Lifting Definition Option*

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Abstract

We implemented a command, \texttt{lift-definition-option}, which can be used to easily generate elements of a restricted type \{\(x :: \alpha. P x\}\), provided the definition is of the form \(\lambda y_1 \ldots y_n. \text{if check } y_1 \ldots y_n \text{ then Some } (\text{generate } y_1 \ldots y_n :: \alpha) \text{ else None }\) and \(\text{check } y_1 \ldots y_n \Rightarrow P (\text{generate } y_1 \ldots y_n)\) can be proven.

In principle, such a definition is also directly possible using one invocation of \texttt{lift-definition}. However, then this definition will not be suitable for code-generation. To this end, we automated a more complex construction of Joachim Breitner which is amenable for code-generation, and where the test \(\text{check } y_1 \ldots y_n\) will only be performed once. In the automation, one auxiliary type is created, and Isabelle’s lifting- and transfer-package is invoked several times.

This entry is outdated as in the meantime the lifting- and transfer-package has the desired functionality in an even more general way. Therefore, only the examples are kept.

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\begin{isabelle}
theory Lifting-Definition-Option-Examples
imports
  Main
begin
\end{isabelle}

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1 Examples

1.1 A simple restricted type without type-parameters

typedef restricted = { i :: int. i mod 2 = 0} morphisms base restricted
(proof)
setup-lifting type-definition-restricted

Let us start with just using a sufficient criterion for testing for even numbers, without actually generating them, i.e., where the generator is just the identity function.
lift-definition(code-dt) restricted-of-simple :: int ⇒ restricted option is
λ x :: int. if x ∈ {0, 2, 4, 6} then Some x else None (proof)

We can also take several input arguments for the test, and generate a more complex value.
lift-definition(code-dt) restricted-of-many-args :: nat ⇒ int ⇒ bool ⇒ restricted option is
λ x y (b :: bool). if int x + y = 5 then Some ((int x + 1) * (y + 1)) else None (proof)

No problem to use type parameters.
lift-definition(code-dt) restricted-of-poly :: 'b list ⇒ restricted option is
λ xs :: 'b list. if length xs = 2 then Some (int (length (xs))) else None (proof)

1.2 Examples with type-parameters in the restricted type.
typedef 'f restrictedf = { xs :: 'f list. length xs < 3} morphisms base restrictedf
(proof)
setup-lifting type-definition-restrictedf

It does not matter, if we take the same or different type-parameters in the lift-definition.
lift-definition(code-dt) test1 :: 'g ⇒ nat ⇒ 'g restrictedf option is
λ (e :: 'g) x. if x < 2 then Some (replicate x e) else None (proof)

lift-definition(code-dt) test2 :: 'f ⇒ nat ⇒ 'f restrictedf option is
λ (e :: 'f) x. if x < 2 then Some (replicate x e) else None (proof)

Tests with multiple type-parameters.
typedef ('a,'f) restr = { (xs :: 'a list,ys :: 'f list) . length xs = length ys} morphisms base' restr
(proof)
setup-lifting type-definition-restr

lift-definition(code-dt) restr-of-pair :: 'g ⇒ 'e list ⇒ nat ⇒ nat ⇒ ('e,nat) restr option is
λ (z :: 'g) (xs :: 'e list) (y :: nat) n. if length xs = n then Some (xs,replicate n y) else None
1.3 Example from IsaFoR/CeTA

An argument filter is a mapping $\pi$ from $n$-ary function symbols into lists of positions, i.e., where each position is between 0 and $n-1$. In IsaFoR, (Isabelle’s Formalization of Rewriting) and CeTA [1], the corresponding certifier for term rewriting related properties, this is modelled as follows, where a partial argument filter in a map is extended to a full one by means of a default filter.

```isar
typedef '$af' = { ($\pi :: 'f \times \text{nat} \Rightarrow \text{nat list}$. (\forall f n. set ($\pi (f,n)$) \subseteq \{0 ..< n\}) )}
morphisms $af$ Abs-$af$ 

setup-lifting type-definition-$af$

type-synonym '$af$-impl = (('f $\times$ nat) $\times$ nat list)

fun fun-of-map-fun :: ('$a \Rightarrow 'b option') $\Rightarrow$ ('$a \Rightarrow 'b') $\Rightarrow$ ('$a \Rightarrow 'b') where
fun-of-map-fun $m$ $a$ = (case $m$ $a$ of Some $b$ $\Rightarrow$ $b$ | None $\Rightarrow$ $f$ $a$)

lift-definition (code-dt) $af$-of :: '$af$-impl $\Rightarrow$ '$af$ option is
$\lambda$ $s$ :: '$af$-impl. if $($\forall$ $fidx$ $\in$ set $s$.$($\forall$ $i$ $\in$ set ($snd$ $fidx$). $i$ $<$ $snd$ ($fst$ $fidx$))$)$ then Some $(fun-of-map-fun$ ($map-of$ $s$) $($$\lambda$ $(f,n)$. $[0 ..< n]$)) else None

(proof)
```

1.4 Code generation tests and derived theorems

```isar
export-code
restricted-of-many-args
restricted-of-simple
restricted-of-poly
test1
test2
restr-of-pair
af-of
in Haskell

lemma restricted-of-simple-Some:
restricted-of-simple $x$ = Some $r$ $\Rightarrow$ base $r$ = $x$
(proof)
```

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We thank Andreas Lochbihler for pointing us to Joachim’s solution, and we thank Makarius Wenzel for explaining us, how we can go back from states to local theories within Isabelle/ML.
References