

Lifting Definition Option*

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Abstract

We implemented a command, **lift-definition-option**, which can be used to easily generate elements of a restricted type $\{x :: 'a. P\ x\}$, provided the definition is of the form $\lambda y_1 \dots y_n. \text{if check } y_1 \dots y_n \text{ then Some (generate } y_1 \dots y_n :: 'a) \text{ else None}$ and $\text{check } y_1 \dots y_n \implies P \text{ (generate } y_1 \dots y_n)$ can be proven.

In principle, such a definition is also directly possible using one invocation of **lift-definition**. However, then this definition will not be suitable for code-generation. To this end, we automated a more complex construction of Joachim Breitner which is amenable for code-generation, and where the test $\text{check } y_1 \dots y_n$ will only be performed once. In the automation, one auxiliary type is created, and Isabelle's lifting- and transfer-package is invoked several times.

This entry is outdated as in the meantime the lifting- and transfer-package has the desired functionality in an even more general way. Therefore, only the examples are kept.

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```
theory Lifting-Definition-Option-Examples
imports
  Main
begin
```

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1 Examples

1.1 A simple restricted type without type-parameters

```
typedef restricted = { i :: int. i mod 2 = 0 } morphisms base restricted
by (intro exI[of - 4]) auto
setup-lifting type-definition-restricted
```

Let us start with just using a sufficient criterion for testing for even numbers, without actually generating them, i.e., where the generator is just the identity function.

```
lift-definition(code-dt) restricted-of-simple :: int ⇒ restricted option is
λ x :: int. if x ∈ {0, 2, 4, 6} then Some x else None by auto
```

We can also take several input arguments for the test, and generate a more complex value.

```
lift-definition(code-dt) restricted-of-many-args :: nat ⇒ int ⇒ bool ⇒ restricted
option is
λ x y (b :: bool). if int x + y = 5 then Some ((int x + 1) * (y + 1)) else None
by clarsimp presburger
```

No problem to use type parameters.

```
lift-definition(code-dt) restricted-of-poly :: 'b list ⇒ restricted option is
λ xs :: 'b list. if length xs = 2 then Some (int (length (xs))) else None by auto
```

1.2 Examples with type-parameters in the restricted type.

```
typedef 'f restrictedf = { xs :: 'f list. length xs < 3 } morphisms basef restrictedf
by (intro exI[of - Nil]) auto
setup-lifting type-definition-restrictedf
```

It does not matter, if we take the same or different type-parameters in the lift-definition.

```
lift-definition(code-dt) test1 :: 'g ⇒ nat ⇒ 'g restrictedf option is
λ (e :: 'g) x. if x < 2 then Some (replicate x e) else None by auto
```

```
lift-definition(code-dt) test2 :: 'f ⇒ nat ⇒ 'f restrictedf option is
λ (e :: 'f) x. if x < 2 then Some (replicate x e) else None by auto
```

Tests with multiple type-parameters.

```
typedef ('a,'f) restr = { (xs :: 'a list,ys :: 'f list) . length xs = length ys }
morphisms base' restr
by (rule exI[of - ([], [])], auto)
setup-lifting type-definition-restr
```

```
lift-definition(code-dt) restr-of-pair :: 'g ⇒ 'e list ⇒ nat ⇒ nat ⇒ ('e,nat) restr
option is
λ (z :: 'g) (xs :: 'e list) (y :: nat) n. if length xs = n then Some (xs,replicate n
y) else None
```

by *auto*

1.3 Example from IsaFoR/CeTA

An argument filter is a mapping π from n-ary function symbols into lists of positions, i.e., where each position is between 0 and n-1. In IsaFoR, (Isabelle's Formalization of Rewriting) and CeTA [1], the corresponding certifier for term rewriting related properties, this is modelled as follows, where a partial argument filter in a map is extended to a full one by means of an default filter.

```
typedef 'f af = { ( $\pi :: 'f \times \text{nat} \Rightarrow \text{nat list}$ ). ( $\forall f n. \text{set} (\pi (f,n)) \subseteq \{0 ..< n\}$ )}
```

morphisms *af Abs-af* **by** (*rule exI[of - λ -. []*], *auto*)

setup-lifting *type-definition-af*

type-synonym 'f af-impl = (('f \times nat) \times nat list)list

```
fun fun-of-map-fun :: ('a  $\Rightarrow$  'b option)  $\Rightarrow$  ('a  $\Rightarrow$  'b)  $\Rightarrow$  ('a  $\Rightarrow$  'b) where  
  fun-of-map-fun m f a = (case m a of Some b  $\Rightarrow$  b | None  $\Rightarrow$  f a)
```

```
lift-definition(code-dt) af-of :: 'f af-impl  $\Rightarrow$  'f af option is  
   $\lambda s :: 'f \text{ af-impl. if } (\forall \text{ fid}x \in \text{set } s. (\forall i \in \text{set} (\text{snd } \text{fid}x). i < \text{snd} (\text{fst } \text{fid}x)))$   
    then Some (fun-of-map-fun (map-of s) ( $\lambda (f,n). [0 ..< n]$ )) else None
```

using *map-of-SomeD* **by** (*fastforce split: option.splits*)

1.4 Code generation tests and derived theorems

export-code

```
  restricted-of-many-args  
  restricted-of-simple  
  restricted-of-poly  
  test1  
  test2  
  restr-of-pair  
  af-of
```

in *Haskell*

lemma *restricted-of-simple-Some*:

```
  restricted-of-simple x = Some r  $\Longrightarrow$  base r = x
```

using *restricted-of-simple.rep-eq*[of x]

apply (*split if-splits*)

apply (*simp-all only: option.map option.inject option.simps(3)*)

done

end

Acknowledgements

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References

- [1] R. Thiemann and C. Sternagel. Certification of termination proofs using CeTA. In S. Berghofer, T. Nipkow, C. Urban, and M. Wenzel, editors, *Theorem Proving in Higher Order Logics, 22nd International Conference, TPHOLs 2009, Munich, Germany, August 17-20, 2009. Proceedings*, volume 5674 of *Lecture Notes in Computer Science*, pages 452–468. Springer, 2009.