Lifting Definition Option*

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Abstract

We implemented a command, lift-definition-option, which can be used to easily generate elements of a restricted type \( \{ x :: 'a. P x \} \), provided the definition is of the form \( \lambda y_1 \ldots y_n. \text{if} \ check y_1 \ldots y_n \ \text{then} \ Some (generate y_1 \ldots y_n :: 'a) \ \text{else} \ None \) and \( check y_1 \ldots y_n \implies P (generate y_1 \ldots y_n) \) can be proven.

In principle, such a definition is also directly possible using one invocation of lift-definition. However, then this definition will not be suitable for code-generation. To this end, we automated a more complex construction of Joachim Breitner which is amenable for code-generation, and where the test \( check y_1 \ldots y_n \) will only be performed once. In the automation, one auxiliary type is created, and Isabelle’s lifting- and transfer-package is invoked several times.

This entry is outdated as in the meantime the lifting- and transfer-package has the desired functionality in an even more general way. Therefore, only the examples are kept.

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theory Lifting-Definition-Option-Examples
imports
  Main
begin

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1 Examples

1.1 A simple restricted type without type-parameters

typedef restricted = { i :: int. i mod 2 = 0} morphisms base restricted
(proof)

Let us start with just using a sufficient criterion for testing for even numbers, without actually generating them, i.e., where the generator is just the identity function.

lift-definition(code-dt) restricted-of-simple :: int ⇒ restricted option is
λ x :: int. if x ∈ {0, 2, 4, 6} then Some x else None (proof)

We can also take several input arguments for the test, and generate a more complex value.

lift-definition(code-dt) restricted-of-many-args :: nat ⇒ int ⇒ bool ⇒ restricted option is
λ x y (b :: bool). if int x + y = 5 then Some ((int x + 1) * (y + 1)) else None (proof)

No problem to use type parameters.

lift-definition(code-dt) restricted-of-poly :: 'b list ⇒ restricted option is
λ xs :: 'b list. if length xs = 2 then Some (int (length (xs))) else None (proof)

1.2 Examples with type-parameters in the restricted type.

typedef 'f restrictedf = { xs :: 'f list. length xs < 3} morphisms base restrictedf
(proof)

It does not matter, if we take the same or different type-parameters in the lift-definition.

lift-definition(code-dt) test1 :: 'g ⇒ nat ⇒ 'g restrictedf option is
λ (e :: 'g) x. if x < 2 then Some (replicate x e) else None (proof)

lift-definition(code-dt) test2 :: 'f ⇒ nat ⇒ 'f restrictedf option is
λ (e :: 'f) x. if x < 2 then Some (replicate x e) else None (proof)

Tests with multiple type-parameters.

typedef ('a,'f) restr = { (xs :: 'a list, ys :: 'f list). length xs = length ys} morphisms base’ restr
(proof)

lift-definition(code-dt) restr-of-pair :: 'g ⇒ 'e list ⇒ nat ⇒ nat ⇒ ('e,nat) restr option is
λ (z :: 'g) (xs :: 'e list) (y :: nat). if length xs = n then Some (xs,replicate n y) else None
1.3 Example from IsaFoR/CeTA

An argument filter is a mapping $\pi$ from n-ary function symbols into lists of positions, i.e., where each position is between 0 and n-1. In IsaFoR, (Isabelle’s Formalization of Rewriting) and CeTA [1], the corresponding certifier for term rewriting related properties, this is modelled as follows, where a partial argument filter in a map is extended to a full one by means of a default filter.

```
typedef 'f af = { (\pi :: 'f \times nat \Rightarrow nat list). (\forall f n. set (\pi (f,n)) \subseteq \{0 ..< n\})}
morphisms af Abs-af {proof}
```

```
setup-lifting type-definition-af
```

```
type-synonym 'f af-impl = (('f \times nat) \times nat list)
```

```
fun fun-of-map-fun :: ('a \Rightarrow 'b option) \Rightarrow ('a \Rightarrow 'b) \Rightarrow ('a \Rightarrow 'b) where
  fun-of-map-fun m f a = (case m a of Some b \Rightarrow b | None \Rightarrow f a)
```

```
lift-definition (code-dt) af-of :: 'f af-impl \Rightarrow 'f af option is
  \lambda s :: 'f af-impl. if (\forall fidx \in set s. (\forall i \in set (snd fidx). i < snd (fst fidx)))
    then Some (fun-of-map-fun (map-of s) (\lambda (f,n). [0 ..< n])) else None
{proof}
```

1.4 Code generation tests and derived theorems

```
export-code
  restricted-of-many-args
  restricted-of-simple
  restricted-of-poly
  test1
  test2
  restr-of-pair
  af-of
in Haskell

lemma restricted-of-simple-Some:
  restricted-of-simple x = Some r \Rightarrow base r = x
{proof}
```

end

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References