A Verified Decision Procedure for a Quantifier-Free Fragment of Set Theory

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Abstract

This formalization verifies a decision procedure due to Cantone and Zarba for a quantifier-free fragment of set theory. The fragment is called multi-level syllogistic with singleton, or MLSS for short. Its syntax syntax includes the usual set operations union, intersection, difference, membership, equality as well as the construction of a set containing a single element. We specify the semantics of MLSS in terms of hereditarily finite sets and provide a sound and complete tableau calculus for it. We also provide an executable specification of a decision procedure that applies the rules of the calculus exhaustively and prove its termination. Furthermore, we extend the calculus with a light-weight type system that paves the way for an integration of the procedure into Isabelle/HOL.

Contents

References

[1] D. Cantone and C. G. Zarba. A new fast tableau-based decision procedure for an unquantified fragment of set theory. In R. Caferra and G. Salzer, editors, *Automated Deduction in Classical and Non-Classical Logics, Selected Papers*, volume 1761 of *Lecture Notes in Computer Science*, pages 126–136. Springer, 1998.