

# Jive Data and Store Model

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## **Abstract**

This document presents the formalization of an object-oriented data and store model in ISABELLE/HOL. This model is being used in the **J**ava **I**nteractive **V**erification **E**nvironment, JIVE.



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# 1 Introduction

JIVE [MPH00, Jiv] is a verification system that is being developed at the University of Kaiserslautern and at the ETH Zürich. It is an interactive special-purpose theorem prover for the verification of object-oriented programs on the basis of a partial-correctness Hoare-style programming logic. JIVE operates on JAVA-KE [PHGR05], a desugared subset of sequential Java which contains all important features of object-oriented languages (subtyping, exceptions, static and dynamic method invocation, etc.). JIVE is written in Java and currently has a size of about 40,000 lines of code.

JIVE is able to operate on completely unannotated programs, allowing the user to dynamically add specifications. It is also possible to preliminarily annotate programs with invariants, pre- and postconditions using the specification language JML [LBR99]. In practice, a mixture of both techniques is employed, in which the user extends and refines the pre-annotated specifications during the verification process. The program to be verified, together with the specifications, is translated to Hoare sequents. Program and pre-annotated specifications are translated during startup, while the dynamically added specifications are translated whenever they are entered by the user. Hoare sequents have the shape  $\mathcal{A} \triangleright \{ \mathbf{P} \} \text{pp} \{ \mathbf{Q} \}$  and express that for all states  $S$  that fulfill  $\mathbf{P}$ , if the execution of the program part  $\text{pp}$  terminates, the state that is reached when  $\text{pp}$  has been evaluated in  $S$  must fulfill  $\mathbf{Q}$ . The so-called assumptions  $\mathcal{A}$  are used to prove recursive methods.

JIVE's logic contains so-called Hoare rules and axioms. The rules consist of one or more Hoare sequents that represent the assumptions of the rule, and a Hoare sequent which is the conclusion of the rule. Axioms consist of only one Hoare sequent; they do not have assumptions. Therefore, axioms represent the known facts of the Hoare logic.

To prove a program specification, the user directly works on the program source code. Proofs can be performed in backward direction and in forward direction. In backward direction, an initial open proof goal is reduced to new, smaller open subgoals by applying a rule. This process is repeated for the smaller subgoals until eventually each open subgoal can be closed by the application of an axiom. If all open subgoals are proven by axioms, the initial goal is proven as well.

In forward direction, the axioms can be used to establish known facts about the statements of a given program. The rules are then used to produce new facts from these already known facts. This way, facts can be constructed for parts of the program.

A large number of the rules and axioms of the Hoare logic is related to the structure of the program part that is currently being examined. Besides these, the logic also contains rules that manipulate the pre- or postcondition of the examined subgoal without affecting the current program part selection. A prominent member of this kind of rules is the rule of consequence<sup>1</sup>:

$$\frac{\mathbf{PP} \Rightarrow \mathbf{P} \quad \mathcal{A} \triangleright \{ \mathbf{P} \} \text{pp} \{ \mathbf{Q} \} \quad \mathbf{Q} \Rightarrow \mathbf{QQ}}{\mathcal{A} \triangleright \{ \mathbf{PP} \} \text{pp} \{ \mathbf{QQ} \}}$$

It plays a special role in the Hoare logic because it additionally requires implications between stronger and weaker conditions to be proven. If a JIVE proof contains an application of the rule of consequence, the implication is attached to the proof tree node that documents this rule application; these attachments are called lemmas. JIVE sends these lemmas to an associated

<sup>1</sup>In JIVE, the rule of consequence is part of a larger rule which serves several purposes at once. Since we want to focus on the rule of consequence, we left out the parts that are irrelevant in this context.

general purpose theorem prover where the user is required to prove them. Currently, JIVE supports ISABELLE/HOL as associated prover. It is required that all lemmas that are attached to any node of a proof tree are proven before the initial goal of the proof tree is accepted as being proven.

In order to prove these logical predicates, ISABELLE/HOL needs a data and store model of JAVA-KE. This model acts as an interface between JIVE and ISABELLE/HOL.

The first paper-and-pencil formalization of the data and store model was given in Arnd Poetzsch-Heffter's habilitation thesis [PH97, Sect. 3.1.2]. The first machine-supported formalization was performed in PVS by Peter Müller, by translating the axioms given in [PH97] to axioms in PVS. The formalization presented in this report extends the PVS formalization. The axioms have been replaced by conservative extensions and proven lemmas, thus there is no longer any possibility to accidentally introduce unsoundness.

Some changes were made to the PVS theories during the conversion. Some were caused due to the differences in the tools ISABELLE/HOL and PVS, but some are more conceptual. Here is a list of the major changes.

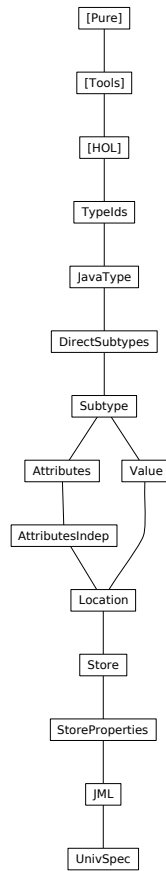
- In PVS, function arguments were sometimes restricted to subtypes. In ISABELLE/HOL, unintended usage of functions is left unspecified.
- In PVS, the program-independent theories were parameterized by the datatypes that were generated for the program to be verified. In ISABELLE/HOL, we just build on the generated theories. This makes the whole setting easier. The drawback is that we have to run the theories for each program we want to verify. But the proof scripts are designed in a way that they will work if the basic program-dependent theories are generated in the proper way. Since we can create an image of a proof session before starting actual verification we do not run into time problems either.
- The subtype relation is based on the direct subtype relation between classes and interfaces. We prove that subtyping forms a partial order. In the PVS version subtyping was expressed by axioms that described the subtype relation for the types appearing in the Java program to be verified.

Besides these changes we also added new concepts to the model. We can now deal with static fields and arrays. This way, the model supports programming languages that are much richer than JAVA-KE to allow for future extensions of JIVE.

Please note that although the typographic conventions in Isabelle suggest that constructors start with a capital letter while types do not, we kept the capitalization as it was before (which means that types start with a capital letter while constructors usually do not) to keep the naming more uniform across the various JIVE-related publications.

The theories presented in this report require the use of ISABELLE 2005. The proofs of lemmas are skipped in the presentation to keep it compact. The full proofs can be found in the original ISABELLE theories.

## 2 Theory Dependencies



The theories “TypeIds”, “DirectSubtypes”, “Attributes” and “UnivSpec” are program-dependent and are generated by the Jive tool. The program-dependent theories presented in this report are just examples and act as placeholders. The theories are stored in four different directories:

Isabelle:

- JavaType.thy
- Subtype.thy
- Value.thy
- JML.thy

Isabelle\_Store:

- AttributesIndep.thy
- Location.thy
- Store.thy
- StoreProperties.thy

Isa\_⟨Prog⟩:

- TypeIds.thy
- DirectSubtypes.thy
- UnivSpec.thy

Isa\_⟨Prog⟩\_Store:

- Attributes.thy

In this naming convention, the suffix “\_Store” denotes those theories that depend on the actual realization of the Store. They have been separated in order to allow for easy exchanging of the Store realization. The midfix “<Prog>” denotes the name of the program for which the program-dependent theories have been generated. This way, different program-dependent theories can reside side-by-side without conflicts.

These four directories have to be added to the ML path before loading UnivSpec. This can be done in a setup theory with the following command (here applied to a program called `Counter`):

```
ML {*
add_path "<PATH_TO_THEORIES>/Isabelle";
add_path "<PATH_TO_THEORIES>/Isabelle_Store";
add_path "<PATH_TO_THEORIES>/Isa_Counter";
add_path "<PATH_TO_THEORIES>/Isa_Counter_Store";
*}
```

This way, one can select the program-dependent theories for the program that currently is to be proven.

### 3 The Example Program

The program-dependent theories are generated for the following example program:

```
interface Counter {
    public int incr();
    public int reset();
}

class CounterImpl implements Counter {
    protected int value;

    public int incr()
    {
        int dummy;
        res = this.value;
        res = (int) res + 1;
        this.value = res;
    }

    public int reset()
    {
        int dummy;
        this.value=0;
        res = (int) 0;
    }
}

class UndoCounter extends CounterImpl {
    private int save;
```



```

public int incr()
{
    int dummy;
    res = this.value;
    this.save = res;
    res = res + 1;
    this.value = res;
}

public int un_do()
{
    int res2;
    res = this.save;
    res2 = this.value;
    this.value = res;
    this.save = res2;
}
}

```

## 4 TypeIds

**theory** *TypeIds* **imports** *Main* **begin**

This theory contains the program specific names of abstract and concrete classes and interfaces. It has to be generated for each program we want to verify. The following classes are an example taken from the program given in Sect. 3. They are complemented by the classes that are known to exist in each Java program implicitly, namely `Object`, `Exception`, `ClassCastException` and `NullPointerException`. The example program does not contain any abstract classes, but since we cannot formalize datatypes without constructors, we have to insert a dummy class which we call `Dummy`.

The datatype `CTypeId` must contain a constructor called `Object` because subsequent proofs in the `Subtype` theory rely on it.

**datatype** *CTypeId* = *CounterImpl* | *UndoCounter*  
| *Object* | *Exception* | *ClassCastException* | *NullPointerException*

— The last line contains the classes that exist in every program by default.

**datatype** *ITypeId* = *Counter*

**datatype** *ATypeId* = *Dummy*

— we cannot have an empty type.

Why do we need different datatypes for the different type identifiers? Because we want to be able to distinguish the different identifier kinds. This has a practical reason: If we formalize objects as "`ObjectId × TypeId`" and if we quantify over all objects, we get a lot of objects that do not exist, namely all objects that bear an interface type identifier or abstract class identifier. This is not very helpful. Therefore, we separate the three identifier kinds from each other.

**end**

## 5 Java-Type

**theory** *JavaType* **imports** *../Isa-Counter/TypeIds*  
**begin**

This theory formalizes the types that appear in a Java program. Note that the types defined by the classes and interfaces are formalized via their identifiers. This way, this theory is program-independent.

We only want to formalize one-dimensional arrays. Therefore, we describe the types that can be used as element types of arrays. This excludes the `null` type and array types themselves. This way, we get a finite number of types in our type hierarchy, and the subtype relations can be given explicitly (see Sec. 6). If desired, this can be extended in the future by using `Javatype` as argument type of the `ArrT` type constructor. This will yield infinitely many types.

**datatype**  $Arraytype = BoolAT \mid IntgAT \mid ShortAT \mid ByteAT$   
 $\mid CClassAT \ CTypeId \mid AClassAT \ ATypeId$   
 $\mid InterfaceAT \ ITypeId$

**datatype**  $Javatype = BoolT \mid IntgT \mid ShortT \mid ByteT \mid NullT \mid ArrT \ Arraytype$   
 $\mid CClassT \ CTypeId \mid AClassT \ ATypeId$   
 $\mid InterfaceT \ ITypeId$

We need a function that widens `Arraytype` to `Javatype`.

**definition**

$at2jt :: Arraytype \Rightarrow Javatype$

**where**

$at2jt \ at = (case \ at \ of$   
 $\ \ \ BoolAT \ \ \ \ \ \Rightarrow BoolT$   
 $\ \ \ IntgAT \ \ \ \ \ \Rightarrow IntgT$   
 $\ \ \ ShortAT \ \ \ \ \ \Rightarrow ShortT$   
 $\ \ \ ByteAT \ \ \ \ \ \Rightarrow ByteT$   
 $\ \ \ CClassAT \ CTypeId \ \Rightarrow CClassT \ CTypeId$   
 $\ \ \ AClassAT \ ATypeId \ \Rightarrow AClassT \ ATypeId$   
 $\ \ \ InterfaceAT \ ITypeId \Rightarrow InterfaceT \ ITypeId)$

We define two predicates that separate the primitive types and the class types.

**primrec**  $isprimitive :: Javatype \Rightarrow bool$

**where**

$isprimitive \ BoolT = True \mid$   
 $isprimitive \ IntgT = True \mid$   
 $isprimitive \ ShortT = True \mid$   
 $isprimitive \ ByteT = True \mid$   
 $isprimitive \ NullT = False \mid$   
 $isprimitive \ (ArrT \ T) = False \mid$   
 $isprimitive \ (CClassT \ c) = False \mid$   
 $isprimitive \ (AClassT \ c) = False \mid$   
 $isprimitive \ (InterfaceT \ i) = False$

**primrec**  $isclass :: Javatype \Rightarrow bool$

**where**

$isclass \ BoolT = False \mid$   
 $isclass \ IntgT = False \mid$   
 $isclass \ ShortT = False \mid$   
 $isclass \ ByteT = False \mid$   
 $isclass \ NullT = False \mid$   
 $isclass \ (ArrT \ T) = False \mid$   
 $isclass \ (CClassT \ c) = True \mid$   
 $isclass \ (AClassT \ c) = True \mid$

---

```
isclass (InterfaceT i) = False
```

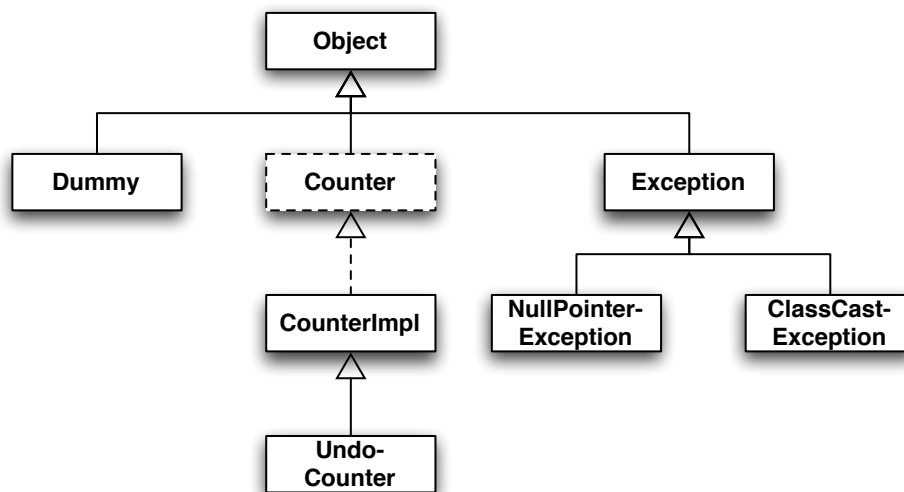
```
end
```

## 6 The Direct Subtype Relation of Java Types

```
theory DirectSubtypes
imports ../Isabelle/JavaType
begin
```

In this theory, we formalize the direct subtype relations of the Java types (as defined in Sec. 4) that appear in the program to be verified. Thus, this theory has to be generated for each program.

We have the following type hierarchy:



We need to describe all direct subtype relations of this type hierarchy. As you can see in the picture, all unnecessary direct subtype relations can be ignored, e.g. the subclass relation between `CounterImpl` and `Object`, because it is added transitively by the widening relation of types (see Sec. 7.2).

We have to specify the direct subtype relation between

- each “leaf” class or interface and its subtype `NullT`
- each “root” class or interface and its supertype `Object`
- each two types that are direct subtypes as specified in the code by `extends` or `implements`
- each array type of a primitive type and its subtype `NullT`
- each array type of a primitive type and its supertype `Object`
- each array type of a “leaf” class or interface and its subtype `NullT`
- the array type `Object []` and its supertype `Object`

- two array types if their element types are in a subtype hierarchy

**definition** *direct-subtype* :: (Javatyp \* Javatyp) set **where**

*direct-subtype* =

```
{ (NullT, AClassT Dummy),
  (NullT, CClassT UndoCounter),
  (NullT, CClassT NullPointerException),
  (NullT, CClassT ClassCastException),

  (AClassT Dummy, CClassT Object),
  (InterfaceT Counter, CClassT Object),
  (CClassT Exception, CClassT Object),

  (CClassT UndoCounter, CClassT CounterImpl),
  (CClassT CounterImpl, InterfaceT Counter),
  (CClassT NullPointerException, CClassT Exception),
  (CClassT ClassCastException, CClassT Exception),

  (NullT, ArrT BoolAT),
  (NullT, ArrT IntgAT),
  (NullT, ArrT ShortAT),
  (NullT, ArrT ByteAT),
  (ArrT BoolAT, CClassT Object),
  (ArrT IntgAT, CClassT Object),
  (ArrT ShortAT, CClassT Object),
  (ArrT ByteAT, CClassT Object),

  (NullT, ArrT (AClassAT Dummy)),
  (NullT, ArrT (CClassAT UndoCounter)),
  (NullT, ArrT (CClassAT NullPointerException)),
  (NullT, ArrT (CClassAT ClassCastException)),

  (ArrT (CClassAT Object), CClassT Object),

  (ArrT (AClassAT Dummy), ArrT (CClassAT Object)),
  (ArrT (CClassAT CounterImpl), ArrT (InterfaceAT Counter)),
  (ArrT (InterfaceAT Counter), ArrT (CClassAT Object)),
  (ArrT (CClassAT Exception), ArrT (CClassAT Object)),
  (ArrT (CClassAT UndoCounter), ArrT (CClassAT CounterImpl)),
  (ArrT (CClassAT NullPointerException), ArrT (CClassAT Exception)),
  (ArrT (CClassAT ClassCastException), ArrT (CClassAT Exception))
}
```

This lemma is used later in the Simplifier.

**lemma** *direct-subtype*:

```
(NullT, AClassT Dummy) ∈ direct-subtype
(NullT, CClassT UndoCounter) ∈ direct-subtype
(NullT, CClassT NullPointerException) ∈ direct-subtype
(NullT, CClassT ClassCastException) ∈ direct-subtype

(AClassT Dummy, CClassT Object) ∈ direct-subtype
(InterfaceT Counter, CClassT Object) ∈ direct-subtype
(CClassT Exception, CClassT Object) ∈ direct-subtype
```

---

```

(CClassT UndoCounter, CClassT CounterImpl) ∈ direct-subtype
(CClassT CounterImpl, InterfaceT Counter) ∈ direct-subtype
(CClassT NullPointerException, CClassT Exception) ∈ direct-subtype
(CClassT ClassCastException, CClassT Exception) ∈ direct-subtype

```

```

(NullT, ArrT BoolAT) ∈ direct-subtype
(NullT, ArrT IntgAT) ∈ direct-subtype
(NullT, ArrT ShortAT) ∈ direct-subtype
(NullT, ArrT ByteAT) ∈ direct-subtype
(ArrT BoolAT, CClassT Object) ∈ direct-subtype
(ArrT IntgAT, CClassT Object) ∈ direct-subtype
(ArrT ShortAT, CClassT Object) ∈ direct-subtype
(ArrT ByteAT, CClassT Object) ∈ direct-subtype

```

```

(NullT, ArrT (AClassAT Dummy)) ∈ direct-subtype
(NullT, ArrT (CClassAT UndoCounter)) ∈ direct-subtype
(NullT, ArrT (CClassAT NullPointerException)) ∈ direct-subtype
(NullT, ArrT (CClassAT ClassCastException)) ∈ direct-subtype

```

```

(ArrT (CClassAT Object), CClassT Object) ∈ direct-subtype

```

```

(ArrT (AClassAT Dummy), ArrT (CClassAT Object)) ∈ direct-subtype
(ArrT (CClassAT CounterImpl), ArrT (InterfaceAT Counter)) ∈ direct-subtype
(ArrT (InterfaceAT Counter), ArrT (CClassAT Object)) ∈ direct-subtype
(ArrT (CClassAT Exception), ArrT (CClassAT Object)) ∈ direct-subtype
(ArrT (CClassAT UndoCounter), ArrT (CClassAT CounterImpl)) ∈ direct-subtype
(ArrT (CClassAT NullPointerException), ArrT (CClassAT Exception)) ∈ direct-subtype
(ArrT (CClassAT ClassCastException), ArrT (CClassAT Exception)) ∈ direct-subtype
by (simp-all add: direct-subtype-def)

```

end

## 7 Widening the Direct Subtype Relation

```

theory Subtype
imports ../Isa-Counter/DirectSubtypes
begin

```

In this theory, we define the widening subtype relation of types and prove that it is a partial order.

### 7.1 Auxiliary lemmas

These general lemmas are not especially related to Jive. They capture some useful properties of general relations.

```

lemma distinct-rtrancl-into-trancl:
  assumes neq-x-y: x ≠ y
  assumes x-y-rtrancl: (x,y) ∈ r*
  shows (x,y) ∈ r+
  using x-y-rtrancl neq-x-y
proof (induct)
  assume x≠x thus (x, x) ∈ r+ by simp

```

next

fix  $y z$   
 assume  $x\text{-}y\text{-}r\text{trancl}$ :  $(x, y) \in r^*$   
 assume  $y\text{-}z\text{-}r$ :  $(y, z) \in r$   
 assume  $x \neq y \implies (x, y) \in r^+$   
 assume  $x \neq z$   
 from  $x\text{-}y\text{-}r\text{trancl}$   
 show  $(x, z) \in r^+$   
 proof (cases)  
 assume  $x=y$   
 with  $y\text{-}z\text{-}r$  have  $(x, z) \in r$  by simp  
 thus  $(x, z) \in r^+..$

next

fix  $w$   
 assume  $(x, w) \in r^*$   
 moreover assume  $(w, y) \in r$   
 ultimately have  $(x, y) \in r^+$   
 by (rule  $r\text{trancl}\text{-}into\text{-}trancl1$ )  
 from this  $y\text{-}z\text{-}r$   
 show  $(x, z) \in r^+..$

qed

qed

lemma  $acyclic\text{-}imp\text{-}antisym\text{-}r\text{trancl}$ :  $acyclic\ r \implies antisym\ (r^*)$

proof (clarsimp simp only:  $acyclic\text{-}def$   $antisym\text{-}def$ )

fix  $x y$   
 assume  $acyclic$ :  $\forall x. (x, x) \notin r^+$   
 assume  $x\text{-}y$ :  $(x, y) \in r^*$   
 assume  $y\text{-}x$ :  $(y, x) \in r^*$   
 show  $x=y$   
 proof (cases  $x=y$ )  
 case True thus ?thesis .  
 next  
 case False  
 from False  $x\text{-}y$  have  $(x, y) \in r^+$   
 by (rule  $distinct\text{-}r\text{trancl}\text{-}into\text{-}trancl$ )  
 also  
 from False  $y\text{-}x$  have  $(y, x) \in r^+$   
 by (fastforce intro:  $distinct\text{-}r\text{trancl}\text{-}into\text{-}trancl$ )  
 finally have  $(x, x) \in r^+$ .  
 with  $acyclic$  show ?thesis by simp

qed

qed

lemma  $acyclic\text{-}trancl\text{-}r\text{trancl}$ :

assumes  $acyclic$ :  $acyclic\ r$   
 shows  $(x, y) \in r^+ = ((x, y) \in r^* \wedge x \neq y)$

proof

assume  $x\text{-}y\text{-}trancl$ :  $(x, y) \in r^+$   
 show  $(x, y) \in r^* \wedge x \neq y$   
 proof  
 from  $x\text{-}y\text{-}trancl$  show  $(x, y) \in r^*..$

next

from  $x\text{-}y\text{-}trancl$   $acyclic$  show  $x \neq y$  by (auto simp add:  $acyclic\text{-}def$ )

```

qed
next
  assume  $(x,y) \in r^* \wedge x \neq y$ 
  thus  $(x,y) \in r^+$ 
    by (auto intro: distinct-rtrancl-into-trancl)
qed

```

## 7.2 The Widening (Subtype) Relation of Javatypes

In this section we widen the direct subtype relations specified in Sec. 6. It is done by a calculation of the transitive closure of the direct subtype relation.

This is the concrete syntax that expresses the subtype relations between all types.

### abbreviation

*direct-subtype-syntax* ::  $Javatype \Rightarrow Javatype \Rightarrow bool$  (- <1 - [71,71] 70)

**where** — direct subtype relation

$A <1 B == (A,B) \in \text{direct-subtype}$

### abbreviation

*widen-syntax* ::  $Javatype \Rightarrow Javatype \Rightarrow bool$  (-  $\preceq$  - [71,71] 70)

**where** — reflexive transitive closure of direct subtype relation

$A \preceq B == (A,B) \in \text{direct-subtype}^*$

### abbreviation

*widen-strict-syntax* ::  $Javatype \Rightarrow Javatype \Rightarrow bool$  (- < - [71,71] 70)

**where** — transitive closure of direct subtype relation

$A < B == (A,B) \in \text{direct-subtype}^+$

## 7.3 The Subtype Relation as Partial Order

We prove the axioms required for partial orders, i.e. reflexivity, transitivity and antisymmetry, for the widened subtype relation. The direct subtype relation has been defined in Sec. 6. The reflexivity lemma is added to the Simplifier and to the Classical reasoner (via the attribute iff), and the transitivity and antisymmetry lemmas are made known as transitivity rules (via the attribute trans). This way, these lemmas will be automatically used in subsequent proofs.

**lemma** *acyclic-direct-subtype*: *acyclic direct-subtype*

**proof** (*clarsimp simp add: acyclic-def*)

**fix**  $x$  **show**  $x < x \implies \text{False}$

**by** (*cases x*) (*fastforce elim: tranclE simp add: direct-subtype-def*)<sup>+</sup>

qed

**lemma** *antisym-rtrancl-direct-subtype*: *antisym (direct-subtype\*)*

**using** *acyclic-direct-subtype* **by** (*rule acyclic-imp-antisym-rtrancl*)

**lemma** *widen-strict-to-widen*:  $C < D = (C \preceq D \wedge C \neq D)$

**using** *acyclic-direct-subtype* **by** (*rule acyclic-trancl-rtrancl*)

The widening relation on Javatype is reflexive.

**lemma** *widen-refl [iff]*:  $X \preceq X$  ..

The widening relation on Javatype is transitive.

```

lemma widen-trans [trans] :
  assumes a-b:  $a \preceq b$ 
  shows  $\bigwedge c. b \preceq c \implies a \preceq c$ 
  by (insert a-b, rule rtrancl-trans)

```

The widening relation on *Javatype* is antisymmetric.

```

lemma widen-antisym [trans]:
  assumes a-b:  $a \preceq b$ 
  assumes b-c:  $b \preceq a$ 
  shows  $a = b$ 
  using a-b b-c antisym-rtrancl-direct-subtype
  by (unfold antisym-def) blast

```

## 7.4 Javatype Ordering Properties

The type class *ord* allows us to overwrite the two comparison operators  $<$  and  $\leq$ . These are the two comparison operators on *Javatype* that we want to use subsequently.

We can also prove that *Javatype* is in the type class *order*. For this we have to prove reflexivity, transitivity, antisymmetry and that  $<$  and  $\leq$  are defined in such a way that  $(x < y) = (x \leq y \wedge x \neq y)$  holds. This proof can easily be achieved by using the lemmas proved above and the definition of *less-Javatype-def*.

```

instantiation Javatype:: order
begin

```

```

definition
  le-Javatype-def:  $A \leq B \equiv A \preceq B$ 

```

```

definition
  less-Javatype-def:  $A < B \equiv A \leq B \wedge \neg B \leq (A::\text{Javatype})$ 

```

```

instance proof
  fix x y z:: Javatype
  {
    show  $x \leq x$ 
    by (simp add: le-Javatype-def )
  next
    assume  $x \leq y \ y \leq z$ 
    then show  $x \leq z$ 
    by (unfold le-Javatype-def) (rule rtrancl-trans)
  next
    assume  $x \leq y \ y \leq x$ 
    then show  $x = y$ 
    apply (unfold le-Javatype-def)
    apply (rule widen-antisym)
    apply assumption +
    done
  next
    show  $(x < y) = (x \leq y \wedge \neg y \leq x)$ 
    by (simp add: less-Javatype-def)
  }
qed

```



end

## 7.5 Enhancing the Simplifier

**lemmas** *subtype-defs* = *le-Javatype-def less-Javatype-def*  
*direct-subtype-def*

**lemmas** *subtype-ok-simps* = *subtype-defs*

**lemmas** *subtype-wrong-elim* = *rtranclE*

During verification we will often have to solve the goal that one type widens to the other. So we equip the simplifier with a special solver-tactic.

**lemma** *widen-asm*:  $(a::\text{Javatype}) \leq b \implies a \leq b$   
**by** *simp*

**lemmas** *direct-subtype-widened* = *direct-subtype[THEN r-into-rtrancl]*

**ML** <

*local val ss = simpset-of @{context} in*

*fun widen-tac ctxt =*

*resolve-tac ctxt @{thms widen-asm} THEN'*

*simp-tac (put-simpset ss ctxt addsimps @{thms le-Javatype-def}) THEN'*

*Method.insert-tac ctxt @{thms direct-subtype-widened} THEN'*

*simp-tac (put-simpset (simpset-of @{theory-context Transitive-Closure}) ctxt)*

*end*

>

**declaration** <*fn - =>*

*Simplifier.map-ss (fn ss => ss addSolver (mk-solver widen widen-tac))*

>

In this solver-tactic, we first try the trivial resolution with *widen-asm* to check if the actual subgoal really is a request to solve a subtyping problem. If so, we unfold the comparison operator, insert the direct subtype relations and call the simplifier.

## 7.6 Properties of the Subtype Relation

The class *Object* has to be the root of the class hierarchy, i.e. it is supertype of each concrete class, abstract class, interface and array type. The proof scripts should run on every correctly generated type hierarchy.

**lemma** *Object-root*:  $CClassT\ C \leq CClassT\ Object$   
**by** (*cases C, simp-all*)

**lemma** *Object-root-abs*:  $AClassT\ C \leq CClassT\ Object$   
**by** (*cases C, simp-all*)

**lemma** *Object-root-int*:  $InterfaceT\ C \leq CClassT\ Object$   
**by** (*cases C, simp-all*)

**lemma** *Object-root-array*:  $ArrT\ C \leq CClassT\ Object$

```

proof (cases C)
  fix x
  assume c: C = CClassAT x
  show ArrT C ≤ CClassT Object
    using c by (cases x, simp-all)
next
  fix x
  assume c: C = AClassAT x
  show ArrT C ≤ CClassT Object
    using c by (cases x, simp-all)
next
  fix x
  assume c: C = InterfaceAT x
  show ArrT C ≤ CClassT Object
    using c by (cases x, simp-all)
next
  assume c: C = BoolAT
  show ArrT C ≤ CClassT Object
    using c by simp
next
  assume c: C = IntgAT
  show ArrT C ≤ CClassT Object
    using c by simp
next
  assume c: C = ShortAT
  show ArrT C ≤ CClassT Object
    using c by simp
next
  assume c: C = ByteAT
  show ArrT C ≤ CClassT Object
    using c by simp
qed

```

If another type is (non-strict) supertype of Object, then it must be the type Object itself.

**lemma** *Object-rootD*:

**assumes** p: CClassT Object ≤ c

**shows** CClassT Object = c

**using** p

**apply** (cases c)

**apply** (fastforce elim: subtype-wrong-elim simp add: subtype-defs) +

— In this lemma, we only get contradictory cases except for Object itself.

**done**

The type NullT has to be the leaf of each branch of the class hierarchy, i.e. it is subtype of each type.

**lemma** *NullT-leaf* [simp]: NullT ≤ CClassT C

**by** (cases C, simp-all)

**lemma** *NullT-leaf-abs* [simp]: NullT ≤ AClassT C

**by** (cases C, simp-all)

**lemma** *NullT-leaf-int* [simp]: NullT ≤ InterfaceT C

**by** (cases C, simp-all)

---

**lemma** *NullT-leaf-array*:  $NullT \leq ArrT\ C$

```

proof (cases C)
  fix x
  assume c: C = CClassAT x
  show  $NullT \leq ArrT\ C$ 
    using c by (cases x, simp-all)
next
  fix x
  assume c: C = AClassAT x
  show  $NullT \leq ArrT\ C$ 
    using c by (cases x, simp-all)
next
  fix x
  assume c: C = InterfaceAT x
  show  $NullT \leq ArrT\ C$ 
    using c by (cases x, simp-all)
next
  assume c: C = BoolAT
  show  $NullT \leq ArrT\ C$ 
    using c by simp
next
  assume c: C = IntgAT
  show  $NullT \leq ArrT\ C$ 
    using c by simp
next
  assume c: C = ShortAT
  show  $NullT \leq ArrT\ C$ 
    using c by simp
next
  assume c: C = ByteAT
  show  $NullT \leq ArrT\ C$ 
    using c by simp

```

**qed**

**end**

## 8 Attributes

```

theory Attributes
imports ../Isabelle/Subtype
begin

```

This theory has to be generated as well for each program under verification. It defines the attributes of the classes and various functions on them.

```

datatype AttId = CounterImpl'value | UndoCounter'save
  | Dummy'dummy | Counter'dummy

```

The last two entries are only added to demonstrate what is to happen with attributes of abstract classes and interfaces.

It would be nice if attribute names were generated in a way that keeps them short, so that the proof state does not get unreadable because of fancy long names. The generation of attribute names that is performed by the Jive tool should only add the definition class if necessary,

i.e. if there would be a name clash otherwise. For the example above, the class names are not necessary. One must be careful, though, not to generate names that might clash with names of free variables that are used subsequently.

The domain type of an attribute is the definition class (or interface) of the attribute.

**definition**  $dtype:: AttId \Rightarrow Javatype$  **where**  
 $dtype f = (case f of$   
      $CounterImpl'value \Rightarrow CClassT CounterImpl$   
      $| UndoCounter'save \Rightarrow CClassT UndoCounter$   
      $| Dummy'dummy \Rightarrow AClassT Dummy$   
      $| Counter'dummy \Rightarrow InterfaceT Counter)$

**lemma**  $dtype-simps [simp]$ :  
 $dtype CounterImpl'value = CClassT CounterImpl$   
 $dtype UndoCounter'save = CClassT UndoCounter$   
 $dtype Dummy'dummy = AClassT Dummy$   
 $dtype Counter'dummy = InterfaceT Counter$   
**by** ( $simp-all add: dtype-def dtype-def dtype-def$ )

For convenience, we add some functions that directly apply the selectors of the datatype *Javatype*.

**definition**  $cDTypeId :: AttId \Rightarrow CTypeId$  **where**  
 $cDTypeId f = (case f of$   
      $CounterImpl'value \Rightarrow CounterImpl$   
      $| UndoCounter'save \Rightarrow UndoCounter$   
      $| Dummy'dummy \Rightarrow undefined$   
      $| Counter'dummy \Rightarrow undefined )$

**definition**  $aDTypeId:: AttId \Rightarrow ATypeId$  **where**  
 $aDTypeId f = (case f of$   
      $CounterImpl'value \Rightarrow undefined$   
      $| UndoCounter'save \Rightarrow undefined$   
      $| Dummy'dummy \Rightarrow Dummy$   
      $| Counter'dummy \Rightarrow undefined )$

**definition**  $iDTypeId:: AttId \Rightarrow ITypeId$  **where**  
 $iDTypeId f = (case f of$   
      $CounterImpl'value \Rightarrow undefined$   
      $| UndoCounter'save \Rightarrow undefined$   
      $| Dummy'dummy \Rightarrow undefined$   
      $| Counter'dummy \Rightarrow Counter )$

**lemma**  $DTypeId-simps [simp]$ :  
 $cDTypeId CounterImpl'value = CounterImpl$   
 $cDTypeId UndoCounter'save = UndoCounter$   
 $aDTypeId Dummy'dummy = Dummy$   
 $iDTypeId Counter'dummy = Counter$   
**by** ( $simp-all add: cDTypeId-def aDTypeId-def iDTypeId-def$ )

The range type of an attribute is the type of the value stored in that attribute.

**definition**  $rtype:: AttId \Rightarrow Javatype$  **where**  
 $rtype f = (case f of$   
      $CounterImpl'value \Rightarrow IntgT$

---

```

| UndoCounter'save ⇒ IntgT
| Dummy'dummy ⇒ NullT
| Counter'dummy ⇒ NullT)

```

**lemma** *rtype-simps* [*simp*]:  
*rtype* CounterImpl'value = IntgT  
*rtype* UndoCounter'save = IntgT  
*rtype* Dummy'dummy = NullT  
*rtype* Counter'dummy = NullT  
**by** (*simp-all add: rtype-def rtype-def rtype-def*)

With the datatype *CAttId* we describe the possible locations in memory for instance fields. We rule out the impossible combinations of class names and field names. For example, a *CounterImpl* cannot have a *save* field. A store model which provides locations for all possible combinations of the Cartesian product of class name and field name works out fine as well, because we cannot express modification of such “wrong” locations in a Java program. So we can only prove useful properties about reasonable combinations. The only drawback in such a model is that we cannot prove a property like *not-treach-ref-impl-not-reach* in theory *StoreProperties*. If the store provides locations for every combination of class name and field name, we cannot rule out reachability of certain pointer chains that go through “wrong” locations. That is why we decided to introduce the new type *CAttId*.

While *AttId* describes which fields are declared in which classes and interfaces, *CAttId* describes which objects of which classes may contain which fields at run-time. Thus, *CAttId* makes the inheritance of fields visible in the formalization.

There is only one such datatype because only objects of concrete classes can be created at run-time, thus only instance fields of concrete classes can occupy memory.

```

datatype CAttId = CounterImpl'CounterImpl'value | UndoCounter'CounterImpl'value
| UndoCounter'UndoCounter'save
| CounterImpl'Counter'dummy | UndoCounter'Counter'dummy

```

Function *catt* builds a *CAttId* from a class name and a field name. In case of the illegal combinations we just return *undefined*. We can also filter out static fields in *catt*.

**definition** *catt*:: CTypeId ⇒ AttId ⇒ CAttId **where**

```

catt C f =
(case C of
  CounterImpl ⇒ (case f of
    CounterImpl'value ⇒ CounterImpl'CounterImpl'value
    | UndoCounter'save ⇒ undefined
    | Dummy'dummy ⇒ undefined
    | Counter'dummy ⇒ CounterImpl'Counter'dummy)
  | UndoCounter ⇒ (case f of
    CounterImpl'value ⇒ UndoCounter'CounterImpl'value
    | UndoCounter'save ⇒ UndoCounter'UndoCounter'save
    | Dummy'dummy ⇒ undefined
    | Counter'dummy ⇒ UndoCounter'Counter'dummy)
  | Object ⇒ undefined
  | Exception ⇒ undefined
  | ClassCastException ⇒ undefined
  | NullPointerException ⇒ undefined
)

```

```

lemma catt-simps [simp]:
catt CounterImpl CounterImpl'value = CounterImpl'CounterImpl'value
catt UndoCounter CounterImpl'value = UndoCounter'CounterImpl'value
catt UndoCounter UndoCounter'save = UndoCounter'UndoCounter'save
catt CounterImpl Counter'dummy = CounterImpl'Counter'dummy
catt UndoCounter Counter'dummy = UndoCounter'Counter'dummy
  by (simp-all add: catt-def)

```

Selection of the class name of the type of the object in which the field lives. The field can only be located in a concrete class.

```

definition cls:: CAttId  $\Rightarrow$  CTypeId where
cls cf = (case cf of
  | CounterImpl'CounterImpl'value  $\Rightarrow$  CounterImpl
  | UndoCounter'CounterImpl'value  $\Rightarrow$  UndoCounter
  | UndoCounter'UndoCounter'save  $\Rightarrow$  UndoCounter
  | CounterImpl'Counter'dummy  $\Rightarrow$  CounterImpl
  | UndoCounter'Counter'dummy  $\Rightarrow$  UndoCounter
)

```

```

lemma cls-simps [simp]:
cls CounterImpl'CounterImpl'value = CounterImpl
cls UndoCounter'CounterImpl'value = UndoCounter
cls UndoCounter'UndoCounter'save = UndoCounter
cls CounterImpl'Counter'dummy = CounterImpl
cls UndoCounter'Counter'dummy = UndoCounter
  by (simp-all add: cls-def)

```

Selection of the field name.

```

definition att:: CAttId  $\Rightarrow$  AttId where
att cf = (case cf of
  | CounterImpl'CounterImpl'value  $\Rightarrow$  CounterImpl'value
  | UndoCounter'CounterImpl'value  $\Rightarrow$  CounterImpl'value
  | UndoCounter'UndoCounter'save  $\Rightarrow$  UndoCounter'save
  | CounterImpl'Counter'dummy  $\Rightarrow$  Counter'dummy
  | UndoCounter'Counter'dummy  $\Rightarrow$  Counter'dummy
)

```

```

lemma att-simps [simp]:
att CounterImpl'CounterImpl'value = CounterImpl'value
att UndoCounter'CounterImpl'value = CounterImpl'value
att UndoCounter'UndoCounter'save = UndoCounter'save
att CounterImpl'Counter'dummy = Counter'dummy
att UndoCounter'Counter'dummy = Counter'dummy
  by (simp-all add: att-def)

```

**end**

## 9 Program-Independent Lemmas on Attributes

```

theory AttributesIndep
imports ../Isa-Counter-Store/Attributes
begin

```

The following lemmas validate the functions defined in the Attributes theory. They also aid in subsequent proving tasks. Since they are program-independent, it is of no use to add them to the generation process of Attributes.thy. Therefore, they have been extracted to this theory.

```
lemma cls-catt [simp]:
  CClassT c ≤ dtype f ⇒ cls (catt c f) = c
apply (case-tac c)
apply (case-tac [!] f)
apply simp-all
  — solves all goals where CClassT c ≤ dtype f
apply (fastforce elim: subtype-wrong-elims simp add: subtype-defs)+
  — solves all the rest where ¬ CClassT c ≤ dtype f can be derived
done
```

```
lemma att-catt [simp]:
  CClassT c ≤ dtype f ⇒ att (catt c f) = f
apply (case-tac c)
apply (case-tac [!] f)
apply simp-all
  — solves all goals where CClassT c ≤ dtype f
apply (fastforce elim: subtype-wrong-elims simp add: subtype-defs)+
  — solves all the rest where ¬ CClassT c ≤ dtype f can be derived
done
```

The following lemmas are just a demonstration of simplification.

```
lemma rtype-att-catt:
  CClassT c ≤ dtype f ⇒ rtype (att (catt c f)) = rtype f
by simp
```

```
lemma widen-cls-dtype-att [simp,intro]:
  (CClassT (cls cf) ≤ dtype (att cf))
by (cases cf, simp-all)
```

**end**

## 10 Value

**theory** *Value* **imports** *Subtype* **begin**

This theory contains our model of the values in the store. The store is untyped, therefore all types that exist in Java are wrapped into one type *Value*.

In a first approach, the primitive Java types supported in this formalization are mapped to similar Isabelle types. Later, we will have proper formalizations of the Java types in Isabelle, which will then be used here.

```
type-synonym JavaInt = int
type-synonym JavaShort = int
type-synonym JavaByte = int
type-synonym JavaBoolean = bool
```

The objects of each class are identified by a unique ID. We use elements of type *nat* here, but in general it is sufficient to use an infinite type with a successor function and a comparison predicate.

**type-synonym**  $ObjectId = nat$

The definition of the datatype  $Value$ . Values can be of the Java types boolean, int, short and byte. Additionally, they can be an object reference, an array reference or the value null.

**datatype**  $Value = boolV\ JavaBoolean$   
 $| intgV\ JavaInt$   
 $| shortV\ JavaShort$   
 $| byteV\ JavaByte$   
 $| objV\ CTypeId\ ObjectId$  — typed object reference  
 $| arrV\ Arraytype\ ObjectId$  — typed array reference  
 $| nullV$

Arrays are modeled as references just like objects. So they can be viewed as special kinds of objects, like in Java.

## 10.1 Discriminator Functions

To test values, we define the following discriminator functions.

**definition**  $isBoolV :: Value \Rightarrow bool$  **where**

$isBoolV\ v = (case\ v\ of$   
 $\quad boolV\ b \Rightarrow True$   
 $\quad | intgV\ i \Rightarrow False$   
 $\quad | shortV\ s \Rightarrow False$   
 $\quad | byteV\ by \Rightarrow False$   
 $\quad | objV\ C\ a \Rightarrow False$   
 $\quad | arrV\ T\ a \Rightarrow False$   
 $\quad | nullV \Rightarrow False)$

**lemma**  $isBoolV$ -simps  $[simp]$ :

$isBoolV\ (boolV\ b) = True$   
 $isBoolV\ (intgV\ i) = False$   
 $isBoolV\ (shortV\ s) = False$   
 $isBoolV\ (byteV\ by) = False$   
 $isBoolV\ (objV\ C\ a) = False$   
 $isBoolV\ (arrV\ T\ a) = False$   
 $isBoolV\ (nullV) = False$   
**by**  $(simp$ -all  $add: isBoolV$ -def)

**definition**  $isIntgV :: Value \Rightarrow bool$  **where**

$isIntgV\ v = (case\ v\ of$   
 $\quad boolV\ b \Rightarrow False$   
 $\quad | intgV\ i \Rightarrow True$   
 $\quad | shortV\ s \Rightarrow False$   
 $\quad | byteV\ by \Rightarrow False$   
 $\quad | objV\ C\ a \Rightarrow False$   
 $\quad | arrV\ T\ a \Rightarrow False$   
 $\quad | nullV \Rightarrow False)$

**lemma**  $isIntgV$ -simps  $[simp]$ :

$isIntgV\ (boolV\ b) = False$   
 $isIntgV\ (intgV\ i) = True$   
 $isIntgV\ (shortV\ s) = False$



$isIntgV (byteV by) = False$   
 $isIntgV (objV C a) = False$   
 $isIntgV (arrV T a) = False$   
 $isIntgV (nullV) = False$   
**by** (*simp-all add: isIntgV-def*)

**definition**  $isShortV :: Value \Rightarrow bool$  **where**

$isShortV v = (case\ v\ of$   
 $\quad boolV\ b \Rightarrow False$   
 $\quad | intgV\ i \Rightarrow False$   
 $\quad | shortV\ s \Rightarrow True$   
 $\quad | byteV\ by \Rightarrow False$   
 $\quad | objV\ C\ a \Rightarrow False$   
 $\quad | arrV\ T\ a \Rightarrow False$   
 $\quad | nullV \Rightarrow False)$

**lemma**  $isShortV-simps [simp]:$

$isShortV (boolV b) = False$   
 $isShortV (intgV i) = False$   
 $isShortV (shortV s) = True$   
 $isShortV (byteV by) = False$   
 $isShortV (objV C a) = False$   
 $isShortV (arrV T a) = False$   
 $isShortV (nullV) = False$   
**by** (*simp-all add: isShortV-def*)

**definition**  $isByteV :: Value \Rightarrow bool$  **where**

$isByteV v = (case\ v\ of$   
 $\quad boolV\ b \Rightarrow False$   
 $\quad | intgV\ i \Rightarrow False$   
 $\quad | shortV\ s \Rightarrow False$   
 $\quad | byteV\ by \Rightarrow True$   
 $\quad | objV\ C\ a \Rightarrow False$   
 $\quad | arrV\ T\ a \Rightarrow False$   
 $\quad | nullV \Rightarrow False)$

**lemma**  $isByteV-simps [simp]:$

$isByteV (boolV b) = False$   
 $isByteV (intgV i) = False$   
 $isByteV (shortV s) = False$   
 $isByteV (byteV by) = True$   
 $isByteV (objV C a) = False$   
 $isByteV (arrV T a) = False$   
 $isByteV (nullV) = False$   
**by** (*simp-all add: isByteV-def*)

**definition**  $isRefV :: Value \Rightarrow bool$  **where**

$isRefV v = (case\ v\ of$   
 $\quad boolV\ b \Rightarrow False$   
 $\quad | intgV\ i \Rightarrow False$   
 $\quad | shortV\ s \Rightarrow False$

```

| byteV by ⇒ False
| objV C a ⇒ True
| arrV T a ⇒ True
| nullV   ⇒ True)

```

**lemma** *isRefV-simps* [simp]:  
*isRefV* (boolV b) = False  
*isRefV* (intgV i) = False  
*isRefV* (shortV s) = False  
*isRefV* (byteV by) = False  
*isRefV* (objV C a) = True  
*isRefV* (arrV T a) = True  
*isRefV* (nullV) = True  
**by** (simp-all add: *isRefV-def*)

**definition** *isObjV* :: Value ⇒ bool **where**  
*isObjV* v = (case v of  
 boolV b ⇒ False  
 | intgV i ⇒ False  
 | shortV s ⇒ False  
 | byteV by ⇒ False  
 | objV C a ⇒ True  
 | arrV T a ⇒ False  
 | nullV ⇒ False)

**lemma** *isObjV-simps* [simp]:  
*isObjV* (boolV b) = False  
*isObjV* (intgV i) = False  
*isObjV* (shortV s) = False  
*isObjV* (byteV by) = False  
*isObjV* (objV c a) = True  
*isObjV* (arrV T a) = False  
*isObjV* nullV = False  
**by** (simp-all add: *isObjV-def*)

**definition** *isArrV* :: Value ⇒ bool **where**  
*isArrV* v = (case v of  
 boolV b ⇒ False  
 | intgV i ⇒ False  
 | shortV s ⇒ False  
 | byteV by ⇒ False  
 | objV C a ⇒ False  
 | arrV T a ⇒ True  
 | nullV ⇒ False)

**lemma** *isArrV-simps* [simp]:  
*isArrV* (boolV b) = False  
*isArrV* (intgV i) = False  
*isArrV* (shortV s) = False  
*isArrV* (byteV by) = False  
*isArrV* (objV c a) = False  
*isArrV* (arrV T a) = True

*isArrV nullV* = *False*  
**by** (*simp-all add: isArrV-def*)

**definition** *isNullV* :: *Value*  $\Rightarrow$  *bool* **where**

*isNullV v* = (case *v* of  
     *boolV b*  $\Rightarrow$  *False*  
     | *intgV i*  $\Rightarrow$  *False*  
     | *shortV s*  $\Rightarrow$  *False*  
     | *byteV by*  $\Rightarrow$  *False*  
     | *objV C a*  $\Rightarrow$  *False*  
     | *arrV T a*  $\Rightarrow$  *False*  
     | *nullV*  $\Rightarrow$  *True*)

**lemma** *isNullV-simps* [*simp*]:

*isNullV (boolV b)* = *False*  
*isNullV (intgV i)* = *False*  
*isNullV (shortV s)* = *False*  
*isNullV (byteV by)* = *False*  
*isNullV (objV c a)* = *False*  
*isNullV (arrV T a)* = *False*  
*isNullV nullV* = *True*  
**by** (*simp-all add: isNullV-def*)

## 10.2 Selector Functions

**definition** *aI* :: *Value*  $\Rightarrow$  *JavaInt* **where**

*aI v* = (case *v* of  
     *boolV b*  $\Rightarrow$  *undefined*  
     | *intgV i*  $\Rightarrow$  *i*  
     | *shortV sh*  $\Rightarrow$  *undefined*  
     | *byteV by*  $\Rightarrow$  *undefined*  
     | *objV C a*  $\Rightarrow$  *undefined*  
     | *arrV T a*  $\Rightarrow$  *undefined*  
     | *nullV*  $\Rightarrow$  *undefined*)

**lemma** *aI-simps* [*simp*]:

*aI (intgV i)* = *i*  
**by** (*simp add: aI-def*)

**definition** *aB* :: *Value*  $\Rightarrow$  *JavaBoolean* **where**

*aB v* = (case *v* of  
     *boolV b*  $\Rightarrow$  *b*  
     | *intgV i*  $\Rightarrow$  *undefined*  
     | *shortV sh*  $\Rightarrow$  *undefined*  
     | *byteV by*  $\Rightarrow$  *undefined*  
     | *objV C a*  $\Rightarrow$  *undefined*  
     | *arrV T a*  $\Rightarrow$  *undefined*  
     | *nullV*  $\Rightarrow$  *undefined*)

**lemma** *aB-simps* [*simp*]:

*aB (boolV b)* = *b*  
**by** (*simp add: aB-def*)

**definition**  $aSh :: Value \Rightarrow JavaShort$  **where**

$aSh\ v = (case\ v\ of$   
 $\quad boolV\ b \Rightarrow undefined$   
 $\quad | intgV\ i \Rightarrow undefined$   
 $\quad | shortV\ sh \Rightarrow sh$   
 $\quad | byteV\ by \Rightarrow undefined$   
 $\quad | objV\ C\ a \Rightarrow undefined$   
 $\quad | arrV\ T\ a \Rightarrow undefined$   
 $\quad | nullV \Rightarrow undefined)$

**lemma**  $aSh-simps$  [ $simp$ ]:

$aSh\ (shortV\ sh) = sh$

**by** ( $simp\ add: aSh-def$ )

**definition**  $aBy :: Value \Rightarrow JavaByte$  **where**

$aBy\ v = (case\ v\ of$   
 $\quad boolV\ b \Rightarrow undefined$   
 $\quad | intgV\ i \Rightarrow undefined$   
 $\quad | shortV\ s \Rightarrow undefined$   
 $\quad | byteV\ by \Rightarrow by$   
 $\quad | objV\ C\ a \Rightarrow undefined$   
 $\quad | arrV\ T\ a \Rightarrow undefined$   
 $\quad | nullV \Rightarrow undefined)$

**lemma**  $aBy-simps$  [ $simp$ ]:

$aBy\ (byteV\ by) = by$

**by** ( $simp\ add: aBy-def$ )

**definition**  $tid :: Value \Rightarrow CTypeId$  **where**

$tid\ v = (case\ v\ of$   
 $\quad boolV\ b \Rightarrow undefined$   
 $\quad | intgV\ i \Rightarrow undefined$   
 $\quad | shortV\ s \Rightarrow undefined$   
 $\quad | byteV\ by \Rightarrow undefined$   
 $\quad | objV\ C\ a \Rightarrow C$   
 $\quad | arrV\ T\ a \Rightarrow undefined$   
 $\quad | nullV \Rightarrow undefined)$

**lemma**  $tid-simps$  [ $simp$ ]:

$tid\ (objV\ C\ a) = C$

**by** ( $simp\ add: tid-def$ )

**definition**  $oid :: Value \Rightarrow ObjectId$  **where**

$oid\ v = (case\ v\ of$   
 $\quad boolV\ b \Rightarrow undefined$   
 $\quad | intgV\ i \Rightarrow undefined$   
 $\quad | shortV\ s \Rightarrow undefined$   
 $\quad | byteV\ by \Rightarrow undefined$   
 $\quad | objV\ C\ a \Rightarrow a$   
 $\quad | arrV\ T\ a \Rightarrow undefined$   
 $\quad | nullV \Rightarrow undefined)$

**lemma**  $oid-simps$  [ $simp$ ]:

$oid (objV C a) = a$   
**by** (*simp add: oid-def*)

**definition**  $jt :: Value \Rightarrow Javatype$  **where**

$jt v = (case v of$   
      $boolV b \Rightarrow undefined$   
   |  $intgV i \Rightarrow undefined$   
   |  $shortV s \Rightarrow undefined$   
   |  $byteV by \Rightarrow undefined$   
   |  $objV C a \Rightarrow undefined$   
   |  $arrV T a \Rightarrow at2jt T$   
   |  $nullV \Rightarrow undefined)$

**lemma**  $jt-simps$  [*simp*]:

$jt (arrV T a) = at2jt T$   
**by** (*simp add: jt-def*)

**definition**  $aid :: Value \Rightarrow ObjectId$  **where**

$aid v = (case v of$   
      $boolV b \Rightarrow undefined$   
   |  $intgV i \Rightarrow undefined$   
   |  $shortV s \Rightarrow undefined$   
   |  $byteV by \Rightarrow undefined$   
   |  $objV C a \Rightarrow undefined$   
   |  $arrV T a \Rightarrow a$   
   |  $nullV \Rightarrow undefined)$

**lemma**  $aid-simps$  [*simp*]:

$aid (arrV T a) = a$   
**by** (*simp add: aid-def*)

### 10.3 Determining the Type of a Value

To determine the type of a value, we define the function *typeof*. This function is often written as  $\tau$  in theoretical texts, therefore we add the appropriate syntax support.

**definition**  $typeof :: Value \Rightarrow Javatype$  **where**

$typeof v = (case v of$   
      $boolV b \Rightarrow BoolT$   
   |  $intgV i \Rightarrow IntgT$   
   |  $shortV sh \Rightarrow ShortT$   
   |  $byteV by \Rightarrow ByteT$   
   |  $objV C a \Rightarrow CClassT C$   
   |  $arrV T a \Rightarrow ArrT T$   
   |  $nullV \Rightarrow NullT)$

**abbreviation**  $tau-syntax :: Value \Rightarrow Javatype$  ( $\tau -$ )

**where**  $\tau v == typeof v$

**lemma**  $typeof-simps$  [*simp*]:

$(\tau (boolV b)) = BoolT$   
 $(\tau (intgV i)) = IntgT$

```

( $\tau$  (shortV sh)) = ShortT
( $\tau$  (byteV by)) = ByteT
( $\tau$  (objV c a)) = CClassT c
( $\tau$  (arrV t a)) = ArrT t
( $\tau$  (nullV)) = NullT
  by (simp-all add: typeof-def)

```

## 10.4 Default Initialization Values for Types

The function *init* yields the default initialization values for each type. For boolean, the default value is False, for the integral types, it is 0, and for the reference types, it is nullV.

**definition** *init* :: Javatype  $\Rightarrow$  Value **where**

```

init T = (case T of
  BoolT       $\Rightarrow$  boolV False
| IntgT       $\Rightarrow$  intgV 0
| ShortT      $\Rightarrow$  shortV 0
| ByteT       $\Rightarrow$  byteV 0
| NullT       $\Rightarrow$  nullV
| ArrT T      $\Rightarrow$  nullV
| CClassT C   $\Rightarrow$  nullV
| AClassT C   $\Rightarrow$  nullV
| InterfaceT I  $\Rightarrow$  nullV)

```

**lemma** *init-simps* [simp]:

```

init BoolT      = boolV False
init IntgT      = intgV 0
init ShortT     = shortV 0
init ByteT      = byteV 0
init NullT      = nullV
init (ArrT T)   = nullV
init (CClassT c) = nullV
init (AClassT a) = nullV
init (InterfaceT i) = nullV
  by (simp-all add: init-def)

```

**lemma** *typeof-init-widen* [simp,intro]: *typeof* (init T)  $\leq$  T

**proof** (cases T)

assume c: T = BoolT

show ( $\tau$  (init T))  $\leq$  T

using c by simp

next

assume c: T = IntgT

show ( $\tau$  (init T))  $\leq$  T

using c by simp

next

assume c: T = ShortT

show ( $\tau$  (init T))  $\leq$  T

using c by simp

next

assume c: T = ByteT

show ( $\tau$  (init T))  $\leq$  T

using c by simp

next

```

assume  $c: T = NullT$ 
show  $(\tau (init T)) \leq T$ 
  using  $c$  by simp
next
  fix  $x$ 
  assume  $c: T = CClassT x$ 
  show  $(\tau (init T)) \leq T$ 
    using  $c$  by (cases x, simp-all)
next
  fix  $x$ 
  assume  $c: T = AClassT x$ 
  show  $(\tau (init T)) \leq T$ 
    using  $c$  by (cases x, simp-all)
next
  fix  $x$ 
  assume  $c: T = InterfaceT x$ 
  show  $(\tau (init T)) \leq T$ 
    using  $c$  by (cases x, simp-all)
next
  fix  $x$ 
  assume  $c: T = ArrT x$ 
  show  $(\tau (init T)) \leq T$ 
    using  $c$ 
  proof (cases x)
    fix  $y$ 
    assume  $c2: x = CClassAT y$ 
    show  $(\tau (init T)) \leq T$ 
      using  $c c2$  by (cases y, simp-all)
    next
    fix  $y$ 
    assume  $c2: x = AClassAT y$ 
    show  $(\tau (init T)) \leq T$ 
      using  $c c2$  by (cases y, simp-all)
    next
    fix  $y$ 
    assume  $c2: x = InterfaceAT y$ 
    show  $(\tau (init T)) \leq T$ 
      using  $c c2$  by (cases y, simp-all)
    next
    assume  $c2: x = BoolAT$ 
    show  $(\tau (init T)) \leq T$ 
      using  $c c2$  by simp
    next
    assume  $c2: x = IntgAT$ 
    show  $(\tau (init T)) \leq T$ 
      using  $c c2$  by simp
    next
    assume  $c2: x = ShortAT$ 
    show  $(\tau (init T)) \leq T$ 
      using  $c c2$  by simp
    next
    assume  $c2: x = ByteAT$ 
    show  $(\tau (init T)) \leq T$ 
      using  $c c2$  by simp

```

```
qed
qed
end
```

## 11 Location

```
theory Location
imports AttributesIndep ../Isabelle/Value
begin
```

A storage location can be a field of an object, a static field, the length of an array, or the contents of an array.

```
datatype Location = objLoc   CAttId ObjectId  — field in object
                  | staticLoc AttId           — static field in concrete class
                  | arrLenLoc Arraytype ObjectId — length of an array
                  | arrLoc   Arraytype ObjectId nat — contents of an array
```

We only directly support one-dimensional arrays. Multidimensional arrays can be simulated by arrays of references to arrays.

The function *ltype* yields the content type of a location.

```
definition ltype:: Location  $\Rightarrow$  Javatype where
ltype l = (case l of
  objLoc cf a  $\Rightarrow$  rtype (att cf)
| staticLoc f  $\Rightarrow$  rtype f
| arrLenLoc T a  $\Rightarrow$  IntgT
| arrLoc T a i  $\Rightarrow$  at2jt T)
```

```
lemma ltype-simps [simp]:
ltype (objLoc cf a) = rtype (att cf)
ltype (staticLoc f) = rtype f
ltype (arrLenLoc T a) = IntgT
ltype (arrLoc T a i) = at2jt T
by (simp-all add: ltype-def)
```

Discriminator functions to test whether a location denotes an array length or whether it denotes a static object. Currently, the discriminator functions for object and array locations are not specified. They can be added if they are needed.

```
definition isArrLenLoc:: Location  $\Rightarrow$  bool where
isArrLenLoc l = (case l of
  objLoc cf a  $\Rightarrow$  False
| staticLoc f  $\Rightarrow$  False
| arrLenLoc T a  $\Rightarrow$  True
| arrLoc T a i  $\Rightarrow$  False)
```

```
lemma isArrLenLoc-simps [simp]:
isArrLenLoc (objLoc cf a) = False
isArrLenLoc (staticLoc f) = False
isArrLenLoc (arrLenLoc T a) = True
isArrLenLoc (arrLoc T a i) = False
by (simp-all add: isArrLenLoc-def)
```



**definition** *isStaticLoc*:: *Location*  $\Rightarrow$  *bool* **where**

*isStaticLoc* *l* = (case *l* of  
     *objLoc* *cf* *a*  $\Rightarrow$  *False*  
   | *staticLoc* *f*  $\Rightarrow$  *True*  
   | *arrLenLoc* *T* *a*  $\Rightarrow$  *False*  
   | *arrLoc* *T* *a* *i*  $\Rightarrow$  *False*)

**lemma** *isStaticLoc-simps* [*simp*]:

*isStaticLoc* (*objLoc* *cf* *a*) = *False*  
*isStaticLoc* (*staticLoc* *f*) = *True*  
*isStaticLoc* (*arrLenLoc* *T* *a*) = *False*  
*isStaticLoc* (*arrLoc* *T* *a* *i*) = *False*  
**by** (*simp-all* add: *isStaticLoc-def*)

The function *ref* yields the object or array containing the location that is passed as argument (see the function *obj* in [PH97, p. 43 f.]). Note that for static locations the result is *nullV* since static locations are not associated to any object.

**definition** *ref*:: *Location*  $\Rightarrow$  *Value* **where**

*ref* *l* = (case *l* of  
     *objLoc* *cf* *a*  $\Rightarrow$  *objV* (*cls* *cf*) *a*  
   | *staticLoc* *f*  $\Rightarrow$  *nullV*  
   | *arrLenLoc* *T* *a*  $\Rightarrow$  *arrV* *T* *a*  
   | *arrLoc* *T* *a* *i*  $\Rightarrow$  *arrV* *T* *a*)

**lemma** *ref-simps* [*simp*]:

*ref* (*objLoc* *cf* *a*) = *objV* (*cls* *cf*) *a*  
*ref* (*staticLoc* *f*) = *nullV*  
*ref* (*arrLenLoc* *T* *a*) = *arrV* *T* *a*  
*ref* (*arrLoc* *T* *a* *i*) = *arrV* *T* *a*  
**by** (*simp-all* add: *ref-def*)

The function *loc* denotes the subscription of an object reference with an attribute.

**primrec** *loc*:: *Value*  $\Rightarrow$  *AttId*  $\Rightarrow$  *Location* (-.- [80,80] 80)

**where** *loc* (*objV* *c* *a*) *f* = *objLoc* (*catt* *c* *f*) *a*

Note that we only define subscription properly for object references. For all other values we do not provide any defining equation, so they will internally be mapped to *arbitrary*.

The length of an array can be selected with the function *arr-len*.

**primrec** *arr-len*:: *Value*  $\Rightarrow$  *Location*

**where** *arr-len* (*arrV* *T* *a*) = *arrLenLoc* *T* *a*

Arrays can be indexed by the function *arr-loc*.

**primrec** *arr-loc*:: *Value*  $\Rightarrow$  *nat*  $\Rightarrow$  *Location* (-.[-] [80,80] 80)

**where** *arr-loc* (*arrV* *T* *a*) *i* = *arrLoc* *T* *a* *i*

The functions *loc*, *arr-len* and *arr-loc* define the interface between the basic store model (based on locations) and the programming language Java. Instance field access *obj.x* is modelled as *obj..x* or *loc obj x* (without the syntactic sugar), array length *a.length* with *arr-len a*, array indexing *a[i]* with *a.[i]* or *arr-loc a i*. The accessing of a static field *C.f* can be expressed by the location itself *staticLoc C'f*. Of course one can build more infrastructure to make access to instance fields and static fields more uniform. We could for example define a function *static*

which indicates whether a field is static or not and based on that create an *objLoc* location or a *staticLoc* location. But this will only complicate the actual proofs and we can already easily perform the distinction whether a field is static or not in the JIVE-frontend and therefore keep the verification simpler.

```
lemma ref-loc [simp]:  $\llbracket isObjV\ r; typeof\ r \leq\ dtype\ f \rrbracket \implies ref\ (r..f) = r$ 
  apply (case-tac r)
  apply (case-tac  $\llbracket ! \rrbracket f$ )
  apply (simp-all)
done
```

```
lemma obj-arr-loc [simp]:  $isArrV\ r \implies ref\ (r.[i]) = r$ 
  by (cases r) simp-all
```

```
lemma obj-arr-len [simp]:  $isArrV\ r \implies ref\ (arr-len\ r) = r$ 
  by (cases r) simp-all
```

**end**

## 12 Store

```
theory Store
imports Location
begin
```

### 12.1 New

The store provides a uniform interface to allocate new objects and new arrays. The constructors of this datatype distinguish both cases.

```
datatype New = new-instance CTypeId — New object, can only be of a concrete class type
  | new-array ArrayType nat — New array with given size
```

The discriminator *isNewArr* can be used to distinguish both kinds of newly created elements.

```
definition isNewArr :: New  $\Rightarrow$  bool where
isNewArr t = (case t of
  new-instance C  $\Rightarrow$  False
  | new-array T l  $\Rightarrow$  True)
```

```
lemma isNewArr-simps [simp]:
isNewArr (new-instance C) = False
isNewArr (new-array T l) = True
  by (simp-all add: isNewArr-def)
```

The function *typeofNew* yields the type of the newly created element.

```
definition typeofNew :: New  $\Rightarrow$  Javatype where
typeofNew n = (case n of
  new-instance C  $\Rightarrow$  CClassT C
  | new-array T l  $\Rightarrow$  ArrT T)
```

```
lemma typeofNew-simps:
typeofNew (new-instance C) = CClassT C
typeofNew (new-array T l) = ArrT T
  by (simp-all add: typeofNew-def)
```

## 12.2 The Definition of the Store

In our store model, all objects<sup>2</sup> of all classes exist at all times, but only those objects that have already been allocated are alive. Objects cannot be deallocated, thus an object that once gained the aliveness status cannot lose it later on.

To model the store, we need two functions that give us fresh object Id's for the allocation of new objects (function *newOID*) and arrays (function *newAID*) as well as a function that maps locations to their contents (function *vals*).

```
record StoreImpl = newOID :: CTypeId ⇒ ObjectId
                newAID :: Arraytype ⇒ ObjectId
                vals   :: Location ⇒ Value
```

The function *aliveImpl* determines for a given value whether it is alive in a given store.

```
definition aliveImpl :: Value ⇒ StoreImpl ⇒ bool where
aliveImpl x s = (case x of
  | boolV b ⇒ True
  | intgV i ⇒ True
  | shortV s ⇒ True
  | byteV by ⇒ True
  | objV C a ⇒ (a < newOID s C)
  | arrV T a ⇒ (a < newAID s T)
  | nullV   ⇒ True)
```

The store itself is defined as new type. The store ensures and maintains the following properties: All stored values are alive; for all locations whose values are not alive, the store yields the location type's init value; and all stored values are of the correct type (i.e. of the type of the location they are stored in).

```
definition Store = {s. (∀ l. aliveImpl (vals s l) s) ∧
  (∀ l. ¬ aliveImpl (ref l) s → vals s l = init (ltype l)) ∧
  (∀ l. typeof (vals s l) ≤ ltype l)}
```

```
typedef Store = Store
unfolding Store-def
apply (rule exI [where ?x=(| newOID = (λC. 0),
  newAID = (λT. 0),
  vals = (λl. init (ltype l)) |)])
apply (auto simp add: aliveImpl-def init-def NullT-leaf-array split: Javatypе.splits)
done
```

One might also model the Store as axiomatic type class and prove that the type StoreImpl belongs to this type class. This way, a clearer separation between the axiomatic description of the store and its properties on the one hand and the realization that has been chosen in this formalization on the other hand could be achieved. Additionally, it would be easier to make use of different store implementations that might have different additional features. This separation remains to be performed as future work.

<sup>2</sup>In the following, the term “objects” includes arrays. This keeps the explanations compact.

### 12.3 The Store Interface

The Store interface consists of five functions: *access* to read the value that is stored at a location; *alive* to test whether a value is alive in the store; *alloc* to allocate a new element in the store; *new* to read the value of a newly allocated element; *update* to change the value that is stored at a location.

```

consts access:: Store ⇒ Location ⇒ Value (-@@- [71,71] 70)
         alive:: Value ⇒ Store ⇒ bool
         alloc:: Store ⇒ New ⇒ Store
         new:: Store ⇒ New ⇒ Value
         update:: Store ⇒ Location ⇒ Value ⇒ Store

```

**nonterminal** *smodifybinds* and *smodifybind*

**syntax**

```

-smodifybind :: ['a, 'a] ⇒ smodifybind ((?- :=/ -))
              :: smodifybind ⇒ smodifybinds (-)
              :: CTypeId ⇒ smodifybind (-)
-smodifybinds:: [smodifybind, smodifybinds] => smodifybinds (-,/ -)
-sModify :: ['a, smodifybinds] ⇒ 'a (-/⟨(-)⟩ [900,0] 900)

```

**translations**

```

-sModify s (-smodifybinds b bs) == -sModify (-sModify s b) bs
s⟨x:=y⟩ == CONST update s x y
s⟨c⟩ == CONST alloc s c

```

With this syntactic setup we can write chains of (array) updates and allocations like in the following term  $s\langle \text{new-instance Node}, x := y, z := \text{intgV } 3, \text{new-array IntgAT } 3, a.[i] := \text{intgV } 4, k := \text{boolV True} \rangle$ .

In the following, the definitions of the five store interface functions and some lemmas about them are given.

**overloading** *alive*  $\equiv$  *alive*

**begin**

**definition** *alive where* *alive*  $x$   $s \equiv$  *aliveImpl*  $x$  (*Rep-Store*  $s$ )

**end**

**lemma** *alive-trivial-simps* [*simp,intro*]:

```

alive (boolV b) s
alive (intgV i) s
alive (shortV sh) s
alive (byteV by) s
alive nullV s
by (simp-all add: alive-def aliveImpl-def)

```

**overloading**

```

access ≡ access
update ≡ update
alloc ≡ alloc
new ≡ new

```

**begin**

**definition** *access*

**where** *access*  $s$   $l \equiv$  *vals* (*Rep-Store*  $s$ )  $l$

**definition** *update*

**where**  $update\ s\ l\ v \equiv$   
 if  $alive\ (ref\ l)\ s \wedge alive\ v\ s \wedge typeof\ v \leq ltype\ l$   
 then  $Abs-Store\ ((Rep-Store\ s)\ (vals := (vals\ (Rep-Store\ s))\ (l := v)))$   
 else  $s$

**definition** *alloc*

**where**  $alloc\ s\ t \equiv$   
 (case  $t$  of  
 new-instance  $C$   
 $\Rightarrow Abs-Store$   
 $((Rep-Store\ s)\ (newOID := \lambda D. if\ C=D$   
 $then\ Suc\ (newOID\ (Rep-Store\ s)\ C)$   
 $else\ newOID\ (Rep-Store\ s)\ D))$   
 | new-array  $T\ l$   
 $\Rightarrow Abs-Store$   
 $((Rep-Store\ s)\ (newAID := \lambda S. if\ T=S$   
 $then\ Suc\ (newAID\ (Rep-Store\ s)\ T)$   
 $else\ newAID\ (Rep-Store\ s)\ S,$   
 $vals := (vals\ (Rep-Store\ s))$   
 $(arrLenLoc\ T\ (newAID\ (Rep-Store\ s)\ T)$   
 $:=\ intqV\ (int\ l))))$

**definition** *new*

**where**  $new\ s\ t \equiv$   
 (case  $t$  of  
 new-instance  $C \Rightarrow objV\ C\ (newOID\ (Rep-Store\ s)\ C)$   
 | new-array  $T\ l \Rightarrow arrV\ T\ (newAID\ (Rep-Store\ s)\ T)$

**end**

The predicate *wts* tests whether the store is well-typed.

**definition**

$wts :: Store \Rightarrow bool$  **where**  
 $wts\ OS = (\forall\ (l::Location) . (typeof\ (OS@@l)) \leq (ltype\ l))$

## 12.4 Derived Properties of the Store

In this subsection, a number of lemmas formalize various properties of the Store. Especially the 13 axioms are proven that must hold for a modelling of a Store (see [PH97, p. 45]). They are labeled with Store1 to Store13.

**lemma** *alive-init* [*simp,intro*]:  $alive\ (init\ T)\ s$   
**by** (cases  $T$ ) (*simp-all add: alive-def aliveImpl-def*)

**lemma** *alive-loc* [*simp*]:  
 $\llbracket isObjV\ x; typeof\ x \leq dtype\ f \rrbracket \Longrightarrow alive\ (ref\ (x..f))\ s = alive\ x\ s$   
**by** (cases  $x$ ) (*simp-all*)

**lemma** *alive-arr-loc* [*simp*]:  
 $isArrV\ x \Longrightarrow alive\ (ref\ (x.[i]))\ s = alive\ x\ s$   
**by** (cases  $x$ ) (*simp-all*)

**lemma** *alive-arr-len* [*simp*]:

$isArrV\ x \implies alive\ (ref\ (arr-len\ x))\ s = alive\ x\ s$   
 by (cases x) (simp-all)

**lemma** *ref-arr-len-new* [simp]:  
 $ref\ (arr-len\ (new\ s\ (new-array\ T\ n))) = new\ s\ (new-array\ T\ n)$   
 by (simp add: new-def)

**lemma** *ref-arr-loc-new* [simp]:  
 $ref\ ((new\ s\ (new-array\ T\ n)).[i]) = new\ s\ (new-array\ T\ n)$   
 by (simp add: new-def)

**lemma** *ref-loc-new* [simp]:  $CClassT\ C \leq dtype\ f$   
 $\implies ref\ ((new\ s\ (new-instance\ C)).f) = new\ s\ (new-instance\ C)$   
 by (simp add: new-def)

**lemma** *access-type-safe* [simp,intro]:  $typeof\ (s@@l) \leq ltype\ l$

**proof** –

have *Rep-Store*  $s \in Store$   
 by (rule *Rep-Store*)  
 thus ?thesis  
 by (auto simp add: access-def *Store-def*)

qed

The store is well-typed by construction.

**lemma** *always-welltyped-store*:  $wts\ OS$   
 by (simp add: wts-def access-type-safe)

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**lemma** *alive-access* [simp,intro]:  $alive\ (s@@l)\ s$

**proof** –

have *Rep-Store*  $s \in Store$   
 by (rule *Rep-Store*)  
 thus ?thesis  
 by (auto simp add: access-def *Store-def* *alive-def* *aliveImpl-def*)

qed

Store3

**lemma** *access-unalive* [simp]:  
 assumes *unalive*:  $\neg alive\ (ref\ l)\ s$   
 shows  $s@@l = init\ (ltype\ l)$

**proof** –

have *Rep-Store*  $s \in Store$   
 by (rule *Rep-Store*)  
 with *unalive* show ?thesis  
 by (simp add: access-def *Store-def* *alive-def* *aliveImpl-def*)

qed

**lemma** *update-induct*:

assumes *skip*:  $P\ s$   
 assumes *update*:  $\llbracket alive\ (ref\ l)\ s; alive\ v\ s; typeof\ v \leq ltype\ l \rrbracket \implies$   
 $P\ (Abs-Store\ ((Rep-Store\ s)(vals:= (vals\ (Rep-Store\ s))(l:=v))))$   
 shows  $P\ (s\langle l:=v \rangle)$

**using** *update skip*  
**by** (*simp add: update-def*)

**lemma** *vals-update-in-Store*:

**assumes** *alive-l*: *alive (ref l) s*  
**assumes** *alive-y*: *alive y s*  
**assumes** *type-conform*: *typeof y ≤ ltype l*  
**shows**  $(\text{Rep-Store } s \setminus \text{vals} := (\text{vals } (\text{Rep-Store } s))(l := y)) \in \text{Store}$   
*(is ?s-upd ∈ Store)*

**proof** –

**have** *s*: *Rep-Store s ∈ Store*  
**by** (*rule Rep-Store*)  
**have** *alloc-eq*: *newOID ?s-upd = newOID (Rep-Store s)*  
**by** *simp*  
**have**  $\forall l. \text{aliveImpl } (\text{vals } ?s\text{-upd } l) ?s\text{-upd}$

**proof**

**fix** *k*  
**show** *aliveImpl (vals ?s-upd k) ?s-upd*  
**proof** (*cases k=l*)  
**case** *True*  
**with** *alive-y* **show** *?thesis*  
**by** (*simp add: alloc-eq alive-def aliveImpl-def split: Value.splits*)  
**next**  
**case** *False*  
**from** *s* **have**  $\forall l. \text{aliveImpl } (\text{vals } (\text{Rep-Store } s) l) (\text{Rep-Store } s)$   
**by** (*simp add: Store-def*)  
**with** *False* **show** *?thesis*  
**by** (*simp add: aliveImpl-def split: Value.splits*)

**qed**

**qed**

**moreover**

**have**  $\forall l. \neg \text{aliveImpl } (\text{ref } l) ?s\text{-upd} \longrightarrow \text{vals } ?s\text{-upd } l = \text{init } (ltype l)$

**proof** (*intro allI impI*)

**fix** *k*  
**assume** *unalive*:  $\neg \text{aliveImpl } (\text{ref } k) ?s\text{-upd}$   
**show** *vals ?s-upd k = init (ltype k)*  
**proof** –  
**from** *unalive* *alive-l*  
**have**  $k \neq l$   
**by** (*auto simp add: alive-def aliveImpl-def split: Value.splits*)  
**hence** *vals ?s-upd k = vals (Rep-Store s) k*  
**by** *simp*  
**moreover from** *unalive*  
**have**  $\neg \text{aliveImpl } (\text{ref } k) (\text{Rep-Store } s)$   
**by** (*simp add: aliveImpl-def split: Value.splits*)  
**ultimately show** *?thesis*  
**using** *s* **by** (*simp add: Store-def*)

**qed**

**qed**

**moreover**

**have**  $\forall l. \text{typeof } (\text{vals } ?s\text{-upd } l) \leq ltype l$

**proof**

**fix** *k* **show** *typeof (vals ?s-upd k) ≤ ltype k*  
**proof** (*cases k=l*)

```

case True
with type-conform show ?thesis
  by simp
next
case False
hence vals ?s-upd k = vals (Rep-Store s) k
  by simp
with s show ?thesis
  by (simp add: Store-def)
qed
qed
ultimately show ?thesis
  by (simp add: Store-def)
qed

```

Store6

```

lemma alive-update-invariant [simp]: alive x (s⟨l:=y⟩) = alive x s
proof (rule update-induct)
  show alive x s = alive x s..
next
assume alive (ref l) s alive y s typeof y ≤ ltype l
hence Rep-Store
  (Abs-Store (Rep-Store s⟨vals := (vals (Rep-Store s))(l := y)⟩))
  = Rep-Store s⟨vals := (vals (Rep-Store s))(l := y)⟩
  by (rule vals-update-in-Store [THEN Abs-Store-inverse])
thus alive x
  (Abs-Store (Rep-Store s⟨vals := (vals (Rep-Store s))(l := y)⟩)) =
  alive x s
  by (simp add: alive-def aliveImpl-def split: Value.split)
qed

```

Store1

```

lemma access-update-other [simp]:
  assumes neq-l-m: l ≠ m
  shows s⟨l:=x⟩@@m = s@@m
proof (rule update-induct)
  show s@@m = s@@m ..
next
assume alive (ref l) s alive x s typeof x ≤ ltype l
hence Rep-Store
  (Abs-Store (Rep-Store s⟨vals := (vals (Rep-Store s))(l := x)⟩))
  = Rep-Store s⟨vals := (vals (Rep-Store s))(l := x)⟩
  by (rule vals-update-in-Store [THEN Abs-Store-inverse])
with neq-l-m
show Abs-Store (Rep-Store s⟨vals := (vals (Rep-Store s))(l := x)⟩)@@m = s@@m
  by (auto simp add: access-def)
qed

```

Store2

```

lemma update-access-same [simp]:
  assumes alive-l: alive (ref l) s
  assumes alive-x: alive x s
  assumes widen-x-l: typeof x ≤ ltype l

```



shows  $s\langle l:=x \rangle @ @ l = x$   
**proof** –  
**from** *alive-l alive-x widen-x-l*  
**have** *Rep-Store*  
     (*Abs-Store* (*Rep-Store*  $s\langle \text{vals} := (\text{vals} (\text{Rep-Store } s))(l := x) \rangle$ ))  
     = *Rep-Store*  $s\langle \text{vals} := (\text{vals} (\text{Rep-Store } s))(l := x) \rangle$   
**by** (*rule vals-update-in-Store [THEN Abs-Store-inverse]*)  
**hence** *Abs-Store* (*Rep-Store*  $s\langle \text{vals} := (\text{vals} (\text{Rep-Store } s))(l := x) \rangle$ ) @ @ l =  $x$   
**by** (*simp add: access-def*)  
**with** *alive-l alive-x widen-x-l*  
**show** *?thesis*  
**by** (*simp add: update-def*)  
**qed**

Store4

**lemma** *update-unalive-val* [*simp,intro*]:  $\neg \text{alive } x \ s \implies s\langle l:=x \rangle = s$   
**by** (*simp add: update-def*)

**lemma** *update-unalive-loc* [*simp,intro*]:  $\neg \text{alive } (\text{ref } l) \ s \implies s\langle l:=x \rangle = s$   
**by** (*simp add: update-def*)

**lemma** *update-type-mismatch* [*simp,intro*]:  $\neg \text{typeof } x \leq \text{ltype } l \implies s\langle l:=x \rangle = s$   
**by** (*simp add: update-def*)

Store9

**lemma** *alive-primitive* [*simp,intro*]: *isprimitive* (*typeof*  $x$ )  $\implies \text{alive } x \ s$   
**by** (*cases*  $x$ ) (*simp-all*)

Store10

**lemma** *new-unalive-old-Store* [*simp*]:  $\neg \text{alive } (\text{new } s \ t) \ s$   
**by** (*cases*  $t$ ) (*simp-all add: alive-def aliveImpl-def new-def*)

**lemma** *alloc-new-instance-in-Store*:

(*Rep-Store*  $s\langle \text{newOID} := \lambda D. \text{if } C = D$   
     *then* *Suc* (*newOID* (*Rep-Store*  $s$ )  $C$ )  
     *else* *newOID* (*Rep-Store*  $s$ )  $D \rangle$ )  $\in \text{Store}$

(*is* *?s-alloc*  $\in \text{Store}$ )

**proof** –

**have**  $s: \text{Rep-Store } s \in \text{Store}$

**by** (*rule Rep-Store*)

**hence**  $\forall l. \text{aliveImpl } (\text{vals } (\text{Rep-Store } s) \ l) \ (\text{Rep-Store } s)$

**by** (*simp add: Store-def*)

**then**

**have**  $\forall l. \text{aliveImpl } (\text{vals } ?s\text{-alloc } l) \ ?s\text{-alloc}$

**by** (*auto intro: less-SucI simp add: aliveImpl-def split: Value.splits*)

**moreover**

**have**  $\forall l. \neg \text{aliveImpl } (\text{ref } l) \ ?s\text{-alloc} \longrightarrow \text{vals } ?s\text{-alloc } l = \text{init } (\text{ltype } l)$

**proof** (*intro allI impI*)

**fix**  $l$

**assume**  $\neg \text{aliveImpl } (\text{ref } l) \ ?s\text{-alloc}$

**hence**  $\neg \text{aliveImpl } (\text{ref } l) \ (\text{Rep-Store } s)$

**by** (*simp add: aliveImpl-def split: Value.splits if-split-asm*)

**with**  $s$  **have**  $\text{vals } (\text{Rep-Store } s) \ l = \text{init } (\text{ltype } l)$

```

    by (simp add: Store-def)
  thus vals ?s-alloc l = init (ltype l)
    by simp
qed
moreover
from s have  $\forall l. \text{typeof } (vals ?s-alloc l) \leq \text{ltype } l$ 
  by (simp add: Store-def)
ultimately
show ?thesis
  by (simp add: Store-def)
qed

```

**lemma** *alloc-new-array-in-Store*:

```

(Rep-Store s (newAID :=
   $\lambda S. \text{if } T = S$ 
    then Suc (newAID (Rep-Store s) T)
    else newAID (Rep-Store s) S,
  vals := (vals (Rep-Store s)
    (arrLenLoc T
      (newAID (Rep-Store s) T) :=
        intgV (int n))))  $\in$  Store

```

(is ?s-alloc  $\in$  Store)

**proof** –

have  $s: \text{Rep-Store } s \in \text{Store}$

by (rule Rep-Store)

have  $\forall l. \text{aliveImpl } (vals ?s-alloc l) ?s-alloc$

**proof**

fix l show  $\text{aliveImpl } (vals ?s-alloc l) ?s-alloc$

**proof** (cases  $l = \text{arrLenLoc } T \text{ (newAID (Rep-Store } s) T)$ )

case True

thus ?thesis

by (simp add: aliveImpl-def split: Value.splits)

next

case False

from s have  $\forall l. \text{aliveImpl } (vals (Rep-Store s) l) (Rep-Store s)$

by (simp add: Store-def)

with False show ?thesis

by (auto intro: less-SucI simp add: aliveImpl-def split: Value.splits)

qed

qed

moreover

have  $\forall l. \neg \text{aliveImpl } (\text{ref } l) ?s-alloc \longrightarrow \text{vals } ?s-alloc l = \text{init } (\text{ltype } l)$

**proof** (intro allI impI)

fix l

assume unalive:  $\neg \text{aliveImpl } (\text{ref } l) ?s-alloc$

show  $\text{vals } ?s-alloc l = \text{init } (\text{ltype } l)$

**proof** (cases  $l = \text{arrLenLoc } T \text{ (newAID (Rep-Store } s) T)$ )

case True

with unalive show ?thesis by (simp add: aliveImpl-def)

next

case False

from unalive

have  $\neg \text{aliveImpl } (\text{ref } l) (Rep-Store s)$

by (simp add: aliveImpl-def split: Value.splits if-split-asm)

```

with  $s$  have  $vals$  ( $Rep\text{-}Store\ s$ )  $l = init$  ( $ltype\ l$ )
  by ( $simp\ add: Store\text{-}def$ )
with  $False$  show  $?thesis$ 
  by  $simp$ 
qed
qed
moreover
from  $s$  have  $\forall l. typeof\ (vals\ ?s\text{-}alloc\ l) \leq ltype\ l$ 
  by ( $simp\ add: Store\text{-}def$ )
ultimately
show  $?thesis$ 
  by ( $simp\ add: Store\text{-}def$ )
qed

```

```

lemma  $new\text{-}alive\text{-}alloc$  [ $simp, intro$ ]:  $alive\ (new\ s\ t)\ (s(t))$ 
proof ( $cases\ t$ )
  case  $new\text{-}instance$  thus  $?thesis$ 
    by ( $simp\ add: alive\text{-}def\ aliveImpl\text{-}def\ new\text{-}def\ alloc\text{-}def$ 
       $alloc\text{-}new\text{-}instance\text{-}in\text{-}Store\ [THEN\ Abs\text{-}Store\text{-}inverse]$ )
next
  case  $new\text{-}array$  thus  $?thesis$ 
    by ( $simp\ add: alive\text{-}def\ aliveImpl\text{-}def\ new\text{-}def\ alloc\text{-}def$ 
       $alloc\text{-}new\text{-}array\text{-}in\text{-}Store\ [THEN\ Abs\text{-}Store\text{-}inverse]$ )
qed

```

```

lemma  $value\text{-}class\text{-}inhabitants$ :
 $(\forall x. typeof\ x = CClassT\ typeId \longrightarrow P\ x) = (\forall a. P\ (objV\ typeId\ a))$ 
 $(is\ (\forall x. ?A\ x) = ?B)$ 
proof
  assume  $\forall x. ?A\ x$  thus  $?B$ 
    by  $simp$ 
next
  assume  $B: ?B$  show  $\forall x. ?A\ x$ 
  proof
    fix  $x$  from  $B$  show  $?A\ x$ 
    by ( $cases\ x$ )  $auto$ 
  qed
qed

```

```

lemma  $value\text{-}array\text{-}inhabitants$ :
 $(\forall x. typeof\ x = ArrT\ typeId \longrightarrow P\ x) = (\forall a. P\ (arrV\ typeId\ a))$ 
 $(is\ (\forall x. ?A\ x) = ?B)$ 
proof
  assume  $\forall x. ?A\ x$  thus  $?B$ 
    by  $simp$ 
next
  assume  $B: ?B$  show  $\forall x. ?A\ x$ 
  proof
    fix  $x$  from  $B$  show  $?A\ x$ 
    by ( $cases\ x$ )  $auto$ 
  qed
qed

```

The following three lemmas are helper lemmas that are not related to the store theory. They might as well be stored in a separate helper theory.

**lemma** *le-Suc-eq*:  $(\forall a. (a < \text{Suc } n) = (a < \text{Suc } m)) = (\forall a. (a < n) = (a < m))$   
*(is*  $(\forall a. ?A a) = (\forall a. ?B a)$ )

**proof**

**assume**  $\forall a. ?A a$  **thus**  $\forall a. ?B a$   
 by *fastforce*

**next**

**assume**  $B: \forall a. ?B a$   
**show**  $\forall a. ?A a$

**proof**

**fix**  $a$

**from**  $B$  **show**  $?A a$

by *(cases a) simp-all*

**qed**

**qed**

**lemma** *all-le-eq-imp-eq*:  $\bigwedge c::\text{nat}. (\forall a. (a < d) = (a < c)) \longrightarrow (d = c)$

**proof** *(induct d)*

**case**  $0$  **thus**  $?case$  by *fastforce*

**next**

**case**  $(\text{Suc } n \ c)$

**thus**  $?case$

by *(cases c) (auto simp add: le-Suc-eq)*

**qed**

**lemma** *all-le-eq*:  $(\forall a::\text{nat}. (a < d) = (a < c)) = (d = c)$

**using** *all-le-eq-imp-eq* by *auto*

Store11

**lemma** *typeof-new*:  $\text{typeof } (\text{new } s \ t) = \text{typeofNew } t$

by *(cases t) (simp-all add: new-def typeofNew-def)*

Store12

**lemma** *new-eq*:  $(\text{new } s1 \ t = \text{new } s2 \ t) =$

$(\forall x. \text{typeof } x = \text{typeofNew } t \longrightarrow \text{alive } x \ s1 = \text{alive } x \ s2)$

by *(cases t)*

*(auto simp add: new-def typeofNew-def alive-def aliveImpl-def  
 value-class-inhabitants value-array-inhabitants all-le-eq)*

**lemma** *new-update [simp]*:  $\text{new } (s[l:=x]) \ t = \text{new } s \ t$

by *(simp add: new-eq)*

**lemma** *alive-alloc-propagation*:

**assumes** *alive-s*:  $\text{alive } x \ s$  **shows**  $\text{alive } x \ (s\langle t \rangle)$

**proof** *(cases t)*

**case** *new-instance* **with** *alive-s* **show**  $?thesis$

by *(cases x)*

*(simp-all add: alive-def aliveImpl-def alloc-def*

*alloc-new-instance-in-Store [THEN Abs-Store-inverse])*

**next**

**case** *new-array* **with** *alive-s* **show**  $?thesis$

by *(cases x)*

```

      (simp-all add: alive-def aliveImpl-def alloc-def
        alloc-new-array-in-Store [THEN Abs-Store-inverse])
qed

Store7

lemma alive-alloc-exhaust: alive x (s⟨t⟩) = (alive x s ∨ (x = new s t))
proof
  assume alive-alloc: alive x (s⟨t⟩)
  show alive x s ∨ x = new s t
  proof (cases t)
    case (new-instance C)
    with alive-alloc show ?thesis
    by (cases x) (auto split: if-split-asm
      simp add: alive-def new-def alloc-def aliveImpl-def
      alloc-new-instance-in-Store [THEN Abs-Store-inverse])
  next
  case (new-array T l)
  with alive-alloc show ?thesis
  by (cases x) (auto split: if-split-asm
    simp add: alive-def new-def alloc-def aliveImpl-def
    alloc-new-array-in-Store [THEN Abs-Store-inverse])

  qed
next
assume alive x s ∨ x = new s t
then show alive x (s⟨t⟩)
proof
  assume alive x s thus ?thesis by (rule alive-alloc-propagation)
next
assume new: x=new s t show ?thesis
proof (cases t)
  case new-instance with new show ?thesis
  by (simp add: alive-def aliveImpl-def new-def alloc-def
    alloc-new-instance-in-Store [THEN Abs-Store-inverse])
  next
  case new-array with new show ?thesis
  by (simp add: alive-def aliveImpl-def new-def alloc-def
    alloc-new-array-in-Store [THEN Abs-Store-inverse])
  qed
qed
qed
qed

lemma alive-alloc-cases [consumes 1]:
  ⟦alive x (s⟨t⟩); alive x s ⟹ P; x=new s t ⟹ P⟧
  ⟹ P
  by (auto simp add: alive-alloc-exhaust)

lemma aliveImpl-vals-independent: aliveImpl x (s⟨vals := z⟩) = aliveImpl x s
  by (cases x) (simp-all add: aliveImpl-def)

lemma access-arr-len-new-alloc [simp]:
  s⟨new-array T l⟩@@arr-len (new s (new-array T l)) = intGV (int l)
  by (subst access-def)
  (simp add: new-def alloc-def alive-def
    alloc-new-array-in-Store [THEN Abs-Store-inverse] access-def)

```

**lemma** *access-new* [*simp*]:  
**assumes** *ref-new*:  $\text{ref } l = \text{new } s \ t$   
**assumes** *no-arr-len*:  $\text{isNewArr } t \longrightarrow l \neq \text{arr-len } (\text{new } s \ t)$   
**shows**  $s\langle t \rangle @ @ l = \text{init } (\text{ltype } l)$   
**proof** –  
**from** *ref-new*  
**have**  $\neg \text{alive } (\text{ref } l) \ s$   
  **by** *simp*  
**hence**  $s @ @ l = \text{init } (\text{ltype } l)$   
  **by** *simp*  
**moreover**  
**from** *ref-new*  
**have**  $\text{alive } (\text{ref } l) \ (s\langle t \rangle)$   
  **by** *simp*  
**moreover**  
**from** *no-arr-len*  
**have**  $\text{vals } (\text{Rep-Store } (s\langle t \rangle)) \ l = s @ @ l$   
  **by** (*cases t*)  
  (*simp-all add: alloc-def new-def access-def*  
  *alloc-new-instance-in-Store [THEN Abs-Store-inverse]*  
  *alloc-new-array-in-Store [THEN Abs-Store-inverse]* )  
**ultimately show**  $s\langle t \rangle @ @ l = \text{init } (\text{ltype } l)$   
  **by** (*subst access-def*) (*simp*)  
**qed**

Store5. We have to take into account that the length of an array is changed during allocation.

**lemma** *access-alloc* [*simp*]:  
**assumes** *no-arr-len-new*:  $\text{isNewArr } t \longrightarrow l \neq \text{arr-len } (\text{new } s \ t)$   
**shows**  $s\langle t \rangle @ @ l = s @ @ l$   
**proof** –  
**show** *?thesis*  
**proof** (*cases alive (ref l) (s⟨t⟩)*)  
  **case** *True*  
  **then**  
  **have** *access-alloc-vals*:  $s\langle t \rangle @ @ l = \text{vals } (\text{Rep-Store } (s\langle t \rangle)) \ l$   
  **by** (*simp add: access-def alloc-def*)  
  **from** *True* **show** *?thesis*  
  **proof** (*cases rule: alive-alloc-cases*)  
  **assume** *alive-l-s*:  $\text{alive } (\text{ref } l) \ s$   
  **with** *new-unalive-old-Store*  
  **have** *l-not-new*:  $\text{ref } l \neq \text{new } s \ t$   
  **by** *fastforce*  
  **hence**  $\text{vals } (\text{Rep-Store } (s\langle t \rangle)) \ l = s @ @ l$   
  **by** (*cases t*)  
  (*auto simp add: alloc-def new-def access-def*  
  *alloc-new-instance-in-Store [THEN Abs-Store-inverse]*  
  *alloc-new-array-in-Store [THEN Abs-Store-inverse]*)  
  **with** *access-alloc-vals*  
  **show** *?thesis*  
  **by** *simp*  
**next**  
**assume** *ref-new*:  $\text{ref } l = \text{new } s \ t$   
**with** *no-arr-len-new*

```

have  $s(t)@@l = \text{init } (ltype\ l)$ 
  by (simp add: access-new)
moreover
from ref-new have  $s@@l = \text{init } (ltype\ l)$ 
  by simp
ultimately
show ?thesis by simp
qed
next
case False
hence  $s(t)@@l = \text{init } (ltype\ l)$ 
  by (simp)
moreover
from False have  $\neg \text{alive } (ref\ l)\ s$ 
  by (auto simp add: alive-alloc-propagation)
hence  $s@@l = \text{init } (ltype\ l)$ 
  by simp
ultimately show ?thesis by simp
qed
qed

```

Store13

**lemma** *Store-eqI*:

```

assumes eq-alive:  $\forall x. \text{alive } x\ s1 = \text{alive } x\ s2$ 
assumes eq-access:  $\forall l. s1@@l = s2@@l$ 
shows  $s1 = s2$ 
proof (cases s1=s2)
  case True thus ?thesis .
next
case False note neq-s1-s2 = this
show ?thesis
proof (cases newOID (Rep-Store s1) = newOID (Rep-Store s2))
  case False
have  $\exists C. \text{newOID } (Rep\text{-Store } s1)\ C \neq \text{newOID } (Rep\text{-Store } s2)\ C$ 
proof (rule ccontr)
  assume  $\neg (\exists C. \text{newOID } (Rep\text{-Store } s1)\ C \neq \text{newOID } (Rep\text{-Store } s2)\ C)$ 
then have  $\text{newOID } (Rep\text{-Store } s1) = \text{newOID } (Rep\text{-Store } s2)$ 
  by (blast intro: ext)
with False show False ..
qed
with eq-alive obtain C
  where  $\text{newOID } (Rep\text{-Store } s1)\ C \neq \text{newOID } (Rep\text{-Store } s2)\ C$ 
   $\forall a. \text{alive } (objV\ C\ a)\ s1 = \text{alive } (objV\ C\ a)\ s2$  by auto
then show ?thesis
  by (simp add: all-le-eq alive-def aliveImpl-def)
next
case True note eq-newOID = this
show ?thesis
proof (cases newAID (Rep-Store s1) = newAID (Rep-Store s2))
  case False
have  $\exists T. \text{newAID } (Rep\text{-Store } s1)\ T \neq \text{newAID } (Rep\text{-Store } s2)\ T$ 
proof (rule ccontr)
  assume  $\neg (\exists T. \text{newAID } (Rep\text{-Store } s1)\ T \neq \text{newAID } (Rep\text{-Store } s2)\ T)$ 
then have  $\text{newAID } (Rep\text{-Store } s1) = \text{newAID } (Rep\text{-Store } s2)$ 

```

```

    by (blast intro: ext)
  with False show False ..
qed
with eq-alive obtain T
  where newAID (Rep-Store s1) T ≠ newAID (Rep-Store s2) T
    ∀ a. alive (arrV T a) s1 = alive (arrV T a) s2 by auto
then show ?thesis
  by (simp add: all-le-eq alive-def aliveImpl-def)
next
case True note eq-newAID = this
show ?thesis
proof (cases vals (Rep-Store s1) = vals (Rep-Store s2))
  case True
  with eq-newOID eq-newAID
  have (Rep-Store s1) = (Rep-Store s2)
    by (cases Rep-Store s1, cases Rep-Store s2) simp
  hence s1=s2
    by (simp add: Rep-Store-inject)
  with neq-s1-s2 show ?thesis
    by simp
next
case False
have ∃ l. vals (Rep-Store s1) l ≠ vals (Rep-Store s2) l
proof (rule ccontr)
  assume ¬ (∃ l. vals (Rep-Store s1) l ≠ vals (Rep-Store s2) l)
  hence vals (Rep-Store s1) = vals (Rep-Store s2)
    by (blast intro: ext)
  with False show False ..
qed
then obtain l
  where vals (Rep-Store s1) l ≠ vals (Rep-Store s2) l
    by auto
with eq-access have False
  by (simp add: access-def)
thus ?thesis ..
qed
qed
qed
qed

```

Lemma 3.1 in [Poetzsch-Heffter97]. The proof of this lemma is quite an impressive demonstration of readable Isar proofs since it closely follows the textual proof.

**lemma comm:**

```

  assumes neq-l-new: ref l ≠ new s t
  assumes neq-x-new: x ≠ new s t
  shows s⟨t⟩⟨l:=x⟩ = s⟨l:=x⟩⟨t⟩
proof (rule Store-eqI [rule-format])
  fix y
  show alive y (s⟨t⟩⟨l:=x⟩) = alive y (s⟨l:=x⟩⟨t⟩)
proof -
  have alive y (s⟨t⟩⟨l:=x⟩) = alive y (s⟨t⟩)
    by (rule alive-update-invariant)
  also have ... = (alive y s ∨ (y = new s t))
    by (rule alive-alloc-exhaust)

```



```

also have ... = (alive y (s⟨l:=x⟩) ∨ y = new s t)
  by (simp only: alive-update-invariant)
also have ... = (alive y (s⟨l:=x⟩) ∨ y = new (s⟨l:=x⟩) t)
proof –
  have new s t = new (s⟨l:=x⟩) t
    by simp
  thus ?thesis by simp
qed
also have ... = alive y (s⟨l:=x⟩⟨t⟩)
  by (simp add: alive-alloc-exhaust)
finally show ?thesis .
qed
next
fix k
show s⟨t⟩⟨l := x⟩@@k = s⟨l := x⟩⟨t⟩@@k
proof (cases l=k)
  case False note neq-l-k = this
  show ?thesis
proof (cases isNewArr t → k ≠ arr-len (new s t))
  case True
  from neq-l-k
  have s⟨t⟩⟨l := x⟩@@k = s⟨t⟩@@k by simp
  also from True
  have ... = s@@k by simp
  also from neq-l-k
  have ... = s⟨l:=x⟩@@k by simp
  also from True
  have ... = s⟨l := x⟩⟨t⟩@@k by simp
  finally show ?thesis .
next
case False
then obtain T n where
  t: t=new-array T n and k: k=arr-len (new s (new-array T n))
  by (cases t) auto
from k have k': k=arr-len (new (s⟨l := x⟩) (new-array T n))
  by simp
from neq-l-k
have s⟨t⟩⟨l := x⟩@@k = s⟨t⟩@@k by simp
also from t k
have ... = intgV (int n)
  by simp
also from t k'
have ... = s⟨l := x⟩⟨t⟩@@k
  by (simp del: new-update)
finally show ?thesis .
qed
next
case True note eq-l-k = this
have lemma-3-1:
  refl ≠ new s t ⇒ alive (ref l) (s⟨t⟩) = alive (ref l) s
  by (simp add: alive-alloc-exhaust)
have lemma-3-2:
  x ≠ new s t ⇒ alive x (s⟨t⟩) = alive x s
  by (simp add: alive-alloc-exhaust)

```

---

```

have lemma-3-3: s⟨l:=x,t⟩@@l = s⟨l:=x⟩@@l
proof -
  from neq-l-new have refl l ≠ new (s⟨l:=x⟩) t
    by simp
  hence isNewArr t ⟶ l ≠ arr-len (new (s⟨l:=x⟩) t)
    by (cases t) auto
  thus ?thesis
    by (simp)
qed
show ?thesis
proof (cases alive x s)
  case True note alive-x = this
  show ?thesis
  proof (cases alive (ref l) s)
    case True note alive-l = this
    show ?thesis
    proof (cases typeof x ≤ ltype l)
      case True
      with alive-l alive-x
      have s⟨l:=x⟩@@l = x
        by (rule update-access-same)
      moreover
      have s⟨t⟩⟨l:=x⟩@@l = x
      proof -
        from alive-l neq-l-new have alive (ref l) (s⟨t⟩)
          by (simp add: lemma-3-1)
        moreover
        from alive-x neq-x-new have alive x (s⟨t⟩)
          by (simp add: lemma-3-2)
        ultimately
        show s⟨t⟩⟨l:=x⟩@@l = x
          using True by (rule update-access-same)
      qed
      ultimately show ?thesis
        using eq-l-k lemma-3-3 by simp
    next
    case False
    thus ?thesis by simp
  qed
next
  case False note not-alive-l = this
  from not-alive-l neq-l-new have ¬ alive (ref l) (s⟨t⟩)
    by (simp add: lemma-3-1)
  then have s⟨t⟩⟨l:=x⟩@@l = init (ltype l)
    by simp
  also from not-alive-l have ... = s⟨l:=x⟩@@l
    by simp
  also have ... = s⟨l:=x⟩⟨t⟩@@l
    by (simp add: lemma-3-3)
  finally show ?thesis by (simp add: eq-l-k)
qed
next
  case False note not-alive-x = this
  from not-alive-x neq-x-new have ¬ alive x (s⟨t⟩)

```

---

```

    by (simp add: lemma-3-2)
  then have  $s\langle t\rangle\langle l:=x\rangle@@l = s\langle t\rangle@@l$ 
    by (simp)
  also have  $\dots = s@@l$ 
  proof -
    from neg-l-new
    have  $isNewArr\ t \longrightarrow l \neq arr-len\ (new\ s\ t)$ 
      by (cases t) auto
    thus ?thesis
      by (simp)
  qed
  also from not-alive-x have  $\dots = s\langle l:=x\rangle@@l$ 
    by (simp)
  also have  $\dots = s\langle l:=x\rangle\langle t\rangle@@l$ 
    by (simp add: lemma-3-3)
  finally show ?thesis by (simp add: eq-l-k)
qed
qed
qed

end

```

## 13 Store Properties

```

theory StoreProperties
imports Store
begin

```

This theory formalizes advanced concepts and properties of stores.

### 13.1 Reachability of a Location from a Reference

For a given store, the function *reachS* yields the set of all pairs  $(l, v)$  where  $l$  is a location that is reachable from the value  $v$  (which must be a reference) in the given store. The predicate *reach* decides whether a location is reachable from a value in a store.

**inductive**

```

  reach :: Store  $\Rightarrow$  Location  $\Rightarrow$  Value  $\Rightarrow$  bool
  ( $\vdash$  - reachable'-from - [91,91,91]90)

```

```

  for  $s :: Store$ 

```

**where**

```

  Immediate:  $ref\ l \neq nullV \implies s\vdash\ l\ reachable-from\ (ref\ l)$ 
| Indirect:  $\llbracket s\vdash\ l\ reachable-from\ (s@@k); ref\ k \neq nullV \rrbracket$ 
   $\implies s\vdash\ l\ reachable-from\ (ref\ k)$ 

```

Note that we explicitly exclude *nullV* as legal reference for reachability. Keep in mind that static fields are not associated to any object, therefore *ref* yields *nullV* if invoked on static fields (see the definition of the function *ref*, Sect. 11). Reachability only describes the locations directly reachable from the object or array by following the pointers and should not include the static fields if we encounter a *nullV* reference in the pointer chain.

We formalize some properties of reachability. Especially, Lemma 3.2 as given in [PH97, p. 53] is proven.

**lemma** *unreachable-Null*:

**assumes** *reach*:  $s \vdash l$  *reachable-from*  $x$  **shows**  $x \neq \text{null}V$   
**using** *reach* **by** (*induct*) *auto*

**corollary** *unreachable-Null-simp* [*simp*]:

$\neg s \vdash l$  *reachable-from*  $\text{null}V$   
**by** (*iprover* *dest*: *unreachable-Null*)

**corollary** *unreachable-NullE* [*elim*]:

$s \vdash l$  *reachable-from*  $\text{null}V \implies P$   
**by** (*simp*)

**lemma** *reachObjLoc* [*simp,intro*]:

$C = \text{cls } cf \implies s \vdash \text{objLoc } cf \ a$  *reachable-from*  $\text{obj}V \ C \ a$   
**by** (*iprover* *intro*: *reach.Immediate* [*of objLoc cf a,simplified*])

**lemma** *reachArrLoc* [*simp,intro*]:  $s \vdash \text{arrLoc } T \ a \ i$  *reachable-from*  $\text{arr}V \ T \ a$

**by** (*rule* *reach.Immediate* [*of arrLoc T a i,simplified*])

**lemma** *reachArrLen* [*simp,intro*]:  $s \vdash \text{arrLenLoc } T \ a$  *reachable-from*  $\text{arr}V \ T \ a$

**by** (*rule* *reach.Immediate* [*of arrLenLoc T a,simplified*])

**lemma** *unreachStatic* [*simp*]:  $\neg s \vdash \text{staticLoc } f$  *reachable-from*  $x$

**proof** –

{  
**fix**  $y$  **assume**  $s \vdash y$  *reachable-from*  $x$   $y = \text{staticLoc } f$   
**then have** *False*  
**by** *induct auto*

}  
**thus** *?thesis*

**by** *auto*

**qed**

**lemma** *unreachStaticE* [*elim*]:  $s \vdash \text{staticLoc } f$  *reachable-from*  $x \implies P$

**by** (*simp* *add*: *unreachStatic*)

**lemma** *reachable-from-ArrLoc-impl-Arr* [*simp,intro*]:

**assumes** *reach-loc*:  $s \vdash l$  *reachable-from*  $(s@@\text{arrLoc } T \ a \ i)$

**shows**  $s \vdash l$  *reachable-from*  $(\text{arr}V \ T \ a)$

**using** *reach.Indirect* [*OF reach-loc*]

**by** *simp*

**lemma** *reachable-from-ObjLoc-impl-Obj* [*simp,intro*]:

**assumes** *reach-loc*:  $s \vdash l$  *reachable-from*  $(s@@\text{objLoc } cf \ a)$

**assumes**  $C$ :  $C = \text{cls } cf$

**shows**  $s \vdash l$  *reachable-from*  $(\text{obj}V \ C \ a)$

**using**  $C$  *reach.Indirect* [*OF reach-loc*]

**by** *simp*

Lemma 3.2 (i)

**lemma** *reach-update* [*simp*]:

**assumes** *unreachable-l-x*:  $\neg s \vdash l$  *reachable-from*  $x$

**shows**  $s \langle l := y \rangle \vdash k$  *reachable-from*  $x = s \vdash k$  *reachable-from*  $x$

**proof**

```

assume  $s \vdash k$  reachable-from  $x$ 
from this unreachable-l-x
show  $s \langle l := y \rangle \vdash k$  reachable-from  $x$ 
proof (induct)
  case (Immediate  $k$ )
  have  $\text{ref } k \neq \text{nullV}$  by fact
  then show  $s \langle l := y \rangle \vdash k$  reachable-from ( $\text{ref } k$ )
    by (rule reach.Immediate)
next
  case (Indirect  $k$   $m$ )
  have hyp:  $\neg s \vdash l$  reachable-from ( $s @ @ m$ )
     $\implies s \langle l := y \rangle \vdash k$  reachable-from ( $s @ @ m$ ) by fact
  have  $\text{ref } m \neq \text{nullV}$  and  $\neg s \vdash l$  reachable-from ( $\text{ref } m$ ) by fact+
  hence  $l \neq m$   $\neg s \vdash l$  reachable-from ( $s @ @ m$ )
    by (auto intro: reach.intros)
  with hyp have  $s \langle l := y \rangle \vdash k$  reachable-from ( $s \langle l := y \rangle @ @ m$ )
    by simp
  then show  $s \langle l := y \rangle \vdash k$  reachable-from ( $\text{ref } m$ )
    by (rule reach.Indirect) (rule Indirect.hyps)
qed
next
assume  $s \langle l := y \rangle \vdash k$  reachable-from  $x$ 
from this unreachable-l-x
show  $s \vdash k$  reachable-from  $x$ 
proof (induct)
  case (Immediate  $k$ )
  have  $\text{ref } k \neq \text{nullV}$  by fact
  then show  $s \vdash k$  reachable-from ( $\text{ref } k$ )
    by (rule reach.Immediate)
next
  case (Indirect  $k$   $m$ )
  with Indirect.hyps
  have hyp:  $\neg s \vdash l$  reachable-from ( $s \langle l := y \rangle @ @ m$ )
     $\implies s \vdash k$  reachable-from ( $s \langle l := y \rangle @ @ m$ ) by simp
  have  $\text{ref } m \neq \text{nullV}$  and  $\neg s \vdash l$  reachable-from ( $\text{ref } m$ ) by fact+
  hence  $l \neq m$   $\neg s \vdash l$  reachable-from ( $s @ @ m$ )
    by (auto intro: reach.intros)
  with hyp have  $s \vdash k$  reachable-from ( $s @ @ m$ )
    by simp
  thus  $s \vdash k$  reachable-from ( $\text{ref } m$ )
    by (rule reach.Indirect) (rule Indirect.hyps)
qed
qed

```

Lemma 3.2 (ii)

**lemma** *reach2*:

$\neg s \vdash l$  *reachable-from*  $x \implies \neg s \langle l := y \rangle \vdash l$  *reachable-from*  $x$   
**by** (*simp*)

Lemma 3.2 (iv)

**lemma** *reach4*:  $\neg s \vdash l$  *reachable-from* ( $\text{ref } k$ )  $\implies k \neq l \vee (\text{ref } k) = \text{nullV}$   
**by** (*auto intro: reach.intros*)

**lemma** *reachable-isRef*:

```

assumes reach:  $s \vdash l$  reachable-from  $x$ 
shows isRefV  $x$ 
using reach
proof (induct)
  case (Immediate  $l$ )
  show isRefV (ref  $l$ )
    by (cases  $l$ ) simp-all
next
  case (Indirect  $l$   $k$ )
  show isRefV (ref  $k$ )
    by (cases  $k$ ) simp-all
qed

```

```

lemma val-ArrLen-IntgT: isArrLenLoc  $l \implies \text{typeof } (s@@l) = \text{IntgT}$ 
proof –
  assume isArrLen: isArrLenLoc  $l$ 
  have  $T$ :  $\text{typeof } (s@@l) \leq \text{ltype } l$ 
    by (simp)
  also from isArrLen have  $I$ :  $\text{ltype } l = \text{IntgT}$ 
    by (cases  $l$ ) simp-all
  finally show ?thesis
    by (auto elim: rtranclE simp add: le-Javatypedef subtype-defs)
qed

```

```

lemma access-alloc' [simp]:
  assumes no-arr-len:  $\neg \text{isArrLenLoc } l$ 
  shows  $s(t)@@l = s@@l$ 
proof –
  from no-arr-len
  have isNewArr  $t \longrightarrow l \neq \text{arr-len } (\text{new } s \ t)$ 
    by (cases  $t$ ) (auto simp add: new-def isArrLenLoc-def split: Location.splits)
  thus ?thesis
    by (rule access-alloc)
qed

```

Lemma 3.2 (v)

```

lemma reach-alloc [simp]:  $s(t) \vdash l$  reachable-from  $x = s \vdash l$  reachable-from  $x$ 
proof
  assume  $s(t) \vdash l$  reachable-from  $x$ 
  thus  $s \vdash l$  reachable-from  $x$ 
  proof (induct)
    case (Immediate  $l$ )
    thus  $s \vdash l$  reachable-from ref  $l$ 
      by (rule reach.intros)
  next
    case (Indirect  $l$   $k$ )
    have reach- $k$ :  $s \vdash l$  reachable-from  $(s(t)@@k)$  by fact
    moreover
    have  $s(t)@@k = s@@k$ 
    proof –
      from reach- $k$  have isRef: isRefV  $(s(t)@@k)$ 
        by (rule reachable-isRef)
      have  $\neg \text{isArrLenLoc } k$ 

```

```

proof (rule ccontr,simp)
  assume isArrLenLoc k
  then have typeof (s⟨t⟩@@k) = IntgT
    by (rule val-ArrLen-IntgT)
  with isRef
  show False
    by (cases (s⟨t⟩@@k)) simp-all
qed
thus ?thesis
  by (rule access-alloc')
qed
ultimately have s⊢ l reachable-from (s@@k)
  by simp
thus s⊢ l reachable-from ref k
  by (rule reach.intros) (rule Indirect.hyps)
qed
next
assume s⊢ l reachable-from x
thus s⟨t⟩⊢ l reachable-from x
proof (induct)
  case (Immediate l)
thus s⟨t⟩⊢ l reachable-from ref l
  by (rule reach.intros)
next
case (Indirect l k)
have reach-k: s⟨t⟩⊢ l reachable-from (s@@k) by fact
moreover
have s⟨t⟩@@k = s@@k
proof –
  from reach-k have isRef: isRefV (s@@k)
    by (rule reachable-isRef)
  have ¬ isArrLenLoc k
  proof (rule ccontr,simp)
    assume isArrLenLoc k
    then have typeof (s@@k) = IntgT
      by (rule val-ArrLen-IntgT)
    with isRef
    show False
      by (cases (s@@k)) simp-all
  qed
thus ?thesis
  by (rule access-alloc')
qed
ultimately have s⟨t⟩⊢ l reachable-from (s⟨t⟩@@k)
  by simp
thus s⟨t⟩⊢ l reachable-from ref k
  by (rule reach.intros) (rule Indirect.hyps)
qed
qed

```

Lemma 3.2 (vi)

**lemma** reach6: isprimitive(typeof x)  $\implies$  ¬ s ⊢ l reachable-from x

**proof**

**assume** prim: isprimitive(typeof x)

```

assume  $s \vdash l \text{ reachable-from } x$ 
hence  $\text{isRefV } x$ 
  by (rule reachable-isRef)
with prim show False
  by (cases x) simp-all
qed

```

Lemma 3.2 (iii)

**lemma** *reach3*:

```

assumes  $k\text{-}y: \neg s \vdash k \text{ reachable-from } y$ 
assumes  $k\text{-}x: \neg s \vdash k \text{ reachable-from } x$ 
shows  $\neg s \langle l := y \rangle \vdash k \text{ reachable-from } x$ 
proof
assume  $s \langle l := y \rangle \vdash k \text{ reachable-from } x$ 
from this  $k\text{-}y$   $k\text{-}x$ 
show False
proof (induct)
  case (Immediate l)
    have  $\neg s \vdash l \text{ reachable-from ref } l$  and  $\text{ref } l \neq \text{nullV}$  by fact+
    thus False
    by (iprover intro: reach.intros)
  next
    case (Indirect m k)
    have  $k\text{-not-Null: ref } k \neq \text{nullV}$  by fact
    have  $\text{not-}m\text{-}y: \neg s \vdash m \text{ reachable-from } y$  by fact
    have  $\text{not-}m\text{-}k: \neg s \vdash m \text{ reachable-from ref } k$  by fact
    have hyp:  $\llbracket \neg s \vdash m \text{ reachable-from } y; \neg s \vdash m \text{ reachable-from } (s \langle l := y \rangle @ @ k) \rrbracket$ 
       $\implies$  False by fact
    have  $m\text{-upd-}k: s \langle l := y \rangle \vdash m \text{ reachable-from } (s \langle l := y \rangle @ @ k)$  by fact
    show False
    proof (cases l=k)
      case False
        then have  $s \langle l := y \rangle @ @ k = s @ @ k$  by simp
        moreover
        from  $\text{not-}m\text{-}k$   $k\text{-not-Null}$  have  $\neg s \vdash m \text{ reachable-from } (s @ @ k)$ 
          by (iprover intro: reach.intros)
        ultimately show False
          using  $\text{not-}m\text{-}y$  hyp by simp
      next
        case True note  $\text{eq-}l\text{-}k = \text{this}$ 
        show ?thesis
        proof (cases alive (ref l) s \wedge alive y s \wedge \text{typeof } y \leq \text{ltype } l)
          case True
            with  $\text{eq-}l\text{-}k$  have  $s \langle l := y \rangle @ @ k = y$ 
              by simp
            with  $\text{not-}m\text{-}y$  hyp show False by simp
          next
            case False
            hence  $s \langle l := y \rangle = s$ 
              by auto
            moreover
            from  $\text{not-}m\text{-}k$   $k\text{-not-Null}$  have  $\neg s \vdash m \text{ reachable-from } (s @ @ k)$ 
              by (iprover intro: reach.intros)
            ultimately show False

```



```

      using not-m-y hyp by simp
    qed
  qed
  qed
  qed

```

Lemma 3.2 (vii).

```

lemma unreachable-from-init [simp,intro]:  $\neg s \vdash l$  reachable-from (init T)
  using reach6 by (cases T) simp-all

```

**lemma** ref-reach-unalive:

```

  assumes unalive-x:  $\neg$  alive x s
  assumes l-x:  $s \vdash l$  reachable-from x
  shows x = ref l
using l-x unalive-x
proof induct
  case (Immediate l)
  show ref l = ref l
    by simp
next
  case (Indirect l k)
  have ref k  $\neq$  nullV by fact
  have  $\neg$  alive (ref k) s by fact
  hence s@@k = init (ltype k) by simp
  moreover have  $s \vdash l$  reachable-from (s@@k) by fact
  ultimately have False by simp
  thus ?case ..
qed

```

**lemma** loc-new-reach:

```

  assumes l: ref l = new s t
  assumes l-x:  $s \vdash l$  reachable-from x
  shows x = new s t
using l-x l
proof induct
  case (Immediate l)
  show ref l = new s t by fact
next
  case (Indirect l k)
  hence s@@k = new s t by iprover
  moreover
  have  $\neg$  alive (new s t) s
    by simp
  moreover
  have alive (s@@k) s
    by simp
  ultimately have False by simp
  thus ?case ..
qed

```

Lemma 3.2 (viii)

**lemma** alive-reach-alive:

```

  assumes alive-x: alive x s
  assumes reach-l:  $s \vdash l$  reachable-from x

```

```

  shows alive (ref l) s
using reach-l alive-x
proof (induct)
  case (Immediate l)
  show ?case by fact
next
  case (Indirect l k)
  have hyp: alive (s@@k) s  $\implies$  alive (ref l) s by fact
  moreover have alive (s@@k) s by simp
  ultimately
  show alive (ref l) s
    by iprover
qed

```

Lemma 3.2 (ix)

```

lemma reach9:
  assumes reach-impl-access-eq:  $\forall l. s1 \vdash l \text{ reachable-from } x \longrightarrow (s1@@l = s2@@l)$ 
  shows  $s1 \vdash l \text{ reachable-from } x = s2 \vdash l \text{ reachable-from } x$ 
proof
  assume  $s1 \vdash l \text{ reachable-from } x$ 
  from this reach-impl-access-eq
  show  $s2 \vdash l \text{ reachable-from } x$ 
  proof (induct)
    case (Immediate l)
    show  $s2 \vdash l \text{ reachable-from ref l}$ 
      by (rule reach.intros) (rule Immediate.hyps)
  next
    case (Indirect l k)
    have hyp:  $\forall l. s1 \vdash l \text{ reachable-from } (s1@@k) \longrightarrow s1@@l = s2@@l$ 
       $\implies s2 \vdash l \text{ reachable-from } (s1@@k)$  by fact
    have k-not-Null:  $\text{ref } k \neq \text{nullV}$  by fact
    have reach-impl-access-eq:
       $\forall l. s1 \vdash l \text{ reachable-from ref } k \longrightarrow s1@@l = s2@@l$  by fact
    have  $s1 \vdash l \text{ reachable-from } (s1@@k)$  by fact
    with k-not-Null
    have  $s1@@k = s2@@k$ 
      by (iprover intro: reach-impl-access-eq [rule-format] reach.intros)
    moreover from reach-impl-access-eq k-not-Null
    have  $\forall l. s1 \vdash l \text{ reachable-from } (s1@@k) \longrightarrow s1@@l = s2@@l$ 
      by (iprover intro: reach.intros)
    then have  $s2 \vdash l \text{ reachable-from } (s1@@k)$ 
      by (rule hyp)
    ultimately have  $s2 \vdash l \text{ reachable-from } (s2@@k)$ 
      by simp
    thus  $s2 \vdash l \text{ reachable-from ref } k$ 
      by (rule reach.intros) (rule Indirect.hyps)
  qed
next
  assume  $s2 \vdash l \text{ reachable-from } x$ 
  from this reach-impl-access-eq
  show  $s1 \vdash l \text{ reachable-from } x$ 
  proof (induct)
    case (Immediate l)
    show  $s1 \vdash l \text{ reachable-from ref l}$ 

```

```

  by (rule reach.intros) (rule Immediate.hyps)
next
case (Indirect l k)
have hyp:  $\forall l. s1 \vdash l \text{ reachable-from } (s2@@k) \longrightarrow s1@@l = s2@@l$ 
   $\implies s1 \vdash l \text{ reachable-from } (s2@@k)$  by fact
have k-not-Null:  $\text{ref } k \neq \text{nullV}$  by fact
have reach-impl-access-eq:
   $\forall l. s1 \vdash l \text{ reachable-from ref } k \longrightarrow s1@@l = s2@@l$  by fact
have s1- $\vdash$  k reachable-from ref k
  by (rule reach.intros) (rule Indirect.hyps)
with reach-impl-access-eq
have eq-k:  $s1@@k = s2@@k$ 
  by simp
from reach-impl-access-eq k-not-Null
have  $\forall l. s1 \vdash l \text{ reachable-from } (s1@@k) \longrightarrow s1@@l = s2@@l$ 
  by (iprover intro: reach.intros)
then
have  $\forall l. s1 \vdash l \text{ reachable-from } (s2@@k) \longrightarrow s1@@l = s2@@l$ 
  by (simp add: eq-k)
with eq-k hyp have  $s1 \vdash l \text{ reachable-from } (s1@@k)$ 
  by simp
thus  $s1 \vdash l \text{ reachable-from ref } k$ 
  by (rule reach.intros) (rule Indirect.hyps)
qed
qed

```

## 13.2 Reachability of a Reference from a Reference

The predicate *rreach* tests whether a value is reachable from another value. This is an extension of the predicate *oreach* as described in [PH97, p. 54] because now arrays are handled as well.

**definition** *rreach*::  $\text{Store} \Rightarrow \text{Value} \Rightarrow \text{Value} \Rightarrow \text{bool}$   
 $(\vdash \text{Ref} - \text{reachable}'\text{-from} - [91,91,91]90)$  **where**  
 $s \vdash \text{Ref } y \text{ reachable-from } x = (\exists l. s \vdash l \text{ reachable-from } x \wedge y = \text{ref } l)$

## 13.3 Disjointness of Reachable Locations

The predicate *disj* tests whether two values are disjoint in a given store. Its properties as given in [PH97, Lemma 3.3, p. 54] are then proven.

**definition** *disj*::  $\text{Value} \Rightarrow \text{Value} \Rightarrow \text{Store} \Rightarrow \text{bool}$  **where**  
 $\text{disj } x y s = (\forall l. \neg s \vdash l \text{ reachable-from } x \vee \neg s \vdash l \text{ reachable-from } y)$

**lemma** *disjI1*:  $\llbracket \bigwedge l. s \vdash l \text{ reachable-from } x \implies \neg s \vdash l \text{ reachable-from } y \rrbracket$   
 $\implies \text{disj } x y s$   
 by (simp add: disj-def)

**lemma** *disjI2*:  $\llbracket \bigwedge l. s \vdash l \text{ reachable-from } y \implies \neg s \vdash l \text{ reachable-from } x \rrbracket$   
 $\implies \text{disj } x y s$   
 by (auto simp add: disj-def)

**lemma** *disj-cases* [consumes 1]:  
 assumes  $\text{disj } x y s$

**assumes**  $\bigwedge l. \neg s \vdash l \text{ reachable-from } x \implies P$   
**assumes**  $\bigwedge l. \neg s \vdash l \text{ reachable-from } y \implies P$   
**shows**  $P$   
**using** *assms* **by** (*auto simp add: disj-def*)

Lemma 3.3 (i) in [PH97]

**lemma** *disj1*:  $\llbracket \text{disj } x \ y \ s; \neg s \vdash l \text{ reachable-from } x; \neg s \vdash l \text{ reachable-from } y \rrbracket$   
 $\implies \text{disj } x \ y \ (s \langle l := z \rangle)$   
**by** (*auto simp add: disj-def*)

Lemma 3.3 (ii)

**lemma** *disj2*:

**assumes** *disj-x-y*:  $\text{disj } x \ y \ s$   
**assumes** *disj-x-z*:  $\text{disj } x \ z \ s$   
**assumes** *unreach-l-x*:  $\neg s \vdash l \text{ reachable-from } x$   
**shows**  $\text{disj } x \ y \ (s \langle l := z \rangle)$

**proof** (*rule disjI1*)

**fix**  $k$   
**assume** *reach-k-x*:  $s \langle l := z \rangle \vdash k \text{ reachable-from } x$   
**show**  $\neg s \langle l := z \rangle \vdash k \text{ reachable-from } y$

**proof** –

**from** *unreach-l-x reach-k-x*  
**have** *reach-s-k-x*:  $s \vdash k \text{ reachable-from } x$   
**by** *simp*

**with** *disj-x-z*

**have**  $\neg s \vdash k \text{ reachable-from } z$

**by** (*simp add: disj-def*)

**moreover from** *reach-s-k-x disj-x-y*

**have**  $\neg s \vdash k \text{ reachable-from } y$

**by** (*simp add: disj-def*)

**ultimately show** *?thesis*

**by** (*rule reach3*)

**qed**

**qed**

Lemma 3.3 (iii)

**lemma** *disj3*: **assumes** *alive-x-s*:  $\text{alive } x \ s$   
**shows**  $\text{disj } x \ (\text{new } s \ t) \ (s \langle t \rangle)$

**proof** (*rule disjI1, simp only: reach-alloc*)

**fix**  $l$

**assume** *reach-l-x*:  $s \vdash l \text{ reachable-from } x$

**show**  $\neg s \vdash l \text{ reachable-from } \text{new } s \ t$

**proof**

**assume** *reach-l-new*:  $s \vdash l \text{ reachable-from } \text{new } s \ t$

**have** *unalive-new*:  $\neg \text{alive } (\text{new } s \ t) \ s$  **by** *simp*

**from** *this reach-l-new*

**have**  $\text{new } s \ t = \text{ref } l$

**by** (*rule ref-reach-unalive*)

**moreover from** *alive-x-s reach-l-x*

**have**  $\text{alive } (\text{ref } l) \ s$

**by** (*rule alive-reach-alive*)

**ultimately show** *False*

**using** *unalive-new*

by *simp*  
 qed  
 qed

Lemma 3.3 (iv)

**lemma** *disj4*:  $\llbracket \text{disj } (\text{objV } C \ a) \ y \ s; \ C\text{ClassT } C \leq \text{dtype } f \rrbracket$   
 $\implies \text{disj } (s@@( \text{objV } C \ a) ..f) \ y \ s$   
 by (*auto simp add: disj-def*)

**lemma** *disj4'*:  $\llbracket \text{disj } (\text{arrV } T \ a) \ y \ s \rrbracket$   
 $\implies \text{disj } (s@@( \text{arrV } T \ a) .[i]) \ y \ s$   
 by (*auto simp add: disj-def*)

## 13.4 X-Equivalence

We call two stores  $s_1$  and  $s_2$  equivalent wrt. a given value  $X$  (which is called X-equivalence) iff  $X$  and all values reachable from  $X$  in  $s_1$  or  $s_2$  have the same state [PH97, p. 55]. This is tested by the predicate *xeq*. Lemma 3.4 of [PH97] is then proven for *xeq*.

**definition** *xeq*:  $\text{Value} \Rightarrow \text{Store} \Rightarrow \text{Store} \Rightarrow \text{bool}$  **where**  
 $\text{xeq } x \ s \ t = (\text{alive } x \ s = \text{alive } x \ t \wedge$   
 $(\forall \ l. \ s \vdash l \ \text{reachable-from } x \longrightarrow s@@l = t@@l))$

**abbreviation** *xeq-syntax* ::  $\text{Store} \Rightarrow \text{Value} \Rightarrow \text{Store} \Rightarrow \text{bool}$   
 $(-/ (\equiv[-]) / - [900,0,900] \ 900)$   
**where**  $s \equiv[x] \ t == \text{xeq } x \ s \ t$

**lemma** *xeqI*:  $\llbracket \text{alive } x \ s = \text{alive } x \ t;$   
 $\bigwedge \ l. \ s \vdash l \ \text{reachable-from } x \implies s@@l = t@@l$   
 $\rrbracket \implies s \equiv[x] \ t$   
 by (*auto simp add: xeq-def*)

Lemma 3.4 (i) in [PH97].

**lemma** *xeq1-refl*:  $s \equiv[x] \ s$   
 by (*simp add: xeq-def*)

Lemma 3.4 (i)

**lemma** *xeq1-sym'*:  
**assumes**  $s \vdash t: s \equiv[x] \ t$   
**shows**  $t \equiv[x] \ s$

**proof** –

**from**  $s \vdash t$  **have**  $\text{alive } x \ s = \text{alive } x \ t$  **by** (*simp add: xeq-def*)

**moreover**

**from**  $s \vdash t$  **have**  $\forall \ l. \ s \vdash l \ \text{reachable-from } x \longrightarrow s@@l = t@@l$   
 by (*simp add: xeq-def*)

**with** *reach9* [OF *this*]

**have**  $\forall \ l. \ t \vdash l \ \text{reachable-from } x \longrightarrow t@@l = s@@l$   
 by *simp*

**ultimately show** *?thesis*

by (*simp add: xeq-def*)

qed

**lemma** *req1-sym*:  $s \equiv[x] t = t \equiv[x] s$   
 by (*auto intro: req1-sym*)

Lemma 3.4 (i)

**lemma** *req1-trans* [*trans*]:  
**assumes** *s-t*:  $s \equiv[x] t$   
**assumes** *t-r*:  $t \equiv[x] r$   
**shows**  $s \equiv[x] r$   
**proof** –  
**from** *s-t t-r*  
**have** *alive x s = alive x r*  
 by (*simp add: req-def*)  
**moreover**  
**have**  $\forall l. s \vdash l \text{ reachable-from } x \longrightarrow s@@l = r@@l$   
**proof** (*intro allI impI*)  
**fix** *l*  
**assume** *reach-l*:  $s \vdash l \text{ reachable-from } x$   
**show**  $s@@l = r@@l$   
**proof** –  
**from** *reach-l s-t* **have**  $s@@l = t@@l$   
 by (*simp add: req-def*)  
**also have**  $t@@l = r@@l$   
**proof** –  
**from** *s-t* **have**  $\forall l. s \vdash l \text{ reachable-from } x \longrightarrow s@@l = t@@l$   
 by (*simp add: req-def*)  
**from** *reach9 [OF this] reach-l* **have**  $t \vdash l \text{ reachable-from } x$   
 by *simp*  
**with** *t-r* **show** *?thesis*  
 by (*simp add: req-def*)  
**qed**  
**finally show** *?thesis* .  
**qed**  
**qed**  
**ultimately show** *?thesis*  
 by (*simp add: req-def*)  
**qed**

Lemma 3.4 (ii)

**lemma** *req2*:  
**assumes** *req*:  $\forall x. s \equiv[x] t$   
**assumes** *static-eq*:  $\forall f. s@@(\text{staticLoc } f) = t@@(\text{staticLoc } f)$   
**shows**  $s = t$   
**proof** (*rule Store-eqI*)  
**from** *req*  
**show**  $\forall x. \text{alive } x \text{ } s = \text{alive } x \text{ } t$   
 by (*simp add: req-def*)  
**next**  
**show**  $\forall l. s@@l = t@@l$   
**proof**  
**fix** *l*  
**show**  $s@@l = t@@l$   
**proof** (*cases l*)  
**case** (*objLoc cf a*)  
**have**  $l = \text{objLoc } cf \text{ } a$  **by** *fact*

```

hence  $s \vdash l \text{ reachable-from } (\text{objV } (\text{cls } cf) a)$ 
  by simp
with xeq show ?thesis
  by (simp add: xeq-def)
next
case (staticLoc f)
have  $l = \text{staticLoc } f$  by fact
with static-eq show ?thesis
  by (simp add: xeq-def)
next
case (arrLenLoc T a)
have  $l = \text{arrLenLoc } T a$  by fact
hence  $s \vdash l \text{ reachable-from } (\text{arrV } T a)$ 
  by simp
with xeq show ?thesis
  by (simp add: xeq-def)
next
case (arrLoc T a i)
have  $l = \text{arrLoc } T a i$  by fact
hence  $s \vdash l \text{ reachable-from } (\text{arrV } T a)$ 
  by simp
with xeq show ?thesis
  by (simp add: xeq-def)
qed
qed
qed

```

Lemma 3.4 (iii)

**lemma** *xeq3*:

**assumes** *unreach-l*:  $\neg s \vdash l \text{ reachable-from } x$   
**shows**  $s \equiv[x] s\langle l := y \rangle$

**proof** (*rule xeqI*)

**show** *alive x s = alive x (s⟨l := y⟩)*  
**by** *simp*

**next**

**fix** *k*  
**assume** *reach-k*:  $s \vdash k \text{ reachable-from } x$   
**with** *unreach-l* **have**  $l \neq k$  **by** *auto*  
**then show**  $s@@k = s\langle l := y \rangle@@k$   
**by** *simp*

**qed**

Lemma 3.4 (iv)

**lemma** *xeq4*: **assumes** *not-new*:  $x \neq \text{new } s t$

**shows**  $s \equiv[x] s\langle t \rangle$

**proof** (*rule xeqI*)

**from** *not-new*

**show** *alive x s = alive x (s⟨t⟩)*  
**by** (*simp add: alive-alloc-exhaust*)

**next**

**fix** *l*  
**assume** *reach-l*:  $s \vdash l \text{ reachable-from } x$   
**show**  $s@@l = s\langle t \rangle@@l$   
**proof** (*cases isNewArr t*  $\longrightarrow l \neq \text{arr-len } (\text{new } s t)$ )

```

case True
with reach-l show ?thesis
  by simp
next
case False
then obtain T n where t = new-array T n and
  l = arr-len (new s t)
  by (cases t) auto
hence ref l = new s t
  by simp
from this reach-l
have x = new s t
  by (rule loc-new-reach)
with not-new show ?thesis ..
qed
qed

```

Lemma 3.4 (v)

```

lemma xeq5:  $s \equiv[x] t \implies s \vdash l \text{ reachable-from } x = t \vdash l \text{ reachable-from } x$ 
  by (rule reach9) (simp add: xeq-def)

```

### 13.5 T-Equivalence

T-equivalence is the extension of X-equivalence from values to types. Two stores are T-equivalent iff they are X-equivalent for all values of type T. This is formalized by the predicate *teq* [PH97, p. 55].

```

definition teq:: Javatype  $\Rightarrow$  Store  $\Rightarrow$  Store  $\Rightarrow$  bool where
teq t s1 s2 = ( $\forall x. \text{typeof } x \leq t \longrightarrow s1 \equiv[x] s2$ )

```

### 13.6 Less Alive

To specify that methods have no side-effects, the following binary relation on stores plays a prominent role. It expresses that the two stores differ only in values that are alive in the store passed as first argument. This is formalized by the predicate *lessalive* [PH97, p. 55]. The stores have to be X-equivalent for the references of the first store that are alive, and the values of the static fields have to be the same in both stores.

```

definition lessalive:: Store  $\Rightarrow$  Store  $\Rightarrow$  bool (-  $\ll$  - [70,71] 70)
  where lessalive s t = ( $(\forall x. \text{alive } x s \longrightarrow s \equiv[x] t) \wedge (\forall f. s@@\text{staticLoc } f = t@@\text{staticLoc } f)$ )

```

We define an introduction rule for the new operator.

```

lemma lessaliveI:
 $\llbracket \bigwedge x. \text{alive } x s \implies s \equiv[x] t; \bigwedge f. s@@\text{staticLoc } f = t@@\text{staticLoc } f \rrbracket$ 
 $\implies s \ll t$ 
by (simp add: lessalive-def)

```

It can be shown that *lessalive* is reflexive, transitive and antisymmetric.

```

lemma lessalive-refl:  $s \ll s$ 
  by (simp add: lessalive-def xeq1-refl)

```

```

lemma lessalive-trans [trans]:
  assumes s-t:  $s \ll t$ 

```



```

assumes t-w:  $t \ll w$ 
shows  $s \ll w$ 
proof (rule lessaliveI)
  fix x
  assume alive-x-s: alive x s
  with s-t have  $s \equiv[x] t$ 
    by (simp add: lessalive-def)
  also
  have  $t \equiv[x] w$ 
  proof –
    from alive-x-s s-t have alive x t by (simp add: lessalive-def xeq-def)
    with t-w show ?thesis
      by (simp add: lessalive-def)
  qed
  finally show  $s \equiv[x] w$ .
next
  fix f
  from s-t t-w show  $s@@\text{staticLoc } f = w@@\text{staticLoc } f$ 
    by (simp add: lessalive-def)
qed

lemma lessalive-antisym:
  assumes s-t:  $s \ll t$ 
  assumes t-s:  $t \ll s$ 
  shows  $s = t$ 
proof (rule xeq2)
  show  $\forall x. s \equiv[x] t$ 
  proof
    fix x show  $s \equiv[x] t$ 
    proof (cases alive x s)
      case True
        with s-t show ?thesis by (simp add: lessalive-def)
    next
      case False note unalive-x-s = this
      show ?thesis
      proof (cases alive x t)
        case True
          with t-s show ?thesis
            by (subst xeq1-sym) (simp add: lessalive-def)
        next
          case False
          show ?thesis
          proof (rule xeqI)
            from False unalive-x-s show alive x s = alive x t by simp
          next
            fix l assume reach-s-x:  $s \vdash l$  reachable-from x
            with unalive-x-s have  $x: x = \text{ref } l$ 
              by (rule ref-reach-unalive)
            with unalive-x-s have  $s@@l = \text{init } (\text{ltype } l)$ 
              by simp
            also from reach-s-x x have  $t \vdash l$  reachable-from x
              by (auto intro: reach.Immediate unreachable-Null)
            with False x have  $t@@l = \text{init } (\text{ltype } l)$ 
              by simp

```

```

      finally show  $s@@l = t@@l$ 
        by simp
    qed
  qed
  qed
  qed
next
from  $s-t$  show  $\forall f. s@@staticLoc f = t@@staticLoc f$ 
  by (simp add: lessalive-def)
qed

```

This gives us a partial ordering on the store. Thus, the type *Store* can be added to the appropriate type class *ord* which lets us define the  $<$  and  $\leq$  symbols, and to the type class *order* which axiomatizes partial orderings.

```

instantiation Store :: order
begin

```

```

definition
  le-Store-def:  $s \leq t \iff s \ll t$ 

```

```

definition
  less-Store-def:  $(s::Store) < t \iff s \leq t \wedge \neg t \leq s$ 

```

We prove Lemma 3.5 of [PH97, p. 56] for this relation.

Lemma 3.5 (i)

```

instance proof
  fix  $s t w:: Store$ 
  {
    show  $s \leq s$ 
      by (simp add: le-Store-def lessalive-refl)
  next
    assume  $s \leq t \ t \leq w$ 
    then show  $s \leq w$ 
      by (unfold le-Store-def) (rule lessalive-trans)
  next
    assume  $s \leq t \ t \leq s$ 
    then show  $s = t$ 
      by (unfold le-Store-def) (rule lessalive-antisym)
  next
    show  $(s < t) = (s \leq t \wedge \neg t \leq s)$ 
      by (simp add: less-Store-def)
  }
qed

end

```

Lemma 3.5 (ii)

```

lemma lessalive2:  $\llbracket s \ll t; \text{alive } x \ s \rrbracket \implies \text{alive } x \ t$ 
  by (simp add: lessalive-def req-def)

```

Lemma 3.5 (iii)

```

lemma lessalive3:

```

```

assumes s-t:  $s \ll t$ 
assumes alive:  $alive\ x\ s \vee \neg\ alive\ x\ t$ 
shows  $s \equiv[x]\ t$ 
proof (cases alive x s)
  case True
    with s-t show ?thesis
      by (simp add: lessalive-def)
  next
    case False
    note unalive-x-s = this
    with alive have unalive-x-t:  $\neg\ alive\ x\ t$ 
      by simp
    show ?thesis
    proof (rule xeqI)
      from False alive show  $alive\ x\ s = alive\ x\ t$ 
        by simp
    next
      fix l assume reach-s-x:  $s \vdash l\ reachable\text{-from}\ x$ 
      with unalive-x-s have x:  $x = ref\ l$ 
        by (rule ref-reach-unalive)
      with unalive-x-s have  $s@@l = init\ (ltype\ l)$ 
        by simp
      also from reach-s-x x have  $t \vdash l\ reachable\text{-from}\ x$ 
        by (auto intro: reach.Immediate unreachable-Null)
      with unalive-x-t x have  $t@@l = init\ (ltype\ l)$ 
        by simp
      finally show  $s@@l = t@@l$ 
        by simp
    qed
  qed

```

Lemma 3.5 (iv)

```

lemma lessalive-update [simp,intro]:
  assumes s-t:  $s \ll t$ 
  assumes unalive-l:  $\neg\ alive\ (ref\ l)\ t$ 
  shows  $s \ll t\langle l:=x \rangle$ 
proof –
  from unalive-l have  $t\langle l:=x \rangle = t$ 
    by simp
  with s-t show ?thesis by simp
qed

```

```

lemma Xequ4':
  assumes alive:  $alive\ x\ s$ 
  shows  $s \equiv[x]\ s\langle t \rangle$ 
proof –
  from alive have  $x \neq new\ s\ t$ 
    by auto
  thus ?thesis
    by (rule xeq4)
qed

```

Lemma 3.5 (v)

```

lemma lessalive-alloc [simp,intro]:  $s \ll s\langle t \rangle$ 

```

by (*simp add: lessalive-def Xequ4'*)

### 13.7 Reachability of Types from Types

The predicate *treach* denotes the fact that the first type reaches the second type by stepping finitely many times from a type to the range type of one of its fields. This formalization diverges from [PH97, p. 106] in that it does not include the number of steps that are allowed to reach the second type. Reachability of types is a static approximation of reachability in the store. If I cannot reach the type of a location from the type of a reference, I cannot reach the location from the reference. See lemma *not-treach-ref-impl-not-reach* below.

**inductive**

*treach* :: *Javatype*  $\Rightarrow$  *Javatype*  $\Rightarrow$  *bool*

**where**

*Subtype*:  $U \leq T \implies \text{treach } T \ U$

| *Attribute*:  $\llbracket \text{treach } T \ S; S \leq \text{dtype } f; U \leq \text{rtype } f \rrbracket \implies \text{treach } T \ U$

| *ArrLength*:  $\text{treach } (\text{Arr } T \ AT) \ \text{Int } T$

| *ArrElem*:  $\text{treach } (\text{Arr } T \ AT) \ (\text{at2jt } AT)$

| *Trans [trans]*:  $\llbracket \text{treach } T \ U; \text{treach } U \ V \rrbracket \implies \text{treach } T \ V$

**lemma** *treach-ref-l* [*simp,intro*]:

**assumes** *not-Null*: *ref l*  $\neq$  *nullV*

**shows** *treach* (*typeof* (*ref l*)) (*ltype l*)

**proof** (*cases l*)

**case** (*objLoc cf a*)

**have** *l=objLoc cf a* **by fact**

**moreover**

**have** *treach* (*CClassT* (*cls cf*)) (*rtype* (*att cf*))

**by** (*rule treach.Attribute* [**where** *?f=att cf* **and** *?S=CClassT* (*cls cf*)])  
(*auto intro: treach.Subtype*)

**ultimately show** *?thesis*

**by** *simp*

**next**

**case** (*staticLoc f*)

**have** *l=staticLoc f* **by fact**

**hence** *ref l = nullV* **by** *simp*

**with** *not-Null* **show** *?thesis*

**by** *simp*

**next**

**case** (*arrLenLoc T a*)

**have** *l=arrLenLoc T a* **by fact**

**then show** *?thesis*

**by** (*auto intro: treach.ArrLength*)

**next**

**case** (*arrLoc T a i*)

**have** *l=arrLoc T a i* **by fact**

**then show** *?thesis*

**by** (*auto intro: treach.ArrElem*)

**qed**

**lemma** *treach-ref-l'* [*simp,intro*]:

**assumes** *not-Null*: *ref l*  $\neq$  *nullV*

**shows** *treach* (*typeof* (*ref l*)) (*typeof* (*s@@l*))

**proof** –

**from** *not-Null* **have** *treach* (*typeof* (*ref l*)) (*ltype l*) **by** (*rule treach-ref-l*)  
**also have** *typeof* (*s@@l*)  $\leq$  *ltype l*  
**by** *simp*  
**hence** *treach* (*ltype l*) (*typeof* (*s@@l*))  
**by** (*rule treach.intros*)  
**finally show** *?thesis* .  
**qed**

**lemma** *reach-impl-treach*:

**assumes** *reach-l*:  $s \vdash l$  *reachable-from x*  
**shows** *treach* (*typeof x*) (*ltype l*)

**using** *reach-l*

**proof** (*induct*)

**case** (*Immediate l*)  
**have** *ref l*  $\neq$  *nullV* **by** *fact*  
**then show** *treach* (*typeof* (*ref l*)) (*ltype l*)  
**by** (*rule treach-ref-l*)

**next**

**case** (*Indirect l k*)  
**have** *treach* (*typeof* (*s@@k*)) (*ltype l*) **by** *fact*  
**moreover**  
**have** *ref k*  $\neq$  *nullV* **by** *fact*  
**hence** *treach* (*typeof* (*ref k*)) (*typeof* (*s@@k*))  
**by** *simp*  
**ultimately show** *treach* (*typeof* (*ref k*)) (*ltype l*)  
**by** (*iprover intro: treach.Trans*)

**qed**

**lemma** *not-treach-ref-impl-not-reach*:

**assumes** *not-treach*:  $\neg$  *treach* (*typeof x*) (*typeof* (*ref l*))  
**shows**  $\neg$   $s \vdash l$  *reachable-from x*

**proof**

**assume** *reach-l*:  $s \vdash l$  *reachable-from x*  
**from** *this not-treach*  
**show** *False*

**proof** (*induct*)

**case** (*Immediate l*)  
**have**  $\neg$  *treach* (*typeof* (*ref l*)) (*typeof* (*ref l*)) **by** *fact*  
**thus** *False* **by** (*iprover intro: treach.intros order-refl*)

**next**

**case** (*Indirect l k*)  
**have** *hyp*:  $\neg$  *treach* (*typeof* (*s@@k*)) (*typeof* (*ref l*))  $\implies$  *False* **by** *fact*  
**have** *not-Null*: *ref k*  $\neq$  *nullV* **by** *fact*  
**have** *not-k-l*:  $\neg$  *treach* (*typeof* (*ref k*)) (*typeof* (*ref l*)) **by** *fact*  
**show** *False*  
**proof** (*cases treach* (*typeof* (*s@@k*)) (*typeof* (*ref l*)))  
**case** *False* **thus** *False* **by** (*rule hyp*)

**next**

**case** *True*  
**from** *not-Null* **have** *treach* (*typeof* (*ref k*)) (*typeof* (*s@@k*))  
**by** (*rule treach-ref-l'*)  
**also note** *True*

```

    finally have treach (typeof (ref k)) (typeof (ref l)) .
    with not-k-l show False ..
  qed
qed
qed

```

Lemma 4.6 in [PH97, p. 107].

```

lemma treach1:
  assumes x-t: typeof x ≤ T
  assumes not-treach: ¬ treach T (typeof (ref l))
  shows ¬ s ⊢ l reachable-from x
proof -
  have ¬ treach (typeof x) (typeof (ref l))
  proof
    from x-t have treach T (typeof x) by (rule treach.intros)
    also assume treach (typeof x) (typeof (ref l))
    finally have treach T (typeof (ref l)) .
    with not-treach show False ..
  qed
  thus ?thesis
  by (rule not-treach-ref-impl-not-reach)
qed

```

end

## 14 The Formalization of JML Operators

```
theory JML imports ../Isabelle-Store/StoreProperties begin
```

JML operators that are to be used in Hoare formulae can be formalized here.

**definition**

```

instanceof :: Value ⇒ Javatype ⇒ bool (- @instanceof -)
where
  instanceof v t = (typeof v ≤ t)

```

end

## 15 The Universal Specification

```
theory UnivSpec imports ../Isabelle/JML begin
```

This theory contains the Isabelle formalization of the program-dependent specification. This theory has to be provided by the user. In later versions of Jive, one may be able to generate it from JML model classes.

**definition**

```

aCounter :: Value ⇒ Store ⇒ JavaInt where
aCounter x s =
  (if x ≈ nullV & (alive x s) & typeof x = CClassT CounterImpl then
    aI ( s@@(x..CounterImpl'value) )
  else undefined)

```

end

---

## References

- [Jiv] Jive project webpage. [http://softech.informatik.uni-kl.de/softech/content/eforschung/e3490/index\\_ger.html](http://softech.informatik.uni-kl.de/softech/content/eforschung/e3490/index_ger.html).
- [LBR99] Gary T. Leavens, Albert L. Baker, and Clyde Ruby. JML: A notation for detailed design. In Haim Kilov, Bernhard Rumpe, and Ian Simmonds, editors, *Behavioral Specifications of Businesses and Systems*, chapter 12, pages 175–188. Kluwer, 1999.
- [MPH00] Jörg Meyer and Arnd Poetzsch-Heffter. An architecture for interactive program provers. In S. Graf and M. Schwartzbach, editors, *TACAS00, Tools and Algorithms for the Construction and Analysis of Systems*, volume 1785 of *Lecture Notes in Computer Science*, pages 63–77. Springer-Verlag, 2000.
- [PH97] Arnd Poetzsch-Heffter. Specification and verification of object-oriented programs. Habilitationsschrift, Technische Universität München, 1997.
- [PHGR05] Arnd Poetzsch-Heffter, Jean-Marie Gaillourdet, and Nicole Rauch. A Hoare Logic for a Java Subset and its Proof of Soundness and Completeness. Internal report, University of Kaiserslautern, Germany, 2005. To appear.