A Machine-Checked Model for a Java-like Language, Virtual Machine and Compiler

Gerwin Klein Tobias Nipkow

March 19, 2025

Contents

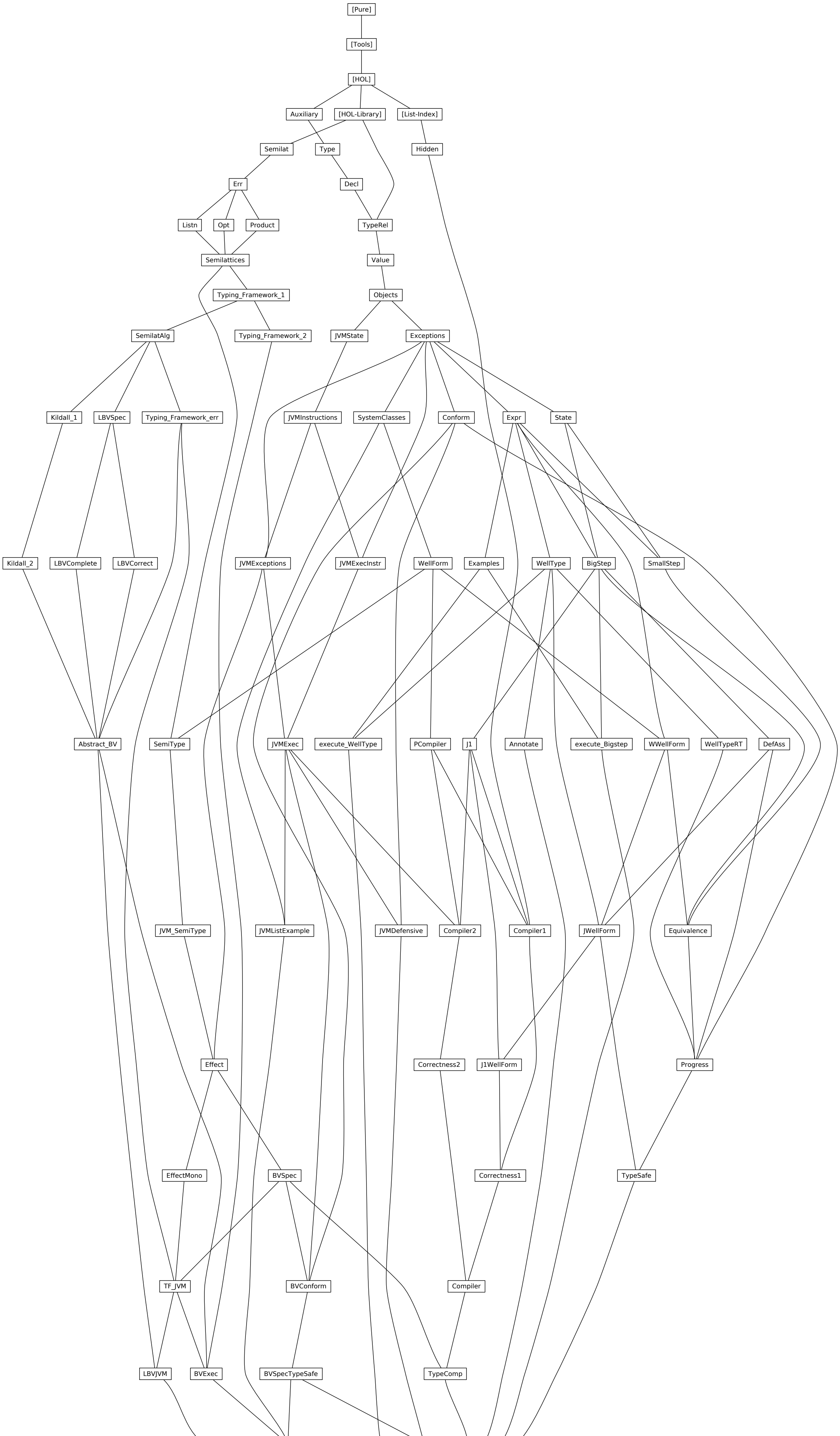
Chapter 1

Preface

This document contains the automatically generated listings of the Isabelle sources for the theories defining and analysing Jinja (a Java-like programming language), the Jinja Virtual Machine, and the compiler. To shorten the document, all proofs have been hidden. For a detailed exposition of these theories see the paper by Klein and Nipkow [1, 2].

1.1 Theory Dependencies

Figure 1.1 shows the dependencies between the Isabelle theories in the following sections.


[Pure]
[Tools]
[HOL]
Auxiliary
[HOL-Library]
[List-Index]
Semilat
Type
Hidden
Err
Decl
Listn
Opt
Product
TypeRel
Semilattices
Value
Typing_Framework_1
Objects
SemilatAlg
Typing_Framework_2
JVMState
Exceptions
Kildall_1
LBVSpec
Typing_Framework_err
JVMInstructions
SystemClasses
Conform
Expr
State
Kildall_2
LBVComplete
LBVCorrect
JVMExceptions
JVMExecInstr
WellForm
Examples
WellType
BigStep
SmallStep
Abstract_BV
SemiType
JVMExec
execute_WellType
PCompiler
J1
Annotate
execute_Bigstep
WWellForm
WellTypeRT
DefAss
JVM_SemiType
JVMListExample
JVMDefensive
Compiler2
Compiler1
JWellForm
Equivalence
Effect
Correctness2
J1WellForm
Progress
EffectMono
BVSpec
Correctness1
TypeSafe
TF_JVM
BVConform
Compiler
LBVJVM
BVExec
BVSpecTypeSafe
TypeComp

Chapter 2

Jinja Source Language

2.1 Auxiliary Definitions

theory *Auxiliary* **imports** *Main* **begin**

lemma *nat-add-max-le*[*simp*]:
$((n{::}nat) + max\ i\ j \leq m) = (n + i \leq m \wedge n + j \leq m)$
⟨*proof*⟩
lemma *Suc-add-max-le*[*simp*]:
$(Suc(n + max\ i\ j) \leq m) = (Suc(n + i) \leq m \wedge Suc(n + j) \leq m)$⟨*proof*⟩

notation *Some* (‹(⌊-⌋)›)

2.1.1 *distinct-fst*

definition *distinct-fst* :: $('a \times 'b)$ *list* ⇒ *bool*
where
distinct-fst ≡ *distinct* ∘ *map fst*

lemma *distinct-fst-Nil* [*simp*]:
distinct-fst []
⟨*proof*⟩
lemma *distinct-fst-Cons* [*simp*]:
distinct-fst ((*k*,*x*)#*kxs*) = (*distinct-fst kxs* ∧ (∀ *y*. (*k*,*y*) ∉ *set kxs*))⟨*proof*⟩

lemma *map-of-SomeI*:
⟦ *distinct-fst kxs*; (*k*,*x*) ∈ *set kxs* ⟧ ⟹ *map-of kxs k* = *Some x*⟨*proof*⟩

2.1.2 Using *list-all2* for relations

definition *fun-of* :: $('a \times 'b)$ *set* ⇒ *'a* ⇒ *'b* ⇒ *bool*
where
fun-of S ≡ λ*x y*. (*x*,*y*) ∈ *S*

Convenience lemmas

lemma *rel-list-all2-Cons* [*iff*]:
list-all2 (*fun-of S*) (*x*#*xs*) (*y*#*ys*) =
((*x*,*y*) ∈ *S* ∧ *list-all2* (*fun-of S*) *xs ys*)
⟨*proof*⟩

lemma *rel-list-all2-Cons1*:
list-all2 (*fun-of S*) (*x#xs*) *ys* =
(∃ *z zs*. *ys* = *z#zs* ∧ (*x*,*z*) ∈ *S* ∧ *list-all2* (*fun-of S*) *xs zs*)
⟨*proof*⟩
lemma *rel-list-all2-Cons2*:
list-all2 (*fun-of S*) *xs* (*y#ys*) =
(∃ *z zs*. *xs* = *z#zs* ∧ (*z*,*y*) ∈ *S* ∧ *list-all2* (*fun-of S*) *zs ys*)
⟨*proof*⟩
lemma *rel-list-all2-refl*:
(⋀*x*. (*x*,*x*) ∈ *S*) ⟹ *list-all2* (*fun-of S*) *xs xs*
⟨*proof*⟩
lemma *rel-list-all2-antisym*:
⟦ (⋀*x y*. ⟦(*x*,*y*) ∈ *S*; (*y*,*x*) ∈ *T*⟧ ⟹ *x* = *y*);
list-all2 (*fun-of S*) *xs ys*; *list-all2* (*fun-of T*) *ys xs* ⟧ ⟹ *xs* = *ys*
⟨*proof*⟩
lemma *rel-list-all2-trans*:
⟦ ⋀*a b c*. ⟦(*a*,*b*) ∈ *R*; (*b*,*c*) ∈ *S*⟧ ⟹ (*a*,*c*) ∈ *T*;
list-all2 (*fun-of R*) *as bs*; *list-all2* (*fun-of S*) *bs cs*⟧
⟹ *list-all2* (*fun-of T*) *as cs*
⟨*proof*⟩
lemma *rel-list-all2-update-cong*:
⟦ *i*<*size xs*; *list-all2* (*fun-of S*) *xs ys*; (*x*,*y*) ∈ *S* ⟧
⟹ *list-all2* (*fun-of S*) (*xs*[*i*:=*x*]) (*ys*[*i*:=*y*])
⟨*proof*⟩
lemma *rel-list-all2-nthD*:
⟦ *list-all2* (*fun-of S*) *xs ys*; *p* < *size xs* ⟧ ⟹ (*xs*!*p*,*ys*!*p*) ∈ *S*
⟨*proof*⟩
lemma *rel-list-all2I*:
⟦ *length a* = *length b*; ⋀*n*. *n* < *length a* ⟹ (*a*!*n*,*b*!*n*) ∈ *S* ⟧ ⟹ *list-all2* (*fun-of S*) *a b*
⟨*proof*⟩
end

2.2 Jinja types

theory *Type* **imports** *Auxiliary* **begin**

type-synonym *cname* = *string* — class names
type-synonym *mname* = *string* — method name
type-synonym *vname* = *string* — names for local/field variables

definition *Object* :: *cname*
where
Object ≡ ''*Object*''

definition *this* :: *vname*
where
this ≡ ''*this*''

— types
datatype *ty*
= *Void* — type of statements
| *Boolean*
| *Integer*

| *NT* — null type
| *Class cname* — class type

definition *is-refT* :: *ty* ⇒ *bool*
where
is-refT T ≡ $T = NT \vee (\exists C.\ T = Class\ C)$

lemma [*iff*]: *is-refT NT*⟨*proof*⟩
lemma [*iff*]: *is-refT*(*Class C*)⟨*proof*⟩
lemma *refTE*:
$[\![is\text{-}refT\ T;\ T = NT \Longrightarrow P;\ \bigwedge C.\ T = Class\ C \Longrightarrow P\ ]\!] \Longrightarrow P$⟨*proof*⟩
lemma *not-refTE*:
$[\![\ \neg is\text{-}refT\ T;\ T = Void \vee T = Boolean \vee T = Integer \Longrightarrow P\ ]\!] \Longrightarrow P$⟨*proof*⟩
end

2.3 Class Declarations and Programs

theory *Decl* **imports** *Type* **begin**

type-synonym
fdecl = *vname* × *ty* — field declaration
type-synonym
′m mdecl = *mname* × *ty list* × *ty* × *′m* — method = name, arg. types, return type, body
type-synonym
′m class = *cname* × *fdecl list* × *′m mdecl list* — class = superclass, fields, methods
type-synonym
′m cdecl = *cname* × *′m class* — class declaration
type-synonym
′m prog = *′m cdecl list* — program

definition *class* :: *′m prog* ⇒ *cname* ⇀ *′m class*
where
class ≡ *map-of*

definition *is-class* :: *′m prog* ⇒ *cname* ⇒ *bool*
where
is-class P C ≡ *class P C* ≠ *None*

lemma *finite-is-class*: *finite* {*C*. *is-class P C*}
⟨*proof*⟩
definition *is-type* :: *′m prog* ⇒ *ty* ⇒ *bool*
where
is-type P T ≡
(*case T of Void* ⇒ *True* | *Boolean* ⇒ *True* | *Integer* ⇒ *True* | *NT* ⇒ *True*
| *Class C* ⇒ *is-class P C*)

lemma *is-type-simps* [*simp*]:
is-type P Void ∧ *is-type P Boolean* ∧ *is-type P Integer* ∧
is-type P NT ∧ *is-type P* (*Class C*) = *is-class P C*⟨*proof*⟩

abbreviation
types P == *Collect* (*is-type P*)

end

2.4 Relations between Jinja Types

theory *TypeRel* **imports**
HOL−Library.Transitive-Closure-Table
Decl
begin

2.4.1 The subclass relations

inductive-set
$subcls1 :: {'m}\ prog \Rightarrow (cname \times cname)\ set$
and $subcls1' :: {'m}\ prog \Rightarrow [cname, cname] \Rightarrow bool$ (‹- ⊢ - $\prec^1$ -› [71,71,71] 70)
for $P :: {'m}\ prog$
where
$P \vdash C \prec^1 D \equiv (C,D) \in subcls1\ P$
| *subcls1I*: $[\![class\ P\ C = Some\ (D,rest);\ C \neq Object]\!] \Longrightarrow P \vdash C \prec^1 D$

abbreviation
$subcls :: {'m}\ prog \Rightarrow [cname, cname] \Rightarrow bool$ (‹- ⊢ - $\preceq^*$ -› [71,71,71] 70)
where $P \vdash C \preceq^* D \equiv (C,D) \in (subcls1\ P)^*$

lemma *subcls1D*: $P \vdash C \prec^1 D \Longrightarrow C \neq Object \wedge (\exists fs\ ms.\ class\ P\ C = Some\ (D,fs,ms))\langle proof\rangle$
lemma [*iff*]: $\neg\ P \vdash Object \prec^1 C\langle proof\rangle$
lemma [*iff*]: $(P \vdash Object \preceq^* C) = (C = Object)\langle proof\rangle$
lemma *subcls1-def2*:
$subcls1\ P =$
$(SIGMA\ C{:}\{C.\ is\text{-}class\ P\ C\}.\ \{D.\ C{\neq}Object \wedge fst\ (the\ (class\ P\ C)){=}D\})\langle proof\rangle$
lemma *finite-subcls1*: $finite\ (subcls1\ P)\langle proof\rangle$

2.4.2 The subtype relations

inductive
$widen :: {'m}\ prog \Rightarrow ty \Rightarrow ty \Rightarrow bool$ (‹- ⊢ - ≤ -› [71,71,71] 70)
for $P :: {'m}\ prog$
where
widen-refl[*iff*]: $P \vdash T \le T$
| *widen-subcls*: $P \vdash C \preceq^* D \Longrightarrow P \vdash Class\ C \le Class\ D$
| *widen-null*[*iff*]: $P \vdash NT \le Class\ C$

abbreviation
$widens :: {'m}\ prog \Rightarrow ty\ list \Rightarrow ty\ list \Rightarrow bool$
(‹- ⊢ - [≤] -› [71,71,71] 70) **where**
$widens\ P\ Ts\ Ts' \equiv list\text{-}all2\ (widen\ P)\ Ts\ Ts'$

lemma [*iff*]: $(P \vdash T \le Void) = (T = Void)\langle proof\rangle$
lemma [*iff*]: $(P \vdash T \le Boolean) = (T = Boolean)\langle proof\rangle$
lemma [*iff*]: $(P \vdash T \le Integer) = (T = Integer)\langle proof\rangle$
lemma [*iff*]: $(P \vdash Void \le T) = (T = Void)\langle proof\rangle$
lemma [*iff*]: $(P \vdash Boolean \le T) = (T = Boolean)\langle proof\rangle$
lemma [*iff*]: $(P \vdash Integer \le T) = (T = Integer)\langle proof\rangle$

lemma *Class-widen*: $P \vdash Class\ C \le T \Longrightarrow \exists D.\ T = Class\ D\langle proof\rangle$

lemma [*iff*]: $(P \vdash T \leq NT) = (T = NT)$⟨*proof*⟩
lemma *Class-widen-Class* [*iff*]: $(P \vdash Class\ C \leq Class\ D) = (P \vdash C \preceq^* D)$⟨*proof*⟩
lemma *widen-Class*: $(P \vdash T \leq Class\ C) = (T = NT \vee (\exists D.\ T = Class\ D \wedge P \vdash D \preceq^* C))$⟨*proof*⟩

lemma *widen-trans*[*trans*]: $[\![P \vdash S \leq U;\ P \vdash U \leq T]\!] \Longrightarrow P \vdash S \leq T$⟨*proof*⟩
lemma *widens-trans* [*trans*]: $[\![P \vdash Ss\ [\leq]\ Ts;\ P \vdash Ts\ [\leq]\ Us]\!] \Longrightarrow P \vdash Ss\ [\leq]\ Us$⟨*proof*⟩

2.4.3 Method lookup

inductive
  Methods :: $['m\ prog,\ cname,\ mname \rightharpoonup (ty\ list \times ty \times 'm) \times cname] \Rightarrow bool$
    (‹- ⊢ - sees′-methods -› [51,51,51] 50)
  for P :: $'m\ prog$
where
  sees-methods-Object:
  $[\![$ class P Object = Some(D,fs,ms); Mm = map-option ($\lambda m.$ (m,Object)) ∘ map-of ms $]\!]$
  $\Longrightarrow P \vdash$ Object sees-methods Mm
| *sees-methods-rec*:
  $[\![$ class P C = Some(D,fs,ms); $C \neq$ Object; $P \vdash D$ sees-methods Mm;
    Mm′ = Mm ++ (map-option ($\lambda m.$ (m,C)) ∘ map-of ms) $]\!]$
  $\Longrightarrow P \vdash C$ sees-methods Mm′

lemma *sees-methods-fun*:
assumes *1*: $P \vdash C$ sees-methods Mm
shows $\bigwedge Mm'.\ P \vdash C$ sees-methods $Mm' \Longrightarrow Mm' = Mm$
⟨*proof*⟩
lemma *visible-methods-exist*:
  $P \vdash C$ sees-methods Mm $\Longrightarrow$ Mm M = Some(m,D) $\Longrightarrow$
  ($\exists D'$ fs ms. class P D = Some(D′,fs,ms) $\wedge$ map-of ms M = Some m)
⟨*proof*⟩
lemma *sees-methods-decl-above*:
assumes *Csees*: $P \vdash C$ sees-methods Mm
shows Mm M = Some(m,D) $\Longrightarrow P \vdash C \preceq^* D$
⟨*proof*⟩
lemma *sees-methods-idemp*:
assumes *Cmethods*: $P \vdash C$ sees-methods Mm
shows $\bigwedge m\ D.$ Mm M = Some(m,D) $\Longrightarrow$
    $\exists Mm'.$ ($P \vdash D$ sees-methods Mm′) $\wedge$ Mm′ M = Some(m,D)⟨*proof*⟩

lemma *sees-methods-decl-mono*:
assumes *sub*: $P \vdash C' \preceq^* C$
shows $P \vdash C$ sees-methods Mm $\Longrightarrow$
  $\exists Mm'\ Mm_2.\ P \vdash C'$ sees-methods $Mm' \wedge Mm' = Mm\ {+}{+}\ Mm_2 \wedge$
    $(\forall M\ m\ D.\ Mm_2\ M = Some(m,D) \longrightarrow P \vdash D \preceq^* C)$⟨*proof*⟩

definition *Method* :: $'m\ prog \Rightarrow cname \Rightarrow mname \Rightarrow ty\ list \Rightarrow ty \Rightarrow 'm \Rightarrow cname \Rightarrow bool$
  (‹- ⊢ - sees -: -→- = - in -› [51,51,51,51,51,51,51] 50)
where
  $P \vdash C$ sees M: Ts→T = m in D ≡
  $\exists Mm.\ P \vdash C$ sees-methods Mm $\wedge$ Mm M = Some((Ts,T,m),D)

definition *has-method* :: $'m\ prog \Rightarrow cname \Rightarrow mname \Rightarrow bool$ (‹- ⊢ - has -› [51,0,51] 50)
where

$P \vdash C$ *has* $M \equiv \exists$ *Ts T m D*. $P \vdash C$ *sees* M:*Ts*$\rightarrow T = m$ *in* D

lemma *sees-method-fun*:
$[\![P \vdash C$ *sees* M:$TS \rightarrow T = m$ *in* D; $P \vdash C$ *sees* M:$TS' \rightarrow T' = m'$ *in* D' $]\!]$
$\Longrightarrow TS' = TS \wedge T' = T \wedge m' = m \wedge D' = D$
⟨*proof*⟩
lemma *sees-method-decl-above*:
$P \vdash C$ *sees* M:*Ts*$\rightarrow T = m$ *in* $D \Longrightarrow P \vdash C \preceq^* D$
⟨*proof*⟩
lemma *visible-method-exists*:
$P \vdash C$ *sees* M:*Ts*$\rightarrow T = m$ *in* $D \Longrightarrow$
$\exists D'$ *fs ms*. *class* P D = *Some*(D',*fs*,*ms*) $\wedge$ *map-of ms* M = *Some*(*Ts*,T,m)⟨*proof*⟩

lemma *sees-method-idemp*:
$P \vdash C$ *sees* M:*Ts*$\rightarrow T$=m *in* $D \Longrightarrow P \vdash D$ *sees* M:*Ts*$\rightarrow T$=m *in* D
⟨*proof*⟩
lemma *sees-method-decl-mono*:
assumes *sub*: $P \vdash C' \preceq^* C$ **and**
C-sees: $P \vdash C$ *sees* M:*Ts*$\rightarrow T$=m *in* D **and**
C′-sees: $P \vdash C'$ *sees* M:*Ts*$'\rightarrow T'$=m' *in* D'
shows $P \vdash D' \preceq^* D$
⟨*proof*⟩
lemma *sees-method-is-class*:
$[\![$ $P \vdash C$ *sees* M:*Ts*$\rightarrow T = m$ *in* D $]\!] \Longrightarrow$ *is-class* P C⟨*proof*⟩

2.4.4 Field lookup

inductive
Fields :: [′*m prog*, *cname*, ((*vname* × *cname*) × *ty*) *list*] ⇒ *bool*
(‹- ⊢ - *has*′*-fields* -› [*51*,*51*,*51*] *50*)
for P :: ′*m prog*
where
has-fields-rec:
$[\![$ *class* P C = *Some*(D,*fs*,*ms*); $C \neq$ *Object*; $P \vdash D$ *has-fields FDTs*;
FDTs′ = *map* (λ(F,T). ((F,C),T)) *fs* @ *FDTs* $]\!]$
$\Longrightarrow P \vdash C$ *has-fields FDTs*′
| *has-fields-Object*:
$[\![$ *class* P *Object* = *Some*(D,*fs*,*ms*); *FDTs* = *map* (λ(F,T). ((F,*Object*),T)) *fs* $]\!]$
$\Longrightarrow P \vdash$ *Object has-fields FDTs*

lemma *has-fields-fun*:
assumes *1*: $P \vdash C$ *has-fields FDTs*
shows $\bigwedge$*FDTs*′. $P \vdash C$ *has-fields FDTs*′ $\Longrightarrow$ *FDTs*′ = *FDTs*
⟨*proof*⟩
lemma *all-fields-in-has-fields*:
assumes *sub*: $P \vdash C$ *has-fields FDTs*
shows $[\![$ $P \vdash C \preceq^* D$; *class* P D = *Some*(D',*fs*,*ms*); (F,T) ∈ *set fs* $]\!]$
$\Longrightarrow$ ((F,D),T) ∈ *set FDTs*⟨*proof*⟩

lemma *has-fields-decl-above*:
assumes *fields*: $P \vdash C$ *has-fields FDTs*
shows ((F,D),T) ∈ *set FDTs* $\Longrightarrow P \vdash C \preceq^* D$⟨*proof*⟩

lemma *subcls-notin-has-fields*:

assumes *fields*: $P \vdash C$ *has-fields FDTs*
shows $((F,D),T) \in$ *set FDTs* $\Longrightarrow (D,C) \notin (\mathit{subcls1}\ P)^+\langle proof\rangle$

lemma *has-fields-mono-lem*:
assumes *sub*: $P \vdash D \preceq^* C$
shows $P \vdash C$ *has-fields FDTs*
$\Longrightarrow \exists\, pre.\ P \vdash D$ *has-fields pre@FDTs* $\wedge$ *dom*(*map-of pre*) $\cap$ *dom*(*map-of FDTs*) $= \{\}\langle proof\rangle$

definition *has-field* :: $'m$ *prog* $\Rightarrow$ *cname* $\Rightarrow$ *vname* $\Rightarrow$ *ty* $\Rightarrow$ *cname* $\Rightarrow$ *bool*
(‹- ⊢ - has -:- in -› [51,51,51,51,51] 50)
where
$P \vdash C$ *has* $F{:}T$ *in* $D \equiv$
$\exists FDTs.\ P \vdash C$ *has-fields FDTs* $\wedge$ *map-of FDTs* $(F,D) =$ *Some* T

lemma *has-field-mono*:
assumes *has*: $P \vdash C$ *has* $F{:}T$ *in* D **and** *sub*: $P \vdash C' \preceq^* C$
shows $P \vdash C'$ *has* $F{:}T$ *in* $D\langle proof\rangle$

definition *sees-field* :: $'m$ *prog* $\Rightarrow$ *cname* $\Rightarrow$ *vname* $\Rightarrow$ *ty* $\Rightarrow$ *cname* $\Rightarrow$ *bool*
(‹- ⊢ - sees -:- in -› [51,51,51,51,51] 50)
where
$P \vdash C$ *sees* $F{:}T$ *in* $D \equiv$
$\exists FDTs.\ P \vdash C$ *has-fields FDTs* $\wedge$
map-of (*map* $(\lambda((F,D),T).\ (F,(D,T)))$ *FDTs*) $F =$ *Some*(D,T)

lemma *map-of-remap-SomeD*:
map-of (*map* $(\lambda((k,k'),x).\ (k,(k',x)))$ t) $k =$ *Some* $(k',x) \Longrightarrow$ *map-of* t $(k, k') =$ *Some* $x\langle proof\rangle$

lemma *has-visible-field*:
$P \vdash C$ *sees* $F{:}T$ *in* $D \Longrightarrow P \vdash C$ *has* $F{:}T$ *in* $D\langle proof\rangle$

lemma *sees-field-fun*:
$[\![P \vdash C$ *sees* $F{:}T$ *in* D; $P \vdash C$ *sees* $F{:}T'$ *in* $D']\!] \Longrightarrow T' = T \wedge D' = D\langle proof\rangle$

lemma *sees-field-decl-above*:
$P \vdash C$ *sees* $F{:}T$ *in* $D \Longrightarrow P \vdash C \preceq^* D\langle proof\rangle$
lemma *sees-field-idemp*:
assumes *sees*: $P \vdash C$ *sees* $F{:}T$ *in* D
shows $P \vdash D$ *sees* $F{:}T$ *in* $D\langle proof\rangle$

2.4.5 Functional lookup

definition *method* :: $'m$ *prog* $\Rightarrow$ *cname* $\Rightarrow$ *mname* $\Rightarrow$ *cname* $\times$ *ty list* $\times$ *ty* $\times\ 'm$
where
method P C $M \equiv$ *THE* $(D,Ts,T,m).\ P \vdash C$ *sees* $M{:}Ts \rightarrow T = m$ *in* D

definition *field* :: $'m$ *prog* $\Rightarrow$ *cname* $\Rightarrow$ *vname* $\Rightarrow$ *cname* $\times$ *ty*
where
field P C $F \equiv$ *THE* $(D,T).\ P \vdash C$ *sees* $F{:}T$ *in* D

definition *fields* :: $'m$ *prog* $\Rightarrow$ *cname* $\Rightarrow$ ((*vname* $\times$ *cname*) $\times$ *ty*) *list*
where
fields P $C \equiv$ *THE FDTs*. $P \vdash C$ *has-fields FDTs*

lemma *fields-def2* [*simp*]: $P \vdash C$ *has-fields FDTs* $\Longrightarrow$ *fields P C* = *FDTs*⟨*proof*⟩
lemma *field-def2* [*simp*]: $P \vdash C$ *sees F*:*T in D* $\Longrightarrow$ *field P C F* = (*D*,*T*)⟨*proof*⟩
lemma *method-def2* [*simp*]: $P \vdash C$ *sees M*: *Ts*→*T* = *m in D* $\Longrightarrow$ *method P C M* = (*D*,*Ts*,*T*,*m*)⟨*proof*⟩

2.4.6 Code generator setup

code-pred
(*modes*: *i* ⇒ *i* ⇒ *i* ⇒ *bool*, *i* ⇒ *i* ⇒ *o* ⇒ *bool*)
subcls1p
⟨*proof*⟩
declare *subcls1-def* [*code-pred-def*]

code-pred
(*modes*: *i* ⇒ *i* × *o* ⇒ *bool*, *i* ⇒ *i* × *i* ⇒ *bool*)
[*inductify*]
subcls1
⟨*proof*⟩
definition *subcls′* **where** *subcls′ G* = (*subcls1p G*)^**
code-pred
(*modes*: *i* ⇒ *i* ⇒ *i* ⇒ *bool*, *i* ⇒ *i* ⇒ *o* ⇒ *bool*)
[*inductify*]
subcls′
⟨*proof*⟩

lemma *subcls-conv-subcls′* [*code-unfold*]:
(*subcls1 G*)^* = {(*C*, *D*). *subcls′ G C D*}
⟨*proof*⟩

code-pred
(*modes*: *i* ⇒ *i* ⇒ *i* ⇒ *bool*)
widen
⟨*proof*⟩

code-pred
(*modes*: *i* ⇒ *i* ⇒ *o* ⇒ *bool*)
Fields
⟨*proof*⟩

lemma *has-field-code* [*code-pred-intro*]:
⟦ $P \vdash C$ *has-fields FDTs*; *map-of FDTs* (*F*, *D*) = ⌊*T*⌋ ⟧
$\Longrightarrow$ $P \vdash C$ *has F*:*T in D*
⟨*proof*⟩

code-pred
(*modes*: *i* ⇒ *i* ⇒ *i* ⇒ *o* ⇒ *i* ⇒ *bool*, *i* ⇒ *i* ⇒ *i* ⇒ *i* ⇒ *i* ⇒ *bool*)
has-field
⟨*proof*⟩

lemma *sees-field-code* [*code-pred-intro*]:
⟦ $P \vdash C$ *has-fields FDTs*; *map-of* (*map* (λ((*F*, *D*), *T*). (*F*, *D*, *T*)) *FDTs*) *F* = ⌊(*D*, *T*)⌋ ⟧
$\Longrightarrow$ $P \vdash C$ *sees F*:*T in D*
⟨*proof*⟩

code-pred

(*modes*: $i \Rightarrow i \Rightarrow i \Rightarrow o \Rightarrow o \Rightarrow bool$, $i \Rightarrow i \Rightarrow i \Rightarrow o \Rightarrow i \Rightarrow bool$,
$i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow o \Rightarrow bool$, $i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow bool$)
sees-field
⟨*proof*⟩

code-pred
(*modes*: $i \Rightarrow i \Rightarrow o \Rightarrow bool$)
Methods
⟨*proof*⟩

lemma *Method-code* [*code-pred-intro*]:
⟦ *P* ⊢ *C sees-methods Mm*; *Mm M* = ⌊((*Ts*, *T*, *m*), *D*)⌋ ⟧
⟹ *P* ⊢ *C sees M*: *Ts*→*T* = *m in D*
⟨*proof*⟩

code-pred
(*modes*: $i \Rightarrow i \Rightarrow i \Rightarrow o \Rightarrow o \Rightarrow o \Rightarrow o \Rightarrow bool$,
$i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow bool$)
Method
⟨*proof*⟩

lemma *eval-Method-i-i-i-o-o-o-o-conv*:
Predicate.eval (*Method-i-i-i-o-o-o-o P C M*) = (λ(*Ts*, *T*, *m*, *D*). *P* ⊢ *C sees M*:*Ts*→*T*=*m in D*)
⟨*proof*⟩

lemma *method-code* [*code*]:
method P C M =
Predicate.the (*Predicate.bind* (*Method-i-i-i-o-o-o-o P C M*) (λ(*Ts*, *T*, *m*, *D*). *Predicate.single* (*D*, *Ts*, *T*, *m*)))
⟨*proof*⟩

lemma *eval-Fields-conv*:
Predicate.eval (*Fields-i-i-o P C*) = (λ*FDTs*. *P* ⊢ *C has-fields FDTs*)
⟨*proof*⟩

lemma *fields-code* [*code*]:
fields P C = *Predicate.the* (*Fields-i-i-o P C*)
⟨*proof*⟩

lemma *eval-sees-field-i-i-i-o-o-conv*:
Predicate.eval (*sees-field-i-i-i-o-o P C F*) = (λ(*T*, *D*). *P* ⊢ *C sees F*:*T in D*)
⟨*proof*⟩

lemma *eval-sees-field-i-i-i-o-i-conv*:
Predicate.eval (*sees-field-i-i-i-o-i P C F D*) = (λ*T*. *P* ⊢ *C sees F*:*T in D*)
⟨*proof*⟩

lemma *field-code* [*code*]:
field P C F = *Predicate.the* (*Predicate.bind* (*sees-field-i-i-i-o-o P C F*) (λ(*T*, *D*). *Predicate.single* (*D*, *T*)))
⟨*proof*⟩

2.5 Jinja Values

theory *Value* **imports** *TypeRel* **begin**

type-synonym *addr* = *nat*

datatype *val*
  = *Unit* — dummy result value of void expressions
  | *Null* — null reference
  | *Bool bool* — Boolean value
  | *Intg int* — integer value
  | *Addr addr* — addresses of objects in the heap

primrec *the-Intg* :: *val* ⇒ *int* **where**
  the-Intg (*Intg i*) = *i*

primrec *the-Addr* :: *val* ⇒ *addr* **where**
  the-Addr (*Addr a*) = *a*

primrec *default-val* :: *ty* ⇒ *val* — default value for all types **where**
  default-val Void = *Unit*
| *default-val Boolean* = *Bool False*
| *default-val Integer* = *Intg 0*
| *default-val NT* = *Null*
| *default-val* (*Class C*) = *Null*

end

2.6 Objects and the Heap

theory *Objects* **imports** *TypeRel Value* **begin**

2.6.1 Objects

type-synonym
  fields = *vname* × *cname* ⇀ *val* — field name, defining class, value
type-synonym
  obj = *cname* × *fields* — class instance with class name and fields

definition *obj-ty* :: *obj* ⇒ *ty*
where
  obj-ty obj ≡ *Class* (*fst obj*)

definition *init-fields* :: ((*vname* × *cname*) × *ty*) *list* ⇒ *fields*
where
  init-fields ≡ *map-of* ∘ *map* (λ(*F*,*T*). (*F*,*default-val T*))

  — a new, blank object with default values in all fields:
definition *blank* :: *'m prog* ⇒ *cname* ⇒ *obj*
where
  blank P C ≡ (*C*,*init-fields* (*fields P C*))

lemma [*simp*]: *obj-ty* (*C*,*fs*) = *Class C*⟨*proof*⟩

2.6.2 Heap

type-synonym *heap* = *addr* ⇀ *obj*

abbreviation
  cname-of :: *heap* ⇒ *addr* ⇒ *cname* **where**
  cname-of hp a == *fst* (*the* (*hp a*))

definition *new-Addr* :: *heap* ⇒ *addr option*
where
  new-Addr h ≡ *if* ∃ *a*. *h a* = *None then Some*(*LEAST a*. *h a* = *None*) *else None*

definition *cast-ok* :: ′*m prog* ⇒ *cname* ⇒ *heap* ⇒ *val* ⇒ *bool*
where
  cast-ok P C h v ≡ *v* = *Null* ∨ *P* ⊢ *cname-of h* (*the-Addr v*) ⪯* *C*

definition *hext* :: *heap* ⇒ *heap* ⇒ *bool* (‹- ⊴ -› [*51*,*51*] *50*)
where
  h ⊴ *h*′ ≡ ∀ *a C fs*. *h a* = *Some*(*C*,*fs*) ⟶ (∃ *fs*′. *h*′ *a* = *Some*(*C*,*fs*′))

primrec *typeof-h* :: *heap* ⇒ *val* ⇒ *ty option* (‹*typeof*_›)
where
  $typeof_h$ *Unit* = *Some Void*
| $typeof_h$ *Null* = *Some NT*
| $typeof_h$ (*Bool b*) = *Some Boolean*
| $typeof_h$ (*Intg i*) = *Some Integer*
| $typeof_h$ (*Addr a*) = (*case h a of None* ⇒ *None* | *Some*(*C*,*fs*) ⇒ *Some*(*Class C*))

lemma *new-Addr-SomeD*:
  new-Addr h = *Some a* ⟹ *h a* = *None*
⟨*proof*⟩
lemma [*simp*]: ($typeof_h$ *v* = *Some Boolean*) = (∃ *b*. *v* = *Bool b*)
⟨*proof*⟩
lemma [*simp*]: ($typeof_h$ *v* = *Some Integer*) = (∃ *i*. *v* = *Intg i*)⟨*proof*⟩
lemma [*simp*]: ($typeof_h$ *v* = *Some NT*) = (*v* = *Null*)
⟨*proof*⟩
lemma [*simp*]: ($typeof_h$ *v* = *Some*(*Class C*)) = (∃ *a fs*. *v* = *Addr a* ∧ *h a* = *Some*(*C*,*fs*))
⟨*proof*⟩
lemma [*simp*]: *h a* = *Some*(*C*,*fs*) ⟹ $typeof_{(h(a \mapsto (C,fs')))}$ *v* = $typeof_h$ *v*
⟨*proof*⟩

For literal values the first parameter of *typeof* can be set to $\lambda x.$ *None* because they do not contain addresses:

abbreviation
  typeof :: *val* ⇒ *ty option* **where**
  typeof v == *typeof-h Map.empty v*

lemma *typeof-lit-typeof*:
  typeof v = *Some T* ⟹ $typeof_h$ *v* = *Some T*
⟨*proof*⟩
lemma *typeof-lit-is-type*:
  typeof v = *Some T* ⟹ *is-type P T*
⟨*proof*⟩

2.6.3 Heap extension $\trianglelefteq$

lemma *hextI*: $\forall a\ C\ fs.\ h\ a = Some(C,fs) \longrightarrow (\exists fs'.\ h'\ a = Some(C,fs')) \Longrightarrow h \trianglelefteq h' \langle proof \rangle$
lemma *hext-objD*: $[\![h \trianglelefteq h';\ h\ a = Some(C,fs)]\!] \Longrightarrow \exists fs'.\ h'\ a = Some(C,fs') \langle proof \rangle$
lemma *hext-refl* [*iff*]: $h \trianglelefteq h \langle proof \rangle$
lemma *hext-new* [*simp*]: $h\ a = None \Longrightarrow h \trianglelefteq h(a \mapsto x) \langle proof \rangle$
lemma *hext-trans*: $[\![h \trianglelefteq h';\ h' \trianglelefteq h'']\!] \Longrightarrow h \trianglelefteq h'' \langle proof \rangle$
lemma *hext-upd-obj*: $h\ a = Some\ (C,fs) \Longrightarrow h \trianglelefteq h(a \mapsto (C,fs')) \langle proof \rangle$
lemma *hext-typeof-mono*: $[\![h \trianglelefteq h';\ typeof_h\ v = Some\ T]\!] \Longrightarrow typeof_{h'}\ v = Some\ T \langle proof \rangle$

Code generator setup for *new-Addr*

definition *gen-new-Addr* :: *heap* ⇒ *addr* ⇒ *addr option*
where *gen-new-Addr h n* ≡ *if* $\exists a.\ a \geq n \wedge h\ a = None$ *then Some*(*LEAST* $a.\ a \geq n \wedge h\ a = None$) *else None*

lemma *new-Addr-code-code* [*code*]:
new-Addr h = *gen-new-Addr h 0*
⟨*proof*⟩

lemma *gen-new-Addr-code* [*code*]:
gen-new-Addr h n = (*if h n* = *None then Some n else gen-new-Addr h* (*Suc n*))
⟨*proof*⟩

end

2.7 Exceptions

theory *Exceptions* **imports** *Objects* **begin**

definition *NullPointer* :: *cname*
where
NullPointer ≡ *''NullPointer''*

definition *ClassCast* :: *cname*
where
ClassCast ≡ *''ClassCast''*

definition *OutOfMemory* :: *cname*
where
OutOfMemory ≡ *''OutOfMemory''*

definition *sys-xcpts* :: *cname set*
where
sys-xcpts ≡ {*NullPointer*, *ClassCast*, *OutOfMemory*}

definition *addr-of-sys-xcpt* :: *cname* ⇒ *addr*
where
addr-of-sys-xcpt s ≡ *if s* = *NullPointer then 0 else*
if s = *ClassCast then 1 else*
if s = *OutOfMemory then 2 else undefined*

definition *start-heap* :: *'c prog* ⇒ *heap*
where
start-heap G ≡ *Map.empty* (*addr-of-sys-xcpt NullPointer* ↦ *blank G NullPointer*,

addr-of-sys-xcpt ClassCast ↦ *blank G ClassCast*,
addr-of-sys-xcpt OutOfMemory ↦ *blank G OutOfMemory*)

definition *preallocated* :: *heap* ⇒ *bool*
where
preallocated h ≡ ∀ *C* ∈ *sys-xcpts*. ∃ *fs*. *h*(*addr-of-sys-xcpt C*) = *Some* (*C*,*fs*)

2.7.1 System exceptions

lemma [*simp*]: *NullPointer* ∈ *sys-xcpts* ∧ *OutOfMemory* ∈ *sys-xcpts* ∧ *ClassCast* ∈ *sys-xcpts*⟨*proof*⟩

lemma *sys-xcpts-cases* [*consumes 1*, *cases set*]:
⟦ *C* ∈ *sys-xcpts*; *P NullPointer*; *P OutOfMemory*; *P ClassCast*⟧ ⟹ *P C*⟨*proof*⟩

2.7.2 *preallocated*

lemma *preallocated-dom* [*simp*]:
⟦ *preallocated h*; *C* ∈ *sys-xcpts* ⟧ ⟹ *addr-of-sys-xcpt C* ∈ *dom h*⟨*proof*⟩

lemma *preallocatedD*:
⟦ *preallocated h*; *C* ∈ *sys-xcpts* ⟧ ⟹ ∃ *fs*. *h*(*addr-of-sys-xcpt C*) = *Some* (*C*, *fs*)⟨*proof*⟩

lemma *preallocatedE* [*elim?*]:
⟦ *preallocated h*; *C* ∈ *sys-xcpts*; ⋀*fs*. *h*(*addr-of-sys-xcpt C*) = *Some*(*C*,*fs*) ⟹ *P h C*⟧
⟹ *P h C*⟨*proof*⟩

lemma *cname-of-xcp* [*simp*]:
⟦ *preallocated h*; *C* ∈ *sys-xcpts* ⟧ ⟹ *cname-of h* (*addr-of-sys-xcpt C*) = *C*⟨*proof*⟩

lemma *typeof-ClassCast* [*simp*]:
preallocated h ⟹ $typeof_h$ (*Addr*(*addr-of-sys-xcpt ClassCast*)) = *Some*(*Class ClassCast*)⟨*proof*⟩

lemma *typeof-OutOfMemory* [*simp*]:
preallocated h ⟹ $typeof_h$ (*Addr*(*addr-of-sys-xcpt OutOfMemory*)) = *Some*(*Class OutOfMemory*)⟨*proof*⟩

lemma *typeof-NullPointer* [*simp*]:
preallocated h ⟹ $typeof_h$ (*Addr*(*addr-of-sys-xcpt NullPointer*)) = *Some*(*Class NullPointer*)⟨*proof*⟩

lemma *preallocated-hext*:
⟦ *preallocated h*; *h* ⊴ *h′* ⟧ ⟹ *preallocated h′*⟨*proof*⟩

lemma *preallocated-start*:
preallocated (*start-heap P*)⟨*proof*⟩

end

2.8 Expressions

theory *Expr*
imports ../*Common*/*Exceptions*
begin

datatype *bop* = *Eq* | *Add* — names of binary operations

datatype *'a exp*
= *new cname* — class instance creation
| *Cast cname* (*'a exp*) — type cast
| *Val val* — value
| *BinOp* (*'a exp*) *bop* (*'a exp*) (‹- «-» -› [*80*,*0*,*81*] *80*) — binary operation
| *Var 'a* — local variable (incl. parameter)
| *LAss 'a* (*'a exp*) (‹-:=-› [*90*,*90*]*90*) — local assignment
| *FAcc* (*'a exp*) *vname cname* (‹-∙-{-}› [*10*,*90*,*99*]*90*) — field access
| *FAss* (*'a exp*) *vname cname* (*'a exp*) (‹-∙-{-} := -› [*10*,*90*,*99*,*90*]*90*) — field assignment
| *Call* (*'a exp*) *mname* (*'a exp list*) (‹-∙-'(-')› [*90*,*99*,*0*] *90*) — method call
| *Block 'a ty* (*'a exp*) (‹'{-:-; -}›)
| *Seq* (*'a exp*) (*'a exp*) (‹-;;/ -› [*61*,*60*]*60*)
| *Cond* (*'a exp*) (*'a exp*) (*'a exp*) (‹*if* '(-') -/ *else* -› [*80*,*79*,*79*]*70*)
| *While* (*'a exp*) (*'a exp*) (‹*while* '(-') -› [*80*,*79*]*70*)
| *throw* (*'a exp*)
| *TryCatch* (*'a exp*) *cname 'a* (*'a exp*) (‹*try* -/ *catch*'(- -') -› [*0*,*99*,*80*,*79*] *70*)

type-synonym
expr = *vname exp* — Jinja expression
type-synonym
J-mb = *vname list* × *expr* — Jinja method body: parameter names and expression
type-synonym
J-prog = *J-mb prog* — Jinja program

The semantics of binary operators:

fun *binop* :: *bop* × *val* × *val* ⇒ *val option* **where**
binop(*Eq*,v_1,v_2) = *Some*(*Bool* (v_1 = v_2))
| *binop*(*Add*,*Intg* i_1,*Intg* i_2) = *Some*(*Intg*(i_1+i_2))
| *binop*(*bop*,v_1,v_2) = *None*

lemma [*simp*]:
(*binop*(*Add*,v_1,v_2) = *Some v*) = (∃ i_1 i_2. v_1 = *Intg* i_1 ∧ v_2 = *Intg* i_2 ∧ *v* = *Intg*(i_1+i_2))⟨*proof*⟩

2.8.1 Syntactic sugar

abbreviation (*input*)
InitBlock:: *'a* ⇒ *ty* ⇒ *'a exp* ⇒ *'a exp* ⇒ *'a exp* (‹(*1*'{-:- := -;/ -})›) **where**
InitBlock V T e1 e2 == {*V*:*T*; *V* := *e1*;; *e2*}

abbreviation *unit* **where** *unit* == *Val Unit*
abbreviation *null* **where** *null* == *Val Null*
abbreviation *addr a* == *Val*(*Addr a*)
abbreviation *true* == *Val*(*Bool True*)
abbreviation *false* == *Val*(*Bool False*)

abbreviation
Throw :: *addr* ⇒ *'a exp* **where**
Throw a == *throw*(*Val*(*Addr a*))

abbreviation
THROW :: *cname* ⇒ *'a exp* **where**
THROW xc == *Throw*(*addr-of-sys-xcpt xc*)

2.8.2 Free Variables

primrec *fv* :: *expr* ⇒ *vname set* **and** *fvs* :: *expr list* ⇒ *vname set* **where**
fv(*new C*) = {}
| *fv*(*Cast C e*) = *fv e*
| *fv*(*Val v*) = {}
| *fv*(e_1 «*bop*» e_2) = *fv* e_1 ∪ *fv* e_2
| *fv*(*Var V*) = {*V*}
| *fv*(*LAss V e*) = {*V*} ∪ *fv e*
| *fv*(*e*•*F*{*D*}) = *fv e*
| *fv*(e_1•*F*{*D*}:=e_2) = *fv* e_1 ∪ *fv* e_2
| *fv*(*e*•*M*(*es*)) = *fv e* ∪ *fvs es*
| *fv*({*V*:*T*; *e*}) = *fv e* − {*V*}
| *fv*(e_1;;e_2) = *fv* e_1 ∪ *fv* e_2
| *fv*(*if* (*b*) e_1 *else* e_2) = *fv b* ∪ *fv* e_1 ∪ *fv* e_2
| *fv*(*while* (*b*) *e*) = *fv b* ∪ *fv e*
| *fv*(*throw e*) = *fv e*
| *fv*(*try* e_1 *catch*(*C V*) e_2) = *fv* e_1 ∪ (*fv* e_2 − {*V*})
| *fvs*([]) = {}
| *fvs*(*e*#*es*) = *fv e* ∪ *fvs es*

lemma [*simp*]: *fvs*(es_1 @ es_2) = *fvs* es_1 ∪ *fvs* es_2⟨*proof*⟩
lemma [*simp*]: *fvs*(*map Val vs*) = {}⟨*proof*⟩
end

2.9 Program State

theory *State* **imports** ../*Common*/*Exceptions* **begin**

type-synonym
locals = *vname* ⇀ *val* — local vars, incl. params and "this"
type-synonym
state = *heap* × *locals*

definition *hp* :: *state* ⇒ *heap*
where
hp ≡ *fst*
definition *lcl* :: *state* ⇒ *locals*
where
lcl ≡ *snd*
end

2.10 Big Step Semantics

theory *BigStep* **imports** *Expr State* **begin**

inductive
eval :: *J-prog* ⇒ *expr* ⇒ *state* ⇒ *expr* ⇒ *state* ⇒ *bool*
(‹- ⊢ ((*1*⟨-,/-⟩) ⇒/ (*1*⟨-,/-⟩))› [*51*,*0*,*0*,*0*,*0*] *81*)
and *evals* :: *J-prog* ⇒ *expr list* ⇒ *state* ⇒ *expr list* ⇒ *state* ⇒ *bool*
(‹- ⊢ ((*1*⟨-,/-⟩) [⇒]/ (*1*⟨-,/-⟩))› [*51*,*0*,*0*,*0*,*0*] *81*)
for *P* :: *J-prog*
where

New:
$[\![$ *new-Addr* h = *Some* a; $P \vdash C$ *has-fields FDTs*; $h' = h(a \mapsto (C,$*init-fields FDTs*$))$ $]\!]$
$\Longrightarrow P \vdash \langle$*new* $C,(h,l)\rangle \Rightarrow \langle$*addr* $a,(h',l)\rangle$

| *NewFail*:
new-Addr h = *None* $\Longrightarrow$
$P \vdash \langle$*new* $C,\ (h,l)\rangle \Rightarrow \langle$*THROW OutOfMemory*$,(h,l)\rangle$

| *Cast*:
$[\![\ P \vdash \langle e,s_0\rangle \Rightarrow \langle$*addr* $a,(h,l)\rangle$; $h\ a$ = *Some*(D,fs); $P \vdash D \preceq^* C$ $]\!]$
$\Longrightarrow P \vdash \langle$*Cast* $C\ e,s_0\rangle \Rightarrow \langle$*addr* $a,(h,l)\rangle$

| *CastNull*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle$*null*$,s_1\rangle \Longrightarrow$
$P \vdash \langle$*Cast* $C\ e,s_0\rangle \Rightarrow \langle$*null*$,s_1\rangle$

| *CastFail*:
$[\![\ P \vdash \langle e,s_0\rangle \Rightarrow \langle$*addr* $a,(h,l)\rangle$; $h\ a$ = *Some*(D,fs); $\neg\ P \vdash D \preceq^* C$ $]\!]$
$\Longrightarrow P \vdash \langle$*Cast* $C\ e,s_0\rangle \Rightarrow \langle$*THROW ClassCast*$,(h,l)\rangle$

| *CastThrow*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle$*throw* $e',s_1\rangle \Longrightarrow$
$P \vdash \langle$*Cast* $C\ e,s_0\rangle \Rightarrow \langle$*throw* $e',s_1\rangle$

| *Val*:
$P \vdash \langle$*Val* $v,s\rangle \Rightarrow \langle$*Val* $v,s\rangle$

| *BinOp*:
$[\![\ P \vdash \langle e_1,s_0\rangle \Rightarrow \langle$*Val* $v_1,s_1\rangle$; $P \vdash \langle e_2,s_1\rangle \Rightarrow \langle$*Val* $v_2,s_2\rangle$; *binop*(bop,v_1,v_2) = *Some* v $]\!]$
$\Longrightarrow P \vdash \langle e_1$ «*bop*» $e_2,s_0\rangle \Rightarrow \langle$*Val* $v,s_2\rangle$

| *BinOpThrow1*:
$P \vdash \langle e_1,s_0\rangle \Rightarrow \langle$*throw* $e,s_1\rangle \Longrightarrow$
$P \vdash \langle e_1$ «*bop*» $e_2,\ s_0\rangle \Rightarrow \langle$*throw* $e,s_1\rangle$

| *BinOpThrow2*:
$[\![\ P \vdash \langle e_1,s_0\rangle \Rightarrow \langle$*Val* $v_1,s_1\rangle$; $P \vdash \langle e_2,s_1\rangle \Rightarrow \langle$*throw* $e,s_2\rangle$ $]\!]$
$\Longrightarrow P \vdash \langle e_1$ «*bop*» $e_2,s_0\rangle \Rightarrow \langle$*throw* $e,s_2\rangle$

| *Var*:
$l\ V$ = *Some* $v \Longrightarrow$
$P \vdash \langle$*Var* $V,(h,l)\rangle \Rightarrow \langle$*Val* $v,(h,l)\rangle$

| *LAss*:
$[\![\ P \vdash \langle e,s_0\rangle \Rightarrow \langle$*Val* $v,(h,l)\rangle$; $l' = l(V \mapsto v)$ $]\!]$
$\Longrightarrow P \vdash \langle V{:=}e,s_0\rangle \Rightarrow \langle$*unit*$,(h,l')\rangle$

| *LAssThrow*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle$*throw* $e',s_1\rangle \Longrightarrow$
$P \vdash \langle V{:=}e,s_0\rangle \Rightarrow \langle$*throw* $e',s_1\rangle$

| *FAcc*:
$[\![\ P \vdash \langle e,s_0\rangle \Rightarrow \langle$*addr* $a,(h,l)\rangle$; $h\ a$ = *Some*(C,fs); $fs(F,D)$ = *Some* v $]\!]$

$\Longrightarrow P \vdash \langle e \cdot F\{D\}, s_0\rangle \Rightarrow \langle Val\ v, (h, l)\rangle$

| *FAccNull*:
$P \vdash \langle e, s_0\rangle \Rightarrow \langle null, s_1\rangle \Longrightarrow$
$P \vdash \langle e \cdot F\{D\}, s_0\rangle \Rightarrow \langle THROW\ NullPointer, s_1\rangle$

| *FAccThrow*:
$P \vdash \langle e, s_0\rangle \Rightarrow \langle throw\ e', s_1\rangle \Longrightarrow$
$P \vdash \langle e \cdot F\{D\}, s_0\rangle \Rightarrow \langle throw\ e', s_1\rangle$

| *FAss*:
$[\![\ P \vdash \langle e_1, s_0\rangle \Rightarrow \langle addr\ a, s_1\rangle;\ P \vdash \langle e_2, s_1\rangle \Rightarrow \langle Val\ v, (h_2, l_2)\rangle;$
$h_2\ a = Some(C, fs);\ fs' = fs((F, D) \mapsto v);\ h_2' = h_2(a \mapsto (C, fs'))\ ]\!]$
$\Longrightarrow P \vdash \langle e_1 \cdot F\{D\} := e_2, s_0\rangle \Rightarrow \langle unit, (h_2', l_2)\rangle$

| *FAssNull*:
$[\![\ P \vdash \langle e_1, s_0\rangle \Rightarrow \langle null, s_1\rangle;\ \ P \vdash \langle e_2, s_1\rangle \Rightarrow \langle Val\ v, s_2\rangle\ ]\!] \Longrightarrow$
$P \vdash \langle e_1 \cdot F\{D\} := e_2, s_0\rangle \Rightarrow \langle THROW\ NullPointer, s_2\rangle$

| *FAssThrow1*:
$P \vdash \langle e_1, s_0\rangle \Rightarrow \langle throw\ e', s_1\rangle \Longrightarrow$
$P \vdash \langle e_1 \cdot F\{D\} := e_2, s_0\rangle \Rightarrow \langle throw\ e', s_1\rangle$

| *FAssThrow2*:
$[\![\ P \vdash \langle e_1, s_0\rangle \Rightarrow \langle Val\ v, s_1\rangle;\ P \vdash \langle e_2, s_1\rangle \Rightarrow \langle throw\ e', s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle e_1 \cdot F\{D\} := e_2, s_0\rangle \Rightarrow \langle throw\ e', s_2\rangle$

| *CallObjThrow*:
$P \vdash \langle e, s_0\rangle \Rightarrow \langle throw\ e', s_1\rangle \Longrightarrow$
$P \vdash \langle e \cdot M(ps), s_0\rangle \Rightarrow \langle throw\ e', s_1\rangle$

| *CallParamsThrow*:
$[\![\ P \vdash \langle e, s_0\rangle \Rightarrow \langle Val\ v, s_1\rangle;\ P \vdash \langle es, s_1\rangle\ [\Rightarrow]\ \langle map\ Val\ vs\ @\ throw\ ex\ \#\ es', s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle e \cdot M(es), s_0\rangle \Rightarrow \langle throw\ ex, s_2\rangle$

| *CallNull*:
$[\![\ P \vdash \langle e, s_0\rangle \Rightarrow \langle null, s_1\rangle;\ \ P \vdash \langle ps, s_1\rangle\ [\Rightarrow]\ \langle map\ Val\ vs, s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle e \cdot M(ps), s_0\rangle \Rightarrow \langle THROW\ NullPointer, s_2\rangle$

| *Call*:
$[\![\ P \vdash \langle e, s_0\rangle \Rightarrow \langle addr\ a, s_1\rangle;\ \ P \vdash \langle ps, s_1\rangle\ [\Rightarrow]\ \langle map\ Val\ vs, (h_2, l_2)\rangle;$
$h_2\ a = Some(C, fs);\ \ P \vdash C\ sees\ M: Ts \rightarrow T = (pns, body)\ in\ D;$
$length\ vs = length\ pns;\ \ l_2' = [this \mapsto Addr\ a,\ pns[\mapsto] vs];$
$P \vdash \langle body, (h_2, l_2')\rangle \Rightarrow \langle e', (h_3, l_3)\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle e \cdot M(ps), s_0\rangle \Rightarrow \langle e', (h_3, l_2)\rangle$

| *Block*:
$P \vdash \langle e_0, (h_0, l_0(V := None))\rangle \Rightarrow \langle e_1, (h_1, l_1)\rangle \Longrightarrow$
$P \vdash \langle \{V : T;\ e_0\}, (h_0, l_0)\rangle \Rightarrow \langle e_1, (h_1, l_1(V := l_0\ \ V))\rangle$

| *Seq*:
$[\![\ P \vdash \langle e_0, s_0\rangle \Rightarrow \langle Val\ v, s_1\rangle;\ P \vdash \langle e_1, s_1\rangle \Rightarrow \langle e_2, s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle e_0;; e_1, s_0\rangle \Rightarrow \langle e_2, s_2\rangle$

| *SeqThrow*:
$P \vdash \langle e_0,s_0\rangle \Rightarrow \langle throw\ e,s_1\rangle \Longrightarrow$
$P \vdash \langle e_0;;e_1,s_0\rangle \Rightarrow \langle throw\ e,s_1\rangle$

| *CondT*:
$[\![\ P \vdash \langle e,s_0\rangle \Rightarrow \langle true,s_1\rangle;\ P \vdash \langle e_1,s_1\rangle \Rightarrow \langle e',s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle if\ (e)\ e_1\ else\ e_2,s_0\rangle \Rightarrow \langle e',s_2\rangle$

| *CondF*:
$[\![\ P \vdash \langle e,s_0\rangle \Rightarrow \langle false,s_1\rangle;\ P \vdash \langle e_2,s_1\rangle \Rightarrow \langle e',s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle if\ (e)\ e_1\ else\ e_2,s_0\rangle \Rightarrow \langle e',s_2\rangle$

| *CondThrow*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash \langle if\ (e)\ e_1\ else\ e_2,\ s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$

| *WhileF*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle false,s_1\rangle \Longrightarrow$
$P \vdash \langle while\ (e)\ c,s_0\rangle \Rightarrow \langle unit,s_1\rangle$

| *WhileT*:
$[\![\ P \vdash \langle e,s_0\rangle \Rightarrow \langle true,s_1\rangle;\ P \vdash \langle c,s_1\rangle \Rightarrow \langle Val\ v_1,s_2\rangle;\ P \vdash \langle while\ (e)\ c,s_2\rangle \Rightarrow \langle e_3,s_3\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle while\ (e)\ c,s_0\rangle \Rightarrow \langle e_3,s_3\rangle$

| *WhileCondThrow*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle\ throw\ e',s_1\rangle \Longrightarrow$
$P \vdash \langle while\ (e)\ c,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$

| *WhileBodyThrow*:
$[\![\ P \vdash \langle e,s_0\rangle \Rightarrow \langle true,s_1\rangle;\ P \vdash \langle c,s_1\rangle \Rightarrow \langle throw\ e',s_2\rangle]\!]$
$\Longrightarrow P \vdash \langle while\ (e)\ c,s_0\rangle \Rightarrow \langle throw\ e',s_2\rangle$

| *Throw*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle addr\ a,s_1\rangle \Longrightarrow$
$P \vdash \langle throw\ e,s_0\rangle \Rightarrow \langle Throw\ a,s_1\rangle$

| *ThrowNull*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle null,s_1\rangle \Longrightarrow$
$P \vdash \langle throw\ e,s_0\rangle \Rightarrow \langle THROW\ NullPointer,s_1\rangle$

| *ThrowThrow*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash \langle throw\ e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$

| *Try*:
$P \vdash \langle e_1,s_0\rangle \Rightarrow \langle Val\ v_1,s_1\rangle \Longrightarrow$
$P \vdash \langle try\ e_1\ catch(C\ V)\ e_2,s_0\rangle \Rightarrow \langle Val\ v_1,s_1\rangle$

| *TryCatch*:
$[\![\ P \vdash \langle e_1,s_0\rangle \Rightarrow \langle Throw\ a,(h_1,l_1)\rangle;\ \ h_1\ a = Some(D,fs);\ \ P \vdash D \preceq^*\ C;$
$P \vdash \langle e_2,(h_1,l_1(V \mapsto Addr\ a))\rangle \Rightarrow \langle e_2',(h_2,l_2)\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle try\ e_1\ catch(C\ V)\ e_2,s_0\rangle \Rightarrow \langle e_2',(h_2,l_2(V:=l_1\ \ V))\rangle$

| *TryThrow*:

$[\![P \vdash \langle e_1,s_0\rangle \Rightarrow \langle Throw\ a,(h_1,l_1)\rangle;\ \ h_1\ a = Some(D,fs);\ \ \neg\ P \vdash D \preceq^* C\ ]\!]$
$\Longrightarrow P \vdash \langle try\ e_1\ catch(C\ V)\ e_2,s_0\rangle \Rightarrow \langle Throw\ a,(h_1,l_1)\rangle$

| *Nil*:
$P \vdash \langle [],s\rangle\ [\Rightarrow]\ \langle [],s\rangle$

| *Cons*:
$[\![P \vdash \langle e,s_0\rangle \Rightarrow \langle Val\ v,s_1\rangle;\ P \vdash \langle es,s_1\rangle\ [\Rightarrow]\ \langle es',s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle e\#es,s_0\rangle\ [\Rightarrow]\ \langle Val\ v\ \#\ es',s_2\rangle$

| *ConsThrow*:
$P \vdash \langle e,\ s_0\rangle \Rightarrow \langle throw\ e',\ s_1\rangle \Longrightarrow$
$P \vdash \langle e\#es,\ s_0\rangle\ [\Rightarrow]\ \langle throw\ e'\ \#\ es,\ s_1\rangle$

2.10.1 Final expressions

definition *final* :: $'a\ exp \Rightarrow bool$
where
$final\ e\ \equiv\ (\exists\, v.\ e = Val\ v) \lor (\exists\, a.\ e = Throw\ a)$

definition *finals*:: $'a\ exp\ list \Rightarrow bool$
where
$finals\ es\ \equiv\ (\exists\, vs.\ es = map\ Val\ vs) \lor (\exists\, vs\ a\ es'.\ es = map\ Val\ vs\ @\ Throw\ a\ \#\ es')$

lemma [*simp*]: $final(Val\ v)\langle proof\rangle$
lemma [*simp*]: $final(throw\ e) = (\exists\, a.\ e = addr\ a)\langle proof\rangle$
lemma *finalE*: $[\![\ final\ e;\ \ \bigwedge v.\ e = Val\ v \Longrightarrow R;\ \ \bigwedge a.\ e = Throw\ a \Longrightarrow R\ ]\!] \Longrightarrow R\langle proof\rangle$
lemma [*iff*]: $finals\ []\langle proof\rangle$
lemma [*iff*]: $finals\ (Val\ v\ \#\ es) = finals\ es\langle proof\rangle$
lemma *finals-app-map*[*iff*]: $finals\ (map\ Val\ vs\ @\ es) = finals\ es\langle proof\rangle$
lemma [*iff*]: $finals\ (map\ Val\ vs)\langle proof\rangle$
lemma [*iff*]: $finals\ (throw\ e\ \#\ es) = (\exists\, a.\ e = addr\ a)\langle proof\rangle$
lemma *not-finals-ConsI*: $\neg\ final\ e \Longrightarrow \neg\ finals(e\#es)\langle proof\rangle$

lemma *eval-final*: $P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle \Longrightarrow final\ e'$
and *evals-final*: $P \vdash \langle es,s\rangle\ [\Rightarrow]\ \langle es',s'\rangle \Longrightarrow finals\ es'\langle proof\rangle$

lemma *eval-lcl-incr*: $P \vdash \langle e,(h_0,l_0)\rangle \Rightarrow \langle e',(h_1,l_1)\rangle \Longrightarrow dom\ l_0 \subseteq dom\ l_1$
and *evals-lcl-incr*: $P \vdash \langle es,(h_0,l_0)\rangle\ [\Rightarrow]\ \langle es',(h_1,l_1)\rangle \Longrightarrow dom\ l_0 \subseteq dom\ l_1\langle proof\rangle$

Only used later, in the small to big translation, but is already a good sanity check:

lemma *eval-finalId*: $final\ e \Longrightarrow P \vdash \langle e,s\rangle \Rightarrow \langle e,s\rangle\langle proof\rangle$

lemma *eval-finalsId*:
assumes *finals*: *finals es* **shows** $P \vdash \langle es,s\rangle\ [\Rightarrow]\ \langle es,s\rangle\langle proof\rangle$

theorem *eval-hext*: $P \vdash \langle e,(h,l)\rangle \Rightarrow \langle e',(h',l')\rangle \Longrightarrow h \trianglelefteq h'$
and *evals-hext*: $P \vdash \langle es,(h,l)\rangle\ [\Rightarrow]\ \langle es',(h',l')\rangle \Longrightarrow h \trianglelefteq h'\langle proof\rangle$

end

2.11 Small Step Semantics

theory *SmallStep*
imports *Expr State*
begin

fun *blocks* :: *vname list* $*$ *ty list* $*$ *val list* $*$ *expr* $\Rightarrow$ *expr*
where
blocks($V\#Vs$, $T\#Ts$, $v\#vs$, e) = {V:T := *Val* v; *blocks*(Vs,Ts,vs,e)}
|*blocks*([],[],[],e) = e

lemmas *blocks-induct* = *blocks.induct*[*split-format* (*complete*)]

lemma [*simp*]:
⟦ *size vs* = *size Vs*; *size Ts* = *size Vs* ⟧ $\Longrightarrow$ *fv*(*blocks*(Vs,Ts,vs,e)) = *fv e* − *set Vs*⟨*proof*⟩

definition *assigned* :: *vname* $\Rightarrow$ *expr* $\Rightarrow$ *bool*
where
assigned V e ≡ ∃ v e'. e = (V := *Val* v;; e')

inductive-set
red :: *J-prog* $\Rightarrow$ ((*expr* × *state*) × (*expr* × *state*)) *set*
and *reds* :: *J-prog* $\Rightarrow$ ((*expr list* × *state*) × (*expr list* × *state*)) *set*
and *red'* :: *J-prog* $\Rightarrow$ *expr* $\Rightarrow$ *state* $\Rightarrow$ *expr* $\Rightarrow$ *state* $\Rightarrow$ *bool*
(‹- ⊢ ((*1*⟨-,/-⟩) →/ (*1*⟨-,/-⟩))› [*51*,*0*,*0*,*0*,*0*] *81*)
and *reds'* :: *J-prog* $\Rightarrow$ *expr list* $\Rightarrow$ *state* $\Rightarrow$ *expr list* $\Rightarrow$ *state* $\Rightarrow$ *bool*
(‹- ⊢ ((*1*⟨-,/-⟩) [→]/ (*1*⟨-,/-⟩))› [*51*,*0*,*0*,*0*,*0*] *81*)
for *P* :: *J-prog*
where

$P \vdash \langle e,s\rangle \to \langle e',s'\rangle \equiv ((e,s),\ e',s') \in$ *red P*
| $P \vdash \langle es,s\rangle$ [→] $\langle es',s'\rangle \equiv ((es,s),\ es',s') \in$ *reds P*

| *RedNew*:
⟦ *new-Addr h* = *Some a*; $P \vdash$ *C has-fields FDTs*; $h' = h(a \mapsto (C,$*init-fields FDTs*$))$ ⟧
$\Longrightarrow P \vdash \langle$*new C*, $(h,l)\rangle \to \langle$*addr a*, $(h',l)\rangle$

| *RedNewFail*:
new-Addr h = *None* $\Longrightarrow$
$P \vdash \langle$*new C*, $(h,l)\rangle \to \langle$*THROW OutOfMemory*, $(h,l)\rangle$

| *CastRed*:
$P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle$*Cast C e*, $s\rangle \to \langle$*Cast C e'*, $s'\rangle$

| *RedCastNull*:
$P \vdash \langle$*Cast C null*, $s\rangle \to \langle$*null*,$s\rangle$

| *RedCast*:
⟦ *hp s a* = *Some*(*D*,*fs*); $P \vdash D \preceq^* C$ ⟧
$\Longrightarrow P \vdash \langle$*Cast C* (*addr a*), $s\rangle \to \langle$*addr a*, $s\rangle$

| *RedCastFail*:
⟦ *hp s a* = *Some*(*D*,*fs*); ¬ $P \vdash D \preceq^* C$ ⟧

$\Longrightarrow P \vdash \langle \textit{Cast } C \ (\textit{addr } a),\ s\rangle \to \langle \textit{THROW ClassCast},\ s\rangle$

| *BinOpRed1*:
$P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle e \ \text{«}bop\text{»}\ e_2,\ s\rangle \to \langle e' \ \text{«}bop\text{»}\ e_2,\ s'\rangle$

| *BinOpRed2*:
$P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle (\textit{Val } v_1)\ \text{«}bop\text{»}\ e,\ s\rangle \to \langle (\textit{Val } v_1)\ \text{«}bop\text{»}\ e',\ s'\rangle$

| *RedBinOp*:
$\textit{binop}(bop,v_1,v_2) = \textit{Some } v \Longrightarrow$
$P \vdash \langle (\textit{Val } v_1)\ \text{«}bop\text{»}\ (\textit{Val } v_2),\ s\rangle \to \langle \textit{Val } v,s\rangle$

| *RedVar*:
$\textit{lcl } s\ V = \textit{Some } v \Longrightarrow$
$P \vdash \langle \textit{Var } V,s\rangle \to \langle \textit{Val } v,s\rangle$

| *LAssRed*:
$P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle V{:=}e,s\rangle \to \langle V{:=}e',s'\rangle$

| *RedLAss*:
$P \vdash \langle V{:=}(\textit{Val } v),\ (h,l)\rangle \to \langle \textit{unit},\ (h,l(V{\mapsto}v))\rangle$

| *FAccRed*:
$P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle e{\bullet}F\{D\},\ s\rangle \to \langle e'{\bullet}F\{D\},\ s'\rangle$

| *RedFAcc*:
$[\![\ hp\ s\ a = \textit{Some}(C,fs);\ fs(F,D) = \textit{Some } v\ ]\!]$
$\Longrightarrow P \vdash \langle (\textit{addr } a){\bullet}F\{D\},\ s\rangle \to \langle \textit{Val } v,s\rangle$

| *RedFAccNull*:
$P \vdash \langle \textit{null}{\bullet}F\{D\},\ s\rangle \to \langle \textit{THROW NullPointer},\ s\rangle$

| *FAssRed1*:
$P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle e{\bullet}F\{D\}{:=}e_2,\ s\rangle \to \langle e'{\bullet}F\{D\}{:=}e_2,\ s'\rangle$

| *FAssRed2*:
$P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle \textit{Val } v{\bullet}F\{D\}{:=}e,\ s\rangle \to \langle \textit{Val } v{\bullet}F\{D\}{:=}e',\ s'\rangle$

| *RedFAss*:
$h\ a = \textit{Some}(C,fs) \Longrightarrow$
$P \vdash \langle (\textit{addr } a){\bullet}F\{D\}{:=}(\textit{Val } v),\ (h,l)\rangle \to \langle \textit{unit},\ (h(a \mapsto (C,fs((F,D) \mapsto v))),l)\rangle$

| *RedFAssNull*:
$P \vdash \langle \textit{null}{\bullet}F\{D\}{:=}\textit{Val } v,\ s\rangle \to \langle \textit{THROW NullPointer},\ s\rangle$

| *CallObj*:
$P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle e{\bullet}M(es),s\rangle \to \langle e'{\bullet}M(es),s'\rangle$

| *CallParams*:
$P \vdash \langle es,s\rangle\ [\rightarrow]\ \langle es',s'\rangle \Longrightarrow$
$P \vdash \langle (Val\ v)\bullet M(es),s\rangle \rightarrow \langle (Val\ v)\bullet M(es'),s'\rangle$

| *RedCall*:
$[\![\ hp\ s\ a = Some(C,fs);\ P \vdash C\ sees\ M{:}Ts{\rightarrow}T = (pns,body)\ in\ D;\ size\ vs = size\ pns;\ size\ Ts = size\ pns\ ]\!]$
$\Longrightarrow P \vdash \langle (addr\ a)\bullet M(map\ Val\ vs),\ s\rangle \rightarrow \langle blocks(this\#pns,\ Class\ D\#Ts,\ Addr\ a\#vs,\ body),\ s\rangle$

| *RedCallNull*:
$P \vdash \langle null\bullet M(map\ Val\ vs),s\rangle \rightarrow \langle THROW\ NullPointer,s\rangle$

| *BlockRedNone*:
$[\![\ P \vdash \langle e,\ (h,l(V{:=}None))\rangle \rightarrow \langle e',\ (h',l')\rangle;\ l'\ V = None;\ \neg\ assigned\ V\ e\ ]\!]$
$\Longrightarrow P \vdash \langle \{V{:}T;\ e\},\ (h,l)\rangle \rightarrow \langle \{V{:}T;\ e'\},\ (h',l'(V := l\ V))\rangle$

| *BlockRedSome*:
$[\![\ P \vdash \langle e,\ (h,l(V{:=}None))\rangle \rightarrow \langle e',\ (h',l')\rangle;\ l'\ V = Some\ v;\neg\ assigned\ V\ e\ ]\!]$
$\Longrightarrow P \vdash \langle \{V{:}T;\ e\},\ (h,l)\rangle \rightarrow \langle \{V{:}T := Val\ v;\ e'\},\ (h',l'(V := l\ V))\rangle$

| *InitBlockRed*:
$[\![\ P \vdash \langle e,\ (h,l(V{\mapsto}v))\rangle \rightarrow \langle e',\ (h',l')\rangle;\ l'\ V = Some\ v'\ ]\!]$
$\Longrightarrow P \vdash \langle \{V{:}T := Val\ v;\ e\},\ (h,l)\rangle \rightarrow \langle \{V{:}T := Val\ v';\ e'\},\ (h',l'(V := l\ V))\rangle$

| *RedBlock*:
$P \vdash \langle \{V{:}T;\ Val\ u\},\ s\rangle \rightarrow \langle Val\ u,\ s\rangle$

| *RedInitBlock*:
$P \vdash \langle \{V{:}T := Val\ v;\ Val\ u\},\ s\rangle \rightarrow \langle Val\ u,\ s\rangle$

| *SeqRed*:
$P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle e;;e_2,\ s\rangle \rightarrow \langle e';;e_2,\ s'\rangle$

| *RedSeq*:
$P \vdash \langle (Val\ v);;e_2,\ s\rangle \rightarrow \langle e_2,\ s\rangle$

| *CondRed*:
$P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle if\ (e)\ e_1\ else\ e_2,\ s\rangle \rightarrow \langle if\ (e')\ e_1\ else\ e_2,\ s'\rangle$

| *RedCondT*:
$P \vdash \langle if\ (true)\ e_1\ else\ e_2,\ s\rangle \rightarrow \langle e_1,\ s\rangle$

| *RedCondF*:
$P \vdash \langle if\ (false)\ e_1\ else\ e_2,\ s\rangle \rightarrow \langle e_2,\ s\rangle$

| *RedWhile*:
$P \vdash \langle while(b)\ c,\ s\rangle \rightarrow \langle if(b)\ (c;;while(b)\ c)\ else\ unit,\ s\rangle$

| *ThrowRed*:
$P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle throw\ e,\ s\rangle \rightarrow \langle throw\ e',\ s'\rangle$

| *RedThrowNull*:
$P \vdash \langle throw\ null,\ s\rangle \to \langle THROW\ NullPointer,\ s\rangle$

| *TryRed*:
$P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle try\ e\ catch(C\ V)\ e_2,\ s\rangle \to \langle try\ e'\ catch(C\ V)\ e_2,\ s'\rangle$

| *RedTry*:
$P \vdash \langle try\ (Val\ v)\ catch(C\ V)\ e_2,\ s\rangle \to \langle Val\ v,\ s\rangle$

| *RedTryCatch*:
$[\![\ hp\ s\ a = Some(D,fs);\ P \vdash D \preceq^* C\ ]\!]$
$\Longrightarrow P \vdash \langle try\ (Throw\ a)\ catch(C\ V)\ e_2,\ s\rangle \to \langle \{V{:}Class\ C := addr\ a;\ e_2\},\ s\rangle$

| *RedTryFail*:
$[\![\ hp\ s\ a = Some(D,fs);\ \neg\ P \vdash D \preceq^* C\ ]\!]$
$\Longrightarrow P \vdash \langle try\ (Throw\ a)\ catch(C\ V)\ e_2,\ s\rangle \to \langle Throw\ a,\ s\rangle$

| *ListRed1*:
$P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle e\#es,s\rangle\ [\to]\ \langle e'\#es,s'\rangle$

| *ListRed2*:
$P \vdash \langle es,s\rangle\ [\to]\ \langle es',s'\rangle \Longrightarrow$
$P \vdash \langle Val\ v\ \#\ es,s\rangle\ [\to]\ \langle Val\ v\ \#\ es',s'\rangle$

— Exception propagation

| *CastThrow*: $P \vdash \langle Cast\ C\ (throw\ e),\ s\rangle \to \langle throw\ e,\ s\rangle$
| *BinOpThrow1*: $P \vdash \langle (throw\ e)\ «bop»\ e_2,\ s\rangle \to \langle throw\ e,\ s\rangle$
| *BinOpThrow2*: $P \vdash \langle (Val\ v_1)\ «bop»\ (throw\ e),\ s\rangle \to \langle throw\ e,\ s\rangle$
| *LAssThrow*: $P \vdash \langle V{:=}(throw\ e),\ s\rangle \to \langle throw\ e,\ s\rangle$
| *FAccThrow*: $P \vdash \langle (throw\ e){\cdot}F\{D\},\ s\rangle \to \langle throw\ e,\ s\rangle$
| *FAssThrow1*: $P \vdash \langle (throw\ e){\cdot}F\{D\}{:=}e_2,\ s\rangle \to \langle throw\ e,s\rangle$
| *FAssThrow2*: $P \vdash \langle Val\ v{\cdot}F\{D\}{:=}(throw\ e),\ s\rangle \to \langle throw\ e,\ s\rangle$
| *CallThrowObj*: $P \vdash \langle (throw\ e){\cdot}M(es),\ s\rangle \to \langle throw\ e,\ s\rangle$
| *CallThrowParams*: $[\![\ es = map\ Val\ vs\ @\ throw\ e\ \#\ es'\ ]\!] \Longrightarrow P \vdash \langle (Val\ v){\cdot}M(es),\ s\rangle \to \langle throw\ e,\ s\rangle$
| *BlockThrow*: $P \vdash \langle \{V{:}T;\ Throw\ a\},\ s\rangle \to \langle Throw\ a,\ s\rangle$
| *InitBlockThrow*: $P \vdash \langle \{V{:}T := Val\ v;\ Throw\ a\},\ s\rangle \to \langle Throw\ a,\ s\rangle$
| *SeqThrow*: $P \vdash \langle (throw\ e);;e_2,\ s\rangle \to \langle throw\ e,\ s\rangle$
| *CondThrow*: $P \vdash \langle if\ (throw\ e)\ e_1\ else\ e_2,\ s\rangle \to \langle throw\ e,\ s\rangle$
| *ThrowThrow*: $P \vdash \langle throw(throw\ e),\ s\rangle \to \langle throw\ e,\ s\rangle$

2.11.1 The reflexive transitive closure

abbreviation
Step :: *J-prog* ⇒ *expr* ⇒ *state* ⇒ *expr* ⇒ *state* ⇒ *bool*
(‹- ⊢ ((1⟨-,/-⟩) →*/ (1⟨-,/-⟩))› [51,0,0,0,0] 81)
where $P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \equiv ((e,s),\ e',s') \in (red\ P)^*$

abbreviation
Steps :: *J-prog* ⇒ *expr list* ⇒ *state* ⇒ *expr list* ⇒ *state* ⇒ *bool*
(‹- ⊢ ((1⟨-,/-⟩) [→]*/ (1⟨-,/-⟩))› [51,0,0,0,0] 81)

where $P \vdash \langle es,s\rangle\ [\rightarrow]{*}\ \langle es',s'\rangle \equiv ((es,s),\ es',s') \in (reds\ P)^*$

lemma *converse-rtrancl-induct-red*[*consumes 1*]:
assumes $P \vdash \langle e,(h,l)\rangle \rightarrow{*}\ \langle e',(h',l')\rangle$
and $\bigwedge e\ h\ l.\ R\ e\ h\ l\ e\ h\ l$
and $\bigwedge e_0\ h_0\ l_0\ e_1\ h_1\ l_1\ e'\ h'\ l'.$
$[\![\ P \vdash \langle e_0,(h_0,l_0)\rangle \rightarrow \langle e_1,(h_1,l_1)\rangle;\ R\ e_1\ h_1\ l_1\ e'\ h'\ l'\ ]\!] \Longrightarrow R\ e_0\ h_0\ l_0\ e'\ h'\ l'$
shows $R\ e\ h\ l\ e'\ h'\ l'$⟨*proof*⟩

2.11.2 Some easy lemmas

lemma [*iff*]: $\neg\ P \vdash \langle [],s\rangle\ [\rightarrow]\ \langle es',s'\rangle$⟨*proof*⟩
lemma [*iff*]: $\neg\ P \vdash \langle Val\ v,s\rangle \rightarrow \langle e',s'\rangle$⟨*proof*⟩
lemma [*iff*]: $\neg\ P \vdash \langle Throw\ a,s\rangle \rightarrow \langle e',s'\rangle$⟨*proof*⟩

lemma *red-hext-incr*: $P \vdash \langle e,(h,l)\rangle \rightarrow \langle e',(h',l')\rangle \Longrightarrow h \trianglelefteq h'$
and *reds-hext-incr*: $P \vdash \langle es,(h,l)\rangle\ [\rightarrow]\ \langle es',(h',l')\rangle \Longrightarrow h \trianglelefteq h'$⟨*proof*⟩

lemma *red-lcl-incr*: $P \vdash \langle e,(h_0,l_0)\rangle \rightarrow \langle e',(h_1,l_1)\rangle \Longrightarrow dom\ l_0 \subseteq dom\ l_1$
and $P \vdash \langle es,(h_0,l_0)\rangle\ [\rightarrow]\ \langle es',(h_1,l_1)\rangle \Longrightarrow dom\ l_0 \subseteq dom\ l_1$⟨*proof*⟩

lemma *red-lcl-add*: $P \vdash \langle e,(h,l)\rangle \rightarrow \langle e',(h',l')\rangle \Longrightarrow (\bigwedge l_0.\ P \vdash \langle e,(h,l_0{+}{+}l)\rangle \rightarrow \langle e',(h',l_0{+}{+}l')\rangle)$
and $P \vdash \langle es,(h,l)\rangle\ [\rightarrow]\ \langle es',(h',l')\rangle \Longrightarrow (\bigwedge l_0.\ P \vdash \langle es,(h,l_0{+}{+}l)\rangle\ [\rightarrow]\ \langle es',(h',l_0{+}{+}l')\rangle)$⟨*proof*⟩

lemma *Red-lcl-add*:
assumes $P \vdash \langle e,(h,l)\rangle \rightarrow{*}\ \langle e',(h',l')\rangle$ **shows** $P \vdash \langle e,(h,l_0{+}{+}l)\rangle \rightarrow{*}\ \langle e',(h',l_0{+}{+}l')\rangle$⟨*proof*⟩

end

2.12 System Classes

theory *SystemClasses*
imports *Decl Exceptions*
begin

This theory provides definitions for the *Object* class, and the system exceptions.

definition *ObjectC* :: *'m cdecl*
where
ObjectC ≡ (*Object*, (*undefined*,[],[]))

definition *NullPointerC* :: *'m cdecl*
where
NullPointerC ≡ (*NullPointer*, (*Object*,[],[]))

definition *ClassCastC* :: *'m cdecl*
where
ClassCastC ≡ (*ClassCast*, (*Object*,[],[]))

definition *OutOfMemoryC* :: *'m cdecl*
where
OutOfMemoryC ≡ (*OutOfMemory*, (*Object*,[],[]))

definition *SystemClasses* :: *'m cdecl list*
where

$SystemClasses \equiv [ObjectC,\ NullPointerC,\ ClassCastC,\ OutOfMemoryC]$

end

2.13 Generic Well-formedness of programs

theory *WellForm* **imports** *TypeRel SystemClasses* **begin**

This theory defines global well-formedness conditions for programs but does not look inside method bodies. Hence it works for both Jinja and JVM programs. Well-typing of expressions is defined elsewhere (in theory *WellType*).

Because Jinja does not have method overloading, its policy for method overriding is the classical one: *covariant in the result type but contravariant in the argument types.* This means the result type of the overriding method becomes more specific, the argument types become more general.

type-synonym $'m\ wf\text{-}mdecl\text{-}test = {}'m\ prog \Rightarrow cname \Rightarrow {}'m\ mdecl \Rightarrow bool$

definition $wf\text{-}fdecl :: {}'m\ prog \Rightarrow fdecl \Rightarrow bool$
where
$wf\text{-}fdecl\ P \equiv \lambda(F,T).\ is\text{-}type\ P\ T$

definition $wf\text{-}mdecl :: {}'m\ wf\text{-}mdecl\text{-}test \Rightarrow {}'m\ wf\text{-}mdecl\text{-}test$
where
$wf\text{-}mdecl\ wf\text{-}md\ P\ C \equiv \lambda(M,Ts,T,mb).$
$(\forall T\in set\ Ts.\ is\text{-}type\ P\ T) \wedge is\text{-}type\ P\ T \wedge wf\text{-}md\ P\ C\ (M,Ts,T,mb)$

definition $wf\text{-}cdecl :: {}'m\ wf\text{-}mdecl\text{-}test \Rightarrow {}'m\ prog \Rightarrow {}'m\ cdecl \Rightarrow bool$
where
$wf\text{-}cdecl\ wf\text{-}md\ P \equiv \lambda(C,(D,fs,ms)).$
$(\forall f\in set\ fs.\ wf\text{-}fdecl\ P\ f) \wedge\ distinct\text{-}fst\ fs \wedge$
$(\forall m\in set\ ms.\ wf\text{-}mdecl\ wf\text{-}md\ P\ C\ m) \wedge\ distinct\text{-}fst\ ms \wedge$
$(C \neq Object \longrightarrow$
$is\text{-}class\ P\ D \wedge \neg\ P \vdash D \preceq^* C \wedge$
$(\forall (M,Ts,T,m)\in set\ ms.$
$\forall D'\ Ts'\ T'\ m'.\ P \vdash D\ sees\ M{:}Ts' \rightarrow T' = m'\ in\ D' \longrightarrow$
$P \vdash Ts'\ [\leq]\ Ts \wedge P \vdash T \leq T'))$

definition $wf\text{-}syscls :: {}'m\ prog \Rightarrow bool$
where
$wf\text{-}syscls\ P \equiv \{Object\} \cup sys\text{-}xcpts \subseteq set(map\ fst\ P)$

definition $wf\text{-}prog :: {}'m\ wf\text{-}mdecl\text{-}test \Rightarrow {}'m\ prog \Rightarrow bool$
where
$wf\text{-}prog\ wf\text{-}md\ P \equiv wf\text{-}syscls\ P \wedge (\forall c \in set\ P.\ wf\text{-}cdecl\ wf\text{-}md\ P\ c) \wedge distinct\text{-}fst\ P$

2.13.1 Well-formedness lemmas

lemma *class-wf*:
$[\![class\ P\ C = Some\ c;\ wf\text{-}prog\ wf\text{-}md\ P]\!] \Longrightarrow wf\text{-}cdecl\ wf\text{-}md\ P\ (C,c)\langle proof\rangle$

lemma *class-Object* [*simp*]:
$wf\text{-}prog\ wf\text{-}md\ P \Longrightarrow \exists\, C\ fs\ ms.\ class\ P\ Object = Some\ (C,fs,ms)\langle proof\rangle$

lemma *is-class-Object* [*simp*]:
wf-prog wf-md P $\Longrightarrow$ *is-class P Object*⟨*proof*⟩

lemma *is-class-xcpt*:
$[\![$ $C \in$ *sys-xcpts*; *wf-prog wf-md P* $]\!]$ $\Longrightarrow$ *is-class P C*⟨*proof*⟩

lemma *subcls1-wfD*:
assumes *sub1*: $P \vdash C \prec^1 D$ **and** *wf*: *wf-prog wf-md P*
shows $D \neq C \wedge (D,C) \notin (subcls1\ P)^+$⟨*proof*⟩

lemma *wf-cdecl-supD*:
$[\![$*wf-cdecl wf-md P* (C,D,r); $C \neq Object]\!]$ $\Longrightarrow$ *is-class P D*⟨*proof*⟩

lemma *subcls-asym*:
$[\![$ *wf-prog wf-md P*; $(C,D) \in (subcls1\ P)^+$ $]\!]$ $\Longrightarrow$ $(D,C) \notin (subcls1\ P)^+$⟨*proof*⟩

lemma *subcls-irrefl*:
$[\![$ *wf-prog wf-md P*; $(C,D) \in (subcls1\ P)^+$ $]\!]$ $\Longrightarrow$ $C \neq D$⟨*proof*⟩

lemma *acyclic-subcls1*:
wf-prog wf-md P $\Longrightarrow$ *acyclic* (*subcls1 P*)⟨*proof*⟩

lemma *wf-subcls1*:
wf-prog wf-md P $\Longrightarrow$ *wf* $((subcls1\ P)^{-1})$⟨*proof*⟩

lemma *single-valued-subcls1*:
wf-prog wf-md G $\Longrightarrow$ *single-valued* (*subcls1 G*)⟨*proof*⟩

lemma *subcls-induct*:
$[\![$ *wf-prog wf-md P*; $\bigwedge C.\ \forall D.\ (C,D) \in (subcls1\ P)^+ \longrightarrow Q\ D \Longrightarrow Q\ C$ $]\!]$ $\Longrightarrow$ $Q\ C$⟨*proof*⟩

lemma *subcls1-induct-aux*:
assumes *is-class P C* **and** *wf*: *wf-prog wf-md P* **and** *QObj*: *Q Object*
shows
$[\![$ $\bigwedge C\ D\ fs\ ms.$
$[\![$ $C \neq Object$; *is-class P C*; *class P C* = *Some* $(D,fs,ms) \wedge$
wf-cdecl wf-md P $(C,D,fs,ms) \wedge P \vdash C \prec^1 D \wedge$ *is-class P D* $\wedge\ Q\ D]\!]$ $\Longrightarrow Q\ C$ $]\!]$
$\Longrightarrow$ $Q\ C$⟨*proof*⟩

lemma *subcls1-induct* [*consumes 2*, *case-names Object Subcls*]:
$[\![$ *wf-prog wf-md P*; *is-class P C*; *Q Object*;
$\bigwedge C\ D.\ [\![C \neq Object;\ P \vdash C \prec^1 D;$ *is-class P D*; $Q\ D]\!] \Longrightarrow Q\ C$ $]\!]$
$\Longrightarrow$ $Q\ C$⟨*proof*⟩

lemma *subcls-C-Object*:
assumes *class*: *is-class P C* **and** *wf*: *wf-prog wf-md P*
shows $P \vdash C \preceq^* Object$⟨*proof*⟩

lemma *is-type-pTs*:
assumes *wf-prog wf-md P* **and** $(C,S,fs,ms) \in$ *set P* **and** $(M,Ts,T,m) \in$ *set ms*
shows *set Ts* $\subseteq$ *types P*⟨*proof*⟩

2.13.2 Well-formedness and method lookup

lemma *sees-wf-mdecl*:
assumes *wf*: *wf-prog wf-md P* **and** *sees*: $P \vdash C$ *sees* $M{:}Ts{\to}T = m$ *in D*
shows *wf-mdecl wf-md P D* $(M,Ts,T,m)\langle proof\rangle$

lemma *sees-method-mono* [*rule-format* (*no-asm*)]:
assumes *sub*: $P \vdash C' \preceq^* C$ **and** *wf*: *wf-prog wf-md P*
shows $\forall D\ Ts\ T\ m.\ P \vdash C$ *sees* $M{:}Ts{\to}T = m$ *in* $D \longrightarrow$
$(\exists D'\ Ts'\ T'\ m'.\ P \vdash C'$ *sees* $M{:}Ts'{\to}T' = m'$ *in* $D' \wedge P \vdash Ts\ [\le]\ Ts' \wedge P \vdash T' \le T)\langle proof\rangle$

lemma *sees-method-mono2*:
$[\![\ P \vdash C' \preceq^* C$; *wf-prog wf-md P*;
$P \vdash C$ *sees* $M{:}Ts{\to}T = m$ *in D*; $P \vdash C'$ *sees* $M{:}Ts'{\to}T' = m'$ *in* $D'\ ]\!]$
$\Longrightarrow P \vdash Ts\ [\le]\ Ts' \wedge P \vdash T' \le T\langle proof\rangle$

lemma *mdecls-visible*:
assumes *wf*: *wf-prog wf-md P* **and** *class*: *is-class P C*
shows $\bigwedge D\ fs\ ms.$ *class P C* $= Some(D,fs,ms)$
$\Longrightarrow \exists Mm.\ P \vdash C$ *sees-methods Mm* $\wedge (\forall (M,Ts,T,m) \in set\ ms.\ Mm\ M = Some((Ts,T,m),C))\langle proof\rangle$

lemma *mdecl-visible*:
assumes *wf*: *wf-prog wf-md P* **and** *C*: $(C,S,fs,ms) \in set\ P$ **and** *m*: $(M,Ts,T,m) \in set\ ms$
shows $P \vdash C$ *sees* $M{:}Ts{\to}T = m$ *in* $C\langle proof\rangle$

lemma *Call-lemma*:
assumes *sees*: $P \vdash C$ *sees* $M{:}Ts{\to}T = m$ *in D* **and** *sub*: $P \vdash C' \preceq^* C$ **and** *wf*: *wf-prog wf-md P*
shows $\exists D'\ Ts'\ T'\ m'.$
$P \vdash C'$ *sees* $M{:}Ts'{\to}T' = m'$ *in* $D' \wedge P \vdash Ts\ [\le]\ Ts' \wedge P \vdash T' \le T \wedge P \vdash C' \preceq^* D'$
$\wedge$ *is-type P T'* $\wedge (\forall T{\in}set\ Ts'.$ *is-type P T*$) \wedge$ *wf-md P D'* $(M,Ts',T',m')\langle proof\rangle$

lemma *wf-prog-lift*:
assumes *wf*: *wf-prog* $(\lambda P\ C\ bd.\ A\ P\ C\ bd)\ P$
and *rule*:
$\bigwedge$*wf-md C M Ts C T m bd.*
wf-prog wf-md P $\Longrightarrow$
$P \vdash C$ *sees* $M{:}Ts{\to}T = m$ *in* $C \Longrightarrow$
set Ts $\subseteq$ *types P* $\Longrightarrow$
$bd = (M,Ts,T,m) \Longrightarrow$
$A\ P\ C\ bd \Longrightarrow$
$B\ P\ C\ bd$
shows *wf-prog* $(\lambda P\ C\ bd.\ B\ P\ C\ bd)\ P\langle proof\rangle$

2.13.3 Well-formedness and field lookup

lemma *wf-Fields-Ex*:
assumes *wf*: *wf-prog wf-md P* **and** *is-class P C*
shows $\exists FDTs.\ P \vdash C$ *has-fields* $FDTs\langle proof\rangle$

lemma *has-fields-types*:
$[\![\ P \vdash C$ *has-fields FDTs*; $(FD,T) \in set\ FDTs$; *wf-prog wf-md P* $]\!] \Longrightarrow$ *is-type P* $T\langle proof\rangle$

lemma *sees-field-is-type*:
$[\![\ P \vdash C$ *sees* $F{:}T$ *in D*; *wf-prog wf-md P* $]\!] \Longrightarrow$ *is-type P* $T\langle proof\rangle$
lemma *wf-syscls*:

set SystemClasses $\subseteq$ *set P* $\Longrightarrow$ *wf-syscls P*$\langle proof\rangle$
end

2.14 Weak well-formedness of Jinja programs

theory *WWellForm* **imports** ../*Common*/*WellForm Expr* **begin**

definition *wwf-J-mdecl* :: *J-prog* $\Rightarrow$ *cname* $\Rightarrow$ *J-mb mdecl* $\Rightarrow$ *bool*
where
wwf-J-mdecl P C $\equiv$ $\lambda(M,Ts,T,(pns,body))$.
length Ts = *length pns* $\wedge$ *distinct pns* $\wedge$ *this* $\notin$ *set pns* $\wedge$ *fv body* $\subseteq$ $\{this\}$ $\cup$ *set pns*

lemma *wwf-J-mdecl*[*simp*]:
wwf-J-mdecl P C $(M,Ts,T,pns,body)$ =
(*length Ts* = *length pns* $\wedge$ *distinct pns* $\wedge$ *this* $\notin$ *set pns* $\wedge$ *fv body* $\subseteq$ $\{this\}$ $\cup$ *set pns*)$\langle proof\rangle$
abbreviation
wwf-J-prog :: *J-prog* $\Rightarrow$ *bool* **where**
wwf-J-prog == *wf-prog wwf-J-mdecl*

end

2.15 Equivalence of Big Step and Small Step Semantics

theory *Equivalence* **imports** *BigStep SmallStep WWellForm* **begin**

2.15.1 Small steps simulate big step

Cast

lemma *CastReds*:
$P \vdash \langle e,s\rangle \rightarrow* \langle e',s'\rangle \Longrightarrow P \vdash \langle Cast\ C\ e,s\rangle \rightarrow* \langle Cast\ C\ e',s'\rangle\langle proof\rangle$
lemma *CastRedsNull*:
$P \vdash \langle e,s\rangle \rightarrow* \langle null,s'\rangle \Longrightarrow P \vdash \langle Cast\ C\ e,s\rangle \rightarrow* \langle null,s'\rangle\langle proof\rangle$
lemma *CastRedsAddr*:
$[\![\ P \vdash \langle e,s\rangle \rightarrow* \langle addr\ a,s'\rangle;\ hp\ s'\ a = Some(D,fs);\ P \vdash D \preceq^* C\ ]\!] \Longrightarrow$
$P \vdash \langle Cast\ C\ e,s\rangle \rightarrow* \langle addr\ a,s'\rangle\langle proof\rangle$
lemma *CastRedsFail*:
$[\![\ P \vdash \langle e,s\rangle \rightarrow* \langle addr\ a,s'\rangle;\ hp\ s'\ a = Some(D,fs);\ \neg\ P \vdash D \preceq^* C\ ]\!] \Longrightarrow$
$P \vdash \langle Cast\ C\ e,s\rangle \rightarrow* \langle THROW\ ClassCast,s'\rangle\langle proof\rangle$
lemma *CastRedsThrow*:
$[\![\ P \vdash \langle e,s\rangle \rightarrow* \langle throw\ a,s'\rangle\ ]\!] \Longrightarrow P \vdash \langle Cast\ C\ e,s\rangle \rightarrow* \langle throw\ a,s'\rangle\langle proof\rangle$

LAss

lemma *LAssReds*:
$P \vdash \langle e,s\rangle \rightarrow* \langle e',s'\rangle \Longrightarrow P \vdash \langle\ V:=e,s\rangle \rightarrow* \langle\ V:=e',s'\rangle\langle proof\rangle$
lemma *LAssRedsVal*:
$[\![\ P \vdash \langle e,s\rangle \rightarrow* \langle Val\ v,(h',l')\rangle\ ]\!] \Longrightarrow P \vdash \langle\ V:=e,s\rangle \rightarrow* \langle unit,(h',l'(V\mapsto v))\rangle\langle proof\rangle$
lemma *LAssRedsThrow*:
$[\![\ P \vdash \langle e,s\rangle \rightarrow* \langle throw\ a,s'\rangle\ ]\!] \Longrightarrow P \vdash \langle\ V:=e,s\rangle \rightarrow* \langle throw\ a,s'\rangle\langle proof\rangle$

BinOp

lemma *BinOp1Reds*:

$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle\ e \ «bop»\ e_2,\ s\rangle \to* \langle e' \ «bop»\ e_2,\ s'\rangle\langle proof\rangle$
lemma *BinOp2Reds*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle(Val\ v)\ «bop»\ e,\ s\rangle \to* \langle(Val\ v)\ «bop»\ e',\ s'\rangle\langle proof\rangle$
lemma *BinOpRedsVal*:
assumes e_1*-steps*: $P \vdash \langle e_1,s_0\rangle \to* \langle Val\ v_1,s_1\rangle$
and e_2*-steps*: $P \vdash \langle e_2,s_1\rangle \to* \langle Val\ v_2,s_2\rangle$
and *op*: $binop(bop,v_1,v_2) = Some\ v$
shows $P \vdash \langle e_1\ «bop»\ e_2,\ s_0\rangle \to* \langle Val\ v,s_2\rangle\langle proof\rangle$
lemma *BinOpRedsThrow1*:
$P \vdash \langle e,s\rangle \to* \langle throw\ e',s'\rangle \Longrightarrow P \vdash \langle e\ «bop»\ e_2,\ s\rangle \to* \langle throw\ e',\ s'\rangle\langle proof\rangle$
lemma *BinOpRedsThrow2*:
assumes e_1*-steps*: $P \vdash \langle e_1,s_0\rangle \to* \langle Val\ v_1,s_1\rangle$
and e_2*-steps*: $P \vdash \langle e_2,s_1\rangle \to* \langle throw\ e,s_2\rangle$
shows $P \vdash \langle e_1\ «bop»\ e_2,\ s_0\rangle \to* \langle throw\ e,s_2\rangle\langle proof\rangle$

FAcc

lemma *FAccReds*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle e{\cdot}F\{D\},\ s\rangle \to* \langle e'{\cdot}F\{D\},\ s'\rangle\langle proof\rangle$
lemma *FAccRedsVal*:
$[\![P \vdash \langle e,s\rangle \to* \langle addr\ a,s'\rangle;\ hp\ s'\ a = Some(C,fs);\ fs(F,D) = Some\ v\ ]\!]$
$\Longrightarrow P \vdash \langle e{\cdot}F\{D\},s\rangle \to* \langle Val\ v,s'\rangle\langle proof\rangle$
lemma *FAccRedsNull*:
$P \vdash \langle e,s\rangle \to* \langle null,s'\rangle \Longrightarrow P \vdash \langle e{\cdot}F\{D\},s\rangle \to* \langle THROW\ NullPointer,s'\rangle\langle proof\rangle$
lemma *FAccRedsThrow*:
$P \vdash \langle e,s\rangle \to* \langle throw\ a,s'\rangle \Longrightarrow P \vdash \langle e{\cdot}F\{D\},s\rangle \to* \langle throw\ a,s'\rangle\langle proof\rangle$

FAss

lemma *FAssReds1*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle e{\cdot}F\{D\}{:=}e_2,\ s\rangle \to* \langle e'{\cdot}F\{D\}{:=}e_2,\ s'\rangle\langle proof\rangle$
lemma *FAssReds2*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle Val\ v{\cdot}F\{D\}{:=}e,\ s\rangle \to* \langle Val\ v{\cdot}F\{D\}{:=}e',\ s'\rangle\langle proof\rangle$
lemma *FAssRedsVal*:
assumes e_1*-steps*: $P \vdash \langle e_1,s_0\rangle \to* \langle addr\ a,s_1\rangle$
and e_2*-steps*: $P \vdash \langle e_2,s_1\rangle \to* \langle Val\ v,(h_2,l_2)\rangle$
and *hC*: $Some(C,fs) = h_2\ a$
shows $P \vdash \langle e_1{\cdot}F\{D\}{:=}e_2,\ s_0\rangle \to* \langle unit,\ (h_2(a \mapsto (C,fs((F,D) \mapsto v))),l_2)\rangle\langle proof\rangle$
lemma *FAssRedsNull*:
assumes e_1*-steps*: $P \vdash \langle e_1,s_0\rangle \to* \langle null,s_1\rangle$
and e_2*-steps*: $P \vdash \langle e_2,s_1\rangle \to* \langle Val\ v,s_2\rangle$
shows $P \vdash \langle e_1{\cdot}F\{D\}{:=}e_2,\ s_0\rangle \to* \langle THROW\ NullPointer,\ s_2\rangle\langle proof\rangle$
lemma *FAssRedsThrow1*:
$P \vdash \langle e,s\rangle \to* \langle throw\ e',s'\rangle \Longrightarrow P \vdash \langle e{\cdot}F\{D\}{:=}e_2,\ s\rangle \to* \langle throw\ e',\ s'\rangle\langle proof\rangle$
lemma *FAssRedsThrow2*:
assumes e_1*-steps*: $P \vdash \langle e_1,s_0\rangle \to* \langle Val\ v,s_1\rangle$
and e_2*-steps*: $P \vdash \langle e_2,s_1\rangle \to* \langle throw\ e,s_2\rangle$
shows $P \vdash \langle e_1{\cdot}F\{D\}{:=}e_2,s_0\rangle \to* \langle throw\ e,s_2\rangle\langle proof\rangle$

;;

lemma *SeqReds*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle e;;e_2,\ s\rangle \to* \langle e';;e_2,\ s'\rangle\langle proof\rangle$
lemma *SeqRedsThrow*:
$P \vdash \langle e,s\rangle \to* \langle throw\ e',s'\rangle \Longrightarrow P \vdash \langle e;;e_2,\ s\rangle \to* \langle throw\ e',\ s'\rangle\langle proof\rangle$

lemma *SeqReds2*:
assumes *e_1-steps*: $P \vdash \langle e_1,s_0\rangle \to* \langle Val\ v_1,s_1\rangle$
and *e_2-steps*: $P \vdash \langle e_2,s_1\rangle \to* \langle e_2{}',s_2\rangle$
shows $P \vdash \langle e_1;;e_2,\ s_0\rangle \to* \langle e_2{}',s_2\rangle\langle proof\rangle$

If

lemma *CondReds*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle if\ (e)\ e_1\ else\ e_2,s\rangle \to* \langle if\ (e')\ e_1\ else\ e_2,s'\rangle\langle proof\rangle$
lemma *CondRedsThrow*:
$P \vdash \langle e,s\rangle \to* \langle throw\ a,s'\rangle \Longrightarrow P \vdash \langle if\ (e)\ e_1\ else\ e_2,\ s\rangle \to* \langle throw\ a,s'\rangle\langle proof\rangle$
lemma *CondReds2T*:
assumes *e-steps*: $P \vdash \langle e,s_0\rangle \to* \langle true,s_1\rangle$
and *e_1-steps*: $P \vdash \langle e_1,\ s_1\rangle \to* \langle e',s_2\rangle$
shows $P \vdash \langle if\ (e)\ e_1\ else\ e_2,\ s_0\rangle \to* \langle e',s_2\rangle\langle proof\rangle$
lemma *CondReds2F*:
assumes *e-steps*: $P \vdash \langle e,s_0\rangle \to* \langle false,s_1\rangle$
and *e_2-steps*: $P \vdash \langle e_2,\ s_1\rangle \to* \langle e',s_2\rangle$
shows $P \vdash \langle if\ (e)\ e_1\ else\ e_2,\ s_0\rangle \to* \langle e',s_2\rangle\langle proof\rangle$

While

lemma *WhileFReds*:
assumes *b-steps*: $P \vdash \langle b,s\rangle \to* \langle false,s'\rangle$
shows $P \vdash \langle while\ (b)\ c,s\rangle \to* \langle unit,s'\rangle\langle proof\rangle$
lemma *WhileRedsThrow*:
assumes *b-steps*: $P \vdash \langle b,s\rangle \to* \langle throw\ e,s'\rangle$
shows $P \vdash \langle while\ (b)\ c,s\rangle \to* \langle throw\ e,s'\rangle\langle proof\rangle$
lemma *WhileTReds*:
assumes *b-steps*: $P \vdash \langle b,s_0\rangle \to* \langle true,s_1\rangle$
and *c-steps*: $P \vdash \langle c,s_1\rangle \to* \langle Val\ v_1,s_2\rangle$
and *while-steps*: $P \vdash \langle while\ (b)\ c,s_2\rangle \to* \langle e,s_3\rangle$
shows $P \vdash \langle while\ (b)\ c,s_0\rangle \to* \langle e,s_3\rangle\langle proof\rangle$
lemma *WhileTRedsThrow*:
assumes *b-steps*: $P \vdash \langle b,s_0\rangle \to* \langle true,s_1\rangle$
and *c-steps*: $P \vdash \langle c,s_1\rangle \to* \langle throw\ e,s_2\rangle$
shows $P \vdash \langle while\ (b)\ c,s_0\rangle \to* \langle throw\ e,s_2\rangle\langle proof\rangle$

Throw

lemma *ThrowReds*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle throw\ e,s\rangle \to* \langle throw\ e',s'\rangle\langle proof\rangle$
lemma *ThrowRedsNull*:
$P \vdash \langle e,s\rangle \to* \langle null,s'\rangle \Longrightarrow P \vdash \langle throw\ e,s\rangle \to* \langle THROW\ NullPointer,s'\rangle\langle proof\rangle$
lemma *ThrowRedsThrow*:
$P \vdash \langle e,s\rangle \to* \langle throw\ a,s'\rangle \Longrightarrow P \vdash \langle throw\ e,s\rangle \to* \langle throw\ a,s'\rangle\langle proof\rangle$

InitBlock

lemma *InitBlockReds-aux*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow$
$\forall h\ l\ h'\ l'\ v.\ s = (h,l(V\mapsto v)) \longrightarrow s' = (h',l') \longrightarrow$
$P \vdash \langle\{V{:}T := Val\ v;\ e\},(h,l)\rangle \to* \langle\{V{:}T := Val(the(l'\ V));\ e'\},(h',l'(V{:}=(l\ V)))\rangle\langle proof\rangle$
lemma *InitBlockReds*:
$P \vdash \langle e,\ (h,l(V\mapsto v))\rangle \to* \langle e',\ (h',l')\rangle \Longrightarrow$

$P \vdash \langle\{V{:}T := Val\ v;\ e\},\ (h,l)\rangle \rightarrow* \langle\{V{:}T := Val(the(l'\ V));\ e'\},\ (h',l'(V{:=}(l\ V)))\rangle\langle proof\rangle$
lemma *InitBlockRedsFinal*:
$[\![\ P \vdash \langle e,(h,l(V{\mapsto}v))\rangle \rightarrow* \langle e',(h',l')\rangle;\ final\ e'\ ]\!] \Longrightarrow$
$P \vdash \langle\{V{:}T := Val\ v;\ e\},(h,l)\rangle \rightarrow* \langle e',(h',\ l'(V := l\ V))\rangle\langle proof\rangle$

Block

lemma *BlockRedsFinal*:
assumes *reds*: $P \vdash \langle e_0,s_0\rangle \rightarrow* \langle e_2,(h_2,l_2)\rangle$ **and** *fin*: *final* e_2
shows $\bigwedge h_0\ l_0.\ s_0 = (h_0,l_0(V{:=}None)) \Longrightarrow P \vdash \langle\{V{:}T;\ e_0\},(h_0,l_0)\rangle \rightarrow* \langle e_2,(h_2,l_2(V{:=}l_0\ V))\rangle\langle proof\rangle$

try-catch

lemma *TryReds*:
$P \vdash \langle e,s\rangle \rightarrow* \langle e',s'\rangle \Longrightarrow P \vdash \langle try\ e\ catch(C\ V)\ e_2,s\rangle \rightarrow* \langle try\ e'\ catch(C\ V)\ e_2,s'\rangle\langle proof\rangle$
lemma *TryRedsVal*:
$P \vdash \langle e,s\rangle \rightarrow* \langle Val\ v,s'\rangle \Longrightarrow P \vdash \langle try\ e\ catch(C\ V)\ e_2,s\rangle \rightarrow* \langle Val\ v,s'\rangle\langle proof\rangle$
lemma *TryCatchRedsFinal*:
assumes e_1*-steps*: $P \vdash \langle e_1,s_0\rangle \rightarrow* \langle Throw\ a,(h_1,l_1)\rangle$
and h_1a: $h_1\ a = Some(D,fs)$ **and** *sub*: $P \vdash D \preceq^* C$
and e_2*-steps*: $P \vdash \langle e_2,\ (h_1,\ l_1(V \mapsto Addr\ a))\rangle \rightarrow* \langle e_2',\ (h_2,l_2)\rangle$
and *final*: *final* e_2'
shows $P \vdash \langle try\ e_1\ catch(C\ V)\ e_2,\ s_0\rangle \rightarrow* \langle e_2',\ (h_2,\ (l_2{::}locals)(V := l_1\ V))\rangle\langle proof\rangle$
lemma *TryRedsFail*:
$[\![\ P \vdash \langle e_1,s\rangle \rightarrow* \langle Throw\ a,(h,l)\rangle;\ h\ a = Some(D,fs);\ \neg\ P \vdash D \preceq^* C\ ]\!]$
$\Longrightarrow P \vdash \langle try\ e_1\ catch(C\ V)\ e_2,s\rangle \rightarrow* \langle Throw\ a,(h,l)\rangle\langle proof\rangle$

List

lemma *ListReds1*:
$P \vdash \langle e,s\rangle \rightarrow* \langle e',s'\rangle \Longrightarrow P \vdash \langle e\#es,s\rangle\ [\rightarrow]*\ \langle e'\ \#\ es,s'\rangle\langle proof\rangle$
lemma *ListReds2*:
$P \vdash \langle es,s\rangle\ [\rightarrow]*\ \langle es',s'\rangle \Longrightarrow P \vdash \langle Val\ v\ \#\ es,s\rangle\ [\rightarrow]*\ \langle Val\ v\ \#\ es',s'\rangle\langle proof\rangle$
lemma *ListRedsVal*:
$[\![\ P \vdash \langle e,s_0\rangle \rightarrow* \langle Val\ v,s_1\rangle;\ P \vdash \langle es,s_1\rangle\ [\rightarrow]*\ \langle es',s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle e\#es,s_0\rangle\ [\rightarrow]*\ \langle Val\ v\ \#\ es',s_2\rangle\langle proof\rangle$

Call

First a few lemmas on what happens to free variables during redction.

lemma assumes *wf*: *wwf-J-prog P*
shows *Red-fv*: $P \vdash \langle e,(h,l)\rangle \rightarrow \langle e',(h',l')\rangle \Longrightarrow fv\ e' \subseteq fv\ e$
and $P \vdash \langle es,(h,l)\rangle\ [\rightarrow]\ \langle es',(h',l')\rangle \Longrightarrow fvs\ es' \subseteq fvs\ es\langle proof\rangle$

lemma *Red-dom-lcl*:
$P \vdash \langle e,(h,l)\rangle \rightarrow \langle e',(h',l')\rangle \Longrightarrow dom\ l' \subseteq dom\ l \cup fv\ e$ **and**
$P \vdash \langle es,(h,l)\rangle\ [\rightarrow]\ \langle es',(h',l')\rangle \Longrightarrow dom\ l' \subseteq dom\ l \cup fvs\ es\langle proof\rangle$
lemma *Reds-dom-lcl*:
assumes *wf*: *wwf-J-prog P*
shows $P \vdash \langle e,(h,l)\rangle \rightarrow* \langle e',(h',l')\rangle \Longrightarrow dom\ l' \subseteq dom\ l \cup fv\ e\langle proof\rangle$

Now a few lemmas on the behaviour of blocks during reduction.

lemma *override-on-upd-lemma*:
$(override\text{-}on\ f\ (g(a{\mapsto}b))\ A)(a := g\ a) = override\text{-}on\ f\ g\ (insert\ a\ A)\langle proof\rangle$

lemma *blocksReds*:
$\bigwedge l.$ ⟦ *length Vs = length Ts*; *length vs = length Ts*; *distinct Vs*;
$P \vdash \langle e, (h,l(Vs [\mapsto] vs))\rangle \to* \langle e', (h',l')\rangle$ ⟧
$\Longrightarrow P \vdash \langle blocks(Vs,Ts,vs,e), (h,l)\rangle \to* \langle blocks(Vs,Ts,map\ (the \circ l')\ Vs,e'), (h',override\text{-}on\ l'\ l\ (set\ Vs))\rangle$⟨*proof*⟩

lemma *blocksFinal*:
$\bigwedge l.$ ⟦ *length Vs = length Ts*; *length vs = length Ts*; *final e* ⟧ $\Longrightarrow$
$P \vdash \langle blocks(Vs,Ts,vs,e), (h,l)\rangle \to* \langle e, (h,l)\rangle$⟨*proof*⟩

lemma *blocksRedsFinal*:
assumes *wf*: *length Vs = length Ts length vs = length Ts distinct Vs*
and *reds*: $P \vdash \langle e, (h,l(Vs [\mapsto] vs))\rangle \to* \langle e', (h',l')\rangle$
and *fin*: *final e'* **and** *l''*: $l'' = override\text{-}on\ l'\ l\ (set\ Vs)$
shows $P \vdash \langle blocks(Vs,Ts,vs,e), (h,l)\rangle \to* \langle e', (h',l'')\rangle$⟨*proof*⟩

An now the actual method call reduction lemmas.

lemma *CallRedsObj*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle e{\cdot}M(es),s\rangle \to* \langle e'{\cdot}M(es),s'\rangle$⟨*proof*⟩

lemma *CallRedsParams*:
$P \vdash \langle es,s\rangle\ [\to]*\ \langle es',s'\rangle \Longrightarrow P \vdash \langle (Val\ v){\cdot}M(es),s\rangle \to* \langle (Val\ v){\cdot}M(es'),s'\rangle$⟨*proof*⟩

lemma *CallRedsFinal*:
assumes *wwf*: *wwf-J-prog P*
and $P \vdash \langle e,s_0\rangle \to* \langle addr\ a,s_1\rangle$
$P \vdash \langle es,s_1\rangle\ [\to]*\ \langle map\ Val\ vs,(h_2,l_2)\rangle$
$h_2\ a = Some(C,fs)\ P \vdash C\ sees\ M{:}Ts{\to}T = (pns,body)\ in\ D$
$size\ vs = size\ pns$
and l_2': $l_2' = [this \mapsto Addr\ a,\ pns[\mapsto]vs]$
and *body*: $P \vdash \langle body,(h_2,l_2')\rangle \to* \langle ef,(h_3,l_3)\rangle$
and *final ef*
shows $P \vdash \langle e{\cdot}M(es),\ s_0\rangle \to* \langle ef,(h_3,l_2)\rangle$⟨*proof*⟩

lemma *CallRedsThrowParams*:
assumes *e-steps*: $P \vdash \langle e,s_0\rangle \to* \langle Val\ v,s_1\rangle$
and *es-steps*: $P \vdash \langle es,s_1\rangle\ [\to]*\ \langle map\ Val\ vs_1\ @\ throw\ a\ \#\ es_2,s_2\rangle$
shows $P \vdash \langle e{\cdot}M(es),s_0\rangle \to* \langle throw\ a,s_2\rangle$⟨*proof*⟩

lemma *CallRedsThrowObj*:
$P \vdash \langle e,s0\rangle \to* \langle throw\ a,s_1\rangle \Longrightarrow P \vdash \langle e{\cdot}M(es),s0\rangle \to* \langle throw\ a,s_1\rangle$⟨*proof*⟩

lemma *CallRedsNull*:
assumes *e-steps*: $P \vdash \langle e,s_0\rangle \to* \langle null,s_1\rangle$
and *es-steps*: $P \vdash \langle es,s_1\rangle\ [\to]*\ \langle map\ Val\ vs,s_2\rangle$
shows $P \vdash \langle e{\cdot}M(es),s_0\rangle \to* \langle THROW\ NullPointer,s_2\rangle$⟨*proof*⟩

The main Theorem

lemma assumes *wwf*: *wwf-J-prog P*
shows *big-by-small*: $P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle \Longrightarrow P \vdash \langle e,s\rangle \to* \langle e',s'\rangle$
and *bigs-by-smalls*: $P \vdash \langle es,s\rangle\ [\Rightarrow]\ \langle es',s'\rangle \Longrightarrow P \vdash \langle es,s\rangle\ [\to]*\ \langle es',s'\rangle$⟨*proof*⟩

2.15.2 Big steps simulates small step

This direction was carried out by Norbert Schirmer and Daniel Wasserrab.

The big step equivalent of *RedWhile*:

lemma *unfold-while*:
$P \vdash \langle while(b)\ c,s\rangle \Rightarrow \langle e',s'\rangle\ =\ P \vdash \langle if(b)\ (c;;while(b)\ c)\ else\ (unit),s\rangle \Rightarrow \langle e',s'\rangle \langle proof\rangle$

lemma *blocksEval*:
$\bigwedge Ts\ vs\ l\ l'.\ [\![size\ ps = size\ Ts;\ size\ ps = size\ vs;\ P \vdash \langle blocks(ps,Ts,vs,e),(h,l)\rangle \Rightarrow \langle e',(h',l')\rangle\ ]\!]$
$\Longrightarrow \exists\ l''.\ P \vdash \langle e,(h,l(ps[\mapsto]vs))\rangle \Rightarrow \langle e',(h',l'')\rangle \langle proof\rangle$

lemma
assumes *wf*: *wwf-J-prog P*
shows *eval-restrict-lcl*:
$P \vdash \langle e,(h,l)\rangle \Rightarrow \langle e',(h',l')\rangle \Longrightarrow (\bigwedge W.\ fv\ e \subseteq W \Longrightarrow P \vdash \langle e,(h,l|`W)\rangle \Rightarrow \langle e',(h',l'|`W)\rangle)$
and $P \vdash \langle es,(h,l)\rangle\ [\Rightarrow]\ \langle es',(h',l')\rangle \Longrightarrow (\bigwedge W.\ fvs\ es \subseteq W \Longrightarrow P \vdash \langle es,(h,l|`W)\rangle\ [\Rightarrow]\ \langle es',(h',l'|`W)\rangle)\langle proof\rangle$

lemma *eval-notfree-unchanged*:
$P \vdash \langle e,(h,l)\rangle \Rightarrow \langle e',(h',l')\rangle \Longrightarrow (\bigwedge V.\ V \notin fv\ e \Longrightarrow l'\ V = l\ V)$
and $P \vdash \langle es,(h,l)\rangle\ [\Rightarrow]\ \langle es',(h',l')\rangle \Longrightarrow (\bigwedge V.\ V \notin fvs\ es \Longrightarrow l'\ V = l\ V)\langle proof\rangle$

lemma *eval-closed-lcl-unchanged*:
$[\![\ P \vdash \langle e,(h,l)\rangle \Rightarrow \langle e',(h',l')\rangle;\ fv\ e = \{\}\ ]\!] \Longrightarrow l' = l\langle proof\rangle$

lemma *list-eval-Throw*:
assumes *eval-e*: $P \vdash \langle throw\ x,s\rangle \Rightarrow \langle e',s'\rangle$
shows $P \vdash \langle map\ Val\ vs\ @\ throw\ x\ \#\ es',s\rangle\ [\Rightarrow]\ \langle map\ Val\ vs\ @\ e'\ \#\ es',s'\rangle\langle proof\rangle$

The key lemma:

lemma
assumes *wf*: *wwf-J-prog P*
shows *extend-1-eval*:
$P \vdash \langle e,s\rangle \to \langle e'',s''\rangle \Longrightarrow\ (\bigwedge s'\ e'.\ P \vdash \langle e'',s''\rangle \Rightarrow \langle e',s'\rangle \Longrightarrow P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle)$
and *extend-1-evals*:
$P \vdash \langle es,t\rangle\ [\to]\ \langle es'',t''\rangle \Longrightarrow (\bigwedge t'\ es'.\ P \vdash \langle es'',t''\rangle\ [\Rightarrow]\ \langle es',t'\rangle \Longrightarrow P \vdash \langle es,t\rangle\ [\Rightarrow]\ \langle es',t'\rangle)\langle proof\rangle$

Its extension to $\to*$:

lemma *extend-eval*:
assumes *wf*: *wwf-J-prog P*
and *reds*: $P \vdash \langle e,s\rangle \to* \langle e'',s''\rangle$ **and** *eval-rest*: $P \vdash \langle e'',s''\rangle \Rightarrow \langle e',s'\rangle$
shows $P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle\langle proof\rangle$

lemma *extend-evals*:
assumes *wf*: *wwf-J-prog P*
and *reds*: $P \vdash \langle es,s\rangle\ [\to]*\ \langle es'',s''\rangle$ **and** *eval-rest*: $P \vdash \langle es'',s''\rangle\ [\Rightarrow]\ \langle es',s'\rangle$
shows $P \vdash \langle es,s\rangle\ [\Rightarrow]\ \langle es',s'\rangle\langle proof\rangle$

Finally, small step semantics can be simulated by big step semantics:

theorem
assumes *wf*: *wwf-J-prog P*
shows *small-by-big*: $[\![P \vdash \langle e,s\rangle \to* \langle e',s'\rangle;\ final\ e']\!] \Longrightarrow P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle$
and $[\![P \vdash \langle es,s\rangle\ [\to]*\ \langle es',s'\rangle;\ finals\ es']\!] \Longrightarrow P \vdash \langle es,s\rangle\ [\Rightarrow]\ \langle es',s'\rangle\langle proof\rangle$

2.15.3 Equivalence

And now, the crowning achievement:

corollary *big-iff-small*:
wwf-J-prog P $\Longrightarrow$
$P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle \;=\; (P \vdash \langle e,s\rangle \rightarrow\!* \langle e',s'\rangle \wedge \mathit{final}\ e')\langle \mathit{proof}\rangle$

end

2.16 Well-typedness of Jinja expressions

theory *WellType*
imports *../Common/Objects Expr*
begin

type-synonym
env = *vname* ⇀ *ty*

inductive
WT :: [*J-prog*,*env*, *expr* , *ty*] ⇒ *bool*
(‹-,- ⊢ - :: -› [*51*,*51*,*51*]*50*)
and *WTs* :: [*J-prog*,*env*, *expr list*, *ty list*] ⇒ *bool*
(‹-,- ⊢ - [::] -› [*51*,*51*,*51*]*50*)
for *P* :: *J-prog*
where

WTNew:
is-class P C $\Longrightarrow$
$P,E \vdash \mathit{new}\ C :: \mathit{Class}\ C$

| *WTCast*:
$[\![\, P,E \vdash e :: \mathit{Class}\ D;\ \ \mathit{is\text{-}class}\ P\ C;\ \ P \vdash C \preceq^* D \vee P \vdash D \preceq^* C \,]\!]$
$\Longrightarrow P,E \vdash \mathit{Cast}\ C\ e :: \mathit{Class}\ C$

| *WTVal*:
$\mathit{typeof}\ v = \mathit{Some}\ T \Longrightarrow$
$P,E \vdash \mathit{Val}\ v :: T$

| *WTVar*:
$E\ V = \mathit{Some}\ T \Longrightarrow$
$P,E \vdash \mathit{Var}\ V :: T$

| *WTBinOpEq*:
$[\![\, P,E \vdash e_1 :: T_1;\ \ P,E \vdash e_2 :: T_2;\ P \vdash T_1 \leq T_2 \vee P \vdash T_2 \leq T_1 \,]\!]$
$\Longrightarrow P,E \vdash e_1$ «*Eq*» $e_2 :: \mathit{Boolean}$

| *WTBinOpAdd*:
$[\![\, P,E \vdash e_1 :: \mathit{Integer};\ \ P,E \vdash e_2 :: \mathit{Integer} \,]\!]$
$\Longrightarrow P,E \vdash e_1$ «*Add*» $e_2 :: \mathit{Integer}$

| *WTLAss*:
$[\![\, E\ V = \mathit{Some}\ T;\ \ P,E \vdash e :: T';\ \ P \vdash T' \leq T;\ \ V \neq \mathit{this} \,]\!]$
$\Longrightarrow P,E \vdash V{:=}e :: \mathit{Void}$

| *WTFAcc*:
$[\![\ P,E \vdash e :: Class\ C;\ \ P \vdash C\ sees\ F{:}T\ in\ D\ ]\!]$
$\Longrightarrow P,E \vdash e{\cdot}F\{D\} :: T$

| *WTFAss*:
$[\![\ P,E \vdash e_1 :: Class\ C;\ \ P \vdash C\ sees\ F{:}T\ in\ D;\ \ P,E \vdash e_2 :: T';\ \ P \vdash T' \le T\ ]\!]$
$\Longrightarrow P,E \vdash e_1{\cdot}F\{D\}{:=}e_2 :: Void$

| *WTCall*:
$[\![\ P,E \vdash e :: Class\ C;\ \ P \vdash C\ sees\ M{:}Ts \rightarrow T = (pns,body)\ in\ D;$
$P,E \vdash es\ [::]\ Ts';\ \ P \vdash Ts'\ [\le]\ Ts\ ]\!]$
$\Longrightarrow P,E \vdash e{\cdot}M(es) :: T$

| *WTBlock*:
$[\![\ is\text{-}type\ P\ T;\ \ P,E(V \mapsto T) \vdash e :: T'\ ]\!]$
$\Longrightarrow\ \ P,E \vdash \{V{:}T;\ e\} :: T'$

| *WTSeq*:
$[\![\ P,E \vdash e_1{::}T_1;\ \ P,E \vdash e_2{::}T_2\ ]\!]$
$\Longrightarrow\ \ P,E \vdash e_1;;e_2 :: T_2$
| *WTCond*:
$[\![\ P,E \vdash e :: Boolean;\ \ P,E \vdash e_1{::}T_1;\ \ P,E \vdash e_2{::}T_2;$
$P \vdash T_1 \le T_2 \lor P \vdash T_2 \le T_1;\ \ P \vdash T_1 \le T_2 \longrightarrow T = T_2;\ \ P \vdash T_2 \le T_1 \longrightarrow T = T_1\ ]\!]$
$\Longrightarrow P,E \vdash if\ (e)\ e_1\ else\ e_2 :: T$

| *WTWhile*:
$[\![\ P,E \vdash e :: Boolean;\ \ P,E \vdash c{::}T\ ]\!]$
$\Longrightarrow P,E \vdash while\ (e)\ c :: Void$

| *WTThrow*:
$P,E \vdash e :: Class\ C\ \ \Longrightarrow$
$P,E \vdash throw\ e :: Void$

| *WTTry*:
$[\![\ P,E \vdash e_1 :: T;\ \ P,E(V \mapsto Class\ C) \vdash e_2 :: T;\ is\text{-}class\ P\ C\ ]\!]$
$\Longrightarrow P,E \vdash try\ e_1\ catch(C\ V)\ e_2 :: T$

— well-typed expression lists

| *WTNil*:
$P,E \vdash []\ [::]\ []$

| *WTCons*:
$[\![\ P,E \vdash e :: T;\ \ P,E \vdash es\ [::]\ Ts\ ]\!]$
$\Longrightarrow\ \ P,E \vdash e\#es\ [::]\ T\#Ts$

lemma [*iff*]: $(P,E \vdash []\ [::]\ Ts) = (Ts = [])\langle proof\rangle$
lemma [*iff*]: $(P,E \vdash e\#es\ [::]\ T\#Ts) = (P,E \vdash e :: T \land P,E \vdash es\ [::]\ Ts)\langle proof\rangle$
lemma [*iff*]: $(P,E \vdash (e\#es)\ [::]\ Ts) =$
$(\exists\ U\ Us.\ Ts = U\#Us \land P,E \vdash e :: U \land P,E \vdash es\ [::]\ Us)\langle proof\rangle$
lemma [*iff*]: $\bigwedge Ts.\ (P,E \vdash es_1\ @\ es_2\ [::]\ Ts) =$
$(\exists\ Ts_1\ Ts_2.\ Ts = Ts_1\ @\ Ts_2 \land P,E \vdash es_1\ [::]\ Ts_1 \land P,E \vdash es_2[::]Ts_2)\langle proof\rangle$
lemma [*iff*]: $P,E \vdash Val\ v :: T = (typeof\ v = Some\ T)\langle proof\rangle$

lemma [*iff*]: $P,E \vdash Var\ V :: T = (E\ V = Some\ T)\langle proof\rangle$
lemma [*iff*]: $P,E \vdash e_1;;e_2 :: T_2 = (\exists\, T_1.\ P,E \vdash e_1::T_1 \wedge P,E \vdash e_2::T_2)\langle proof\rangle$
lemma [*iff*]: $(P,E \vdash \{V{:}T;\ e\} :: T') = (is\text{-}type\ P\ T \wedge P,E(V \mapsto T) \vdash e :: T')\langle proof\rangle$

lemma *wt-env-mono*:
$P,E \vdash e :: T \Longrightarrow (\bigwedge E'.\ E \subseteq_m E' \Longrightarrow P,E' \vdash e :: T)$ **and**
$P,E \vdash es\ [::]\ Ts \Longrightarrow (\bigwedge E'.\ E \subseteq_m E' \Longrightarrow P,E' \vdash es\ [::]\ Ts)\langle proof\rangle$

lemma *WT-fv*: $P,E \vdash e :: T \Longrightarrow fv\ e \subseteq dom\ E$
and $P,E \vdash es\ [::]\ Ts \Longrightarrow fvs\ es \subseteq dom\ E\langle proof\rangle$

2.17 Runtime Well-typedness

theory *WellTypeRT*
imports *WellType*
begin

inductive
WTrt :: *J-prog* ⇒ *heap* ⇒ *env* ⇒ *expr* ⇒ *ty* ⇒ *bool*
and *WTrts* :: *J-prog* ⇒ *heap* ⇒ *env* ⇒ *expr list* ⇒ *ty list* ⇒ *bool*
and *WTrt2* :: [*J-prog*,*env*,*heap*,*expr*,*ty*] ⇒ *bool*
(‹-,-,- ⊢ - : -› [*51*,*51*,*51*]*50*)
and *WTrts2* :: [*J-prog*,*env*,*heap*,*expr list*, *ty list*] ⇒ *bool*
(‹-,-,- ⊢ - [:] -› [*51*,*51*,*51*]*50*)
for *P* :: *J-prog* **and** *h* :: *heap*
where

$P,E,h \vdash e : T \equiv WTrt\ P\ h\ E\ e\ T$
| $P,E,h \vdash es[:]Ts \equiv WTrts\ P\ h\ E\ es\ Ts$

| *WTrtNew*:
$is\text{-}class\ P\ C \Longrightarrow$
$P,E,h \vdash new\ C : Class\ C$

| *WTrtCast*:
$[\![\ P,E,h \vdash e : T;\ is\text{-}refT\ T;\ is\text{-}class\ P\ C\ ]\!]$
$\Longrightarrow P,E,h \vdash Cast\ C\ e : Class\ C$

| *WTrtVal*:
$typeof_h\ v = Some\ T \Longrightarrow$
$P,E,h \vdash Val\ v : T$

| *WTrtVar*:
$E\ V = Some\ T \Longrightarrow$
$P,E,h \vdash Var\ V : T$

| *WTrtBinOpEq*:
$[\![\ P,E,h \vdash e_1 : T_1;\ \ P,E,h \vdash e_2 : T_2\ ]\!]$
$\Longrightarrow P,E,h \vdash e_1$ «*Eq*» $e_2 : Boolean$

| *WTrtBinOpAdd*:
$[\![\ P,E,h \vdash e_1 : Integer;\ \ P,E,h \vdash e_2 : Integer\ ]\!]$
$\Longrightarrow P,E,h \vdash e_1$ «*Add*» $e_2 : Integer$

| *WTrtLAss*:
$\llbracket$ $E\ V = Some\ T;\ \ P,E,h \vdash e : T';\ \ P \vdash T' \le T$ $\rrbracket$
$\Longrightarrow P,E,h \vdash V{:=}e : Void$

| *WTrtFAcc*:
$\llbracket$ $P,E,h \vdash e : Class\ C;\ P \vdash C\ has\ F{:}T\ in\ D$ $\rrbracket$ $\Longrightarrow$
$P,E,h \vdash e{\cdot}F\{D\} : T$

| *WTrtFAccNT*:
$P,E,h \vdash e : NT \Longrightarrow$
$P,E,h \vdash e{\cdot}F\{D\} : T$

| *WTrtFAss*:
$\llbracket$ $P,E,h \vdash e_1 : Class\ C;\ \ P \vdash C\ has\ F{:}T\ in\ D;\ P,E,h \vdash e_2 : T_2;\ \ P \vdash T_2 \le T$ $\rrbracket$
$\Longrightarrow P,E,h \vdash e_1{\cdot}F\{D\}{:=}e_2 : Void$

| *WTrtFAssNT*:
$\llbracket$ $P,E,h \vdash e_1{:}NT;\ P,E,h \vdash e_2 : T_2$ $\rrbracket$
$\Longrightarrow P,E,h \vdash e_1{\cdot}F\{D\}{:=}e_2 : Void$

| *WTrtCall*:
$\llbracket$ $P,E,h \vdash e : Class\ C;\ P \vdash C\ sees\ M{:}Ts \rightarrow T = (pns,body)\ in\ D;$
$P,E,h \vdash es\ [:]\ Ts';\ P \vdash Ts'\ [\le]\ Ts$ $\rrbracket$
$\Longrightarrow P,E,h \vdash e{\cdot}M(es) : T$

| *WTrtCallNT*:
$\llbracket$ $P,E,h \vdash e : NT;\ P,E,h \vdash es\ [:]\ Ts$ $\rrbracket$
$\Longrightarrow P,E,h \vdash e{\cdot}M(es) : T$

| *WTrtBlock*:
$P,E(V{\mapsto}T),h \vdash e : T' \Longrightarrow$
$P,E,h \vdash \{V{:}T;\ e\} : T'$

| *WTrtSeq*:
$\llbracket$ $P,E,h \vdash e_1{:}T_1;\ \ P,E,h \vdash e_2{:}T_2$ $\rrbracket$
$\Longrightarrow P,E,h \vdash e_1;;e_2 : T_2$

| *WTrtCond*:
$\llbracket$ $P,E,h \vdash e : Boolean;\ \ P,E,h \vdash e_1{:}T_1;\ \ P,E,h \vdash e_2{:}T_2;$
$P \vdash T_1 \le T_2 \lor P \vdash T_2 \le T_1;\ P \vdash T_1 \le T_2 \longrightarrow T = T_2;\ P \vdash T_2 \le T_1 \longrightarrow T = T_1$ $\rrbracket$
$\Longrightarrow P,E,h \vdash if\ (e)\ e_1\ else\ e_2 : T$

| *WTrtWhile*:
$\llbracket$ $P,E,h \vdash e : Boolean;\ \ P,E,h \vdash c{:}T$ $\rrbracket$
$\Longrightarrow\ \ P,E,h \vdash while(e)\ c : Void$

| *WTrtThrow*:
$\llbracket$ $P,E,h \vdash e : T_r;\ is\text{-}refT\ T_r$ $\rrbracket$ $\Longrightarrow$
$P,E,h \vdash throw\ e : T$

| *WTrtTry*:
$\llbracket$ $P,E,h \vdash e_1 : T_1;\ \ P,E(V \mapsto Class\ C),h \vdash e_2 : T_2;\ P \vdash T_1 \le T_2$ $\rrbracket$
$\Longrightarrow P,E,h \vdash try\ e_1\ catch(C\ V)\ e_2 : T_2$

— well-typed expression lists

| *WTrtNil*:
  $P,E,h \vdash [] \; [:] \; []$

| *WTrtCons*:
  $[\![\; P,E,h \vdash e : T; \;\; P,E,h \vdash es \; [:] \; Ts \;]\!]$
  $\Longrightarrow \;\; P,E,h \vdash e\#es \; [:] \; T\#Ts$

2.17.1 Easy consequences

lemma [*iff*]: $(P,E,h \vdash [] \; [:] \; Ts) = (Ts = [])\langle proof\rangle$
lemma [*iff*]: $(P,E,h \vdash e\#es \; [:] \; T\#Ts) = (P,E,h \vdash e : T \wedge P,E,h \vdash es \; [:] \; Ts)\langle proof\rangle$
lemma [*iff*]: $(P,E,h \vdash (e\#es) \; [:] \; Ts) =$
  $(\exists \, U \; Us. \; Ts = U\#Us \wedge P,E,h \vdash e : U \wedge P,E,h \vdash es \; [:] \; Us)\langle proof\rangle$
lemma [*simp*]: $\forall \, Ts. \; (P,E,h \vdash es_1 \; @ \; es_2 \; [:] \; Ts) =$
  $(\exists \, Ts_1 \; Ts_2. \; Ts = Ts_1 \; @ \; Ts_2 \wedge P,E,h \vdash es_1 \; [:] \; Ts_1 \; \& \; P,E,h \vdash es_2[:]Ts_2)\langle proof\rangle$
lemma [*iff*]: $P,E,h \vdash Val \; v : T = (typeof_h \; v = Some \; T)\langle proof\rangle$
lemma [*iff*]: $P,E,h \vdash Var \; v : T = (E \; v = Some \; T)\langle proof\rangle$
lemma [*iff*]: $P,E,h \vdash e_1;;e_2 : T_2 = (\exists \, T_1. \; P,E,h \vdash e_1{:}T_1 \wedge P,E,h \vdash e_2{:}T_2)\langle proof\rangle$
lemma [*iff*]: $P,E,h \vdash \{V{:}T; \; e\} : T' \; = \; (P,E(V \mapsto T),h \vdash e : T')\langle proof\rangle$

2.17.2 Some interesting lemmas

lemma *WTrts-Val*[*simp*]:
$\bigwedge Ts. \; (P,E,h \vdash map \; Val \; vs \; [:] \; Ts) = (map \; (typeof_h) \; vs = map \; Some \; Ts)\langle proof\rangle$

lemma *WTrts-same-length*: $\bigwedge Ts. \; P,E,h \vdash es \; [:] \; Ts \Longrightarrow length \; es = length \; Ts\langle proof\rangle$

lemma *WTrt-env-mono*:
  $P,E,h \vdash e : T \Longrightarrow (\bigwedge E'. \; E \subseteq_m E' \Longrightarrow P,E',h \vdash e : T)$ **and**
  $P,E,h \vdash es \; [:] \; Ts \Longrightarrow (\bigwedge E'. \; E \subseteq_m E' \Longrightarrow P,E',h \vdash es \; [:] \; Ts)\langle proof\rangle$

lemma *WTrt-hext-mono*: $P,E,h \vdash e : T \Longrightarrow h \trianglelefteq h' \Longrightarrow P,E,h' \vdash e : T$
and *WTrts-hext-mono*: $P,E,h \vdash es \; [:] \; Ts \Longrightarrow h \trianglelefteq h' \Longrightarrow P,E,h' \vdash es \; [:] \; Ts\langle proof\rangle$

lemma *WT-implies-WTrt*: $P,E \vdash e :: T \Longrightarrow P,E,h \vdash e : T$
and *WTs-implies-WTrts*: $P,E \vdash es \; [::] \; Ts \Longrightarrow P,E,h \vdash es \; [:] \; Ts\langle proof\rangle$

end

2.18 Definite assignment

theory *DefAss* **imports** *BigStep* **begin**

2.18.1 Hypersets

type-synonym *'a hyperset = 'a set option*

definition *hyperUn* :: *'a hyperset ⇒ 'a hyperset ⇒ 'a hyperset* (**infixl** ‹⊔› *65*)
where
  $A \sqcup B \; \equiv \; case \; A \; of \; None \Rightarrow None$

$\mid \lfloor A \rfloor \Rightarrow (\mathit{case}\ B\ \mathit{of}\ \mathit{None} \Rightarrow \mathit{None} \mid \lfloor B \rfloor \Rightarrow \lfloor A \cup B \rfloor)$

definition *hyperInt* :: *'a hyperset* ⇒ *'a hyperset* ⇒ *'a hyperset* (**infixl** ‹⊓› *70*)
where
$A \sqcap B \equiv \mathit{case}\ A\ \mathit{of}\ \mathit{None} \Rightarrow B$
$\mid \lfloor A \rfloor \Rightarrow (\mathit{case}\ B\ \mathit{of}\ \mathit{None} \Rightarrow \lfloor A \rfloor \mid \lfloor B \rfloor \Rightarrow \lfloor A \cap B \rfloor)$

definition *hyperDiff1* :: *'a hyperset* ⇒ *'a* ⇒ *'a hyperset* (**infixl** ‹⊖› *65*)
where
$A \ominus a \equiv \mathit{case}\ A\ \mathit{of}\ \mathit{None} \Rightarrow \mathit{None} \mid \lfloor A \rfloor \Rightarrow \lfloor A - \{a\} \rfloor$

definition *hyper-isin* :: *'a* ⇒ *'a hyperset* ⇒ *bool* (**infix** ‹∈∈› *50*)
where
$a \in\in A \equiv \mathit{case}\ A\ \mathit{of}\ \mathit{None} \Rightarrow \mathit{True} \mid \lfloor A \rfloor \Rightarrow a \in A$

definition *hyper-subset* :: *'a hyperset* ⇒ *'a hyperset* ⇒ *bool* (**infix** ‹⊑› *50*)
where
$A \sqsubseteq B \equiv \mathit{case}\ B\ \mathit{of}\ \mathit{None} \Rightarrow \mathit{True}$
$\mid \lfloor B \rfloor \Rightarrow (\mathit{case}\ A\ \mathit{of}\ \mathit{None} \Rightarrow \mathit{False} \mid \lfloor A \rfloor \Rightarrow A \subseteq B)$

lemmas *hyperset-defs* =
hyperUn-def hyperInt-def hyperDiff1-def hyper-isin-def hyper-subset-def

lemma [*simp*]: $\lfloor\{\}\rfloor \sqcup A = A \ \wedge\ A \sqcup \lfloor\{\}\rfloor = A$⟨*proof*⟩
lemma [*simp*]: $\lfloor A \rfloor \sqcup \lfloor B \rfloor = \lfloor A \cup B \rfloor \wedge \lfloor A \rfloor \ominus a = \lfloor A - \{a\} \rfloor$⟨*proof*⟩
lemma [*simp*]: $\mathit{None} \sqcup A = \mathit{None} \wedge A \sqcup \mathit{None} = \mathit{None}$⟨*proof*⟩
lemma [*simp*]: $a \in\in \mathit{None} \wedge \mathit{None} \ominus a = \mathit{None}$⟨*proof*⟩
lemma *hyperUn-assoc*: $(A \sqcup B) \sqcup C = A \sqcup (B \sqcup C)$⟨*proof*⟩
lemma *hyper-insert-comm*: $A \sqcup \lfloor\{a\}\rfloor = \lfloor\{a\}\rfloor \sqcup A \wedge A \sqcup (\lfloor\{a\}\rfloor \sqcup B) = \lfloor\{a\}\rfloor \sqcup (A \sqcup B)$⟨*proof*⟩

2.18.2 Definite assignment

primrec
$\mathcal{A}$ :: *'a exp* ⇒ *'a hyperset*
and $\mathcal{A}s$ :: *'a exp list* ⇒ *'a hyperset*
where
$\mathcal{A}\ (\mathit{new}\ C) = \lfloor\{\}\rfloor$
$\mid \mathcal{A}\ (\mathit{Cast}\ C\ e) = \mathcal{A}\ e$
$\mid \mathcal{A}\ (\mathit{Val}\ v) = \lfloor\{\}\rfloor$
$\mid \mathcal{A}\ (e_1\ «bop»\ e_2) = \mathcal{A}\ e_1 \sqcup \mathcal{A}\ e_2$
$\mid \mathcal{A}\ (\mathit{Var}\ V) = \lfloor\{\}\rfloor$
$\mid \mathcal{A}\ (\mathit{LAss}\ V\ e) = \lfloor\{V\}\rfloor \sqcup \mathcal{A}\ e$
$\mid \mathcal{A}\ (e{\cdot}F\{D\}) = \mathcal{A}\ e$
$\mid \mathcal{A}\ (e_1{\cdot}F\{D\}{:=}e_2) = \mathcal{A}\ e_1 \sqcup \mathcal{A}\ e_2$
$\mid \mathcal{A}\ (e{\cdot}M(es)) = \mathcal{A}\ e \sqcup \mathcal{A}s\ es$
$\mid \mathcal{A}\ (\{V{:}T;\ e\}) = \mathcal{A}\ e \ominus V$
$\mid \mathcal{A}\ (e_1;;e_2) = \mathcal{A}\ e_1 \sqcup \mathcal{A}\ e_2$
$\mid \mathcal{A}\ (\mathit{if}\ (e)\ e_1\ \mathit{else}\ e_2) = \mathcal{A}\ e \sqcup (\mathcal{A}\ e_1 \sqcap \mathcal{A}\ e_2)$
$\mid \mathcal{A}\ (\mathit{while}\ (b)\ e) = \mathcal{A}\ b$
$\mid \mathcal{A}\ (\mathit{throw}\ e) = \mathit{None}$
$\mid \mathcal{A}\ (\mathit{try}\ e_1\ \mathit{catch}(C\ V)\ e_2) = \mathcal{A}\ e_1 \sqcap (\mathcal{A}\ e_2 \ominus V)$

$\mid \mathcal{A}s\ ([]) = \lfloor\{\}\rfloor$
$\mid \mathcal{A}s\ (e\#es) = \mathcal{A}\ e \sqcup \mathcal{A}s\ es$

primrec
$\mathcal{D}$ :: *'a exp ⇒ 'a hyperset ⇒ bool*
and $\mathcal{D}s$ :: *'a exp list ⇒ 'a hyperset ⇒ bool*
where
$\mathcal{D}$ (*new C*) *A* = *True*
| $\mathcal{D}$ (*Cast C e*) *A* = $\mathcal{D}$ *e A*
| $\mathcal{D}$ (*Val v*) *A* = *True*
| $\mathcal{D}$ (e_1 «*bop*» e_2) *A* = ($\mathcal{D}$ e_1 *A* ∧ $\mathcal{D}$ e_2 (*A* ⊔ $\mathcal{A}$ e_1))
| $\mathcal{D}$ (*Var V*) *A* = (*V* ∈∈ *A*)
| $\mathcal{D}$ (*LAss V e*) *A* = $\mathcal{D}$ *e A*
| $\mathcal{D}$ (*e*•*F*{*D*}) *A* = $\mathcal{D}$ *e A*
| $\mathcal{D}$ (e_1•*F*{*D*}:=e_2) *A* = ($\mathcal{D}$ e_1 *A* ∧ $\mathcal{D}$ e_2 (*A* ⊔ $\mathcal{A}$ e_1))
| $\mathcal{D}$ (*e*•*M*(*es*)) *A* = ($\mathcal{D}$ *e A* ∧ $\mathcal{D}s$ *es* (*A* ⊔ $\mathcal{A}$ *e*))
| $\mathcal{D}$ ({*V*:*T*; *e*}) *A* = $\mathcal{D}$ *e* (*A* ⊖ *V*)
| $\mathcal{D}$ (e_1;;e_2) *A* = ($\mathcal{D}$ e_1 *A* ∧ $\mathcal{D}$ e_2 (*A* ⊔ $\mathcal{A}$ e_1))
| $\mathcal{D}$ (*if* (*e*) e_1 *else* e_2) *A* =
($\mathcal{D}$ *e A* ∧ $\mathcal{D}$ e_1 (*A* ⊔ $\mathcal{A}$ *e*) ∧ $\mathcal{D}$ e_2 (*A* ⊔ $\mathcal{A}$ *e*))
| $\mathcal{D}$ (*while* (*e*) *c*) *A* = ($\mathcal{D}$ *e A* ∧ $\mathcal{D}$ *c* (*A* ⊔ $\mathcal{A}$ *e*))
| $\mathcal{D}$ (*throw e*) *A* = $\mathcal{D}$ *e A*
| $\mathcal{D}$ (*try* e_1 *catch*(*C V*) e_2) *A* = ($\mathcal{D}$ e_1 *A* ∧ $\mathcal{D}$ e_2 (*A* ⊔ ⌊{*V*}⌋))

| $\mathcal{D}s$ ([]) *A* = *True*
| $\mathcal{D}s$ (*e*#*es*) *A* = ($\mathcal{D}$ *e A* ∧ $\mathcal{D}s$ *es* (*A* ⊔ $\mathcal{A}$ *e*))

lemma *As-map-Val*[*simp*]: $\mathcal{A}s$ (*map Val vs*) = ⌊{}⌋⟨*proof*⟩
lemma *D-append*[*iff*]: ⋀*A*. $\mathcal{D}s$ (*es* @ *es'*) *A* = ($\mathcal{D}s$ *es A* ∧ $\mathcal{D}s$ *es'* (*A* ⊔ $\mathcal{A}s$ *es*))⟨*proof*⟩

lemma *A-fv*: ⋀*A*. $\mathcal{A}$ *e* = ⌊*A*⌋ ⟹ *A* ⊆ *fv e*
and ⋀*A*. $\mathcal{A}s$ *es* = ⌊*A*⌋ ⟹ *A* ⊆ *fvs es*⟨*proof*⟩

lemma *sqUn-lem*: *A* ⊑ *A'* ⟹ *A* ⊔ *B* ⊑ *A'* ⊔ *B*⟨*proof*⟩
lemma *diff-lem*: *A* ⊑ *A'* ⟹ *A* ⊖ *b* ⊑ *A'* ⊖ *b*⟨*proof*⟩

lemma *D-mono*: ⋀*A A'*. *A* ⊑ *A'* ⟹ $\mathcal{D}$ *e A* ⟹ $\mathcal{D}$ (*e*::*'a exp*) *A'*
and *Ds-mono*: ⋀*A A'*. *A* ⊑ *A'* ⟹ $\mathcal{D}s$ *es A* ⟹ $\mathcal{D}s$ (*es*::*'a exp list*) *A'*⟨*proof*⟩

lemma *D-mono'*: $\mathcal{D}$ *e A* ⟹ *A* ⊑ *A'* ⟹ $\mathcal{D}$ *e A'*
and *Ds-mono'*: $\mathcal{D}s$ *es A* ⟹ *A* ⊑ *A'* ⟹ $\mathcal{D}s$ *es A'*⟨*proof*⟩

end

2.19 Conformance Relations for Type Soundness Proofs

theory *Conform*
imports *Exceptions*
begin

definition *conf* :: *'m prog ⇒ heap ⇒ val ⇒ ty ⇒ bool* (‹-,- ⊢ - :≤ -› [*51*,*51*,*51*,*51*] *50*)
where
P,*h* ⊢ *v* :≤ *T* ≡
∃ *T'*. $typeof_h$ *v* = *Some T'* ∧ *P* ⊢ *T'* ≤ *T*

definition *oconf* :: *'m prog* ⇒ *heap* ⇒ *obj* ⇒ *bool* (‹-,- ⊢ - √› [51,51,51] 50)
where
P,*h* ⊢ *obj* √ ≡
let (*C*,*fs*) = *obj in* ∀ *F D T*. *P* ⊢ *C has F*:*T in D* ⟶
(∃ *v*. *fs*(*F*,*D*) = *Some v* ∧ *P*,*h* ⊢ *v* :≤ *T*)

definition *hconf* :: *'m prog* ⇒ *heap* ⇒ *bool* (‹- ⊢ - √› [51,51] 50)
where
P ⊢ *h* √ ≡
(∀ *a obj*. *h a* = *Some obj* ⟶ *P*,*h* ⊢ *obj* √) ∧ *preallocated h*

definition *lconf* :: *'m prog* ⇒ *heap* ⇒ (*vname* ⇀ *val*) ⇒ (*vname* ⇀ *ty*) ⇒ *bool* (‹-,- ⊢ - '(:≤') -› [51,51,51,51] 50)
where
P,*h* ⊢ *l* (:≤) *E* ≡
∀ *V v*. *l V* = *Some v* ⟶ (∃ *T*. *E V* = *Some T* ∧ *P*,*h* ⊢ *v* :≤ *T*)

abbreviation
confs :: *'m prog* ⇒ *heap* ⇒ *val list* ⇒ *ty list* ⇒ *bool*
(‹-,- ⊢ - [:≤] -› [51,51,51,51] 50) **where**
P,*h* ⊢ *vs* [:≤] *Ts* ≡ *list-all2* (*conf P h*) *vs Ts*

2.19.1 Value conformance :≤

lemma *conf-Null* [*simp*]: *P*,*h* ⊢ *Null* :≤ *T* = *P* ⊢ *NT* ≤ *T*⟨*proof*⟩
lemma *typeof-conf*[*simp*]: $typeof_h$ *v* = *Some T* ⟹ *P*,*h* ⊢ *v* :≤ *T*⟨*proof*⟩
lemma *typeof-lit-conf*[*simp*]: *typeof v* = *Some T* ⟹ *P*,*h* ⊢ *v* :≤ *T*⟨*proof*⟩
lemma *defval-conf*[*simp*]: *P*,*h* ⊢ *default-val T* :≤ *T*⟨*proof*⟩
lemma *conf-upd-obj*: *h a* = *Some*(*C*,*fs*) ⟹ (*P*,*h*(*a*↦(*C*,*fs'*)) ⊢ *x* :≤ *T*) = (*P*,*h* ⊢ *x* :≤ *T*)⟨*proof*⟩
lemma *conf-widen*: *P*,*h* ⊢ *v* :≤ *T* ⟹ *P* ⊢ *T* ≤ *T'* ⟹ *P*,*h* ⊢ *v* :≤ *T'*⟨*proof*⟩
lemma *conf-hext*: *h* ⊴ *h'* ⟹ *P*,*h* ⊢ *v* :≤ *T* ⟹ *P*,*h'* ⊢ *v* :≤ *T*⟨*proof*⟩
lemma *conf-ClassD*: *P*,*h* ⊢ *v* :≤ *Class C* ⟹
v = *Null* ∨ (∃ *a obj T*. *v* = *Addr a* ∧ *h a* = *Some obj* ∧ *obj-ty obj* = *T* ∧ *P* ⊢ *T* ≤ *Class C*)⟨*proof*⟩
lemma *conf-NT* [*iff*]: *P*,*h* ⊢ *v* :≤ *NT* = (*v* = *Null*)⟨*proof*⟩
lemma *non-npD*: ⟦ *v* ≠ *Null*; *P*,*h* ⊢ *v* :≤ *Class C* ⟧
⟹ ∃ *a C' fs*. *v* = *Addr a* ∧ *h a* = *Some*(*C'*,*fs*) ∧ *P* ⊢ *C'* ≼* *C*⟨*proof*⟩

2.19.2 Value list conformance [:≤]

lemma *confs-widens* [*trans*]: ⟦*P*,*h* ⊢ *vs* [:≤] *Ts*; *P* ⊢ *Ts* [≤] *Ts'*⟧ ⟹ *P*,*h* ⊢ *vs* [:≤] *Ts'*⟨*proof*⟩
lemma *confs-rev*: *P*,*h* ⊢ *rev s* [:≤] *t* = (*P*,*h* ⊢ *s* [:≤] *rev t*)⟨*proof*⟩
lemma *confs-conv-map*:
⋀*Ts'*. *P*,*h* ⊢ *vs* [:≤] *Ts'* = (∃ *Ts*. *map* $typeof_h$ *vs* = *map Some Ts* ∧ *P* ⊢ *Ts* [≤] *Ts'*)⟨*proof*⟩
lemma *confs-hext*: *P*,*h* ⊢ *vs* [:≤] *Ts* ⟹ *h* ⊴ *h'* ⟹ *P*,*h'* ⊢ *vs* [:≤] *Ts*⟨*proof*⟩
lemma *confs-Cons2*: *P*,*h* ⊢ *xs* [:≤] *y*#*ys* = (∃ *z zs*. *xs* = *z*#*zs* ∧ *P*,*h* ⊢ *z* :≤ *y* ∧ *P*,*h* ⊢ *zs* [:≤] *ys*)⟨*proof*⟩

2.19.3 Object conformance

lemma *oconf-hext*: *P*,*h* ⊢ *obj* √ ⟹ *h* ⊴ *h'* ⟹ *P*,*h'* ⊢ *obj* √⟨*proof*⟩
lemma *oconf-init-fields*:
P ⊢ *C has-fields FDTs* ⟹ *P*,*h* ⊢ (*C*, *init-fields FDTs*) √
⟨*proof*⟩

lemma *oconf-fupd* [*intro?*]:

$[\![\ P \vdash C\ has\ F{:}T\ in\ D;\ P,h \vdash v :\leq\ T;\ P,h \vdash (C,fs)\ \surd\ ]\!]$
$\Longrightarrow P,h \vdash (C,\ fs((F,D)\mapsto v))\ \surd\langle proof\rangle$

2.19.4 Heap conformance

lemma *hconfD*: $[\![\ P \vdash h\ \surd;\ h\ a = Some\ obj\ ]\!] \Longrightarrow P,h \vdash obj\ \surd\langle proof\rangle$
lemma *hconf-new*: $[\![\ P \vdash h\ \surd;\ h\ a = None;\ P,h \vdash obj\ \surd\ ]\!] \Longrightarrow P \vdash h(a\mapsto obj)\ \surd\langle proof\rangle$
lemma *hconf-upd-obj*: $[\![\ P \vdash h\surd;\ h\ a = Some(C,fs);\ P,h \vdash (C,fs')\surd\ ]\!] \Longrightarrow P \vdash h(a\mapsto(C,fs'))\surd\langle proof\rangle$

2.19.5 Local variable conformance

lemma *lconf-hext*: $[\![\ P,h \vdash l\ (:\leq)\ E;\ h \trianglelefteq h'\ ]\!] \Longrightarrow P,h' \vdash l\ (:\leq)\ E\langle proof\rangle$
lemma *lconf-upd*:
$[\![\ P,h \vdash l\ (:\leq)\ E;\ P,h \vdash v :\leq\ T;\ E\ V = Some\ T\ ]\!] \Longrightarrow P,h \vdash l(V\mapsto v)\ (:\leq)\ E\langle proof\rangle$
lemma *lconf-empty*[*iff*]: $P,h \vdash Map.empty\ (:\leq)\ E\langle proof\rangle$
lemma *lconf-upd2*: $[\![P,h \vdash l\ (:\leq)\ E;\ P,h \vdash v :\leq\ T]\!] \Longrightarrow P,h \vdash l(V\mapsto v)\ (:\leq)\ E(V\mapsto T)\langle proof\rangle$

end

2.20 Progress of Small Step Semantics

theory *Progress*
imports *Equivalence WellTypeRT DefAss ../Common/Conform*
begin

lemma *final-addrE*:
$[\![\ P,E,h \vdash e : Class\ C;\ final\ e;$
$\bigwedge a.\ e = addr\ a \Longrightarrow R;$
$\bigwedge a.\ e = Throw\ a \Longrightarrow R\ ]\!] \Longrightarrow R\langle proof\rangle$

lemma *finalRefE*:
$[\![\ P,E,h \vdash e : T;\ is\text{-}refT\ T;\ final\ e;$
$e = null \Longrightarrow R;$
$\bigwedge a\ C.\ [\![\ e = addr\ a;\ T = Class\ C\ ]\!] \Longrightarrow R;$
$\bigwedge a.\ e = Throw\ a \Longrightarrow R\ ]\!] \Longrightarrow R\langle proof\rangle$

Derivation of new induction scheme for well typing:

inductive
WTrt' :: [*J-prog*,*heap*,*env*,*expr*,*ty*] ⇒ *bool*
and *WTrts'* :: [*J-prog*,*heap*,*env*,*expr list*, *ty list*] ⇒ *bool*
and *WTrt2'* :: [*J-prog*,*env*,*heap*,*expr*,*ty*] ⇒ *bool*
(‹-,-,- ⊢ - :'' -› [*51*,*51*,*51*]*50*)
and *WTrts2'* :: [*J-prog*,*env*,*heap*,*expr list*, *ty list*] ⇒ *bool*
(‹-,-,- ⊢ - [:''] -› [*51*,*51*,*51*]*50*)
for *P* :: *J-prog* **and** *h* :: *heap*
where
$P,E,h \vdash e :' T \equiv WTrt'\ P\ h\ E\ e\ T$
$|\ P,E,h \vdash es\ [:']\ Ts \equiv WTrts'\ P\ h\ E\ es\ Ts$

$|\ is\text{-}class\ P\ C\ \Longrightarrow\ P,E,h \vdash new\ C :'\ Class\ C$
$|\ [\![\ P,E,h \vdash e :'\ T;\ is\text{-}refT\ T;\ is\text{-}class\ P\ C\ ]\!]$
$\Longrightarrow P,E,h \vdash Cast\ C\ e :'\ Class\ C$
$|\ typeof_h\ v = Some\ T \Longrightarrow P,E,h \vdash Val\ v :'\ T$
$|\ E\ v = Some\ T\ \Longrightarrow\ P,E,h \vdash Var\ v :'\ T$

$| \llbracket\ P,E,h \vdash e_1 ::' T_1;\ \ P,E,h \vdash e_2 ::' T_2\ \rrbracket$
$\ \ \Longrightarrow P,E,h \vdash e_1 \ll Eq \gg e_2 ::' Boolean$
$| \llbracket\ P,E,h \vdash e_1 ::' Integer;\ \ P,E,h \vdash e_2 ::' Integer\ \rrbracket$
$\ \ \Longrightarrow P,E,h \vdash e_1 \ll Add \gg e_2 ::' Integer$
$| \llbracket\ P,E,h \vdash Var\ V ::' T;\ \ P,E,h \vdash e ::' T';\ \ P \vdash T' \le T$ ~~$V \ne This$~~ $\rrbracket$
$\ \ \Longrightarrow P,E,h \vdash V{:=}e ::' Void$
$| \llbracket\ P,E,h \vdash e ::' Class\ C;\ P \vdash C\ has\ F{:}T\ in\ D\ \rrbracket \Longrightarrow P,E,h \vdash e{\bullet}F\{D\} ::' T$
$| P,E,h \vdash e ::' NT \Longrightarrow P,E,h \vdash e{\bullet}F\{D\} ::' T$
$| \llbracket\ P,E,h \vdash e_1 ::' Class\ C;\ \ P \vdash C\ has\ F{:}T\ in\ D;$
$\ \ \ \ P,E,h \vdash e_2 ::' T_2;\ \ P \vdash T_2 \le T\ \rrbracket$
$\ \ \Longrightarrow P,E,h \vdash e_1{\bullet}F\{D\}{:=}e_2 ::' Void$
$| \llbracket\ P,E,h \vdash e_1{::}'NT;\ P,E,h \vdash e_2 ::' T_2\ \rrbracket \Longrightarrow P,E,h \vdash e_1{\bullet}F\{D\}{:=}e_2 ::' Void$
$| \llbracket\ P,E,h \vdash e ::' Class\ C;\ P \vdash C\ sees\ M{:}Ts \rightarrow T = (pns,body)\ in\ D;$
$\ \ \ \ P,E,h \vdash es\ [::']\ Ts';\ P \vdash Ts'\ [\le]\ Ts\ \rrbracket$
$\ \ \Longrightarrow P,E,h \vdash e{\bullet}M(es) ::' T$
$| \llbracket\ P,E,h \vdash e ::' NT;\ P,E,h \vdash es\ [::']\ Ts\ \rrbracket \Longrightarrow P,E,h \vdash e{\bullet}M(es) ::' T$
$| P,E,h \vdash []\ [::']\ []$
$| \llbracket\ P,E,h \vdash e ::' T;\ \ P,E,h \vdash es\ [::']\ Ts\ \rrbracket \Longrightarrow\ \ P,E,h \vdash e\#es\ [::']\ T\#Ts$
$| \llbracket\ typeof_h\ v = Some\ T_1;\ P \vdash T_1 \le T;\ P,E(V{\mapsto}T),h \vdash e_2 ::' T_2\ \rrbracket$
$\ \ \Longrightarrow\ \ P,E,h \vdash \{V{:}T := Val\ v;\ e_2\} ::' T_2$
$| \llbracket\ P,E(V{\mapsto}T),h \vdash e ::' T';\ \neg\ assigned\ V\ e\ \rrbracket \Longrightarrow\ \ P,E,h \vdash \{V{:}T;\ e\} ::' T'$
$| \llbracket\ P,E,h \vdash e_1{::}'\ T_1;\ \ P,E,h \vdash e_2{::}'T_2\ \rrbracket\ \ \Longrightarrow\ \ P,E,h \vdash e_1;;e_2 ::' T_2$
$| \llbracket\ P,E,h \vdash e ::' Boolean;\ \ P,E,h \vdash e_1{::}'\ T_1;\ \ P,E,h \vdash e_2{::}'\ T_2;$
$\ \ \ \ P \vdash T_1 \le T_2 \lor P \vdash T_2 \le T_1;$
$\ \ \ \ P \vdash T_1 \le T_2 \longrightarrow T = T_2;\ P \vdash T_2 \le T_1 \longrightarrow T = T_1\ \rrbracket$
$\ \ \Longrightarrow P,E,h \vdash if\ (e)\ e_1\ else\ e_2 ::' T$

$| \llbracket\ P,E,h \vdash e ::' Boolean;\ \ P,E,h \vdash c{::}'\ T\ \rrbracket$
$\ \ \Longrightarrow\ \ P,E,h \vdash while(e)\ c ::' Void$
$| \llbracket\ P,E,h \vdash e ::' T_r;\ is\text{-}refT\ T_r\ \rrbracket\ \ \Longrightarrow\ \ P,E,h \vdash throw\ e ::' T$
$| \llbracket\ P,E,h \vdash e_1 ::' T_1;\ \ P,E(V \mapsto Class\ C),h \vdash e_2 ::' T_2;\ P \vdash T_1 \le T_2\ \rrbracket$
$\ \ \Longrightarrow P,E,h \vdash try\ e_1\ catch(C\ V)\ e_2 ::' T_2$

lemma [*iff*]: $P,E,h \vdash e_1;;e_2 ::' T_2 = (\exists T_1.\ P,E,h \vdash e_1{::}'\ T_1 \land P,E,h \vdash e_2{::}'\ T_2)\langle proof\rangle$
lemma [*iff*]: $P,E,h \vdash Val\ v ::' T = (typeof_h\ v = Some\ T)\langle proof\rangle$
lemma [*iff*]: $P,E,h \vdash Var\ v ::' T = (E\ v = Some\ T)\langle proof\rangle$

lemma *wt-wt'*: $P,E,h \vdash e :: T \Longrightarrow P,E,h \vdash e ::' T$
and *wts-wts'*: $P,E,h \vdash es\ [::]\ Ts \Longrightarrow P,E,h \vdash es\ [::']\ Ts\langle proof\rangle$

lemma *wt'-wt*: $P,E,h \vdash e ::' T \Longrightarrow P,E,h \vdash e :: T$
and *wts'-wts*: $P,E,h \vdash es\ [::']\ Ts \Longrightarrow P,E,h \vdash es\ [::]\ Ts\langle proof\rangle$

corollary *wt'-iff-wt*: $(P,E,h \vdash e ::' T) = (P,E,h \vdash e :: T)\langle proof\rangle$

corollary *wts'-iff-wts*: $(P,E,h \vdash es\ [::']\ Ts) = (P,E,h \vdash es\ [::]\ Ts)\langle proof\rangle$
theorem assumes *wf*: *wwf-J-prog P* **and** *hconf*: $P \vdash h\ \surd$
shows *progress*: $P,E,h \vdash e :: T \Longrightarrow$
$(\bigwedge l.\ \llbracket\ \mathcal{D}\ e\ \lfloor dom\ l\rfloor;\ \neg\ final\ e\ \rrbracket \Longrightarrow \exists e'\ s'.\ P \vdash \langle e,(h,l)\rangle \rightarrow \langle e',s'\rangle)$
and $P,E,h \vdash es\ [::]\ Ts \Longrightarrow$
$(\bigwedge l.\ \llbracket\ \mathcal{D}s\ es\ \lfloor dom\ l\rfloor;\ \neg\ finals\ es\ \rrbracket \Longrightarrow \exists es'\ s'.\ P \vdash \langle es,(h,l)\rangle\ [\rightarrow]\ \langle es',s'\rangle)\langle proof\rangle$

end

2.21 Well-formedness Constraints

theory *JWellForm*
imports *../Common/WellForm WWellForm WellType DefAss*
begin

definition *wf-J-mdecl* :: *J-prog* ⇒ *cname* ⇒ *J-mb mdecl* ⇒ *bool*
where
  wf-J-mdecl P C ≡ λ(*M*,*Ts*,*T*,(*pns*,*body*)).
  length Ts = *length pns* ∧
  distinct pns ∧
  this ∉ *set pns* ∧
  (∃ *T′*. *P*,[*this*↦*Class C*,*pns*[↦]*Ts*] ⊢ *body* :: *T′* ∧ *P* ⊢ *T′* ≤ *T*) ∧
  𝒟 *body* ⌊{*this*} ∪ *set pns*⌋

lemma *wf-J-mdecl*[*simp*]:
  wf-J-mdecl P C (*M*,*Ts*,*T*,*pns*,*body*) ≡
  (*length Ts* = *length pns* ∧
  distinct pns ∧
  this ∉ *set pns* ∧
  (∃ *T′*. *P*,[*this*↦*Class C*,*pns*[↦]*Ts*] ⊢ *body* :: *T′* ∧ *P* ⊢ *T′* ≤ *T*) ∧
  𝒟 *body* ⌊{*this*} ∪ *set pns*⌋)⟨*proof*⟩

abbreviation
  wf-J-prog :: *J-prog* ⇒ *bool* **where**
  wf-J-prog == *wf-prog wf-J-mdecl*

lemma *wf-J-prog-wf-J-mdecl*:
  ⟦ *wf-J-prog P*; (*C*, *D*, *fds*, *mths*) ∈ *set P*; *jmdcl* ∈ *set mths* ⟧
  ⟹ *wf-J-mdecl P C jmdcl*⟨*proof*⟩

lemma *wf-mdecl-wwf-mdecl*: *wf-J-mdecl P C Md* ⟹ *wwf-J-mdecl P C Md*⟨*proof*⟩

lemma *wf-prog-wwf-prog*: *wf-J-prog P* ⟹ *wwf-J-prog P*⟨*proof*⟩

end

2.22 Type Safety Proof

theory *TypeSafe*
imports *Progress JWellForm*
begin

2.22.1 Basic preservation lemmas

First two easy preservation lemmas.

theorem *red-preserves-hconf*:
  P ⊢ ⟨*e*,(*h*,*l*)⟩ → ⟨*e′*,(*h′*,*l′*)⟩ ⟹ (⋀*T E*. ⟦ *P*,*E*,*h* ⊢ *e* : *T*; *P* ⊢ *h* √ ⟧ ⟹ *P* ⊢ *h′* √)
and *reds-preserves-hconf*:
  P ⊢ ⟨*es*,(*h*,*l*)⟩ [→] ⟨*es′*,(*h′*,*l′*)⟩ ⟹ (⋀*Ts E*. ⟦ *P*,*E*,*h* ⊢ *es* [:] *Ts*; *P* ⊢ *h* √ ⟧ ⟹ *P* ⊢ *h′* √)⟨*proof*⟩

theorem *red-preserves-lconf*:
  P ⊢ ⟨*e*,(*h*,*l*)⟩ → ⟨*e′*,(*h′*,*l′*)⟩ ⟹

$(\bigwedge T\ E.\ [\![\ P,E,h \vdash e{:}T;\ P,h \vdash l\ (:\leq)\ E\ ]\!] \Longrightarrow P,h' \vdash l'\ (:\leq)\ E)$
and *reds-preserves-lconf*:
$P \vdash \langle es,(h,l)\rangle\ [\rightarrow]\ \langle es',(h',l')\rangle \Longrightarrow$
$(\bigwedge Ts\ E.\ [\![\ P,E,h \vdash es[:]Ts;\ P,h \vdash l\ (:\leq)\ E\ ]\!] \Longrightarrow P,h' \vdash l'\ (:\leq)\ E)\langle proof\rangle$

Preservation of definite assignment more complex and requires a few lemmas first.

lemma [*iff*]: $\bigwedge A.\ [\![\ length\ Vs = length\ Ts;\ length\ vs = length\ Ts]\!] \Longrightarrow$
$\mathcal{D}\ (blocks\ (Vs,Ts,vs,e))\ A = \mathcal{D}\ e\ (A \sqcup \lfloor set\ Vs\rfloor)\langle proof\rangle$

lemma *red-lA-incr*: $P \vdash \langle e,(h,l)\rangle \rightarrow \langle e',(h',l')\rangle \Longrightarrow \lfloor dom\ l\rfloor \sqcup \mathcal{A}\ e \sqsubseteq\ \lfloor dom\ l'\rfloor \sqcup \mathcal{A}\ e'$
and *reds-lA-incr*: $P \vdash \langle es,(h,l)\rangle\ [\rightarrow]\ \langle es',(h',l')\rangle \Longrightarrow \lfloor dom\ l\rfloor \sqcup \mathcal{A}s\ es \sqsubseteq\ \lfloor dom\ l'\rfloor \sqcup \mathcal{A}s\ es'\langle proof\rangle$

Now preservation of definite assignment.

lemma assumes *wf*: *wf-J-prog P*
shows *red-preserves-defass*:
$P \vdash \langle e,(h,l)\rangle \rightarrow \langle e',(h',l')\rangle \Longrightarrow \mathcal{D}\ e\ \lfloor dom\ l\rfloor \Longrightarrow \mathcal{D}\ e'\ \lfloor dom\ l'\rfloor$
and $P \vdash \langle es,(h,l)\rangle\ [\rightarrow]\ \langle es',(h',l')\rangle \Longrightarrow \mathcal{D}s\ es\ \lfloor dom\ l\rfloor \Longrightarrow \mathcal{D}s\ es'\ \lfloor dom\ l'\rfloor\langle proof\rangle$

Combining conformance of heap and local variables:

definition *sconf* :: *J-prog* ⇒ *env* ⇒ *state* ⇒ *bool* (‹-,- ⊢ - √› [*51*,*51*,*51*]*50*)
where
$P,E \vdash s\ \surd\ \equiv\ let\ (h,l) = s\ in\ P \vdash h\ \surd \wedge P,h \vdash l\ (:\leq)\ E$

lemma *red-preserves-sconf*:
$[\![\ P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle;\ P,E,hp\ s \vdash e : T;\ P,E \vdash s\ \surd\ ]\!] \Longrightarrow P,E \vdash s'\ \surd\langle proof\rangle$
lemma *reds-preserves-sconf*:
$[\![\ P \vdash \langle es,s\rangle\ [\rightarrow]\ \langle es',s'\rangle;\ P,E,hp\ s \vdash es\ [:]\ Ts;\ P,E \vdash s\ \surd\ ]\!] \Longrightarrow P,E \vdash s'\ \surd\langle proof\rangle$

2.22.2 Subject reduction

lemma *wt-blocks*:
$\bigwedge E.\ [\![\ length\ Vs = length\ Ts;\ length\ vs = length\ Ts\ ]\!] \Longrightarrow$
$(P,E,h \vdash blocks(Vs,Ts,vs,e) : T) =$
$(P,E(Vs[\mapsto]Ts),h \vdash e{:}T \wedge (\exists\ Ts'.\ map\ (typeof_h)\ vs = map\ Some\ Ts' \wedge P \vdash Ts'\ [\leq]\ Ts))\langle proof\rangle$

theorem assumes *wf*: *wf-J-prog P*
shows *subject-reduction2*: $P \vdash \langle e,(h,l)\rangle \rightarrow \langle e',(h',l')\rangle \Longrightarrow$
$(\bigwedge E\ T.\ [\![\ P,E \vdash (h,l)\ \surd;\ P,E,h \vdash e{:}T\ ]\!]$
$\Longrightarrow \exists\ T'.\ P,E,h' \vdash e'{:}T' \wedge P \vdash T' \leq T)$
and *subjects-reduction2*: $P \vdash \langle es,(h,l)\rangle\ [\rightarrow]\ \langle es',(h',l')\rangle \Longrightarrow$
$(\bigwedge E\ Ts.\ [\![\ P,E \vdash (h,l)\ \surd;\ P,E,h \vdash es\ [:]\ Ts\ ]\!]$
$\Longrightarrow \exists\ Ts'.\ P,E,h' \vdash es'\ [:]\ Ts' \wedge P \vdash Ts'\ [\leq]\ Ts)\langle proof\rangle$

corollary *subject-reduction*:
$[\![\ wf\text{-}J\text{-}prog\ P;\ P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle;\ P,E \vdash s\ \surd;\ P,E,hp\ s \vdash e{:}T\ ]\!]$
$\Longrightarrow \exists\ T'.\ P,E,hp\ s' \vdash e'{:}T' \wedge P \vdash T' \leq T\langle proof\rangle$
corollary *subjects-reduction*:
$[\![\ wf\text{-}J\text{-}prog\ P;\ P \vdash \langle es,s\rangle\ [\rightarrow]\ \langle es',s'\rangle;\ P,E \vdash s\ \surd;\ P,E,hp\ s \vdash es[:]Ts\ ]\!]$
$\Longrightarrow \exists\ Ts'.\ P,E,hp\ s' \vdash es'[:]Ts' \wedge P \vdash Ts'\ [\leq]\ Ts\langle proof\rangle$

2.22.3 Lifting to →*

Now all these preservation lemmas are first lifted to the transitive closure . . .

lemma *Red-preserves-sconf*:

assumes *wf*: *wf-J-prog P* **and** *Red*: $P \vdash \langle e,s\rangle \to\!* \langle e',s'\rangle$
shows $\bigwedge T.$ $[\![$ $P,E,hp\ s \vdash e : T;\ P,E \vdash s\ \surd$ $]\!] \Longrightarrow P,E \vdash s'\ \surd$⟨*proof*⟩

lemma *Red-preserves-defass*:
assumes *wf*: *wf-J-prog P* **and** *reds*: $P \vdash \langle e,s\rangle \to\!* \langle e',s'\rangle$
shows $\mathcal{D}\ e\ \lfloor dom(lcl\ s)\rfloor \Longrightarrow \mathcal{D}\ e'\ \lfloor dom(lcl\ s')\rfloor$
⟨*proof*⟩

lemma *Red-preserves-type*:
assumes *wf*: *wf-J-prog P* **and** *Red*: $P \vdash \langle e,s\rangle \to\!* \langle e',s'\rangle$
shows $!!T.$ $[\![$ $P,E \vdash s\surd;\ P,E,hp\ s \vdash e{:}T$ $]\!]$
$\Longrightarrow \exists T'.\ P \vdash T' \le T \wedge P,E,hp\ s' \vdash e'{:}T'$⟨*proof*⟩

2.22.4 Lifting to $\Rightarrow$

. . . and now to the big step semantics, just for fun.

lemma *eval-preserves-sconf*:
$[\![$ *wf-J-prog P*; $P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle;\ P,E \vdash e{::}T;\ P,E \vdash s\surd$ $]\!] \Longrightarrow P,E \vdash s'\surd$⟨*proof*⟩

lemma *eval-preserves-type*: **assumes** *wf*: *wf-J-prog P*
shows $[\![$ $P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle;\ P,E \vdash s\surd;\ P,E \vdash e{::}T$ $]\!]$
$\Longrightarrow \exists T'.\ P \vdash T' \le T \wedge P,E,hp\ s' \vdash e'{:}T'$⟨*proof*⟩

2.22.5 The final polish

The above preservation lemmas are now combined and packed nicely.

definition *wf-config* :: *J-prog* $\Rightarrow$ *env* $\Rightarrow$ *state* $\Rightarrow$ *expr* $\Rightarrow$ *ty* $\Rightarrow$ *bool* (‹-,-,- ⊢ - : - √› [*51,0,0,0,0*]*50*)
where
$P,E,s \vdash e{:}T\ \surd\ \equiv\ P,E \vdash s\ \surd \wedge P,E,hp\ s \vdash e{:}T$

theorem *Subject-reduction*: **assumes** *wf*: *wf-J-prog P*
shows $P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow P,E,s \vdash e : T\ \surd$
$\Longrightarrow \exists T'.\ P,E,s' \vdash e' : T'\ \surd \wedge P \vdash T' \le T$⟨*proof*⟩

theorem *Subject-reductions*:
assumes *wf*: *wf-J-prog P* **and** *reds*: $P \vdash \langle e,s\rangle \to\!* \langle e',s'\rangle$
shows $\bigwedge T.\ P,E,s \vdash e{:}T\ \surd \Longrightarrow \exists T'.\ P,E,s' \vdash e'{:}T'\ \surd \wedge P \vdash T' \le T$⟨*proof*⟩

corollary *Progress*: **assumes** *wf*: *wf-J-prog P*
shows $[\![$ $P,E,s\ \vdash e : T\ \surd;\ \mathcal{D}\ e\ \lfloor dom(lcl\ s)\rfloor;\ \neg\ final\ e$ $]\!] \Longrightarrow \exists e'\ s'.\ P \vdash \langle e,s\rangle \to \langle e',s'\rangle$⟨*proof*⟩

corollary *TypeSafety*:
fixes *s*::*state*
assumes *wf*: *wf-J-prog P* **and** *sconf*: $P,E \vdash s\ \surd$ **and** *wt*: $P,E \vdash e{::}T$
and $\mathcal{D}$: $\mathcal{D}\ e\ \lfloor dom(lcl\ s)\rfloor$
and *steps*: $P \vdash \langle e,s\rangle \to\!* \langle e',s'\rangle$
and *nstep*: $\neg(\exists e''\ s''.\ P \vdash \langle e',s'\rangle \to \langle e'',s''\rangle)$
shows $(\exists v.\ e' = Val\ v \wedge P,hp\ s' \vdash v :\le T) \vee$
$(\exists a.\ e' = Throw\ a \wedge a \in dom(hp\ s'))$⟨*proof*⟩

end

2.23 Program annotation

```
theory Annotate imports WellType begin

inductive
  Anno :: [J-prog,env, expr     , expr] ⇒ bool
        (‹-,- ⊢ - ⇝ -›    [51,0,0,51]50)
  and Annos :: [J-prog,env, expr list, expr list] ⇒ bool
        (‹-,- ⊢ - [⇝] -› [51,0,0,51]50)
  for P :: J-prog
where

  AnnoNew: P,E ⊢ new C ⇝ new C
| AnnoCast: P,E ⊢ e ⇝ e′ ⟹ P,E ⊢ Cast C e ⇝ Cast C e′
| AnnoVal: P,E ⊢ Val v ⇝ Val v
| AnnoVarVar: E V = ⌊T⌋ ⟹ P,E ⊢ Var V ⇝ Var V
| AnnoVarField: ⟦ E V = None; E this = ⌊Class C⌋; P ⊢ C sees V:T in D ⟧
               ⟹ P,E ⊢ Var V ⇝ Var this·V{D}
| AnnoBinOp:
  ⟦ P,E ⊢ e1 ⇝ e1′;  P,E ⊢ e2 ⇝ e2′ ⟧
   ⟹ P,E ⊢ e1 «bop» e2 ⇝ e1′ «bop» e2′
| AnnoLAssVar:
  ⟦ E V = ⌊T⌋; P,E ⊢ e ⇝ e′ ⟧ ⟹ P,E ⊢ V:=e ⇝ V:=e′
| AnnoLAssField:
  ⟦ E V = None; E this = ⌊Class C⌋; P ⊢ C sees V:T in D; P,E ⊢ e ⇝ e′ ⟧
   ⟹ P,E ⊢ V:=e ⇝ Var this·V{D} := e′
| AnnoFAcc:
  ⟦ P,E ⊢ e ⇝ e′;  P,E ⊢ e′ :: Class C;  P ⊢ C sees F:T in D ⟧
   ⟹ P,E ⊢ e·F{[]} ⇝ e′·F{D}
| AnnoFAss: ⟦ P,E ⊢ e1 ⇝ e1′;  P,E ⊢ e2 ⇝ e2′;
              P,E ⊢ e1′ :: Class C; P ⊢ C sees F:T in D ⟧
          ⟹ P,E ⊢ e1·F{[]} := e2 ⇝ e1′·F{D} := e2′
| AnnoCall:
  ⟦ P,E ⊢ e ⇝ e′;  P,E ⊢ es [⇝] es′ ⟧
   ⟹ P,E ⊢ Call e M es ⇝ Call e′ M es′
| AnnoBlock:
  P,E(V ↦ T) ⊢ e ⇝ e′  ⟹  P,E ⊢ {V:T; e} ⇝ {V:T; e′}
| AnnoComp: ⟦ P,E ⊢ e1 ⇝ e1′;  P,E ⊢ e2 ⇝ e2′ ⟧
           ⟹  P,E ⊢ e1;;e2 ⇝ e1′;;e2′
| AnnoCond: ⟦ P,E ⊢ e ⇝ e′; P,E ⊢ e1 ⇝ e1′;  P,E ⊢ e2 ⇝ e2′ ⟧
          ⟹ P,E ⊢ if (e) e1 else e2 ⇝ if (e′) e1′ else e2′
| AnnoLoop: ⟦ P,E ⊢ e ⇝ e′;  P,E ⊢ c ⇝ c′ ⟧
          ⟹ P,E ⊢ while (e) c ⇝ while (e′) c′
| AnnoThrow: P,E ⊢ e ⇝ e′  ⟹  P,E ⊢ throw e ⇝ throw e′
| AnnoTry: ⟦ P,E ⊢ e1 ⇝ e1′;  P,E(V ↦ Class C) ⊢ e2 ⇝ e2′ ⟧
         ⟹ P,E ⊢ try e1 catch(C V) e2 ⇝ try e1′ catch(C V) e2′

| AnnoNil: P,E ⊢ [] [⇝] []
| AnnoCons: ⟦ P,E ⊢ e ⇝ e′;  P,E ⊢ es [⇝] es′ ⟧
           ⟹  P,E ⊢ e#es [⇝] e′#es′

end
```

2.24 Example Expressions

```
theory Examples imports Expr begin

definition classObject::J-mb cdecl
where
  classObject == (''Object'','''',[],[])


definition classI :: J-mb cdecl
where
  classI ==
  (''I'', Object,
  [],
  [(''mult'',[Integer,Integer],Integer,[''i'',''j''],
   if (Var ''i'' «Eq» Val(Intg 0)) (Val(Intg 0))
   else Var ''j'' «Add»
       Var this · ''mult''([Var ''i'' «Add» Val(Intg (− 1)),Var ''j'']))
  ])


definition classL :: J-mb cdecl
where
  classL ==
  (''L'', Object,
  [(''F'',Integer), (''N'',Class ''L'')],
  [(''app'',[Class ''L''],Void,[''l''],
   if (Var this · ''N''{''L''} «Eq» null)
      (Var this · ''N''{''L''} := Var ''l'')
   else (Var this · ''N''{''L''}) · ''app''([Var ''l'']))
  ])


definition testExpr-BuildList :: expr
where
  testExpr-BuildList ==
  {''l1'':Class ''L'' := new ''L'';
   Var ''l1''·''F''{''L''} := Val(Intg 1);;
  {''l2'':Class ''L'' := new ''L'';
   Var ''l2''· ''F''{''L''} := Val(Intg 2);;
  {''l3'':Class ''L'' := new ''L'';
   Var ''l3''· ''F''{''L''} := Val(Intg 3);;
  {''l4'':Class ''L'' := new ''L'';
   Var ''l4''· ''F''{''L''} := Val(Intg 4);;
   Var ''l1''·''app''([Var ''l2'']);;
   Var ''l1''·''app''([Var ''l3'']);;
   Var ''l1''·''app''([Var ''l4''])}}}}

definition testExpr1 ::expr
where
  testExpr1 == Val(Intg 5)
definition testExpr2 ::expr
where
  testExpr2 == BinOp (Val(Intg 5)) Add (Val(Intg 6))
```

definition *testExpr3* ::*expr*
where
testExpr3 == *BinOp* (*Var* ''*V*'') *Add* (*Val*(*Intg 6*))
definition *testExpr4* ::*expr*
where
testExpr4 == ''*V*'' := *Val*(*Intg 6*)
definition *testExpr5* ::*expr*
where
testExpr5 == *new* ''*Object*'';; {''*V*'':(*Class* ''*C*'') := *new* ''*C*''; *Var* ''*V*''•''*F*''{''*C*''} := *Val*(*Intg 42*)}
definition *testExpr6* ::*expr*
where
testExpr6 == {''*V*'':(*Class* ''*I*'') := *new* ''*I*''; *Var* ''*V*''•''*mult*''([*Val*(*Intg 40*),*Val*(*Intg 4*)])}

definition *mb-isNull*:: *expr*
where
mb-isNull == *Var this* · ''*test*''{''*A*''} «*Eq*» *null*

definition *mb-add*:: *expr*
where
mb-add == (*Var this* · ''*int*''{''*A*''} :=(*Var this* · ''*int*''{''*A*''} «*Add*» *Var* ''*i*''));; (*Var this* · ''*int*''{''*A*''})

definition *mb-mult-cond*:: *expr*
where
mb-mult-cond == (*Var* ''*j*'' «*Eq*» *Val* (*Intg 0*)) «*Eq*» *Val* (*Bool False*)

definition *mb-mult-block*:: *expr*
where
mb-mult-block == ''*temp*'':=(*Var* ''*temp*'' «*Add*» *Var* ''*i*'');;''*j*'':=(*Var* ''*j*'' «*Add*» *Val* (*Intg* (− *1*)))

definition *mb-mult*:: *expr*
where
mb-mult == {''*temp*'':*Integer*:=*Val* (*Intg 0*); *While* (*mb-mult-cond*) *mb-mult-block*;; (*Var this* · ''*int*''{''*A*''} := *Var* ''*temp*'';; *Var* ''*temp*'')}

definition *classA*:: *J-mb cdecl*
where
classA ==
(''*A*'', *Object*,
[(''*int*'',*Integer*),
(''*test*'',*Class* ''*A*'')],
[(''*isNull*'',[],*Boolean*,[], *mb-isNull*),
(''*add*'',[*Integer*],*Integer*,[''*i*''], *mb-add*),
(''*mult*'',[*Integer*,*Integer*],*Integer*,[''*i*'',''*j*''], *mb-mult*)])

definition *testExpr-ClassA*:: *expr*
where
testExpr-ClassA ==
{''*A1*'':*Class* ''*A*'':= *new* ''*A*'';
{''*A2*'':*Class* ''*A*'':= *new* ''*A*'';
{''*testint*'':*Integer*:= *Val* (*Intg 5*);
(*Var* ''*A2*''• ''*int*''{''*A*''} := (*Var* ''*A1*''• ''*add*''([*Var* ''*testint*'']));;

```
  (Var ''A2''• ''int''{''A''} := (Var ''A1''• ''add''([Var ''testint''])));;
   Var ''A2''• ''mult''([Var ''A2''• ''int''{''A''}, Var ''testint'']) }}}
```

end

2.25 Code Generation For BigStep

```
theory execute-Bigstep
imports
  BigStep Examples
  HOL−Library.Code-Target-Numeral
begin

inductive map-val :: expr list ⇒ val list ⇒ bool
where
  Nil: map-val [] []
| Cons: map-val xs ys ⟹ map-val (Val y # xs) (y # ys)

inductive map-val2 :: expr list ⇒ val list ⇒ expr list ⇒ bool
where
  Nil: map-val2 [] [] []
| Cons: map-val2 xs ys zs ⟹ map-val2 (Val y # xs) (y # ys) zs
| Throw: map-val2 (throw e # xs) [] (throw e # xs)

theorem map-val-conv: (xs = map Val ys) = map-val xs ys⟨proof⟩
theorem map-val2-conv:
 (xs = map Val ys @ throw e # zs) = map-val2 xs ys (throw e # zs)⟨proof⟩
lemma CallNull2:
  ⟦ P ⊢ ⟨e,s0⟩ ⇒ ⟨null,s1⟩;  P ⊢ ⟨ps,s1⟩ [⇒] ⟨evs,s2⟩; map-val evs vs ⟧
  ⟹ P ⊢ ⟨e•M(ps),s0⟩ ⇒ ⟨THROW NullPointer,s2⟩
⟨proof⟩


lemma CallParamsThrow2:
  ⟦ P ⊢ ⟨e,s0⟩ ⇒ ⟨Val v,s1⟩; P ⊢ ⟨es,s1⟩ [⇒] ⟨evs,s2⟩;
     map-val2 evs vs (throw ex # es'') ⟧
   ⟹ P ⊢ ⟨e•M(es),s0⟩ ⇒ ⟨throw ex,s2⟩
⟨proof⟩

lemma Call2:
  ⟦ P ⊢ ⟨e,s0⟩ ⇒ ⟨addr a,s1⟩;  P ⊢ ⟨ps,s1⟩ [⇒] ⟨evs,(h2,l2)⟩;
     map-val evs vs;
     h2 a = Some(C,fs);  P ⊢ C sees M:Ts→T = (pns,body) in D;
     length vs = length pns;  l2' = [this↦Addr a, pns[↦]vs];
     P ⊢ ⟨body,(h2,l2')⟩ ⇒ ⟨e',(h3,l3)⟩ ⟧
  ⟹ P ⊢ ⟨e•M(ps),s0⟩ ⇒ ⟨e',(h3,l2)⟩
⟨proof⟩

code-pred
  (modes: i ⇒ o ⇒ bool)
  map-val
⟨proof⟩
```

code-pred
  (*modes*: *i* ⇒ *o* ⇒ *o* ⇒ *bool*)
  map-val2
⟨*proof*⟩

lemmas [*code-pred-intro*] =
 eval-evals.New eval-evals.NewFail
 eval-evals.Cast eval-evals.CastNull eval-evals.CastFail eval-evals.CastThrow
 eval-evals.Val eval-evals.Var
 eval-evals.BinOp eval-evals.BinOpThrow1 eval-evals.BinOpThrow2
 eval-evals.LAss eval-evals.LAssThrow
 eval-evals.FAcc eval-evals.FAccNull eval-evals.FAccThrow
 eval-evals.FAss eval-evals.FAssNull
 eval-evals.FAssThrow1 eval-evals.FAssThrow2
 eval-evals.CallObjThrow

declare *CallNull2* [*code-pred-intro CallNull2*]
declare *CallParamsThrow2* [*code-pred-intro CallParamsThrow2*]
declare *Call2* [*code-pred-intro Call2*]

lemmas [*code-pred-intro*] =
 eval-evals.Block
 eval-evals.Seq eval-evals.SeqThrow
 eval-evals.CondT eval-evals.CondF eval-evals.CondThrow
 eval-evals.WhileF eval-evals.WhileT
 eval-evals.WhileCondThrow

declare *eval-evals.WhileBodyThrow* [*code-pred-intro WhileBodyThrow2*]

lemmas [*code-pred-intro*] =
 eval-evals.Throw eval-evals.ThrowNull
 eval-evals.ThrowThrow
 eval-evals.Try eval-evals.TryCatch eval-evals.TryThrow
 eval-evals.Nil eval-evals.Cons eval-evals.ConsThrow

code-pred
  (*modes*: *i* ⇒ *i* ⇒ *i* ⇒ *o* ⇒ *o* ⇒ *bool as execute*)
  eval
⟨*proof*⟩

notation *execute* (‹- ⊢ ((*1*⟨-,/-⟩) ⇒/ ⟨′-, ′-⟩)› [*51*,*0*,*0*] *81*)

definition *test1* = [] ⊢ ⟨*testExpr1*,(*Map.empty*,*Map.empty*)⟩ ⇒ ⟨-,-⟩
definition *test2* = [] ⊢ ⟨*testExpr2*,(*Map.empty*,*Map.empty*)⟩ ⇒ ⟨-,-⟩
definition *test3* = [] ⊢ ⟨*testExpr3*,(*Map.empty*,*Map.empty*(″*V*″↦*Intg 77*))⟩ ⇒ ⟨-,-⟩
definition *test4* = [] ⊢ ⟨*testExpr4*,(*Map.empty*,*Map.empty*)⟩ ⇒ ⟨-,-⟩
definition *test5* = [(″*Object*″,(″″,[],[])),(″*C*″,(″*Object*″,[(″*F*″,*Integer*)],[]))] ⊢ ⟨*testExpr5*,(*Map.empty*,*Map.empty*)⟩ ⇒ ⟨-,-⟩
definition *test6* = [(″*Object*″,(″″,[],[])), *classI*] ⊢ ⟨*testExpr6*,(*Map.empty*,*Map.empty*)⟩ ⇒ ⟨-,-⟩

definition *V* = ″*V*″
definition *C* = ″*C*″
definition *F* = ″*F*″

⟨*ML*⟩

definition *test7* = [*classObject*, *classL*] ⊢ ⟨*testExpr-BuildList*, (*Map.empty*,*Map.empty*)⟩ ⇒ ⟨-,-⟩

definition *L* = ''*L*''
definition *N* = ''*N*''

⟨*ML*⟩

definition *test8* = [*classObject*, *classA*] ⊢ ⟨*testExpr-ClassA*, (*Map.empty*,*Map.empty*)⟩ ⇒ ⟨-,-⟩
definition *i* = ''*int*''
definition *t* = ''*test*''
definition *A* = ''*A*''

⟨*ML*⟩

end

2.26 Code Generation For WellType

theory *execute-WellType*
imports
  WellType Examples
begin

lemma *WTCond1*:
  $[\![P,E \vdash e :: Boolean;\ \ P,E \vdash e_1{::}T_1;\ \ P,E \vdash e_2{::}T_2;\ P \vdash T_1 \leq T_2;$
    $P \vdash T_2 \leq T_1 \longrightarrow T_2 = T_1\ ]\!] \Longrightarrow P,E \vdash if\ (e)\ e_1\ else\ e_2 :: T_2$
⟨*proof*⟩

lemma *WTCond2*:
  $[\![P,E \vdash e :: Boolean;\ \ P,E \vdash e_1{::}T_1;\ \ P,E \vdash e_2{::}T_2;\ P \vdash T_2 \leq T_1;$
    $P \vdash T_1 \leq T_2 \longrightarrow T_1 = T_2\ ]\!] \Longrightarrow P,E \vdash if\ (e)\ e_1\ else\ e_2 :: T_1$
⟨*proof*⟩

lemmas [*code-pred-intro*] =
  WT-WTs.WTNew
  WT-WTs.WTCast
  WT-WTs.WTVal
  WT-WTs.WTVar
  WT-WTs.WTBinOpEq
  WT-WTs.WTBinOpAdd
  WT-WTs.WTLAss
  WT-WTs.WTFAcc
  WT-WTs.WTFAss
  WT-WTs.WTCall
  WT-WTs.WTBlock
  WT-WTs.WTSeq

declare
  WTCond1 [*code-pred-intro WTCond1*]

WTCond2 [*code-pred-intro WTCond2*]

lemmas [*code-pred-intro*] =
WT-WTs.WTWhile
WT-WTs.WTThrow
WT-WTs.WTTry
WT-WTs.WTNil
WT-WTs.WTCons

code-pred
(*modes*: $i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow$ *bool as type-check*, $i \Rightarrow i \Rightarrow i \Rightarrow o \Rightarrow$ *bool as infer-type*)
WT
⟨*proof*⟩

notation *infer-type* (‹-,- ⊢ - :: ′-› [*51*,*51*,*51*]*100*)

definition *test1* **where** *test1* = [],*Map.empty* ⊢ *testExpr1* :: -
definition *test2* **where** *test2* = [], *Map.empty* ⊢ *testExpr2* :: -
definition *test3* **where** *test3* = [], *Map.empty*(''*V*'' ↦ *Integer*) ⊢ *testExpr3* :: -
definition *test4* **where** *test4* = [], *Map.empty*(''*V*'' ↦ *Integer*) ⊢ *testExpr4* :: -
definition *test5* **where** *test5* = [*classObject*, (''*C*'',(''*Object*'',[(''*F*'',*Integer*)],[]))], *Map.empty* ⊢ *testExpr5* :: -
definition *test6* **where** *test6* = [*classObject*, *classI*], *Map.empty* ⊢ *testExpr6* :: -

⟨*ML*⟩

definition *testmb-isNull* **where** *testmb-isNull* = [*classObject*, *classA*], *Map.empty*([*this*] [↦] [*Class* ''*A*'']) ⊢ *mb-isNull* :: -
definition *testmb-add* **where** *testmb-add* = [*classObject*, *classA*], *Map.empty*([*this*,''*i*''] [↦] [*Class* ''*A*'',*Integer*]) ⊢ *mb-add* :: -
definition *testmb-mult-cond* **where** *testmb-mult-cond* = [*classObject*, *classA*], *Map.empty*([*this*,''*j*''] [↦] [*Class* ''*A*'',*Integer*]) ⊢ *mb-mult-cond* :: -
definition *testmb-mult-block* **where** *testmb-mult-block* = [*classObject*, *classA*], *Map.empty*([*this*,''*i*'',''*j*'',''*temp*''] [↦] [*Class* ''*A*'',*Integer*,*Integer*,*Integer*]) ⊢ *mb-mult-block* :: -
definition *testmb-mult* **where** *testmb-mult* = [*classObject*, *classA*], *Map.empty*([*this*,''*i*'',''*j*''] [↦] [*Class* ''*A*'',*Integer*,*Integer*]) ⊢ *mb-mult* :: -

⟨*ML*⟩

definition *test* **where** *test* = [*classObject*, *classA*], *Map.empty* ⊢ *testExpr-ClassA* :: -

⟨*ML*⟩

end

Chapter 3

Jinja Virtual Machine

3.1 State of the JVM

theory *JVMState* **imports** *../Common/Objects* **begin**

3.1.1 Frame Stack

type-synonym
pc = nat

type-synonym
frame = val list × val list × cname × mname × pc
— operand stack
— registers (including this pointer, method parameters, and local variables)
— name of class where current method is defined
— parameter types
— program counter within frame

3.1.2 Runtime State

type-synonym
jvm-state = addr option × heap × frame list
— exception flag, heap, frames

end

3.2 Instructions of the JVM

theory *JVMInstructions* **imports** *JVMState* **begin**

datatype
instr = Load nat — load from local variable
| *Store nat* — store into local variable
| *Push val* — push a value (constant)
| *New cname* — create object
| *Getfield vname cname* — Fetch field from object
| *Putfield vname cname* — Set field in object
| *Checkcast cname* — Check whether object is of given type
| *Invoke mname nat* — inv. instance meth of an object

| *Return* — return from method
| *Pop* — pop top element from opstack
| *IAdd* — integer addition
| *Goto int* — goto relative address
| *CmpEq* — equality comparison
| *IfFalse int* — branch if top of stack false
| *Throw* — throw top of stack as exception

type-synonym
bytecode = *instr list*

type-synonym
ex-entry = *pc* × *pc* × *cname* × *pc* × *nat*
— start-pc, end-pc, exception type, handler-pc, remaining stack depth

type-synonym
ex-table = *ex-entry list*

type-synonym
jvm-method = *nat* × *nat* × *bytecode* × *ex-table*
— max stacksize
— number of local variables. Add 1 + no. of parameters to get no. of registers
— instruction sequence
— exception handler table

type-synonym
jvm-prog = *jvm-method prog*

end

3.3 JVM Instruction Semantics

theory *JVMExecInstr*
imports *JVMInstructions JVMState ../Common/Exceptions*
begin

primrec
exec-instr :: [*instr*, *jvm-prog*, *heap*, *val list*, *val list*,
cname, *mname*, *pc*, *frame list*] => *jvm-state*
where
exec-instr-Load:
exec-instr (*Load n*) *P h stk loc* C_0 M_0 *pc frs* =
(*None*, *h*, ((*loc* ! *n*) # *stk*, *loc*, C_0, M_0, *pc+1*)#*frs*)

| *exec-instr* (*Store n*) *P h stk loc* C_0 M_0 *pc frs* =
(*None*, *h*, (*tl stk*, *loc*[*n*:=*hd stk*], C_0, M_0, *pc+1*)#*frs*)

| *exec-instr-Push*:
exec-instr (*Push v*) *P h stk loc* C_0 M_0 *pc frs* =
(*None*, *h*, (*v* # *stk*, *loc*, C_0, M_0, *pc+1*)#*frs*)

| *exec-instr-New*:
exec-instr (*New C*) *P h stk loc* C_0 M_0 *pc frs* =

```
  (case new-Addr h of
    None ⇒   (Some (addr-of-sys-xcpt OutOfMemory), h, (stk, loc, C₀, M₀, pc)#frs)
  | Some a ⇒ (None, h(a↦blank P C), (Addr a#stk, loc, C₀, M₀, pc+1)#frs))

| exec-instr (Getfield F C) P h stk loc C₀ M₀ pc frs =
  (let v      = hd stk;
       xp'    = if v=Null then ⌊addr-of-sys-xcpt NullPointer⌋ else None;
       (D,fs) = the(h(the-Addr v))
   in (xp', h, (the(fs(F,C))#(tl stk), loc, C₀, M₀, pc+1)#frs))

| exec-instr (Putfield F C) P h stk loc C₀ M₀ pc frs =
  (let v    = hd stk;
       r    = hd (tl stk);
       xp'  = if r=Null then ⌊addr-of-sys-xcpt NullPointer⌋ else None;
       a    = the-Addr r;
       (D,fs) = the (h a);
       h'  = h(a ↦ (D, fs((F,C) ↦ v)))
   in (xp', h', (tl (tl stk), loc, C₀, M₀, pc+1)#frs))

| exec-instr (Checkcast C) P h stk loc C₀ M₀ pc frs =
  (let v   = hd stk;
       xp' = if ¬cast-ok P C h v then ⌊addr-of-sys-xcpt ClassCast⌋ else None
   in (xp', h, (stk, loc, C₀, M₀, pc+1)#frs))

| exec-instr-Invoke:
 exec-instr (Invoke M n) P h stk loc C₀ M₀ pc frs =
  (let ps  = take n stk;
       r   = stk!n;
       xp' = if r=Null then ⌊addr-of-sys-xcpt NullPointer⌋ else None;
       C   = fst(the(h(the-Addr r)));
       (D,M',Ts,mxs,mxl₀,ins,xt)= method P C M;
       f'  = ([],[r]@(rev ps)@(replicate mxl₀ undefined),D,M,0)
   in (xp', h, f'#(stk, loc, C₀, M₀, pc)#frs))

| exec-instr Return P h stk₀ loc₀ C₀ M₀ pc frs =
  (if frs=[] then (None, h, []) else
   let v = hd stk₀;
       (stk,loc,C,m,pc) = hd frs;
       n = length (fst (snd (method P C₀ M₀)))
   in (None, h, (v#(drop (n+1) stk),loc,C,m,pc+1)#tl frs))

| exec-instr Pop P h stk loc C₀ M₀ pc frs =
     (None, h, (tl stk, loc, C₀, M₀, pc+1)#frs)

| exec-instr IAdd P h stk loc C₀ M₀ pc frs =
  (let i₂ = the-Intg (hd stk);
       i₁ = the-Intg (hd (tl stk))
   in (None, h, (Intg (i₁+i₂)#(tl (tl stk)), loc, C₀, M₀, pc+1)#frs))

| exec-instr (IfFalse i) P h stk loc C₀ M₀ pc frs =
  (let pc' = if hd stk = Bool False then nat(int pc+i) else pc+1
   in (None, h, (tl stk, loc, C₀, M₀, pc')#frs))

| exec-instr CmpEq P h stk loc C₀ M₀ pc frs =
```

(*let* v_2 = *hd stk*;
v_1 = *hd* (*tl stk*)
in (*None*, *h*, (*Bool* (v_1=v_2) # *tl* (*tl stk*), *loc*, C_0, M_0, *pc*+*1*)#*frs*))

| *exec-instr-Goto*:
exec-instr (*Goto i*) *P h stk loc* C_0 M_0 *pc frs* =
(*None*, *h*, (*stk*, *loc*, C_0, M_0, *nat*(*int pc*+*i*))#*frs*)

| *exec-instr Throw P h stk loc* C_0 M_0 *pc frs* =
(*let xp′* = *if hd stk* = *Null then* ⌊*addr-of-sys-xcpt NullPointer*⌋ *else* ⌊*the-Addr*(*hd stk*)⌋
in (*xp′*, *h*, (*stk*, *loc*, C_0, M_0, *pc*)#*frs*))

lemma *exec-instr-Store*:
exec-instr (*Store n*) *P h* (*v*#*stk*) *loc* C_0 M_0 *pc frs* =
(*None*, *h*, (*stk*, *loc*[*n*:=*v*], C_0, M_0, *pc*+*1*)#*frs*)
⟨*proof*⟩

lemma *exec-instr-Getfield*:
exec-instr (*Getfield F C*) *P h* (*v*#*stk*) *loc* C_0 M_0 *pc frs* =
(*let xp′* = *if v*=*Null then* ⌊*addr-of-sys-xcpt NullPointer*⌋ *else None*;
(*D*,*fs*) = *the*(*h*(*the-Addr v*))
in (*xp′*, *h*, (*the*(*fs*(*F*,*C*))#*stk*, *loc*, C_0, M_0, *pc*+*1*)#*frs*))
⟨*proof*⟩

lemma *exec-instr-Putfield*:
exec-instr (*Putfield F C*) *P h* (*v*#*r*#*stk*) *loc* C_0 M_0 *pc frs* =
(*let xp′* = *if r*=*Null then* ⌊*addr-of-sys-xcpt NullPointer*⌋ *else None*;
a = *the-Addr r*;
(*D*,*fs*) = *the* (*h a*);
h′ = *h*(*a* ↦ (*D*, *fs*((*F*,*C*) ↦ *v*)))
in (*xp′*, *h′*, (*stk*, *loc*, C_0, M_0, *pc*+*1*)#*frs*))
⟨*proof*⟩

lemma *exec-instr-Checkcast*:
exec-instr (*Checkcast C*) *P h* (*v*#*stk*) *loc* C_0 M_0 *pc frs* =
(*let xp′* = *if* ¬*cast-ok P C h v then* ⌊*addr-of-sys-xcpt ClassCast*⌋ *else None*
in (*xp′*, *h*, (*v*#*stk*, *loc*, C_0, M_0, *pc*+*1*)#*frs*))
⟨*proof*⟩

lemma *exec-instr-Return*:
exec-instr Return P h (*v*#stk_0) loc_0 C_0 M_0 *pc frs* =
(*if frs*=[] *then* (*None*, *h*, []) *else*
let (*stk*,*loc*,*C*,*m*,*pc*) = *hd frs*;
n = *length* (*fst* (*snd* (*method P* C_0 M_0)))
in (*None*, *h*, (*v*#(*drop* (*n*+*1*) *stk*),*loc*,*C*,*m*,*pc*+*1*)#*tl frs*))
⟨*proof*⟩

lemma *exec-instr-IPop*:
exec-instr Pop P h (*v*#*stk*) *loc* C_0 M_0 *pc frs* =
(*None*, *h*, (*stk*, *loc*, C_0, M_0, *pc*+*1*)#*frs*)
⟨*proof*⟩

lemma *exec-instr-IAdd*:

exec-instr IAdd P h (Intg i_2 *# Intg* i_1 *# stk) loc* C_0 M_0 *pc frs =*
(None, h, (Intg (i_1+i_2)*#stk, loc,* C_0, M_0, *pc+1)#frs)*
⟨*proof*⟩

lemma *exec-instr-IfFalse*:
exec-instr (IfFalse i) P h (v#stk) loc C_0 M_0 *pc frs =*
(let pc′ = if v = Bool False then nat(int pc+i) else pc+1
in (None, h, (stk, loc, C_0, M_0, *pc′)#frs))*
⟨*proof*⟩

lemma *exec-instr-CmpEq*:
exec-instr CmpEq P h $(v_2\#v_1\#stk)$ *loc* C_0 M_0 *pc frs =*
(None, h, (Bool $(v_1=v_2)$ *# stk, loc,* C_0, M_0, *pc+1)#frs)*
⟨*proof*⟩

lemma *exec-instr-Throw*:
exec-instr Throw P h (v#stk) loc C_0 M_0 *pc frs =*
(let xp′ = if v = Null then ⌊addr-of-sys-xcpt NullPointer⌋ else ⌊the-Addr v⌋
in (xp′, h, (v#stk, loc, C_0, M_0, *pc)#frs))*
⟨*proof*⟩

end

3.4 Exception handling in the JVM

theory *JVMExceptions* **imports** *JVMInstructions ../Common/Exceptions* **begin**

definition *matches-ex-entry* :: *′m prog ⇒ cname ⇒ pc ⇒ ex-entry ⇒ bool*
where
matches-ex-entry P C pc xcp ≡
let (s, e, C′, h, d) = xcp in
$s \le pc \wedge pc < e \wedge P \vdash C \preceq^* C'$

primrec *match-ex-table* :: *′m prog ⇒ cname ⇒ pc ⇒ ex-table ⇒ (pc × nat) option*
where
match-ex-table P C pc [] = None
| *match-ex-table P C pc (e#es) = (if matches-ex-entry P C pc e*
then Some (snd(snd(snd e)))
else match-ex-table P C pc es)

abbreviation
ex-table-of :: *jvm-prog ⇒ cname ⇒ mname ⇒ ex-table* **where**
ex-table-of P C M == snd (snd (snd (snd (snd (snd(method P C M))))))

primrec *find-handler* :: *jvm-prog ⇒ addr ⇒ heap ⇒ frame list ⇒ jvm-state*
where
find-handler P a h [] = (Some a, h, [])
| *find-handler P a h (fr#frs) =*
(let (stk,loc,C,M,pc) = fr in
case match-ex-table P (cname-of h a) pc (ex-table-of P C M) of
None ⇒ find-handler P a h frs

| *Some pc-d* ⇒ (*None*, *h*, (*Addr a* # *drop* (*size stk* − *snd pc-d*) *stk*, *loc*, *C*, *M*, *fst pc-d*)#*frs*))

end

3.5 Program Execution in the JVM

theory *JVMExec*
imports *JVMExecInstr JVMExceptions*
begin

abbreviation
instrs-of :: *jvm-prog* ⇒ *cname* ⇒ *mname* ⇒ *instr list* **where**
instrs-of P C M == *fst*(*snd*(*snd*(*snd*(*snd*(*snd*(*method P C M*))))))

fun *exec* :: *jvm-prog* × *jvm-state* => *jvm-state option* **where** — single step execution
exec (*P*, *xp*, *h*, []) = *None*

| *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*) =
(*let*
i = *instrs-of P C M* ! *pc*;
(*xcpt'*, *h'*, *frs'*) = *exec-instr i P h stk loc C M pc frs*
in Some(*case xcpt' of*
None ⇒ (*None*,*h'*,*frs'*)
| *Some a* ⇒ *find-handler P a h* ((*stk*,*loc*,*C*,*M*,*pc*)#*frs*)))

| *exec* (*P*, *Some xa*, *h*, *frs*) = *None*

— relational view
inductive-set
exec-1 :: *jvm-prog* ⇒ (*jvm-state* × *jvm-state*) *set*
and *exec-1'* :: *jvm-prog* ⇒ *jvm-state* ⇒ *jvm-state* ⇒ *bool*
(‹- ⊢/ - −*jvm*→$_1$/ -› [*61*,*61*,*61*] *60*)
for *P* :: *jvm-prog*
where
P ⊢ σ −*jvm*→$_1$ σ' ≡ (σ,σ') ∈ *exec-1 P*
| *exec-1I*: *exec* (*P*,σ) = *Some* σ' ⟹ *P* ⊢ σ −*jvm*→$_1$ σ'

— reflexive transitive closure:
definition *exec-all* :: *jvm-prog* ⇒ *jvm-state* ⇒ *jvm-state* ⇒ *bool*
(‹(- ⊢/ - −*jvm*→/ -)› [*61*,*61*,*61*]*60*) **where**

exec-all-def1: *P* ⊢ σ −*jvm*→ σ' ⟷ (σ,σ') ∈ (*exec-1 P*)*

notation (*ASCII*)
exec-all (‹- |−/ - −*jvm*−>/ -› [*61*,*61*,*61*]*60*)

lemma *exec-1-eq*:
exec-1 P = {(σ,σ'). *exec* (*P*,σ) = *Some* σ'}⟨*proof*⟩
lemma *exec-1-iff*:
P ⊢ σ −*jvm*→$_1$ σ' = (*exec* (*P*,σ) = *Some* σ')⟨*proof*⟩
lemma *exec-all-def*:
P ⊢ σ −*jvm*→ σ' = ((σ,σ') ∈ {(σ,σ'). *exec* (*P*,σ) = *Some* σ'}*)⟨*proof*⟩

lemma *jvm-refl*[*iff*]: $P \vdash \sigma$ *−jvm→* $\sigma\langle proof\rangle$
lemma *jvm-trans*[*trans*]:
$[\![P \vdash \sigma$ *−jvm→* σ'; $P \vdash \sigma'$ *−jvm→* $\sigma'']\!] \Longrightarrow P \vdash \sigma$ *−jvm→* $\sigma''\langle proof\rangle$
lemma *jvm-one-step1* [*trans*]:
$[\![P \vdash \sigma$ *−jvm→*$_1$ σ'; $P \vdash \sigma'$ *−jvm→* $\sigma'']\!] \Longrightarrow P \vdash \sigma$ *−jvm→* $\sigma''\langle proof\rangle$
lemma *jvm-one-step2* [*trans*]:
$[\![P \vdash \sigma$ *−jvm→* σ'; $P \vdash \sigma'$ *−jvm→*$_1$ $\sigma'']\!] \Longrightarrow P \vdash \sigma$ *−jvm→* $\sigma''\langle proof\rangle$
lemma *exec-all-conf*:
$[\![P \vdash \sigma$ *−jvm→* σ'; $P \vdash \sigma$ *−jvm→* $\sigma'']\!]$
$\Longrightarrow P \vdash \sigma'$ *−jvm→* $\sigma'' \vee P \vdash \sigma''$ *−jvm→* $\sigma'\langle proof\rangle$

lemma *exec-all-finalD*: $P \vdash (x, h, [])$ *−jvm→* $\sigma \Longrightarrow \sigma = (x, h, [])\langle proof\rangle$
lemma *exec-all-deterministic*:
$[\![P \vdash \sigma$ *−jvm→* $(x,h,[])$; $P \vdash \sigma$ *−jvm→* $\sigma']\!] \Longrightarrow P \vdash \sigma'$ *−jvm→* $(x,h,[])\langle proof\rangle$

The start configuration of the JVM: in the start heap, we call a method m of class C in program P. The *this* pointer of the frame is set to *Null* to simulate a static method invokation.

definition *start-state* :: *jvm-prog* ⇒ *cname* ⇒ *mname* ⇒ *jvm-state* **where**
start-state P C M =
(*let* (D,Ts,T,mxs,mxl_0,b) = *method P C M in*
(*None*, *start-heap P*, [([], *Null* # *replicate* mxl_0 *undefined*, *C*, *M*, *0*)]))

end

3.6 A Defensive JVM

theory *JVMDefensive*
imports *JVMExec ../Common/Conform*
begin

Extend the state space by one element indicating a type error (or other abnormal termination)

datatype *'a type-error* = *TypeError* | *Normal 'a*

fun *is-Addr* :: *val* ⇒ *bool* **where**
is-Addr (*Addr a*) ⟷ *True*
| *is-Addr v* ⟷ *False*

fun *is-Intg* :: *val* ⇒ *bool* **where**
is-Intg (*Intg i*) ⟷ *True*
| *is-Intg v* ⟷ *False*

fun *is-Bool* :: *val* ⇒ *bool* **where**
is-Bool (*Bool b*) ⟷ *True*
| *is-Bool v* ⟷ *False*

definition *is-Ref* :: *val* ⇒ *bool* **where**
is-Ref v ⟷ *v* = *Null* ∨ *is-Addr v*

primrec *check-instr* :: [*instr*, *jvm-prog*, *heap*, *val list*, *val list*,
cname, *mname*, *pc*, *frame list*] ⇒ *bool* **where**
check-instr-Load:
check-instr (*Load n*) *P h stk loc C* M_0 *pc frs* =

$(n < length\ loc)$

| *check-instr-Store*:
$check\text{-}instr\ (Store\ n)\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ frs =$
$(0 < length\ stk \wedge n < length\ loc)$

| *check-instr-Push*:
$check\text{-}instr\ (Push\ v)\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ frs =$
$(\neg is\text{-}Addr\ v)$

| *check-instr-New*:
$check\text{-}instr\ (New\ C)\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ frs =$
$is\text{-}class\ P\ C$

| *check-instr-Getfield*:
$check\text{-}instr\ (Getfield\ F\ C)\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ frs =$
$(0 < length\ stk \wedge (\exists C'\ T.\ P \vdash C\ sees\ F{:}T\ in\ C') \wedge$
$(let\ (C',\ T) = field\ P\ C\ F;\ ref = hd\ stk\ in$
$C' = C \wedge is\text{-}Ref\ ref \wedge (ref \neq Null \longrightarrow$
$h\ (the\text{-}Addr\ ref) \neq None \wedge$
$(let\ (D,vs) = the\ (h\ (the\text{-}Addr\ ref))\ in$
$P \vdash D \preceq^* C \wedge vs\ (F,C) \neq None \wedge P,h \vdash the\ (vs\ (F,C)) :\leq T))))$

| *check-instr-Putfield*:
$check\text{-}instr\ (Putfield\ F\ C)\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ frs =$
$(1 < length\ stk \wedge (\exists C'\ T.\ P \vdash C\ sees\ F{:}T\ in\ C') \wedge$
$(let\ (C',\ T) = field\ P\ C\ F;\ v = hd\ stk;\ ref = hd\ (tl\ stk)\ in$
$C' = C \wedge is\text{-}Ref\ ref \wedge (ref \neq Null \longrightarrow$
$h\ (the\text{-}Addr\ ref) \neq None \wedge$
$(let\ D = fst\ (the\ (h\ (the\text{-}Addr\ ref)))\ in$
$P \vdash D \preceq^* C \wedge P,h \vdash v :\leq T))))$

| *check-instr-Checkcast*:
$check\text{-}instr\ (Checkcast\ C)\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ frs =$
$(0 < length\ stk \wedge is\text{-}class\ P\ C \wedge is\text{-}Ref\ (hd\ stk))$

| *check-instr-Invoke*:
$check\text{-}instr\ (Invoke\ M\ n)\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ frs =$
$(n < length\ stk \wedge is\text{-}Ref\ (stk!n) \wedge$
$(stk!n \neq Null \longrightarrow$
$(let\ a = the\text{-}Addr\ (stk!n);$
$C = cname\text{-}of\ h\ a;$
$Ts = fst\ (snd\ (method\ P\ C\ M))$
$in\ h\ a \neq None \wedge P \vdash C\ has\ M \wedge$
$P,h \vdash rev\ (take\ n\ stk)\ [:\leq]\ Ts)))$

| *check-instr-Return*:
$check\text{-}instr\ Return\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ frs =$
$(0 < length\ stk \wedge ((0 < length\ frs) \longrightarrow$
$(P \vdash C_0\ has\ M_0) \wedge$
$(let\ v = hd\ stk;$
$T = fst\ (snd\ (snd\ (method\ P\ C_0\ M_0)))$
$in\ P,h \vdash v :\leq T)))$

| *check-instr-Pop*:
check-instr Pop P h stk loc C_0 M_0 *pc frs* =
(*0* < *length stk*)

| *check-instr-IAdd*:
check-instr IAdd P h stk loc C_0 M_0 *pc frs* =
(*1* < *length stk* ∧ *is-Intg* (*hd stk*) ∧ *is-Intg* (*hd* (*tl stk*)))

| *check-instr-IfFalse*:
check-instr (*IfFalse b*) *P h stk loc* C_0 M_0 *pc frs* =
(*0* < *length stk* ∧ *is-Bool* (*hd stk*) ∧ *0* ≤ *int pc*+*b*)

| *check-instr-CmpEq*:
check-instr CmpEq P h stk loc C_0 M_0 *pc frs* =
(*1* < *length stk*)

| *check-instr-Goto*:
check-instr (*Goto b*) *P h stk loc* C_0 M_0 *pc frs* =
(*0* ≤ *int pc*+*b*)

| *check-instr-Throw*:
check-instr Throw P h stk loc C_0 M_0 *pc frs* =
(*0* < *length stk* ∧ *is-Ref* (*hd stk*))

definition *check* :: *jvm-prog* ⇒ *jvm-state* ⇒ *bool* **where**
check P σ = (*let* (*xcpt*, *h*, *frs*) = σ *in*
(*case frs of* [] ⇒ *True* | (*stk*,*loc*,*C*,*M*,*pc*)#*frs*′ ⇒
P ⊢ *C has M* ∧
(*let* (*C*′,*Ts*,*T*,*mxs*,mxl_0,*ins*,*xt*) = *method P C M*; *i* = *ins*!*pc in*
pc < *size ins* ∧ *size stk* ≤ *mxs* ∧
check-instr i P h stk loc C M pc frs′)))

definition *exec-d* :: *jvm-prog* ⇒ *jvm-state* ⇒ *jvm-state option type-error* **where**
exec-d P σ = (*if check P* σ *then Normal* (*exec* (*P*, σ)) *else TypeError*)

inductive-set
exec-1-d :: *jvm-prog* ⇒ (*jvm-state type-error* × *jvm-state type-error*) *set*
and *exec-1-d*′ :: *jvm-prog* ⇒ *jvm-state type-error* ⇒ *jvm-state type-error* ⇒ *bool*
(‹- ⊢ - −*jvmd*→$_1$ -› [*61*,*61*,*61*]*60*)
for *P* :: *jvm-prog*
where
P ⊢ σ −*jvmd*→$_1$ σ′ ≡ (σ,σ′) ∈ *exec-1-d P*
| *exec-1-d-ErrorI*: *exec-d P* σ = *TypeError* ⟹ *P* ⊢ *Normal* σ −*jvmd*→$_1$ *TypeError*
| *exec-1-d-NormalI*: *exec-d P* σ = *Normal* (*Some* σ′) ⟹ *P* ⊢ *Normal* σ −*jvmd*→$_1$ *Normal* σ′

— reflexive transitive closure:
definition *exec-all-d* :: *jvm-prog* ⇒ *jvm-state type-error* ⇒ *jvm-state type-error* ⇒ *bool*
(‹- ⊢ - −*jvmd*→ -› [*61*,*61*,*61*]*60*) **where**
exec-all-d-def1: *P* ⊢ σ −*jvmd*→ σ′ ⟷ (σ,σ′) ∈ (*exec-1-d P*)*

notation (*ASCII*)
exec-all-d (‹- |− - −*jvmd*−> -› [*61*,*61*,*61*]*60*)

lemma *exec-1-d-eq*:
$exec\text{-}1\text{-}d\ P = \{(s,t).\ \exists\sigma.\ s = Normal\ \sigma \wedge t = TypeError \wedge exec\text{-}d\ P\ \sigma = TypeError\} \cup$
$\{(s,t).\ \exists\sigma\ \sigma'.\ s = Normal\ \sigma \wedge t = Normal\ \sigma' \wedge exec\text{-}d\ P\ \sigma = Normal\ (Some\ \sigma')\}$
⟨*proof*⟩

declare *split-paired-All* [*simp del*]
declare *split-paired-Ex* [*simp del*]

lemma *if-neq* [*dest*!]:
$(if\ P\ then\ A\ else\ B) \neq B \Longrightarrow P$
⟨*proof*⟩

lemma *exec-d-no-errorI* [*intro*]:
$check\ P\ \sigma \Longrightarrow exec\text{-}d\ P\ \sigma \neq TypeError$
⟨*proof*⟩

theorem *no-type-error-commutes*:
$exec\text{-}d\ P\ \sigma \neq TypeError \Longrightarrow exec\text{-}d\ P\ \sigma = Normal\ (exec\ (P,\ \sigma))$
⟨*proof*⟩

lemma *defensive-imp-aggressive*:
$P \vdash (Normal\ \sigma)\ {-jvmd\rightarrow}\ (Normal\ \sigma') \Longrightarrow P \vdash \sigma\ {-jvm\rightarrow}\ \sigma'$⟨*proof*⟩
end

3.7 Example for generating executable code from JVM semantics

theory *JVMListExample*
imports
../Common/SystemClasses
JVMExec
HOL−Library.Code-Target-Numeral
begin

definition *list-name* :: *string*
where
list-name == ''*list*''

definition *test-name* :: *string*
where
test-name == ''*test*''

definition *val-name* :: *string*
where
val-name == ''*val*''

definition *next-name* :: *string*
where
next-name == ''*next*''

definition *append-name* :: *string*
where
append-name == ''*append*''

definition *makelist-name* :: *string*
where
makelist-name == ''*makelist*''

definition *append-ins* :: *bytecode*
where
append-ins ==
[*Load 0*,
Getfield next-name list-name,
Load 0,
Getfield next-name list-name,
Push Null,
CmpEq,
IfFalse 7,
Pop,
Load 0,
Load 1,
Putfield next-name list-name,
Push Unit,
Return,
Load 1,
Invoke append-name 1,
Return]

definition *list-class* :: *jvm-method class*
where
list-class ==
(*Object*,
[(*val-name*, *Integer*), (*next-name*, *Class list-name*)],
[(*append-name*, [*Class list-name*], *Void*,
(*3*, *0*, *append-ins*, [(*1*, *2*, *NullPointer*, *7*, *0*)]))])

definition *make-list-ins* :: *bytecode*
where
make-list-ins ==
[*New list-name*,
Store 0,
Load 0,
Push (*Intg 1*),
Putfield val-name list-name,
New list-name,
Store 1,
Load 1,
Push (*Intg 2*),
Putfield val-name list-name,
New list-name,
Store 2,
Load 2,
Push (*Intg 3*),
Putfield val-name list-name,

Load 0,
Load 1,
Invoke append-name 1,
Pop,
Load 0,
Load 2,
Invoke append-name 1,
Return]

definition *test-class* :: *jvm-method class*
where
test-class ==
(*Object*, [],
[(*makelist-name*, [], *Void*, (*3*, *2*, *make-list-ins*, []))])

definition *E* :: *jvm-prog*
where
E == *SystemClasses* @ [(*list-name*, *list-class*), (*test-name*, *test-class*)]

definition *undefined-cname* :: *cname*
where [*code del*]: *undefined-cname* = *undefined*
declare *undefined-cname-def*[*symmetric*, *code-unfold*]
code-printing constant *undefined-cname* ⇀ (*SML*) *object*

definition *undefined-val* :: *val*
where [*code del*]: *undefined-val* = *undefined*
declare *undefined-val-def*[*symmetric*, *code-unfold*]
code-printing constant *undefined-val* ⇀ (*SML*) *Unit*

lemmas [*code-unfold*] = *SystemClasses-def* [*unfolded ObjectC-def NullPointerC-def ClassCastC-def OutOfMemoryC-def*]

definition *test* = *exec* (*E*, *start-state E test-name makelist-name*)

⟨*ML*⟩

end

Chapter 4

Bytecode Verifier

4.1 Semilattices

theory *Semilat*
imports *Main HOL−Library.While-Combinator*
begin

type-synonym *'a ord* = *'a* ⇒ *'a* ⇒ *bool*
type-synonym *'a binop* = *'a* ⇒ *'a* ⇒ *'a*
type-synonym *'a sl* = *'a set* × *'a ord* × *'a binop*

definition *lesub* :: *'a* ⇒ *'a ord* ⇒ *'a* ⇒ *bool*
  where *lesub x r y* ⟷ *r x y*

definition *lesssub* :: *'a* ⇒ *'a ord* ⇒ *'a* ⇒ *bool*
  where *lesssub x r y* ⟷ *lesub x r y* ∧ *x* ≠ *y*

definition *plussub* :: *'a* ⇒ (*'a* ⇒ *'b* ⇒ *'c*) ⇒ *'b* ⇒ *'c*
  where *plussub x f y* = *f x y*

notation (*ASCII*)
  lesub (‹(- /<='-- -)› [*50*, *1000*, *51*] *50*) **and**
  lesssub (‹(- /<'-- -)› [*50*, *1000*, *51*] *50*) **and**
  plussub (‹(- /+'-- -)› [*65*, *1000*, *66*] *65*)

notation
  lesub (‹(- /⊑- -)› [*50*, *0*, *51*] *50*) **and**
  lesssub (‹(- /⊏- -)› [*50*, *0*, *51*] *50*) **and**
  plussub (‹(- /⊔- -)› [*65*, *0*, *66*] *65*)

abbreviation (*input*)
  lesub1 :: *'a* ⇒ *'a ord* ⇒ *'a* ⇒ *bool* (‹(- /⊑- -)› [*50*, *1000*, *51*] *50*)
  where $x \sqsubseteq_r y == x \sqsubseteq_r y$

abbreviation (*input*)
  lesssub1 :: *'a* ⇒ *'a ord* ⇒ *'a* ⇒ *bool* (‹(- /⊏- -)› [*50*, *1000*, *51*] *50*)
  where $x \sqsubset_r y == x \sqsubset_r y$

abbreviation (*input*)

plussub1 :: $'a \Rightarrow ('a \Rightarrow 'b \Rightarrow 'c) \Rightarrow 'b \Rightarrow 'c$ (‹(- /⊔- -)› [*65*, *1000*, *66*] *65*)
where $x \sqcup_f y == x \sqcup_f y$

definition *ord* :: $('a \times 'a)$ *set* $\Rightarrow$ $'a$ *ord*
where
ord r = $(\lambda x\ y.\ (x,y) \in r)$

definition *order* :: $'a$ *ord* $\Rightarrow$ $'a$ *set* $\Rightarrow$ *bool*
where
order r A $\longleftrightarrow$ $(\forall x{\in}A.\ x \sqsubseteq_r x) \wedge (\forall x{\in}A.\ \forall y{\in}A.\ x \sqsubseteq_r y \wedge y \sqsubseteq_r x \longrightarrow x{=}y) \wedge (\forall x{\in}A.\ \forall y{\in}A.\ \forall z{\in}A.\ x \sqsubseteq_r y \wedge y \sqsubseteq_r z \longrightarrow x \sqsubseteq_r z)$

definition *top* :: $'a$ *ord* $\Rightarrow$ $'a \Rightarrow$ *bool*
where
top r T $\longleftrightarrow$ $(\forall x.\ x \sqsubseteq_r T)$

definition *acc* :: $'a$ *ord* $\Rightarrow$ *bool*
where
acc r $\longleftrightarrow$ *wf* $\{(y,x).\ x \sqsubset_r y\}$

definition *closed* :: $'a$ *set* $\Rightarrow$ $'a$ *binop* $\Rightarrow$ *bool*
where
closed A f $\longleftrightarrow$ $(\forall x{\in}A.\ \forall y{\in}A.\ x \sqcup_f y \in A)$

definition *semilat* :: $'a$ *sl* $\Rightarrow$ *bool*
where
semilat = $(\lambda(A,r,f).$ *order r A* $\wedge$ *closed A f* $\wedge$
$(\forall x{\in}A.\ \forall y{\in}A.\ x \sqsubseteq_r x \sqcup_f y) \wedge$
$(\forall x{\in}A.\ \forall y{\in}A.\ y \sqsubseteq_r x \sqcup_f y) \wedge$
$(\forall x{\in}A.\ \forall y{\in}A.\ \forall z{\in}A.\ x \sqsubseteq_r z \wedge y \sqsubseteq_r z \longrightarrow x \sqcup_f y \sqsubseteq_r z))$

definition *is-ub* :: $('a \times 'a)$ *set* $\Rightarrow$ $'a \Rightarrow 'a \Rightarrow 'a \Rightarrow$ *bool*
where
is-ub r x y u $\longleftrightarrow$ $(x,u){\in}r \wedge (y,u){\in}r$

definition *is-lub* :: $('a \times 'a)$ *set* $\Rightarrow$ $'a \Rightarrow 'a \Rightarrow 'a \Rightarrow$ *bool*
where
is-lub r x y u $\longleftrightarrow$ *is-ub r x y u* $\wedge$ $(\forall z.$ *is-ub r x y z* $\longrightarrow (u,z){\in}r)$

definition *some-lub* :: $('a \times 'a)$ *set* $\Rightarrow$ $'a \Rightarrow 'a \Rightarrow 'a$
where
some-lub r x y = (*SOME z. is-lub r x y z*)

locale *Semilat* =
fixes *A* :: $'a$ *set*
fixes *r* :: $'a$ *ord*
fixes *f* :: $'a$ *binop*
assumes *semilat*: *semilat* (*A*, *r*, *f*)

lemma *order-refl* [*simp*, *intro*]: *order r A* $\Longrightarrow x \in A \Longrightarrow x \sqsubseteq_r x$
⟨*proof*⟩
lemma *order-antisym*: ⟦ *order r A*; $x \sqsubseteq_r y$; $y \sqsubseteq_r x$; $x \in A$; $y \in A$ ⟧ $\Longrightarrow x = y$
⟨*proof*⟩
lemma *order-trans*: ⟦ *order r A*; $x \sqsubseteq_r y$; $y \sqsubseteq_r z$; $x \in A$; $y \in A$; $z \in A$ ⟧ $\Longrightarrow x \sqsubseteq_r z$

⟨*proof*⟩
lemma *order-less-irrefl* [*intro*, *simp*]: *order r A* $\Longrightarrow$ $x \in A$ $\Longrightarrow$ $\neg\ x \sqsubset_r x$
⟨*proof*⟩
lemma *order-less-trans*: $[\![$ *order r A*; $x \sqsubset_r y$; $y \sqsubset_r z$; $x \in A$; $y \in A$; $z \in A$ $]\!]$ $\Longrightarrow$ $x \sqsubset_r z$
⟨*proof*⟩
lemma *topD* [*simp*, *intro*]: *top r T* $\Longrightarrow$ $x \sqsubseteq_r T$
⟨*proof*⟩
lemma *top-le-conv* [*simp*]: $[\![$ *order r A*; *top r T*; $x \in A$; $T \in A]\!]$ $\Longrightarrow$ $(T \sqsubseteq_r x) = (x = T)$
⟨*proof*⟩
lemma *semilat-Def*:
semilat(*A*,*r*,*f*) $\longleftrightarrow$ *order r A* $\wedge$ *closed A f* $\wedge$
$(\forall x{\in}A.\ \forall y{\in}A.\ x \sqsubseteq_r x \sqcup_f y)$ $\wedge$
$(\forall x{\in}A.\ \forall y{\in}A.\ y \sqsubseteq_r x \sqcup_f y)$ $\wedge$
$(\forall x{\in}A.\ \forall y{\in}A.\ \forall z{\in}A.\ x \sqsubseteq_r z \wedge y \sqsubseteq_r z \longrightarrow x \sqcup_f y \sqsubseteq_r z)$
⟨*proof*⟩
lemma (**in** *Semilat*) *orderI* [*simp*, *intro*]: *order r A*
⟨*proof*⟩
lemma (**in** *Semilat*) *closedI* [*simp*, *intro*]: *closed A f*
⟨*proof*⟩
lemma *closedD*: $[\![$ *closed A f*; $x{\in}A$; $y{\in}A$ $]\!]$ $\Longrightarrow$ $x \sqcup_f y \in A$
⟨*proof*⟩
lemma *closed-UNIV* [*simp*]: *closed UNIV f*
⟨*proof*⟩
lemma (**in** *Semilat*) *closed-f* [*simp*, *intro*]: $[\![x \in A;\ y \in A]\!]$ $\Longrightarrow$ $x \sqcup_f y \in A$
⟨*proof*⟩
lemma (**in** *Semilat*) *refl-r* [*intro*, *simp*]: $x \in A$ $\Longrightarrow$ $x \sqsubseteq_r x$ ⟨*proof*⟩

lemma (**in** *Semilat*) *antisym-r* [*intro?*]: $[\![$ $x \sqsubseteq_r y$; $y \sqsubseteq_r x$; $x \in A$; $y \in A$ $]\!]$ $\Longrightarrow$ $x = y$
⟨*proof*⟩
lemma (**in** *Semilat*) *trans-r* [*trans*, *intro?*]: $[\![x \sqsubseteq_r y;\ y \sqsubseteq_r z;\ x \in A;\ y \in A;\ z \in A\ ]\!]$ $\Longrightarrow$ $x \sqsubseteq_r z$
⟨*proof*⟩
lemma (**in** *Semilat*) *ub1* [*simp*, *intro?*]: $[\![$ $x \in A$; $y \in A$ $]\!]$ $\Longrightarrow$ $x \sqsubseteq_r x \sqcup_f y$
⟨*proof*⟩
lemma (**in** *Semilat*) *ub2* [*simp*, *intro?*]: $[\![$ $x \in A$; $y \in A$ $]\!]$ $\Longrightarrow$ $y \sqsubseteq_r x \sqcup_f y$
⟨*proof*⟩
lemma (**in** *Semilat*) *lub* [*simp*, *intro?*]:
$[\![$ $x \sqsubseteq_r z$; $y \sqsubseteq_r z$; $x \in A$; $y \in A$; $z \in A$ $]\!]$ $\Longrightarrow$ $x \sqcup_f y \sqsubseteq_r z$
⟨*proof*⟩
lemma (**in** *Semilat*) *plus-le-conv* [*simp*]:
$[\![$ $x \in A$; $y \in A$; $z \in A$ $]\!]$ $\Longrightarrow$ $(x \sqcup_f y \sqsubseteq_r z) = (x \sqsubseteq_r z \wedge y \sqsubseteq_r z)$
⟨*proof*⟩
lemma (**in** *Semilat*) *le-iff-plus-unchanged*:
assumes $x \in A$ **and** $y \in A$
shows $x \sqsubseteq_r y \longleftrightarrow x \sqcup_f y = y$ (**is** *?P* $\longleftrightarrow$ *?Q*)⟨*proof*⟩
lemma (**in** *Semilat*) *le-iff-plus-unchanged2*:
assumes $x \in A$ **and** $y \in A$
shows $x \sqsubseteq_r y \longleftrightarrow y \sqcup_f x = y$ (**is** *?P* $\longleftrightarrow$ *?Q*)⟨*proof*⟩
lemma (**in** *Semilat*) *plus-assoc* [*simp*]:
assumes *a*: $a \in A$ **and** *b*: $b \in A$ **and** *c*: $c \in A$
shows $a \sqcup_f (b \sqcup_f c) = a \sqcup_f b \sqcup_f c$⟨*proof*⟩
lemma (**in** *Semilat*) *plus-com-lemma*:
$[\![a \in A;\ b \in A]\!]$ $\Longrightarrow$ $a \sqcup_f b \sqsubseteq_r b \sqcup_f a$⟨*proof*⟩
lemma (**in** *Semilat*) *plus-commutative*:
$[\![a \in A;\ b \in A]\!]$ $\Longrightarrow$ $a \sqcup_f b = b \sqcup_f a$

⟨*proof*⟩
lemma *is-lubD*:
is-lub r x y u $\Longrightarrow$ *is-ub r x y u* $\wedge$ ($\forall$ *z. is-ub r x y z* $\longrightarrow$ (*u*,*z*) $\in$ *r*)
⟨*proof*⟩
lemma *is-ubI*:
⟦ (*x*,*u*) $\in$ *r*; (*y*,*u*) $\in$ *r* ⟧ $\Longrightarrow$ *is-ub r x y u*
⟨*proof*⟩
lemma *is-ubD*:
is-ub r x y u $\Longrightarrow$ (*x*,*u*) $\in$ *r* $\wedge$ (*y*,*u*) $\in$ *r*
⟨*proof*⟩

lemma *is-lub-bigger1* [*iff*]:
is-lub (*r*^*) *x y y* = ((*x*,*y*)∈*r*^*)⟨*proof*⟩
lemma *is-lub-bigger2* [*iff*]:
is-lub (*r*^*) *x y x* = ((*y*,*x*)∈*r*^*)⟨*proof*⟩
lemma *extend-lub*:
⟦ *single-valued r*; *is-lub* (*r*^*) *x y u*; (*x*′,*x*) $\in$ *r* ⟧
$\Longrightarrow$ ∃ *v. is-lub* (*r*^*) *x*′ *y v*⟨*proof*⟩
lemma *single-valued-has-lubs* [*rule-format*]:
⟦ *single-valued r*; (*x*,*u*) $\in$ *r*^* ⟧ $\Longrightarrow$ (∀ *y*. (*y*,*u*) $\in$ *r*^* $\longrightarrow$
(∃ *z. is-lub* (*r*^*) *x y z*))⟨*proof*⟩
lemma *some-lub-conv*:
⟦ *acyclic r*; *is-lub* (*r*^*) *x y u* ⟧ $\Longrightarrow$ *some-lub* (*r*^*) *x y* = *u*⟨*proof*⟩
lemma *is-lub-some-lub*:
⟦ *single-valued r*; *acyclic r*; (*x*,*u*)∈*r*^*; (*y*,*u*)∈*r*^* ⟧
$\Longrightarrow$ *is-lub* (*r*^*) *x y* (*some-lub* (*r*^*) *x y*)
⟨*proof*⟩

4.1.1 An executable lub-finder

definition *exec-lub* :: ('*a* ∗ '*a*) *set* ⇒ ('*a* ⇒ '*a*) ⇒ '*a binop*
where
exec-lub r f x y = *while* (λ*z*. (*x*,*z*) $\notin$ r^*) *f y*

lemma *exec-lub-refl*: *exec-lub r f T T* = *T*
⟨*proof*⟩

lemma *acyclic-single-valued-finite*:
⟦*acyclic r*; *single-valued r*; (*x*,*y*) $\in$ r^*⟧
$\Longrightarrow$ *finite* (*r* ∩ {*a*. (*x*, *a*) $\in$ r^*} × {*b*. (*b*, *y*) $\in$ r^*})⟨*proof*⟩

lemma *exec-lub-conv*:
⟦ *acyclic r*; ∀ *x y*. (*x*,*y*) $\in$ *r* $\longrightarrow$ *f x* = *y*; *is-lub* (r^*) *x y u* ⟧ $\Longrightarrow$
exec-lub r f x y = *u*⟨*proof*⟩
lemma *is-lub-exec-lub*:
⟦ *single-valued r*; *acyclic r*; (*x*,*u*):*r*^*; (*y*,*u*):*r*^*; ∀ *x y*. (*x*,*y*) $\in$ *r* $\longrightarrow$ *f x* = *y* ⟧
$\Longrightarrow$ *is-lub* (*r*^*) *x y* (*exec-lub r f x y*)
⟨*proof*⟩
end

4.2 The Error Type

theory *Err*
imports *Semilat*

begin

datatype $'a\ err = Err \mid OK\ 'a$

type-synonym $'a\ ebinop = 'a \Rightarrow 'a \Rightarrow 'a\ err$
type-synonym $'a\ esl = 'a\ set \times 'a\ ord \times 'a\ ebinop$

primrec $ok\text{-}val :: 'a\ err \Rightarrow 'a$
where
$ok\text{-}val\ (OK\ x) = x$

definition $lift :: ('a \Rightarrow 'b\ err) \Rightarrow ('a\ err \Rightarrow 'b\ err)$
where
$lift\ f\ e = (case\ e\ of\ Err \Rightarrow Err \mid OK\ x \Rightarrow f\ x)$

definition $lift2 :: ('a \Rightarrow 'b \Rightarrow 'c\ err) \Rightarrow 'a\ err \Rightarrow 'b\ err \Rightarrow 'c\ err$
where
$lift2\ f\ e_1\ e_2 =$
$(case\ e_1\ of\ Err \Rightarrow Err \mid OK\ x \Rightarrow (case\ e_2\ of\ Err \Rightarrow Err \mid OK\ y \Rightarrow f\ x\ y))$

definition $le :: 'a\ ord \Rightarrow 'a\ err\ ord$
where
$le\ r\ e_1\ e_2 =$
$(case\ e_2\ of\ Err \Rightarrow True \mid OK\ y \Rightarrow (case\ e_1\ of\ Err \Rightarrow False \mid OK\ x \Rightarrow x \sqsubseteq_r y))$

definition $sup :: ('a \Rightarrow 'b \Rightarrow 'c) \Rightarrow ('a\ err \Rightarrow 'b\ err \Rightarrow 'c\ err)$
where
$sup\ f = lift2\ (\lambda x\ y.\ OK\ (x \sqcup_f y))$

definition $err :: 'a\ set \Rightarrow 'a\ err\ set$
where
$err\ A = insert\ Err\ \{OK\ x|x.\ x{\in}A\}$

definition $esl :: 'a\ sl \Rightarrow 'a\ esl$
where
$esl = (\lambda(A,r,f).\ (A,\ r,\ \lambda x\ y.\ OK(f\ x\ y)))$

definition $sl :: 'a\ esl \Rightarrow 'a\ err\ sl$
where
$sl = (\lambda(A,r,f).\ (err\ A,\ le\ r,\ lift2\ f))$

abbreviation
$err\text{-}semilat :: 'a\ esl \Rightarrow bool$ **where**
$err\text{-}semilat\ L == semilat(sl\ L)$

primrec $strict :: ('a \Rightarrow 'b\ err) \Rightarrow ('a\ err \Rightarrow 'b\ err)$
where
$strict\ f\ Err = Err$
$|\ strict\ f\ (OK\ x) = f\ x$

lemma $err\text{-}def'$:
$err\ A = insert\ Err\ \{x.\ \exists\, y{\in}A.\ x = OK\ y\}\langle proof\rangle$
lemma $strict\text{-}Some$ $[simp]$:
$(strict\ f\ x = OK\ y) = (\exists\, z.\ x = OK\ z \wedge f\ z = OK\ y)\langle proof\rangle$

lemma *not-Err-eq*: $(x \neq Err) = (\exists\, a.\ x = OK\ a)\langle proof\rangle$
lemma *not-OK-eq*: $(\forall\, y.\ x \neq OK\ y) = (x = Err)\langle proof\rangle$
lemma *unfold-lesub-err*: $e1 \sqsubseteq_{le\ r}\ e2 = le\ r\ e1\ e2\langle proof\rangle$
lemma *le-err-refl*: $\forall\, x.\ x \sqsubseteq_r x \Longrightarrow e \sqsubseteq_{le\ r} e\langle proof\rangle$
lemma *le-err-refl'*: $(\forall\, x{\in}A.\ x \sqsubseteq_r x) \Longrightarrow e \in err\ A \Longrightarrow\ e \sqsubseteq_{le\ r} e\langle proof\rangle\langle proof\rangle\langle proof\rangle\langle proof\rangle\langle proof\rangle\langle proof\rangle\langle proof\rangle$

4.3 More about Options

theory *Opt* **imports** *Err* **begin**

definition $le :: {}'a\ ord \Rightarrow {}'a\ option\ ord$
where
$le\ r\ o_1\ o_2 =$
$(case\ o_2\ of\ None \Rightarrow o_1{=}None \mid Some\ y \Rightarrow (case\ o_1\ of\ None \Rightarrow True \mid Some\ x \Rightarrow x \sqsubseteq_r y))$

definition $opt :: {}'a\ set \Rightarrow {}'a\ option\ set$
where
$opt\ A = insert\ None\ \{Some\ y\ |y.\ y \in A\}$

definition $sup :: {}'a\ ebinop \Rightarrow {}'a\ option\ ebinop$
where
$sup\ f\ o_1\ o_2 =$
$(case\ o_1\ of\ None \Rightarrow OK\ o_2$
$\mid Some\ x \Rightarrow (case\ o_2\ of\ None \Rightarrow OK\ o_1$
$\mid Some\ y \Rightarrow (case\ f\ x\ y\ of\ Err \Rightarrow Err \mid OK\ z \Rightarrow OK\ (Some\ z))))$

definition $esl :: {}'a\ esl \Rightarrow {}'a\ option\ esl$
where
$esl = (\lambda(A,r,f).\ (opt\ A,\ le\ r,\ sup\ f))$

lemma *unfold-le-opt*:
$o_1 \sqsubseteq_{le\ r} o_2 =$
$(case\ o_2\ of\ None \Rightarrow o_1{=}None \mid$
$Some\ y \Rightarrow (case\ o_1\ of\ None \Rightarrow True \mid Some\ x \Rightarrow x \sqsubseteq_r y))\langle proof\rangle$
lemma *le-opt-refl*: $order\ r\ A \Longrightarrow\ x \in opt\ A \Longrightarrow x \sqsubseteq_{le\ r} x\langle proof\rangle\langle proof\rangle\langle proof\rangle\langle proof\rangle\langle proof\rangle\langle proof\rangle\langle proof\rangle\langle proof\rangle$

4.4 Products as Semilattices

theory *Product*
imports *Err*
begin

definition $le :: {}'a\ ord \Rightarrow {}'b\ ord \Rightarrow ({}'a \times {}'b)\ ord$
where
$le\ r_A\ r_B = (\lambda(a_1,b_1)\ (a_2,b_2).\ a_1 \sqsubseteq_{r_A} a_2 \wedge b_1 \sqsubseteq_{r_B} b_2)$

definition $sup :: {}'a\ ebinop \Rightarrow {}'b\ ebinop \Rightarrow ({}'a \times {}'b)\ ebinop$
where
$sup\ f\ g = (\lambda(a_1,b_1)(a_2,b_2).\ Err.sup\ Pair\ (a_1 \sqcup_f a_2)\ (b_1 \sqcup_g b_2))$

definition $esl :: {}'a\ esl \Rightarrow {}'b\ esl \Rightarrow ({}'a \times {}'b\ )\ esl$
where

esl = (λ(A,r_A,f_A) (B,r_B,f_B). (A × B, *le* r_A r_B, *sup* f_A f_B))

abbreviation
lesubprod :: 'a × 'b ⇒ ('a ⇒ 'a ⇒ bool) ⇒ ('b ⇒ 'b ⇒ bool) ⇒ 'a × 'b ⇒ bool
(‹(- /⊑'(-,-') -)› [50, 0, 0, 51] 50) **where**
p ⊑(*rA*,*rB*) *q* == *p* $\sqsubseteq_{Product.le\ rA\ rB}$ *q*

lemma *unfold-lesub-prod*: $x \sqsubseteq(r_A,r_B)\ y$ = *le* r_A r_B *x y*⟨*proof*⟩
lemma *le-prod-Pair-conv* [*iff*]: $((a_1,b_1) \sqsubseteq(r_A,r_B)\ (a_2,b_2)) = (a_1 \sqsubseteq_{r_A} a_2\ \&\ b_1 \sqsubseteq_{r_B} b_2)$⟨*proof*⟩
lemma *less-prod-Pair-conv*:
$((a_1,b_1) \sqsubset_{Product.le\ r_A\ r_B} (a_2,b_2)) =$
$(a_1 \sqsubset_{r_A} a_2\ \&\ b_1 \sqsubseteq_{r_B} b_2 \mid a_1 \sqsubseteq_{r_A} a_2\ \&\ b_1 \sqsubset_{r_B} b_2)$⟨*proof*⟩
lemma *order-le-prodI* [*iff*]: (*order* r_A A & *order* r_B B) ⟹ *order* (*Product.le* r_A r_B) (A × B)
⟨*proof*⟩

lemma *order-le-prodE*: $A \neq \{\}$ ⟹ $B \neq \{\}$ ⟹ *order* (*Product.le* r_A r_B) (A × B) ⟹ (*order* r_A A & *order* r_B B)
⟨*proof*⟩

lemma *order-le-prod* [*iff*]: $A \neq \{\}$ ⟹ $B \neq \{\}$ ⟹ *order*(*Product.le* r_A r_B) (A × B) = (*order* r_A A & *order* r_B B)⟨*proof*⟩
lemma *acc-le-prodI* [*intro*!]:
⟦ *acc* r_A; *acc* r_B ⟧ ⟹ *acc*(*Product.le* r_A r_B)⟨*proof*⟩

lemma *closed-lift2-sup*:
⟦ *closed* (*err A*) (*lift2 f*); *closed* (*err B*) (*lift2 g*) ⟧ ⟹
closed (*err*(*A*×*B*)) (*lift2*(*sup f g*))⟨*proof*⟩
lemma *unfold-plussub-lift2*: $e_1 \sqcup_{lift2\ f} e_2$ = *lift2 f* e_1 e_2⟨*proof*⟩

lemma *plus-eq-Err-conv* [*simp*]:
assumes *x*∈*A* *y*∈*A* *semilat*(*err A*, *Err.le r*, *lift2 f*)
shows $(x \sqcup_f y = Err) = (\neg(\exists z \in A.\ x \sqsubseteq_r z \wedge y \sqsubseteq_r z))$⟨*proof*⟩
lemma *err-semilat-Product-esl*:
$\bigwedge L_1\ L_2$. ⟦ *err-semilat* L_1; *err-semilat* L_2 ⟧ ⟹ *err-semilat*(*Product.esl* L_1 L_2)⟨*proof*⟩
end

4.5 Fixed Length Lists

theory *Listn*
imports *Err HOL−Library.NList*
begin

definition *le* :: 'a ord ⇒ ('a list)ord
where
le r = *list-all2* (λ*x y*. $x \sqsubseteq_r y$)

abbreviation
lesublist :: 'a list ⇒ 'a ord ⇒ 'a list ⇒ bool (‹(- /[⊑-] -)› [50, 0, 51] 50) **where**
x [$\sqsubseteq_r$] *y* == *x* <=-(*Listn.le r*) *y*

abbreviation
lesssublist :: 'a list ⇒ 'a ord ⇒ 'a list ⇒ bool (‹(- /[⊏-] -)› [50, 0, 51] 50) **where**
x [$\sqsubset_r$] *y* == *x* <-(*Listn.le r*) *y*

abbreviation
plussublist :: $'a$ *list* ⇒ ($'a$ ⇒ $'b$ ⇒ $'c$) ⇒ $'b$ *list* ⇒ $'c$ *list*
(‹(- /[⊔-] -)› [*65*, *0*, *66*] *65*) **where**
x [$\sqcup_f$] y == x $\sqcup_{map2\ f}$ y

primrec *coalesce* :: $'a$ *err list* ⇒ $'a$ *list err*
where
coalesce [] = *OK*[]
| *coalesce* (*ex*#*exs*) = *Err.sup* (#) *ex* (*coalesce exs*)

definition *sl* :: *nat* ⇒ $'a$ *sl* ⇒ $'a$ *list sl*
where
sl n = (λ(*A*,*r*,*f*). (*nlists n A*, *le r*, *map2 f*))

definition *sup* :: ($'a$ ⇒ $'b$ ⇒ $'c$ *err*) ⇒ $'a$ *list* ⇒ $'b$ *list* ⇒ $'c$ *list err*
where
sup f = (λ*xs ys*. *if size xs* = *size ys then coalesce*(*xs* [$\sqcup_f$] *ys*) *else Err*)

definition *upto-esl* :: *nat* ⇒ $'a$ *esl* ⇒ $'a$ *list esl*
where
upto-esl m = (λ(*A*,*r*,*f*). (*Union*{*nlists n A* |*n*. *n* ≤ *m*}, *le r*, *sup f*))

lemmas [*simp*] = *set-update-subsetI*

lemma *unfold-lesub-list*: *xs* [$\sqsubseteq_r$] *ys* = *Listn.le r xs ys*⟨*proof*⟩
lemma *Nil-le-conv* [*iff*]: ([] [$\sqsubseteq_r$] *ys*) = (*ys* = [])⟨*proof*⟩
lemma *Cons-notle-Nil* [*iff*]: ¬ *x*#*xs* [$\sqsubseteq_r$] []⟨*proof*⟩
lemma *Cons-le-Cons* [*iff*]: *x*#*xs* [$\sqsubseteq_r$] *y*#*ys* = (*x* $\sqsubseteq_r$ *y* ∧ *xs* [$\sqsubseteq_r$] *ys*)⟨*proof*⟩
lemma *list-update-le-cong*:
⟦ *i*<*size xs*; *xs* [$\sqsubseteq_r$] *ys*; *x* $\sqsubseteq_r$ *y* ⟧ ⟹ *xs*[*i*:=*x*] [$\sqsubseteq_r$] *ys*[*i*:=*y*]⟨*proof*⟩

lemma *le-listD*: ⟦ *xs* [$\sqsubseteq_r$] *ys*; *p* < *size xs* ⟧ ⟹ *xs*!*p* $\sqsubseteq_r$ *ys*!*p*⟨*proof*⟩
lemma *le-list-refl*: ∀ *x*. *x* $\sqsubseteq_r$ *x* ⟹ *xs* [$\sqsubseteq_r$] *xs*⟨*proof*⟩

lemma *le-list-trans*:
assumes *ord*: *order r A*
and *xs*: *xs* ∈ *nlists n A* **and** *ys*: *ys* ∈ *nlists n A* **and** *zs*: *zs* ∈ *nlists n A*
and *xs* [$\sqsubseteq_r$] *ys* **and** *ys* [$\sqsubseteq_r$] *zs*
shows *xs* [$\sqsubseteq_r$] *zs*⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proo*

4.6 Typing and Dataflow Analysis Framework

theory *Typing-Framework-1* **imports** *Semilattices* **begin**

The relationship between dataflow analysis and a welltyped-instruction predicate.

type-synonym
$'s$ *step-type* = *nat* ⇒ $'s$ ⇒ (*nat* × $'s$) *list*

definition *stable* :: $'s$ *ord* ⇒ $'s$ *step-type* ⇒ $'s$ *list* ⇒ *nat* ⇒ *bool*
where
stable r step τs p ⟷ (∀ (*q*,*τ*) ∈ *set* (*step p* (*τs*!*p*)). *τ* $\sqsubseteq_r$ *τs*!*q*)

definition *stables* :: *'s ord* ⇒ *'s step-type* ⇒ *'s list* ⇒ *bool*
where
stables r step τs ⟷ (∀ *p* < *size τs*. *stable r step τs p*)

definition *wt-step* :: *'s ord* ⇒ *'s* ⇒ *'s step-type* ⇒ *'s list* ⇒ *bool*
where
wt-step r T step τs ⟷ (∀ *p*<*size τs*. *τs*!*p* ≠ *T* ∧ *stable r step τs p*)

end

4.7 More on Semilattices

theory *SemilatAlg*
imports *Typing-Framework-1*
begin

definition *lesubstep-type* :: (*nat* × *'s*) *set* ⇒ *'s ord* ⇒ (*nat* × *'s*) *set* ⇒ *bool*
(‹(- /{⊑_} -)› [*50*, *0*, *51*] *50*)
where *A* {⊑$_r$} *B* ≡ ∀ (*p*,*τ*) ∈ *A*. ∃ *τ'*. (*p*,*τ'*) ∈ *B* ∧ *τ* ⊑$_r$ *τ'*

notation (*ASCII*)
lesubstep-type (‹(- /{<='--} -)› [*50*, *0*, *51*] *50*)

primrec *pluslussub* :: *'a list* ⇒ (*'a* ⇒ *'a* ⇒ *'a*) ⇒ *'a* ⇒ *'a* (‹(- /⨆_ -)› [*65*, *0*, *66*] *65*)
where
pluslussub [] *f y* = *y*
| *pluslussub* (*x*#*xs*) *f y* = *pluslussub xs f* (*x* ⊔$_f$ *y*)
definition *bounded* :: *'s step-type* ⇒ *nat* ⇒ *bool*
where
bounded step n ⟷ (∀ *p*<*n*. ∀ *τ*. ∀ (*q*,*τ'*) ∈ *set* (*step p τ*). *q*<*n*)

definition *pres-type* :: *'s step-type* ⇒ *nat* ⇒ *'s set* ⇒ *bool*
where
pres-type step n A ⟷ (∀ *τ*∈*A*. ∀ *p*<*n*. ∀ (*q*,*τ'*) ∈ *set* (*step p τ*). *τ'* ∈ *A*)

definition *mono* :: *'s ord* ⇒ *'s step-type* ⇒ *nat* ⇒ *'s set* ⇒ *bool*
where
mono r step n A ⟷
(∀ *τ p τ'*. *τ* ∈ *A* ∧ *p*<*n* ∧ *τ* ⊑$_r$ *τ'* ⟶ *set* (*step p τ*) {⊑$_r$} *set* (*step p τ'*))

lemma [*iff*]: {} {⊑$_r$} *B*
⟨*proof*⟩
lemma [*iff*]: (*A* {⊑$_r$} {}) = (*A* = {})
⟨*proof*⟩
lemma *lesubstep-union*:
⟦ A_1 {⊑$_r$} B_1; A_2 {⊑$_r$} B_2 ⟧ ⟹ A_1 ∪ A_2 {⊑$_r$} B_1 ∪ B_2
⟨*proof*⟩
lemma *pres-typeD*:
⟦ *pres-type step n A*; *s*∈*A*; *p*<*n*; (*q*,*s'*)∈*set* (*step p s*) ⟧ ⟹ *s'* ∈ *A*⟨*proof*⟩
lemma *monoD*:
⟦ *mono r step n A*; *p* < *n*; *s*∈*A*; *s* ⊑$_r$ *t* ⟧ ⟹ *set* (*step p s*) {⊑$_r$} *set* (*step p t*)⟨*proof*⟩
lemma *boundedD*:

$\llbracket$ *bounded step n*; $p < n$; $(q,t) \in$ *set* (*step p xs*) $\rrbracket \Longrightarrow q < n\langle proof\rangle$
lemma *lesubstep-type-refl* [*simp*, *intro*]:
$(\bigwedge x.\ x \sqsubseteq_r x) \Longrightarrow A\ \{\sqsubseteq_r\}\ A\langle proof\rangle$
lemma *lesub-step-typeD*:
$A\ \{\sqsubseteq_r\}\ B \Longrightarrow (x,y) \in A \Longrightarrow \exists y'.\ (x,\ y') \in B \wedge y \sqsubseteq_r y'\langle proof\rangle$

lemma *list-update-le-listI* [*rule-format*]:
set xs $\subseteq A \longrightarrow$ *set ys* $\subseteq A \longrightarrow xs\ [\sqsubseteq_r]\ ys \longrightarrow p <$ *size xs* $\longrightarrow$
$x \sqsubseteq_r ys!p \longrightarrow$ *semilat*$(A,r,f) \longrightarrow x{\in}A \longrightarrow$
$xs[p := x \sqcup_f xs!p]\ [\sqsubseteq_r]\ ys\langle proof\rangle$
lemma *plusplus-closed*: **assumes** *Semilat A r f* **shows**
$\bigwedge y.\ \llbracket$ *set* $x \subseteq A$; $y \in A\rrbracket \Longrightarrow x \bigsqcup_f y \in A\langle proof\rangle$

lemma (**in** *Semilat*) *pp-ub2*:
$\bigwedge y.\ \llbracket$ *set* $x \subseteq A$; $y \in A\rrbracket \Longrightarrow y \sqsubseteq_r x \bigsqcup_f y\langle proof\rangle$

lemma (**in** *Semilat*) *pp-ub1*:
shows $\bigwedge y.\ \llbracket$*set ls* $\subseteq A$; $y \in A$; $x \in$ *set ls*$\rrbracket \Longrightarrow x \sqsubseteq_r ls \bigsqcup_f y\langle proof\rangle$

lemma (**in** *Semilat*) *pp-lub*:
assumes z: $z \in A$
shows
$\bigwedge y.\ y \in A \Longrightarrow$ *set xs* $\subseteq A \Longrightarrow \forall x \in$ *set xs*. $x \sqsubseteq_r z \Longrightarrow y \sqsubseteq_r z \Longrightarrow xs \bigsqcup_f y \sqsubseteq_r z\langle proof\rangle$

lemma *ub1'*: **assumes** *Semilat A r f*
shows $\llbracket\forall (p,s) \in$ *set S*. $s \in A$; $y \in A$; $(a,b) \in$ *set S*$\rrbracket$
$\Longrightarrow b \sqsubseteq_r$ *map snd* $[(p',\ t') \leftarrow S.\ p' = a] \bigsqcup_f y\langle proof\rangle$

lemma *plusplus-empty*:
$\forall s'.\ (q,\ s') \in$ *set S* $\longrightarrow s' \sqcup_f ss\ !\ q = ss\ !\ q \Longrightarrow$
(*map snd* $[(p',\ t') \leftarrow S.\ p' = q] \bigsqcup_f ss\ !\ q) = ss\ !\ q\langle proof\rangle$

end

4.8 Lifting the Typing Framework to err, app, and eff

theory *Typing-Framework-err* **imports** *SemilatAlg* **begin**

definition *wt-err-step* :: *'s ord* $\Rightarrow$ *'s err step-type* $\Rightarrow$ *'s err list* $\Rightarrow$ *bool*
where
wt-err-step r step $\tau s \longleftrightarrow$ *wt-step* (*Err.le r*) *Err step* τs

definition *wt-app-eff* :: *'s ord* $\Rightarrow$ (*nat* $\Rightarrow$ *'s* $\Rightarrow$ *bool*) $\Rightarrow$ *'s step-type* $\Rightarrow$ *'s list* $\Rightarrow$ *bool*
where
wt-app-eff r app step $\tau s \longleftrightarrow$
$(\forall p <$ *size* τs. *app p* $(\tau s!p) \wedge (\forall (q,\tau) \in$ *set* (*step p* $(\tau s!p)$). τ <=-r $\tau s!q))$

definition *map-snd* :: $('b \Rightarrow 'c) \Rightarrow ('a \times 'b)$ *list* $\Rightarrow ('a \times 'c)$ *list*
where
map-snd f = *map* $(\lambda(x,y).\ (x,\ f\ y))$

definition *error* :: *nat* $\Rightarrow$ (*nat* $\times$ *'a err*) *list*
where

error n = *map* (λ*x*. (*x*,*Err*)) [*0*..<*n*]

definition *err-step* :: *nat* ⇒ (*nat* ⇒ ′*s* ⇒ *bool*) ⇒ ′*s step-type* ⇒ ′*s err step-type*
where
err-step n app step p t =
(*case t of*
Err ⇒ *error n*
| *OK* τ ⇒ *if app p* τ *then map-snd OK* (*step p* τ) *else error n*)

definition *app-mono* :: ′*s ord* ⇒ (*nat* ⇒ ′*s* ⇒ *bool*) ⇒ *nat* ⇒ ′*s set* ⇒ *bool*
where
app-mono r app n A ⟷
(∀ *s p t*. *s* ∈ *A* ∧ *p* < *n* ∧ *s* ⊑$_r$ *t* ⟶ *app p t* ⟶ *app p s*)

lemmas *err-step-defs* = *err-step-def map-snd-def error-def*

lemma *bounded-err-stepD*:
⟦ *bounded* (*err-step n app step*) *n*;
p < *n*; *app p a*; (*q*,*b*) ∈ *set* (*step p a*) ⟧ ⟹ *q* < *n*⟨*proof*⟩

lemma *in-map-sndD*: (*a*,*b*) ∈ *set* (*map-snd f xs*) ⟹ ∃ *b*′. (*a*,*b*′) ∈ *set xs*⟨*proof*⟩

lemma *bounded-err-stepI*:
∀ *p*. *p* < *n* ⟶ (∀ *s*. *ap p s* ⟶ (∀ (*q*,*s*′) ∈ *set* (*step p s*). *q* < *n*))
⟹ *bounded* (*err-step n ap step*) *n*⟨*proof*⟩

lemma *bounded-lift*:
bounded step n ⟹ *bounded* (*err-step n app step*) *n*⟨*proof*⟩

lemma *le-list-map-OK* [*simp*]:
⋀*b*. (*map OK a* [⊑$_{Err.le\ r}$] *map OK b*) = (*a* [⊑$_r$] *b*)⟨*proof*⟩

lemma *map-snd-lessI*:
set xs {⊑$_r$} *set ys* ⟹ *set* (*map-snd OK xs*) {⊑$_{Err.le\ r}$} *set* (*map-snd OK ys*)⟨*proof*⟩

lemma *mono-lift*:
⟦ *order r A*; *app-mono r app n A*; *bounded* (*err-step n app step*) *n*;
∀ *s p t*. *s* ∈ *A* ∧ *p* < *n* ∧ *s* ⊑$_r$ *t* ⟶ *app p t* ⟶ *set* (*step p s*) {⊑$_r$} *set* (*step p t*) ⟧
⟹ *mono* (*Err.le r*) (*err-step n app step*) *n* (*err A*)⟨*proof*⟩
lemma *in-errorD*: (*x*,*y*) ∈ *set* (*error n*) ⟹ *y* = *Err*⟨*proof*⟩
lemma *pres-type-lift*:
∀ *s*∈*A*. ∀ *p*. *p* < *n* ⟶ *app p s* ⟶ (∀ (*q*, *s*′)∈*set* (*step p s*). *s*′ ∈ *A*)
⟹ *pres-type* (*err-step n app step*) *n* (*err A*)⟨*proof*⟩

lemma *wt-err-imp-wt-app-eff*:
assumes *wt*: *wt-err-step r* (*err-step* (*size ts*) *app step*) *ts*
assumes *b*: *bounded* (*err-step* (*size ts*) *app step*) (*size ts*)
shows *wt-app-eff r app step* (*map ok-val ts*)⟨*proof*⟩

lemma *wt-app-eff-imp-wt-err*:
assumes *app-eff*: *wt-app-eff r app step ts*
assumes *bounded*: *bounded* (*err-step* (*size ts*) *app step*) (*size ts*)

shows *wt-err-step r* (*err-step* (*size ts*) *app step*) (*map OK ts*)⟨*proof*⟩
end

4.9 Kildall's Algorithm

theory *Kildall-1*
imports *SemilatAlg*
begin

primrec *merges* :: *'s binop* ⇒ (*nat* × *'s*) *list* ⇒ *'s list* ⇒ *'s list*
where
merges f [] $\tau s = \tau s$
| *merges f* (*p'*#*ps*) τs = (*let* (*p*,τ) = *p'* *in merges f ps* (τs[*p* := $\tau \sqcup_f \tau s$!*p*]))

lemmas [*simp*] = *Let-def Semilat.le-iff-plus-unchanged* [*OF Semilat.intro*, *symmetric*]

lemma (**in** *Semilat*) *nth-merges*:
⋀*ss*. ⟦*p* < *length ss*; *ss* ∈ *nlists n A*; ∀ (*p*,*t*)∈*set ps*. *p*<*n* ∧ *t*∈*A* ⟧ ⟹
(*merges f ps ss*)!*p* = *map snd* [(*p'*,*t'*) ← *ps*. *p'*=*p*] $\bigsqcup_f$ *ss*!*p*
(**is** ⋀*ss*. ⟦-; -; *?steptype ps*⟧ ⟹ *?P ss ps*)⟨*proof*⟩

lemma *length-merges* [*simp*]:
⋀*ss*. *size*(*merges f ps ss*) = *size ss*⟨*proof*⟩
lemma (**in** *Semilat*) *merges-preserves-type-lemma*:
shows ∀ *xs*. *xs* ∈ *nlists n A* ⟶ (∀ (*p*,*x*) ∈ *set ps*. *p*<*n* ∧ *x*∈*A*)
⟶ *merges f ps xs* ∈ *nlists n A*⟨*proof*⟩
lemma (**in** *Semilat*) *merges-preserves-type* [*simp*]:
⟦ *xs* ∈ *nlists n A*; ∀ (*p*,*x*) ∈ *set ps*. *p*<*n* ∧ *x*∈*A* ⟧
⟹ *merges f ps xs* ∈ *nlists n A*
⟨*proof*⟩

lemma (**in** *Semilat*) *list-update-le-listI* [*rule-format*]:
set xs ⊆ *A* ⟶ *set ys* ⊆ *A* ⟶ *xs* [$\sqsubseteq_r$] *ys* ⟶ *p* < *size xs* ⟶
x $\sqsubseteq_r$ *ys*!*p* ⟶ *x*∈*A* ⟶ *xs*[*p* := *x* $\sqcup_f$ *xs*!*p*] [$\sqsubseteq_r$] *ys*⟨*proof*⟩
lemma (**in** *Semilat*) *merges-pres-le-ub*:
assumes *set ts* ⊆ *A* *set ss* ⊆ *A*
∀ (*p*,*t*)∈*set ps*. *t* $\sqsubseteq_r$ *ts*!*p* ∧ *t* ∈ *A* ∧ *p* < *size ts* *ss* [$\sqsubseteq_r$] *ts*
shows *merges f ps ss* [$\sqsubseteq_r$] *ts*⟨*proof*⟩

end

4.10 Kildall's Algorithm

theory *Kildall-2*
imports *SemilatAlg Kildall-1*
begin

primrec *propa* :: $'s$ *binop* ⇒ (*nat* × $'s$) *list* ⇒ $'s$ *list* ⇒ *nat set* ⇒ $'s$ *list* ∗ *nat set*
where
  propa f [] τ*s w* = (τ*s*,*w*)
| *propa f* (q'#*qs*) τ*s w* = (*let* (*q*,τ) = q';
      $u = \tau \sqcup_f \tau s!q$;
      w' = (*if* $u = \tau s!q$ *then w else insert q w*)
    in propa f qs (τ*s*[*q* := *u*]) w')

definition *iter* :: $'s$ *binop* ⇒ $'s$ *step-type* ⇒
  $'s$ *list* ⇒ *nat set* ⇒ $'s$ *list* × *nat set*
where
  iter f step τ*s w* =
   while (λ(τ*s*,*w*). *w* ≠ {})
    (λ(τ*s*,*w*). *let p* = *some-elem w*
      in propa f (*step p* (τ*s*!*p*)) τ*s* (*w*−{*p*}))
    (τ*s*,*w*)

definition *unstables* :: $'s$ *ord* ⇒ $'s$ *step-type* ⇒ $'s$ *list* ⇒ *nat set*
where
  unstables r step τ*s* = {*p*. *p* < *size* τ*s* ∧ ¬*stable r step* τ*s p*}

definition *kildall* :: $'s$ *ord* ⇒ $'s$ *binop* ⇒ $'s$ *step-type* ⇒ $'s$ *list* ⇒ $'s$ *list*
where
  kildall r f step τ*s* = *fst*(*iter f step* τ*s* (*unstables r step* τ*s*))

lemma (**in** *Semilat*) *merges-incr-lemma*:
  ∀ *xs*. *xs* ∈ *nlists n A* ⟶ (∀ (*p*,*x*)∈*set ps*. *p*<*size xs* ∧ *x* ∈ *A*) ⟶ *xs* $[\sqsubseteq_r]$ *merges f ps xs*
  ⟨*proof*⟩

lemma (**in** *Semilat*) *merges-incr*:
 ⟦ *xs* ∈ *nlists n A*; ∀ (*p*,*x*)∈*set ps*. *p*<*size xs* ∧ *x* ∈ *A* ⟧
  ⟹ *xs* $[\sqsubseteq_r]$ *merges f ps xs*
  ⟨*proof*⟩

lemma (**in** *Semilat*) *merges-same-conv* [*rule-format*]:
 (∀ *xs*. *xs* ∈ *nlists n A* ⟶ (∀ (*p*,*x*)∈*set ps*. *p*<*size xs* ∧ *x*∈*A*) ⟶
   (*merges f ps xs* = *xs*) = (∀ (*p*,*x*)∈*set ps*. $x \sqsubseteq_r xs!p$))⟨*proof*⟩
lemma *decomp-propa*:
  ⋀*ss w*. (∀ (*q*,*t*)∈*set qs*. *q* < *size ss*) ⟹
   propa f qs ss w =
   (*merges f qs ss*, {*q*. ∃ *t*.(*q*,*t*)∈*set qs* ∧ $t \sqcup_f ss!q \neq ss!q$} ∪ *w*)⟨*proof*⟩
lemma (**in** *Semilat*) *stable-pres-lemma*:
shows ⟦*pres-type step n A*; *bounded step n*;
   ss ∈ *nlists n A*; *p* ∈ *w*; ∀ *q*∈*w*. *q* < *n*;
   ∀ *q*. *q* < *n* ⟶ *q* ∉ *w* ⟶ *stable r step ss q*; *q* < *n*;
   ∀ s'. (*q*,s') ∈ *set* (*step p* (*ss*!*p*)) ⟶ $s' \sqcup_f ss!q = ss!q$;
   q ∉ *w* ∨ *q* = *p* ⟧

$\Longrightarrow$ *stable r step (merges f (step p (ss!p)) ss) q*⟨*proof*⟩

lemma (**in** *Semilat*) *merges-bounded-lemma*:
⟦ *mono r step n A*; *bounded step n*; *pres-type step n A*;
$\forall (p',s') \in$ *set* (*step p* (*ss!p*)). $s' \in A$; $ss \in$ *nlists n A*; $ts \in$ *nlists n A*; $p < n$;
ss $[\sqsubseteq_r]$ *ts*; $\forall p.\ p < n \longrightarrow$ *stable r step ts p* ⟧
$\Longrightarrow$ *merges f* (*step p* (*ss!p*)) *ss* $[\sqsubseteq_r]$ *ts*⟨*proof*⟩

lemma *termination-lemma*: **assumes** *Semilat A r f*
shows ⟦ $ss \in$ *nlists n A*; $\forall (q,t) \in$ *set qs*. $q<n \wedge t \in A$; $p \in w$ ⟧ $\Longrightarrow$
ss $[\sqsubset_r]$ *merges f qs ss* $\vee$
merges f qs ss $=$ *ss* $\wedge$ $\{q.\ \exists t.\ (q,t) \in$ *set qs* $\wedge\ t \sqcup_f ss!q \neq ss!q\} \cup (w - \{p\}) \subset w$⟨*proof*⟩
end

4.11 The Lightweight Bytecode Verifier

theory *LBVSpec*
imports *SemilatAlg Opt*
begin

type-synonym
's certificate = *'s list*

primrec *merge* :: *'s certificate* ⇒ *'s binop* ⇒ *'s ord* ⇒ *'s* ⇒ *nat* ⇒ (*nat* × *'s*) *list* ⇒ *'s* ⇒ *'s*
where
merge cert f r T pc [] *x* = *x*
| *merge cert f r T pc* (*s#ss*) *x* = *merge cert f r T pc ss* (*let* (*pc'*,*s'*) = *s in*
if pc'=pc+1 then $s' \sqcup_f x$
else if $s' \sqsubseteq_r$ *cert!pc' then x*
else T)

definition *wtl-inst* :: *'s certificate* ⇒ *'s binop* ⇒ *'s ord* ⇒ *'s* ⇒
's step-type ⇒ *nat* ⇒ *'s* ⇒ *'s*
where
wtl-inst cert f r T step pc s = *merge cert f r T pc* (*step pc s*) (*cert!*(*pc+1*))

definition *wtl-cert* :: *'s certificate* ⇒ *'s binop* ⇒ *'s ord* ⇒ *'s* ⇒ *'s* ⇒
's step-type ⇒ *nat* ⇒ *'s* ⇒ *'s*
where
wtl-cert cert f r T B step pc s =
(*if cert!pc* = *B then*
wtl-inst cert f r T step pc s
else
if $s \sqsubseteq_r$ *cert!pc then wtl-inst cert f r T step pc* (*cert!pc*) *else T*)

primrec *wtl-inst-list* :: *'a list* ⇒ *'s certificate* ⇒ *'s binop* ⇒ *'s ord* ⇒ *'s* ⇒ *'s* ⇒
's step-type ⇒ *nat* ⇒ *'s* ⇒ *'s*
where
wtl-inst-list [] *cert f r T B step pc s* = *s*
| *wtl-inst-list* (*i#is*) *cert f r T B step pc s* =
(*let s'* = *wtl-cert cert f r T B step pc s in*

if $s' = T \vee s = T$ *then* T *else* *wtl-inst-list is cert f r T B step* $(pc+1)$ s')

definition *cert-ok* :: *'s certificate* ⇒ *nat* ⇒ *'s* ⇒ *'s* ⇒ *'s set* ⇒ *bool*
where
cert-ok cert n T B A ⟷ $(\forall\, i < n.\ cert!i \in A \wedge cert!i \neq T) \wedge (cert!n = B)$

definition *bottom* :: *'a ord* ⇒ *'a* ⇒ *bool*
where
bottom r B ⟷ $(\forall\, x.\ B \sqsubseteq_r x)$

locale *lbv* = *Semilat* +
fixes *T* :: *'a* (‹⊤›)
fixes *B* :: *'a* (‹⊥›)
fixes *step* :: *'a step-type*
assumes *top*: *top r* ⊤
assumes *T-A*: ⊤ ∈ *A*
assumes *bot*: *bottom r* ⊥
assumes *B-A*: ⊥ ∈ *A*

fixes *merge* :: *'a certificate* ⇒ *nat* ⇒ (*nat* × *'a*) *list* ⇒ *'a* ⇒ *'a*
defines *mrg-def*: *merge cert* ≡ *LBVSpec.merge cert f r* ⊤

fixes *wti* :: *'a certificate* ⇒ *nat* ⇒ *'a* ⇒ *'a*
defines *wti-def*: *wti cert* ≡ *wtl-inst cert f r* ⊤ *step*

fixes *wtc* :: *'a certificate* ⇒ *nat* ⇒ *'a* ⇒ *'a*
defines *wtc-def*: *wtc cert* ≡ *wtl-cert cert f r* ⊤ ⊥ *step*

fixes *wtl* :: *'b list* ⇒ *'a certificate* ⇒ *nat* ⇒ *'a* ⇒ *'a*
defines *wtl-def*: *wtl ins cert* ≡ *wtl-inst-list ins cert f r* ⊤ ⊥ *step*

lemma (**in** *lbv*) *wti*:
wti c pc s = *merge c pc* (*step pc s*) ($c!(pc+1)$)
⟨*proof*⟩
lemma (**in** *lbv*) *wtc*:
wtc c pc s = (*if* $c!pc = \bot$ *then* *wti c pc s* *else if* $s \sqsubseteq_r c!pc$ *then* *wti c pc* ($c!pc$) *else* ⊤)
⟨*proof*⟩
lemma *cert-okD1* [*intro?*]:
cert-ok c n T B A ⟹ $pc < n$ ⟹ $c!pc \in A$
⟨*proof*⟩
lemma *cert-okD2* [*intro?*]:
cert-ok c n T B A ⟹ $c!n = B$
⟨*proof*⟩
lemma *cert-okD3* [*intro?*]:
cert-ok c n T B A ⟹ $B \in A$ ⟹ $pc < n$ ⟹ $c!Suc\ pc \in A$
⟨*proof*⟩
lemma *cert-okD4* [*intro?*]:
cert-ok c n T B A ⟹ $pc < n$ ⟹ $c!pc \neq T$
⟨*proof*⟩
declare *Let-def* [*simp*]

4.11.1 more semilattice lemmas

lemma (**in** *lbv*) *sup-top* [*simp*, *elim*]:
assumes *x*: $x \in A$
shows $x \sqcup_f \top = \top$⟨*proof*⟩
lemma (**in** *lbv*) *plusplussup-top* [*simp*, *elim*]:
$set\ xs \subseteq A \Longrightarrow xs \bigsqcup_f \top = \top$
⟨*proof*⟩

lemma (**in** *Semilat*) *pp-ub1′*:
assumes *S*: $snd`set\ S \subseteq A$
assumes *y*: $y \in A$ **and** *ab*: $(a,\ b) \in set\ S$
shows $b \sqsubseteq_r map\ snd\ [(p',\ t') \leftarrow S\ .\ p' = a] \bigsqcup_f y$⟨*proof*⟩
lemma (**in** *lbv*) *bottom-le* [*simp*, *intro*!]: $\bot \sqsubseteq_r x$
⟨*proof*⟩

lemma (**in** *lbv*) *le-bottom* [*simp*]: $x \in A \Longrightarrow x \sqsubseteq_r \bot = (x = \bot)$
⟨*proof*⟩

4.11.2 merge

lemma (**in** *lbv*) *merge-Nil* [*simp*]:
$merge\ c\ pc\ []\ x = x$ ⟨*proof*⟩

lemma (**in** *lbv*) *merge-Cons* [*simp*]:
$merge\ c\ pc\ (l\#ls)\ x = merge\ c\ pc\ ls\ (if\ fst\ l{=}pc{+}1\ then\ snd\ l\ {+}{-}f\ x$
$else\ if\ snd\ l \sqsubseteq_r c!fst\ l\ then\ x$
$else\ \top)$
⟨*proof*⟩

lemma (**in** *lbv*) *merge-Err* [*simp*]:
$snd`set\ ss \subseteq A \Longrightarrow merge\ c\ pc\ ss\ \top = \top$
⟨*proof*⟩

lemma (**in** *lbv*) *merge-not-top*:
$\bigwedge x.\ snd`set\ ss \subseteq A \Longrightarrow merge\ c\ pc\ ss\ x \neq \top \Longrightarrow$
$\forall (pc',s') \in set\ ss.\ (pc' \neq pc{+}1 \longrightarrow s' \sqsubseteq_r c!pc')$
(**is** $\bigwedge x.\ ?set\ ss \Longrightarrow ?merge\ ss\ x \Longrightarrow ?P\ ss$)⟨*proof*⟩

lemma (**in** *lbv*) *merge-def*:
shows
$\bigwedge x.\ x \in A \Longrightarrow snd`set\ ss \subseteq A \Longrightarrow$
$merge\ c\ pc\ ss\ x =$
$(if\ \forall (pc',s') \in set\ ss.\ pc' {\neq} pc{+}1 \longrightarrow s' \sqsubseteq_r c!pc'\ then$
$map\ snd\ [(p',t') \leftarrow ss.\ p'{=}pc{+}1] \bigsqcup_f x$
$else\ \top)$
(**is** $\bigwedge x.\ \text{-} \Longrightarrow \text{-} \Longrightarrow ?merge\ ss\ x = ?if\ ss\ x$ **is** $\bigwedge x.\ \text{-} \Longrightarrow \text{-} \Longrightarrow ?P\ ss\ x$)⟨*proof*⟩
lemma (**in** *lbv*) *merge-not-top-s*:
assumes *x*: $x \in A$ **and** *ss*: $snd`set\ ss \subseteq A$
assumes *m*: $merge\ c\ pc\ ss\ x \neq \top$
shows $merge\ c\ pc\ ss\ x = (map\ snd\ [(p',t') \leftarrow ss.\ p'{=}pc{+}1] \bigsqcup_f x)$⟨*proof*⟩

4.11.3 wtl-inst-list

lemmas [*iff*] = *not-Err-eq*

lemma (**in** *lbv*) *wtl-Nil* [*simp*]: *wtl* [] *c pc s* = *s*
⟨*proof*⟩

lemma (**in** *lbv*) *wtl-Cons* [*simp*]:
wtl (*i*#*is*) *c pc s* =
(*let s′* = *wtc c pc s in if s′* = ⊤ ∨ *s* = ⊤ *then* ⊤ *else wtl is c* (*pc*+*1*) *s′*)
⟨*proof*⟩

lemma (**in** *lbv*) *wtl-Cons-not-top*:
wtl (*i*#*is*) *c pc s* ≠ ⊤ =
(*wtc c pc s* ≠ ⊤ ∧ *s* ≠ *T* ∧ *wtl is c* (*pc*+*1*) (*wtc c pc s*) ≠ ⊤)
⟨*proof*⟩

lemma (**in** *lbv*) *wtl-top* [*simp*]: *wtl ls c pc* ⊤ = ⊤
⟨*proof*⟩

lemma (**in** *lbv*) *wtl-not-top*:
wtl ls c pc s ≠ ⊤ ⟹ *s* ≠ ⊤
⟨*proof*⟩

lemma (**in** *lbv*) *wtl-append* [*simp*]:
⋀*pc s*. *wtl* (*a*@*b*) *c pc s* = *wtl b c* (*pc*+*length a*) (*wtl a c pc s*)
⟨*proof*⟩

lemma (**in** *lbv*) *wtl-take*:
wtl is c pc s ≠ ⊤ ⟹ *wtl* (*take pc′ is*) *c pc s* ≠ ⊤
(**is** *?wtl is* ≠ - ⟹ -)⟨*proof*⟩
lemma *take-Suc*:
∀ *n*. *n* < *length l* ⟶ *take* (*Suc n*) *l* = (*take n l*)@[*l*!*n*] (**is** *?P l*)⟨*proof*⟩
lemma (**in** *lbv*) *wtl-Suc*:
assumes *suc*: *pc*+*1* < *length is*
assumes *wtl*: *wtl* (*take pc is*) *c 0 s* ≠ ⊤
shows *wtl* (*take* (*pc*+*1*) *is*) *c 0 s* = *wtc c pc* (*wtl* (*take pc is*) *c 0 s*)⟨*proof*⟩
lemma (**in** *lbv*) *wtl-all*:
assumes *all*: *wtl is c 0 s* ≠ ⊤ (**is** *?wtl is* ≠ -)
assumes *pc*: *pc* < *length is*
shows *wtc c pc* (*wtl* (*take pc is*) *c 0 s*) ≠ ⊤⟨*proof*⟩

4.11.4 preserves-type

lemma (**in** *lbv*) *merge-pres*:
assumes *s0*: *snd‘set ss* ⊆ *A* **and** *x*: *x* ∈ *A*
shows *merge c pc ss x* ∈ *A*⟨*proof*⟩
lemma *pres-typeD2*:
pres-type step n A ⟹ *s* ∈ *A* ⟹ *p* < *n* ⟹ *snd‘set* (*step p s*) ⊆ *A*
⟨*proof*⟩

lemma (**in** *lbv*) *wti-pres* [*intro?*]:
assumes *pres*: *pres-type step n A*
assumes *cert*: *c*!(*pc*+*1*) ∈ *A*
assumes *s-pc*: *s* ∈ *A pc* < *n*

shows *wti c pc s* ∈ *A*⟨*proof*⟩
lemma (**in** *lbv*) *wtc-pres*:
assumes *pres-type step n A*
assumes *c*!*pc* ∈ *A* **and** *c*!(*pc*+*1*) ∈ *A*
assumes *s* ∈ *A* **and** *pc* < *n*
shows *wtc c pc s* ∈ *A*⟨*proof*⟩
lemma (**in** *lbv*) *wtl-pres*:
assumes *pres*: *pres-type step* (*length is*) *A*
assumes *cert*: *cert-ok c* (*length is*) ⊤ ⊥ *A*
assumes *s*: *s* ∈ *A*
assumes *all*: *wtl is c 0 s* ≠ ⊤
shows *pc* < *length is* ⟹ *wtl* (*take pc is*) *c 0 s* ∈ *A*
(**is** *?len pc* ⟹ *?wtl pc* ∈ *A*)⟨*proof*⟩
end

4.12 Correctness of the LBV

theory *LBVCorrect*
imports *LBVSpec Typing-Framework-1*
begin

locale *lbvs* = *lbv* +
fixes s_0 :: *'a*
fixes *c* :: *'a list*
fixes *ins* :: *'b list*
fixes *τs* :: *'a list*
defines *phi-def*:
τs ≡ *map* (λ*pc*. *if c*!*pc* = ⊥ *then wtl* (*take pc ins*) *c 0* s_0 *else c*!*pc*)
[*0*..<*size ins*]

assumes *bounded*: *bounded step* (*size ins*)
assumes *cert*: *cert-ok c* (*size ins*) ⊤ ⊥ *A*
assumes *pres*: *pres-type step* (*size ins*) *A*

lemma (**in** *lbvs*) *phi-None* [*intro?*]:
⟦ *pc* < *size ins*; *c*!*pc* = ⊥ ⟧ ⟹ *τs*!*pc* = *wtl* (*take pc ins*) *c 0* s_0⟨*proof*⟩
lemma (**in** *lbvs*) *phi-Some* [*intro?*]:
⟦ *pc* < *size ins*; *c*!*pc* ≠ ⊥ ⟧ ⟹ *τs*!*pc* = *c*!*pc*⟨*proof*⟩
lemma (**in** *lbvs*) *phi-len* [*simp*]: *size τs* = *size ins*⟨*proof*⟩
lemma (**in** *lbvs*) *wtl-suc-pc*:
assumes *all*: *wtl ins c 0* s_0 ≠ ⊤
assumes *pc*: *pc*+*1* < *size ins*
assumes *sA*: s_0 ∈ *A*
shows *wtl* (*take* (*pc*+*1*) *ins*) *c 0* s_0 $\sqsubseteq_r$ *τs*!(*pc*+*1*)⟨*proof*⟩
lemma (**in** *lbvs*) *wtl-stable*:
assumes *wtl*: *wtl ins c 0* s_0 ≠ ⊤
assumes s_0: s_0 ∈ *A* **and** *pc*: *pc* < *size ins*
shows *stable r step τs pc*⟨*proof*⟩
lemma (**in** *lbvs*) *phi-not-top*:
assumes *wtl*: *wtl ins c 0* s_0 ≠ ⊤ **and** *pc*: *pc* < *size ins*
shows *τs*!*pc* ≠ ⊤⟨*proof*⟩
lemma (**in** *lbvs*) *phi-in-A*:
assumes *wtl*: *wtl ins c 0* s_0 ≠ ⊤ **and** s_0: s_0 ∈ *A*

shows $\tau s \in \mathit{nlists}\ (\mathit{size}\ \mathit{ins})\ A\langle\mathit{proof}\rangle$
lemma (**in** *lbvs*) *phi0*:
assumes *wtl*: $\mathit{wtl}\ \mathit{ins}\ c\ 0\ s_0 \neq \top$ **and** *0*: $0 < \mathit{size}\ \mathit{ins}$ **and** s_0: $s_0 \in A$
shows $s_0 \sqsubseteq_r \tau s!0\langle\mathit{proof}\rangle$

theorem (**in** *lbvs*) *wtl-sound*:
assumes *wtl*: $\mathit{wtl}\ \mathit{ins}\ c\ 0\ s_0 \neq \top$ **and** s_0: $s_0 \in A$
shows $\exists \tau s.\ \mathit{wt\text{-}step}\ r\ \top\ \mathit{step}\ \tau s\langle\mathit{proof}\rangle$

theorem (**in** *lbvs*) *wtl-sound-strong*:
assumes *wtl*: $\mathit{wtl}\ \mathit{ins}\ c\ 0\ s_0 \neq \top$
assumes s_0: $s_0 \in A$ **and** *ins*: $0 < \mathit{size}\ \mathit{ins}$
shows $\exists \tau s \in \mathit{nlists}\ (\mathit{size}\ \mathit{ins})\ A.\ \mathit{wt\text{-}step}\ r\ \top\ \mathit{step}\ \tau s \wedge s_0 \sqsubseteq_r \tau s!0\langle\mathit{proof}\rangle$
end

4.13 Completeness of the LBV

theory *LBVComplete*
imports *LBVSpec Typing-Framework-1*
begin

definition *is-target* :: $'s\ \mathit{step\text{-}type} \Rightarrow 's\ \mathit{list} \Rightarrow \mathit{nat} \Rightarrow \mathit{bool}$ **where**
$\mathit{is\text{-}target}\ \mathit{step}\ \tau s\ pc' \longleftrightarrow (\exists pc\ s'.\ pc' \neq pc{+}1 \wedge pc < \mathit{size}\ \tau s \wedge (pc',s') \in \mathit{set}\ (\mathit{step}\ pc\ (\tau s!pc)))$

definition *make-cert* :: $'s\ \mathit{step\text{-}type} \Rightarrow 's\ \mathit{list} \Rightarrow 's \Rightarrow 's\ \mathit{certificate}$ **where**
$\mathit{make\text{-}cert}\ \mathit{step}\ \tau s\ B = \mathit{map}\ (\lambda pc.\ \mathit{if}\ \mathit{is\text{-}target}\ \mathit{step}\ \tau s\ pc\ \mathit{then}\ \tau s!pc\ \mathit{else}\ B)\ [0..<\mathit{size}\ \tau s]\ @\ [B]$

lemma [*code*]:
$\mathit{is\text{-}target}\ \mathit{step}\ \tau s\ pc' =$
$\mathit{list\text{-}ex}\ (\lambda pc.\ pc' \neq pc{+}1 \wedge \mathit{List.member}\ (\mathit{map}\ \mathit{fst}\ (\mathit{step}\ pc\ (\tau s!pc)))\ pc')\ [0..<\mathit{size}\ \tau s]\langle\mathit{proof}\rangle$
locale $\mathit{lbvc} = \mathit{lbv}\ +$
fixes $\tau s :: 'a\ \mathit{list}$
fixes $c :: 'a\ \mathit{list}$
defines *cert-def*: $c \equiv \mathit{make\text{-}cert}\ \mathit{step}\ \tau s\ \bot$

assumes *mono*: $\mathit{mono}\ r\ \mathit{step}\ (\mathit{size}\ \tau s)\ A$
assumes *pres*: $\mathit{pres\text{-}type}\ \mathit{step}\ (\mathit{size}\ \tau s)\ A$
assumes τs: $\forall pc < \mathit{size}\ \tau s.\ \tau s!pc \in A \wedge \tau s!pc \neq \top$
assumes *bounded*: $\mathit{bounded}\ \mathit{step}\ (\mathit{size}\ \tau s)$

assumes *B-neq-T*: $\bot \neq \top$

lemma (**in** *lbvc*) *cert*: $\mathit{cert\text{-}ok}\ c\ (\mathit{size}\ \tau s)\ \top\ \bot\ A\langle\mathit{proof}\rangle$
lemmas [*simp del*] = *split-paired-Ex*

lemma (**in** *lbvc*) *cert-target* [*intro?*]:
$[\![\ (pc',s') \in \mathit{set}\ (\mathit{step}\ pc\ (\tau s!pc));$
$pc' \neq pc{+}1;\ pc < \mathit{size}\ \tau s;\ pc' < \mathit{size}\ \tau s\ ]\!]$
$\Longrightarrow c!pc' = \tau s!pc'\langle\mathit{proof}\rangle$
lemma (**in** *lbvc*) *cert-approx* [*intro?*]:
$[\![\ pc < \mathit{size}\ \tau s;\ c!pc \neq \bot\ ]\!] \Longrightarrow c!pc = \tau s!pc\langle\mathit{proof}\rangle$
lemma (**in** *lbv*) *le-top* [*simp*, *intro*]: $x <\!=\!\text{-}r\ \top\langle\mathit{proof}\rangle$

lemma (**in** *lbv*) *merge-mono*:
assumes *less*: *set* ss_2 $\{\sqsubseteq_r\}$ *set* ss_1
assumes *x*: $x \in A$
assumes ss_1: *snd'set* $ss_1 \subseteq A$
assumes ss_2: *snd'set* $ss_2 \subseteq A$
assumes *boun*: $\forall x \in (fst`set\ ss_1).\ x < size\ \tau s$
assumes *cert*: *cert-ok c* (*size* τs) *T B A*
shows *merge c pc* ss_2 $x \sqsubseteq_r$ *merge c pc* ss_1 *x* (**is** $?s_2 \sqsubseteq_r ?s_1$)⟨*proof*⟩
lemma (**in** *lbvc*) *wti-mono*:
assumes *less*: $s_2 \sqsubseteq_r s_1$
assumes *pc*: $pc < size\ \tau s$ **and** s_1: $s_1 \in A$ **and** s_2: $s_2 \in A$
shows *wti c pc* $s_2 \sqsubseteq_r$ *wti c pc* s_1 (**is** $?s_2' \sqsubseteq_r ?s_1'$)⟨*proof*⟩
lemma (**in** *lbvc*) *wtc-mono*:
assumes *less*: $s_2 \sqsubseteq_r s_1$
assumes *pc*: $pc < size\ \tau s$ **and** s_1: $s_1 \in A$ **and** s_2: $s_2 \in A$
shows *wtc c pc* $s_2 \sqsubseteq_r$ *wtc c pc* s_1 (**is** $?s_2' \sqsubseteq_r ?s_1'$)⟨*proof*⟩
lemma (**in** *lbv*) *top-le-conv* [*simp*]: $x \in A \Longrightarrow \top \sqsubseteq_r x = (x = \top)$⟨*proof*⟩
lemma (**in** *lbv*) *neq-top* [*simp*, *elim*]: $[\![x \sqsubseteq_r y;\ y \neq \top;\ y \in A]\!] \Longrightarrow x \neq \top$⟨*proof*⟩
lemma (**in** *lbvc*) *stable-wti*:
assumes *stable*: *stable r step* τs *pc* **and** *pc*: $pc < size\ \tau s$
shows *wti c pc* ($\tau s!pc$) $\neq \top$⟨*proof*⟩
lemma (**in** *lbvc*) *wti-less*:
assumes *stable*: *stable r step* τs *pc* **and** *suc-pc*: $Suc\ pc < size\ \tau s$
shows *wti c pc* ($\tau s!pc$) $\sqsubseteq_r \tau s!Suc\ pc$ (**is** $?wti \sqsubseteq_r$ -)⟨*proof*⟩
lemma (**in** *lbvc*) *stable-wtc*:
assumes *stable*: *stable r step* τs *pc* **and** *pc*: $pc < size\ \tau s$
shows *wtc c pc* ($\tau s!pc$) $\neq \top$⟨*proof*⟩
lemma (**in** *lbvc*) *wtc-less*:
assumes *stable*: *stable r step* τs *pc* **and** *suc-pc*: $Suc\ pc < size\ \tau s$
shows *wtc c pc* ($\tau s!pc$) $\sqsubseteq_r \tau s!Suc\ pc$ (**is** $?wtc \sqsubseteq_r$ -)⟨*proof*⟩
lemma (**in** *lbvc*) *wt-step-wtl-lemma*:
assumes *wt-step*: *wt-step r* $\top$ *step* τs
shows $\bigwedge pc\ s.\ pc{+}size\ ls = size\ \tau s \Longrightarrow s \sqsubseteq_r \tau s!pc \Longrightarrow s \in A \Longrightarrow s{\neq}\top \Longrightarrow$
$wtl\ ls\ c\ pc\ s \neq \top$
(**is** $\bigwedge pc\ s.$ - $\Longrightarrow$ - $\Longrightarrow$ - $\Longrightarrow$ - $\Longrightarrow$ $?wtl\ ls\ pc\ s \neq$ -)⟨*proof*⟩
theorem (**in** *lbvc*) *wtl-complete*:
assumes *wt*: *wt-step r* $\top$ *step* τs
assumes *s*: $s \sqsubseteq_r \tau s!0$ $s \in A$ $s \neq \top$ **and** *eq*: $size\ ins = size\ \tau s$
shows *wtl ins c 0 s* $\neq \top$⟨*proof*⟩
end

4.14 The Jinja Type System as a Semilattice

theory *SemiType*
imports ../*Common*/*WellForm* ../*DFA*/*Semilattices*
begin

definition *super* :: *'a prog* ⇒ *cname* ⇒ *cname*
where *super P C* ≡ *fst* (*the* (*class P C*))

lemma *superI*:
(*C*,*D*) ∈ *subcls1 P* ⟹ *super P C* = *D*
⟨*proof*⟩

primrec *the-Class* :: *ty* ⇒ *cname*
where
the-Class (*Class C*) = *C*

definition *sup* :: *'c prog* ⇒ *ty* ⇒ *ty* ⇒ *ty err*
where
$sup\ P\ T_1\ T_2 \equiv$
$if\ is\text{-}refT\ T_1 \wedge is\text{-}refT\ T_2\ then$
$OK\ (if\ T_1 = NT\ then\ T_2\ else$
$if\ T_2 = NT\ then\ T_1\ else$
$(Class\ (exec\text{-}lub\ (subcls1\ P)\ (super\ P)\ (the\text{-}Class\ T_1)\ (the\text{-}Class\ T_2))))$
$else$
$(if\ T_1 = T_2\ then\ OK\ T_1\ else\ Err)$

lemma *sup-def'*:
$sup\ P = (\lambda T_1\ T_2.$
$if\ is\text{-}refT\ T_1 \wedge is\text{-}refT\ T_2\ then$
$OK\ (if\ T_1 = NT\ then\ T_2\ else$
$if\ T_2 = NT\ then\ T_1\ else$
$(Class\ (exec\text{-}lub\ (subcls1\ P)\ (super\ P)\ (the\text{-}Class\ T_1)\ (the\text{-}Class\ T_2))))$
$else$
$(if\ T_1 = T_2\ then\ OK\ T_1\ else\ Err))$
⟨*proof*⟩

abbreviation
subtype :: *'c prog* ⇒ *ty* ⇒ *ty* ⇒ *bool*
where *subtype P* ≡ *widen P*

definition *esl* :: *'c prog* ⇒ *ty esl*
where
esl P ≡ (*types P*, *subtype P*, *sup P*)

lemma *is-class-is-subcls*:
wf-prog m P ⟹ *is-class P C* = $P \vdash C \preceq^* Object$⟨*proof*⟩

lemma *subcls-antisym*:
⟦*wf-prog m P*; $P \vdash C \preceq^* D$; $P \vdash D \preceq^* C$⟧ ⟹ *C* = *D*
⟨*proof*⟩

lemma *widen-antisym*:
⟦ *wf-prog m P*; $P \vdash T \leq U$; $P \vdash U \leq T$ ⟧ ⟹ *T* = *U*⟨*proof*⟩
lemma *order-widen* [*intro*,*simp*]:
wf-prog m P ⟹ *order* (*subtype P*) (*types P*)⟨*proof*⟩

lemma *NT-widen*:
$P \vdash NT \leq T = (T = NT \vee (\exists C.\ T = Class\ C))$⟨*proof*⟩

lemma *Class-widen2*: $P \vdash Class\ C \leq T = (\exists D.\ T = Class\ D \wedge P \vdash C \preceq^* D)$⟨*proof*⟩
lemma *wf-converse-subcls1-impl-acc-subtype*:

wf $((subcls1\ P)\hat{}{-1}) \Longrightarrow$ *acc* (*subtype P*)⟨*proof*⟩
lemma *wf-subtype-acc* [*intro*, *simp*]:
wf-prog wf-mb P $\Longrightarrow$ *acc* (*subtype P*)⟨*proof*⟩
lemma *exec-lub-refl* [*simp*]: *exec-lub r f T T* = *T*⟨*proof*⟩
lemma *closed-err-types*:
wf-prog wf-mb P $\Longrightarrow$ *closed* (*err* (*types P*)) (*lift2* (*sup P*))⟨*proof*⟩

lemma *sup-subtype-greater*:
⟦ *wf-prog wf-mb P*; *is-type P t1*; *is-type P t2*; *sup P t1 t2* = *OK s* ⟧
$\Longrightarrow$ *subtype P t1 s* ∧ *subtype P t2 s*⟨*proof*⟩
lemma *sup-subtype-smallest*:
⟦ *wf-prog wf-mb P*; *is-type P a*; *is-type P b*; *is-type P c*;
subtype P a c; *subtype P b c*; *sup P a b* = *OK d* ⟧
$\Longrightarrow$ *subtype P d c*⟨*proof*⟩
lemma *sup-exists*:
⟦ *subtype P a c*; *subtype P b c* ⟧ $\Longrightarrow$ ∃ *T*. *sup P a b* = *OK T*⟨*proof*⟩
lemma *err-semilat-JType-esl*:
wf-prog wf-mb P $\Longrightarrow$ *err-semilat* (*esl P*)⟨*proof*⟩

end

4.15 The JVM Type System as Semilattice

theory *JVM-SemiType* **imports** *SemiType* **begin**

type-synonym ty_l = *ty err list*
type-synonym ty_s = *ty list*
type-synonym $ty_i = ty_s \times ty_l$
type-synonym $ty_i{}'$ = ty_i *option*
type-synonym ty_m = $ty_i{}'$ *list*
type-synonym ty_P = *mname* ⇒ *cname* ⇒ ty_m

definition *stk-esl* :: *'c prog* ⇒ *nat* ⇒ ty_s *esl*
where
stk-esl P mxs ≡ *upto-esl mxs* (*SemiType.esl P*)

definition *loc-sl* :: *'c prog* ⇒ *nat* ⇒ ty_l *sl*
where
loc-sl P mxl ≡ *Listn.sl mxl* (*Err.sl* (*SemiType.esl P*))

definition *sl* :: *'c prog* ⇒ *nat* ⇒ *nat* ⇒ $ty_i{}'$ *err sl*
where
sl P mxs mxl ≡
Err.sl(*Opt.esl*(*Product.esl* (*stk-esl P mxs*) (*Err.esl*(*loc-sl P mxl*))))

definition *states* :: *'c prog* ⇒ *nat* ⇒ *nat* ⇒ $ty_i{}'$ *err set*
where *states P mxs mxl* ≡ *fst*(*sl P mxs mxl*)

definition *le* :: *'c prog* ⇒ *nat* ⇒ *nat* ⇒ $ty_i{}'$ *err ord*
where
le P mxs mxl ≡ *fst*(*snd*(*sl P mxs mxl*))

definition *sup* :: $'c$ *prog* ⇒ *nat* ⇒ *nat* ⇒ ty_i' *err binop*
where
sup P mxs mxl ≡ *snd*(*snd*(*sl P mxs mxl*))

definition *sup-ty-opt* :: [$'c$ *prog*,*ty err*,*ty err*] ⇒ *bool*
(‹- ⊢ - $\leq_\top$ -› [*71*,*71*,*71*] *70*)
where
sup-ty-opt P ≡ *Err.le* (*subtype P*)

definition *sup-state* :: [$'c$ *prog*,ty_i,ty_i] ⇒ *bool*
(‹- ⊢ - $\leq_i$ -› [*71*,*71*,*71*] *70*)
where
sup-state P ≡ *Product.le* (*Listn.le* (*subtype P*)) (*Listn.le* (*sup-ty-opt P*))

definition *sup-state-opt* :: [$'c$ *prog*,ty_i',ty_i'] ⇒ *bool*
(‹- ⊢ - $\leq''$ -› [*71*,*71*,*71*] *70*)
where
sup-state-opt P ≡ *Opt.le* (*sup-state P*)

abbreviation
sup-loc :: [$'c$ *prog*,ty_l,ty_l] ⇒ *bool* (‹- ⊢ - [$\leq_\top$] -› [*71*,*71*,*71*] *70*)
where *P* ⊢ *LT* [$\leq_\top$] *LT'* ≡ *list-all2* (*sup-ty-opt P*) *LT LT'*

notation (*ASCII*)
sup-ty-opt (‹- |− - <=*T* -› [*71*,*71*,*71*] *70*) **and**
sup-state (‹- |− - <=*i* -› [*71*,*71*,*71*] *70*) **and**
sup-state-opt (‹- |− - <=′ -› [*71*,*71*,*71*] *70*) **and**
sup-loc (‹- |− - [<=*T*] -› [*71*,*71*,*71*] *70*)

4.15.1 Unfolding

lemma *JVM-states-unfold*:
states P mxs mxl ≡ *err*(*opt*((*Union* {*nlists n* (*types P*) |*n*. *n* <= *mxs*}) ×
nlists mxl (*err*(*types P*))))⟨*proof*⟩
lemma *JVM-le-unfold*:
le P m n ≡
Err.le(*Opt.le*(*Product.le*(*Listn.le*(*subtype P*))(*Listn.le*(*Err.le*(*subtype P*)))))⟨*proof*⟩
lemma *sl-def2*:
JVM-SemiType.sl P mxs mxl ≡
(*states P mxs mxl*, *JVM-SemiType.le P mxs mxl*, *JVM-SemiType.sup P mxs mxl*)⟨*proof*⟩

lemma *JVM-le-conv*:
le P m n (*OK t1*) (*OK t2*) = *P* ⊢ *t1* ≤′ *t2*⟨*proof*⟩
lemma *JVM-le-Err-conv*:
le P m n = *Err.le* (*sup-state-opt P*)⟨*proof*⟩
lemma *err-le-unfold* [*iff*]:
Err.le r (*OK a*) (*OK b*) = *r a b*⟨*proof*⟩

4.15.2 Semilattice

lemma *order-sup-state-opt′* [*intro*, *simp*]:
wf-prog wf-mb P ⟹

order (*sup-state-opt P*) (*opt* (($\bigcup$ {*nlists n* (*types P*) |*n*. $n \leq mxs$}) × *nlists* (*Suc* (*length Ts* + mxl_0)) (*err* (*types P*))))⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*pro*

4.16 Effect of Instructions on the State Type

theory *Effect*
imports *JVM-SemiType ../JVM/JVMExceptions*
begin

— FIXME
locale *prog* =
  fixes *P* :: *'a prog*

locale *jvm-method* = *prog* +
  fixes *mxs* :: *nat*
  fixes mxl_0 :: *nat*
  fixes *Ts* :: *ty list*
  fixes T_r :: *ty*
  fixes *is* :: *instr list*
  fixes *xt* :: *ex-table*

  fixes *mxl* :: *nat*
  defines *mxl-def*: *mxl* ≡ *1*+*size Ts*+mxl_0

Program counter of successor instructions:

primrec *succs* :: *instr* ⇒ ty_i ⇒ *pc* ⇒ *pc list* **where**
  succs (*Load idx*) τ *pc* = [*pc*+*1*]
| *succs* (*Store idx*) τ *pc* = [*pc*+*1*]
| *succs* (*Push v*) τ *pc* = [*pc*+*1*]
| *succs* (*Getfield F C*) τ *pc* = [*pc*+*1*]
| *succs* (*Putfield F C*) τ *pc* = [*pc*+*1*]
| *succs* (*New C*) τ *pc* = [*pc*+*1*]
| *succs* (*Checkcast C*) τ *pc* = [*pc*+*1*]
| *succs Pop* τ *pc* = [*pc*+*1*]
| *succs IAdd* τ *pc* = [*pc*+*1*]
| *succs CmpEq* τ *pc* = [*pc*+*1*]
| *succs-IfFalse*:
    succs (*IfFalse b*) τ *pc* = [*pc*+*1*, *nat* (*int pc* + *b*)]
| *succs-Goto*:
    succs (*Goto b*) τ *pc* = [*nat* (*int pc* + *b*)]
| *succs-Return*:
    succs Return τ *pc* = []
| *succs-Invoke*:
    succs (*Invoke M n*) τ *pc* = (*if* (*fst* τ)!*n* = *NT then* [] *else* [*pc*+*1*])
| *succs-Throw*:
    succs Throw τ *pc* = []

Effect of instruction on the state type:

fun *the-class*:: *ty* ⇒ *cname* **where**
  the-class (*Class C*) = *C*

fun eff_i :: *instr* × *'m prog* × ty_i ⇒ ty_i **where**
  eff_i*-Load*:
    eff_i (*Load n*, *P*, (*ST*, *LT*)) = (*ok-val* (*LT* ! *n*) # *ST*, *LT*)

| eff_i*-Store*:
  eff_i (*Store n*, *P*, (*T*#*ST*, *LT*)) = (*ST*, *LT*[*n*:= *OK T*])
| eff_i*-Push*:
  eff_i (*Push v*, *P*, (*ST*, *LT*)) = (*the* (*typeof v*) # *ST*, *LT*)
| eff_i*-Getfield*:
  eff_i (*Getfield F C*, *P*, (*T*#*ST*, *LT*)) = (*snd* (*field P C F*) # *ST*, *LT*)
| eff_i*-Putfield*:
  eff_i (*Putfield F C*, *P*, (T_1#T_2#*ST*, *LT*)) = (*ST*,*LT*)
| eff_i*-New*:
  eff_i (*New C*, *P*, (*ST*,*LT*)) = (*Class C* # *ST*, *LT*)
| eff_i*-Checkcast*:
  eff_i (*Checkcast C*, *P*, (*T*#*ST*,*LT*)) = (*Class C* # *ST*,*LT*)
| eff_i*-Pop*:
  eff_i (*Pop*, *P*, (*T*#*ST*,*LT*)) = (*ST*,*LT*)
| eff_i*-IAdd*:
  eff_i (*IAdd*, *P*,(T_1#T_2#*ST*,*LT*)) = (*Integer*#*ST*,*LT*)
| eff_i*-CmpEq*:
  eff_i (*CmpEq*, *P*, (T_1#T_2#*ST*,*LT*)) = (*Boolean*#*ST*,*LT*)
| eff_i*-IfFalse*:
  eff_i (*IfFalse b*, *P*, (T_1#*ST*,*LT*)) = (*ST*,*LT*)
| eff_i*-Invoke*:
  eff_i (*Invoke M n*, *P*, (*ST*,*LT*)) =
  (*let C* = *the-class* (*ST*!*n*); (*D*,*Ts*,T_r,*b*) = *method P C M*
   in (T_r # *drop* (*n*+*1*) *ST*, *LT*))
| eff_i*-Goto*:
  eff_i (*Goto n*, *P*, *s*) = *s*

fun *is-relevant-class* :: *instr* ⇒ ′*m prog* ⇒ *cname* ⇒ *bool* **where**
  rel-Getfield:
  is-relevant-class (*Getfield F D*) = (λ*P C*. *P* ⊢ *NullPointer* $\preceq^*$ *C*)
| *rel-Putfield*:
  is-relevant-class (*Putfield F D*) = (λ*P C*. *P* ⊢ *NullPointer* $\preceq^*$ *C*)
| *rel-Checcast*:
  is-relevant-class (*Checkcast D*) = (λ*P C*. *P* ⊢ *ClassCast* $\preceq^*$ *C*)
| *rel-New*:
  is-relevant-class (*New D*) = (λ*P C*. *P* ⊢ *OutOfMemory* $\preceq^*$ *C*)
| *rel-Throw*:
  is-relevant-class Throw = (λ*P C*. *True*)
| *rel-Invoke*:
  is-relevant-class (*Invoke M n*) = (λ*P C*. *True*)
| *rel-default*:
  is-relevant-class i = (λ*P C*. *False*)

definition *is-relevant-entry* :: ′*m prog* ⇒ *instr* ⇒ *pc* ⇒ *ex-entry* ⇒ *bool* **where**
  is-relevant-entry P i pc e ⟷ (*let* (*f*,*t*,*C*,*h*,*d*) = *e in is-relevant-class i P C* ∧ *pc* ∈ {*f*..<*t*})

definition *relevant-entries* :: ′*m prog* ⇒ *instr* ⇒ *pc* ⇒ *ex-table* ⇒ *ex-table* **where**
  relevant-entries P i pc = *filter* (*is-relevant-entry P i pc*)

definition *xcpt-eff* :: *instr* ⇒ ′*m prog* ⇒ *pc* ⇒ ty_i
  ⇒ *ex-table* ⇒ (*pc* × ty_i′) *list* **where**
  xcpt-eff i P pc τ et = (*let* (*ST*,*LT*) = *τ in*
  map (λ(*f*,*t*,*C*,*h*,*d*). (*h*, *Some* (*Class C*#*drop* (*size ST* − *d*) *ST*, *LT*))) (*relevant-entries P i pc et*))

definition *norm-eff* :: *instr* $\Rightarrow$ *'m prog* $\Rightarrow$ *nat* $\Rightarrow$ ty_i $\Rightarrow$ (*pc* $\times$ ty_i') *list* **where**
norm-eff i P pc τ = *map* ($\lambda pc'$. (pc',*Some* (eff_i (i,P,τ)))) (*succs i* τ *pc*)

definition *eff* :: *instr* $\Rightarrow$ *'m prog* $\Rightarrow$ *pc* $\Rightarrow$ *ex-table* $\Rightarrow$ ty_i' $\Rightarrow$ (*pc* $\times$ ty_i') *list* **where**
eff i P pc et t = (*case t of*
None $\Rightarrow$ []
| *Some* τ $\Rightarrow$ (*norm-eff i P pc* τ) @ (*xcpt-eff i P pc* τ *et*))

lemma *eff-None*:
eff i P pc xt None = []
⟨*proof*⟩

lemma *eff-Some*:
eff i P pc xt (*Some* τ) = *norm-eff i P pc* τ @ *xcpt-eff i P pc* τ *xt*
⟨*proof*⟩

Conditions under which eff is applicable:

fun app_i :: *instr* $\times$ *'m prog* $\times$ *pc* $\times$ *nat* $\times$ *ty* $\times$ ty_i $\Rightarrow$ *bool* **where**
app_i-*Load*:
app_i (*Load n*, *P*, *pc*, *mxs*, T_r, (*ST*,*LT*)) =
(n < *length LT* $\wedge$ *LT* ! $n \neq$ *Err* $\wedge$ *length ST* < *mxs*)
| app_i-*Store*:
app_i (*Store n*, *P*, *pc*, *mxs*, T_r, (*T*#*ST*, *LT*)) =
(n < *length LT*)
| app_i-*Push*:
app_i (*Push v*, *P*, *pc*, *mxs*, T_r, (*ST*,*LT*)) =
(*length ST* < *mxs* $\wedge$ *typeof v* $\neq$ *None*)
| app_i-*Getfield*:
app_i (*Getfield F C*, *P*, *pc*, *mxs*, T_r, (*T*#*ST*, *LT*)) =
($\exists\, T_f$. $P \vdash C$ *sees* F:T_f *in* C $\wedge$ $P \vdash T \leq$ *Class C*)
| app_i-*Putfield*:
app_i (*Putfield F C*, *P*, *pc*, *mxs*, T_r, (T_1#T_2#*ST*, *LT*)) =
($\exists\, T_f$. $P \vdash C$ *sees* F:T_f *in* C $\wedge$ $P \vdash T_2 \leq$ (*Class C*) $\wedge$ $P \vdash T_1 \leq T_f$)
| app_i-*New*:
app_i (*New C*, *P*, *pc*, *mxs*, T_r, (*ST*,*LT*)) =
(*is-class P C* $\wedge$ *length ST* < *mxs*)
| app_i-*Checkcast*:
app_i (*Checkcast C*, *P*, *pc*, *mxs*, T_r, (*T*#*ST*,*LT*)) =
(*is-class P C* $\wedge$ *is-refT T*)
| app_i-*Pop*:
app_i (*Pop*, *P*, *pc*, *mxs*, T_r, (*T*#*ST*,*LT*)) =
True
| app_i-*IAdd*:
app_i (*IAdd*, *P*, *pc*, *mxs*, T_r, (T_1#T_2#*ST*,*LT*)) = ($T_1 = T_2$ $\wedge$ T_1 = *Integer*)
| app_i-*CmpEq*:
app_i (*CmpEq*, *P*, *pc*, *mxs*, T_r, (T_1#T_2#*ST*,*LT*)) =
($T_1 = T_2$ $\vee$ *is-refT* T_1 $\wedge$ *is-refT* T_2)
| app_i-*IfFalse*:
app_i (*IfFalse b*, *P*, *pc*, *mxs*, T_r, (*Boolean*#*ST*,*LT*)) =
(*0* $\leq$ *int pc* + *b*)
| app_i-*Goto*:
app_i (*Goto b*, *P*, *pc*, *mxs*, T_r, *s*) =
(*0* $\leq$ *int pc* + *b*)

| app_i-*Return*:
  app_i (*Return*, *P*, *pc*, *mxs*, T_r, (T#ST,LT)) =
  ($P \vdash T \leq T_r$)
| app_i-*Throw*:
  app_i (*Throw*, *P*, *pc*, *mxs*, T_r, (T#ST,LT)) =
  is-refT T
| app_i-*Invoke*:
  app_i (*Invoke M n*, *P*, *pc*, *mxs*, T_r, (ST,LT)) =
  ($n <$ *length ST* $\wedge$
  (*ST*!*n* $\neq$ *NT* $\longrightarrow$
   ($\exists$ *C D Ts T m*. *ST*!*n* = *Class C* $\wedge$ $P \vdash$ *C sees M*:*Ts* $\rightarrow$ *T* = *m in D* $\wedge$
      $P \vdash$ *rev* (*take n ST*) [$\leq$] *Ts*)))

| app_i-*default*:
  app_i (i,P, pc,mxs,T_r,s) = *False*

definition *xcpt-app* :: *instr* $\Rightarrow$ '*m prog* $\Rightarrow$ *pc* $\Rightarrow$ *nat* $\Rightarrow$ *ex-table* $\Rightarrow$ ty_i $\Rightarrow$ *bool* **where**
  xcpt-app i P pc mxs xt τ $\longleftrightarrow$ ($\forall$ (f,t,C,h,d) $\in$ *set* (*relevant-entries P i pc xt*). *is-class P C* $\wedge$ $d \leq$ *size* (*fst* τ) $\wedge$ $d <$ *mxs*)

definition *app* :: *instr* $\Rightarrow$ '*m prog* $\Rightarrow$ *nat* $\Rightarrow$ *ty* $\Rightarrow$ *nat* $\Rightarrow$ *nat* $\Rightarrow$ *ex-table* $\Rightarrow$ ty_i' $\Rightarrow$ *bool* **where**
  app i P mxs T_r *pc mpc xt t* = (*case t of None* $\Rightarrow$ *True* | *Some* τ $\Rightarrow$
  app_i (i,P,pc,mxs,T_r,τ) $\wedge$ *xcpt-app i P pc mxs xt* τ $\wedge$
  ($\forall$ (pc',τ') $\in$ *set* (*eff i P pc xt t*). $pc' < mpc$))

lemma *app-Some*:
  app i P mxs T_r *pc mpc xt* (*Some* τ) =
  (app_i (i,P,pc,mxs,T_r,τ) $\wedge$ *xcpt-app i P pc mxs xt* τ $\wedge$
  ($\forall$ (pc',s') $\in$ *set* (*eff i P pc xt* (*Some* τ)). $pc' < mpc$))
⟨*proof*⟩

locale *eff* = *jvm-method* +
  fixes eff_i **and** app_i **and** *eff* **and** *app*
  fixes *norm-eff* **and** *xcpt-app* **and** *xcpt-eff*

  fixes *mpc*
  defines *mpc* $\equiv$ *size is*

  defines eff_i *i* τ $\equiv$ *Effect*.eff_i (i,P,τ)
  notes eff_i-*simps* [*simp*] = *Effect*.eff_i.*simps* [**where** *P* = *P*, *folded* eff_i-*def*]

  defines app_i *i pc* τ $\equiv$ *Effect*.app_i (i, P, pc, mxs, T_r, τ)
   notes app_i-*simps* [*simp*] = *Effect*.app_i.*simps* [**where** *P*=*P* **and** *mxs*=*mxs* **and** T_r=T_r, *folded* app_i-*def*]

  defines *xcpt-eff i pc* τ $\equiv$ *Effect.xcpt-eff i P pc* τ *xt*
  notes *xcpt-eff* = *Effect.xcpt-eff-def* [*of - P - - xt*, *folded xcpt-eff-def*]

  defines *norm-eff i pc* τ $\equiv$ *Effect.norm-eff i P pc* τ
  notes *norm-eff* = *Effect.norm-eff-def* [*of - P*, *folded norm-eff-def* eff_i-*def*]

  defines *eff i pc* $\equiv$ *Effect.eff i P pc xt*

notes *eff* = *Effect.eff-def* [*of* - *P* - *xt*, *folded eff-def norm-eff-def xcpt-eff-def*]

defines *xcpt-app i pc* $\tau \equiv$ *Effect.xcpt-app i P pc mxs xt* τ
notes *xcpt-app* = *Effect.xcpt-app-def* [*of* - *P* - *mxs xt*, *folded xcpt-app-def*]

defines *app i pc* $\equiv$ *Effect.app i P mxs* T_r *pc mpc xt*
notes *app* = *Effect.app-def* [*of* - *P mxs* T_r - *mpc xt*, *folded app-def xcpt-app-def* app_i*-def eff-def*]

lemma *length-cases2*:
assumes $\bigwedge LT.\ P\ ([],LT)$
assumes $\bigwedge l\ ST\ LT.\ P\ (l\#ST,LT)$
shows $P\ s$
⟨*proof*⟩

lemma *length-cases3*:
assumes $\bigwedge LT.\ P\ ([],LT)$
assumes $\bigwedge l\ LT.\ P\ ([l],LT)$
assumes $\bigwedge l\ ST\ LT.\ P\ (l\#ST,LT)$
shows $P\ s$⟨*proof*⟩
lemma *length-cases4*:
assumes $\bigwedge LT.\ P\ ([],LT)$
assumes $\bigwedge l\ LT.\ P\ ([l],LT)$
assumes $\bigwedge l\ l'\ LT.\ P\ ([l,l'],LT)$
assumes $\bigwedge l\ l'\ ST\ LT.\ P\ (l\#l'\#ST,LT)$
shows $P\ s$⟨*proof*⟩

simp rules for *app*

lemma *appNone*[*simp*]: *app i P mxs* T_r *pc mpc et None* = *True*
⟨*proof*⟩

lemma *appLoad*[*simp*]:
app_i (*Load idx*, *P*, T_r, *mxs*, *pc*, *s*) = ($\exists$ *ST LT*. *s* = (*ST*,*LT*) $\wedge$ *idx* < *length LT* $\wedge$ *LT*!*idx* $\neq$ *Err* $\wedge$ *length ST* < *mxs*)
⟨*proof*⟩

lemma *appStore*[*simp*]:
app_i (*Store idx*,*P*,*pc*,*mxs*,T_r,*s*) = ($\exists$ *ts ST LT*. *s* = (*ts*#*ST*,*LT*) $\wedge$ *idx* < *length LT*)
⟨*proof*⟩

lemma *appPush*[*simp*]:
app_i (*Push v*,*P*,*pc*,*mxs*,T_r,*s*) =
($\exists$ *ST LT*. *s* = (*ST*,*LT*) $\wedge$ *length ST* < *mxs* $\wedge$ *typeof v* $\neq$ *None*)
⟨*proof*⟩

lemma *appGetField*[*simp*]:
app_i (*Getfield F C*,*P*,*pc*,*mxs*,T_r,*s*) =
($\exists$ *oT vT ST LT*. *s* = (*oT*#*ST*, *LT*) $\wedge$
$P \vdash$ *C sees F*:*vT in C* $\wedge$ $P \vdash oT \leq$ (*Class C*))
⟨*proof*⟩

lemma *appPutField*[*simp*]:

app_i (*Putfield F C*,*P*,*pc*,*mxs*,T_r,*s*) =
(∃ *vT vT′ oT ST LT*. *s* = (*vT*#*oT*#*ST*, *LT*) ∧
P ⊢ *C sees F*:*vT′ in C* ∧ *P* ⊢ *oT* ≤ (*Class C*) ∧ *P* ⊢ *vT* ≤ *vT′*)
⟨*proof*⟩

lemma *appNew*[*simp*]:
app_i (*New C*,*P*,*pc*,*mxs*,T_r,*s*) =
(∃ *ST LT*. *s*=(*ST*,*LT*) ∧ *is-class P C* ∧ *length ST* < *mxs*)
⟨*proof*⟩

lemma *appCheckcast*[*simp*]:
app_i (*Checkcast C*,*P*,*pc*,*mxs*,T_r,*s*) =
(∃ *T ST LT*. *s* = (*T*#*ST*,*LT*) ∧ *is-class P C* ∧ *is-refT T*)
⟨*proof*⟩

lemma app_i*Pop*[*simp*]:
app_i (*Pop*,*P*,*pc*,*mxs*,T_r,*s*) = (∃ *ts ST LT*. *s* = (*ts*#*ST*,*LT*))
⟨*proof*⟩

lemma *appIAdd*[*simp*]:
app_i (*IAdd*,*P*,*pc*,*mxs*,T_r,*s*) = (∃ *ST LT*. *s* = (*Integer*#*Integer*#*ST*,*LT*))⟨*proof*⟩

lemma *appIfFalse* [*simp*]:
app_i (*IfFalse b*,*P*,*pc*,*mxs*,T_r,*s*) =
(∃ *ST LT*. *s* = (*Boolean*#*ST*,*LT*) ∧ *0* ≤ *int pc* + *b*)⟨*proof*⟩
lemma *appCmpEq*[*simp*]:
app_i (*CmpEq*,*P*,*pc*,*mxs*,T_r,*s*) =
(∃ T_1 T_2 *ST LT*. *s* = (T_1#T_2#*ST*,*LT*) ∧ (¬*is-refT* T_1 ∧ T_2 = T_1 ∨ *is-refT* T_1 ∧ *is-refT* T_2))
⟨*proof*⟩

lemma *appReturn*[*simp*]:
app_i (*Return*,*P*,*pc*,*mxs*,T_r,*s*) = (∃ *T ST LT*. *s* = (*T*#*ST*,*LT*) ∧ *P* ⊢ *T* ≤ T_r)
⟨*proof*⟩

lemma *appThrow*[*simp*]:
app_i (*Throw*,*P*,*pc*,*mxs*,T_r,*s*) = (∃ *T ST LT*. *s*=(*T*#*ST*,*LT*) ∧ *is-refT T*)
⟨*proof*⟩

lemma *effNone*:
(*pc′*, *s′*) ∈ *set* (*eff i P pc et None*) ⟹ *s′* = *None*
⟨*proof*⟩

some helpers to make the specification directly executable:

lemma *relevant-entries-append* [*simp*]:
relevant-entries P i pc (*xt* @ *xt′*) = *relevant-entries P i pc xt* @ *relevant-entries P i pc xt′*
⟨*proof*⟩

lemma *xcpt-app-append* [*iff*]:
xcpt-app i P pc mxs (*xt*@*xt′*) τ = (*xcpt-app i P pc mxs xt* τ ∧ *xcpt-app i P pc mxs xt′* τ)
⟨*proof*⟩

lemma *xcpt-eff-append* [*simp*]:
xcpt-eff i P pc τ (*xt*@*xt′*) = *xcpt-eff i P pc* τ *xt* @ *xcpt-eff i P pc* τ *xt′*
⟨*proof*⟩

lemma *app-append* [*simp*]:
$app\ i\ P\ pc\ T\ mxs\ mpc\ (xt@xt')\ \tau = (app\ i\ P\ pc\ T\ mxs\ mpc\ xt\ \tau \wedge app\ i\ P\ pc\ T\ mxs\ mpc\ xt'\ \tau)$
$\langle proof \rangle$

end

4.17 Monotonicity of eff and app

theory *EffectMono* **imports** *Effect* **begin**

declare *not-Err-eq* [*iff*]

lemma app_i*-mono*:
assumes *wf*: *wf-prog p P*
assumes *less*: $P \vdash \tau \leq_i \tau'$
shows $app_i\ (i,P,mxs,mpc,rT,\tau') \Longrightarrow app_i\ (i,P,mxs,mpc,rT,\tau)\langle proof \rangle$
lemma *succs-mono*:
assumes *wf*: *wf-prog p P* **and** app_i: $app_i\ (i,P,mxs,mpc,rT,\tau')$
shows $P \vdash \tau \leq_i \tau' \Longrightarrow set\ (succs\ i\ \tau\ pc) \subseteq set\ (succs\ i\ \tau'\ pc)\langle proof \rangle$

lemma *app-mono*:
assumes *wf*: *wf-prog p P*
assumes *less'*: $P \vdash \tau \leq' \tau'$
shows $app\ i\ P\ m\ rT\ pc\ mpc\ xt\ \tau' \Longrightarrow app\ i\ P\ m\ rT\ pc\ mpc\ xt\ \tau\langle proof \rangle$

lemma eff_i*-mono*:
assumes *wf*: *wf-prog p P*
assumes *less*: $P \vdash \tau \leq_i \tau'$
assumes app_i: $app\ i\ P\ m\ rT\ pc\ mpc\ xt\ (Some\ \tau')$
assumes *succs*: $succs\ i\ \tau\ pc \neq []\ \ succs\ i\ \tau'\ pc \neq []$
shows $P \vdash eff_i\ (i,P,\tau) \leq_i eff_i\ (i,P,\tau')\langle proof \rangle$
end

4.18 The Bytecode Verifier

theory *BVSpec*
imports *Effect*
begin

This theory contains a specification of the BV. The specification describes correct typings of method bodies; it corresponds to type *checking*.

definition
— The method type only contains declared classes:
$check\text{-}types :: {'m}\ prog \Rightarrow nat \Rightarrow nat \Rightarrow ty_i{'}\ err\ list \Rightarrow bool$
where
$check\text{-}types\ P\ mxs\ mxl\ \tau s \equiv set\ \tau s \subseteq states\ P\ mxs\ mxl$

— An instruction is welltyped if it is applicable and its effect
— is compatible with the type at all successor instructions:
definition
$wt\text{-}instr :: [{'m}\ prog,ty,nat,pc,ex\text{-}table,instr,pc,ty_m] \Rightarrow bool$
(‹-,-,-,-,- ⊢ -,- :: -› [60,0,0,0,0,0,0,61] 60)

where
$P,T,mxs,mpc,xt \vdash i,pc :: \tau s \equiv$
$app\ i\ P\ mxs\ T\ pc\ mpc\ xt\ (\tau s!pc) \wedge$
$(\forall (pc',\tau') \in set\ (eff\ i\ P\ pc\ xt\ (\tau s!pc)).\ P \vdash \tau' \leq' \tau s!pc')$

— The type at $pc{=}0$ conforms to the method calling convention:
definition *wt-start* :: $['m\ prog,cname,ty\ list,nat,ty_m] \Rightarrow bool$
where
$wt\text{-}start\ P\ C\ Ts\ mxl_0\ \tau s \equiv$
$P \vdash Some\ ([],OK\ (Class\ C)\#map\ OK\ Ts@replicate\ mxl_0\ Err) \leq' \tau s!0$

— A method is welltyped if the body is not empty,
— if the method type covers all instructions and mentions
— declared classes only, if the method calling convention is respected, and
— if all instructions are welltyped.
definition *wt-method* :: $['m\ prog,cname,ty\ list,ty,nat,nat,instr\ list,$
$ex\text{-}table,ty_m] \Rightarrow bool$
where
$wt\text{-}method\ P\ C\ Ts\ T_r\ mxs\ mxl_0\ is\ xt\ \tau s \equiv$
$0 < size\ is \wedge size\ \tau s = size\ is \wedge$
$check\text{-}types\ P\ mxs\ (1{+}size\ Ts{+}mxl_0)\ (map\ OK\ \tau s) \wedge$
$wt\text{-}start\ P\ C\ Ts\ mxl_0\ \tau s \wedge$
$(\forall pc < size\ is.\ P,T_r,mxs,size\ is,xt \vdash is!pc,pc :: \tau s)$

— A program is welltyped if it is wellformed and all methods are welltyped
definition *wf-jvm-prog-phi* :: $ty_P \Rightarrow jvm\text{-}prog \Rightarrow bool$ (‹*wf'-jvm'-prog_*›)
where
$wf\text{-}jvm\text{-}prog_\Phi \equiv$
$wf\text{-}prog\ (\lambda P\ C\ (M,Ts,T_r,(mxs,mxl_0,is,xt)).$
$wt\text{-}method\ P\ C\ Ts\ T_r\ mxs\ mxl_0\ is\ xt\ (\Phi\ C\ M))$

definition *wf-jvm-prog* :: *jvm-prog* $\Rightarrow$ *bool*
where
$wf\text{-}jvm\text{-}prog\ P \equiv \exists \Phi.\ wf\text{-}jvm\text{-}prog_\Phi\ P$

lemma *wt-jvm-progD*:
$wf\text{-}jvm\text{-}prog_\Phi\ P \Longrightarrow \exists wt.\ wf\text{-}prog\ wt\ P$⟨*proof*⟩
lemma *wt-jvm-prog-impl-wt-instr*:
assumes *wf*: $wf\text{-}jvm\text{-}prog_\Phi\ P$ **and**
sees: $P \vdash C\ sees\ M{:}Ts \rightarrow T = (mxs,mxl_0,ins,xt)\ in\ C$ **and**
pc: $pc < size\ ins$
shows $P,T,mxs,size\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M$⟨*proof*⟩
lemma *wt-jvm-prog-impl-wt-start*:
assumes *wf*: $wf\text{-}jvm\text{-}prog_\Phi\ P$ **and**
sees: $P \vdash C\ sees\ M{:}Ts \rightarrow T = (mxs,mxl_0,ins,xt)\ in\ C$
shows $0 < size\ ins \wedge wt\text{-}start\ P\ C\ Ts\ mxl_0\ (\Phi\ C\ M)$⟨*proof*⟩

4.19 The Typing Framework for the JVM

theory *TF-JVM*
imports *../DFA/Typing-Framework-err EffectMono BVSpec*
begin

definition *exec* :: *jvm-prog* ⇒ *nat* ⇒ *ty* ⇒ *ex-table* ⇒ *instr list* ⇒ ty_i' *err step-type*
where
exec G maxs rT et bs ≡
err-step (*size bs*) (λ*pc*. *app* (*bs*!*pc*) *G maxs rT pc* (*size bs*) *et*)
(λ*pc*. *eff* (*bs*!*pc*) *G pc et*)

locale *JVM-sl* =
fixes *P* :: *jvm-prog* **and** *mxs* **and** mxl_0 **and** *n*
fixes *Ts* :: *ty list* **and** *is* **and** *xt* **and** T_r

fixes *mxl* **and** *A* **and** *r* **and** *f* **and** *app* **and** *eff* **and** *step*
defines [*simp*]: *mxl* ≡ *1*+*size Ts*+mxl_0
defines [*simp*]: *A* ≡ *states P mxs mxl*
defines [*simp*]: *r* ≡ *JVM-SemiType.le P mxs mxl*
defines [*simp*]: *f* ≡ *JVM-SemiType.sup P mxs mxl*

defines [*simp*]: *app* ≡ λ*pc*. *Effect.app* (*is*!*pc*) *P mxs* T_r *pc* (*size is*) *xt*
defines [*simp*]: *eff* ≡ λ*pc*. *Effect.eff* (*is*!*pc*) *P pc xt*
defines [*simp*]: *step* ≡ *err-step* (*size is*) *app eff*

defines [*simp*]: *n* ≡ *size is*

locale *start-context* = *JVM-sl* +
fixes *p* **and** *C*
assumes *wf*: *wf-prog p P*
assumes *C*: *is-class P C*
assumes *Ts*: *set Ts* ⊆ *types P*

fixes *first* :: ty_i' **and** *start*
defines [*simp*]:
first ≡ *Some* ([],*OK* (*Class C*) # *map OK Ts* @ *replicate* mxl_0 *Err*)
defines [*simp*]:
start ≡ *OK first* # *replicate* (*size is* − *1*) (*OK None*)

4.19.1 Connecting JVM and Framework

lemma (**in** *start-context*) *semi*: *semilat* (*A*, *r*, *f*)
⟨*proof*⟩

lemma (**in** *JVM-sl*) *step-def-exec*: *step* ≡ *exec P mxs* T_r *xt is*
⟨*proof*⟩

lemma *special-ex-swap-lemma* [*iff*]:
(*? X*. (*? n*. *X* = *A n* & *P n*) & *Q X*) = (*? n*. *Q*(*A n*) & *P n*)
⟨*proof*⟩

lemma *ex-in-nlists* [*iff*]:
(∃ *n*. *ST* ∈ *nlists n A* ∧ *n* ≤ *mxs*) = (*set ST* ⊆ *A* ∧ *size ST* ≤ *mxs*)
⟨*proof*⟩

lemma *singleton-nlists*:
(∃ *n*. [*Class C*] ∈ *nlists n* (*types P*) ∧ *n* ≤ *mxs*) = (*is-class P C* ∧ *0* < *mxs*)
⟨*proof*⟩

lemma *set-drop-subset*:
$set\ xs \subseteq A \Longrightarrow set\ (drop\ n\ xs) \subseteq A$
⟨*proof*⟩

lemma *Suc-minus-minus-le*:
$n < mxs \Longrightarrow Suc\ (n - (n - b)) \leq mxs$
⟨*proof*⟩

lemma *in-nlistsE*:
$[\![\ xs \in nlists\ n\ A;\ [\![size\ xs = n;\ set\ xs \subseteq A]\!] \Longrightarrow P\ ]\!] \Longrightarrow P$
⟨*proof*⟩

declare *is-relevant-entry-def* [*simp*]
declare *set-drop-subset* [*simp*]

theorem (**in** *start-context*) *exec-pres-type*:
pres-type step (*size is*) *A*⟨*proof*⟩
declare *is-relevant-entry-def* [*simp del*]
declare *set-drop-subset* [*simp del*]

lemma *lesubstep-type-simple*:
$xs\ [\sqsubseteq_{Product.le\ (=)\ r}]\ ys \Longrightarrow set\ xs\ \{\sqsubseteq_r\}\ set\ ys$⟨*proof*⟩
declare *is-relevant-entry-def* [*simp del*]

lemma *conjI2*: $[\![\ A;\ A \Longrightarrow B\ ]\!] \Longrightarrow A \wedge B$ ⟨*proof*⟩

lemma (**in** *JVM-sl*) *eff-mono*:
$[\![wf\text{-}prog\ p\ P;\ pc < length\ is;\ s \sqsubseteq_{sup\text{-}state\text{-}opt\ P}\ t;\ app\ pc\ t]\!]$
$\Longrightarrow set\ (eff\ pc\ s)\ \{\sqsubseteq_{sup\text{-}state\text{-}opt\ P}\}\ set\ (eff\ pc\ t)$⟨*proof*⟩
lemma (**in** *JVM-sl*) *bounded-step*: *bounded step* (*size is*)⟨*proof*⟩
theorem (**in** *JVM-sl*) *step-mono*:
$wf\text{-}prog\ wf\text{-}mb\ P \Longrightarrow mono\ r\ step\ (size\ is)\ A$⟨*proof*⟩

lemma (**in** *start-context*) *first-in-A* [*iff*]: $OK\ first \in A$
⟨*proof*⟩

lemma (**in** *JVM-sl*) *wt-method-def2*:
$wt\text{-}method\ P\ C'\ Ts\ T_r\ mxs\ mxl_0\ is\ xt\ \tau s =$
$(is \neq [] \wedge$
$size\ \tau s = size\ is \wedge$
$OK\ `\ set\ \tau s \subseteq states\ P\ mxs\ mxl \wedge$
$wt\text{-}start\ P\ C'\ Ts\ mxl_0\ \tau s \wedge$
$wt\text{-}app\text{-}eff\ (sup\text{-}state\text{-}opt\ P)\ app\ eff\ \tau s)$⟨*proof*⟩

end

4.20 Typing and Dataflow Analysis Framework

theory *Typing-Framework-2* **imports** *Typing-Framework-1* **begin**

The relationship between dataflow analysis and a welltyped-instruction predicate.

definition *is-bcv* :: $'s$ *ord* ⇒ $'s$ ⇒ $'s$ *step-type* ⇒ *nat* ⇒ $'s$ *set* ⇒ ($'s$ *list* ⇒ $'s$ *list*) ⇒ *bool*
where
is-bcv r T step n A bcv ⟷ ($\forall \tau s_0 \in$ *nlists n A*.
($\forall p<n.$ (*bcv* τs_0)!$p \neq T$) = ($\exists \tau s \in$ *nlists n A*. τs_0 $[\sqsubseteq_r]$ τs ∧ *wt-step r T step* τs))

end

4.21 Kildall for the JVM

theory *BVExec*
imports *../DFA/Abstract-BV TF-JVM ../DFA/Typing-Framework-2*
begin

definition *kiljvm* :: *jvm-prog* ⇒ *nat* ⇒ *nat* ⇒ *ty* ⇒
instr list ⇒ *ex-table* ⇒ ty_i' *err list* ⇒ ty_i' *err list*
where
kiljvm P mxs mxl T_r *is xt* ≡
kildall (*JVM-SemiType.le P mxs mxl*) (*JVM-SemiType.sup P mxs mxl*)
(*exec P mxs* T_r *xt is*)

definition *wt-kildall* :: *jvm-prog* ⇒ *cname* ⇒ *ty list* ⇒ *ty* ⇒ *nat* ⇒ *nat* ⇒
instr list ⇒ *ex-table* ⇒ *bool*
where
wt-kildall P C' Ts T_r *mxs* mxl_0 *is xt* ≡
0 < *size is* ∧
(*let first* = *Some* ([],[*OK* (*Class C'*)]@(*map OK Ts*)@(*replicate* mxl_0 *Err*));
start = *OK first*#(*replicate* (*size is* − *1*) (*OK None*));
result = *kiljvm P mxs* (*1*+*size Ts*+mxl_0) T_r *is xt start*
in ∀ *n* < *size is*. *result*!*n* ≠ *Err*)

definition $wf\text{-}jvm\text{-}prog_k$ :: *jvm-prog* ⇒ *bool*
where
$wf\text{-}jvm\text{-}prog_k$ *P* ≡
wf-prog (λ*P C'* (*M*,*Ts*,T_r,(*mxs*,mxl_0,*is*,*xt*)). *wt-kildall P C' Ts* T_r *mxs* mxl_0 *is xt*) *P*

context *start-context*
begin

lemma *Cons-less-Conss3* [*simp*]:
x#*xs* $[\sqsubset_r]$ *y*#*ys* = (*x* $\sqsubset_r$ *y* ∧ *xs* $[\sqsubseteq_r]$ *ys* ∨ *x* = *y* ∧ *xs* $[\sqsubset_r]$ *ys*)
⟨*proof*⟩

lemma *acc-le-listI3* [*intro*!]:
acc r ⟹ *acc* (*Listn.le r*)
⟨*proof*⟩

lemma *wf-jvm*: *wf* {(*ss'*, *ss*). *ss* $[\sqsubset_r]$ *ss'*}
⟨*proof*⟩

lemma *iter-properties-bv*[*rule-format*]:
shows ⟦ ∀ *p*∈*w0*. *p* < *n*; *ss0* ∈ *nlists n A*; ∀ *p*<*n*. *p* ∉ *w0* ⟶ *stable r step ss0 p* ⟧ ⟹
iter f step ss0 w0 = (*ss'*,*w'*) ⟶

$ss' \in nlists\ n\ A \wedge stables\ r\ step\ ss' \wedge ss0\ [\sqsubseteq_r]\ ss' \wedge$
$(\forall ts \in nlists\ n\ A.\ ss0\ [\sqsubseteq_r]\ ts \wedge stables\ r\ step\ ts \longrightarrow ss'\ [\sqsubseteq_r]\ ts)$
$\langle proof \rangle$

lemma *kildall-properties-bv*:
shows $[\![\ ss0 \in nlists\ n\ A\ ]\!] \Longrightarrow$
$kildall\ r\ f\ step\ ss0 \in nlists\ n\ A \wedge$
$stables\ r\ step\ (kildall\ r\ f\ step\ ss0) \wedge$
$ss0\ [\sqsubseteq_r]\ kildall\ r\ f\ step\ ss0 \wedge$
$(\forall ts \in nlists\ n\ A.\ ss0\ [\sqsubseteq_r]\ ts \wedge stables\ r\ step\ ts \longrightarrow$
$kildall\ r\ f\ step\ ss0\ [\sqsubseteq_r]\ ts)\langle proof \rangle\langle proof \rangle\langle proof \rangle\langle proof \rangle\langle proof \rangle\langle proof \rangle\langle proof \rangle\langle proof \rangle$

4.22 LBV for the JVM

theory *LBVJVM*
imports *../DFA/Abstract-BV TF-JVM*
begin

type-synonym $prog\text{-}cert = cname \Rightarrow mname \Rightarrow ty_i'\ err\ list$

definition $check\text{-}cert :: jvm\text{-}prog \Rightarrow nat \Rightarrow nat \Rightarrow nat \Rightarrow ty_i'\ err\ list \Rightarrow bool$
where
$check\text{-}cert\ P\ mxs\ mxl\ n\ cert \equiv check\text{-}types\ P\ mxs\ mxl\ cert \wedge size\ cert = n+1 \wedge$
$(\forall i<n.\ cert!i \neq Err) \wedge cert!n = OK\ None$

definition $lbvjvm :: jvm\text{-}prog \Rightarrow nat \Rightarrow nat \Rightarrow ty \Rightarrow ex\text{-}table \Rightarrow$
$ty_i'\ err\ list \Rightarrow instr\ list \Rightarrow ty_i'\ err \Rightarrow ty_i'\ err$
where
$lbvjvm\ P\ mxs\ maxr\ T_r\ et\ cert\ bs \equiv$
$wtl\text{-}inst\text{-}list\ bs\ cert\ (JVM\text{-}SemiType.sup\ P\ mxs\ maxr)\ (JVM\text{-}SemiType.le\ P\ mxs\ maxr)\ Err\ (OK\ None)\ (exec\ P\ mxs\ T_r\ et\ bs)\ 0$

definition $wt\text{-}lbv :: jvm\text{-}prog \Rightarrow cname \Rightarrow ty\ list \Rightarrow ty \Rightarrow nat \Rightarrow nat \Rightarrow$
$ex\text{-}table \Rightarrow ty_i'\ err\ list \Rightarrow instr\ list \Rightarrow bool$
where
$wt\text{-}lbv\ P\ C\ Ts\ T_r\ mxs\ mxl_0\ et\ cert\ ins \equiv$
$check\text{-}cert\ P\ mxs\ (1+size\ Ts+mxl_0)\ (size\ ins)\ cert \wedge$
$0 < size\ ins \wedge$
$(let\ start\ = Some\ ([],(OK\ (Class\ C))\#((map\ OK\ Ts))@(replicate\ mxl_0\ Err));$
$result = lbvjvm\ P\ mxs\ (1+size\ Ts+mxl_0)\ T_r\ et\ cert\ ins\ (OK\ start)$
$in\ result \neq Err)$

definition $wt\text{-}jvm\text{-}prog\text{-}lbv :: jvm\text{-}prog \Rightarrow prog\text{-}cert \Rightarrow bool$
where
$wt\text{-}jvm\text{-}prog\text{-}lbv\ P\ cert \equiv$
$wf\text{-}prog\ (\lambda P\ C\ (mn,Ts,T_r,(mxs,mxl_0,b,et)).\ wt\text{-}lbv\ P\ C\ Ts\ T_r\ mxs\ mxl_0\ et\ (cert\ C\ mn)\ b)\ P$

definition $mk\text{-}cert :: jvm\text{-}prog \Rightarrow nat \Rightarrow ty \Rightarrow ex\text{-}table \Rightarrow instr\ list$
$\Rightarrow ty_m \Rightarrow ty_i'\ err\ list$
where
$mk\text{-}cert\ P\ mxs\ T_r\ et\ bs\ phi \equiv make\text{-}cert\ (exec\ P\ mxs\ T_r\ et\ bs)\ (map\ OK\ phi)\ (OK\ None)$

definition $prg\text{-}cert :: jvm\text{-}prog \Rightarrow ty_P \Rightarrow prog\text{-}cert$

where
prg-cert P phi C mn ≡ *let* $(C, Ts, T_r, (mxs, mxl_0, ins, et))$ = *method P C mn*
in *mk-cert P mxs* T_r *et ins* (*phi C mn*)

lemma *check-certD* [*intro?*]:
check-cert P mxs mxl n cert ⟹ *cert-ok cert n Err* (*OK None*) (*states P mxs mxl*)
⟨*proof*⟩

lemma (**in** *start-context*) *wt-lbv-wt-step*:
assumes *lbv*: *wt-lbv P C Ts* T_r *mxs* mxl_0 *xt cert is*
shows ∃ τ*s* ∈ *nlists* (*size is*) *A*. *wt-step r Err step* τ*s* ∧ *OK first* $\sqsubseteq_r$ τ*s*!*0*⟨*proof*⟩

lemma (**in** *start-context*) *wt-lbv-wt-method*:
assumes *lbv*: *wt-lbv P C Ts* T_r *mxs* mxl_0 *xt cert is*
shows ∃ τ*s*. *wt-method P C Ts* T_r *mxs* mxl_0 *is xt* τ*s*⟨*proof*⟩

lemma (**in** *start-context*) *wt-method-wt-lbv*:
assumes *wt*: *wt-method P C Ts* T_r *mxs* mxl_0 *is xt* τ*s*
defines [*simp*]: *cert* ≡ *mk-cert P mxs* T_r *xt is* τ*s*

shows *wt-lbv P C Ts* T_r *mxs* mxl_0 *xt cert is*⟨*proof*⟩

theorem *jvm-lbv-correct*:
wt-jvm-prog-lbv P Cert ⟹ *wf-jvm-prog P*⟨*proof*⟩
theorem *jvm-lbv-complete*:
assumes *wt*: $\textit{wf-jvm-prog}_\Phi$ *P*
shows *wt-jvm-prog-lbv P* (*prg-cert P* Φ)⟨*proof*⟩
end

4.23 BV Type Safety Invariant

theory *BVConform*
imports *BVSpec ../JVM/JVMExec ../Common/Conform*
begin

definition *confT* :: *'c prog* ⇒ *heap* ⇒ *val* ⇒ *ty err* ⇒ *bool*
(‹-,- ⊢ - :≤⊤ -› [*51*,*51*,*51*,*51*] *50*)
where
P,*h* ⊢ *v* :≤⊤ *E* ≡ *case E of Err* ⇒ *True* | *OK T* ⇒ *P*,*h* ⊢ *v* :≤ *T*

notation (*ASCII*)
confT (‹-,- |– - :<=T -› [*51*,*51*,*51*,*51*] *50*)

abbreviation
confTs :: *'c prog* ⇒ *heap* ⇒ *val list* ⇒ ty_l ⇒ *bool*
(‹-,- ⊢ - [:≤⊤] -› [*51*,*51*,*51*,*51*] *50*) **where**
P,*h* ⊢ *vs* [:≤⊤] *Ts* ≡ *list-all2* (*confT P h*) *vs Ts*

notation (*ASCII*)
confTs (‹-,- |– - [:<=T] -› [*51*,*51*,*51*,*51*] *50*)

definition *conf-f* :: *jvm-prog* ⇒ *heap* ⇒ ty_i ⇒ *bytecode* ⇒ *frame* ⇒ *bool*
where
  conf-f P h ≡ λ(*ST*,*LT*) *is* (*stk*,*loc*,*C*,*M*,*pc*).
  P,*h* ⊢ *stk* [:≤] *ST* ∧ *P*,*h* ⊢ *loc* [:$\leq_\top$] *LT* ∧ *pc* < *size is*

lemma *conf-f-def2*:
  conf-f P h (*ST*,*LT*) *is* (*stk*,*loc*,*C*,*M*,*pc*) ≡
  P,*h* ⊢ *stk* [:≤] *ST* ∧ *P*,*h* ⊢ *loc* [:$\leq_\top$] *LT* ∧ *pc* < *size is*
  ⟨*proof*⟩

primrec *conf-fs* :: [*jvm-prog*,*heap*,ty_P,*mname*,*nat*,*ty*,*frame list*] ⇒ *bool*
where
  conf-fs P h Φ M_0 n_0 T_0 [] = *True*
| *conf-fs P h* Φ M_0 n_0 T_0 (*f*#*frs*) =
  (*let* (*stk*,*loc*,*C*,*M*,*pc*) = *f in*
  (∃ *ST LT Ts T mxs* mxl_0 *is xt*.
    Φ *C M* ! *pc* = *Some* (*ST*,*LT*) ∧
    (*P* ⊢ *C sees M*:*Ts* → *T* = (*mxs*,mxl_0,*is*,*xt*) *in C*) ∧
    (∃ *D Ts*′ *T*′ *m D*′.
      is!*pc* = (*Invoke* M_0 n_0) ∧ *ST*!n_0 = *Class D* ∧
      P ⊢ *D sees* M_0:*Ts*′ → *T*′ = *m in D*′ ∧ *P* ⊢ T_0 ≤ *T*′) ∧
    conf-f P h (*ST*, *LT*) *is f* ∧ *conf-fs P h* Φ *M* (*size Ts*) *T frs*))

definition *correct-state* :: [*jvm-prog*,ty_P,*jvm-state*] ⇒ *bool* (‹-,- ⊢ - √› [*61*,*0*,*0*] *61*)
where
  correct-state P Φ ≡ λ(*xp*,*h*,*frs*).
  case xp of
    None ⇒ (*case frs of*
        [] ⇒ *True*
        | (*f*#*fs*) ⇒ *P*⊢ *h*√ ∧
        (*let* (*stk*,*loc*,*C*,*M*,*pc*) = *f*
         in ∃ *Ts T mxs* mxl_0 *is xt* τ.
            (*P* ⊢ *C sees M*:*Ts*→*T* = (*mxs*,mxl_0,*is*,*xt*) *in C*) ∧
            Φ *C M* ! *pc* = *Some* τ ∧
            conf-f P h τ *is f* ∧ *conf-fs P h* Φ *M* (*size Ts*) *T fs*))
  | *Some x* ⇒ *frs* = []

notation
  correct-state (‹-,- |− - [*ok*]› [*61*,*0*,*0*] *61*)

4.23.1 Values and ⊤

lemma *confT-Err* [*iff*]: *P*,*h* ⊢ *x* :$\leq_\top$ *Err*
  ⟨*proof*⟩

lemma *confT-OK* [*iff*]: *P*,*h* ⊢ *x* :$\leq_\top$ *OK T* = (*P*,*h* ⊢ *x* :≤ *T*)
  ⟨*proof*⟩

lemma *confT-cases*:
  P,*h* ⊢ *x* :$\leq_\top$ *X* = (*X* = *Err* ∨ (∃ *T*. *X* = *OK T* ∧ *P*,*h* ⊢ *x* :≤ *T*))
  ⟨*proof*⟩

lemma *confT-hext* [*intro?*, *trans*]:
$[\![P,h \vdash x :\leq_\top T; h \trianglelefteq h']\!] \Longrightarrow P,h' \vdash x :\leq_\top T$
⟨*proof*⟩

lemma *confT-widen* [*intro?*, *trans*]:
$[\![P,h \vdash x :\leq_\top T; P \vdash T \leq_\top T']\!] \Longrightarrow P,h \vdash x :\leq_\top T'$
⟨*proof*⟩

4.23.2 Stack and Registers

lemmas *confTs-Cons1* [*iff*] = *list-all2-Cons1* [*of confT P h*] **for** *P h*

lemma *confTs-confT-sup*:
$[\![P,h \vdash loc [:\leq_\top] LT; n < size\ LT; LT!n = OK\ T; P \vdash T \leq T']\!]$
$\Longrightarrow P,h \vdash (loc!n) :\leq T'$⟨*proof*⟩
lemma *confTs-hext* [*intro?*]:
$P,h \vdash loc [:\leq_\top] LT \Longrightarrow h \trianglelefteq h' \Longrightarrow P,h' \vdash loc [:\leq_\top] LT$
⟨*proof*⟩

lemma *confTs-widen* [*intro?*, *trans*]:
$P,h \vdash loc [:\leq_\top] LT \Longrightarrow P \vdash LT [\leq_\top] LT' \Longrightarrow P,h \vdash loc [:\leq_\top] LT'$
⟨*proof*⟩

lemma *confTs-map* [*iff*]:
$\bigwedge vs. (P,h \vdash vs [:\leq_\top] map\ OK\ Ts) = (P,h \vdash vs [:\leq] Ts)$
⟨*proof*⟩

lemma *reg-widen-Err* [*iff*]:
$\bigwedge LT. (P \vdash replicate\ n\ Err [\leq_\top] LT) = (LT = replicate\ n\ Err)$
⟨*proof*⟩

lemma *confTs-Err* [*iff*]:
$P,h \vdash replicate\ n\ v [:\leq_\top] replicate\ n\ Err$
⟨*proof*⟩

4.23.3 correct-frames

lemmas [*simp del*] = *fun-upd-apply*

lemma *conf-fs-hext*:
$\bigwedge M\ n\ T_r.$
$[\![conf\text{-}fs\ P\ h\ \Phi\ M\ n\ T_r\ frs; h \trianglelefteq h']\!] \Longrightarrow conf\text{-}fs\ P\ h'\ \Phi\ M\ n\ T_r\ frs$⟨*proof*⟩
end

4.24 BV Type Safety Proof

theory *BVSpecTypeSafe*
imports *BVConform*
begin

This theory contains proof that the specification of the bytecode verifier only admits type safe programs.

4.24.1 Preliminaries

Simp and intro setup for the type safety proof:

lemmas *defs1* = *correct-state-def conf-f-def wt-instr-def eff-def norm-eff-def app-def xcpt-app-def*

lemmas *widen-rules* [*intro*] = *conf-widen confT-widen confs-widens confTs-widen*

4.24.2 Exception Handling

For the *Invoke* instruction the BV has checked all handlers that guard the current *pc*.

lemma *Invoke-handlers*:
match-ex-table P C pc xt = *Some* (*pc*′,*d*′) ⟹
∃(*f*,*t*,*D*,*h*,*d*) ∈ *set* (*relevant-entries P* (*Invoke n M*) *pc xt*).
P ⊢ *C* ⪯* *D* ∧ *pc* ∈ {*f*..<*t*} ∧ *pc*′ = *h* ∧ *d*′ = *d*
⟨*proof*⟩

We can prove separately that the recursive search for exception handlers (*find-handler*) in the frame stack results in a conforming state (if there was no matching exception handler in the current frame). We require that the exception is a valid heap address, and that the state before the exception occurred conforms.

term *find-handler*
lemma *uncaught-xcpt-correct*:
assumes *wt*: *wf-jvm-prog*$_\Phi$ *P*
assumes *h*: *h xcp* = *Some obj*
shows ⋀*f*. *P*,Φ ⊢ (*None*, *h*, *f*#*frs*)√ ⟹ *P*,Φ ⊢ (*find-handler P xcp h frs*) √
(**is** ⋀*f*. *?correct* (*None*, *h*, *f*#*frs*) ⟹ *?correct* (*?find frs*))⟨*proof*⟩

The requirement of lemma *uncaught-xcpt-correct* (that the exception is a valid reference on the heap) is always met for welltyped instructions and conformant states:

lemma *exec-instr-xcpt-h*:
⟦ *fst* (*exec-instr* (*ins*!*pc*) *P h stk vars Cl M pc frs*) = *Some xcp*;
P,*T*,*mxs*,*size ins*,*xt* ⊢ *ins*!*pc*,*pc* :: Φ *C M*;
P,Φ ⊢ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)√ ⟧
⟹ ∃ *obj*. *h xcp* = *Some obj*
(**is** ⟦ *?xcpt*; *?wt*; *?correct* ⟧ ⟹ *?thesis*)⟨*proof*⟩
lemma *conf-sys-xcpt*:
⟦*preallocated h*; *C* ∈ *sys-xcpts*⟧ ⟹ *P*,*h* ⊢ *Addr* (*addr-of-sys-xcpt C*) :≤ *Class C*
⟨*proof*⟩

lemma *match-ex-table-SomeD*:
match-ex-table P C pc xt = *Some* (*pc*′,*d*′) ⟹
∃(*f*,*t*,*D*,*h*,*d*) ∈ *set xt*. *matches-ex-entry P C pc* (*f*,*t*,*D*,*h*,*d*) ∧ *h* = *pc*′ ∧ *d*=*d*′
⟨*proof*⟩

Finally we can state that, whenever an exception occurs, the next state always conforms:

lemma *xcpt-correct*:
fixes σ′ :: *jvm-state*
assumes *wtp*: *wf-jvm-prog*$_\Phi$ *P*
assumes *meth*: *P* ⊢ *C sees M*:*Ts*→*T*=(*mxs*,mxl_0,*ins*,*xt*) *in C*
assumes *wt*: *P*,*T*,*mxs*,*size ins*,*xt* ⊢ *ins*!*pc*,*pc* :: Φ *C M*
assumes *xp*: *fst* (*exec-instr* (*ins*!*pc*) *P h stk loc C M pc frs*) = *Some xcp*
assumes *s*′: *Some* σ′ = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)
assumes *correct*: *P*,Φ ⊢ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)√
shows *P*,Φ ⊢ σ′√⟨*proof*⟩

4.24.3 Single Instructions

In this section we prove for each single (welltyped) instruction that the state after execution of the instruction still conforms. Since we have already handled exceptions above, we can now assume that no exception occurs in this step.

declare *defs1* [*simp*]

lemma *Invoke-correct*:
  fixes σ' :: *jvm-state*
  assumes *wtprog*: *wf-jvm-prog*$_\Phi$ *P*
  assumes *meth-C*: $P \vdash C$ *sees* M:$Ts \rightarrow T$=(*mxs*,mxl_0,*ins*,*xt*) *in* *C*
  assumes *ins*: *ins* ! *pc* = *Invoke* M' *n*
  assumes *wti*: *P*,*T*,*mxs*,*size ins*,*xt* ⊢ *ins*!*pc*,*pc* :: Φ *C M*
  assumes σ': *Some* σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)
  assumes *approx*: *P*,Φ ⊢ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)√
  assumes *no-xcp*: *fst* (*exec-instr* (*ins*!*pc*) *P h stk loc C M pc frs*) = *None*
  shows *P*,Φ ⊢ σ'√⟨*proof*⟩
declare *list-all2-Cons2* [*iff*]

lemma *Return-correct*:
  fixes σ' :: *jvm-state*
  assumes *wt-prog*: *wf-jvm-prog*$_\Phi$ *P*
  assumes *meth*: $P \vdash C$ *sees* M:$Ts \rightarrow T$=(*mxs*,mxl_0,*ins*,*xt*) *in* *C*
  assumes *ins*: *ins* ! *pc* = *Return*
  assumes *wt*: *P*,*T*,*mxs*,*size ins*,*xt* ⊢ *ins*!*pc*,*pc* :: Φ *C M*
  assumes s': *Some* σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)
  assumes *correct*: *P*,Φ ⊢ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)√

  shows *P*,Φ ⊢ σ'√⟨*proof*⟩
declare *sup-state-opt-any-Some* [*iff*]
declare *not-Err-eq* [*iff*]

lemma *Load-correct*:
⟦ *wf-prog wt P*;
    $P \vdash C$ *sees* M:$Ts \rightarrow T$=(*mxs*,mxl_0,*ins*,*xt*) *in* *C*;
    ins!*pc* = *Load idx*;
    P,*T*,*mxs*,*size ins*,*xt* ⊢ *ins*!*pc*,*pc* :: Φ *C M*;
    Some σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*);
    P,Φ ⊢ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)√ ⟧
⟹ *P*,Φ ⊢ σ'√
  ⟨*proof*⟩

declare [[*simproc del*: *list-to-set-comprehension*]]

lemma *Store-correct*:
⟦ *wf-prog wt P*;
  $P \vdash C$ *sees* M:$Ts \rightarrow T$=(*mxs*,mxl_0,*ins*,*xt*) *in* *C*;
  ins!*pc* = *Store idx*;
  P,*T*,*mxs*,*size ins*,*xt* ⊢ *ins*!*pc*,*pc* :: Φ *C M*;
  Some σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*);
  P,Φ ⊢ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)√ ⟧
⟹ *P*,Φ ⊢ σ'√⟨*proof*⟩

lemma *Push-correct*:

⟦ *wf-prog wt P*;
  $P \vdash C$ *sees* $M{:}Ts \rightarrow T = (mxs, mxl_0, ins, xt)$ *in* C;
  ins!*pc* = *Push v*;
  $P, T, mxs, size\ ins, xt \vdash ins!pc, pc :: \Phi\ C\ M$;
  Some σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*);
  $P,\Phi \vdash$ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)$\surd$ ⟧
$\Longrightarrow P,\Phi \vdash \sigma'\surd$⟨*proof*⟩

lemma *Cast-conf2*:
⟦ *wf-prog ok P*; $P,h \vdash v :\leq T$; *is-refT T*; *cast-ok P C h v*;
  $P \vdash$ *Class* $C \leq T'$; *is-class P C*⟧
$\Longrightarrow P,h \vdash v :\leq T'$⟨*proof*⟩

lemma *Checkcast-correct*:
⟦ *wf-jvm-prog*$_\Phi$ *P*;
  $P \vdash C$ *sees* $M{:}Ts \rightarrow T = (mxs, mxl_0, ins, xt)$ *in* C;
  ins!*pc* = *Checkcast D*;
  $P, T, mxs, size\ ins, xt \vdash ins!pc, pc :: \Phi\ C\ M$;
  Some σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*) ;
  $P,\Phi \vdash$ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)$\surd$;
  fst (*exec-instr* (*ins*!*pc*) *P h stk loc C M pc frs*) = *None* ⟧
$\Longrightarrow P,\Phi \vdash \sigma'\surd$⟨*proof*⟩
declare *split-paired-All* [*simp del*]

lemmas *widens-Cons* [*iff*] = *list-all2-Cons1* [*of widen P*] **for** *P*

lemma *Getfield-correct*:
fixes σ' :: *jvm-state*
assumes *wf*: *wf-prog wt P*
assumes *mC*: $P \vdash C$ *sees* $M{:}Ts \rightarrow T = (mxs, mxl_0, ins, xt)$ *in* C
assumes *i*: *ins*!*pc* = *Getfield F D*
assumes *wt*: $P, T, mxs, size\ ins, xt \vdash ins!pc, pc :: \Phi\ C\ M$
assumes *s'*: *Some* σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)
assumes *cf*: $P,\Phi \vdash$ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)$\surd$
assumes *xc*: *fst* (*exec-instr* (*ins*!*pc*) *P h stk loc C M pc frs*) = *None*

shows $P,\Phi \vdash \sigma'\surd$⟨*proof*⟩
lemma *Putfield-correct*:
fixes σ' :: *jvm-state*
assumes *wf*: *wf-prog wt P*
assumes *mC*: $P \vdash C$ *sees* $M{:}Ts \rightarrow T = (mxs, mxl_0, ins, xt)$ *in* C
assumes *i*: *ins*!*pc* = *Putfield F D*
assumes *wt*: $P, T, mxs, size\ ins, xt \vdash ins!pc, pc :: \Phi\ C\ M$
assumes *s'*: *Some* σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)
assumes *cf*: $P,\Phi \vdash$ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)$\surd$
assumes *xc*: *fst* (*exec-instr* (*ins*!*pc*) *P h stk loc C M pc frs*) = *None*

shows $P,\Phi \vdash \sigma'\surd$⟨*proof*⟩

lemma *has-fields-b-fields*:
$P \vdash C$ *has-fields FDTs* $\Longrightarrow$ *fields P C* = *FDTs*⟨*proof*⟩

lemma *oconf-blank* [*intro*, *simp*]:
⟦*is-class P C*; *wf-prog wt P*⟧ $\Longrightarrow P,h \vdash$ *blank P C* $\surd$⟨*proof*⟩

lemma *obj-ty-blank* [*iff*]: *obj-ty* (*blank P C*) = *Class C*
⟨*proof*⟩

lemma *New-correct*:
fixes σ' :: *jvm-state*
assumes *wf*: *wf-prog wt P*
assumes *meth*: $P \vdash C$ *sees* M:$Ts \rightarrow T$=(mxs,mxl_0,ins,xt) *in C*
assumes *ins*: *ins*!*pc* = *New X*
assumes *wt*: $P,T,mxs,size\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M$
assumes *exec*: *Some* σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)
assumes *conf*: $P,\Phi \vdash$ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)$\surd$
assumes *no-x*: *fst* (*exec-instr* (*ins*!*pc*) *P h stk loc C M pc frs*) = *None*
shows $P,\Phi \vdash \sigma'\surd$⟨*proof*⟩

lemma *Goto-correct*:
⟦ *wf-prog wt P*;
$P \vdash C$ *sees* M:$Ts \rightarrow T$=(mxs,mxl_0,ins,xt) *in C*;
ins ! *pc* = *Goto branch*;
$P,T,mxs,size\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M$;
Some σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*) ;
$P,\Phi \vdash$ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)$\surd$ ⟧
$\Longrightarrow P,\Phi \vdash \sigma'\surd$⟨*proof*⟩

lemma *IfFalse-correct*:
⟦ *wf-prog wt P*;
$P \vdash C$ *sees* M:$Ts \rightarrow T$=(mxs,mxl_0,ins,xt) *in C*;
ins ! *pc* = *IfFalse branch*;
$P,T,mxs,size\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M$;
Some σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*) ;
$P,\Phi \vdash$ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)$\surd$ ⟧
$\Longrightarrow P,\Phi \vdash \sigma'\surd$⟨*proof*⟩
lemma *CmpEq-correct*:
⟦ *wf-prog wt P*;
$P \vdash C$ *sees* M:$Ts \rightarrow T$=(mxs,mxl_0,ins,xt) *in C*;
ins ! *pc* = *CmpEq*;
$P,T,mxs,size\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M$;
Some σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*) ;
$P,\Phi \vdash$ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)$\surd$ ⟧
$\Longrightarrow P,\Phi \vdash \sigma'\surd$⟨*proof*⟩
lemma *Pop-correct*:
⟦ *wf-prog wt P*;
$P \vdash C$ *sees* M:$Ts \rightarrow T$=(mxs,mxl_0,ins,xt) *in C*;
ins ! *pc* = *Pop*;
$P,T,mxs,size\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M$;
Some σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*) ;
$P,\Phi \vdash$ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)$\surd$ ⟧
$\Longrightarrow P,\Phi \vdash \sigma'\surd$⟨*proof*⟩

lemma *IAdd-correct*:
⟦ *wf-prog wt P*;
$P \vdash C$ *sees* M:$Ts \rightarrow T$=(mxs,mxl_0,ins,xt) *in C*;
ins ! *pc* = *IAdd*;
$P,T,mxs,size\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M$;
Some σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*) ;

$P,\Phi \vdash (None,\ h,\ (stk,loc,C,M,pc)\#frs)\surd$ ⟧
$\Longrightarrow P,\Phi \vdash \sigma'\surd$⟨*proof*⟩

lemma *Throw-correct*:
⟦ *wf-prog wt P*;
$P \vdash C$ *sees* $M{:}Ts{\rightarrow}T{=}(mxs,mxl_0,ins,xt)$ *in* C;
ins ! *pc* = *Throw*;
Some σ' = *exec* $(P,\ None,\ h,\ (stk,loc,C,M,pc)\#frs)$;
$P,\Phi \vdash (None,\ h,\ (stk,loc,C,M,pc)\#frs)\surd$;
fst (*exec-instr* (*ins*!*pc*) *P h stk loc C M pc frs*) = *None* ⟧
$\Longrightarrow P,\Phi \vdash \sigma'\surd$
⟨*proof*⟩

The next theorem collects the results of the sections above, i.e. exception handling and the execution step for each instruction. It states type safety for single step execution: in welltyped programs, a conforming state is transformed into another conforming state when one instruction is executed.

theorem *instr-correct*:
⟦ $wf\text{-}jvm\text{-}prog_{\Phi}$ P;
$P \vdash C$ *sees* $M{:}Ts{\rightarrow}T{=}(mxs,mxl_0,ins,xt)$ *in* C;
Some σ' = *exec* $(P,\ None,\ h,\ (stk,loc,C,M,pc)\#frs)$;
$P,\Phi \vdash (None,\ h,\ (stk,loc,C,M,pc)\#frs)\surd$ ⟧
$\Longrightarrow P,\Phi \vdash \sigma'\surd$⟨*proof*⟩

4.24.4 Main

lemma *correct-state-impl-Some-method*:
$P,\Phi \vdash (None,\ h,\ (stk,loc,C,M,pc)\#frs)\surd$
$\Longrightarrow \exists\, m\ Ts\ T.\ P \vdash C$ *sees* $M{:}Ts{\rightarrow}T = m$ *in* C
⟨*proof*⟩

lemma *BV-correct-1* [*rule-format*]:
$\bigwedge\sigma.$ ⟦ $wf\text{-}jvm\text{-}prog_{\Phi}$ P; $P,\Phi \vdash \sigma\surd$⟧ $\Longrightarrow P \vdash \sigma\ {-jvm{\rightarrow}_1}\ \sigma' \longrightarrow P,\Phi \vdash \sigma'\surd$⟨*proof*⟩

theorem *progress*:
⟦ $xp{=}None$; $frs{\neq}[]$ ⟧ $\Longrightarrow \exists\sigma'.\ P \vdash (xp,h,frs)\ {-jvm{\rightarrow}_1}\ \sigma'$
⟨*proof*⟩

lemma *progress-conform*:
⟦$wf\text{-}jvm\text{-}prog_{\Phi}$ P; $P,\Phi \vdash (xp,h,frs)\surd$; $xp{=}None$; $frs{\neq}[]$⟧
$\Longrightarrow \exists\sigma'.\ P \vdash (xp,h,frs)\ {-jvm{\rightarrow}_1}\ \sigma' \wedge P,\Phi \vdash \sigma'\surd$⟨*proof*⟩
theorem *BV-correct* [*rule-format*]:
⟦ $wf\text{-}jvm\text{-}prog_{\Phi}$ P; $P \vdash \sigma\ {-jvm{\rightarrow}}\ \sigma'$ ⟧ $\Longrightarrow P,\Phi \vdash \sigma\surd \longrightarrow P,\Phi \vdash \sigma'\surd$⟨*proof*⟩
lemma *hconf-start*:
assumes *wf*: *wf-prog wf-mb P*
shows $P \vdash$ (*start-heap P*) $\surd$⟨*proof*⟩
lemma *BV-correct-initial*:
shows ⟦ $wf\text{-}jvm\text{-}prog_{\Phi}$ P; $P \vdash C$ *sees* $M{:}[]{\rightarrow}T = m$ *in* C ⟧
$\Longrightarrow P,\Phi \vdash$ *start-state P C M* $\surd$⟨*proof*⟩
theorem *typesafe*:
assumes *welltyped*: $wf\text{-}jvm\text{-}prog_{\Phi}$ P
assumes *main-method*: $P \vdash C$ *sees* $M{:}[]{\rightarrow}T = m$ *in* C
shows $P \vdash$ *start-state P C M* $-jvm{\rightarrow}\ \sigma \Longrightarrow P,\Phi \vdash \sigma\ \surd$⟨*proof*⟩
end

4.25 Welltyped Programs produce no Type Errors

theory *BVNoTypeError*
imports *../JVM/JVMDefensive BVSpecTypeSafe*
begin

lemma *has-methodI*:
$P \vdash C$ *sees* $M{:}Ts{\rightarrow}T = m$ *in* $D \Longrightarrow P \vdash C$ *has* M
⟨*proof*⟩

Some simple lemmas about the type testing functions of the defensive JVM:

lemma *typeof-NoneD* [*simp*,*dest*]: *typeof* v = *Some* $x \Longrightarrow \neg$*is-Addr* v
⟨*proof*⟩

lemma *is-Ref-def2*:
is-Ref $v = (v =$ *Null* $\vee (\exists a.\ v =$ *Addr* $a))$
⟨*proof*⟩

lemma [*iff*]: *is-Ref Null* ⟨*proof*⟩

lemma *is-RefI* [*intro*, *simp*]: $P,h \vdash v :\leq T \Longrightarrow$ *is-refT* $T \Longrightarrow$ *is-Ref* v⟨*proof*⟩
lemma *is-IntgI* [*intro*, *simp*]: $P,h \vdash v :\leq$ *Integer* $\Longrightarrow$ *is-Intg* v⟨*proof*⟩
lemma *is-BoolI* [*intro*, *simp*]: $P,h \vdash v :\leq$ *Boolean* $\Longrightarrow$ *is-Bool* v⟨*proof*⟩
declare *defs1* [*simp del*]

lemma *wt-jvm-prog-states*:
⟦ *wf-jvm-prog*$_\Phi$ P; $P \vdash C$ *sees* M: $Ts{\rightarrow}T = (mxs, mxl, ins, et)$ *in* C;
Φ C M ! $pc = \tau$; $pc <$ *size ins* ⟧
$\Longrightarrow$ *OK* $\tau \in$ *states* P *mxs* (1+*size* Ts+*mxl*)⟨*proof*⟩

The main theorem: welltyped programs do not produce type errors if they are started in a conformant state.

theorem *no-type-error*:
fixes σ :: *jvm-state*
assumes *welltyped*: *wf-jvm-prog*$_\Phi$ P **and** *conforms*: $P,\Phi \vdash \sigma\ \surd$
shows *exec-d* P $\sigma \neq$ *TypeError*⟨*proof*⟩

The theorem above tells us that, in welltyped programs, the defensive machine reaches the same result as the aggressive one (after arbitrarily many steps).

theorem *welltyped-aggressive-imp-defensive*:
wf-jvm-prog$_\Phi$ $P \Longrightarrow P,\Phi \vdash \sigma\ \surd \Longrightarrow P \vdash \sigma$ −*jvm*→ σ'
$\Longrightarrow P \vdash$ (*Normal* σ) −*jvmd*→ (*Normal* σ')⟨*proof*⟩

As corollary we get that the aggressive and the defensive machine are equivalent for welltyped programs (if started in a conformant state or in the canonical start state)

corollary *welltyped-commutes*:
fixes σ :: *jvm-state*
assumes *wf*: *wf-jvm-prog*$_\Phi$ P **and** *conforms*: $P,\Phi \vdash \sigma\ \surd$
shows $P \vdash$ (*Normal* σ) −*jvmd*→ (*Normal* σ') = $P \vdash \sigma$ −*jvm*→ σ'
⟨*proof*⟩

corollary *welltyped-initial-commutes*:
assumes *wf*: *wf-jvm-prog* P
assumes *meth*: $P \vdash C$ *sees* $M{:}[]{\rightarrow}T = b$ *in* C

defines *start*: $\sigma \equiv$ *start-state P C M*
shows $P \vdash (Normal\ \sigma)\ {-jvmd\rightarrow}\ (Normal\ \sigma') = P \vdash \sigma\ {-jvm\rightarrow}\ \sigma'$
⟨*proof*⟩

lemma *not-TypeError-eq* [*iff*]:
$x \neq TypeError = (\exists\, t.\ x = Normal\ t)$
⟨*proof*⟩

locale *cnf* =
fixes *P* **and** Φ **and** σ
assumes *wf*: *wf-jvm-prog*$_\Phi$ *P*
assumes *cnf*: *correct-state* $P\ \Phi\ \sigma$

theorem (**in** *cnf*) *no-type-errors*:
$P \vdash (Normal\ \sigma)\ {-jvmd\rightarrow}\ \sigma' \Longrightarrow \sigma' \neq TypeError$
⟨*proof*⟩

locale *start* =
fixes *P* **and** *C* **and** *M* **and** σ **and** *T* **and** *b*
assumes *wf*: *wf-jvm-prog P*
assumes *sees*: $P \vdash C$ *sees* M:[]$\rightarrow T = b$ *in* C
defines $\sigma \equiv Normal\ (start\text{-}state\ P\ C\ M)$

corollary (**in** *start*) *bv-no-type-error*:
shows $P \vdash \sigma\ {-jvmd\rightarrow}\ \sigma' \Longrightarrow \sigma' \neq TypeError$
⟨*proof*⟩

end

4.26 Example Welltypings

theory *BVExample*
imports *../JVM/JVMListExample BVSpecTypeSafe BVExec*
HOL−Library.Code-Target-Numeral
begin

This theory shows type correctness of the example program in section 3.7 (p. 70) by explicitly providing a welltyping. It also shows that the start state of the program conforms to the welltyping; hence type safe execution is guaranteed.

4.26.1 Setup

lemma *distinct-classes'*:
list-name ≠ *test-name*
list-name ≠ *Object*
list-name ≠ *ClassCast*
list-name ≠ *OutOfMemory*
list-name ≠ *NullPointer*
test-name ≠ *Object*
test-name ≠ *OutOfMemory*
test-name ≠ *ClassCast*

test-name ≠ *NullPointer*
ClassCast ≠ *NullPointer*
ClassCast ≠ *Object*
NullPointer ≠ *Object*
OutOfMemory ≠ *ClassCast*
OutOfMemory ≠ *NullPointer*
OutOfMemory ≠ *Object*
⟨*proof*⟩

lemmas *distinct-classes* = *distinct-classes′ distinct-classes′* [*symmetric*]

lemma *distinct-fields*:
val-name ≠ *next-name*
next-name ≠ *val-name*
⟨*proof*⟩

Abbreviations for definitions we will have to use often in the proofs below:

lemmas *system-defs* = *SystemClasses-def ObjectC-def NullPointerC-def*
OutOfMemoryC-def ClassCastC-def
lemmas *class-defs* = *list-class-def test-class-def*

These auxiliary proofs are for efficiency: class lookup, subclass relation, method and field lookup are computed only once:

lemma *class-Object* [*simp*]:
class E Object = *Some* (*undefined*, [],[])
⟨*proof*⟩

lemma *class-NullPointer* [*simp*]:
class E NullPointer = *Some* (*Object*, [], [])
⟨*proof*⟩

lemma *class-OutOfMemory* [*simp*]:
class E OutOfMemory = *Some* (*Object*, [], [])
⟨*proof*⟩

lemma *class-ClassCast* [*simp*]:
class E ClassCast = *Some* (*Object*, [], [])
⟨*proof*⟩

lemma *class-list* [*simp*]:
class E list-name = *Some list-class*
⟨*proof*⟩

lemma *class-test* [*simp*]:
class E test-name = *Some test-class*
⟨*proof*⟩

lemma *E-classes* [*simp*]:
{*C*. *is-class E C*} = {*list-name*, *test-name*, *NullPointer*,
ClassCast, *OutOfMemory*, *Object*}
⟨*proof*⟩

The subclass releation spelled out:

lemma *subcls1*:

subcls1 E = {(*list-name*,*Object*), (*test-name*,*Object*), (*NullPointer*, *Object*),
(*ClassCast*, *Object*), (*OutOfMemory*, *Object*)}⟨*proof*⟩

The subclass relation is acyclic; hence its converse is well founded:

lemma *notin-rtrancl*:
$(a,b) \in r^* \Longrightarrow a \neq b \Longrightarrow (\bigwedge y.\ (a,y) \notin r) \Longrightarrow False$
⟨*proof*⟩

lemma *acyclic-subcls1-E*: *acyclic* (*subcls1 E*)⟨*proof*⟩
lemma *wf-subcls1-E*: *wf* $((subcls1\ E)^{-1})$⟨*proof*⟩

Method and field lookup:

lemma *method-append* [*simp*]:
method E list-name append-name =
(*list-name*, [*Class list-name*], *Void*, *3*, *0*, *append-ins*, [(*1*, *2*, *NullPointer*, *7*, *0*)])⟨*proof*⟩
lemma *method-makelist* [*simp*]:
method E test-name makelist-name =
(*test-name*, [], *Void*, *3*, *2*, *make-list-ins*, [])⟨*proof*⟩
lemma *field-val* [*simp*]:
field E list-name val-name = (*list-name*, *Integer*)⟨*proof*⟩
lemma *field-next* [*simp*]:
field E list-name next-name = (*list-name*, *Class list-name*)⟨*proof*⟩
lemma [*simp*]: *fields E Object* = []
⟨*proof*⟩

lemma [*simp*]: *fields E NullPointer* = []
⟨*proof*⟩

lemma [*simp*]: *fields E ClassCast* = []
⟨*proof*⟩

lemma [*simp*]: *fields E OutOfMemory* = []
⟨*proof*⟩

lemma [*simp*]: *fields E test-name* = []⟨*proof*⟩
lemmas [*simp*] = *is-class-def*

4.26.2 Program structure

The program is structurally wellformed:

lemma *wf-struct*:
wf-prog (λ*G C mb*. *True*) *E* (**is** *wf-prog ?mb E*)⟨*proof*⟩

4.26.3 Welltypings

We show welltypings of the methods *append-name* in class *list-name*, and *makelist-name* in class *test-name*:

lemmas *eff-simps* [*simp*] = *eff-def norm-eff-def xcpt-eff-def*

definition *phi-append* :: ty_m (‹φ_a›)
where
$\varphi_a \equiv$ *map* (λ(*x*,*y*). *Some* (*x*, *map OK y*)) [

([], [*Class list-name*, *Class list-name*]),
([*Class list-name*], [*Class list-name*, *Class list-name*]),
([*Class list-name*], [*Class list-name*, *Class list-name*]),
([*Class list-name*, *Class list-name*], [*Class list-name*, *Class list-name*]),
([*Class list-name*, *Class list-name*], [*Class list-name*, *Class list-name*]),
([*NT*, *Class list-name*, *Class list-name*], [*Class list-name*, *Class list-name*]),
([*Boolean*, *Class list-name*], [*Class list-name*, *Class list-name*]),

([*Class Object*], [*Class list-name*, *Class list-name*]),
([], [*Class list-name*, *Class list-name*]),
([*Class list-name*], [*Class list-name*, *Class list-name*]),
([*Class list-name*, *Class list-name*], [*Class list-name*, *Class list-name*]),
([], [*Class list-name*, *Class list-name*]),
([*Void*], [*Class list-name*, *Class list-name*]),

([*Class list-name*], [*Class list-name*, *Class list-name*]),
([*Class list-name*, *Class list-name*], [*Class list-name*, *Class list-name*]),
([*Void*], [*Class list-name*, *Class list-name*])]

The next definition and three proof rules implement an algorithm to enumarate natural numbers. The command *apply* (*elim pc-end pc-next pc-0* transforms a goal of the form

pc < *n* ⟹ *P pc*

into a series of goals

P 0

P (*Suc 0*)

. . .

P n

definition *intervall* :: *nat* ⇒ *nat* ⇒ *nat* ⇒ *bool* (‹- ∈ [-, -′)›)
where
x ∈ [*a*, *b*) ≡ *a* ≤ *x* ∧ *x* < *b*

lemma *pc-0*: *x* < *n* ⟹ (*x* ∈ [*0*, *n*) ⟹ *P x*) ⟹ *P x*
⟨*proof*⟩

lemma *pc-next*: *x* ∈ [*n0*, *n*) ⟹ *P n0* ⟹ (*x* ∈ [*Suc n0*, *n*) ⟹ *P x*) ⟹ *P x*⟨*proof*⟩
lemma *pc-end*: *x* ∈ [*n*,*n*) ⟹ *P x*
⟨*proof*⟩

lemma *types-append* [*simp*]: *check-types E 3* (*Suc* (*Suc 0*)) (*map OK* φ_a)⟨*proof*⟩
lemma *wt-append* [*simp*]:
wt-method E list-name [*Class list-name*] *Void 3 0 append-ins*
[(*Suc 0*, *2*, *NullPointer*, *7*, *0*)] φ_a⟨*proof*⟩

Some abbreviations for readability

abbreviation *Clist* == *Class list-name*
abbreviation *Ctest* == *Class test-name*

definition *phi-makelist* :: ty_m (‹φ_m›)
where
$\varphi_m \equiv$ *map* ($\lambda(x,y)$. *Some* (x, y)) [
([], [*OK Ctest*, *Err* , *Err*]),
([*Clist*], [*OK Ctest*, *Err* , *Err*]),
([], [*OK Clist*, *Err* , *Err*]),
([*Clist*], [*OK Clist*, *Err* , *Err*]),
([*Integer*, *Clist*], [*OK Clist*, *Err* , *Err*]),

([], [*OK Clist*, *Err* , *Err*]),
([*Clist*], [*OK Clist*, *Err* , *Err*]),
([], [*OK Clist*, *OK Clist*, *Err*]),
([*Clist*], [*OK Clist*, *OK Clist*, *Err*]),
([*Integer*, *Clist*], [*OK Clist*, *OK Clist*, *Err*]),

([], [*OK Clist*, *OK Clist*, *Err*]),
([*Clist*], [*OK Clist*, *OK Clist*, *Err*]),
([], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Clist*], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Integer*, *Clist*], [*OK Clist*, *OK Clist*, *OK Clist*]),

([], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Clist*], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Clist*, *Clist*], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Void*], [*OK Clist*, *OK Clist*, *OK Clist*]),
([], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Clist*], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Clist*, *Clist*], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Void*], [*OK Clist*, *OK Clist*, *OK Clist*])]

lemma *types-makelist* [*simp*]: *check-types E 3* (*Suc* (*Suc* (*Suc 0*))) (*map OK* φ_m)⟨*proof*⟩
lemma *wt-makelist* [*simp*]:
wt-method E test-name [] *Void 3 2 make-list-ins* [] φ_m⟨*proof*⟩
lemma *wf-md'E*:
⟦ *wf-prog wf-md P*;
$\bigwedge$*C S fs ms m*.⟦(*C*,*S*,*fs*,*ms*) ∈ *set P*; *m* ∈ *set ms*⟧ ⟹ *wf-md' P C m* ⟧
⟹ *wf-prog wf-md' P*⟨*proof*⟩

The whole program is welltyped:

definition *Phi* :: ty_P (‹Φ›)
where
Φ *C mn* ≡ *if C* = *test-name* ∧ *mn* = *makelist-name then* φ_m *else*
if C = *list-name* ∧ *mn* = *append-name then* φ_a *else* []

lemma *wf-prog*:
$\textit{wf-jvm-prog}_\Phi$ *E*⟨*proof*⟩

4.26.4 Conformance

Execution of the program will be typesafe, because its start state conforms to the welltyping:

lemma *E*,Φ ⊢ *start-state E test-name makelist-name* √⟨*proof*⟩

4.26.5 Example for code generation: inferring method types

definition *test-kil* :: *jvm-prog* ⇒ *cname* ⇒ *ty list* ⇒ *ty* ⇒ *nat* ⇒ *nat* ⇒
ex-table ⇒ *instr list* ⇒ *ty*$_i$′ *err list*
where
test-kil G C pTs rT mxs mxl et instr ≡
(*let first* = *Some* ([],(*OK* (*Class C*))#(*map OK pTs*)@(*replicate mxl Err*));
start = *OK first*#(*replicate* (*size instr* − *1*) (*OK None*))
in kiljvm G mxs (*1*+*size pTs*+*mxl*) *rT instr et start*)

lemma [*code*]:
unstables r step ss =
fold (λ*p A*. *if* ¬*stable r step ss p then insert p A else A*) [*0*..<*size ss*] {}
⟨*proof*⟩

declare *some-elem-def* [*code del*]
code-printing
constant *some-elem* ⇀ (*SML*) (*case/ - of/ Set/ xs/ =>/ hd/ xs*)

This code setup is just a demonstration and *not* sound!

notepad begin
⟨*proof*⟩
end

lemma [*code*]:
iter f step ss w = *while* (λ(*ss*, *w*). ¬ *Set.is-empty w*)
(λ(*ss*, *w*).
let p = *some-elem w in propa f* (*step p* (*ss* ! *p*)) *ss* (*w* − {*p*}))
(*ss*, *w*)
⟨*proof*⟩

lemma *JVM-sup-unfold* [*code*]:
JVM-SemiType.sup S m n = *lift2* (*Opt.sup*
(*Product.sup* (*Listn.sup* (*SemiType.sup S*))
(λ*x y*. *OK* (*map2* (*lift2* (*SemiType.sup S*)) *x y*))))
⟨*proof*⟩

lemmas [*code*] = *SemiType.sup-def* [*unfolded exec-lub-def*] *JVM-le-unfold*

lemmas [*code*] = *lesub-def plussub-def*

lemma [*code*]:
is-refT T = (*case T of NT* ⇒ *True* | *Class C* ⇒ *True* | - ⇒ *False*)
⟨*proof*⟩

declare *app*$_i$.*simps* [*code*]

lemma [*code*]:
app$_i$ (*Getfield F C*, *P*, *pc*, *mxs*, T_r, (*T*#*ST*, *LT*)) =
Predicate.holds (*Predicate.bind* (*sees-field-i-i-i-o-i P C F C*) (λT_f. *if P* ⊢ *T* ≤ *Class C then Predicate.single* () *else bot*))
⟨*proof*⟩

lemma [*code*]:

app$_i$ (*Putfield F C*, *P*, *pc*, *mxs*, T_r, (T_1#T_2#*ST*, *LT*)) =
Predicate.holds (*Predicate.bind* (*sees-field-i-i-i-o-i P C F C*) (λT_f. *if* $P \vdash T_2 \leq$ (*Class C*) $\wedge$ $P \vdash$ $T_1 \leq T_f$ *then Predicate.single* () *else bot*))
⟨*proof*⟩

lemma [*code*]:
app$_i$ (*Invoke M n*, *P*, *pc*, *mxs*, T_r, (*ST*,*LT*)) =
($n <$ *length ST* $\wedge$
(*ST*!*n* $\neq$ *NT* $\longrightarrow$
(*case ST*!*n of*
Class C $\Rightarrow$ *Predicate.holds* (*Predicate.bind* (*Method-i-i-i-o-o-o-o P C M*) (λ(*Ts*, *T*, *m*, *D*). *if* $P \vdash$ *rev* (*take n ST*) [$\leq$] *Ts then Predicate.single* () *else bot*))
| - $\Rightarrow$ *False*)))
⟨*proof*⟩

lemmas [*code*] =
SemiType.sup-def [*unfolded exec-lub-def*]
widen.equation
is-relevant-class.simps

definition *test1* **where**
test1 = *test-kil E list-name* [*Class list-name*] *Void 3 0*
[(*Suc 0*, *2*, *NullPointer*, *7*, *0*)] *append-ins*
definition *test2* **where**
test2 = *test-kil E test-name* [] *Void 3 2* [] *make-list-ins*
definition *test3* **where** *test3* = φ_a
definition *test4* **where** *test4* = φ_m

⟨*ML*⟩

end

Chapter 5

Compilation

5.1 An Intermediate Language

theory *J1* **imports** *../J/BigStep* **begin**

type-synonym $expr_1$ = *nat exp*
type-synonym J_1-*prog* = $expr_1$ *prog*
type-synonym $state_1$ = *heap* × (*val list*)

primrec
  max-vars :: *'a exp* ⇒ *nat*
  and *max-varss* :: *'a exp list* ⇒ *nat*
where
  max-vars(*new C*) = *0*
| *max-vars*(*Cast C e*) = *max-vars e*
| *max-vars*(*Val v*) = *0*
| *max-vars*(e_1 «*bop*» e_2) = *max* (*max-vars* e_1) (*max-vars* e_2)
| *max-vars*(*Var V*) = *0*
| *max-vars*(*V*:=*e*) = *max-vars e*
| *max-vars*(*e*•*F*{*D*}) = *max-vars e*
| *max-vars*(*FAss* e_1 *F D* e_2) = *max* (*max-vars* e_1) (*max-vars* e_2)
| *max-vars*(*e*•*M*(*es*)) = *max* (*max-vars e*) (*max-varss es*)
| *max-vars*({*V*:*T*; *e*}) = *max-vars e* + *1*
| *max-vars*(e_1;;e_2) = *max* (*max-vars* e_1) (*max-vars* e_2)
| *max-vars*(*if* (*e*) e_1 *else* e_2) =
   max (*max-vars e*) (*max* (*max-vars* e_1) (*max-vars* e_2))
| *max-vars*(*while* (*b*) *e*) = *max* (*max-vars b*) (*max-vars e*)
| *max-vars*(*throw e*) = *max-vars e*
| *max-vars*(*try* e_1 *catch*(*C V*) e_2) = *max* (*max-vars* e_1) (*max-vars* e_2 + *1*)

| *max-varss* [] = *0*
| *max-varss* (*e*#*es*) = *max* (*max-vars e*) (*max-varss es*)

inductive
  $eval_1$ :: J_1-*prog* ⇒ $expr_1$ ⇒ $state_1$ ⇒ $expr_1$ ⇒ $state_1$ ⇒ *bool*
      (‹- $\vdash_1$ ((*1*⟨-,/-⟩) ⇒/ (*1*⟨-,/-⟩))› [*51*,*0*,*0*,*0*,*0*] *81*)
  and $evals_1$ :: J_1-*prog* ⇒ $expr_1$ *list* ⇒ $state_1$ ⇒ $expr_1$ *list* ⇒ $state_1$ ⇒ *bool*
      (‹- $\vdash_1$ ((*1*⟨-,/-⟩) [⇒]/ (*1*⟨-,/-⟩))› [*51*,*0*,*0*,*0*,*0*] *81*)
  for *P* :: J_1-*prog*
where

New_1:
$[\![\ \textit{new-Addr}\ h = \textit{Some}\ a;\ P \vdash C\ \textit{has-fields}\ FDTs;\ h' = h(a \mapsto (C, \textit{init-fields}\ FDTs))\ ]\!]$
$\Longrightarrow P \vdash_1 \langle \textit{new}\ C,(h,l)\rangle \Rightarrow \langle \textit{addr}\ a,(h',l)\rangle$
| $\textit{NewFail}_1$:
$\textit{new-Addr}\ h = \textit{None} \Longrightarrow$
$P \vdash_1 \langle \textit{new}\ C,\ (h,l)\rangle \Rightarrow \langle \textit{THROW OutOfMemory},(h,l)\rangle$

| $\textit{Cast}_1$:
$[\![\ P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle \textit{addr}\ a,(h,l)\rangle;\ h\ a = \textit{Some}(D,fs);\ P \vdash D \preceq^* C\ ]\!]$
$\Longrightarrow P \vdash_1 \langle \textit{Cast}\ C\ e,s_0\rangle \Rightarrow \langle \textit{addr}\ a,(h,l)\rangle$
| $\textit{CastNull}_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle \textit{null},s_1\rangle \Longrightarrow$
$P \vdash_1 \langle \textit{Cast}\ C\ e,s_0\rangle \Rightarrow \langle \textit{null},s_1\rangle$
| $\textit{CastFail}_1$:
$[\![\ P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle \textit{addr}\ a,(h,l)\rangle;\ h\ a = \textit{Some}(D,fs);\ \neg\ P \vdash D \preceq^* C\ ]\!]$
$\Longrightarrow P \vdash_1 \langle \textit{Cast}\ C\ e,s_0\rangle \Rightarrow \langle \textit{THROW ClassCast},(h,l)\rangle$
| $\textit{CastThrow}_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle \textit{throw}\ e',s_1\rangle \Longrightarrow$
$P \vdash_1 \langle \textit{Cast}\ C\ e,s_0\rangle \Rightarrow \langle \textit{throw}\ e',s_1\rangle$

| $\textit{Val}_1$:
$P \vdash_1 \langle \textit{Val}\ v,s\rangle \Rightarrow \langle \textit{Val}\ v,s\rangle$

| $\textit{BinOp}_1$:
$[\![\ P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle \textit{Val}\ v_1,s_1\rangle;\ P \vdash_1 \langle e_2,s_1\rangle \Rightarrow \langle \textit{Val}\ v_2,s_2\rangle;\ \textit{binop}(\textit{bop},v_1,v_2) = \textit{Some}\ v\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e_1\ «\textit{bop}»\ e_2,s_0\rangle \Rightarrow \langle \textit{Val}\ v,s_2\rangle$
| $\textit{BinOpThrow}_{11}$:
$P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle \textit{throw}\ e,s_1\rangle \Longrightarrow$
$P \vdash_1 \langle e_1\ «\textit{bop}»\ e_2,\ s_0\rangle \Rightarrow \langle \textit{throw}\ e,s_1\rangle$
| $\textit{BinOpThrow}_{21}$:
$[\![\ P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle \textit{Val}\ v_1,s_1\rangle;\ P \vdash_1 \langle e_2,s_1\rangle \Rightarrow \langle \textit{throw}\ e,s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e_1\ «\textit{bop}»\ e_2,s_0\rangle \Rightarrow \langle \textit{throw}\ e,s_2\rangle$

| $\textit{Var}_1$:
$[\![\ ls!i = v;\ i < \textit{size}\ ls\ ]\!] \Longrightarrow$
$P \vdash_1 \langle \textit{Var}\ i,(h,ls)\rangle \Rightarrow \langle \textit{Val}\ v,(h,ls)\rangle$

| $\textit{LAss}_1$:
$[\![\ P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle \textit{Val}\ v,(h,ls)\rangle;\ i < \textit{size}\ ls;\ ls' = ls[i := v]\ ]\!]$
$\Longrightarrow P \vdash_1 \langle i{:=}\ e,s_0\rangle \Rightarrow \langle \textit{unit},(h,ls')\rangle$
| $\textit{LAssThrow}_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle \textit{throw}\ e',s_1\rangle \Longrightarrow$
$P \vdash_1 \langle i{:=}\ e,s_0\rangle \Rightarrow \langle \textit{throw}\ e',s_1\rangle$

| $\textit{FAcc}_1$:
$[\![\ P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle \textit{addr}\ a,(h,ls)\rangle;\ h\ a = \textit{Some}(C,fs);\ fs(F,D) = \textit{Some}\ v\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e{\bullet}F\{D\},s_0\rangle \Rightarrow \langle \textit{Val}\ v,(h,ls)\rangle$
| $\textit{FAccNull}_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle \textit{null},s_1\rangle \Longrightarrow$
$P \vdash_1 \langle e{\bullet}F\{D\},s_0\rangle \Rightarrow \langle \textit{THROW NullPointer},s_1\rangle$
| $\textit{FAccThrow}_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle \textit{throw}\ e',s_1\rangle \Longrightarrow$
$P \vdash_1 \langle e{\bullet}F\{D\},s_0\rangle \Rightarrow \langle \textit{throw}\ e',s_1\rangle$

| $FAss_1$:
$[\![P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle addr\ a,s_1\rangle;\ P \vdash_1 \langle e_2,s_1\rangle \Rightarrow \langle Val\ v,(h_2,l_2)\rangle;$
$h_2\ a = Some(C,fs);\ fs' = fs((F,D)\mapsto v);\ h_2' = h_2(a\mapsto(C,fs'))\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e_1{\cdot}F\{D\}{:=}\ e_2,s_0\rangle \Rightarrow \langle unit,(h_2',l_2)\rangle$
| $FAssNull_1$:
$[\![P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle null,s_1\rangle;\ \ P \vdash_1 \langle e_2,s_1\rangle \Rightarrow \langle Val\ v,s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e_1{\cdot}F\{D\}{:=}\ e_2,s_0\rangle \Rightarrow \langle THROW\ NullPointer,s_2\rangle$
| $FAssThrow_{11}$:
$P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash_1 \langle e_1{\cdot}F\{D\}{:=}\ e_2,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$
| $FAssThrow_{21}$:
$[\![P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle Val\ v,s_1\rangle;\ P \vdash_1 \langle e_2,s_1\rangle \Rightarrow \langle throw\ e',s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e_1{\cdot}F\{D\}{:=}\ e_2,s_0\rangle \Rightarrow \langle throw\ e',s_2\rangle$

| $CallObjThrow_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash_1 \langle e{\cdot}M(es),s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$
| $CallNull_1$:
$[\![P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle null,s_1\rangle;\ P \vdash_1 \langle es,s_1\rangle\ [\Rightarrow]\ \langle map\ Val\ vs,s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e{\cdot}M(es),s_0\rangle \Rightarrow \langle THROW\ NullPointer,s_2\rangle$
| $Call_1$:
$[\![P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle addr\ a,s_1\rangle;\ P \vdash_1 \langle es,s_1\rangle\ [\Rightarrow]\ \langle map\ Val\ vs,(h_2,ls_2)\rangle;$
$h_2\ a = Some(C,fs);\ P \vdash C\ sees\ M{:}Ts{\rightarrow}T = body\ in\ D;$
$size\ vs = size\ Ts;\ ls_2' = (Addr\ a)\ \#\ vs\ @\ replicate\ (max\text{-}vars\ body)\ undefined;$
$P \vdash_1 \langle body,(h_2,ls_2')\rangle \Rightarrow \langle e',(h_3,ls_3)\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e{\cdot}M(es),s_0\rangle \Rightarrow \langle e',(h_3,ls_2)\rangle$
| $CallParamsThrow_1$:
$[\![P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle Val\ v,s_1\rangle;\ P \vdash_1 \langle es,s_1\rangle\ [\Rightarrow]\ \langle es',s_2\rangle;$
$es' = map\ Val\ vs\ @\ throw\ ex\ \#\ es_2\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e{\cdot}M(es),s_0\rangle \Rightarrow \langle throw\ ex,s_2\rangle$

| $Block_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle e',s_1\rangle \Longrightarrow P \vdash_1 \langle Block\ i\ T\ e,s_0\rangle \Rightarrow \langle e',s_1\rangle$

| Seq_1:
$[\![P \vdash_1 \langle e_0,s_0\rangle \Rightarrow \langle Val\ v,s_1\rangle;\ P \vdash_1 \langle e_1,s_1\rangle \Rightarrow \langle e_2,s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e_0;;e_1,s_0\rangle \Rightarrow \langle e_2,s_2\rangle$
| $SeqThrow_1$:
$P \vdash_1 \langle e_0,s_0\rangle \Rightarrow \langle throw\ e,s_1\rangle \Longrightarrow$
$P \vdash_1 \langle e_0;;e_1,s_0\rangle \Rightarrow \langle throw\ e,s_1\rangle$

| $CondT_1$:
$[\![P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle true,s_1\rangle;\ P \vdash_1 \langle e_1,s_1\rangle \Rightarrow \langle e',s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle if\ (e)\ e_1\ else\ e_2,s_0\rangle \Rightarrow \langle e',s_2\rangle$
| $CondF_1$:
$[\![P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle false,s_1\rangle;\ P \vdash_1 \langle e_2,s_1\rangle \Rightarrow \langle e',s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle if\ (e)\ e_1\ else\ e_2,s_0\rangle \Rightarrow \langle e',s_2\rangle$
| $CondThrow_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash_1 \langle if\ (e)\ e_1\ else\ e_2,\ s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$

| $WhileF_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle false,s_1\rangle \Longrightarrow$

$P \vdash_1 \langle while\ (e)\ c,s_0\rangle \Rightarrow \langle unit,s_1\rangle$
| $WhileT_1$:
$[\![\ P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle true,s_1\rangle;\ P \vdash_1 \langle c,s_1\rangle \Rightarrow \langle Val\ v_1,s_2\rangle;$
$P \vdash_1 \langle while\ (e)\ c,s_2\rangle \Rightarrow \langle e_3,s_3\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle while\ (e)\ c,s_0\rangle \Rightarrow \langle e_3,s_3\rangle$
| $WhileCondThrow_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash_1 \langle while\ (e)\ c,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$
| $WhileBodyThrow_1$:
$[\![\ P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle true,s_1\rangle;\ P \vdash_1 \langle c,s_1\rangle \Rightarrow \langle throw\ e',s_2\rangle]\!]$
$\Longrightarrow P \vdash_1 \langle while\ (e)\ c,s_0\rangle \Rightarrow \langle throw\ e',s_2\rangle$

| $Throw_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle addr\ a,s_1\rangle \Longrightarrow$
$P \vdash_1 \langle throw\ e,s_0\rangle \Rightarrow \langle Throw\ a,s_1\rangle$
| $ThrowNull_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle null,s_1\rangle \Longrightarrow$
$P \vdash_1 \langle throw\ e,s_0\rangle \Rightarrow \langle THROW\ NullPointer,s_1\rangle$
| $ThrowThrow_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash_1 \langle throw\ e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$

| Try_1:
$P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle Val\ v_1,s_1\rangle \Longrightarrow$
$P \vdash_1 \langle try\ e_1\ catch(C\ i)\ e_2,s_0\rangle \Rightarrow \langle Val\ v_1,s_1\rangle$
| $TryCatch_1$:
$[\![\ P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle Throw\ a,(h_1,ls_1)\rangle;$
$h_1\ a = Some(D,fs);\ P \vdash D \preceq^* C;\ i < length\ ls_1;$
$P \vdash_1 \langle e_2,(h_1,ls_1[i:=Addr\ a])\rangle \Rightarrow \langle e_2',(h_2,ls_2)\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle try\ e_1\ catch(C\ i)\ e_2,s_0\rangle \Rightarrow \langle e_2',(h_2,ls_2)\rangle$
| $TryThrow_1$:
$[\![\ P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle Throw\ a,(h_1,ls_1)\rangle;\ h_1\ a = Some(D,fs);\ \neg\ P \vdash D \preceq^* C\ ]\!]$
$\Longrightarrow P \vdash_1 \langle try\ e_1\ catch(C\ i)\ e_2,s_0\rangle \Rightarrow \langle Throw\ a,(h_1,ls_1)\rangle$

| Nil_1:
$P \vdash_1 \langle [],s\rangle\ [\Rightarrow]\ \langle [],s\rangle$

| $Cons_1$:
$[\![\ P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle Val\ v,s_1\rangle;\ P \vdash_1 \langle es,s_1\rangle\ [\Rightarrow]\ \langle es',s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e\#es,s_0\rangle\ [\Rightarrow]\ \langle Val\ v\ \#\ es',s_2\rangle$
| $ConsThrow_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash_1 \langle e\#es,s_0\rangle\ [\Rightarrow]\ \langle throw\ e'\ \#\ es,\ s_1\rangle$

lemma $eval_1$-$preserves$-len:
$P \vdash_1 \langle e_0,(h_0,ls_0)\rangle \Rightarrow \langle e_1,(h_1,ls_1)\rangle \Longrightarrow length\ ls_0 = length\ ls_1$
and $evals_1$-$preserves$-len:
$P \vdash_1 \langle es_0,(h_0,ls_0)\rangle\ [\Rightarrow]\ \langle es_1,(h_1,ls_1)\rangle \Longrightarrow length\ ls_0 = length\ ls_1\langle proof\rangle$

lemma $evals_1$-$preserves$-$elen$:
$\bigwedge es'\ s\ s'.\ P \vdash_1 \langle es,s\rangle\ [\Rightarrow]\ \langle es',s'\rangle \Longrightarrow length\ es = length\ es'\langle proof\rangle$

lemma $eval_1$-$final$: $P \vdash_1 \langle e,s\rangle \Rightarrow \langle e',s'\rangle \Longrightarrow final\ e'$
and $evals_1$-$final$: $P \vdash_1 \langle es,s\rangle\ [\Rightarrow]\ \langle es',s'\rangle \Longrightarrow finals\ es'\langle proof\rangle$

end

5.2 Well-Formedness of Intermediate Language

theory *J1WellForm*
imports *../J/JWellForm J1*
begin

5.2.1 Well-Typedness

type-synonym
  env_1 = *ty list* — type environment indexed by variable number

inductive
  WT_1 :: [J_1-*prog*,env_1, $expr_1$, *ty*] ⇒ *bool*
    (‹(-,- $\vdash_1$/ - :: -)› [51,51,51]50)
  and WTs_1 :: [J_1-*prog*,env_1, $expr_1$ *list*, *ty list*] ⇒ *bool*
    (‹(-,- $\vdash_1$/ - [::] -)› [51,51,51]50)
  for *P* :: J_1-*prog*
where

  $WTNew_1$:
  is-class P C ⟹
  $P,E \vdash_1$ *new C* :: *Class C*

| $WTCast_1$:
  ⟦ $P,E \vdash_1 e$ :: *Class D*; *is-class P C*; $P \vdash C \preceq^* D \vee P \vdash D \preceq^* C$ ⟧
  ⟹ $P,E \vdash_1$ *Cast C e* :: *Class C*

| $WTVal_1$:
  typeof v = *Some T* ⟹
  $P,E \vdash_1$ *Val v* :: *T*

| $WTVar_1$:
  ⟦ $E!i = T$; $i <$ *size E* ⟧
  ⟹ $P,E \vdash_1$ *Var i* :: *T*

| $WTBinOp_1$:
  ⟦ $P,E \vdash_1 e_1 :: T_1$; $P,E \vdash_1 e_2 :: T_2$;
    case bop of Eq ⇒ $(P \vdash T_1 \le T_2 \vee P \vdash T_2 \le T_1) \wedge T$ = *Boolean*
          | *Add* ⇒ T_1 = *Integer* ∧ T_2 = *Integer* ∧ *T* = *Integer* ⟧
  ⟹ $P,E \vdash_1 e_1$ «*bop*» e_2 :: *T*

| $WTLAss_1$:
  ⟦ $E!i = T$; $i <$ *size E*; $P,E \vdash_1 e :: T'$; $P \vdash T' \le T$ ⟧
  ⟹ $P,E \vdash_1$ *i:=e* :: *Void*

| $WTFAcc_1$:
  ⟦ $P,E \vdash_1 e$ :: *Class C*; $P \vdash C$ *sees F:T in D* ⟧
  ⟹ $P,E \vdash_1$ *e•F{D}* :: *T*

| $WTFAss_1$:
  ⟦ $P,E \vdash_1 e_1$ :: *Class C*; $P \vdash C$ *sees F:T in D*; $P,E \vdash_1 e_2 :: T'$; $P \vdash T' \le T$ ⟧

$\Longrightarrow P,E \vdash_1 e_1{\cdot}F\{D\} := e_2 :: Void$

| $WTCall_1$:
$[\![P,E \vdash_1 e :: Class\ C;\ P \vdash C\ sees\ M{:}Ts' \to T = m\ in\ D;$
$P,E \vdash_1 es\ [::]\ Ts;\ \ P \vdash Ts\ [\le]\ Ts']\!]$
$\Longrightarrow P,E \vdash_1 e{\cdot}M(es) :: T$

| $WTBlock_1$:
$[\![is\text{-}type\ P\ T;\ P,E@[T] \vdash_1 e{::}T']\!]$
$\Longrightarrow\ P,E \vdash_1 \{i{:}T;\ e\} :: T'$

| $WTSeq_1$:
$[\![P,E \vdash_1 e_1{::}T_1;\ \ P,E \vdash_1 e_2{::}T_2]\!]$
$\Longrightarrow\ P,E \vdash_1 e_1;;e_2 :: T_2$

| $WTCond_1$:
$[\![P,E \vdash_1 e :: Boolean;\ \ P,E \vdash_1 e_1{::}T_1;\ \ P,E \vdash_1 e_2{::}T_2;$
$P \vdash T_1 \le T_2 \vee P \vdash T_2 \le T_1;\ \ P \vdash T_1 \le T_2 \longrightarrow T = T_2;\ P \vdash T_2 \le T_1 \longrightarrow T = T_1]\!]$
$\Longrightarrow P,E \vdash_1 if\ (e)\ e_1\ else\ e_2 :: T$

| $WTWhile_1$:
$[\![P,E \vdash_1 e :: Boolean;\ \ P,E \vdash_1 c{::}T]\!]$
$\Longrightarrow P,E \vdash_1 while\ (e)\ c :: Void$

| $WTThrow_1$:
$P,E \vdash_1 e :: Class\ C\ \Longrightarrow$
$P,E \vdash_1 throw\ e :: Void$

| $WTTry_1$:
$[\![P,E \vdash_1 e_1 :: T;\ \ P,E@[Class\ C] \vdash_1 e_2 :: T;\ is\text{-}class\ P\ C]\!]$
$\Longrightarrow P,E \vdash_1 try\ e_1\ catch(C\ i)\ e_2 :: T$

| $WTNil_1$:
$P,E \vdash_1 []\ [::]\ []$

| $WTCons_1$:
$[\![P,E \vdash_1 e :: T;\ \ P,E \vdash_1 es\ [::]\ Ts]\!]$
$\Longrightarrow\ P,E \vdash_1 e\#es\ [::]\ T\#Ts$

lemma $WTs_1\text{-}same\text{-}size$: $\bigwedge Ts.\ P,E \vdash_1 es\ [::]\ Ts \Longrightarrow size\ es = size\ Ts \langle proof\rangle$

lemma $WT_1\text{-}unique$:
$P,E \vdash_1 e :: T_1 \Longrightarrow (\bigwedge T_2.\ P,E \vdash_1 e :: T_2 \Longrightarrow T_1 = T_2)$ **and**
$P,E \vdash_1 es\ [::]\ Ts_1 \Longrightarrow (\bigwedge Ts_2.\ P,E \vdash_1 es\ [::]\ Ts_2 \Longrightarrow Ts_1 = Ts_2)\langle proof\rangle$

lemma assumes *wf*: *wf-prog p P*
shows $WT_1\text{-}is\text{-}type$: $P,E \vdash_1 e :: T \Longrightarrow set\ E \subseteq types\ P \Longrightarrow is\text{-}type\ P\ T$
and $P,E \vdash_1 es\ [::]\ Ts \Longrightarrow True\langle proof\rangle$

5.2.2 Well-formedness

primrec $\mathcal{B} :: expr_1 \Rightarrow nat \Rightarrow bool$
and $\mathcal{B}s :: expr_1\ list \Rightarrow nat \Rightarrow bool$ **where**
$\mathcal{B}\ (new\ C)\ i = True$ |

$\mathcal{B}$ (*Cast C e*) *i* = $\mathcal{B}$ *e i* |
$\mathcal{B}$ (*Val v*) *i* = *True* |
$\mathcal{B}$ (e_1 «*bop*» e_2) *i* = ($\mathcal{B}$ e_1 *i* ∧ $\mathcal{B}$ e_2 *i*) |
$\mathcal{B}$ (*Var j*) *i* = *True* |
$\mathcal{B}$ (*e*•*F*{*D*}) *i* = $\mathcal{B}$ *e i* |
$\mathcal{B}$ (*j*:=*e*) *i* = $\mathcal{B}$ *e i* |
$\mathcal{B}$ (e_1•*F*{*D*} := e_2) *i* = ($\mathcal{B}$ e_1 *i* ∧ $\mathcal{B}$ e_2 *i*) |
$\mathcal{B}$ (*e*•*M*(*es*)) *i* = ($\mathcal{B}$ *e i* ∧ $\mathcal{B}$*s es i*) |
$\mathcal{B}$ ({*j*:*T* ; *e*}) *i* = (*i* = *j* ∧ $\mathcal{B}$ *e* (*i*+*1*)) |
$\mathcal{B}$ (e_1;;e_2) *i* = ($\mathcal{B}$ e_1 *i* ∧ $\mathcal{B}$ e_2 *i*) |
$\mathcal{B}$ (*if* (*e*) e_1 *else* e_2) *i* = ($\mathcal{B}$ *e i* ∧ $\mathcal{B}$ e_1 *i* ∧ $\mathcal{B}$ e_2 *i*) |
$\mathcal{B}$ (*throw e*) *i* = $\mathcal{B}$ *e i* |
$\mathcal{B}$ (*while* (*e*) *c*) *i* = ($\mathcal{B}$ *e i* ∧ $\mathcal{B}$ *c i*) |
$\mathcal{B}$ (*try* e_1 *catch*(*C j*) e_2) *i* = ($\mathcal{B}$ e_1 *i* ∧ *i*=*j* ∧ $\mathcal{B}$ e_2 (*i*+*1*)) |

$\mathcal{B}$*s* [] *i* = *True* |
$\mathcal{B}$*s* (*e*#*es*) *i* = ($\mathcal{B}$ *e i* ∧ $\mathcal{B}$*s es i*)

definition *wf-*J_1*-mdecl* :: J_1*-prog* ⇒ *cname* ⇒ *expr*$_1$ *mdecl* ⇒ *bool*
where
wf-J_1*-mdecl P C* ≡ λ(*M*,*Ts*,*T*,*body*).
(∃ *T*′. *P*,*Class C*#*Ts* ⊢$_1$ *body* :: *T*′ ∧ *P* ⊢ *T*′ ≤ *T*) ∧
$\mathcal{D}$ *body* ⌊{..*size Ts*}⌋ ∧ $\mathcal{B}$ *body* (*size Ts* + *1*)

lemma *wf-*J_1*-mdecl*[*simp*]:
wf-J_1*-mdecl P C* (*M*,*Ts*,*T*,*body*) ≡
((∃ *T*′. *P*,*Class C*#*Ts* ⊢$_1$ *body* :: *T*′ ∧ *P* ⊢ *T*′ ≤ *T*) ∧
$\mathcal{D}$ *body* ⌊{..*size Ts*}⌋ ∧ $\mathcal{B}$ *body* (*size Ts* + *1*))⟨*proof*⟩
abbreviation *wf-*J_1*-prog* == *wf-prog wf-*J_1*-mdecl*

end

5.3 Program Compilation

theory *PCompiler*
imports ../*Common*/*WellForm*
begin

definition *compM* :: (′*a* ⇒ ′*b*) ⇒ ′*a mdecl* ⇒ ′*b mdecl*
where
compM f ≡ λ(*M*, *Ts*, *T*, *m*). (*M*, *Ts*, *T*, *f m*)

definition *compC* :: (′*a* ⇒ ′*b*) ⇒ ′*a cdecl* ⇒ ′*b cdecl*
where
compC f ≡ λ(*C*,*D*,*Fdecls*,*Mdecls*). (*C*,*D*,*Fdecls*, *map* (*compM f*) *Mdecls*)

definition *compP* :: (′*a* ⇒ ′*b*) ⇒ ′*a prog* ⇒ ′*b prog*
where
compP f ≡ *map* (*compC f*)

Compilation preserves the program structure. Therfore lookup functions either commute with compilation (like method lookup) or are preserved by it (like the subclass relation).

lemma *map-of-map4*:

map-of (*map* ($\lambda(x,a,b,c).(x,a,b,f\ c)$) *ts*) =
map-option ($\lambda(a,b,c).(a,b,f\ c)$) $\circ$ (*map-of ts*)⟨*proof*⟩

lemma *class-compP*:
class P C = *Some* (*D*, *fs*, *ms*)
$\Longrightarrow$ *class* (*compP f P*) *C* = *Some* (*D*, *fs*, *map* (*compM f*) *ms*)⟨*proof*⟩

lemma *class-compPD*:
class (*compP f P*) *C* = *Some* (*D*, *fs*, *cms*)
$\Longrightarrow$ $\exists$ *ms*. *class P C* = *Some*(*D*,*fs*,*ms*) $\wedge$ *cms* = *map* (*compM f*) *ms*⟨*proof*⟩

lemma [*simp*]: *is-class* (*compP f P*) *C* = *is-class P C*⟨*proof*⟩

lemma [*simp*]: *class* (*compP f P*) *C* = *map-option* (λc. *snd*(*compC f* (*C*,*c*))) (*class P C*)⟨*proof*⟩

lemma *sees-methods-compP*:
P $\vdash$ *C sees-methods Mm* $\Longrightarrow$
compP f P $\vdash$ *C sees-methods* (*map-option* (λ((*Ts*,*T*,*m*),*D*). ((*Ts*,*T*,*f m*),*D*)) $\circ$ *Mm*)⟨*proof*⟩

lemma *sees-method-compP*:
P $\vdash$ *C sees M*: *Ts*$\rightarrow$*T* = *m in D* $\Longrightarrow$
compP f P $\vdash$ *C sees M*: *Ts*$\rightarrow$*T* = (*f m*) *in D*⟨*proof*⟩

lemma [*simp*]:
P $\vdash$ *C sees M*: *Ts*$\rightarrow$*T* = *m in D* $\Longrightarrow$
method (*compP f P*) *C M* = (*D*,*Ts*,*T*,*f m*)⟨*proof*⟩

lemma *sees-methods-compPD*:
⟦ *cP* $\vdash$ *C sees-methods Mm'*; *cP* = *compP f P* ⟧ $\Longrightarrow$
$\exists$ *Mm*. *P* $\vdash$ *C sees-methods Mm* $\wedge$
Mm' = (*map-option* (λ((*Ts*,*T*,*m*),*D*). ((*Ts*,*T*,*f m*),*D*)) $\circ$ *Mm*)⟨*proof*⟩

lemma *sees-method-compPD*:
compP f P $\vdash$ *C sees M*: *Ts*$\rightarrow$*T* = *fm in D* $\Longrightarrow$
$\exists$ *m*. *P* $\vdash$ *C sees M*: *Ts*$\rightarrow$*T* = *m in D* $\wedge$ *f m* = *fm*⟨*proof*⟩

lemma [*simp*]: *subcls1*(*compP f P*) = *subcls1 P*⟨*proof*⟩

lemma *compP-widen*[*simp*]: (*compP f P* $\vdash$ *T* $\leq$ *T'*) = (*P* $\vdash$ *T* $\leq$ *T'*)⟨*proof*⟩

lemma [*simp*]: (*compP f P* $\vdash$ *Ts* [$\leq$] *Ts'*) = (*P* $\vdash$ *Ts* [$\leq$] *Ts'*)⟨*proof*⟩

lemma [*simp*]: *is-type* (*compP f P*) *T* = *is-type P T*⟨*proof*⟩

lemma [*simp*]: (*compP* (*f*::'*a*$\Rightarrow$'*b*) *P* $\vdash$ *C has-fields FDTs*) = (*P* $\vdash$ *C has-fields FDTs*)⟨*proof*⟩

lemma [*simp*]: *fields* (*compP f P*) *C* = *fields P C*⟨*proof*⟩

lemma [*simp*]: (*compP f P* $\vdash$ *C sees F*:*T in D*) = (*P* $\vdash$ *C sees F*:*T in D*)⟨*proof*⟩

lemma [*simp*]: *field* (*compP f P*) *F D* = *field P F D*⟨*proof*⟩

5.3.1 Invariance of *wf-prog* under compilation

lemma [*iff*]: *distinct-fst* (*compP f P*) = *distinct-fst P*⟨*proof*⟩

lemma [*iff*]: *distinct-fst* (*map* (*compM f*) *ms*) = *distinct-fst ms*⟨*proof*⟩

lemma [*iff*]: *wf-syscls* (*compP f P*) = *wf-syscls P*⟨*proof*⟩

lemma [*iff*]: *wf-fdecl* (*compP f P*) = *wf-fdecl P*⟨*proof*⟩

lemma *set-compP*:
((*C*,*D*,*fs*,*ms*′) ∈ *set*(*compP f P*)) =
(∃ *ms*. (*C*,*D*,*fs*,*ms*) ∈ *set P* ∧ *ms*′ = *map* (*compM f*) *ms*)⟨*proof*⟩

lemma *wf-cdecl-compPI*:
⟦ ⋀*C M Ts T m*.
⟦ *wf-mdecl* wf_1 *P C* (*M*,*Ts*,*T*,*m*); *P* ⊢ *C sees M*:*Ts*→*T* = *m in C* ⟧
⟹ *wf-mdecl* wf_2 (*compP f P*) *C* (*M*,*Ts*,*T*, *f m*);
∀ *x*∈*set P*. *wf-cdecl* wf_1 *P x*; *x* ∈ *set* (*compP f P*); *wf-prog p P* ⟧
⟹ *wf-cdecl* wf_2 (*compP f P*) *x*⟨*proof*⟩

lemma *wf-prog-compPI*:
assumes *lift*:
⋀*C M Ts T m*.
⟦ *P* ⊢ *C sees M*:*Ts*→*T* = *m in C*; *wf-mdecl* wf_1 *P C* (*M*,*Ts*,*T*,*m*) ⟧
⟹ *wf-mdecl* wf_2 (*compP f P*) *C* (*M*,*Ts*,*T*, *f m*)
and *wf*: *wf-prog* wf_1 *P*
shows *wf-prog* wf_2 (*compP f P*)⟨*proof*⟩

end
theory *Hidden*
imports *List−Index.List-Index*
begin

definition *hidden* :: ′*a list* ⇒ *nat* ⇒ *bool* **where**
hidden xs i ≡ *i* < *size xs* ∧ *xs*!*i* ∈ *set*(*drop* (*i*+*1*) *xs*)

lemma *hidden-last-index*: *x* ∈ *set xs* ⟹ *hidden* (*xs* @ [*x*]) (*last-index xs x*)
⟨*proof*⟩

lemma *hidden-inacc*: *hidden xs i* ⟹ *last-index xs x* ≠ *i*
⟨*proof*⟩

lemma [*simp*]: *hidden xs i* ⟹ *hidden* (*xs*@[*x*]) *i*
⟨*proof*⟩

lemma *fun-upds-apply*:
(*m*(*xs*[↦]*ys*)) *x* =
(*let xs*′ = *take* (*size ys*) *xs*
in if x ∈ *set xs*′ *then Some*(*ys* ! *last-index xs*′ *x*) *else m x*)
⟨*proof*⟩

lemma *map-upds-apply-eq-Some*:
$((m(xs[\mapsto]ys))\ x = Some\ y) =$
$(let\ xs' = take\ (size\ ys)\ xs$
$in\ if\ x \in set\ xs'\ then\ ys\ !\ last\text{-}index\ xs'\ x = y\ else\ m\ x = Some\ y)$
$\langle proof \rangle$

lemma *map-upds-upd-conv-last-index*:
$[\![x \in set\ xs;\ size\ xs \leq size\ ys\ ]\!]$
$\Longrightarrow m(xs[\mapsto]ys,\ x\mapsto y) = m(xs[\mapsto]ys[last\text{-}index\ xs\ x := y])$
$\langle proof \rangle$

end

5.4 Compilation Stage 1

theory *Compiler1* **imports** *PCompiler J1 Hidden* **begin**

Replacing variable names by indices.

primrec $compE_1$ :: $vname\ list \Rightarrow expr \Rightarrow expr_1$
and $compEs_1$ :: $vname\ list \Rightarrow expr\ list \Rightarrow expr_1\ list$ **where**
$compE_1\ Vs\ (new\ C) = new\ C$
$|\ compE_1\ Vs\ (Cast\ C\ e) = Cast\ C\ (compE_1\ Vs\ e)$
$|\ compE_1\ Vs\ (Val\ v) = Val\ v$
$|\ compE_1\ Vs\ (e_1\ «bop»\ e_2) = (compE_1\ Vs\ e_1)\ «bop»\ (compE_1\ Vs\ e_2)$
$|\ compE_1\ Vs\ (Var\ V) = Var(last\text{-}index\ Vs\ V)$
$|\ compE_1\ Vs\ (V{:=}e) = (last\text{-}index\ Vs\ V){:=}\ (compE_1\ Vs\ e)$
$|\ compE_1\ Vs\ (e{\cdot}F\{D\}) = (compE_1\ Vs\ e){\cdot}F\{D\}$
$|\ compE_1\ Vs\ (e_1{\cdot}F\{D\}{:=}e_2) = (compE_1\ Vs\ e_1){\cdot}F\{D\} := (compE_1\ Vs\ e_2)$
$|\ compE_1\ Vs\ (e{\cdot}M(es)) = (compE_1\ Vs\ e){\cdot}M(compEs_1\ Vs\ es)$
$|\ compE_1\ Vs\ \{V{:}T;\ e\} = \{(size\ Vs){:}T;\ compE_1\ (Vs@[V])\ e\}$
$|\ compE_1\ Vs\ (e_1;;e_2) = (compE_1\ Vs\ e_1);;(compE_1\ Vs\ e_2)$
$|\ compE_1\ Vs\ (if\ (e)\ e_1\ else\ e_2) = if\ (compE_1\ Vs\ e)\ (compE_1\ Vs\ e_1)\ else\ (compE_1\ Vs\ e_2)$
$|\ compE_1\ Vs\ (while\ (e)\ c) = while\ (compE_1\ Vs\ e)\ (compE_1\ Vs\ c)$
$|\ compE_1\ Vs\ (throw\ e) = throw\ (compE_1\ Vs\ e)$
$|\ compE_1\ Vs\ (try\ e_1\ catch(C\ V)\ e_2) =$
$try(compE_1\ Vs\ e_1)\ catch(C\ (size\ Vs))\ (compE_1\ (Vs@[V])\ e_2)$

$|\ compEs_1\ Vs\ []\ \ \ = []$
$|\ compEs_1\ Vs\ (e\#es) = compE_1\ Vs\ e\ \#\ compEs_1\ Vs\ es$

lemma [*simp*]: $compEs_1\ Vs\ es = map\ (compE_1\ Vs)\ es\langle proof \rangle$

primrec fin_1:: $expr \Rightarrow expr_1$ **where**
$fin_1(Val\ v) = Val\ v$
$|\ fin_1(throw\ e) = throw(fin_1\ e)$

lemma *comp-final*: $final\ e \Longrightarrow compE_1\ Vs\ e = fin_1\ e\langle proof \rangle$

lemma [*simp*]:
$\bigwedge Vs.\ max\text{-}vars\ (compE_1\ Vs\ e) = max\text{-}vars\ e$
and $\bigwedge Vs.\ max\text{-}varss\ (compEs_1\ Vs\ es) = max\text{-}varss\ es\langle proof \rangle$

Compiling programs:

definition $compP_1$ :: *J-prog* ⇒ J_1*-prog*
where
$compP_1$ ≡ *compP* (λ(*pns*,*body*). $compE_1$ (*this#pns*) *body*)

end

5.5 Correctness of Stage 1

theory *Correctness1*
imports *J1WellForm Compiler1*
begin

5.5.1 Correctness of program compilation

primrec *unmod* :: $expr_1$ ⇒ *nat* ⇒ *bool*
and *unmods* :: $expr_1$ *list* ⇒ *nat* ⇒ *bool* **where**
unmod (*new C*) *i* = *True* |
unmod (*Cast C e*) *i* = *unmod e i* |
unmod (*Val v*) *i* = *True* |
unmod (e_1 «*bop*» e_2) *i* = (*unmod* e_1 *i* ∧ *unmod* e_2 *i*) |
unmod (*Var i*) *j* = *True* |
unmod (*i*:=*e*) *j* = (*i* ≠ *j* ∧ *unmod e j*) |
unmod (*e*•*F*{*D*}) *i* = *unmod e i* |
unmod (e_1•*F*{*D*}:=e_2) *i* = (*unmod* e_1 *i* ∧ *unmod* e_2 *i*) |
unmod (*e*•*M*(*es*)) *i* = (*unmod e i* ∧ *unmods es i*) |
unmod {*j*:*T*; *e*} *i* = *unmod e i* |
unmod (e_1;;e_2) *i* = (*unmod* e_1 *i* ∧ *unmod* e_2 *i*) |
unmod (*if* (*e*) e_1 *else* e_2) *i* = (*unmod e i* ∧ *unmod* e_1 *i* ∧ *unmod* e_2 *i*) |
unmod (*while* (*e*) *c*) *i* = (*unmod e i* ∧ *unmod c i*) |
unmod (*throw e*) *i* = *unmod e i* |
unmod (*try* e_1 *catch*(*C i*) e_2) *j* = (*unmod* e_1 *j* ∧ (*if i*=*j then False else unmod* e_2 *j*)) |

unmods ([]) *i* = *True* |
unmods (*e#es*) *i* = (*unmod e i* ∧ *unmods es i*)

lemma *hidden-unmod*: ⋀*Vs*. *hidden Vs i* ⟹ *unmod* ($compE_1$ *Vs e*) *i* **and**
⋀*Vs*. *hidden Vs i* ⟹ *unmods* ($compEs_1$ *Vs es*) *i*⟨*proof*⟩

lemma $eval_1$*-preserves-unmod*:
⟦ *P* $\vdash_1$ ⟨*e*,(*h*,*ls*)⟩ ⇒ ⟨*e*′,(*h*′,*ls*′)⟩; *unmod e i*; *i* < *size ls* ⟧
⟹ *ls* ! *i* = *ls*′ ! *i*
and ⟦ *P* $\vdash_1$ ⟨*es*,(*h*,*ls*)⟩ [⇒] ⟨*es*′,(*h*′,*ls*′)⟩; *unmods es i*; *i* < *size ls* ⟧
⟹ *ls* ! *i* = *ls*′ ! *i*⟨*proof*⟩

lemma *LAss-lem*:
⟦*x* ∈ *set xs*; *size xs* ≤ *size ys* ⟧
⟹ m_1 $\subseteq_m$ m_2(*xs*[↦]*ys*) ⟹ m_1(*x*↦*y*) $\subseteq_m$ m_2(*xs*[↦]*ys*[*last-index xs x* := *y*])⟨*proof*⟩**lemma** *Block-lem*:
fixes *l* :: ′*a* ⇀ ′*b*
assumes *0*: *l* $\subseteq_m$ [*Vs* [↦] *ls*]
and *1*: *l*′ $\subseteq_m$ [*Vs* [↦] *ls*′, *V*↦*v*]
and *hidden*: *V* ∈ *set Vs* ⟹ *ls* ! *last-index Vs V* = *ls*′ ! *last-index Vs V*
and *size*: *size ls* = *size ls*′ *size Vs* < *size ls*′

shows $l'(V := l\ V) \subseteq_m [Vs\ [\mapsto]\ ls']\langle proof\rangle$

The main theorem:

theorem assumes *wf*: *wwf-J-prog P*
shows $eval_1\text{-}eval$: $P \vdash \langle e,(h,l)\rangle \Rightarrow \langle e',(h',l')\rangle$
$\Longrightarrow (\bigwedge Vs\ ls.\ [\![\ fv\ e \subseteq set\ Vs;\ \ l \subseteq_m [Vs[\mapsto]ls];\ size\ Vs + max\text{-}vars\ e \le size\ ls\ ]\!]$
$\Longrightarrow \exists ls'.\ compP_1\ P \vdash_1 \langle compE_1\ Vs\ e,(h,ls)\rangle \Rightarrow \langle fin_1\ e',(h',ls')\rangle \wedge l' \subseteq_m [Vs[\mapsto]ls'])$
and $evals_1\text{-}evals$: $P \vdash \langle es,(h,l)\rangle\ [\Rightarrow]\ \langle es',(h',l')\rangle$
$\Longrightarrow (\bigwedge Vs\ ls.\ [\![\ fvs\ es \subseteq set\ Vs;\ \ l \subseteq_m [Vs[\mapsto]ls];\ size\ Vs + max\text{-}varss\ es \le size\ ls\ ]\!]$
$\Longrightarrow \exists ls'.\ compP_1\ P \vdash_1 \langle compEs_1\ Vs\ es,(h,ls)\rangle\ [\Rightarrow]\ \langle compEs_1\ Vs\ es',(h',ls')\rangle \wedge$
$l' \subseteq_m [Vs[\mapsto]ls'])\langle proof\rangle$

5.5.2 Preservation of well-formedness

The compiler preserves well-formedness. Is less trivial than it may appear. We start with two simple properties: preservation of well-typedness

lemma $compE_1\text{-}pres\text{-}wt$: $\bigwedge Vs\ Ts\ U.$
$[\![\ P,[Vs[\mapsto]Ts] \vdash e :: U;\ size\ Ts = size\ Vs\ ]\!]$
$\Longrightarrow compP\ f\ P,Ts \vdash_1 compE_1\ Vs\ e :: U$
and $\bigwedge Vs\ Ts\ Us.$
$[\![\ P,[Vs[\mapsto]Ts] \vdash es\ [::]\ Us;\ size\ Ts = size\ Vs\ ]\!]$
$\Longrightarrow compP\ f\ P,Ts \vdash_1 compEs_1\ Vs\ es\ [::]\ Us\langle proof\rangle$

and the correct block numbering:

lemma $\mathcal{B}$: $\bigwedge Vs\ n.\ size\ Vs = n \Longrightarrow \mathcal{B}\ (compE_1\ Vs\ e)\ n$
and $\mathcal{B}s$: $\bigwedge Vs\ n.\ size\ Vs = n \Longrightarrow \mathcal{B}s\ (compEs_1\ Vs\ es)\ n\langle proof\rangle$

The main complication is preservation of definite assignment $\mathcal{D}$.

lemma *image-last-index*: $A \subseteq set(xs@[x]) \Longrightarrow last\text{-}index\ (xs\ @\ [x])\ `\ A =$
$(if\ x \in A\ then\ insert\ (size\ xs)\ (last\text{-}index\ xs\ `\ (A-\{x\}))\ else\ last\text{-}index\ xs\ `\ A)\langle proof\rangle$

lemma $A\text{-}compE_1\text{-}None[simp]$:
$\bigwedge Vs.\ \mathcal{A}\ e = None \Longrightarrow \mathcal{A}\ (compE_1\ Vs\ e) = None$
and $\bigwedge Vs.\ \mathcal{A}s\ es = None \Longrightarrow \mathcal{A}s\ (compEs_1\ Vs\ es) = None\langle proof\rangle$

lemma $A\text{-}compE_1$:
$\bigwedge A\ Vs.\ [\![\ \mathcal{A}\ e = \lfloor A\rfloor;\ fv\ e \subseteq set\ Vs\ ]\!] \Longrightarrow \mathcal{A}\ (compE_1\ Vs\ e) = \lfloor last\text{-}index\ Vs\ `\ A\rfloor$
and $\bigwedge A\ Vs.\ [\![\ \mathcal{A}s\ es = \lfloor A\rfloor;\ fvs\ es \subseteq set\ Vs\ ]\!] \Longrightarrow \mathcal{A}s\ (compEs_1\ Vs\ es) = \lfloor last\text{-}index\ Vs\ `\ A\rfloor\langle proof\rangle$

lemma *D-None*[*iff*]: $\mathcal{D}\ (e::'a\ exp)\ None$ **and** [*iff*]: $\mathcal{D}s\ (es::'a\ exp\ list)\ None\langle proof\rangle$

lemma $D\text{-}last\text{-}index\text{-}compE_1$:
$\bigwedge A\ Vs.\ [\![\ A \subseteq set\ Vs;\ fv\ e \subseteq set\ Vs\ ]\!] \Longrightarrow$
$\mathcal{D}\ e\ \lfloor A\rfloor \Longrightarrow \mathcal{D}\ (compE_1\ Vs\ e)\ \lfloor last\text{-}index\ Vs\ `\ A\rfloor$
and $\bigwedge A\ Vs.\ [\![\ A \subseteq set\ Vs;\ fvs\ es \subseteq set\ Vs\ ]\!] \Longrightarrow$
$\mathcal{D}s\ es\ \lfloor A\rfloor \Longrightarrow \mathcal{D}s\ (compEs_1\ Vs\ es)\ \lfloor last\text{-}index\ Vs\ `\ A\rfloor\langle proof\rangle$

lemma *last-index-image-set*: $distinct\ xs \Longrightarrow last\text{-}index\ xs\ `\ set\ xs = \{..<size\ xs\}\langle proof\rangle$

lemma $D\text{-}compE_1$:
$[\![\ \mathcal{D}\ e\ \lfloor set\ Vs\rfloor;\ fv\ e \subseteq set\ Vs;\ distinct\ Vs\ ]\!] \Longrightarrow \mathcal{D}\ (compE_1\ Vs\ e)\ \lfloor\{..<length\ Vs\}\rfloor\langle proof\rangle$

lemma $D\text{-}compE_1{}'$:

assumes $\mathcal{D}$ *e* ⌊*set*(*V#Vs*)⌋ **and** *fv e* ⊆ *set*(*V#Vs*) **and** *distinct*(*V#Vs*)
shows $\mathcal{D}$ (*compE*$_1$ (*V#Vs*) *e*) ⌊{..*length Vs*}⌋⟨*proof*⟩

lemma *compP*$_1$*-pres-wf*: *wf-J-prog P* ⟹ *wf-J*$_1$*-prog* (*compP*$_1$ *P*)⟨*proof*⟩

end

5.6 Compilation Stage 2

theory *Compiler2*
imports *PCompiler J1 ../JVM/JVMExec*
begin

```
primrec compE2 :: expr1 ⇒ instr list
  and compEs2 :: expr1 list ⇒ instr list where
  compE2 (new C) = [New C]
| compE2 (Cast C e) = compE2 e @ [Checkcast C]
| compE2 (Val v) = [Push v]
| compE2 (e1 «bop» e2) = compE2 e1 @ compE2 e2 @
  (case bop of Eq ⇒ [CmpEq]
             | Add ⇒ [IAdd])
| compE2 (Var i) = [Load i]
| compE2 (i:=e) = compE2 e @ [Store i, Push Unit]
| compE2 (e•F{D}) = compE2 e @ [Getfield F D]
| compE2 (e1•F{D} := e2) =
      compE2 e1 @ compE2 e2 @ [Putfield F D, Push Unit]
| compE2 (e•M(es)) = compE2 e @ compEs2 es @ [Invoke M (size es)]
| compE2 ({i:T; e}) = compE2 e
| compE2 (e1;;e2) = compE2 e1 @ [Pop] @ compE2 e2
| compE2 (if (e) e1 else e2) =
       (let cnd   = compE2 e;
            thn   = compE2 e1;
            els   = compE2 e2;
            test  = IfFalse (int(size thn + 2));
            thnex = Goto (int(size els + 1))
        in cnd @ [test] @ thn @ [thnex] @ els)
| compE2 (while (e) c) =
       (let cnd   = compE2 e;
            bdy   = compE2 c;
            test  = IfFalse (int(size bdy + 3));
            loop  = Goto (−int(size bdy + size cnd + 2))
        in cnd @ [test] @ bdy @ [Pop] @ [loop] @ [Push Unit])
| compE2 (throw e) = compE2 e @ [instr.Throw]
| compE2 (try e1 catch(C i) e2) =
   (let catch = compE2 e2
    in compE2 e1 @ [Goto (int(size catch)+2), Store i] @ catch)

| compEs2 []     = []
| compEs2 (e#es) = compE2 e @ compEs2 es
```

Compilation of exception table. Is given start address of code to compute absolute addresses necessary in exception table.

```
primrec compxE2 :: expr1      ⇒ pc ⇒ nat ⇒ ex-table
```

and *compxEs*$_2$ :: *expr*$_1$ *list* ⇒ *pc* ⇒ *nat* ⇒ *ex-table* **where**
compxE$_2$ (*new C*) *pc d* = []
| *compxE*$_2$ (*Cast C e*) *pc d* = *compxE*$_2$ *e pc d*
| *compxE*$_2$ (*Val v*) *pc d* = []
| *compxE*$_2$ (e_1 «*bop*» e_2) *pc d* =
compxE$_2$ e_1 *pc d* @ *compxE*$_2$ e_2 (*pc* + *size*(*compE*$_2$ e_1)) (*d*+*1*)
| *compxE*$_2$ (*Var i*) *pc d* = []
| *compxE*$_2$ (*i*:=*e*) *pc d* = *compxE*$_2$ *e pc d*
| *compxE*$_2$ (*e*•*F*{*D*}) *pc d* = *compxE*$_2$ *e pc d*
| *compxE*$_2$ (e_1•*F*{*D*} := e_2) *pc d* =
compxE$_2$ e_1 *pc d* @ *compxE*$_2$ e_2 (*pc* + *size*(*compE*$_2$ e_1)) (*d*+*1*)
| *compxE*$_2$ (*e*•*M*(*es*)) *pc d* =
compxE$_2$ *e pc d* @ *compxEs*$_2$ *es* (*pc* + *size*(*compE*$_2$ *e*)) (*d*+*1*)
| *compxE*$_2$ ({*i*:*T*; *e*}) *pc d* = *compxE*$_2$ *e pc d*
| *compxE*$_2$ (e_1;;e_2) *pc d* =
compxE$_2$ e_1 *pc d* @ *compxE*$_2$ e_2 (*pc*+*size*(*compE*$_2$ e_1)+*1*) *d*
| *compxE*$_2$ (*if* (*e*) e_1 *else* e_2) *pc d* =
(*let* pc_1 = *pc* + *size*(*compE*$_2$ *e*) + *1*;
pc_2 = pc_1 + *size*(*compE*$_2$ e_1) + *1*
in *compxE*$_2$ *e pc d* @ *compxE*$_2$ e_1 pc_1 *d* @ *compxE*$_2$ e_2 pc_2 *d*)
| *compxE*$_2$ (*while* (*b*) *e*) *pc d* =
compxE$_2$ *b pc d* @ *compxE*$_2$ *e* (*pc*+*size*(*compE*$_2$ *b*)+*1*) *d*
| *compxE*$_2$ (*throw e*) *pc d* = *compxE*$_2$ *e pc d*
| *compxE*$_2$ (*try* e_1 *catch*(*C i*) e_2) *pc d* =
(*let* pc_1 = *pc* + *size*(*compE*$_2$ e_1)
in *compxE*$_2$ e_1 *pc d* @ *compxE*$_2$ e_2 (pc_1+*2*) *d* @ [(*pc*,pc_1,*C*,pc_1+*1*,*d*)])

| *compxEs*$_2$ [] *pc d* = []
| *compxEs*$_2$ (*e*#*es*) *pc d* = *compxE*$_2$ *e pc d* @ *compxEs*$_2$ *es* (*pc*+*size*(*compE*$_2$ *e*)) (*d*+*1*)

primrec *max-stack* :: *expr*$_1$ ⇒ *nat*
and *max-stacks* :: *expr*$_1$ *list* ⇒ *nat* **where**
max-stack (*new C*) = *1*
| *max-stack* (*Cast C e*) = *max-stack e*
| *max-stack* (*Val v*) = *1*
| *max-stack* (e_1 «*bop*» e_2) = *max* (*max-stack* e_1) (*max-stack* e_2) + *1*
| *max-stack* (*Var i*) = *1*
| *max-stack* (*i*:=*e*) = *max-stack e*
| *max-stack* (*e*•*F*{*D*}) = *max-stack e*
| *max-stack* (e_1•*F*{*D*} := e_2) = *max* (*max-stack* e_1) (*max-stack* e_2) + *1*
| *max-stack* (*e*•*M*(*es*)) = *max* (*max-stack e*) (*max-stacks es*) + *1*
| *max-stack* ({*i*:*T*; *e*}) = *max-stack e*
| *max-stack* (e_1;;e_2) = *max* (*max-stack* e_1) (*max-stack* e_2)
| *max-stack* (*if* (*e*) e_1 *else* e_2) =
max (*max-stack e*) (*max* (*max-stack* e_1) (*max-stack* e_2))
| *max-stack* (*while* (*e*) *c*) = *max* (*max-stack e*) (*max-stack c*)
| *max-stack* (*throw e*) = *max-stack e*
| *max-stack* (*try* e_1 *catch*(*C i*) e_2) = *max* (*max-stack* e_1) (*max-stack* e_2)

| *max-stacks* [] = *0*
| *max-stacks* (*e*#*es*) = *max* (*max-stack e*) (*1* + *max-stacks es*)

lemma *max-stack1*: *1* ≤ *max-stack e*⟨*proof*⟩

definition $compMb_2$ :: $expr_1 \Rightarrow$ *jvm-method*
where
$compMb_2 \equiv \lambda body.$
let $ins = compE_2$ *body* @ [*Return*];
$xt = compxE_2$ *body 0 0*
in (*max-stack body*, *max-vars body*, *ins*, *xt*)

definition $compP_2$ :: J_1*-prog* $\Rightarrow$ *jvm-prog*
where
$compP_2 \equiv$ *compP* $compMb_2$

lemma $compMb_2$ [*simp*]:
$compMb_2$ *e* = (*max-stack e*, *max-vars e*, $compE_2$ *e* @ [*Return*], $compxE_2$ *e 0 0*)⟨*proof*⟩

end

5.7 Correctness of Stage 2

theory *Correctness2*
imports *HOL−Library.Sublist Compiler2*
begin

5.7.1 Instruction sequences

How to select individual instructions and subsequences of instructions from a program given the class, method and program counter.

definition *before* :: *jvm-prog* ⇒ *cname* ⇒ *mname* ⇒ *nat* ⇒ *instr list* ⇒ *bool*
(‹(-,-,-,-/ ⊳ -)› [*51*,*0*,*0*,*0*,*51*] *50*) **where**
P,*C*,*M*,*pc* ⊳ *is* ⟷ *prefix is* (*drop pc* (*instrs-of P C M*))

definition *at* :: *jvm-prog* ⇒ *cname* ⇒ *mname* ⇒ *nat* ⇒ *instr* ⇒ *bool*
(‹(-,-,-,-/ ▹ -)› [*51*,*0*,*0*,*0*,*51*] *50*) **where**
P,*C*,*M*,*pc* ▹ *i* ⟷ (∃ *is*. *drop pc* (*instrs-of P C M*) = *i*#*is*)

lemma [*simp*]: *P*,*C*,*M*,*pc* ⊳ []⟨*proof*⟩

lemma [*simp*]: *P*,*C*,*M*,*pc* ⊳ (*i*#*is*) = (*P*,*C*,*M*,*pc* ▹ *i* ∧ *P*,*C*,*M*,*pc* + *1* ⊳ *is*)⟨*proof*⟩

lemma [*simp*]: *P*,*C*,*M*,*pc* ⊳ (is_1 @ is_2) = (*P*,*C*,*M*,*pc* ⊳ is_1 ∧ *P*,*C*,*M*,*pc* + *size* is_1 ⊳ is_2)⟨*proof*⟩

lemma [*simp*]: *P*,*C*,*M*,*pc* ▹ *i* ⟹ *instrs-of P C M* ! *pc* = *i*⟨*proof*⟩

lemma *beforeM*:
P ⊢ *C sees M*: *Ts*→*T* = *body in D* ⟹
$compP_2$ *P*,*D*,*M*,*0* ⊳ $compE_2$ *body* @ [*Return*]⟨*proof*⟩

This lemma executes a single instruction by rewriting:

lemma [*simp*]:
P,*C*,*M*,*pc* ▹ *instr* ⟹
(*P* ⊢ (*None*, *h*, (*vs*,*ls*,*C*,*M*,*pc*) # *frs*) −*jvm*→ σ′) =
((*None*, *h*, (*vs*,*ls*,*C*,*M*,*pc*) # *frs*) = σ′ ∨
(∃ σ. *exec*(*P*,(*None*, *h*, (*vs*,*ls*,*C*,*M*,*pc*) # *frs*)) = *Some* σ ∧ *P* ⊢ σ −*jvm*→ σ′))⟨*proof*⟩

5.7.2 Exception tables

definition *pcs* :: *ex-table* ⇒ *nat set*
where
$pcs\ xt\ \equiv\ \bigcup(f,t,C,h,d) \in set\ xt.\ \{f\ ..<\ t\}$

lemma *pcs-subset*:
shows $\bigwedge pc\ d.\ pcs(compxE_2\ e\ pc\ d) \subseteq \{pc..<pc+size(compE_2\ e)\}$
and $\bigwedge pc\ d.\ pcs(compxEs_2\ es\ pc\ d) \subseteq \{pc..<pc+size(compEs_2\ es)\}\langle proof\rangle$

lemma [*simp*]: $pcs\ [] = \{\}\langle proof\rangle$

lemma [*simp*]: $pcs\ (x\#xt) = \{fst\ x\ ..<\ fst(snd\ x)\} \cup pcs\ xt\langle proof\rangle$

lemma [*simp*]: $pcs(xt_1\ @\ xt_2) = pcs\ xt_1 \cup pcs\ xt_2\langle proof\rangle$

lemma [*simp*]: $pc < pc_0 \lor pc_0+size(compE_2\ e) \leq pc \Longrightarrow pc \notin pcs(compxE_2\ e\ pc_0\ d)\langle proof\rangle$

lemma [*simp*]: $pc < pc_0 \lor pc_0+size(compEs_2\ es) \leq pc \Longrightarrow pc \notin pcs(compxEs_2\ es\ pc_0\ d)\langle proof\rangle$

lemma [*simp*]: $pc_1 + size(compE_2\ e_1) \leq pc_2 \Longrightarrow pcs(compxE_2\ e_1\ pc_1\ d_1) \cap pcs(compxE_2\ e_2\ pc_2\ d_2) = \{\}\langle proof\rangle$

lemma [*simp*]: $pc_1 + size(compE_2\ e) \leq pc_2 \Longrightarrow pcs(compxE_2\ e\ pc_1\ d_1) \cap pcs(compxEs_2\ es\ pc_2\ d_2) = \{\}\langle proof\rangle$

lemma [*simp*]:
$pc \notin pcs\ xt_0 \Longrightarrow match\text{-}ex\text{-}table\ P\ C\ pc\ (xt_0\ @\ xt_1) = match\text{-}ex\text{-}table\ P\ C\ pc\ xt_1\langle proof\rangle$

lemma [*simp*]: $[\![\ x \in set\ xt;\ pc \notin pcs\ xt\ ]\!] \Longrightarrow \neg\ matches\text{-}ex\text{-}entry\ P\ D\ pc\ x\langle proof\rangle$

lemma [*simp*]:
assumes *xe*: $xe \in set(compxE_2\ e\ pc\ d)$ **and** *outside*: $pc' < pc \lor pc+size(compE_2\ e) \leq pc'$
shows $\neg\ matches\text{-}ex\text{-}entry\ P\ C\ pc'\ xe\langle proof\rangle$

lemma [*simp*]:
assumes *xe*: $xe \in set(compxEs_2\ es\ pc\ d)$ **and** *outside*: $pc' < pc \lor pc+size(compEs_2\ es) \leq pc'$
shows $\neg\ matches\text{-}ex\text{-}entry\ P\ C\ pc'\ xe\langle proof\rangle$

lemma *match-ex-table-app*[*simp*]:
$\forall xte \in set\ xt_1.\ \neg\ matches\text{-}ex\text{-}entry\ P\ D\ pc\ xte \Longrightarrow$
$match\text{-}ex\text{-}table\ P\ D\ pc\ (xt_1\ @\ xt) = match\text{-}ex\text{-}table\ P\ D\ pc\ xt\langle proof\rangle$

lemma [*simp*]:
$\forall x \in set\ xtab.\ \neg\ matches\text{-}ex\text{-}entry\ P\ C\ pc\ x \Longrightarrow$
$match\text{-}ex\text{-}table\ P\ C\ pc\ xtab = None\langle proof\rangle$

lemma *match-ex-entry*:
$matches\text{-}ex\text{-}entry\ P\ C\ pc\ (start,\ end,\ catch\text{-}type,\ handler) =$
$(start \leq pc \land pc < end \land\ \ P \vdash C \preceq^*\ catch\text{-}type)\langle proof\rangle$

definition *caught* :: *jvm-prog* ⇒ *pc* ⇒ *heap* ⇒ *addr* ⇒ *ex-table* ⇒ *bool* **where**
$caught\ P\ pc\ h\ a\ xt \longleftrightarrow$
$(\exists entry \in set\ xt.\ matches\text{-}ex\text{-}entry\ P\ (cname\text{-}of\ h\ a)\ pc\ entry)$

definition *beforex* :: *jvm-prog* ⇒ *cname* ⇒ *mname* ⇒ *ex-table* ⇒ *nat set* ⇒ *nat* ⇒ *bool*
(‹(2-,/-,/- ▷/ - /′/ -,/-)› [51,0,0,0,0,51] 50) **where**
$P,C,M \rhd xt\ /\ I,d \longleftrightarrow$
$(\exists xt_0\ xt_1.\ ex\text{-}table\text{-}of\ P\ C\ M = xt_0\ @\ xt\ @\ xt_1 \wedge pcs\ xt_0 \cap I = \{\} \wedge pcs\ xt \subseteq I \wedge$
$(\forall pc \in I.\ \forall C\ pc'\ d'.\ match\text{-}ex\text{-}table\ P\ C\ pc\ xt_1 = \lfloor(pc',d')\rfloor \longrightarrow d' \leq d))$

definition *dummyx* :: *jvm-prog* ⇒ *cname* ⇒ *mname* ⇒ *ex-table* ⇒ *nat set* ⇒ *nat* ⇒ *bool* (‹(2-,-,- ▷/ - ′/-,-)› [51,0,0,0,0,51] 50) **where**
$P,C,M \rhd xt/I,d \longleftrightarrow P,C,M \rhd xt/I,d$

abbreviation
$beforex_0\ P\ C\ M\ d\ I\ xt\ xt_0\ xt_1$
$\equiv ex\text{-}table\text{-}of\ P\ C\ M = xt_0\ @\ xt\ @\ xt_1 \wedge pcs\ xt_0 \cap I = \{\}$
$\wedge\ pcs\ xt \subseteq I \wedge (\forall pc \in I.\ \forall C\ pc'\ d'.\ match\text{-}ex\text{-}table\ P\ C\ pc\ xt_1 = \lfloor(pc',d')\rfloor \longrightarrow d' \leq d)$

lemma *beforex-beforex*$_0$*-eq*:
$P,C,M \rhd xt\ /\ I,d \equiv \exists xt_0\ xt_1.\ beforex_0\ P\ C\ M\ d\ I\ xt\ xt_0\ xt_1$
⟨*proof*⟩

lemma *beforexD1*: $P,C,M \rhd xt\ /\ I,d \Longrightarrow pcs\ xt \subseteq I$⟨*proof*⟩

lemma *beforex-mono*: $[\![\ P,C,M \rhd xt/I,d';\ d' \leq d\ ]\!] \Longrightarrow P,C,M \rhd xt/I,d$⟨*proof*⟩

lemma [*simp*]: $P,C,M \rhd xt/I,d \Longrightarrow P,C,M \rhd xt/I,Suc\ d$⟨*proof*⟩

lemma *beforex-append*[*simp*]:
$pcs\ xt_1 \cap pcs\ xt_2 = \{\} \Longrightarrow$
$P,C,M \rhd xt_1\ @\ xt_2/I,d =$
$(P,C,M \rhd xt_1/I-pcs\ xt_2,d\ \wedge\ P,C,M \rhd xt_2/I-pcs\ xt_1,d \wedge P,C,M \rhd xt_1@xt_2/I,d)$⟨*proof*⟩

lemma *beforex-appendD1*:
assumes *bx*: $P,C,M \rhd xt_1\ @\ xt_2\ @\ [(f,t,D,h,d)]\ /\ I,d$
and *pcs*: $pcs\ xt_1 \subseteq J$ **and** *JI*: $J \subseteq I$ **and** *Jpcs*: $J \cap pcs\ xt_2 = \{\}$
shows $P,C,M \rhd xt_1\ /\ J,d$⟨*proof*⟩

lemma *beforex-appendD2*:
assumes *bx*: $P,C,M \rhd xt_1\ @\ xt_2\ @\ [(f,t,D,h,d)]\ /\ I,d$
and *pcs*: $pcs\ xt_2 \subseteq J$ **and** *JI*: $J \subseteq I$ **and** *Jpcs*: $J \cap pcs\ xt_1 = \{\}$
shows $P,C,M \rhd xt_2\ /\ J,d$⟨*proof*⟩

lemma *beforexM*:
$P \vdash C\ sees\ M{:}\ Ts{\rightarrow}T = body\ in\ D \Longrightarrow$
$compP_2\ P,D,M \rhd compxE_2\ body\ 0\ 0/\{..{<}size(compE_2\ body)\},0$⟨*proof*⟩

lemma *match-ex-table-SomeD2*:
assumes *met*: $match\text{-}ex\text{-}table\ P\ D\ pc\ (ex\text{-}table\text{-}of\ P\ C\ M) = \lfloor(pc',d')\rfloor$
and *bx*: $P,C,M \rhd xt/I,d$
and *nmet*: $\forall x \in set\ xt.\ \neg\ matches\text{-}ex\text{-}entry\ P\ D\ pc\ x$ **and** *pcI*: $pc \in I$
shows $d' \leq d$⟨*proof*⟩

lemma *match-ex-table-SomeD1*:
$[\![\ match\text{-}ex\text{-}table\ P\ D\ pc\ (ex\text{-}table\text{-}of\ P\ C\ M) = \lfloor(pc',d')\rfloor;$

$P,C,M \rhd xt\ /\ I,d;\ pc \in I;\ pc \notin pcs\ xt\ \rrbracket \Longrightarrow d' \le d\langle proof\rangle$

5.7.3 The correctness proof

definition
handle :: *jvm-prog* ⇒ *cname* ⇒ *mname* ⇒ *addr* ⇒ *heap* ⇒ *val list* ⇒ *val list* ⇒ *nat* ⇒ *frame list* ⇒ *jvm-state* **where**
handle P C M a h vs ls pc frs = *find-handler P a h* ((*vs*,*ls*,*C*,*M*,*pc*) # *frs*)

lemma *handle-Cons*:
$\llbracket P,C,M \rhd xt/I,d;\ d \le size\ vs;\ pc \in I;$
$\forall x \in set\ xt.\ \neg\ matches\text{-}ex\text{-}entry\ P\ (cname\text{-}of\ h\ xa)\ pc\ x\ \rrbracket \Longrightarrow$
$handle\ P\ C\ M\ xa\ h\ (v\ \#\ vs)\ ls\ pc\ frs = handle\ P\ C\ M\ xa\ h\ vs\ ls\ pc\ frs\langle proof\rangle$
lemma *handle-append*:
assumes *bx*: $P,C,M \rhd xt/I,d$ **and** *d*: $d \le size\ vs$
and *pcI*: $pc \in I$ **and** *pc-not*: $pc \notin pcs\ xt$
shows $handle\ P\ C\ M\ xa\ h\ (ws\ @\ vs)\ ls\ pc\ frs = handle\ P\ C\ M\ xa\ h\ vs\ ls\ pc\ frs\langle proof\rangle$

lemma *aux-isin*[*simp*]: $\llbracket B \subseteq A;\ a \in B\ \rrbracket \Longrightarrow a \in A\langle proof\rangle$

lemma fixes P_1 **defines** [*simp*]: $P \equiv compP_2\ P_1$
shows *Jcc*:
$P_1 \vdash_1 \langle e,(h_0,ls_0)\rangle \Rightarrow \langle ef,(h_1,ls_1)\rangle \Longrightarrow$
$(\bigwedge C\ M\ pc\ v\ xa\ vs\ frs\ I.$
$\llbracket P,C,M,pc \rhd compE_2\ e;\ P,C,M \rhd compxE_2\ e\ pc\ (size\ vs)/I,size\ vs;$
$\{pc..<pc+size(compE_2\ e)\} \subseteq I\ \rrbracket \Longrightarrow$
$(ef = Val\ v \longrightarrow$
$P \vdash (None,h_0,(vs,ls_0,C,M,pc)\#frs)\ -jvm\rightarrow$
$(None,h_1,(v\#vs,ls_1,C,M,pc+size(compE_2\ e))\#frs))$
$\wedge$
$(ef = Throw\ xa \longrightarrow$
$(\exists pc_1.\ pc \le pc_1 \wedge pc_1 < pc + size(compE_2\ e) \wedge$
$\neg\ caught\ P\ pc_1\ h_1\ xa\ (compxE_2\ e\ pc\ (size\ vs)) \wedge$
$P \vdash (None,h_0,(vs,ls_0,C,M,pc)\#frs)\ -jvm\rightarrow handle\ P\ C\ M\ xa\ h_1\ vs\ ls_1\ pc_1\ frs)))$
and $P_1 \vdash_1 \langle es,(h_0,ls_0)\rangle\ [\Rightarrow]\ \langle fs,(h_1,ls_1)\rangle \Longrightarrow$
$(\bigwedge C\ M\ pc\ ws\ xa\ es'\ vs\ frs\ I.$
$\llbracket P,C,M,pc \rhd compEs_2\ es;\ P,C,M \rhd compxEs_2\ es\ pc\ (size\ vs)/I,size\ vs;$
$\{pc..<pc+size(compEs_2\ es)\} \subseteq I\ \rrbracket \Longrightarrow$
$(fs = map\ Val\ ws \longrightarrow$
$P \vdash (None,h_0,(vs,ls_0,C,M,pc)\#frs)\ -jvm\rightarrow$
$(None,h_1,(rev\ ws\ @\ vs,ls_1,C,M,pc+size(compEs_2\ es))\#frs))$
$\wedge$
$(fs = map\ Val\ ws\ @\ Throw\ xa\ \#\ es' \longrightarrow$
$(\exists pc_1.\ pc \le pc_1 \wedge pc_1 < pc + size(compEs_2\ es) \wedge$
$\neg\ caught\ P\ pc_1\ h_1\ xa\ (compxEs_2\ es\ pc\ (size\ vs)) \wedge$
$P \vdash (None,h_0,(vs,ls_0,C,M,pc)\#frs)\ -jvm\rightarrow handle\ P\ C\ M\ xa\ h_1\ vs\ ls_1\ pc_1\ frs)))\langle proof\rangle$

lemma *atLeast0AtMost*[*simp*]: $\{0::nat..n\} = \{..n\}$
$\langle proof\rangle$

lemma *atLeast0LessThan*[*simp*]: $\{0::nat..<n\} = \{..<n\}$

⟨*proof*⟩

fun *exception* :: *'a exp* ⇒ *addr option* **where**
  exception (*Throw a*) = *Some a*
| *exception e* = *None*

lemma *comp*$_2$-*correct*:
assumes *method*: $P_1 \vdash C$ *sees* $M{:}Ts{\to}T$ = *body in C*
  and *eval*: $P_1 \vdash_1 \langle body,(h,ls)\rangle \Rightarrow \langle e',(h',ls')\rangle$
shows *compP*$_2$ $P_1 \vdash (None,h,[([],ls,C,M,0)])$ −*jvm*→ (*exception* $e',h',[]$)⟨*proof*⟩
end

5.8 Combining Stages 1 and 2

theory *Compiler*
imports *Correctness1 Correctness2*
begin

definition *J2JVM* :: *J-prog* ⇒ *jvm-prog*
where
  J2JVM ≡ *compP*$_2$ ∘ *compP*$_1$

theorem *comp-correct*:
assumes *wwf*: *wwf-J-prog P*
and *method*: $P \vdash C$ *sees* $M{:}Ts{\to}T$ = (*pns*,*body*) *in C*
and *eval*: $P \vdash \langle body,(h,[this\#pns\ [\mapsto]\ vs])\rangle \Rightarrow \langle e',(h',l')\rangle$
and *sizes*: *size vs* = *size pns* + *1*   *size rest* = *max-vars body*
shows *J2JVM P* ⊢ (*None*,*h*,[([],*vs*@*rest*,*C*,*M*,*0*)]) −*jvm*→ (*exception* $e',h',[]$)⟨*proof*⟩

end

5.9 Preservation of Well-Typedness

theory *TypeComp*
imports *Compiler ../BV/BVSpec*
begin

locale *TC0* =
  fixes *P* :: J_1-*prog* **and** *mxl* :: *nat*
begin

definition *ty E e* = (*THE T*. $P,E \vdash_1 e :: T$)

definition ty_l $E\ A'$ = *map* (λ*i*. *if* $i \in A' \wedge i <$ *size E then OK*(*E*!*i*) *else Err*) [*0*..<*mxl*]

definition $ty_i{}'$ *ST E A* = (*case A of None* ⇒ *None* | ⌊*A'*⌋ ⇒ *Some*(*ST*, ty_l *E A'*))

definition *after E A ST e* = $ty_i{}'$ (*ty E e* # *ST*) *E* (*A* ⊔ $\mathcal{A}$ *e*)

end

lemma (**in** *TC0*) *ty-def2* [*simp*]: $P,E \vdash_1 e :: T \Longrightarrow$ *ty E e* = *T*⟨*proof*⟩

lemma (**in** *TC0*) [*simp*]: $ty_i{}'$ *ST E None* = *None*⟨*proof*⟩
lemma (**in** *TC0*) ty_l*-app-diff* [*simp*]:
ty_l (*E*@[*T*]) (*A* − {*size E*}) = ty_l *E A*⟨*proof*⟩

lemma (**in** *TC0*) $ty_i{}'$*-app-diff* [*simp*]:
$ty_i{}'$ *ST* (*E* @ [*T*]) (*A* ⊖ *size E*) = $ty_i{}'$ *ST E A*⟨*proof*⟩

lemma (**in** *TC0*) ty_l*-antimono*:
$A \subseteq A' \Longrightarrow P \vdash ty_l\ E\ A'\ [\le_\top]\ ty_l\ E\ A$⟨*proof*⟩

lemma (**in** *TC0*) $ty_i{}'$*-antimono*:
$A \subseteq A' \Longrightarrow P \vdash ty_i{}'\ ST\ E\ \lfloor A' \rfloor \le' ty_i{}'\ ST\ E\ \lfloor A \rfloor$⟨*proof*⟩

lemma (**in** *TC0*) ty_l*-env-antimono*:
$P \vdash ty_l\ (E@[T])\ A\ [\le_\top]\ ty_l\ E\ A$⟨*proof*⟩

lemma (**in** *TC0*) $ty_i{}'$*-env-antimono*:
$P \vdash ty_i{}'\ ST\ (E@[T])\ A \le' ty_i{}'\ ST\ E\ A$⟨*proof*⟩

lemma (**in** *TC0*) $ty_i{}'$*-incr*:
$P \vdash ty_i{}'\ ST\ (E\ @\ [T])\ \lfloor insert\ (size\ E)\ A \rfloor \le' ty_i{}'\ ST\ E\ \lfloor A \rfloor$⟨*proof*⟩

lemma (**in** *TC0*) ty_l*-incr*:
$P \vdash ty_l\ (E\ @\ [T])\ (insert\ (size\ E)\ A)\ [\le_\top]\ ty_l\ E\ A$⟨*proof*⟩

lemma (**in** *TC0*) ty_l*-in-types*:
$set\ E \subseteq types\ P \Longrightarrow ty_l\ E\ A \in nlists\ mxl\ (err\ (types\ P))$⟨*proof*⟩
locale *TC1* = *TC0*
begin

primrec $compT :: ty\ list \Rightarrow nat\ hyperset \Rightarrow ty\ list \Rightarrow expr_1 \Rightarrow ty_i{}'\ list$ **and**
$compTs :: ty\ list \Rightarrow nat\ hyperset \Rightarrow ty\ list \Rightarrow expr_1\ list \Rightarrow ty_i{}'\ list$ **where**
compT E A ST (*new C*) = []
| *compT E A ST* (*Cast C e*) =
compT E A ST e @ [*after E A ST e*]
| *compT E A ST* (*Val v*) = []
| $compT\ E\ A\ ST\ (e_1 «bop» e_2)$ =
$(let\ ST_1 = ty\ E\ e_1\#ST;\ A_1 = A \sqcup \mathcal{A}\ e_1\ in$
$compT\ E\ A\ ST\ e_1\ @\ [after\ E\ A\ ST\ e_1]\ @$
$compT\ E\ A_1\ ST_1\ e_2\ @\ [after\ E\ A_1\ ST_1\ e_2])$
| *compT E A ST* (*Var i*) = []
| *compT E A ST* (*i* := *e*) = *compT E A ST e* @
$[after\ E\ A\ ST\ e,\ ty_i{}'\ ST\ E\ (A \sqcup \mathcal{A}\ e \sqcup \lfloor \{i\} \rfloor)]$
| *compT E A ST* (*e*•*F*{*D*}) =
compT E A ST e @ [*after E A ST e*]
| $compT\ E\ A\ ST\ (e_1{\bullet}F\{D\} := e_2)$ =
$(let\ ST_1 = ty\ \ E\ e_1\#ST;\ A_1 = A \sqcup \mathcal{A}\ e_1;\ A_2 = A_1 \sqcup \mathcal{A}\ e_2\ in$
$compT\ E\ A\ ST\ e_1\ @\ [after\ E\ A\ ST\ e_1]\ @$
$compT\ E\ A_1\ ST_1\ e_2\ @\ [after\ E\ A_1\ ST_1\ e_2]\ @$
$[ty_i{}'\ ST\ E\ A_2])$
| *compT E A ST* {*i*:*T*; *e*} = *compT* (*E*@[*T*]) (*A*⊖*i*) *ST e*
| $compT\ E\ A\ ST\ (e_1;;e_2)$ =
$(let\ A_1 = A \sqcup \mathcal{A}\ e_1\ in$
$compT\ E\ A\ ST\ e_1\ @\ [after\ E\ A\ ST\ e_1,\ ty_i{}'\ ST\ E\ A_1]\ @$

$compT\ E\ A_1\ ST\ e_2)$
$|\ compT\ E\ A\ ST\ (if\ (e)\ e_1\ else\ e_2) =$
$(let\ A_0 = A \sqcup \mathcal{A}\ e;\ \tau = ty_i'\ ST\ E\ A_0\ in$
$compT\ E\ A\ ST\ e\ @\ [after\ E\ A\ ST\ e,\ \tau]\ @$
$compT\ E\ A_0\ ST\ e_1\ @\ [after\ E\ A_0\ ST\ e_1,\ \tau]\ @$
$compT\ E\ A_0\ ST\ e_2)$
$|\ compT\ E\ A\ ST\ (while\ (e)\ c) =$
$(let\ A_0 = A \sqcup \mathcal{A}\ e;\ \ A_1 = A_0 \sqcup \mathcal{A}\ c;\ \tau = ty_i'\ ST\ E\ A_0\ in$
$compT\ E\ A\ ST\ e\ @\ [after\ E\ A\ ST\ e,\ \tau]\ @$
$compT\ E\ A_0\ ST\ c\ @\ [after\ E\ A_0\ ST\ c,\ ty_i'\ ST\ E\ A_1,\ ty_i'\ ST\ E\ A_0])$
$|\ compT\ E\ A\ ST\ (throw\ e) = compT\ E\ A\ ST\ e\ @\ [after\ E\ A\ ST\ e]$
$|\ compT\ E\ A\ ST\ (e \cdot M(es)) =$
$compT\ E\ A\ ST\ e\ @\ [after\ E\ A\ ST\ e]\ @$
$compTs\ E\ (A \sqcup \mathcal{A}\ e)\ (ty\ \ E\ e\ \#\ ST)\ es$
$|\ compT\ E\ A\ ST\ (try\ e_1\ catch(C\ i)\ e_2) =$
$compT\ E\ A\ ST\ e_1\ @\ [after\ E\ A\ ST\ e_1]\ @$
$[ty_i'\ (Class\ C\#ST)\ E\ A,\ ty_i'\ ST\ (E@[Class\ C])\ (A \sqcup \lfloor\{i\}\rfloor)]\ @$
$compT\ (E@[Class\ C])\ (A \sqcup \lfloor\{i\}\rfloor)\ ST\ e_2$
$|\ compTs\ E\ A\ ST\ [] = []$
$|\ compTs\ \ E\ A\ ST\ (e\#es) = compT\ E\ A\ ST\ e\ @\ [after\ E\ A\ ST\ e]\ @$
$compTs\ E\ (A \sqcup (\mathcal{A}\ e))\ (ty\ E\ e\ \#\ ST)\ es$

definition $compT_a :: ty\ list \Rightarrow nat\ hyperset \Rightarrow ty\ list \Rightarrow expr_1 \Rightarrow ty_i'\ list$ **where**
$compT_a\ E\ A\ ST\ e = compT\ E\ A\ ST\ e\ @\ [after\ E\ A\ ST\ e]$

end

lemma $compE_2$*-not-Nil*[*simp*]: $compE_2\ e \neq []\langle proof\rangle$
lemma (**in** *TC1*) *compT-sizes*[*simp*]:
shows $\bigwedge E\ A\ ST.\ size(compT\ E\ A\ ST\ e) = size(compE_2\ e) - 1$
and $\bigwedge E\ A\ ST.\ size(compTs\ E\ A\ ST\ es) = size(compEs_2\ es)\langle proof\rangle$

lemma (**in** *TC1*) [*simp*]: $\bigwedge ST\ E.\ \lfloor\tau\rfloor \notin set\ (compT\ E\ None\ ST\ e)$
and [*simp*]: $\bigwedge ST\ E.\ \lfloor\tau\rfloor \notin set\ (compTs\ E\ None\ ST\ es)\langle proof\rangle$

lemma (**in** *TC0*) *pair-eq-*ty_i'*-conv*:
$(\lfloor(ST,\ LT)\rfloor = ty_i'\ ST_0\ E\ A) =$
$(case\ A\ of\ None \Rightarrow False\ |\ Some\ A \Rightarrow (ST = ST_0 \wedge LT = ty_l\ E\ A))\langle proof\rangle$

lemma (**in** *TC0*) *pair-conv-*ty_i':
$\lfloor(ST,\ ty_l\ E\ A)\rfloor = ty_i'\ ST\ E\ \lfloor A\rfloor\langle proof\rangle$

lemma (**in** *TC1*) *compT-LT-prefix*:
$\bigwedge E\ A\ ST_0.\ [\![\ \lfloor(ST,LT)\rfloor \in set(compT\ E\ A\ ST_0\ e);\ \mathcal{B}\ e\ (size\ E)\ ]\!]$
$\Longrightarrow P \vdash \lfloor(ST,LT)\rfloor \leq'\ ty_i'\ ST\ E\ A$
and
$\bigwedge E\ A\ ST_0.\ [\![\ \lfloor(ST,LT)\rfloor \in set(compTs\ E\ A\ ST_0\ es);\ \mathcal{B}s\ es\ (size\ E)\ ]\!]$
$\Longrightarrow P \vdash \lfloor(ST,LT)\rfloor \leq'\ ty_i'\ ST\ E\ A\langle proof\rangle$

lemma [*iff*]: $OK\ None \in states\ P\ mxs\ mxl\langle proof\rangle$
lemma (**in** *TC0*) *after-in-states*:
assumes *wf*: *wf-prog p P* **and** *wt*: $P,E \vdash_1 e :: T$
and *Etypes*: $set\ E \subseteq types\ P$ **and** *STtypes*: $set\ ST \subseteq types\ P$
and *stack*: $size\ ST + max\text{-}stack\ e \leq mxs$

shows *OK* (*after E A ST e*) ∈ *states P mxs mxl*⟨*proof*⟩

lemma (**in** *TC0*) *OK-*ty_i*′-in-statesI*[*simp*]:
⟦ *set E* ⊆ *types P*; *set ST* ⊆ *types P*; *size ST* ≤ *mxs* ⟧
⟹ *OK* (ty_i*′ ST E A*) ∈ *states P mxs mxl*⟨*proof*⟩

lemma *is-class-type-aux*: *is-class P C* ⟹ *is-type P* (*Class C*)⟨*proof*⟩
theorem (**in** *TC1*) *compT-states*:
assumes *wf*: *wf-prog p P*
shows ⋀*E T A ST*.
⟦ *P*,*E* $\vdash_1$ *e* :: *T*; *set E* ⊆ *types P*; *set ST* ⊆ *types P*;
size ST + *max-stack e* ≤ *mxs*; *size E* + *max-vars e* ≤ *mxl* ⟧
⟹ *OK* ‘ *set*(*compT E A ST e*) ⊆ *states P mxs mxl*
and ⋀*E Ts A ST*.
⟦ *P*,*E* $\vdash_1$ *es*[::]*Ts*; *set E* ⊆ *types P*; *set ST* ⊆ *types P*;
size ST + *max-stacks es* ≤ *mxs*; *size E* + *max-varss es* ≤ *mxl* ⟧
⟹ *OK* ‘ *set*(*compTs E A ST es*) ⊆ *states P mxs mxl*⟨*proof*⟩

definition *shift* :: *nat* ⇒ *ex-table* ⇒ *ex-table*
where
shift n xt ≡ *map* (λ(*from*,*to*,*C*,*handler*,*depth*). (*from*+*n*,*to*+*n*,*C*,*handler*+*n*,*depth*)) *xt*

lemma [*simp*]: *shift 0 xt* = *xt*⟨*proof*⟩
lemma [*simp*]: *shift n* [] = []⟨*proof*⟩
lemma [*simp*]: *shift n* (xt_1 @ xt_2) = *shift n* xt_1 @ *shift n* xt_2⟨*proof*⟩
lemma [*simp*]: *shift m* (*shift n xt*) = *shift* (*m*+*n*) *xt*⟨*proof*⟩
lemma [*simp*]: *pcs* (*shift n xt*) = {*pc*+*n*|*pc*. *pc* ∈ *pcs xt*}⟨*proof*⟩

lemma *shift-compx*E_2:
shows ⋀*pc pc′ d*. *shift pc* (*compx*E_2 *e pc′ d*) = *compx*E_2 *e* (*pc′* + *pc*) *d*
and ⋀*pc pc′ d*. *shift pc* (*compx*Es_2 *es pc′ d*) = *compx*Es_2 *es* (*pc′* + *pc*) *d*⟨*proof*⟩

lemma *compx*E_2*-size-convs*[*simp*]:
shows *n* ≠ *0* ⟹ *compx*E_2 *e n d* = *shift n* (*compx*E_2 *e 0 d*)
and *n* ≠ *0* ⟹ *compx*Es_2 *es n d* = *shift n* (*compx*Es_2 *es 0 d*)⟨*proof*⟩
locale *TC2* = *TC1* +
fixes T_r :: *ty* **and** *mxs* :: *pc*
begin

definition
wt-instrs :: *instr list* ⇒ *ex-table* ⇒ ty_i*′ list* ⇒ *bool*
(‹(⊢ -, - /[::]/ -)› [*0*,*0*,*51*] *50*) **where**
⊢ *is*,*xt* [::] *τs* ⟷ *size is* < *size τs* ∧ *pcs xt* ⊆ {*0*..<*size is*} ∧
(∀ *pc*< *size is*. *P*,T_r,*mxs*,*size τs*,*xt* ⊢ *is*!*pc*,*pc* :: *τs*)

end

notation *TC2.wt-instrs* (‹(-,-,- ⊢/ -, - /[::]/ -)› [*50*,*50*,*50*,*50*,*50*,*51*] *50*)

lemma (**in** *TC2*) [*simp*]: *τs* ≠ [] ⟹ ⊢ [],[] [::] *τs*⟨*proof*⟩
lemma [*simp*]: *eff i P pc et None* = []⟨*proof*⟩
lemma *wt-instr-appR*:
⟦ *P*,*T*,*m*,*mpc*,*xt* ⊢ *is*!*pc*,*pc* :: *τs*;

$pc < size\ is$; $size\ is < size\ \tau s$; $mpc \le size\ \tau s$; $mpc \le mpc'$ ⟧
$\Longrightarrow P,T,m,mpc',xt \vdash is!pc,pc :: \tau s@\tau s'$⟨*proof*⟩

lemma *relevant-entries-shift* [*simp*]:
relevant-entries P i (pc+n) (shift n xt) = shift n (relevant-entries P i pc xt)⟨*proof*⟩

lemma [*simp*]:
xcpt-eff i P (pc+n) τ *(shift n xt) =*
map $(\lambda(pc,\tau).\ (pc + n,\ \tau))$ *(xcpt-eff i P pc* τ *xt)*⟨*proof*⟩

lemma [*simp*]:
$app_i\ (i,\ P,\ pc,\ m,\ T,\ \tau) \Longrightarrow$
eff i P (pc+n) (shift n xt) (Some τ*) =*
map $(\lambda(pc,\tau).\ (pc+n,\tau))$ *(eff i P pc xt (Some* τ*))*⟨*proof*⟩

lemma [*simp*]:
xcpt-app i P (pc+n) mxs (shift n xt) τ *= xcpt-app i P pc mxs xt* τ⟨*proof*⟩

lemma *wt-instr-appL*:
assumes $P,T,m,mpc,xt \vdash i,pc :: \tau s$ **and** $pc < size\ \tau s$ **and** $mpc \le size\ \tau s$
shows $P,T,m,mpc + size\ \tau s',shift\ (size\ \tau s')\ xt \vdash i,pc+size\ \tau s' :: \tau s'@\tau s$⟨*proof*⟩

lemma *wt-instr-Cons*:
assumes *wti*: $P,T,m,mpc - 1,[] \vdash i,pc - 1 :: \tau s$
and *pcl*: $0 < pc$ **and** *mpcl*: $0 < mpc$
and *pcu*: $pc < size\ \tau s + 1$ **and** *mpcu*: $mpc \le size\ \tau s + 1$
shows $P,T,m,mpc,[] \vdash i,pc :: \tau\#\tau s$⟨*proof*⟩

lemma *wt-instr-append*:
assumes *wti*: $P,T,m,mpc - size\ \tau s',[] \vdash i,pc - size\ \tau s' :: \tau s$
and *pcl*: $size\ \tau s' \le pc$ **and** *mpcl*: $size\ \tau s' \le mpc$
and *pcu*: $pc < size\ \tau s + size\ \tau s'$ **and** *mpcu*: $mpc \le size\ \tau s + size\ \tau s'$
shows $P,T,m,mpc,[] \vdash i,pc :: \tau s'@\tau s$⟨*proof*⟩

lemma *xcpt-app-pcs*:
$pc \notin pcs\ xt \Longrightarrow$ *xcpt-app i P pc mxs xt* τ⟨*proof*⟩

lemma *xcpt-eff-pcs*:
$pc \notin pcs\ xt \Longrightarrow$ *xcpt-eff i P pc* τ *xt = []*⟨*proof*⟩

lemma *pcs-shift*:
$pc < n \Longrightarrow pc \notin pcs\ (shift\ n\ xt)$⟨*proof*⟩

lemma *wt-instr-appRx*:
⟦ $P,T,m,mpc,xt \vdash is!pc,pc :: \tau s$; $pc < size\ is$; $size\ is < size\ \tau s$; $mpc \le size\ \tau s$ ⟧
$\Longrightarrow P,T,m,mpc,xt\ @\ shift\ (size\ is)\ xt' \vdash is!pc,pc :: \tau s$⟨*proof*⟩

lemma *wt-instr-appLx*:
⟦ $P,T,m,mpc,xt \vdash i,pc :: \tau s$; $pc \notin pcs\ xt'$ ⟧
$\Longrightarrow P,T,m,mpc,xt'@xt \vdash i,pc :: \tau s$⟨*proof*⟩

lemma (**in** *TC2*) *wt-instrs-extR*:
$\vdash is,xt\ [::]\ \tau s \Longrightarrow \vdash is,xt\ [::]\ \tau s\ @\ \tau s'$⟨*proof*⟩

lemma (**in** *TC2*) *wt-instrs-ext*:
assumes wt_1: $\vdash is_1,xt_1$ [::] $\tau s_1 @ \tau s_2$ **and** wt_2: $\vdash is_2,xt_2$ [::] τs_2
and τs-*size*: *size* τs_1 = *size* is_1
shows $\vdash is_1 @ is_2$, xt_1 @ *shift* (*size* is_1) xt_2 [::] $\tau s_1 @ \tau s_2$⟨*proof*⟩
corollary (**in** *TC2*) *wt-instrs-ext2*:
⟦ $\vdash is_2,xt_2$ [::] τs_2; $\vdash is_1,xt_1$ [::] $\tau s_1 @ \tau s_2$; *size* τs_1 = *size* is_1 ⟧
⟹ $\vdash is_1 @ is_2$, xt_1 @ *shift* (*size* is_1) xt_2 [::] $\tau s_1 @ \tau s_2$⟨*proof*⟩

corollary (**in** *TC2*) *wt-instrs-ext-prefix* [*trans*]:
⟦ $\vdash is_1,xt_1$ [::] $\tau s_1 @ \tau s_2$; $\vdash is_2,xt_2$ [::] τs_3;
size τs_1 = *size* is_1; *prefix* τs_3 τs_2 ⟧
⟹ $\vdash is_1 @ is_2$, xt_1 @ *shift* (*size* is_1) xt_2 [::] $\tau s_1 @ \tau s_2$⟨*proof*⟩

corollary (**in** *TC2*) *wt-instrs-app*:
assumes is_1: $\vdash is_1,xt_1$ [::] $\tau s_1 @ [\tau]$
assumes is_2: $\vdash is_2,xt_2$ [::] $\tau \# \tau s_2$
assumes *s*: *size* τs_1 = *size* is_1
shows $\vdash is_1 @ is_2$, xt_1@*shift* (*size* is_1) xt_2 [::] $\tau s_1 @ \tau \# \tau s_2$⟨*proof*⟩

corollary (**in** *TC2*) *wt-instrs-app-last*[*trans*]:
assumes $\vdash is_2,xt_2$ [::] $\tau \# \tau s_2$ $\vdash is_1,xt_1$ [::] τs_1
last $\tau s_1 = \tau$ *size* τs_1 = *size* is_1+*1*
shows $\vdash is_1 @ is_2$, xt_1@*shift* (*size* is_1) xt_2 [::] $\tau s_1 @ \tau s_2$⟨*proof*⟩

corollary (**in** *TC2*) *wt-instrs-append-last*[*trans*]:
assumes *wtis*: $\vdash is,xt$ [::] τs **and** *wti*: $P,T_r,mxs,mpc,[] \vdash i,pc :: \tau s$
and *pc*: *pc* = *size is* **and** *mpc*: *mpc* = *size* τs **and** *is*-τs: *size is* + *1* < *size* τs
shows $\vdash is @ [i],xt$ [::] τs⟨*proof*⟩

corollary (**in** *TC2*) *wt-instrs-app2*:
⟦ $\vdash is_2,xt_2$ [::] $\tau' \# \tau s_2$; $\vdash is_1,xt_1$ [::] $\tau \# \tau s_1 @ [\tau']$;
$xt' = xt_1$ @ *shift* (*size* is_1) xt_2; *size* τs_1+*1* = *size* is_1 ⟧
⟹ $\vdash is_1 @ is_2,xt'$ [::] $\tau \# \tau s_1 @ \tau' \# \tau s_2$⟨*proof*⟩

corollary (**in** *TC2*) *wt-instrs-app2-simp*[*trans*,*simp*]:
⟦ $\vdash is_2,xt_2$ [::] $\tau' \# \tau s_2$; $\vdash is_1,xt_1$ [::] $\tau \# \tau s_1 @ [\tau']$; *size* τs_1+*1* = *size* is_1 ⟧
⟹ $\vdash is_1 @ is_2$, xt_1@*shift* (*size* is_1) xt_2 [::] $\tau \# \tau s_1 @ \tau' \# \tau s_2$⟨*proof*⟩

corollary (**in** *TC2*) *wt-instrs-Cons*[*simp*]:
⟦ $\tau s \neq []$; $\vdash [i],[]$ [::] $[\tau,\tau']$; $\vdash is,xt$ [::] $\tau' \# \tau s$ ⟧
⟹ $\vdash i \# is$,*shift 1 xt* [::] $\tau \# \tau' \# \tau s$⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨
theory *Jinja*
imports
J/TypeSafe
J/Annotate

J/execute-Bigstep
J/execute-WellType
JVM/JVMDefensive
JVM/JVMListExample
BV/BVExec
BV/LBVJVM
BV/BVNoTypeError
BV/BVExample

Compiler/TypeComp
begin

end

Bibliography

[1] G. Klein and T. Nipkow. A machine-checked model for a Java-like language, virtual machine and compiler. Technical report, National ICT Australia, Sydney, Mar. 2004.

[2] G. Klein and T. Nipkow. A machine-checked model for a Java-like language, virtual machine and compiler. *ACM Trans. Prog. Lang. Syst.*, 28(4):619–695, 2006.