

# LOFT — Verified Migration of Linux Firewalls to SDN

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## Abstract

We present LOFT — *Linux firewall OpenFlow Translator*, a system that transforms the main routing table and **FORWARD** chain of iptables of a Linux-based firewall into a set of static OpenFlow rules. Our implementation is verified against a model of a simplified Linux-based router and we can directly show how much of the original functionality is preserved.

Please note that this document is organized in two distinct parts. The first part contains the necessary definitions, helper lemmas and proofs in all their technicality as made in the theory code. The second part reiterates the most important definitions and proofs in a manner that is more suitable for human readers and enriches them with detailed explanations in natural language. Any interested reader should start from there.

Many of the considerations that have led to the definitions made here have been explained in [8].

## Contents

<b>I</b>	<b>Code</b>	<b>2</b>
<b>II</b>	<b>Documentation</b>	<b>44</b>
<b>1</b>	<b>Configuration Translation</b>	<b>44</b>
1.1	Linux Firewall Model . . . . .	44
1.1.1	Routing Table . . . . .	45
1.1.2	iptables Firewall . . . . .	46
1.2	OpenFlow Switch Model . . . . .	46
1.2.1	Matching Flow Table entries . . . . .	47
1.2.2	Evaluating a Flow Table . . . . .	47
1.3	Translation Implementation . . . . .	49
1.3.1	Chaining Firewalls . . . . .	49
1.3.2	Translation Implementation . . . . .	50
1.3.3	Comparison to Exodus . . . . .	52
<b>2</b>	<b>Evaluation</b>	<b>52</b>
2.1	Mininet Examples . . . . .	52
2.2	Performance Evaluation . . . . .	54
<b>3</b>	<b>Conclusion and Future Work</b>	<b>55</b>

# Part I

## Code

```
theory OpenFlow-Matches
imports IP-Addresses.Prefix-Match
       Simple-Firewall.Simple-Packet
       HOL-Library.Monad-Syntax

       HOL-Library.List-Lexorder
       HOL-Library.Char-ord
begin

datatype of-match-field =
  IngressPort string
| EtherSrc 48 word
| EtherDst 48 word
| EtherType 16 word
| VlanId 16 word
| VlanPriority 16 word

| IPv4Src 32 prefix-match
| IPv4Dst 32 prefix-match
| IPv4Proto 8 word

| L4Src 16 word 16 word
| L4Dst 16 word 16 word

schematic-goal of-match-field-typeset: (field-match :: of-match-field) ∈ {
  IngressPort (?s::string),
  EtherSrc (?as::48 word), EtherDst (?ad::48 word),
  EtherType (?t::16 word),
  VlanId (?i::16 word), VlanPriority (?p::16 word),
  IPv4Src (?pms::32 prefix-match),
  IPv4Dst (?pmd::32 prefix-match),
  IPv4Proto (?ipp :: 8 word),
  L4Src (?ps :: 16 word) (?ms :: 16 word),
  L4Dst (?pd :: 16 word) (?md :: 16 word)
}
proof((cases field-match;clarsimp),goal-cases)
  next case (IngressPort s) thus s = (case field-match of IngressPort s ⇒ s) unfolding IngressPort of-match-field.simps
by rule
  next case (EtherSrc s) thus s = (case field-match of EtherSrc s ⇒ s) unfolding EtherSrc of-match-field.simps
by rule
  next case (EtherDst s) thus s = (case field-match of EtherDst s ⇒ s) unfolding EtherDst of-match-field.simps
by rule
  next case (EtherType s) thus s = (case field-match of EtherType s ⇒ s) unfolding EtherType of-match-field.simps
by rule
  next case (VlanId s) thus s = (case field-match of VlanId s ⇒ s) unfolding VlanId of-match-field.simps
by rule
  next case (VlanPriority s) thus s = (case field-match of VlanPriority s ⇒ s) unfolding VlanPriority of-match-field.simps
by rule
```

```

next case (IPv4Src s)      thus s = (case field-match of IPv4Src s  $\Rightarrow$  s)      unfolding IPv4Src of-match-field.simps by
rule
next case (IPv4Dst s)      thus s = (case field-match of IPv4Dst s  $\Rightarrow$  s)      by simp
next case (IPv4Proto s)    thus s = (case field-match of IPv4Proto s  $\Rightarrow$  s)    by simp
next case (L4Src p l)      thus p = (case field-match of L4Src p m  $\Rightarrow$  p)  $\wedge$  l = (case field-match of L4Src p m  $\Rightarrow$  m) by
simp
next case (L4Dst p l)      thus p = (case field-match of L4Dst p m  $\Rightarrow$  p)  $\wedge$  l = (case field-match of L4Dst p m  $\Rightarrow$  m) by
simp
qed

```

```

function prerequisites :: of-match-field  $\Rightarrow$  of-match-field set  $\Rightarrow$  bool where
prerequisites (IngressPort -) - = True |

```

```

prerequisites (EtherDst -) - = True |

```

```

prerequisites (EtherSrc -) - = True |

```

```

prerequisites (EtherType -) - = True |

```

```

prerequisites (VlanId -) - = True |

```

```

prerequisites (VlanPriority -) m = ( $\exists$  id. let v = VlanId id in v  $\in$  m  $\wedge$  prerequisites v m) |

```

```

prerequisites (IPv4Proto -) m = (let v = EtherType 0x0800 in v  $\in$  m  $\wedge$  prerequisites v m) |

```

```

prerequisites (IPv4Src -) m = (let v = EtherType 0x0800 in v  $\in$  m  $\wedge$  prerequisites v m) |

```

```

prerequisites (IPv4Dst -) m = (let v = EtherType 0x0800 in v  $\in$  m  $\wedge$  prerequisites v m) |

```

```

prerequisites (L4Src - -) m = ( $\exists$  proto  $\in$  {TCP,UDP,L4-Protocol.SCTP}. let v = IPv4Proto proto in v  $\in$  m  $\wedge$  prerequisites
v m) |

```

```

prerequisites (L4Dst - -) m = prerequisites (L4Src undefined undefined) m

```

```

by pat-completeness auto

```

```

fun match-sorter :: of-match-field  $\Rightarrow$  nat where

```

```

match-sorter (IngressPort -) = 1 |

```

```

match-sorter (VlanId -) = 2 |

```

```

match-sorter (VlanPriority -) = 3 |

```

```

match-sorter (EtherType -) = 4 |

```

```

match-sorter (EtherSrc -) = 5 |

```

```

match-sorter (EtherDst -) = 6 |

```

```

match-sorter (IPv4Proto -) = 7 |

```

```

match-sorter (IPv4Src -) = 8 |

```

```

match-sorter (IPv4Dst -) = 9 |

```

```

match-sorter (L4Src - -) = 10 |

```

```

match-sorter (L4Dst - -) = 11

```

```

termination prerequisites by(relation measure (match-sorter  $\circ$  fst), simp-all)

```

```

definition less-eq-of-match-field1 :: of-match-field  $\Rightarrow$  of-match-field  $\Rightarrow$  bool

```

**where** *less-eq-of-match-field1* (*a*::*of-match-field*) (*b*::*of-match-field*)  $\longleftrightarrow$  (*case* (*a*, *b*) *of*  
 (*IngressPort* *a*, *IngressPort* *b*)  $\Rightarrow a \leq b$  |  
 (*VlanId* *a*, *VlanId* *b*)  $\Rightarrow a \leq b$  |  
 (*EtherDst* *a*, *EtherDst* *b*)  $\Rightarrow a \leq b$  |  
 (*EtherSrc* *a*, *EtherSrc* *b*)  $\Rightarrow a \leq b$  |  
 (*EtherType* *a*, *EtherType* *b*)  $\Rightarrow a \leq b$  |  
 (*VlanPriority* *a*, *VlanPriority* *b*)  $\Rightarrow a \leq b$  |  
 (*IPv4Proto* *a*, *IPv4Proto* *b*)  $\Rightarrow a \leq b$  |  
 (*IPv4Src* *a*, *IPv4Src* *b*)  $\Rightarrow a \leq b$  |  
 (*IPv4Dst* *a*, *IPv4Dst* *b*)  $\Rightarrow a \leq b$  |  
 (*L4Src* *a1* *a2*, *L4Src* *b1* *b2*)  $\Rightarrow$  *if* *a2* = *b2* *then* *a1*  $\leq$  *b1* *else* *a2*  $\leq$  *b2* |  
 (*L4Dst* *a1* *a2*, *L4Dst* *b1* *b2*)  $\Rightarrow$  *if* *a2* = *b2* *then* *a1*  $\leq$  *b1* *else* *a2*  $\leq$  *b2* |  
 (*a*, *b*)  $\Rightarrow$  *match-sorter* *a* < *match-sorter* *b*)

**instantiation** *of-match-field* :: *linorder*  
**begin**

**definition**

*less-eq-of-match-field* (*a*::*of-match-field*) (*b*::*of-match-field*)  $\longleftrightarrow$  *less-eq-of-match-field1* *a* *b*

**definition**

*less-of-match-field* (*a*::*of-match-field*) (*b*::*of-match-field*)  $\longleftrightarrow$  *a*  $\neq$  *b*  $\wedge$  *less-eq-of-match-field1* *a* *b*

**instance**

**by** *standard* (*auto simp add: less-eq-of-match-field-def less-of-match-field-def less-eq-of-match-field1-def split: prod.splits of-match-field.splits if-splits*)

**end**

**fun** *match-no-prereq* :: *of-match-field*  $\Rightarrow$  (*32*, 'a) *simple-packet-ext-scheme*  $\Rightarrow$  *bool* **where**

*match-no-prereq* (*IngressPort* *i*) *p* = (*p-iface* *p* = *i*) |  
*match-no-prereq* (*EtherDst* *i*) *p* = (*p-l2src* *p* = *i*) |  
*match-no-prereq* (*EtherSrc* *i*) *p* = (*p-l2dst* *p* = *i*) |  
*match-no-prereq* (*EtherType* *i*) *p* = (*p-l2type* *p* = *i*) |  
*match-no-prereq* (*VlanId* *i*) *p* = (*p-vlanid* *p* = *i*) |  
*match-no-prereq* (*VlanPriority* *i*) *p* = (*p-vlanprio* *p* = *i*) |  
*match-no-prereq* (*IPv4Proto* *i*) *p* = (*p-proto* *p* = *i*) |  
*match-no-prereq* (*IPv4Src* *i*) *p* = (*prefix-match-semantics* *i* (*p-src* *p*)) |  
*match-no-prereq* (*IPv4Dst* *i*) *p* = (*prefix-match-semantics* *i* (*p-dst* *p*)) |  
*match-no-prereq* (*L4Src* *i* *m*) *p* = (*p-sport* *p* && *m* = *i*) |  
*match-no-prereq* (*L4Dst* *i* *m*) *p* = (*p-dport* *p* && *m* = *i*)

**definition** *match-prereq* :: *of-match-field*  $\Rightarrow$  *of-match-field set*  $\Rightarrow$  (*32*, 'a) *simple-packet-ext-scheme*  $\Rightarrow$  *bool option* **where**  
*match-prereq* *i* *s* *p* = (*if prerequisites* *i* *s* *then* *Some* (*match-no-prereq* *i* *p*) *else* *None*)

**definition** *set-seq* *s*  $\equiv$  *if* ( $\forall x \in s. x \neq \text{None}$ ) *then* *Some* (*the* ' *s*) *else* *None*

**definition** *all-true* *s*  $\equiv \forall x \in s. x$

**term** *map-option*

**definition** *OF-match-fields* :: *of-match-field set*  $\Rightarrow$  (*32*, 'a) *simple-packet-ext-scheme*  $\Rightarrow$  *bool option* **where** *OF-match-fields* *m* *p* = *map-option all-true* (*set-seq* (( $\lambda f. \text{match-prereq } f \text{ } m \text{ } p$ ) ' *m*))

**definition** *OF-match-fields-unsafe* :: *of-match-field set*  $\Rightarrow$  (*32*, 'a) *simple-packet-ext-scheme*  $\Rightarrow$  *bool* **where**  
*OF-match-fields-unsafe* *m* *p* = ( $\forall f \in m. \text{match-no-prereq } f \text{ } p$ )

**definition** *OF-match-fields-safe* *m*  $\equiv$  *the*  $\circ$  *OF-match-fields* *m*

**definition** *all-prerequisites*  $m \equiv \forall f \in m. \text{prerequisites } f \ m$

**lemma**

*all-prerequisites*  $p \implies$

$L4Src \ x \ y \in p \implies$

$IPv4Proto \ \{TCP, UDP, L4-Protocol.SCTP\} \cap p \neq \{\}$

**unfolding** *all-prerequisites-def* **by** *auto*

**lemma** *of-safe-unsafe-match-eq*: *all-prerequisites*  $m \implies OF\text{-match-fields } m \ p = \text{Some } (OF\text{-match-fields-unsafe } m \ p)$

**unfolding** *OF-match-fields-def* *OF-match-fields-unsafe-def* *comp-def* *set-seq-def* *match-prereq-def* *all-prerequisites-def*

**proof** *goal-cases*

**case** 1

**have** 2:  $(\lambda f. \text{if prerequisites } f \ m \text{ then Some (match-no-prereq } f \ p) \text{ else None}) \ 'm = (\lambda f. \text{Some (match-no-prereq } f \ p)) \ 'm$

**using** 1 **by** *fastforce*

**have** 3:  $\forall x \in (\lambda f. \text{Some (match-no-prereq } f \ p)) \ 'm. x \neq \text{None}$  **by** *blast*

**show** ?*case*

**unfolding** 2 **unfolding** *eqTrueI[OF 3]* **unfolding** *if-True* **unfolding** *image-comp* *comp-def* **unfolding** *option.sel* **by** (*simp* *add: all-true-def*)

**qed**

**lemma** *of-match-fields-safe-eq*: **assumes** *all-prerequisites*  $m$  **shows** *OF-match-fields-safe*  $m = OF\text{-match-fields-unsafe } m$

**unfolding** *OF-match-fields-safe-def[abs-def]* *fun-eq-iff* *comp-def* **unfolding** *of-safe-unsafe-match-eq[OF assms]* **unfolding** *option.sel* **by** *clarify*

**lemma** *OF-match-fields-alt*: *OF-match-fields*  $m \ p =$

*(if*  $\exists f \in m. \neg \text{prerequisites } f \ m$  *then* *None* *else*

*if*  $\forall f \in m. \text{match-no-prereq } f \ p$  *then* *Some True* *else* *Some False*)

**unfolding** *OF-match-fields-def* *all-true-def[abs-def]* *set-seq-def* *match-prereq-def*

**by** (*auto simp add: ball-Un*)

**lemma** *of-match-fields-safe-eq2*: **assumes** *all-prerequisites*  $m$  **shows** *OF-match-fields-safe*  $m \ p \longleftrightarrow OF\text{-match-fields } m \ p = \text{Some True}$

**unfolding** *OF-match-fields-safe-def[abs-def]* *fun-eq-iff* *comp-def* **unfolding** *of-safe-unsafe-match-eq[OF assms]* **unfolding** *option.sel* **by** *simp*

**end**

**theory** *OpenFlow-Action*

**imports**

*OpenFlow-Matches*

**begin**

**datatype** *of-action* = *Forward* (*oiface-sel*: *string*) | *ModifyField-l2dst* 48 *word*

**fun** *of-action-semantics* **where**

*of-action-semantics*  $p \ [] = \{\}$  |

*of-action-semantics*  $p \ (a \# as) = (\text{case } a \text{ of}$

*Forward*  $i \Rightarrow \text{insert } (i, p) \text{ (of-action-semantic} s \text{ p as) |}$   
*ModifyField-l2dst*  $a \Rightarrow \text{of-action-semantic} s \text{ (p(p-l2dst := a)) as)$

**value** *of-action-semantic*  $p \ []$   
**value** *of-action-semantic*  $p \ [\text{ModifyField-l2dst } 66, \text{Forward } "oif"]$

**end**  
**theory** *Semantics-OpenFlow*  
**imports** *List-Group Sort-Descending*  
*IP-Addresses.IPv4*  
*OpenFlow-Helpers*  
**begin**

**datatype** *'a flowtable-behavior* = *Action 'a | NoAction | Undefined*

**definition** *option-to-ftb*  $b \equiv \text{case } b \text{ of Some } a \Rightarrow \text{Action } a \mid \text{None} \Rightarrow \text{NoAction}$

**definition** *ftb-to-option*  $b \equiv \text{case } b \text{ of Action } a \Rightarrow \text{Some } a \mid \text{NoAction} \Rightarrow \text{None}$

**datatype** (*'m, 'a*) *flow-entry-match* = *OFEntry (ofe-prio: 16 word) (ofe-fields: 'm set) (ofe-action: 'a)*

**find-consts** ( $('a \times 'b) \Rightarrow 'c \Rightarrow 'a \Rightarrow 'b \Rightarrow 'c$

**find-consts** ( $'a \Rightarrow 'b \Rightarrow 'c \Rightarrow ('a \times 'b) \Rightarrow 'c$

**definition** *split3*  $f \ p \equiv \text{case } p \text{ of } (a, b, c) \Rightarrow f \ a \ b \ c$

**find-consts** ( $'a \Rightarrow 'b \Rightarrow 'c \Rightarrow 'd \Rightarrow ('a \times 'b \times 'c) \Rightarrow 'd$

**type-synonym** (*'m, 'a*) *flowtable* = (*'m, 'a*) *flow-entry-match* *list*

**type-synonym** (*'m, 'p*) *field-matcher* = (*'m set*  $\Rightarrow 'p \Rightarrow \text{bool}$ )

**definition** *OF-same-priority-match2*  $:: ('m, 'p) \text{ field-matcher} \Rightarrow ('m, 'a) \text{ flowtable} \Rightarrow 'p \Rightarrow 'a \text{ flowtable-behavior}$  **where**

*OF-same-priority-match2*  $\gamma \text{ flow-entries packet} \equiv \text{let } s =$

$\{ \text{ofe-action } f \mid f. f \in \text{set flow-entries} \wedge \gamma (\text{ofe-fields } f) \text{ packet} \wedge$

$(\forall fo \in \text{set flow-entries. ofe-prio } fo > \text{ofe-prio } f \longrightarrow \neg \gamma (\text{ofe-fields } fo) \text{ packet}) \}$  *in*

*case card s of 0*  $\Rightarrow \text{NoAction}$

*| (Suc 0)*  $\Rightarrow \text{Action (the-elem } s)$

*| -*  $\Rightarrow \text{Undefined}$

**definition** *check-no-overlap*  $\gamma$  *ft* =  $(\forall a \in \text{set } ft. \forall b \in \text{set } ft. \forall p \in UNIV. (\text{ofe-prio } a = \text{ofe-prio } b \wedge \gamma (\text{ofe-fields } a) p \wedge a \neq b) \longrightarrow \neg \gamma (\text{ofe-fields } b) p)$

**definition** *check-no-overlap2*  $\gamma$  *ft* =  $(\forall a \in \text{set } ft. \forall b \in \text{set } ft. (a \neq b \wedge \text{ofe-prio } a = \text{ofe-prio } b) \longrightarrow \neg(\exists p \in UNIV. \gamma (\text{ofe-fields } a) p \wedge \gamma (\text{ofe-fields } b) p))$

**lemma** *check-no-overlap-alt*: *check-no-overlap*  $\gamma$  *ft* = *check-no-overlap2*  $\gamma$  *ft*

**unfolding** *check-no-overlap2-def* *check-no-overlap-def*

**by** *blast*

**lemma** *no-overlap-not-undefined*: *check-no-overlap*  $\gamma$  *ft*  $\implies$  *OF-same-priority-match2*  $\gamma$  *ft*  $p \neq Undefined$

**proof**

**assume** *goal1*: *check-no-overlap*  $\gamma$  *ft* *OF-same-priority-match2*  $\gamma$  *ft*  $p = Undefined$

**let** *?as* =  $\{f. f \in \text{set } ft \wedge \gamma (\text{ofe-fields } f) p \wedge (\forall fo \in \text{set } ft. \text{ofe-prio } f < \text{ofe-prio } fo \longrightarrow \neg \gamma (\text{ofe-fields } fo) p)\}$

**have** *fin*: *finite* *?as* **by** *simp*

**note** *goal1* (2)[*unfolded OF-same-priority-match2-def*]

**then have**  $2 \leq \text{card } (\text{ofe-action } ' ?as)$  **unfolding** *f-Img-ex-set*

**unfolding** *Let-def*

**by**(*cases* *card* (*ofe-action* ' *?as*), *simp*) (*rename-tac* *nat1*, *case-tac* *nat1*, *simp* *add*: *image-Collect*, *presburger*)

**then have**  $2 \leq \text{card } ?as$  **using** *card-image-le*[*OF fin*, *of ofe-action*] **by** *linarith*

**then obtain** *a b* **where** *ab*:  $a \neq b \wedge a \in ?as \wedge b \in ?as$  **using** *card2-eI* **by** *blast*

**then have** *ab2*:  $a \in \text{set } ft \wedge \gamma (\text{ofe-fields } a) p \wedge (\forall fo \in \text{set } ft. \text{ofe-prio } a < \text{ofe-prio } fo \longrightarrow \neg \gamma (\text{ofe-fields } fo) p)$

$b \in \text{set } ft \wedge \gamma (\text{ofe-fields } b) p \wedge (\forall fo \in \text{set } ft. \text{ofe-prio } b < \text{ofe-prio } fo \longrightarrow \neg \gamma (\text{ofe-fields } fo) p)$  **by** *simp-all*

**then have** *ofe-prio* *a* = *ofe-prio* *b*

**by** *fastforce*

**note** *goal1* (1)[*unfolded check-no-overlap-def*] *ab2* (1) *ab2* (4) *this* *ab2* (2) *ab* (1) *ab2* (5)

**then show** *False* **by** *blast*

**qed**

**fun** *OF-match-linear* ::  $(\text{'m}, \text{'p}) \text{ field-matcher} \Rightarrow (\text{'m}, \text{'a}) \text{ flowtable} \Rightarrow \text{'p} \Rightarrow \text{'a} \text{ flowtable-behavior}$  **where**

*OF-match-linear* - [] - = *NoAction* |

*OF-match-linear*  $\gamma$  (*a* # *as*)  $p = (\text{if } \gamma (\text{ofe-fields } a) p \text{ then } \text{Action } (\text{ofe-action } a) \text{ else } \text{OF-match-linear } \gamma \text{ as } p)$

**lemma** *OF-match-linear-ne-Undefined*: *OF-match-linear*  $\gamma$  *ft*  $p \neq Undefined$

**by**(*induction* *ft*) *auto*

**lemma** *OF-match-linear-append*: *OF-match-linear*  $\gamma$  (*a* @ *b*)  $p = (\text{case } \text{OF-match-linear } \gamma \text{ a } p \text{ of } \text{NoAction} \Rightarrow \text{OF-match-linear } \gamma \text{ b } p \mid x \Rightarrow x)$

**by**(*induction* *a*) *simp-all*

**lemma** *OF-match-linear-match-allsameaction*:  $\llbracket gr \in \text{set } oms; \gamma \text{ gr } p = \text{True} \rrbracket$

$\implies \text{OF-match-linear } \gamma (\text{map } (\lambda x. \text{split3 } \text{OFEntry } (pri, x, act)) oms) p = \text{Action } act$

**by**(*induction* *oms*) (*auto* *simp* *add*: *split3-def*)

**lemma** *OF-lm-noa-none-iff*: *OF-match-linear*  $\gamma$  *ft*  $p = \text{NoAction} \longleftrightarrow (\forall e \in \text{set } ft. \neg \gamma (\text{ofe-fields } e) p)$

**by**(*induction* *ft*) (*simp-all* *split*: *if-splits*)

**lemma** *set-eq-rule*:  $(\bigwedge x. x \in a \implies x \in b) \implies (\bigwedge x. x \in b \implies x \in a) \implies a = b$  **by**(*rule antisym*[*OF subsetI subsetI*])

**lemma** *unmatching-insert-agnostic*:  $\neg \gamma (\text{ofe-fields } a) p \implies \text{OF-same-priority-match2 } \gamma (a \# ft) p = \text{OF-same-priority-match2 } \gamma ft p$

**proof** –

**let** *?as* =  $\{f. f \in \text{set } ft \wedge \gamma (\text{ofe-fields } f) p \wedge (\forall fo \in \text{set } ft. \text{ofe-prio } f < \text{ofe-prio } fo \longrightarrow \neg \gamma (\text{ofe-fields } fo) p)\}$

**let** *?aas* =  $\{f \mid f. f \in \text{set } (a \# ft) \wedge \gamma (\text{ofe-fields } f) p \wedge (\forall fo \in \text{set } (a \# ft). \text{ofe-prio } f < \text{ofe-prio } fo \longrightarrow \neg \gamma (\text{ofe-fields } fo) p)\}$

**assume** *nm*:  $\neg \gamma (\text{ofe-fields } a) p$

```

have aa: ?aas = ?as
proof(rule set-eq-rule)
  fix x
  assume x ∈ {f | f. f ∈ set (a # ft) ∧ γ (ofe-fields f) p ∧ (∀ fo ∈ set (a # ft). ofe-prio f < ofe-prio fo → ¬ γ (ofe-fields fo) p)}
  hence as: x ∈ set (a # ft) ∧ γ (ofe-fields x) p ∧ (∀ fo ∈ set (a # ft). ofe-prio x < ofe-prio fo → ¬ γ (ofe-fields fo) p) by simp
  with nm have x ∈ set ft by fastforce
  moreover from as have (∀ fo ∈ set ft. ofe-prio x < ofe-prio fo → ¬ γ (ofe-fields fo) p) by simp
  ultimately show x ∈ {f ∈ set ft. γ (ofe-fields f) p ∧ (∀ fo ∈ set ft. ofe-prio f < ofe-prio fo → ¬ γ (ofe-fields fo) p)} using as by force
next
  fix x
  assume x ∈ {f ∈ set ft. γ (ofe-fields f) p ∧ (∀ fo ∈ set ft. ofe-prio f < ofe-prio fo → ¬ γ (ofe-fields fo) p)}
  hence as: x ∈ set ft ∧ γ (ofe-fields x) p ∧ (∀ fo ∈ set ft. ofe-prio x < ofe-prio fo → ¬ γ (ofe-fields fo) p) by simp-all
  from as(1) have x ∈ set (a # ft) by simp
  moreover from as(3) have (∀ fo ∈ set (a # ft). ofe-prio x < ofe-prio fo → ¬ γ (ofe-fields fo) p) using nm by simp
  ultimately show x ∈ {f | f. f ∈ set (a # ft) ∧ γ (ofe-fields f) p ∧ (∀ fo ∈ set (a # ft). ofe-prio f < ofe-prio fo → ¬ γ (ofe-fields fo) p)} using as(2) by blast
qed
note uf = arg-cong[OF aa, of (·) ofe-action, unfolded image-Collect]
show ?thesis unfolding OF-same-priority-match2-def using uf by presburger
qed

lemma OF-match-eq: sorted-descending (map ofe-prio ft) ⇒ check-no-overlap γ ft ⇒
  OF-same-priority-match2 γ ft p = OF-match-linear γ ft p
proof(induction ft)
  case (Cons a ft)
  have 1: sorted-descending (map ofe-prio ft) using Cons(2) by simp
  have 2: check-no-overlap γ ft using Cons(3) unfolding check-no-overlap-def using set-subset-Cons by fast
  note mIH = Cons(1)[OF 1 2]
  show ?case (is ?kees)
  proof(cases γ (ofe-fields a) p)
    case False thus ?kees
      by(simp only: OF-match-linear.simps if-False mIH[symmetric] unmatching-insert-agnostic[of γ, OF False])
  next
    note sorted-descending-split[OF Cons(2)]
    then obtain m n where mn: a # ft = m @ n ∧ ∀ e ∈ set m. ofe-prio a = ofe-prio e ∧ ∀ e ∈ set n. ofe-prio e < ofe-prio a
      unfolding list.sel by blast
    hence aem: a ∈ set m
      by (metis UnE less-imp-neq list.set-intros(1) set-append)
    have mover: check-no-overlap γ m using Cons(3) unfolding check-no-overlap-def
      by (metis Un-iff mn(1) set-append)
    let ?fc = (λs.
      {f. f ∈ set s ∧ γ (ofe-fields f) p ∧
        (∀ fo ∈ set (a # ft). ofe-prio f < ofe-prio fo → ¬ γ (ofe-fields fo) p)})
    case True
    have ?fc (m @ n) = ?fc m ∪ ?fc n by auto
    moreover have ?fc n = {}
    proof(rule set-eq-rule, rule ccontr, goal-cases)
      case (1 x)
      hence g1: x ∈ set n ∧ γ (ofe-fields x) p
        (∀ fo ∈ set m. ofe-prio x < ofe-prio fo → ¬ γ (ofe-fields fo) p)
        (∀ fo ∈ set n. ofe-prio x < ofe-prio fo → ¬ γ (ofe-fields fo) p)

```



```

  unfolding mn(1) by(simp-all)
  from g1(1) mn(3) have le: ofe-prio x < ofe-prio a by simp
  note le g1(3) aem True
  then show False by blast
qed simp
ultimately have cc: ?fc (m @ n) = ?fc m by blast
have cm: ?fc m = {a}
proof -
  have  $\forall f \in \text{set } m. (\forall fo \in \text{set } (a \# ft). \text{ ofe-prio } f < \text{ ofe-prio } fo \longrightarrow \neg \gamma (\text{ ofe-fields } fo) p)$ 
  by (metis UnE less-asm mn set-append)
  hence 1: ?fc m = {f ∈ set m.  $\gamma (\text{ ofe-fields } f) p$ } by blast
  show {f ∈ set m.  $\gamma (\text{ ofe-fields } f) p \wedge (\forall fo \in \text{set } (a \# ft). \text{ ofe-prio } f < \text{ ofe-prio } fo \longrightarrow \neg \gamma (\text{ ofe-fields } fo) p)$ } = {a}
unfolding 1
proof(rule set-eq-rule, goal-cases fwd bwd)
  case (bwd x)
  have a ∈ {f ∈ set m.  $\gamma (\text{ ofe-fields } f) p$ } using True aem by simp
  thus ?case using bwd by simp
next
  case (fwd x) show ?case proof(rule ccontr)
    assume  $x \notin \{a\}$  hence ne:  $x \neq a$  by simp
    from fwd have 1:  $x \in \text{set } m \wedge \gamma (\text{ ofe-fields } x) p$  by simp-all
    have 2: ofe-prio x = ofe-prio a using 1(1) mn(2) by simp
    show False using 1 ne mover aem True 2 unfolding check-no-overlap-def by blast
  qed
qed
qed
show ?kees
  unfolding mn(1)
  unfolding OF-same-priority-match2-def
  unfolding f-Img-ex-set
  unfolding cc[unfolded mn(1)]
  unfolding cm[unfolded mn(1)]
  unfolding Let-def
  by(simp only: mn(1)[symmetric] OF-match-linear.simps True if-True, simp)
qed
qed (simp add: OF-same-priority-match2-def)

lemma overlap-sort-invar[simp]: check-no-overlap  $\gamma (\text{ sort-descending-key } k \text{ } ft) = \text{ check-no-overlap } \gamma \text{ } ft$ 
  unfolding check-no-overlap-def
  unfolding sort-descending-set-inv
  ..

lemma OF-match-eq2:
  assumes check-no-overlap  $\gamma \text{ } ft$ 
  shows OF-same-priority-match2  $\gamma \text{ } ft \text{ } p = \text{ OF-match-linear } \gamma (\text{ sort-descending-key ofe-prio } ft) \text{ } p$ 
proof -
  have sorted-descending (map ofe-prio (sort-descending-key ofe-prio ft)) by (simp add: sorted-descending-sort-descending-key)
  note ceq = OF-match-eq[OF this, unfolded overlap-sort-invar, OF  $\langle \text{ check-no-overlap } \gamma \text{ } ft \rangle$ , symmetric]
  show ?thesis
    unfolding ceq
    unfolding OF-same-priority-match2-def
    unfolding sort-descending-set-inv
    ..
qed

```

**lemma** prio-match-matcher-alt:  $\{f. f \in \text{set flow-entries} \wedge \gamma (\text{ofe-fields } f) \text{ packet} \wedge$   
 $(\forall fo \in \text{set flow-entries. ofe-prio } fo > \text{ofe-prio } f \longrightarrow \neg \gamma (\text{ofe-fields } fo) \text{ packet})\}$   
 $=$  (  
 $\text{let matching} = \{f. f \in \text{set flow-entries} \wedge \gamma (\text{ofe-fields } f) \text{ packet}\}$   
 $\text{in } \{f. f \in \text{matching} \wedge (\forall fo \in \text{matching. ofe-prio } fo \leq \text{ofe-prio } f)\}$   
 $)$

**by**(auto simp add: Let-def)

**lemma** prio-match-matcher-alt2: (  
 $\text{let matching} = \{f. f \in \text{set flow-entries} \wedge \gamma (\text{ofe-fields } f) \text{ packet}\}$   
 $\text{in } \{f. f \in \text{matching} \wedge (\forall fo \in \text{matching. ofe-prio } fo \leq \text{ofe-prio } f)\}$   
 $) = \text{set } ($   
 $\text{let matching} = \text{filter } (\lambda f. \gamma (\text{ofe-fields } f) \text{ packet}) \text{ flow-entries}$   
 $\text{in filter } (\lambda f. \forall fo \in \text{set matching. ofe-prio } fo \leq \text{ofe-prio } f) \text{ matching}$   
 $)$

**by**(auto simp add: Let-def)

**definition** OF-priority-match **where**

$OF\text{-priority-match } \gamma \text{ flow-entries packet} \equiv$   
 $\text{let } m = \text{filter } (\lambda f. \gamma (\text{ofe-fields } f) \text{ packet}) \text{ flow-entries};$   
 $m' = \text{filter } (\lambda f. \forall fo \in \text{set } m. \text{ofe-prio } fo \leq \text{ofe-prio } f) m \text{ in}$   
 $\text{case } m' \text{ of } [] \Rightarrow \text{NoAction}$   
 $\quad | [s] \Rightarrow \text{Action } (\text{ofe-action } s)$   
 $\quad | - \Rightarrow \text{Undefined}$

**definition** OF-priority-match-ana **where**

$OF\text{-priority-match-ana } \gamma \text{ flow-entries packet} \equiv$   
 $\text{let } m = \text{filter } (\lambda f. \gamma (\text{ofe-fields } f) \text{ packet}) \text{ flow-entries};$   
 $m' = \text{filter } (\lambda f. \forall fo \in \text{set } m. \text{ofe-prio } fo \leq \text{ofe-prio } f) m \text{ in}$   
 $\text{case } m' \text{ of } [] \Rightarrow \text{NoAction}$   
 $\quad | [s] \Rightarrow \text{Action } s$   
 $\quad | - \Rightarrow \text{Undefined}$

**lemma** filter-singleton:  $[x \leftarrow s. f \ x] = [y] \implies f \ y \wedge y \in \text{set } s \text{ by } (\text{metis filter-eq-Cons-iff in-set-conv-decomp})$

**lemma** OF-spm3-get-fe:  $OF\text{-priority-match } \gamma \text{ ft } p = \text{Action } a \implies \exists fe. \text{ofe-action } fe = a \wedge fe \in \text{set } ft \wedge OF\text{-priority-match-ana}$   
 $\gamma \text{ ft } p = \text{Action } fe$

**unfolding** OF-priority-match-def OF-priority-match-ana-def

**by**(clarsimp split: flowtable-behavior.splits list.splits) (drule filter-singleton; simp)

**fun** no-overlaps **where**

$\text{no-overlaps } [] = \text{True} \mid$   
 $\text{no-overlaps } \gamma (a \# as) = (\text{no-overlaps } \gamma as \wedge ($   
 $\forall b \in \text{set } as. \text{ofe-prio } a = \text{ofe-prio } b \longrightarrow \neg (\exists p \in UNIV. \gamma (\text{ofe-fields } a) p \wedge \gamma (\text{ofe-fields } b) p)))$

**lemma** no-overlap-ConsI:  $\text{check-no-overlap2 } \gamma (x \# xs) \implies \text{check-no-overlap2 } \gamma xs$

**unfolding** check-no-overlap2-def **by** simp

**lemma** no-overlapsI:  $\text{check-no-overlap } \gamma t \implies \text{distinct } t \implies \text{no-overlaps } \gamma t$

**unfolding** check-no-overlap-alt

**proof**(induction t)

**case** (Cons a t)

**from** no-overlap-ConsI[OF Cons(2)] Cons(3,1)

```

have no-overlaps  $\gamma$  t by simp
thus ?case using Cons(2,3) unfolding check-no-overlap2-def by auto
qed (simp add: check-no-overlap2-def)

```

```

lemma check-no-overlapI: no-overlaps  $\gamma$  t  $\implies$  check-no-overlap  $\gamma$  t
unfolding check-no-overlap-alt
proof(induction t)
case (Cons a t)
from Cons(1)[OF conjunct1[OF Cons(2)[unfolded no-overlaps.simps]]]
show ?case
using conjunct2[OF Cons(2)[unfolded no-overlaps.simps]]
unfolding check-no-overlap2-def
by auto
qed (simp add: check-no-overlap2-def)

```

```

lemma ( $\bigwedge e p. e \in \text{set } t \implies \neg \gamma (\text{ofe-fields } e) p$ )  $\implies$  no-overlaps  $\gamma$  t
by(induction t) simp-all

```

```

lemma no-overlaps-append: no-overlaps  $\gamma$  (x @ y)  $\implies$  no-overlaps  $\gamma$  y
by(induction x) simp-all

```

```

lemma no-overlaps-ne1: no-overlaps  $\gamma$  (x @ a # y @ b # z)  $\implies$  (( $\exists p. \gamma (\text{ofe-fields } a) p$ )  $\vee$  ( $\exists p. \gamma (\text{ofe-fields } b) p$ ))  $\implies$  a
 $\neq$  b

```

```

proof (rule notI, goal-cases contr)
case contr
from contr(1) no-overlaps-append have no-overlaps  $\gamma$  (a # y @ b # z) by blast
note this[unfolded no-overlaps.simps]
with contr(3) have  $\neg (\exists p \in \text{UNIV}. \gamma (\text{ofe-fields } a) p \wedge \gamma (\text{ofe-fields } b) p)$  by simp
with contr(2) show False unfolding contr(3) by simp
qed

```

```

lemma no-overlaps-defeq: no-overlaps  $\gamma$  fe  $\implies$  OF-same-priority-match2  $\gamma$  fe p = OF-priority-match  $\gamma$  fe p
unfolding OF-same-priority-match2-def OF-priority-match-def

```

```

unfolding f-Img-ex-set
unfolding prio-match-matcher-alt
unfolding prio-match-matcher-alt2

```

```

proof (goal-cases uf)
case uf
let ?m' = let m = [f  $\leftarrow$  fe .  $\gamma$  (ofe-fields f) p] in [f  $\leftarrow$  m .  $\forall fo \in \text{set } m. \text{ofe-prio } fo \leq \text{ofe-prio } f$ ]
let ?s = ofe-action 'set ?m'
from uf show ?case
proof(cases ?m')
case Nil
moreover then have card ?s = 0 by force
ultimately show ?thesis by(simp add: Let-def)

```

```

next

```

```

case (Cons a as)
have as = []
proof(rule ccontr)
assume as  $\neq$  []
then obtain b bs where bbs: as = b # bs by (meson neq-Nil-conv)
note no = Cons[unfolded Let-def filter-filter]
have f1: a  $\in$  set ?m' b  $\in$  set ?m' unfolding bbs local.Cons by simp-all
hence ofe-prio a = ofe-prio b by (simp add: antisym)
moreover have ms:  $\gamma$  (ofe-fields a) p  $\gamma$  (ofe-fields b) p using no[symmetric] unfolding bbs by(blast dest: Cons-eq-filterD)+
moreover have abis: a  $\in$  set fe b  $\in$  set fe using f1 by auto

```

```

moreover have  $a \neq b$  proof(cases  $\exists x y z. fe = x @ a \# y @ b \# z$ )
  case True
    then obtain  $x y z$  where  $xyz: fe = x @ a \# y @ b \# z$  by blast
    from no-overlaps-ne1 ms(1) uf[unfolded xyz]
    show ?thesis by blast
  next
    case False
    then obtain  $x y z$  where  $xyz: fe = x @ b \# y @ a \# z$ 
    using no unfolding bbs
    by (metis (no-types, lifting) Cons-eq-filterD)
    from no-overlaps-ne1 ms(1) uf[unfolded xyz]
    show ?thesis by blast
  qed
ultimately show False using check-no-overlapI[OF uf, unfolded check-no-overlap-def] by blast
qed
then have  $oe: a \# as = [a]$  by simp
show ?thesis using Cons[unfolded oe] by force
qed
qed

lemma distinct fe  $\implies$  check-no-overlap  $\gamma$  fe  $\implies$  OF-same-priority-match2  $\gamma$  fe p = OF-priority-match  $\gamma$  fe p
by(rule no-overlaps-defeq) (drule (2) no-overlapsI)

theorem OF-eq:
assumes no: no-overlaps  $\gamma$  f
and so: sorted-descending (map ofe-prio f)
shows OF-match-linear  $\gamma$  f p = OF-priority-match  $\gamma$  f p
unfolding no-overlaps-defeq[symmetric, OF no] OF-match-eq[OF so check-no-overlapI[OF no]]
..

corollary OF-eq-sort:
assumes no: no-overlaps  $\gamma$  f
shows OF-priority-match  $\gamma$  f p = OF-match-linear  $\gamma$  (sort-descending-key ofe-prio f) p
using OF-match-eq2 check-no-overlapI no no-overlaps-defeq by fastforce

lemma OF-lm-noa-none: OF-match-linear  $\gamma$  ft p = NoAction  $\implies \forall e \in \text{set } ft. \neg \gamma$  (ofe-fields e) p
by(induction ft) (simp-all split: if-splits)

lemma OF-spm3-noa-none:
assumes no: no-overlaps  $\gamma$  ft
shows OF-priority-match  $\gamma$  ft p = NoAction  $\implies \forall e \in \text{set } ft. \neg \gamma$  (ofe-fields e) p
unfolding OF-eq-sort[OF no] by(drule OF-lm-noa-none) simp

lemma no-overlaps-not-undefined: no-overlaps  $\gamma$  ft  $\implies$  OF-priority-match  $\gamma$  ft p  $\neq$  Undefined
using check-no-overlapI no-overlap-not-undefined no-overlaps-defeq by fastforce

end
theory OpenFlow-Serialize
imports OpenFlow-Matches
         OpenFlow-Action
         Semantics-OpenFlow
         Simple-Firewall.Primitives-toString

```

*IP-Addresses.Lib-Word-toString*

**begin**

**definition** *serialization-test-entry*  $\equiv$  *OFEntry* 7 {*EtherDst* 0x1, *IPv4Dst* (*PrefixMatch* 0xA000201 32), *IngressPort* "s1-lan", *L4Dst* 0x50 0, *L4Src* 0x400 0x3FF, *IPv4Proto* 6, *EtherType* 0x800} [*ModifyField-l2dst* 0xA641F185E862, *Forward* "s1-wan"]

**value** (*map* ((<<) (1::48 word)  $\circ$  (\*) 8)  $\circ$  rev) [0..<6]

**definition** *serialize-mac* (*m*::48 word)  $\equiv$  (*intersperse* (CHR "'")  $\circ$  *map* (*hex-string-of-word* 1  $\circ$  ( $\lambda h.$  (*m* >> *h* \* 8) && 0xFF))  $\circ$  rev) [0..<6]

**lemma** *serialize-mac* 0xdeadbeefcafe = "de:ad:be:ef:ca:fe" by eval

**definition** *serialize-action pids* *a*  $\equiv$  (case *a* of  
*Forward* *oif*  $\Rightarrow$  "output:" @ *pids* *oif* |  
*ModifyField-l2dst* *na*  $\Rightarrow$  "mod-dl-dst:" @ *serialize-mac* *na*)

**definition** *serialize-actions pids* *a*  $\equiv$  if length *a* = 0 then "drop" else (*intersperse* (CHR "'")  $\circ$  *map* (*serialize-action pids*)) *a*

**lemma** *serialize-actions* ( $\lambda oif.$  "42") (*ofe-action serialization-test-entry*) =  
"mod-dl-dst:a6:41:f1:85:e8:62,output:42" by eval

**lemma** *serialize-actions* anything [] = "drop"  
by (simp add: *serialize-actions-def*)

**definition** *prefix-to-string pfx*  $\equiv$  *ipv4-cidr-toString* (*pfxm-prefix* *pfx*, *pfxm-length* *pfx*)

**primrec** *serialize-of-match* **where**

*serialize-of-match pids* (*IngressPort* *p*) = "in-port=" @ *pids* *p* |  
*serialize-of-match* - (*VlanId* *i*) = "dl-vlan=" @ *dec-string-of-word0* *i* |  
*serialize-of-match* - (*VlanPriority* -) = undefined |  
*serialize-of-match* - (*EtherType* *i*) = "dl-type=0x" @ *hex-string-of-word0* *i* |  
*serialize-of-match* - (*EtherSrc* *m*) = "dl-src=" @ *serialize-mac* *m* |  
*serialize-of-match* - (*EtherDst* *m*) = "dl-dst=" @ *serialize-mac* *m* |  
*serialize-of-match* - (*IPv4Proto* *i*) = "nw-proto=" @ *dec-string-of-word0* *i* |  
*serialize-of-match* - (*IPv4Src* *p*) = "nw-src=" @ *prefix-to-string* *p* |  
*serialize-of-match* - (*IPv4Dst* *p*) = "nw-dst=" @ *prefix-to-string* *p* |  
*serialize-of-match* - (*L4Src* *i* *m*) = "tp-src=" @ *dec-string-of-word0* *i* @ (if *m* = - 1 then [] else "/0x" @ *hex-string-of-word* 3 *m*) |  
*serialize-of-match* - (*L4Dst* *i* *m*) = "tp-dst=" @ *dec-string-of-word0* *i* @ (if *m* = - 1 then [] else "/0x" @ *hex-string-of-word* 3 *m*)

**definition** *serialize-of-matches* :: (string  $\Rightarrow$  string)  $\Rightarrow$  of-match-field set  $\Rightarrow$  string

**where**

*serialize-of-matches pids*  $\equiv$  (@) "hard-timeout=0,idle-timeout=0,"  $\circ$  *intersperse* (CHR "'")  $\circ$  *map* (*serialize-of-match pids*)  
 $\circ$  *sorted-list-of-set*

**lemma** *serialize-of-matches pids of-matches* =  
(*List.append* "hard-timeout=0,idle-timeout=0,"  
(*intersperse* (CHR "'") (*map* (*serialize-of-match pids*) (*sorted-list-of-set of-matches*))))  
by (simp add: *serialize-of-matches-def*)

**export-code** *serialize-of-matches* **checking** *SML*

**lemma** *serialize-of-matches* ( $\lambda oif. "42"$ ) (*ofe-fields serialization-test-entry*) =  
 $"hard-timeout=0, idle-timeout=0, in-port=42, dl-type=0x800, dl-dst=00:00:00:00:00:01, nw-proto=6, nw-dst=10.0.2.1/32, tp-src=1024,$   
**by** *eval*

**definition** *serialize-of-entry pids e*  $\equiv$  (case *e* of (*OFEntry p f a*)  $\Rightarrow$  "priority=" @ *dec-string-of-word0 p* @ ", " @ *serialize-of-matches pids f* @ ", " @ "action=" @ *serialize-actions pids a*)

**lemma** *serialize-of-entry* (*the*  $\circ$  *map-of* [(*s1-lan*,"42"),(*s1-wan*,"1337")]) *serialization-test-entry* =  
 $"priority=7, hard-timeout=0, idle-timeout=0, in-port=42, dl-type=0x800, dl-dst=00:00:00:00:00:01, nw-proto=6, nw-dst=10.0.2.1/32, tp-$   
**by** *eval*

**end**

**theory** *Featherweight-OpenFlow-Comparison*

**imports** *Semantics-OpenFlow*

**begin**

**inductive** *guha-table-semantics* :: ('*m*, '*p*) *field-matcher*  $\Rightarrow$  ('*m*, '*a*) *flowtable*  $\Rightarrow$  '*p*  $\Rightarrow$  '*a* *option*  $\Rightarrow$  **bool** **where**  
*guha-matched*:  $\gamma$  (*ofe-fields fe*) *p* = **True**  $\Longrightarrow$   
 $\forall fe' \in \text{set } (ft1 @ ft2). \text{ ofe-prio } fe' > \text{ ofe-prio } fe \longrightarrow \gamma$  (*ofe-fields fe'*) *p* = **False**  $\Longrightarrow$   
*guha-table-semantics*  $\gamma$  (*ft1* @ *fe* # *ft2*) *p* (*Some* (*ofe-action fe*)) |  
*guha-unmatched*:  $\forall fe \in \text{set } ft. \gamma$  (*ofe-fields fe*) *p* = **False**  $\Longrightarrow$   
*guha-table-semantics*  $\gamma$  *ft p* **None**

**lemma** *guha-table-semantics-ex2res*:

**assumes** *ta*: *CARD*('a)  $\geq 2$

**assumes** *ms*:  $\exists ff. \gamma ff p$

**shows**  $\exists ft (a1 :: 'a) (a2 :: 'a). a1 \neq a2 \wedge \text{guha-table-semantics } \gamma ft p (\text{Some } a1) \wedge \text{guha-table-semantics } \gamma ft p (\text{Some } a2)$

**proof** –

**from** *ms* **obtain** *ff* **where** *m*:  $\gamma ff p$  ..

**from** *ta* **obtain** *a1 a2* :: '*a* **where** *as*: *a1*  $\neq$  *a2* **using** *card2-eI* **by** *blast*

**let** *?fe1* = *OFEntry 0 ff a1*

**let** *?fe2* = *OFEntry 0 ff a2*

**let** *?ft* = [*?fe1*, *?fe2*]

**have** *guha-table-semantics*  $\gamma ?ft p (\text{Some } a1)$  *guha-table-semantics*  $\gamma ?ft p (\text{Some } a2)$

**by**(*rule guha-table-semantics.intros(1)[of*  $\gamma ?fe1 p$  [] [*?fe2*], *unfolded append-Nil flow-entry-match.sel* |

*rule guha-table-semantics.intros(1)[of*  $\gamma ?fe2 p$  [*?fe1*] [], *unfolded append-Nil2 flow-entry-match.sel append.simps*] |

*simp add: m*)+

**thus** *?thesis* **using** *as* **by**(*intro exI conjI*)

**qed**

**lemma** *guha-umstaendlich*:

**assumes** *ae*: *a* = *ofe-action fe*

**assumes** *ele*: *fe*  $\in$  *set ft*

**assumes** *rest*:  $\gamma$  (*ofe-fields fe*) *p*

$\forall fe' \in \text{set } ft. \text{ ofe-prio } fe' > \text{ ofe-prio } fe \longrightarrow \neg \gamma$  (*ofe-fields fe'*) *p*

**shows** *guha-table-semantics*  $\gamma ft p (\text{Some } a)$

**proof** –

**from** *ele* **obtain** *ft1 ft2* **where** *ftspl*: *ft* = *ft1* @ *fe* # *ft2* **using** *split-list* **by** *fastforce*

**show** *?thesis* **unfolding** *ae ftspl*

```

apply(rule guha-table-semantics.intros(1))
using rest(1) apply(simp)
using rest(2)[unfolded ftspl] apply simp
done
qed

```

**lemma** *guha-matched-rule-inversion*:

```

assumes guha-table-semantics  $\gamma$  ft p (Some a)
shows  $\exists fe \in \text{set ft. } a = \text{ofe-action } fe \wedge \gamma (\text{ofe-fields } fe) p \wedge (\forall fe' \in \text{set ft. } \text{ofe-prio } fe' > \text{ofe-prio } fe \longrightarrow \neg \gamma (\text{ofe-fields } fe'))$ 
p)
proof –
{
  fix d
  assume guha-table-semantics  $\gamma$  ft p d
  hence Some a = d  $\implies (\exists fe \in \text{set ft. } a = \text{ofe-action } fe \wedge \gamma (\text{ofe-fields } fe) p \wedge (\forall fe' \in \text{set ft. } \text{ofe-prio } fe' > \text{ofe-prio } fe \longrightarrow \neg \gamma (\text{ofe-fields } fe')) p)$ 
  by(induction rule: guha-table-semantics.induct) simp-all
}
from this[OF assms refl]
show ?thesis .
qed

```

**lemma** *guha-equal-Action*:

```

assumes no: no-overlaps  $\gamma$  ft
assumes spm: OF-priority-match  $\gamma$  ft p = Action a
shows guha-table-semantics  $\gamma$  ft p (Some a)
proof –
  note spm[THEN OF-spm3-get-fe] then obtain fe where a: ofe-action fe = a and fein: fe  $\in$  set ft and feana:
  OF-priority-match-ana  $\gamma$  ft p = Action fe by blast
  show ?thesis
  apply(rule guha-umstaendlich)
  apply(rule a[symmetric])
  apply(rule fein)
  using feana unfolding OF-priority-match-ana-def
  apply(auto dest!: filter-singleton split: list.splits)
done
qed

```

**lemma** *guha-equal-NoAction*:

```

assumes no: no-overlaps  $\gamma$  ft
assumes spm: OF-priority-match  $\gamma$  ft p = NoAction
shows guha-table-semantics  $\gamma$  ft p None
using spm unfolding OF-priority-match-def
by(auto simp add: filter-empty-conv OF-spm3-noa-none[OF no spm] intro: guha-table-semantics.intros(2) split: list.splits)

```

**lemma** *guha-equal-hlp*:

```

assumes no: no-overlaps  $\gamma$  ft
shows guha-table-semantics  $\gamma$  ft p (ftb-to-option (OF-priority-match  $\gamma$  ft p))
unfolding ftb-to-option-def
apply(cases (OF-priority-match  $\gamma$  ft p))
apply(simp add: guha-equal-Action[OF no])
apply(simp add: guha-equal-NoAction[OF no])
apply(subgoal-tac False, simp)
apply(simp add: no no-overlaps-not-unefined)

```

done

**lemma** *guha-deterministic1*: *guha-table-semantics*  $\gamma$  *ft* *p* (Some *x1*)  $\implies \neg$  *guha-table-semantics*  $\gamma$  *ft* *p* None  
**by**(*auto simp add: guha-table-semantics.simps*)

**lemma** *guha-deterministic2*:  $\llbracket \text{no-overlaps } \gamma \text{ ft}; \text{guha-table-semantics } \gamma \text{ ft } p \text{ (Some } x1); \text{guha-table-semantics } \gamma \text{ ft } p \text{ (Some } a) \rrbracket$   
 $\implies x1 = a$

**proof**(*rule ccontr, goal-cases*)

**case** 1

**note** 1(2-3)[*THEN guha-matched-rule-inversion*] **then obtain** *fe1 fe2* **where** *fes*:

$fe1 \in \text{set ft} \rightarrow x1 = \text{ofe-action } fe1 \wedge \gamma (\text{ofe-fields } fe1) \wedge p \wedge (\forall fe' \in \text{set ft. ofe-prio } fe1 < \text{ofe-prio } fe' \longrightarrow \neg \gamma (\text{ofe-fields } fe') \wedge p)$   
 $fe2 \in \text{set ft} \rightarrow a = \text{ofe-action } fe2 \wedge \gamma (\text{ofe-fields } fe2) \wedge p \wedge (\forall fe' \in \text{set ft. ofe-prio } fe2 < \text{ofe-prio } fe' \longrightarrow \neg \gamma (\text{ofe-fields } fe') \wedge p)$   
**by** *blast*

**from**  $\langle x1 \neq a \rangle$  **have** *fene*: *fe1*  $\neq$  *fe2* **using** *fes*(2,6) **by** *blast*

**have** *pe*: *ofe-prio* *fe1* = *ofe-prio* *fe2* **using** *fes*(1,3-4,5,7-8) *less-linear* **by** *blast*

**note**  $\langle \text{no-overlaps } \gamma \text{ ft} \rangle$  [*THEN check-no-overlapI, unfolded check-no-overlap-def*]

**note** *this*[*unfolded Ball-def, THEN spec, THEN mp, OF fes*(1), *THEN spec, THEN mp, OF fes*(5), *THEN spec, THEN mp, OF UNIV-I, of p*] *pe fene fes*(3,7)

**thus** *False* **by** *blast*

**qed**

**lemma** *guha-equal*:

**assumes** *no*: *no-overlaps*  $\gamma$  *ft*

**shows** *OF-priority-match*  $\gamma$  *ft* *p* = *option-to-ftb* *d*  $\longleftrightarrow$  *guha-table-semantics*  $\gamma$  *ft* *p* *d*

**using** *guha-equal-hlp*[*OF no, of p*] **unfolding** *ftb-to-option-def option-to-ftb-def*

**apply**(*cases OF-priority-match*  $\gamma$  *ft* *p*; *cases d*)

**apply**(*simp-all*)

**using** *guha-deterministic1* **apply** *fast*

**using** *guha-deterministic2*[*OF no*] **apply** *blast*

**using** *guha-deterministic1* **apply** *fast*

**using** *no-overlaps-not-uneffined*[*OF no*] **apply** *fastforce*

**using** *no-overlaps-not-uneffined*[*OF no*] **apply** *fastforce*

done

**lemma** *guha-nondeterministicD*:

**assumes**  $\neg \text{check-no-overlap } \gamma \text{ ft}$

**shows**  $\exists fe1 fe2 p. fe1 \in \text{set ft} \wedge fe2 \in \text{set ft}$

$\wedge \text{guha-table-semantics } \gamma \text{ ft } p \text{ (Some (ofe-action } fe1))$

$\wedge \text{guha-table-semantics } \gamma \text{ ft } p \text{ (Some (ofe-action } fe2))$

**using** *assms*

**apply**(*unfold check-no-overlap-def*)

**apply**(*clarsimp*)

**apply**(*rename-tac* *fe1 fe2 p*)

**apply**(*rule-tac* *x = fe1 in exI*)

**apply**(*simp*)

**apply**(*rule-tac* *x = fe2 in exI*)

**apply**(*simp*)

**apply**(*rule-tac* *x = p in exI*)

**apply**(*rule conjI*)

**apply**(*subst guha-table-semantics.simps*)

**apply**(*rule disjI1*)

**apply**(*clarsimp*)

**apply**(*rule-tac* *x = fe1 in exI*)

**apply**(*drule split-list*)



```

apply(clarify)
apply(rename-tac ft1 ft2)
apply(rule-tac x = ft1 in exI)
apply(rule-tac x = ft2 in exI)
apply(simp)
oops

```

The above lemma does indeed not hold, the reason for this are (possibly partially) shadowed overlaps. This is exemplified below: If there are at least three different possible actions (necessary assumption) and a match expression that matches all packets (convenience assumption), it is possible to construct a flow table that is admonished by *check-no-overlap* but still will never run into undefined behavior.

```

lemma
  assumes CARD('action) ≥ 3
  assumes ∀ p. γ x p
shows ∃ ft:('a, 'action) flow-entry-match list. ¬check-no-overlap γ ft ∧
  ¬(∃ fe1 fe2 p. fe1 ∈ set ft ∧ fe2 ∈ set ft ∧ fe1 ≠ fe2 ∧ ofe-prio fe1 = ofe-prio fe2
  ∧ guha-table-semantics γ ft p (Some (ofe-action fe1)))
  ∧ guha-table-semantics γ ft p (Some (ofe-action fe2)))
proof -
  obtain adef aa ab :: 'action where anb[simp]: aa ≠ ab adef ≠ aa adef ≠ ab using assms(1) card3-eI by blast
  let ?cex = [OFEntry 1 x adef, OFEntry 0 x aa, OFEntry 0 x ab]
  have ol: ¬check-no-overlap γ ?cex
    unfolding check-no-overlap-def ball-simps
    apply(rule bexI[where x = OFEntry 0 x aa, rotated], (simp;fail))
    apply(rule bexI[where x = OFEntry 0 x ab, rotated], (simp;fail))
    apply(simp add: assms)
  done
  have df: guha-table-semantics γ ?cex p oc ⇒ oc = Some adef for p oc
  unfolding guha-table-semantics.simps
  apply(elim disjE; clarsimp simp: assms)
  subgoal for fe ft1 ft2
  apply(cases ft1 = [])
  apply(fastforce)
  apply(cases ft2 = [])
  apply(fastforce)
  apply(subgoal-tac ft1 = [OFEntry 1 x adef] ∧ fe = OFEntry 0 x aa ∧ ft2 = [OFEntry 0 x ab])
  apply(simp;fail)
  apply(clarsimp simp add: List.neq-Nil-conv)
  apply(rename-tac ya ys yz)
  apply(case-tac ys; clarsimp simp add: List.neq-Nil-conv)
  done done
  show ?thesis
    apply(intro exI[where x = ?cex], intro conjI, fact ol)
    apply(clarify)
    apply(unfold set-simps)
    apply(elim insertE; clarsimp)
    apply((drule df)+; unfold option.inject; (elim anb[symmetric, THEN notE] | (simp;fail))?)
  done
qed

```

```

end
theory LinuxRouter-OpenFlow-Translation

```

```

imports IP-Addresses.CIDR-Split
         Automatic-Refinement.Misc
         Simple-Firewall.Generic-SimpleFw
         Semantics-OpenFlow
         OpenFlow-Matches
         OpenFlow-Action
         Routing.Linux-Router
         Pure-ex.Guess
begin
hide-const Misc.uncurry
hide-fact Misc.uncurry-def

```

```

definition route2match r =
  (iiface = ifaceAny, oiface = ifaceAny,
   src = (0,0), dst=(pfxm-prefix (routing-match r),pfxm-length (routing-match r)),
   proto=ProtoAny, sports=(0,- 1), ports=(0,- 1))

```

```

definition toprefixmatch where
toprefixmatch m  $\equiv$  (let pm = PrefixMatch (fst m) (snd m) in if pm = PrefixMatch 0 0 then None else Some pm)

```

```

lemma prefix-match-semantics-simple-match:

```

```

  assumes some: toprefixmatch m = Some pm
  assumes vld: valid-prefix pm
  shows prefix-match-semantics pm = simple-match-ip m

```

```

using some

```

```

  by (cases m)
    (clarsimp
     simp add: toprefixmatch-def ipset-from-cidr-def pfxm-mask-def fun-eq-iff
              prefix-match-semantics-ipset-from-netmask[OF vld] NOT-mask-shifted-lenword[symmetric]
     split: if-splits)

```

```

definition simple-match-to-of-match-single ::

```

```

  (32, 'a) simple-match-scheme
   $\Rightarrow$  char list option  $\Rightarrow$  protocol  $\Rightarrow$  (16 word  $\times$  16 word) option  $\Rightarrow$  (16 word  $\times$  16 word) option  $\Rightarrow$  of-match-field set
  where

```

```

simple-match-to-of-match-single m iif prot sport dport  $\equiv$ 
  uncurry L4Src ' option2set sport  $\cup$  uncurry L4Dst ' option2set dport
 $\cup$  IPv4Proto ' (case prot of ProtoAny  $\Rightarrow$  {} | Proto p  $\Rightarrow$  {p}) — protocol is an 8 word option anyway...
 $\cup$  IngressPort ' option2set iif
 $\cup$  IPv4Src ' option2set (toprefixmatch (src m))  $\cup$  IPv4Dst ' option2set (toprefixmatch (dst m))
 $\cup$  {EtherType 0x0800}

```

```

definition simple-match-to-of-match :: 32 simple-match  $\Rightarrow$  string list  $\Rightarrow$  of-match-field set list where

```

```

simple-match-to-of-match m ifs  $\equiv$  (let
  npm = ( $\lambda p$ . fst p = 0  $\wedge$  snd p = - 1);
  sb = ( $\lambda p$ . (if npm p then [None] else if fst p  $\leq$  snd p
    then map (Some  $\circ$  ( $\lambda pfx$ . (pfxm-prefix pfx, Bit-Operations.not (pfxm-mask pfx)))) (wordinterval-CIDR-split-prefixmatch
      (WordInterval (fst p) (snd p))) else []))
  in [simple-match-to-of-match-single m iif (proto m) sport dport.
    iif  $\leftarrow$  (if iiface m = ifaceAny then [None] else [Some i. i  $\leftarrow$  ifs, match-iface (iiface m) i]),
    sport  $\leftarrow$  sb (sports m),
    dport  $\leftarrow$  sb (dports m)]
)

```

**lemma** *smtoms-eq-hlp*: *simple-match-to-of-match-single*  $r\ a\ b\ c\ d = \text{simple-match-to-of-match-single}\ r\ f\ g\ h\ i \iff (a = f \wedge b = g \wedge c = h \wedge d = i)$

**proof**(*rule iffI,goal-cases*)

**case** 1

**thus** ?*case* **proof**(*intro conjI*)

**have** \*:  $\bigwedge P\ z\ x. \llbracket \forall x :: \text{of-match-field}. P\ x; z = \text{Some}\ x \rrbracket \implies P\ (\text{IngressPort}\ x)$  **by** *simp*

**show**  $a = f$  **using** 1 **by**(*cases a; cases f*)

(*simp add: option2set-None simple-match-to-of-match-single-def toprefixmatch-def option2set-def;*

*subst(asm) set-eq-iff; drule (1) \*; simp split: option.splits uncurry-splits protocol.splits*)+

**next**

**have** \*:  $\bigwedge P\ z\ x. \llbracket \forall x :: \text{of-match-field}. P\ x; z = \text{Proto}\ x \rrbracket \implies P\ (\text{IPv4Proto}\ x)$  **by** *simp*

**show**  $b = g$  **using** 1 **by**(*cases b; cases g*)

(*simp add: option2set-None simple-match-to-of-match-single-def toprefixmatch-def option2set-def;*

*subst(asm) set-eq-iff; drule (1) \*; simp split: option.splits uncurry-splits protocol.splits*)+

**next**

**have** \*:  $\bigwedge P\ z\ x. \llbracket \forall x :: \text{of-match-field}. P\ x; z = \text{Some}\ x \rrbracket \implies P\ (\text{uncurry}\ L4\text{Src}\ x)$  **by** *simp*

**show**  $c = h$  **using** 1 **by**(*cases c; cases h*)

(*simp add: option2set-None simple-match-to-of-match-single-def toprefixmatch-def option2set-def;*

*subst(asm) set-eq-iff; drule (1) \*; simp split: option.splits uncurry-splits protocol.splits*)+

**next**

**have** \*:  $\bigwedge P\ z\ x. \llbracket \forall x :: \text{of-match-field}. P\ x; z = \text{Some}\ x \rrbracket \implies P\ (\text{uncurry}\ L4\text{Dst}\ x)$  **by** *simp*

**show**  $d = i$  **using** 1 **by**(*cases d; cases i*)

(*simp add: option2set-None simple-match-to-of-match-single-def toprefixmatch-def option2set-def;*

*subst(asm) set-eq-iff; drule (1) \*; simp split: option.splits uncurry-splits protocol.splits*)+

**qed**

**qed** *simp*

**lemma** *simple-match-to-of-match-generates-prereqs*: *simple-match-valid*  $m \implies r \in \text{set}(\text{simple-match-to-of-match}\ m\ \text{ifs}) \implies \text{all-prerequisites}\ r$

**unfolding** *simple-match-to-of-match-def Let-def*

**proof**(*clarsimp, goal-cases*)

**case** (1 *xiface xsrccp xdstp*)

**note**  $o = \text{this}$

**show** ?*case* **unfolding** *simple-match-to-of-match-single-def all-prerequisites-def*

**unfolding** *ball-Un*

**proof**((*intro conjI; ((simp;fail)| - )*), *goal-cases*)

**case** 1

**have**  $e: (\text{fst}(\text{sports}\ m) = 0 \wedge \text{snd}(\text{sports}\ m) = -1) \vee \text{proto}\ m = \text{Proto}\ \text{TCP} \vee \text{proto}\ m = \text{Proto}\ \text{UDP} \vee \text{proto}\ m = \text{Proto}\ L4\text{-Protocol.SCTP}$

**using**  $o(1)$

**unfolding** *simple-match-valid-alt Let-def*

**by**(*clarsimp split: if-splits*)

**show** ?*case*

**using**  $o(3)\ e$

**by**(*elim disjE; simp add: option2set-def split: if-splits prod.splits uncurry-splits*)

**next**

**case** 2

**have**  $e: (\text{fst}(\text{dports}\ m) = 0 \wedge \text{snd}(\text{dports}\ m) = -1) \vee \text{proto}\ m = \text{Proto}\ \text{TCP} \vee \text{proto}\ m = \text{Proto}\ \text{UDP} \vee \text{proto}\ m = \text{Proto}\ L4\text{-Protocol.SCTP}$

**using**  $o(1)$

**unfolding** *simple-match-valid-alt Let-def*

```

  by(clarsimp split: if-splits)
show ?case
  using o(4) e
  by(elim disjE; simp add: option2set-def split: if-splits prod.splits uncurry-splits)
qed
qed

```

**lemma** *and-assoc*:  $a \wedge b \wedge c \longleftrightarrow (a \wedge b) \wedge c$  **by** *simp*

**lemmas** *custom-simpset = Let-def set-concat set-map map-map comp-def concat-map-maps set-maps UN-iff fun-app-def Set.image-iff*

**abbreviation** *simple-fw-prefix-to-wordinterval*  $\equiv$  *prefix-to-wordinterval*  $\circ$  *uncurry PrefixMatch*

**lemma** *simple-match-port-alt*: *simple-match-port*  $m\ p \longleftrightarrow p \in \text{wordinterval-to-set } (\text{uncurry } \text{WordInterval } m)$  **by**(*simp split: uncurry-splits*)

**lemma** *simple-match-src-alt*: *simple-match-valid*  $r \implies$   
*simple-match-ip*  $(\text{src } r)\ p \longleftrightarrow \text{prefix-match-semantic} (\text{PrefixMatch } (\text{fst } (\text{src } r)) (\text{snd } (\text{src } r)))\ p$   
**by**(*cases (src r) (simp add: prefix-match-semantic-ipset-from-netmask2 prefix-to-wordset-ipset-from-cidr simple-match-valid-def valid-prefix-fw-def)*)

**lemma** *simple-match-dst-alt*: *simple-match-valid*  $r \implies$   
*simple-match-ip*  $(\text{dst } r)\ p \longleftrightarrow \text{prefix-match-semantic} (\text{PrefixMatch } (\text{fst } (\text{dst } r)) (\text{snd } (\text{dst } r)))\ p$   
**by**(*cases (dst r) (simp add: prefix-match-semantic-ipset-from-netmask2 prefix-to-wordset-ipset-from-cidr simple-match-valid-def valid-prefix-fw-def)*)

**lemma**  $x \in \text{set } (\text{wordinterval-CIDR-split-prefixmatch } w) \implies \text{valid-prefix } x$   
**using** *wordinterval-CIDR-split-prefixmatch-all-valid-Ball[THEN bspec, THEN conjunct1]* .

**lemma** *simple-match-to-of-matchI*:  
**assumes** *mv*: *simple-match-valid*  $r$   
**assumes** *mm*: *simple-matches*  $r\ p$   
**assumes** *ii*: *p-iiface*  $p \in \text{set ifs}$   
**assumes** *ippkt*: *p-l2type*  $p = 0x800$   
**shows** *eq*:  $\exists gr \in \text{set } (\text{simple-match-to-of-match } r\ ifs). \text{OF-match-fields } gr\ p = \text{Some True}$

**proof** –

**let** *?npm*  $= \lambda p. \text{fst } p = 0 \wedge \text{snd } p = -1$   
**let** *?sb*  $= \lambda p\ r. (\text{if } ?npm\ p\ \text{then None else Some } r)$   
**obtain** *si* **where** *si*: *case si of Some ssi  $\Rightarrow$  p-sport*  $p \in \text{prefix-to-wordset } ssi \mid \text{None} \Rightarrow \text{True}$   
*case si of None  $\Rightarrow$  True  $\mid$  Some ssi  $\Rightarrow$  ssi  $\in \text{set } (\text{wordinterval-CIDR-split-prefixmatch } (\text{uncurry } \text{WordInterval } (\text{sports } r)))$*   
*si = None  $\longleftrightarrow$  ?npm (sports r)*

**proof**(*cases ?npm (sports r), goal-cases*)

**case** *1*

**hence** (*case None of None  $\Rightarrow$  True  $\mid$  Some ssi  $\Rightarrow$  p-sport*  $p \in \text{prefix-to-wordset } ssi$ )  $\wedge$   
(*case None of None  $\Rightarrow$  True*  
 $\mid$  *Some ssi  $\Rightarrow$  ssi  $\in \text{set } (\text{wordinterval-CIDR-split-prefixmatch } (\text{uncurry } \text{WordInterval } (\text{sports } r)))$* ) **by** *simp*  
**with** *1* **show** *?thesis* **by** *blast*

**next**

**case** *2*

**from** *mm* **have** *p-sport*  $p \in \text{wordinterval-to-set } (\text{uncurry } \text{WordInterval } (\text{sports } r))$

**by**(*simp only: simple-matches.simps simple-match-port-alt*)

**then obtain** *ssi* **where** *ssi*:

*ssi  $\in \text{set } (\text{wordinterval-CIDR-split-prefixmatch } (\text{uncurry } \text{WordInterval } (\text{sports } r)))$*

```

p-sport p ∈ prefix-to-wordset ssi
using wordinterval-CIDR-split-existential by fast
hence (case Some ssi of None ⇒ True | Some ssi ⇒ p-sport p ∈ prefix-to-wordset ssi) ∧
      (case Some ssi of None ⇒ True
       | Some ssi ⇒ ssi ∈ set (wordinterval-CIDR-split-prefixmatch (uncurry WordInterval (sports r)))) by simp
with 2 show ?thesis by blast
qed
obtain di where di: case di of Some ddi ⇒ p-dport p ∈ prefix-to-wordset ddi | None ⇒ True
case di of None ⇒ True | Some ddi ⇒ ddi ∈ set (
wordinterval-CIDR-split-prefixmatch (uncurry WordInterval (dports r)))
di = None ⇔ ?npm (dports r)
proof(cases ?npm (dports r), goal-cases)
case 1
hence (case None of None ⇒ True | Some ssi ⇒ p-dport p ∈ prefix-to-wordset ssi) ∧
      (case None of None ⇒ True
       | Some ssi ⇒ ssi ∈ set (wordinterval-CIDR-split-prefixmatch (uncurry WordInterval (dports r)))) by simp
with 1 show ?thesis by blast
next
case 2
from mm have p-dport p ∈ wordinterval-to-set (uncurry WordInterval (dports r))
by(simp only: simple-matches.simps simple-match-port-alt)
then obtain ddi where ddi:
ddi ∈ set (wordinterval-CIDR-split-prefixmatch (uncurry WordInterval (dports r)))
p-dport p ∈ prefix-to-wordset ddi
using wordinterval-CIDR-split-existential by fast
hence (case Some ddi of None ⇒ True | Some ssi ⇒ p-dport p ∈ prefix-to-wordset ssi) ∧
      (case Some ddi of None ⇒ True
       | Some ssi ⇒ ssi ∈ set (wordinterval-CIDR-split-prefixmatch (uncurry WordInterval (dports r)))) by simp
with 2 show ?thesis by blast
qed
show ?thesis
proof
let ?mf = map-option (apsnd (wordNOT ∘ mask ∘ (−) 16) ∘ prefix-match-dtor)
let ?gr = simple-match-to-of-match-single r
(if iiface r = iifaceAny then None else Some (p-iiface p))
(if proto r = ProtoAny then ProtoAny else Proto (p-protocol p))
(?mf si) (?mf di)
note mfu = simple-match-port.simps[of fst (sports r) snd (sports r), unfolded surjective-pairing[of sports r, symmetric]]
simple-match-port.simps[of fst (dports r) snd (dports r), unfolded surjective-pairing[of dports r, symmetric]]
note u = mm[unfolded simple-matches.simps mfu ord-class.atLeastAtMost-iff simple-packet-unext-def simple-packet.simps]
note of-safe-unsafe-match-eq[OF simple-match-to-of-match-generates-prereqs]
from u have ple: fst (sports r) ≤ snd (sports r) fst (dports r) ≤ snd (dports r) by force+
show eg: ?gr ∈ set (simple-match-to-of-match r ifs)
unfolding simple-match-to-of-match-def
unfolding custom-simpset
unfolding smtoms-eq-hlp
proof(intro beqI, (intro conjI; ((rule refl)?)), goal-cases)
case 2 thus ?case using ple(2) di
apply(simp add: pfrm-mask-def prefix-match-dtor-def Set.image-iff
split: option.splits prod.splits uncurry-splits)
apply(erule beqI[rotated])
apply(simp split: prefix-match.splits)
done
next

```

```

case 3 thus ?case using ple(1) si
  apply(simp add: pfxm-mask-def prefix-match-dtor-def Set.image-iff
    split: option.splits prod.splits uncurry-splits)
  apply(erule bexI[rotated])
  apply(simp split: prefix-match.splits)
done
next
case 4 thus ?case
  using u ii by(clarsimp simp: set-maps split: if-splits)
next
case 1 thus ?case using ii u by simp-all (metis match-proto.elims(2))
qed
have dpm: di = Some (PrefixMatch x1 x2)  $\implies$  p-dport p &&  $\sim\sim$  (mask (16 - x2)) = x1 for x1 x2
proof -
  have *: di = Some (PrefixMatch x1 x2)  $\implies$  prefix-match-antics (the di) (p-dport p)  $\implies$  p-dport p &&  $\sim\sim$  (mask
(16 - x2)) = x1
  by(clarsimp simp: prefix-match-antics-def pfxm-mask-def word-bw-comms;fail)
  have **: pfx  $\in$  set (wordinterval-CIDR-split-prefixmatch ra)  $\implies$  prefix-match-antics pfx a = (a  $\in$  prefix-to-wordset
pfx) for pfx ra and a :: 16 word
  by (fact prefix-match-antics-wordset[OF wordinterval-CIDR-split-prefixmatch-all-valid-Ball[THEN bspec, THEN
conjunctI]])
  have  $\llbracket$ di = Some (PrefixMatch x1 x2); p-dport p  $\in$  prefix-to-wordset (PrefixMatch x1 x2); PrefixMatch x1 x2  $\in$  set
(wordinterval-CIDR-split-prefixmatch (uncurry WordInterval (dports r))) $\rrbracket$ 
 $\implies$  p-dport p &&  $\sim\sim$  (mask (16 - x2)) = x1
  using di(1,2)
  using * ** by auto
  thus di = Some (PrefixMatch x1 x2)  $\implies$  p-dport p &&  $\sim\sim$  (mask (16 - x2)) = x1 using di(1,2) by auto
qed
have spm: si = Some (PrefixMatch x1 x2)  $\implies$  p-sport p &&  $\sim\sim$  (mask (16 - x2)) = x1 for x1 x2
using si
proof -
  have *: si = Some (PrefixMatch x1 x2)  $\implies$  prefix-match-antics (the si) (p-sport p)  $\implies$  p-sport p &&  $\sim\sim$  (mask
(16 - x2)) = x1
  by(clarsimp simp: prefix-match-antics-def pfxm-mask-def word-bw-comms;fail)
  have **: pfx  $\in$  set (wordinterval-CIDR-split-prefixmatch ra)  $\implies$  prefix-match-antics pfx a = (a  $\in$  prefix-to-wordset
pfx) for pfx ra and a :: 16 word
  by (fact prefix-match-antics-wordset[OF wordinterval-CIDR-split-prefixmatch-all-valid-Ball[THEN bspec, THEN
conjunctI]])
  have  $\llbracket$ si = Some (PrefixMatch x1 x2); p-sport p  $\in$  prefix-to-wordset (PrefixMatch x1 x2); PrefixMatch x1 x2  $\in$  set
(wordinterval-CIDR-split-prefixmatch (uncurry WordInterval (sports r))) $\rrbracket$ 
 $\implies$  p-sport p &&  $\sim\sim$  (mask (16 - x2)) = x1
  using si(1,2)
  using * ** by auto
  thus si = Some (PrefixMatch x1 x2)  $\implies$  p-sport p &&  $\sim\sim$  (mask (16 - x2)) = x1 using si(1,2) by auto
qed
show OF-match-fields ?gr p = Some True
unfolding of-safe-unsafe-match-eq[OF simple-match-to-of-match-generates-prereqs[OF mv eg]]
by(cases si; cases di)
  (simp-all
  add: simple-match-to-of-match-single-def OF-match-fields-unsafe-def spm
  option2set-def u ippkt prefix-match-dtor-def toprefixmatch-def dpm
  simple-match-dst-alt[OF mv, symmetric] simple-match-src-alt[OF mv, symmetric]
  split: prefix-match.splits)
qed

```

qed

**lemma** *prefix-match-00*[simp,intro!]: *prefix-match-semantics* (*PrefixMatch* 0 0) *p*  
 by (simp add: *valid-prefix-def* *zero-prefix-match-all*)

**lemma** *simple-match-to-of-matchD*:

**assumes** *eg*: *gr* ∈ *set* (*simple-match-to-of-match* *r* *ifs*)

**assumes** *mo*: *OF-match-fields* *gr* *p* = *Some True*

**assumes** *me*: *match-iface* (*oiface* *r*) (*p-oiface* *p*)

**assumes** *mv*: *simple-match-valid* *r*

**shows** *simple-matches* *r* *p*

**proof** –

**from** *mv* **have** *validpfx*:

*valid-prefix* (*uncurry PrefixMatch* (*src* *r*)) *valid-prefix* (*uncurry PrefixMatch* (*dst* *r*))

$\wedge pm. \text{toprefixmatch } (src\ r) = Some\ pm \implies \text{valid-prefix } pm$

$\wedge pm. \text{toprefixmatch } (dst\ r) = Some\ pm \implies \text{valid-prefix } pm$

**unfolding** *simple-match-valid-def* *valid-prefix-fw-def* *toprefixmatch-def*

by(*simp-all* *split*: *uncurry-splits* *if-splits*)

**from** *mo* **have** *mo*: *OF-match-fields-unsafe* *gr* *p*

**unfolding** *of-safe-unsafe-match-eq*[*OF simple-match-to-of-match-generates-prereqs*[*OF mv eg*]]

by *simp*

**note** *this*[*unfolded OF-match-fields-unsafe-def*]

**note** *eg*[*unfolded simple-match-to-of-match-def simple-match-to-of-match-single-def custom-simpset option2set-def*]

**then** **guess** *x* .. **moreover** **from** *this*(2) **guess** *xa* .. **moreover** **from** *this*(2) **guess** *xb* ..

**note** *xx* = *calculation*(1,3) *this*

{ **fix** *a b xc xa*  
**fix** *pp* :: 16 *word*  
**have**  $\llbracket pp \ \&\& \ \sim\sim \ (pfxm\text{-}mask\ xc) = pfxm\text{-}prefix\ xc \rrbracket$   
 $\implies \text{prefix-match-semantics } xc\ (pp) \text{ for } xc$   
 by(*simp* add: *prefix-match-semantics-def* *word-bw-comms*;fail)  
**moreover** **have** *pp* ∈ *wordinterval-to-set* (*WordInterval* *a b*)  $\implies a \leq pp \wedge pp \leq b$  **by** *simp*  
**moreover** **have** *xc* ∈ *set* (*wordinterval-CIDR-split-prefixmatch* (*WordInterval* *a b*))  $\implies pp \in \text{prefix-to-wordset } xc \implies$   
 $pp \in \text{wordinterval-to-set } (WordInterval\ a\ b)$   
**by**(*subst* *wordinterval-CIDR-split-prefixmatch*) *blast*  
**moreover** **have**  $\llbracket xc \in \text{set } (wordinterval\text{-}CIDR\text{-}split\text{-}prefixmatch\ (WordInterval\ a\ b)); xa = Some\ (pfxm\text{-}prefix\ xc, \sim\sim$   
 $(pfxm\text{-}mask\ xc)); \text{prefix-match-semantics } xc\ (pp) \rrbracket \implies pp \in \text{prefix-to-wordset } xc$   
**apply** (*subst* (*asm*)(1) *prefix-match-semantics-wordset*)  
**apply** (*erule* *wordinterval-CIDR-split-prefixmatch-all-valid-Ball*[*THEN* *bspec*, *THEN* *conjunct1*];fail)  
**apply** *assumption*  
**done**  
**ultimately** **have**  $\llbracket xc \in \text{set } (wordinterval\text{-}CIDR\text{-}split\text{-}prefixmatch\ (WordInterval\ a\ b)); xa = Some\ (pfxm\text{-}prefix\ xc, \sim\sim$   
 $(pfxm\text{-}mask\ xc));$   
 $pp \ \&\& \ \sim\sim \ (pfxm\text{-}mask\ xc) = pfxm\text{-}prefix\ xc \rrbracket$   
 $\implies a \leq pp \wedge pp \leq b$   
**by** *metis*  
**}** **note** *l4port-logic* = *this*

**show** ?*thesis* **unfolding** *simple-matches.simps*

**proof**(*unfold and-assoc*, (*rule*)+)

**show** *match-iface* (*iiface* *r*) (*p-iiface* *p*)

**apply**(*cases* *iiface* *r* = *ifaceAny*)

**apply** (*simp* add: *match-ifaceAny*)

**using** *xx*(1) *mo* **unfolding** *xx*(4) *OF-match-fields-unsafe-def*

```

  apply(simp only: if-False set-maps UN-iff)
  apply(clarify)
  apply(rename-tac a; subgoal-tac match-iface (iface r) a)
  apply(clarsimp simp add: option2set-def; fail)
  apply(rule ccontr, simp; fail)
done
next
show match-iface (oiface r) (p-oiface p) using me .
next
show simple-match-ip (src r) (p-src p)
  using mo unfolding xx(4) OF-match-fields-unsafe-def toprefixmatch-def
  by(clarsimp
    simp add: simple-packet-unext-def option2set-def validpfx simple-match-src-alt[OF mv] toprefixmatch-def
    split: if-splits)
next
show simple-match-ip (dst r) (p-dst p)
  using mo unfolding xx(4) OF-match-fields-unsafe-def toprefixmatch-def
  by(clarsimp
    simp add: simple-packet-unext-def option2set-def validpfx simple-match-dst-alt[OF mv] toprefixmatch-def
    split: if-splits)
next
show match-proto (proto r) (p-proto p)
  using mo unfolding xx(4) OF-match-fields-unsafe-def
  using xx(1) by(clarsimp
    simp add: singleton-iff simple-packet-unext-def option2set-def prefix-match-semantics-simple-match ball-Un
    split: if-splits protocol.splits)
next
show simple-match-port (sports r) (p-sport p)
  using mo xx(2) unfolding xx(4) OF-match-fields-unsafe-def
  by(cases sports r) (clarsimp simp add: l4port-logic simple-packet-unext-def option2set-def
    prefix-match-semantics-simple-match split: if-splits)
next
show simple-match-port (dports r) (p-dport p)
  using mo xx(3) unfolding xx(4) OF-match-fields-unsafe-def
  by(cases dports r) (clarsimp simp add: l4port-logic simple-packet-unext-def option2set-def
    prefix-match-semantics-simple-match split: if-splits)
qed
qed

primrec annotate-rlen where
  annotate-rlen [] = [] |
  annotate-rlen (a#as) = (length as, a) # annotate-rlen as
lemma annotate-rlen "asdf" = [(3, CHR "a"), (2, CHR "s"), (1, CHR "d"), (0, CHR "f")] by simp

lemma fst-annotate-rlen-le:  $(k, a) \in \text{set } (\text{annotate-rlen } l) \implies k < \text{length } l$ 
  by(induction l arbitrary: k; simp; force)

lemma distinct-fst-annotate-rlen: distinct (map fst (annotate-rlen l))
  using fst-annotate-rlen-le by(induction l) (simp, fastforce)
lemma distinct-annotate-rlen: distinct (annotate-rlen l)
  using distinct-fst-annotate-rlen unfolding distinct-map by blast
lemma in-annotate-rlen:  $(a, x) \in \text{set } (\text{annotate-rlen } l) \implies x \in \text{set } l$ 
  by(induction l) (simp-all, blast)
lemma map-snd-annotate-rlen: map snd (annotate-rlen l) = l

```



```

  by(induction l) simp-all
lemma sorted-descending (map fst (annotate-rlen l))
  by(induction l; clarsimp) (force dest: fst-annotate-rlen-le)
lemma annotate-rlen l = zip (rev [0..<length l]) l
  by(induction l; simp)

primrec annotate-rlen-code where
  annotate-rlen-code [] = (0,[]) |
  annotate-rlen-code (a#as) = (case annotate-rlen-code as of (r,aas) ⇒ (Suc r, (r, a) # aas))
lemma annotate-rlen-len: fst (annotate-rlen-code r) = length r
  by(induction r) (clarsimp split: prod.splits)+
lemma annotate-rlen-code[code]: annotate-rlen s = snd (annotate-rlen-code s)
proof(induction s)
  case (Cons s ss) thus ?case using annotate-rlen-len[of ss] by(clarsimp split: prod.split)
qed simp

lemma suc2plus-inj-on: inj-on (of-nat :: nat ⇒ ('l :: len) word) {0..unat (max-word :: 'l word)}
proof(rule inj-onI)
  let ?mmw = (max-word :: 'l word)
  let ?mstp = (of-nat :: nat ⇒ 'l word)
  fix x y :: nat
  assume x ∈ {0..unat ?mmw} y ∈ {0..unat ?mmw}
  hence se: x ≤ unat ?mmw y ≤ unat ?mmw by simp-all
  assume eq: ?mstp x = ?mstp y
  note f = le-unat-uoι[OF se(1)] le-unat-uoι[OF se(2)]
  show x = y using eq le-unat-uoι se by metis
qed

lemma distinct-of-nat-list:
  distinct l ⇒ ∀ e ∈ set l. e ≤ unat (max-word :: ('l::len) word) ⇒ distinct (map (of-nat :: nat ⇒ 'l word) l)
proof(induction l)
  let ?mmw = (max-word :: 'l word)
  let ?mstp = (of-nat :: nat ⇒ 'l word)
  case (Cons a as)
  have distinct as ∀ e ∈ set as. e ≤ unat ?mmw using Cons.premis by simp-all
  note mIH = Cons.IH[OF this]
  moreover have ?mstp a ∉ ?mstp `set as
  proof
    have representable-set: set as ⊆ {0..unat ?mmw} using ⟨∀ e ∈ set (a # as). e ≤ unat max-word⟩ by fastforce
    have a-reprbl: a ∈ {0..unat ?mmw} using ⟨∀ e ∈ set (a # as). e ≤ unat max-word⟩ by simp
    assume ?mstp a ∈ ?mstp `set as
    with inj-on-image-mem-iff[OF suc2plus-inj-on a-reprbl representable-set]
    have a ∈ set as by simp
    with ⟨distinct (a # as)⟩ show False by simp
  qed
  ultimately show ?case by simp
qed simp

lemma annotate-first-le-hlp:
  length l < unat (max-word :: ('l :: len) word) ⇒ ∀ e ∈ set (map fst (annotate-rlen l)). e ≤ unat (max-word :: 'l word)
  by(clarsimp) (meson fst-annotate-rlen-le less-trans nat-less-le)
lemmas distinct-of-prio-hlp = distinct-of-nat-list[OF distinct-fst-annotate-rlen annotate-first-le-hlp]

```

**lemma** *fst-annotate-rlen*:  $\text{map } \text{fst } (\text{annotate-rlen } l) = \text{rev } [0..<\text{length } l]$   
**by**(*induction l*) (*simp-all*)

**lemma** *sorted-word-upt*:

**defines**[*simp*]:  $\text{won} \equiv (\text{of-nat} :: \text{nat} \Rightarrow ('l :: \text{len}) \text{ word})$

**assumes**  $\text{length } l \leq \text{unat } (\text{max-word} :: 'l \text{ word})$

**shows** *sorted-descending* ( $\text{map } \text{won } (\text{rev } [0..<\text{Suc } (\text{length } l)])$ )

**using** *assms*

**by**(*induction l rule: rev-induct; clarsimp*)

(*metis* (*mono-tags*, *opaque-lifting*) *le-SucI le-unat-uoI of-nat-Suc order-refl word-le-nat-alt*)

**lemma** *sorted-annotated*:

**assumes**  $\text{length } l \leq \text{unat } (\text{max-word} :: ('l :: \text{len}) \text{ word})$

**shows** *sorted-descending* ( $\text{map } \text{fst } (\text{map } (\text{apfst } (\text{of-nat} :: \text{nat} \Rightarrow 'l \text{ word})) (\text{annotate-rlen } l))$ )

**proof** –

**let**  $?won = (\text{of-nat} :: \text{nat} \Rightarrow 'l \text{ word})$

**have** *sorted-descending* ( $\text{map } ?won (\text{rev } [0..<\text{Suc } (\text{length } l)])$ )

**using** *sorted-word-upt*[*OF assms*] .

**hence** *sorted-descending* ( $\text{map } ?won (\text{map } \text{fst } (\text{annotate-rlen } l))$ ) **by**(*simp add: fst-annotate-rlen*)

**thus** *sorted-descending* ( $\text{map } \text{fst } (\text{map } (\text{apfst } ?won) (\text{annotate-rlen } l))$ ) **by** *simp*

**qed**

l3 device to l2 forwarding

**definition** *lr-of-tran-s3* *ifs ard* = (

$[(p, b, \text{case } a \text{ of } \text{simple-action.Accept} \Rightarrow [\text{Forward } c] \mid \text{simple-action.Drop} \Rightarrow []).$

$(p, r, (c, a)) \leftarrow \text{ard}, b \leftarrow \text{simple-match-to-of-match } r \text{ ifs}]$ )

**definition** *oif-ne-iif-p1* *ifs*  $\equiv [( \text{simple-match-any}(\text{oiface} := \text{Iface } \text{oif}, \text{iiface} := \text{Iface } \text{iif}), \text{simple-action.Accept}). \text{oif} \leftarrow \text{ifs},$   
 $\text{iif} \leftarrow \text{ifs}, \text{oif} \neq \text{iif}]$

**definition** *oif-ne-iif-p2* *ifs* =  $[( \text{simple-match-any}(\text{oiface} := \text{Iface } i, \text{iiface} := \text{Iface } i), \text{simple-action.Drop}). i \leftarrow \text{ifs}]$

**definition** *oif-ne-iif* *ifs* = *oif-ne-iif-p2* *ifs* @ *oif-ne-iif-p1* *ifs*

**definition** *lr-of-tran-s4* *ard ifs*  $\equiv \text{generalized-fw-join } \text{ard } (\text{oif-ne-iif } \text{ifs})$

**definition** *lr-of-tran-s1* *rt* =  $[(\text{route2match } r, \text{output-iface } (\text{routing-action } r)). r \leftarrow \text{rt}]$

**definition** *lr-of-tran-fbs* *rt fw ifs*  $\equiv \text{let}$

*gfw* =  $\text{map } \text{simple-rule-dtor } \text{fw}$ ; — generalized simple fw, hopefully for FORWARD

*frt* = *lr-of-tran-s1* *rt*; — *rt* as fw

*prd* = *generalized-fw-join* *frt gfw*

*in prd*

**definition** *pack-OF-entries* *ifs ard*  $\equiv (\text{map } (\text{split3 } \text{OFEntry}) (\text{lr-of-tran-s3 } \text{ifs } \text{ard}))$

**definition** *no-oif-match*  $\equiv \text{list-all } (\lambda m. \text{oiface } (\text{match-sel } m) = \text{iifaceAny})$

**definition** *lr-of-tran* *rt fw ifs*  $\equiv$

*if*  $\neg (\text{no-oif-match } \text{fw} \wedge \text{has-default-policy } \text{fw} \wedge \text{simple-fw-valid } \text{fw} \wedge \text{valid-prefixes } \text{rt} \wedge \text{has-default-route } \text{rt} \wedge \text{distinct } \text{ifs})$   
 $\text{then Inl } \text{"Error in creating OpenFlow table: prerequisites not satisfied"}$

*else* (

$\text{let } \text{nrd} = \text{lr-of-tran-fbs } \text{rt } \text{fw } \text{ifs}$ ;

*ard* =  $\text{map } (\text{apfst } \text{of-nat}) (\text{annotate-rlen } \text{nrd})$  — give them a priority

```

in
if length nrd < unat (- 1 :: 16 word)
then Inr (pack-OF-entries ifs ard)
else Inl "Error in creating OpenFlow table: priority number space exhausted"

```

**definition** *is-iface-name*  $i \equiv i \neq [] \wedge \neg \text{Iface.iface-name-is-wildcard } i$

**definition** *is-iface-list*  $\text{ifs} \equiv \text{distinct ifs} \wedge \text{list-all is-iface-name ifs}$

**lemma** *max-16-word-max*[simp]:  $(a :: 16 \text{ word}) \leq 0\text{xffff}$

**proof** –

have  $0\text{xFFFF} = (- 1 :: 16 \text{ word})$  **by** simp

**then show** ?thesis **by** (simp only:) simp

**qed**

**lemma** *replicate-FT-hlp*:  $x \leq 16 \wedge y \leq 16 \implies \text{replicate } (16 - x) \text{ False} @ \text{replicate } x \text{ True} = \text{replicate } (16 - y) \text{ False} @ \text{replicate } y \text{ True} \implies x = y$

**proof** –

**let** ?ns = {0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16}

**assume**  $x \leq 16 \wedge y \leq 16$

**hence**  $x \in ?ns \ y \in ?ns$  **by** (simp; presburger)+

**moreover assume**  $\text{replicate } (16 - x) \text{ False} @ \text{replicate } x \text{ True} = \text{replicate } (16 - y) \text{ False} @ \text{replicate } y \text{ True}$

**ultimately show**  $x = y$  **by** simp (elim disjE; simp-all add: numeral-eq-Suc)

**qed**

**lemma** *mask-inj-hlp1*:  $\text{inj-on } (\text{mask} :: \text{nat} \Rightarrow 16 \text{ word}) \{0..16\}$

**proof**(intro inj-onI, goal-cases)

**case** (1  $x \ y$ )

**from** 1(3)

**have**  $\text{oe: of-bl } (\text{replicate } (16 - x) \text{ False} @ \text{replicate } x \text{ True}) = (\text{of-bl } (\text{replicate } (16 - y) \text{ False} @ \text{replicate } y \text{ True})) :: 16 \text{ word}$

**unfolding** mask-bl of-bl-rep-False .

**have**  $\bigwedge z. z \leq 16 \implies \text{length } (\text{replicate } (16 - z) \text{ False} @ \text{replicate } z \text{ True}) = 16$  **by** auto

**with** 1(1,2)

**have**  $\text{ps: replicate } (16 - x) \text{ False} @ \text{replicate } x \text{ True} \in \{\text{bl. length bl} = \text{LENGTH}(16)\} \ \text{replicate } (16 - y) \text{ False} @ \text{replicate } y \text{ True} \in \{\text{bl. length bl} = \text{LENGTH}(16)\}$  **by** simp-all

**from** inj-onD[OF word-bl.Abs-inj-on, OF oe ps]

**show** ?case **using** 1(1,2) **by** (fastforce intro: replicate-FT-hlp)

**qed**

**lemma** *distinct-simple-match-to-of-match-portlist-hlp*:

**fixes**  $\text{ps} :: (16 \text{ word} \times 16 \text{ word})$

**shows**  $\text{distinct ifs} \implies$

*distinct*

(if  $\text{fst ps} = 0 \wedge \text{snd ps} = \text{max-word}$  then [None]

else if  $\text{fst ps} \leq \text{snd ps}$

then  $\text{map } (\text{Some} \circ (\lambda \text{pfx. } (\text{pfxm-prefix pfx, } \sim\sim (\text{pfxm-mask pfx})))$

(wordinterval-CIDR-split-prefixmatch (WordInterval (fst ps) (snd ps)))

else [])

**proof** –

**assume** *di*: distinct ifs

**{ define** *wis* **where**  $\text{wis} = \text{set } (\text{wordinterval-CIDR-split-prefixmatch } (\text{WordInterval } (\text{fst ps}) (\text{snd ps})))$

**fix**  $x \ y :: 16 \text{ prefix-match}$

**obtain**  $\text{xm xn ym yn}$  **where**  $\text{xyd[simp]: } x = \text{PrefixMatch xm xn} \ y = \text{PrefixMatch ym yn}$  **by** (cases x; cases y)

```

    assume iw:  $x \in \text{wis}$   $y \in \text{wis}$  and et: ( $\text{pfm-prefix } x, \sim\sim (\text{pfm-mask } x) = (\text{pfm-prefix } y, \sim\sim (\text{pfm-mask } y))$ )
    hence le16:  $x_n \leq 16$   $y_n \leq 16$  unfolding wis-def using wordinterval-CIDR-split-prefixmatch-all-valid-Ball[unfolded
Ball-def, THEN spec, THEN mp] by force+
    with et have  $16 - x_n = 16 - y_n$  unfolding pfm-mask-def by (auto intro: mask-inj-hlp1[THEN inj-onD])
    hence  $x = y$  using et le16 using diff-diff-cancel by simp
  } note * = this
show ?thesis
  apply (clarsimp simp add: smtoms-eq-hlp distinct-map wordinterval-CIDR-split-distinct)
  apply (subst comp-inj-on-iff[symmetric]; intro inj-onI)
using * by simp-all
qed

```

```

lemma distinct-simple-match-to-of-match:  $\text{distinct } \text{ifs} \implies \text{distinct } (\text{simple-match-to-of-match } m \text{ ifs})$ 
apply (unfold simple-match-to-of-match-def Let-def)
apply (rule distinct-3lcomprI)
subgoal by (induction ifs; clarsimp)
subgoal by (fact distinct-simple-match-to-of-match-portlist-hlp)
subgoal by (fact distinct-simple-match-to-of-match-portlist-hlp)
subgoal by (simp-all add: smtoms-eq-hlp)
done

```

```

lemma inj-inj-on:  $\text{inj } F \implies \text{inj-on } F \text{ } A$  using subset-inj-on by auto

```

```

lemma no-overlaps-lroft-hlp2:  $\text{distinct } (\text{map fst } \text{amr}) \implies (\bigwedge r. \text{distinct } (\text{fm } r)) \implies$ 
 $\text{distinct } (\text{concat } (\text{map } (\lambda(p, r, c, a). \text{map } (\lambda b. (p, b, \text{fs } a \text{ } c)) (\text{fm } r)) \text{amr}))$ 
by (induction amr; force intro: injI inj-onI simp add: distinct-map split: prod.splits)

```

```

lemma distinct-lroft-s3:  $\llbracket \text{distinct } (\text{map fst } \text{amr}); \text{distinct } \text{ifs} \rrbracket \implies \text{distinct } (\text{lr-of-tran-s3 } \text{ifs } \text{amr})$ 
unfolding lr-of-tran-s3-def
by (erule no-overlaps-lroft-hlp2, simp add: distinct-simple-match-to-of-match)

```

```

lemma no-overlaps-lroft-hlp3:  $\text{distinct } (\text{map fst } \text{amr}) \implies$ 
 $(aa, ab, ac) \in \text{set } (\text{lr-of-tran-s3 } \text{ifs } \text{amr}) \implies (ba, bb, bc) \in \text{set } (\text{lr-of-tran-s3 } \text{ifs } \text{amr}) \implies$ 
 $ac \neq bc \implies aa \neq ba$ 
apply (unfold lr-of-tran-s3-def)
apply (clarsimp)
apply (clarsimp split: simple-action.splits)
apply (metis map-of-eq-Some-iff old.prod.inject option.inject)
apply (metis map-of-eq-Some-iff old.prod.inject option.inject simple-action.distinct(2))+
done

```

```

lemma no-overlaps-lroft-s3-hlp-hlp:
 $\llbracket \text{distinct } (\text{map fst } \text{amr}); \text{OF-match-fields-unsafe } ab \text{ } p; ab \neq ad \vee ba \neq bb; \text{OF-match-fields-unsafe } ad \text{ } p;$ 
 $(ac, ab, ba) \in \text{set } (\text{lr-of-tran-s3 } \text{ifs } \text{amr}); (ac, ad, bb) \in \text{set } (\text{lr-of-tran-s3 } \text{ifs } \text{amr}) \rrbracket$ 
 $\implies \text{False}$ 
proof (elim disjE, goal-cases)
case 1
have 4:  $\llbracket \text{distinct } (\text{map fst } \text{amr}); (ac, ab, x1, x2) \in \text{set } \text{amr}; (ac, bb, x4, x5) \in \text{set } \text{amr}; ab \neq bb \rrbracket$ 
 $\implies \text{False}$  for  $ab \text{ } x1 \text{ } x2 \text{ } bb \text{ } x4 \text{ } x5$ 
by (meson distinct-map-fstD old.prod.inject)
have conjunctSomeProtoAnyD:  $\text{Some } \text{ProtoAny} = \text{simple-proto-conjunct } a \text{ } (\text{Proto } b) \implies \text{False}$  for  $a \text{ } b$ 
using conjunctProtoD by force
have 5:
 $\llbracket \text{OF-match-fields-unsafe } am \text{ } p; \text{OF-match-fields-unsafe } bm \text{ } p; am \neq bm; \rrbracket$ 

```

```

  am ∈ set (simple-match-to-of-match ab ifs); bm ∈ set (simple-match-to-of-match bb ifs); ¬ ab ≠ bb]
  ⇒ False for ab bb am bm
by (clarify | unfold
  simple-match-to-of-match-def smtoms-eq-hlp Let-def set-concat set-map de-Morgan-conj not-False-eq-True)+
(auto dest: conjunctSomeProtoAnyD cidrsplit-no-overlaps
  simp add: OF-match-fields-unsafe-def simple-match-to-of-match-single-def option2set-def comp-def
  split: if-splits
  cong: smtoms-eq-hlp)
from 1 show ?case
using 4 5 by (clarsimp simp add: lr-of-tran-s3-def) blast
qed (metis no-overlaps-lroft-hlp3)

```

**lemma** no-overlaps-lroft-s3-hlp: *distinct (map fst amr) ⇒ distinct ifs ⇒ no-overlaps OF-match-fields-unsafe (map (split3 OFEntry) (lr-of-tran-s3 ifs amr))*

```

  apply (rule no-overlapsI[rotated])
  apply (subst distinct-map, rule conjI)
  subgoal by (erule (1) distinct-lroft-s3)
  subgoal
    apply (rule inj-inj-on)
    apply (rule injI)
    apply (rename-tac x y, case-tac x, case-tac y)
    apply (simp add: split3-def; fail)
  done
  subgoal
    apply (unfold check-no-overlap-def)
    apply (clarify)
    apply (unfold set-map)
    apply (clarify)
    apply (unfold split3-def prod.simps flow-entry-match.simps flow-entry-match.sel de-Morgan-conj)
    apply (clarsimp simp only:)
    apply (erule (1) no-overlaps-lroft-s3-hlp-hlp)
    apply simp
    apply assumption
    apply assumption
    apply simp
  done
done

```

**lemma** lr-of-tran-no-overlaps: *assumes distinct ifs shows Inr t = (lr-of-tran rt fw ifs) ⇒ no-overlaps OF-match-fields-unsafe t*

```

  apply (unfold lr-of-tran-def Let-def pack-OF-entries-def)
  apply (simp split: if-splits)
  apply (thin-tac t = -)
  apply (drule distinct-of-prio-hlp)
  apply (rule no-overlaps-lroft-s3-hlp[rotated])
  subgoal by (simp add: asms)
  subgoal by (simp add: o-assoc)
done

```

**lemma** sorted-lr-of-tran-s3-hlp: *∀ x ∈ set f. fst x ≤ a ⇒ b ∈ set (lr-of-tran-s3 s f) ⇒ fst b ≤ a*  
 by (auto simp add: lr-of-tran-s3-def)

**lemma** lr-of-tran-s3-Cons: *lr-of-tran-s3 ifs (a#ard) = (*

$[(p, b, \text{case } a \text{ of simple-action.Accept} \Rightarrow [\text{Forward } c] \mid \text{simple-action.Drop} \Rightarrow []).$   
 $(p, r, (c, a)) \leftarrow [a], b \leftarrow \text{simple-match-to-of-match } r \text{ ifs}] \text{ @ } \text{lr-of-tran-s3 ifs and}$   
 $\text{by}(\text{clarsimp simp: lr-of-tran-s3-def})$

**lemma** *sorted-lr-of-tran-s3*: *sorted-descending* (map fst f)  $\implies$  *sorted-descending* (map fst (lr-of-tran-s3 s f))  
**apply**(*induction* f)  
**subgoal** **by**(*simp add: lr-of-tran-s3-def*)  
**apply**(*clarsimp simp: lr-of-tran-s3-Cons map-concat comp-def*)  
**apply**(*unfold sorted-descending-append*)  
**apply**(*simp add: sorted-descending-alt rev-map sorted-lr-of-tran-s3-hlp sorted-const*)  
**done**

**lemma** *sorted-lr-of-tran-hlp*: (*ofe-prio*  $\circ$  *split3 OFEntry*) = fst **by**(*simp add: fun-eq-iff comp-def split3-def*)

**lemma** *lr-of-tran-sorted-descending*: *Inr* r = *lr-of-tran* rt fw ifs  $\implies$  *sorted-descending* (map *ofe-prio* r)  
**apply**(*unfold lr-of-tran-def Let-def*)  
**apply**(*simp split: if-splits*)  
**apply**(*thin-tac* r = -)  
**apply**(*unfold sorted-lr-of-tran-hlp pack-OF-entries-def split3-def[abs-def] fun-app-def map-map comp-def prod.case-distrib*)  
**apply**(*simp add: fst-def[symmetric]*)  
**apply**(*rule sorted-lr-of-tran-s3*)  
**apply**(*drule sorted-annotated[OF less-or-eq-imp-le, OF disjI1]*)  
**apply**(*simp add: o-assoc*)  
**done**

**lemma** *lr-of-tran-s1-split*: *lr-of-tran-s1* (a # rt) = (*route2match* a, *output-iface* (*routing-action* a)) # *lr-of-tran-s1* rt  
**by**(*unfold lr-of-tran-s1-def list.map, rule*)

**lemma** *route2match-correct*: *valid-prefix* (*routing-match* a)  $\implies$  *prefix-match-semantics* (*routing-match* a) (*p-dst* p)  $\longleftrightarrow$  *simple-matches* (*route2match* a) (p)  
**by**(*simp add: route2match-def simple-matches.simps match-ifaceAny match-iface-refl ipset-from-cidr-0 prefix-match-semantics-ipset-from-netmask2*)

**lemma** *s1-correct*: *valid-prefixes* rt  $\implies$  *has-default-route* (rt::('i::len) *prefix-routing*)  $\implies$   
 $\exists$  rm ra. *generalized-sfw* (*lr-of-tran-s1* rt) p = *Some* (rm,ra)  $\wedge$  ra = *output-iface* (*routing-table-semantics* rt (p-dst p))  
**apply**(*induction* rt)  
**apply**(*simp;fail*)  
**apply**(*drule valid-prefixes-split*)  
**apply**(*clarsimp*)  
**apply**(*erule disjE*)  
**subgoal** **for** a rt  
**apply**(*case-tac* a)  
**apply**(*rename-tac routing-m metric routing-action*)  
**apply**(*case-tac routing-m*)  
**apply**(*simp add: valid-prefix-def pfxm-mask-def prefix-match-semantics-def generalized-sfw-def*  
*lr-of-tran-s1-def route2match-def simple-matches.simps match-ifaceAny match-iface-refl ipset-from-cidr-0*  
*max-word-mask[where 'a = 'i, symmetric, simplified]*)  
**done**  
**subgoal**  
**apply**(*rule conjI*)  
**apply**(*simp add: generalized-sfw-def lr-of-tran-s1-def route2match-correct;fail*)  
**apply**(*simp add: route2match-def simple-matches.simps prefix-match-semantics-ipset-from-netmask2*  
*lr-of-tran-s1-split generalized-sfw-simps*)  
**done**

done

**definition** *to-OF-action*  $a \equiv (\text{case } a \text{ of } (p, d) \Rightarrow (\text{case } d \text{ of } \text{simple-action.Accept} \Rightarrow [\text{Forward } p] \mid \text{simple-action.Drop} \Rightarrow []))$

**definition** *from-OF-action*  $a = (\text{case } a \text{ of } [] \Rightarrow (""', \text{simple-action.Drop}) \mid [\text{Forward } p] \Rightarrow (p, \text{simple-action.Accept}))$

**lemma** *OF-match-linear-not-noD*:  $\text{OF-match-linear } \gamma \text{ oms } p \neq \text{NoAction} \implies \exists \text{ome. } \text{ome} \in \text{set oms} \wedge \gamma \text{ (ofe-fields ome) } p$

**apply**(*induction oms*)

**apply**(*simp*)

**apply**(*simp split: if-splits*)

**apply** *blast+*

done

**lemma** *s3-noaction-hlp*:  $[\text{simple-match-valid } ac; \neg \text{simple-matches } ac \text{ } p; \text{match-iface (oiface } ac) (p\text{-oiface } p)] \implies$

*OF-match-linear OF-match-fields-safe (map ( $\lambda x. \text{split3 OFEntry } (x1, x, \text{case } ba \text{ of } \text{simple-action.Accept} \Rightarrow [\text{Forward } ad] \mid \text{simple-action.Drop} \Rightarrow [])) (\text{simple-match-to-of-match } ac \text{ ifs}))$   $p = \text{NoAction}$*

**apply**(*rule ccontr*)

**apply**(*drule OF-match-linear-not-noD*)

**apply**(*clarsimp*)

**apply**(*rename-tac x*)

**apply**(*subgoal-tac all-prerequisites x*)

**apply**(*drule simple-match-to-of-matchD*)

**apply**(*simp add: split3-def*)

**apply**(*subst(asm) of-match-fields-safe-eq2*)

**apply**(*simp;fail*)+

**using** *simple-match-to-of-match-generates-prereqs* **by** *blast*

**lemma** *aux*:

$\langle v = \text{Some } x \implies \text{the } v = x \rangle$

**by** *simp*

**lemma** *s3-correct*:

**assumes** *vsfwm*: *list-all simple-match-valid (map (fst  $\circ$  snd) ard)*

**assumes** *ippkt*:  $p\text{-l2type } p = 0x800$

**assumes** *iiifs*:  $p\text{-iiface } p \in \text{set ifs}$

**assumes** *oiifs*: *list-all ( $\lambda m. \text{oiface (fst (snd } m)) = \text{ifaceAny}$ ) ard*

**shows** *OF-match-linear OF-match-fields-safe (pack-OF-entries ifs ard)  $p = \text{Action } ao \longleftrightarrow (\exists r \text{ af. generalized-sfw (map snd ard) } p = (\text{Some } (r, \text{af})) \wedge (\text{if } \text{snd } \text{af} = \text{simple-action.Drop then } ao = [] \text{ else } ao = [\text{Forward } (\text{fst } \text{af})]))$*

**unfolding** *pack-OF-entries-def lr-of-tran-s3-def fun-app-def*

**using** *vsfwm oiifs*

**apply**(*induction ard*)

**subgoal by**(*simp add: generalized-sfw-simps*)

**apply** *simp*

**apply**(*clarsimp simp add: generalized-sfw-simps split: prod.splits*)

**apply**(*intro conjI*)

**subgoal for** *ard x1 ac ad ba*

**apply**(*clarsimp simp add: OF-match-linear-append split: prod.splits*)

**apply**(*drule simple-match-to-of-matchI[rotated]*)

**apply**(*rule iiifs*)

**apply**(*rule ippkt*)

**apply** *blast*

**apply**(*clarsimp simp add: comp-def*)

**apply**(*drule*

*OF-match-linear-match-allsameaction*[**where**

$\gamma = \text{OF-match-fields-safe}$  **and**  $\text{pri} = x1$  **and**

```

    oms = simple-match-to-of-match ac ifs and
    act = case ba of simple-action.Accept ⇒ [Forward ad] | simple-action.Drop ⇒ []
  apply(unfold OF-match-fields-safe-def comp-def)
  apply(erule aux)
  apply(clarsimp)
  apply(intro iffI)
  subgoal
    apply(rule exI[where x = ac])
    apply(rule exI[where x = ad])
    apply(rule exI[where x = ba])
    apply(clarsimp simp: split3-def split: simple-action.splits flowtable-behavior.splits if-splits)
  done
  subgoal
    apply(clarsimp)
    apply(rename-tac b)
    apply(case-tac b)
    apply(simp-all)
  done
done
subgoal for ard x1 ac ad ba
  apply(simp add: OF-match-linear-append OF-match-fields-safe-def comp-def)
  apply(clarify)
  apply(subgoal-tac OF-match-linear OF-match-fields-safe (map (λx. split3 OFEntry (x1, x, case ba of simple-action.Accept
⇒ [Forward ad] | simple-action.Drop ⇒ [])) (simple-match-to-of-match ac ifs)) p = NoAction)
    apply(simp;fail)
    apply(erule (1) s3-noaction-hlp)
    apply(simp add: match-ifaceAny;fail)
  done
done

context
  notes valid-prefix-00[simp, intro!]
begin
  lemma lr-of-tran-s1-valid: valid-prefixes rt ⇒ gsfw-valid (lr-of-tran-s1 rt)
    unfolding lr-of-tran-s1-def route2match-def gsfw-valid-def list-all-iff
    apply(clarsimp simp: simple-match-valid-def valid-prefix-fw-def)
    apply(intro conjI)
    apply force
    apply(simp add: valid-prefixes-alt-def)
  done
end

lemma simple-match-valid-fbs-rlen: [valid-prefixes rt; simple-fw-valid fw; (a, aa, ab, b) ∈ set (annotate-rlen (lr-of-tran-fbs rt
fw ifs))] ⇒ simple-match-valid aa
proof(goal-cases)
  case 1
  note 1[unfolded lr-of-tran-fbs-def Let-def]
  have gsfw-valid (map simple-rule-dtor fw) using gsfw-validI 1 by blast
  moreover have gsfw-valid (lr-of-tran-s1 rt) using 1 lr-of-tran-s1-valid by blast
  ultimately have gsfw-valid (generalized-fw-join (lr-of-tran-s1 rt) (map simple-rule-dtor fw)) using gsfw-join-valid by blast
  moreover have (aa, ab, b) ∈ set (lr-of-tran-fbs rt fw ifs) using 1 using in-annotate-rlen by fast
  ultimately show ?thesis unfolding lr-of-tran-fbs-def Let-def gsfw-valid-def list-all-iff by fastforce
qed

```



**lemma** *simple-match-valid-fbs*:  $\llbracket \text{valid-prefixes } rt; \text{simple-fw-valid } fw \rrbracket \implies \text{list-all simple-match-valid (map fst (lr-of-tran-fbs } rt \text{ fw ifs))}$   
**proof**(*goal-cases*)  
  **case** 1  
  **note** 1[*unfolded lr-of-tran-fbs-def Let-def*]  
  **have** *gsfw-valid* (map *simple-rule-dtor* fw) **using** *gsfw-validI* 1 **by** *blast*  
  **moreover** **have** *gsfw-valid* (lr-of-tran-s1 rt) **using** 1 *lr-of-tran-s1-valid* **by** *blast*  
  **ultimately** **have** *gsfw-valid* (*generalized-fw-join* (lr-of-tran-s1 rt) (map *simple-rule-dtor* fw)) **using** *gsfw-join-valid* **by** *blast*  
  **thus** ?thesis **unfolding** *lr-of-tran-fbs-def Let-def gsfw-valid-def list-all-iff* **by** *fastforce*  
**qed**

**lemma** *lr-of-tran-prereqs*:  $\text{valid-prefixes } rt \implies \text{simple-fw-valid } fw \implies \text{lr-of-tran } rt \text{ fw ifs} = \text{Inr oft} \implies \text{list-all (all-prerequisites } \circ \text{ ofe-fields) oft}$   
**unfolding** *lr-of-tran-def pack-OF-entries-def lr-of-tran-s3-def Let-def*  
  **apply**(*simp add: map-concat comp-def prod.case-distrib split3-def split: if-splits*)  
  **apply**(*simp add: list-all-iff*)  
  **apply**(*clarsimp*)  
  **apply**(*drule simple-match-valid-fbs-rlen[rotated]*)  
  **apply**(*simp add: list-all-iff;fail*)  
  **apply**(*simp add: list-all-iff;fail*)  
  **apply**(*rule simple-match-to-of-match-generates-prereqs; assumption*)  
**done**

**lemma** *OF-unsafe-safe-match3-eq*:  
   $\text{list-all (all-prerequisites } \circ \text{ ofe-fields) oft} \implies \text{OF-priority-match OF-match-fields-unsafe oft} = \text{OF-priority-match OF-match-fields-safe oft}$   
**unfolding** *OF-priority-match-def[abs-def]*  
**proof**(*goal-cases*)  
  **case** 1  
  **from** 1 **have**  $\bigwedge \text{packet. [f} \leftarrow \text{oft . OF-match-fields-unsafe (ofe-fields f) packet]} = \text{[f} \leftarrow \text{oft . OF-match-fields-safe (ofe-fields f) packet]}$   
  **apply**(*clarsimp simp add: list-all-iff of-match-fields-safe-eq*)  
  **using** *of-match-fields-safe-eq* **by**(*metis (mono-tags, lifting) filter-cong*)  
  **thus** ?case **by** *metis*  
**qed**

**lemma** *OF-unsafe-safe-match-linear-eq*:  
   $\text{list-all (all-prerequisites } \circ \text{ ofe-fields) oft} \implies \text{OF-match-linear OF-match-fields-unsafe oft} = \text{OF-match-linear OF-match-fields-safe oft}$   
**unfolding** *fun-eq-iff*  
**by**(*induction oft*) (*clarsimp simp add: list-all-iff of-match-fields-safe-eq*)**+**

**lemma** *simple-action-ne[simp]*:  
   $b \neq \text{simple-action.Accept} \longleftrightarrow b = \text{simple-action.Drop}$   
   $b \neq \text{simple-action.Drop} \longleftrightarrow b = \text{simple-action.Accept}$   
**using** *simple-action.exhaust* **by** *blast+*

**lemma** *map-snd-apfst*:  $\text{map snd (map (apfst x) l)} = \text{map snd l}$   
  **unfolding** *map-map comp-def snd-apfst ..*

**lemma** *match-ifaceAny-eq*:  $\text{iface } m = \text{ifaceAny} \implies \text{simple-matches } m \text{ } p = \text{simple-matches } m \text{ (p(p-oiface := any))}$   
**by**(*cases m*) (*simp add: simple-matches.simps match-ifaceAny*)  
**lemma** *no-oif-matchD*:  $\text{no-oif-match } fw \implies \text{simple-fw } fw \text{ } p = \text{simple-fw } fw \text{ (p(p-oiface := any))}$

**by**(*induction fw*)  
 (*auto simp add: no-oif-match-def simple-fw-alt dest: match-ifaceAny-eq*)

**lemma** *lr-of-tran-fbs-acceptD:*

**assumes** *s1: valid-prefixes rt has-default-route rt*

**assumes** *s2: no-oif-match fw*

**shows** *generalized-sfw (lr-of-tran-fbs rt fw ifs) p = Some (r, oif, simple-action.Accept)  $\implies$  simple-linux-router-nol12 rt fw p = Some (p\p-oiface := oif\))*

**proof**(*goal-cases*)

**case** *1*

**note** *1[unfolded lr-of-tran-fbs-def Let-def, THEN generalized-fw-joinD]*

**then guess** *r1 .. then guess r2 .. note r12 = this*

**note** *s1-correct[OF s1, of p]*

**then guess** *rm .. then guess ra .. note rmra = this*

**from** *r12 rmra have oifra: oif = ra by simp*

**from** *r12 have sfw: simple-fw fw p = Decision.FinalAllow using simple-fw-iff-generalized-fw-accept by blast*

**note** *ifupdateirrel = no-oif-matchD[OF s2, where any = output-iface (routing-table-semantic rt (p-dst p)) and p = p, symmetric]*

**show** *?case unfolding simple-linux-router-nol12-def by(simp add: Let-def ifupdateirrel sfw oifra rmra split: Option.bind-splits option.splits)*

**qed**

**lemma** *lr-of-tran-fbs-acceptI:*

**assumes** *s1: valid-prefixes rt has-default-route rt*

**assumes** *s2: no-oif-match fw has-default-policy fw*

**shows** *simple-linux-router-nol12 rt fw p = Some (p\p-oiface := oif\))  $\implies$*

*$\exists r. generalized-sfw (lr-of-tran-fbs rt fw ifs) p = Some (r, oif, simple-action.Accept)$*

**proof**(*goal-cases*)

**from** *s2 have nud:  $\bigwedge p. simple-fw fw p \neq Undecided$  by (metis has-default-policy state.distinct(1))*

**note** *ifupdateirrel = no-oif-matchD[OF s2(1), symmetric]*

**case** *1*

**from** *1 have simple-fw fw p = Decision.FinalAllow by(simp add: simple-linux-router-nol12-def Let-def nud ifupdateirrel split: Option.bind-splits state.splits final-decision.splits)*

**then obtain** *r where r: generalized-sfw (map simple-rule-dtor fw) p = Some (r, simple-action.Accept) using simple-fw-iff-generalized-fw-accept by blast*

**have** *oif-def: oif = output-iface (routing-table-semantic rt (p-dst p)) using 1 by(cases p) (simp add: simple-linux-router-nol12-def Let-def nud ifupdateirrel split: Option.bind-splits state.splits final-decision.splits)*

**note** *s1-correct[OF s1, of p] then guess rm .. then guess ra .. note rmra = this*

**show** *?case unfolding lr-of-tran-fbs-def Let-def*

*apply(rule exI)*

*apply(rule generalized-fw-joinI)*

*unfolding oif-def using rmra apply simp*

*apply(rule r)*

**done**

**qed**

**lemma** *lr-of-tran-fbs-dropD:*

**assumes** *s1: valid-prefixes rt has-default-route rt*

**assumes** *s2: no-oif-match fw*

**shows** *generalized-sfw (lr-of-tran-fbs rt fw ifs) p = Some (r, oif, simple-action.Drop)  $\implies$  simple-linux-router-nol12 rt fw p = None*

**proof**(*goal-cases*)

**note** *ifupdateirrel = no-oif-matchD[OF s2(1), symmetric]*

**case** *1*

```

from 1[unfolded lr-of-tran-fbs-def Let-def, THEN generalized-fw-joinD]
obtain rr fr where generalized-sfw (lr-of-tran-s1 rt) p = Some (rr, oif) ∧
    generalized-sfw (map simple-rule-dtor fw) p = Some (fr, simple-action.Drop) ∧ Some r = simple-match-and rr fr by
presburger
hence fd: ∧u. simple-fw fw (p(p-oiface := u)) = Decision FinalDeny unfolding ifupdateirrel
using simple-fw-iff-generalized-fw-drop by blast
show ?thesis
by(clarsimp simp: simple-linux-router-nol12-def Let-def fd split: Option.bind-splits)
qed

```

```

lemma lr-of-tran-fbs-dropI:
assumes s1: valid-prefixes rt has-default-route rt
assumes s2: no-oif-match fw has-default-policy fw
shows simple-linux-router-nol12 rt fw p = None  $\implies$ 
 $\exists r$  oif. generalized-sfw (lr-of-tran-fbs rt fw ifs) p = Some (r, oif, simple-action.Drop)
proof(goal-cases)
from s2 have nud: ∧p. simple-fw fw p ≠ Undecided by (metis has-default-policy state.distinct(1))
note ifupdateirrel = no-oif-matchD[OF s2(1), symmetric]
case 1
from 1 have simple-fw fw p = Decision FinalDeny by(simp add: simple-linux-router-nol12-def Let-def nud ifupdateirrel
split: Option.bind-splits state.splits final-decision.splits)
then obtain r where r: generalized-sfw (map simple-rule-dtor fw) p = Some (r, simple-action.Drop) using
simple-fw-iff-generalized-fw-drop by blast
note s1-correct[OF s1, of p] then guess rm .. then guess ra .. note rmra = this
show ?case unfolding lr-of-tran-fbs-def Let-def
    apply(rule exI)
    apply(rule exI[where x = ra])
    apply(rule generalized-fw-joinI)
    using rmra apply simp
    apply(rule r)
done
qed

```

```

lemma no-oif-match-fbs:
no-oif-match fw  $\implies$  list-all (λm. oiface (fst (snd m)) = ifaceAny) (map (apfst of-nat) (annotate-rlen (lr-of-tran-fbs rt fw
ifs)))
proof(goal-cases)
case 1
have c: ∧mr ar mf af f a. [(mr, ar) ∈ set (lr-of-tran-s1 rt); (mf, af) ∈ simple-rule-dtor ‘ set fw; simple-match-and mr mf
= Some a]  $\implies$  oiface a = ifaceAny
proof(goal-cases)
case (1 mr ar mf af f a)
have oiface mr = ifaceAny using 1(1) unfolding lr-of-tran-s1-def route2match-def by(clarsimp simp add: Set.image-iff)
moreover have oiface mf = ifaceAny using 1(2) ⟨no-oif-match fw⟩ unfolding no-oif-match-def
simple-rule-dtor-def[abs-def]
by(clarsimp simp: list-all-iff split: simple-rule.splits) fastforce
ultimately show ?case using 1(3) by(cases a; cases mr; cases mf) (simp add: iface-conjunct-ifaceAny split: option.splits)
qed
have la: list-all (λm. oiface (fst m) = ifaceAny) (lr-of-tran-fbs rt fw ifs)
unfolding lr-of-tran-fbs-def Let-def list-all-iff
apply(clarify)
apply(subst(asm) generalized-sfw-join-set)
apply(clarsimp)

```

```

using c by blast
thus ?case
proof(goal-cases)
  case 1
  have *: (λm. oiface (fst (snd m)) = ifaceAny) = (λm. oiface (fst m) = ifaceAny) ∘ snd unfolding comp-def ..
  show ?case unfolding * list-all-map[symmetric] map-snd-apfst map-snd-annotate-rlen using la .
qed
qed

lemma lr-of-tran-correct:
  fixes p :: (32, 'a) simple-packet-ext-scheme
  assumes nerr: lr-of-tran rt fw ifs = Inr oft
  and ippkt: p-l2type p = 0x800
  and ifvld: p-iface p ∈ set ifs
  shows OF-priority-match OF-match-fields-safe oft p = Action [Forward oif] ⟷ simple-linux-router-nol12 rt fw p = (Some
    (p(p-oiface := oif)))
    OF-priority-match OF-match-fields-safe oft p = Action [] ⟷ simple-linux-router-nol12 rt fw p = None

    OF-priority-match OF-match-fields-safe oft p ≠ NoAction OF-priority-match OF-match-fields-safe oft p ≠ Undefined
    OF-priority-match OF-match-fields-safe oft p = Action ls ⟶ length ls ≤ 1
    ∃ ls. length ls ≤ 1 ∧ OF-priority-match OF-match-fields-safe oft p = Action ls
proof -
  have s1: valid-prefixes rt has-default-route rt
  and s2: has-default-policy fw simple-fw-valid fw no-oif-match fw
  and difs: distinct ifs
  using nerr unfolding lr-of-tran-def by(simp-all split: if-splits)
  have no-oif-match fw using nerr unfolding lr-of-tran-def by(simp split: if-splits)
  note s2 = s2 this
  have unsafe-safe-eq:
    OF-priority-match OF-match-fields-unsafe oft = OF-priority-match OF-match-fields-safe oft
    OF-match-linear OF-match-fields-unsafe oft = OF-match-linear OF-match-fields-safe oft
    apply(subst OF-unsafe-safe-match3-eq; (rule lr-of-tran-prereqs s1 s2 nerr refl)+)
    apply(subst OF-unsafe-safe-match-linear-eq; (rule lr-of-tran-prereqs s1 s2 nerr refl)+)
  done
  have lin: OF-priority-match OF-match-fields-safe oft = OF-match-linear OF-match-fields-safe oft
  using OF-eq[OF lr-of-tran-no-overlaps lr-of-tran-sorted-descending, OF difs nerr[symmetric] nerr[symmetric]] unfolding
  fun-eq-iff unsafe-safe-eq by metis
  let ?ard = map (apfst of-nat) (annotate-rlen (lr-of-tran-fbs rt fw ifs))
  have oft-def: oft = pack-OF-entries ifs ?ard using nerr unfolding lr-of-tran-def Let-def by(simp split: if-splits)
  have vld: list-all simple-match-valid (map (fst ∘ snd) ?ard)
    unfolding fun-app-def map-map[symmetric] snd-apfst map-snd-apfst map-snd-annotate-rlen using
  simple-match-valid-fbs[OF s1(1) s2(2)] .
  have *: list-all (λm. oiface (fst (snd m)) = ifaceAny) ?ard using no-oif-match-fbs[OF s2(3)] .
  have not-undec: ∧p. simple-fw fw p ≠ Undecided by (metis has-default-policy s2(1) state.simps(3))
  have w1-1: ∧oif. OF-match-linear OF-match-fields-safe oft p = Action [Forward oif] ⟹ simple-linux-router-nol12 rt fw
  p = Some (p(p-oiface := oif))
  ∧ oif = output-iface (routing-table-semantics rt (p-dst p))
proof(intro conjI, goal-cases)
  case (1 oif)
  note s3-correct[OF vld ippkt ifvld(1) *, THEN iffD1, unfolded oft-def[symmetric], OF 1]
  hence ∃ r. generalized-sfw (map snd (map (apfst of-nat) (annotate-rlen (lr-of-tran-fbs rt fw ifs)))) p = Some (r, (oif,
  simple-action.Accept))
  by(clarsimp split: if-splits)

```

```

then obtain r where generalized-sfw (lr-of-tran-fbs rt fw ifs) p = Some (r, (oif, simple-action.Accept))
  unfolding map-map comp-def snd-apfst map-snd-annotate-rlen by blast
thus ?case using lr-of-tran-fbs-acceptD[OF s1 s2(3)] by metis
thus oif = output-iface (routing-table-semantics rt (p-dst p))
  by(cases p) (clarsimp simp: simple-linux-router-nol12-def Let-def not-undec split: Option.bind-splits state.splits fi-
nal-decision.splits)
qed
have w1-2:  $\bigwedge oif. \text{simple-linux-router-nol12 rt fw } p = \text{Some } (p \langle p\text{-oiface} := oif \rangle) \implies \text{OF-match-linear OF-match-fields-safe}$ 
oft p = Action [Forward oif]
proof(goal-cases)
  case (1 oif)
  note lr-of-tran-fbs-acceptI[OF s1 s2(3) s2(1) this, of ifs] then guess r .. note r = this
  hence generalized-sfw (map snd (map (apfst of-nat) (annotate-rlen (lr-of-tran-fbs rt fw ifs)))) p = Some (r, (oif, sim-
ple-action.Accept))
  unfolding map-snd-apfst map-snd-annotate-rlen .
  moreover note s3-correct[OF vld ippkt ifold(1) *, THEN iffD2, unfolded oft-def[symmetric], of [Forward oif]]
  ultimately show ?case by simp
qed
show w1:  $\bigwedge oif. (\text{OF-priority-match OF-match-fields-safe oft } p = \text{Action [Forward oif]}) = (\text{simple-linux-router-nol12 rt fw}$ 
 $p = \text{Some } (p \langle p\text{-oiface} := oif \rangle))$ 
  unfolding lin using w1-1 w1-2 by blast
show w2:  $(\text{OF-priority-match OF-match-fields-safe oft } p = \text{Action []}) = (\text{simple-linux-router-nol12 rt fw } p = \text{None})$ 
  unfolding lin
proof(rule iffI, goal-cases)
  case 1
  note s3-correct[OF vld ippkt ifold(1) *, THEN iffD1, unfolded oft-def[symmetric], OF 1]
  then obtain r oif where roif: generalized-sfw (lr-of-tran-fbs rt fw ifs) p = Some (r, oif, simple-action.Drop)
  unfolding map-snd-apfst map-snd-annotate-rlen by(clarsimp split: if-splits)
  note lr-of-tran-fbs-dropD[OF s1 s2(3) this]
  thus ?case .
next
  case 2
  note lr-of-tran-fbs-dropI[OF s1 s2(3) s2(1) this, of ifs] then
  obtain r oif where generalized-sfw (lr-of-tran-fbs rt fw ifs) p = Some (r, oif, simple-action.Drop) by blast
  hence generalized-sfw (map snd (map (apfst of-nat) (annotate-rlen (lr-of-tran-fbs rt fw ifs)))) p = Some (r, oif, sim-
ple-action.Drop)
  unfolding map-snd-apfst map-snd-annotate-rlen .
  moreover note s3-correct[OF vld ippkt ifold(1) *, THEN iffD2, unfolded oft-def[symmetric], of []]
  ultimately show ?case by force
qed
have lr-determ:  $\bigwedge a. \text{simple-linux-router-nol12 rt fw } p = \text{Some } a \implies a = p \langle p\text{-oiface} := \text{output-iface (routing-table-semantics}$ 
 $\text{rt (p-dst p))} \rangle$ 
  by(clarsimp simp: simple-linux-router-nol12-def Let-def not-undec split: Option.bind-splits state.splits final-decision.splits)
show notno:  $\text{OF-priority-match OF-match-fields-safe oft } p \neq \text{NoAction}$ 
  apply(cases simple-linux-router-nol12 rt fw p)
  using w2 apply(simp)
  using w1 [of output-iface (routing-table-semantics rt (p-dst p))] apply(simp)
  apply(drule lr-determ)
  apply(simp)
done
show notub:  $\text{OF-priority-match OF-match-fields-safe oft } p \neq \text{Undefined}$  unfolding lin using OF-match-linear-ne-Undefined
.
show notmult:  $\bigwedge ls. \text{OF-priority-match OF-match-fields-safe oft } p = \text{Action } ls \longrightarrow \text{length } ls \leq 1$ 
  apply(cases simple-linux-router-nol12 rt fw p)

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```

    using w2 apply(simp)
    using w1[of output-iface (routing-table-semantic rt (p-dst p))] apply(simp)
    apply(drule lr-determ)
    apply(clarsimp)
  done
show  $\exists ls. \text{length } ls \leq 1 \wedge \text{OF-priority-match OF-match-fields-safe of } p = \text{Action } ls$ 
  apply(cases OF-priority-match OF-match-fields-safe of p)
  using notmult apply blast
  using notno apply blast
  using notub apply blast
done
qed

end
theory OF-conv-test
imports
  Iptables-Semantics.Parser
  Simple-Firewall.SimpleFw-toString
  Routing.IpRoute-Parser
  ../../LinuxRouter-OpenFlow-Translation
  ../../OpenFlow-Serialize
begin

parse-iptables-save SQRL-fw=iptables-save

term SQRL-fw
thm SQRL-fw-def
thm SQRL-fw-FORWARD-default-policy-def

value[code] map ( $\lambda(c,rs). (c, \text{map } (\text{quote-rewrite} \circ \text{common-primitive-rule-toString}) \text{ rs})$ ) SQRL-fw
definition unfolded = unfold-ruleset-FORWARD SQRL-fw-FORWARD-default-policy (map-of-string-ipv4 SQRL-fw)
lemma map (quote-rewrite  $\circ$  common-primitive-rule-toString) unfolded =
  ["-p icmp -j ACCEPT",
   "-i s1-lan -p tcp -m tcp --spts [1024:65535] -m tcp --dpts [80] -j ACCEPT",
   "-i s1-wan -p tcp -m tcp --spts [80] -m tcp --dpts [1024:65535] -j ACCEPT",
   "-j DROP"] by eval

lemma length unfolded = 4 by eval

value[code] map (quote-rewrite  $\circ$  common-primitive-rule-toString) (upper-closure unfolded)
lemma length (upper-closure unfolded) = 4 by eval

value[code] upper-closure (packet-assume-new unfolded)

lemma length (lower-closure unfolded) = 4 by eval

lemma check-simple-fw-preconditions (upper-closure unfolded) = True by eval
lemma  $\forall m \in \text{get-match'set } (\text{upper-closure } (\text{packet-assume-new unfolded})). \text{normalized-nnf-match } m$  by eval

```

```

lemma  $\forall m \in \text{get-match'set } (\text{optimize-matches abstract-for-simple-firewall } (\text{upper-closure } (\text{packet-assume-new unfolded})))$ .
normalized-nnf-match m by eval
lemma check-simple-fw-preconditions (upper-closure (optimize-matches abstract-for-simple-firewall (upper-closure
(packet-assume-new unfolded)))) by eval
lemma length (to-simple-firewall (upper-closure (packet-assume-new unfolded))) = 4 by eval

lemma (lower-closure (optimize-matches abstract-for-simple-firewall (lower-closure (packet-assume-new unfolded)))) =
lower-closure unfolded
  lower-closure unfolded = upper-closure unfolded
  (upper-closure (optimize-matches abstract-for-simple-firewall (upper-closure (packet-assume-new unfolded)))) = up-
per-closure unfolded by eval+

value[code] (getParts (to-simple-firewall (lower-closure (optimize-matches abstract-for-simple-firewall (lower-closure
(packet-assume-new unfolded))))))

definition SQRL-fw-simple  $\equiv \text{remdups-rev } (\text{to-simple-firewall } (\text{upper-closure } (\text{optimize-matches abstract-for-simple-firewall }
(\text{upper-closure } (\text{packet-assume-new unfolded}))))))$ 
value[code] SQRL-fw-simple
lemma simple-fw-valid SQRL-fw-simple by eval

parse-ip-route SQRL-rtbl-main = ip-route
value SQRL-rtbl-main
lemma SQRL-rtbl-main = [(routing-match = PrefixMatch 0xA000100 24, metric = 0, routing-action = (output-iface =
"s1-lan", next-hop = None)),
  (routing-match = PrefixMatch 0xA000200 24, metric = 0, routing-action = (output-iface = "s1-wan", next-hop = None)),
  (routing-match = PrefixMatch 0 0, metric = 0, routing-action = (output-iface = "s1-wan", next-hop = Some
0xA000201))] by eval
value dotdecimal-of-ipv4addr 0xA0D2500
lemma SQRL-rtbl-main = [
  rr-ctor (10,0,1,0) 24 "s1-lan" None 0,
  rr-ctor (10,0,2,0) 24 "s1-wan" None 0,
  rr-ctor (0,0,0,0) 0 "s1-wan" (Some (10,0,2,1)) 0
]
by eval

definition SQRL-rtbl-main-sorted  $\equiv \text{rev } (\text{sort-key } (\lambda r. \text{pfm-length } (\text{routing-match } r)) \text{ SQRL-rtbl-main})$ 
value SQRL-rtbl-main-sorted
definition SQRL-ifs  $\equiv [$ 
  (iface-name = "s1-lan", iface-mac = 0x10001),
  (iface-name = "s1-wan", iface-mac = 0x10002)
]
value SQRL-ifs

definition SQRL-macs  $\equiv [$ 
  (/"s1-lan"/(ipv4addr-of-dotdecimal(10,0,1,2)/0x1)/0x9),
  (/"s1-lan"/(ipv4addr-of-dotdecimal(10,0,1,3)/0x2)),
  (/"s1-wan"/(ipv4addr-of-dotdecimal(10,0,2,1)/0x3))
  (/"s1-wan"/(ipv4addr-of-dotdecimal(10,0,2,4)/0x66d0152059))
]

definition SQRL-ports  $\equiv [$ 

```

```

("s1-lan", "1"),
("s1-wan", "2")
]

```

**lemma** *let fw = SQRL-fw-simple in no-oif-match fw ∧ has-default-policy fw ∧ simple-fw-valid fw* **by** eval

**lemma** *let rt = SQRL-rtbl-main-sorted in valid-prefixes rt ∧ has-default-route rt* **by** eval

**lemma** *let ifs = (map iface-name SQRL-ifs) in distinct ifs* **by** eval

**definition** *ofi* ≡

case (lr-of-tran SQRL-rtbl-main-sorted SQRL-fw-simple (map iface-name SQRL-ifs))  
of (Inr openflow-rules) ⇒ map (serialize-of-entry (the ◦ map-of SQRL-ports)) openflow-rules

**lemma** *ofi* =

```

["priority=11,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-proto=1,nw-dst=10.0.2.0/24,action=output:2",
 "priority=10,hard-timeout=0,idle-timeout=0,in-port=1,dl-type=0x800,nw-proto=6,nw-dst=10.0.2.0/24,tp-src=1024/0xfc00,tp-dst=8",
 "priority=10,hard-timeout=0,idle-timeout=0,in-port=1,dl-type=0x800,nw-proto=6,nw-dst=10.0.2.0/24,tp-src=2048/0xf800,tp-dst=8",
 "priority=10,hard-timeout=0,idle-timeout=0,in-port=1,dl-type=0x800,nw-proto=6,nw-dst=10.0.2.0/24,tp-src=4096/0xf000,tp-dst=8",
 "priority=10,hard-timeout=0,idle-timeout=0,in-port=1,dl-type=0x800,nw-proto=6,nw-dst=10.0.2.0/24,tp-src=8192/0xe000,tp-dst=8",
 "priority=10,hard-timeout=0,idle-timeout=0,in-port=1,dl-type=0x800,nw-proto=6,nw-dst=10.0.2.0/24,tp-src=16384/0xc000,tp-dst=8",
 "priority=10,hard-timeout=0,idle-timeout=0,in-port=1,dl-type=0x800,nw-proto=6,nw-dst=10.0.2.0/24,tp-src=32768/0x8000,tp-dst=8",
 "priority=9,hard-timeout=0,idle-timeout=0,in-port=2,dl-type=0x800,nw-proto=6,nw-dst=10.0.2.0/24,tp-src=80,tp-dst=1024/0xfc00",
 "priority=9,hard-timeout=0,idle-timeout=0,in-port=2,dl-type=0x800,nw-proto=6,nw-dst=10.0.2.0/24,tp-src=80,tp-dst=2048/0xf800",
 "priority=9,hard-timeout=0,idle-timeout=0,in-port=2,dl-type=0x800,nw-proto=6,nw-dst=10.0.2.0/24,tp-src=80,tp-dst=4096/0xf000",
 "priority=9,hard-timeout=0,idle-timeout=0,in-port=2,dl-type=0x800,nw-proto=6,nw-dst=10.0.2.0/24,tp-src=80,tp-dst=8192/0xe000",
 "priority=9,hard-timeout=0,idle-timeout=0,in-port=2,dl-type=0x800,nw-proto=6,nw-dst=10.0.2.0/24,tp-src=80,tp-dst=16384/0xc000",
 "priority=9,hard-timeout=0,idle-timeout=0,in-port=2,dl-type=0x800,nw-proto=6,nw-dst=10.0.2.0/24,tp-src=80,tp-dst=32768/0x8000",
 "priority=8,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-dst=10.0.2.0/24,action=drop",
 "priority=7,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-proto=1,nw-dst=10.0.1.0/24,action=output:1",
 "priority=6,hard-timeout=0,idle-timeout=0,in-port=1,dl-type=0x800,nw-proto=6,nw-dst=10.0.1.0/24,tp-src=1024/0xfc00,tp-dst=8",
 "priority=6,hard-timeout=0,idle-timeout=0,in-port=1,dl-type=0x800,nw-proto=6,nw-dst=10.0.1.0/24,tp-src=2048/0xf800,tp-dst=8",
 "priority=6,hard-timeout=0,idle-timeout=0,in-port=1,dl-type=0x800,nw-proto=6,nw-dst=10.0.1.0/24,tp-src=4096/0xf000,tp-dst=8",
 "priority=6,hard-timeout=0,idle-timeout=0,in-port=1,dl-type=0x800,nw-proto=6,nw-dst=10.0.1.0/24,tp-src=8192/0xe000,tp-dst=8",
 "priority=6,hard-timeout=0,idle-timeout=0,in-port=1,dl-type=0x800,nw-proto=6,nw-dst=10.0.1.0/24,tp-src=16384/0xc000,tp-dst=8",
 "priority=6,hard-timeout=0,idle-timeout=0,in-port=1,dl-type=0x800,nw-proto=6,nw-dst=10.0.1.0/24,tp-src=32768/0x8000,tp-dst=8",
 "priority=5,hard-timeout=0,idle-timeout=0,in-port=2,dl-type=0x800,nw-proto=6,nw-dst=10.0.1.0/24,tp-src=80,tp-dst=1024/0xfc00",
 "priority=5,hard-timeout=0,idle-timeout=0,in-port=2,dl-type=0x800,nw-proto=6,nw-dst=10.0.1.0/24,tp-src=80,tp-dst=2048/0xf800",
 "priority=5,hard-timeout=0,idle-timeout=0,in-port=2,dl-type=0x800,nw-proto=6,nw-dst=10.0.1.0/24,tp-src=80,tp-dst=4096/0xf000",
 "priority=5,hard-timeout=0,idle-timeout=0,in-port=2,dl-type=0x800,nw-proto=6,nw-dst=10.0.1.0/24,tp-src=80,tp-dst=8192/0xe000",
 "priority=5,hard-timeout=0,idle-timeout=0,in-port=2,dl-type=0x800,nw-proto=6,nw-dst=10.0.1.0/24,tp-src=80,tp-dst=16384/0xc000",
 "priority=5,hard-timeout=0,idle-timeout=0,in-port=2,dl-type=0x800,nw-proto=6,nw-dst=10.0.1.0/24,tp-src=80,tp-dst=32768/0x8000",
 "priority=4,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-dst=10.0.1.0/24,action=drop",
 "priority=3,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-proto=1,action=output:2",
 "priority=2,hard-timeout=0,idle-timeout=0,in-port=1,dl-type=0x800,nw-proto=6,tp-src=1024/0xfc00,tp-dst=80,action=output:2",
 "priority=2,hard-timeout=0,idle-timeout=0,in-port=1,dl-type=0x800,nw-proto=6,tp-src=2048/0xf800,tp-dst=80,action=output:2",
 "priority=2,hard-timeout=0,idle-timeout=0,in-port=1,dl-type=0x800,nw-proto=6,tp-src=4096/0xf000,tp-dst=80,action=output:2",
 "priority=2,hard-timeout=0,idle-timeout=0,in-port=1,dl-type=0x800,nw-proto=6,tp-src=8192/0xe000,tp-dst=80,action=output:2",
 "priority=2,hard-timeout=0,idle-timeout=0,in-port=1,dl-type=0x800,nw-proto=6,tp-src=16384/0xc000,tp-dst=80,action=output:2",
 "priority=2,hard-timeout=0,idle-timeout=0,in-port=1,dl-type=0x800,nw-proto=6,tp-src=32768/0x8000,tp-dst=80,action=output:2",
 "priority=1,hard-timeout=0,idle-timeout=0,in-port=2,dl-type=0x800,nw-proto=6,tp-src=80,tp-dst=1024/0xfc00,action=output:2",
 "priority=1,hard-timeout=0,idle-timeout=0,in-port=2,dl-type=0x800,nw-proto=6,tp-src=80,tp-dst=2048/0xf800,action=output:2",
 "priority=1,hard-timeout=0,idle-timeout=0,in-port=2,dl-type=0x800,nw-proto=6,tp-src=80,tp-dst=4096/0xf000,action=output:2",
 "priority=1,hard-timeout=0,idle-timeout=0,in-port=2,dl-type=0x800,nw-proto=6,tp-src=80,tp-dst=8192/0xe000,action=output:2",
 "priority=1,hard-timeout=0,idle-timeout=0,in-port=2,dl-type=0x800,nw-proto=6,tp-src=80,tp-dst=16384/0xc000,action=output:2",
 "priority=1,hard-timeout=0,idle-timeout=0,in-port=2,dl-type=0x800,nw-proto=6,tp-src=80,tp-dst=32768/0x8000,action=output:2"]

```



```
"priority=0,hard-timeout=0,idle-timeout=0,dl-type=0x800,action=drop"] by eval
```

```
value[code] ofi
```

```
end
```

```
theory RFC2544
```

```
imports
```

```
  Iptables-Semantics.Parser
```

```
  Routing.IpRoute-Parser
```

```
  ../../LinuxRouter-OpenFlow-Translation
```

```
  ../../OpenFlow-Serialize
```

```
begin
```

```
parse-iptables-save SQRL-fw=iptables-save
```

```
term SQRL-fw
```

```
thm SQRL-fw-def
```

```
thm SQRL-fw-FORWARD-default-policy-def
```

```
value[code] map (λ(c,rs). (c, map (quote-rewrite ∘ common-primitive-rule-toString) rs)) SQRL-fw
```

```
definition unfolded = unfold-ruleset-FORWARD SQRL-fw-FORWARD-default-policy (map-of-string-ipv4 SQRL-fw)
```

```
lemma length unfolded = 26 by eval
```

```
value[code] unfolded
```

```
value[code] (upper-closure unfolded)
```

```
value[code] map (quote-rewrite ∘ common-primitive-rule-toString) (upper-closure unfolded)
```

```
lemma length (upper-closure unfolded) = 26 by eval
```

```
value[code] upper-closure (packet-assume-new unfolded)
```

```
lemma length (lower-closure unfolded) = 26 by eval
```

```
lemma check-simple-fw-preconditions (upper-closure unfolded) by eval
```

```
lemma ∀ m ∈ get-match'set (upper-closure (packet-assume-new unfolded)). normalized-nnf-match m by eval
```

```
lemma ∀ m ∈ get-match'set (optimize-matches abstract-for-simple-firewall (upper-closure (packet-assume-new unfolded))).  
normalized-nnf-match m by eval
```

```
lemma check-simple-fw-preconditions (upper-closure (optimize-matches abstract-for-simple-firewall (upper-closure  
(packet-assume-new unfolded)))) by eval
```

```
lemma length (to-simple-firewall (upper-closure (packet-assume-new unfolded))) = 26 by eval
```

```
lemma (lower-closure (optimize-matches abstract-for-simple-firewall (lower-closure (packet-assume-new unfolded)))) =  
lower-closure unfolded
```

```
  lower-closure unfolded = upper-closure unfolded
```

```
  (upper-closure (optimize-matches abstract-for-simple-firewall (upper-closure (packet-assume-new unfolded)))) = up-  
per-closure unfolded by eval+
```

**value**[code] (getParts (to-simple-firewall (lower-closure (optimize-matches abstract-for-simple-firewall (lower-closure (packet-assume-new unfolded))))))

**definition** SQRL-fw-simple  $\equiv$  remdups-rev (to-simple-firewall (upper-closure (optimize-matches abstract-for-simple-firewall (upper-closure (packet-assume-new unfolded))))))

**value**[code] SQRL-fw-simple

**lemma** simple-fw-valid SQRL-fw-simple **by** eval

**parse-ip-route** SQRL-rtbl-main = ip-route

**value** SQRL-rtbl-main

**lemma** SQRL-rtbl-main = [(routing-match = PrefixMatch 0xC6120100 24, metric = 0, routing-action = (output-iface = "ip1", next-hop = None)),  
 (routing-match = PrefixMatch 0xC6130100 24, metric = 0, routing-action = (output-iface = "op1", next-hop = None)),  
 (routing-match = PrefixMatch 0 0, metric = 0, routing-action = (output-iface = "op1", next-hop = Some 0xC6130102))]

**by** eval

**lemma** SQRL-rtbl-main = [  
 rr-ctor (198,18,1,0) 24 "ip1" None 0,  
 rr-ctor (198,19,1,0) 24 "op1" None 0,  
 rr-ctor (0,0,0,0) 0 "op1" (Some (198,19,1,2)) 0  
 ]

**by** eval

**definition** SQRL-ports  $\equiv$  [  
 ("ip1", "1"),  
 ("op1", "2")  
 ]

**definition** ofi  $\equiv$

case (lr-of-tran SQRL-rtbl-main SQRL-fw-simple (map fst SQRL-ports))  
 of (Inr openflow-rules)  $\Rightarrow$  map (serialize-of-entry (the  $\circ$  map-of SQRL-ports)) openflow-rules

**lemma** ofi =

["priority=27,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-dst=198.18.1.0/24,action=drop",  
 "priority=26,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-dst=198.19.1.0/24,action=drop",  
 "priority=25,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-src=192.18.1.1/32,nw-dst=192.18.101.1/32,action=drop",  
 "priority=24,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-src=192.18.2.2/32,nw-dst=192.18.102.2/32,action=drop",  
 "priority=23,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-src=192.18.3.3/32,nw-dst=192.18.103.3/32,action=drop",  
 "priority=22,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-src=192.18.4.4/32,nw-dst=192.18.104.4/32,action=drop",  
 "priority=21,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-src=192.18.5.5/32,nw-dst=192.18.105.5/32,action=drop",  
 "priority=20,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-src=192.18.6.6/32,nw-dst=192.18.106.6/32,action=drop",  
 "priority=19,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-src=192.18.7.7/32,nw-dst=192.18.107.7/32,action=drop",  
 "priority=18,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-src=192.18.8.8/32,nw-dst=192.18.108.8/32,action=drop",  
 "priority=17,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-src=192.18.9.9/32,nw-dst=192.18.109.9/32,action=drop",  
 "priority=16,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-src=192.18.10.10/32,nw-dst=192.18.110.10/32,action=drop",  
 "priority=15,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-src=192.18.11.11/32,nw-dst=192.18.111.11/32,action=drop",  
 "priority=14,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-src=192.18.12.12/32,nw-dst=192.18.112.12/32,action=drop",  
 "priority=13,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-src=192.19.1.2/32,nw-dst=192.19.65.1/32,action=output:2",  
 "priority=12,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-src=192.18.13.13/32,nw-dst=192.18.113.13/32,action=drop",  
 "priority=11,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-src=192.18.14.14/32,nw-dst=192.18.114.14/32,action=drop",  
 "priority=10,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-src=192.18.15.15/32,nw-dst=192.18.115.15/32,action=drop",  
 "priority=9,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-src=192.18.16.16/32,nw-dst=192.18.116.16/32,action=drop",  
 "priority=8,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-src=192.18.17.17/32,nw-dst=192.18.117.17/32,action=drop",  
 "priority=7,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-src=192.18.18.18/32,nw-dst=192.18.118.18/32,action=drop",  
 "priority=6,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-src=192.18.19.19/32,nw-dst=192.18.119.19/32,action=drop",  
 "priority=5,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-src=192.18.20.20/32,nw-dst=192.18.120.20/32,action=drop",

```

"priority=4,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-src=192.18.21.21/32,nw-dst=192.18.121.21/32,action=drop",
"priority=3,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-src=192.18.22.22/32,nw-dst=192.18.122.22/32,action=drop",
"priority=2,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-src=192.18.23.23/32,nw-dst=192.18.123.23/32,action=drop",
"priority=1,hard-timeout=0,idle-timeout=0,dl-type=0x800,nw-src=192.18.24.24/32,nw-dst=192.18.124.24/32,action=drop",
"priority=0,hard-timeout=0,idle-timeout=0,dl-type=0x800,action=drop"] by eval
value[code] length ofi

```

```

end

```

## Part II

# Documentation

### 1 Configuration Translation

All the results we present in this section are formalized and verified in Isabelle/HOL [11]. This means that their formal correctness can be trusted a level close to absolute certainty. The definitions and lemmas stated here are merely a repetition of lemmas stated in other theory files. This means that they have been directly set to this document from Isabelle and no typos or hidden assumptions are possible. Additionally, it allows us to omit various helper lemmas that do not help the understanding. However, it causes some notation inaccuracy, as type and function definitions are stated as lemmas or schematic goals.

**theory** *OpenFlow-Documentation*

#### 1.1 Linux Firewall Model

We want to write a program that translates the configuration of a linux firewall to that of an OpenFlow switch. We furthermore want to verify that translation. For this purpose, we need a clear definition of the behavior of the two device types – we need their models and semantics. In case of a linux firewall, this is problematic because a linux firewall is a highly complex device that is ultimately capable of general purpose computation. Creating a comprehensive semantics that encompasses all possible configuration types of a linux firewall is thus highly non-trivial and not useful for the purpose of analysis. We decided to approach the problem from the other side: we created a model that includes only the most basic features. (This implies neglecting IPv6.) Fortunately, many of the highly complex features are rarely essential and even our basic model is still of some use.

We first divided the firewall into subsystems. Given a routing table  $rt$ , the firewall rules  $fw$ , the routing decision for a packet  $p$  can be obtained by *routing-table-semantics*  $rt$  ( $p\text{-dst } p$ ), the firewall decision by *simple-fw*  $fw$   $p$ . We draft the first description of our linux router model:

1. The destination MAC address of an arriving packet is checked: Does it match the MAC

address of the ingress port? If it does, we continue, otherwise, the packet is discarded.

2. The routing decision  $rd \equiv \text{routing-table-semantics } rt \ p$  is obtained.
3. The packet's output interface is updated based on  $rd$ <sup>1</sup>.
4. The firewall is queried for a decision: *simple-fw*  $fw$   $p$ . If the decision is to *Drop*, the packet is discarded.
5. The next hop is computed: If  $rd$  provides a next hop, that is used. Otherwise, the destination address of the packet is used.
6. The MAC address of the next hop is looked up; the packet is updated with it and sent.

We decided that this description is best formalized as an abortable program in the option monad:

```
lemma simple-linux-router  $rt \ fw \ mlf \ ifl \ p \equiv do \{$ 
   $- \leftarrow iface\text{-}packet\text{-}check \ ifl \ p;$ 
   $let \ rd \equiv (routing\text{-}decision) = routing\text{-}table\text{-}semantics \ rt$ 
   $(p\text{-}dst \ p);$ 
   $let \ p = p(p\text{-}oiface := output\text{-}iface \ rd);$ 
   $let \ fd \equiv (firewall\text{-}decision) = simple\text{-}fw \ fw \ p;$ 
   $- \leftarrow (case \ fd \ of \ Decision \ FinalAllow \Rightarrow Some \ () \mid Decision$ 
   $FinalDeny \Rightarrow None);$ 
   $let \ nh = (case \ next\text{-}hop \ rd \ of \ None \Rightarrow p\text{-}dst \ p \mid Some \ a \Rightarrow$ 
   $a);$ 
   $ma \leftarrow mlf \ nh;$ 
   $Some \ (p(p\text{-}l2dst := ma))$ 
 $\}$ 
unfolding  $fromMaybe\text{-}def[symmetric]$  by  $(fact$ 
 $simple\text{-}linux\text{-}router\text{-}def)$ 
```

where  $mlf$  is a function that looks up the MAC address for an IP address.

There are already a few important aspects that have not been modelled, but they are not core essential for the functionality of a firewall. Namely, there is no local traffic from/to the firewall. This is problematic since this model can not generate ARP replies — thus, an equivalent OpenFlow device will not do so, either. Furthermore, this model is problematic because it requires access to a function that looks up a MAC address, something that may not be known at the time of time running a translation to an OpenFlow configuration.

<sup>1</sup>Note that we assume a packet model with input and output interfaces. The origin of this is explained in Section 1.1.2

It is possible to circumvent these problems by inserting static ARP table entries in the directly connected devices and looking up their MAC addresses *a priori*. A test-wise implementation of the translation based on this model showed acceptable results. However, we deemed the *a priori* lookup of the MAC addresses to be rather inelegant and built a second model.

**definition** *simple-linux-router-altered*  $rt\ fw\ ifl\ p \equiv do \{$   
 $let\ rd = routing-table-semantic\ rt\ (p-dst\ p);$   
 $let\ p = p(p-oiface := output-iface\ rd);$   
 $- \leftarrow if\ p-oiface\ p = p-iiface\ p\ then\ None\ else\ Some\ ();$   
 $let\ fd = simple-fw\ fw\ p;$   
 $- \leftarrow (case\ fd\ of\ Decision\ FinalAllow \Rightarrow Some\ () \mid Decision\ FinalDeny \Rightarrow None);$   
 $Some\ p$   
 $\}$

In this model, all access to the MAC layer has been eliminated. This is done by the approximation that the firewall will be asked to route a packet (i.e. be addressed on the MAC layer) iff the destination IP address of the packet causes it to be routed out on a different interface. Because this model does not insert destination MAC addresses, the destination MAC address has to be already correct when the packet is sent. This can only be achieved by changing the subnet of all connected device, moving them into one common subnet<sup>2</sup>.

While a test-wise implementation based on this model also showed acceptable results, the model is still problematic. The check  $p-oiface\ p = p-iiface\ p$  and the firewall require access to the output interface. The details of why this cannot be provided are elaborated in Section 1.3. The intuitive explanation is that an OpenFlow match can not have a field for the output interface. We thus simplified the model even further:

**lemma** *simple-linux-router-nol12*  $rt\ fw\ p \equiv do \{$   
 $let\ rd = routing-table-semantic\ rt\ (p-dst\ p);$   
 $let\ p = p(p-oiface := output-iface\ rd);$   
 $let\ fd = simple-fw\ fw\ p;$   
 $- \leftarrow (case\ fd\ of\ Decision\ FinalAllow \Rightarrow Some\ () \mid Decision\ FinalDeny \Rightarrow None);$   
 $Some\ p$   
 $\}$  **by** (*fact simple-linux-router-nol12-def*)

We continue with this definition as a basis for our translation. Even this strongly altered version and the original linux firewall still behave the same in a substantial amount of cases:

<sup>2</sup>There are cases where this is not possible — A limitation of our system.

## theorem

$\llbracket iface-packet-check\ ifl\ pii \neq None;$   
 $mlf\ (case\ next-hop\ (routing-table-semantic\ rt\ (p-dst\ pii))$   
 $of\ None \Rightarrow p-dst\ pii \mid Some\ a \Rightarrow a) \neq None \rrbracket \implies$   
 $\exists x. \quad map-option\ (\lambda p. \quad p(p-l2dst := x))$   
 $(simple-linux-router-nol12\ rt\ fw\ pii) = simple-linux-router$   
 $rt\ fw\ mlf\ ifl\ pii$   
**by** (*fact rtr-nomac-eq[unfolded fromMaybe-def]*)

The conditions are to be read as “The check whether a received packet has the correct destination MAC never returns *False*” and “The next hop MAC address for all packets can be looked up”. Obviously, these conditions do not hold for all packets. We will show an example where this makes a difference in Section 2.1.

### 1.1.1 Routing Table

The routing system in linux features multiple tables and a system that can use the iptables firewall and an additional match language to select a routing table. Based on our directive, we only focused on the single most used **main** routing table.

We define a routing table entry to be a record (named tuple) of a prefix match, a metric and the routing action, which in turn is a record of an output interface and an optional next-hop address.

**schematic-goal** ( $?rtbl-entry :: ('a::len)\ routing-rule = ()$   
 $routing-match = PrefixMatch\ pfx\ len, metric = met, routing-action = ()$   
 $output-iface = oif-string, next-hop = (h :: 'a\ word\ option) \mid () ..$

A routing table is then a list of these entries:

**lemma** ( $rtbl :: ('a :: len)\ prefix-routing) = (rtbl :: 'a\ routing-rule\ list)$  **by** *rule*

Not all members of the type *prefix-routing* are sane routing tables. There are three different validity criteria that we require so that our definitions are adequate.

- The prefixes have to be 0 in bits exceeding their length.
- There has to be a default rule, i.e. one with prefix length 0. With the condition above, that implies that all its prefix bits are zero and it thus matches any address.
- The entries have to be sorted by prefix length and metric.

The first two are set into code in the following way:

```

lemma valid-prefix (PrefixMatch pfx len)  $\equiv$  pfx && (2 ^
(32 - len) - 1) = (0 :: 32 word)
  by (simp add: valid-prefix-def pfxm-mask-def
mask-eq-decr-exp and commute)
lemma has-default-route rt  $\longleftrightarrow$  ( $\exists r \in \text{set } rt. \text{pfxm-length}$ 
(routing-match r) = 0)
by(fact has-default-route-alt)

```

The third is not needed in any of the further proofs, so we omit it.

The semantics of a routing table is to simply traverse the list until a matching entry is found.

```

schematic-goal routing-table-semantics (rt-entry # rt)
dst-addr = (if prefix-match-semantics (routing-match
rt-entry) dst-addr then routing-action rt-entry else
routing-table-semantics rt dst-addr) by(fact
routing-table-semantics.simps)

```

If no matching entry is found, the behavior is undefined.

### 1.1.2 iptables Firewall

The firewall subsystem in a linux router is not any less complex than any of the other systems. Fortunately, this complexity has been dealt with in [6, 5] already and we can directly use the result.

In short, one of the results is that a complex *iptables* configuration can be simplified to be represented by a single list of matches that only support the following match conditions:

- (String) prefix matches on the input and output interfaces.
- A *prefix-match* on the source and destination IP address.
- An exact match on the layer 4 protocol.
- Interval matches on the source or destination port, e.g.  $p_d \in \{1..1023\}$

The model/type of the packet is adjusted to fit that: it is a record of the fields matched on. This also means that input and output interface are coded to the packet. Given that this information is usually stored alongside the packet content, this can be deemed a reasonable model. In case the output interface is not needed (e.g., when evaluating an OpenFlow table), it can simply be left blank.

Obviously, a simplification into the above match type cannot always produce an equivalent firewall, and the set of accepted packets has to be over- or underapproximated. The reader interested in the details of this is strongly referred to [6]; we are simply going to continue with the result: *simple-fw*.

One property of the simplification is worth noting here: The simplified firewall does not know state and the simplification approximates stateful matches by stateless ones. Thus, the overapproximation of a stateful firewall ruleset that begins with accepting packets of established connections usually begins with a rule that accepts all packets. Dealing with this by writing a meaningful simplification of stateful firewalls is future work.

## 1.2 OpenFlow Switch Model

In this section, we present our model of an OpenFlow switch. The requirements for this model are derived from the fact that it models devices that are the target of a configuration translation. This has two implications:

- All configurations that are representable in our model should produce the correct behavior wrt. their semantics. The problem is that correct here means that the behavior is the same that any real device would produce. Since we cannot possibly account for all device types, we instead focus on those that conform to the OpenFlow specifications. To account for the multiple different versions of the specification (e.g. [2, 3]), we tried making our model a subset of both the oldest stable version 1.0 [2] and the newest available specification version 1.5.1 [3].
- Conversely, our model does not need to represent all possible behavior of an OpenFlow switch, just the behavior that can be invoked by the result of our translation. This is especially useful regarding for controller interaction, but also for MPLS or VLANs, which we did not model in Section 1.1.

More concretely, we set the following rough outline for our model.

- A switch consists of a single flow table.
- A flow table entry consists of a priority, a match condition and an action list.

- The only possible action (we require) is to forward the packet on a port.
- We do not model controller interaction.

Additionally, we decided that we wanted to be able to ensure the validity of the flow table in all qualities, i.e. we want to model the conditions ‘no overlapping flow entries appear’, ‘all match conditions have their necessary preconditions’. The details of this are explained in the following sections.

### 1.2.1 Matching Flow Table entries

Table 3 of Section 3.1 of [2] gives a list of required packet fields that can be used to match packets. This directly translates into the type for a match expression on a single field:

```
schematic-goal (field-match :: of-match-field) ∈ {
  IngressPort (?s::string),
  EtherSrc (?as::48 word), EtherDst (?ad::48 word),
  EtherType (?t::16 word),
  VlanId (?i::16 word), VlanPriority (?p::16 word),
  IPv4Src (?pms::32 prefix-match),
  IPv4Dst (?pmd::32 prefix-match),
  IPv4Proto (?ipp :: 8 word),
  L4Src (?ps :: 16 word) (?ms :: 16 word),
  L4Dst (?pd :: 16 word) (?md :: 16 word)
} by(fact of-match-field-typeset)
```

Two things are worth additional mention: L3 and L4 “addressess”. The *IPv4Src* and *IPv4Dst* matches are specified as “can be subnet masked” in [2], whereas [3] states clearly that arbitrary bitmasks can be used. We took the conservative approach here. Our alteration of *L4Src* and *L4Dst* is more grave. While [2] does not state anything about layer 4 ports and masks, [3] specifically forbids using masks on them. Nevertheless, OpenVSwitch [1] and some other implementations support them. We will explain in detail why we must include bitmasks on layer 4 ports to obtain a meaningful translation in Section 1.3.

One *of-match-field* is not enough to classify a packet. To match packets, we thus use entire sets of match fields. As Guha *et al.* [7] noted<sup>3</sup>, executing a set of given *of-match-fields* on a packet requires careful consideration. For example, it is not meaningful to use *IPv4Dst* if the given packet is not actually an IP packet,

<sup>3</sup>See also: [8, §section>2.3]

i.e. *IPv4Dst* has the prerequisite of *EtherType 2048* being among the match fields. Guha *et al.* decided to use the fact that the preconditions can be arranged on a directed acyclic graph (or rather: an acyclic forest). They evaluated match conditions in a manner following that graph: first, all field matches without preconditions are evaluated. Upon evaluating a field match (e.g., *EtherType 2048*), the matches that had their precondition fulfilled by it (e.g., *IPv4Src* and *IPv4Dst* in this example) are evaluated. This mirrors the faulty behavior of some implementations (see [7]). Adopting that behavior into our model would mean that any packet matches against the field match set  $\{IPv4Dst (PrefixMatch 134744072\ 32)\}$  instead of just those destined for 8.8.8.8 or causing an error. We found this to be unsatisfactory.

To solve this problem, we made three definitions. The first, *match-no-prereq* matches an *of-match-field* against a packet without considering prerequisites. The second, *prerequisites*, checks for a given *of-match-field* whether its prerequisites are in a set of given match fields. Especially:

#### lemma

```
prerequisites (VlanPriority pri) m = (∃ id. let v = VlanId
id in v ∈ m ∧ prerequisites v m)
prerequisites (IPv4Proto pr) m = (let v = EtherType
0x0800 in v ∈ m ∧ prerequisites v m)
prerequisites (IPv4Src a) m = (let v = EtherType 0x0800
in v ∈ m ∧ prerequisites v m)
prerequisites (IPv4Dst a) m = (let v = EtherType 0x0800
in v ∈ m ∧ prerequisites v m)
prerequisites (L4Src p msk) m = (∃ proto ∈
{TCP,UDP,L4-Protocol.SCTP}. let v = IPv4Proto proto
in v ∈ m ∧ prerequisites v m)
prerequisites (L4Dst p msk) m = prerequisites (L4Src un-
defined undefined) m
by(fact prerequisites.simps)+
```

Then, to actually match a set of *of-match-field* against a packet, we use the option type:

#### lemma OF-match-fields m p =

```
(if ∃ f ∈ m. ¬prerequisites f m then None else
if ∀ f ∈ m. match-no-prereq f p then Some True else
Some False)
by(fact OF-match-fields-alt)
```

### 1.2.2 Evaluating a Flow Table

In the previous section, we explained how we match the set of match fields belonging to a single flow entry against a packet. This section explains how the correct



flow entry from a table can be selected. To prevent to much entanglement with the previous section, we assume an arbitrary match function  $\gamma$ . This function  $\gamma$  takes the match condition  $m$  from a flow entry *OFEntry* *priority*  $m$  *action* and decides whether a packet matches those.

The flow table is simply a list of flow table entries *flow-entry-match*. Deciding the right flow entry to use for a given packet is explained in the OpenFlow specification [2], Section 3.4:

Packets are matched against flow entries based on prioritization. An entry that specifies an exact match (i.e., has no wildcards) is always the highest priority<sup>4</sup>. All wildcard entries have a priority associated with them. Higher priority entries must match before lower priority ones. If multiple entries have the same priority, the switch is free to choose any ordering.

We use the term “overlapping” for the flow entries that can cause a packet to match multiple flow entries with the same priority. Guha *et al.* [7] have dealt with overlapping. However, the semantics for a flow table they presented [7, Figure 5] is slightly different from what they actually used in their theory files. We have tried to reproduce the original inductive definition (while keeping our abstraction  $\gamma$ ), in Isabelle/HOL<sup>5</sup>:

**lemma**  $\gamma$  (ofe-fields fe) p = True  $\implies$   
 $\forall fe' \in \text{set } (ft1 @ ft2). \text{ ofe-prio } fe' > \text{ ofe-prio } fe \longrightarrow \gamma$   
 $(\text{ofe-fields } fe') p = \text{False} \implies$   
 $\text{guha-table-semantics } \gamma (ft1 @ fe \# ft2) p (\text{Some } (\text{ofe-action } fe))$   
 $\forall fe \in \text{set } ft. \gamma (\text{ofe-fields } fe) p = \text{False} \implies$   
 $\text{guha-table-semantics } \gamma ft p \text{ None } \text{by}(\text{fact } \text{guha-matched } \text{guha-unmatched})+$

Guha *et al.* have deliberately made their semantics non-deterministic, to match the fact that the switch “may choose any ordering”. This can lead to undesired results:

**lemma**  $\text{CARD}('action) \geq 2 \implies \exists ff. \gamma ff p \implies \exists ft (a1 :: 'action) (a2 :: 'action). a1 \neq a2 \wedge \text{guha-table-semantics } \gamma$   
 $ft p (\text{Some } a1) \wedge \text{guha-table-semantics } \gamma ft p (\text{Some } a2)$   
 $\text{by}(\text{fact } \text{guha-table-semantics-ex2res})$

<sup>4</sup>This behavior has been deprecated.

<sup>5</sup>The original is written in Coq [4] and we can not use it directly.

This means that, given at least two distinct actions exist and our matcher  $\gamma$  is not false for all possible match conditions, we can say that a flow table and two actions exist such that both actions are executed. This can be misleading, as the switch might choose an ordering on some flow table and never execute some of the (overlapped) actions.

Instead, we decided to follow Section 5.3 of the specification [3], which states:

If there are multiple matching flow entries, the selected flow entry is explicitly undefined.

This still leaves some room for interpretation, but it clearly states that overlapping flow entries are undefined behavior, and undefined behavior should not be invoked. Thus, we came up with a semantics that clearly indicates when undefined behavior has been invoked:

**lemma**

*OF-priority-match*  $\gamma$  *flow-entries* *packet* = (  
 $\text{let } m = \text{filter } (\lambda f. \gamma (\text{ofe-fields } f) \text{ packet}) \text{ flow-entries};$   
 $m' = \text{filter } (\lambda f. \forall fo \in \text{set } m. \text{ ofe-prio } fo \leq \text{ ofe-prio } f)$   
 $m \text{ in}$   
 $\text{case } m' \text{ of } [] \Rightarrow \text{NoAction}$   
 $| [s] \Rightarrow \text{Action } (\text{ofe-action } s)$   
 $| - \Rightarrow \text{Undefined})$

**unfolding** *OF-priority-match-def* ..

The definition works the following way<sup>6</sup>:

1. The flow table is filtered for those entries that match, the result is called  $m$ .
2.  $m$  is filtered again, leaving only those entries for which no entries with lower priority could be found, i.e. the matching flow table entries with minimal priority. The result is called  $m'$ .
3. A case distinction on  $m'$  is made. If only one matching entry was found, its action is returned for execution. If  $m$  is empty, the flow table semantics returns *NoAction* to indicate that the flow table does not decide an action for the packet. If, not zero or one entry is found, but more, the special value *Undefined* for indicating undefined behavior is returned.

<sup>6</sup>Note that the order of the flow table entries is irrelevant. We could have made this definition on sets but chose not to for consistency.



The use of *Undefined* immediately raises the question in which condition it cannot occur. We give the following definition:

**lemma** *check-no-overlap*  $\gamma$  *ft* =  $(\forall a \in \text{set } ft. \forall b \in \text{set } ft. (a \neq b \wedge \text{ofe-prio } a = \text{ofe-prio } b) \longrightarrow \neg(\exists p. \gamma (\text{ofe-fields } a) p \wedge \gamma (\text{ofe-fields } b) p))$  **unfolding** *check-no-overlap-alt* *check-no-overlap2-def* **by** *force*

Together with distinctness of the flow table, this provides the absence of *Undefined*<sup>7</sup>:

**lemma**  $\llbracket \text{check-no-overlap } \gamma \text{ ft}; \text{distinct } ft \rrbracket \implies \text{OF-priority-match } \gamma \text{ ft } p \neq \text{Undefined}$  **by** (*simp add: no-overlapsI no-overlaps-not-undefined*)

Given the absence of overlapping or duplicate flow entries, we can show two interesting equivalences. the first is the equality to the semantics defined by Guha *et al.*:

**lemma**  $\llbracket \text{check-no-overlap } \gamma \text{ ft}; \text{distinct } ft \rrbracket \implies \text{OF-priority-match } \gamma \text{ ft } p = \text{option-to-ftb } d \longleftrightarrow \text{guha-table-semantics } \gamma \text{ ft } p \text{ } d$  **by** (*simp add: guha-equal no-overlapsI*)

where *option-to-ftb* maps between the return type of *OF-priority-match* and an option type as one would expect.

The second equality for *OF-priority-match* is one that helps reasoning about flow tables. We define a simple recursive traversal for flow tables:

**lemma**  
 $\text{OF-match-linear } \gamma \llbracket p = \text{NoAction}$   
 $\text{OF-match-linear } \gamma (a \# as) p = (\text{if } \gamma (\text{ofe-fields } a) p \text{ then}$   
 $\text{Action } (\text{ofe-action } a) \text{ else OF-match-linear } \gamma \text{ as } p)$   
**by**(*fact OF-match-linear.simps*)+

For this definition to be equivalent, we need the flow table to be sorted:

**lemma**  
 $\llbracket \text{no-overlaps } \gamma f ; \text{sorted-descending } (\text{map ofe-prio } f) \rrbracket \implies$   
 $\text{OF-match-linear } \gamma f p = \text{OF-priority-match } \gamma f p$   
**by**(*fact OF-eq*)

As the last step, we implemented a serialization function for flow entries; it has to remain unverified. The serialization function deals with one little inaccuracy: We have modelled the *IngressPort* match to use the interface name, but OpenFlow requires numerical interface IDs instead. We deemed that pulling this translation step into the main translation would only make the

<sup>7</sup>It is slightly stronger than necessary, overlapping rules might be shadowed and thus never influence the behavior.

correctness lemma of the translation more complicated while not increasing the confidence in the correctness significantly. We thus made replacing interface names by their ID part of the serialization.

Having collected all important definitions and models, we can move on to the conversion.

### 1.3 Translation Implementation

This section explains how the functions that are executed sequentially in a linux firewall can be compressed into a single OpenFlow table. Creating this flow table in a single step would be immensely complicated. We thus divided the task into several steps using the following key insights:

- All steps that are executed in the linux router can be formulated as a firewall, more specifically, a generalization of *simple-fw* that allows arbitrary actions instead of just accept and drop.
- A function that computes the conjunction of two *simple-fw* matches is already present. Extending this to a function that computes the join of two firewalls is relatively simple. This is explained in Section 1.3.1

#### 1.3.1 Chaining Firewalls

This section explains how to compute the join of two firewalls.

The basis of this is a generalization of *simple-fw*. Instead of only allowing *Accept* or *Drop* as actions, it allows arbitrary actions. The type of the function that evaluates this generalized simple firewall is *generalized-sfw*. The definition is straightforward:

**lemma**  
 $\text{generalized-sfw } \llbracket p = \text{None}$   
 $\text{generalized-sfw } (a \# as) p = (\text{if } (\text{case } a \text{ of } (m, -) \Rightarrow \text{simple-matches } m) p) \text{ then Some } a \text{ else generalized-sfw } as \text{ } p)$   
**by**(*fact generalized-sfw.simps*)+

Based on that, we asked: if  $fw_1$  makes the decision  $a$  (where  $a$  is the second element of the result tuple from *generalized-sfw*) and  $fw_2$  makes the decision  $b$ , how can we compute the firewall that makes the decision  $(a, b)$ <sup>8</sup>. One possible answer is given by the following definition:

<sup>8</sup>Note that tuples are right-associative in Isabelle/HOL, i.e.,  $(a, b, c)$  is a pair of  $a$  and the pair  $(b, c)$

**lemma** *generalized-fw-join*  $l1\ l2 \equiv [(u,a,b). (m1,a) \leftarrow l1, (m2,b) \leftarrow l2, u \leftarrow (\text{case } \text{simple-match-and } m1\ m2 \text{ of } \text{None} \Rightarrow [] \mid \text{Some } s \Rightarrow [s])]$   
**by** (*fact generalized-fw-join-def* [*unfolded option2list-def*]) +

This definition validates the following lemma:

**lemma** *generalized-sfw* (*generalized-fw-join*  $fw_1\ fw_2$ )  $p = \text{Some } (u, d_1, d_2) \longleftrightarrow (\exists r_1\ r_2. \text{generalized-sfw } fw_1\ p = \text{Some } (r_1, d_1) \wedge \text{generalized-sfw } fw_2\ p = \text{Some } (r_2, d_2) \wedge \text{Some } u = \text{simple-match-and } r_1\ r_2)$   
**by** (*auto dest: generalized-fw-joinD sym simp add: generalized-fw-joinI*)

Thus, *generalized-fw-join* has a number of applications. For example, it could be used to compute a firewall ruleset that represents two firewalls that are executed in sequence.

**definition** *simple-action-conj*  $a\ b \equiv (\text{if } a = \text{simple-action.Accept} \wedge b = \text{simple-action.Accept} \text{ then } \text{simple-action.Accept} \text{ else } \text{simple-action.Drop})$

**definition** *simple-rule-conj*  $\equiv (\text{uncurry SimpleRule} \circ \text{apsnd} \circ (\text{uncurry simple-action-conj}))$

**theorem** *simple-fw*  $rs_1\ p = \text{Decision FinalAllow} \wedge \text{simple-fw } rs_2\ p = \text{Decision FinalAllow} \longleftrightarrow \text{simple-fw } (\text{map simple-rule-conj } (\text{generalized-fw-join } (\text{map simple-rule-dtor } rs_1) (\text{map simple-rule-dtor } rs_2)))\ p = \text{Decision FinalAllow}$

**unfolding** *simple-rule-conj-def*  
*simple-action-conj-def* [*abs-def*] **using** *simple-fw-join*  
**by** (*force simp add: comp-def apsnd-def map-prod-def case-prod-unfold uncurry-def* [*abs-def*])

Using the join, it should be possible to compute any  $n$ -ary logical operation on firewalls. We will use it for something somewhat different in the next section.

### 1.3.2 Translation Implementation

This section shows the actual definition of the translation function, in Figure 1. Before beginning the translation, the definition checks whether the necessary preconditions are valid. This first two steps are to convert  $fw$  and  $rt$  to lists that can be evaluated by *generalized-sfw*. For  $fw$ , this is done by *map simple-rule-dtor*, which just deconstructs *simple-rules* into tuples of match and action. For  $rt$ , we made a firewall ruleset with rules that use prefix matches on the destination IP address. The next step is to join the two rulesets. The result of the join is a ruleset with rules  $r$  that only match if both, the corresponding firewall rule  $fwr$  and the corresponding routing rule  $rr$  matches. The data accompanying  $r$  is the port from  $rr$

and the firewall decision from  $fwr$ . Next, descending priorities are added to the rules using *map (apfst word-of-nat) \circ annotate-rlen*. If the number of rules is too large to fit into the  $2^{16}$  priority classes, an error is returned. Otherwise, the function *pack-OF-entries* is used to convert the  $(16\ \text{word} \times 32\ \text{simple-match} \times \text{char list} \times \text{simple-action})\ \text{list}$  to an OpenFlow table. While converting the  $\text{char list} \times \text{simple-action}$  tuple is straightforward, converting the *simple-match* to an equivalent list of *of-match-field set* is non-trivial. This is done by the function *simple-match-to-of-match*.

The main difficulties for *simple-match-to-of-match* lie in making sure that the prerequisites are satisfied and in the fact that a *simple-match* operates on slightly stronger match expressions.

- A *simple-match* allows a (string) prefix match on the input and output interfaces. Given a list of existing interfaces on the router *ifs*, the function has to insert flow entries for each interface matching the prefix.
- A *simple-match* can match ports by an interval. Now it becomes obvious why Section 1.2.1 added bitmasks to *L4Src* and *L4Dst*. Using the algorithm to split word intervals into intervals that can be represented by prefix matches from [6], we can efficiently represent the original interval by a few (32 in the worst case) prefix matches and insert flow entries for each of them.<sup>9</sup>

The following lemma characterizes *simple-match-to-of-match*:

**lemma** *simple-match-to-of-match*:

**assumes**

*simple-match-valid*  $r$   
 $p\text{-iface } p \in \text{set } ifs$   
 $\text{match-iface } (oiface\ r)\ (p\text{-oiface } p)$   
 $p\text{-l2type } p = 0x800$

**shows**

$\text{simple-matches } r\ p \longleftrightarrow (\exists gr \in \text{set } (\text{simple-match-to-of-match } r\ ifs). \text{OF-match-fields } gr\ p = \text{Some True})$

**using** *assms* *simple-match-to-of-matchD*  
*simple-match-to-of-matchI* **by** *blast*

The assumptions are to be read as follows:

<sup>9</sup>It might be possible to represent the interval match more efficiently than a split into prefixes. However, that would produce overlapping matches (which is not a problem if we assign separate priorities) and we did not have a verified implementation of an algorithm that does so.

```

lemma lr-of-tran rt fw ifs  $\equiv$ 
if  $\neg$  (no-oif-match fw  $\wedge$  has-default-policy fw  $\wedge$  simple-fw-valid fw  $\wedge$  valid-prefixes rt  $\wedge$  has-default-route rt  $\wedge$ 
distinct ifs)
  then Inl "Error in creating OpenFlow table: prerequisites not satisfied"
  else (
let
  nfw = map simple-rule-dtor fw;
  frt = map ( $\lambda r$ . (route2match r, output-iface (routing-action r))) rt;
  nrd = generalized-fw-join frt nfw;
  ard = (map (apfst of-nat)  $\circ$  annotate-rlen) nrd
in
  if length nrd < unat (- 1 :: 16 word)
  then Inr (pack-OF-entries ifs ard)
  else Inl "Error in creating OpenFlow table: priority number space exhausted"
)
unfolding Let-def lr-of-tran-def lr-of-tran-fbs-def lr-of-tran-s1-def comp-def route2match-def by force

```

Figure 1: Function for translating a '*i* simple-rule list, a '*i* routing-rule list, and a list of interfaces to a flow table.

- The match  $r$  has to be valid, i.e. it has to use *valid-prefix* matches, and it cannot use anything other than 0-65535 for the port matches unless its protocol match ensures *TCP*, *UDP* or *L4-Protocol.SCTP*.
- *simple-match-to-of-match* cannot produce rules for packets that have input interfaces that are not named in the interface list.
- The output interface of  $p$  has to match the output interface match of  $r$ . This is a weakened formulation of *oiface r = ifaceAny*, since

*match-iface ifaceAny i*

. We require this because OpenFlow field matches cannot be used to match on the output port — they are supposed to match a packet and decide an output port.

- The *simple-match* type was designed for IP(v4) packets, we limit ourselves to them.

The conclusion then states that the *simple-match*  $r$  matches iff an element of the result of *simple-match-to-of-match* matches. The third assumption is part of the explanation why we did not use *simple-linux-router-altered: simple-match-to-of-match* cannot deal with output

interface matches. Thus, before passing a generalized simple firewall to *pack-OF-entries*, we would have to set the output ports to *ifaceAny*. A system replace output interface matches with destination IP addresses has already been formalized and will be published in a future version of [5]. For now, we limit ourselves to firewalls that do not do output port matching, i.e., we require *no-oif-match fw*.

Given discussed properties, we present the central theorem for our translation in Figure 2. The first two assumptions are limitations on the traffic we make a statement about. Obviously, we will never see any packets with an input interface that is not in the interface list. Furthermore, we do not state anything about non-IPv4 traffic. (The traffic will remain unmatched in by the flow table, but we have not verified that.) The last assumption is that the translation does not return a run-time error. The translation will return a run-time error if the rules can not be assigned priorities from a 16 bit integer, or when one of the following conditions on the input data is not satisfied:

**lemma**

```

 $\neg$  no-oif-match fw  $\vee$ 
 $\neg$  has-default-policy fw  $\vee$ 
 $\neg$  simple-fw-valid fw  $\vee$ 
 $\neg$  valid-prefixes rt  $\vee$ 
 $\neg$  has-default-route rt  $\vee$ 
 $\neg$  distinct ifs  $\implies$ 

```

```

theorem
fixes
  p :: (32, 'a) simple-packet-ext-scheme
assumes
  p-iiface p ∈ set ifs and p-l2type p = 0x800
  lr-of-tran rt fw ifs = Inr oft
shows
  OF-priority-match OF-match-fields-safe oft p = Action [Forward oif]  $\longleftrightarrow$  simple-linux-router-nol12 rt fw p =
  (Some (p(p-oiface := oif)))
  OF-priority-match OF-match-fields-safe oft p = Action []  $\longleftrightarrow$  simple-linux-router-nol12 rt fw p = None
  OF-priority-match OF-match-fields-safe oft p  $\neq$  NoAction  $\longleftrightarrow$  OF-priority-match OF-match-fields-safe oft p  $\neq$ 
  Undefined
  OF-priority-match OF-match-fields-safe oft p = Action ls  $\longrightarrow$  length ls  $\leq$  1
   $\exists$  ls. length ls  $\leq$  1  $\wedge$  OF-priority-match OF-match-fields-safe oft p = Action ls
using assms lr-of-tran-correct by simp-all

```

Figure 2: Central theorem on *lr-of-tran*

$\exists err. lr-of-tran rt fw ifs = Inl err$  **unfolding** *lr-of-tran-def*  
**by**(*simp split: if-splits*)

### 1.3.3 Comparison to Exodus

We are not the first researchers to attempt automated static migration to SDN. The (only) other attempt we are aware of is *Exodus* by Nelson *et al.* [10].

There are some fundamental differences between Exodus and our work:

- Exodus focuses on Cisco IOS instead of linux.
- Exodus does not produce OpenFlow rulesets, but FlowLog [9] controller programs.
- Exodus is not limited to using a single flow table.
- Exodus requires continuous controller interaction for some of its functions.
- Exodus attempts to support as much functionality as possible and has implemented support for dynamic routing, VLANs and NAT.
- Nelson *et al.* reject the idea that the translation could or should be proven correct.

## 2 Evaluation

In Section 1, we have made lots of definitions and created lots of models. How far these models are in accordance with the real world has been up to the vigilance of the reader. This section attempts to alleviate this burden by providing some examples.

### 2.1 Mininet Examples

The first example is designed to be minimal while still showing the most important properties of our conversion. For this purpose, we used a linux firewall F, that we want to convert. We gave it two interfaces, and connected one client each. Its original configuration and the ruleset resulting from the translation is shown in Figure 3. (The list of interfaces can be extracted from the routing table; **s1-lan** received port number 1.) While the configuration does not fulfil any special function (especially, no traffic from the interface **s1-wan** is permitted), it is small enough to let us have a detailed look. More specifically, we can see how the only firewall rule (Line 2) got combined with the first rule of the routing table to form Line 1 of the OpenFlow rules. This also shows why the bitmasks on the layer 4 ports are necessary. If we only allowed exact matches, we would have  $2^{15}$  rules instead of just one. Line 2 of the OpenFlow ruleset has been formed by combining the default drop policy with Line 1 of the routing table.

```

1 :FORWARD DROP [0:0]
2 -A FORWARD -d 10.0.2.0/24 -i s1-lan -p tcp -
  m tcp --sport 32768:65535 --dport 80 -j
  ACCEPT

```

(a) FORWARD chain

```

1 10.0.2.0/24 dev s1-wan proto kernel scope
  link src 10.0.2.4
2 10.0.1.0/24 dev s1-lan proto kernel scope
  link src 10.0.1.1
3 default via 10.0.2.1 dev s1-wan

```

(b) Routing table (sorted)

```

1 priority=4,hard_timeout=0,idle_timeout=0,in_port=1,dl_type=0x800,nw_proto=6,nw_dst=10.0.2.0/24,
  tp_src=32768/0x8000,tp_dst=80,action=output:2
2 priority=3,hard_timeout=0,idle_timeout=0,dl_type=0x800,nw_dst=10.0.2.0/24,action=drop
3 priority=2,hard_timeout=0,idle_timeout=0,dl_type=0x800,nw_dst=10.0.1.0/24,action=drop
4 priority=1,hard_timeout=0,idle_timeout=0,in_port=1,dl_type=0x800,nw_proto=6,nw_dst=10.0.2.0/24,
  tp_src=32768/0x8000,tp_dst=80,action=output:2
5 priority=0,hard_timeout=0,idle_timeout=0,dl_type=0x800,action=drop

```

(c) Resulting OpenFlow rules

Figure 3: Example Network 1 – Configuration

In a similar fashion, Line 2 of the routing rules has also been combined with the two firewall rules. However, as 10.0.2.0/24 from the firewall and 10.0.1.0/24 from the routing table have no common elements, no rule results from combining Line 2 and Line 2. In a similar fashion, the rest of the OpenFlow ruleset can be explained.

We feel that it is also worth noting again that it is necessary to change the IP configuration of the two devices attached to F. Assuming they are currently configured with, e.g., 10.0.1.100/24 and 10.0.2.1/24, the subnet would have to be changed from 24 to 22 or lower to ensure that a common subnet is formed and the MAC layer can function properly.

Next, we show a somewhat more evolved example. Its topology is depicted in Figure 4a. As before, we called the device to be replaced F. It is supposed to implement the simple policy that the clients H1 and H2 are allowed to communicate with the outside world via HTTP, ICMP is generally allowed, any other traffic is to be dropped (we neglected DNS for this example). We used the iptables configuration that is shown in Figure 4b. The routing table is the same as in the first example network.

The topology has been chosen for a number of reasons: we wanted one device which is not inside a common subnet with F and thus requires no reconfiguration for the translation. Moreover, we wanted two devices in a network that can communicate with each other while being overheard by F. For this purpose, we added two

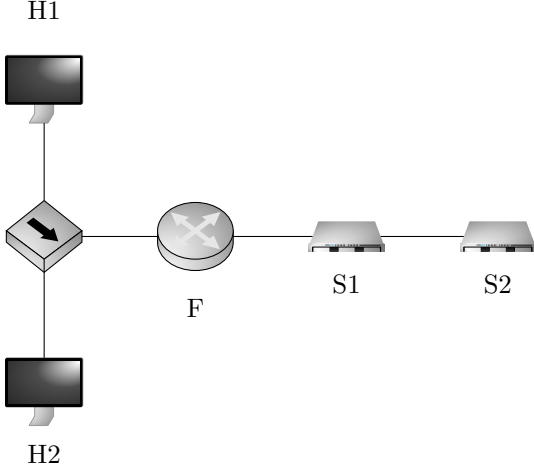
clients H1 and H2 instead of just one. We connected them with a broadcasting device.<sup>10</sup>

Executing our conversion function results in 36 rules<sup>11</sup>, we decided not to include them here. Comparing to the first example network, the size of the ruleset seems relatively high. This can be explained by the port matches: 1024-65535 has to be expressed by 6 different matches, `tp_src=1024/0xfc00`, `tp_src=2048/0xf800`, ..., `tp_src=32768/0x8000` (or `tp_dst` respectively). When installing these rules, we also have to move all of H1, H2 and S1 into a common subnet. We chose 10.0.0.0/16 and updated the IP configuration of the three hosts accordingly. As discussed, the configuration of S2 did not have to be updated, as it does not share any subnet with F. We then tested reachability for TCP 22 and 80 and ICMP. The connectivity between all pairs of hosts (H1,H2,S1 and S2) remained the same compared to before the conversion. This shows that the concept can be made to work.

However, the example also reveals a flaw: When substituting the more complete model of a linux firewall with the simple one in Section 1.1, we assumed that the check whether the correct MAC address is set and the packets are destined for the modelled device would never fail — we assumed that all traffic arriving at a device is actually destined for it. Obviously, this network violates

<sup>10</sup>For the lack of a hub in mininet, we emulated one with an OpenFlow switch.

<sup>11</sup>If we had implemented some spoofing protection by adding `! -s 10.0.1.0/24` to the respective rule, the number of rules would have been increased to 312. This is because a cross product of two prefix splits would occur.



(a) Topology

```

1 :FORWARD DROP [0:0]
2 -A FORWARD -p icmp -j ACCEPT
3 -A FORWARD -i s1-lan -p tcp -m tcp --sport
  1024:65535 --dport 80 -j ACCEPT
4 -A FORWARD -d 10.0.1.0/24 -i s1-wan -p tcp -m tcp
  --sport 80 --dport 1024:65535 -j ACCEPT

```

(b) FORWARD chain

Figure 4: Example Network 2

this assumption. We can trigger this in many ways, for example by sending an ICMP ping from H1 to H2. This will cause the generated rule `priority=7, icmp, nw_dst=10.0.1.0/24 actions=output:1` (where port 1 is the port facing H1 and H2) to be activated twice. This is obviously not desired behavior. Dealing with this is, as mentioned, future work.

## 2.2 Performance Evaluation

Unfortunately, we do not have any real-world data that does not use output port matches as required in Section 1.3. There is thus no way to run the translation on the real-world firewall rulesets we have available and obtain a meaningful result. Nevertheless, we can use a real-world ruleset to evaluate the performance of our translation. For this purpose, we picked the largest firewall from the firewall collection from [6]. A significant amount of time is necessary to convert its **FORWARD** chain including 4946 rules<sup>12</sup> to the required simplified firewall form. Additionally to the simplified firewall, we acquired the routing table (26 entries) from the same machine. We then evaluated the time necessary to complete the translation and the size of the resulting rule-set when using only the first  $n$  simple firewall rules and the full routing table. The result is shown in Figure 5.

<sup>12</sup>In the pre-parsed and already normalized version we used for this benchmark, it took 45s. The full required time lies closer to 11min as stated in [6].

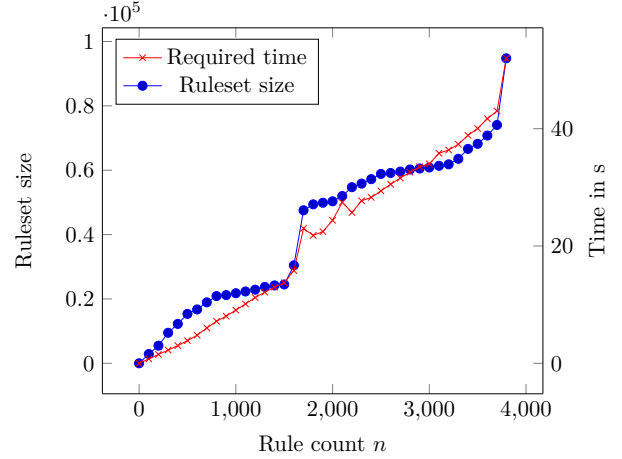


Figure 5: Benchmark

Given the time necessary to complete the conversion of the iptables firewall to a simple firewall, it is reasonable to say that the translation function is efficient enough. At first glance, size of the resulting ruleset seems high. This can be explained by two facts:

- The firewall contains a large number of rules with port matches that allow the ports 1-65535, which requires 16 OpenFlow rules.
- Some combinations of matches from the firewall and the routing table cannot be ruled out, since



the firewall match might only contain an output port and the rule can thus only apply for the packets matching a few routing table entries. However, the translation is not aware of that and can thus not remove the combination of the firewall rule and other routing table entries.

In some rules, the conditions above coincide, resulting in 416 ( $= 16 \cdot 26$ ) rules. To avoid the high number of rules resulting from the port matches, rules that forbid packets with source or destination port 0 could be added to the start of the firewall and the 1-65535 could be removed; dealing with the firewall / routing table problem is part of the future work on output interfaces.

### 3 Conclusion and Future Work

We believe that we have shown that it is possible to translate at least basic configurations of a linux firewall into OpenFlow rulesets while preserving the most important aspects of the behavior. We recognize that our system has limited practical applicability. One possible example would be a router or firewall inside a company network whose state tables have been polluted by special attack traffic. Our translation could provide an OpenFlow based stateless replacement. However, given the current prerequisites the implementation has on the configuration, this application is relatively unlikely.

For the configuration translation, we have contributed formal models of a linux firewall and of an OpenFlow switch. Furthermore, the function that joins two firewalls and the function that translates a simplified match from [6] to a list of equivalent OpenFlow field match sets are contributions that we think are likely to be of further use.

We want to explicitly formulate the following two goals for our future work:

- We want to deal with output interface matches. The idea is to formulate and verify a destination interface / destination IP address rewriting that can exchange output interfaces and destination IP addressed in a firewall, based on the information from the routing table.<sup>13</sup>
- We want to develop a system that can provide a stricter approximation of stateful matches so our translation will be applicable in more cases.

<sup>13</sup>As of now this has already been implemented, but is not yet fully ready.

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