACS-def get-spec-def get-impl-def TopoS-Composition-Theory.get-ACS-def*)
apply(*induction M*)
apply(*simp*)
apply(*simp*)
apply(*rule conjI*)
apply(*clarify*)
using *SecurityInvariant-complies-formal-def-def* **apply** (*auto*)[*1*]
apply(*clarify*)
using *SecurityInvariant-complies-formal-def-def* **apply** (*auto*)[*1*]
done
next
from *a1* **show** (*get-spec ?zippedACS*) = *TopoS-Composition-Theory.get-ACS* (*get-spec M*)
apply(*simp add*: *TopoS-Composition-Theory-impl.get-ACS-def get-spec-def get-impl-def TopoS-Composition-Theory.get-ACS-def*)
apply(*induction M*)
apply(*simp*)
apply(*simp*)
apply(*rule conjI*)
apply(*clarify*)
using *SecurityInvariant-complies-formal-def-def* **apply** (*auto*)[*1*]
apply(*clarify*)
using *SecurityInvariant-complies-formal-def-def* **apply** (*auto*)[*1*]
done
qed

thm *get-IFS-get-ACS-select-simps*

9.5 All security requirements fulfilled

definition *all-security-requirements-fulfilled* :: *'v SecurityInvariant list* $\Rightarrow$ *'v list-graph* $\Rightarrow$ *bool* **where**
all-security-requirements-fulfilled M G $\equiv \forall$ *m* $\in$ *set M*. (*implc-sinvar m*) *G*

lemma *all-security-requirements-fulfilled-complies*:
$[\![\forall$ (*m-impl*, *m-spec*) $\in$ *set M*. *SecurityInvariant-complies-formal-def m-impl m-spec*;
wf-list-graph (*G*::(*'v*::*vertex*) *list-graph*) $]\!] \Longrightarrow$
all-security-requirements-fulfilled (*get-impl M*) *G* $\longleftrightarrow$ *TopoS-Composition-Theory.all-security-requirements-fulfilled* (*get-spec M*) (*list-graph-to-graph G*)
apply(*simp add*: *all-security-requirements-fulfilled-def TopoS-Composition-Theory.all-security-requirements-fulfilled-de*
apply(*simp add*: *get-impl-def get-spec-def*)
using *SecurityInvariant-complies-formal-def-def* **by** *fastforce*

9.6 generate valid topology

value *concat* [[*1*::*int*,*2*,*3*], [*4*,*6*,*5*]]

fun *generate-valid-topology* :: *'v SecurityInvariant list* $\Rightarrow$ *'v list-graph* $\Rightarrow$ (*'v list-graph*) **where**
generate-valid-topology M G = *delete-edges G* (*concat* (*implc-get-offending-flows M G*))

lemma *generate-valid-topology-complies*:
⟦ ∀ (*m-impl*, *m-spec*) ∈ *set M*. *SecurityInvariant-complies-formal-def m-impl m-spec*;
wf-list-graph (*G*::(*'v list-graph*)) ⟧ ⟹
list-graph-to-graph (*generate-valid-topology* (*get-impl M*) *G*) =
TopoS-Composition-Theory.generate-valid-topology (*get-spec M*) (*list-graph-to-graph G*)
apply (*subst generate-valid-topology-def-alt*)
apply (*drule*(*1*) *implc-get-offending-flows-complies*)
apply (*simp add*: *delete-edges-correct* [*symmetric*])
done

9.7 generate valid topology

tuned for invariants where we don't want to calculate all offending flows

Theoretic foundations: The algorithm *generate-valid-topology-SOME* picks ONE offending flow non-deterministically. This is sound: ⟦*valid-reqs ?M*; *wf-graph* (⦇*nodes* = *?V*, *edges* = *?E*⦈)⟧ ⟹ *TopoS-Composition-Theory.all-security-requirements-fulfilled ?M* (*generate-valid-topology-SOME ?M* (⦇*nodes* = *?V*, *edges* = *?E*⦈)). However, this non-deterministic choice is hard to implement. To pick one offending flow deterministically, we have implemented *TopoS-Interface-impl.minimalize-offending-o* It gives back one offending flow: ⟦*SecurityInvariant-preliminaries ?sinvar*; *wf-graph ?G*; *SecurityInvariant-with* *?sinvar* (*set ?ff*) *?G ?nP*; *set ?ff* ⊆ *edges ?G*; *distinct ?ff*⟧ ⟹ *set* (*SecurityInvariant-withOffendingFlows.min* *?sinvar ?ff* [] *?G ?nP*) ∈ *SecurityInvariant-withOffendingFlows.set-offending-flows ?sinvar ?G ?nP* The good thing about this function is, that it does not need to construct the complete *SecurityInvariant-withOffendingFlows.set-offending-flows*. Therefore, it can be used for security invariants which may have an exponential number of offending flows. The corresponding algorithm that uses this function is *generate-valid-topology-some*. It is also sound: ⟦*valid-reqs ?M*; *wf-graph* (⦇*nodes* = *?V*, *edges* = *?E*⦈); *set ?Es* = *?E*; *distinct ?Es*⟧ ⟹ *TopoS-Composition-Theory.all-security-requirements-fulfilled ?M* (*generate-valid-topology-some ?M ?Es* (⦇*nodes* = *?V*, *edges* = *?E*⦈)).

fun *generate-valid-topology-some* :: *'v SecurityInvariant list* ⇒ *'v list-graph* ⇒ (*'v list-graph*) **where**
generate-valid-topology-some [] *G* = *G* |
generate-valid-topology-some (*m*#*Ms*) *G* = (*if implc-sinvar m G*
then generate-valid-topology-some Ms G
else delete-edges (*generate-valid-topology-some Ms G*) (*minimalize-offending-overapprox* (*implc-sinvar m*) (*edgesL G*) [] *G*)
)

thm *TopoS-Composition-Theory.generate-valid-topology-some-sound*

lemma *generate-valid-topology-some-complies*:
⟦ ∀ (*m-impl*, *m-spec*) ∈ *set M*. *SecurityInvariant-complies-formal-def m-impl m-spec*;
wf-list-graph (*G*::(*'v*::*vertex list-graph*)) ⟧ ⟹
list-graph-to-graph (*generate-valid-topology-some* (*get-impl M*) *G*) =
TopoS-Composition-Theory.generate-valid-topology-some (*get-spec M*) (*edgesL G*) (*list-graph-to-graph G*)
proof(*induction M*)
case *Nil* **thus** *?case* **by**(*simp add*: *get-spec-def get-impl-def*)
next
case (*Cons m M*)
obtain *m-impl m-spec* **where** *m*: *m* = (*m-impl*, *m-spec*) **by**(*cases m*) *blast*

from *m* **have** *m-impl*: *get-impl* ((*m-impl*, *m-spec*) # *M*) = *m-impl* # (*get-impl M*) **by** (*simp add*: *get-impl-def*)
from *m* **have** *m-spec*: *get-spec* ((*m-impl*, *m-spec*) # *M*) = *m-spec* # (*get-spec M*) **by** (*simp add*: *get-spec-def*)

from *Cons.prems*(*1*) *m* **have** *complies-formal-def*: *SecurityInvariant-complies-formal-def m-impl m-spec* **by** *simp*
with *Cons.prems*(*2*) **have** *impl-spec*: *implc-sinvar m-impl G* ⟷ *c-sinvar m-spec* (*list-graph-to-graph G*)
by (*simp add*: *SecurityInvariant-complies-formal-def-def*)

from *complies-formal-def*
have ⋀*G nP*. *wf-list-graph G* ⟹
(λ*G nP*. (*c-sinvar m-spec*) *G*) (*list-graph-to-graph G*) *nP* = (λ*G nP*. (*implc-sinvar m-impl*) *G*) *G nP*
by (*simp add*: *SecurityInvariant-complies-formal-def-def*)

from *minimalize-offending-overapprox-spec-impl*[*OF Cons.prems*(*2*),
of (λ*G nP*. (*c-sinvar m-spec*) *G*) (λ*G nP*. (*implc-sinvar m-impl*) *G*), *OF this*]

have *TopoS-Interface-impl.minimalize-offending-overapprox* (*implc-sinvar m-impl*) *fs keeps G* =
TopoS-withOffendingFlows.minimalize-offending-overapprox (*c-sinvar m-spec*) *fs keeps* (*list-graph-to-graph G*)
for *fs keeps* **by** *simp*
from *this*[*of* (*edgesL G*) []] **have** *minimalize-offending-overapprox-spec*:
TopoS-Interface-impl.minimalize-offending-overapprox (*implc-sinvar m-impl*) (*edgesL G*) [] *G* =
TopoS-withOffendingFlows.minimalize-offending-overapprox (*c-sinvar m-spec*) (*edgesL G*) [] (*list-graph-to-graph G*) **.**

from *Cons* **show** *?case*
apply(*simp*)
apply(*simp add*: *m m-impl m-spec*)
apply(*intro conjI impI*)
apply (*simp add*: *impl-spec*; *fail*)
apply (*simp add*: *impl-spec*; *fail*)
apply(*simp add*: *delete-edges-correct*[*symmetric*])
apply(*simp add*: *list-graph-to-graph-def FiniteGraph.delete-edges-simp2*)
apply(*simp add*: *minimalize-offending-overapprox-spec*)
by (*simp add*: *list-graph-to-graph-def*)
qed

end
theory *TopoS-Stateful-Policy-Algorithm*
imports *TopoS-Stateful-Policy TopoS-Composition-Theory*
begin

10 Stateful Policy – Algorithm

10.1 Some unimportant lemmata

lemma *False-set*: $\{(r, s).\ False\} = \{\}$ **by** *simp*
lemma *valid-reqs-ACS-D*: *valid-reqs M* $\Longrightarrow$ *valid-reqs* (*get-ACS M*)
by(*simp add*: *valid-reqs-def get-ACS-def*)
lemma *valid-reqs-IFS-D*: *valid-reqs M* $\Longrightarrow$ *valid-reqs* (*get-IFS M*)
by(*simp add*: *valid-reqs-def get-IFS-def*)
lemma *all-security-requirements-fulfilled-ACS-D*: *all-security-requirements-fulfilled M G* $\Longrightarrow$
all-security-requirements-fulfilled (*get-ACS M*) *G*
by(*simp add*: *all-security-requirements-fulfilled-def get-ACS-def*)
lemma *all-security-requirements-fulfilled-IFS-D*: *all-security-requirements-fulfilled M G* $\Longrightarrow$
all-security-requirements-fulfilled (*get-IFS M*) *G*
by(*simp add*: *all-security-requirements-fulfilled-def get-IFS-def*)
lemma *all-security-requirements-fulfilled-mono-stateful-policy-to-network-graph*:
$[\![$ *valid-reqs M*; $E' \subseteq E$; *wf-graph* $(\!|$ *nodes* = *V*, *edges* = *Efix* $\cup$ *E* $|\!)$ $]\!] \Longrightarrow$
all-security-requirements-fulfilled M
(*stateful-policy-to-network-graph* $(\!|$ *hosts* = *V*, *flows-fix* = *Efix*, *flows-state* = *E* $|\!)) \Longrightarrow$
all-security-requirements-fulfilled M
(*stateful-policy-to-network-graph* $(\!|$ *hosts* = *V*, *flows-fix* = *Efix*, *flows-state* = *E'* $|\!)$)
apply(*simp add*: *stateful-policy-to-network-graph-def all-flows-def*)
apply(*drule all-security-requirements-fulfilled-mono*[**where** *E*=*Efix* $\cup$ *E* $\cup$ *backflows E* **and** *E'*=*Efix* $\cup$ *E'* $\cup$ *backflows E'* **and** *V*=*V*])
apply(*thin-tac wf-graph G* **for** *G*)
apply(*thin-tac all-security-requirements-fulfilled M G* **for** *M G*)
apply(*simp add*: *backflows-def*, *blast*)
apply(*thin-tac all-security-requirements-fulfilled M G* **for** *M G*)
apply(*simp add*: *wf-graph-def*)
apply(*simp add*: *backflows-def*)
using [[*simproc add*: *finite-Collect*]] **apply**(*auto*)[*1*]
apply(*simp-all*)
done

10.2 Sketch for generating a stateful policy from a simple directed policy

Having no stateful flows, we trivially get a valid stateful policy.

lemma *trivial-stateful-policy-compliance*:
$[\![$ *wf-graph* $(\!|$ *nodes* = *V*, *edges* = *E* $|\!)$; *valid-reqs M*; *all-security-requirements-fulfilled M* $(\!|$ *nodes* = *V*, *edges* = *E* $|\!)$ $]\!] \Longrightarrow$
stateful-policy-compliance $(\!|$ *hosts* = *V*, *flows-fix* = *E*, *flows-state* = $\{\}$ $|\!)$ $(\!|$ *nodes* = *V*, *edges* = *E* $|\!)$ *M*
apply(*unfold-locales*)
apply(*simp-all add*: *wf-graph-def stateful-policy-to-network-graph-def all-flows-def backflows-def False-set*)
apply(*simp add*: *get-IFS-def get-ACS-def all-security-requirements-fulfilled-def*)
apply(*clarify*)
apply(*drule valid-reqs-ACS-D*)
apply(*drule all-security-requirements-fulfilled-ACS-D*)
apply(*drule*(*1*) *all-security-requirements-fulfilled-imp-get-offending-empty*)
by *force*

trying better

First, filtering flows that cause no IFS violations

fun *filter-IFS-no-violations-accu* :: *'v::vertex graph* ⇒ *'v SecurityInvariant-configured list* ⇒ (*'v* × *'v*) *list* ⇒ (*'v* × *'v*) *list* ⇒ (*'v* × *'v*) *list* **where**
filter-IFS-no-violations-accu G M accu [] = *accu* |
filter-IFS-no-violations-accu G M accu (*e*#*Es*) = (*if*
all-security-requirements-fulfilled (*get-IFS M*) (*stateful-policy-to-network-graph* (| *hosts* = *nodes G*, *flows-fix* = *edges G*, *flows-state* = *set* (*e*#*accu*) |))
then filter-IFS-no-violations-accu G M (*e*#*accu*) *Es*
else filter-IFS-no-violations-accu G M accu Es)
definition *filter-IFS-no-violations* :: *'v::vertex graph* ⇒ *'v SecurityInvariant-configured list* ⇒ (*'v* × *'v*) *list* ⇒ (*'v* × *'v*) *list* **where**
filter-IFS-no-violations G M Es = *filter-IFS-no-violations-accu G M* [] *Es*

lemma *filter-IFS-no-violations-subseteq-input*: *set* (*filter-IFS-no-violations G M Es*) ⊆ *set Es*
apply(*subgoal-tac* ∀ *accu*. *set* (*filter-IFS-no-violations-accu G M accu Es*) ⊆ *set Es* ∪ *set accu*)
apply(*erule-tac x*=[] **in** *allE*)
apply(*simp add*: *filter-IFS-no-violations-def*)
unfolding *filter-IFS-no-violations-def*
apply(*induct-tac Es*)
apply(*simp-all*)
apply *force*
done
lemma *filter-IFS-no-violations-accu-correct-induction*: *valid-reqs* (*get-IFS M*) ⟹ *wf-graph* (| *nodes* = *V*, *edges* = *E* |) ⟹
all-security-requirements-fulfilled (*get-IFS M*) (*stateful-policy-to-network-graph* (| *hosts* = *V*, *flows-fix* = *E*, *flows-state* = *set* (*accu*) |)) ⟹
(*set accu*) ∪ (*set edgesList*) ⊆ *E* ⟹
all-security-requirements-fulfilled (*get-IFS M*) (*stateful-policy-to-network-graph* (| *hosts* = *V*, *flows-fix* = *E*, *flows-state* = *set* (*filter-IFS-no-violations-accu* (| *nodes* = *V*, *edges* = *E* |) *M accu edgesList*) |))
apply(*induction edgesList arbitrary*: *accu*)
by(*simp-all*)
lemma *filter-IFS-no-violations-correct*: ⟦*valid-reqs* (*get-IFS M*); *wf-graph G*;
all-security-requirements-fulfilled (*get-IFS M*) *G*;
(*set edgesList*) ⊆ *edges G* ⟧ ⟹
all-security-requirements-fulfilled (*get-IFS M*) (*stateful-policy-to-network-graph* (| *hosts* = *nodes G*, *flows-fix* = *edges G*, *flows-state* = *set* (*filter-IFS-no-violations G M edgesList*) |))
unfolding *filter-IFS-no-violations-def*
apply(*case-tac G*, *simp*)
apply(*drule*(*1*) *filter-IFS-no-violations-accu-correct-induction*[**where** *accu*=[], *simplified*])
apply(*simp-all*)
by(*simp add*: *stateful-policy-to-network-graph-def all-flows-def backflows-def False-set*)
lemma *filter-IFS-no-violations-accu-no-IFS*: *valid-reqs* (*get-IFS M*) ⟹ *wf-graph G* ⟹ *get-IFS M* = [] ⟹
(*set accu*) ∪ (*set edgesList*) ⊆ *edges G* ⟹
filter-IFS-no-violations-accu G M accu edgesList = *rev*(*edgesList*)@*accu*
apply(*induction edgesList arbitrary*: *accu*)
by(*simp-all add*: *all-security-requirements-fulfilled-def*)

lemma *filter-IFS-no-violations-accu-maximal-induction*: *valid-reqs* (*get-IFS M*) ⟹ *wf-graph* (| *nodes* = *V*, *edges* = *E* |) ⟹
set accu ⊆ *E* ⟹ *set edgesList* ⊆ *E* ⟹
∀ *e* ∈ *E* − (*set accu* ∪ *set edgesList*).

¬ *all-security-requirements-fulfilled* (*get-IFS M*) (*stateful-policy-to-network-graph* ⦇ *hosts* = *V*, *flows-fix* = *E*, *flows-state* = {*e*} ∪ (*set accu*) ⦈)
⟹
let stateful = *set* (*filter-IFS-no-violations-accu* ⦇ *nodes* = *V*, *edges* = *E* ⦈ *M accu edgesList*) *in*
(∀ *e* ∈ *E* − *stateful*.
¬ *all-security-requirements-fulfilled* (*get-IFS M*) (*stateful-policy-to-network-graph* ⦇ *hosts* = *V*, *flows-fix* = *E*, *flows-state* = {*e*} ∪ *stateful* ⦈))
proof(*induction edgesList arbitrary*: *accu*)
case *Nil* **thus** *?case* **by**(*simp add*: *Let-def*)
next
case(*Cons e Es*)
from *Cons.prems*(*3*) *Cons.prems*(*2*) **have** *fst* ‘ *set accu* ⊆ *V* **and** *snd* ‘ *set accu* ⊆ *V*
by(*auto simp add*: *wf-graph-def*)
— *wf-graph* for some complicated structures
from *Cons.prems*(*2*) *this Cons.prems*(*4*) **have** ⋀*ea*. *ea*∈*E* ⟹ *wf-graph* ⦇ *nodes* = *V*, *edges* = *insert e* (*insert ea* (*set accu*)) ⦈
by(*auto simp add*: *wf-graph-def*)
from *backflows-wf*[*OF this*] *wf-graph-union-edges*[*OF Cons.prems*(*2*)]
have ⋀*ea*. *ea*∈*E* ⟹ *wf-graph* ⦇ *nodes* = *V*, *edges* = *E* ∪ *backflows* (*insert e* (*insert ea* (*set accu*))) ⦈ **by** (*simp*)
hence ⋀*ea*. *ea*∈*E* ⟹ *wf-graph* ⦇ *nodes* = *V*, *edges* = *E* ∪ *set accu* ∪ *backflows* (*insert e* (*insert ea* (*set accu*))) ⦈
by (*metis Cons.prems*(*3*) *sup.order-iff*)
from *this Cons.prems*(*4*)
have ⋀*ea*. *ea*∈*E* ⟹ *wf-graph* ⦇ *nodes* = *V*, *edges* = *insert e* (*insert ea* (*E* ∪ *set accu* ∪ *backflows* (*insert e* (*insert ea* (*set accu*))))) ⦈
by(*simp add*: *insert-absorb*)
hence *validgraph1*: ⋀*ea*. *ea*∈*E* − (*set* (*e* # *accu*) ∪ *set Es*) ⟹
wf-graph ⦇ *nodes* = *V*, *edges* = *insert e* (*insert ea* (*E* ∪ *set accu* ∪ *backflows* (*insert e* (*insert ea* (*set accu*))))) ⦈ **by**(*simp*)

have *validgraph2*: ⋀ *ea*.
insert ea (*E* ∪ *set accu* ∪ *backflows* (*insert ea* (*set accu*))) ⊆ *insert e* (*insert ea* (*E* ∪ *set accu* ∪ *backflows* (*insert e* (*insert ea* (*set accu*)))))
apply(*simp add*: *backflows-def*)
by *blast*

from *all-security-requirements-fulfilled-mono*[*OF Cons.prems*(*1*) *validgraph2 validgraph1*] **have** *neg-mono*:
⋀*ea*. *ea* ∈ *E* − (*set* (*e* # *accu*) ∪ *set Es*) ⟹
¬ *all-security-requirements-fulfilled* (*get-IFS M*)
⦇*nodes* = *V*, *edges* = *insert ea* (*E* ∪ *set accu* ∪ *backflows* (*insert ea* (*set accu*)))⦈
⟹
¬ *all-security-requirements-fulfilled* (*get-IFS M*)
⦇*nodes* = *V*, *edges* = *insert e* (*insert ea* (*E* ∪ *set accu* ∪ *backflows* (*insert e* (*insert ea* (*set accu*))))⦈
apply(*simp*)
by *blast*

from *Cons.prems*(*5*) **have** ⋀*ea*. *ea*∈*E* − (*set* (*e* # *accu*) ∪ *set Es*) ⟹
¬ *all-security-requirements-fulfilled* (*get-IFS M*) (*stateful-policy-to-network-graph*
⦇*hosts* = *V*, *flows-fix* = *E*, *flows-state* = {*ea*} ∪ *set* (*e* # *accu*)⦈)
apply(*erule-tac x*=*ea* **in** *ballE*)

```
        prefer 2
        apply simp
       apply(simp only: stateful-policy-to-network-graph-def all-flows-def stateful-policy.select-convs)
        apply(simp)
        apply(frule(1) neg-mono[simplified])
        by(simp)
       hence goalTrue:
       ∀ ea∈E − (set (e # accu) ∪ set Es).
         ¬ all-security-requirements-fulfilled (get-IFS M)
            (stateful-policy-to-network-graph (|hosts = V, flows-fix = E, flows-state = {ea} ∪ set (e
# accu)|))
       by simp

      show ?case
      apply(simp add: Let-def)
      apply(rule conjI)

       apply(rule impI)
       apply(thin-tac -)
         using Cons.IH[where accu=e # accu, OF Cons.prems(1) Cons.prems(2) - - goalTrue,
simplified Let-def] Cons.prems(3) Cons.prems(4)
       apply(auto) [1]

      apply(rule impI)
        using Cons.IH[where accu=accu, OF Cons.prems(1) Cons.prems(2), simplified Let-def]
Cons.prems(5) Cons.prems(3) Cons.prems(4)
      apply(auto)
      done
    qed
   lemma filter-IFS-no-violations-maximal: ⟦valid-reqs (get-IFS M); wf-graph G;
         (set edgesList) = edges G ⟧ ⟹
         let stateful = set (filter-IFS-no-violations G M edgesList) in
         ∀ e ∈ edges G − stateful.
           ¬ all-security-requirements-fulfilled (get-IFS M) (stateful-policy-to-network-graph (| hosts
= nodes G, flows-fix = edges G, flows-state = {e} ∪ stateful |))
   unfolding filter-IFS-no-violations-def
   apply(case-tac G, simp)
  apply(drule(1) filter-IFS-no-violations-accu-maximal-induction[where accu=[] and edgesList=edgesList])
     by(simp-all)

   — It is not only maximal for single flows but all non-empty subsets
   corollary filter-IFS-no-violations-maximal-allsubsets:
   assumes a1: valid-reqs (get-IFS M)
   and     a2: wf-graph G
   and     a4: (set edgesList) = edges G
   shows   let stateful = set (filter-IFS-no-violations G M edgesList) in
         ∀ E ⊆ edges G − stateful. E ≠ {} ⟶
           ¬ all-security-requirements-fulfilled (get-IFS M) (stateful-policy-to-network-graph (| hosts
= nodes G, flows-fix = edges G, flows-state = E ∪ stateful |))
   proof −
     let ?stateful = set (filter-IFS-no-violations G M edgesList)
     from filter-IFS-no-violations-maximal[OF a1 a2 a4] have not-fulfilled-single:
      ∀ e∈edges G − ?stateful. ¬ all-security-requirements-fulfilled (get-IFS M)
            (stateful-policy-to-network-graph (|hosts = nodes G, flows-fix = edges G, flows-state =
```

$\{e\} \cup$ *?stateful*$\|)$)
by(*simp add*: *Let-def*)
have *neg-mono*:
$\bigwedge$ *e E*. $e \in E \Longrightarrow E \subseteq$ *edges G* $-$ *?stateful* $\Longrightarrow E \neq \{\} \Longrightarrow$
$\neg$ *all-security-requirements-fulfilled* (*get-IFS M*)
(*stateful-policy-to-network-graph* $(\|$*hosts* = *nodes G*, *flows-fix* = *edges G*, *flows-state* = $\{e\}$
$\cup$ *?stateful*$\|)$) $\Longrightarrow$
$\neg$ *all-security-requirements-fulfilled* (*get-IFS M*)
(*stateful-policy-to-network-graph* $(\|$*hosts* = *nodes G*, *flows-fix* = *edges G*, *flows-state* = *E*
$\cup$ *?stateful*$\|)$)
proof $-$
fix *e E*
assume *h1*: $e \in E$
and *h2*: $E \subseteq$ *edges G* $-$ *?stateful*
and *h3*: $E \neq \{\}$
and *h4*: $\neg$ *all-security-requirements-fulfilled* (*get-IFS M*)
(*stateful-policy-to-network-graph* $(\|$*hosts* = *nodes G*, *flows-fix* = *edges G*, *flows-state* = $\{e\}$
$\cup$ *?stateful*$\|)$)

from *filter-IFS-no-violations-subseteq-input a4* **have** *?stateful* $\subseteq$ *edges G* **by** *blast*
hence *edges G* $\cup$ (*E* $\cup$ *?stateful*) = *edges G* **using** *h2* **by** *blast*
from *a2 this* **have** *validgraph1*: *wf-graph* $(\|$*nodes* = *nodes G*, *edges* = *edges G* $\cup$ (*E* $\cup$
?stateful)$\|)$
by(*case-tac G*, *simp*)

from *h1 h2 h3* **have** *subseteq*: ($\{e\} \cup$ *?stateful*) $\subseteq$ (*E* $\cup$ *?stateful*) **by** *blast*

have *revimp*: $\bigwedge A\ B.\ (A \Longrightarrow B) \Longrightarrow (\neg\ B \Longrightarrow \neg\ A)$ **by** *fast*

from *all-security-requirements-fulfilled-mono-stateful-policy-to-network-graph*[*OF a1 subseteq*
validgraph1] *h4*
show $\neg$ *all-security-requirements-fulfilled* (*get-IFS M*)
(*stateful-policy-to-network-graph* $(\|$*hosts* = *nodes G*, *flows-fix* = *edges G*, *flows-state* = *E* $\cup$
?stateful$\|)$)
apply(*rule revimp*)
by *assumption*
qed

show *?thesis*
proof(*simp add*: *Let-def*,*rule allI*,*rule impI*, *rule impI*)
fix *E*
assume *h1*: $E \subseteq$ *edges G* $-$ *?stateful*
and *h2*: $E \neq \{\}$

from *h1 h2* **obtain** *e* **where** *e-prop1*: $e \in E$ **by** *blast*
from *this h1* **have** $e \in$ *edges G* $-$ *?stateful* **by** *blast*
from *this not-fulfilled-single* **have** *e-prop2*: $\neg$ *all-security-requirements-fulfilled* (*get-IFS M*)
(*stateful-policy-to-network-graph* $(\|$*hosts* = *nodes G*, *flows-fix* = *edges G*, *flows-state* = $\{e\} \cup$
?stateful$\|)$)
by *simp*

from *neg-mono*[*OF e-prop1 h1 h2 e-prop2*]
show $\neg$ *all-security-requirements-fulfilled* (*get-IFS M*)
(*stateful-policy-to-network-graph* $(\|$*hosts* = *nodes G*, *flows-fix* = *edges G*, *flows-state* = *E*

∪ *set* (*filter-IFS-no-violations G M edgesList*)⦈))
.
qed
qed

— soundness and completeness
thm *filter-IFS-no-violations-correct filter-IFS-no-violations-maximal*

Next

fun *filter-compliant-stateful-ACS-accu* :: *'v*::*vertex graph* ⇒ *'v SecurityInvariant-configured list* ⇒ (*'v* × *'v*) *list* ⇒ (*'v* × *'v*) *list* ⇒ (*'v* × *'v*) *list* **where**
filter-compliant-stateful-ACS-accu G M accu [] = *accu* |
filter-compliant-stateful-ACS-accu G M accu (*e*#*Es*) = (*if*
e ∉ *backflows* (*edges G*) ∧ (∀ *F* ∈ *get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* ⦇ *hosts* = *nodes G*, *flows-fix* = *edges G*, *flows-state* = *set* (*e*#*accu*) ⦈). *F* ⊆ *backflows* (*set* (*e*#*accu*)))
then filter-compliant-stateful-ACS-accu G M (*e*#*accu*) *Es*
else filter-compliant-stateful-ACS-accu G M accu Es)
definition *filter-compliant-stateful-ACS* :: *'v*::*vertex graph* ⇒ *'v SecurityInvariant-configured list* ⇒ (*'v* × *'v*) *list* ⇒ (*'v* × *'v*) *list* **where**
filter-compliant-stateful-ACS G M Es = *filter-compliant-stateful-ACS-accu G M* [] *Es*

lemma *filter-compliant-stateful-ACS-subseteq-input*: *set* (*filter-compliant-stateful-ACS G M Es*) ⊆ *set Es*
apply(*subgoal-tac* ∀ *accu*. *set* (*filter-compliant-stateful-ACS-accu G M accu Es*) ⊆ *set Es* ∪ *set accu*)
apply(*erule-tac x*=[] **in** *allE*)
apply(*simp add*: *filter-compliant-stateful-ACS-def*)
apply(*induct-tac Es*)
apply(*simp-all*)
apply (*metis Un-insert-right set-simps*(2) *set-subset-Cons set-union subset-trans*)
done
lemma *filter-compliant-stateful-ACS-accu-correct-induction*: *valid-reqs* (*get-ACS M*) ⟹ *wf-graph* ⦇ *nodes* = *V*, *edges* = *E* ⦈ ⟹
(*set accu*) ∪ (*set edgesList*) ⊆ *E* ⟹
∀ *F* ∈ *get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* ⦇ *hosts* = *V*, *flows-fix* = *E*, *flows-state* = *set* (*accu*) ⦈). *F* ⊆ *backflows* (*set accu*) ⟹
(∀ *a* ∈ *set accu*. *a* ∉ (*backflows E*)) ⟹
𝒯 = (⦇ *hosts* = *V*, *flows-fix* = *E*, *flows-state* = *set* (*filter-compliant-stateful-ACS-accu* (⦇ *nodes* = *V*, *edges* = *E* ⦈) *M accu edgesList*) ⦈) ⟹
∀ *F* ∈ *get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* 𝒯). *F* ⊆ *backflows* (*filternew-flows-state* 𝒯)
proof(*induction edgesList arbitrary*: *accu*)
case *Nil*
from *Nil*(5) **have** *backflows* (*set accu*) = *backflows* {*e* ∈ *set accu*. *e* ∉ *backflows E*} **by** (*metis* (*lifting*) *Collect-cong Collect-mem-eq*)
from *this Nil*(4) **have** ∀ *F*∈*get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* (⦇*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *set accu*⦈). *F* ⊆ *backflows* {*e* ∈ *set accu*. *e* ∉ *backflows E*} **by** *simp*
from *this Nil*(6) **show** *?case* **by**(*simp add*: *filternew-flows-state-alt2*)
next
case (*Cons e Es*)
from *Cons.IH*[*OF Cons.prems*(1) *Cons.prems*(2)] *Cons.prems*(3) *Cons.prems*(4) *Cons.prems*(5) *Cons.prems*(6)

show *?case* **by**(*simp add*: *filternew-flows-state-alt2 split*: *if-split-asm*)
qed

lemma *filter-compliant-stateful-ACS-accu-no-side-effects*: *valid-reqs* (*get-ACS M*) $\Longrightarrow$ *wf-graph G* $\Longrightarrow$
$\forall F \in$ *get-offending-flows* (*get-ACS M*) (|*nodes* = *nodes G*, *edges* = *edges G* $\cup$ *backflows* (*edges G*)|). $F \subseteq$ (*backflows* (*edges G*)) $-$ (*edges G*) $\Longrightarrow$
(*set accu*) $\cup$ (*set edgesList*) $\subseteq$ *edges G* $\Longrightarrow$
($\forall a \in$ *set accu*. $a \notin$ (*backflows* (*edges G*))) $\Longrightarrow$
filter-compliant-stateful-ACS-accu G M accu edgesList = *rev*([$e \leftarrow$ *edgesList*. $e \notin$ *backflows* (*edges G*)])@*accu*
apply(*simp add*: *backflows-minus-backflows*)
apply(*induction edgesList arbitrary*: *accu*)
apply(*simp*)
apply(*simp add*: *stateful-policy-to-network-graph-def all-flows-def*)
apply(*rule impI*)
apply(*case-tac G*, *simp*, *rename-tac V E*)
thm *Un-set-offending-flows-bound-minus-subseteq'*[**where** *X*=*backflows E* $-$ *E* **and** *E*=*E* $\cup$ *backflows E*]
apply(*drule-tac X*=*backflows E* $-$ *E* **and** *E*=*E* $\cup$ *backflows E* **and** *E'*=(*E* $\cup$ *backflows E*) $-$ (*insert a* (*E* $\cup$ *set accu* $\cup$ *backflows* (*insert a* (*set accu*)))) **in** *Un-set-offending-flows-bound-minus-subseteq'*)
defer
prefer *2*
apply *blast*
apply *auto*[*1*]
apply(*subgoal-tac E* $\cup$ *backflows E* $-$ (*E* $\cup$ *backflows E* $-$ *insert a* (*E* $\cup$ *set accu* $\cup$ *backflows* (*insert a* (*set accu*)))) = *insert a* (*E* $\cup$ *set accu* $\cup$ *backflows* (*insert a* (*set accu*))))
apply(*simp*)
prefer *2*
apply (*metis Un-assoc Un-least Un-mono backflows-subseteq double-diff insert-def insert-subset subset-refl*)
apply(*subgoal-tac backflows* (*insert a* (*set accu*)) $\subseteq$ *backflows E* $-$ *E* $-$ (*E* $\cup$ *backflows E* $-$ *insert a* (*E* $\cup$ *set accu* $\cup$ *backflows* (*insert a* (*set accu*)))))
apply(*blast*)
apply(*simp add*: *backflows-def*)
apply *fast*
using *FiniteGraph.backflows-wf FiniteGraph.wf-graph-union-edges* **by** *metis*

lemma *filter-compliant-stateful-ACS-correct*:
assumes *a1*: *valid-reqs* (*get-ACS M*)
and *a2*: *wf-graph G*
and *a3*: *set edgesList* $\subseteq$ *edges G*
and *a4*: *all-security-requirements-fulfilled* (*get-ACS M*) *G*
and *a5*: $\mathcal{T}$ = (| *hosts* = *nodes G*, *flows-fix* = *edges G*, *flows-state* = *set* (*filter-compliant-stateful-ACS G M edgesList*) |)
shows $\forall F \in$ *get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* $\mathcal{T}$). $F \subseteq$ *backflows* (*filternew-flows-state* $\mathcal{T}$)
proof $-$
obtain *V E* **where** *VE*: *G* = (| *nodes* = *V*, *edges* = *E* |) **by**(*case-tac G*, *blast*)
from *VE a2* **have** *wfVE*: *wf-graph* (| *nodes* = *V*, *edges* = *E* |) **by** *simp*
from *VE a3* **have** *set edgesList* $\subseteq$ *E* **by** *simp*

from *a5 VE* **have** *a5'*: $\mathcal{T} = (\!|hosts = V, flows\text{-}fix = E, flows\text{-}state = set\ (filter\text{-}compliant\text{-}stateful\text{-}ACS\text{-}accu\ (\!|nodes = V,\ edges = E|\!)\ M\ []\ edgesList)|\!)$
unfolding *filter-compliant-stateful-ACS-def*
by(*simp*)

from *all-security-requirements-fulfilled-imp-get-offending-empty*[*OF a1 a4*] *VE*
have $\forall F{\in}$*get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* $(\!|hosts = V, flows\text{-}fix = E, flows\text{-}state = \{\}|\!)$). $F \subseteq$ *backflows* $\{\}$
by(*simp add*: *stateful-policy-to-network-graph-def all-flows-def backflows-def False-set*)

from *filter-compliant-stateful-ACS-accu-correct-induction*[**where** *accu*=[] **and** *edgesList*=*edgesList*, *simplified*, *OF a1 wfVE* ‹*set edgesList* ⊆ *E*› *this a5'*]
show *?thesis* **.**
qed

lemma *filter-compliant-stateful-ACS-accu-induction-maximal*:$[\![$ *valid-reqs* (*get-ACS M*); *wf-graph* $(\!|$ *nodes* = *V*, *edges* = *E* $|\!)$;
(*set edgesList*) ⊆ *E*;
(*set accu*) ⊆ *E*;
stateful = *set* (*filter-compliant-stateful-ACS-accu* $(\!|$ *nodes* = *V*, *edges* = *E* $|\!)$ *M accu edgesList*);
$\forall e \in E - (set\ edgesList \cup set\ accu \cup \{e \in E.\ e \in backflows\ E\})$.
$\neg \bigcup$(*get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* $(\!|$ *hosts* = *V*, *flows-fix* = *E*, *flows-state* = *set accu* ∪ {*e*} $|\!)$))
⊆ *backflows* (*filternew-flows-state* $(\!|$ *hosts* = *V*, *flows-fix* = *E*, *flows-state* = *set accu* ∪ {*e*} $|\!)$)
$]\!] \Longrightarrow$
$\forall e \in E - (stateful \cup \{e \in E.\ e \in backflows\ E\})$. ~~*E' = (computed stateful flows plus triple) stateful flows*~~
$\neg \bigcup$(*get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* $(\!|$ *hosts* = *V*, *flows-fix* = *E*, *flows-state* = *stateful* ∪ {*e*} $|\!)$))
⊆ *backflows* (*filternew-flows-state* $(\!|$ *hosts* = *V*, *flows-fix* = *E*, *flows-state* = *stateful* ∪ {*e*} $|\!)$)
proof(*induction edgesList arbitrary*: *accu E*)
case *Nil* **from** *Nil*(*5*)[*simplified*] *Nil*(*6*) **show** *?case* **by**(*simp*)
next
case (*Cons a Es*)
— case distinction
let *?caseDistinction*=$a \notin backflows\ (E) \wedge\ (\forall F{\in}$*get-offending-flows* (*get-ACS M*)
(*stateful-policy-to-network-graph* $(\!|hosts = V, flows\text{-}fix = E, flows\text{-}state = set\ (a\ \#\ accu)|\!)$).
$F \subseteq backflows\ (set\ (a\ \#\ accu))$)
from *Cons.prems*(*3*) **have** *set Es* ⊆ *E* **by** *simp*

show *?case*
proof(*cases ?caseDistinction*)
assume *CaseTrue*: *?caseDistinction*
from *CaseTrue* **have**
set (*filter-compliant-stateful-ACS-accu* $(\!|nodes = V, edges = E|\!)$ *M accu* (*a* # *Es*)) =
set (*filter-compliant-stateful-ACS-accu* $(\!|nodes = V, edges = E|\!)$ *M* (*a* # *accu*) *Es*)
by(*simp*)
from *this Cons.prems*(*5*) **have** *statefulsimp*:

stateful = *set* (*filter-compliant-stateful-ACS-accu* (|*nodes* = *V*, *edges* = *E*|) *M* (*a* # *accu*) *Es*) **by** *simp*
from *Cons.prems*(*3*) *Cons.prems*(*4*) **have** *set* (*a* # *accu*) ⊆ *E* **by** *simp*

have ∀ *e*∈*E* − (*set Es* ∪ *set* (*a* # *accu*) ∪ {*e* ∈ *E*. *e* ∈ *backflows E*}).
¬ ⋃(*get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *set* (*a* # *accu*) ∪ {*e*}|)))
⊆ *backflows* (*filternew-flows-state* (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *set* (*a* # *accu*) ∪ {*e*}|))
proof(*rule ballI*)
fix *e*
assume *h1*: *e* ∈ *E* − (*set Es* ∪ *set* (*a* # *accu*) ∪ {*e* ∈ *E*. *e* ∈ *backflows E*})

from *conjunct1*[*OF CaseTrue*] **have** *filternew-flows-state-moveout-a*:
filternew-flows-state (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *set* (*a* # *accu*) ∪ {*e*}|) =
{*a*} ∪ *filternew-flows-state* (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *set accu* ∪ {*e*}|)
apply(*simp add*: *filternew-flows-state-alt*) **by** *blast*

have *backflowssubseta*: ⋀*X*. *backflows X* ⊆ *backflows* ({*a*} ∪ *X*) **by**(*simp add*: *backflows-def*, *blast*)

from *Cons.prems*(*6*) *h1* **have**
¬ ⋃(*get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *set accu* ∪ {*e*}|)))
⊆ *backflows* (*filternew-flows-state* (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *set accu* ∪ {*e*}|)) **by** *simp*
from *this* **obtain** *dat-offender* **where**
dat-in: *dat-offender* ∈ ⋃(*get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *set accu* ∪ {*e*}|)))
and *dat-offends*: *dat-offender* ∉ *backflows* (*filternew-flows-state* (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *set accu* ∪ {*e*}|)) **by** *blast*

have *wfGraphA*: *wf-graph* (*stateful-policy-to-network-graph* (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *set* (*a* # *accu*) ∪ {*e*}|))
proof(*simp add*: *stateful-policy-to-network-graph-def all-flows-def*)
from *Cons.prems*(*2*) *h1 Cons.prems*(*3*) *Cons.prems*(*4*)
have *wf-graph* (|*nodes*=*V*, *edges* = *insert e* (*insert a* (*set accu*)) |)
apply(*auto simp add*: *wf-graph-def*) **by** *force*
from *this backflows-wf*
have *vgh1*: *wf-graph* (|*nodes* = *V*, *edges* = *backflows* (*insert e* (*insert a* (*set accu*)))|) **by** *auto*
from *Cons.prems*(*2*) *wf-graph-add-subset-edges h1* *Cons.prems*(*3*) *Cons.prems*(*4*)
have *vgh2*: *wf-graph* (|*nodes* = *V*, *edges* = *insert e* ((*insert a E*) ∪ *set accu*)|)
proof −
have *f1*: *e* ∈ *E* − (*set Es* ∪ *insert a* (*set accu*) ∪ {*R* ∈ *E*. *R* ∈ *backflows E*})
using *h1* **by** *simp*
have *f2*: *insert a* (*set accu*) ⊆ *E*
using ‹*set* (*a* # *accu*) ⊆ *E*› **by** *simp*
have *f3*: *e* ∈ *E*
using *f1* **by** *fastforce*
have *E* ∪ *insert a* (*set accu*) = *E*
using *f2* **by** *fastforce*
thus *wf-graph* (|*nodes* = *V*, *edges* = *insert e* (*insert a E* ∪ *set accu*)|)
using *f3 Cons.prems*(*2*) *Un-insert-right insert-absorb sup-commute* **by** *fastforce*

qed
from *vgh1 vgh2 wf-graph-union-edges*
show *wf-graph* (|*nodes* = *V*, *edges* = *insert e* (*insert a* (*E* ∪ *set accu* ∪ *backflows* (*insert e* (*insert a* (*set accu*)))))|) **by** *fastforce*
qed

from *dat-in* **have** *dat-in-simplified*:
dat-offender ∈ ⋃(*get-offending-flows* (*get-ACS M*) (|*nodes* = *V*, *edges* = *insert e* (*E* ∪ *set accu* ∪ *backflows* (*insert e* (*set accu*)))|))
by(*simp add*: *stateful-policy-to-network-graph-def all-flows-def*)

have *subsethlp*: *insert e* (*E* ∪ *set accu* ∪ *backflows* (*insert e* (*set accu*))) ⊆ *E* ∪ (*set* (*a* # *accu*) ∪ {*e*}) ∪ *backflows* (*set* (*a* # *accu*) ∪ {*e*})
apply(*simp*)
apply(*rule*, *blast*)
apply(*rule*, *blast*)
apply(*rule*)
apply(*simp add*: *backflows-def*, *fast*)
done

from *get-offending-flows-union-mono*[*OF*
Cons.prems(*1*)
wfGraphA[*simplified stateful-policy-to-network-graph-def all-flows-def graph.select-convs stateful-policy.select-convs*],
OF subsethlp]
dat-in-simplified **have** *dat-in-a*: *dat-offender* ∈ ⋃(*get-offending-flows* (*get-ACS M*)
(*stateful-policy-to-network-graph* (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *set* (*a* # *accu*) ∪ {*e*}|)))
by(*simp add*: *stateful-policy-to-network-graph-def all-flows-def*, *fast*)

have *dat-offender* ≠ (*snd a*, *fst a*)
proof(*rule ccontr*)
assume ¬ *dat-offender* ≠ (*snd a*, *fst a*)
hence *hlpassm*: *dat-offender* = (*snd a*, *fst a*) **by** *simp*
from *this* **obtain** *a1 a2* **where** *dat-offender* = (*a2*, *a1*) **by** *blast*

have ⋃(*get-offending-flows* (*get-ACS M*) (|*nodes* = *V*, *edges* = *insert e* (*E* ∪ *set accu* ∪ *backflows* (*insert e* (*set accu*)))|)) ⊆
insert e (*E* ∪ *set accu* ∪ *backflows* (*insert e* (*set accu*)))
by (*metis Cons.prems*(*1*) *Sup-le-iff get-offending-flows-subseteq-edges*)
from *this h1* **have** *UN-get-subset*:
⋃(*get-offending-flows* (*get-ACS M*) (|*nodes* = *V*, *edges* = *insert e* (*E* ∪ *set accu* ∪ *backflows* (*insert e* (*set accu*)))|)) ⊆
(*E* ∪ *set accu* ∪ *backflows* (*insert e* (*set accu*)))
by *blast*

from *dat-offends* **have** *dat-offends-simplified*:
dat-offender ∉ *backflows* (*insert e* (*set accu*)) − *E*
by(*simp only*: *filternew-flows-state-alt stateful-policy.select-convs backflows-minus-backflows*, *simp*)

from *conjunct1*[*OF CaseTrue*] *hlpassm* **have** *dat-offender* ∉ *E*
by(*simp add*: *backflows-def*, *fastforce*)

from *dat-in-simplified UN-get-subset this* **have** *dat-offender* ∈ *set accu* ∪ *backflows* (*insert e* (*set accu*)) **by** *blast*
from *this Cons.prems*(*4*) ‹*dat-offender* ∉ *E*› **have** *dat-offender* ∈ *backflows* (*insert e* (*set accu*)) **by** *blast*
from *dat-offends-simplified*[*simplified*] *this* **have** *dat-offender* ∈ *E* **by** *simp*
from ‹*dat-offender* ∉ *E*› ‹*dat-offender* ∈ *E*› **show** *False* **by** *simp*
qed

from *this dat-offends* **have**
dat-offender ∉ *backflows* ({*a*} ∪ *filternew-flows-state* (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *set accu* ∪ {*e*}|))
apply(*simp add*: *backflows-def*) **by** *force*

from *dat-in-a this*
show ¬ ⋃(*get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *set* (*a* # *accu*) ∪ {*e*}|)))
⊆ *backflows* (*filternew-flows-state* (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *set* (*a* # *accu*) ∪ {*e*}|))
apply(*subst filternew-flows-state-moveout-a*) **by** *blast*
qed

from *Cons.IH*[*OF Cons.prems*(*1*) *Cons.prems*(*2*) ‹*set Es* ⊆ *E*› ‹*set* (*a* # *accu*) ⊆ *E*› *statefulsimp this*] **show** *?case*
by(*simp*)
next
assume *CaseFalse*: ¬ *?caseDistinction*

from *CaseFalse* **have** *funapplysimp*:
set (*filter-compliant-stateful-ACS-accu* (|*nodes* = *V*, *edges* = *E*|) *M accu* (*a* # *Es*)) =
set (*filter-compliant-stateful-ACS-accu* (|*nodes* = *V*, *edges* = *E*|) *M accu Es*)
by *auto*
from *this Cons.prems*(*5*) **have** *statefulsimp*:
stateful = *set* (*filter-compliant-stateful-ACS-accu* (|*nodes* = *V*, *edges* = *E*|) *M accu Es*) **by** *simp*
from *Cons.prems*(*4*) **have** *set accu* ⊆ *E* **.**

have *a* ∈ *E* − (*set Es* ∪ *set accu* ∪ {*e* ∈ *E*. *e* ∈ *backflows E*}) ⟹ ¬ ⋃(*get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *set accu* ∪ {*a*}|)))
⊆ *backflows* (*filternew-flows-state* (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *set accu* ∪ {*a*}|))
proof(*rule ccontr*)
assume *h1*: *a* ∈ *E* − (*set Es* ∪ *set accu* ∪ {*e* ∈ *E*. *e* ∈ *backflows E*})
and ¬ ¬⋃(*get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *set accu* ∪ {*a*}|))) ⊆ *backflows* (*filternew-flows-state* (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *set accu* ∪ {*a*}|))
hence *hccontr*: ⋃(*get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *set accu* ∪ {*a*}|))) ⊆ *backflows* (*filternew-flows-state* (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *set accu* ∪ {*a*}|)) **by** *simp*

moreover from *h1* **have** *stateful-to-graph*: *stateful-policy-to-network-graph* (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *set accu* ∪ {*a*}|) = (|*nodes* = *V*, *edges* = *E* ∪ *set accu* ∪ *backflows* (*insert a* (*set accu*))|)
by(*simp add*: *stateful-policy-to-network-graph-def all-flows-def*, *blast*)
moreover have *backflows* (*filternew-flows-state* (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* =

set accu ∪ {*a*}⦈) = *backflows* (*insert a* (*set accu*)) − *E*
by(*simp add*: *filternew-flows-state-alt backflows-minus-backflows*)
ultimately have *hccontr-simp*:
⋃(*get-offending-flows* (*get-ACS M*) (⦇*nodes* = *V*, *edges* = *E* ∪ *set accu* ∪ *backflows* (*insert a* (*set accu*))⦈) ⊆ *backflows* (*insert a* (*set accu*)) − *E* **by** *simp*

from *Cons.prems*(*3*) *Cons.prems*(*4*) **have** *backaaccusubE*: *backflows* (*set* (*a* # *accu*)) ⊆ *backflows E* **by**(*simp add*: *backflows-def*, *fastforce*)
from *h1* **have** *a* ∉ *backflows E* **by** *fastforce*
from *backaaccusubE* ‹*a* ∉ *backflows E*› **have** *a* ∉ *backflows* (*insert a* (*set accu*)) **by** *auto*

from ‹*a* ∉ *backflows E*› *CaseFalse* **have** ¬ (∀ *F*∈*get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* (⦇*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *set* (*a* # *accu*)⦈). *F* ⊆ *backflows* (*set* (*a* # *accu*))) **by**(*simp*)
from *this stateful-to-graph* **have** ¬ (∀ *F*∈*get-offending-flows* (*get-ACS M*) (⦇*nodes* = *V*, *edges* = *E* ∪ *set accu* ∪ *backflows* (*insert a* (*set accu*))⦈). *F* ⊆ *backflows* (*insert a* (*set accu*))) **by**(*simp*)
from *this hccontr-simp* **show** *False* **by** *blast*
qed
from *Cons.prems*(*6*)[*simplified funapplysimp statefulsimp*] *this*
have ∀ *e*∈*E* − (*set Es* ∪ *set accu* ∪ {*e* ∈ *E*. *e* ∈ *backflows E*}).
¬ ⋃(*get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* (⦇*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *set accu* ∪ {*e*}⦈)))
⊆ *backflows* (*filternew-flows-state* (⦇*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *set accu* ∪ {*e*}⦈)) **by** *auto*

from *Cons.IH*[*OF Cons.prems*(*1*) *Cons.prems*(*2*) ‹*set Es* ⊆ *E*› ‹*set accu* ⊆ *E*› *statefulsimp this*]
show *?case* **by** *simp*
qed
qed

lemma *filter-compliant-stateful-ACS-maximal*: ⟦ *valid-reqs* (*get-ACS M*); *wf-graph* (⦇ *nodes* = *V*, *edges* = *E* ⦈);
(*set edgesList*) = *E*;
stateful = *set* (*filter-compliant-stateful-ACS* (⦇ *nodes* = *V*, *edges* = *E* ⦈ *M edgesList*)
⟧ ⟹
∀ *e* ∈ *E* − (*stateful* ∪ {*e* ∈ *E*. *e* ∈ *backflows E*}). ~~*E* ∪ (*computed stateful flows plus this*) ∉ *stateful flows*~~
¬ ⋃ (*get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* (⦇ *hosts* = *V*, *flows-fix* = *E*, *flows-state* = *stateful* ∪ {*e*} ⦈)))
⊆ *backflows* (*filternew-flows-state* (⦇ *hosts* = *V*, *flows-fix* = *E*, *flows-state* = *stateful* ∪ {*e*} ⦈)
apply(*drule*(*1*) *filter-compliant-stateful-ACS-accu-induction-maximal*[**where** *accu*=[], *simplified*])
apply(*blast*)
apply(*simp add*: *filter-compliant-stateful-ACS-def*)
apply(*simp*)

apply *fastforce*
apply(*simp add*: *filter-compliant-stateful-ACS-def*)
done

lemma *filter-compliant-stateful-ACS-maximal-allsubsets*:
assumes *a1*: *valid-reqs* (*get-ACS M*) **and** *a2*: *wf-graph* (| *nodes* = *V*, *edges* = *E* |)
and *a3*: (*set edgesList*) = *E*
and *a4*: *stateful* = *set* (*filter-compliant-stateful-ACS* (| *nodes* = *V*, *edges* = *E* |) *M edgesList*)
and *a5*: *X* ⊆ *E* − (*stateful* ∪ *backflows E*) **and** *a6*: *X* ≠ {}
shows
¬ ⋃(*get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* (| *hosts* = *V*, *flows-fix* = *E*, *flows-state* = *stateful* ∪ *X* |)))
⊆ *backflows* (*filternew-flows-state* (| *hosts* = *V*, *flows-fix* = *E*, *flows-state* = *stateful* ∪ *X* |))
proof(*rule ccontr*, *simp*)
from *a5* **have** *X* ⊆ *E* **by** *blast*
assume *accontr*: ⋃(*get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *stateful* ∪ *X*|))) ⊆ *backflows* (*filternew-flows-state* (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *stateful* ∪ *X*|))
hence ⋃(*get-offending-flows* (*get-ACS M*) (|*nodes* = *V*, *edges* = *E* ∪ (*stateful* ∪ *X*) ∪ *backflows* (*stateful* ∪ *X*)|)) ⊆ *backflows* (*stateful* ∪ *X*) − *E*
by(*simp add*: *stateful-policy-to-network-graph-def all-flows-def filternew-flows-state-alt backflows-minus-backflows*)
hence ⋃(*get-offending-flows* (*get-ACS M*) (|*nodes* = *V*, *edges* = *E* ∪ *X* ∪ *backflows* (*stateful* ∪ *X*)|)) ⊆ *backflows* (*stateful* ∪ *X*) − *E*
using *a4 a3 filter-compliant-stateful-ACS-subseteq-input* **by** (*metis Diff-subset-conv Un-Diff-cancel Un-assoc a3 bot.extremum-unique sup-bot-right*)
hence *accontr-simp*: ⋃(*get-offending-flows* (*get-ACS M*) (|*nodes* = *V*, *edges* = *E* ∪ (*backflows stateful*) ∪ (*backflows X*)|)) ⊆ *backflows* (*stateful* ∪ *X*) − *E*
using *Set.Un-absorb2*[*OF* ‹*X* ⊆ *E*›] *backflows-un*[*of stateful X*] **by** (*metis Un-assoc*)

from *a2 a5* **have** *finite X* **apply**(*simp add*: *wf-graph-def*) **by** (*metis* (*full-types*) *finite-Diff finite-subset*)
from *a6* **obtain** *x* **where** *x* ∈ *X* **by** *blast*

from ‹*x* ∈ *X*› *a5* **have** *xX-simp1*: (*backflows X*) − (*backflows* (*X* − {*x*}) − *E*) = *backflows* {*x*}
apply(*simp add*: *backflows-def*) **by** *fast*
from *a5* **have** *X* ∩ *stateful* = {} **by** *auto*
from ‹*x* ∈ *X*› *this* **have** *xX-simp2*: (*backflows stateful*) − (*backflows* (*X* − {*x*}) − *E*) = *backflows stateful*
apply(*simp add*: *backflows-def*) **by** *fast*
have *xX-simp3*:*backflows* (*stateful* ∪ *X*) − (*backflows* (*X* − {*x*}) − *E*) = *backflows* (*stateful* ∪ {*x*})
apply(*simp only*: *backflows-un*)
using *xX-simp1 xX-simp2* **by** *blast*

have *xX-simp4*: *backflows* (*stateful* ∪ *X*) − *E* − (*backflows* (*X* − {*x*}) − *E*) = *backflows* (*filternew-flows-state* (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *stateful* ∪ {*x*}|))
apply(*simp add*: *filternew-flows-state-alt backflows-minus-backflows*)
using *xX-simp3* **by** *auto*

have *xX-simp5*: (*E* ∪ *backflows stateful* ∪ *backflows X*) − (*backflows* (*X* − {*x*}) − *E*) = *E* ∪

backflows stateful ∪ *backflows* {*x*}
using *xX-simp3*[*simplified backflows-un*] **by** *blast*

have *Eexpand*: *E* ∪ *stateful* ∪ {*x*} = *E*
using *a4 a3 filter-compliant-stateful-ACS-subseteq-input a5* ‹*x*∈*X*› **by** *blast*

have *backflows* (*stateful* ∪ *X*) − *E* − *backflows* (*X* − {*x*}) = (*backflows* (*stateful* ∪ *X*) − *E*) − *backflows* (*X* − {*x*}) **by** *simp*
from ‹*finite X*› *backflows-finite* **have** *finite*: *finite* (*backflows* (*X* − {*x*}) − *E*) **by** *auto*
from *a2 a4 a3 filter-compliant-stateful-ACS-subseteq-input* **have** *wf-graph* (|*nodes* = *V*, *edges* = *stateful*|) **by** (*metis Diff-partition wf-graph-remove-edges-union*)
from *backflows-wf*[*OF this*] **have** *wf-graph* (|*nodes* = *V*, *edges* = *backflows stateful*|) **.**
from *a2* ‹*X* ⊆ *E*› **have** *wf-graph* (|*nodes* = *V*, *edges* = *X*|) **by** (*metis double-diff dual-order.refl wf-graph-remove-edges*)
from *backflows-wf*[*OF this*] **have** *wf-graph* (|*nodes* = *V*, *edges* = *backflows X*|) **.**
from *this wf-graph-union-edges* ‹*wf-graph* (|*nodes* = *V*, *edges* = *backflows stateful*|)› *a2* **have** *wfG*:
wf-graph (|*nodes* = *V*, *edges* = *E* ∪ *backflows stateful* ∪ *backflows X*|) **by** *metis*

from ‹*x*∈*X*› **have** *subset*: *backflows* (*X* − {*x*}) − *E* ⊆ *E* ∪ *backflows stateful* ∪ *backflows X* **apply**(*simp add*: *backflows-def*) **by** *fast*

from *Un-set-offending-flows-bound-minus-subseteq'*[*OF a1 wfG subset accontr-simp*] **have**
⋃(*get-offending-flows* (*get-ACS M*) (|*nodes* = *V*, *edges* = (*E* ∪ *backflows stateful* ∪ *backflows X*) − (*backflows* (*X* − {*x*}) − *E*)|)) ⊆ (*backflows* (*stateful* ∪ *X*) − *E*) − (*backflows* (*X* − {*x*}) − *E*) **by** *simp*
from *this xX-simp4 xX-simp5* **have** *trans1*:
⋃(*get-offending-flows* (*get-ACS M*) (|*nodes* = *V*, *edges* = *E* ∪ *backflows stateful* ∪ *backflows* {*x*}|)) ⊆ *backflows* (*filternew-flows-state* (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *stateful* ∪ {*x*}|)) **by** *simp*

hence ⋃(*get-offending-flows* (*get-ACS M*) (|*nodes* = *V*, *edges* = *E* ∪ *backflows* (*stateful* ∪ {*x*})|)) ⊆ *backflows* (*filternew-flows-state* (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *stateful* ∪ {*x*}|))
apply(*simp only*: *backflows-un*) **by** (*metis Un-assoc*)
hence *contr1*: ⋃(*get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *stateful* ∪ {*x*}|))) ⊆ *backflows* (*filternew-flows-state* (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *stateful* ∪ {*x*}|))
apply(*simp only*: *stateful-policy-to-network-graph-def all-flows-def stateful-policy.select-convs*)
using *Eexpand* **by** (*metis Un-assoc*)

from *filter-compliant-stateful-ACS-maximal*[*OF a1 a2 a3 a4*] **have**
∀ *e*∈*E* − (*stateful* ∪ {*e* ∈ *E*. *e* ∈ *backflows E*}). ¬ ⋃(*get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *stateful* ∪ {*e*}|))) ⊆ *backflows* (*filternew-flows-state* (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *stateful* ∪ {*e*}|)) **.**
from *this a5* ‹*x* ∈ *X*› **have** *contr2*: ¬ ⋃(*get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *stateful* ∪ {*x*}|))) ⊆ *backflows* (*filternew-flows-state* (|*hosts* = *V*, *flows-fix* = *E*, *flows-state* = *stateful* ∪ {*x*}|)) **by** *blast*

from *contr1 contr2*
show *False* **by** *simp*
qed

filter-compliant-stateful-ACS is correct and maximal

thm *filter-compliant-stateful-ACS-correct filter-compliant-stateful-ACS-maximal*

Getting those together. We cannot say *edgesList* = *E* here because one filters first. I guess filtering ACS first is easier, ...

definition *generate-valid-stateful-policy-IFSACS* :: *'v*::*vertex graph* ⇒ *'v SecurityInvariant-configured list* ⇒ (*'v* × *'v*) *list* ⇒ *'v stateful-policy* **where**
generate-valid-stateful-policy-IFSACS G M edgesList ≡ (*let filterIFS* = *filter-IFS-no-violations G M edgesList in*
(*let filterACS* = *filter-compliant-stateful-ACS G M filterIFS in* (| *hosts* = *nodes G*, *flows-fix* = *edges G*, *flows-state* = *set filterACS* |)))

lemma *generate-valid-stateful-policy-IFSACS-wf-stateful-policy*: **assumes** *wfG*: *wf-graph G*
and *edgesList*: (*set edgesList*) = *edges G*
shows *wf-stateful-policy* (*generate-valid-stateful-policy-IFSACS G M edgesList*)
proof −
from *wfG* **show** *?thesis*
apply(*simp add*: *generate-valid-stateful-policy-IFSACS-def wf-stateful-policy-def*)
apply(*auto simp add*: *wf-graph-def*)
using *edgesList filter-IFS-no-violations-subseteq-input filter-compliant-stateful-ACS-subseteq-input* **by** (*metis rev-subsetD*)
qed

lemma *generate-valid-stateful-policy-IFSACS-select-simps*:
shows *hosts* (*generate-valid-stateful-policy-IFSACS G M edgesList*) = *nodes G*
and *flows-fix* (*generate-valid-stateful-policy-IFSACS G M edgesList*) = *edges G*
and *flows-state* (*generate-valid-stateful-policy-IFSACS G M edgesList*) ⊆ *set edgesList*
proof −
show *hosts* (*generate-valid-stateful-policy-IFSACS G M edgesList*) = *nodes G*
by(*simp add*: *generate-valid-stateful-policy-IFSACS-def*)
show *flows-fix* (*generate-valid-stateful-policy-IFSACS G M edgesList*) = *edges G*
by(*simp add*: *generate-valid-stateful-policy-IFSACS-def*)
show *flows-state* (*generate-valid-stateful-policy-IFSACS G M edgesList*) ⊆ *set edgesList*
apply(*simp add*: *generate-valid-stateful-policy-IFSACS-def*)
using *filter-IFS-no-violations-subseteq-input filter-compliant-stateful-ACS-subseteq-input* **by** (*metis subset-trans*)
qed

lemma *generate-valid-stateful-policy-IFSACS-all-security-requirements-fulfilled-IFS*: **assumes** *validReqs*: *valid-reqs M*
and *wfG*: *wf-graph G*
and *high-level-policy-valid*: *all-security-requirements-fulfilled M G*
and *edgesList*: (*set edgesList*) ⊆ *edges G*
shows *all-security-requirements-fulfilled* (*get-IFS M*) (*stateful-policy-to-network-graph* (*generate-valid-stateful-polic*
G M edgesList))
proof −
have *simp3*: *flows-state* (*generate-valid-stateful-policy-IFSACS G M edgesList*) ⊆ *edges G* **using** *generate-valid-stateful-policy-IFSACS-select-simps*(*3*) *edgesList* **by** *fast*

have *set* (*filter-compliant-stateful-ACS G M* (*filter-IFS-no-violations G M edgesList*)) ⊆ *set* (*filter-IFS-no-violations G M edgesList*)
using *filter-compliant-stateful-ACS-subseteq-input edgesList* **by** (*metis*)
from *backflows-subseteq this* **have**
backflows (*set* (*filter-compliant-stateful-ACS G M* (*filter-IFS-no-violations G M edgesList*))) ⊆ *backflows* (*set* (*filter-IFS-no-violations G M edgesList*)) **by** *metis*

hence *subseteqhlp1*:
edges G ∪ backflows (set (filter-compliant-stateful-ACS G M (filter-IFS-no-violations G M edges-List))) ⊆ edges G ∪ backflows (set (filter-IFS-no-violations G M edgesList)) **by** *blast*

from *high-level-policy-valid* **have** *all-security-requirements-fulfilled (get-IFS M) G* **by**(*simp add*: *all-security-requirements-fulfilled-def get-IFS-def*)
from *filter-IFS-no-violations-correct*[*OF valid-reqs-IFS-D*[*OF validReqs*] *wfG this edgesList*] **have**
all-security-requirements-fulfilled (get-IFS M) (stateful-policy-to-network-graph (|hosts = nodes G, flows-fix = edges G, flows-state = set (filter-IFS-no-violations G M edgesList)|)) .
from *this edgesList* **have** *goalIFS*:
all-security-requirements-fulfilled (get-IFS M) (|nodes = nodes G, edges = edges G ∪ backflows (set (filter-IFS-no-violations G M edgesList))|)
apply(*simp add*: *stateful-policy-to-network-graph-def all-flows-def*)
by (*metis Un-absorb2 filter-IFS-no-violations-subseteq-input order-trans*)

from *wfG filter-IFS-no-violations-subseteq-input*[**where** *Es=edgesList* **and** *G=G* **and** *M=M*] *edgesList* **have**
wf-graph (|nodes = nodes G, edges = set (filter-IFS-no-violations G M edgesList)|)
apply(*case-tac G*, *simp*)
by (*metis le-iff-sup wf-graph-remove-edges-union*)
from *backflows-wf*[*OF this*] **have**
wf-graph (|nodes = nodes G, edges = backflows (set (filter-IFS-no-violations G M edgesList))|) **by**(*simp*)
from *this wf-graph-union-edges wfG* **have**
wf-graph (|nodes = nodes G, edges = edges G ∪ backflows (set (filter-IFS-no-violations G M edgesList))|)
by (*metis graph.cases graph.select-convs*(*1*) *graph.select-convs*(*2*))

from *all-security-requirements-fulfilled-mono*[*OF valid-reqs-IFS-D*[*OF validReqs*] *subseteqhlp1 this goalIFS*]
have *all-security-requirements-fulfilled (get-IFS M) (|nodes = nodes G, edges = edges G ∪ backflows (set (filter-compliant-stateful-ACS G M (filter-IFS-no-violations G M edgesList)))|)*
.
thus *?thesis*
apply(*simp add*: *stateful-policy-to-network-graph-def all-flows-def generate-valid-stateful-policy-IFSACS-select-simp simp3 Un-absorb2*)
by(*simp add*: *generate-valid-stateful-policy-IFSACS-def*)
qed

theorem *generate-valid-stateful-policy-IFSACS-stateful-policy-compliance*:
assumes *validReqs*: *valid-reqs M*
and *wfG*: *wf-graph G*
and *high-level-policy-valid*: *all-security-requirements-fulfilled M G*
and *edgesList*: (*set edgesList*) = *edges G*
and *Tau*: $\mathcal{T}$ = *generate-valid-stateful-policy-IFSACS G M edgesList*
shows *stateful-policy-compliance* $\mathcal{T}$ *G M*
proof −
have *1*: *wf-stateful-policy* $\mathcal{T}$
apply(*simp add*: *Tau*)
by(*simp add*: *generate-valid-stateful-policy-IFSACS-wf-stateful-policy*[*OF wfG edgesList*])
have *2*: *wf-stateful-policy* (*generate-valid-stateful-policy-IFSACS G M edgesList*)
by(*simp add*: *generate-valid-stateful-policy-IFSACS-wf-stateful-policy*[*OF wfG edgesList*])
have *3*: *hosts* $\mathcal{T}$ = *nodes G*

apply(*simp add*: *Tau*)
by(*simp add*: *generate-valid-stateful-policy-IFSACS-select-simps*(*1*))
have *4*: *flows-fix* $\mathcal{T} \subseteq$ *edges G*
apply(*simp add*: *Tau*)
by(*simp add*: *generate-valid-stateful-policy-IFSACS-select-simps*(*2*))
have *5*: *all-security-requirements-fulfilled* (*get-IFS M*) (*stateful-policy-to-network-graph* $\mathcal{T}$)
apply(*simp add*: *Tau*)
using *generate-valid-stateful-policy-IFSACS-all-security-requirements-fulfilled-IFS*[*OF validReqs wfG high-level-policy-valid*] *edgesList* **by** *blast*
have *6*: $\forall$ *F*$\in$*get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* $\mathcal{T}$). *F* $\subseteq$ *backflows* (*filternew-flows-state* $\mathcal{T}$)
using *filter-compliant-stateful-ACS-correct*[*OF valid-reqs-ACS-D*[*OF validReqs*] *wfG - - Tau*[*simplified generate-valid-stateful-policy-IFSACS-def Let-def*]] *all-security-requirements-fulfilled-ACS-D*[*OF high-level-policy-valid*] *edgesList filter-IFS-no-violations-subseteq-input* **by** *metis*

from *1 2 3 4 5 6 validReqs high-level-policy-valid wfG*
show *?thesis*
unfolding *stateful-policy-compliance-def* **by** *simp*
qed

definition *generate-valid-stateful-policy-IFSACS-2* :: *'v*::*vertex graph* $\Rightarrow$ *'v SecurityInvariant-configured list* $\Rightarrow$ (*'v* $\times$ *'v*) *list* $\Rightarrow$ *'v stateful-policy* **where**
generate-valid-stateful-policy-IFSACS-2 G M edgesList $\equiv$
$(\!|$ *hosts* = *nodes G*, *flows-fix* = *edges G*, *flows-state* = *set* (*filter-IFS-no-violations G M edgesList*) $\cap$ *set* (*filter-compliant-stateful-ACS G M edgesList*) $|\!)$

lemma *generate-valid-stateful-policy-IFSACS-2-wf-stateful-policy*: **assumes** *wfG*: *wf-graph G*
and *edgesList*: (*set edgesList*) = *edges G*
shows *wf-stateful-policy* (*generate-valid-stateful-policy-IFSACS-2 G M edgesList*)
proof −
from *wfG* **show** *?thesis*
apply(*simp add*: *generate-valid-stateful-policy-IFSACS-2-def wf-stateful-policy-def*)
apply(*auto simp add*: *wf-graph-def*)
using *edgesList filter-IFS-no-violations-subseteq-input* **by** (*metis rev-subsetD*)
qed

lemma *generate-valid-stateful-policy-IFSACS-2-select-simps*:
shows *hosts* (*generate-valid-stateful-policy-IFSACS-2 G M edgesList*) = *nodes G*
and *flows-fix* (*generate-valid-stateful-policy-IFSACS-2 G M edgesList*) = *edges G*
and *flows-state* (*generate-valid-stateful-policy-IFSACS-2 G M edgesList*) $\subseteq$ *set edgesList*
proof −
show *hosts* (*generate-valid-stateful-policy-IFSACS-2 G M edgesList*) = *nodes G*
by(*simp add*: *generate-valid-stateful-policy-IFSACS-2-def*)
show *flows-fix* (*generate-valid-stateful-policy-IFSACS-2 G M edgesList*) = *edges G*
by(*simp add*: *generate-valid-stateful-policy-IFSACS-2-def*)
show *flows-state* (*generate-valid-stateful-policy-IFSACS-2 G M edgesList*) $\subseteq$ *set edgesList*
apply(*simp add*: *generate-valid-stateful-policy-IFSACS-2-def*)
using *filter-compliant-stateful-ACS-subseteq-input* **by** (*metis inf.coboundedI2*)
qed

lemma *generate-valid-stateful-policy-IFSACS-2-all-security-requirements-fulfilled-IFS*: **assumes** *validReqs*: *valid-reqs M*
and *wfG*: *wf-graph G*
and *high-level-policy-valid*: *all-security-requirements-fulfilled M G*
and *edgesList*: (*set edgesList*) ⊆ *edges G*
shows *all-security-requirements-fulfilled* (*get-IFS M*) (*stateful-policy-to-network-graph* (*generate-valid-stateful-polic*
G M edgesList))
proof −
have *subseteq*: *set* (*filter-IFS-no-violations G M edgesList*) ∩ *set* (*filter-compliant-stateful-ACS G*
M edgesList) ⊆ *set* (*filter-IFS-no-violations G M edgesList*) **by** *blast*

from *wfG filter-IFS-no-violations-subseteq-input edgesList*
have *wfG'*: *wf-graph* (|*nodes* = *nodes G*, *edges* = *edges G* ∪ *set* (*filter-IFS-no-violations G M*
edgesList)|)
by (*metis graph-eq-intro Un-absorb2 graph.select-convs*(*1*) *graph.select-convs*(*2*) *order.trans*)

from *high-level-policy-valid* **have** *all-security-requirements-fulfilled* (*get-IFS M*) *G* **by**(*simp add*:
all-security-requirements-fulfilled-def get-IFS-def)
from *filter-IFS-no-violations-correct*[*OF valid-reqs-IFS-D*[*OF validReqs*] *wfG this edgesList*] **have**
all-security-requirements-fulfilled (*get-IFS M*) (*stateful-policy-to-network-graph* (|*hosts* = *nodes G*,
flows-fix = *edges G*, *flows-state* = *set* (*filter-IFS-no-violations G M edgesList*)|)) **.**

from *all-security-requirements-fulfilled-mono-stateful-policy-to-network-graph*[*OF valid-reqs-IFS-D*[*OF*
validReqs] *subseteq wfG' this*]
have *all-security-requirements-fulfilled* (*get-IFS M*) (*stateful-policy-to-network-graph* (*generate-valid-stateful-policy-IF*
G M edgesList))
by(*simp add*: *generate-valid-stateful-policy-IFSACS-2-def*)
thus *?thesis* **.**
qed

lemma *generate-valid-stateful-policy-IFSACS-2-filter-compliant-stateful-ACS*:
assumes *validReqs*: *valid-reqs M*
and *wfG*: *wf-graph G*
and *high-level-policy-valid*: *all-security-requirements-fulfilled M G*
and *edgesList*: (*set edgesList*) ⊆ *edges G*
and *Tau*: $\mathcal{T}$ = *generate-valid-stateful-policy-IFSACS-2 G M edgesList*
shows ∀ *F*∈*get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* $\mathcal{T}$). *F* ⊆ *backflows*
(*filternew-flows-state* $\mathcal{T}$)
proof−
let *?filterACS* = *set* (*filter-compliant-stateful-ACS G M edgesList*)
let *?filterIFS* = *set* (*filter-IFS-no-violations G M edgesList*)
from *all-security-requirements-fulfilled-ACS-D*[*OF high-level-policy-valid*] **have** *all-security-requirements-fulfilled*
(*get-ACS M*) *G* **.**

from *filter-compliant-stateful-ACS-correct*[*OF valid-reqs-ACS-D*[*OF validReqs*] *wfG edgesList this*]
have
∀ *F*∈*get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* (|*hosts* = *nodes G*,
flows-fix = *edges G*, *flows-state* = *?filterACS*|)).
F ⊆ *backflows* (*?filterACS*) − *edges G*
apply(*simp*)
apply(*simp add*: *backflows-minus-backflows*[*symmetric*])

by(*simp add*: *filternew-flows-state-alt*)
hence $\forall F{\in}$*get-offending-flows* (*get-ACS M*) (|*nodes* = *nodes G*, *edges* = *edges G* ∪ *backflows* (*?filterACS*)|). *F* ⊆ *backflows* (*?filterACS*) − *edges G*
apply(*simp add*: *stateful-policy-to-network-graph-def all-flows-def*)
using *filter-compliant-stateful-ACS-subseteq-input* **by** (*metis* (*lifting*, *no-types*) *Un-absorb2 edgesList order-trans*)
from *this validReqs* **have** *offending-filterACS-upperbound*:
⋀*m*. *m* ∈ *set* (*get-ACS M*) ⟹
⋃(*c-offending-flows m* (|*nodes* = *nodes G*, *edges* = *edges G* ∪ *backflows* (*?filterACS*)|)) ⊆
backflows (*?filterACS*) − *edges G*
by(*simp add*: *valid-reqs-def get-offending-flows-def*, *blast*)

from *wfG filter-compliant-stateful-ACS-subseteq-input edgesList* **have** *wf-graph* (|*nodes* = *nodes G*, *edges* = *?filterACS*|)
by (*metis graph.cases graph.select-convs*(*1*) *graph.select-convs*(*2*) *le-iff-sup wf-graph-remove-edges-union*)
from *this backflows-wf* **have** *wf-graph* (|*nodes* = *nodes G*, *edges* = *backflows* (*?filterACS*)|) **by** *blast*
moreover have *wf-graph* (|*nodes* = *nodes G*, *edges* = *edges G*|) **using** *wfG* **by**(*case-tac G*, *simp*)
ultimately have *wfG1*: *wf-graph* (|*nodes* = *nodes G*, *edges* = *edges G* ∪ *backflows* (*?filterACS*)|)
using *wf-graph-union-edges* **by** *blast*

from *edgesList* **have** *edgesUnsimp*: *edges G* ∪ (*?filterACS* ∩ *?filterIFS*) = *edges G*
using *filter-IFS-no-violations-subseteq-input filter-compliant-stateful-ACS-subseteq-input* **by** *blast*

— We set up a ?REM that we use in the ⟦*configured-SecurityInvariant ?m*; *wf-graph* (|*nodes* = *?V*, *edges* = *?E*|); ⋃ (*c-offending-flows ?m* (|*nodes* = *?V*, *edges* = *?E*|)) ⊆ *?X*⟧ ⟹ ⋃ (*c-offending-flows ?m* (|*nodes* = *?V*, *edges* = *?E* − *?E'*|)) ⊆ *?X* − *?E'* lemma
let *?REM* = (*backflows* (*?filterACS*) − *backflows* (*?filterIFS*)) − *edges G*

have *REM-gives-desired-upper-bound*: (*backflows* (*?filterACS*) − *edges G*) − *?REM* = *backflows* (*?filterACS* ∩ *?filterIFS*) − *edges G*
by(*simp add*: *backflows-def*, *blast*)

have *REM-gives-desired-edges*: (*edges G* ∪ *backflows* (*?filterACS*)) − *?REM* = *edges G* ∪ (*backflows* (*?filterACS* ∩ *?filterIFS*))
by(*simp add*: *backflows-def*, *blast*)

from *wfG* **have** *finite* (*edges G*) **using** *wf-graph-def* **by** *blast*
hence *finite* (*backflows ?filterACS*) **using** *backflows-finite* **by** (*metis List.finite-set*)
hence *finite1*: *finite* (*backflows* (*?filterACS*) − *backflows* (*?filterIFS*) − *edges G*) **by** *fast*

from *configured-SecurityInvariant.Un-set-offending-flows-bound-minus-subseteq*[**where** *E'*=*?REM* **and** *X*=(*backflows* (*?filterACS*) − *edges G*),
OF - wfG1 offending-filterACS-upperbound, *simplified REM-gives-desired-upper-bound REM-gives-desired-edges*
] *valid-reqs-ACS-D*[*OF validReqs*, *unfolded valid-reqs-def*]
have ⋀*m*. *m* ∈ *set* (*get-ACS M*) ⟹
∀ *F*∈*c-offending-flows m* (|*nodes* = *nodes G*, *edges* = *edges G* ∪ *backflows* (*?filterACS* ∩ *?filterIFS*)|).
F ⊆ *backflows* (*?filterACS* ∩ *?filterIFS*) − *edges G* **by** *blast*
hence ∀ *F*∈*get-offending-flows* (*get-ACS M*)
(|*nodes* = *nodes G*, *edges* = *edges G* ∪ (*backflows* (*?filterACS* ∩ *?filterIFS*))|). *F* ⊆ *backflows* (*?filterACS* ∩ *?filterIFS*) − *edges G*
using *get-offending-flows-def* **by** *fast*
hence ∀ *F*∈*get-offending-flows* (*get-ACS M*)

(|*nodes* = *nodes G*, *edges* = *edges G* ∪ (*?filterACS* ∩ *?filterIFS*) ∪ (*backflows* (*?filterACS* ∩ *?filterIFS*))|).
F ⊆ *backflows* (*?filterACS* ∩ *?filterIFS*) − *edges G*
by(*simp add*: *edgesUnsimp*)
hence ∀ *F*∈*get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* (|*hosts* = *nodes G*, *flows-fix* = *edges G*, *flows-state* = *?filterACS* ∩ *?filterIFS*|)).
F ⊆ *backflows* (*?filterACS* ∩ *?filterIFS*) − *edges G*
by(*simp add*: *stateful-policy-to-network-graph-def all-flows-def*)

thus *?thesis*
apply(*simp add*: *Tau generate-valid-stateful-policy-IFSACS-2-def*)
apply(*simp add*: *filternew-flows-state-alt backflows-minus-backflows*)
by (*metis inf-commute*)
qed

theorem *generate-valid-stateful-policy-IFSACS-2-stateful-policy-compliance*:
assumes *validReqs*: *valid-reqs M*
and *wfG*: *wf-graph G*
and *high-level-policy-valid*: *all-security-requirements-fulfilled M G*
and *edgesList*: (*set edgesList*) = *edges G*
and *Tau*: $\mathcal{T}$ = *generate-valid-stateful-policy-IFSACS-2 G M edgesList*
shows *stateful-policy-compliance* $\mathcal{T}$ *G M*
proof −
have *1*: *wf-stateful-policy* $\mathcal{T}$
apply(*simp add*: *Tau*)
by(*simp add*: *generate-valid-stateful-policy-IFSACS-2-wf-stateful-policy*[*OF wfG edgesList*])
have *2*: *wf-stateful-policy* (*generate-valid-stateful-policy-IFSACS G M edgesList*)
by(*simp add*: *generate-valid-stateful-policy-IFSACS-wf-stateful-policy*[*OF wfG edgesList*])
have *3*: *hosts* $\mathcal{T}$ = *nodes G*
apply(*simp add*: *Tau*)
by(*simp add*: *generate-valid-stateful-policy-IFSACS-2-select-simps*(*1*))
have *4*: *flows-fix* $\mathcal{T}$ ⊆ *edges G*
apply(*simp add*: *Tau*)
by(*simp add*: *generate-valid-stateful-policy-IFSACS-2-select-simps*(*2*))
have *5*: *all-security-requirements-fulfilled* (*get-IFS M*) (*stateful-policy-to-network-graph* $\mathcal{T}$)
apply(*simp add*: *Tau*)
using *generate-valid-stateful-policy-IFSACS-2-all-security-requirements-fulfilled-IFS*[*OF validReqs wfG high-level-policy-valid*] *edgesList* **by** *blast*
have *6*: ∀ *F*∈*get-offending-flows* (*get-ACS M*) (*stateful-policy-to-network-graph* $\mathcal{T}$). *F* ⊆ *backflows* (*filternew-flows-state* $\mathcal{T}$)
using *generate-valid-stateful-policy-IFSACS-2-filter-compliant-stateful-ACS*[*OF validReqs wfG high-level-policy-valid*]
Tau edgesList **by** *auto*

from *1 2 3 4 5 6 validReqs high-level-policy-valid wfG*
show *?thesis*
unfolding *stateful-policy-compliance-def* **by** *simp*
qed

If there are no IFS requirements and the ACS requirements cause no side effects, effectively,

the graph can be considered as undirected graph!

lemma *generate-valid-stateful-policy-IFSACS-2-noIFS-noACSsideeffects-imp-fullgraph*:
assumes *validReqs*: *valid-reqs M*
and *wfG*: *wf-graph G*
and *high-level-policy-valid*: *all-security-requirements-fulfilled M G*
and *edgesList*: (*set edgesList*) = *edges G*
and *no-ACS-sideeffects*: ∀ *F* ∈ *get-offending-flows* (*get-ACS M*) (⟦*nodes* = *nodes G*, *edges* = *edges G* ∪ *backflows* (*edges G*)⟧). *F* ⊆ (*backflows* (*edges G*)) − (*edges G*)
and *no-IFS*: *get-IFS M* = []
shows *stateful-policy-to-network-graph* (*generate-valid-stateful-policy-IFSACS-2 G M edgesList*) = *undirected G*
proof −
from *filter-IFS-no-violations-accu-no-IFS*[*OF valid-reqs-IFS-D*[*OF validReqs*] *wfG no-IFS*] *edgesList*
have *filter-IFS-no-violations G M edgesList* = *rev edgesList*
by(*simp add*: *filter-IFS-no-violations-def*)
from *this filter-compliant-stateful-ACS-subseteq-input* **have** *flows-state-IFS*: *flows-state* (*generate-valid-stateful-policy-G M edgesList*) = *set* (*filter-compliant-stateful-ACS G M edgesList*)
by(*auto simp add*: *generate-valid-stateful-policy-IFSACS-2-def*)

have *flowsfix*: *flows-fix* (*generate-valid-stateful-policy-IFSACS-2 G M edgesList*) = *edges G* **by**(*simp add*: *generate-valid-stateful-policy-IFSACS-2-def*)

have *hosts*: *hosts* (*generate-valid-stateful-policy-IFSACS-2 G M edgesList*) = *nodes G* **by**(*simp add*: *generate-valid-stateful-policy-IFSACS-2-def*)

from *filter-compliant-stateful-ACS-accu-no-side-effects*[*OF valid-reqs-ACS-D*[*OF validReqs*] *wfG no-ACS-sideeffects*] **have**
filter-compliant-stateful-ACS G M edgesList = *rev* [*e*←*edgesList* . *e* ∉ *backflows* (*edges G*)]
by(*simp add*: *filter-compliant-stateful-ACS-def edgesList*)
hence *filterACS*: *set* (*filter-compliant-stateful-ACS G M edgesList*) = *edges G* − (*backflows* (*edges G*)) **using** *edgesList* **by** *force*

show *?thesis*
apply(*simp add*: *undirected-backflows stateful-policy-to-network-graph-def all-flows-def*)
apply(*simp add*: *hosts filterACS flows-state-IFS flowsfix*)
apply(*simp add*: *backflows-minus-backflows*)
by *fast*
qed
lemma *generate-valid-stateful-policy-IFSACS-noIFS-noACSsideeffects-imp-fullgraph*:
assumes *validReqs*: *valid-reqs M*
and *wfG*: *wf-graph G*
and *high-level-policy-valid*: *all-security-requirements-fulfilled M G*
and *edgesList*: (*set edgesList*) = *edges G*
and *no-ACS-sideeffects*: ∀ *F* ∈ *get-offending-flows* (*get-ACS M*) (⟦*nodes* = *nodes G*, *edges* = *edges G* ∪ *backflows* (*edges G*)⟧). *F* ⊆ (*backflows* (*edges G*)) − (*edges G*)
and *no-IFS*: *get-IFS M* = []
shows *stateful-policy-to-network-graph* (*generate-valid-stateful-policy-IFSACS G M edgesList*) = *undirected G*
proof −
from *filter-IFS-no-violations-accu-no-IFS*[*OF valid-reqs-IFS-D*[*OF validReqs*] *wfG no-IFS*] *edgesList*
have *filter-IFS-no-violations G M edgesList* = *rev edgesList*
by(*simp add*: *filter-IFS-no-violations-def*)
from *this filter-compliant-stateful-ACS-subseteq-input* **have** *flows-state-IFS*: *flows-state* (*generate-valid-stateful-policy-G M edgesList*) = *set* (*filter-compliant-stateful-ACS G M* (*rev edgesList*))

by(*simp add*: *generate-valid-stateful-policy-IFSACS-def*)

have *flowsfix*: *flows-fix* (*generate-valid-stateful-policy-IFSACS G M edgesList*) = *edges G* **by**(*simp add*: *generate-valid-stateful-policy-IFSACS-def*)

have *hosts*: *hosts* (*generate-valid-stateful-policy-IFSACS G M edgesList*) = *nodes G* **by**(*simp add*: *generate-valid-stateful-policy-IFSACS-def*)

from *filter-compliant-stateful-ACS-accu-no-side-effects*[*OF valid-reqs-ACS-D*[*OF validReqs*] *wfG no-ACS-sideeffects*] **have**
filter-compliant-stateful-ACS G M (*rev edgesList*) = [*e*←*edgesList* . *e* ∉ *backflows* (*edges G*)]
apply(*simp add*: *filter-compliant-stateful-ACS-def edgesList*) **by** (*metis rev-filter rev-swap*)
hence *filterACS*: *set* (*filter-compliant-stateful-ACS G M* (*rev edgesList*)) = *edges G* − (*backflows* (*edges G*)) **using** *edgesList* **by** *force*

show *?thesis*
apply(*simp add*: *undirected-backflows stateful-policy-to-network-graph-def all-flows-def*)
apply(*simp add*: *hosts filterACS flows-state-IFS flowsfix*)
apply(*simp add*: *backflows-minus-backflows*)
by *fast*
qed

end
theory *TopoS-Stateful-Policy-impl*
imports *TopoS-Composition-Theory-impl TopoS-Stateful-Policy-Algorithm*
begin

11 Stateful Policy – List Implementaion

record *'v stateful-list-policy* =
hostsL :: *'v list*
flows-fixL :: (*'v* × *'v*) *list*
flows-stateL :: (*'v* × *'v*) *list*

definition *stateful-list-policy-to-list-graph* :: *'v stateful-list-policy* ⇒ *'v list-graph* **where**
stateful-list-policy-to-list-graph $\mathcal{T}$ = (| *nodesL* = *hostsL* $\mathcal{T}$, *edgesL* = (*flows-fixL* $\mathcal{T}$) @ [*e* ← *flows-stateL* $\mathcal{T}$. *e* ∉ *set* (*flows-fixL* $\mathcal{T}$)] @ [*e* ← *backlinks* (*flows-stateL* $\mathcal{T}$). *e* ∉ *set* (*flows-fixL* $\mathcal{T}$)] |)

lemma *stateful-list-policy-to-list-graph-complies*:
list-graph-to-graph (*stateful-list-policy-to-list-graph* (| *hostsL* = *V*, *flows-fixL* = E_f, *flows-stateL* = E_σ |)) =
stateful-policy-to-network-graph (| *hosts* = *set V*, *flows-fix* = *set* E_f, *flows-state* = *set* E_σ |)
by(*simp add*: *stateful-list-policy-to-list-graph-def stateful-policy-to-network-graph-def all-flows-def list-graph-to-graph-def backlinks-correct*, *blast*)

lemma *wf-list-graph-stateful-list-policy-to-list-graph*:
wf-list-graph G ⟹ *distinct E* ⟹ *set E* ⊆ *set* (*edgesL G*) ⟹ *wf-list-graph* (*stateful-list-policy-to-list-graph* (|*hostsL* = *nodesL G*, *flows-fixL* = *edgesL G*, *flows-stateL* = *E*|))
apply(*simp add*: *wf-list-graph-def stateful-list-policy-to-list-graph-def*)
apply(*rule conjI*)

apply(*simp add*: *backlinks-distinct*)
apply(*rule conjI*)
apply(*simp add*: *backlinks-set*)
apply(*blast*)
apply(*rule conjI*)
apply(*simp add*: *backlinks-set*)
apply(*blast*)
apply(*simp add*: *wf-list-graph-axioms-def*)
apply(*rule conjI*)
apply(*simp add*: *backlinks-set*)
apply(*force*)
apply(*simp add*: *backlinks-set*)
apply(*clarsimp*)
apply(*erule disjE*)
apply(*auto*)[*1*]
apply(*erule disjE*)
apply(*auto*)[*1*]
by *force*

11.1 Algorithms

fun *filter-IFS-no-violations-accu* :: *'v list-graph* ⇒ *'v SecurityInvariant list* ⇒ (*'v* × *'v*) *list* ⇒ (*'v* × *'v*) *list* ⇒ (*'v* × *'v*) *list* **where**
filter-IFS-no-violations-accu G M accu [] = *accu* |
filter-IFS-no-violations-accu G M accu (*e*#*Es*) = (*if*
all-security-requirements-fulfilled (*TopoS-Composition-Theory-impl.get-IFS M*) (*stateful-list-policy-to-list-graph* (| *hostsL* = *nodesL G*, *flows-fixL* = *edgesL G*, *flows-stateL* = (*e*#*accu*) |))
then filter-IFS-no-violations-accu G M (*e*#*accu*) *Es*
else filter-IFS-no-violations-accu G M accu Es)
definition *filter-IFS-no-violations* :: *'v list-graph* ⇒ *'v SecurityInvariant list* ⇒ (*'v* × *'v*) *list* **where**
filter-IFS-no-violations G M = *filter-IFS-no-violations-accu G M* [] (*edgesL G*)

lemma *filter-IFS-no-violations-accu-distinct*: ⟦ *distinct* (*Es*@*accu*) ⟧ ⟹ *distinct* (*filter-IFS-no-violations-accu G M accu Es*)
apply(*induction Es arbitrary*: *accu*)
by(*simp-all*)

lemma *filter-IFS-no-violations-accu-complies*:
⟦∀ (*m-impl*, *m-spec*) ∈ *set M*. *SecurityInvariant-complies-formal-def m-impl m-spec*;
wf-list-graph G; *set Es* ⊆ *set* (*edgesL G*); *set accu* ⊆ *set* (*edgesL G*); *distinct* (*Es*@*accu*) ⟧ ⟹
filter-IFS-no-violations-accu G (*get-impl M*) *accu Es* = *TopoS-Stateful-Policy-Algorithm.filter-IFS-no-violations-accu* (*list-graph-to-graph G*) (*get-spec M*) *accu Es*
proof(*induction Es arbitrary*: *accu*)
case *Nil*
thus *?case* **by**(*simp add*: *get-impl-def get-spec-def*)
next
case (*Cons e Es*)
— ⟦*set Es* ⊆ *set* (*edgesL G*); *set ?accu* ⊆ *set* (*edgesL G*); *distinct* (*Es* @ *?accu*)⟧ ⟹ *TopoS-Stateful-Policy-impl.filter-IFS-no-violations-accu G* (*get-impl M*) *?accu Es* = *TopoS-Stateful-Policy-Algorithm.filter-IFS-no-violations-accu* (*list-graph-to-graph G*) (*get-spec M*) *?accu Es*
let *?caseDistinction* = *all-security-requirements-fulfilled* (*TopoS-Composition-Theory-impl.get-IFS* (*get-impl M*)) (*stateful-list-policy-to-list-graph* (| *hostsL* = *nodesL G*, *flows-fixL* = *edgesL G*, *flows-stateL* = (*e*#*accu*) |))

from *get-IFS-get-ACS-select-simps(2)*[*OF Cons.prems(1)*] **have** *get-impl-zip-simp*: (*get-impl* (*zip* (*TopoS-Composition-Theory-impl.get-IFS* (*get-impl M*)) (*TopoS-Composition-Theory.get-IFS* (*get-spec M*)))) = *TopoS-Composition-Theory-impl.get-IFS* (*get-impl M*) **by** *simp*

from *get-IFS-get-ACS-select-simps(3)*[*OF Cons.prems(1)*] **have** *get-spec-zip-simp*: (*get-spec* (*zip* (*TopoS-Composition-Theory-impl.get-IFS* (*get-impl M*)) (*TopoS-Composition-Theory.get-IFS* (*get-spec M*)))) = *TopoS-Composition-Theory.get-IFS* (*get-spec M*) **by** *simp*

from *Cons.prems(3) Cons.prems(4)* **have** *set* (*e # accu*) ⊆ *set* (*edgesL G*) **by** *simp*
from *Cons.prems(4)* **have** *set* (*accu*) ⊆ *set* (*edgesL G*) **by** *simp*
from *Cons.prems(5)* **have** *distinct* (*e # accu*) **by** *simp*
from *Cons.prems(3)* **have** *set Es* ⊆ *set* (*edgesL G*) **by** *simp*
from *Cons.prems(5)* **have** *distinct* (*Es @ accu*) **by** *simp*
from *Cons.prems(5)* **have** *distinct* (*Es @* (*e # accu*)) **by** *simp*

from *Cons.prems(2)* **have** *validLG*: *wf-list-graph* (*stateful-list-policy-to-list-graph* (|*hostsL* = *nodesL G*, *flows-fixL* = *edgesL G*, *flows-stateL* = *e # accu*|))
apply(*rule wf-list-graph-stateful-list-policy-to-list-graph*)
apply(*fact* ‹*distinct* (*e # accu*)›)
apply(*fact* ‹*set* (*e # accu*) ⊆ *set* (*edgesL G*)›)
done

from *get-IFS-get-ACS-select-simps(1)*[*OF Cons.prems(1)*]
have ∀ (*m-impl*, *m-spec*) ∈ *set* (*zip* (*get-IFS* (*get-impl M*)) (*TopoS-Composition-Theory.get-IFS* (*get-spec M*))). *SecurityInvariant-complies-formal-def m-impl m-spec* .
from *all-security-requirements-fulfilled-complies*[*OF this*] **have** *all-security-requirements-fulfilled-eq-rule*:

⋀*G*. *wf-list-graph G* ⟹
TopoS-Composition-Theory-impl.all-security-requirements-fulfilled (*TopoS-Composition-Theory-impl.get-IFS* (*get-impl M*)) *G* =
TopoS-Composition-Theory.all-security-requirements-fulfilled (*TopoS-Composition-Theory.get-IFS* (*get-spec M*)) (*list-graph-to-graph G*)
by(*simp add*: *get-impl-zip-simp get-spec-zip-simp*)

have *case-impl-spec*: *?caseDistinction* ⟷ *TopoS-Composition-Theory.all-security-requirements-fulfilled* (*TopoS-Composition-Theory.get-IFS* (*get-spec M*)) (*stateful-policy-to-network-graph* (| *hosts* = *set* (*nodesL G*), *flows-fix* = *set* (*edgesL G*), *flows-state* = *set* (*e#accu*) |))
apply(*subst all-security-requirements-fulfilled-eq-rule*[*OF validLG*])
by(*simp add*: *stateful-list-policy-to-list-graph-complies*)

show *?case*
proof(*case-tac ?caseDistinction*)
assume *cTrue*: *?caseDistinction*

from *cTrue* **have** *g1*: *TopoS-Stateful-Policy-impl.filter-IFS-no-violations-accu G* (*get-impl M*) *accu* (*e # Es*) = *TopoS-Stateful-Policy-impl.filter-IFS-no-violations-accu G* (*get-impl M*) (*e # accu*) *Es* **by** *simp*

from *cTrue*[*simplified case-impl-spec*] **have** *g2*: *TopoS-Stateful-Policy-Algorithm.filter-IFS-no-violations-accu* (*list-graph-to-graph G*) (*get-spec M*) *accu* (*e # Es*) =
TopoS-Stateful-Policy-Algorithm.filter-IFS-no-violations-accu (*list-graph-to-graph G*) (*get-spec M*) (*e#accu*)*Es*
by(*simp add*: *list-graph-to-graph-def*)

show *?case*
apply(*simp only*: *g1 g2*)
using *Cons.IH*[*OF Cons.prems*(*1*) *Cons.prems*(*2*) ‹*set Es* ⊆ *set* (*edgesL G*)› ‹*set* (*e* # *accu*) ⊆ *set* (*edgesL G*)› ‹*distinct* (*Es* @ (*e* # *accu*))›] **by** *simp*
next
assume *cFalse*: ¬ *?caseDistinction*

from *cFalse* **have** *g1*: *TopoS-Stateful-Policy-impl.filter-IFS-no-violations-accu G* (*get-impl M*) *accu* (*e* # *Es*) = *TopoS-Stateful-Policy-impl.filter-IFS-no-violations-accu G* (*get-impl M*) *accu Es* **by** *simp*

from *cFalse*[*simplified case-impl-spec*] **have** *g2*: *TopoS-Stateful-Policy-Algorithm.filter-IFS-no-violations-accu* (*list-graph-to-graph G*) (*get-spec M*) *accu* (*e* # *Es*) =
TopoS-Stateful-Policy-Algorithm.filter-IFS-no-violations-accu (*list-graph-to-graph G*) (*get-spec M*) *accu Es*
by(*simp add*: *list-graph-to-graph-def*)

show *?case*
apply(*simp only*: *g1 g2*)
using *Cons.IH*[*OF Cons.prems*(*1*) *Cons.prems*(*2*) ‹*set Es* ⊆ *set* (*edgesL G*)› ‹*set accu* ⊆ *set* (*edgesL G*)› ‹*distinct* (*Es* @ *accu*)›] **by** *simp*
qed
qed

lemma *filter-IFS-no-violations-complies*:
⟦ ∀ (*m-impl*, *m-spec*) ∈ *set M*. *SecurityInvariant-complies-formal-def m-impl m-spec*; *wf-list-graph G* ⟧ ⟹
filter-IFS-no-violations G (*get-impl M*) = *TopoS-Stateful-Policy-Algorithm.filter-IFS-no-violations* (*list-graph-to-graph G*) (*get-spec M*) (*edgesL G*)
apply(*unfold filter-IFS-no-violations-def TopoS-Stateful-Policy-Algorithm.filter-IFS-no-violations-def*)

apply(*rule filter-IFS-no-violations-accu-complies*)
apply(*simp-all*)
apply(*simp add*: *wf-list-graph-def*)
done

fun *filter-compliant-stateful-ACS-accu* :: *'v list-graph* ⇒ *'v SecurityInvariant list* ⇒ (*'v* × *'v*) *list* ⇒ (*'v* × *'v*) *list* ⇒ (*'v* × *'v*) *list* **where**
filter-compliant-stateful-ACS-accu G M accu [] = *accu* |
filter-compliant-stateful-ACS-accu G M accu (*e*#*Es*) = (*if*
e ∉ *set* (*backlinks* (*edgesL G*)) ∧ (∀ *F* ∈ *set* (*implc-get-offending-flows* (*get-ACS M*) (*stateful-list-policy-to-list-graph* ⦇ *hostsL* = *nodesL G*, *flows-fixL* = *edgesL G*, *flows-stateL* = (*e*#*accu*) ⦈)). *set F* ⊆ *set* (*backlinks* (*e*#*accu*)))
then filter-compliant-stateful-ACS-accu G M (*e*#*accu*) *Es*
else filter-compliant-stateful-ACS-accu G M accu Es)
definition *filter-compliant-stateful-ACS* :: *'v list-graph* ⇒ *'v SecurityInvariant list* ⇒ (*'v* × *'v*) *list* **where**

filter-compliant-stateful-ACS G M = filter-compliant-stateful-ACS-accu G M [] (edgesL G)

lemma *filter-compliant-stateful-ACS-accu-complies*:
⟦∀ (*m-impl*, *m-spec*) ∈ *set M*. *SecurityInvariant-complies-formal-def m-impl m-spec*;
wf-list-graph G; *set Es* ⊆ *set* (*edgesL G*); *set accu* ⊆ *set* (*edgesL G*); *distinct* (*Es*@*accu*) ⟧ ⟹
filter-compliant-stateful-ACS-accu G (*get-impl M*) *accu Es* = *TopoS-Stateful-Policy-Algorithm.filter-compliant-statefu*
(*list-graph-to-graph G*) (*get-spec M*) *accu Es*
proof(*induction Es arbitrary*: *accu*)
case *Nil*
thus *?case* **by**(*simp add*: *get-impl-def get-spec-def*)
next
case (*Cons e Es*)
— ⟦*set Es* ⊆ *set* (*edgesL G*); *set ?accu* ⊆ *set* (*edgesL G*); *distinct* (*Es* @ *?accu*)⟧ ⟹
TopoS-Stateful-Policy-impl.filter-compliant-stateful-ACS-accu G (*get-impl M*) *?accu Es* = *TopoS-Stateful-Policy-Algorith*
(*list-graph-to-graph G*) (*get-spec M*) *?accu Es*
let *?caseDistinction* = *e* ∉ *set* (*backlinks* (*edgesL G*)) ∧ (∀ *F* ∈ *set* (*implc-get-offending-flows*
(*get-ACS* (*get-impl M*)) (*stateful-list-policy-to-list-graph* (| *hostsL* = *nodesL G*, *flows-fixL* = *edgesL G*,
flows-stateL = (*e*#*accu*) |)). *set F* ⊆ *set* (*backlinks* (*e*#*accu*)))

have *backlinks-simp*: (*e* ∉ *set* (*backlinks* (*edgesL G*))) ⟷ (*e* ∉ *backflows* (*set* (*edgesL G*)))
by(*simp add*: *backlinks-correct*)

have ⋀ *G X*. (∀ *F*∈*set* (*implc-get-offending-flows* (*TopoS-Composition-Theory-impl.get-ACS*
(*get-impl M*)) *G*). *set F* ⊆ *X*) =
(∀ *F*∈*set* ‘ *set* (*implc-get-offending-flows* (*TopoS-Composition-Theory-impl.get-ACS* (*get-impl*
M)) *G*). *F* ⊆ *X*) **by** *blast*
also have ⋀ *G X*. *wf-list-graph G* ⟹ (∀ *F*∈*set* ‘ *set* (*implc-get-offending-flows* (*TopoS-Composition-Theory-impl.g*
(*get-impl M*)) *G*). *F* ⊆ *X*) =
(∀ *F*∈*get-offending-flows* (*TopoS-Composition-Theory.get-ACS* (*get-spec M*)) (*list-graph-to-graph*
G). *F* ⊆ *X*)
using *implc-get-offending-flows-complies*[*OF get-IFS-get-ACS-select-simps*(*4*)[*OF Cons.prems*(*1*)],
simplified get-IFS-get-ACS-select-simps[*OF Cons.prems*(*1*)]] **by** *simp*
finally have *implc-get-offending-flows-simp-rule*: ⋀*G X*. *wf-list-graph G* ⟹
(∀ *F*∈*set* (*implc-get-offending-flows* (*TopoS-Composition-Theory-impl.get-ACS* (*get-impl*
M)) *G*). *set F* ⊆ *X*) = (∀ *F*∈*get-offending-flows* (*TopoS-Composition-Theory.get-ACS* (*get-spec M*))
(*list-graph-to-graph G*). *F* ⊆ *X*) **.**

from *Cons.prems*(*3*) *Cons.prems*(*4*) **have** *set* (*e* # *accu*) ⊆ *set* (*edgesL G*) **by** *simp*
from *Cons.prems*(*4*) **have** *set* (*accu*) ⊆ *set* (*edgesL G*) **by** *simp*
from *Cons.prems*(*5*) **have** *distinct* (*e* # *accu*) **by** *simp*
from *Cons.prems*(*3*) **have** *set Es* ⊆ *set* (*edgesL G*) **by** *simp*
from *Cons.prems*(*5*) **have** *distinct* (*Es* @ *accu*) **by** *simp*
from *Cons.prems*(*5*) **have** *distinct* (*Es* @ (*e* # *accu*)) **by** *simp*
from *Cons.prems*(*2*) **have** *validLG*: *wf-list-graph* (*stateful-list-policy-to-list-graph* (|*hostsL* =
nodesL G, *flows-fixL* = *edgesL G*, *flows-stateL* = *e* # *accu*|))
apply(*rule wf-list-graph-stateful-list-policy-to-list-graph*)
apply(*fact* ‹*distinct* (*e* # *accu*)›)
apply(*fact* ‹*set* (*e* # *accu*) ⊆ *set* (*edgesL G*)›)
done

have *set* (*backlinks* (*e* # *accu*)) = *backflows* (*insert e* (*set accu*))
by(*simp add*: *backlinks-set backflows-def*)

— (∀ F∈*set* (*implc-get-offending-flows* (*TopoS-Composition-Theory-impl.get-ACS* (*get-impl* *M*)) (*stateful-list-policy-to-list-graph* (|*hostsL* = *nodesL G*, *flows-fixL* = *edgesL G*, *flows-stateL* = *e* # *accu*|)). *set F* ⊆ *?X*) = (∀ F∈*get-offending-flows* (*TopoS-Composition-Theory.get-ACS* (*get-spec* *M*)) (*list-graph-to-graph* (*stateful-list-policy-to-list-graph* (|*hostsL* = *nodesL G*, *flows-fixL* = *edgesL G*, *flows-stateL* = *e* # *accu*|)). *F* ⊆ *?X*)

have *case-impl-spec*: *?caseDistinction* ⟷ (
e ∉ *backflows* (*set* (*edgesL G*)) ∧ (∀ *F* ∈ *get-offending-flows* (*TopoS-Composition-Theory.get-ACS* (*get-spec M*)) (*stateful-policy-to-network-graph* (| *hosts* = *set* (*nodesL G*), *flows-fix* = *set* (*edgesL G*), *flows-state* = *set* (*e*#*accu*) |)). *F* ⊆ (*backflows* (*set* (*e*#*accu*)))))

apply(*simp add*: *backlinks-simp*)

apply(*simp add*: *implc-get-offending-flows-simp-rule*[*OF validLG*])

apply(*simp add*: *stateful-list-policy-to-list-graph-complies*)

by(*simp add*: ‹*set* (*backlinks* (*e* # *accu*)) = *backflows* (*insert e* (*set accu*))›)

show *?case*

proof(*case-tac ?caseDistinction*)

assume *cTrue*: *?caseDistinction*

from *cTrue* **have** *g1*: *TopoS-Stateful-Policy-impl.filter-compliant-stateful-ACS-accu G* (*get-impl* *M*) *accu* (*e* # *Es*) = *TopoS-Stateful-Policy-impl.filter-compliant-stateful-ACS-accu G* (*get-impl M*) (*e*#*accu*) *Es* **by** *simp*

from *cTrue*[*simplified case-impl-spec*] **have** *g2*: *TopoS-Stateful-Policy-Algorithm.filter-compliant-stateful-ACS-accu* (*list-graph-to-graph G*) (*get-spec M*) *accu* (*e* # *Es*) =
TopoS-Stateful-Policy-Algorithm.filter-compliant-stateful-ACS-accu (*list-graph-to-graph G*) (*get-spec M*) (*e*#*accu*) *Es*
by(*simp add*: *list-graph-to-graph-def*)

show *?case*

apply(*simp only*: *g1 g2*)

using *Cons.IH*[*OF Cons.prems*(*1*) *Cons.prems*(*2*) ‹*set Es* ⊆ *set* (*edgesL G*)› ‹*set* (*e* # *accu*) ⊆ *set* (*edgesL G*)› ‹*distinct* (*Es* @ (*e* # *accu*))›] **by** *simp*

next

assume *cFalse*: ¬ (*?caseDistinction*)

from *cFalse* **have** *g1*: *TopoS-Stateful-Policy-impl.filter-compliant-stateful-ACS-accu G* (*get-impl* *M*) *accu* (*e* # *Es*) = *TopoS-Stateful-Policy-impl.filter-compliant-stateful-ACS-accu G* (*get-impl M*) *accu Es* **by** *force*

from *cFalse*[*simplified case-impl-spec*] **have** *g2*: *TopoS-Stateful-Policy-Algorithm.filter-compliant-stateful-ACS-accu* (*list-graph-to-graph G*) (*get-spec M*) *accu* (*e* # *Es*) =
TopoS-Stateful-Policy-Algorithm.filter-compliant-stateful-ACS-accu (*list-graph-to-graph G*) (*get-spec M*) *accu Es*
apply(*simp add*: *list-graph-to-graph-def*) **by** *fast*

show *?case*

apply(*simp only*: *g1 g2*)

using *Cons.IH*[*OF Cons.prems*(*1*) *Cons.prems*(*2*) ‹*set Es* ⊆ *set* (*edgesL G*)› ‹*set accu* ⊆ *set* (*edgesL G*)› ‹*distinct* (*Es* @ *accu*)›] **by** *simp*

qed

qed

lemma *filter-compliant-stateful-ACS-cont-complies*:
⟦ ∀ (*m-impl*, *m-spec*) ∈ *set M*. *SecurityInvariant-complies-formal-def m-impl m-spec*; *wf-list-graph G*; *set Es* ⊆ *set* (*edgesL G*); *distinct Es* ⟧ ⟹
filter-compliant-stateful-ACS-accu G (*get-impl M*) [] *Es* = *TopoS-Stateful-Policy-Algorithm.filter-compliant-stateful-A* (*list-graph-to-graph G*) (*get-spec M*) *Es*
apply(*unfold filter-compliant-stateful-ACS-def TopoS-Stateful-Policy-Algorithm.filter-compliant-stateful-ACS-def*)

apply(*rule filter-compliant-stateful-ACS-accu-complies*)
apply(*simp-all*)
done

lemma *filter-compliant-stateful-ACS-complies*:
⟦ ∀ (*m-impl*, *m-spec*) ∈ *set M*. *SecurityInvariant-complies-formal-def m-impl m-spec*; *wf-list-graph G* ⟧ ⟹
filter-compliant-stateful-ACS G (*get-impl M*) = *TopoS-Stateful-Policy-Algorithm.filter-compliant-stateful-ACS* (*list-graph-to-graph G*) (*get-spec M*) (*edgesL G*)
apply(*unfold filter-compliant-stateful-ACS-def TopoS-Stateful-Policy-Algorithm.filter-compliant-stateful-ACS-def*)

apply(*rule filter-compliant-stateful-ACS-accu-complies*)
apply(*simp-all*)
apply(*simp add*: *wf-list-graph-def*)
done

definition *generate-valid-stateful-policy-IFSACS* :: *'v list-graph* ⇒ *'v SecurityInvariant list* ⇒ *'v stateful-list-policy* **where**
generate-valid-stateful-policy-IFSACS G M = (*let filterIFS* = *filter-IFS-no-violations G M in*
(*let filterACS* = *filter-compliant-stateful-ACS-accu G M* [] *filterIFS in* (| *hostsL* = *nodesL G*, *flows-fixL* = *edgesL G*, *flows-stateL* = *filterACS* |)))

fun *inefficient-list-intersect* :: *'a list* ⇒ *'a list* ⇒ *'a list* **where**
inefficient-list-intersect [] *bs* = [] |
inefficient-list-intersect (*a*#*as*) *bs* = (*if a* ∈ *set bs then a*#(*inefficient-list-intersect as bs*) *else inefficient-list-intersect as bs*)
lemma *inefficient-list-intersect-correct*: *set* (*inefficient-list-intersect a b*) = (*set a*) ∩ (*set b*)
apply(*induction a*)
by(*simp-all*)

definition *generate-valid-stateful-policy-IFSACS-2* :: *'v list-graph* ⇒ *'v SecurityInvariant list* ⇒ *'v stateful-list-policy* **where**
generate-valid-stateful-policy-IFSACS-2 G M =
(| *hostsL* = *nodesL G*, *flows-fixL* = *edgesL G*, *flows-stateL* = *inefficient-list-intersect* (*filter-IFS-no-violations G M*) (*filter-compliant-stateful-ACS G M*) |)

lemma *generate-valid-stateful-policy-IFSACS-2-complies*: ⟦∀ (*m-impl*, *m-spec*) ∈ *set M*. *SecurityInvariant-complies-fo m-impl m-spec*;
wf-list-graph G;

valid-reqs (*get-spec M*);
TopoS-Composition-Theory.all-security-requirements-fulfilled (*get-spec M*) (*list-graph-to-graph G*);
$\mathcal{T}$ = (*generate-valid-stateful-policy-IFSACS-2 G* (*get-impl M*))⟧ ⟹
stateful-policy-compliance (⦇*hosts* = *set* (*hostsL* $\mathcal{T}$), *flows-fix* = *set* (*flows-fixL* $\mathcal{T}$), *flows-state* = *set* (*flows-stateL* $\mathcal{T}$) ⦈ (*list-graph-to-graph G*) (*get-spec M*)
apply(*rule-tac edgesList=edgesL G* **in** *generate-valid-stateful-policy-IFSACS-2-stateful-policy-compliance*)
apply(*simp*)
apply (*metis wf-list-graph-def wf-list-graph-iff-wf-graph*)
apply(*simp*)
apply(*simp add*: *list-graph-to-graph-def*)
apply(*simp add*: *TopoS-Stateful-Policy-Algorithm.generate-valid-stateful-policy-IFSACS-2-def TopoS-Stateful-Policy-i*
apply(*simp add*: *list-graph-to-graph-def inefficient-list-intersect-correct*)
apply(*thin-tac* $\mathcal{T}$ = -)
apply(*frule*(*1*) *filter-compliant-stateful-ACS-complies*)
apply(*frule*(*1*) *filter-IFS-no-violations-complies*)
apply(*thin-tac* -)
apply(*thin-tac* -)
apply(*thin-tac* -)
apply(*thin-tac* -)
apply(*simp*)
by (*metis list-graph-to-graph-def*)

end
theory *METASINVAR-SystemBoundary*
imports *SINVAR-BLPtrusted-impl*
SINVAR-SubnetsInGW-impl
../TopoS-Composition-Theory-impl
begin

11.1.1 Meta SecurityInvariant: System Boundaries

datatype *system-components* = *SystemComponent*
| *SystemBoundaryInput*
| *SystemBoundaryOutput*
| *SystemBoundaryInputOutput*

fun *system-components-to-subnets* :: *system-components* ⇒ *subnets* **where**
system-components-to-subnets SystemComponent = *Member* |
system-components-to-subnets SystemBoundaryInput = *InboundGateway* |
system-components-to-subnets SystemBoundaryOutput = *Member* |
system-components-to-subnets SystemBoundaryInputOutput = *InboundGateway*

fun *system-components-to-blp* :: *system-components* ⇒ *SINVAR-BLPtrusted.node-config* **where**
system-components-to-blp SystemComponent = ⦇ *security-level* = *1*, *trusted* = *False* ⦈ |
system-components-to-blp SystemBoundaryInput = ⦇ *security-level* = *1*, *trusted* = *False* ⦈ |
system-components-to-blp SystemBoundaryOutput = ⦇ *security-level* = *0*, *trusted* = *True* ⦈ |
system-components-to-blp SystemBoundaryInputOutput = ⦇ *security-level* = *0*, *trusted* = *True* ⦈

definition *new-meta-system-boundary* :: ('*v*::*vertex* × *system-components*) *list* ⇒ *string* ⇒ ('*v Secu-*

```
rityInvariant) list where
  new-meta-system-boundary C description = [
      new-configured-list-SecurityInvariant SINVAR-LIB-SubnetsInGW (|
          node-properties = map-of (map (λ(v,c). (v, system-components-to-subnets c)) C)
          |) (description @ ''(ACS)'')
      ,
      new-configured-list-SecurityInvariant SINVAR-LIB-BLPtrusted (|
          node-properties = map-of (map (λ(v,c). (v, system-components-to-blp c)) C)
          |) (description @ ''(IFS)'')
      ]


lemma system-components-to-subnets:
      SINVAR-SubnetsInGW.allowed-subnet-flow
        SINVAR-SubnetsInGW.default-node-properties
        (system-components-to-subnets c) ⟷
       c = SystemBoundaryInput ∨ c = SystemBoundaryInputOutput
by(cases c)(simp-all add: SINVAR-SubnetsInGW.default-node-properties-def)

lemma system-components-to-blp:
      (¬ trusted SINVAR-BLPtrusted.default-node-properties ⟶
     security-level (system-components-to-blp c) ≤ security-level SINVAR-BLPtrusted.default-node-properties)
       ⟷
       c = SystemBoundaryOutput ∨ c = SystemBoundaryInputOutput
by(cases c)(simp-all add: SINVAR-BLPtrusted.default-node-properties-def)


lemma all-security-requirements-fulfilled (new-meta-system-boundary C description) G ⟷
      (∀(v1, v2) ∈ set (edgesL G). case ((map-of C) v1, (map-of C) v2)
         of
         — No restrictions outside of the component
           (None, None) ⇒ True

         — no restrictions inside the component
         | (Some c1, Some c2) ⇒ True

         — System Boundaries Input
         | (None, Some SystemBoundaryInputOutput) ⇒ True
         | (None, Some SystemBoundaryInput) ⇒ True

         — System Boundaries Output
         | (Some SystemBoundaryOutput, None) ⇒ True
         | (Some SystemBoundaryInputOutput, None) ⇒ True

         — everything else is prohibited
         | - ⇒ False
      )
apply(simp)
apply(simp add: new-meta-system-boundary-def)
apply(simp add: all-security-requirements-fulfilled-def)
apply(simp add: Let-def)
apply(simp add: SINVAR-LIB-SubnetsInGW-def SINVAR-LIB-BLPtrusted-def)
apply(simp add: SINVAR-SubnetsInGW-impl.NetModel-node-props-def SINVAR-BLPtrusted-impl.NetModel-node-props
apply(rule iffI)
```

```
 apply(clarsimp)
 subgoal for a b
 apply(erule-tac x=(a,b) in ballE)+
  apply(simp-all)
 apply(case-tac map-of C a)
  apply(case-tac map-of C b)
   apply(simp-all)
  apply(simp add: map-of-map)
  apply(simp split: system-components.split)
  apply(simp add: system-components-to-subnets)
  apply blast
 apply(case-tac map-of C b)
  apply(simp add: map-of-map)
  apply(simp split: system-components.split)
  apply(simp add: system-components-to-blp)
  apply blast
 apply(simp add: map-of-map)
 apply(simp split: system-components.split; fail)
 done
apply(intro conjI)
 apply(simp add: map-of-map)
 apply(clarsimp)
 subgoal for a b
 apply(erule-tac x=(a,b) in ballE)+
  apply(simp-all)
 apply(simp split: option.split-asm system-components.split-asm)
    by(simp-all add: SINVAR-SubnetsInGW.default-node-properties-def)
apply(clarsimp)
subgoal for a b
apply(erule-tac x=(a,b) in ballE)+
 apply(simp-all)
apply(simp add: map-of-map)
apply(simp split: option.split-asm system-components.split-asm)
  apply(simp add: SINVAR-BLPtrusted.default-node-properties-def; fail)
 apply(rename-tac x, case-tac x, simp-all)+
done
done


value[code] let nodes = [1,2,3,4,8,9,10];
          sinvars = new-meta-system-boundary
            [(1::int, SystemBoundaryInput),
             (2, SystemComponent),
             (3, SystemBoundaryOutput),
             (4, SystemBoundaryInputOutput)
             ] ''foobar''
      in generate-valid-topology sinvars (|nodesL = nodes, edgesL = List.product nodes nodes |)


end
theory TopoS-Impl
imports TopoS-Library TopoS-Composition-Theory-impl

    Security-Invariants/METASINVAR-SystemBoundary
```

Lib/ML-GraphViz
TopoS-Stateful-Policy-impl
begin

12 ML Visualization Interface

definition *print-offending-flows-debug* ::
'v SecurityInvariant list ⇒ 'v list-graph ⇒ (string × ('v × 'v) list list) list **where**
print-offending-flows-debug M G = map
(λ*m*.
(*implc-description m* @ *''* (*''* @ *implc-type m* @ *''*)*''*
, *implc-offending-flows m G*)
) *M*

ML‹
fun pretty-assoclist ctxt header t = let
val ls : (*term* ∗ *term*) *list* = *t* |> *HOLogic.dest-list* |> *map HOLogic.dest-prod*;
val pretty = *fn t* => *Pretty.string-of* (*Syntax.pretty-term ctxt t*);
in ls
|> *map* (*fn* (*x*, *y*) => ^*pretty x*^: ^*pretty y*)
|> *space-implode* *n*
|> (*fn s* => *header*^*s*)
|> *writeln end*
›

12.1 Utility Functions

fun *rembiflowdups* :: (*'a* × *'a*) *list* ⇒ (*'a* × *'a*) *list* **where**
rembiflowdups [] = [] |
rembiflowdups ((*s*,*r*)#*as*) = (*if* (*s*,*r*) ∈ *set as* ∨ (*r*,*s*) ∈ *set as then rembiflowdups as else* (*s*,*r*)#*rembiflowdups as*)

lemma *rembiflowdups-complete*: ⟦ ∀(*s*,*r*) ∈ *set x*. (*r*,*s*) ∈ *set x* ⟧ ⟹ *set* (*rembiflowdups x*) ∪ *set* (*backlinks* (*rembiflowdups x*)) = *set x*
proof
assume *a*: ∀(*s*,*r*) ∈ *set x*. (*r*,*s*) ∈ *set x*
have *subset1*: *set* (*rembiflowdups x*) ⊆ *set x*
apply(*induction x*)
apply(*simp*)
apply(*clarsimp*)
apply(*simp split*: *if-split-asm*)
by(*blast*)+
have *set-backlinks-simp*: ⋀ *x*. ∀(*s*,*r*) ∈ *set x*. (*r*,*s*) ∈ *set x* ⟹ *set* (*backlinks x*) = *set x*
apply(*simp add*: *backlinks-set*)
apply(*rule*)
by *fast*+
have *subset2*: *set* (*backlinks* (*rembiflowdups x*)) ⊆ *set x*
apply(*subst set-backlinks-simp*[*OF a*, *symmetric*])
by(*simp add*: *backlinks-subset subset1*)

from *subset1 subset2*

show *set* (*rembiflowdups x*) ∪ *set* (*backlinks* (*rembiflowdups x*)) ⊆ *set x* **by** *blast*
next
show *set x* ⊆ *set* (*rembiflowdups x*) ∪ *set* (*backlinks* (*rembiflowdups x*))
apply(*rule*)
apply(*induction x*)
apply(*simp*)
apply(*rename-tac a as e*)
apply(*simp*)
apply(*erule disjE*)
apply(*simp*)
defer
apply *fastforce*
apply(*case-tac a*)
apply(*rename-tac s r*)
apply(*case-tac* (*s*,*r*) ∉ *set as* ∧ (*r*,*s*) ∉ *set as*)
apply(*simp*)
apply(*simp add*: *backlinks-set*)
by *blast*
qed

only for prettyprinting

definition *filter-for-biflows*:: ('*a* × '*a*) *list* ⇒ ('*a* × '*a*) *list* **where**
filter-for-biflows E ≡ [*e* ← *E*. (*snd e*, *fst e*) ∈ *set E*]

definition *filter-for-uniflows*:: ('*a* × '*a*) *list* ⇒ ('*a* × '*a*) *list* **where**
filter-for-uniflows E ≡ [*e* ← *E*. (*snd e*, *fst e*) ∉ *set E*]

lemma *filter-for-biflows-correct*: ∀ (*s*,*r*) ∈ *set* (*filter-for-biflows E*). (*r*,*s*) ∈ *set* (*filter-for-biflows E*)
unfolding *filter-for-biflows-def*
by(*induction E*, *auto*)

lemma *filter-for-biflows-un-filter-for-uniflows*: *set* (*filter-for-biflows E*) ∪ *set* (*filter-for-uniflows E*) = *set E*
apply(*simp add*: *filter-for-biflows-def filter-for-uniflows-def*) **by** *blast*

definition *partition-by-biflows* :: ('*a* × '*a*) *list* ⇒ (('*a* × '*a*) *list* × ('*a* × '*a*) *list*) **where**
partition-by-biflows E ≡ (*rembiflowdups* (*filter-for-biflows E*), *remdups* (*filter-for-uniflows E*))

lemma *partition-by-biflows-correct*: *case partition-by-biflows E of* (*biflows*, *uniflows*) ⇒ *set biflows* ∪ *set* (*backlinks* (*biflows*)) ∪ *set uniflows* = *set E*
apply(*simp add*: *partition-by-biflows-def*)
by(*simp add*: *filter-for-biflows-un-filter-for-uniflows filter-for-biflows-correct rembiflowdups-complete*)

lemma *partition-by-biflows* [(*1*::*int*, *1*::*int*), (*1*,*2*), (*2*, *1*), (*1*,*3*)] = ([(*1*, *1*), (*2*, *1*)], [(*1*, *3*)]) **by** *eval*

ML‹
(∗*apply args to f*. *f ist best supplied using* @{*const-name name-of-function*} ∗)
fun apply-function (*ctxt*: *Proof.context*) (*f*: *string*) (*args*: *term list*) : *term* =
let

```
  val - = writeln (applying ^f^ to  ^ (fold (fn t => fn acc => acc^(Pretty.string-of (Syntax.pretty-term
(Config.put show-types true ctxt) t))^ ) args ));
    (*val t-eval = Code-Evaluation.dynamic-value-strict thy t;*)
    (* $ associates to the left, give f its arguments*)
    val applied-untyped-uneval: term = list-comb (Const (f, dummyT), args);
    val applied-uneval: term = Syntax.check-term ctxt applied-untyped-uneval;
  in
    applied-uneval |> Code-Evaluation.dynamic-value-strict ctxt
  end;


(*ctxt -> edges -> (biflows, uniflows)*)
fun partition-by-biflows ctxt (t: term) : (term * term) =
  apply-function ctxt @{const-name partition-by-biflows} [t] |> HOLogic.dest-prod


local
  fun get-tune-node-format (edges: term) : term -> string -> string =
    if (fastype-of edges) = @{typ (string × string) list}
    then
      tune-string-vertex-format
    else
      Graphviz.default-tune-node-format;

  fun evalutae-term ctxt (edges: term) : term =
    case Code-Evaluation.dynamic-value ctxt edges
      of SOME x => x
      | NONE => raise TERM (could not evaluate, []);
in
  fun visualize-edges ctxt (edges: term) (coloredges: (string * term) list) (graphviz-header: string) =
    let
      val - = writeln(visualize-edges);
      val (biflows, uniflows) = partition-by-biflows ctxt edges;
    in
      Graphviz.visualize-graph-pretty ctxt (get-tune-node-format edges) ([
      (, uniflows),
      (edge [dir=\none\, color=\#000000\], biflows)] @ coloredges) (*dir=none, dir=both*)
       graphviz-header
    end

  (*iterate over the edges in ML, useful for printing them in certain formats*)
  fun iterate-edges-ML ctxt (edges: term) (all: (string*string) -> unit) (bi: (string*string) -> unit)
(uni: (string*string) -> unit): unit =
    let
      val - = writeln(iterate-edges-ML);
      val tune-node-format = (get-tune-node-format edges);
      val node-to-string = Graphviz.node-to-string ctxt tune-node-format;
      val evaluated-edges : term = evalutae-term ctxt edges;
      val (biflows, uniflows) = partition-by-biflows ctxt evaluated-edges;
    in
      let
          fun edge-to-list (es: term) : (term * term) list = es |> HOLogic.dest-list |> map HO-
Logic.dest-prod;
        fun edge-to-string (es: (term * term) list) : (string * string) list =
```

```
      map (fn (v1, v2) => (node-to-string v1, node-to-string v2)) es
    in
      edge-to-list evaluated-edges |> edge-to-string |> map all;
      edge-to-list biflows |> edge-to-string |> map bi;
      edge-to-list uniflows |> edge-to-string |> map uni;
      ()
    end
    handle Subscript => writeln (Subscript Exception in iterate-edges-ML)
  end;

end
›

ML-val‹
local
  val (biflows, uniflows) = partition-by-biflows @{context} @{term [(1::int, 1::int), (1,2), (2, 1),
(1,3)]};
in
  val - = Pretty.writeln (Syntax.pretty-term (Config.put show-types true @{context}) biflows);
  val - = Pretty.writeln (Syntax.pretty-term (Config.put show-types true @{context}) uniflows);
end;

val t = fastype-of @{term [(''x'', 2::nat)]};

›
ML-val‹(*
visualize-edges @{context}  @{term [(1::int, 1::int), (1,2), (2, 1), (1,3)]} []; *)
›



definition internal-get-invariant-types-list:: 'a SecurityInvariant list ⇒ string list where
  internal-get-invariant-types-list M ≡ map implc-type M


definition internal-node-configs :: 'a list-graph ⇒ ('a ⇒ 'b) ⇒ ('a ×'b) list where
  internal-node-configs G config ≡ zip (nodesL G) (map config (nodesL G))

ML ‹
local
  fun get-graphivz-node-desc ctxt (node-config: term): string =
   let
     val (node, config) = HOLogic.dest-prod node-config;
     (*TODO: tune node format? There must be a better way ...*)
     val tune-node-format = if (fastype-of node) = @{typ string}
      then
        tune-string-vertex-format
      else
        Graphviz.default-tune-node-format;
     val node-str = Graphviz.node-to-string ctxt tune-node-format node;
     val config-str = Graphviz.term-to-string-html ctxt config;
   in
    node-str ^ [label=<<TABLE BORDER=\0\ CELLSPACING=\0\><TR><TD><FONT face=\Verdana
Bold\> ^node-str ^</FONT></TD></TR><TR><TD> ^config-str ^</TD></TR></TABLE>>]\n
```

```
  end;
in
 fun generate-graphviz-header ctxt (G: term) (configs: term): string =
   let
     val configlist: term list = apply-function ctxt @{const-name internal-node-configs} [G, configs]
|> HOLogic.dest-list;
   in
     fold (fn c => fn acc => acc^get-graphivz-node-desc ctxt c) configlist
   end;
end;

(* Convenience function. Use whenever possible!
 M: security requirements, list
 G: list-graph*)
fun visualize-graph-header ctxt (M: term) (G: term) (Config: term): unit =
 let
   val wf-list-graph = apply-function ctxt @{const-name wf-list-graph} [G];
   val all-fulfilled = apply-function ctxt @{const-name all-security-requirements-fulfilled} [M, G];
   val edges = apply-function ctxt @{const-name edgesL} [G];
   val invariants = apply-function ctxt @{const-name internal-get-invariant-types-list} [M];
   val - = writeln(Invariants: ^ Pretty.string-of (Syntax.pretty-term ctxt invariants));
   val header = if Config = @{term []} then #header else generate-graphviz-header ctxt G Config;
 in
   if wf-list-graph = @{term False} then
     error (The supplied graph is syntactically invalid. Check wf-list-graph.)
   else if all-fulfilled = @{term False} then
     (let
       val offending = apply-function ctxt @{const-name implc-get-offending-flows} [M, G];
          val offending-flat = apply-function ctxt @{const-name List.remdups} [apply-function ctxt
@{const-name List.concat} [offending]];
       val offending-debug = apply-function ctxt @{const-name print-offending-flows-debug} [M, G];
      in
       writeln(offending flows:);
       Pretty.writeln (Syntax.pretty-term ctxt offending);
       pretty-assoclist ctxt Offending flows per invariant:\n offending-debug;
     visualize-edges ctxt edges [(edge [dir=\arrow\, style=dashed, color=\#FF0000\, constraint=false],
offending-flat)] header;
     () end)
   else if all-fulfilled <> @{term True} then raise ERROR all-fulfilled neither False nor True else (
     writeln(All valid:);
     visualize-edges ctxt edges [] header;
     ())
 end;


fun visualize-graph ctxt (M: term) (G: term): unit = visualize-graph-header ctxt M G @{term []};
›
```

end

13 Network Security Policy Verification

theory *Network-Security-Policy-Verification*

imports
TopoS-Interface
TopoS-Interface-impl
TopoS-Library
TopoS-Composition-Theory
TopoS-Stateful-Policy
TopoS-Composition-Theory-impl
TopoS-Stateful-Policy-Algorithm
TopoS-Stateful-Policy-impl
TopoS-Impl
begin

14 A small Tutorial

We demonstrate usage of the executable theory.

Everything that is indented and starts with 'Interlude:' summarizes the main correctness proofs and can be skipped if only the implementation is concerned

14.1 Policy

The secuity policy is a directed graph.

definition *policy* :: *nat list-graph* **where**
policy ≡ (| *nodesL* = [*1*,*2*,*3*],
edgesL = [(*1*,*2*), (*2*,*2*), (*2*,*3*)] |)

It is syntactically well-formed

lemma *wf-list-graph-policy*: *wf-list-graph policy* **by** *eval*

In contrast, this is not a syntactically well-formed graph.

lemma ¬ *wf-list-graph* (| *nodesL* = [*1*,*2*]::*nat list*, *edgesL* = [(*1*,*2*), (*2*,*2*), (*2*,*3*)] |) **by** *eval*

Our *policy* has three rules.

lemma *length* (*edgesL policy*) = *3* **by** *eval*

14.2 Security Invariants

We construct a security invariant. Node *2* has confidential data

definition *BLP-security-levels* :: *nat* ⇀ *SINVAR-BLPtrusted.node-config***where**
BLP-security-levels ≡ [*2* ↦ (| *security-level* = *1*, *trusted* = *False* |)]

definition *BLP-m*::(*nat SecurityInvariant*) **where**
BLP-m ≡ *new-configured-list-SecurityInvariant SINVAR-LIB-BLPtrusted* (|
node-properties = *BLP-security-levels*
|) *''Two has confidential information''*

Interlude: *BLP-m* is a valid implementation of a SecurityInvariant

definition *BLP-m-spec* :: *nat SecurityInvariant-configured option***where**
BLP-m-spec ≡ *new-configured-SecurityInvariant* (
SINVAR-BLPtrusted.sinvar,

```
    SINVAR-BLPtrusted.default-node-properties,
    SINVAR-BLPtrusted.receiver-violation,
    SecurityInvariant.node-props SINVAR-BLPtrusted.default-node-properties (|
      node-properties = BLP-security-levels
    |))
```

Fist, we need to show that the formal definition obeys all requirements, *new-configured-SecurityInvariant* verifies this. To double check, we manually give the configuration.

```
  lemma BLP-m-spec: assumes nP = (λ v. (case BLP-security-levels v of Some c ⇒ c | None ⇒
SINVAR-BLPtrusted.default-node-properties))
      shows BLP-m-spec = Some (|
            c-sinvar = (λG. SINVAR-BLPtrusted.sinvar G nP),
                  c-offending-flows = (λG. SecurityInvariant-withOffendingFlows.set-offending-flows
SINVAR-BLPtrusted.sinvar G nP),
            c-isIFS = SINVAR-BLPtrusted.receiver-violation
          |) (is BLP-m-spec = Some ?Spec)
  proof −
    have NetModelLib: TopoS-modelLibrary SINVAR-LIB-BLPtrusted SINVAR-BLPtrusted.sinvar
    by(unfold-locales)
    from assms have nP: nP = nm-node-props SINVAR-LIB-BLPtrusted (|
            node-properties = BLP-security-levels
        |) by(simp add: fun-eq-iff SINVAR-LIB-BLPtrusted-def SINVAR-BLPtrusted-impl.NetModel-node-props-def)

    have BLP-m-spec = new-configured-SecurityInvariant (SINVAR-BLPtrusted.sinvar, SINVAR-BLPtrusted.default-noo
SINVAR-BLPtrusted.receiver-violation, nP)
      unfolding BLP-m-spec-def nP by(simp add: SINVAR-BLPtrusted-impl.NetModel-node-props-def
SINVAR-LIB-BLPtrusted-def)
    also with TopoS-modelLibrary-yields-new-configured-SecurityInvariant[OF NetModelLib nP]
    have ... = Some ?Spec by (simp add: SINVAR-LIB-BLPtrusted-def)
    finally show ?thesis by blast
  qed
  lemma valid-reqs-BLP: valid-reqs [the BLP-m-spec]
   by(simp add: valid-reqs-def)(metis BLP-m-spec-def BLPtrusted-impl.spec new-configured-SecurityInvariant.simps
new-configured-SecurityInvariant-sound option.distinct(1) option.exhaust-sel)
```

Interlude: While *BLP-m* is executable code, we will now show that this executable code complies with its formal definition.

```
  lemma complies-BLP: SecurityInvariant-complies-formal-def BLP-m (the BLP-m-spec)
    unfolding BLP-m-def
    apply(rule new-configured-list-SecurityInvariant-complies)
      apply(simp-all add: BLP-m-spec-def)
     apply(unfold-locales)
   by(simp add: fun-eq-iff SINVAR-LIB-BLPtrusted-def SINVAR-BLPtrusted-impl.NetModel-node-props-def)
```

We define the list of all security invariants of type *nat SecurityInvariant list*. The type *nat* is because the policy's nodes are of type *nat*.

```
definition security-invariants = [BLP-m]
```

We can see that the policy does not fulfill the security invariants.

```
lemma ¬ all-security-requirements-fulfilled security-invariants policy by eval
```

We ask why. Obviously, node 2 leaks confidential data to node 3.

```
value implc-get-offending-flows security-invariants policy
```

lemma *implc-get-offending-flows security-invariants policy = [[(2, 3)]]* **by** *eval*

Interlude: the implementation *implc-get-offending-flows* corresponds to the formal definition *get-offending-flows*

lemma *set ' set (implc-get-offending-flows (get-impl [(BLP-m, the BLP-m-spec)]) policy) = get-offending-flows (get-spec [(BLP-m, the BLP-m-spec)]) (list-graph-to-graph policy)*
apply(*rule implc-get-offending-flows-complies*)
by(*simp-all add*: *complies-BLP wf-list-graph-policy*)

Visualization of the violation (only in interactive mode)

ML-val‹
visualize-graph @{*context*} @{*term security-invariants*} @{*term policy*};
›

Experimental: the config (only one) can be added to the end.

ML-val‹
visualize-graph-header @{*context*} @{*term security-invariants*} @{*term policy*} @{*term BLP-security-levels*};
›

The policy has a flaw. We throw it away and generate a new one which fulfills the invariants.

definition *max-policy = generate-valid-topology security-invariants (|nodesL = nodesL policy, edgesL = List.product (nodesL policy) (nodesL policy) |)*

Interlude: the implementation *implc-get-offending-flows* corresponds to the formal definition *get-offending-flows*

thm *generate-valid-topology-complies*

Interlude: the formal definition is sound

thm *generate-valid-topology-sound*

Here, it is also complete

lemma *wf-graph G ⟹ max-topo [the BLP-m-spec] (TopoS-Composition-Theory.generate-valid-topology [the BLP-m-spec] (fully-connected G))*
apply(*rule generate-valid-topology-max-topo*[*OF valid-reqs-BLP*])
apply(*assumption*)
apply(*simp add*: *BLP-m-spec*)
by *blast*

Calculating the maximum policy

value *max-policy*
lemma *max-policy = (|nodesL = [1, 2, 3], edgesL = [(1, 1), (1, 2), (1, 3), (2, 2), (3, 1), (3, 2), (3, 3)]|)* **by** *eval*

Visualizing the maximum policy (only in interactive mode)

ML‹
visualize-graph @{*context*} @{*term security-invariants*} @{*term max-policy*};
›

Of course, all security invariants hold for the maximum policy.

lemma *all-security-requirements-fulfilled security-invariants max-policy* **by** *eval*

14.3 A stateful implementation

We generate a stateful policy

definition *stateful-policy = generate-valid-stateful-policy-IFSACS-2 policy security-invariants*

When thinking about it carefully, no flow can be stateful without introducing an information leakage here!

value *stateful-policy*
lemma *stateful-policy = (⦇hostsL = [1, 2, 3], flows-fixL = [(1, 2), (2, 2), (2, 3)], flows-stateL = []⦈)*
by *eval*

Interlude: the stateful policy we are computing fulfills all the necessary properties

thm *generate-valid-stateful-policy-IFSACS-2-complies*

thm *filter-compliant-stateful-ACS-correct filter-compliant-stateful-ACS-maximal*
thm *filter-IFS-no-violations-correct filter-IFS-no-violations-maximal*

Visualizing the stateful policy (only in interactive mode)

ML-val‹
visualize-edges @{context} @{term flows-fixL stateful-policy}
[(edge [dir=\arrow\, style=dashed, color=\#FF8822\, constraint=false], @{term flows-stateL stateful-policy})] ;
›

This is how it would look like if $(3::'a, 1::'b)$ were a stateful flow

ML-val‹
visualize-edges @{context} @{term flows-fixL stateful-policy}
[(edge [dir=\arrow\, style=dashed, color=\#FF8822\, constraint=false], @{term [(3::nat,1::nat)]})]
;
›

hide-const *policy security-invariants max-policy stateful-policy*

end
theory *Example-BLP*
imports *TopoS-Library*
begin

definition *BLPexample1::bool* **where**
BLPexample1 ≡ (nm-eval SINVAR-LIB-BLPbasic) fabNet (⦇ node-properties = [''PresenceSensor'' ↦ 2,
''Webcam'' ↦ 3,
''SensorSink'' ↦ 3,
''Statistics'' ↦ 3] ⦈)
definition *BLPexample3::(string × string) list list* **where**
BLPexample3 ≡ (nm-offending-flows SINVAR-LIB-BLPbasic) fabNet ((nm-node-props SINVAR-LIB-BLPbasic) sensorProps-NMParams-try3)

value *BLPexample1*
value *BLPexample3*

end
theory *TopoS-generateCode*
imports
TopoS-Library
Example-BLP
begin

setup ‹*fn thy* =>
let
val package = *package tum.in.net.psn.log-topo.SecurityInvariants.GENERATED*;
val date = *Date.toString* (*Date.fromTimeLocal* (*Time.now* ()));
val export-file = *Context.theory-name thy* ^ *.thy*;
val header = *package* ^ *n* ^ *// Generated by Isabelle* (^ *Distribution.version* ^) *on* ^ *date* ^ *n*
^ *// src:* ^ *export-file* ^ *n*;
in
Code-Target.set-printings (*Code-Symbol.Module* (, [(*Scala*, *SOME* (*header*, []))])) *thy*
end
›

export-code
— generic network security invariants
SINVAR-LIB-BLPbasic
SINVAR-LIB-Dependability
SINVAR-LIB-DomainHierarchyNG
SINVAR-LIB-Subnets
SINVAR-LIB-BLPtrusted
SINVAR-LIB-PolEnforcePointExtended
SINVAR-LIB-Sink
SINVAR-LIB-NonInterference
SINVAR-LIB-SubnetsInGW
SINVAR-LIB-CommunicationPartners
— accessors to the packed invariants
nm-eval
nm-node-props
nm-offending-flows
nm-sinvar
nm-default
nm-receiver-violation nm-name
— TopoS Params
node-properties
— Finite Graph functions
FiniteListGraph.wf-list-graph
FiniteListGraph.add-node
FiniteListGraph.delete-node
FiniteListGraph.add-edge
FiniteListGraph.delete-edge
FiniteListGraph.delete-edges
— Examples
BLPexample1 BLPexample3
in *Scala*

end
theory *SINVAR-Examples*
imports
TopoS-Interface
TopoS-Interface-impl
TopoS-Library
TopoS-Composition-Theory
TopoS-Stateful-Policy
TopoS-Composition-Theory-impl
TopoS-Stateful-Policy-Algorithm
TopoS-Stateful-Policy-impl
TopoS-Impl
begin

ML‹
case !*Graphviz.open-viewer of*
OpenImmediately => Graphviz.open-viewer := *AskTimeouted 3.0*
| *AskTimeouted* - => ()
| *DoNothing* => ()
›

definition *make-policy* :: ('*a SecurityInvariant*) *list* ⇒ '*a list* ⇒ '*a list-graph* **where**
make-policy sinvars V ≡ *generate-valid-topology sinvars* (|*nodesL* = *V*, *edgesL* = *List.product V V* |)

context begin
private definition *SINK-m* ≡ *new-configured-list-SecurityInvariant SINVAR-LIB-Sink* (|
node-properties = [''*Bot1*'' ↦ *Sink*,
''*Bot2*'' ↦ *Sink*,
''*MissionControl1*'' ↦ *SinkPool*,
''*MissionControl2*'' ↦ *SinkPool*
]
|) ''*bots and control are infromation sink*''
value[*code*] *make-policy* [*SINK-m*] [''*INET*'', ''*Supervisor*'', ''*Bot1*'', ''*Bot2*'', ''*MissionControl1*'', ''*MissionControl2*'']
ML-val‹
visualize-graph-header @{*context*} @{*term* [*SINK-m*]}
@{*term make-policy* [*SINK-m*] [''*INET*'', ''*Supervisor*'', ''*Bot1*'', ''*Bot2*'', ''*MissionControl1*'', ''*MissionControl2*'']}
@{*term* [''*Bot1*'' ↦ *Sink*,
''*Bot2*'' ↦ *Sink*,
''*MissionControl1*'' ↦ *SinkPool*,
''*MissionControl2*'' ↦ *SinkPool*
]};
›
end

```
context begin
 private definition ACL-m ≡ new-configured-list-SecurityInvariant SINVAR-LIB-CommunicationPartners
(⦇
        node-properties = [''db1'' ↦ Master [''h1'', ''h2''],
                           ''db2'' ↦ Master [''db1''],
                           ''h1'' ↦ Care,
                           ''h2'' ↦ Care
                           ]
        ⦈ ''ACL for databases''
  value[code] make-policy [ACL-m] [''db1'', ''db2'', ''h1'', ''h2'', ''h3'']
  ML-val‹
  visualize-graph-header @{context} @{term [ACL-m]}
    @{term make-policy [ACL-m] [''db1'', ''db2'', ''h1'', ''h2'', ''h3'']}
    @{term [''db1'' ↦ Master [''h1'', ''h2''],
            ''db2'' ↦ Master [''db1''],
            ''h1'' ↦ Care,
            ''h2'' ↦ Care
            ]};
›
end
```

```
definition CommWith-m::(nat SecurityInvariant) where
    CommWith-m ≡ new-configured-list-SecurityInvariant SINVAR-LIB-ACLcommunicateWith (⦇
        node-properties = [
              1 ↦ [2,3],
              2 ↦ [3]]
        ⦈ ''One can only talk to 2,3''
```

Experimental: the config (only one) can be added to the end.

```
ML-val‹
visualize-graph-header @{context} @{term [CommWith-m]}
      @{term (⦇ nodesL = [1::nat, 2, 3],
             edgesL = [(1,2), (2,3)]⦈)} @{term [
               (1::nat) ↦ [2::nat,3],
               2 ↦ [3]]};
›
```

```
value[code] make-policy [CommWith-m] [1,2,3]
value[code] implc-offending-flows CommWith-m (⦇nodesL = [1,2,3,4], edgesL = List.product [1,2,3,4]
[1,2,3,4] ⦈)
value[code] make-policy [CommWith-m] [1,2,3,4]
```

```
ML-val‹
visualize-graph @{context} @{term [ new-configured-list-SecurityInvariant SINVAR-LIB-ACLcommunicateWith
(⦇
        node-properties = [
```

```
        1::nat ↦ [1,2,3],
        2 ↦ [1,2,3,4],
        3 ↦ [1,2,3,4],
        4 ↦ [1,2,3,4]]
    ⦇ ''usefull description here''⦈} @{term (⦇nodesL = [1::nat,2,3,4], edgesL = [(1,2), (1,3), (2,3),
(3, 4)] ⦈)};
›

lemma implc-offending-flows (new-configured-list-SecurityInvariant SINVAR-LIB-ACLcommunicateWith
(⦇
    node-properties = [
        1::nat ↦ [1,2,3],
        2 ↦ [1,2,3,4],
        3 ↦ [1,2,3,4],
        4 ↦ [1,2,3,4]]
    ⦇ ''usefull description here'') (⦇nodesL = [1::nat,2,3,4], edgesL = [(1,2), (1,3), (2,3), (3, 4)]
⦈) =
    [[(1, 2), (1, 3)], [(1, 3), (2, 3)], [(3, 4)]] by eval


context begin
  private definition G-dep :: nat list-graph where
    G-dep ≡ (⦇nodesL = [1::nat,2,3,4,5,6,7], edgesL = [(1,2), (2,1), (2,3),
                                                       (4,5), (5,6), (6,7)] ⦈)
  private lemma wf-list-graph G-dep by eval

  private definition DEP-m ≡ new-configured-list-SecurityInvariant SINVAR-LIB-Dependability (⦇
      node-properties = Some ∘ dependability-fix-nP G-dep (λ-. 0)
      ⦇ ''automatically computed dependability invariant''
  ML-val‹
  visualize-graph-header @{context} @{term [DEP-m]}
    @{term G-dep}
    @{term Some ∘ dependability-fix-nP G-dep (λ-. 0)};
›
```

Connecting $(3::'a, 4::'b)$. This causes only one offedning flow at $(3::'a, 4::'b)$.

```
  ML-val‹
  visualize-graph-header @{context} @{term [DEP-m]}
    @{term G-dep(⦇edgesL := (3,4)#edgesL G-dep⦈)}
    @{term Some ∘ dependability-fix-nP G-dep (λ-. 0)};
›
```

We try to increase the dependability level at $3::'a$. Suddenly, offending flows everywhere.

```
  ML-val‹
  visualize-graph-header @{context} @{term [new-configured-list-SecurityInvariant SINVAR-LIB-Dependability
(⦇
      node-properties = Some ∘ ((dependability-fix-nP G-dep (λ-. 0))(3 := 2))
      ⦇ ''changed deps''⦈}
    @{term G-dep(⦇edgesL := (3,4)#edgesL G-dep⦈)}
    @{term Some ∘ ((dependability-fix-nP G-dep (λ-. 0))(3 := 2))};
›
  lemma implc-offending-flows (new-configured-list-SecurityInvariant SINVAR-LIB-Dependability (⦇
```

```
                node-properties = Some ∘ ((dependability-fix-nP G-dep (λ-. 0))(3 := 2))
                ⦇ ''changed deps'')
        (G-dep⦇edgesL := (3,4)#edgesL G-dep⦈) =
    [[(3, 4)], [(1, 2), (2, 1), (5, 6)], [(1, 2), (4, 5)], [(2, 1), (4, 5)], [(2, 3), (4, 5)], [(2, 3), (5,
6)]]
     by eval
```

If we recompute the dependability levels for the changed graph, we see that suddenly, The level at $1::'a$ and $2::'a$ increased, though we only added the edge $(3::'a, 4::'b)$. This hints that we connected the graph. If an attacker can now compromise $1::'a$, she may be able to peek much deeper into the network.

```
ML-val‹
visualize-graph-header @{context} @{term [new-configured-list-SecurityInvariant SINVAR-LIB-Dependability
⦇
        node-properties = Some ∘ dependability-fix-nP (G-dep⦇edgesL := (3,4)#edgesL G-dep⦈) (λ-.
0)
        ⦈ ''changed deps'']}
    @{term G-dep⦇edgesL := (3,4)#edgesL G-dep⦈}
    @{term Some ∘ dependability-fix-nP (G-dep⦇edgesL := (3,4)#edgesL G-dep⦈) (λ-. 0)};
›
```

Dependability is reflexive, a host can depend on itself.

```
ML-val‹
visualize-graph-header @{context} @{term [new-configured-list-SecurityInvariant SINVAR-LIB-Dependability
⦇
        node-properties = Some ∘ dependability-fix-nP ⦇nodesL = [1::nat], edgesL = [(1,1)] ⦈ (λ-. 0)
        ⦈ ''changed deps'']}
    @{term ⦇nodesL = [1::nat], edgesL = [(1,1)] ⦈}
    @{term Some ∘ dependability-fix-nP ⦇nodesL = [1::nat], edgesL = [(1,1)] ⦈ (λ-. 0)};
›
ML-val‹
visualize-graph-header @{context} @{term [new-configured-list-SecurityInvariant SINVAR-LIB-Dependability-noreflexivity
⦇
        node-properties = (λ-::nat. Some 0)
        ⦈ ''changed deps'']}
    @{term ⦇nodesL = [1::nat], edgesL = [(1,1)] ⦈}
    @{term (λ-::nat. Some (0::nat))};
›

end
```

```
context begin
  private definition G-noninter :: nat list-graph where
    G-noninter ≡ ⦇nodesL = [1::nat,2,3,4], edgesL = [(1,2), (1,3), (2,3), (3, 4)] ⦈
  private lemma wf-list-graph G-noninter by eval

  private definition NonI-m ≡ new-configured-list-SecurityInvariant SINVAR-LIB-NonInterference
⦇
        node-properties = [
                1::nat ↦ Interfering,
                2 ↦ Unrelated,
```

```
            3 ↦ Unrelated,
            4 ↦ Interfering]
        ⦇ ''One and Four interfere''
  ML-val‹
  visualize-graph @{context} @{term [ NonI-m ]} @{term G-noninter};
›

  lemma implc-offending-flows NonI-m G-noninter = [[(1, 2), (1, 3)], [(1, 3), (2, 3)], [(3, 4)]]
        by eval

  ML-val‹
  visualize-graph @{context} @{term [ NonI-m ]} @{term (⦇nodesL = [1::nat,2,3,4], edgesL = [(1,2),
(1,3), (2,3), (4, 3)] ⦈};
›

  lemma implc-offending-flows NonI-m (⦇nodesL = [1::nat,2,3,4], edgesL = [(1,2), (1,3), (2,3), (4,
3)] ⦈) =
    [[(1, 2), (1, 3)], [(1, 3), (2, 3)], [(4, 3)]]
        by eval
```

In comparison, *SINVAR-LIB-ACLcommunicateWith* is less strict. Changing the direction of the edge (*3*::'*a*, *4*::'*b*) removes the access from *1*::'*a* to *4*::'*a* and the invariant holds.

```
  lemma implc-offending-flows (new-configured-list-SecurityInvariant SINVAR-LIB-ACLcommunicateWith
(⦇
        node-properties = [
            1::nat ↦ [1,2,3],
            2 ↦ [1,2,3,4],
            3 ↦ [1,2,3,4],
            4 ↦ [1,2,3,4]]
        ⦈ ''One must not access Four'') (⦇nodesL = [1::nat,2,3,4], edgesL = [(1,2), (1,3), (2,3), (4,
3)] ⦈) = [] by eval
end


context begin
  private definition subnets-host-attributes ≡ [
                        ''v11'' ↦ Subnet 1,
                        ''v12'' ↦ Subnet 1,
                        ''v13'' ↦ Subnet 1,
                        ''v1b'' ↦ BorderRouter 1,
                        ''v21'' ↦ Subnet 2,
                        ''v22'' ↦ Subnet 2,
                        ''v23'' ↦ Subnet 2,
                        ''v2b'' ↦ BorderRouter 2,
                        ''v3b'' ↦ BorderRouter 3
                       ]
  private definition Subnets-m ≡ new-configured-list-SecurityInvariant SINVAR-LIB-Subnets (⦇
        node-properties = subnets-host-attributes
        ⦈ ''Collaborating hosts''
  private definition subnet-hosts ≡ [''v11'', ''v12'', ''v13'', ''v1b'',
                                    ''v21'', ''v22'', ''v23'', ''v2b'',
```

```
                                    ''v3b'', ''vo'']

  private lemma dom (subnets-host-attributes) ⊆ set (subnet-hosts)
    by(simp add: subnet-hosts-def subnets-host-attributes-def)
  value[code] make-policy [Subnets-m] subnet-hosts
  ML-val‹
  visualize-graph-header @{context} @{term [Subnets-m]}
    @{term make-policy [Subnets-m] subnet-hosts}
    @{term subnets-host-attributes};
›
```

Emulating the same but with accessible members with SubnetsInGW and ACLs

```
  private definition SubnetsInGW-ACL-ms ≡ [new-configured-list-SecurityInvariant SINVAR-LIB-SubnetsInGW
(⦇
          node-properties = [''v11'' ↦ Member, ''v12'' ↦ Member, ''v13'' ↦ Member, ''v1b'' ↦
InboundGateway]
        ⦈ ''v1 subnet'',
        new-configured-list-SecurityInvariant SINVAR-LIB-CommunicationPartners (⦇
        node-properties = [''v1b'' ↦ Master [''v11'', ''v12'', ''v13'', ''v2b'', ''v3b''],
                           ''v11'' ↦ Care,
                           ''v12'' ↦ Care,
                           ''v13'' ↦ Care,
                           ''v2b'' ↦ Care,
                           ''v3b'' ↦ Care
                           ]
        ⦈ ''v1b ACL'',
        new-configured-list-SecurityInvariant SINVAR-LIB-SubnetsInGW (⦇
           node-properties = [''v21'' ↦ Member, ''v22'' ↦ Member, ''v23'' ↦ Member, ''v2b'' ↦
InboundGateway]
        ⦈ ''v2 subnet'',
        new-configured-list-SecurityInvariant SINVAR-LIB-CommunicationPartners (⦇
        node-properties = [''v2b'' ↦ Master [''v21'', ''v22'', ''v23'', ''v1b'', ''v3b''],
                           ''v21'' ↦ Care,
                           ''v22'' ↦ Care,
                           ''v23'' ↦ Care,
                           ''v1b'' ↦ Care,
                           ''v3b'' ↦ Care
                           ]
        ⦈ ''v2b ACL'',
        new-configured-list-SecurityInvariant SINVAR-LIB-SubnetsInGW (⦇ node-properties =
[''v3b'' ↦ InboundGateway] ⦈ ''v3b'',
        new-configured-list-SecurityInvariant SINVAR-LIB-CommunicationPartners (⦇
        node-properties = [''v3b'' ↦ Master [''v1b'', ''v2b''],
                           ''v1b'' ↦ Care,
                           ''v2b'' ↦ Care
                           ]
        ⦈ ''v3b ACL'']
  value[code] make-policy SubnetsInGW-ACL-ms subnet-hosts
  lemma set (edgesL (make-policy [Subnets-m] subnet-hosts)) ⊆ set (edgesL (make-policy SubnetsInGW-ACL-ms
subnet-hosts)) by eval
  lemma [e <− edgesL (make-policy SubnetsInGW-ACL-ms subnet-hosts). e ∉ set (edgesL (make-policy
[Subnets-m] subnet-hosts))] =
    [(''v1b'', ''v11''), (''v1b'', ''v12''), (''v1b'', ''v13''), (''v2b'', ''v21''), (''v2b'', ''v22''), (''v2b'', ''v23'')]
by eval
```

```
  ML-val‹
  visualize-graph @{context} @{term SubnetsInGW-ACL-ms}
    @{term make-policy SubnetsInGW-ACL-ms subnet-hosts};
›
end




context begin
  private definition secgwext-host-attributes ≡ [
                              ''hypervisor'' ↦ PolEnforcePoint,
                              ''securevm1'' ↦ DomainMember,
                              ''securevm2'' ↦ DomainMember,
                              ''publicvm1'' ↦ AccessibleMember,
                              ''publicvm2'' ↦ AccessibleMember
                             ]
  private definition SecGwExt-m ≡ new-configured-list-SecurityInvariant SINVAR-LIB-PolEnforcePointExtended
(⦇
        node-properties = secgwext-host-attributes
        ⦈ ''secure hypervisor mediates accesses between secure VMs''
  private definition secgwext-hosts ≡ [''hypervisor'', ''securevm1'', ''securevm2'',
                                       ''publicvm1'', ''publicvm2'',
                                       ''INET'']

  private lemma dom (secgwext-host-attributes) ⊆ set (secgwext-hosts)
    by(simp add: secgwext-hosts-def secgwext-host-attributes-def)
  value[code] make-policy [SecGwExt-m] secgwext-hosts
  ML-val‹
  visualize-graph-header @{context} @{term [SecGwExt-m]}
    @{term make-policy [SecGwExt-m] secgwext-hosts}
    @{term secgwext-host-attributes};
›

  ML-val‹
  visualize-graph-header @{context} @{term [SecGwExt-m, new-configured-list-SecurityInvariant SINVAR-LIB-BLPtrust
(⦇
        node-properties = [''hypervisor'' ↦ ⦇ security-level = 0, trusted = True ⦈,
                           ''securevm1'' ↦ ⦇ security-level = 1, trusted = False ⦈,
                           ''securevm2'' ↦ ⦇ security-level = 1, trusted = False ⦈
                          ] ⦈ ''secure vms are confidential'']}
    @{term make-policy [SecGwExt-m, new-configured-list-SecurityInvariant SINVAR-LIB-BLPtrusted
(⦇
        node-properties = [''hypervisor'' ↦ ⦇ security-level = 0, trusted = True ⦈,
                           ''securevm1'' ↦ ⦇ security-level = 1, trusted = False ⦈,
                           ''securevm2'' ↦ ⦇ security-level = 1, trusted = False ⦈
                          ] ⦈ ''secure vms are confidential''] secgwext-hosts}
    @{term secgwext-host-attributes};
›
end


end
```

15 Example: Imaginary Factory Network

theory *Imaginary-Factory-Network*
imports *../TopoS-Impl*
begin

In this theory, we give an an example of an imaginary factory network. The example was chosen to show the interplay of several security invariants and to demonstrate their configuration effort.

The specified security invariants deliberately include some minor specification problems. These problems will be used to demonstrate the inner workings of the algorithms and to visualize why some computed results will deviate from the expected results.

The described scenario is an imaginary factory network. It consists of sensors and actuators in a cyber-physical system. The on-site production units of the factory are completely automated and there are no humans in the production area. Sensors are monitoring the building. The production units are two robots (abbreviated bots) which manufacture the actual goods. The robots are controlled by two control systems.

The network consists of the following hosts which are responsible for monitoring the building.

- Statistics: A server which collects, processes, and stores all data from the sensors.
- SensorSink: A device which receives the data from the PresenceSensor, Webcam, TempSensor, and FireSensor. It sends the data to the Statistics server.
- PresenceSensor: A sensor which detects whether a human is in the building.
- Webcam: A camera which monitors the building indoors.
- TempSensor: A sensor which measures the temperature in the building.
- FireSensor: A sensor which detects fire and smoke.

The following hosts are responsible for the production line.

- MissionControl1: An automation device which drives and controls the robots.
- MissionControl2: An automation device which drives and controls the robots. It contains the logic for a secret production step, carried out only by Robot2.
- Watchdog: Regularly checks the health and technical readings of the robots.
- Robot1: Production robot unit 1.
- Robot2: Production robot unit 2. Does a secret production step.
- AdminPc: A human administrator can log into this machine to supervise or troubleshoot the production.

We model one additional special host.

- INET: A symbolic host which represents all hosts which are not part of this network.

The security policy is defined below.

definition *policy* :: *string list-graph* **where**
policy ≡ (| *nodesL* = [''*Statistics*'',
''*SensorSink*'',
''*PresenceSensor*'',
''*Webcam*'',
''*TempSensor*'',
''*FireSensor*'',
''*MissionControl1*'',
''*MissionControl2*'',
''*Watchdog*'',
''*Robot1*'',
''*Robot2*'',
''*AdminPc*'',
''*INET*''],
edgesL = [(''*PresenceSensor*'', ''*SensorSink*''),
(''*Webcam*'', ''*SensorSink*''),
(''*TempSensor*'', ''*SensorSink*''),
(''*FireSensor*'', ''*SensorSink*''),
(''*SensorSink*'', ''*Statistics*''),
(''*MissionControl1*'', ''*Robot1*''),
(''*MissionControl1*'', ''*Robot2*''),
(''*MissionControl2*'', ''*Robot2*''),
(''*AdminPc*'', ''*MissionControl2*''),
(''*AdminPc*'', ''*MissionControl1*''),
(''*Watchdog*'', ''*Robot1*''),
(''*Watchdog*'', ''*Robot2*'')
] |)

lemma *wf-list-graph policy* **by** *eval*

ML-val‹
visualize-graph @{*context*} @{*term* []::*string SecurityInvariant list*} @{*term policy*};
›

The idea behind the policy is the following. The sensors on the left can all send their readings in an unidirectional fashion to the sensor sink, which forwards the data to the statistics server. In the production line, on the right, all devices will set up stateful connections. This means, once a connection is established, packet exchange can be bidirectional. This makes sure that the watchdog will receive the health information from the robots, the mission control machines will receive the current state of the robots, and the administrator can actually log into the mission control machines. The policy should only specify who is allowed to set up the connections. We will elaborate on the stateful implementation in `../TopoS_Stateful_Policy.thy` and `../TopoS_Stateful_Policy_Algorithm.thy`.

15.1 Specification of Security Invariants

Several security invariants are specified.

Privacy for employees. The sensors in the building may record any employee. Due to privacy requirements, the sensor readings, processing, and storage of the data is treated with high

security levels. The presence sensor does not allow do identify an individual employee, hence produces less critical data, hence has a lower level.

```
context begin
  private definition BLP-privacy-host-attributes ≡ [''Statistics'' ↦ 3,
                     ''SensorSink'' ↦ 3,
                     ''PresenceSensor'' ↦ 2, — less critical data
                     ''Webcam'' ↦ 3
                     ]
  private lemma dom (BLP-privacy-host-attributes) ⊆ set (nodesL policy)
    by(simp add: BLP-privacy-host-attributes-def policy-def)
  definition BLP-privacy-m ≡ new-configured-list-SecurityInvariant SINVAR-LIB-BLPbasic (|
      node-properties = BLP-privacy-host-attributes |) ''confidential sensor data''
end
```

Secret corporate knowledge and intellectual property: The production process is a corporate trade secret. The mission control devices have the trade secretes in their program. The important and secret step is done by MissionControl2.

```
context begin
  private definition BLP-tradesecrets-host-attributes ≡ [''MissionControl1'' ↦ 1,
                     ''MissionControl2'' ↦ 2,
                     ''Robot1'' ↦ 1,
                     ''Robot2'' ↦ 2
                     ]
  private lemma dom (BLP-tradesecrets-host-attributes) ⊆ set (nodesL policy)
    by(simp add: BLP-tradesecrets-host-attributes-def policy-def)
  definition BLP-tradesecrets-m ≡ new-configured-list-SecurityInvariant SINVAR-LIB-BLPbasic (|
      node-properties = BLP-tradesecrets-host-attributes |) ''trade secrets''
end
```

Note that Invariant 1 and Invariant 2 are two distinct specifications. They specify individual security goals independent of each other. For example, in Invariant 1,*''MissionControl2''* has the security level $\bot$ and in Invariant 2, *''PresenceSensor''* has security level $\bot$. Consequently, both cannot interact.

Privacy for employees, exporting aggregated data: Monitoring the building while both ensuring privacy of the employees is an important goal for the company. While the presence sensor only collects the single-bit information whether a human is present, the webcam allows to identify individual employees. The data collected by the presence sensor is classified as secret while the data produced by the webcam is top secret. The sensor sink only has the secret security level, hence it is not allowed to process the data generated by the webcam. However, the sensor sink aggregates all data and only distributes a statistical average which does not allow to identify individual employees. It does not store the data over long periods. Therefore, it is marked as trusted and may thus receive the webcam's data. The statistics server, which archives all the data, is considered top secret.

```
context begin
  private definition BLP-employee-export-host-attributes ≡
                     [''Statistics'' ↦ (| security-level = 3, trusted = False |),
                      ''SensorSink'' ↦ (| security-level = 2, trusted = True |),
                      ''PresenceSensor'' ↦ (| security-level = 2, trusted = False |),
                      ''Webcam'' ↦ (| security-level = 3, trusted = False |)
                      ]
```

```
  private lemma dom (BLP-employee-export-host-attributes) ⊆ set (nodesL policy)
    by(simp add: BLP-employee-export-host-attributes-def policy-def)
 definition BLP-employee-export-m ≡ new-configured-list-SecurityInvariant SINVAR-LIB-BLPtrusted
(⌊
       node-properties = BLP-employee-export-host-attributes ⌋ ''employee data (privacy)''
```

end

Who can access bot2? Robot2 carries out a mission-critical production step. It must be made sure that Robot2 only receives packets from Robot1, the two mission control devices and the watchdog.

```
context begin
  private definition ACL-bot2-host-attributues ≡
                      [''Robot2'' ↦ Master [''Robot1'',
                                            ''MissionControl1'',
                                            ''MissionControl2'',
                                            ''Watchdog'']],
                       ''MissionControl1'' ↦ Care,
                       ''MissionControl2'' ↦ Care,
                       ''Watchdog'' ↦ Care
                      ]
  private lemma dom (ACL-bot2-host-attributues) ⊆ set (nodesL policy)
    by(simp add: ACL-bot2-host-attributues-def policy-def)
 definition ACL-bot2-m ≡ new-configured-list-SecurityInvariant SINVAR-LIB-CommunicationPartners
                        (⌊node-properties = ACL-bot2-host-attributues ⌋ ''Robot2 ACL''
```

Note that Robot1 is in the access list of Robot2 but it does not have the *Care* attribute. This means, Robot1 can never access Robot2. A tool could automatically detect such inconsistencies and emit a warning. However, a tool should only emit a warning (not an error) because this setting can be desirable.

In our factory, this setting is currently desirable: Three months ago, Robot1 had an irreparable hardware error and needed to be removed from the production line. When removing Robot1 physically, all its host attributes were also deleted. The access list of Robot2 was not changed. It was planned that Robot1 will be replaced and later will have the same access rights again. A few weeks later, a replacement for Robot1 arrived. The replacement is also called Robot1. The new robot arrived neither configured nor tested for the production. After carefully testing Robot1, Robot1 has been given back the host attributes for the other security invariants. Despite the ACL entry of Robot2, when Robot1 was added to the network, because of its missing *Care* attribute, it was not given automatically access to Robot2. This prevented that Robot1 would accidentally impact Robot2 without being fully configured. In our scenario, once Robot1 will be fully configured, tested, and verified, it will be given the *Care* attribute back.

In general, this design choice of the invariant template prevents that a newly added host may inherit access rights due to stale entries in access lists. At the same time, it does not force administrators to clean up their access lists because a host may only be removed temporarily and wants to be given back its access rights later on. Note that managing access lists scales quadratically in the number of hosts. In contrast, the *Care* attribute can be considered as a Boolean flag which allows to temporarily enable or disable the access rights of a host locally without touching the carefully constructed access lists of other hosts. It also prevents that new hosts which have the name of hosts removed long ago (but where stale access rights were

not cleaned up) accidentally inherit their access rights.

end

Hierarchy of fab robots: The production line is designed according to a strict command hierarchy. On top of the hierarchy are control terminals which allow a human operator to intervene and supervise the production process. On the level below, one distinguishes between supervision devices and control devices. The watchdog is a typical supervision device whereas the mission control devices are control devices. Directly below the control devices are the robots. This is the structure that is necessary for the example. However, the company defined a few more sub-departments for future use. The full domain hierarchy tree is visualized below.

Apart from the watchdog, only the following linear part of the tree is used: $''Robots'' \sqsubseteq ''ControlDevices'' \sqsubseteq ''ControlTerminal''$. Because the watchdog is in a different domain, it needs a trust level of 1 to access the robots it is monitoring.

```
context begin
 private definition DomainHierarchy-host-attributes ≡
            [(''MissionControl1'',
               DN (''ControlTerminal''--''ControlDevices''--Leaf, 0)),
             (''MissionControl2'',
               DN (''ControlTerminal''--''ControlDevices''--Leaf, 0)),
             (''Watchdog'',
               DN (''ControlTerminal''--''Supervision''--Leaf, 1)),
             (''Robot1'',
               DN (''ControlTerminal''--''ControlDevices''--''Robots''--Leaf, 0)),
             (''Robot2'',
               DN (''ControlTerminal''--''ControlDevices''--''Robots''--Leaf, 0)),
             (''AdminPc'',
               DN (''ControlTerminal''--Leaf, 0))
             ]
 private lemma dom (map-of DomainHierarchy-host-attributes) ⊆ set (nodesL policy)
  by(simp add: DomainHierarchy-host-attributes-def policy-def)

 lemma DomainHierarchyNG-sanity-check-config
  (map snd DomainHierarchy-host-attributes)
      (
      Department ''ControlTerminal'' [
        Department ''ControlDevices'' [
          Department ''Robots'' [],
          Department ''OtherStuff'' [],
          Department ''ThirdSubDomain'' []
        ],
        Department ''Supervision'' [
          Department ''S1'' [],
          Department ''S2'' []
        ]
      ]) by eval

 definition Control-hierarchy-m ≡ new-configured-list-SecurityInvariant
                                  SINVAR-LIB-DomainHierarchyNG
                                  (| node-properties = map-of DomainHierarchy-host-attributes |)
                                  ''Production device hierarchy''
end
```

Sensor Gateway: The sensors should not communicate among each other; all accesses must be mediated by the sensor sink.

```
context begin
  private definition PolEnforcePoint-host-attributes ≡
              [''SensorSink'' ↦ PolEnforcePoint,
               ''PresenceSensor'' ↦ DomainMember,
               ''Webcam'' ↦ DomainMember,
               ''TempSensor'' ↦ DomainMember,
               ''FireSensor'' ↦ DomainMember
              ]
  private lemma dom PolEnforcePoint-host-attributes ⊆ set (nodesL policy)
    by(simp add: PolEnforcePoint-host-attributes-def policy-def)
  definition PolEnforcePoint-m ≡ new-configured-list-SecurityInvariant
                                 SINVAR-LIB-PolEnforcePointExtended
                                   (| node-properties = PolEnforcePoint-host-attributes |)
                                 ''sensor slaves''
end
```

Production Robots are an information sink: The actual control program of the robots is a corporate trade secret. The control commands must not leave the robots. Therefore, they are declared information sinks. In addition, the control command must not leave the mission control devices. However, the two devices could possibly interact to synchronize and they must send their commands to the robots. Therefore, they are labeled as sink pools.

```
context begin
  private definition SinkRobots-host-attributes ≡
              [''MissionControl1'' ↦ SinkPool,
               ''MissionControl2'' ↦ SinkPool,
               ''Robot1'' ↦ Sink,
               ''Robot2'' ↦ Sink
              ]
  private lemma dom SinkRobots-host-attributes ⊆ set (nodesL policy)
    by(simp add: SinkRobots-host-attributes-def policy-def)
  definition SinkRobots-m ≡ new-configured-list-SecurityInvariant
                                 SINVAR-LIB-Sink
                                   (| node-properties = SinkRobots-host-attributes |)
                            ''non−leaking production units''
end
```

Subnet of the fab: The sensors, including their sink and statistics server are located in their own subnet and must not be accessible from elsewhere. Also, the administrator's PC is in its own subnet. The production units (mission control and robots) are already isolated by the DomainHierarchy and are not added to a subnet explicitly.

```
context begin
  private definition Subnets-host-attributes ≡
              [''Statistics'' ↦ Subnet 1,
               ''SensorSink'' ↦ Subnet 1,
               ''PresenceSensor'' ↦ Subnet 1,
               ''Webcam'' ↦ Subnet 1,
               ''TempSensor'' ↦ Subnet 1,
               ''FireSensor'' ↦ Subnet 1,
               ''AdminPc'' ↦ Subnet 4
              ]
```

```
  private lemma dom Subnets-host-attributes ⊆ set (nodesL policy)
    by(simp add: Subnets-host-attributes-def policy-def)
  definition Subnets-m ≡ new-configured-list-SecurityInvariant
                                SINVAR-LIB-Subnets
                                  ⦇ node-properties = Subnets-host-attributes ⦈
                             ''network segmentation''
end
```

Access Gateway for the Statistics server: The statistics server is further protected from external accesses. Another, smaller subnet is defined with the only member being the statistics server. The only way it may be accessed is via that sensor sink.

```
context begin
  private definition SubnetsInGW-host-attributes ≡
              [''Statistics'' ↦ Member,
               ''SensorSink'' ↦ InboundGateway
               ]
  private lemma dom SubnetsInGW-host-attributes ⊆ set (nodesL policy)
    by(simp add: SubnetsInGW-host-attributes-def policy-def)
  definition SubnetsInGW-m ≡ new-configured-list-SecurityInvariant
                                SINVAR-LIB-SubnetsInGW
                                  ⦇ node-properties = SubnetsInGW-host-attributes ⦈
                             ''Protectting statistics srv''
end
```

NonInterference (for the sake of example): The fire sensor is managed by an external company and has a built-in GSM module to call the fire fighters in case of an emergency. This additional, out-of-band connectivity is not modeled. However, the contract defines that the company's administrator must not interfere in any way with the fire sensor.

```
context begin
  private definition NonInterference-host-attributes ≡
              [''Statistics'' ↦ Unrelated,
               ''SensorSink'' ↦ Unrelated,
               ''PresenceSensor'' ↦ Unrelated,
               ''Webcam'' ↦ Unrelated,
               ''TempSensor'' ↦ Unrelated,
               ''FireSensor'' ↦ Interfering, — (!)
               ''MissionControl1'' ↦ Unrelated,
               ''MissionControl2'' ↦ Unrelated,
               ''Watchdog'' ↦ Unrelated,
               ''Robot1'' ↦ Unrelated,
               ''Robot2'' ↦ Unrelated,
               ''AdminPc'' ↦ Interfering, — (!)
               ''INET'' ↦ Unrelated
               ]
  private lemma dom NonInterference-host-attributes ⊆ set (nodesL policy)
    by(simp add: NonInterference-host-attributes-def policy-def)
 definition NonInterference-m ≡ new-configured-list-SecurityInvariant SINVAR-LIB-NonInterference
                                  ⦇ node-properties = NonInterference-host-attributes ⦈
                                ''for the sake of an acdemic example!''
end
```

As discussed, this invariant is very strict and rather theoretical. It is not ENF-structured and may produce an exponential number of offending flows. Therefore, we exclude it by default

from our algorithms.

definition *invariants* ≡ [*BLP-privacy-m*, *BLP-tradesecrets-m*, *BLP-employee-export-m*,
ACL-bot2-m, *Control-hierarchy-m*,
PolEnforcePoint-m, *SinkRobots-m*, *Subnets-m*, *SubnetsInGW-m*]

We have excluded *NonInterference-m* because of its infeasible runtime.

lemma *length invariants = 9* **by** *eval*

15.2 Policy Verification

The given policy fulfills all the specified security invariants. Also with *NonInterference-m*, the policy fulfills all security invariants.

lemma *all-security-requirements-fulfilled* (*NonInterference-m#invariants*) *policy* **by** *eval*
ML‹
visualize-graph @{*context*} @{*term invariants*} @{*term policy*};
›

definition *make-policy* :: ('*a SecurityInvariant*) *list* ⇒ '*a list* ⇒ '*a list-graph* **where**
make-policy sinvars Vs ≡ *generate-valid-topology sinvars* (|*nodesL = Vs*, *edgesL = List.product Vs Vs* |)

definition *make-policy-efficient* :: ('*a SecurityInvariant*) *list* ⇒ '*a list* ⇒ '*a list-graph* **where**
make-policy-efficient sinvars Vs ≡ *generate-valid-topology-some sinvars* (|*nodesL = Vs*, *edgesL = List.product Vs Vs* |)

The question, "how good are the specified security invariants?" remains. Therefore, we use the algorithm from *make-policy* to generate a policy. Then, we will compare our policy with the automatically generated one. If we exclude the NonInterference invariant from the policy construction, we know that the resulting policy must be maximal. Therefore, the computed policy reflects the view of the specified security invariants. By maximality of the computed policy and monotonicity, we know that our manually specified policy must be a subset of the computed policy. This allows to compare the manually-specified policy to the policy implied by the security invariants: If there are too many flows which are allowed according to the computed policy but which are not in our manually-specified policy, we can conclude that our security invariants are not strict enough.

value[*code*] *make-policy invariants* (*nodesL policy*)
lemma *make-policy invariants* (*nodesL policy*) =
(|*nodesL* =
[''*Statistics*'', ''*SensorSink*'', ''*PresenceSensor*'', ''*Webcam*'', ''*TempSensor*'',
''*FireSensor*'', ''*MissionControl1*'', ''*MissionControl2*'', ''*Watchdog*'', ''*Robot1*'',
''*Robot2*'', ''*AdminPc*'', ''*INET*''],
edgesL =
[(''*Statistics*'', ''*Statistics*''), (''*SensorSink*'', ''*Statistics*''),
(''*SensorSink*'', ''*SensorSink*''), (''*SensorSink*'', ''*Webcam*''),
(''*PresenceSensor*'', ''*SensorSink*''), (''*PresenceSensor*'', ''*PresenceSensor*''),
(''*Webcam*'', ''*SensorSink*''), (''*Webcam*'', ''*Webcam*''),
(''*TempSensor*'', ''*SensorSink*''), (''*TempSensor*'', ''*TempSensor*''),
(''*TempSensor*'', ''*INET*''), (''*FireSensor*'', ''*SensorSink*''),
(''*FireSensor*'', ''*FireSensor*''), (''*FireSensor*'', ''*INET*''),

(''MissionControl1'', ''MissionControl1''),
(''MissionControl1'', ''MissionControl2''), (''MissionControl1'', ''Robot1''),
(''MissionControl1'', ''Robot2''), (''MissionControl2'', ''MissionControl2''),
(''MissionControl2'', ''Robot2''), (''Watchdog'', ''MissionControl1''),
(''Watchdog'', ''MissionControl2''), (''Watchdog'', ''Watchdog''),
(''Watchdog'', ''Robot1''), (''Watchdog'', ''Robot2''), (''Watchdog'', ''INET''),
(''Robot1'', ''Robot1''), (''Robot2'', ''Robot2''), (''AdminPc'', ''MissionControl1''),
(''AdminPc'', ''MissionControl2''), (''AdminPc'', ''Watchdog''),
(''AdminPc'', ''Robot1''), (''AdminPc'', ''AdminPc''), (''AdminPc'', ''INET''),
(''INET'', ''INET'')]⟫ **by** *eval*

Additional flows which would be allowed but which are not in the policy

lemma *set* [*e* ← *edgesL* (*make-policy invariants* (*nodesL policy*)). *e* ∉ *set* (*edgesL policy*)] =
set [(*v*,*v*). *v* ← (*nodesL policy*)] ∪
set [(''SensorSink'', ''Webcam''),
(''TempSensor'', ''INET''),
(''FireSensor'', ''INET''),
(''MissionControl1'', ''MissionControl2''),
(''Watchdog'', ''MissionControl1''),
(''Watchdog'', ''MissionControl2''),
(''Watchdog'', ''INET''),
(''AdminPc'', ''Watchdog''),
(''AdminPc'', ''Robot1''),
(''AdminPc'', ''INET'')] **by** *eval*

We visualize this comparison below. The solid edges correspond to the manually-specified policy. The dotted edges correspond to the flow which would be additionally permitted by the computed policy.

ML-val⟨
visualize-edges @{*context*} @{*term edgesL policy*}
[(*edge* [*dir*=*arrow*\, *style*=*dashed*, *color*=\#*FF8822*\, *constraint*=*false*],
@{*term* [*e* ← *edgesL* (*make-policy invariants* (*nodesL policy*)).
e ∉ *set* (*edgesL policy*)]})] ;
⟩

The comparison reveals that the following flows would be additionally permitted. We will discuss whether this is acceptable or if the additional permission indicates that we probably forgot to specify an additional security goal.

- All reflexive flows, i.e. all host can communicate with themselves. Since each host in the policy corresponds to one physical entity, there is no need to explicitly prohibit or allow in-host communication.
- The ''*SensorSink*'' may access the ''*Webcam*''. Both share the same security level, there is no problem with this possible information flow. Technically, a bi-directional connection may even be desirable, since this allows the sensor sink to influence the video stream, e.g. request a lower bit rate if it is overloaded.
- Both the ''*TempSensor*'' and the ''*FireSensor*'' may access the Internet. No security levels or other privacy concerns are specified for them. This may raise the question whether this data is indeed public. It is up to the company to decide that this data should also be considered confidential.

- *"MissionControl1"* can send to *"MissionControl2"*. This may be desirable since it was stated anyway that the two may need to cooperate. Note that the opposite direction is definitely prohibited since the critical and secret production step only known to *"MissionControl2"* must not leak.
- The *"Watchdog"* may access *"MissionControl1"*, *"MissionControl2"*, and the *"INET"*. While it may be acceptable that the watchdog which monitors the robots may also access the control devices, it should raise a concern that the watchdog may freely send data to the Internet. Indeed, the watchdog can access devices which have corporate trade secrets stored but it was never specified that the watchdog should be treated confidentially. Note that in the current setting, the trade secrets will never leave the robots. This is because the policy only specifies a unidirectional information flow from the watchdog to the robots; the robots will not leak any information back to the watchdog. This also means that the watchdog cannot actually monitor the robots. Later, when implementing the scenario, we will see that the simple, hand-waving argument "the watchdog connects to the robots and the robots send back their data over the established connection" will not work because of this possible information leak.
- The *"AdminPc"* is allowed to access the *"Watchdog"*, *"Robot1"*, and the *"INET"*. Since this machine is trusted anyway, the company does not see a problem with this.

without *NonInterference-m*

lemma *all-security-requirements-fulfilled invariants* (*make-policy invariants* (*nodesL policy*)) **by** *eval*

Side note: what if we exclude subnets?

ML-val ‹
visualize-edges @{*context*} @{*term edgesL* (*make-policy invariants* (*nodesL policy*))}
[(*edge* [*dir*=*arrow*\, *style*=*dashed*, *color*=*#FF8822*\, *constraint*=*false*],
@{*term* ‹[*e* ← *edgesL* (*make-policy* [*BLP-privacy-m*, *BLP-tradesecrets-m*, *BLP-employee-export-m*,
ACL-bot2-m, *Control-hierarchy-m*,
PolEnforcePoint-m, *SinkRobots-m*, ~~*SubnetsInGW-m*~~ *SubnetsInGW-m*] (*nodesL policy*)).
e ∉ *set* (*edgesL* (*make-policy invariants* (*nodesL policy*)))]›})] ;
›

15.3 About NonInterference

The NonInterference template was deliberately selected for our scenario as one of the 'problematic' and rather theoretical invariants. Our framework allows to specify almost arbitrary invariant templates. We concluded that all non-ENF-structured invariants which may produce an exponential number of offending flows are problematic for practical use. This includes "Comm. With" (`../Security_Invariants/SINVAR_ACLcommunicateWith.thy`), "Not Comm. With" (`../Security_Invariants/SINVAR_ACLnotCommunicateWith.thy`), Dependability (`../Security_Invariants/SINVAR_Dependability.thy`), and NonInterference (`../Security_Invariants/SINVAR_NonInterference.thy`). In this section, we discuss the consequences of the NonInterference invariant for automated policy construction. We will conclude that, though we can solve all technical challenges, said invariants are —due to their inherent ambiguity— not very well suited for automated policy construction.

The computed maximum policy does not fulfill invariant 10 (NonInterference). This is because the fire sensor and the administrator's PC may be indirectly connected over the Internet.

lemma ¬ *all-security-requirements-fulfilled* (*NonInterference-m#invariants*) (*make-policy invariants* (*nodesL policy*)) **by** *eval*

Since the NonInterference template may produce an exponential number of offending flows, it is infeasible to try our automated policy construction algorithm with it. We have tried to do so on a machine with 128GB of memory but after a few minutes, the computation ran out of memory. On said machine, we were unable to run our policy construction algorithm with the NonInterference invariant for more that five hosts.

Algorithm *make-policy-efficient* improves the policy construction algorithm. The new algorithm instantly returns a solution for this scenario with a very small memory footprint.

The more efficient algorithm does not need to construct the complete set of offending flows

value[*code*] *make-policy-efficient* (*invariants*@[*NonInterference-m*]) (*nodesL policy*)
value[*code*] *make-policy-efficient* (*NonInterference-m#invariants*) (*nodesL policy*)

lemma *make-policy-efficient* (*invariants*@[*NonInterference-m*]) (*nodesL policy*) =
make-policy-efficient (*NonInterference-m#invariants*) (*nodesL policy*) **by** *eval*

But *NonInterference-m* insists on removing something, which would not be necessary.

lemma *make-policy invariants* (*nodesL policy*) ≠ *make-policy-efficient* (*NonInterference-m#invariants*) (*nodesL policy*) **by** *eval*

lemma *set* (*edgesL* (*make-policy-efficient* (*NonInterference-m#invariants*) (*nodesL policy*)))
⊆
set (*edgesL* (*make-policy invariants* (*nodesL policy*))) **by** *eval*

This is what it wants to be gone.

lemma [*e* ← *edgesL* (*make-policy invariants* (*nodesL policy*)).
e ∉ *set* (*edgesL* (*make-policy-efficient* (*NonInterference-m#invariants*) (*nodesL policy*)))]
=
[(*''AdminPc''*, *''MissionControl1''*), (*''AdminPc''*, *''MissionControl2''*),
(*''AdminPc''*, *''Watchdog''*), (*''AdminPc''*, *''Robot1''*), (*''AdminPc''*, *''INET''*)]
by *eval*

lemma [*e* ← *edgesL* (*make-policy invariants* (*nodesL policy*)).
e ∉ *set* (*edgesL* (*make-policy-efficient* (*NonInterference-m#invariants*) (*nodesL policy*)))]
=
[*e* ← *edgesL* (*make-policy invariants* (*nodesL policy*)). *fst e* = *''AdminPc''* ∧ *snd e* ≠ *''AdminPc''*]
by *eval*
ML-val⟨
visualize-edges @{*context*} @{*term edgesL policy*}
[(*edge* [*dir*=*arrow*\, *style*=*dashed*, *color*=*#FF8822*\, *constraint*=*false*],
@{*term* [*e* ← *edgesL* (*make-policy invariants* (*nodesL policy*)).
e ∉ *set* (*edgesL* (*make-policy-efficient* (*NonInterference-m#invariants*) (*nodesL policy*)))]})]
;
⟩

However, it is an inherent property of the NonInterferance template (and similar templates), that the set of offending flows is not uniquely defined. Consequently, since several solutions are possible, even our new algorithm may not be able to compute one maximum solution. It would

be possible to construct some maximal solution, however, this would require to enumerate all offending flows, which is infeasible. Therefore, our algorithm can only return some (valid but probably not maximal) solution for non-END-structured invariants.

As a human, we know the scenario and the intention behind the policy. Probably, the best solution for policy construction with the NonInterferance property would be to restrict outgoing edges from the fire sensor. If we consider the policy above which was constructed without NonInterference, if we cut off the fire sensor from the Internet, we get a valid policy for the NonInterference property. Unfortunately, an algorithm does not have the information of which flows we would like to cut first and the algorithm needs to make some choice. In this example, the algorithm decides to isolate the administrator's PC from the rest of the world. This is also a valid solution. We could change the order of the elements to tell the algorithm which edges we would rather sacrifice than others. This may help but requires some additional input. The author personally prefers to construct only maximum policies with Φ-structured invariants and afterwards fix the policy manually for the remaining non-Φ-structured invariants. Though our new algorithm gives better results and returns instantly, the very nature of invariant templates with an exponential number of offending flows tells that these invariants are problematic for automated policy construction.

15.4 Stateful Implementation

In this section, we will implement the policy and deploy it in a network. As the scenario description stated, all devices in the production line should establish stateful connections which allows – once the connection is established – packets to travel in both directions. This is necessary for the watchdog, the mission control devices, and the administrator's PC to actually perform their task.

We compute a stateful implementation. Below, the stateful implementation is visualized. It consists of the policy as visualized above. In addition, dotted edges visualize where answer packets are permitted.

```
definition stateful-policy = generate-valid-stateful-policy-IFSACS policy invariants
lemma stateful-policy =
 (⟦hostsL = nodesL policy,
    flows-fixL = edgesL policy,
    flows-stateL =
      [(''Webcam'', ''SensorSink''),
       (''SensorSink'', ''Statistics'')]⟧) by eval

ML-val‹
visualize-edges @{context} @{term flows-fixL stateful-policy}
    [(edge [dir=\arrow\, style=dashed, color=\#FF8822\, constraint=false], @{term flows-stateL
stateful-policy})] ;
›
```

As can be seen, only the flows (*''Webcam''*, *''SensorSink''*) and (*''SensorSink''*, *''Statistics''*) are allowed to be stateful. This setup cannot be practically deployed because the watchdog, the mission control devices, and the administrator's PC also need to set up stateful connections. Previous section's discussion already hinted at this problem. The reason why the desired stateful connections are not permitted is due to information leakage. In detail: *BLP-tradesecrets-m* and *SinkRobots-m* are responsible. Both invariants prevent that any data leaves the robots and the mission control devices. To verify this suspicion, the two invariants

are removed and the stateful flows are computed again. The result visualized is below.

lemma *generate-valid-stateful-policy-IFSACS policy*
[*BLP-privacy-m*, *BLP-employee-export-m*,
ACL-bot2-m, *Control-hierarchy-m*,
PolEnforcePoint-m, *Subnets-m*, *SubnetsInGW-m*] =
(|*hostsL* = *nodesL policy*,
flows-fixL = *edgesL policy*,
flows-stateL =
[(''*Webcam*'', ''*SensorSink*''),
(''*SensorSink*'', ''*Statistics*''),
(''*MissionControl1*'', ''*Robot1*''),
(''*MissionControl1*'', ''*Robot2*''),
(''*MissionControl2*'', ''*Robot2*''),
(''*AdminPc*'', ''*MissionControl2*''),
(''*AdminPc*'', ''*MissionControl1*''),
(''*Watchdog*'', ''*Robot1*''),
(''*Watchdog*'', ''*Robot2*'')]|) **by** *eval*

This stateful policy could be transformed into a fully functional implementation. However, there would be no security invariants specified which protect the trade secrets. Without those two invariants, the invariant specification is too permissive. For example, if we recompute the maximum policy, we can see that the robots and mission control can leak any data to the Internet. Even without the maximum policy, in the stateful policy above, it can be seen that MissionControl1 can exfiltrate information from robot 2, once it establishes a stateful connection.

Without the two invariants, the security goals are way too permissive!

lemma *set* [*e* ← *edgesL* (*make-policy* [*BLP-privacy-m*, *BLP-employee-export-m*,
ACL-bot2-m, *Control-hierarchy-m*,
PolEnforcePoint-m, *Subnets-m*, *SubnetsInGW-m*] (*nodesL policy*)). *e* ∉ *set* (*edgesL policy*)] =
set [(*v*,*v*). *v* ← (*nodesL policy*)] ∪
set [(''*SensorSink*'', ''*Webcam*''),
(''*TempSensor*'', ''*INET*''),
(''*FireSensor*'', ''*INET*''),
(''*MissionControl1*'', ''*MissionControl2*''),
(''*Watchdog*'', ''*MissionControl1*''),
(''*Watchdog*'', ''*MissionControl2*''),
(''*Watchdog*'', ''*INET*''),
(''*AdminPc*'', ''*Watchdog*''),
(''*AdminPc*'', ''*Robot1*''),
(''*AdminPc*'', ''*INET*'')] ∪
set [(''*MissionControl1*'', ''*INET*''),
(''*MissionControl2*'', ''*MissionControl1*''),
(''*MissionControl2*'', ''*Robot1*''),
(''*MissionControl2*'', ''*INET*''),
(''*Robot1*'', ''*INET*''),
(''*Robot2*'', ''*Robot1*''),
(''*Robot2*'', ''*INET*'')] **by** *eval*

ML-val⟨
visualize-edges @{*context*} @{*term flows-fixL* (*generate-valid-stateful-policy-IFSACS policy* [*BLP-privacy-m*, *BLP-employee-export-m*,

```
                    ACL-bot2-m, Control-hierarchy-m,
                    PolEnforcePoint-m, Subnets-m, SubnetsInGW-m])}
  [(edge [dir=\arrow\, style=dashed, color=\#FF8822\, constraint=false],
  @{term flows-stateL (generate-valid-stateful-policy-IFSACS policy [BLP-privacy-m, BLP-employee-export-m,
                    ACL-bot2-m, Control-hierarchy-m,
                    PolEnforcePoint-m, Subnets-m, SubnetsInGW-m])})] ;
›
```

Therefore, the two invariants are not removed but repaired. The goal is to allow the watchdog, administrator's pc, and the mission control devices to set up stateful connections without leaking corporate trade secrets to the outside.

First, we repair *BLP-tradesecrets-m*. On the one hand, the watchdog should be able to send packets both *"Robot1"* and *"Robot2"*. *"Robot1"* has a security level of *1* and *"Robot2"* has a security level of *2*. Consequently, in order to be allowed to send packets to both, *"Watchdog"* must have a security level not higher than *1*. On the other hand, the *"Watchdog"* should be able to receive packets from both. By the same argument, it must have a security level of at least *2*. Consequently, it is impossible to express the desired meaning in the BLP basic template. There are only two solutions to the problem: Either the company installs one watchdog for each security level, or the watchdog must be trusted. We decide for the latter option and upgrade the template to the Bell LaPadula model with trust. We define the watchdog as trusted with a security level of *1*. This means, it can receive packets from and send packets to both robots but it cannot leak information to the outside world. We do the same for the *"AdminPc"*.

Then, we repair *SinkRobots-m*. We realize that the following set set of hosts forms one big pool of devices which must all somehow interact but where information must not leave the pool: The administrator's PC, the mission control devices, the robots, and the watchdog. Therefore, all those devices are configured to be in the same *SinkPool*.

```
definition invariants-tuned ≡ [BLP-privacy-m, BLP-employee-export-m,
          ACL-bot2-m, Control-hierarchy-m,
          PolEnforcePoint-m, Subnets-m, SubnetsInGW-m,
          new-configured-list-SecurityInvariant SINVAR-LIB-Sink
            (| node-properties = ["MissionControl1" ↦ SinkPool,
                                  "MissionControl2" ↦ SinkPool,
                                  "Robot1" ↦ SinkPool,
                                  "Robot2" ↦ SinkPool,
                                  "Watchdog" ↦ SinkPool,
                                  "AdminPc" ↦ SinkPool
                                 ] |)
            "non−leaking production units",
          new-configured-list-SecurityInvariant SINVAR-LIB-BLPtrusted
            (| node-properties = ["MissionControl1" ↦ (| security-level = 1, trusted = False |),
                                  "MissionControl2" ↦ (| security-level = 2, trusted = False |),
                                  "Robot1" ↦ (| security-level = 1, trusted = False |),
                                  "Robot2" ↦ (| security-level = 2, trusted = False |),
                                  "Watchdog" ↦ (| security-level = 1, trusted = True |),
                                  — trust because bot2 must send to it. security-level 1 to interact with
bot 1
                                  "AdminPc" ↦ (| security-level = 1, trusted = True |)
                                 ] |)
            "trade secrets"
          ]
```

lemma *all-security-requirements-fulfilled invariants-tuned policy* **by** *eval*

definition *stateful-policy-tuned = generate-valid-stateful-policy-IFSACS policy invariants-tuned*

The computed stateful policy is visualized below.

lemma *stateful-policy-tuned*
=
(|*hostsL = nodesL policy*,
flows-fixL = edgesL policy,
flows-stateL =
[(''*Webcam*'', ''*SensorSink*''),
(''*SensorSink*'', ''*Statistics*''),
(''*MissionControl1*'', ''*Robot1*''),
(''*MissionControl2*'', ''*Robot2*''),
(''*AdminPc*'', ''*MissionControl2*''),
(''*AdminPc*'', ''*MissionControl1*''),
(''*Watchdog*'', ''*Robot1*''),
(''*Watchdog*'', ''*Robot2*'')]|) **by** *eval*

We even get a better (i.e. stricter) maximum policy

lemma *set* (*edgesL* (*make-policy invariants-tuned* (*nodesL policy*))) ⊂
set (*edgesL* (*make-policy invariants* (*nodesL policy*))) **by** *eval*
lemma *set* [*e* ← *edgesL* (*make-policy invariants-tuned* (*nodesL policy*)). *e* ∉ *set* (*edgesL policy*)] =
set [(*v*,*v*). *v* ← (*nodesL policy*)] ∪
set [(''*SensorSink*'', ''*Webcam*''),
(''*TempSensor*'', ''*INET*''),
(''*FireSensor*'', ''*INET*''),
(''*MissionControl1*'', ''*MissionControl2*''),
(''*Watchdog*'', ''*MissionControl1*''),
(''*Watchdog*'', ''*MissionControl2*''),
(''*AdminPc*'', ''*Watchdog*''),
(''*AdminPc*'', ''*Robot1*'')] **by** *eval*

It can be seen that all connections which should be stateful are now indeed stateful. In addition, it can be seen that MissionControl1 cannot set up a stateful connection to Bot2. This is because MissionControl1 was never declared a trusted device and the confidential information in MissionControl2 and Robot2 must not leak.

The improved invariant definition even produces a better (i.e. stricter) maximum policy.

15.5 Iptables Implementation

firewall – classical use case

ML-val⟨

(∗*header*∗)
writeln (∗(*echo 1 > /proc/sys/net/ipv4/ip-forward*^*n*^
 # flush all rules^*n*^
iptables −F^*n*^
#default policy for FORWARD chain:^*n*^
iptables −P FORWARD DROP);∗)

```
(*filter\n^
 :INPUT ACCEPT [0:0]\n^
 :FORWARD ACCEPT [0:0]\n^
 :OUTPUT ACCEPT [0:0]);

iterate-edges-ML @{context}  @{term flows-fixL stateful-policy-tuned}
 (fn (v1,v2) => writeln (-A FORWARD -i $ ^v1^-iface -s $ ^v1^-ipv4 -o $ ^v2^-iface -d $ ^v2^-ipv4
-j ACCEPT) )
                        (*(iptables -A FORWARD -i $\\$\\mathit{ ^v1^\\-iface}$ -s $\\$\\mathit{ ^
                         v1^\\-ipv4}$ -o $\\$\\mathit{ ^v2^\\-iface}$ -d $\\$\\mathit{ ^v2^\\-ipv4}$
-j ACCEPT) )*)
 (fn - => () )
 (fn - => () );

iterate-edges-ML @{context} @{term flows-stateL stateful-policy-tuned}
 (fn (v1,v2) => writeln (-I FORWARD -m state --state ESTABLISHED -i $ ^v2^-iface -s $ ^
                          v2^-ipv4 -o $ ^v1^-iface -d $ ^v1^-ipv4 -j ACCEPT) )
                       (*(iptables -I FORWARD -m state --state ESTABLISHED -i $\\$\\mathit{ ^
                          v2^\\-iface}$ -s $\\$\\mathit{ ^v2^\\-ipv4}$ -o $\\$\\mathit{ ^
                            v1^\\-iface}$ -d $\\$\\mathit{ ^v1^-ipv4}$ -j ACCEPT # ^v2^ -> ^v1^
(answer)) )*)
 (fn - => () )
 (fn - => () );

writeln COMMIT;
```

Using, https://github.com/diekmann/Iptables_Semantics, the iptables ruleset is indeed correct.

end

References

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