Lifting Definition Option*

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May 26, 2024

Abstract

We implemented a command, **lift-definition-option**, which can be used to easily generate elements of a restricted type $\{x :: 'a.\ P\ x\}$, provided the definition is of the form $\lambda\ y_1 \ldots y_n.\ \mathit{if}\ \mathit{check}\ y_1 \ldots y_n\ \mathit{then}\ \mathit{Some}\ (\mathit{generate}\ y_1 \ldots y_n :: 'a)\ \mathit{else}\ \mathit{None}$ and $\mathit{check}\ y_1 \ldots y_n \Longrightarrow P\ (\mathit{generate}\ y_1 \ldots y_n)$ can be proven.

In principle, such a definition is also directly possible using one invocation of **lift-definition**. However, then this definition will not be suitable for code-generation. To this end, we automated a more complex construction of Joachim Breitner which is amenable for code-generation, and where the test $\mathit{check}\ y_1 \ldots y_n$ will only be performed once. In the automation, one auxiliary type is created, and Isabelle's lifting- and transfer-package is invoked several times.

This entry is outdated as in the meantime the lifting- and transfer-package has the desired functionality in an even more general way. Therefore, only the examples are kept.

Contents

theory *Lifting-Definition-Option-Examples*
imports
Main
begin

*This research is supported by FWF (Austrian Science Fund) project Y 757.

1 Examples

1.1 A simple restricted type without type-parameters

typedef *restricted* = { *i* :: *int*. *i mod 2* = *0*} **morphisms** *base restricted*
⟨*proof*⟩
setup-lifting *type-definition-restricted*

Let us start with just using a sufficient criterion for testing for even numbers, without actually generating them, i.e., where the generator is just the identity function.

lift-definition(*code-dt*) *restricted-of-simple* :: *int* ⇒ *restricted option* **is**
λ *x* :: *int*. *if x* ∈ {*0*, *2*, *4*, *6*} *then Some x else None* ⟨*proof*⟩

We can also take several input arguments for the test, and generate a more complex value.

lift-definition(*code-dt*) *restricted-of-many-args* :: *nat* ⇒ *int* ⇒ *bool* ⇒ *restricted option* **is**
λ *x y* (*b* :: *bool*). *if int x* + *y* = *5 then Some* ((*int x* + *1*) ∗ (*y* + *1*)) *else None* ⟨*proof*⟩

No problem to use type parameters.

lift-definition(*code-dt*) *restricted-of-poly* :: *'b list* ⇒ *restricted option* **is**
λ *xs* :: *'b list*. *if length xs* = *2 then Some* (*int* (*length* (*xs*))) *else None* ⟨*proof*⟩

1.2 Examples with type-parameters in the restricted type.

typedef *'f restrictedf* = { *xs* :: *'f list*. *length xs* < *3*} **morphisms** *basef restrictedf*

⟨*proof*⟩
setup-lifting *type-definition-restrictedf*

It does not matter, if we take the same or different type-parameters in the lift-definition.

lift-definition(*code-dt*) *test1* :: *'g* ⇒ *nat* ⇒ *'g restrictedf option* **is**
λ (*e* :: *'g*) *x*. *if x* < *2 then Some* (*replicate x e*) *else None* ⟨*proof*⟩

lift-definition(*code-dt*) *test2* :: *'f* ⇒ *nat* ⇒ *'f restrictedf option* **is**
λ (*e* :: *'f*) *x*. *if x* < *2 then Some* (*replicate x e*) *else None* ⟨*proof*⟩

Tests with multiple type-parameters.

typedef (*'a*,*'f*) *restr* = { (*xs* :: *'a list*,*ys* :: *'f list*) . *length xs* = *length ys*}
morphisms *base' restr*
⟨*proof*⟩
setup-lifting *type-definition-restr*

lift-definition(*code-dt*) *restr-of-pair* :: *'g* ⇒ *'e list* ⇒ *nat* ⇒ *nat* ⇒ (*'e*,*nat*) *restr option* **is**
λ (*z* :: *'g*) (*xs* :: *'e list*) (*y* :: *nat*) *n*. *if length xs* = *n then Some* (*xs*,*replicate n y*) *else None*

⟨*proof*⟩

1.3 Example from IsaFoR/CeTA

An argument filter is a mapping π from n-ary function symbols into lists of positions, i.e., where each position is between 0 and n-1. In IsaFoR, (Isabelle's Formalization of Rewriting) and CeTA [1], the corresponding certifier for term rewriting related properties, this is modelled as follows, where a partial argument filter in a map is extended to a full one by means of an default filter.

typedef $'f\ af = \{\ (\pi :: 'f \times nat \Rightarrow nat\ list).\ (\forall\ f\ n.\ set\ (\pi\ (f,n)) \subseteq \{0\ ..< n\})\}$
  morphisms *af Abs-af* ⟨*proof*⟩

setup-lifting *type-definition-af*

type-synonym $'f\ af\text{-}impl = (('f \times nat) \times nat\ list)list$

fun *fun-of-map-fun* :: $('a \Rightarrow 'b\ option) \Rightarrow ('a \Rightarrow 'b) \Rightarrow ('a \Rightarrow 'b)$ **where**
  fun-of-map-fun m f a = (*case m a of Some b* $\Rightarrow$ *b* | *None* $\Rightarrow$ *f a*)

lift-definition(*code-dt*) *af-of* :: $'f\ af\text{-}impl \Rightarrow 'f\ af\ option$ **is**
  $\lambda\ s :: 'f\ af\text{-}impl.\ if\ (\forall\ fidx \in set\ s.\ (\forall\ i \in set\ (snd\ fidx).\ i < snd\ (fst\ fidx)))$
    then Some (*fun-of-map-fun* (*map-of s*) (λ (f,n). $[0\ ..< n]$)) *else None*
⟨*proof*⟩

1.4 Code generation tests and derived theorems

export-code
  restricted-of-many-args
  restricted-of-simple
  restricted-of-poly
  test1
  test2
  restr-of-pair
  af-of
in *Haskell*

lemma *restricted-of-simple-Some*:
  restricted-of-simple x = *Some r* $\Longrightarrow$ *base r* = *x*
⟨*proof*⟩

end

Acknowledgements

We thank Andreas Lochbihler for pointing us to Joachim's solution, and we thank Makarius Wenzel for explaining us, how we can go back from states to local theories within Isabelle/ML.

References

[1] R. Thiemann and C. Sternagel. Certification of termination proofs using CeTA. In S. Berghofer, T. Nipkow, C. Urban, and M. Wenzel, editors, *Theorem Proving in Higher Order Logics, 22nd International Conference, TPHOLs 2009, Munich, Germany, August 17-20, 2009. Proceedings*, volume 5674 of *Lecture Notes in Computer Science*, pages 452–468. Springer, 2009.