A Meta-Model for the Isabelle API

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Abstract

We represent a theory of (a fragment of) Isabelle/HOL in Isabelle/HOL. The purpose of this exercise is to write packages for domain-specific specifications such as class models, B-machines, . . . , and generally speaking, any domain-specific languages whose abstract syntax can be defined by a HOL “datatype”. On this basis, the Isabelle code-generator can then be used to generate code for global context transformations as well as tactic code.

Consequently the package is geared towards parsing, printing and code-generation to the Isabelle API. It is at the moment not sufficiently rich for doing meta theory on Isabelle itself. Extensions in this direction are possible though.

Moreover, the chosen fragment is fairly rudimentary. However it should be easily adapted to one’s needs if a package is written on top of it. The supported API contains types, terms, transformation of global context like definitions and data-type declarations as well as infrastructure for Isar-setups.

This theory is drawn from the Featherweight OCL\cite{1} project where it is used to construct a package for object-oriented data-type theories generated from UML class diagrams. The Featherweight OCL, for example, allows for both the direct execution of compiled tactic code by the Isabelle API as well as the generation of .thy-files for debugging purposes.

Gained experience from this project shows that the compiled code is sufficiently efficient for practical purposes while being based on a formal model on which properties of the package can be proven such as termination of certain transformations, correctness, etc.
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Part I.

A Meta-Model for the Isabelle API
1. Initialization

theory Init
  imports isabelle-home/src/HOL/Isabelle-Main
begin

1.1. Optimization on the String Datatype

The following types will allow to delay all concatenations on integer list, until we reach the end. As optimization, we also consider the use of String.literal besides integer list.

type-notation natural (nat)
definition Succ x = x + 1
datatype string_base = ST String.literal |
  ST' integer list
datatype abr-string = SS-base string_base |
  String-concatWith abr-string abr-string list

syntax -string1 :: - ⇒ abr-string (⟨ ⟨-⟩ ⟩ )
translations ⟨x⟩ = CONST SS-base (CONST ST x)
syntax -string3 :: - ⇒ abr-string (≪ ⟨-⟩ ≫ )
translations ≪x≫ = CONST SS-base (CONST ST’ x)
syntax -integer1 :: - ⇒ abr-string (" ⟨-⟩ ")
translations "x" = CONST SS-base (CONST ST’ ((CONST Cons) x (CONST Nil)))
type-notation abr-string (string)

1.2. Basic Extension of the Standard Library

1.2.1. Polymorphic Cartouches

We generalize the construction of cartouches for them to be used “polymorphically”, however the type inference is not automatic: types of all cartouche expressions will need to be specified earlier before their use (we will however provide a default type).
ML:
structure Cartouche-Grammar = struct
fun list-comb-mk cst n c = list-comb (Syntax.const cst, String.Syntax.mk-bits-syntax n c)
val nil1 = Syntax.const @{const-syntax String.empty-literal}
fun cons1 c l = list-comb-mk @{const-syntax String.Literal} 7 c $ l
val default = 
  [ (char list
    , (Const @{const-syntax Nil}, @{typ char list})
    , fn c => fn l => Syntax.const @{const-syntax Cons} $ list-comb-mk @{const-syntax Char} 8 c $ l
    , snd))
  , (String.literal, (nil1, cons1, snd))
  , (abr-string
    , nil1
    , cons1
    , fn (_, x) => Syntax.const @{const-syntax SS-base} $ (Syntax.const @{const-syntax ST} $ x))] end

fun parse-translation-cartouche binding l f-integer accu =
  let val cartouche-type = Attrib.setup-config-string binding (K (fst (hd l)))
  (* if there is no type specified, by default we set the first element
to be the default type of cartouches *) in
  fn ctxt =>
  let val cart-type = Config.get ctxt cartouche-type in
  case List.find (fn (s, _) => s = cart-type) l of
    NONE => error (Unregistered return type for the cartouche: \^ cart-type ^\)
  | SOME (c, (nil0, cons, f)) =>
    string-tr f (f-integer, cons, nil0) accu (Symbol.Pos.cartouche-content o Symbol.Pos.explode)
  end
  end
end
parse-translation ( [
  @{syntax-const -cartouche-string}
  , parse-translation-cartouche @{binding cartouche-type} Cartouche-Grammar.default (K I) ()])

This is the special command which sets the type of subsequent cartouches. Note: here
the given type is currently parsed as a string, one should extend it to be a truly “typed”
type...

declare[[cartouche-type = abr-string]]
1.2.2. Operations on List

datatype ('a, 'b) nsplit = Nsplit-text 'a | Nsplit-sep 'b

locale L

begin
definition map where \( \text{map } f \ l = \text{rev} (\text{foldl} (\lambda l \ x. f x \# l) \ [] \ l) \)
definition flatten l = \text{foldl} (\lambda acc \ l. \text{foldl} (\lambda acc \ x. x \# acc) acc (\text{rev} l)) \ [] \ (\text{rev} l)
definition mapf f l = \text{rev} (\text{fst} (\text{foldl} (\lambda l.\text{cpt} x. (f \text{cpt} x \# l, \text{Succ} \text{cpt})) ([], 0::\text{nat}) \ l))
definition iter f = \text{foldl} (\lambda r. f) ()
definition maps f x = \text{map} f \ x
definition replace-gen f-res l c0 lby = \text{let } \text{assoc } x1 l = \text{List} \cdot \text{fold} (\lambda x2, v. \lambda \text{None }\Rightarrow \text{if } x1 = x2 \text{ then } \text{Some } v \text{ else } \text{None } | x \Rightarrow x) l \text{ None} \text{ definition split where } \text{split } l = (\text{L.map} \ fst \ l, \text{L.map} \ snd \ l)\text{ definition upto where } \text{upto } i j = (\text{let } \text{to-i }= \text{Ln. int-of-integer} (\text{integer-of-natural } n) \text{ in } \text{L.map} (\text{natural-of-integer } o \text{ integer-of-int}) \ (\text{List} \cdot \text{upto} \ (\text{to-i } i) \ (\text{to-i } j)))\text{ definition take-first } = \text{take-rev}\text{ definition split-at } f l = (\text{let } f = \lambda x. \text{\( \sim f x \in \)})\text{ definition replace-gen } f-res l c0 lby = (\text{let } \text{Nsplit-text }= \lambda l \ \text{gen}. \text{if } l = [] \text{ then } \text{gen} \ \text{else } \text{Nsplit-text} l \# l \text{gen in} \text{case } \text{List} \cdot \text{fold} \ \text{of} \ \begin{cases} (\lambda c1 \ (l, \text{gen}). & \text{if } c0 \ c1 \text{ then} \ (lby, \text{Nsplit-sep } c1 \# \text{Nsplit-text } l \ \text{gen}) \text{ else} \ (c1 \# l, \text{gen})) \text{ of } (l, \text{gen}) \Rightarrow f-res (\text{Nsplit-text } l \ \text{gen}) \end{cases})\text{ definition nsplit-f } l c0 = \text{replace-gen id } l \ c0 []\text{ definition replace } = \text{replace-gen} (\text{L.flatten } o \text{L.map} (\lambda \text{Nsplit-text } l \Rightarrow l | - \Rightarrow []))

fun map-find-aux where

\text{map-find-aux accu} f l = (\lambda [] \Rightarrow \text{List} \cdot \text{rev} \ accu)n x \# xs \Rightarrow \text{case } f x \text{ of } \text{Some } x \Rightarrow \text{List} \cdot \text{fold} \ \text{Cons} \ accu (x \# xs) \text{ None } \Rightarrow \text{map-find-aux} (x \# accu) f x s) \ l\text{ definition map-find } = \text{map-find-aux} []
notation \texttt{L.append (infixr \texttt{@@} 65)}

lemmas \[\text{code} =
  \- \text{def}
  \text{L.map-def}
  \text{L.flatten-def}
  \text{L.mapi-def}
  \text{L.iter-def}
  \text{L.maps-def}
  \text{L.append-def}
  \text{L.filter-def}
  \text{L.rev-map-def}
  \text{L.mapM-def}
  \text{L.assoc-def}
  \text{L.split-def}
  \text{L.upto-def}
  \text{L.split-at-def}
  \text{L.take-def}
  \text{L.take-last-def}
  \text{L.take-first-def}
  \text{L.replace-gen-def}
  \text{L.nsplit-f-def}
  \text{L.replace-def}
  \text{L.map-find-def}
\]

  \- \text{fun}
  \text{L.map-find-aux.simps}

1.2.3. Operations on Char

definition \texttt{ascii-of-literal (INT) where}
  \texttt{ascii-of-literal = hd o String.ascii-of-literal}

definition \texttt{(integer-escape :: integer) = 0x09}
definition \texttt{ST0 c = \langle\langle c\rangle\rangle}
definition \texttt{ST0-base c = ST' [c]}

1.2.4. Operations on String (I)

notation \texttt{String.ascii-of-literal (INTS)}

locale \texttt{S}
locale \texttt{String}
locale \texttt{Stringbase}

definition \texttt{(in S) flatten = String-concatWith \langle\rangle}
definition \texttt{(in String) flatten a b = S.flatten [a, b]}
notation \texttt{String.flatten (infixr \texttt{@@} 65)}
definition \texttt{(in String) make n c = \langle\langle L.map (\lambda-. c) (L.upto 1 n)\rangle\rangle}
definition (in String_base) map-replace g = (\lambda ST s ⇒ replace (Some s) (∧) ST' s ⇒ S.flatten (L.map g s))

fun (in String) map-gen where
map-gen replace g e =
(\lambda SS-base s ⇒ String_base.map-gen replace g s
| String-concatWith abr l ⇒ String-concatWith (map-gen replace g abr) (List.map (map-gen replace g l) e)

definition (in String) foldl-one f accu = foldl f accu o INTS

definition (in String_base) foldl where foldl f accu = (\lambda ST s ⇒ String.foldl-one f accu s
| ST' s ⇒ List.foldl f accu s)

fun (in String) foldl where
foldl f accu e =
(\lambda SS-base s ⇒ String_base.foldl f accu s
| String-concatWith abr l ⇒
  (case l of [] ⇒ accu
   | x # xs ⇒ List.foldl ((\lambda accu. foldl f (foldl f accu abr)) (foldl f accu x) xs)) e)

definition (in S) replace-integers f s1 s2 =
s1 @@ (case s of None ⇒ () | Some s ⇒ flatten (L.map f (INTS s))) @@ s2

definition (in String) map where map f = map-gen (S.replace-integers (\lambda c. °f c°)) (\lambda x. °f x°)

definition (in String) replace-integers f = map-gen (S.replace-integers (\lambda c. f c)) f

definition (in String) all f = foldl (\lambda b s. b & f s) True

definition (in String) length where length = foldl (\lambda n -. Suc n) 0

definition (in String_base) to-list = (\lambda ST s ⇒ INTS s | ST' l ⇒ l)

definition (in String) meta-of-logic = String.literal-of-asciis o to-list

definition (in String) to-String_base = (\lambda SS-base s ⇒ s | s ⇒ ST' (to-list s))

definition (in String_base) to-String = SS-base

definition (in String_base) is-empty = (\lambda ST s ⇒ s = STR ""
| ST' s ⇒ s = [])

fun (in String) is-empty where
is-empty e = (\lambda SS-base s ⇒ String_base.is-empty s | String-concatWith - l ⇒ list-all is-empty l) e

definition (in String) equal s1 s2 = (to-list s1 = to-list s2)

notation String.equal (infixl \triangleq 50)

definition (in String) assoc x l = L.assoc (to-list x) (L.map (map-prod String_base.to-list id) l)

definition (in String) member l x = L.member (L.map String_base.to-list l) (to-list x)

definition (in String_base) flatten l = String.to-String_base (S.flatten (L.map to-String l))

lemmas [code] =
  -- def
  S.flatten-def
  String.flatten-def
  String.make-def
  String_base.map-gen-def
  String.foldl-one-def
  String_base.foldl-def
  S.replace-integers-def
  String.map-def
  String.replace-integers-def
1.2.5. Operations on String (II)

**definition** wildcard = (∗)

**context** String

**begin**

**definition** lowercase = map (λ n. if n < 97 then n + 32 else n)

**definition** uppercase = map (λ n. if n < 97 then n else n − 32)

**definition** to-bold-number = replace-integers (λ n. [0, 1, 2, 3, 4, 5, 6, 7, 8, 9] ! nat-of-integer (n − 48))

**fun** nat-to-digit10-aux where

| nat-to-digit10-aux l (n :: Nat.nat) = (if n < 10 then n # l else nat-to-digit10-aux (n mod 10 # l) (n div 10)) |
|---|---|
| **definition** nat-to-digit10 n = |
| (let nat-raw-to-str = L.map (integer-of-nat o (+) 0x30) in |
| ≪ nat-raw-to-str (nat-to-digit10-aux [] n) ≫ ) |
| **definition** natural-to-digit10 = nat-to-digit10 o nat-of-natural |

**declare** [cartouche-type = String.literal]

**definition** integer-to-digit16 = |
| (let f = nth (INTS ⟨0123456789ABCDEF⟩) o nat-of-integer in |
| λn ⇒ ≪ f (n div 16), f (n mod 16) ≫ ) |

**end**

**lemmas** [code] =

| def |
|---|---|
| String.lowercase-def |
| String.uppercase-def |
| String.to-bold-number-def |
| String.nat-to-digit10-def |
| String.natural-to-digit10-def |
definition add-0 n =
(let n = nat-of-integer n in
S.flatten (L.map (λ-. ⟨0⟩) (upt 0 (if n < 10 then 2 else if n < 100 then 1 else 0))))
@ String.nat-to-digit10 n)
definition is-letter =
(let int-A = INT ⟨A⟩; int-Z = INT ⟨Z⟩; int-a = INT ⟨a⟩; int-z = INT ⟨z⟩ in
(λn. n ≥ int-A & n ≤ int-Z | n ≥ int-a & n ≤ int-z))
definition is-digit =
(let int-0 = INT ⟨0⟩; int-9 = INT ⟨9⟩ in
(λn. n ≥ int-0 & n ≤ int-9))
definition is-special = List.member (INTS {<=>-./}{})
context String
begin
definition base255 = replace-integers (λc. if is-letter c then °c° else add-0 c)
declare[cartouche-type = abr-string]
definition sub =
replace-integers (let is-und = List.member (INTS STR "-")) in
(λc. if is-letter c | is-digit c | is-und c then ⟨⟩ else °c° else add-0 c))
definition isup s = ⟨-⟩ s
declare[cartouche-type = abr-string]
definition text-of-str str =
(let s = ⟨c⟩;
ap = ⟨#⟩ in
S.flatten [(λlet, s, ( = char-of :: nat ⇒ char in )
, String.replace-integers (λc.
if is-letter c then
S.flatten [CHR "",c",",ap]
else
S.flatten [s, ⟨⟩, add-0 c, ap])
str , ⟨[]⟩)]
deinition text2-of-str = String.replace-integers (λc. S.flatten [\, ⟨⟩, ⟨",c",⟩])
definition textstr-of-str f-flatten f-integer f-str str =
(let str0 = String.to-list str
; f-letter = λc. is-letter c | is-digit c | is-special c
; s = ⟨c⟩
; f-text = λ Nsplit-text l ⇒ S.flatten [f-str (S.flatten [STR "", ≪l≫,'"'])]
   | Nsplit-sep c ⇒ S.flatten [f-integer c]
; str = case L.nsplit-f str0 (Not o f-letter) of
   [] ⇒ S.flatten [f-str ⟨STR ""⟩]
   | l ⇒ S.flatten (L.map (λx. ⟨(⟩ @ f-text x @⟩ @@) # ) l) @ @ ⟨⟩, in
if list-all f-letter str0 then
  str
else
  f-flatten (S.flatten [ ⟨⟨⟩, str, ⟨⟩] )
)
definition escape-sml = String.replace-integers (λn. if n = 0x22 then ⟨⟩ else "n")
definition mk-constr-name name = (λ x. S.flatten [String.isub name, ⟨-⟩, String.isub x])
definition mk-dot s1 s2 = S.flatten [⟨⟩, s1, s2]
definition mk-dot-par-gen dot l-s = S.flatten [dot, ⟨⟩, case l-s of [] ⇒ ⟨⟩ | x # xs ⇒ S.flatten [x, S.flatten (L.map (λs. ⟨⟩ @ @ s) xs)], ⟨⟩]]
definition mk-dot-par dot s = mk-dot-par-gen dot [s]
definition mk-dot-comment s1 s2 s3 = mk-dot s1 (S.flatten [s2, ⟨/*⟩, s3, ⟨*/⟩])
definition hol-definition s = S.flatten [s, ⟨-def⟩]
definition hol-split s = S.flatten [s, ⟨split⟩]
end
2. Defining Meta-Models

2.1. (Pure) Term Meta-Model aka. AST definition of (Pure) Term

theory Meta-Pure
imports ..../Init
begin

2.1.1. Type Definition

type-synonym indexname = string × nat
type-synonym class = string
type-synonym sort = class list
datatype typ =
  Type string typ list |
  TFree string sort |
  TVar indexname sort
datatype term =
  Const string typ |
  Free string typ |
  Var indexname typ |
  Bound nat |
  Abs string typ term |
  App term term (infixl $\times 200$)

2.1.2. Operations of Fold, Map, ..., on the Meta-Model

fun map-Const where
  map-Const f expr = (λ Const s ty ⇒ Const (f s ty) ty
                     | Free s ty ⇒ Free s ty
                     | Var i ty ⇒ Var i ty
                     | Bound n ⇒ Bound n
                     | Abs s ty term ⇒ Abs s ty (map-Const f term)
                     | App term1 term2 ⇒ App (map-Const f term1)
                                       (map-Const f term2))
expr

fun fold-Const where
  fold-Const f accu expr = (λ Const s ⇒ f accu s
                           | Abs - - term ⇒ fold-Const f accu term
                           | App term1 term2 ⇒ fold-Const f (fold-Const f accu term1) term2
                           | - ⇒ accu)
fun fold-Free where
  fold-Free f accu expr = (λ Free s - ⇒ f accu s
  | Abs - - term ⇒ fold-Free f accu term
  | App term1 term2 ⇒ fold-Free f (fold-Free f accu term1) term2
  | - ⇒ accu)
  end

2.2. SML Meta-Model aka. AST definition of SML

theory Meta-SML
imports ..../Init
begin

2.2.1. Type Definition

The following datatypes beginning with semi__ represent semi-concrete syntax, deliberately not minimal abstract syntax like (Pure) Term, this is for example to facilitate the pretty-printing process, or for manipulating recursively data-structures through an abstract and typed API.

datatype semi--val-fun = Sval
  | Sfun

datatype semi--term' = SML-string string
  | SML-rewrite semi--val-fun semi--term' — left string — symb rewriting
  semi--term' — right
  | SML-basic string list
  | SML-binop semi--term' string semi--term'
  | SML-annot semi--term' string — type
  | SML-function (semi--term' — pattern × semi--term' — to return) list
  | SML-apply semi--term' semi--term' list
  | SML-paren string — left string — right semi--term'
  | SML-let-open string semi--term'

2.2.2. Extending the Meta-Model

locale SML
begin

no-type-notation abr-string (string) definition string = SML-string
definition rewrite = SML-rewrite
definition basic = SML-basic
definition binop = SML-binop
definition annot = SML-annot
definition function = SML-function
definition apply = SML-apply
definition paren = SML-paren

definition let-open = SML-let-open

definition app s = apply (basic [s])
definition none = basic [⟨NONE⟩]
definition some s = app (⟨SOME⟩) [s]
definition option' f l = (case map-option f l of None ⇒ none | Some s ⇒ some s)
definition option = option' id

definition parenthesis — mandatory parenthesis = paren (⟨⟩ ⟨⟩)
definition binop-l s l = (case rev l of x # xs ⇒ List.fold (λ x. binop x s) xs x)
definition list l = (case l of [] ⇒ basic [⟨[]⟩] | - ⇒ paren [⟨|⟩] (binop-l ⟨⟩ l))
definition list' f l = list (L.map f l)
definition pair e1 e2 = parenthesis (binop e1 (⟨⟩ e2))
definition pair' f1 f2 = (λ (e1, e2) ⇒ parenthesis (binop (f1 e1) (⟨⟩ (f2 e2))))
definition rewrite-val = rewrite Sval
definition rewrite-fun = rewrite Sfun
end

lemmas [code] =
  — def
SML.string-def
SML.rewrite-def
SML.basic-def
SML.binop-def
SML.annot-def
SML.function-def
SML.apply-def
SML.paren-def
SML.let-open-def
SML.app-def
SML.none-def
SML.some-def
SML.option'-def
SML.option-def
SML.parenthesis-def
SML.binop-l-def
SML.list-def
SML.list'-def
SML.pair-def
SML.pair'-def
SML.rewrite-val-def
SML.rewrite-fun-def

end

2.3. Isabelle Meta-Model aka. AST definition of Isabelle

theory Meta-Isabelle
imports Meta-Pure

end
Meta-SML

begin

2.3.1. Type Definition

The following datatypes beginning with `semi__` represent semi-concrete syntax, deliberately not minimal abstract syntax like (Pure) Term, this is for example to facilitate the pretty-printing process, or for manipulating recursively data-structures through an abstract and typed API.

```plaintext
datatype semi--typ = Typ-apply semi--typ semi--typ list
  | Typ-apply-bin string — binop semi--typ semi--typ
  | Typ-apply-paren string — left string — right semi--typ
  | Typ-base string

datatype datatype = Datatype string — name
  (string — name × semi--typ list — arguments) list — constructors

datatype type-synonym = Type-synonym string — name
  string list — parametric variables
  semi--typ — content

datatype semi--term = Term-rewrite semi--term — left string — symb rewriting semi--term — right
  | Term-basic string list
  | Term-annot semi--term semi--typ
  | Term-bind string — symbol semi--term — arg semi--term
  | Term-fun-case semi--term — value option — none: function (semi--term — pattern × semi--term — to return) list
  | Term-apply semi--term semi--term list
  | Term-paren string — left string — right semi--term
  | Term-if-then-else semi--term semi--term semi--term
  | Term-term string list — simulate a pre-initialized context (de bruijn variables under 'lam')

  term — usual continuation of inner syntax term

 datatype type-notation = Type-notation string — name
  string — content

 datatype instantiation = Instantiation string — name
  string — name in definition
  semi--term

 datatype overloading = Overloading string — name consts semi--term
  string — name def semi--term — content

 datatype consts = Consts string — name
  semi--typ
  string — expression in 'post' mixfix
```
**datatype** definition = Definition semi-term
  | Definition-where1 string — name semi-term — syntax extension × nat —
  priority semi-term
  | Definition-where2 string — name semi-term — syntax extension semi-term

**datatype** semi-thm-attribute = Thm-thm string — represents a single thm
  | Thm-thms string — represents several thms
  | Thm-THEN semi-thm-attribute semi-thm-attribute
  | Thm-simplified semi-thm-attribute semi-thm-attribute
  | Thm-symmetric semi-thm-attribute
  | Thm-where semi-thm-attribute (string × semi-term) list
  | Thm-of semi-thm-attribute semi-term list
  | Thm-OF semi-thm-attribute semi-thm-attribute

**datatype** semi-thm = Thms-single semi-thm-attribute
  | Thms-mult semi-thm-attribute

**type-synonym** semi-thm-l = semi-thm list

**datatype** lemmas = Lemmas-simp-thm bool — True : simp
  string — name
  semi-thm-attribute list
  | Lemmas-simp-thms string — name
  string — thms list

**datatype** semi-method-simp = Method-simp-only semi-thm-l
  — split

**datatype** semi-method = Method-rule semi-thm-attribute option
  | Method-drule semi-thm-attribute
  | Method-erule semi-thm-attribute
  | Method-intro semi-thm-attribute list
  | Method-elim semi-thm-attribute
  | Method-subst bool — asm
    string — nat list — pos
    semi-thm-attribute
  | Method-insert semi-thm-l
  | Method-plus semi-method list
  | Method-option semi-method list
  | Method-or semi-method list
  | Method-one semi-method-simp
  | Method-all semi-method-simp
  | Method-auto-simp-add-split semi-thm-l string list
  | Method-rename-tac string list
  | Method-case-tac semi-term
  | Method-blast nat option
  | Method-clarify
Method-metis string list — e.g. no-types (override-type-encs)

semi-thm-attribute list

datatype semi-command-final = Command-done
| Command-by semi-method list
| Command-sorry

datatype semi-command-state = Command-apply-end semi-method list — apply-end (., . . .)

datatype semi-command-proof = Command-apply semi-method list — apply (., . . .)
| Command-using semi-thm-l — using . . .
| Command-unfolding semi-thm-l — unfolding . . .
| Command-let semi-term — name semi-term
| Command-have string — name
  bool — true: add [simp]
  semi-term
  semi-command-final
| Command-fix-let string list
  (semi-term — name × semi-term) list — let statements
  (semi-term list — show . . . ⇒ . . .)
  × semi-term list — when . . . . . .
  option — None ⇒

?thesis

semi-command-state list — qed apply-end . . .

datatype lemma = Lemma string — name semi-term list — specification to prove
semi-method list list — tactics: apply (., . . .) apply . . .
semi-command-final
| Lemma-assumes string — name
  (string — name × bool — true: add [simp] × semi-term) list — specification to prove (assms)
  semi-term — specification to prove (conclusion)
  semi-command-proof list
  semi-command-final

datatype axiomatization = Axiomatization string — name

semi-term

datatype section = Section nat — nesting level

string — content

datatype text = Text string

datatype ML = SML semi-term'

datatype setup = Setup semi-term'

datatype thm = Thm semi-thm-attribute list

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datatype interpretation = Interpretation string — name
  string — locale name
  semi-term list — locale param
  semi-command-final

datatype semi-theory = Theory-datatype datatype
  Theory-type-synonym type-synonym
  Theory-type-notation type-notation
  Theory-instantiation instantiation
  Theory-overloading overloading
  Theory-consts consts
  Theory-definition definition
  Theory-lemmas lemmas
  Theory-lemma lemma
  Theory-axiomatization axiomatization
  Theory-section section
  Theory-text text
  Theory-ML ML
  Theory-setup setup
  Theory-thm thm
  Theory-interpretation interpretation

record semi-locale =
  HolThyLocale-name :: string
  HolThyLocale-header :: ( (semi-term — name × semi-typ — fix statement) list
                      × (string — name × semi-term — assumes statement) option — None:
                        no assumes to generate) list

datatype semi-theories = Theories-one semi-theory
  | Theories-locale semi-locale semi-theory list — positioning comments can
    occur before and after this group of commands list

2.3.2. Extending the Meta-Model

locale T
begin
  definition thm = Thm-thm
  definition thms = Thm-thms
  definition THEN = Thm-THEN
  definition simplified = Thm-simplified
  definition symmetric = Thm-symmetric
  definition where = Thm-where
  definition of' = Thm-of
  definition OF = Thm-OF
  definition OF-l s l = List.fold (\x acc. Thm-OF acc x) l s
  definition simplified-l s l = List.fold (\x acc. Thm-simplified acc x) l s
end

lemmas [code] =
— def
T.thm-def
T.thms-def
T.THEN-def
T.simplified-def
T.symmetric-def
T.where-def
T.of’-def
T.OF-def
T.OF-l-def
T.simplified-l-def

definition Opt s = Typ-apply (Typ-base (option)) [Typ-base s]
definition Raw = Typ-base
definition Type-synonym’ n = Type-synonym n []
definition Type-synonym” n l f = Type-synonym n l (f l)
definition Term-annot e s = Term-annot e (Typ-base s)
definition Term-lambdas s = Term-bind (λ) (Term-basic s)
definition Term-lambda x = Term-lambdas [x]
definition Term-lambdas0 = Term-bind (λ)
definition Term-empty = Term-parenthesis — mandatory parenthesis = Term-paren (⟨⟩ ⟨⟩)
definition Term-empty’ — optional parenthesis that can be removed but a warning will be raised = Term-parenthesis
definition Term-pat b = Term-basic [⟨?⟩ @@ b]
definition Term-And x f = Term-bind (⋀) (Term-basic [x]) (f x)
definition Term-exists x f = Term-bind (∃) (Term-basic [x]) (f x)
definition Term-lambdas0 = Term-bind (λ)
definition Term-empty = Term-parenthesis — mandatory parenthesis = Term-paren (⟨⟩ ⟨⟩)
definition Term-empty’ — optional parenthesis that can be removed but a warning will be raised = Term-parenthesis
definition Term-lambdas0 = Term-bind (λ)
definition Term-empty = Term-parenthesis — mandatory parenthesis = Term-paren (⟨⟩ ⟨⟩)
definition Term-empty’ — optional parenthesis that can be removed but a warning will be raised = Term-parenthesis
definition Term-empty = Term-parenthesis — mandatory parenthesis = Term-paren (⟨⟩ ⟨⟩)
definition Term-empty’ — optional parenthesis that can be removed but a warning will be raised = Term-parenthesis
definition Term-empty = Term-parenthesis — mandatory parenthesis = Term-paren (⟨⟩ ⟨⟩)
definition Term-empty’ — optional parenthesis that can be removed but a warning will be raised = Term-parenthesis

definition Term-set l = (case l of [] ⇒ Term-basic [⟨{}⟩] | - ⇒ Term-paren ⟨|⟩ ⟨⟩ (term-binop ⟨,⟩ l))
definition Term-list l = (case l of [] ⇒ Term-basic [⟨|⟩] | - ⇒ Term-paren ⟨|⟩ ⟨⟩ (term-binop ⟨,⟩ l))
definition Term-list’ f l = Term-list (L.map f l)
definition Term-pair e1 e2 = Term-parenthesis (Term-binop e1 ⟨,⟩ e2)
definition Term-pair’ l = (case l of [] ⇒ Term-basic [⟨⟩] | - ⇒ Term-paren ⟨|⟩ ⟨⟩ (term-binop ⟨,⟩ l))
definition Term-empty = Term-parenthesis — mandatory parenthesis = Term-paren (⟨⟩ ⟨⟩)
definition Term-empty’ — optional parenthesis that can be removed but a warning will be raised = Term-parenthesis
definition Term-empty = Term-parenthesis — mandatory parenthesis = Term-paren (⟨⟩ ⟨⟩)
definition Term-empty’ — optional parenthesis that can be removed but a warning will be raised = Term-parenthesis

definition Term-case = Term-fun-case o Some

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definition Term-function = Term-fun-case None

definition Term-term′ = Term-term []

definition Lemmas-simp = Lemmas-simp-thm True

definition Lemmas-nosimp = Lemmas-simp-thm False

definition Consts-value = (·)

definition Consts-raw0 s l e o-arg = Consts s l (String.replace-integers (λn. if n = 0x5F then (·) else "n") e @@ (case o-arg of None ⇒ ⟨⟩ | Some arg ⇒ let ap = λs. (·) @@ s @@ (·) in ap (if arg = 0 then ⟨⟩ else Consts-value @@ (S.flatten (L.map (λ·. (·) @@ Consts-value) (L.upto 2 arg)))))

definition Ty-arrow = Typ-apply-bin (⇒)

definition Ty-times = Typ-apply-bin (×)

definition Ty-arrow′ x = Ty-arrow x (Typ-base (·))

definition Ty-paren = Typ-apply-paren (⟨⟩ ⟨⟩)

definition Consts′ s l e = Consts-raw0 s (Ty-arrow (Typ-base (·)) l) e None

definition Overloading′ n ty = Overloading n (Term-annot (Term-basic [n]) ty)

locale M
begin

definition Method-simp-add-del l-a l-d = Method-simp-add-del-split l-a l-d []

definition Method-subst-l = Method-subst False

definition rule′ = Method-rule None

definition rule = Method-rule o Some

definition drule = Method-drule

definition erule = Method-erule

definition intro = Method-intro

definition elim = Method-elim

definition subst-l0 = Method-subst

definition subst-l = Method-subst-l

definition insert where insert = Method-insert o L.map Thms-single

definition plus where plus = Method-plus

definition option = Method-option

definition or = Method-or

definition meth-gen-simp = Method-simp-add-del [] []

definition meth-gen-simp-add2 l1 l2 = Method-simp-add-del (L.flatten [ L.map Thms-mult l1, L.map (Thms-single o Thm-thm) l2])

[]

definition meth-gen-simp-add-del l1 l2 = Method-simp-add-del (L.map (Thms-single o Thm-thm) l1) (L.map (Thms-single o Thm-thm) l2)

definition meth-gen-simp-add-del-split l1 l2 l3 = Method-simp-add-del-split (L.map Thms-single l1) (L.map Thms-single l2) (L.map Thms-single l3)
definition \texttt{meth-gen-simp-add-split} \texttt{l1 l2} = \texttt{Method-simp-add-del-split (L.map Thms-single l1)} \\
\texttt{(L.map Thms-single l2)}

definition \texttt{meth-gen-simp-only} \texttt{l} = \texttt{Method-simp-only (L.map Thms-single l)}

definition \texttt{meth-gen-simp-add0} \texttt{l} = \texttt{Method-simp-add-del (L.map Thms-single l)}

definition \texttt{simp} = \texttt{Method-one \texttt{meth-gen-simp}}

definition \texttt{simp-add2} \texttt{l1 l2} = \texttt{Method-one (meth-gen-simp-add2 l1 l2)}

definition \texttt{simp-add-del} \texttt{l1 l2} = \texttt{Method-one (meth-gen-simp-add-del l1 l2)}

definition \texttt{simp-add-split} \texttt{l1 l2} = \texttt{Method-one (meth-gen-simp-add-split l1 l2)}

definition \texttt{simp-only} \texttt{l} = \texttt{Method-one (meth-gen-simp-only l)}

definition \texttt{simp-only'} \texttt{l} = \texttt{Method-one (meth-gen-simp-only' l)}

definition \texttt{simp-add0} \texttt{l} = \texttt{Method-one (meth-gen-simp-add0 l)}

definition \texttt{simp-add} = \texttt{simp-add2}

definition \texttt{simp-all} = \texttt{Method-all \texttt{meth-gen-simp}}

definition \texttt{simp-all-add} \texttt{l} = \texttt{Method-all (meth-gen-simp-add2 l l)}

definition \texttt{simp-all-only} \texttt{l} = \texttt{Method-all (meth-gen-simp-only l)}

definition \texttt{simp-all-only'} \texttt{l} = \texttt{Method-all (meth-gen-simp-only' l)}

definition \texttt{auto-simp-add2} \texttt{l1 l2} = \texttt{Method-auto-simp-add-split (L.flatten [L.map Thms-mult l1, L.map (Thms-single o Thm-thm) l2])}

definition \texttt{auto-simp-add-split} \texttt{l} = \texttt{Method-auto-simp-add-split (L.map Thms-single l)}

definition \texttt{rename-tac} = \texttt{Method-rename-tac}

definition \texttt{case-tac} = \texttt{Method-case-tac}

definition \texttt{blast} = \texttt{Method-blast}

definition \texttt{clarify} = \texttt{Method-clarify}

definition \texttt{metis} = \texttt{Method-metis}

definition \texttt{metis0} = \texttt{Method-metis}

definition \texttt{subst-asm b} = \texttt{subst-l0 b [\{0\]}

definition \texttt{subst} = \texttt{subst-l [\{0\]}

definition \texttt{auto-simp-add} = \texttt{auto-simp-add2}

definition \texttt{auto} = \texttt{auto-simp-add}

end

lemmas [code] =
  — def
  \texttt{M.Method-simp-add-del-def}
  \texttt{M.Method-subst-l-def}
  \texttt{M.rule '-'-def}
  \texttt{M.rule-def}
  \texttt{M.drule-def}
  \texttt{Merule-def}
  \texttt{M.intro-def}
  \texttt{M.elim-def}
  \texttt{M.subst-l0-def}
  \texttt{M.subst-l-def}
  \texttt{M.insert-def}
  \texttt{M.plus-def}

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definition ty-arrow l = (case rev l of x # xs ⇒ List.fold Ty-arrow xs x)

locale C
begin
  definition done = Command-done
  definition by = Command-by
  definition sorry = Command-sorry
  definition apply-end = Command-apply-end
  definition apply = Command-apply
  definition using = Command-using o L.map Thms-single
  definition unfolding = Command-unfolding o L.map Thms-single
  definition let' = Command-let
  definition fix-let = Command-fix-let
definition fix l = Command-fix-let l [] None []
definition have n = Command-have n False
definition have0 = Command-have end

lemmas [code] =
  — def C.
  done-def C.
  by-def C.
  sorry-def C.
  apply-end-def C.
  apply-def C.
  using-def C.
  unfolding-def C.
  let'-def C.
  fix-let-def C.
  fix-def C.
  have-def C.
  have0-def C.

fun cross-abs-aux where
cross-abs-aux f l x = (λ (Suc n, Abs s - t) ⇒ f s (cross-abs-aux f (s ≠ l) (n, t))
  | (-, e) ⇒ Term-term l e)
  x
definition cross-abs f n l = cross-abs-aux f [] (n, l)

2.3.3. Operations of Fold, Map, ..., on the Meta-Model
definition map-lemma f = (λ Theory-lemma x ⇒ Theory-lemma (f x)
  | x ⇒ x)
end
3. Parsing Meta-Models

3.1. Initializing the Parser

theory Parser-init
imports ../Init
begin

3.1.1. Some Generic Combinators

definition K x - = x

definition co1 = (a)
definition co2 f g x1 x2 = f (g x1 x2)
definition co3 f g x1 x2 x3 = f (g x1 x2 x3)
definition co4 f g x1 x2 x3 x4 = f (g x1 x2 x3 x4)
definition co5 f g x1 x2 x3 x4 x5 = f (g x1 x2 x3 x4 x5)
definition co6 f g x1 x2 x3 x4 x5 x6 = f (g x1 x2 x3 x4 x5 x6)
definition co7 f g x1 x2 x3 x4 x5 x6 x7 = f (g x1 x2 x3 x4 x5 x6 x7)
definition co8 f g x1 x2 x3 x4 x5 x6 x7 x8 = f (g x1 x2 x3 x4 x5 x6 x7 x8)
definition co9 f g x1 x2 x3 x4 x5 x6 x7 x8 x9 = f (g x1 x2 x3 x4 x5 x6 x7 x8 x9)
definition co10 f g x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 = f (g x1 x2 x3 x4 x5 x6 x7 x8 x9 x10)
definition co11 f g x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11 = f (g x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11)
definition co12 f g x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11 x12 = f (g x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11 x12)
definition co13 f g x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11 x12 x13 = f (g x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11 x12 x13)
definition co14 f g x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11 x12 x13 x14 = f (g x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11 x12 x13 x14)
definition co15 f g x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11 x12 x13 x14 x15 = f (g x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11 x12 x13 x14 x15)
definition ap1 a v0 f1 v1 = a v0 [f1 v1]
definition ap2 a v0 f1 f2 v1 v2 = a v0 [f1 v1, f2 v2]
definition ap3 a v0 f1 f2 f3 v1 v2 v3 = a v0 [f1 v1, f2 v2, f3 v3]
definition ap4 a v0 f1 f2 f3 f4 v1 v2 v3 v4 = a v0 [f1 v1, f2 v2, f3 v3, f4 v4]
definition ap5 a v0 f1 f2 f3 f4 f5 v1 v2 v3 v4 v5 = a v0 [f1 v1, f2 v2, f3 v3, f4 v4, f5 v5]
definition ap6 a v0 f1 f2 f3 f4 f5 f6 v1 v2 v3 v4 v5 v6 = a v0 [f1 v1, f2 v2, f3 v3, f4 v4, f5 v5, f6 v6]
definition ap7 a v0 f1 f2 f3 f4 f5 f6 f7 v1 v2 v3 v4 v5 v6 v7 = a v0 [f1 v1, f2 v2, f3 v3, f4 v4, f5 v5, f6 v6, f7 v7]
definition ap8 a v0 f1 f2 f3 f4 f5 f6 f7 f8 v1 v2 v3 v4 v5 v6 v7 v8 = a v0 [f1 v1, f2 v2, f3 v3, f4 v4, f5 v5, f6 v6, f7 v7]
\[ f_4 v_4, f_5 v_5, f_6 v_6, f_7 v_7, f_8 v_8 \]
definition \text{ap9} a v_0 \{ f_1 f_2 f_3 f_4 f_5 f_6 f_7 f_8 f_9 v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8 v_9 = a v_0 \} [f_1 v_1, f_2 v_2, f_3 v_3, f_4 v_4, f_5 v_5, f_6 v_6, f_7 v_7, f_8 v_8, f_9 v_9]
definition \text{ap10} a v_0 \{ f_1 f_2 f_3 f_4 f_5 f_6 f_7 f_8 f_9 f_{10} v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8 v_9 v_{10} = a v_0 \} [f_1 v_1, f_2 v_2, f_3 v_3, f_4 v_4, f_5 v_5, f_6 v_6, f_7 v_7, f_8 v_8, f_9 v_9, f_{10} v_{10}]
definition \text{ap11} a v_0 \{ f_1 f_2 f_3 f_4 f_5 f_6 f_7 f_8 f_9 f_{10} f_{11} v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8 v_9 v_{10} v_{11} = a v_0 \} [f_1 v_1, f_2 v_2, f_3 v_3, f_4 v_4, f_5 v_5, f_6 v_6, f_7 v_7, f_8 v_8, f_9 v_9, f_{10} v_{10}, f_{11} v_{11}]
definition \text{ap12} a v_0 \{ f_1 f_2 f_3 f_4 f_5 f_6 f_7 f_8 f_9 f_{10} f_{11} f_{12} v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8 v_9 v_{10} v_{11} v_{12} = a v_0 \} [f_1 v_1, f_2 v_2, f_3 v_3, f_4 v_4, f_5 v_5, f_6 v_6, f_7 v_7, f_8 v_8, f_9 v_9, f_{10} v_{10}, f_{11} v_{11}, f_{12} v_{12}]
definition \text{ap13} a v_0 \{ f_1 f_2 f_3 f_4 f_5 f_6 f_7 f_8 f_9 f_{10} f_{11} f_{12} f_{13} v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8 v_9 v_{10} v_{11} v_{12} v_{13} = a v_0 \} [f_1 v_1, f_2 v_2, f_3 v_3, f_4 v_4, f_5 v_5, f_6 v_6, f_7 v_7, f_8 v_8, f_9 v_9, f_{10} v_{10}, f_{11} v_{11}, f_{12} v_{12}, f_{13} v_{13}]
definition \text{ap14} a v_0 \{ f_1 f_2 f_3 f_4 f_5 f_6 f_7 f_8 f_9 f_{10} f_{11} f_{12} f_{13} f_{14} v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8 v_{9} v_{10} v_{11} v_{12} v_{13} v_{14} = a v_0 \} [f_1 v_1, f_2 v_2, f_3 v_3, f_4 v_4, f_5 v_5, f_6 v_6, f_7 v_7, f_8 v_8, f_9 v_9, f_{10} v_{10}, f_{11} v_{11}, f_{12} v_{12}, f_{13} v_{13}, f_{14} v_{14}]
definition \text{ap15} a v_0 \{ f_1 f_2 f_3 f_4 f_5 f_6 f_7 f_8 f_{9} f_{10} f_{11} f_{12} f_{13} f_{14} f_{15} v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8 v_9 v_{10} v_{11} v_{12} v_{13} v_{14} v_{15} = a v_0 \} [f_1 v_1, f_2 v_2, f_3 v_3, f_4 v_4, f_5 v_5, f_6 v_6, f_7 v_7, f_8 v_8, f_9 v_9, f_{10} v_{10}, f_{11} v_{11}, f_{12} v_{12}, f_{13} v_{13}, f_{14} v_{14}, f_{15} v_{15}]
3.1.2. Generic Locale for Parsing

locale Parse =
  fixes ext :: string ⇒ string

— (effective) first order

fixes of-string :: (′a ⇒ ′a list ⇒ ′a) ⇒ (string ⇒ ′a) ⇒ string ⇒ ′a
fixes of-string_base :: (′a ⇒ ′a list ⇒ ′a) ⇒ (string ⇒ ′a) ⇒ string_base ⇒ ′a
fixes of-nat :: (′a ⇒ ′a list ⇒ ′a) ⇒ (string ⇒ ′a) ⇒ natural ⇒ ′a
fixes of-unit :: (string ⇒ ′a) ⇒ unit ⇒ ′a
fixes of-bool :: (string ⇒ ′a) ⇒ bool ⇒ ′a

— (simulation) higher order

fixes Of-Pair Of-Nil Of-Cons Of-None Of-Some :: string

begin

definition of-pair a b f1 f2 = (λf. λ(c, d) ⇒ f c d)
(ap2 a (b Of-Pair) f1 f2)

definition of-list a b f = (λf0. rec-list f0 o co1 K)
(b Of-Nil)
(ar2 a (b Of-Cons) f)

definition of-option a b f = rec-option
(b Of-None)
(ap1 a (b Of-Some) f)

end

lemmas [code] =
  Parse.of-pair-def
  Parse.of-list-def
  Parse.of-option-def

This theory and all the deriving one could also be prefixed by “print” instead of “parse”. In any case, we are converting (or printing) the above datatypes to another format, and finally this format will be “parsed” by Isabelle!

end

3.2. Instantiating the Parser of (Pure) Term

theory Parser-Pure
imports Meta-Pure
  Parser-init
begin

3.2.1. Main

context Parse
begin

definition \texttt{of-pure-indexname} \ a \ b = \texttt{of-pair} \ a \ b \ (\texttt{of-string} \ a \ b) \ (\texttt{of-nat} \ a \ b)
definition \texttt{of-pure-class} = \texttt{of-string}
definition \texttt{of-pure-sort} \ a \ b = \texttt{of-list} \ a \ b \ (\texttt{of-pure-class} \ a \ b)
definition \texttt{of-pure-typ} \ a \ b = \texttt{rec-typ}
  (\texttt{ap2} \ a \ (b \ : \ Type)) \ (\texttt{of-string} \ a \ b) \ (\texttt{of-list} \ a \ b) \ \texttt{snd})
  (\texttt{ap2} \ a \ (b \ : \ TFree) \ (\texttt{of-string} \ a \ b) \ (\texttt{of-pure-sort} \ a \ b))
  (\texttt{ap2} \ a \ (b \ : \ TVar) \ (\texttt{of-pure-indexname} \ a \ b) \ (\texttt{of-pure-sort} \ a \ b))
definition \texttt{of-pure-term} \ a \ b = (\lambda f0 \ f1 \ f2 \ f3 \ f4 \ f5. \ \texttt{rec-term} \ f0 \ f1 \ f2 \ f3 \ (co2 \ K \ f4) \ (\lambda- \ . \ f5))
  (\texttt{ap2} \ a \ (b \ : \ Const) \ (\texttt{of-string} \ a \ b) \ (\texttt{of-pure-typ} \ a \ b))
  (\texttt{ap2} \ a \ (b \ : \ Free) \ (\texttt{of-string} \ a \ b) \ (\texttt{of-pure-typ} \ a \ b))
  (\texttt{ap2} \ a \ (b \ : \ Var) \ (\texttt{of-pure-indexname} \ a \ b) \ (\texttt{of-pure-typ} \ a \ b))
  (\texttt{ap1} \ a \ (b \ : \ Bound) \ (\texttt{of-nat} \ a \ b))
  (\texttt{ar3} \ a \ (b \ : \ Abs) \ (\texttt{of-string} \ a \ b) \ (\texttt{of-pure-typ} \ a \ b))
  (\texttt{ar2} \ a \ (b \ : \ App) \ \texttt{id})
end

lemmas \texttt{[code]} =
  \texttt{Parse.of-pure-indexname-def}
  \texttt{Parse.of-pure-class-def}
  \texttt{Parse.of-pure-sort-def}
  \texttt{Parse.of-pure-typ-def}
  \texttt{Parse.of-pure-term-def}
end
4. Printing Meta-Models

4.1. Initializing the Printer

```plaintext
theory Printer-init
imports ../Init
     ../isabelle-home/src/HOL/Isabelle-Main1
begin

At the time of writing, the following target languages supported by Isabelle are also supported by the meta-compiler: Haskell, OCaml, Scala, SML.

4.1.1. Kernel Code for Target Languages

```haskell
lazy-code-printing code-module CodeType ↦ (Haskell) :
module CodeType where {
  type MlInt = Integer
  ; type MlMonad a = IO a
} |

code-module CodeConst ↦ (Haskell) :
module CodeConst where {
  import System.Directory
  ; import System.IO
  ; import qualified CodeConstPrintf
  outFile1 f file = (do
    fileExists <- doesFileExist file
    if fileExists then error (File exists ++ file ++ \n) else do
      h <- openFile file WriteMode
      f (\pat -> hPutStr h . CodeConstPrintf.sprintf1 pat)
      hClose h)

  ; outStand1 :: ((String -> String -> IO ()) -> IO ()) -> IO ()
  ; outStand1 f = f (\pat -> putStrLn . CodeConstPrintf.sprintf1 pat)
} |

code-module CodeConst.Monad ↦ (Haskell) :
module CodeConst.Monad where {
  bind a = (>>=) a
  ; return :: a -> IO a
  ; return = Prelude.return
} |

code-module CodeConst.Printf ↦ (Haskell) :
module CodeConst.Printf where {
  import Text.Printf
  ; sprintf0 = id
```
; sprintf1 :: PrintfArg a => String -> a -> String
; sprintf1 = printf

; sprintf2 :: PrintfArg a => PrintfArg b => String -> a -> b -> String
; sprintf2 = printf

; sprintf3 :: PrintfArg a => PrintfArg b => PrintfArg c => String -> a -> b -> c -> String
; sprintf3 = printf

; sprintf4 :: PrintfArg a => PrintfArg b => PrintfArg c => PrintfArg d => String -> a -> b -> c -> d -> String
; sprintf4 = printf

; sprintf5 :: PrintfArg a => PrintfArg b => PrintfArg c => PrintfArg d => PrintfArg e => String -> a -> b -> c -> d -> e -> String
; sprintf5 = printf
let bind = function
  None -> fun _ -> None
  Some a -> fun f -> f a
let return a = Some a
end

module Printf = struct
  include Printf
  let sprintf0 = sprintf
  let sprintf1 = sprintf
  let sprintf2 = sprintf
  let sprintf3 = sprintf
  let sprintf4 = sprintf
  let sprintf5 = sprintf
end

module String = String

module Sys = struct
  open Sys

  let isDirectory2 s = try Some (is-directory s) with _ -> None

end

module To = struct
  let nat big-int x = Big-int.int-of-big-int (big-int x)
end

⟩

<table>
<thead>
<tr>
<th>code-module</th>
<th>(Scala)</th>
</tr>
</thead>
<tbody>
<tr>
<td>object CodeType</td>
<td></td>
</tr>
<tr>
<td>type mlMonad [A] = Option [A]</td>
<td></td>
</tr>
<tr>
<td>type mlInt = Int</td>
<td></td>
</tr>
<tr>
<td>object CodeConst</td>
<td></td>
</tr>
<tr>
<td>def outFile1 [A] (f : (String =&gt; A =&gt; Option [Unit]) =&gt; Option [Unit], file0 : String) : Option [Unit] =</td>
<td></td>
</tr>
<tr>
<td>val file = new java.io.File (file0)</td>
<td></td>
</tr>
<tr>
<td>if (file.isFile) {</td>
<td></td>
</tr>
<tr>
<td>None } else {</td>
<td></td>
</tr>
<tr>
<td>val writer = new java.io.PrintWriter (file)</td>
<td></td>
</tr>
<tr>
<td>f ((fmt : String) =&gt; (s : A) =&gt; Some (writer.write (fmt.format (s))))</td>
<td></td>
</tr>
<tr>
<td>Some (writer.close ())</td>
<td></td>
</tr>
<tr>
<td>}</td>
<td></td>
</tr>
<tr>
<td>def outStand1 [A] (f : (String =&gt; A =&gt; Option [Unit]) =&gt; Option [Unit]) : Option[Unit] =</td>
<td></td>
</tr>
</tbody>
</table>
|     f ((fmt : String) => (s : A) => Some (print (fmt.format (s))))}
object Monad {
    def bind [A, B] (x : Option [A], f : A => Option [B]) : Option [B] = x match {
        case None => None
        case Some (a) => f (a)
    }
    def Return [A] (a : A) = Some (a)
}

object Printf {
    def sprintf0 (x0 : String) = x0
    def sprintf1 [A1] (fmt : String, x1 : A1) = fmt . format (x1)
    def sprintf2 [A1, A2] (fmt : String, x1 : A1, x2 : A2) = fmt . format (x1, x2)
    def sprintf3 [A1, A2, A3] (fmt : String, x1 : A1, x2 : A2, x3 : A3) = fmt . format (x1, x2, x3)
    def sprintf4 [A1, A2, A3, A4] (fmt : String, x1 : A1, x2 : A2, x3 : A3, x4 : A4) = fmt . format (x1, x2, x3, x4)
    def sprintf5 [A1, A2, A3, A4, A5] (fmt : String, x1 : A1, x2 : A2, x3 : A3, x4 : A4, x5 : A5) = fmt . format (x1, x2, x3, x4, x5)
}

object String {
    def concat (s : String, l : List [String]) = l filter (- . nonEmpty) mkString s
}

object Sys {
    def isDirectory2 (s : String) = Some (new java . io . File (s) . isDirectory)
}

object To {
    def nat [A] (f : A => BigInt, x : A) = f (x) . intValue ()
}

structure CodeType = struct
    type mlInt = string
    type 'a mlMonad = 'a option
end

structure CodeConst = struct
structure Monad = struct
    val bind = fn
        NONE => (fn _ => NONE)
    | SOME a => fn f => f a
    val return = SOME
end

structure Printf = struct
    local
        fun sprintf s l =
            case String . fields (fn #% => true | _ => false) s of

let fun aux acc l-pat l-s =
    case l-pat of
      [] => rev acc
    | x :: xs => aux (String.extract (x, 1, NONE) :: hd l-s :: acc) xs (tl l-s) in
      String.concat (x :: aux [] xs l)
end

fun sprintf0 s-pat = s-pat
fun sprintf1 s-pat s1 = sprintf s-pat [s1]
fun sprintf2 s-pat s1 s2 = sprintf s-pat [s1, s2]
fun sprintf3 s-pat s1 s2 s3 = sprintf s-pat [s1, s2, s3]
fun sprintf4 s-pat s1 s2 s3 s4 = sprintf s-pat [s1, s2, s3, s4]
fun sprintf5 s-pat s1 s2 s3 s4 s5 = sprintf s-pat [s1, s2, s3, s4, s5]
end

structure String = struct
val concat = String.concatWith
end

structure Sys = struct
val isDirectory2 = SOME o File.is-dir o Path.explode handle ERROR - => K NONE
end

structure To = struct
fun nat f = Int.toString o f
end

fun outFile1 f file =
    let
      val pfile = Path.explode file
      val () = if File.exists pfile then error (File.exists \ file \ (n) else ()
      val oc = Unsynchronized.ref []
      val - = f (fn a => fn b => SOME (oc := Printf sprintf1 a b :: (Unsynchronized! oc))) in
        SOME (File.write-list pfile (rev (Unsynchronized! oc))) handle - => NONE
    end

fun outStand1 f = outFile1 f (Unsynchronized! stdout-file)
end

4.1.2. Interface with Types

datatype ml-int = ML-int

code-printing type-constructor ml-int => (Haskell) CodeType.MlInt — syntax!
type-constructor ml-int \rightarrow (\text{OCaml}) \text{CodeType.mlInt}
| type-constructor ml-int \rightarrow (\text{Scala}) \text{CodeType.mlInt}
| type-constructor ml-int \rightarrow (\text{SML}) \text{CodeType.mlInt}

datatype 'a ml-monad = ML-monad 'a
code-printing type-constructor ml-monad \rightarrow (\text{Haskell}) \text{CodeType.MlMonad} — syntax!
| type-constructor ml-monad \rightarrow (\text{OCaml}) - \text{CodeType.mlMonad}
| type-constructor ml-monad \rightarrow (\text{Scala}) \text{CodeType.mlMonad} [:]
| type-constructor ml-monad \rightarrow (\text{SML}) - \text{CodeType.mlMonad}

type-synonym ml-string = \text{String.literal}

4.1.3. Interface with Constants

module CodeConst

consts out-file1 :: ((\text{ml-string} \Rightarrow 'a \Rightarrow \text{unit ml-monad}) — \text{fprintf} \Rightarrow \text{unit ml-monad}) \Rightarrow \text{ml-string} \Rightarrow \text{unit ml-monad}

code-printing constant out-file1 \rightarrow (\text{Haskell}) \text{CodeConst.outFile1}
| constant out-file1 \rightarrow (\text{OCaml}) \text{CodeConst.outFile1}
| constant out-file1 \rightarrow (\text{Scala}) \text{CodeConst.outFile1}
| constant out-file1 \rightarrow (\text{SML}) \text{CodeConst.outFile1}

consts out-stand1 :: ((\text{ml-string} \Rightarrow 'a \Rightarrow \text{unit ml-monad}) — \text{fprintf} \Rightarrow \text{unit ml-monad}) \Rightarrow \text{unit ml-monad}

code-printing constant out-stand1 \rightarrow (\text{Haskell}) \text{CodeConst.outStand1}
| constant out-stand1 \rightarrow (\text{OCaml}) \text{CodeConst.outStand1}
| constant out-stand1 \rightarrow (\text{Scala}) \text{CodeConst.outStand1}
| constant out-stand1 \rightarrow (\text{SML}) \text{CodeConst.outStand1}

module Monad

consts bind :: 'a ml-monad \Rightarrow ('a \Rightarrow 'b ml-monad) \Rightarrow 'b ml-monad

code-printing constant bind \rightarrow (\text{Haskell}) \text{CodeConst.Monad.bind}
| constant bind \rightarrow (\text{OCaml}) \text{CodeConst.Monad.bind}
| constant bind \rightarrow (\text{Scala}) \text{CodeConst.Monad.bind}
| constant bind \rightarrow (\text{SML}) \text{CodeConst.Monad.bind}

consts return :: 'a \Rightarrow 'a ml-monad

code-printing constant return \rightarrow (\text{Haskell}) \text{CodeConst.Monad.return}
| constant return \rightarrow (\text{OCaml}) \text{CodeConst.Monad.return}
| constant return \rightarrow (\text{Scala}) \text{CodeConst.Monad.Return} — syntax!
| constant return \rightarrow (\text{SML}) \text{CodeConst.Monad.return}

module Printf

consts sprintf0 :: ml-string \Rightarrow ml-string

code-printing constant sprintf0 \rightarrow (\text{Haskell}) \text{CodeConst_Printfsprintf0}
| constant sprintf0 \rightarrow (\text{OCaml}) \text{CodeConst_Printfsprintf0}
| constant sprintf0 \rightarrow (\text{Scala}) \text{CodeConst_Printfsprintf0}
| constant sprintf0 \rightarrow (\text{SML}) \text{CodeConst_Printfsprintf0}
consts $\text{sprintf1} :: \text{ml-string} \Rightarrow 'a1 \Rightarrow \text{ml-string}$
code-printing constant $\text{sprintf1} \rightarrow \text{(Haskell) CodeConst.Printfsprintf1}$
  | constant $\text{sprintf1} \rightarrow \text{(OCaml) CodeConst.Printfsprintf1}$
  | constant $\text{sprintf1} \rightarrow \text{(Scala) CodeConst.Printfsprintf1}$
  | constant $\text{sprintf1} \rightarrow \text{(SML) CodeConst.Printfsprintf1}$

consts $\text{sprintf2} :: \text{ml-string} \Rightarrow 'a1 \Rightarrow 'a2 \Rightarrow \text{ml-string}$
code-printing constant $\text{sprintf2} \rightarrow \text{(Haskell) CodeConst.Printfsprintf2}$
  | constant $\text{sprintf2} \rightarrow \text{(OCaml) CodeConst.Printfsprintf2}$
  | constant $\text{sprintf2} \rightarrow \text{(Scala) CodeConst.Printfsprintf2}$
  | constant $\text{sprintf2} \rightarrow \text{(SML) CodeConst.Printfsprintf2}$

consts $\text{sprintf3} :: \text{ml-string} \Rightarrow 'a1 \Rightarrow 'a2 \Rightarrow 'a3 \Rightarrow \text{ml-string}$
code-printing constant $\text{sprintf3} \rightarrow \text{(Haskell) CodeConst.Printfsprintf3}$
  | constant $\text{sprintf3} \rightarrow \text{(OCaml) CodeConst.Printfsprintf3}$
  | constant $\text{sprintf3} \rightarrow \text{(Scala) CodeConst.Printfsprintf3}$
  | constant $\text{sprintf3} \rightarrow \text{(SML) CodeConst.Printfsprintf3}$

consts $\text{sprintf4} :: \text{ml-string} \Rightarrow 'a1 \Rightarrow 'a2 \Rightarrow 'a3 \Rightarrow 'a4 \Rightarrow \text{ml-string}$
code-printing constant $\text{sprintf4} \rightarrow \text{(Haskell) CodeConst.Printfsprintf4}$
  | constant $\text{sprintf4} \rightarrow \text{(OCaml) CodeConst.Printfsprintf4}$
  | constant $\text{sprintf4} \rightarrow \text{(Scala) CodeConst.Printfsprintf4}$
  | constant $\text{sprintf4} \rightarrow \text{(SML) CodeConst.Printfsprintf4}$

consts $\text{sprintf5} :: \text{ml-string} \Rightarrow 'a1 \Rightarrow 'a2 \Rightarrow 'a3 \Rightarrow 'a4 \Rightarrow 'a5 \Rightarrow \text{ml-string}$
code-printing constant $\text{sprintf5} \rightarrow \text{(Haskell) CodeConst.Printfsprintf5}$
  | constant $\text{sprintf5} \rightarrow \text{(OCaml) CodeConst_Printfsprintf5}$
  | constant $\text{sprintf5} \rightarrow \text{(Scala) CodeConst_Printfsprintf5}$
  | constant $\text{sprintf5} \rightarrow \text{(SML) CodeConst_Printfsprintf5}$

module String
consts $\text{String-concat} :: \text{ml-string} \Rightarrow \text{ml-string list} \Rightarrow \text{ml-string}$
code-printing constant $\text{String-concat} \rightarrow \text{(Haskell) CodeConst.Stringconcat}$
  | constant $\text{String-concat} \rightarrow \text{(OCaml) CodeConst.Stringconcat}$
  | constant $\text{String-concat} \rightarrow \text{(Scala) CodeConst.Stringconcat}$
  | constant $\text{String-concat} \rightarrow \text{(SML) CodeConst.Stringconcat}$

module Sys
consts $\text{Sys-is-directory2} :: \text{ml-string} \Rightarrow \text{bool ml-monad}$
code-printing constant $\text{Sys-is-directory2} \rightarrow \text{(Haskell) CodeConst.Sys.isDirectory2}$
  | constant $\text{Sys-is-directory2} \rightarrow \text{(OCaml) CodeConst.Sys.isDirectory2}$
  | constant $\text{Sys-is-directory2} \rightarrow \text{(Scala) CodeConst.Sys.isDirectory2}$
  | constant $\text{Sys-is-directory2} \rightarrow \text{(SML) CodeConst.Sys.isDirectory2}$

module To
consts $\text{ToNat} :: (\text{nat} \Rightarrow \text{integer}) \Rightarrow \text{nat} \Rightarrow \text{ml-int}$
code-printing constant $\text{ToNat} \rightarrow \text{(Haskell) CodeConst.To.nat}$
  | constant $\text{ToNat} \rightarrow \text{(OCaml) CodeConst.To.nat}$

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4.1.4. Some Notations (I): Raw Translations

**Syntax**
- `sprint0 :: - ⇒ ml-string (sprint0 (-)')`
- `sprint1 :: - ⇒ ml-string (sprint1 (-)')`
- `sprint2 :: - ⇒ ml-string (sprint2 (-)')`
- `sprint3 :: - ⇒ ml-string (sprint3 (-)')`
- `sprint4 :: - ⇒ ml-string (sprint4 (-)')`
- `sprint5 :: - ⇒ ml-string (sprint5 (-)')`

**Translations**
- `sprint0 x' ⇔ CONST sprintf0 x`
- `sprint1 x' ⇔ CONST sprintf1 x`
- `sprint2 x' ⇔ CONST sprintf2 x`
- `sprint3 x' ⇔ CONST sprintf3 x`
- `sprint4 x' ⇔ CONST sprintf4 x`
- `sprint5 x' ⇔ CONST sprintf5 x`

4.1.5. Some Notations (II): Polymorphic Cartouches

**Syntax**
- `cartouche-string' :: String.literal`

**Translations**
- `cartouche-string' ⇔ -cartouche-string'`

```
parsed-translation (  
  let fun f c x = Syntax.const c $ x in
  fn (theta, x) => x 
  | (1, x) => f @{ const-syntax sprintf1 } x 
  | (2, x) => f @{ const-syntax sprintf2 } x 
  | (3, x) => f @{ const-syntax sprintf3 } x 
  | (4, x) => f @{ const-syntax sprintf4 } x 
  | (5, x) => f @{ const-syntax sprintf5 } x
  end)

  :: Cartouche-Grammar.default
  (fn 37 — #*%* ⇒ (fn x => x + 1)
   | - ⇒ I
   | 0])
```

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4.1.6. Generic Locale for Printing

locale Print = 
  fixes To-string :: string ⇒ ml-string 
  fixes To-nat :: nat ⇒ ml-int 
begin 
  declare[[cartouche-type ′ = fun_printf]]
end

As remark, printing functions (like sprintf...) are currently weakly typed in Isabelle, 
we will continue the typing using the type system of target languages.
end

4.2. Instantiating the Printer for (Pure) Term

theory Printer-Pure
imports Meta-Pure Printer-init
begin

context Print
begin

fun of-pure-term where of-pure-term l e = (λ 
  Const s - ⇒ To-string s
  | Free s - ⇒ To-string s
  | App t1 t2 ⇒ (%(s) (%(s)) (of-pure-term l t1) (of-pure-term l t2)
  | Abs s - t ⇒
      let s = To-string s in
      (%(λ %s. %s) s (of-pure-term (s ≠ l) t)
  | Bound n ⇒ (%s (l ! nat-of-natural n)) e

end

lemmas [code] =
  — def
  — fun
  Print.of-pure-term.simps
end

4.3. Instantiating the Printer for SML

theory Printer-SML
imports Meta-SML Printer-init
begin
**context Print**

**begin**

**definition** of-semi--val-fun = (λ Sval ⇒ ⟨val⟩ | Sfun ⇒ ⟨fun⟩)

**fun** of-semi--term' **where** of-semi--term' e = (λ

| SML-string s ⇒ ⟨str⟩ (To-string (escape-sml s))
| SML-rewrite val-fun e1 symb e2 ⇒ ⟨str %s %s %s⟩ (of-semi--val-fun val-fun) (of-semi--term' e1) (To-string symb) (of-semi--term' e2)
| SML-basic l ⇒ ⟨str⟩ (String-concat ♯ (List.map To-string l))
| SML-binop e1 s e2 ⇒ ⟨str %s %s %s⟩ (of-semi--term' e1) (of-semi--term' e2) (of-semi--term' e1) (of-semi--term' e2)
| SML-annot e s ⇒ ⟨str⟩ (String-concat ♯ (List.map (λ e ⇒ ⟨str⟩ (of-semi--term' e)) l))
| SML-apply e l ⇒ ⟨str %s %s⟩ (of-semi--term' e) (To-string p-left) (of-semi--term' e) (To-string p-right)
| SML-let-open s e ⇒ ⟨str⟩ (let open %s in %s end) (To-string s) (of-semi--term' e)) e)

**end**

**lemmas** [code] =

— def
Print.of-semi--val-fun-def
— fun
Print.of-semi--term'.simps

**end**

### 4.4. Instantiating the Printer for Isabelle

**theory** Printer-Isabelle

**imports** Meta-Isabelle

Printer-Pure

Printer-SML

**begin**

**context** Print

**begin**

**fun** of-semi--typ where of-semi--typ e = (λ

| Typ-base s ⇒ To-string s
| Typ-apply name l ⇒ ⟨%s %s⟩ (let s = String-concat ♯ (List.map of-semi--typ l) in case l of [l] ⇒ s | _ ⇒ ⟨%s %s⟩ s) (of-semi--typ name)
| Typ-apply-bin s ty1 ty2 ⇒ ⟨%s %s %s⟩ (of-semi--typ ty1) (To-string s) (of-semi--typ ty2)

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definition of-datatype - = (λ Datatype n l ⇒
(String-concat ⟨
(λ(n,l).
⟨Datatype %s = %s⟩
(To-string n)
⟩
(L.map λx. ⟨%s %s⟩ (of-semi--typ x) l))) l))

definition of-type-synonym - = (λ Type-synonym n v l ⇒
⟨Type-synonym %s = %s⟩
if v = [] then
(To-string n)
else
(of-semi--typ (Typ-apply (Typ-base n) (L.map Typ-base v)))
(of-semi--typ l))

fun of-semi--term where
of-semi--term e = (λ
Term-rewrite e1 symb e2 ⇒ ⟨%s %s %s⟩ (of-semi--term e1) (To-string symb) (of-semi--term e2)
Term-basic l ⇒ ⟨%s⟩ (String-concat ⟨⟩ (L.map To-string l))
Term-annot e s ⇒ ⟨%s s %s⟩ (To-string symb) (of-semi--term e1) (of-semi--term e2)
Term-bind symb e1 e2 ⇒ ⟨%s %s . %s⟩ (To-string symb) (of-semi--term e1) (of-semi--term e2)
Term-fun-case e-case l ⇒ ⟨%s %s⟩
(case e-case of None ⇒ ⟨⟩
| Some e ⇒ ⟨case %s of %s⟩ (of-semi--term e))
| String-concat (λ (s1, s2) ⇒ ⟨%s %s %s⟩ (of-semi--term s1) (of-semi--term s2) l))
Term-apply e l ⇒ ⟨%s %s⟩ (of-semi--term e) (String-concat ⟨⟩ (List.map (λ e ⇒ ⟨%s %s⟩ (of-semi--term e) l)))
Term-paren p-left p-right e ⇒ ⟨%s %s %s⟩ (To-string p-left) (of-semi--term e) (To-string p-right)
Term-if-then-else e-if e-then e-else ⇒ ⟨if %s then %s else %s⟩ (of-semi--term e-if) (of-semi--term e-then) (of-semi--term e-else)
| Term-term l pure ⇒ of-pure-term (L.map To-string l) pure e

definition of-type-notation - = (λ Type-notation n e ⇒
⟨Type-notation %s (%s)⟩ (To-string n) (To-string e))

definition of-instantiation - = (λ Instantiation n n-def expr ⇒
let name = To-string n in
⟨instantiation %s :: object
begin
definition %s-%s-def : %s
instance ..
end:
  name
  (To-string n-def)
  name
  (of-semi--term expr))

definition of-overloading - = (\ Overloading n-c e-c n e \Rightarrow
  overloading %s \equiv %s
begin
  definition %s : %s
end: (To-string n-c) (of-semi--term e-c) (To-string n) (of-semi--term e))

definition of-consts - = (\ Consts n ty symb \Rightarrow
  consts %s :: %s (%s %s): (To-string n) (of-semi--typ ty) (To-string Consts-value) (To-string symb))

definition of-definition - = (\ Definition e \Rightarrow
  definition %s
  | Definition-where1 name (abbrev, prio) e \Rightarrow
  definition %s
  where %s: (To-string name) (of-semi--term abbrev) (To-nat prio) (of-semi--term e)
  | Definition-where2 name abbrev e \Rightarrow
  definition %s
  where %s: (To-string name) (of-semi--term abbrev) (of-semi--term e))

definition (of-semi--thm-attribute-aux-gen :: String.literal \times String.literal \Rightarrow \ - \Rightarrow \ - \Rightarrow \ -) m lacc
s =
  \let s-base = (\lambda s lacc. \%s[\%s]\) (To-string s) (String-concat \ and \ (L.map (\(s, x). \%s \%s s x) lacc)))\ in
  s-base s (m \# lacc)

definition of-semi--thm-attribute-aux-gen-where l =
  (\where, String-concat \ and \ (L.map (\(var, expr). \%s = \%s
    (To-string var)
    (of-semi--term expr)) l))

definition of-semi--thm-attribute-aux-gen-of l =
  (\of, String-concat \ and \ (L.map (\(expr. \%s (of-semi--term expr)) l))

fun of-semi--thm-attribute-aux where of-semi--thm-attribute-aux lacc e =
  (\ Thm-thm s \Rightarrow (To-string s
  | Thm-thms s \Rightarrow (To-string s
  | Thm-THEN (Thm-thms s) e2 \Rightarrow of-semi--thm-attribute-aux-gen (\THEN, of-semi--thm-attribute-aux
e2) lacc s
  | Thm-THEN (Thm-thms s) e2 \Rightarrow of-semi--thm-attribute-aux-gen (\THEN, of-semi--thm-attribute-aux
e2) lacc s
  | Thm-THEN e1 e2 \Rightarrow of-semi--thm-attribute-aux ((\THEN, of-semi--thm-attribute-aux []
e2) \# lacc) e1

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\[ \text{Thm-simplified} (\text{Thm-thm} \ s) \ e2 \Rightarrow \text{of-semi-thm-attribute-aux-gen} ((\text{simplified}), \text{of-semi-thm-attribute-aux} \ [\ ] \ e2) \ lacc \ s \]

\[ \text{Thm-simplified} (\text{Thm-thms} \ s) \ e2 \Rightarrow \text{of-semi-thm-attribute-aux-gen} ((\text{simplified}), \text{of-semi-thm-attribute-aux} \ [\ ] \ e2) \ lacc \ s \]

\[ \text{Thm-simplified} \ e1 \ e2 \Rightarrow \text{of-semi-thm-attribute-aux} ((\text{simplified}), \text{of-semi-thm-attribute-aux} \ [\ ] \ e2) \ # \ lacc \ e1 \]

\[ \text{Thm-symmetric} (\text{Thm-thm} \ s) \Rightarrow \text{of-semi-thm-attribute-aux-gen} ((\text{symmetric}), \text{of-semi-thm-attribute-aux} \ [\ ] \ lacc \ s) \]

\[ \text{Thm-symmetric} (\text{Thm-thms} \ s) \Rightarrow \text{of-semi-thm-attribute-aux-gen} ((\text{symmetric}), \text{of-semi-thm-attribute-aux} \ [\ ] \ lacc \ s) \]

\[ \text{Thm-symmetric} \ e1 \Rightarrow \text{of-semi-thm-attribute-aux} ((\text{symmetric}), \text{of-semi-thm-attribute-aux} \ [\ ] \ lacc \ e1) \]

\[ \text{Thm-where} (\text{Thm-thm} \ s) \ l \Rightarrow \text{of-semi-thm-attribute-aux-gen} ((\text{of-semi-thm-attribute-aux-gen-where} \ l), \text{of-semi-thm-attribute-aux} \ [\ ] \ lacc \ s) \]

\[ \text{Thm-where} (\text{Thm-thms} \ s) \ l \Rightarrow \text{of-semi-thm-attribute-aux-gen} ((\text{of-semi-thm-attribute-aux-gen-where} \ l), \text{of-semi-thm-attribute-aux} \ [\ ] \ lacc \ s) \]

\[ \text{Thm-where} \ e1 \ l \Rightarrow \text{of-semi-thm-attribute-aux} ((\text{of-semi-thm-attribute-aux-gen-where} \ l), \text{of-semi-thm-attribute-aux} \ [\ ] \ lacc \ e1) \]

\[ \text{Thm-of} (\text{Thm-thm} \ s) \ l \Rightarrow \text{of-semi-thm-attribute-aux-gen} ((\text{of-semi-thm-attribute-aux-gen-of} \ l), \text{of-semi-thm-attribute-aux} \ [\ ] \ lacc \ s) \]

\[ \text{Thm-of} (\text{Thm-thms} \ s) \ l \Rightarrow \text{of-semi-thm-attribute-aux-gen} ((\text{of-semi-thm-attribute-aux-gen-of} \ l), \text{of-semi-thm-attribute-aux} \ [\ ] \ lacc \ s) \]

\[ \text{Thm-of} \ e1 \ l \Rightarrow \text{of-semi-thm-attribute-aux} ((\text{of-semi-thm-attribute-aux-gen-of} \ l), \text{of-semi-thm-attribute-aux} \ [\ ] \ lacc \ e1) \]

\[ \text{Thm-OF} (\text{Thm-thm} \ s) \ e2 \Rightarrow \text{of-semi-thm-attribute-aux-gen} ((\text{OF}), \text{of-semi-thm-attribute-aux} \ [\ ] \ e2) \ lacc \ s \]

\[ \text{Thm-OF} (\text{Thm-thms} \ s) \ e2 \Rightarrow \text{of-semi-thm-attribute-aux-gen} ((\text{OF}), \text{of-semi-thm-attribute-aux} \ [\ ] \ lacc \ s) \]

\[ \text{Thm-OF} \ e1 \ e2 \Rightarrow \text{of-semi-thm-attribute-aux} ((\text{OF}), \text{of-semi-thm-attribute-aux} \ [\ ] \ e2) \ # \ lacc \ e1 \) \]

**definition**

\[ \text{of-semi-thm-attribute} = \text{of-semi-thm-attribute-aux} \ [\ ] \]

**definition**

\[ \text{of-semi-thm} = (\lambda \ \text{Thms-single} \ thy \Rightarrow \text{of-semi-thm-attribute} \ thy) \]

\[ \text{Thms-mult} \ thy \Rightarrow \text{of-semi-thm-attribute} \ thy \]

**definition**

\[ \text{of-semi-thm-attribute-l} \ l = \text{String-concat} \ (\text{L.map of-semi-thm-attribute} \ l) \]

**definition**

\[ \text{of-semi-thm-attribute-l} \ l = \text{String-concat} \ (\text{L.map of-semi-thm-attribute} \ l) \]

**definition**

\[ \text{of-semi-thm-l} \ l = \text{String-concat} \ (\text{L.map of-semi-thm} \ l) \]

**definition**

\[ \text{of-lemmas} = (\lambda \ \text{Lemmas-simp-thm} \ simp \ s \ l \Rightarrow \langle \text{lemmas}%s%\%s = %s \rangle)

\[ \text{if} \ \text{String.is-empty} \ s \ \text{then} \ \circ \ \text{else} \ \langle \%s \rangle \ \text{(To-string} \ s))

\[ \text{if} \ \text{simp} \ \text{then} \ \langle \text{simp,code-unfold} \rangle \ \text{else} \ \circ \rangle

\[ \text{of-semi-thm-attribute-l} \ l \]

\[ \text{Lemmas-simp-thms} \ s \ l \Rightarrow \]

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(lemmas% \{ simp,code-unfold \} = %s)
(if \String.is-empty s then \0 else (\%s\ (\String.to-string s))
(\String-concat \0 \ (L.map \String.to-string l)))

definition (of-semi--attrib-genA :: (semi-thm list \Rightarrow \String.literal)
\Rightarrow \String.literal \Rightarrow semi-thm list \Rightarrow \String.literal) f attr l = — error reflection: to be merged
(if l = [] then
\0
else
\ (\%s\ \%s\ attr (f l))

definition (of-semi--attrib-genB :: (string list \Rightarrow \String.literal)
\Rightarrow \String.literal \Rightarrow string list \Rightarrow \String.literal) f attr l = — error reflection: to be merged
(if l = [] then
\0
else
\ (\%s\ \%s\ attr (f l))

definition of-semi--attrib = of-semi--attrib-genA of-semi--thm-l

definition of-semi--attrib1 = of-semi--attrib-genB (\lambda l. \String-concat \0 \ (L.map \String.to-string l))

definition of-semi--method-simp (s :: — polymorphism weakening needed by code-reflect
\String.literal) =
(\lambda Method-simp-only l \Rightarrow \%s only; \%s\ s (of-semi--thm-l l)
| Method-simp-add-del-split l1 l2 [] \Rightarrow \%s\%s\%s\ s
(s (of-semi--attrib (add) l1)
(s (of-semi--attrib (del) l2)
| Method-simp-add-del-split l1 l2 l3 \Rightarrow \%s\%s\%s\ s
(s (of-semi--attrib (add) l1)
(s (of-semi--attrib (del) l2)
(s (of-semi--attrib (split) l3))

fun of-semi--method where of-semi--method expr = (\lambda
Method-rule o-s \Rightarrow \rule\%s\ (case o-s of None \Rightarrow \0
| Some s \Rightarrow \%s\ (of-semi--thm-attribute s))
| Method-drule s \Rightarrow \drule\%s\ (of-semi--thm-attribute s)
| Method-erule s \Rightarrow \erule\%s\ (of-semi--thm-attribute s)
| Method-intro l \Rightarrow \intro\%s\ (of-semi--thm-attribute-l l)
| Method-elim s \Rightarrow \elim\%s\ (of-semi--thm-attribute s)
| Method-subst asm l s =>
let s-asm = if asm then (\asm) \ else \0 in
if L.map \String.meta-of-logic l = [STR "0"] then
\ (\subst\%s\%s\ s-asm (of-semi--thm-attribute s)
else
\ (\subst\%s\%s\%s\ s-asm \String-concat \0 \ (L.map \String.to-string l)) (of-semi--thm-attribute s)


| Method-insert l => (insert %s) (of-semi-thm-l l) |
| Method-plus t => (⟨%s⟩+) (String-concat ⟨,⟩ (List.map of-semi-method t)) |
| Method-option t => (⟨%s⟩?) (String-concat ⟨,⟩ (List.map of-semi-method t)) |
| Method-or t => (⟨%s⟩) (String-concat ⟨|⟩ (List.map of-semi-method t)) |
| Method-one s => of-semi-method-simp (simp) s |
| Method-all s => of-semi-method-simp (simp-all) s |
| Method-auto-simp-add-split l-simp l-split => (⟨auto%ss%ss⟩ (of-semi-attrib ⟨simp⟩ l-simp)) (of-semi-attrib1 ⟨split⟩ l-split) |
| Method-rename-tac l => (rename-tac %ss) (String-concat ⟨⟩ (List.map To-string l)) |
| Method-case-tac e => (case-tac %ss) (of-semi-term e) |
| Method-blast None => (blast) |
| Method-blast (Some n) => (blast %d) (To-nat n) |
| Method-clarify => (clarify) |
| Method-metis l-opt l => (⟨metis%ss%ss⟩ (if l-opt = [] then ⟨⟩ else (⟨%s⟩) (String-concat ⟨,⟩ (List.map To-string l-opt)))) |

\(\text{expr}\)

\textbf{definition} of-semi-command-final = (λ Command-done ⇒ ⟨done⟩) \(\mid\) Command-by-l-apply ⇒ ⟨by(%ss)⟩ (String-concat ⟨,⟩ (List.map of-semi-method l-apply)) \(\mid\) Command-sorry ⇒ ⟨sorry⟩

\textbf{definition} of-semi-command-state = (λ Command-apply-end [] => ⟨⟩ \(\mid\) Command-apply-end l-apply ⇒ ⟨apply-end%ss⟩) (String-concat ⟨,⟩ (List.map of-semi-method l-apply))

\textbf{definition} of-semi-command-proof = (λ thesis = ⟨?thesis⟩
\begin{align*}
\text{let thesis} &= ⟨?thesis⟩ \\
\text{scope-thesis-gen} &= λ\text{proof show when.}⟨\text{proof} - %s show %s%ss⟩ \\
\text{scope-thesis} &= λs. scope-thesis-gen s thesis [] in \\
\text{λ Command-apply-end} [] => ⟨⟩ \\
\text{Command-apply-end} l-apply => ⟨apply-end%ss⟩ \\
\text{String-concat} ⟨,⟩ (List.map of-semi-method l-apply) \\
\text{Command-using} l => ⟨using %ss⟩ \\
\text{of-semi-thm-l l} \\
\text{Command-unfolding} l => ⟨unfolding %ss⟩ \\
\text{of-semi-thm-l l} \\
\text{Command-let} e-name e-body => scope-thesis (⟨let %s = %ss %s (af-semi-term e-name) (af-semi-term e-body)⟩) \\
\text{Command-have} n b e e-last => scope-thesis (⟨have %s%ss%ss %s%ss⟩ (To-string n) (if b then


```plaintext
(defini
tion of-lemma - = 
(λ Lemma n l-spec l-apply tactic-last ⇒
  ⟨lemma %s : %s
  
  (String-concat (⇒) (L.map of-semi--term l-spec)
    (String-concat (λ (n, b, e). %s
        (let (n, b) = if b then (%s[simp] (To-string n), False) else (To-string n, String.is-empty n) in
          if b then (λ l-apply-e. %sqed
            (if l-apply-e = [] then
            
              else
              (λ-apply-e. %sqed
                (⟨
                
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\%(s \rangle
\left. (\text{String-concat} \odot (L.\text{map of-semi--command-state l-apply-e}))) \right) \left. (\text{List.map-filter} \right)
\left. (\lambda \text{Command-let } - - \Rightarrow \text{Some } [] \mid \text{Command-have } - - - \Rightarrow \text{Some } [] \mid \text{Command-fx-let} \right. \left. - - l \Rightarrow \text{Some } l \mid - \Rightarrow \text{None} \right) \left. (\text{rev l-apply})) \right)\rangle
\left. \text{definition of-axiomatization } - = (\lambda \text{Axiomatization n e} \Rightarrow \langle \text{axiomatization where } \%(s:\%(s) \rangle \\
\text{\%(s) (To-string n) (of-semi--term e)} \rangle \right) \left. \text{definition of-section } - = (\lambda \text{Section n section-title} \Rightarrow \langle \%(s) \rangle \\
\text{\%(ssection} \left. \text{\%(s) (if n = 0 then } \langle \rangle \right. \left. \text{else if n = 1 then } \langle \text{sub} \rangle \right. \left. \text{else } \langle \text{subsub} \rangle \rangle \right) \left. (\text{To-string section-title}) \right) \right) \left. \text{definition of-text } - = (\lambda \text{Text s} \Rightarrow \langle \text{text} \%(s) \rangle \left. (\text{To-string s}) \right) \right) \left. \text{definition of-ML } - = (\lambda \text{SML e} \Rightarrow \langle \text{ML} \%(s) \rangle \left. (\text{of-semi--term e}) \right) \right) \left. \text{definition of-setup } - = (\lambda \text{Setup e} \Rightarrow \langle \text{setup} \%(s) \rangle \left. (\text{of-semi--term e}) \right) \right) \left. \text{definition of-thm } - = (\lambda \text{Thm thm} \Rightarrow \langle \text{thm} \%(s) \rangle \left. (\text{of-semi--thm-attribute-l1 thm}) \right) \right) \left. \text{definition of-interpretation } - = (\lambda \text{Interpretation n loc-n loc-param tac} \Rightarrow \langle \text{interpretation} \%(s) \%(s) \%(s) \rangle \\
\text{\%(s) (To-string n) \left. (\text{To-string loc-n}) \right. \left. (\text{String-concat} \odot (L.\text{map (\lambda s. (\%(s) \text{of-semi--term s}) loc-param})) \right. \left. (\text{of-semi--command-final tac}) \rangle \right) \right) \right) \right) \left. \text{definition of-semi--theory env = (\lambda \text{Theory-datatype dataty} \Rightarrow \langle \text{of-datatype env dataty} \rangle \left. \text{\%(s) (of-semi--theory env dataty}) \right) \right) \right) \right) \right) \right)
| Theory-interpretation thm ⇒ of-interpretation env thm |

definition String-concat-map s f l = String-concat s (L.map f l)

definition of-semi--theories env =
(λ Theories-one t ⇒ of-semi--theory env t
  | Theories-locale data l ⇒
    locale %s =
    %s begin
    %s end: (To-string (HolThyLocale-name data))
    (String-concat-map
      
      (λ (l-fix, o-assum).
        %s %s (String-concat-map :)
      )
    )

  ) (λ (e, ty).
    (fixes %s :: %s (of-semi--term e) (of-semi--typ ty)) l-fix)
    (case o-assum of None ⇒ ()
      | Some (name, e) ⇒ (assumes %s: %s (To-string name) (of-semi--term e))
      (HolThyLocale-header data))
    (String-concat-map :

  )
  ) (of-semi--theory env) l))
end

lemmas [code] =
  — def
Print.of-datatype-def
Print.of-type-synonym-def
Print.of-type-notation-def
Print.of-instantiation-def
Print.of-overloading-def
Print.ofconsts-def
Print.of-definition-def
Print.of-semi-thm-attribute-aux-gen-def
Print.of-semi-thm-attribute-aux-gen-where-def
Print.of-semi-thm-attribute-aux-gen-of-def
Print.of-semi-thm-attribute-def
Print.of-semi-thm-def
Print.of-semi-thm-attribute-l-def
Print.of-semi-thm-attribute-lI-def
Print.of-semi-thm-l-def
Print.of-lemmas-def
Print.of-semi-attrib-genA-def

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Print.of-semi--attrib-genB-def
Print.of-semi--attrib-def
Print.of-semi--attrib1-def
Print.of-semi--method-simp-def
Print.of-semi--command-final-def
Print.of-semi--command-state-def
Print.of-semi--command-proof-def
Print.of-lemma-def
Print.of-axiomatization-def
Print.of-section-def
Print.of-text-def
Print.of-ML-def
Print.of-setup-def
Print.of-thm-def
Print.of-interpretation-def
Print.of-semi--theory-def
Print.String-concat-map-def
Print.of-semi--theories-def

— fun
Print.of-semi--typ.simps
Print.of-semi--term.simps
Print.of-semi--thm-attribute-aux.simps
Print.of-semi--method.simps

end
5. Main

We present two solutions for obtaining an Isabelle file.

5.1. Static Meta Embedding with Exportation

definition Design =
(\text{let } n = \lambda n1 n2. \text{ToyTyObj (ToyTyCore-pre n1) (case n2 of None } \Rightarrow | | \text{ Some n2 } \Rightarrow [| (ToyTyCore-pre n2) |])
; mk = \lambda n t. \text{toy-class-raw.make n t} [| False \text{ in}
[\text{mk (n (Galaxy None)} | (\text{sound, ToyTy-raw (unit)}, (\text{moving, ToyTy-raw (bool)}])
, \text{mk (n (Planet (Some (Galaxy))) [|(\text{weight, ToyTy-raw (nat)}])
, \text{mk (n (Person (Some (Planet))) [|(\text{salary, ToyTy-raw (int)})]]"

Since we are in a Isabelle session, at this time, it becomes possible to inspect with the command \textbf{value} the result of the translations applied with \textit{Design}. A suitable environment should nevertheless be provided, one can typically experiment this by copying-pasting the following environment initialized in the above \textit{main}:

definition main =
(\text{let } n = \lambda n1. \text{ToyTyObj (ToyTyCore-pre n1) []}
; ToyMult = \lambda m r. \text{toy-multiplicity.make} [\text{m}] r [\text{Set}] \text{ in}
write-file
(compiler-env-config.extend
 (compiler-env-config-empty True None (oidInit (Oid 0)) Gen-only-design \texttt{(None, False)}
, D-output-disable-thy := False
, D-output-header-thy := Some ((Design-generated:
  [/../Toy-Library]
  [/../embedding/Generator-dynamic-sequential]) [|])
(L.map (META-class-raw Floor1) Design
@\vdash @ \texttt{(META-association (toy-association.make
ToyAssTy-association

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(ToyAssRel [ (n :Person, ToyMult (Mult-star, None) None)
             , (n (Person), ToyMult (Mult-nat 0, Some (Mult-nat 1))
               )]
             , (Some (boss)))))))

5.1.2. Statically Executing the Exportation

apply_code_printing ()
export_code main
(* in Haskell *)
(* in OCaml module_name M *)
(* in Scala module_name M *)
(* in SML module_name M *)

After the exportation and executing the exported, we obtain an Isabelle .thy file containing the generated code associated to the above input.
end

5.2. Dynamic Meta Embedding with Reflection

theory Generator-dynamic-sequential
imports Printer
  ../../../isabelle-home/src/HOL/Isabelle-Main2
begin

In the “dynamic” solution: the exportation is automatically handled inside Isabelle/jEdit. Inputs are provided using the syntax of the Toy Language, and in output we basically have two options:

- The first is to generate an Isabelle file for inspection or debugging. The generated file can interactively be loaded in Isabelle/jEdit, or saved to the hard disk. This mode is called the “deep exportation” mode or shortly the “deep” mode. The aim is to maximally automate the process one is manually performing in Generator_static.thy.
- On the other hand, it is also possible to directly execute in Isabelle/jEdit the generated file from the random access memory. This mode corresponds to the “shallow reflection” mode or shortly “shallow” mode.

In both modes, the reflection is necessary since the main part used by both was defined at Isabelle side. As a consequence, experimentations in “deep” and “shallow” are performed without leaving the editing session, in the same as the one the meta-compiler is actually running.

apply-code-printing-reflect (val stdout-file = Unsynchronized.ref )

This variable is not used in this theory (only in Generator_static.thy), but needed for well typechecking the reflected SML code.
code-reflect' open META
functions
  fold-thy-deep fold-thy-shallow

write-file

compiler-env-config-reset-all
compiler-env-config-update
oidInit
D-output-header-thy-update
map2-ctxt-term
check-export-code

isabelle-apply isabelle-of-compiler-env-config

5.2.1. Interface Between the Reflected and the Native

ML
val To-string0 = META.meta-of-logic

ML
structure From = struct
val string = META.SS-base o META.ST
val binding = string o Binding.name-of
  (*fun term ctxt s = string (XML.content-of (YXML.parse-body (Syntax.string-of-term ctxt s)))*
val internal-oid = META.Oid o Code-Numeral.natural-of-integer
val option = Option.map
val list = List.map
fun pair f1 f2 (x, y) = (f1 x, f2 y)
fun pair3 f1 f2 f3 (x, y, z) = (f1 x, f2 y, f3 z)

structure Pure = struct
val indexname = pair string Code-Numeral.natural-of-integer
val class = string
val sort = list class
fun typ e = (fn
  Type (s, t) => (META.Type o pair string (list typ)) (s, t)
| TFree (s, s0) => (META.TFree o pair string sort) (s, s0)
| TVar (i, s0) => (META.TVar o pair indexname sort) (i, s0)
) e
fun term e = (fn
  Const (s, t) => (META.Const o pair string typ) (s, t)
| Free (s, t) => (META.Free o pair string typ) (s, t)
| Var (i, t) => (META.Var o pair indexname typ) (i, t)
| Bound i => (META.Bound o Code-Numeral.natural-of-integer) i |
| Abs (s, ty, t) => (META.Abs o pair3 string typ term) (s, ty, t) |
| op $ (term1, term2) => (META.App o pair term term) (term1, term2) |
| e end |

fun toy-ctxt-term thy expr =
    META.T-pure (Pure.term (Syntax.read-term (Proof-Context.init-global thy) expr))
end

ML fun List-mapi f = META.mapi (f o Code-Numeral.integer-of-natural): |
ML |
structure Ty' = struct |
fun check l-oid l =
    let val Mp = META.map-prod |
        val Me = String.expplode |
        val Mi = Stringimplode |
        val Ml = map in |
        META.check-export-code |
            (writeln o Mi) |
            (warning o Ml) |
                (fn s => writeln (Markup.markup (Markup.bad ()) (Mi s))) |
            (error o To-string0) |
                (Ml (Mp I Me) l-oid) |
                    ((META.SS-base o META.ST) l) |
    end |
end
|

5.2.2. Binding of the Reflected API to the Native API |
ML |
structure META-overload = struct |
val of-semi--typ = META.of-semi-typ To-string0 |
val of-semi--term = META.of-semi-term To-string0 |
val of-semi--term' = META.of-semi-term To-string0 |
val fold = fold |
end |
|
ML |
structure Bind-Isabelle = struct |
fun To-binding s = Binding.make (s, Position.none) |
val To-sbinding = To-binding o To-string0 |

fun semi-method-simp g f = Method.Basic (fn ctxt => SIMPLE-METHOD (g (asm-full-simp-tac (f ctxt))))
val semi-method-simp-one = semi-method-simp (fn f => f 1)  
val semi-method-simp-all = semi-method-simp (CHANGED-PROP o PARALLEL-ALLGOALS)

datatype semi-thm' = Thms-single' of thm  
  | Thms-mult' of thm list

fun semi-thm-attribute ctxt = let open META open META-overload val S = fn Thms-single' t 
  => t  
  val M = fn Thms-mult' t => t in
fn Thm-thm s => Thms-single' (Proof-Context.get-thm ctxt (To-string0 s))  
| Thm-thms s => Thms-mult' (Proof-Context.get-thms ctxt (To-string0 s))  
| Thm-THEN (e1, e2) =>
  (case (semi-thm-attribute ctxt e1, semi-thm-attribute ctxt e2) of
    (Thms-single' e1, Thms-single' e2) => Thms-single' (e1 RSN (1, e2))
  | (Thms-mult' e1, Thms-mult' e2) => Thms-mult' (e1 RLN (1, e2)))
| Thm-simplified (e1, e2) =>  
  Thms-single' (asm-full-simplify (clear-simpset ctxt addsimps [S (semi-thm-attribute ctxt e2)]))  
| Thm-OF (e1, e2) =>
  Thms-single' ([S (semi-thm-attribute ctxt e2)] MRS (S (semi-thm-attribute ctxt e1))))
| Thm-where (nth, l) =>
  Thms-single' (Rule-Insts.where-rule ctxt
    (List.map (fn (var, expr) =>
      (((To-string0 var, 0), Position.none), of-semi-term expr)) l)
      []
    (S (semi-thm-attribute ctxt nth))))
| Thm-symmetric e1 =>
  let val e2 = S (semi-thm-attribute ctxt (Thm-thm (From.string sym))) in
  case semi-thm-attribute ctxt e1 of
    Thms-single' e1 => Thms-single' (e1 RSN (1, e2))
  | Thms-mult' e1 => Thms-mult' (e1 RLN (1, [e2]))
  end
| Thm-OF (nth, l) =>
  Thms-single' (Rule-Insts.of-rule ctxt
    (List.map (SOME o of-semi-term) l, [])
    []
  (S (semi-thm-attribute ctxt nth))))

end

fun semi-thm-attribute-single ctxt s = case (semi-thm-attribute ctxt s) of Thms-single' t => t

fun semi-thm-mult ctxt =  
  let fun f thy = case (semi-thm-attribute ctxt thy) of Thms-mult' t => t
    | Thms-single' t => [t] in
  fn META.Thms-single thy => f thy  
  | META.Thms-mult thy => f thy
end

fun semi-thm-mult-l ctxt l = List.concat (map (semi-thm-mult ctxt l))

fun semi-method-simp-only l ctxt = clear-simpset ctxt addsimps (semi-thm-mult-l ctxt l)
fun semi-method-simp-add-del-split (l-add, l-del, l-split) ctxt =
  fold Splitter.add-split (semi-thm-mult-l ctxt l-split)
  (ctxt addsimps (semi-thm-mult-l ctxt l-add)
   delsimps (semi-thm-mult-l ctxt l-del))

fun semi-method expr = let open META open Method open META-overload in case expr of
  Method-rule o-s =>
    Basic (fn ctxt =>
      METHOD (HEADGOAL o Classical.rule-tac
      ctxt
      (case o-s of NONE => []
       | SOME s => [semi-thm-attribute-single ctxt s])))
  Method-drule s =>
    Basic (fn ctxt => drule ctxt 0 [semi-thm-attribute-single ctxt s])
  Method-erule s =>
    Basic (fn ctxt =>erule ctxt 0 [semi-thm-attribute-single ctxt s])
  Method-elim s =>
    Basic (fn ctxt => elim ctxt [semi-thm-attribute-single ctxt s])
  Method-intro l =>
    Basic (fn ctxt => intro ctxt (map (semi-thm-attribute-single ctxt l)))
  Method-subst (asm, l, s) =>
    Basic (fn ctxt =>
      SIMPLE-METHOD ((if asm then EqSubst.eqsubst-asm-tac else EqSubst.eqsubst-tac)
      ctxt
      (map (fn s => case Int.fromString (To-string0 s) of
        SOME i => i) l)
      [semi-thm-attribute-single ctxt s]))
  Method-insert l =>
    Basic (fn ctxt => insert (semi-thm-mult-l ctxt l))
  Method-plus t =>
    Combinator (no-combinator-info
      , Repeat1
      , [Combinator (no-combinator-info, Then, List.map semi-method l)])
  Method-option t =>
    Combinator (no-combinator-info
      , Try
      , [Combinator (no-combinator-info, Then, List.map semi-method l)])
  Method-or t =>
    Combinator (no-combinator-info, Orelse, List.map semi-method t)
  Method-one (Method-simp-only l) =>
    semi-method-simp-one (semi-method-simp-only l)
  Method-one (Method-simp-add-del-split l) =>
    semi-method-simp-one (semi-method-simp-add-del-split l)
  Method-all (Method-simp-only l) =>
    semi-method-simp-all (semi-method-simp-only l)
  Method-all (Method-simp-add-del-split l) =>
    semi-method-simp-all (semi-method-simp-add-del-split l)
  Method-auto-simp-add-split (l-simp, l-split) =>
    Basic (fn ctxt =>
      SIMPLE-METHOD (auto-tac (fold (fn (f, l) => fold f l)
        ([Simplifier.add-simp, semi-thm-mult-l ctxt l-simp]
        , (Splitter.add-split, List.map (Proof-Context.get-thm ctxt o To-string0) l-split]
        ctxt))))
  Method-rename-tac l =>
    Basic (K (SIMPLE-METHOD' (Tactic.rename-tac (List.map To-string0 l))))
  Method-case-tac e =>
    Basic (fn ctxt =>
      SIMPLE-METHOD' (Induct-Tacs.case-tac ctxt (of-semi-term e) []]]
NONE))
| Method-blast n =>
|   Basic (case n of NONE => SIMPLE-METHOD' o blast-tac
|   | SOME lim => fn ctxt => SIMPLE-METHOD' (depth-tac ctxt
|   (Code-Numerals.integer-of-natural lim)))
| Method-clarify => Basic (fn ctxt => (SIMPLE-METHOD' (fn i => CHANGED-PROP
| (clarify-tac ctxt i))))
| Method-metis (l-opt, l) =>
|   Basic (fn ctxt => (METHOD oo Metis-Tactic.metis-method)
|     (if l-opt = [] then NONE else SOME (map To-string0 l-opt), NONE)
|     , map (semi-thm-attribute-single ctxt) l) ctxt)
end

fun then-tactic l = let open Method in
  (Combinator (no-combinator-info, Then, map semi--method l), (Position.none, Position.none))
end

fun local-terminal-proof o-by = let open META in case o-by of
  Command-done => Proof.local-done-proof
  | Command-sorry => Proof.local-skip-proof true
  | Command-by l-apply => Proof.local-terminal-proof (then-tactic l-apply, NONE)
end

fun global-terminal-proof o-by = let open META in case o-by of
  Command-done => Proof.global-done-proof
  | Command-sorry => Proof.global-skip-proof true
  | Command-by l-apply => Proof.global-terminal-proof (then-tactic l-apply, NONE)
end

fun proof-show-gen f (thes, thes-when) st = st
| > Proof.proof
|   (SOME (Method.Source [Token.make-string (−, Position.none)]
|     , (Position.none, Position.none)))
| > Seq.the-result
| > f
| > Proof.show-cmd
|   (thes-when = [])
|   NONE
|   (K t)
|   []
|   (if thes-when = [] then [] else [(Binding.empty-atts, map (fn t => (t, [])) thes-when)])
|   [(Binding.empty-atts, [(thes, [])])]
|   true
| > snd

val semi--command-state = let open META-overload in
  fn META.Command-apply-end l => (fn st => st |> Proof.apply-end (then-tactic l)
|   |> Seq.the-result )
val semi-command-proof = let open META-overload
  val thesis = ?thesis
  fun proof-show f = proof-show-gen f (thesis, []) in
  fn META.Command-apply l => (fn st => st |> Proof.apply (then-tactic l)
     |> Seq.the-result )
  | META.Command-using l => (fn st =>
    let val ctxt = Proof.context-of st in
    Proof.using [map (fn s => ([s], [])) (semi-thm-mult-l ctxt l)] st
    end)
  | META.Command-unfolding l => (fn st =>
    let val ctxt = Proof.context-of st in
    Proof.unfolding [map (fn s => ([s], [])) (semi-thm-mult-l ctxt l)] st
    end)
  | META.Command-let (e1, e2) =>
    proof-show (Proof.let-bind-cmd [([of-semi-term e1], of-semi-term e2)])
  | META.Command-have (n, b, e, e-pr) => proof-show (fn st => st
    |> Proof.have-cmd true NONE (K I) [] []
       [( (To-sbinding n, if b then [[Token.make-string (simp, Position.none)]] else []
           , [[of-semi-term e, []]])]
       true
       |> snd
       |> local-terminal-proof e-pr)
  | META.Command-fix-let (l, l-let, o-exp, _) =>
    proof-show-gen (fold (fn (e1, e2) =>
      Proof.let-bind-cmd [([of-semi-term e1], of-semi-term e2)])
      l-let
      o Proof.fix-cmd (List.map (fn i => (To-sbinding i, NONE, NoSyn)) l))
      (case o-exp of NONE => thesis | SOME (l-spec, _) =>>
      (String.concatWith (⇒
        (List.map of-semi-term l-spec))
       , case o-exp of NONE => [] | SOME (_, l-when) => List.map of-semi-term
       l-when)
end

fun semi-theory in-theory in-local = let open META open META-overload in (*let val f = *) fn
  Theory-datatype (Datatype (n, l)) => in-local
  (BNF-FP-Def-Sugar.co-datatype-cmd
   BNF-Util.Least-FP
   BNF-LFP.construct-lfp
   (Ctr-Sugar.default-ctr-options-cmd,
    ([ ( ( (([]), To-sbinding n), NoSyn)
       , List.map (fn (n, l) => ( (To-binding , To-sbinding n)
           , List.map (fn s => (To-binding , of-semi-typ s)) l)
           , NoSyn)) l)
    , (To-binding , To-binding ))
    , [])))

Theory-type-synonym (Type-synonym (n, v, l)) => in-theory
(fn thy =>
  let val s-bind = To-sbinding n in
  (snd o Typedecl.abbrev-global
      (s-bind, map To-string0 v, NoSyn)
      (Isabelle-Typedecl.abbrev-cmd0 (SOME s-bind) thy (of-semi--typ l))) thy
end)

Theory-type-notation (Type-notation (n, e)) => in-local
(Specification.type-notation-cmd true (_, true) [(To-string0 n, Mixfix (Input.string (To-string0 e)), [], 1000, Position.no-range)])

Theory-instantiation (Instantiation (n, n-def, expr)) => in-theory
(fn thy =>
  let val name = To-string0 n
    val tycons =
    [ let val Term.Type (s, -) = (Isabelle-Typedecl.abbrev-cmd0 NONE thy name) in s end
    in
      thy
      |> Class.instantiation (tycons, [], Syntax.read-sort (Proof-Context.init-global thy) object)
      |> fold-map (fn - => fn thy =>
          let val ((-, (-, tyg)), thy) = Specification.definition-cmd NONE [] []
            ((To-binding (To-string0 n-def ^ ^ name ^ ^ -def), []), of-semi--term expr) false thy in
            (ty, thy)
        end)
    end
    |> Class.prove-instantiation-exit-result (map o Morphism.thm) (fn ctxt => fn thms =>
        Class.intro-classes-tac ctxt [] THEN ALLGOALS (Proof-Context факт-tac ctxt thms))
    |> K I
  end)

Theory-overloading (Overloading (n-c, e-c, n, e)) => in-theory
(fn thy =>
  |> Overloading.overloading-cmd [(To-string0 n-c, of-semi--term e-c, true)]
  |> snd o Specification.definition-cmd NONE [] [] ((To-shbinding n, []), of-semi--term e) false
  |> Local-Theory.exit-global)

Theory-consts (Consts (n, ty, symb)) => in-theory
(Sign.add-consts-cmd [(To-shbinding n
    , of-semi--typ ty
    , Mixfix (Input.string ((^ ^ To-string0 symb), [], 1000, Position.no-range)))])

Theory-definition def => in-local
let val (def, e) = case def of
  Definition e => (NONE, e)
  | Definition-where1 (name, (abbrev, prio), e) =>
      (SOME (To-shbinding name
          , NONE
          , Mixfix (Input.string ((^ ^ of-semi--term abbrev ^ ^)), [], Code-Numerical.integer-of-natural
              prio, Position.no-range)), e)
  | Definition-where2 (name, abbrev, e) =>
      (SOME (To-shbinding name
          , NONE
          , NONE
Mixfix (Input.
string (( ~ of--term abbrev ~)), [], 1000, Position.no-range))

| Theory-lemmas | (Lemmas-simp-thm (simp, s, l)) => in-local
| Theory-lemmas | (fn lthy => (snd o Specification.theorems Thm.theoremK)
| Theory-lemmas | ((To-binding s, List.map (fn s => Attrib.check-src lthy [Token.make-string (s, Position.none)]))
| Theory-lemmas | (if simp then [simp, code-unfold] else []),
| Theory-lemmas | List.map (fn x => ([semi--thm-attribute-single lthy x], [])) l)]
| Theory-lemmas | [] false lthy)
| Theory-lemmas | (fn lthy => (snd o Specification.theorems Thm.theoremK)
| Theory-lemmas | ((To-binding s, List.map (fn s => Attrib.check-src lthy [Token.make-string (s, Position.none)]))
| Theory-lemmas | [simp, code-unfold]),
| Theory-lemmas | List.map (fn x => (Proof-Context.get-thms lthy (Token0 x), [])) l)]
| Theory-lemmas | [] false lthy)
| Theory-lemmas | (fn lthy => (Specification.theorem-cmd true Thm.theoremK NONE (K I)
| Theory-lemmas | Binding.empty-atts [] [] (ElementSHOW [((To-binding n, [])
| Theory-lemmas | , [(String.concatWith (⇒) (List.map of--term l-spec)), []])])
| Theory-lemmas | false lthy)
| Theory-lemmas | (fn lthy => lthy)
| Theory-lemmas | Speciation.theorem-cmd true Thm.theoremK NONE (K I)
| Theory-lemmas | (To-binding n, [])
| Theory-lemmas | (List.map (fn (n, (b, e)) =>
| Theory-lemmas | ElementSHOW [((To-binding n
| Theory-lemmas | , if b then [(Token.make-string (simp, Position.none))] else []
| Theory-lemmas | , [(of--term e, [])])]
| Theory-lemmas | l-spec)
| Theory-lemmas | (ElementSHOW [(Binding.empty-atts, [(of--term concl, [])])])
| Theory-lemmas | false)
| Theory-lemmas | (fn lthy => lthy)
| Theory-lemmas | (case map-filter (fn META.Command-let - => SOME []
| Theory-lemmas | META.Command-have - => SOME []
| Theory-lemmas | META.Command-fix-let (-, -, -, l) => SOME l
| Theory-lemmas | - => NONE)
| Theory-lemmas | (fn lthy => lthy)
| Theory-lemmas | let arg = (NONE, true) in fn st => st
local-terminal-proof o-by
> fold (fn l => fold semi--command-state l o Proof.local-qed arg) l
> Proof.global-qed arg end)

Theory-axiomatization (Axiomatization (n, e)) => in-theory
(#2 o Specification.axiomatization-cmd [] [] [(To-sbinding n, []), of-semi--term e])
Theory-section - => in-theory I
Theory-text - => in-theory I
Theory-ML (SML ml) =>
in-theory (Code-printing.reflect-ml (Input.source false (of-semi--term' ml)
(Position.none, Position_none)))

Theory-setup (Setup ml) =>
in-theory (Isar-Cmd.setup (Input.source false (of-semi--term' ml)
(Position.none, Position_none)))

Theory-thm (Thm thm) => in-local
(fn lthy =>
let val () =
  writeln
  (Pretty.string-of
   (Proof-Context.pretty-fact lthy, List.map (semi--thm-attribute-single lthy) thm))) in
lthy end)

Theory-interpretation (Interpretation (n, loc-n, loc-param, o-by)) => in-local
(fn lthy => lthy
> Interpretation.interpretation-cmd ( [] ( (To-string0 loc-n, Position.none)
  , (To-string0 n, true)
  , (if loc-param = [] then
      Expression.Named []
    else
      Expression.Positional (map (SOME o of-semi--term)
        loc-param), [[]])))
  , [])
> global-terminal-proof o-by)
(*in fn t => fn thy => f t thy handle ERROR s => (warning s; thy)
end*)
end
end

structure Bind-META = struct open Bind-Isabelle

fun all-meta aux ret = let open META open META-overload in fn
META-semi-theories thy =>
ret o (case thy of
  Theories-one thy => semi--theory I Named-Target.theory-map thy
  Theories-locale (data, I) => fn thy => thy
  |> ( Expression.add-locale-cmd
    (To-sbinding (META.holThyLocale-name data))
      Binding.empty
    []
  ))
end
(\[; \[\]
(List.concat
(map
(fn (fixes, assumes) => List.concat
[ map (fn (e,ty) => Element.Fixes [(To-binding (of-semi--term e)
, SOME (of-semi--typ ty)
, NoSyn)]) fixes
, case assumes of NONE => []
| SOME (n, e) => [Element.Assumes [((To-sbinding n, [])
, [(of-semi--term e, [])])]])
(META.holThyLocale-header data))
)# > snd)
| > fold (fold (semi--theory Local-Theory.background-theory
(fn f =>
 Local-Theory.new-group
# > f
# > Local-Theory.reset-group
# > (fn lthy =>

# | (Target-Context.switch-named-cmd NONE (Context.Proof

lthy)) lthy
| > Context.the-proof))))) l
| > Local-Theory.exit-global)
| META-boot-generation-syntax - => ret o I
| META-boot-setup-env - => ret o I
| META-all-meta-embedding meta => fn thy =>

aux
(map2-ctxt-term
(fn T-pure x => T-pure x
| e =>

let fun aux e = case e of
 T-to-be-parsed (s, -) => SOME let val t = Syntax.read-term (Proof-Context.init-global

thy)
(To-string0 s) in

(t, Term.add-frees t [])
end
| T-lambda (a, e) =>
Option.map
(fn (e, l-free) =>
let val a = To-string0 a
val (t, l-free) = case List.partition (fn (x, _) => x = a) l-free of
([], l-free) => (Term.TFree ('a, [HOL.type]), l-free)
| ([([], t), l-free) => (t, l-free) in

(lambda (Term.Free (a, t)) e, l-free)
end)
(axu e)
| - => NONE in
case aux e of
NONE => error nested pure expression not expected
| SOME (e, -) => META.T-pure (From.Pure.term e)
end) meta) thy
end
end
}end
Part II.

A Toy Example
5.3. A Toy Library for Objects in a State

theory Toy-Library
imports Main
begin

type-notation option (⟨-⟩⊥)
notation Some (⟨[-]⟩)

fun drop :: ‘α option ⇒ ‘α (⟨[-]⟩)
where drop-lift[simp]: [⟨v⟩] = v

type-synonym oid = nat

type-synonym ‘α val’ = unit ⇒ ‘α

type-notation val’ (-(-))

record (∀)state =
    heap :: oid ⇒ ‘α
    assocs :: oid ⇒ ((oid list) list) list

lemmas [simp,code-unfold] = state.defs
end

5.4. Example: A Class Model Converted into a Theory File

5.4.1. Introduction

theory Design-deep
imports
    ../embedding/Generator-dynamic-sequential
    ../../Antiquote-Setup
begin

In this example, we configure our package to generate a .thy file, without executing the associated generated code contained in this .thy file (c.f. Design_shallow.thy for a direct evaluation). This mode is particularly relevant for debugging purposes: while by default no evaluation occurs, the generated files (and their proofs!) can be executed on a step by step basis, depending on how we interact with the output window (by selectively clicking on what is generated).

After clicking on the generated content, the newly inserted content could depend on some theories which are not loaded by this current one. In this case, it is necessary to manually add all the needed dependencies above after the keyword imports. One should compare this current theory with Design_shallow.thy to see the differences of imported theories, and which ones to manually import (whenever an error happens).
While in theory it is possible to set the \texttt{deep} mode for generating in all target languages, i.e. by writing \texttt{[ in Haskell, in OCaml module-name M, in Scala module-name M, in SML module-name M ]}, usually using only one target is enough, since the task of all target is to generate the same Isabelle content. However in case one language takes too much time to setup, we recommend to try the generation with another target language, because all optimizations are currently not (yet) seemingly implemented for all target languages, or differently activated.

\section*{5.4.2. Designing Class Models (I): Basics}

The following example shows the definitions of a set of classes, called the “universe” of classes. Instead of providing a single command for building all the complete universe of classes directly in one block, we are constructing classes one by one. So globally the universe describing all classes is partial, it will only be fully constructed when all classes will be finished to be defined.

This allows to define classes without having to follow a particular order of definitions. Here \texttt{Atom} is defined before the one of \texttt{Molecule} (\texttt{Molecule} will come after):

\begin{verbatim}
Class Atom < Molecule
  Attributes size : Integer
End
\end{verbatim}

The “blue” color of \texttt{End} indicates that \texttt{End} is not a “green” keyword. \texttt{End} and \texttt{Class} are in fact similar, they belong to the group of meta-commands (all meta-commands are defined in \texttt{Isabelle-Meta-Model Generator-dynamic-sequential}). At run-time and in \texttt{deep} mode, the semantics of all meta-commands are approximately similar: all meta-commands displays some quantity of Isabelle code in the output window (as long as
meta-commands are syntactically correctly formed). However each meta-command is unique because what is displayed in the output window depends on the sequence of all meta-commands already encountered before (and also depends on arguments given to the meta-commands).

One particularity of \texttt{End} is to behave as the identity function when \texttt{End} is called without arguments. As example, here we are calling lots of \texttt{End} without arguments, and no Isabelle code is generated.

\begin{verbatim}
End End End
\end{verbatim}

We remark that, like any meta-commands, \texttt{End} could have been written anywhere in this theory, for example before \texttt{Class} or even before \texttt{generation-syntax}... Something does not have to be specially opened before using an \texttt{End}.

\texttt{Class Molecule < Person}

As example, here no \texttt{End} is written.

The semantics of \texttt{End} is further precised here. We earlier mentioned that the universe of classes is partially constructed, but one can still examine what is partially constructed, and one possibility is to use \texttt{End} for doing so. \texttt{End} can be seen as a lazy meta-command:

- without parameters, no code is generated,
- with some parameters (e.g., the symbol \texttt{!}), it forces the generation of the computation of the universe, by considering all already encountered classes. Then a partial representation of the universe can be interactively inspected.

\texttt{Class Galaxy}

\begin{verbatim}
Attributes wormhole : UnlimitedNatural
is-sound : Void
\end{verbatim}

\texttt{End!}

At this position, in the output window, we can observe for the first time some generated Isabelle code, corresponding to the partial universe of classes being constructed.

Note: By default, \texttt{Atom} and \texttt{Molecule} are not (yet) present in the shown universe because \texttt{Person} has not been defined in a separate line (unlike \texttt{Galaxy} above).

\texttt{Class Person < Galaxy}

\begin{verbatim}
Attributes salary : Integer
 boss : Person
is-meta-thinking : Boolean
\end{verbatim}

There is not only \texttt{End} which forces the computation of the universe, for example \texttt{Instance} declares a set of objects belonging to the classes earlier defined, but the entire universe is needed as knowledge, so there is no choice than forcing the generation of the universe.

\begin{verbatim}
Instance X\texttt{Person}1 :: Person = [ salary = 1300 , boss = X\texttt{Person}2 ]
and X\texttt{Person}2 :: Person = [ salary = 1800 ]
\end{verbatim}
Here we will call Instance again to show that the universe will not be computed again since it was already computed in the previous Instance.

**Instance** \( X_{\text{Person}}^3 :: \text{Person} = [ \text{salary} = 1 ] \)

However at any time, the universe can (or will) automatically be recomputed, whenever we are adding meanwhile another class:

\[
\text{(* Class Big Bang < Atom (* This will force the creation of a new universe. *) *)}
\]

As remark, not only the universe is recomputed, but the recomputation takes also into account all meta-commands already encountered. So in the new setting, \( X_{\text{Person}}^1 \), \( X_{\text{Person}}^2 \) and \( X_{\text{Person}}^3 \) will be resurrected... after the Big-Bang.

### 5.4.3. Designing Class Models (II): Jumping to Another Semantic Floor

Until now, meta-commands was used to generate lines of code, and these lines belong to the Isabelle language. One particularity of meta-commands is to generate pieces of code containing not only Isabelle code but also arbitrary meta-commands. In **deep** mode, this is particularly not a danger for meta-commands to generate themselves (whereas for **shallow** the recursion might not terminate).

In this case, such meta-commands must automatically generate the appropriate call to **generation-syntax** beforehand. However this is not enough, the compiling environment (comprising the history of meta-commands) are changing throughout the interactive evaluations, so the environment must also be taken into account and propagated when meta-commands are generating themselves. For example, the environment is needed for consultation whenever resurrecting objects, recomputing the universe or accessing the hierarchy of classes being defined.

As a consequence, in the next example a line **setup** is added after **generation-syntax** for bootstrapping the state of the compiling environment.

**State** \( \sigma_1 = \)

\[
[ ( [ \text{salary} = 1000 \text{, boss} = \text{self 1} ] :: \text{Person} )
, ( [ \text{salary} = 1200 ] :: \text{Person} )
, ( [ \text{salary} = 2600 \text{, boss} = \text{self 3} ] :: \text{Person} )
, X_{\text{Person}}^1 
, ( [ \text{salary} = 2300 \text{, boss} = \text{self 2} ] :: \text{Person} )
, X_{\text{Person}}^2 ]
\]

**State** \( \sigma_1' = \)

\[
[ X_{\text{Person}}^1
, X_{\text{Person}}^2
, X_{\text{Person}}^3 ]
\]

In certain circumstances, the command **setup** must be added again between some par-
Particular interleaving of two meta-commands and this may not depend on the presence of \texttt{generation-syntax} (which is defined only once when generating the first meta-command). For more details, one can refer to the source code of \texttt{ignore-meta-header} and \texttt{bootstrap-floor}.

PrePost $\sigma_1 \sigma_1'$

The generation of meta-commands allows to perform various extensions on the Toy language being embedded, without altering the semantics of a particular command. \texttt{PrePost} usually only takes “bound variables” as parameters (not arbitrary $\lambda$-terms), however the semantics of \texttt{PrePost} was extended to mimic the support of some particular terms not restricted to variables. This extension was implemented by executing some steps of “$\zeta$-reductions rewriting rules” operating on the meta-level of commands. First, it is at least needed to extend the syntax of expressions accepted by \texttt{PrePost} we then modify the parsing so that a larger subset of $\lambda$-terms can be given as parameters.

Starting from this expression:

\begin{verbatim}
(* PrePost (sigma) (~sub>1 [ ([ salary = 1000 , boss = self 1 ] :: Person) ] *)
\end{verbatim}

the rewriting begins with a first call to the next semantic floor, we obtain the following meta-commands (where \texttt{PrePost[shallow]} is an expression in normal form):

\begin{verbatim}
(* State WFF_10_post = [ ([ "salary" = 1000, "boss" = self 1 ] :: Person) ]
PrePost[shallow] (sigma) (~sub>1 WFF_10_post *) (WFF-10-post is an automatically generated name).
\end{verbatim}

The rewriting of the above \texttt{State} is performed in its turn. Finally the overall ultimately terminates when reaching \texttt{Instance} being already in normal form:

\begin{verbatim}
(* Instance WFF_10_post_object0 :: Person = [ "salary" = 1000, "boss" = [ ] ]
State[shallow] WFF_10_post = [ WFF_10_post_object0 ]
PrePost[shallow] (sigma) (~sub>1 WFF_10_post *)
\end{verbatim}

\subsection*{5.4.4. Designing Class Models (III): Interaction with (Pure) Term}

Meta-commands are obviously not restricted to manipulate expressions in the Outer Syntax level. It is possible to build meta-commands so that Inner Syntax expressions are directly parsed. However the dependencies of this theory have been minimized so that experimentations and debugging can easily occur in deep mode (this file only depends on \texttt{Isabelle-Meta-Model GENERATOR-dynamic-sequential}). Since the Inner Syntax expressions would perhaps manipulate expressions coming from other theories than \texttt{Isabelle-Meta-Model GENERATOR-dynamic-sequential}, it can be desirable to consider the Inner Syntax container as a string and leave the parsing for subsequent semantic floors.

This is what is implemented here:

Context Person :: content ()

Post "><"
Here the expression $\times$ is not well-typed in Isabelle, but an error is not raised because
the above expression is not (yet) parsed as an Inner Syntax element\(^1\).
However, this is not the same for the resulting generated meta-command:

\begin{verbatim}
Context [shallow] Person :: content ()
  Post : "("\lambda\> result self. (\times)")"
\end{verbatim}

and an error is immediately raised because the parsing of Inner Syntax expressions is
activated in this case.

For example, one can put the mouse, with the CTRL gesture, over the variable $a$, $b$ or
$c$ to be convinced that they are free variables compared with above:

\begin{verbatim}
Context[shallow] Person :: content ()
  Post : $a + b = c$
\end{verbatim}

5.4.5. Designing Class Models (IV): Saving the Generated to File

The experimentations usually finish by saving all the universe and generated Isabelle
theory to the hard disk:

\begin{verbatim}
(* generation_syntax deep flush_all *)
\end{verbatim}

5.4.6. Designing Class Models (V): Inspection of Generated Files

According to options given to the (first) command \texttt{generation-syntax} above, we re-
trieve the first generated file in the mentioned directory: \texttt{../document_generated/}
\texttt{Design_generated.thy}.

Because this file still contains meta-commands, we are here executing again a new gen-
erating step inside this file, the new result becomes saved in \texttt{../document_generated/}
\texttt{Design_generated_generated.thy}. As remark, in this last file, the dependency to
Isabelle-Meta-Model.\texttt{Generator-dynamic-sequential} was automatically removed because
the meta-compiler has detected the absence of meta-commands in the generated content.

Note: While the first generated file is intended to be always well-typed, it can hap-
pen that subsequent generations will lead to a not well-typed file. This is because the
meta-compiler only saves the history of meta-commands. In case some “native” Isabelle
declarations are generated among meta-commands, then these Isabelle declarations are
not saved by the meta-compiler, so these declarations will not be again generated. Any-
way, we see potential solutions for solving this and they would perhaps be implemented
in a future version of the meta-compiler...

end

\(^1\)In any case an error will not be raised, because the above code is written in verbatim in the real \texttt{.thy}
file, however one can copy-paste this code out of the verbatim scope to see that no errors are really
raised. For presentation purposes, it was embedded in verbatim because we will later discuss about
meta-commands generating Isabelle code, and then what is generated by this meta-command is of
course not well-typed!
5.5. Example: A Class Model Interactively Executed

5.5.1. Introduction

theory
  Design-shallow
imports
  ../Toy-Library
  ../Toy-Library-Static
  ../embedding/Generator-dynamic-sequential
  ../../../Antiquote-Setup
begin

In this example, we configure our package to execute tactic SML code (corresponding to
some generated .thy file, Design_deep.thy details how to obtain such generated .thy
file). Since SML code are already compiled (or reflected) and bound with the native
Isabelle API in Isabelle-Meta-Model,Generator-dynamic-sequential, nothing is generated
in this theory. The system only parses arguments given to meta-commands and imme-
diately calls the corresponding compiled functions.

The execution time is comparatively similar as if tactics were written by hand, except
that the generated SML code potentially inherits all optimizations performed by the raw
code generation of Isabelle (if any).

generation-syntax [ shallow (generation-semantics [ design ])

]

The configuration in shallow mode is straightforward: in this mode generation-syntax
basically terminates in $O(1)$.

5.5.2. Designing Class Models (I): Basics

Class Atom < Molecule
  Attributes size : Integer
End

End End End

Class Molecule < Person

Class Galaxy
  Attributes wormhole : UnlimitedNatural
    is-sound : Void
End!

Class Person < Galaxy
  Attributes salary : Integer
    boss : Person
    is-meta-thinking: Boolean
Instance $X_{Person}^1 :: Person = [ \text{salary} = 1300 \ , \ boss = X_{Person}^2 ]$
and $X_{Person}^2 :: Person = [ \text{salary} = 1800 ]$

Instance $X_{Person}^3 :: Person = [ \text{salary} = 1 ]$

5.5.3. Designing Class Models (II): Jumping to Another Semantic Floor

State $\sigma_1 =$
\[ (( [ \text{salary} = 1000 , \ boss = self^1 ] :: Person) \]
\[ , ([ \text{salary} = 1200 ] :: Person) \]
\[ , ([ \text{salary} = 2600 , \ boss = self^3 ] :: Person) \]
\[ , X_{Person}^1 \]
\[ , ([ \text{salary} = 2300 , \ boss = self^2 ] :: Person) \]
\[ , X_{Person}^2 \]

State $\sigma_1'$
\[ [ X_{Person}^1 \\
\[ , X_{Person}^2 \\
\[ , X_{Person}^3 ] \]

PrePost $\sigma_1\sigma_1'$

5.5.4. Designing Class Models (III): Interaction with (Pure) Term

Here in [shallow] mode, the following expression is directly rejected:

Context Person :: content ()
Post "="

Context[shallow] Person :: content ()
Post : $a + b = c$

end
Bibliography

Part III.

Appendix
A. Grammars of Commands

A.1. Main Setup of Meta Commands

\[
\text{generation-syntax} : \text{theory} \rightarrow \text{theory}
\]

\[
\text{deep}, \text{flush\_all}
\]

\[
\text{syntax}, \text{syntax\_print}
\]

\[
\text{deep-embedding}, \text{long-or-dirty}, \text{number}
\]

\[
\text{design}, \text{analysis}, \text{oid\_start}, \text{nat}
\]
`export-code` sets the behavior of all incoming meta-commands. By default, without firstly writing `generation-syntax` meta-commands will only print in output what they have parsed, this is similar as giving to `generation-syntax` a non-empty list having only `syntax-print` as elements (on the other hand, nothing is printed when an empty list is received). Additionally `syntax-print` can be followed by an integer indicating the printing depth in output, similar as declaring `ML-print-depth` with an integer, but the global option `syntax-print` is restricted to meta-commands. Besides the printing of syntaxes, several options are provided to further analyze the semantics of languages being embedded, and tell if their evaluation should occur immediately using the `shallow` mode, or to only display what would have been evaluated using the `deep` mode (i.e., to only show the generated Isabelle content in the output window).

Since several occurrences of `deep`, `shallow` or `syntax-print` can appear in the parameterizing list, for each meta-command the overall evaluation respects the order of events given in the list (from head to tail). At the time of writing, it is only possible to evaluate this list sequentially: the execution stops as soon as one first error is raised, thus ignoring remaining events.

`generation-syntax deep flush-all` performs as side effect the writing of all the generated Isabelle contents to the hard disk (all at the calling time), by iterating the saving for each `deep` mode in the list. In particular, this is only effective if there is at least one `deep` mode earlier declared.

As a side note, target languages for the `deep` mode currently supported are: Haskell, OCaml, Scala and SML. So in principle, all these targets generate the same Isabelle content and exit correctly. However, depending on the intended use, exporting with some targets may be more appropriate than other targets:

- For efficiency reasons, the meta-compiler has implemented a particular optimization for accelerating the process of evaluating incoming meta-commands. By default in Haskell and OCaml, the meta-compiler (at HOL side) is exported only
once, during the \texttt{generation-syntax} step. Then all incoming meta-commands are considered as arguments sent to the exported meta-compiler. As a compositionality aspect, these arguments are compiled then linked together with the (already compiled) meta-compiler, but this implies the use of one call of \texttt{unsafeCoerce} in Haskell and one \texttt{Obj.magic} statement in OCaml (otherwise another solution would be to extract the meta-compiler as a functor). Similar optimizations are not yet implemented for Scala and are only half-implemented for the SML target (which basically performs a step of marshalling to string in Isabelle/ML).

- For safety reasons, it simply suffices to extract all the meta-compiler together with the respective arguments in front of each incoming meta-commands everytime, then the overall needs to be newly compiled everytime. This is the current implemented behavior for Scala. For Haskell, OCaml and SML, it was also the default behavior in a prototyping version of the compiler, as a consequence one can restore that functionality for future versions.

Concerning the semantics of generated contents, if lemmas and proofs are generated, \texttt{SORRY} allows to explicitly skip the evaluation of all proofs, irrespective of the presence of \texttt{sorry} or not in generated proofs. In any cases, the semantics of \texttt{sorry} has not been overloaded, e.g., red background may appear as usual.

Finally \texttt{generation-semantics} is a container for specifying various options for varying the semantics of languages being embedded. For example, \texttt{design} and \texttt{analysis} are two options for specifying how the modelling of objects will be represented in the Toy Language. Similarly, this would be a typical place for options like \texttt{eager} or \texttt{lazy} for choosing how the evaluation should happen...

### A.2. All Meta Commands of the Toy Language

\begin{verbatim}
Class : theory → theory
Abstract-class : theory → theory

binding = type-base
  type-object = class
\end{verbatim}
class

Attributes

binding : toy-type

context

Operations

binding : toy-type

= term

post-use-prop

invariant

Constraints

Existential

Aggregation : theory → theory
Association : theory → theory
Composition : theory → theory
Aggregation

Association

Composition

binding

association

Between

association-end

association-end

Associationclass : theory → theory

Abstract-associationclass : theory → theory

Context : theory → theory

Context

1

shallow

1

type-object

context
**Instance**: \( \text{theory} \rightarrow \text{theory} \)

**State**: \( \text{theory} \rightarrow \text{theory} \)
\[
\text{state} = \text{shallow} \circ \text{binding} = \text{state}
\]

\[
\text{PrePost} : \text{theory} \rightarrow \text{theory}
\]

\[
\text{End} : \text{theory} \rightarrow \text{theory}
\]
A.3. Extensions of Isabelle Commands

\texttt{code-reflect'} has the same semantics as \texttt{code-reflect} except that it additionally contains the option \texttt{open} inspired from the command \texttt{export-code} (with the same semantics).
**lazy-code-printing** : theory → theory

**apply-code-printing** : theory → theory

**apply-code-printing-reflect** : local-theory → local-theory

---

**lazy_code_printing**

- printing-const
- printing-typeconstructor
- printing-class
- printing-class-relation
- printing-class-instance
- printing-module

**apply_code_printing**

( )

**apply_code_printing_reflect**

text

**lazy-code-printing** has the same semantics as **code-printing** or **ML** except that no side effects occur until we give more details about its intended future semantics: this will be precised by calling **apply-code-printing** or **apply-code-printing-reflect**.

**apply-code-printing** repeatedly calls **code-printing** to all previously registered elements with **lazy-code-printing** (the order is preserved).

**apply-code-printing-reflect** repeatedly calls **ML** to all previously registered elements with **lazy-code-printing** (the order is preserved). As a consequence, code for other targets (Haskell, OCaml, Scala) are ignored. Moreover before the execution of the overall, it is possible to give an additional piece of SML code as argument to priorly execute.
B. Content of the Directory isabelle_home

B.1. Extensions for Cartouches

- ./src/HOL/ex/Isabelle_Cartouche_Examples.thy
  Main0:
  Some functions have been generalized for supporting cartouches.

B.2. Other Changes

- ./src/Tools/Code/Isabelle_code_runtime.thy
  Main1:
  The option open was introduced in this file for the definition of code_reflect'.

- ./src/Tools/Code/Isabelle_code_target.thy
  Main1:
  Some signatures has removed for exposing the main structure, we have also defined
  at the end the implementation of lazy_code_printing, apply_code_printing and
  apply_code_printing_reflect.

- ./src/Pure/Isar/Isabelle_typedecl.thy
  Main2:
  Short modification of the argument lifting a binding to a binding option with some
  signatures removed.
C. Content of One Generated File (as example)

theory Design-generated-generated imports ../Toy-Library ../Toy-Library-Static begin

For certain concepts like classes and class-types, only a generic definition for its resulting semantics can be given. Generic means, there is a function outside HOL that “compiles” a concrete, closed-world class diagram into a “theory” of this data model, consisting of a bunch of definitions for classes, accessors, method, casts, and tests for actual types, as well as proofs for the fundamental properties of these operations in this concrete data model.

Our data universe consists in the concrete class diagram just of node’s, and implicitly of the class object. Each class implies the existence of a class type defined for the corresponding object representations as follows:

datatype \texttt{tyEXT\_Atom} = \texttt{mkEXT\_Atom\_oid\_oid\_list\_option\_int\_option\_bool\_option\_nat\_option\_unit\_option}
datatype \texttt{ty\_Atom} = \texttt{mk\_Atom\_tyEXT\_Atom\_int\_option}
datatype \texttt{tyEXT\_Molecule} = \texttt{mkEXT\_Molecule\_Atom\_ty\_Atom}
  | \texttt{mkEXT\_Molecule\_oid\_oid\_list\_option\_int\_option\_bool\_option\_nat\_option\_unit\_option}
datatype \texttt{ty\_Molecule} = \texttt{mk\_Molecule\_ty\_EXT\_Molecule}
datatype \texttt{ty\_EXT\_Person} = \texttt{mk\_EXT\_Person\_Molecule\_ty\_Molecule}
  | \texttt{mk\_EXT\_Person\_Atom\_ty\_Atom}
  | \texttt{mk\_EXT\_Person\_oid\_nat\_option\_unit\_option}
datatype \texttt{ty\_Person} = \texttt{mk\_Person\_ty\_EXT\_Person\_oid\_list\_option\_int\_option\_bool\_option}
datatype \texttt{ty\_EXT\_Galaxy} = \texttt{mk\_EXT\_Galaxy\_Person\_ty\_Person}
  | \texttt{mk\_EXT\_Galaxy\_Molecule\_ty\_Molecule}
  | \texttt{mk\_EXT\_Galaxy\_Atom\_ty\_Atom}
  | \texttt{mk\_EXT\_Galaxy\_oid}
datatype \texttt{ty\_Galaxy} = \texttt{mk\_Galaxy\_ty\_EXT\_Galaxy\_nat\_option\_unit\_option}
datatype \texttt{ty\_EXT\_ToyAny} = \texttt{mk\_EXT\_ToyAny\_Galaxy\_ty\_Galaxy}
  | \texttt{mk\_EXT\_ToyAny\_Person\_ty\_Person}
  | \texttt{mk\_EXT\_ToyAny\_Molecule\_ty\_Molecule}
  | \texttt{mk\_EXT\_ToyAny\_Atom\_ty\_Atom}
  | \texttt{mk\_EXT\_ToyAny\_oid}
datatype \texttt{ty\_ToyAny} = \texttt{mk\_ToyAny\_ty\_EXT\_ToyAny}

Now, we construct a concrete “universe of ToyAny types” by injection into a sum type containing the class types. This type of ToyAny will be used as instance for all respective type-variables.
Having fixed the object universe, we can introduce type synonyms that exactly correspond to Toy types. Again, we exploit that our representation of Toy is a “shallow embedding” with a one-to-one correspondence of Toy-types to types of the meta-language HOL.

datatype $\mathfrak{A} = \text{in}_{\text{Atom}} \, \text{ty}_{\text{Atom}}$

[119x747]datatype $\mathfrak{A} = \text{in}_{\text{Molecule}} \, \text{ty}_{\text{Molecule}}$

[119x747]datatype $\mathfrak{A} = \text{in}_{\text{Person}} \, \text{ty}_{\text{Person}}$

[119x747]datatype $\mathfrak{A} = \text{in}_{\text{Galaxy}} \, \text{ty}_{\text{Galaxy}}$

[119x747]datatype $\mathfrak{A} = \text{in}_{\text{ToyAny}} \, \text{ty}_{\text{ToyAny}}$

Having fixed the object universe, we can introduce type synonyms that exactly correspond to Toy types. Again, we exploit that our representation of Toy is a “shallow embedding” with a one-to-one correspondence of Toy-types to types of the meta-language HOL.

type-synonym Atom = $\langle\langle \text{ty}_{\text{Atom}} \rangle\rangle_\bot$

type-synonym Molecule = $\langle\langle \text{ty}_{\text{Molecule}} \rangle\rangle_\bot$

type-synonym Person = $\langle\langle \text{ty}_{\text{Person}} \rangle\rangle_\bot$

type-synonym Galaxy = $\langle\langle \text{ty}_{\text{Galaxy}} \rangle\rangle_\bot$

type-synonym ToyAny = $\langle\langle \text{ty}_{\text{ToyAny}} \rangle\rangle_\bot$

definition oid$_{\text{Atom}}$-0---boss = 0

definition oid$_{\text{Molecule}}$-0---boss = 0

definition oid$_{\text{Person}}$-0---boss = 0


definition switch$_2$-01 = (λ[x0, x1] ⇒ (x0, x1))

definition switch$_2$-10 = (λ[x0, x1] ⇒ (x1, x0))

definition oid1 = 1

definition oid2 = 2

definition inst-assoc1 = (λoid-class to-from oid. ((case (deref-assocs-list ((to-from::oid list list ⇒ oid list × oid list)) ((oid::oid)) ((drop (((map-of-list (((oid$_{\text{Person}}$-0---boss , (List.map ((λ(x, y). [x, y]) o switch$_2$-01) (([[oid1], [oid2]]))] (oid-class::oid)))))))) of Nil ⇒ None
 | l ⇒ (Some (l))::oid list option))

definition oid3 = 3

definition inst-assoc3 = (λoid-class to-from oid. ((case (deref-assocs-list ((to-from::oid list list ⇒ oid list × oid list)) ((oid::oid)) ((drop (((map-of-list ([]) (oid-class::oid)))))))) of Nil ⇒ None
 | l ⇒ (Some (l))::oid list option))

definition oid4 = 4

definition oid5 = 5

definition oid6 = 6

definition oid7 = 7
definition inst-assoc4 = (λoid-class to-from oid. ((case (deref-assocs-list ((to-from::oid list list ⇒ oid list × oid list)) (oid::oid)) ((drop (((map-of-list (((oid\Person·0--boss , (List.map ((λ(x , y). [x , y]) o switch2-01)) (((oid7 , [oid6]) , [oid6], [oid1], [oid4], [oid5]]))))))) ((oid-class::oid)))))))) of Nil ⇒ None
 | l ⇒ (Some (l))::oid list option)

locale state-σ1 =
fixes oid4 :: nat
fixes oid5 :: nat
fixes oid6 :: nat
fixes oid7 :: nat
fixes oid12 :: nat

assumes distinct-oid: (distinct ([oid4 , oid5 , oid6 , oid1 , oid7 , oid2]))
fixes σ1-object0 Person :: tyPerson
fixes σ1-object0 :: •::Person
assumes σ1-object0-def: σ1-object0 = (λ-. [[σ1-object0 Person]])
fixes σ1-object1 Person :: tyPerson
fixes σ1-object1 :: •::Person
assumes σ1-object1-def: σ1-object1 = (λ-. [[σ1-object1 Person]])
fixes σ1-object2 Person :: tyPerson
fixes σ1-object2 :: •::Person
assumes σ1-object2-def: σ1-object2 = (λ-. [[σ1-object2 Person]])
fixes XPerson1 Person :: tyPerson
fixes XPerson1 :: •::Person
assumes XPerson1-def: XPerson1 = (λ-. [[XPerson1 Person]])
fixes σ1-object4 Person :: tyPerson
fixes σ1-object4 :: •::Person
assumes σ1-object4-def: σ1-object4 = (λ-. [[σ1-object4 Person]])
fixes XPerson2 Person :: tyPerson
fixes XPerson2 :: •::Person
assumes XPerson2-def: XPerson2 = (λ-. [[XPerson2 Person]])

begin

definition σ1 = (state.make ((Map.empty (oid4 ⇒ (inPerson (σ1-object0 Person)))) (oid5 ⇒
(inPerson (σ1-object1 Person)))) (oid6 ⇒ (inPerson (σ1-object2 Person)))) (oid7 ⇒ (inPerson (XPerson1 Person)))) (oid8 ⇒ (inPerson (σ1-object14 Person)))) ((map-of-list (((oid\Person·0--boss , (List.map ((λ(x , y). [x , y]) o switch2-01)) (((oid7 , [oid6]) , [oid6], [oid1], [oid4], [oid5]])))))))

lemma perm-σ1 : σ1 = (state.make ((Map.empty (oid2 ⇒ (inPerson (XPerson2 Person)))) (oid7
⇒ (inPerson (σ1-object4 Person)))) (oid1 ⇒ (inPerson (XPerson1 Person)))) (oid6 ⇒ (inPerson (σ1-object2 Person)))) (oid5 ⇒ (inPerson (σ1-object1 Person)))) (oid4 ⇒ (inPerson (σ1-object0 Person))))

apply(simp add: σ1-def)
apply(subst (1) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (2) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (1) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (3) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (2) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (1) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (4) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (3) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (2) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (1) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)

apply(subst (1) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (3) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (2) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (1) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (4) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (3) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (2) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (1) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)

apply(subst (1) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (3) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (2) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (1) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (4) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (3) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (2) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (1) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)

apply(subst (1) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (3) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (2) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (1) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (4) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (3) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (2) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (1) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)

apply(subst (1) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (3) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (2) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (1) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (4) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (3) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (2) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (1) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)

apply(subst (1) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (3) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (2) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (1) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (4) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (3) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (2) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (1) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)

locale state-σ1' =
fixes oid1 :: nat
fixes oid2 :: nat
fixes oid3 :: nat
assumes distinct-oid: (distinct ([oid1 , oid2 , oid3]))
fixes X_person Person :: typerson
fixes X_person1 : :: Person
assumes X_person1-def: X_person1 = (λ-. [[X_person1 Person]])
fixes X_person2 : :: Person
assumes X_person2-def: X_person2 = (λ-. [[X_person2 Person]])
fixes X_person3 : :: Person
assumes X_person3-def: X_person3 = (λ-. [[X_person3 Person]])
begin

definition σ1' = (state.make ((Map.empty (oid1 ↦ (in_person (X_person1 Person)))) (oid2 ↦ (in_person (X_person2 Person)))) (oid3 ↦ (in_person (X_person3 Person)))) ((map-of-list ([[[oid1 , [oid2]]]])))))

lemma perm-σ1' : σ1' = (state.make ((Map.empty (oid3 ↦ (in_person (X_person3 Person)))) (oid2 ↦ (in_person (X_person2 Person)))) (oid1 ↦ (in_person (X_person1 Person)))))) (associates (σ1')))
apply(simp add: σ1'-def)
apply(subst (1) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (2) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)
apply(subst (1) fun-upd-twist, metis distinct-oid distinct-length-2-or-more)

locale pre-post-σ1-σ1' =
fixes oid1 :: nat
fixes oid2 :: nat
fixes oid3 :: nat
fixes oid4 :: nat
fixes oid5 :: nat
fixes oid6 :: nat
fixes oid7 :: nat
assumes distinct-oid: (distinct ([oid1, oid2, oid3, oid4, oid5, oid6, oid7]))
fixes X_person1 :: typerson
fixes X_person1 :: typerson
assumes X_person1-def: X_person1 = (λ-. [X_person1 person])
fixes X_person2 :: typerson
fixes X_person2 :: typerson
assumes X_person2-def: X_person2 = (λ-. [X_person2 person])
fixes X_person3 :: typerson
fixes X_person3 :: typerson
assumes X_person3-def: X_person3 = (λ-. [X_person3 person])
fixes σ1-object0 :: typerson
fixes σ1-object0 :: typerson
assumes σ1-object0-def: σ1-object0 = (λ-. [σ1-object0 person])
fixes σ1-object1 :: typerson
fixes σ1-object1 :: typerson
assumes σ1-object1-def: σ1-object1 = (λ-. [σ1-object1 person])
fixes σ1-object2 :: typerson
fixes σ1-object2 :: typerson
assumes σ1-object2-def: σ1-object2 = (λ-. [σ1-object2 person])
fixes σ1-object4 :: typerson
fixes σ1-object4 :: typerson
assumes σ1-object4-def: σ1-object4 = (λ-. [σ1-object4 person])
assumes σ1: (state-σ1 (oid4) (oid5) (oid6) (oid7) (oid2) (σ1-object0 person) (σ1-object0) (σ1-object1 person) (σ1-object1) (σ1-object2 person) (σ1-object2) (X_person1 person) (X_person1) (σ1-object4 person) (σ1-object4) (X_person2 person) (X_person2))
assumes σ1': (state-σ1' (oid1) (oid2) (oid3) (X_person1 person) (X_person1) (X_person2 person) (X_person3) (X_person2) (X_person3) (X_person3))
begin
interpretation state-σ1: state-σ1 oid4 oid5 oid6 oid7 oid2 σ1-object0 person σ1-object0 σ1-object1 person σ1-object1 σ1-object2 person σ1-object2 person X_person1 person X_person1 person X_person2 person X_person2 person X_person3 person X_person3 person
by (rule σ1)
interpretation state-σ1': state-σ1' oid1 oid2 oid3 X_person1 person X_person1 person X_person2 person X_person2 person X_person3 person X_person3 person
by (rule σ1')
definition heap-σ1 = (heap (state-σ1, σ1))
definition heap-σ1' = (heap (state-σ1', σ1'))
end
end