An Isabelle/HOL Formalization of the Modular Assembly Kit for Security Properties

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Abstract

The "Modular Assembly Kit for Security Properties" (MAKS) is a framework for both the definition and verification of possibilistic information-flow security properties at the specification-level. MAKS supports the uniform representation of a wide range of possibilistic information-flow properties and provides support for the verification of such properties via unwinding results and compositionality results. We provide a formalization of this framework in Isabelle/HOL.

Contents

1 Introduction

This is a formalization of the Modular Assembly Kit for Security Properties (MAKS) [2, 3] in its version from [3]. We provide a more detailed explanation on how key concepts of MAKS are formalized in Isabelle/HOL in [1].

2 Basic Definitions

In the following, we define the notion of prefixes and the notion of projection. These definitions are preliminaries for the remaining parts of the Isabelle/HOL formalization of MAKS.

theory *Prefix*
imports *Main*
begin

definition *prefix* :: *'e list* ⇒ *'e list* ⇒ *bool* (**infixl** ⪯ *100*)
where
$(l1 \preceq l2) \equiv (\exists\, l3.\ l1 \mathbin{@} l3 = l2)$

definition *prefixclosed* :: (*'e list*) *set* ⇒ *bool*
where
$prefixclosed\ tr \equiv (\forall\, l1 \in tr.\ \forall\, l2.\ l2 \preceq l1 \longrightarrow l2 \in tr)$

lemma *empty-prefix-of-all*: $[] \preceq l$
⟨*proof*⟩

lemma *empty-trace-contained*: $[\![\ prefixclosed\ tr\ ;\ tr \neq \{\}\ ]\!] \Longrightarrow [] \in tr$
⟨*proof*⟩

lemma *transitive-prefix*: $[\![\ l1 \preceq l2\ ;\ l2 \preceq l3\ ]\!] \Longrightarrow l1 \preceq l3$
⟨*proof*⟩

end
theory *Projection*
imports *Main*
begin

definition *projection*:: *'e list* ⇒ *'e set* ⇒ *'e list* (**infixl** ↾ *100*)
where
$l \restriction E \equiv filter\ (\lambda x\ .\ x \in E)\ l$

lemma *projection-on-union*:
$l \restriction Y = [] \Longrightarrow l \restriction (X \cup Y) = l \restriction X$
⟨*proof*⟩

lemma *projection-on-empty-trace*: $[] \restriction X = []$ ⟨*proof*⟩

lemma *projection-to-emptyset-is-empty-trace*: $l \restriction \{\} = []$ ⟨*proof*⟩

lemma *projection-idempotent*: $l \restriction X = (l \restriction X) \restriction X$ ⟨*proof*⟩

lemma *projection-empty-implies-absence-of-events*: $l \restriction X = [] \Longrightarrow X \cap (set\ l) = \{\}$
⟨*proof*⟩

lemma *disjoint-projection*: $X \cap Y = \{\} \Longrightarrow (l \restriction X) \restriction Y = []$
⟨*proof*⟩

lemma *projection-concatenation-commute*:
$(l1 \mathbin{@} l2) \restriction X = (l1 \restriction X) \mathbin{@} (l2 \restriction X)$
⟨*proof*⟩

lemma *projection-subset-eq-from-superset-eq*:
$((xs \restriction (X \cup Y)) = (ys \restriction (X \cup Y))) \Longrightarrow ((xs \restriction X) = (ys \restriction X))$
(**is** $(?L1 = ?L2) \Longrightarrow (?L3 = ?L4)$)
⟨*proof*⟩

lemma *list-subset-iff-projection-neutral*: $(set\ l \subseteq X) = ((l \restriction X) = l)$
(**is** $?A = ?B$)
⟨*proof*⟩

lemma *projection-split-last*: $Suc\ n = length\ (\tau \restriction X) \Longrightarrow$
$\exists\ \beta\ x\ \alpha.\ (x \in X \wedge \tau = \beta \mathbin{@} [x] \mathbin{@} \alpha \wedge \alpha \restriction X = [] \wedge n = length\ ((\beta \mathbin{@} \alpha) \restriction X))$
⟨*proof*⟩

lemma *projection-rev-commute*:
$rev\ (l \restriction X) = (rev\ l) \restriction X$
⟨*proof*⟩

lemma *projection-split-first*: $[\![(\tau \restriction X) = x \# xs]\!] \Longrightarrow \exists\ \alpha\ \beta.\ (\tau = \alpha \mathbin{@} [x] \mathbin{@} \beta \wedge \alpha \restriction X = [])$
⟨*proof*⟩

lemma *projection-split-first-with-suffix*:
$[\![(\tau \restriction X) = x \# xs]\!] \Longrightarrow \exists\ \alpha\ \beta.\ (\tau = \alpha \mathbin{@} [x] \mathbin{@} \beta \wedge \alpha \restriction X = [] \wedge \beta \restriction X = xs)$
⟨*proof*⟩

lemma *projection-split-arbitrary-element*:
$[\![\tau \restriction X = (\alpha \mathbin{@} [x] \mathbin{@} \beta) \restriction X;\ x \in X]\!]$
$\Longrightarrow \exists\ \alpha'\ \beta'.\ (\tau = \alpha' \mathbin{@} [x] \mathbin{@} \beta' \wedge \alpha' \restriction X = \alpha \restriction X \wedge \beta' \restriction X = \beta \restriction X)$
⟨*proof*⟩

lemma *projection-on-intersection*: $l \restriction X = [] \Longrightarrow l \restriction (X \cap Y) = []$
(**is** *?L1* $= [] \Longrightarrow$ *?L2* $= []$)
⟨*proof*⟩

lemma *projection-on-subset*: $[\![\ Y \subseteq X;\ l \restriction X = []\ ]\!] \Longrightarrow l \restriction Y = []$
⟨*proof*⟩

lemma *projection-on-subset2*: $[\![\ set\ l \subseteq L;\ l \restriction X' = [];\ X \cap L \subseteq X'\ ]\!] \Longrightarrow l \restriction X = []$
⟨*proof*⟩

lemma *non-empty-projection-on-subset*: $X \subseteq Y \wedge l_1 \restriction Y = l_2 \restriction Y \Longrightarrow\ l_1 \restriction X = l_2 \restriction X$
⟨*proof*⟩

lemma *projection-intersection-neutral*: $(set\ l \subseteq X) \Longrightarrow (l \restriction (X \cap Y) = l \restriction Y)$
⟨*proof*⟩

lemma *projection-commute*:
$(l \restriction X) \restriction Y = (l \restriction Y) \restriction X$
⟨*proof*⟩

lemma *projection-subset-elim*: $Y \subseteq X \Longrightarrow (l \restriction X) \restriction Y = l \restriction Y$
⟨*proof*⟩

lemma *projection-sequence*: $(xs \restriction X) \restriction Y = (xs \restriction (X \cap Y))$
⟨*proof*⟩

fun *merge* :: *'e set* ⇒ *'e set* ⇒ *'e list* ⇒ *'e list* ⇒ *'e list*
where
merge A B [] *t2* = *t2* |
merge A B t1 [] = *t1* |
merge A B (*e1* # *t1'*) (*e2* # *t2'*) = (*if e1* = *e2 then*
e1 # (*merge A B t1' t2'*)
else (*if e1* ∈ (*A* ∩ *B*) *then*
e2 # (*merge A B* (*e1* # *t1'*) *t2'*)
else e1 # (*merge A B t1'* (*e2* # *t2'*))))

lemma *merge-property*: $[\![set\ t1 \subseteq A;\ set\ t2 \subseteq B;\ t1 \upharpoonright B = t2 \upharpoonright A]\!]$
$\Longrightarrow$ *let* $t = (merge\ A\ B\ t1\ t2)$ *in* $(t \upharpoonright A = t1 \wedge t \upharpoonright B = t2 \wedge set\ t \subseteq ((set\ t1) \cup (set\ t2)))$
⟨*proof*⟩

end

3 System Specification

3.1 Event Systems

We define the system model of event systems as well as the parallel composition operator for event systems provided as part of MAKS in [3].

theory *EventSystems*
imports *../Basics/Prefix ../Basics/Projection*
begin

record $'e$ *ES-rec* =
E-ES :: $'e$ *set*
I-ES :: $'e$ *set*
O-ES :: $'e$ *set*
Tr-ES :: $('e\ list)$ *set*

abbreviation *ESrecEES* :: $'e$ *ES-rec* $\Rightarrow$ $'e$ *set*
$(E\_\ [1000]\ 1000)$
where
$E_{ES} \equiv (E\text{-}ES\ ES)$

abbreviation *ESrecIES* :: $'e$ *ES-rec* $\Rightarrow$ $'e$ *set*
$(I\_\ [1000]\ 1000)$
where
$I_{ES} \equiv (I\text{-}ES\ ES)$

abbreviation *ESrecOES* :: $'e$ *ES-rec* $\Rightarrow$ $'e$ *set*
$(O\_\ [1000]\ 1000)$
where
$O_{ES} \equiv (O\text{-}ES\ ES)$

abbreviation *ESrecTrES* :: $'e$ *ES-rec* $\Rightarrow$ $('e\ list)$ *set*
$(Tr\_\ [1000]\ 1000)$
where
$Tr_{ES} \equiv (Tr\text{-}ES\ ES)$

definition *es-inputs-are-events* :: $'e$ *ES-rec* $\Rightarrow$ *bool*
where
es-inputs-are-events ES $\equiv I_{ES} \subseteq E_{ES}$

definition *es-outputs-are-events* :: *'e ES-rec* ⇒ *bool*
where
es-outputs-are-events ES ≡ $O_{ES} \subseteq E_{ES}$

definition *es-inputs-outputs-disjoint* :: *'e ES-rec* ⇒ *bool*
where
es-inputs-outputs-disjoint ES ≡ $I_{ES} \cap O_{ES} = \{\}$

definition *traces-contain-events* :: *'e ES-rec* ⇒ *bool*
where
traces-contain-events ES ≡ ∀ $l \in Tr_{ES}$. ∀ $e \in (set\ l)$. $e \in E_{ES}$

definition *traces-prefixclosed* :: *'e ES-rec* ⇒ *bool*
where
traces-prefixclosed ES ≡ *prefixclosed* Tr_{ES}

definition *ES-valid* :: *'e ES-rec* ⇒ *bool*
where
ES-valid ES ≡
  es-inputs-are-events ES ∧ *es-outputs-are-events ES*
  ∧ *es-inputs-outputs-disjoint ES* ∧ *traces-contain-events ES*
  ∧ *traces-prefixclosed ES*

definition *total* :: *'e ES-rec* ⇒ *'e set* ⇒ *bool*
where
total ES E ≡ $E \subseteq E_{ES}$ ∧ (∀ $\tau \in Tr_{ES}$. ∀ $e \in E$. τ @ $[e] \in Tr_{ES}$)

lemma *totality*: ⟦ *total ES E*; $t \in Tr_{ES}$; *set* $t' \subseteq E$ ⟧ ⟹ t @ $t' \in Tr_{ES}$
  ⟨*proof*⟩

definition *composeES* :: *'e ES-rec* ⇒ *'e ES-rec* ⇒ *'e ES-rec*
where
composeES ES1 ES2 ≡
  (|
    E-ES = $E_{ES1} \cup E_{ES2}$,
    I-ES = $(I_{ES1} - O_{ES2}) \cup (I_{ES2} - O_{ES1})$,
    O-ES = $(O_{ES1} - I_{ES2}) \cup (O_{ES2} - I_{ES1})$,
    Tr-ES = {τ . (τ ⌈ E_{ES1}) ∈ Tr_{ES1} ∧ (τ ⌈ E_{ES2}) ∈ Tr_{ES2}
                ∧ (*set* $\tau \subseteq E_{ES1} \cup E_{ES2}$)}
  |)

abbreviation *composeESAbbrv* :: *'e ES-rec* ⇒ *'e ES-rec* ⇒ *'e ES-rec*
(- ‖ -[*1000*] *1000*)
where
ES1 ‖ *ES2* ≡ (*composeES ES1 ES2*)

definition *composable* :: *'e ES-rec* ⇒ *'e ES-rec* ⇒ *bool*
where
composable ES1 ES2 ≡ $(E_{ES1} \cap E_{ES2}) \subseteq ((O_{ES1} \cap I_{ES2}) \cup (O_{ES2} \cap I_{ES1}))$

lemma *composeES-yields-ES*:
⟦ *ES-valid ES1*; *ES-valid ES2* ⟧ ⟹ *ES-valid* (*ES1* ∥ *ES2*)
⟨*proof*⟩

end

3.2 State-Event Systems

We define the system model of state-event systems as well as the translation from state-event systems to event systems provided as part of MAKS in [3]. State-event systems are the basis for the unwinding theorems that we prove later in this entry.

theory *StateEventSystems*
imports *EventSystems*
begin

record (*'s*, *'e*) *SES-rec* =
S-SES :: *'s set*
s0-SES :: *'s*
E-SES :: *'e set*
I-SES :: *'e set*
O-SES :: *'e set*
T-SES :: *'s* ⇒ *'e* ⇀ *'s*

abbreviation *SESrecSSES* :: (*'s*, *'e*) *SES-rec* ⇒ *'s set*
(*S*_ [*1000*] *1000*)
where
S_{SES} ≡ (*S-SES SES*)

abbreviation *SESrecs0SES* :: (*'s*, *'e*) *SES-rec* ⇒ *'s*
(*s0*_ [*1000*] *1000*)
where
$s0_{SES}$ ≡ (*s0-SES SES*)

abbreviation *SESrecESES* :: (*'s*, *'e*) *SES-rec* ⇒ *'e set*
(*E*_ [*1000*] *1000*)
where
E_{SES} ≡ (*E-SES SES*)

abbreviation *SESrecISES* :: (*'s*, *'e*) *SES-rec* ⇒ *'e set*
(*I*_ [*1000*] *1000*)
where
I_{SES} ≡ (*I-SES SES*)

abbreviation *SESrecOSES* :: ('*s*, '*e*) *SES-rec* ⇒ '*e set*
(*O-* [*1000*] *1000*)
where
$O_{SES} \equiv$ (*O-SES SES*)

abbreviation *SESrecTSES* :: ('*s*, '*e*) *SES-rec* ⇒ ('*s* ⇒ '*e* ⇀ '*s*)
(*T-* [*1000*] *1000*)
where
$T_{SES} \equiv$ (*T-SES SES*)

abbreviation *TSESpred* :: '*s* ⇒ '*e* ⇒ ('*s*, '*e*) *SES-rec* ⇒ '*s* ⇒ *bool*
(- -⟶- - [*100*,*100*,*100*,*100*] *100*)
where
$s\ e \longrightarrow_{SES} s' \equiv (T_{SES}\ s\ e = Some\ s')$

definition *s0-is-state* :: ('*s*, '*e*) *SES-rec* ⇒ *bool*
where
$s0\text{-}is\text{-}state\ SES \equiv s0_{SES} \in S_{SES}$

definition *ses-inputs-are-events* :: ('*s*, '*e*) *SES-rec* ⇒ *bool*
where
$ses\text{-}inputs\text{-}are\text{-}events\ SES \equiv I_{SES} \subseteq E_{SES}$

definition *ses-outputs-are-events* :: ('*s*, '*e*) *SES-rec* ⇒ *bool*
where
$ses\text{-}outputs\text{-}are\text{-}events\ SES \equiv O_{SES} \subseteq E_{SES}$

definition *ses-inputs-outputs-disjoint* :: ('*s*, '*e*) *SES-rec* ⇒ *bool*
where
$ses\text{-}inputs\text{-}outputs\text{-}disjoint\ SES \equiv I_{SES} \cap O_{SES} = \{\}$

definition *correct-transition-relation* :: ('*s*, '*e*) *SES-rec* ⇒ *bool*
where
correct-transition-relation SES ≡
$\forall x\ y\ z.\ x\ y \longrightarrow_{SES} z \longrightarrow ((x \in S_{SES}) \wedge (y \in E_{SES}) \wedge (z \in S_{SES}))$

definition *SES-valid* :: ('*s*, '*e*) *SES-rec* ⇒ *bool*
where
SES-valid SES ≡
s0-is-state SES ∧ *ses-inputs-are-events SES*
∧ *ses-outputs-are-events SES* ∧ *ses-inputs-outputs-disjoint SES* ∧
correct-transition-relation SES

primrec *path* :: ('*s*, '*e*) *SES-rec* ⇒ '*s* ⇒ '*e list* ⇀ '*s*
where
path-empt: *path SES s1* [] = (*Some s1*) |

path-nonempt: *path SES s1* $(e \# t) =$
$(if\ (\exists\, s2.\ s1\ e \longrightarrow_{SES} s2)$
then $(path\ SES\ (the\ (T_{SES}\ s1\ e))\ t)$
else None)

abbreviation *pathpred* :: $'s \Rightarrow 'e\ list \Rightarrow ('s, 'e)\ SES\text{-}rec \Rightarrow 's \Rightarrow bool$
$(\text{-}\ \text{-}\Longrightarrow\text{-}\ \text{-}\ [100, 100, 100, 100]\ 100)$
where
$s\ t\Longrightarrow_{SES} s' \equiv path\ SES\ s\ t = Some\ s'$

definition *reachable* :: $('s, 'e)\ SES\text{-}rec \Rightarrow 's \Rightarrow bool$
where
$reachable\ SES\ s \equiv (\exists\, t.\ s0_{SES}\ t\Longrightarrow_{SES} s)$

definition *enabled* :: $('s, 'e)\ SES\text{-}rec \Rightarrow 's \Rightarrow 'e\ list \Rightarrow bool$
where
$enabled\ SES\ s\ t \equiv (\exists\, s'.\ s\ t\Longrightarrow_{SES} s')$

definition *possible-traces* :: $('s, 'e)\ SES\text{-}rec \Rightarrow ('e\ list)\ set$
where
$possible\text{-}traces\ SES \equiv \{t.\ (enabled\ SES\ s0_{SES}\ t)\}$

definition *induceES* :: $('s, 'e)\ SES\text{-}rec \Rightarrow 'e\ ES\text{-}rec$
where
$induceES\ SES \equiv$
$(\!|$
$E\text{-}ES = E_{SES},$
$I\text{-}ES = I_{SES},$
$O\text{-}ES\ = O_{SES},$
$Tr\text{-}ES = possible\text{-}traces\ SES$
$|\!)$

lemma *none-remains-none* : $\bigwedge s\ e.\ (path\ SES\ s\ t) = None$
$\Longrightarrow (path\ SES\ s\ (t\ @\ [e])) = None$
⟨*proof*⟩

lemma *path-trans-single-neg*: $\bigwedge s1.\ [\![s1\ t\Longrightarrow_{SES} s2;\ \neg\ (s2\ e\longrightarrow_{SES} sn)]\!]$
$\Longrightarrow \neg\ (s1\ (t\ @\ [e])\Longrightarrow_{SES} sn)$
⟨*proof*⟩

lemma *path-split-single*: $s1\ (t@[e])\Longrightarrow_{SES} sn$
$\Longrightarrow \exists\, s'.\ s1\ t\Longrightarrow_{SES} s'\ \wedge\ s'\ e\longrightarrow_{SES} sn$
⟨*proof*⟩

lemma *path-trans-single*: $\bigwedge s.\ [\![\ s\ t{\Longrightarrow}_{SES}\ s';\ s'\ e{\longrightarrow}_{SES}\ sn\ ]\!]$
$\Longrightarrow s\ (t\ @\ [e]){\Longrightarrow}_{SES}\ sn$
⟨*proof*⟩

lemma *path-split*: $\bigwedge\ sn.\ [\![\ s1\ (t1\ @\ t2){\Longrightarrow}_{SES}\ sn\ ]\!]$
$\Longrightarrow (\exists\, s2.\ (s1\ t1{\Longrightarrow}_{SES}\ s2\ \wedge\ s2\ t2{\Longrightarrow}_{SES}\ sn))$
⟨*proof*⟩

lemma *path-trans*:
$\bigwedge sn.\ [\![\ s1\ l1{\Longrightarrow}_{SES}\ s2;\ s2\ l2{\Longrightarrow}_{SES}\ sn\ ]\!] \Longrightarrow s1\ (l1\ @\ l2){\Longrightarrow}_{SES}\ sn$
⟨*proof*⟩

lemma *enabledPrefixSingle* : $[\![\ enabled\ SES\ s\ (t@[e])\ ]\!] \Longrightarrow enabled\ SES\ s\ t$
⟨*proof*⟩

lemma *enabledPrefix* : $[\![\ enabled\ SES\ s\ (t1\ @\ t2)\ ]\!] \Longrightarrow enabled\ SES\ s\ t1$
⟨*proof*⟩

lemma *enabledPrefixSingleFinalStep* : $[\![\ enabled\ SES\ s\ (t@[e])\ ]\!] \Longrightarrow \exists\ t'\ t''.\ t'\ e{\longrightarrow}_{SES}\ t''$
⟨*proof*⟩

lemma *induceES-yields-ES*:
SES-valid SES $\Longrightarrow$ *ES-valid* (*induceES SES*)
⟨*proof*⟩

end

4 Security Specification

4.1 Views & Flow Policies

We define views, flow policies and how views can be derived from a given flow policy.

theory *Views*
imports *Main*
begin

record $'e$ *V-rec* =

$V :: 'e\ set$
$N :: 'e\ set$
$C :: 'e\ set$

abbreviation $VrecV :: 'e\ V\text{-}rec \Rightarrow 'e\ set$
$(V\_\ [100]\ 1000)$
where
$V_v \equiv (V\ v)$

abbreviation $VrecN :: 'e\ V\text{-}rec \Rightarrow 'e\ set$
$(N\_\ [100]\ 1000)$
where
$N_v \equiv (N\ v)$

abbreviation $VrecC :: 'e\ V\text{-}rec \Rightarrow 'e\ set$
$(C\_\ [100]\ 1000)$
where
$C_v \equiv (C\ v)$

definition $VN\text{-}disjoint :: 'e\ V\text{-}rec \Rightarrow bool$
where
$VN\text{-}disjoint\ v \equiv V_v \cap N_v = \{\}$

definition $VC\text{-}disjoint :: 'e\ V\text{-}rec \Rightarrow bool$
where
$VC\text{-}disjoint\ v \equiv V_v \cap C_v = \{\}$

definition $NC\text{-}disjoint :: 'e\ V\text{-}rec \Rightarrow bool$
where
$NC\text{-}disjoint\ v \equiv N_v \cap C_v = \{\}$

definition $V\text{-}valid :: 'e\ V\text{-}rec \Rightarrow bool$
where
$V\text{-}valid\ \ v \equiv VN\text{-}disjoint\ v \wedge VC\text{-}disjoint\ v \wedge NC\text{-}disjoint\ v$

definition $isViewOn :: 'e\ V\text{-}rec \Rightarrow 'e\ set \Rightarrow bool$
where
$isViewOn\ \mathcal{V}\ E \equiv V\text{-}valid\ \mathcal{V} \wedge V_{\mathcal{V}} \cup N_{\mathcal{V}} \cup C_{\mathcal{V}} = E$

end
theory *FlowPolicies*
imports *Views*
begin

record $'domain\ FlowPolicy\text{-}rec =$
$D :: 'domain\ set$
$v\text{-}rel :: ('domain \times 'domain)\ set$

n-rel :: (*'domain* × *'domain*) *set*
c-rel :: (*'domain* × *'domain*) *set*

definition *FlowPolicy* :: *'domain FlowPolicy-rec* ⇒ *bool*
where
FlowPolicy fp ≡
((*v-rel fp*) ∪ (*n-rel fp*) ∪ (*c-rel fp*) = ((*D fp*) × (*D fp*)))
∧ (*v-rel fp*) ∩ (*n-rel fp*) = {}
∧ (*v-rel fp*) ∩ (*c-rel fp*) = {}
∧ (*n-rel fp*) ∩ (*c-rel fp*) = {}
∧ (∀ *d* ∈ (*D fp*). (*d*, *d*) ∈ (*v-rel fp*))

type-synonym (*'e*, *'domain*) *dom-type* = *'e* ⇀ *'domain*

definition *dom* :: (*'e*, *'domain*) *dom-type* ⇒ *'domain set* ⇒ *'e set* ⇒ *bool*
where
dom domas dset es ≡
(∀ *e*. ∀ *d*. ((*domas e* = *Some d*) ⟶ (*e* ∈ *es* ∧ *d* ∈ *dset*)))

definition *view-dom* :: *'domain FlowPolicy-rec* ⇒ *'domain* ⇒ (*'e*, *'domain*) *dom-type* ⇒ *'e V-rec*
where
view-dom fp d domas ≡
(| *V* = {*e*. ∃ *d'*. (*domas e* = *Some d'* ∧ (*d'*, *d*) ∈ (*v-rel fp*))},
N = {*e*. ∃ *d'*. (*domas e* = *Some d'* ∧ (*d'*, *d*) ∈ (*n-rel fp*))},
C = {*e*. ∃ *d'*. (*domas e* = *Some d'* ∧ (*d'*, *d*) ∈ (*c-rel fp*))} |)

end

4.2 Basic Security Predicates

We define all 14 basic security predicates provided as part of MAKS in [3].

theory *BasicSecurityPredicates*
imports *Views ../Basics/Projection*
begin

definition *areTracesOver* :: (*'e list*) *set* ⇒ *'e set* ⇒ *bool*
where
areTracesOver Tr E ≡
∀ *τ* ∈ *Tr*. (*set τ*) ⊆ *E*

type-synonym *'e BSP* = *'e V-rec* ⇒ ((*'e list*) *set*) ⇒ *bool*

definition *BSP-valid* :: *'e BSP* ⇒ *bool*

where
BSP-valid bsp ≡
$\forall \mathcal{V}\ Tr\ E.$ (*isViewOn* $\mathcal{V}\ E \wedge$ *areTracesOver* $Tr\ E$)
$\longrightarrow (\exists\ Tr'.\ Tr' \supseteq Tr\ \wedge$ *bsp* $\mathcal{V}\ Tr')$

definition R :: *'e BSP*
where
$R\ \mathcal{V}\ Tr \equiv$
$\forall \tau \in Tr.\ \exists \tau' \in Tr.\ \tau' \restriction C_{\mathcal{V}} = [] \wedge \tau' \restriction V_{\mathcal{V}} = \tau \restriction V_{\mathcal{V}}$

lemma *BSP-valid-R*: *BSP-valid R*
⟨*proof*⟩

definition D :: *'e BSP*
where
$D\ \mathcal{V}\ Tr \equiv$
$\forall \alpha\ \beta.\ \forall c \in C_{\mathcal{V}}.\ ((\beta\ @\ [c]\ @\ \alpha) \in Tr \wedge \alpha \restriction C_{\mathcal{V}} = [])$
$\longrightarrow (\exists \alpha'\ \beta'.\ ((\beta'\ @\ \alpha') \in Tr \wedge \alpha' \restriction V_{\mathcal{V}} = \alpha \restriction V_{\mathcal{V}} \wedge \alpha' \restriction C_{\mathcal{V}} = []$
$\wedge\ \beta' \restriction (V_{\mathcal{V}} \cup C_{\mathcal{V}}) = \beta \restriction (V_{\mathcal{V}} \cup C_{\mathcal{V}})))$

lemma *BSP-valid-D*: *BSP-valid D*
⟨*proof*⟩

definition I :: *'e BSP*
where
$I\ \mathcal{V}\ Tr \equiv$
$\forall \alpha\ \beta.\ \forall c \in C_{\mathcal{V}}.\ ((\beta\ @\ \alpha) \in Tr \wedge \alpha \restriction C_{\mathcal{V}} = [])$
$\longrightarrow (\exists \alpha'\ \beta'.\ ((\beta'\ @\ [c]\ @\ \alpha') \in Tr \wedge \alpha' \restriction V_{\mathcal{V}} = \alpha \restriction V_{\mathcal{V}} \wedge \alpha' \restriction C_{\mathcal{V}} = []$
$\wedge\ \beta' \restriction (V_{\mathcal{V}} \cup C_{\mathcal{V}}) = \beta \restriction (V_{\mathcal{V}} \cup C_{\mathcal{V}})))$

lemma *BSP-valid-I*: *BSP-valid I*
⟨*proof*⟩

type-synonym *'e Rho* = *'e V-rec* ⇒ *'e set*

definition
Adm :: *'e V-rec* ⇒ *'e Rho* ⇒ (*'e list*) *set* ⇒ *'e list* ⇒ *'e* ⇒ *bool*
where
Adm $\mathcal{V}\ \varrho\ Tr\ \beta\ e \equiv$
$\exists \gamma.\ ((\gamma\ @\ [e]) \in Tr \wedge \gamma \restriction (\varrho\ \mathcal{V}) = \beta \restriction (\varrho\ \mathcal{V}))$

definition *IA* :: *'e Rho* ⇒ *'e BSP*
where
IA $\varrho\ \mathcal{V}\ Tr \equiv$
$\forall \alpha\ \beta.\ \forall c \in C_{\mathcal{V}}.\ ((\beta\ @\ \alpha) \in Tr \wedge \alpha \restriction C_{\mathcal{V}} = [] \wedge (Adm\ \mathcal{V}\ \varrho\ Tr\ \beta\ c))$
$\longrightarrow (\exists\ \alpha'\ \beta'.\ ((\beta'\ @\ [c]\ @\ \alpha') \in Tr) \wedge \alpha' \restriction V_{\mathcal{V}} = \alpha \restriction V_{\mathcal{V}}$

$$\wedge\ \alpha' \restriction C_{\mathcal{V}} = [] \wedge \beta' \restriction (V_{\mathcal{V}} \cup C_{\mathcal{V}}) = \beta \restriction (V_{\mathcal{V}} \cup C_{\mathcal{V}}))$$

lemma *BSP-valid-IA*: *BSP-valid* (*IA* ϱ)
⟨*proof*⟩

definition *BSD* :: *'e BSP*
where
BSD $\mathcal{V}$ *Tr* ≡
$\forall\, \alpha\ \beta.\ \forall\, c \in C_{\mathcal{V}}.\ ((\beta\ @\ [c]\ @\ \alpha) \in Tr \wedge \alpha \restriction C_{\mathcal{V}} = [])$
$\longrightarrow (\exists\, \alpha'.\ ((\beta\ @\ \alpha') \in Tr \wedge \alpha' \restriction V_{\mathcal{V}} = \alpha \restriction V_{\mathcal{V}} \wedge \alpha' \restriction C_{\mathcal{V}} = []))$

lemma *BSP-valid-BSD*: *BSP-valid BSD*
⟨*proof*⟩

definition *BSI* :: *'e BSP*
where
BSI $\mathcal{V}$ *Tr* ≡
$\forall\, \alpha\ \beta.\ \forall\, c \in C_{\mathcal{V}}.\ ((\beta\ @\ \alpha) \in Tr \wedge \alpha \restriction C_{\mathcal{V}} = [])$
$\longrightarrow (\exists\, \alpha'.\ ((\beta\ @\ [c]\ @\ \alpha') \in Tr \wedge \alpha' \restriction V_{\mathcal{V}} = \alpha \restriction V_{\mathcal{V}} \wedge \alpha' \restriction C_{\mathcal{V}} = []))$

lemma *BSP-valid-BSI*: *BSP-valid BSI*
⟨*proof*⟩

definition *BSIA* :: *'e Rho* ⇒ *'e BSP*
where
BSIA ϱ $\mathcal{V}$ *Tr* ≡
$\forall\, \alpha\ \beta.\ \forall\, c \in C_{\mathcal{V}}.\ ((\beta\ @\ \alpha) \in Tr \wedge \alpha \restriction C_{\mathcal{V}} = [] \wedge (Adm\ \mathcal{V}\ \varrho\ Tr\ \beta\ c))$
$\longrightarrow (\exists\, \alpha'.\ ((\beta\ @\ [c]\ @\ \alpha') \in Tr \wedge \alpha' \restriction V_{\mathcal{V}} = \alpha \restriction V_{\mathcal{V}} \wedge \alpha' \restriction C_{\mathcal{V}} = []))$

lemma *BSP-valid-BSIA*: *BSP-valid* (*BSIA* ϱ)
⟨*proof*⟩

record *'e Gamma* =
Nabla :: *'e set*
Delta :: *'e set*
Upsilon :: *'e set*

abbreviation *GammaNabla* :: *'e Gamma* ⇒ *'e set*
(∇- [*100*] *1000*)
where
$\nabla_\Gamma \equiv (Nabla\ \Gamma)$

abbreviation *GammaDelta* :: *'e Gamma* ⇒ *'e set*
(Δ- [*100*] *1000*)
where
$\Delta_\Gamma \equiv (Delta\ \Gamma)$

abbreviation *GammaUpsilon* :: *'e Gamma* ⇒ *'e set*
(ϒ_ [*100*] *1000*)
where
$\Upsilon_\Gamma \equiv (Upsilon\ \Gamma)$

definition *FCD* :: *'e Gamma* ⇒ *'e BSP*
where
FCD Γ $\mathcal{V}$ *Tr* ≡
$\forall\,\alpha\ \beta.\ \forall\, c{\in}(C_{\mathcal{V}} \cap \Upsilon_\Gamma).\ \forall\, v{\in}(V_{\mathcal{V}} \cap \nabla_\Gamma).$
$((\beta\ @\ [c,v]\ @\ \alpha) \in Tr \wedge \alpha \restriction C_{\mathcal{V}} = [])$
$\longrightarrow (\exists\,\alpha'.\ \exists\,\delta'.\ (set\ \delta') \subseteq (N_{\mathcal{V}} \cap \Delta_\Gamma)$
$\wedge\ ((\beta\ @\ \delta'\ @\ [v]\ @\ \alpha') \in Tr$
$\wedge\ \alpha' \restriction V_{\mathcal{V}} = \alpha \restriction V_{\mathcal{V}} \wedge \alpha' \restriction C_{\mathcal{V}} = []))$

lemma *BSP-valid-FCD*: *BSP-valid* (*FCD* Γ)
⟨*proof*⟩

definition *FCI* :: *'e Gamma* ⇒ *'e BSP*
where
FCI Γ $\mathcal{V}$ *Tr* ≡
$\forall\,\alpha\ \beta.\ \forall\, c{\in}(C_{\mathcal{V}} \cap \Upsilon_\Gamma).\ \forall\, v{\in}(V_{\mathcal{V}} \cap \nabla_\Gamma).$
$((\beta\ @\ [v]\ @\ \alpha) \in Tr \wedge \alpha \restriction C_{\mathcal{V}} = [])$
$\longrightarrow (\exists\,\alpha'.\ \exists\,\delta'.\ (set\ \delta') \subseteq (N_{\mathcal{V}} \cap \Delta_\Gamma)$
$\wedge\ ((\beta\ @\ [c]\ @\ \delta'\ @\ [v]\ @\ \alpha') \in Tr$
$\wedge\ \alpha' \restriction V_{\mathcal{V}} = \alpha \restriction V_{\mathcal{V}} \wedge \alpha' \restriction C_{\mathcal{V}} = []))$

lemma *BSP-valid-FCI*: *BSP-valid* (*FCI* Γ)
⟨*proof*⟩

definition *FCIA* :: *'e Rho* ⇒ *'e Gamma* ⇒ *'e BSP*
where
FCIA ϱ Γ $\mathcal{V}$ *Tr* ≡
$\forall\,\alpha\ \beta.\ \forall\, c{\in}(C_{\mathcal{V}} \cap \Upsilon_\Gamma).\ \forall\, v{\in}(V_{\mathcal{V}} \cap \nabla_\Gamma).$
$((\beta\ @\ [v]\ @\ \alpha) \in Tr \wedge \alpha \restriction C_{\mathcal{V}} = [] \wedge (Adm\ \mathcal{V}\ \varrho\ Tr\ \beta\ c))$
$\longrightarrow (\exists\,\alpha'.\ \exists\,\delta'.\ (set\ \delta') \subseteq (N_{\mathcal{V}} \cap \Delta_\Gamma)$
$\wedge\ ((\beta\ @\ [c]\ @\ \delta'\ @\ [v]\ @\ \alpha') \in Tr$
$\wedge\ \alpha' \restriction V_{\mathcal{V}} = \alpha \restriction V_{\mathcal{V}} \wedge \alpha' \restriction C_{\mathcal{V}} = []))$

lemma *BSP-valid-FCIA*: *BSP-valid* (*FCIA* ϱ Γ)
⟨*proof*⟩

definition *SR* :: *'e BSP*
where
SR $\mathcal{V}$ *Tr* ≡ $\forall\,\tau{\in}Tr.\ \tau \restriction (V_{\mathcal{V}} \cup N_{\mathcal{V}}) \in Tr$

lemma *BSP-valid SR*

⟨*proof*⟩

definition *SD* :: *'e BSP*
where
SD $\mathcal{V}$ *Tr* ≡
$\forall \alpha\ \beta.\ \forall c{\in}C_{\mathcal{V}}.\ ((\beta\ @\ [c]\ @\ \alpha) \in Tr \wedge \alpha{\restriction}C_{\mathcal{V}} = []) \longrightarrow \beta\ @\ \alpha \in Tr$

lemma *BSP-valid SD*
⟨*proof*⟩

definition *SI* :: *'e BSP*
where
SI $\mathcal{V}$ *Tr* ≡
$\forall \alpha\ \beta.\ \forall c{\in}C_{\mathcal{V}}.\ ((\beta\ @\ \alpha) \in Tr \wedge \alpha \restriction C_{\mathcal{V}} = []) \longrightarrow \beta\ @\ [c]\ @\ \alpha \in Tr$

lemma *BSP-valid SI*
⟨*proof*⟩

definition *SIA* :: *'e Rho* ⇒ *'e BSP*
where
SIA ϱ $\mathcal{V}$ *Tr* ≡
$\forall \alpha\ \beta.\ \forall c{\in}C_{\mathcal{V}}.\ ((\beta\ @\ \alpha) \in Tr \wedge \alpha \restriction C_{\mathcal{V}} = [] \wedge (Adm\ \mathcal{V}\ \varrho\ Tr\ \beta\ c))$
$\longrightarrow (\beta\ @\ [c]\ @\ \alpha) \in Tr$

lemma *BSP-valid* (*SIA* ϱ)
⟨*proof*⟩

end

4.3 Information-Flow Properties

We define the notion of information-flow properties from [3].

theory *InformationFlowProperties*
imports *BasicSecurityPredicates*
begin

type-synonym *'e SP* = (*'e BSP*) *set*

type-synonym *'e IFP-type* = (*'e V-rec set*) × *'e SP*

definition *IFP-valid* :: *'e set* ⇒ *'e IFP-type* ⇒ *bool*
where
IFP-valid E ifp ≡
$\forall \mathcal{V} \in (fst\ ifp).\ isViewOn\ \mathcal{V}\ E$
$\wedge\ (\forall BSP \in (snd\ ifp).\ BSP\text{-}valid\ BSP)$

definition *IFPIsSatisfied* :: *'e IFP-type* ⇒ (*'e list*) *set* ⇒ *bool*
where
IFPIsSatisfied ifp Tr ≡
∀ $\mathcal{V}$∈(*fst ifp*). ∀ *BSP*∈(*snd ifp*). *BSP* $\mathcal{V}$ *Tr*

end

4.4 Property Library

We define the representations of several possibilistic information-flow properties from the literature that are provided as part of MAKS in [3].

theory *PropertyLibrary*
imports *InformationFlowProperties ../SystemSpecification/EventSystems ../Verification/Basics/BSPTaxonomy*
begin

definition
HighInputsConfidential :: *'e set* ⇒ *'e set* ⇒ *'e set* ⇒ *'e V-rec*
where
HighInputsConfidential L H IE ≡ (| *V*=*L*, *N*=*H*−*IE*, *C*=*H* ∩ *IE* |)

definition *HighConfidential* :: *'e set* ⇒ *'e set* ⇒ *'e V-rec*
where
HighConfidential L H ≡ (| *V*=*L*, *N*={}, *C*=*H* |)

fun *interleaving* :: *'e list* ⇒ *'e list* ⇒ (*'e list*) *set*
where
interleaving t1 [] = {*t1*} |
interleaving [] *t2* = {*t2*} |
interleaving (*e1* # *t1*) (*e2* # *t2*) =
{*t*. (∃ *t'*. *t*=(*e1* # *t'*) ∧ *t'* ∈ *interleaving t1* (*e2* #*t2*))}
∪ {*t*. (∃ *t'*. *t*=(*e2* # *t'*) ∧ *t'* ∈ *interleaving* (*e1* # *t1*) *t2*)}

definition *GNI* :: *'e set* ⇒ *'e set* ⇒ *'e set* ⇒ *'e IFP-type*
where
GNI L H IE ≡ ({*HighInputsConfidential L H IE*}, {*BSD*, *BSI*})

lemma *GNI-valid*: *L* ∩ *H* = {} ⟹ *IFP-valid* (*L* ∪ *H*) (*GNI L H IE*)
⟨*proof*⟩

definition *litGNI* :: *'e set* ⇒ *'e set* ⇒ *'e set* ⇒ (*'e list*) *set* ⇒ *bool*
where
litGNI L H IE Tr ≡
∀ *t1 t2 t3*.

$t1 \mathbin{@} t2 \in Tr \wedge t3 \restriction (L \cup (H - IE)) = t2 \restriction (L \cup (H - IE))$
$\longrightarrow (\exists\ t4.\ t1 \mathbin{@} t4 \in Tr \wedge t4 \restriction (L \cup (H \cap IE)) = t3 \restriction (L \cup (H \cap IE)))$

definition *IBGNI* :: *'e set* ⇒ *'e set* ⇒ *'e set* ⇒ *'e IFP-type*
where *IBGNI L H IE* ≡ ({*HighInputsConfidential L H IE*}, {*D*, *I*})

lemma *IBGNI-valid*: $L \cap H = \{\} \Longrightarrow$ *IFP-valid* $(L \cup H)$ *(IBGNI L H IE)*
⟨*proof*⟩

definition
litIBGNI :: *'e set* ⇒ *'e set* ⇒ *'e set* ⇒ *('e list) set* ⇒ *bool*
where
litIBGNI L H IE Tr ≡
$\forall\ \tau\text{-}l \in Tr.\ \forall\ t\text{-}hi\ t.$
$(set\ t\text{-}hi) \subseteq (H \cap IE)\ \wedge t \in interleaving\ t\text{-}hi\ (\tau\text{-}l \restriction L)$
$\longrightarrow (\exists\ \tau' \in Tr.\ \tau' \restriction (L \cup (H \cap IE)) = t)$

definition *FC* :: *'e set* ⇒ *'e set* ⇒ *'e set* ⇒ *'e IFP-type*
where
FC L H IE ≡
({*HighInputsConfidential L H IE*},
{*BSD*, *BSI*, (*FCD* ⦇ *Nabla*=*IE*, *Delta*={}, *Upsilon*=*IE* ⦈),
(*FCI* ⦇ *Nabla*=*IE*, *Delta*={}, *Upsilon*=*IE* ⦈)})

lemma *FC-valid*: $L \cap H = \{\} \Longrightarrow$ *IFP-valid* $(L \cup H)$ *(FC L H IE)*
⟨*proof*⟩

definition *litFC* :: *'e set* ⇒ *'e set* ⇒ *'e set* ⇒ *('e list) set* ⇒ *bool*
where
litFC L H IE Tr ≡
$\forall\, t\text{-}1\ t\text{-}2.\ \forall\ hi \in (H \cap IE).$
(
$(\forall\ li \in (L \cap IE).$
$t\text{-}1 \mathbin{@} [li] \mathbin{@} t\text{-}2 \in Tr \wedge t\text{-}2 \restriction (H \cap IE) = []$
$\longrightarrow (\exists\ t\text{-}3.\ t\text{-}1 \mathbin{@} [hi] \mathbin{@} [li] \mathbin{@} t\text{-}3 \in Tr$
$\wedge\ t\text{-}3 \restriction L = t\text{-}2 \restriction L \wedge t\text{-}3 \restriction (H \cap IE) = []))$
$\wedge\ (t\text{-}1 \mathbin{@} t\text{-}2 \in Tr \wedge t\text{-}2 \restriction (H \cap IE) = []$
$\longrightarrow (\exists\ t\text{-}3.\ t\text{-}1 \mathbin{@} [hi] \mathbin{@} t\text{-}3 \in Tr$
$\wedge\ t\text{-}3 \restriction L = t\text{-}2 \restriction L \wedge t\text{-}3 \restriction (H \cap IE) = []))$
$\wedge\ (\forall\ li \in (L \cap IE).$
$t\text{-}1 \mathbin{@} [hi] \mathbin{@} [li] \mathbin{@} t\text{-}2 \in Tr \wedge t\text{-}2 \restriction (H \cap IE) = []$
$\longrightarrow (\exists\ t\text{-}3.\ t\text{-}1 \mathbin{@} [li] \mathbin{@} t\text{-}3 \in Tr$
$\wedge\ t\text{-}3 \restriction L = t\text{-}2 \restriction L \wedge t\text{-}3 \restriction (H \cap IE) = []))$
$\wedge\ (t\text{-}1 \mathbin{@} [hi] \mathbin{@} t\text{-}2 \in Tr \wedge t\text{-}2 \restriction (H \cap IE) = []$
$\longrightarrow (\exists\ t\text{-}3.\ t\text{-}1 \mathbin{@} t\text{-}3 \in Tr$
$\wedge\ t\text{-}3 \restriction L = t\text{-}2 \restriction L \wedge t\text{-}3 \restriction (H \cap IE) = []))$

)

definition *NDO* :: *'e set* ⇒ *'e set* ⇒ *'e set* ⇒ *'e IFP-type*
where
NDO UI L H ≡
({*HighConfidential L H*}, {*BSD*, (*BSIA* (λ *V*. $C_{\mathcal{V}}$ ∪ ($V_{\mathcal{V}}$ ∩ *UI*)))})

lemma *NDO-valid*: *L* ∩ *H* = {} ⟹ *IFP-valid* (*L* ∪ *H*) (*NDO UI L H*)
⟨*proof*⟩

definition *litNDO* :: *'e set* ⇒ *'e set* ⇒ *'e set* ⇒ (*'e list*) *set* ⇒ *bool*
where
litNDO UI L H Tr ≡
∀ *τ-l* ∈ *Tr*. ∀ *τ-hlui* ∈ *Tr*. ∀ *t*.
t↾*L* = *τ-l*↾*L* ∧ *t*↾(*H* ∪ (*L* ∩ *UI*)) = *τ-hlui*↾(*H* ∪ (*L* ∩ *UI*)) ⟶ *t* ∈ *Tr*

definition *NF* :: *'e set* ⇒ *'e set* ⇒ *'e IFP-type*
where
NF L H ≡ ({*HighConfidential L H*}, {*R*})

lemma *NF-valid*: *L* ∩ *H* = {} ⟹ *IFP-valid* (*L* ∪ *H*) (*NF L H*)
⟨*proof*⟩

definition *litNF* :: *'e set* ⇒ *'e set* ⇒ (*'e list*) *set* ⇒ *bool*
where
litNF L H Tr ≡ ∀ *τ* ∈ *Tr*. *τ* ↾ *L* ∈ *Tr*

definition *GNF* :: *'e set* ⇒ *'e set* ⇒ *'e set* ⇒ *'e IFP-type*
where
GNF L H IE ≡ ({*HighInputsConfidential L H IE*}, {*R*})

lemma *GNF-valid*: *L* ∩ *H* = {} ⟹ *IFP-valid* (*L* ∪ *H*) (*GNF L H IE*)
⟨*proof*⟩

definition *litGNF* :: *'e set* ⇒ *'e set* ⇒ *'e set* ⇒ (*'e list*) *set* ⇒ *bool*
where
litGNF L H IE Tr ≡
∀ *τ* ∈ *Tr*. ∃ *τ'* ∈ *Tr*. *τ'*↾ (*H* ∩ *IE*) = [] ∧ *τ'*↾ *L* = *τ* ↾ *L*

definition *SEP* :: *'e set* ⇒ *'e set* ⇒ *'e IFP-type*

where
SEP L H ≡ ({*HighConfidential L H*}, {*BSD*, (*BSIA* (λ $\mathcal{V}$. $C_{\mathcal{V}}$))})

lemma *SEP-valid*: $L \cap H = \{\} \Longrightarrow$ *IFP-valid* $(L \cup H)$ (*SEP L H*)
⟨*proof*⟩

definition *litSEP* :: *'e set* ⇒ *'e set* ⇒ (*'e list*) *set* ⇒ *bool*
where
litSEP L H Tr ≡
∀ τ-*l* ∈ *Tr*. ∀ τ-*h* ∈ *Tr*.
interleaving (τ-*l* ↾ *L*) (τ-*h* ↾ *H*) ⊆ {τ ∈ *Tr* . τ ↾ *L* = τ-*l* ↾ *L*}

definition *PSP* :: *'e set* ⇒ *'e set* ⇒ *'e IFP-type*
where
PSP L H ≡
({*HighConfidential L H*}, {*BSD*, (*BSIA* (λ $\mathcal{V}$. $C_{\mathcal{V}} \cup N_{\mathcal{V}} \cup V_{\mathcal{V}}$))})

lemma *PSP-valid*: $L \cap H = \{\} \Longrightarrow$ *IFP-valid* $(L \cup H)$ (*PSP L H*)
⟨*proof*⟩

definition *litPSP* :: *'e set* ⇒ *'e set* ⇒ (*'e list*) *set* ⇒ *bool*
where
litPSP L H Tr ≡
(∀ τ ∈ *Tr*. τ ↾ *L* ∈ *Tr*)
∧ (∀ α β. (β @ α) ∈ *Tr* ∧ (α ↾ *H*) = []
⟶ (∀ *h* ∈ *H*. β @ [*h*] ∈ *Tr* ⟶ β @ [*h*] @ α ∈ *Tr*))

end

5 Verification

5.1 Basic Definitions

We define when an event system and a state-event system are secure given an information-flow property.

theory *SecureSystems*
imports *../../SystemSpecification/StateEventSystems*
../../SecuritySpecification/InformationFlowProperties
begin

locale *SecureESIFP* =
fixes *ES* :: *'e ES-rec*
and *IFP* :: *'e IFP-type*

assumes *validES*: *ES-valid ES*
and *validIFPES*: *IFP-valid* E_{ES} *IFP*

context *SecureESIFP*
begin

definition *ES-sat-IFP* :: *bool*
where
ES-sat-IFP ≡ *IFPIsSatisfied IFP* Tr_{ES}

end

locale *SecureSESIFP* =
fixes *SES* :: ('*s*, '*e*) *SES-rec*
and *IFP* :: '*e IFP-type*

assumes *validSES*: *SES-valid SES*
and *validIFPSES*: *IFP-valid* E_{SES} *IFP*

sublocale *SecureSESIFP* ⊆ *SecureESIFP induceES SES IFP*
⟨*proof*⟩

context *SecureSESIFP*
begin

abbreviation *SES-sat-IFP*
where
SES-sat-IFP ≡ *ES-sat-IFP*

end

end

5.2 Taxonomy Results

We prove the taxonomy results from [3].

theory *BSPTaxonomy*
imports *../../SystemSpecification/EventSystems*
../../SecuritySpecification/BasicSecurityPredicates
begin

locale *BSPTaxonomyDifferentCorrections* =
fixes *ES* :: '*e ES-rec*

and $\mathcal{V}$ :: *'e V-rec*

assumes *validES*: *ES-valid ES*
and *VIsViewOnE*: *isViewOn* $\mathcal{V}$ E_{ES}

locale *BSPTaxonomyDifferentViews* =
fixes *ES* :: *'e ES-rec*
and $\mathcal{V}_1$ :: *'e V-rec*
and $\mathcal{V}_2$ :: *'e V-rec*

assumes *validES*: *ES-valid ES*
and $\mathcal{V}_1$*IsViewOnE*: *isViewOn* $\mathcal{V}_1$ E_{ES}
and $\mathcal{V}_2$*IsViewOnE*: *isViewOn* $\mathcal{V}_2$ E_{ES}

locale *BSPTaxonomyDifferentViewsFirstDim*= *BSPTaxonomyDifferentViews* +
assumes *V2-subset-V1*: $V_{\mathcal{V}_2} \subseteq V_{\mathcal{V}_1}$
and *N2-supset-N1*: $N_{\mathcal{V}_2} \supseteq N_{\mathcal{V}_1}$
and *C2-subset-C1*: $C_{\mathcal{V}_2} \subseteq C_{\mathcal{V}_1}$

sublocale *BSPTaxonomyDifferentViewsFirstDim* $\subseteq$ *BSPTaxonomyDifferentViews*
⟨*proof*⟩

locale *BSPTaxonomyDifferentViewsSecondDim*= *BSPTaxonomyDifferentViews* +
assumes *V2-subset-V1*: $V_{\mathcal{V}_2} \subseteq V_{\mathcal{V}_1}$
and *N2-supset-N1*: $N_{\mathcal{V}_2} \supseteq N_{\mathcal{V}_1}$
and *C2-equals-C1*: $C_{\mathcal{V}_2} = C_{\mathcal{V}_1}$

sublocale *BSPTaxonomyDifferentViewsSecondDim* $\subseteq$ *BSPTaxonomyDifferentViews*
⟨*proof*⟩

context *BSPTaxonomyDifferentCorrections*
begin

lemma *SR-implies-R*:
SR $\mathcal{V}$ $Tr_{ES} \Longrightarrow$ *R* $\mathcal{V}$ Tr_{ES}
⟨*proof*⟩

lemma *SD-implies-BSD* :
(*SD* $\mathcal{V}$ Tr_{ES}) $\Longrightarrow$ *BSD* $\mathcal{V}$ Tr_{ES}
⟨*proof*⟩

lemma *BSD-implies-D*:
BSD $\mathcal{V}$ $Tr_{ES} \Longrightarrow$ *D* $\mathcal{V}$ Tr_{ES}
⟨*proof*⟩

lemma *SD-implies-SR*:
SD $\mathcal{V}$ $Tr_{ES} \Longrightarrow$ *SR* $\mathcal{V}$ Tr_{ES}

⟨*proof*⟩

lemma *D-implies-R*:
$D\ \mathcal{V}\ Tr_{ES} \Longrightarrow R\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *SR-implies-R-for-modified-view* :
$[\![SR\ \mathcal{V}\ Tr_{ES};\ \mathcal{V}' = (\!|\ V = V_{\mathcal{V}} \cup N_{\mathcal{V}}\ ,\ N = \{\}\ ,\ C = C_{\mathcal{V}}\ |\!)]\!] \Longrightarrow R\ \mathcal{V}'\ Tr_{ES}$
⟨*proof*⟩

lemma *R-implies-SR-for-modified-view* :
$[\![R\ \mathcal{V}'\ Tr_{ES};\ \mathcal{V}' = (\!|\ V = V_{\mathcal{V}} \cup N_{\mathcal{V}}\ ,\ N = \{\}\ ,\ C = C_{\mathcal{V}}\ |\!)]\!] \Longrightarrow SR\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *SD-implies-BSD-for-modified-view* :
$[\![SD\ \mathcal{V}\ Tr_{ES};\ \mathcal{V}' = (\!|\ V = V_{\mathcal{V}} \cup N_{\mathcal{V}}\ ,\ N = \{\}\ ,\ C = C_{\mathcal{V}}\ |\!)]\!] \Longrightarrow BSD\ \mathcal{V}'\ Tr_{ES}$
⟨*proof*⟩

lemma *BSD-implies-SD-for-modified-view* :
$[\![BSD\ \mathcal{V}'\ Tr_{ES};\ \mathcal{V}' = (\!|\ V = V_{\mathcal{V}} \cup N_{\mathcal{V}}\ ,\ N = \{\}\ ,\ C = C_{\mathcal{V}}\ |\!)]\!] \Longrightarrow SD\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *SD-implies-FCD*:
$(SD\ \mathcal{V}\ Tr_{ES}) \Longrightarrow FCD\ \Gamma\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *SI-implies-BSI* :
$(SI\ \mathcal{V}\ Tr_{ES}) \Longrightarrow BSI\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *BSI-implies-I*:
$(BSI\ \mathcal{V}\ Tr_{ES}) \Longrightarrow (I\ \mathcal{V}\ Tr_{ES})$
⟨*proof*⟩

lemma *SIA-implies-BSIA*:
$(SIA\ \varrho\ \mathcal{V}\ Tr_{ES}) \Longrightarrow (BSIA\ \varrho\ \mathcal{V}\ Tr_{ES})$
⟨*proof*⟩

lemma *BSIA-implies-IA*:
$(BSIA\ \varrho\ \mathcal{V}\ Tr_{ES}) \Longrightarrow (IA\ \varrho\ \mathcal{V}\ Tr_{ES})$
⟨*proof*⟩

lemma *SI-implies-SIA*:
$SI\ \mathcal{V}\ Tr_{ES} \Longrightarrow SIA\ \varrho\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *BSI-implies-BSIA*:
$BSI\ \mathcal{V}\ Tr_{ES} \Longrightarrow BSIA\ \varrho\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *I-implies-IA*:
$I\ \mathcal{V}\ Tr_{ES} \Longrightarrow IA\ \varrho\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *SI-implies-BSI-for-modified-view* :
$[\![SI\ \mathcal{V}\ Tr_{ES};\ \mathcal{V}' = (\!|\ V = V_{\mathcal{V}} \cup N_{\mathcal{V}}\ ,\ N = \{\}\ ,\ C = C_{\mathcal{V}}\ |\!)]\!] \Longrightarrow BSI\ \mathcal{V}'\ Tr_{ES}$
⟨*proof*⟩

lemma *BSI-implies-SI-for-modified-view* :
$[\![BSI\ \mathcal{V}'\ Tr_{ES};\ \mathcal{V}' = (\!|\ V = V_{\mathcal{V}} \cup N_{\mathcal{V}}\ ,\ N = \{\}\ ,\ C = C_{\mathcal{V}}\ |\!)]\!] \Longrightarrow SI\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *SIA-implies-BSIA-for-modified-view* :
$[\![SIA\ \varrho\ \mathcal{V}\ Tr_{ES};\ \mathcal{V}' = (\!|\ V = V_{\mathcal{V}} \cup N_{\mathcal{V}}\ ,\ N = \{\}\ ,\ C = C_{\mathcal{V}}\ |\!)\ ;\ \varrho\ \mathcal{V} = \varrho'\ \mathcal{V}']\!] \Longrightarrow BSIA\ \varrho'\ \mathcal{V}'\ Tr_{ES}$
⟨*proof*⟩

lemma *BSIA-implies-SIA-for-modified-view* :
$[\![BSIA\ \varrho'\ \mathcal{V}'\ Tr_{ES};\ \mathcal{V}' = (\!|\ V = V_{\mathcal{V}} \cup N_{\mathcal{V}}\ ,\ N = \{\}\ ,\ C = C_{\mathcal{V}}\ |\!);\ \varrho\ \mathcal{V} = \varrho'\ \mathcal{V}']\!] \Longrightarrow SIA\ \varrho\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩
end

lemma *Adm-implies-Adm-for-modified-rho*:
$[\![\ Adm\ \mathcal{V}_2\ \varrho_2\ Tr\ \alpha\ e; \varrho_2(\mathcal{V}_2) \supseteq \varrho_1(\mathcal{V}_1)]\!] \Longrightarrow Adm\ \mathcal{V}_1\ \varrho_1\ Tr\ \alpha\ e$
⟨*proof*⟩

context *BSPTaxonomyDifferentCorrections*
begin

lemma *SI-implies-FCI*:
$(SI\ \mathcal{V}\ Tr_{ES}) \Longrightarrow FCI\ \Gamma\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *SIA-implies-FCIA*:
$(SIA\ \varrho\ \mathcal{V}\ Tr_{ES}) \Longrightarrow FCIA\ \varrho\ \Gamma\ \mathcal{V}\ Tr_{ES}$

⟨*proof*⟩

lemma *FCI-implies-FCIA*:
$(FCI\ \Gamma\ \mathcal{V}\ Tr_{ES}) \Longrightarrow FCIA\ \varrho\ \Gamma\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *Trivially-fulfilled-SR-C-empty*:
$C_{\mathcal{V}} = \{\} \Longrightarrow SR\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *Trivially-fulfilled-R-C-empty*:
$C_{\mathcal{V}} = \{\} \Longrightarrow R\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *Trivially-fulfilled-SD-C-empty*:
$C_{\mathcal{V}} = \{\} \Longrightarrow SD\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *Trivially-fulfilled-BSD-C-empty*:
$C_{\mathcal{V}} = \{\} \Longrightarrow BSD\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *Trivially-fulfilled-D-C-empty*:
$C_{\mathcal{V}} = \{\} \Longrightarrow D\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *Trivially-fulfilled-FCD-C-empty*:
$C_{\mathcal{V}} = \{\} \Longrightarrow FCD\ \Gamma\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *Trivially-fullfilled-R-V-empty*:
$V_{\mathcal{V}}=\{\} \Longrightarrow R\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *Trivially-fulfilled-BSD-V-empty*:
$V_{\mathcal{V}} = \{\} \Longrightarrow BSD\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *Trivially-fulfilled-D-V-empty*:
$V_{\mathcal{V}} = \{\} \Longrightarrow D\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *Trivially-fulfilled-FCD-V-empty*:
$V_{\mathcal{V}} = \{\} \Longrightarrow FCD\ \Gamma\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *Trivially-fulfilled-FCD-Nabla-*Υ*-empty*:
$[\![\nabla_\Gamma=\{\} \vee \Upsilon_\Gamma=\{\}]\!] \Longrightarrow FCD\ \Gamma\ \mathcal{V}\ Tr_{ES}$

⟨*proof*⟩

lemma *Trivially-fulfilled-FCD-N-subseteq-Δ-and-BSD*:
$[\![N_{\mathcal{V}} \subseteq \Delta_\Gamma;\ BSD\ \mathcal{V}\ Tr_{ES}]\!] \Longrightarrow FCD\ \Gamma\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *Trivially-fulfilled-SI-C-empty*:
$C_{\mathcal{V}} = \{\} \Longrightarrow SI\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *Trivially-fulfilled-BSI-C-empty*:
$C_{\mathcal{V}} = \{\} \Longrightarrow BSI\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *Trivially-fulfilled-I-C-empty*:
$C_{\mathcal{V}} = \{\} \Longrightarrow I\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *Trivially-fulfilled-FCI-C-empty*:
$C_{\mathcal{V}} = \{\} \Longrightarrow FCI\ \Gamma\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *Trivially-fulfilled-SIA-C-empty*:
$C_{\mathcal{V}} = \{\} \Longrightarrow SIA\ \varrho\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *Trivially-fulfilled-BSIA-C-empty*:
$C_{\mathcal{V}} = \{\} \Longrightarrow BSIA\ \varrho\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *Trivially-fulfilled-IA-C-empty*:
$C_{\mathcal{V}} = \{\} \Longrightarrow IA\ \varrho\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *Trivially-fulfilled-FCIA-C-empty*:
$C_{\mathcal{V}} = \{\} \Longrightarrow FCIA\ \Gamma\ \varrho\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *Trivially-fulfilled-FCI-V-empty*:
$V_{\mathcal{V}} = \{\} \Longrightarrow FCI\ \Gamma\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *Trivially-fulfilled-FCIA-V-empty*:
$V_{\mathcal{V}} = \{\} \Longrightarrow FCIA\ \varrho\ \Gamma\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *Trivially-fulfilled-BSIA-V-empty-rho-subseteq-C-N*:
$[\![V_{\mathcal{V}} = \{\};\ \varrho\ \mathcal{V} \supseteq (C_{\mathcal{V}} \cup N_{\mathcal{V}})\ ]\!] \Longrightarrow BSIA\ \varrho\ \ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *Trivially-fulfilled-IA-V-empty-rho-subseteq-C-N*:

$[\![V_{\mathcal{V}} = \{\};\ \varrho\ \mathcal{V} \supseteq (C_{\mathcal{V}} \cup N_{\mathcal{V}})\ ]\!] \Longrightarrow IA\ \varrho\ \ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *Trivially-fulfilled-BSI-V-empty-total-ES-C*:
$[\![V_{\mathcal{V}} = \{\};\ total\ ES\ C_{\mathcal{V}}\ ]\!] \Longrightarrow BSI\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *Trivially-fulfilled-I-V-empty-total-ES-C*:
$[\![V_{\mathcal{V}} = \{\};\ total\ ES\ C_{\mathcal{V}}\ ]\!] \Longrightarrow I\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *Trivially-fulfilled-FCI-Nabla-ϒ-empty*:
$[\![\nabla_\Gamma{=}\{\} \vee \Upsilon_\Gamma{=}\{\}]\!] \Longrightarrow FCI\ \Gamma\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *Trivially-fulfilled-FCIA-Nabla-ϒ-empty*:
$[\![\nabla_\Gamma{=}\{\} \vee \Upsilon_\Gamma{=}\{\}]\!] \Longrightarrow FCIA\ \varrho\ \Gamma\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *Trivially-fulfilled-FCI-N-subseteq-Δ-and-BSI*:
$[\![N_{\mathcal{V}} \subseteq \Delta_\Gamma;\ BSI\ \mathcal{V}\ Tr_{ES}]\!] \Longrightarrow FCI\ \Gamma\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

lemma *Trivially-fulfilled-FCIA-N-subseteq-Δ-and-BSIA*:
$[\![N_{\mathcal{V}} \subseteq \Delta_\Gamma;\ BSIA\ \varrho\ \mathcal{V}\ Tr_{ES}]\!] \Longrightarrow FCIA\ \varrho\ \Gamma\ \mathcal{V}\ Tr_{ES}$
⟨*proof*⟩

end

context *BSPTaxonomyDifferentViewsFirstDim*
begin

lemma *R-implies-R-for-modified-view*:
$R\ \mathcal{V}_1\ Tr_{ES} \Longrightarrow R\ \mathcal{V}_2\ Tr_{ES}$
⟨*proof*⟩

lemma *BSD-implies-BSD-for-modified-view*:
$BSD\ \mathcal{V}_1\ Tr_{ES} \Longrightarrow BSD\ \mathcal{V}_2\ Tr_{ES}$
⟨*proof*⟩

lemma *D-implies-D-for-modified-view*:
$D\ \mathcal{V}_1\ Tr_{ES} \Longrightarrow D\ \mathcal{V}_2\ Tr_{ES}$
⟨*proof*⟩
end

context *BSPTaxonomyDifferentViewsSecondDim*
begin

lemma *FCD-implies-FCD-for-modified-view-gamma*:
$[\![FCD\ \Gamma_1\ \mathcal{V}_1\ Tr_{ES};$
$\quad V_{\mathcal{V}_2} \cap \nabla_{\Gamma_2} \subseteq\ V_{\mathcal{V}_1} \cap \nabla_{\Gamma_1};\ \ N_{\mathcal{V}_2} \cap \Delta_{\Gamma_2} \supseteq\ N_{\mathcal{V}_1} \cap \Delta_{\Gamma_1};\ \ C_{\mathcal{V}_2} \cap \Upsilon_{\Gamma_2} \subseteq\ C_{\mathcal{V}_1} \cap \Upsilon_{\Gamma_1}\ ]\!]$

$\Longrightarrow$ *FCD* Γ_2 $\mathcal{V}_2$ Tr_{ES}
⟨*proof*⟩

lemma *SI-implies-SI-for-modified-view* :
SI $\mathcal{V}_1$ Tr_{ES} $\Longrightarrow$ *SI* $\mathcal{V}_2$ Tr_{ES}
⟨*proof*⟩

lemma *BSI-implies-BSI-for-modified-view* :
BSI $\mathcal{V}_1$ Tr_{ES} $\Longrightarrow$ *BSI* $\mathcal{V}_2$ Tr_{ES}
⟨*proof*⟩

lemma *I-implies-I-for-modified-view* :
I $\mathcal{V}_1$ Tr_{ES} $\Longrightarrow$ *I* $\mathcal{V}_2$ Tr_{ES}
⟨*proof*⟩

lemma *SIA-implies-SIA-for-modified-view* :
$[\![$*SIA* ϱ_1 $\mathcal{V}_1$ Tr_{ES}; $\varrho_2(\mathcal{V}_2) \supseteq \varrho_1(\mathcal{V}_1)$ $]\!]$ $\Longrightarrow$ *SIA* ϱ_2 $\mathcal{V}_2$ Tr_{ES}
⟨*proof*⟩

lemma *BSIA-implies-BSIA-for-modified-view* :
$[\![$*BSIA* ϱ_1 $\mathcal{V}_1$ Tr_{ES}; $\varrho_2(\mathcal{V}_2) \supseteq \varrho_1(\mathcal{V}_1)$ $]\!]$ $\Longrightarrow$ *BSIA* ϱ_2 $\mathcal{V}_2$ Tr_{ES}
⟨*proof*⟩

lemma *IA-implies-IA-for-modified-view* :
$[\![$*IA* ϱ_1 $\mathcal{V}_1$ Tr_{ES}; $\varrho_2(\mathcal{V}_2) \supseteq \varrho_1(\mathcal{V}_1)$ $]\!]$ $\Longrightarrow$ *IA* ϱ_2 $\mathcal{V}_2$ Tr_{ES}
⟨*proof*⟩

lemma *FCI-implies-FCI-for-modified-view-gamma*:
$[\![$*FCI* Γ_1 $\mathcal{V}_1$ Tr_{ES};
$V_{\mathcal{V}_2} \cap \nabla_{\Gamma_2} \subseteq V_{\mathcal{V}_1} \cap \nabla_{\Gamma_1}$; $N_{\mathcal{V}_2} \cap \Delta_{\Gamma_2} \supseteq N_{\mathcal{V}_1} \cap \Delta_{\Gamma_1}$; $C_{\mathcal{V}_2} \cap \Upsilon_{\Gamma_2} \subseteq C_{\mathcal{V}_1} \cap \Upsilon_{\Gamma_1}$ $]\!]$
$\Longrightarrow$ *FCI* Γ_2 $\mathcal{V}_2$ Tr_{ES}
⟨*proof*⟩

lemma *FCIA-implies-FCIA-for-modified-view-rho-gamma*:
$[\![$*FCIA* ϱ_1 Γ_1 $\mathcal{V}_1$ Tr_{ES}; $\varrho_2(\mathcal{V}_2) \supseteq \varrho_1(\mathcal{V}_1)$;
$V_{\mathcal{V}_2} \cap \nabla_{\Gamma_2} \subseteq V_{\mathcal{V}_1} \cap \nabla_{\Gamma_1}$; $N_{\mathcal{V}_2} \cap \Delta_{\Gamma_2} \supseteq N_{\mathcal{V}_1} \cap \Delta_{\Gamma_1}$; $C_{\mathcal{V}_2} \cap \Upsilon_{\Gamma_2} \subseteq C_{\mathcal{V}_1} \cap \Upsilon_{\Gamma_1}$ $]\!]$
$\Longrightarrow$ *FCIA* ϱ_2 Γ_2 $\mathcal{V}_2$ Tr_{ES}
⟨*proof*⟩
end

end

5.3 Unwinding

We define the unwinding conditions provided in [3] and prove the unwinding theorems from [3] that use these unwinding conditions.

5.3.1 Unwinding Conditions

theory *UnwindingConditions*
imports *../Basics/BSPTaxonomy*
../../SystemSpecification/StateEventSystems
begin

locale *Unwinding* =
fixes *SES* :: ('*s*, '*e*) *SES-rec*
and $\mathcal{V}$ :: '*e V-rec*

assumes *validSES*: *SES-valid SES*
and *validVU*: *isViewOn* $\mathcal{V}$ E_{SES}

sublocale *Unwinding* $\subseteq$ *BSPTaxonomyDifferentCorrections induceES SES* $\mathcal{V}$
⟨*proof*⟩

context *Unwinding*
begin

definition *osc* :: '*s rel* ⇒ *bool*
where
osc ur ≡
$\forall$ *s1* ∈ S_{SES}. $\forall$ *s1'* ∈ S_{SES}. $\forall$ *s2'* ∈ S_{SES}. $\forall$ *e* ∈ (E_{SES} − $C_{\mathcal{V}}$).
(*reachable SES s1* ∧ *reachable SES s1'*
∧ *s1'* *e*$\longrightarrow_{SES}$ *s2'* ∧ (*s1'*, *s1*) ∈ *ur*)
⟶ (∃ *s2* ∈ S_{SES}. ∃ δ. δ ↾ $C_{\mathcal{V}}$ = [] ∧ δ ↾ $V_{\mathcal{V}}$ = [*e*] ↾ $V_{\mathcal{V}}$
∧ *s1* δ$\Longrightarrow_{SES}$ *s2* ∧ (*s2'*, *s2*) ∈ *ur*)

definition *lrf* :: '*s rel* ⇒ *bool*
where
lrf ur ≡
$\forall$ *s* ∈ S_{SES}. $\forall$ *s'* ∈ S_{SES}. $\forall$ *c* ∈ $C_{\mathcal{V}}$.
((*reachable SES s* ∧ *s c*$\longrightarrow_{SES}$ *s'*) ⟶ (*s'*, *s*) ∈ *ur*)

definition *lrb* :: '*s rel* ⇒ *bool*
where
lrb ur ≡ $\forall$ *s* ∈ S_{SES}. $\forall$ *c* ∈ $C_{\mathcal{V}}$.
(*reachable SES s* ⟶ (∃ *s'* ∈ S_{SES}. (*s c*$\longrightarrow_{SES}$ *s'* ∧ ((*s*, *s'*) ∈ *ur*))))

definition *fcrf* :: *'e Gamma* ⇒ *'s rel* ⇒ *bool*
where
fcrf Γ *ur* ≡
∀ *c* ∈ ($C_{\mathcal{V}}$ ∩ Υ_Γ). ∀ *v* ∈ ($V_{\mathcal{V}}$ ∩ ∇_Γ). ∀ *s* ∈ S_{SES}. ∀ *s'* ∈ S_{SES}.
((*reachable SES s* ∧ *s* ([*c*] @ [*v*])⟹$_{SES}$ *s'*)
⟶ (∃ *s''* ∈ S_{SES}. ∃ δ. (∀ *d* ∈ (*set* δ). *d* ∈ ($N_{\mathcal{V}}$ ∩ Δ_Γ)) ∧
s (δ @ [*v*])⟹$_{SES}$ *s''* ∧ (*s'*, *s''*) ∈ *ur*))

definition *fcrb* :: *'e Gamma* ⇒ *'s rel* ⇒ *bool*
where
fcrb Γ *ur* ≡
∀ *c* ∈ ($C_{\mathcal{V}}$ ∩ Υ_Γ). ∀ *v* ∈ ($V_{\mathcal{V}}$ ∩ ∇_Γ). ∀ *s* ∈ S_{SES}. ∀ *s''* ∈ S_{SES}.
((*reachable SES s* ∧ *s v*⟶$_{SES}$ *s''*)
⟶ (∃ *s'* ∈ S_{SES}. ∃ δ. (∀ *d* ∈ (*set* δ). *d* ∈ ($N_{\mathcal{V}}$ ∩ Δ_Γ)) ∧
s ([*c*] @ δ @ [*v*])⟹$_{SES}$ *s'* ∧ (*s''*, *s'*) ∈ *ur*))

definition *En* :: *'e Rho* ⇒ *'s* ⇒ *'e* ⇒ *bool*
where
En ϱ *s e* ≡
∃ β γ. ∃ *s'* ∈ S_{SES}. ∃ *s''* ∈ S_{SES}.
s0$_{SES}$ β⟹$_{SES}$ *s* ∧ (γ ↾ (ϱ $\mathcal{V}$) = β ↾ (ϱ $\mathcal{V}$))
∧ *s0*$_{SES}$ γ⟹$_{SES}$ *s'* ∧ *s'* *e*⟶$_{SES}$ *s''*

definition *lrbe* :: *'e Rho* ⇒ *'s rel* ⇒ *bool*
where
lrbe ϱ *ur* ≡
∀ *s* ∈ S_{SES}. ∀ *c* ∈ $C_{\mathcal{V}}$.
((*reachable SES s* ∧ (*En* ϱ *s c*))
⟶ (∃ *s'* ∈ S_{SES}. (*s c*⟶$_{SES}$ *s'* ∧ (*s*, *s'*) ∈ *ur*)))

definition *fcrbe* :: *'e Gamma* ⇒ *'e Rho* ⇒ *'s rel* ⇒ *bool*
where
fcrbe Γ ϱ *ur* ≡
∀ *c* ∈ ($C_{\mathcal{V}}$ ∩ Υ_Γ). ∀ *v* ∈ ($V_{\mathcal{V}}$ ∩ ∇_Γ). ∀ *s* ∈ S_{SES}. ∀ *s''* ∈ S_{SES}.
((*reachable SES s* ∧ *s v*⟶$_{SES}$ *s''* ∧ (*En* ϱ *s c*))
⟶ (∃ *s'* ∈ S_{SES}. ∃ δ. (∀ *d* ∈ (*set* δ). *d* ∈ ($N_{\mathcal{V}}$ ∩ Δ_Γ)) ∧
s ([*c*] @ δ @ [*v*])⟹$_{SES}$ *s'* ∧ (*s''*, *s'*) ∈ *ur*))

end

end

5.3.2 Auxiliary Results

theory *AuxiliaryLemmas*
imports *UnwindingConditions*
begin

context *Unwinding*
begin

lemma *osc-property*:
$\bigwedge s1\ s1'.\ [\![\ osc\ ur;\ s1 \in S_{SES};\ s1' \in S_{SES};\ \alpha \restriction C_{\mathcal{V}} = [];$
$reachable\ SES\ s1;\ reachable\ SES\ s1';\ enabled\ SES\ s1'\ \alpha;\ (s1', s1) \in ur\ ]\!]$
$\Longrightarrow (\exists \alpha'.\ \alpha' \restriction C_{\mathcal{V}} = [] \wedge \alpha' \restriction V_{\mathcal{V}} = \alpha \restriction V_{\mathcal{V}} \wedge enabled\ SES\ s1\ \alpha')$
⟨*proof*⟩

lemma *path-state-closure*: $[\![\ s\ \tau \Longrightarrow_{SES} s';\ s \in S_{SES}\ ]\!] \Longrightarrow s' \in S_{SES}$
(**is** $[\![\ ?P\ s\ \tau\ s';\ ?S\ s\ SES\ ]\!] \Longrightarrow ?S\ s'\ SES$)
⟨*proof*⟩

theorem *En-to-Adm*:
$[\![\ reachable\ SES\ s;\ En\ \varrho\ s\ e]\!]$
$\Longrightarrow \exists \beta.\ (\ s0_{SES}\ \beta \Longrightarrow_{SES} s \wedge Adm\ \mathcal{V}\ \varrho\ Tr_{(induceES\ SES)}\ \beta\ e\ )$
⟨*proof*⟩

theorem *Adm-to-En*:
$[\![\ \beta \in Tr_{(induceES\ SES)};\ Adm\ \mathcal{V}\ \varrho\ Tr_{(induceES\ SES)}\ \beta\ e\ ]\!]$
$\Longrightarrow \exists s \in S_{SES}.\ (s0_{SES}\ \beta \Longrightarrow_{SES} s \wedge En\ \varrho\ s\ e)$
⟨*proof*⟩

lemma *state-from-induceES-trace*:
$[\![\ (\beta\ @\ \alpha) \in Tr_{(induceES\ SES)}\ ]\!]$
$\Longrightarrow \exists s \in S_{SES}.\ s0_{SES}\ \beta \Longrightarrow_{SES} s \wedge enabled\ SES\ s\ \alpha \wedge reachable\ SES\ s$
⟨*proof*⟩

lemma *path-split2*:$s0_{SES}\ (\beta\ @\ \alpha) \Longrightarrow_{SES} s$
$\Longrightarrow \exists s' \in S_{SES}.\ (\ s0_{SES}\ \beta \Longrightarrow_{SES} s' \wedge s'\ \alpha \Longrightarrow_{SES} s \wedge reachable\ SES\ s'\ )$
⟨*proof*⟩

lemma *path-split-single2*:
$s0_{SES}\ (\beta\ @\ [x]) \Longrightarrow_{SES} s$
$\Longrightarrow \exists s' \in S_{SES}.\ (\ s0_{SES}\ \beta \Longrightarrow_{SES} s' \wedge s'\ x \longrightarrow_{SES} s \wedge reachable\ SES\ s'\ )$
⟨*proof*⟩

lemma *modified-view-valid*: $isViewOn\ (\!|\ V = (V_{\mathcal{V}} \cup N_{\mathcal{V}}),\ N = \{\},\ C = C_{\mathcal{V}}\ |\!)\ E_{SES}$

⟨*proof*⟩

end

end

5.3.3 Unwinding Theorems

theory *UnwindingResults*
imports *AuxiliaryLemmas*
begin

context *Unwinding*
begin
theorem *unwinding-theorem-BSD*:
⟦ *lrf ur*; *osc ur* ⟧ ⟹ *BSD* $\mathcal{V}$ $Tr_{(induceES\ SES)}$
⟨*proof*⟩

theorem *unwinding-theorem-BSI*:
⟦ *lrb ur*; *osc ur* ⟧ ⟹ *BSI* $\mathcal{V}$ $Tr_{(induceES\ SES)}$
⟨*proof*⟩

theorem *unwinding-theorem-BSIA*:
⟦ *lrbe* ϱ *ur*; *osc ur* ⟧ ⟹ *BSIA* ϱ $\mathcal{V}$ $Tr_{(induceES\ SES)}$
⟨*proof*⟩

theorem *unwinding-theorem-FCD*:
⟦ *fcrf* Γ *ur*; *osc ur* ⟧ ⟹ *FCD* Γ $\mathcal{V}$ $Tr_{(induceES\ SES)}$
⟨*proof*⟩

theorem *unwinding-theorem-FCI*:
⟦ *fcrb* Γ *ur*; *osc ur* ⟧ ⟹ *FCI* Γ $\mathcal{V}$ $Tr_{(induceES\ SES)}$
⟨*proof*⟩

theorem *unwinding-theorem-FCIA*:
⟦ *fcrbe* Γ ϱ *ur*; *osc ur* ⟧ ⟹ *FCIA* ϱ Γ $\mathcal{V}$ $Tr_{(induceES\ SES)}$
⟨*proof*⟩

theorem *unwinding-theorem-SD*:
⟦ $\mathcal{V}' = (\!|\ V = (V_{\mathcal{V}} \cup N_{\mathcal{V}}),\ N = \{\},\ C = C_{\mathcal{V}}\ |\!)$;
Unwinding.lrf SES $\mathcal{V}'$ *ur*; *Unwinding.osc SES* $\mathcal{V}'$ *ur* ⟧
⟹ *SD* $\mathcal{V}$ $Tr_{(induceES\ SES)}$
⟨*proof*⟩

theorem *unwinding-theorem-SI*:
⟦ $\mathcal{V}' = (\!|\ V = (V_{\mathcal{V}} \cup N_{\mathcal{V}}),\ N = \{\},\ C = C_{\mathcal{V}}\ |\!)$;
Unwinding.lrb SES $\mathcal{V}'$ *ur*; *Unwinding.osc SES* $\mathcal{V}'$ *ur* ⟧
⟹ *SI* $\mathcal{V}$ $Tr_{(induceES\ SES)}$
⟨*proof*⟩

theorem *unwinding-theorem-SIA*:
$[\![\mathcal{V}' = (\!| V = (V_{\mathcal{V}} \cup N_{\mathcal{V}}), N = \{\}, C = C_{\mathcal{V}} |\!); \varrho\ \mathcal{V} = \varrho\ \mathcal{V}';$
Unwinding.lrbe SES $\mathcal{V}'$ ϱ *ur*; *Unwinding.osc SES* $\mathcal{V}'$ *ur* $]\!]$
$\Longrightarrow$ *SIA* ϱ $\mathcal{V}$ $Tr_{(induceES\ SES)}$
⟨*proof*⟩

theorem *unwinding-theorem-SR*:
$[\![\mathcal{V}' = (\!| V = (V_{\mathcal{V}} \cup N_{\mathcal{V}}), N = \{\}, C = C_{\mathcal{V}} |\!);$
Unwinding.lrf SES $\mathcal{V}'$ *ur*; *Unwinding.osc SES* $\mathcal{V}'$ *ur* $]\!]$
$\Longrightarrow$ *SR* $\mathcal{V}$ $Tr_{(induceES\ SES)}$
⟨*proof*⟩

theorem *unwinding-theorem-D*:
$[\![$ *lrf ur*; *osc ur* $]\!] \Longrightarrow D\ \mathcal{V}\ Tr_{(induceES\ SES)}$
⟨*proof*⟩

theorem *unwinding-theorem-I*:
$[\![$ *lrb ur*; *osc ur* $]\!] \Longrightarrow I\ \mathcal{V}\ Tr_{(induceES\ SES)}$
⟨*proof*⟩

theorem *unwinding-theorem-IA*:
$[\![$ *lrbe* ϱ *ur*; *osc ur* $]\!] \Longrightarrow$ *IA* ϱ $\mathcal{V}$ $Tr_{(induceES\ SES)}$
⟨*proof*⟩

theorem *unwinding-theorem-R*:
$[\![$ *lrf ur*; *osc ur* $]\!] \Longrightarrow R\ \mathcal{V}\ (Tr_{(induceES\ SES)})$
⟨*proof*⟩

end

end

5.4 Compositionality

We prove the compositionality results from [3].

5.4.1 Auxiliary Definitions & Results

theory *CompositionBase*
imports *../Basics/BSPTaxonomy*
begin

definition
properSeparationOfViews ::
'e ES-rec ⇒ *'e ES-rec* ⇒ *'e V-rec* ⇒ *'e V-rec* ⇒ *'e V-rec* ⇒ *bool*
where
properSeparationOfViews ES1 ES2 $\mathcal{V}$ *V1 V2* ≡
$V_{\mathcal{V}} \cap E_{ES1} = V_{\mathcal{V}1}$
$\wedge\ V_{\mathcal{V}} \cap E_{ES2} = V_{\mathcal{V}2}$
$\wedge\ C_{\mathcal{V}} \cap E_{ES1} \subseteq C_{\mathcal{V}1}$
$\wedge\ C_{\mathcal{V}} \cap E_{ES2} \subseteq C_{\mathcal{V}2}$

$\wedge\ N_{\mathcal{V}1} \cap N_{\mathcal{V}2} = \{\}$

definition
wellBehavedComposition ::
'e ES-rec ⇒ *'e ES-rec* ⇒ *'e V-rec* ⇒ *'e V-rec* ⇒ *'e V-rec* ⇒ *bool*
where
wellBehavedComposition ES1 ES2 $\mathcal{V}$ $\mathcal{V}1$ $\mathcal{V}2$ ≡
($N_{\mathcal{V}1} \cap E_{ES2} = \{\} \wedge N_{\mathcal{V}2} \cap E_{ES1} = \{\}$)
∨ (∃ $\varrho 1$. ($N_{\mathcal{V}1} \cap E_{ES2} = \{\}$ ∧ *total ES1* ($C_{\mathcal{V}1} \cap N_{\mathcal{V}2}$)
∧ *BSIA* $\varrho 1$ $\mathcal{V}1$ Tr_{ES1}))
∨ (∃ $\varrho 2$. ($N_{\mathcal{V}2} \cap E_{ES1} = \{\}$ ∧ *total ES2* ($C_{\mathcal{V}2} \cap N_{\mathcal{V}1}$)
∧ *BSIA* $\varrho 2$ $\mathcal{V}2$ Tr_{ES2}))
∨ (∃ $\varrho 1$ $\varrho 2$ $\Gamma 1$ $\Gamma 2$. (
$\nabla_{\Gamma 1} \subseteq E_{ES1} \wedge \Delta_{\Gamma 1} \subseteq E_{ES1} \wedge \Upsilon_{\Gamma 1} \subseteq E_{ES1}$
$\wedge\ \nabla_{\Gamma 2} \subseteq E_{ES2} \wedge \Delta_{\Gamma 2} \subseteq E_{ES2} \wedge \Upsilon_{\Gamma 2} \subseteq E_{ES2}$
∧ *BSIA* $\varrho 1$ $\mathcal{V}1$ Tr_{ES1} ∧ *BSIA* $\varrho 2$ $\mathcal{V}2$ Tr_{ES2}
∧ *total ES1* ($C_{\mathcal{V}1} \cap N_{\mathcal{V}2}$) ∧ *total ES2* ($C_{\mathcal{V}2} \cap N_{\mathcal{V}1}$)
∧ *FCIA* $\varrho 1$ $\Gamma 1$ $\mathcal{V}1$ Tr_{ES1} ∧ *FCIA* $\varrho 2$ $\Gamma 2$ $\mathcal{V}2$ Tr_{ES2}
$\wedge\ V_{\mathcal{V}1} \cap V_{\mathcal{V}2} \subseteq \nabla_{\Gamma 1} \cup \nabla_{\Gamma 2}$
$\wedge\ C_{\mathcal{V}1} \cap N_{\mathcal{V}2} \subseteq \Upsilon_{\Gamma 1} \wedge C_{\mathcal{V}2} \cap N_{\mathcal{V}1} \subseteq \Upsilon_{\Gamma 2}$
$\wedge\ N_{\mathcal{V}1} \cap \Delta_{\Gamma 1} \cap E_{ES2} = \{\} \wedge N_{\mathcal{V}2} \cap \Delta_{\Gamma 2} \cap E_{ES1} = \{\}$))

locale *Compositionality* =
fixes *ES1* :: *'e ES-rec*
and *ES2* :: *'e ES-rec*
and $\mathcal{V}$ :: *'e V-rec*
and $\mathcal{V}1$ :: *'e V-rec*
and $\mathcal{V}2$ :: *'e V-rec*

assumes *validES1*: *ES-valid ES1*
and *validES2*: *ES-valid ES2*
and *composableES1ES2*: *composable ES1 ES2*

and *validVC*: *isViewOn* $\mathcal{V}$ ($E_{(ES1 \parallel ES2)}$)
and *validV1*: *isViewOn* $\mathcal{V}1$ E_{ES1}
and *validV2*: *isViewOn* $\mathcal{V}2$ E_{ES2}

and *propSepViews*: *properSeparationOfViews ES1 ES2* $\mathcal{V}$ $\mathcal{V}1$ $\mathcal{V}2$

and *well-behaved-composition*: *wellBehavedComposition ES1 ES2* $\mathcal{V}$ $\mathcal{V}1$ $\mathcal{V}2$

sublocale *Compositionality* ⊆ *BSPTaxonomyDifferentCorrections ES1* ∥ *ES2* $\mathcal{V}$
⟨*proof*⟩

context *Compositionality*
begin

lemma *Vv-is-Vv1-union-Vv2*: $V_{\mathcal{V}} = V_{\mathcal{V}1} \cup V_{\mathcal{V}2}$
⟨*proof*⟩

lemma *disjoint-Nv1-Vv2*: $N_{\mathcal{V}1} \cap V_{\mathcal{V}2} = \{\}$
⟨*proof*⟩

lemma *disjoint-Nv2-Vv1*: $N_{\mathcal{V}2} \cap V_{\mathcal{V}1} = \{\}$
⟨*proof*⟩

lemma *merge-property'*: $[\![$ *set t1* $\subseteq E_{ES1}$; *set t2* $\subseteq E_{ES2}$;
t1 $\restriction E_{ES2} =$ *t2* $\restriction E_{ES1}$; *t1* $\restriction V_{\mathcal{V}} = []$; *t2* $\restriction V_{\mathcal{V}} = []$;
t1 $\restriction C_{\mathcal{V}} = []$; *t2* $\restriction C_{\mathcal{V}} = []]\!]$
$\Longrightarrow \exists$ *t*. (*t* $\restriction E_{ES1} =$ *t1* $\wedge$ *t* $\restriction E_{ES2} =$ *t2* $\wedge$ *t* $\restriction V_{\mathcal{V}} = [] \wedge$ *t* $\restriction C_{\mathcal{V}} = [] \wedge$ *set t* $\subseteq (E_{ES1} \cup E_{ES2})$)
⟨*proof*⟩

lemma *Nv1-union-Nv2-subsetof-Nv*: $N_{\mathcal{V}1} \cup N_{\mathcal{V}2} \subseteq N_{\mathcal{V}}$
⟨*proof*⟩

end

end
theory *CompositionSupport*
imports *CompositionBase*
begin

locale *CompositionSupport* =
fixes *ESi* :: *'e ES-rec*
and $\mathcal{V}$ :: *'e V-rec*
and $\mathcal{V}i$ :: *'e V-rec*

assumes *validESi*: *ES-valid ESi*

and *validVi*: *isViewOn* $\mathcal{V}i$ E_{ESi}
and *Vv-inter-Ei-is-Vvi*: $V_{\mathcal{V}} \cap E_{ESi} = V_{\mathcal{V}i}$
and *Cv-inter-Ei-subsetof-Cvi*: $C_{\mathcal{V}} \cap E_{ESi} \subseteq C_{\mathcal{V}i}$

context *CompositionSupport*
begin

lemma *BSD-in-subsystem*:
$[\![\ c \in C_{\mathcal{V}};\ ((\beta\ @\ [c]\ @\ \alpha) \restriction E_{ESi}) \in Tr_{ESi}\ ;\ BSD\ \mathcal{V}i\ Tr_{ESi}\ ]\!]$
$\quad \Longrightarrow \exists\, \alpha\text{-}i'.\ (\ ((\beta \restriction E_{ESi})\ @\ \alpha\text{-}i') \in Tr_{ESi}$
$\quad \wedge\ (\alpha\text{-}i' \restriction V_{\mathcal{V}i}) = (\alpha \restriction V_{\mathcal{V}i}) \wedge \alpha\text{-}i' \restriction C_{\mathcal{V}i} = [\,]\ )$
⟨*proof*⟩

lemma *BSD-in-subsystem2*:
$[\![\ ((\beta\ @\ \alpha) \restriction E_{ESi}) \in Tr_{ESi}\ ;\ BSD\ \mathcal{V}i\ Tr_{ESi}\ ]\!]$
$\quad \Longrightarrow \exists\ \alpha\text{-}i'.\ (\ ((\beta \restriction E_{ESi})\ @\ \alpha\text{-}i') \in Tr_{ESi} \wedge (\alpha\text{-}i' \restriction V_{\mathcal{V}i}) = (\alpha \restriction V_{\mathcal{V}i}) \wedge \alpha\text{-}i' \restriction C_{\mathcal{V}i} = [\,]\ )$
⟨*proof*⟩

end

end

5.4.2 Generalized Zipping Lemma

theory *GeneralizedZippingLemma*
imports *CompositionBase*
begin

context *Compositionality*
begin

lemma *generalized-zipping-lemma1*: $[\![\ N_{\mathcal{V}1} \cap E_{ES2} = \{\};\ N_{\mathcal{V}2} \cap E_{ES1} = \{\}\ ]\!] \Longrightarrow$
$\quad \forall\ \tau\ lambda\ t1\ t2.\ (\ (\ set\ \tau \subseteq E_{(ES1\ \parallel\ ES2)} \wedge set\ lambda \subseteq V_{\mathcal{V}} \wedge set\ t1 \subseteq E_{ES1} \wedge set\ t2 \subseteq E_{ES2}$
$\quad \wedge ((\tau \restriction E_{ES1})\ @\ t1) \in Tr_{ES1} \wedge ((\tau \restriction E_{ES2})\ @\ t2) \in Tr_{ES2} \wedge (lambda \restriction E_{ES1}) = (t1 \restriction V_{\mathcal{V}})$
$\quad \wedge (lambda \restriction E_{ES2}) = (t2 \restriction V_{\mathcal{V}}) \wedge (t1 \restriction C_{\mathcal{V}1}) = [\,] \wedge (t2 \restriction C_{\mathcal{V}2}) = [\,])$
$\quad \longrightarrow (\exists\ t.\ ((\tau\ @\ t) \in Tr_{(ES1\ \parallel\ ES2)} \wedge (t \restriction V_{\mathcal{V}}) = lambda \wedge (t \restriction C_{\mathcal{V}}) = [\,]))\ )$
⟨*proof*⟩

lemma *generalized-zipping-lemma2*: $[\![\ N_{\mathcal{V}1} \cap E_{ES2} = \{\};\ total\ ES1\ (C_{\mathcal{V}1} \cap N_{\mathcal{V}2});\ BSIA\ \varrho 1\ \mathcal{V}1\ Tr_{ES1}\ ]\!]$
$\Longrightarrow$
$\quad \forall\ \tau\ lambda\ t1\ t2.\ (\ (\ set\ \tau \subseteq (E_{(ES1\ \parallel\ ES2)}) \wedge set\ lambda \subseteq V_{\mathcal{V}} \wedge set\ t1 \subseteq E_{ES1} \wedge set\ t2 \subseteq E_{ES2}$
$\quad \wedge ((\tau \restriction E_{ES1})\ @\ t1) \in Tr_{ES1} \wedge ((\tau \restriction E_{ES2})\ @\ t2) \in Tr_{ES2}$
$\quad \wedge (lambda \restriction E_{ES1}) = (t1 \restriction V_{\mathcal{V}}) \wedge (lambda \restriction E_{ES2}) = (t2 \restriction V_{\mathcal{V}})$
$\quad \wedge (t1 \restriction C_{\mathcal{V}1}) = [\,] \wedge (t2 \restriction C_{\mathcal{V}2}) = [\,])$
$\quad \longrightarrow (\exists\ t.\ ((\tau\ @\ t) \in (Tr_{(ES1\ \parallel\ ES2)}) \wedge (t \restriction V_{\mathcal{V}}) = lambda \wedge (t \restriction C_{\mathcal{V}}) = [\,]))\ )$
⟨*proof*⟩

lemma *generalized-zipping-lemma3*: $[\![\ N_{\mathcal{V}2} \cap E_{ES1} = \{\};\ total\ ES2\ (C_{\mathcal{V}2} \cap N_{\mathcal{V}1});\ BSIA\ \varrho 2\ \mathcal{V}2\ Tr_{ES2}\ ]\!]$
$\Longrightarrow$
$\quad \forall\ \tau\ lambda\ t1\ t2.\ (\ (\ set\ \tau \subseteq E_{(ES1\ \parallel\ ES2)} \wedge set\ lambda \subseteq V_{\mathcal{V}} \wedge set\ t1 \subseteq E_{ES1} \wedge set\ t2 \subseteq E_{ES2}$
$\quad \wedge ((\tau \restriction E_{ES1})\ @\ t1) \in Tr_{ES1} \wedge ((\tau \restriction E_{ES2})\ @\ t2) \in Tr_{ES2}$

$\wedge\ (lambda \restriction E_{ES1}) = (t1 \restriction V_{\mathcal{V}}) \wedge (lambda \restriction E_{ES2}) = (t2 \restriction V_{\mathcal{V}})$
$\wedge\ (t1 \restriction C_{\mathcal{V}1}) = [] \wedge (t2 \restriction C_{\mathcal{V}2}) = [])$
$\longrightarrow (\exists\ t.\ ((\tau\ @\ t) \in Tr_{(ES1\ \|\ ES2)} \wedge (t \restriction V_{\mathcal{V}}) = lambda \wedge (t \restriction C_{\mathcal{V}}) = [])))$
⟨*proof*⟩

lemma *generalized-zipping-lemma4*:
$[\![\ \nabla_{\Gamma 1} \subseteq E_{ES1};\ \Delta_{\Gamma 1} \subseteq E_{ES1};\ \Upsilon_{\Gamma 1} \subseteq E_{ES1};\ \nabla_{\Gamma 2} \subseteq E_{ES2};\ \Delta_{\Gamma 2} \subseteq E_{ES2};\ \Upsilon_{\Gamma 2} \subseteq E_{ES2};$
BSIA ϱ1 V1 Tr$_{ES1}$; *BSIA ϱ2 V2 Tr*$_{ES2}$; *total ES1* $(C_{\mathcal{V}1} \cap N_{\mathcal{V}2})$; *total ES2* $(C_{\mathcal{V}2} \cap N_{\mathcal{V}1})$;
FCIA ϱ1 Γ1 V1 Tr$_{ES1}$; *FCIA ϱ2 Γ2 V2 Tr*$_{ES2}$; $V_{\mathcal{V}1} \cap V_{\mathcal{V}2} \subseteq \nabla_{\Gamma 1} \cup \nabla_{\Gamma 2}$;
$C_{\mathcal{V}1} \cap N_{\mathcal{V}2} \subseteq \Upsilon_{\Gamma 1}$; $C_{\mathcal{V}2} \cap N_{\mathcal{V}1} \subseteq \Upsilon_{\Gamma 2}$;
$N_{\mathcal{V}1} \cap \Delta_{\Gamma 1} \cap E_{ES2} = \{\}$; $N_{\mathcal{V}2} \cap \Delta_{\Gamma 2} \cap E_{ES1} = \{\}\ ]\!] \Longrightarrow$
$\forall\ \tau\ lambda\ t1\ t2.\ (\ (\ set\ \tau \subseteq (E_{(ES1\ \|\ ES2)}) \wedge set\ lambda \subseteq V_{\mathcal{V}}\ \wedge set\ t1 \subseteq E_{ES1}$
$\wedge\ set\ t2 \subseteq E_{ES2} \wedge ((\tau \restriction E_{ES1})\ @\ t1) \in Tr_{ES1} \wedge ((\tau \restriction E_{ES2})\ @\ t2) \in Tr_{ES2}$
$\wedge\ (lambda \restriction E_{ES1}) = (t1 \restriction V_{\mathcal{V}}) \wedge (lambda \restriction E_{ES2}) = (t2 \restriction V_{\mathcal{V}})$
$\wedge\ (t1 \restriction C_{\mathcal{V}1}) = [] \wedge (t2 \restriction C_{\mathcal{V}2}) = [])$
$\longrightarrow (\exists\, t.\ ((\tau\ @\ t) \in (Tr_{(ES1\ \|\ ES2)}) \wedge (t \restriction V_{\mathcal{V}}) = lambda \wedge (t \restriction C_{\mathcal{V}}) = [])))$
⟨*proof*⟩

lemma *generalized-zipping-lemma*:
$\forall\ \tau\ lambda\ t1\ t2.\ (\ (\ set\ \tau \subseteq E_{(ES1\ \|\ ES2)}$
$\wedge\ set\ lambda \subseteq V_{\mathcal{V}} \wedge set\ t1 \subseteq E_{ES1} \wedge set\ t2 \subseteq E_{ES2}$
$\wedge\ ((\tau \restriction E_{ES1})\ @\ t1) \in Tr_{ES1} \wedge ((\tau \restriction E_{ES2})\ @\ t2) \in Tr_{ES2}$
$\wedge\ (lambda \restriction E_{ES1}) = (t1 \restriction V_{\mathcal{V}}) \wedge (lambda \restriction E_{ES2}) = (t2 \restriction V_{\mathcal{V}})$
$\wedge\ (t1 \restriction C_{\mathcal{V}1}) = [] \wedge (t2 \restriction C_{\mathcal{V}2}) = [])$
$\longrightarrow (\exists\, t.\ ((\tau\ @\ t) \in Tr_{(ES1\ \|\ ES2)} \wedge (t \restriction V_{\mathcal{V}}) = lambda \wedge (t \restriction C_{\mathcal{V}}) = [])))$
⟨*proof*⟩

end

end

5.4.3 Compositionality Results

theory *CompositionalityResults*
imports *GeneralizedZippingLemma CompositionSupport*
begin

context *Compositionality*
begin

theorem *compositionality-BSD*:
$[\![\ BSD\ \mathcal{V}1\ Tr_{ES1};\ BSD\ \mathcal{V}2\ Tr_{ES2}\ ]\!] \Longrightarrow BSD\ \mathcal{V}\ Tr_{(ES1\ \|\ ES2)}$
⟨*proof*⟩

theorem *compositionality-BSI*:
$[\![\ BSD\ \mathcal{V}1\ Tr_{ES1};\ BSD\ \mathcal{V}2\ Tr_{ES2};\ BSI\ \mathcal{V}1\ Tr_{ES1};\ BSI\ \mathcal{V}2\ Tr_{ES2}\ ]\!]$

$\Longrightarrow$ *BSI* $\mathcal{V}$ $Tr_{(ES1 \parallel ES2)}$

⟨*proof*⟩

theorem *compositionality-BSIA*:

⟦ *BSD* $\mathcal{V}1$ Tr_{ES1}; *BSD* $\mathcal{V}2$ Tr_{ES2}; *BSIA* $\varrho 1$ $\mathcal{V}1$ Tr_{ES1}; *BSIA* $\varrho 2$ $\mathcal{V}2$ Tr_{ES2};
$(\varrho 1\ \mathcal{V}1) \subseteq (\varrho\ \mathcal{V}) \cap E_{ES1}$; $(\varrho 2\ \mathcal{V}2) \subseteq (\varrho\ \mathcal{V}) \cap E_{ES2}$ ⟧
$\Longrightarrow$ *BSIA* ϱ $\mathcal{V}$ $(Tr_{(ES1 \parallel ES2)})$

⟨*proof*⟩

theorem *compositionality-FCD*:

⟦ *BSD* $\mathcal{V}1$ Tr_{ES1}; *BSD* $\mathcal{V}2$ Tr_{ES2};
$\nabla_\Gamma \cap E_{ES1} \subseteq \nabla_{\Gamma 1}$; $\nabla_\Gamma \cap E_{ES2} \subseteq \nabla_{\Gamma 2}$;
$\Upsilon_\Gamma \cap E_{ES1} \subseteq \Upsilon_{\Gamma 1}$; $\Upsilon_\Gamma \cap E_{ES2} \subseteq \Upsilon_{\Gamma 2}$;
$(\ \Delta_{\Gamma 1} \cap N_{\mathcal{V}1} \cup \Delta_{\Gamma 2} \cap N_{\mathcal{V}2}\ ) \subseteq \Delta_\Gamma$;
$N_{\mathcal{V}1} \cap \Delta_{\Gamma 1} \cap E_{ES2} = \{\}$; $N_{\mathcal{V}2} \cap \Delta_{\Gamma 2} \cap E_{ES1} = \{\}$;
FCD $\Gamma 1$ $\mathcal{V}1$ Tr_{ES1}; *FCD* $\Gamma 2$ $\mathcal{V}2$ Tr_{ES2} ⟧
$\Longrightarrow$ *FCD* Γ $\mathcal{V}$ $(Tr_{(ES1 \parallel ES2)})$

⟨*proof*⟩

theorem *compositionality-FCI*:

⟦ *BSD* $\mathcal{V}1$ Tr_{ES1}; *BSD* $\mathcal{V}2$ Tr_{ES2}; *BSIA* $\varrho 1$ $\mathcal{V}1$ Tr_{ES1}; *BSIA* $\varrho 2$ $\mathcal{V}2$ Tr_{ES2};
total ES1 $(C_{\mathcal{V}1} \cap \Upsilon_{\Gamma 1})$; *total ES2* $(C_{\mathcal{V}2} \cap \Upsilon_{\Gamma 2})$;
$\nabla_\Gamma \cap E_{ES1} \subseteq \nabla_{\Gamma 1}$; $\nabla_\Gamma \cap E_{ES2} \subseteq \nabla_{\Gamma 2}$;
$\Upsilon_\Gamma \cap E_{ES1} \subseteq \Upsilon_{\Gamma 1}$; $\Upsilon_\Gamma \cap E_{ES2} \subseteq \Upsilon_{\Gamma 2}$;
$(\ \Delta_{\Gamma 1} \cap N_{\mathcal{V}1} \cup \Delta_{\Gamma 2} \cap N_{\mathcal{V}2}\ ) \subseteq \Delta_\Gamma$;
$(N_{\mathcal{V}1} \cap \Delta_{\Gamma 1} \cap E_{ES2} = \{\} \wedge N_{\mathcal{V}2} \cap \Delta_{\Gamma 2} \cap E_{ES1} \subseteq \Upsilon_{\Gamma 1})$
$\vee\ (\ N_{\mathcal{V}2} \cap \Delta_{\Gamma 2} \cap E_{ES1} = \{\} \wedge N_{\mathcal{V}1} \cap \Delta_{\Gamma 1} \cap E_{ES2} \subseteq \Upsilon_{\Gamma 2})$;
FCI $\Gamma 1$ $\mathcal{V}1$ Tr_{ES1}; *FCI* $\Gamma 2$ $\mathcal{V}2$ Tr_{ES2} ⟧
$\Longrightarrow$ *FCI* Γ $\mathcal{V}$ $(Tr_{(ES1 \parallel ES2)})$

⟨*proof*⟩

theorem *compositionality-FCIA*:

⟦ *BSD* $\mathcal{V}1$ Tr_{ES1}; *BSD* $\mathcal{V}2$ Tr_{ES2}; *BSIA* $\varrho 1$ $\mathcal{V}1$ Tr_{ES1}; *BSIA* $\varrho 2$ $\mathcal{V}2$ Tr_{ES2};
$(\varrho 1\ \mathcal{V}1) \subseteq (\varrho\ \mathcal{V}) \cap E_{ES1}$; $(\varrho 2\ \mathcal{V}2) \subseteq (\varrho\ \mathcal{V}) \cap E_{ES2}$;
total ES1 $(C_{\mathcal{V}1} \cap \Upsilon_{\Gamma 1} \cap N_{\mathcal{V}2} \cap \Delta_{\Gamma 2})$; *total ES2* $(C_{\mathcal{V}2} \cap \Upsilon_{\Gamma 2} \cap N_{\mathcal{V}1} \cap \Delta_{\Gamma 1})$;
$\nabla_\Gamma \cap E_{ES1} \subseteq \nabla_{\Gamma 1}$; $\nabla_\Gamma \cap E_{ES2} \subseteq \nabla_{\Gamma 2}$;
$\Upsilon_\Gamma \cap E_{ES1} \subseteq \Upsilon_{\Gamma 1}$; $\Upsilon_\Gamma \cap E_{ES2} \subseteq \Upsilon_{\Gamma 2}$;
$(\ \Delta_{\Gamma 1} \cap N_{\mathcal{V}1} \cup \Delta_{\Gamma 2} \cap N_{\mathcal{V}2}\ ) \subseteq \Delta_\Gamma$;
$(N_{\mathcal{V}1} \cap \Delta_{\Gamma 1} \cap E_{ES2} = \{\} \wedge N_{\mathcal{V}2} \cap \Delta_{\Gamma 2} \cap E_{ES1} \subseteq \Upsilon_{\Gamma 1})$
$\vee\ (\ N_{\mathcal{V}2} \cap \Delta_{\Gamma 2} \cap E_{ES1} = \{\} \wedge N_{\mathcal{V}1} \cap \Delta_{\Gamma 1} \cap E_{ES2} \subseteq \Upsilon_{\Gamma 2})$;
FCIA $\varrho 1$ $\Gamma 1$ $\mathcal{V}1$ Tr_{ES1}; *FCIA* $\varrho 2$ $\Gamma 2$ $\mathcal{V}2$ Tr_{ES2} ⟧
$\Longrightarrow$ *FCIA* ϱ Γ $\mathcal{V}$ $(Tr_{(ES1 \parallel ES2)})$

⟨*proof*⟩

theorem *compositionality-R*:

$[\![\ R\ \mathcal{V}1\ Tr_{ES1};\ R\ \mathcal{V}2\ Tr_{ES2}\ ]\!] \Longrightarrow R\ \mathcal{V}\ (Tr_{(ES1\ \|\ ES2)})$
⟨*proof*⟩

end

locale *CompositionalityStrictBSPs* = *Compositionality* +

assumes *NV-inter-E1-is-NV1*: $N_{\mathcal{V}} \cap E_{ES1} = N_{\mathcal{V}1}$
and *NV-inter-E2-is-NV2*: $N_{\mathcal{V}} \cap E_{ES2} = N_{\mathcal{V}2}$

sublocale *CompositionalityStrictBSPs* ⊆ *Compositionality*
⟨*proof*⟩

context *CompositionalityStrictBSPs*
begin

theorem *compositionality-SR*:
$[\![\ SR\ \mathcal{V}1\ Tr_{ES1};\ SR\ \mathcal{V}2\ Tr_{ES2}\ ]\!] \Longrightarrow SR\ \mathcal{V}\ (Tr_{(ES1\ \|\ ES2)})$
⟨*proof*⟩

theorem *compositionality-SD*:
$[\![\ SD\ \mathcal{V}1\ Tr_{ES1};\ SD\ \mathcal{V}2\ Tr_{ES2}\ ]\!] \Longrightarrow SD\ \mathcal{V}\ (Tr_{(ES1\ \|\ ES2)})$
⟨*proof*⟩

theorem *compositionality-SI*:
$[\![SD\ \mathcal{V}1\ Tr_{ES1};\ SD\ \mathcal{V}2\ Tr_{ES2};\ SI\ \mathcal{V}1\ Tr_{ES1};\ SI\ \mathcal{V}2\ Tr_{ES2}\ ]\!]$
$\Longrightarrow SI\ \mathcal{V}\ (Tr_{(ES1\ \|\ ES2)})$
⟨*proof*⟩

theorem *compositionality-SIA*:
$[\![SD\ \mathcal{V}1\ Tr_{ES1};\ SD\ \mathcal{V}2\ Tr_{ES2};\ SIA\ \varrho 1\ \mathcal{V}1\ Tr_{ES1};\ SIA\ \varrho 2\ \mathcal{V}2\ Tr_{ES2};$
$(\varrho 1\ \mathcal{V}1) \subseteq (\varrho\ \mathcal{V}) \cap E_{ES1};\ (\varrho 2\ \mathcal{V}2) \subseteq (\varrho\ \mathcal{V}) \cap E_{ES2}\ ]\!]$
$\Longrightarrow SIA\ \varrho\ \mathcal{V}\ (Tr_{(ES1\ \|\ ES2)})$
⟨*proof*⟩
end

end

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