

# Slicing Guarantees Information Flow Noninterference

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## Abstract

In this contribution, we show how correctness proofs for intra- [8] and interprocedural slicing [9] can be used to prove that slicing is able to guarantee information flow noninterference. Moreover, we also illustrate how to lift the control flow graphs of the respective frameworks such that they fulfil the additional assumptions needed in the noninterference proofs. A detailed description of the intraprocedural proof and its interplay with the slicing framework can be found in [10].

## 1 Introduction

Information Flow Control (IFC) encompasses algorithms which determines if a given program leaks secret information to public entities. The major group are so called IFC type systems, where well-typed means that the respective program is secure. Several IFC type systems have been verified in proof assistants, e.g. see [1, 2, 5, 3, 7].

However, type systems have some drawbacks which can lead to false alarms. To overcome this problem, an IFC approach basing on slicing has been developed [4], which can significantly reduce the amount of false alarms. This contribution presents the first machine-checked proof that slicing is able to guarantee IFC noninterference. It bases on previously published machine-checked correctness proofs for slicing [8, 9]. Details for the intraprocedural case can be found in [10].

## 2 Slicing guarantees IFC Noninterference

```
theory NonInterferenceIntra imports  
  Slicing.Slice  
  Slicing.CFGExit-wf  
begin
```

## 2.1 Assumptions of this Approach

Classical IFC noninterference, a special case of a noninterference definition using partial equivalence relations (per) [6], partitions the variables (i.e. locations) into security levels. Usually, only levels for secret or high, written  $H$ , and public or low, written  $L$ , variables are used. Basically, a program that is noninterferent has to fulfil one basic property: executing the program in two different initial states that may differ in the values of their  $H$ -variables yields two final states that again only differ in the values of their  $H$ -variables; thus the values of the  $H$ -variables did not influence those of the  $L$ -variables.

Every per-based approach makes certain assumptions: (i) all  $H$ -variables are defined at the beginning of the program, (ii) all  $L$ -variables are observed (or used in our terms) at the end and (iii) every variable is either  $H$  or  $L$ . This security label is fixed for a variable and can not be altered during a program run. Thus, we have to extend the prerequisites of the slicing framework in [8] accordingly in a new locale:

```

locale NonInterferenceIntraGraph =
  BackwardSlice sourcenode targetnode kind valid-edge Entry Def Use state-val
  backward-slice +
  CFGExit-wf sourcenode targetnode kind valid-edge Entry Def Use state-val Exit
  for sourcenode :: 'edge  $\Rightarrow$  'node and targetnode :: 'edge  $\Rightarrow$  'node
  and kind :: 'edge  $\Rightarrow$  'state edge-kind and valid-edge :: 'edge  $\Rightarrow$  bool
  and Entry :: 'node ('('Entry'-')) and Def :: 'node  $\Rightarrow$  'var set
  and Use :: 'node  $\Rightarrow$  'var set and state-val :: 'state  $\Rightarrow$  'var  $\Rightarrow$  'val
  and backward-slice :: 'node set  $\Rightarrow$  'node set
  and Exit :: 'node ('('Exit'-')) +
  fixes H :: 'var set
  fixes L :: 'var set
  fixes High :: 'node ('('High'-'))
  fixes Low :: 'node ('('Low'-'))
  assumes Entry-edge-Exit-or-High:
   $\llbracket$ valid-edge a; sourcenode a = (-Entry-) $\rrbracket$ 
   $\implies$  targetnode a = (-Exit-)  $\vee$  targetnode a = (-High-)
  and High-target-Entry-edge:
   $\exists$  a. valid-edge a  $\wedge$  sourcenode a = (-Entry-)  $\wedge$  targetnode a = (-High-)  $\wedge$ 
    kind a = ( $\lambda$ s. True) $\surd$ 
  and Entry-predecessor-of-High:
   $\llbracket$ valid-edge a; targetnode a = (-High-) $\rrbracket \implies$  sourcenode a = (-Entry-)
  and Exit-edge-Entry-or-Low:  $\llbracket$ valid-edge a; targetnode a = (-Exit-) $\rrbracket$ 
   $\implies$  sourcenode a = (-Entry-)  $\vee$  sourcenode a = (-Low-)
  and Low-source-Exit-edge:
   $\exists$  a. valid-edge a  $\wedge$  sourcenode a = (-Low-)  $\wedge$  targetnode a = (-Exit-)  $\wedge$ 
    kind a = ( $\lambda$ s. True) $\surd$ 
  and Exit-successor-of-Low:
   $\llbracket$ valid-edge a; sourcenode a = (-Low-) $\rrbracket \implies$  targetnode a = (-Exit-)
  and DefHigh: Def (-High-) = H
  and UseHigh: Use (-High-) = H

```

**and** *UseLow*:  $Use(-Low-) = L$   
**and** *HighLowDistinct*:  $H \cap L = \{\}$   
**and** *HighLowUNIV*:  $H \cup L = UNIV$

**begin**

**lemma** *Low-neq-Exit*: **assumes**  $L \neq \{\}$  **shows**  $(-Low-) \neq (-Exit-)$   
 $\langle proof \rangle$

**lemma** *Entry-path-High-path*:  
**assumes**  $(-Entry-) -as \rightarrow^* n$  **and** *inner-node*  $n$   
**obtains**  $a' as'$  **where**  $as = a' \# as'$  **and**  $(-High-) -as' \rightarrow^* n$   
**and** *kind*  $a' = (\lambda s. True)_{\checkmark}$   
 $\langle proof \rangle$

**lemma** *Exit-path-Low-path*:  
**assumes**  $n -as \rightarrow^* (-Exit-)$  **and** *inner-node*  $n$   
**obtains**  $a' as'$  **where**  $as = as'@[a']$  **and**  $n -as' \rightarrow^* (-Low-)$   
**and** *kind*  $a' = (\lambda s. True)_{\checkmark}$   
 $\langle proof \rangle$

**lemma** *not-Low-High*:  $V \notin L \implies V \in H$   
 $\langle proof \rangle$

**lemma** *not-High-Low*:  $V \notin H \implies V \in L$   
 $\langle proof \rangle$

## 2.2 Low Equivalence

In classical noninterference, an external observer can only see public values, in our case the  $L$ -variables. If two states agree in the values of all  $L$ -variables, these states are indistinguishable for him. *Low equivalence* groups those states in an equivalence class using the relation  $\approx_L$ :

**definition** *lowEquivalence* ::  $'state \Rightarrow 'state \Rightarrow bool$  (**infixl**  $\approx_L$  50)  
**where**  $s \approx_L s' \equiv \forall V \in L. state-val\ s\ V = state-val\ s'\ V$

The following lemmas connect low equivalent states with relevant variables as necessary in the correctness proof for slicing.

**lemma** *relevant-vars-Entry*:  
**assumes**  $V \in rv\ S$   $(-Entry-)$  **and**  $(-High-) \notin backward-slice\ S$   
**shows**  $V \in L$   
 $\langle proof \rangle$

**lemma** *lowEquivalence-relevant-nodes-Entry*:  
**assumes**  $s \approx_L s'$  **and**  $(-High-) \notin \text{backward-slice } S$   
**shows**  $\forall V \in \text{rv } S \ (-Entry-). \text{state-val } s \ V = \text{state-val } s' \ V$   
 $\langle \text{proof} \rangle$

**lemma** *rv-Low-Use-Low*:  
**assumes**  $(-Low-) \in S$   
**shows**  $\llbracket n \ -as \rightarrow^* \ (-Low-); n \ -as' \rightarrow^* \ (-Low-);$   
 $\forall V \in \text{rv } S \ n. \text{state-val } s \ V = \text{state-val } s' \ V;$   
 $\text{preds } (\text{slice-kinds } S \ as) \ s; \text{preds } (\text{slice-kinds } S \ as') \ s^\wedge$   
 $\implies \forall V \in \text{Use } (-Low-). \text{state-val } (\text{transfers } (\text{slice-kinds } S \ as) \ s) \ V =$   
 $\text{state-val } (\text{transfers } (\text{slice-kinds } S \ as') \ s^\wedge) \ V$   
 $\langle \text{proof} \rangle$

### 2.3 The Correctness Proofs

In the following, we present two correctness proofs that slicing guarantees IFC noninterference. In both theorems,  $(-High-) \notin \text{backward-slice } S$ , where  $(-Low-) \in S$ , makes sure that no high variable (which are all defined in  $(-High-)$ ) can influence a low variable (which are all used in  $(-Low-)$ ).

First, a theorem regarding  $(-Entry-) \ -as \rightarrow^* \ (-Exit-)$  paths in the control flow graph (CFG), which agree to a complete program execution:

**lemma** *nonInterference-path-to-Low*:  
**assumes**  $s \approx_L s'$  **and**  $(-High-) \notin \text{backward-slice } S$  **and**  $(-Low-) \in S$   
**and**  $(-Entry-) \ -as \rightarrow^* \ (-Low-)$  **and**  $\text{preds } (\text{kinds } as) \ s$   
**and**  $(-Entry-) \ -as' \rightarrow^* \ (-Low-)$  **and**  $\text{preds } (\text{kinds } as') \ s'$   
**shows**  $\text{transfers } (\text{kinds } as) \ s \approx_L \text{transfers } (\text{kinds } as') \ s'$   
 $\langle \text{proof} \rangle$

**theorem** *nonInterference-path*:  
**assumes**  $s \approx_L s'$  **and**  $(-High-) \notin \text{backward-slice } S$  **and**  $(-Low-) \in S$   
**and**  $(-Entry-) \ -as \rightarrow^* \ (-Exit-)$  **and**  $\text{preds } (\text{kinds } as) \ s$   
**and**  $(-Entry-) \ -as' \rightarrow^* \ (-Exit-)$  **and**  $\text{preds } (\text{kinds } as') \ s'$   
**shows**  $\text{transfers } (\text{kinds } as) \ s \approx_L \text{transfers } (\text{kinds } as') \ s'$   
 $\langle \text{proof} \rangle$

**end**

The second theorem assumes that we have a operational semantics, whose evaluations are written  $\langle c, s \rangle \Rightarrow \langle c', s^\wedge \rangle$  and which conforms to the CFG. The correctness theorem then states that if no high variable influenced a low variable and the initial states were low equivalent, the resulting states are again low equivalent:

**locale** *NonInterferenceIntra* =

```

NonInterferenceIntraGraph sourcenode targetnode kind valid-edge Entry
  Def Use state-val backward-slice Exit H L High Low +
BackwardSlice-wf sourcenode targetnode kind valid-edge Entry Def Use state-val
  backward-slice sem identifies
for sourcenode :: 'edge  $\Rightarrow$  'node and targetnode :: 'edge  $\Rightarrow$  'node
and kind :: 'edge  $\Rightarrow$  'state edge-kind and valid-edge :: 'edge  $\Rightarrow$  bool
and Entry :: 'node (('(-Entry'-)) and Def :: 'node  $\Rightarrow$  'var set
and Use :: 'node  $\Rightarrow$  'var set and state-val :: 'state  $\Rightarrow$  'var  $\Rightarrow$  'val
and backward-slice :: 'node set  $\Rightarrow$  'node set
and sem :: 'com  $\Rightarrow$  'state  $\Rightarrow$  'com  $\Rightarrow$  'state  $\Rightarrow$  bool
  (((1<-,->)  $\Rightarrow$  / (1<-,->)) [0,0,0,0] 81)
and identifies :: 'node  $\Rightarrow$  'com  $\Rightarrow$  bool (-  $\triangleq$  - [51, 0] 80)
and Exit :: 'node (('(-Exit'-))
and H :: 'var set and L :: 'var set
and High :: 'node (('(-High'-)) and Low :: 'node (('(-Low'-)) +
fixes final :: 'com  $\Rightarrow$  bool
assumes final-edge-Low: [final c; n  $\triangleq$  c]
 $\Rightarrow \exists a.$  valid-edge a  $\wedge$  sourcenode a = n  $\wedge$  targetnode a = (-Low-)  $\wedge$  kind a =
 $\uparrow id$ 
begin

```

The following theorem needs the explicit edge from  $(-High-)$  to  $n$ . An approach using a *init* predicate for initial statements, being reachable from  $(-High-)$  via a  $(\lambda s. True)_{\vee}$  edge, does not work as the same statement could be identified by several nodes, some initial, some not. E.g., in the program `while (True) Skip;;Skip` two nodes identify this initial statement: the initial node and the node within the loop (because of loop unrolling).

```

theorem nonInterference:
assumes  $s_1 \approx_L s_2$  and  $(-High-) \notin$  backward-slice  $S$  and  $(-Low-) \in S$ 
and valid-edge a and sourcenode a =  $(-High-)$  and targetnode a = n
and kind a =  $(\lambda s. True)_{\vee}$  and  $n \triangleq c$  and final c'
and  $\langle c, s_1 \rangle \Rightarrow \langle c', s_1' \rangle$  and  $\langle c, s_2 \rangle \Rightarrow \langle c', s_2' \rangle$ 
shows  $s_1' \approx_L s_2'$ 
<proof>

```

**end**

**end**

### 3 Framework Graph Lifting for Noninterference

```

theory LiftingIntra
imports NonInterferenceIntra Slicing.CDepInstantiations
begin

```

In this section, we show how a valid CFG from the slicing framework in [8] can be lifted to fulfil all properties of the *NonInterferenceIntraGraph*

locale. Basically, we redefine the hitherto existing *Entry* and *Exit* nodes as new *High* and *Low* nodes, and introduce two new nodes *NewEntry* and *NewExit*. Then, we have to lift all functions to operate on this new graph.

### 3.1 Liftings

#### 3.1.1 The datatypes

```
datatype 'node LDCFG-node = Node 'node
  | NewEntry
  | NewExit
```

```
type-synonym ('edge,'node,'state) LDCFG-edge =
  'node LDCFG-node × ('state edge-kind) × 'node LDCFG-node
```

#### 3.1.2 Lifting *valid-edge*

```
inductive lift-valid-edge :: ('edge ⇒ bool) ⇒ ('edge ⇒ 'node) ⇒ ('edge ⇒ 'node)
⇒
  ('edge ⇒ 'state edge-kind) ⇒ 'node ⇒ 'node ⇒ ('edge,'node,'state) LDCFG-edge
⇒
  bool
for valid-edge::'edge ⇒ bool and src::'edge ⇒ 'node and trg::'edge ⇒ 'node
and knd::'edge ⇒ 'state edge-kind and E::'node and X::'node
```

**where** *lve-edge*:

```
[[valid-edge a; src a ≠ E ∨ trg a ≠ X;
  e = (Node (src a),knd a,Node (trg a))]]
⇒ lift-valid-edge valid-edge src trg knd E X e
```

| *lve-Entry-edge*:

```
e = (NewEntry,(λs. True)√,Node E)
⇒ lift-valid-edge valid-edge src trg knd E X e
```

| *lve-Exit-edge*:

```
e = (Node X,(λs. True)√,NewExit)
⇒ lift-valid-edge valid-edge src trg knd E X e
```

| *lve-Entry-Exit-edge*:

```
e = (NewEntry,(λs. False)√,NewExit)
⇒ lift-valid-edge valid-edge src trg knd E X e
```

```
lemma [simp]:¬ lift-valid-edge valid-edge src trg knd E X (Node E,et,Node X)
⟨proof⟩
```

#### 3.1.3 Lifting *Def* and *Use* sets

```
inductive-set lift-Def-set :: ('node ⇒ 'var set) ⇒ 'node ⇒ 'node ⇒
```

$'var\ set \Rightarrow 'var\ set \Rightarrow ('node\ LDCFG\text{-}node \times 'var)\ set$

**for**  $Def::('node \Rightarrow 'var\ set)$  **and**  $E::'node$  **and**  $X::'node$   
**and**  $H::'var\ set$  **and**  $L::'var\ set$

**where** *lift-Def-node*:

$V \in Def\ n \Longrightarrow (Node\ n, V) \in lift\text{-}Def\text{-}set\ Def\ E\ X\ H\ L$

| *lift-Def-High*:

$V \in H \Longrightarrow (Node\ E, V) \in lift\text{-}Def\text{-}set\ Def\ E\ X\ H\ L$

**abbreviation** *lift-Def*  $:: ('node \Rightarrow 'var\ set) \Rightarrow 'node \Rightarrow 'node \Rightarrow$

$'var\ set \Rightarrow 'var\ set \Rightarrow 'node\ LDCFG\text{-}node \Rightarrow 'var\ set$

**where** *lift-Def*  $Def\ E\ X\ H\ L\ n \equiv \{V. (n, V) \in lift\text{-}Def\text{-}set\ Def\ E\ X\ H\ L\}$

**inductive-set** *lift-Use-set*  $:: ('node \Rightarrow 'var\ set) \Rightarrow 'node \Rightarrow 'node \Rightarrow$

$'var\ set \Rightarrow 'var\ set \Rightarrow ('node\ LDCFG\text{-}node \times 'var)\ set$

**for**  $Use::'node \Rightarrow 'var\ set$  **and**  $E::'node$  **and**  $X::'node$

**and**  $H::'var\ set$  **and**  $L::'var\ set$

**where**

*lift-Use-node*:

$V \in Use\ n \Longrightarrow (Node\ n, V) \in lift\text{-}Use\text{-}set\ Use\ E\ X\ H\ L$

| *lift-Use-High*:

$V \in H \Longrightarrow (Node\ E, V) \in lift\text{-}Use\text{-}set\ Use\ E\ X\ H\ L$

| *lift-Use-Low*:

$V \in L \Longrightarrow (Node\ X, V) \in lift\text{-}Use\text{-}set\ Use\ E\ X\ H\ L$

**abbreviation** *lift-Use*  $:: ('node \Rightarrow 'var\ set) \Rightarrow 'node \Rightarrow 'node \Rightarrow$

$'var\ set \Rightarrow 'var\ set \Rightarrow 'node\ LDCFG\text{-}node \Rightarrow 'var\ set$

**where** *lift-Use*  $Use\ E\ X\ H\ L\ n \equiv \{V. (n, V) \in lift\text{-}Use\text{-}set\ Use\ E\ X\ H\ L\}$

## 3.2 The lifting lemmas

### 3.2.1 Lifting the basic locales

**abbreviation** *src*  $:: ('edge, 'node, 'state)\ LDCFG\text{-}edge \Rightarrow 'node\ LDCFG\text{-}node$

**where**  $src\ a \equiv fst\ a$

**abbreviation** *trg*  $:: ('edge, 'node, 'state)\ LDCFG\text{-}edge \Rightarrow 'node\ LDCFG\text{-}node$

**where**  $trg\ a \equiv snd(snd\ a)$

**definition** *knd*  $:: ('edge, 'node, 'state)\ LDCFG\text{-}edge \Rightarrow 'state\ edge\text{-}kind$

**where**  $knd\ a \equiv fst(snd\ a)$

**lemma** *lift-CFG*:

**assumes**  $wf:CFGExit\text{-}wf$   $sourcenode$   $targetnode$   $kind$   $valid\text{-}edge$   $Entry$   $Def$   $Use$   
 $state\text{-}val$   $Exit$   
**shows**  $CFG$   $src$   $trg$   
 $(lift\text{-}valid\text{-}edge$   $valid\text{-}edge$   $sourcenode$   $targetnode$   $kind$   $Entry$   $Exit$ )  $NewEntry$   
 $\langle proof \rangle$

**lemma**  $lift\text{-}CFG\text{-}wf$ :  
**assumes**  $wf:CFGExit\text{-}wf$   $sourcenode$   $targetnode$   $kind$   $valid\text{-}edge$   $Entry$   $Def$   $Use$   
 $state\text{-}val$   $Exit$   
**shows**  $CFG\text{-}wf$   $src$   $trg$   $knd$   
 $(lift\text{-}valid\text{-}edge$   $valid\text{-}edge$   $sourcenode$   $targetnode$   $kind$   $Entry$   $Exit$ )  $NewEntry$   
 $(lift\text{-}Def$   $Def$   $Entry$   $Exit$   $H$   $L$ )  $(lift\text{-}Use$   $Use$   $Entry$   $Exit$   $H$   $L$ )  $state\text{-}val$   
 $\langle proof \rangle$

**lemma**  $lift\text{-}CFGExit$ :  
**assumes**  $wf:CFGExit\text{-}wf$   $sourcenode$   $targetnode$   $kind$   $valid\text{-}edge$   $Entry$   $Def$   $Use$   
 $state\text{-}val$   $Exit$   
**shows**  $CFGExit$   $src$   $trg$   $knd$   
 $(lift\text{-}valid\text{-}edge$   $valid\text{-}edge$   $sourcenode$   $targetnode$   $kind$   $Entry$   $Exit$ )  
 $NewEntry$   $NewExit$   
 $\langle proof \rangle$

**lemma**  $lift\text{-}CFGExit\text{-}wf$ :  
**assumes**  $wf:CFGExit\text{-}wf$   $sourcenode$   $targetnode$   $kind$   $valid\text{-}edge$   $Entry$   $Def$   $Use$   
 $state\text{-}val$   $Exit$   
**shows**  $CFGExit\text{-}wf$   $src$   $trg$   $knd$   
 $(lift\text{-}valid\text{-}edge$   $valid\text{-}edge$   $sourcenode$   $targetnode$   $kind$   $Entry$   $Exit$ )  $NewEntry$   
 $(lift\text{-}Def$   $Def$   $Entry$   $Exit$   $H$   $L$ )  $(lift\text{-}Use$   $Use$   $Entry$   $Exit$   $H$   $L$ )  $state\text{-}val$   $NewExit$   
 $\langle proof \rangle$

### 3.2.2 Lifting $wod\text{-}backward\text{-}slice$

**lemma**  $lift\text{-}wod\text{-}backward\text{-}slice$ :  
**fixes**  $valid\text{-}edge$  **and**  $sourcenode$  **and**  $targetnode$  **and**  $kind$  **and**  $Entry$  **and**  $Exit$   
**and**  $Def$  **and**  $Use$  **and**  $H$  **and**  $L$   
**defines**  $lve:lve \equiv lift\text{-}valid\text{-}edge$   $valid\text{-}edge$   $sourcenode$   $targetnode$   $kind$   $Entry$   $Exit$   
**and**  $lDef:lDef \equiv lift\text{-}Def$   $Def$   $Entry$   $Exit$   $H$   $L$   
**and**  $lUse:lUse \equiv lift\text{-}Use$   $Use$   $Entry$   $Exit$   $H$   $L$   
**assumes**  $wf:CFGExit\text{-}wf$   $sourcenode$   $targetnode$   $kind$   $valid\text{-}edge$   $Entry$   $Def$   $Use$   
 $state\text{-}val$   $Exit$   
**and**  $H \cap L = \{\}$  **and**  $H \cup L = UNIV$   
**shows**  $NonInterferenceIntraGraph$   $src$   $trg$   $knd$   $lve$   $NewEntry$   $lDef$   $lUse$   $state\text{-}val$   
 $(CFG\text{-}wf.wod\text{-}backward\text{-}slice$   $src$   $trg$   $lve$   $lDef$   $lUse$ )  
 $NewExit$   $H$   $L$   $(Node$   $Entry)$   $(Node$   $Exit)$   
 $\langle proof \rangle$

### 3.2.3 Lifting PDG-BS with standard-control-dependence

**lemma** *lift-Postdomination*:

**assumes** *wf:CFGExit-wf sourcenode targetnode kind valid-edge Entry Def Use state-val Exit*  
**and** *pd:Postdomination sourcenode targetnode kind valid-edge Entry Exit*  
**and** *inner:CFGExit.inner-node sourcenode targetnode valid-edge Entry Exit nx*  
**shows** *Postdomination src trg kn*  
*(lift-valid-edge valid-edge sourcenode targetnode kind Entry Exit) NewEntry NewExit*  
*<proof>*

**lemma** *lift-PDG-scd*:

**assumes** *PDG:PDG sourcenode targetnode kind valid-edge Entry Def Use state-val Exit*  
*(Postdomination.standard-control-dependence sourcenode targetnode valid-edge Exit)*  
**and** *pd:Postdomination sourcenode targetnode kind valid-edge Entry Exit*  
**and** *inner:CFGExit.inner-node sourcenode targetnode valid-edge Entry Exit nx*  
**shows** *PDG src trg kn*  
*(lift-valid-edge valid-edge sourcenode targetnode kind Entry Exit) NewEntry*  
*(lift-Def Def Entry Exit H L) (lift-Use Use Entry Exit H L) state-val NewExit*  
*(Postdomination.standard-control-dependence src trg*  
*(lift-valid-edge valid-edge sourcenode targetnode kind Entry Exit) NewExit)*  
*<proof>*

**lemma** *lift-PDG-standard-backward-slice*:

**fixes** *valid-edge and sourcenode and targetnode and kind and Entry and Exit*  
**and** *Def and Use and H and L*  
**defines** *lve:lve ≡ lift-valid-edge valid-edge sourcenode targetnode kind Entry Exit*  
**and** *lDef:lDef ≡ lift-Def Def Entry Exit H L*  
**and** *lUse:lUse ≡ lift-Use Use Entry Exit H L*  
**assumes** *PDG:PDG sourcenode targetnode kind valid-edge Entry Def Use state-val Exit*  
*(Postdomination.standard-control-dependence sourcenode targetnode valid-edge Exit)*  
**and** *pd:Postdomination sourcenode targetnode kind valid-edge Entry Exit*  
**and** *inner:CFGExit.inner-node sourcenode targetnode valid-edge Entry Exit nx*  
**and** *H ∩ L = {} and H ∪ L = UNIV*  
**shows** *NonInterferenceIntraGraph src trg kn lve NewEntry lDef lUse state-val*  
*(PDG.PDG-BS src trg lve lDef lUse*  
*(Postdomination.standard-control-dependence src trg lve NewExit))*  
*NewExit H L (Node Entry) (Node Exit)*  
*<proof>*

### 3.2.4 Lifting PDG-BS with weak-control-dependence

**lemma** *lift-StrongPostdomination*:

**assumes** *wf:CFGExit-wf sourcenode targetnode kind valid-edge Entry Def Use*

*state-val Exit*

**and** *spd:StrongPostdomination sourcenode targetnode kind valid-edge Entry Exit*  
**and** *inner:CFGExit.inner-node sourcenode targetnode valid-edge Entry Exit nx*  
**shows** *StrongPostdomination src trg kn*  
*(lift-valid-edge valid-edge sourcenode targetnode kind Entry Exit) NewEntry NewExit*  
 ⟨*proof*⟩

**lemma** *lift-PDG-wcd:*  
**assumes** *PDG:PDG sourcenode targetnode kind valid-edge Entry Def Use state-val Exit*  
*(StrongPostdomination.weak-control-dependence sourcenode targetnode valid-edge Exit)*  
**and** *spd:StrongPostdomination sourcenode targetnode kind valid-edge Entry Exit*  
**and** *inner:CFGExit.inner-node sourcenode targetnode valid-edge Entry Exit nx*  
**shows** *PDG src trg kn*  
*(lift-valid-edge valid-edge sourcenode targetnode kind Entry Exit) NewEntry*  
*(lift-Def Def Entry Exit H L) (lift-Use Use Entry Exit H L) state-val NewExit*  
*(StrongPostdomination.weak-control-dependence src trg*  
*(lift-valid-edge valid-edge sourcenode targetnode kind Entry Exit) NewExit)*  
 ⟨*proof*⟩

**lemma** *lift-PDG-weak-backward-slice:*  
**fixes** *valid-edge and sourcenode and targetnode and kind and Entry and Exit*  
**and** *Def and Use and H and L*  
**defines** *lve:lve ≡ lift-valid-edge valid-edge sourcenode targetnode kind Entry Exit*  
**and** *lDef:lDef ≡ lift-Def Def Entry Exit H L*  
**and** *lUse:lUse ≡ lift-Use Use Entry Exit H L*  
**assumes** *PDG:PDG sourcenode targetnode kind valid-edge Entry Def Use state-val Exit*  
*(StrongPostdomination.weak-control-dependence sourcenode targetnode valid-edge Exit)*  
**and** *spd:StrongPostdomination sourcenode targetnode kind valid-edge Entry Exit*  
**and** *inner:CFGExit.inner-node sourcenode targetnode valid-edge Entry Exit nx*  
**and** *H ∩ L = {} and H ∪ L = UNIV*  
**shows** *NonInterferenceIntraGraph src trg kn lve NewEntry lDef lUse state-val*  
*(PDG.PDG-BS src trg lve lDef lUse*  
*(StrongPostdomination.weak-control-dependence src trg lve NewExit))*  
*NewExit H L (Node Entry) (Node Exit)*  
 ⟨*proof*⟩

**end**

## 4 Information Flow for While

```
theory NonInterferenceWhile imports
  Slicing.SemanticsWellFormed
  Slicing.StaticControlDependences
  LiftingIntra
begin

locale SecurityTypes =
  fixes H :: vname set
  fixes L :: vname set
  assumes HighLowDistinct:  $H \cap L = \{\}$ 
  and HighLowUNIV:  $H \cup L = UNIV$ 
begin
```

### 4.1 Lifting labels-nodes and Defining final

```
fun labels-LDCFG-nodes :: cmd  $\Rightarrow$  w-node LDCFG-node  $\Rightarrow$  cmd  $\Rightarrow$  bool
  where labels-LDCFG-nodes prog (Node n) c = labels-nodes prog n c
  | labels-LDCFG-nodes prog n c = False
```

```
lemmas WCFG-path-induct[consumes 1, case-names empty-path Cons-path]
  = CFG.path.induct[OF While-CFG-aux]
```

```
lemma lift-valid-node:
  assumes CFG.valid-node sourcenode targetnode (valid-edge prog) n
  shows CFG.valid-node src trg
  (lift-valid-edge (valid-edge prog) sourcenode targetnode kind (-Entry-) (-Exit-))
  (Node n)
   $\langle$ proof $\rangle$ 
```

```
lemma lifted-CFG-fund-prop:
  assumes labels-LDCFG-nodes prog n c and  $\langle c, s \rangle \rightarrow^* \langle c', s' \rangle$ 
  shows  $\exists n' \text{ as. } CFG.path \text{ src trg}$ 
  (lift-valid-edge (valid-edge prog) sourcenode targetnode kind (-Entry-) (-Exit-))
   $n \text{ as } n' \wedge \text{transfers } (CFG.kinds \text{ knd as}) \text{ } s = s' \wedge$ 
   $\text{preds } (CFG.kinds \text{ knd as}) \text{ } s \wedge \text{labels-LDCFG-nodes prog } n' \text{ } c'$ 
   $\langle$ proof $\rangle$ 
```

```
fun final :: cmd  $\Rightarrow$  bool
  where final Skip = True
  | final c = False
```

**lemma** *final-edge*:

*labels-nodes prog n Skip*  $\implies$  *prog*  $\vdash$  *n*  $\dashv\vdash id \rightarrow$  (*-Exit-*)  
 ⟨*proof*⟩

## 4.2 Semantic Non-Interference for Weak Order Dependence

**lemmas** *WODNonInterferenceGraph* =

*lift-wod-backward-slice*[*OF While-CFGExit-wf-aux HighLowDistinct HighLowUNIV*]

**lemma** *WODNonInterference*:

*NonInterferenceIntra src trg kno*  
 (*lift-valid-edge (valid-edge prog) sourcenode targetnode kind*  
 (*-Entry-*) (*-Exit-*))  
*NewEntry (lift-Def (Defs prog) (-Entry-) (-Exit-) H L)*  
*(lift-Use (Uses prog) (-Entry-) (-Exit-) H L) id*  
*(CFG-wf.wod-backward-slice src trg*  
*(lift-valid-edge (valid-edge prog) sourcenode targetnode kind*  
 (*-Entry-*) (*-Exit-*))  
*(lift-Def (Defs prog) (-Entry-) (-Exit-) H L)*  
*(lift-Use (Uses prog) (-Entry-) (-Exit-) H L))*  
*reds (labels-LDCFG-nodes prog)*  
*NewExit H L (LDCFG-node.Node (-Entry-)) (LDCFG-node.Node (-Exit-)) final*  
 ⟨*proof*⟩

## 4.3 Semantic Non-Interference for Standard Control Dependence

**lemma** *inner-node-exists*: $\exists n$ . *CFGExit.inner-node sourcenode targetnode*

(*valid-edge prog*) (*-Entry-*) (*-Exit-*) *n*  
 ⟨*proof*⟩

**lemmas** *SCDNonInterferenceGraph* =

*lift-PDG-standard-backward-slice*[*OF WStandardControlDependence.PDG-scd*  
*WhilePostdomination-aux - HighLowDistinct HighLowUNIV*]

**lemma** *SCDNonInterference*:

*NonInterferenceIntra src trg kno*  
 (*lift-valid-edge (valid-edge prog) sourcenode targetnode kind*  
 (*-Entry-*) (*-Exit-*))  
*NewEntry (lift-Def (Defs prog) (-Entry-) (-Exit-) H L)*  
*(lift-Use (Uses prog) (-Entry-) (-Exit-) H L) id*  
*(PDG.PDG-BS src trg*  
*(lift-valid-edge (valid-edge prog) sourcenode targetnode kind*  
 (*-Entry-*) (*-Exit-*))

```

    (lift-Def (Defs prog) (-Entry-) (-Exit-) H L)
    (lift-Use (Uses prog) (-Entry-) (-Exit-) H L)
    (Postdomination.standard-control-dependence src trg
     (lift-valid-edge (valid-edge prog) sourcenode targetnode kind
      (-Entry-) (-Exit-)) NewExit))
    reds (labels-LDCFG-nodes prog)
    NewExit H L (LDCFG-node.Node (-Entry-)) (LDCFG-node.Node (-Exit-)) final
  ⟨proof⟩

```

#### 4.4 Semantic Non-Interference for Weak Control Dependence

**lemmas** *WCDNonInterferenceGraph* =  
*lift-PDG-weak-backward-slice*[*OF WWeakControlDependence.PDG-wcd*  
*WhileStrongPostdomination-aux - HighLowDistinct HighLowUNIV*]

**lemma** *WCDNonInterference*:  
*NonInterferenceIntra src trg kind*  
 (lift-valid-edge (valid-edge prog) sourcenode targetnode kind  
 (-Entry-) (-Exit-))  
 NewEntry (lift-Def (Defs prog) (-Entry-) (-Exit-) H L)  
 (lift-Use (Uses prog) (-Entry-) (-Exit-) H L) *id*  
 (PDG.PDG-BS src trg  
 (lift-valid-edge (valid-edge prog) sourcenode targetnode kind  
 (-Entry-) (-Exit-))  
 (lift-Def (Defs prog) (-Entry-) (-Exit-) H L)  
 (lift-Use (Uses prog) (-Entry-) (-Exit-) H L)  
 (StrongPostdomination.weak-control-dependence src trg  
 (lift-valid-edge (valid-edge prog) sourcenode targetnode kind  
 (-Entry-) (-Exit-)) NewExit))  
 reds (labels-LDCFG-nodes prog)  
 NewExit H L (LDCFG-node.Node (-Entry-)) (LDCFG-node.Node (-Exit-)) final  
 ⟨proof⟩

**end**

**end**

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