

A formal proof of the Chandy–Lamport distributed snapshot algorithm

Ben Fiedler¹ and Dmitriy Traytel¹

¹ETH Zürich

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Abstract

We provide a suitable distributed system model and implementation of the Chandy–Lamport distributed snapshot algorithm [1]. Our main result is a formal termination and correctness proof of the Chandy–Lamport algorithm and its use in stable property detection.

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1 Modelling distributed systems

We assume familiarity with Chandy and Lamport’s paper *Distributed Snapshots: Determining Global States of Distributed Systems* [1].

```

theory Distributed-System

imports Main

begin

type-synonym 'a fifo = 'a list
type-synonym channel-id = nat

```

```

datatype 'm message =
  Marker
  | Msg 'm

datatype recording-state =
  NotStarted
  | Recording
  | Done

```

We characterize distributed systems by three underlying type variables: Type variable 'p captures the processes of the underlying system. Type variable 's describes the possible states of the processes. Finally, type variable 'm describes all possible messages in said system.

Each process is in exactly one state at any point in time of the system. Processes are interconnected by directed channels, which hold messages in-flight between connected processes. There can be an arbitrary number of channels between different processes. The entire state of the system including the (potentially unfinished) snapshot state is called *configuration*.

```

record ('p, 's, 'm) configuration =
  states :: 'p ⇒ 's
  msgs :: channel-id ⇒ 'm message fifo

  process-snapshot :: 'p ⇒ 's option
  channel-snapshot :: channel-id ⇒ 'm fifo * recording-state

```

An event in Chandy and Lamport’s formalization describes a process’ state transition, optionally producing or consuming (but not both) a message on

a channel. Additionally, a process may either initiate a snapshot spontaneously, or is forced to do so by receiving a snapshot *marker* on one of its incoming channels.

datatype ($'p, 's, 'm$) *event* =
 $isTrans: Trans$ (*occurs-on*: $'p$) $'s 's$
 $| isSend: Send$ (*getId*: *channel-id*)
 (*occurs-on*: $'p$)
 (*partner*: $'p$)
 $'s 's$ (*getMsg*: $'m$)
 $| isRecv: Recv$ (*getId*: *channel-id*)
 (*occurs-on*: $'p$)
 (*partner*: $'p$)
 $'s 's$ (*getMsg*: $'m$)

 $| isSnapshot: Snapshot$ (*occurs-on*: $'p$)
 $| isRecvMarker: RecvMarker$ (*getId*: *channel-id*)
 (*occurs-on*: $'p$)
 (*partner*: $'p$)

We introduce abbreviations and type synonyms for commonly used terms.

type-synonym ($'p, 's, 'm$) *trace* = ($'p, 's, 'm$) *event list*

abbreviation *ps* **where** $ps \equiv process_snapshot$

abbreviation *cs* **where** $cs \equiv channel_snapshot$

abbreviation *no-snapshot-change* **where**

$no_snapshot_change\ c\ c' \equiv ((\forall p'. ps\ c\ p' = ps\ c'\ p') \wedge (\forall i'. cs\ c\ i' = cs\ c'\ i'))$

abbreviation *has-snapshot* **where**

$has_snapshot\ c\ p \equiv process_snapshot\ c\ p \neq None$

A regular event is an event as described in Chandy and Lamport's original paper: A state transition accompanied by the emission or receiving of a message. Nonregular events are related to snapshotting and receiving markers along communication channels.

definition *regular-event*[*simp*]:

$regular_event\ ev \equiv (isTrans\ ev \vee isSend\ ev \vee isRecv\ ev)$

lemma *nonregular-event*:

$\sim\ regular_event\ ev = (isSnapshot\ ev \vee isRecvMarker\ ev)$

$\langle proof \rangle$

lemma *event-occurs-on-unique*:

assumes

$p \neq q$

$occurs_on\ ev = p$

shows

$occurs_on\ ev \neq q$

<proof>

1.1 The distributed system locale

In order to capture Chandy and Lamport's computation system we introduce two locales. The distributed system locale describes global truths, such as the mapping from channel IDs to sender and receiver processes, the transition relations for the underlying computation system and the core assumption that no process has a channel to itself. While not explicitly mentioned in Chandy's and Lamport's work, it makes sense to assume that a channel need not communicate to itself via messages, since it shares memory with itself.

locale *distributed-system* =
fixes
 channel :: *channel-id* \Rightarrow (*p* * *p*) *option* **and**
 trans :: *'p* \Rightarrow *'s* \Rightarrow *'s* \Rightarrow *bool* **and**
 send :: *channel-id* \Rightarrow *'p* \Rightarrow *'p* \Rightarrow *'s* \Rightarrow *'s* \Rightarrow *'m* \Rightarrow *bool* **and**
 recv :: *channel-id* \Rightarrow *'p* \Rightarrow *'p* \Rightarrow *'s* \Rightarrow *'s* \Rightarrow *'m* \Rightarrow *bool*
assumes
 no-self-channel:
 $\forall i. \nexists p. \text{channel } i = \text{Some } (p, p)$
begin

1.1.1 State transitions

definition *can-occur* :: (*'p*, *'s*, *'m*) *event* \Rightarrow (*'p*, *'s*, *'m*) *configuration* \Rightarrow *bool* **where**
can-occur ev c \equiv (case *ev* of
 Trans p s s' \Rightarrow *states c p = s*
 \wedge *trans p s s'*
| *Send i p q s s' msg* \Rightarrow *states c p = s*
 \wedge *channel i = Some (p, q)*
 \wedge *send i p q s s' msg*
| *Recv i p q s s' msg* \Rightarrow *states c p = s*
 \wedge *channel i = Some (q, p)*
 \wedge *length (msgs c i) > 0*
 \wedge *hd (msgs c i) = Msg msg*
 \wedge *recv i p q s s' msg*
| *Snapshot p* \Rightarrow \neg *has-snapshotted c p*
| *RecvMarker i p q* \Rightarrow *channel i = Some (q, p)*
 \wedge *length (msgs c i) > 0*
 \wedge *hd (msgs c i) = Marker*)

definition *src* **where**
src i p \equiv ($\exists q. \text{channel } i = \text{Some } (p, q)$)

definition *dest* **where**
dest i q \equiv ($\exists p. \text{channel } i = \text{Some } (p, q)$)

lemma *can-occur-Recv*:

assumes

can-occur (Recv i p q s s' m) c

shows

states c p = s \wedge *channel i = Some (q, p)* \wedge $(\exists xs. \text{msgs } c \ i = \text{Msg } m \ \# \ xs)$ \wedge
recv i p q s s' m
 $\langle \text{proof} \rangle$

abbreviation *check-snapshot-occur where*

check-snapshot-occur c c' p \equiv
 (*can-occur (Snapshot p) c* \wedge
 (*ps c' p = Some (states c p)*)
 \wedge $(\forall p'. \text{states } c \ p' = \text{states } c' \ p')$
 \wedge $(\forall p'. (p' \neq p) \longrightarrow \text{ps } c' \ p' = \text{ps } c \ p')$
 \wedge $(\forall i. (\exists q. \text{channel } i = \text{Some } (p, q)) \longrightarrow \text{msgs } c' \ i = \text{msgs } c \ i \ @ \ [\text{Marker}])$
 \wedge $(\forall i. (\exists q. \text{channel } i = \text{Some } (q, p)) \longrightarrow \text{channel-snapshot } c' \ i = (\text{fst } (\text{channel-snapshot } c \ i), \text{Recording}))$
 \wedge $(\forall i. (\nexists q. \text{channel } i = \text{Some } (p, q)) \longrightarrow \text{msgs } c' \ i = \text{msgs } c \ i)$
 \wedge $(\forall i. (\nexists q. \text{channel } i = \text{Some } (q, p)) \longrightarrow \text{channel-snapshot } c' \ i = \text{channel-snapshot } c \ i))$

abbreviation *check-recv-marker-occur where*

check-recv-marker-occur c c' i p q \equiv
 (*can-occur (RecvMarker i p q) c*
 \wedge $(\forall r. \text{states } c \ r = \text{states } c' \ r)$
 \wedge $(\forall r. (r \neq p) \longrightarrow \text{process-snapshot } c \ r = \text{process-snapshot } c' \ r)$
 \wedge $(\text{Marker} \ \# \ \text{msgs } c' \ i = \text{msgs } c \ i)$
 \wedge $(\text{channel-snapshot } c' \ i = (\text{fst } (\text{channel-snapshot } c \ i), \text{Done}))$
 \wedge $(\text{if has-snapshotted } c \ p$
 then $(\text{process-snapshot } c \ p = \text{process-snapshot } c' \ p)$
 \wedge $(\forall i'. (i' \neq i) \longrightarrow \text{msgs } c' \ i' = \text{msgs } c \ i')$
 \wedge $(\forall i'. (i' \neq i) \longrightarrow \text{channel-snapshot } c \ i' = \text{channel-snapshot } c' \ i')$
 else $(\text{process-snapshot } c' \ p = \text{Some } (\text{states } c \ p))$
 \wedge $(\forall i'. i' \neq i \wedge (\exists r. \text{channel } i' = \text{Some } (p, r))$
 $\longrightarrow \text{msgs } c' \ i' = \text{msgs } c \ i' \ @ \ [\text{Marker}])$
 \wedge $(\forall i'. i' \neq i \wedge (\exists r. \text{channel } i' = \text{Some } (r, p))$
 $\longrightarrow \text{channel-snapshot } c' \ i' = (\text{fst } (\text{channel-snapshot } c \ i'), \text{Recording}))$
 \wedge $(\forall i'. i' \neq i \wedge (\nexists r. \text{channel } i' = \text{Some } (p, r))$
 $\longrightarrow \text{msgs } c' \ i' = \text{msgs } c \ i')$
 \wedge $(\forall i'. i' \neq i \wedge (\nexists r. \text{channel } i' = \text{Some } (r, p))$
 $\longrightarrow \text{channel-snapshot } c' \ i' = \text{channel-snapshot } c \ i'))$

abbreviation *check-trans-occur where*

check-trans-occur c c' p s s' \equiv
 (*can-occur (Trans p s s') c*
 \wedge $(\text{states } c' \ p = s')$
 \wedge $(\forall r. (r \neq p) \longrightarrow \text{states } c' \ r = \text{states } c \ r)$
 \wedge $(\forall i. \text{msgs } c' \ i = \text{msgs } c \ i)$
 \wedge $(\text{no-snapshot-change } c \ c')$

abbreviation *check-send-occur* **where**

$$\begin{aligned}
& \text{check-send-occur } c \ c' \ i \ p \ q \ s \ s' \ \text{msg} \equiv \\
& \quad (\text{can-occur } (\text{Send } i \ p \ q \ s \ s' \ \text{msg}) \ c \\
& \quad \wedge (\text{states } c' \ p = s') \\
& \quad \wedge (\forall r. (r \neq p) \longrightarrow \text{states } c' \ r = \text{states } c \ r) \\
& \quad \wedge (\text{msgs } c' \ i = \text{msgs } c \ i \ @ \ [\text{Msg } \text{msg}]) \\
& \quad \wedge (\forall i'. i \neq i' \longrightarrow \text{msgs } c' \ i' = \text{msgs } c \ i') \\
& \quad \wedge (\text{no-snapshot-change } c \ c')
\end{aligned}$$

abbreviation *check-recv-occur* **where**

$$\begin{aligned}
& \text{check-recv-occur } c \ c' \ i \ p \ q \ s \ s' \ \text{msg} \equiv \\
& \quad (\text{can-occur } (\text{Recv } i \ p \ q \ s \ s' \ \text{msg}) \ c \\
& \quad \wedge (\text{states } c \ p = s \ \wedge \ \text{states } c' \ p = s') \\
& \quad \wedge (\forall r. (r \neq p) \longrightarrow \text{states } c' \ r = \text{states } c \ r) \\
& \quad \wedge (\text{msgs } c \ i = \text{Msg } \text{msg} \ # \ \text{msgs } c' \ i) \\
& \quad \wedge (\forall i'. i \neq i' \longrightarrow \text{msgs } c' \ i' = \text{msgs } c \ i') \\
& \quad \wedge (\forall r. \text{process-snapshot } c \ r = \text{process-snapshot } c' \ r) \\
& \quad \wedge (\forall i'. i' \neq i \longrightarrow \text{channel-snapshot } c \ i' = \text{channel-snapshot } c' \ i') \\
& \quad \wedge (\text{if } \text{snd } (\text{channel-snapshot } c \ i) = \text{Recording} \\
& \quad \quad \text{then } \text{channel-snapshot } c' \ i = (\text{fst } (\text{channel-snapshot } c \ i) \ @ \ [\text{msg}], \ \text{Recording}) \\
& \quad \quad \text{else } \text{channel-snapshot } c \ i = \text{channel-snapshot } c' \ i))
\end{aligned}$$

The *next* predicate lets us express configuration transitions using events. The predicate $\text{next}(s_1, e, s_2)$ denotes the transition of the configuration s_1 to s_2 via the event e . It ensures that e can occur in state s_1 and the state s_2 is correctly constructed from s_1 .

primrec *next* ::

$$\begin{aligned}
& ('p, 's, 'm) \text{ configuration} \\
& \Rightarrow ('p, 's, 'm) \text{ event} \\
& \Rightarrow ('p, 's, 'm) \text{ configuration} \\
& \Rightarrow \text{bool} \\
& (- \vdash - \mapsto - [\gamma_0, \gamma_0, \gamma_0]) \text{ where} \\
& \quad \text{next-snapshot: } c \vdash \text{Snapshot } p \mapsto c' = \\
& \quad \quad \text{check-snapshot-occur } c \ c' \ p \\
& \quad | \text{ next-recv-marker: } c \vdash \text{RecvMarker } i \ p \ q \mapsto c' = \\
& \quad \quad \text{check-recv-marker-occur } c \ c' \ i \ p \ q \\
& \quad | \text{ next-trans: } c \vdash \text{Trans } p \ s \ s' \mapsto c' = \\
& \quad \quad \text{check-trans-occur } c \ c' \ p \ s \ s' \\
& \quad | \text{ next-send: } c \vdash \text{Send } i \ p \ q \ s \ s' \ \text{msg} \mapsto c' = \\
& \quad \quad \text{check-send-occur } c \ c' \ i \ p \ q \ s \ s' \ \text{msg} \\
& \quad | \text{ next-recv: } c \vdash \text{Recv } i \ p \ q \ s \ s' \ \text{msg} \mapsto c' = \\
& \quad \quad \text{check-recv-occur } c \ c' \ i \ p \ q \ s \ s' \ \text{msg}
\end{aligned}$$

Useful lemmas about state transitions

lemma *state-and-event-determine-next*:

assumes

$$\begin{aligned}
& c \vdash \text{ev} \mapsto c' \ \text{and} \\
& c \vdash \text{ev} \mapsto c''
\end{aligned}$$

shows
 $c' = c''$
 $\langle proof \rangle$

lemma *exists-next-if-can-occur*:

assumes
 $can\text{-occur } ev \ c$
shows
 $\exists c'. c \vdash ev \mapsto c'$
 $\langle proof \rangle$

lemma *exists-exactly-one-following-state*:

$can\text{-occur } ev \ c \implies \exists! c'. c \vdash ev \mapsto c'$
 $\langle proof \rangle$

lemma *no-state-change-if-no-event*:

assumes
 $c \vdash ev \mapsto c'$ **and**
 $occurs\text{-on } ev \neq p$
shows
 $states \ c \ p = states \ c' \ p \wedge process\text{-snapshot } c \ p = process\text{-snapshot } c' \ p$
 $\langle proof \rangle$

lemma *no-msgs-change-if-no-channel*:

assumes
 $c \vdash ev \mapsto c'$ **and**
 $channel \ i = None$
shows
 $msgs \ c \ i = msgs \ c' \ i$
 $\langle proof \rangle$

lemma *no-cs-change-if-no-channel*:

assumes
 $c \vdash ev \mapsto c'$ **and**
 $channel \ i = None$
shows
 $cs \ c \ i = cs \ c' \ i$
 $\langle proof \rangle$

lemma *no-msg-change-if-no-event*:

assumes
 $c \vdash ev \mapsto c'$ **and**
 $isSend \ ev \implies getId \ ev \neq i$ **and**
 $isRecv \ ev \implies getId \ ev \neq i$ **and**
 $regular\text{-event } ev$
shows
 $msgs \ c \ i = msgs \ c' \ i$
 $\langle proof \rangle$

lemma *no-cs-change-if-no-event*:

assumes

$c \vdash ev \mapsto c'$ **and**
 $isRecv\ ev \longrightarrow getId\ ev \neq i$ **and**
regular-event ev

shows

$cs\ c\ i = cs\ c'\ i$

<proof>

lemma *happen-implies-can-occur*:

assumes

$c \vdash ev \mapsto c'$

shows

can-occur $ev\ c$

<proof>

lemma *snapshot-increases-message-length*:

assumes

$ev = Snapshot\ p$ **and**
 $c \vdash ev \mapsto c'$ **and**
 $channel\ i = Some\ (q, r)$

shows

$length\ (msgs\ c\ i) \leq length\ (msgs\ c'\ i)$

<proof>

lemma *recv-marker-changes-head-only-at-i*:

assumes

$ev = RecvMarker\ i\ p\ q$ **and**
 $c \vdash ev \mapsto c'$ **and**
 $i' \neq i$

shows

$msgs\ c\ i' = [] \vee hd\ (msgs\ c\ i') = hd\ (msgs\ c'\ i')$

<proof>

lemma *recv-marker-other-channels-not-shrinking*:

assumes

$ev = RecvMarker\ i\ p\ q$ **and**
 $c \vdash ev \mapsto c'$

shows

$length\ (msgs\ c\ i') \leq length\ (msgs\ c'\ i') \longleftrightarrow i \neq i'$

<proof>

lemma *regular-event-cannot-induce-snapshot*:

assumes

$\sim has_snapshotted\ c\ p$ **and**
 $c \vdash ev \mapsto c'$

shows

regular-event $ev \longrightarrow \sim has_snapshotted\ c'\ p$

<proof>

lemma *regular-event-preserves-process-snapshots:*

assumes

$c \vdash ev \mapsto c'$

shows

$regular\text{-}event\ ev \implies ps\ c\ r = ps\ c'\ r$

$\langle proof \rangle$

lemma *no-state-change-if-nonregular-event:*

assumes

$\sim regular\text{-}event\ ev$ **and**

$c \vdash ev \mapsto c'$

shows

$states\ c\ p = states\ c'\ p$

$\langle proof \rangle$

lemma *nonregular-event-induces-snapshot:*

assumes

$\sim has\text{-}snapshotted\ c\ p$ **and**

$c \vdash ev \mapsto c'$ **and**

$occurs\text{-}on\ ev = p$ **and**

$\sim regular\text{-}event\ ev$

shows

$\sim regular\text{-}event\ ev \longrightarrow has\text{-}snapshotted\ c'\ p$

$\langle proof \rangle$

lemma *snapshot-state-unchanged:*

assumes

step: $c \vdash ev \mapsto c'$ **and**

$has\text{-}snapshotted\ c\ p$

shows

$ps\ c\ p = ps\ c'\ p$

$\langle proof \rangle$

lemma *message-must-be-delivered:*

assumes

valid: $c \vdash ev \mapsto c'$ **and**

delivered: $(msgs\ c\ i \neq [] \wedge hd\ (msgs\ c\ i) = m) \wedge (msgs\ c'\ i = [] \vee hd\ (msgs\ c'\ i) \neq m)$

shows

$(\exists p\ q. ev = RecvMarker\ i\ p\ q \wedge m = Marker)$

$\vee (\exists p\ q\ s\ s'\ m'. ev = Recv\ i\ p\ q\ s\ s'\ m' \wedge m = Msg\ m')$

$\langle proof \rangle$

lemma *message-must-be-delivered-2:*

assumes

$c \vdash ev \mapsto c'$

$m : set\ (msgs\ c\ i)$

$m \notin set\ (msgs\ c'\ i)$

shows

$(\exists p q. ev = RecvMarker\ i\ p\ q \wedge m = Marker) \vee (\exists p q s s' m'. ev = Recv\ i\ p\ q\ s\ s'\ m' \wedge m = Msg\ m')$
 $\langle proof \rangle$

lemma *recv-marker-means-snapshotted-1*:

assumes

$ev = RecvMarker\ i\ p\ q$ **and**
 $c \vdash ev \mapsto c'$

shows

$has-snapshotted\ c'\ p$
 $\langle proof \rangle$

lemma *recv-marker-means-snapshotted-2*:

fixes

$c\ c' :: ('p, 's, 'm)$ configuration **and**
 $ev :: ('p, 's, 'm)$ event **and**
 $i :: channel-id$

assumes

$c \vdash ev \mapsto c'$ **and**
 $Marker : set\ (msgs\ c\ i)$ **and**
 $Marker \notin set\ (msgs\ c'\ i)$ **and**
 $channel\ i = Some\ (q, p)$

shows

$has-snapshotted\ c'\ p$
 $\langle proof \rangle$

lemma *event-stays-valid-if-no-occurrence*:

assumes

$c \vdash ev \mapsto c'$ **and**
 $occurs-on\ ev \neq occurs-on\ ev'$ **and**
 $can-occur\ ev'\ c$

shows

$can-occur\ ev'\ c'$
 $\langle proof \rangle$

lemma *msgs-unchanged-for-other-is*:

assumes

$c \vdash ev \mapsto c'$ **and**
 $regular-event\ ev$ **and**
 $getId\ ev = i$ **and**
 $i' \neq i$

shows

$msgs\ c\ i' = msgs\ c'\ i'$
 $\langle proof \rangle$

lemma *msgs-unchanged-if-snapshotted-RecvMarker-for-other-is*:

assumes

$c \vdash ev \mapsto c'$ **and**

$ev = \text{RecvMarker } i \ p \ q$ **and**
 $\text{has-snapshotted } c \ p$ **and**
 $i' \neq i$
shows
 $\text{msgs } c \ i' = \text{msgs } c' \ i'$
 <proof>

lemma *event-can-go-back-if-no-sender:*

assumes
 $c \vdash ev \mapsto c'$ **and**
 $\text{occurs-on } ev \neq \text{occurs-on } ev'$ **and**
 $\text{can-occur } ev' \ c'$ **and**
 $\sim \text{isRecvMarker } ev'$ **and**
 $\sim \text{isSend } ev$
shows
 $\text{can-occur } ev' \ c$
 <proof>

lemma *nonregular-event-can-go-back-if-in-distinct-processes:*

assumes
 $c \vdash ev \mapsto c'$ **and**
 $\text{regular-event } ev$ **and**
 $\sim \text{regular-event } ev'$ **and**
 $\text{can-occur } ev' \ c'$ **and**
 $\text{occurs-on } ev \neq \text{occurs-on } ev'$
shows
 $\text{can-occur } ev' \ c$
 <proof>

lemma *same-state-implies-same-result-state:*

assumes
 $\text{states } c \ p = \text{states } d \ p$
 $c \vdash ev \mapsto c'$ **and**
 $d \vdash ev \mapsto d'$
shows
 $\text{states } d' \ p = \text{states } c' \ p$
 <proof>

lemma *same-snapshot-state-implies-same-result-snapshot-state:*

assumes
 $\text{ps } c \ p = \text{ps } d \ p$ **and**
 $\text{states } c \ p = \text{states } d \ p$ **and**
 $c \vdash ev \mapsto c'$ **and**
 $d \vdash ev \mapsto d'$
shows
 $\text{ps } d' \ p = \text{ps } c' \ p$
 <proof>

lemma *same-messages-imply-same-resulting-messages:*

assumes
 $msgs\ c\ i = msgs\ d\ i$
 $c \vdash ev \mapsto c'$ **and**
 $d \vdash ev \mapsto d'$ **and**
regular-event ev
shows
 $msgs\ c'\ i = msgs\ d'\ i$
⟨*proof*⟩

lemma *Trans-msg:*

assumes
 $c \vdash ev \mapsto c'$ **and**
isTrans ev
shows
 $msgs\ c\ i = msgs\ c'\ i$
⟨*proof*⟩

lemma *new-msg-in-set-implies-occurrence:*

assumes
 $c \vdash ev \mapsto c'$ **and**
 $m \notin set\ (msgs\ c\ i)$ **and**
 $m \in set\ (msgs\ c'\ i)$ **and**
 $channel\ i = Some\ (p, q)$
shows
occurs-on $ev = p$ (**is** $?P$)
⟨*proof*⟩

lemma *new-Marker-in-set-implies-nonregular-occurrence:*

assumes
 $c \vdash ev \mapsto c'$ **and**
 $Marker \notin set\ (msgs\ c\ i)$ **and**
 $Marker \in set\ (msgs\ c'\ i)$ **and**
 $channel\ i = Some\ (p, q)$
shows
 $\sim\ regular\text{-event}\ ev$ (**is** $?P$)
⟨*proof*⟩

lemma *RecvMarker-implies-Marker-in-set:*

assumes
 $c \vdash ev \mapsto c'$ **and**
 $ev = RecvMarker\ cid\ p\ q$
shows
 $Marker \in set\ (msgs\ c\ cid)$
⟨*proof*⟩

lemma *RecvMarker-given-channel:*

assumes
isRecvMarker ev **and**
 $getId\ ev = cid$ **and**

$channel\ cid = Some\ (p, q)$ **and**
 $can\ occur\ ev\ c$
shows
 $ev = RecvMarker\ cid\ q\ p$
 $\langle proof \rangle$

lemma *Recv-given-channel:*

assumes
 $isRecv\ ev$ **and**
 $getId\ ev = cid$ **and**
 $channel\ cid = Some\ (p, q)$ **and**
 $can\ occur\ ev\ c$
shows
 $\exists s\ s'\ m. ev = Recv\ cid\ q\ p\ s\ s'\ m$
 $\langle proof \rangle$

lemma *same-cs-if-not-recv:*

assumes
 $c \vdash ev \mapsto c'$ **and**
 $\sim isRecv\ ev$
shows
 $fst\ (cs\ c\ cid) = fst\ (cs\ c'\ cid)$
 $\langle proof \rangle$

lemma *done-only-from-recv-marker:*

assumes
 $c \vdash ev \mapsto c'$ **and**
 $channel\ cid = Some\ (p, q)$ **and**
 $snd\ (cs\ c\ cid) \neq Done$ **and**
 $snd\ (cs\ c'\ cid) = Done$
shows
 $ev = RecvMarker\ cid\ q\ p$
 $\langle proof \rangle$

lemma *cs-not-not-started-stable:*

assumes
 $c \vdash ev \mapsto c'$ **and**
 $snd\ (cs\ c\ cid) \neq NotStarted$ **and**
 $channel\ cid = Some\ (p, q)$
shows
 $snd\ (cs\ c'\ cid) \neq NotStarted$
 $\langle proof \rangle$

lemma *fst-cs-changed-by-recv-recording:*

assumes
 $step: c \vdash ev \mapsto c'$ **and**
 $fst\ (cs\ c\ cid) \neq fst\ (cs\ c'\ cid)$ **and**
 $channel\ cid = Some\ (p, q)$
shows

$snd (cs\ c\ cid) = Recording \wedge (\exists p\ q\ u\ u'\ m. ev = Recv\ cid\ q\ p\ u\ u'\ m)$
 ⟨proof⟩

lemma *no-marker-and-snapshotted-implies-no-more-markers*:

assumes
 $c \vdash ev \mapsto c'$ **and**
has-snapshotted $c\ p$ **and**
 $Marker \notin set\ (msgs\ c\ cid)$ **and**
 $channel\ cid = Some\ (p, q)$
shows
 $Marker \notin set\ (msgs\ c'\ cid)$
 ⟨proof⟩

lemma *same-messages-if-no-occurrence*:

assumes
 $c \vdash ev \mapsto c'$ **and**
 $\sim\ occurs\ on\ ev = p$ **and**
 $\sim\ occurs\ on\ ev = q$ **and**
 $channel\ cid = Some\ (p, q)$
shows
 $msgs\ c\ cid = msgs\ c'\ cid \wedge cs\ c\ cid = cs\ c'\ cid$
 ⟨proof⟩

end

end

2 Traces

Traces extend transitions to finitely many intermediate events.

theory *Trace*

imports
HOL-Library.Sublist
Distributed-System

begin

context *distributed-system*

begin

We can think of a trace as the transitive closure of the next relation. A trace consists of initial and final configurations c and c' , with an ordered list of events t occurring sequentially on c , yielding c' .

inductive (in *distributed-system*) *trace* **where**

tr-init: $trace\ c\ []\ c$
 | *tr-step*: $\llbracket c \vdash ev \mapsto c'; trace\ c'\ t\ c'' \rrbracket$
 $\implies trace\ c\ (ev\ \# \ t)\ c''$

2.1 Properties of traces

lemma *trace-trans*:

shows

$$\begin{aligned} & \llbracket \text{trace } c \ t \ c'; \\ & \quad \text{trace } c' \ t' \ c'' \\ & \rrbracket \Longrightarrow \text{trace } c \ (t \ @ \ t') \ c'' \end{aligned}$$

<proof>

lemma *trace-decomp-head*:

assumes

$$\text{trace } c \ (ev \ \# \ t) \ c'$$

shows

$$\exists c''. \ c \vdash \ ev \mapsto c'' \wedge \text{trace } c'' \ t \ c'$$

<proof>

lemma *trace-decomp-tail*:

shows

$$\text{trace } c \ (t \ @ \ [ev]) \ c' \Longrightarrow \exists c''. \ \text{trace } c \ t \ c'' \wedge c'' \vdash \ ev \mapsto c'$$

<proof>

lemma *trace-snoc*:

assumes

$$\text{trace } c \ t \ c' \ \mathbf{and}$$

$$c' \vdash \ ev \mapsto c''$$

shows

$$\text{trace } c \ (t \ @ \ [ev]) \ c''$$

<proof>

lemma *trace-rev-induct* [consumes 1, case-names *tr-rev-init tr-rev-step*]:

$$\begin{aligned} & \llbracket \text{trace } c \ t \ c'; \\ & \quad (\bigwedge c. \ P \ c \ \llbracket \ c); \\ & \quad (\bigwedge c \ t \ c' \ ev \ c''. \ \text{trace } c \ t \ c' \Longrightarrow P \ c \ t \ c' \Longrightarrow c' \vdash \ ev \mapsto c'' \Longrightarrow P \ c \ (t \ @ \ [ev]) \\ & \quad c'') \\ & \rrbracket \Longrightarrow P \ c \ t \ c' \end{aligned}$$

<proof>

lemma *trace-and-start-determines-end*:

shows

$$\text{trace } c \ t \ c' \Longrightarrow \text{trace } c \ t \ d' \Longrightarrow c' = d'$$

<proof>

lemma *suffix-split-trace*:

shows

$$\begin{aligned} & \llbracket \text{trace } c \ t \ c'; \\ & \quad \text{suffix } t' \ t \\ & \rrbracket \Longrightarrow \exists c''. \ \text{trace } c'' \ t' \ c' \end{aligned}$$

<proof>

lemma *prefix-split-trace*:

fixes

$c :: ('p, 's, 'm)$ configuration **and**

$t :: ('p, 's, 'm)$ trace

shows

$\llbracket \exists c'. \text{trace } c \ t \ c';$

 prefix $t' \ t$

$\rrbracket \implies \exists c''. \text{trace } c \ t' \ c''$

$\langle \text{proof} \rangle$

lemma *split-trace*:

shows

$\llbracket \text{trace } c \ t \ c';$

$t = t' \ @ \ t''$

$\rrbracket \implies \exists c''. \text{trace } c \ t' \ c'' \wedge \text{trace } c'' \ t'' \ c'$

$\langle \text{proof} \rangle$

2.2 Describing intermediate configurations

definition *construct-fun-from-rel* :: $('a * 'b)$ set $\Rightarrow 'a \Rightarrow 'b$ **where**

construct-fun-from-rel $R \ x = (\text{THE } y. (x, y) \in R)$

definition *trace-rel* **where**

trace-rel $\equiv \{(x, t'), y\}. \text{trace } x \ t' \ y\}$

lemma *fun-must-admit-trace*:

shows

single-valued $R \implies x \in \text{Domain } R$

$\implies (x, \text{construct-fun-from-rel } R \ x) \in R$

$\langle \text{proof} \rangle$

lemma *single-valued-trace-rel*:

shows

single-valued *trace-rel*

$\langle \text{proof} \rangle$

definition *run-trace* **where**

run-trace $\equiv \text{construct-fun-from-rel } \text{trace-rel}$

In order to describe intermediate configurations of a trace we introduce the s function definition, which, given an initial configuration c , a trace t and an index $i \in \mathbb{N}$, determines the unique state after the first i events of t .

definition s **where**

$s \ c \ t \ i = (\text{THE } c'. \text{trace } c \ (\text{take } i \ t) \ c')$

lemma *s-is-partial-execution*:

shows

$s \ c \ t \ i = \text{run-trace } (c, \text{take } i \ t)$

$\langle \text{proof} \rangle$

lemma *exists-trace-for-any-i:*

assumes

$\exists c'. \text{trace } c \ t \ c'$

shows

$\text{trace } c \ (\text{take } i \ t) \ (s \ c \ t \ i)$

$\langle \text{proof} \rangle$

lemma *exists-trace-for-any-i-j:*

assumes

$\exists c'. \text{trace } c \ t \ c'$ **and**

$i \leq j$

shows

$\text{trace } (s \ c \ t \ i) \ (\text{take } (j - i) \ (\text{drop } i \ t)) \ (s \ c \ t \ j)$

$\langle \text{proof} \rangle$

lemma *step-Suc:*

assumes

$i < \text{length } t$ **and**

valid: $\text{trace } c \ t \ c'$

shows $(s \ c \ t \ i) \vdash (t \ ! \ i) \mapsto (s \ c \ t \ (\text{Suc } i))$

$\langle \text{proof} \rangle$

2.3 Trace-related lemmas

lemma *snapshot-state-unchanged-trace:*

assumes

$\text{trace } c \ t \ c'$ **and**

$ps \ c \ p = \text{Some } u$

shows

$ps \ c' \ p = \text{Some } u$

$\langle \text{proof} \rangle$

lemma *no-state-change-if-only-nonregular-events:*

shows

$\llbracket \text{trace } c \ t \ c';$

$\nexists ev. ev \in \text{set } t \wedge \text{regular-event } ev \wedge \text{occurs-on } ev = p;$

$\text{states } c \ p = st$

$\rrbracket \implies \text{states } c' \ p = st$

$\langle \text{proof} \rangle$

lemma *message-must-be-delivered-2-trace:*

assumes

$\text{trace } c \ t \ c'$ **and**

$m : \text{set } (\text{msgs } c \ i)$ **and**

$m \notin \text{set } (\text{msgs } c' \ i)$ **and**

$\text{channel } i = \text{Some } (q, p)$

shows

$\exists ev \in \text{set } t. (\exists p \ q. ev = \text{RecvMarker } i \ p \ q \wedge m = \text{Marker}) \vee (\exists p \ q \ s \ s' \ m'. ev = \text{Recv } i \ q \ p \ s \ s' \ m' \wedge m = \text{Msg } m')$

$\langle proof \rangle$

lemma *marker-must-be-delivered-2-trace*:

assumes

$trace\ c\ t\ c'$ **and**

$Marker\ :\ set\ (msgs\ c\ i)$ **and**

$Marker\ \notin\ set\ (msgs\ c'\ i)$ **and**

$channel\ i = Some\ (p,\ q)$

shows

$\exists ev \in set\ t. (\exists p\ q. ev = RecvMarker\ i\ p\ q)$

$\langle proof \rangle$

lemma *snapshot-stable*:

shows

$\llbracket trace\ c\ t\ c';$

$has-snapshot\ c\ p$

$\rrbracket \implies has-snapshot\ c'\ p$

$\langle proof \rangle$

lemma *snapshot-stable-2*:

shows

$trace\ c\ t\ c' \implies \sim has-snapshot\ c'\ p \implies \sim has-snapshot\ c\ p$

$\langle proof \rangle$

lemma *no-markers-if-all-snapshot*:

shows

$\llbracket trace\ c\ t\ c';$

$\forall p. has-snapshot\ c\ p;$

$Marker\ \notin\ set\ (msgs\ c\ i)$

$\rrbracket \implies Marker\ \notin\ set\ (msgs\ c'\ i)$

$\langle proof \rangle$

lemma *event-stays-valid-if-no-occurrence-trace*:

shows

$\llbracket trace\ c\ t\ c';$

$list-all\ (\lambda ev. occurs-on\ ev \neq occurs-on\ ev')\ t;$

$can-occur\ ev'\ c$

$\rrbracket \implies can-occur\ ev'\ c'$

$\langle proof \rangle$

lemma *event-can-go-back-if-no-sender-trace*:

shows

$\llbracket trace\ c\ t\ c';$

$list-all\ (\lambda ev. occurs-on\ ev \neq occurs-on\ ev')\ t;$

$can-occur\ ev'\ c';$

$\sim isRecvMarker\ ev';$

$list-all\ (\lambda ev. \sim isSend\ ev)\ t$

$\rrbracket \implies can-occur\ ev'\ c$

$\langle proof \rangle$

lemma *done-only-from-recv-marker-trace*:

assumes

trace c t c' **and**

t $\neq []$ **and**

snd (cs c cid) $\neq \text{Done}$ **and**

snd (cs c' cid) = *Done* **and**

channel cid = *Some (p, q)*

shows

RecvMarker cid q p $\in \text{set } t$

<proof>

lemma *cs-not-not-started-stable-trace*:

shows

$\llbracket \text{trace } c t c'; \text{snd } (cs c cid) \neq \text{NotStarted}; \text{channel } cid = \text{Some } (p, q) \rrbracket \implies$
snd (cs c' cid) $\neq \text{NotStarted}$

<proof>

lemma *no-messages-introduced-if-no-channel*:

assumes

trace: trace init t final **and**

no-msgs-if-no-channel: \forall i. channel i = None \longrightarrow msgs init i = []

shows

channel cid = None \implies msgs (s init t i) cid = []

<proof>

end

end

3 Utilities

theory *Util*

imports

Main

HOL-Library.Sublist

HOL-Library.Multiset

begin

abbreviation *swap-events* **where**

swap-events i j t $\equiv \text{take } i t @ [t ! j, t ! i] @ \text{take } (j - (i+1)) (\text{drop } (i+1) t) @$
drop (j+1) t

lemma *swap-neighbors-2*:

shows

i+1 < length t \implies swap-events i (i+1) t = (t[i := t ! (i+1)])[i+1 := t ! i]

<proof>

lemma *swap-identical-length*:

assumes

$i < j$ **and**
 $j < \text{length } t$

shows

$\text{length } t = \text{length } (\text{swap-events } i \ j \ t)$

<proof>

lemma *swap-identical-heads*:

assumes

$i < j$ **and**
 $j < \text{length } t$

shows

$\text{take } i \ t = \text{take } i \ (\text{swap-events } i \ j \ t)$

<proof>

lemma *swap-identical-tails*:

assumes

$i < j$ **and**
 $j < \text{length } t$

shows

$\text{drop } (j+1) \ t = \text{drop } (j+1) \ (\text{swap-events } i \ j \ t)$

<proof>

lemma *swap-neighbors*:

shows

$i+1 < \text{length } l \implies (l[i := l! (i+1)])[i+1 := l! i] = \text{take } i \ l \ @ \ [l! (i+1), l! i] \ @ \ \text{drop } (i+2) \ l$

<proof>

lemma *swap-events-perm*:

assumes

$i < j$ **and**
 $j < \text{length } t$

shows

$\text{mset } (\text{swap-events } i \ j \ t) = \text{mset } t$

<proof>

lemma *sum-eq-if-same-subterms*:

fixes

$i :: \text{nat}$

shows

$\forall k. i \leq k \wedge k < j \implies f \ k = f' \ k \implies \text{sum } f \ \{i..<j\} = \text{sum } f' \ \{i..<j\}$

<proof>

lemma *filter-neq-takeWhile*:

assumes

$\text{filter } ((\neq) \ a) \ l \neq \ \text{takeWhile } ((\neq) \ a) \ l$

shows

$\exists i j. i < j \wedge j < \text{length } l \wedge l ! i = a \wedge l ! j \neq a$ (is ?P)
<proof>

lemma *util-exactly-one-element*:

assumes

$m \notin \text{set } l$ **and**

$l' = l @ [m]$

shows

$\exists ! j. j < \text{length } l' \wedge l' ! j = m$ (is ?P)

<proof>

lemma *exists-one-iff-filter-one*:

shows

$(\exists ! j. j < \text{length } l \wedge l ! j = a) \longleftrightarrow \text{length } (\text{filter } ((=) a) l) = 1$

<proof>

end

4 Swap lemmas

theory *Swap*

imports

Distributed-System

begin

context *distributed-system*

begin

lemma *swap-msgs-Trans-Trans*:

assumes

$c \vdash ev \mapsto d$ **and**

$d \vdash ev' \mapsto e$ **and**

isTrans ev **and**

isTrans ev' **and**

$c \vdash ev' \mapsto d'$ **and**

$d' \vdash ev \mapsto e'$ **and**

occurs-on $ev \neq \text{occurs-on } ev'$

shows

$\text{msgs } e \ i = \text{msgs } e' \ i$

<proof>

lemma *swap-msgs-Send-Send*:

assumes

$c \vdash ev \mapsto d$ **and**

$d \vdash ev' \mapsto e$ **and**

isSend ev **and**

isSend ev' and
c ⊢ ev' ↦ d' and
d' ⊢ ev ↦ e' and
occurs-on ev ≠ occurs-on ev'
shows
msgs e i = msgs e' i
 ⟨proof⟩

lemma *swap-msgs-Recv-Recv:*

assumes
c ⊢ ev ↦ d and
d ⊢ ev' ↦ e and
isRecv ev and
isRecv ev' and
c ⊢ ev' ↦ d' and
d' ⊢ ev ↦ e' and
occurs-on ev ≠ occurs-on ev'
shows
msgs e i = msgs e' i
 ⟨proof⟩

lemma *swap-msgs-Send-Trans:*

assumes
c ⊢ ev ↦ d and
d ⊢ ev' ↦ e and
isSend ev and
isTrans ev' and
c ⊢ ev' ↦ d' and
d' ⊢ ev ↦ e' and
occurs-on ev ≠ occurs-on ev'
shows
msgs e i = msgs e' i
 ⟨proof⟩

lemma *swap-msgs-Trans-Send:*

assumes
c ⊢ ev ↦ d and
d ⊢ ev' ↦ e and
isTrans ev and
isSend ev' and
c ⊢ ev' ↦ d' and
d' ⊢ ev ↦ e' and
occurs-on ev ≠ occurs-on ev'
shows
msgs e i = msgs e' i
 ⟨proof⟩

lemma *swap-msgs-Recv-Trans:*

assumes

$c \vdash ev \mapsto d$ **and**
 $d \vdash ev' \mapsto e$ **and**
 $isRecv\ ev$ **and**
 $isTrans\ ev'$ **and**
 $c \vdash ev' \mapsto d'$ **and**
 $d' \vdash ev \mapsto e'$ **and**
 $occurs-on\ ev \neq occurs-on\ ev'$
shows
 $msgs\ e\ i = msgs\ e'\ i$
 ⟨proof⟩

lemma *swap-msgs-Trans-Recv*:

assumes
 $c \vdash ev \mapsto d$ **and**
 $d \vdash ev' \mapsto e$ **and**
 $isTrans\ ev$ **and**
 $isRecv\ ev'$ **and**
 $c \vdash ev' \mapsto d'$ **and**
 $d' \vdash ev \mapsto e'$ **and**
 $occurs-on\ ev \neq occurs-on\ ev'$
shows
 $msgs\ e\ i = msgs\ e'\ i$
 ⟨proof⟩

lemma *swap-msgs-Send-Recv*:

assumes
 $c \vdash ev \mapsto d$ **and**
 $d \vdash ev' \mapsto e$ **and**
 $isSend\ ev$ **and**
 $isRecv\ ev'$ **and**
 $c \vdash ev' \mapsto d'$ **and**
 $d' \vdash ev \mapsto e'$ **and**
 $occurs-on\ ev \neq occurs-on\ ev'$
shows
 $msgs\ e\ i = msgs\ e'\ i$
 ⟨proof⟩

lemma *swap-msgs-Recv-Send*:

assumes
 $c \vdash ev \mapsto d$ **and**
 $d \vdash ev' \mapsto e$ **and**
 $isRecv\ ev$ **and**
 $isSend\ ev'$ **and**
 $c \vdash ev' \mapsto d'$ **and**
 $d' \vdash ev \mapsto e'$ **and**
 $occurs-on\ ev \neq occurs-on\ ev'$
shows
 $msgs\ e\ i = msgs\ e'\ i$
 ⟨proof⟩

lemma *same-cs-implies-same-resulting-cs*:

assumes

$cs\ c\ i = cs\ d\ i$

$c \vdash ev \mapsto c'$ **and**

$d \vdash ev \mapsto d'$ **and**

regular-event ev

shows

$cs\ c'\ i = cs\ d'\ i$

$\langle proof \rangle$

lemma *regular-event-implies-same-channel-snapshot-Recv-Recv*:

assumes

$c \vdash ev \mapsto d$ **and**

$d \vdash ev' \mapsto e$ **and**

isRecv ev **and**

isRecv ev' **and**

$c \vdash ev' \mapsto d'$ **and**

$d' \vdash ev \mapsto e'$ **and**

occurs-on $ev \neq occurs-on\ ev'$

shows

$cs\ e\ i = cs\ e'\ i$

$\langle proof \rangle$

lemma *regular-event-implies-same-channel-snapshot-Recv*:

assumes

$c \vdash ev \mapsto d$ **and**

$d \vdash ev' \mapsto e$ **and**

$\sim isRecv\ ev$ **and**

regular-event ev **and**

isRecv ev' **and**

$c \vdash ev' \mapsto d'$ **and**

$d' \vdash ev \mapsto e'$ **and**

occurs-on $ev \neq occurs-on\ ev'$

shows

$cs\ e\ i = cs\ e'\ i$

$\langle proof \rangle$

lemma *same-messages-2*:

assumes

$\forall p. has-snapshot\ c\ p = has-snapshot\ d\ p$ **and**

$msgs\ c\ i = msgs\ d\ i$ **and**

$c \vdash ev \mapsto c'$ **and**

$d \vdash ev \mapsto d'$ **and**

$\sim regular-event\ ev$

shows

$msgs\ c'\ i = msgs\ d'\ i$

$\langle proof \rangle$

lemma *same-cs-2*:

assumes

$\forall p. \text{has-snapshotted } c \ p = \text{has-snapshotted } d \ p$ **and**

$\text{cs } c \ i = \text{cs } d \ i$ **and**

$c \vdash \text{ev} \mapsto c'$ **and**

$d \vdash \text{ev} \mapsto d'$

shows

$\text{cs } c' \ i = \text{cs } d' \ i$

<proof>

lemma *swap-Snapshot-Trans*:

assumes

$c \vdash \text{ev} \mapsto d$ **and**

$d \vdash \text{ev}' \mapsto e$ **and**

isSnapshot ev **and**

isTrans ev' **and**

$c \vdash \text{ev}' \mapsto d'$ **and**

$d' \vdash \text{ev} \mapsto e'$ **and**

$\text{occurs-on } \text{ev} \neq \text{occurs-on } \text{ev}'$

shows

$\text{msgs } e \ i = \text{msgs } e' \ i$

<proof>

lemma *swap-msgs-Trans-RecvMarker*:

assumes

$c \vdash \text{ev} \mapsto d$ **and**

$d \vdash \text{ev}' \mapsto e$ **and**

isRecvMarker ev **and**

isTrans ev' **and**

$c \vdash \text{ev}' \mapsto d'$ **and**

$d' \vdash \text{ev} \mapsto e'$ **and**

$\text{occurs-on } \text{ev} \neq \text{occurs-on } \text{ev}'$

shows

$\text{msgs } e' \ i = \text{msgs } e \ i$

<proof>

lemma *swap-Trans-Snapshot*:

assumes

$c \vdash \text{ev} \mapsto d$ **and**

$d \vdash \text{ev}' \mapsto e$ **and**

isTrans ev **and**

isSnapshot ev' **and**

$c \vdash \text{ev}' \mapsto d'$ **and**

$d' \vdash \text{ev} \mapsto e'$ **and**

$\text{occurs-on } \text{ev} \neq \text{occurs-on } \text{ev}'$

shows

$\text{msgs } e \ i = \text{msgs } e' \ i$

<proof>

lemma *swap-Send-Snapshot*:

assumes

$c \vdash ev \mapsto d$ **and**
 $d \vdash ev' \mapsto e$ **and**
isSend ev **and**
isSnapshot ev' **and**
 $c \vdash ev' \mapsto d'$ **and**
 $d' \vdash ev \mapsto e'$ **and**
 $occurs-on\ ev \neq occurs-on\ ev'$

shows

$msgs\ e\ i = msgs\ e'\ i$

<proof>

lemma *swap-Snapshot-Send*:

assumes

$c \vdash ev \mapsto d$ **and**
 $d \vdash ev' \mapsto e$ **and**
isSnapshot ev **and**
isSend ev' **and**
 $c \vdash ev' \mapsto d'$ **and**
 $d' \vdash ev \mapsto e'$ **and**
 $occurs-on\ ev \neq occurs-on\ ev'$

shows

$msgs\ e\ i = msgs\ e'\ i$

<proof>

lemma *swap-Recv-Snapshot*:

assumes

$c \vdash ev \mapsto d$ **and**
 $d \vdash ev' \mapsto e$ **and**
isRecv ev **and**
isSnapshot ev' **and**
 $c \vdash ev' \mapsto d'$ **and**
 $d' \vdash ev \mapsto e'$ **and**
 $occurs-on\ ev \neq occurs-on\ ev'$

shows

$msgs\ e\ i = msgs\ e'\ i$

<proof>

lemma *swap-Snapshot-Recv*:

assumes

$c \vdash ev \mapsto d$ **and**
 $d \vdash ev' \mapsto e$ **and**
isSnapshot ev **and**
isRecv ev' **and**
 $c \vdash ev' \mapsto d'$ **and**
 $d' \vdash ev \mapsto e'$ **and**
 $occurs-on\ ev \neq occurs-on\ ev'$

shows

$msgs\ e\ i = msgs\ e'\ i$
(proof)

lemma *swap-msgs-Recv-RecvMarker*:

assumes

$c \vdash ev \mapsto d$ **and**
 $d \vdash ev' \mapsto e$ **and**
isRecv ev **and**
isRecvMarker ev' **and**
 $c \vdash ev' \mapsto d'$ **and**
 $d' \vdash ev \mapsto e'$ **and**
 $occurs-on\ ev \neq occurs-on\ ev'$

shows

$msgs\ e\ i = msgs\ e'\ i$
(proof)

lemma *swap-RecvMarker-Recv*:

assumes

$c \vdash ev \mapsto d$ **and**
 $d \vdash ev' \mapsto e$ **and**
isRecvMarker ev **and**
isRecv ev' **and**
 $c \vdash ev' \mapsto d'$ **and**
 $d' \vdash ev \mapsto e'$ **and**
 $occurs-on\ ev \neq occurs-on\ ev'$

shows

$msgs\ e\ i = msgs\ e'\ i$
(proof)

lemma *swap-msgs-Send-RecvMarker*:

assumes

$c \vdash ev \mapsto d$ **and**
 $d \vdash ev' \mapsto e$ **and**
isSend ev **and**
isRecvMarker ev' **and**
 $c \vdash ev' \mapsto d'$ **and**
 $d' \vdash ev \mapsto e'$ **and**
 $occurs-on\ ev \neq occurs-on\ ev'$

shows

$msgs\ e\ i = msgs\ e'\ i$
(proof)

lemma *swap-RecvMarker-Send*:

assumes

$c \vdash ev \mapsto d$ **and**
 $d \vdash ev' \mapsto e$ **and**
isRecvMarker ev **and**
isSend ev' **and**
 $c \vdash ev' \mapsto d'$ **and**

$d' \vdash ev \mapsto e'$ **and**
 $occurs\text{-on } ev \neq occurs\text{-on } ev'$
shows
 $msgs\ e\ i = msgs\ e'\ i$
 ⟨*proof*⟩

lemma *swap-cs-Trans-Snapshot:*

assumes
 $c \vdash ev \mapsto d$ **and**
 $d \vdash ev' \mapsto e$ **and**
 $isTrans\ ev$ **and**
 $isSnapshot\ ev'$ **and**
 $c \vdash ev' \mapsto d'$ **and**
 $d' \vdash ev \mapsto e'$
shows
 $cs\ e\ i = cs\ e'\ i$
 ⟨*proof*⟩

lemma *swap-cs-Snapshot-Trans:*

assumes
 $c \vdash ev \mapsto d$ **and**
 $d \vdash ev' \mapsto e$ **and**
 $isSnapshot\ ev$ **and**
 $isTrans\ ev'$ **and**
 $c \vdash ev' \mapsto d'$ **and**
 $d' \vdash ev \mapsto e'$
shows
 $cs\ e\ i = cs\ e'\ i$
 ⟨*proof*⟩

lemma *swap-cs-Send-Snapshot:*

assumes
 $c \vdash ev \mapsto d$ **and**
 $d \vdash ev' \mapsto e$ **and**
 $isSend\ ev$ **and**
 $isSnapshot\ ev'$ **and**
 $c \vdash ev' \mapsto d'$ **and**
 $d' \vdash ev \mapsto e'$
shows
 $cs\ e\ i = cs\ e'\ i$
 ⟨*proof*⟩

lemma *swap-cs-Snapshot-Send:*

assumes
 $c \vdash ev \mapsto d$ **and**
 $d \vdash ev' \mapsto e$ **and**
 $isSnapshot\ ev$ **and**
 $isSend\ ev'$ **and**
 $c \vdash ev' \mapsto d'$ **and**

$d' \vdash ev \mapsto e'$
shows
 $cs\ e\ i = cs\ e'\ i$
 ⟨proof⟩

lemma *swap-cs-Recv-Snapshot*:

assumes
 $c \vdash ev \mapsto d$ **and**
 $d \vdash ev' \mapsto e$ **and**
 $isRecv\ ev$ **and**
 $isSnapshot\ ev'$ **and**
 $c \vdash ev' \mapsto d'$ **and**
 $d' \vdash ev \mapsto e'$ **and**
 $occurs-on\ ev \neq occurs-on\ ev'$
shows
 $cs\ e\ i = cs\ e'\ i$
 ⟨proof⟩

lemma *swap-cs-Snapshot-Recv*:

assumes
 $c \vdash ev \mapsto d$ **and**
 $d \vdash ev' \mapsto e$ **and**
 $isSnapshot\ ev$ **and**
 $isRecv\ ev'$ **and**
 $c \vdash ev' \mapsto d'$ **and**
 $d' \vdash ev \mapsto e'$ **and**
 $occurs-on\ ev \neq occurs-on\ ev'$
shows
 $cs\ e\ i = cs\ e'\ i$
 ⟨proof⟩

lemma *swap-cs-Trans-RecvMarker*:

assumes
 $c \vdash ev \mapsto d$ **and**
 $d \vdash ev' \mapsto e$ **and**
 $isTrans\ ev$ **and**
 $isRecvMarker\ ev'$ **and**
 $c \vdash ev' \mapsto d'$ **and**
 $d' \vdash ev \mapsto e'$
shows
 $cs\ e\ i = cs\ e'\ i$
 ⟨proof⟩

lemma *swap-cs-RecvMarker-Trans*:

assumes
 $c \vdash ev \mapsto d$ **and**
 $d \vdash ev' \mapsto e$ **and**
 $isRecvMarker\ ev$ **and**
 $isTrans\ ev'$ **and**

$c \vdash ev' \mapsto d'$ **and**
 $d' \vdash ev \mapsto e'$
shows
 $cs\ e\ i = cs\ e'\ i$
 ⟨*proof*⟩

lemma *swap-cs-Send-RecvMarker:*

assumes
 $c \vdash ev \mapsto d$ **and**
 $d \vdash ev' \mapsto e$ **and**
 $isSend\ ev$ **and**
 $isRecvMarker\ ev'$ **and**
 $c \vdash ev' \mapsto d'$ **and**
 $d' \vdash ev \mapsto e'$
shows
 $cs\ e\ i = cs\ e'\ i$
 ⟨*proof*⟩

lemma *swap-cs-RecvMarker-Send:*

assumes
 $c \vdash ev \mapsto d$ **and**
 $d \vdash ev' \mapsto e$ **and**
 $isRecvMarker\ ev$ **and**
 $isSend\ ev'$ **and**
 $c \vdash ev' \mapsto d'$ **and**
 $d' \vdash ev \mapsto e'$
shows
 $cs\ e\ i = cs\ e'\ i$
 ⟨*proof*⟩

lemma *swap-cs-Recv-RecvMarker:*

assumes
 $c \vdash ev \mapsto d$ **and**
 $d \vdash ev' \mapsto e$ **and**
 $isRecv\ ev$ **and**
 $isRecvMarker\ ev'$ **and**
 $c \vdash ev' \mapsto d'$ **and**
 $d' \vdash ev \mapsto e'$ **and**
 $occurs-on\ ev \neq occurs-on\ ev'$
shows
 $cs\ e\ i = cs\ e'\ i$
 ⟨*proof*⟩

end

end

5 The Chandy–Lamport algorithm

```
theory Snapshot
imports
  HOL–Library.Sublist
  Distributed-System
  Trace
  Util
  Swap
```

```
begin
```

5.1 The computation locale

We extend the distributed system locale presented earlier: Now we are given a trace t of the distributed system between two configurations, the initial and final configurations of t . Our objective is to show that the Chandy–Lamport algorithm terminated successfully and exhibits the same properties as claimed in [1]. In the initial state no snapshotting must have taken place yet, however the computation itself may have progressed arbitrarily far already.

We assume that there exists at least one process, that the total number of processes in the system is finite, and that there are only finitely many channels between the processes. The process graph is strongly connected. Finally there are Chandy and Lamport’s core assumptions: every process snapshots at some time and no marker may remain in a channel forever.

```
locale computation = distributed-system +
fixes
  init final :: ('a, 'b, 'c) configuration
assumes
  finite-channels:
    finite {i.  $\exists p q$ . channel i = Some (p, q)} and
  strongly-connected-raw:
     $\forall p q$ . (p  $\neq$  q)  $\longrightarrow$ 
      (tranclp ( $\lambda p q$ . ( $\exists i$ . channel i = Some (p, q)))) p q and

  at-least-two-processes:
    card (UNIV :: 'a set) > 1 and
  finite-processes:
    finite (UNIV :: 'a set) and

  no-initial-Marker:
     $\forall i$ . ( $\exists p q$ . channel i = Some (p, q))
       $\longrightarrow$  Marker  $\notin$  set (msgs init i) and
  no-msgs-if-no-channel:
     $\forall i$ . channel i = None  $\longrightarrow$  msgs init i = [] and
  no-initial-process-snapshot:
```

$\forall p. \sim \text{has-snapshotted init } p$ **and**
no-initial-channel-snapshot:
 $\forall i. \text{channel-snapshot init } i = ([], \text{NotStarted})$ **and**

valid: $\exists t. \text{trace init } t \text{ final}$ **and**
l1: $\forall t \ i \ cid. \text{trace init } t \text{ final}$
 $\wedge \text{Marker} \in \text{set } (\text{msgs } (s \text{ init } t \ i) \ cid)$
 $\longrightarrow (\exists j. j \geq i \wedge \text{Marker} \notin \text{set } (\text{msgs } (s \text{ init } t \ j) \ cid))$ **and**
l2: $\forall t \ p. \text{trace init } t \text{ final}$
 $\longrightarrow (\exists i. \text{has-snapshotted } (s \text{ init } t \ i) \ p \wedge i \leq \text{length } t)$
begin

definition *has-channel* **where**
 $\text{has-channel } p \ q \longleftrightarrow (\exists i. \text{channel } i = \text{Some } (p, q))$

lemmas *strongly-connected* = *strongly-connected-raw*[*folded has-channel-def*]

lemma *exists-some-channel:*
shows $\exists i \ p \ q. \text{channel } i = \text{Some } (p, q)$
 $\langle \text{proof} \rangle$

abbreviation *S* **where**
 $S \equiv s \text{ init}$

lemma *no-messages-if-no-channel:*
assumes *trace init t final*
shows $\text{channel } cid = \text{None} \implies \text{msgs } (s \text{ init } t \ i) \ cid = []$
 $\langle \text{proof} \rangle$

lemma *S-induct* [*consumes 3, case-names S-init S-step*]:
 $\llbracket \text{trace init } t \text{ final}; i \leq j; j \leq \text{length } t;$
 $\bigwedge i. P \ i \ i;$
 $\bigwedge i \ j. i < j \implies j \leq \text{length } t \implies (S \ t \ i) \vdash (t \ ! \ i) \mapsto (S \ t \ (\text{Suc } i)) \implies P \ (\text{Suc } i)$
 $\bigwedge j \implies P \ i \ j$
 $\rrbracket \implies P \ i \ j$
 $\langle \text{proof} \rangle$

lemma *exists-index:*
assumes
trace init t final and
 $ev \in \text{set } (\text{take } (j - i) \ (\text{drop } i \ t))$
shows
 $\exists k. i \leq k \wedge k < j \wedge ev = t \ ! \ k$
 $\langle \text{proof} \rangle$

lemma *no-change-if-ge-length-t:*
assumes
trace init t final and
 $i \geq \text{length } t$ **and**

$j \geq i$
shows
 $S\ t\ i = S\ t\ j$
 ⟨proof⟩

lemma *no-marker-if-no-snapshot*:

shows
 $\llbracket \text{trace } \textit{init}\ t\ \textit{final}; \text{channel } \textit{cid} = \textit{Some}\ (p, q);$
 $\sim \textit{has-snapshotted}\ (S\ t\ i)\ p \rrbracket$
 $\implies \textit{Marker} \notin \textit{set}\ (\textit{msgs}\ (S\ t\ i)\ \textit{cid})$
 ⟨proof⟩

5.2 Termination

We prove that the snapshot algorithm terminates, as exhibited by lemma `snapshot_algorithm_must_terminate`. In the final configuration all processes have snapshotted, and no markers remain in the channels.

lemma *must-exist-snapshot*:

assumes
 $\textit{trace}\ \textit{init}\ t\ \textit{final}$
shows
 $\exists p\ i. \textit{Snapshot}\ p = t!\ i$
 ⟨proof⟩

lemma *recv-marker-means-snapshotted*:

assumes
 $\textit{trace}\ \textit{init}\ t\ \textit{final}$ **and**
 $\textit{ev} = \textit{RecvMarker}\ \textit{cid}\ p\ q$ **and**
 $(S\ t\ i) \vdash \textit{ev} \mapsto (S\ t\ (\textit{Suc}\ i))$
shows
 $\textit{has-snapshotted}\ (S\ t\ i)\ q$
 ⟨proof⟩

lemma *recv-marker-means-cs-Done*:

assumes
 $\textit{trace}\ \textit{init}\ t\ \textit{final}$ **and**
 $t!\ i = \textit{RecvMarker}\ \textit{cid}\ p\ q$ **and**
 $i < \textit{length}\ t$
shows
 $\textit{snd}\ (\textit{cs}\ (S\ t\ (i+1))\ \textit{cid}) = \textit{Done}$
 ⟨proof⟩

lemma *snapshot-produces-marker*:

assumes
 $\textit{trace}\ \textit{init}\ t\ \textit{final}$ **and**
 $\sim \textit{has-snapshotted}\ (S\ t\ i)\ p$ **and**
 $\textit{has-snapshotted}\ (S\ t\ (\textit{Suc}\ i))\ p$ **and**
 $\textit{channel}\ \textit{cid} = \textit{Some}\ (p, q)$
shows

$Marker : set (msgs (S t (Suc i)) cid) \vee has-snapshotted (S t i) q$
 $\langle proof \rangle$

lemma *exists-snapshot-for-all-p:*

assumes

$trace\ init\ t\ final$

shows

$\exists i. \sim has-snapshotted (S t i) p \wedge has-snapshotted (S t (Suc i)) p$ (is ?Q)

$\langle proof \rangle$

lemma *all-processes-snapshotted-in-final-state:*

assumes

$trace\ init\ t\ final$

shows

$has-snapshotted\ final\ p$

$\langle proof \rangle$

definition *next-marker-free-state* **where**

$next-marker-free-state\ t\ i\ cid = (LEAST\ j. j \geq i \wedge Marker \notin set (msgs (S t j) cid))$

lemma *exists-next-marker-free-state:*

assumes

$channel\ cid = Some (p, q)$

$trace\ init\ t\ final$

shows

$\exists! j. next-marker-free-state\ t\ i\ cid = j \wedge j \geq i \wedge Marker \notin set (msgs (S t j) cid)$

$\langle proof \rangle$

theorem *snapshot-algorithm-must-terminate:*

assumes

$trace\ init\ t\ final$

shows

$\exists phi. ((\forall p. has-snapshotted (S t phi) p) \wedge (\forall cid. Marker \notin set (msgs (S t phi) cid)))$

$\langle proof \rangle$

5.3 Correctness

The greatest part of this work is spent on the correctness of the Chandy-Lamport algorithm. We prove that the snapshot is consistent, i.e. there exists a permutation t' of the trace t and an intermediate configuration c' of t' such that the configuration recorded in the snapshot corresponds to the snapshot taken during execution of t , which is given as Theorem 1 in [1].

lemma *snapshot-stable-ver-2:*

shows $trace\ init\ t\ final \implies has-snapshotted (S t i) p \implies j \geq i \implies has-snapshotted (S t j) p$

$\langle \text{proof} \rangle$

lemma *snapshot-stable-ver-3*:

shows $\text{trace init } t \text{ final} \implies \sim \text{has-snapshotted } (S \ t \ i) \ p \implies i \geq j \implies \sim$
 $\text{has-snapshotted } (S \ t \ j) \ p$
 $\langle \text{proof} \rangle$

lemma *marker-must-stay-if-no-snapshot*:

assumes

$\text{trace init } t \text{ final}$ **and**
 $\text{has-snapshotted } (S \ t \ i) \ p$ **and**
 $\sim \text{has-snapshotted } (S \ t \ i) \ q$ **and**
 $\text{channel cid} = \text{Some } (p, q)$

shows

$\text{Marker} : \text{set } (\text{msgs } (S \ t \ i) \ \text{cid})$

$\langle \text{proof} \rangle$

5.3.1 Pre- and postrecording events

definition *prerecording-event*:

$\text{prerecording-event } t \ i \equiv$
 $i < \text{length } t \wedge \text{regular-event } (t \ ! \ i)$
 $\wedge \sim \text{has-snapshotted } (S \ t \ i) \ (\text{occurs-on } (t \ ! \ i))$

definition *postrecording-event*:

$\text{postrecording-event } t \ i \equiv$
 $i < \text{length } t \wedge \text{regular-event } (t \ ! \ i)$
 $\wedge \text{has-snapshotted } (S \ t \ i) \ (\text{occurs-on } (t \ ! \ i))$

abbreviation *neighboring where*

$\text{neighboring } t \ i \ j \equiv i < j \wedge j < \text{length } t \wedge \text{regular-event } (t \ ! \ i) \wedge \text{regular-event } (t$
 $\ ! \ j)$
 $\wedge (\forall k. i < k \wedge k < j \implies \sim \text{regular-event } (t \ ! \ k))$

lemma *pre-if-regular-and-not-post*:

assumes

$\text{regular-event } (t \ ! \ i)$ **and**
 $\sim \text{postrecording-event } t \ i$ **and**
 $i < \text{length } t$

shows

$\text{prerecording-event } t \ i$

$\langle \text{proof} \rangle$

lemma *post-if-regular-and-not-pre*:

assumes

$\text{regular-event } (t \ ! \ i)$ **and**
 $\sim \text{prerecording-event } t \ i$ **and**
 $i < \text{length } t$

shows

postrecording-event t i
 ⟨proof⟩

lemma *post-before-pre-different-processes:*

assumes

i < j **and**
j < length t **and**
neighboring: $\forall k. (i < k \wedge k < j) \longrightarrow \sim \text{regular-event } (t ! k)$ **and**
post-ei: postrecording-event t i **and**
pre-ej: prerecording-event t j **and**
valid: trace init t final

shows

occurs-on (t ! i) \neq occurs-on (t ! j)
 ⟨proof⟩

lemma *post-before-pre-neighbors:*

assumes

i < j **and**
j < length t **and**
neighboring: $\forall k. (i < k \wedge k < j) \longrightarrow \sim \text{regular-event } (t ! k)$ **and**
post-ei: postrecording-event t i **and**
pre-ej: prerecording-event t j **and**
valid: trace init t final

shows

Ball (set (take (j - (i+1)) (drop (i+1) t))) (%ev. $\sim \text{regular-event } ev \wedge \sim$
occurs-on ev = occurs-on (t ! j))
 ⟨proof⟩

lemma *can-swap-neighboring-pre-and-postrecording-events:*

assumes

i < j **and**
j < length t **and**
occurs-on (t ! i) = p **and**
occurs-on (t ! j) = q **and**
neighboring: $\forall k. (i < k \wedge k < j)$
 $\longrightarrow \sim \text{regular-event } (t ! k)$ **and**
post-ei: postrecording-event t i **and**
pre-ej: prerecording-event t j **and**
valid: trace init t final

shows

can-occur (t ! j) (S t i)
 ⟨proof⟩

5.3.2 Event swapping

lemma *swap-events:*

shows $\llbracket i < j; j < \text{length } t;$
 $\forall k. (i < k \wedge k < j) \longrightarrow \sim \text{regular-event } (t ! k);$
postrecording-event t i; prerecording-event t j;

$$\begin{aligned}
& \text{trace init } t \text{ final } \mathbb{I} \\
\implies & \text{trace init } (\text{swap-events } i \ j \ t) \text{ final} \\
& \wedge (\forall k. k \geq j + 1 \longrightarrow S (\text{swap-events } i \ j \ t) \ k = S \ t \ k) \\
& \wedge (\forall k. k \leq i \longrightarrow S (\text{swap-events } i \ j \ t) \ k = S \ t \ k) \\
& \wedge \text{prerecording-event } (\text{swap-events } i \ j \ t) \ i \\
& \wedge \text{postrecording-event } (\text{swap-events } i \ j \ t) \ (i+1) \\
& \wedge (\forall k. k > i+1 \wedge k < j+1 \\
& \quad \longrightarrow \sim \text{regular-event } ((\text{swap-events } i \ j \ t) ! \ k)) \\
\langle \text{proof} \rangle
\end{aligned}$$

5.3.3 Relating configurations and the computed snapshot

definition *ps-equal-to-snapshot* where

$$\begin{aligned}
\text{ps-equal-to-snapshot } c \ c' & \equiv \\
& \forall p. \text{Some } (\text{states } c \ p) = \text{process-snapshot } c' \ p
\end{aligned}$$

definition *cs-equal-to-snapshot* where

$$\begin{aligned}
\text{cs-equal-to-snapshot } c \ c' & \equiv \\
& \forall \text{cid}. \text{channel } \text{cid} \neq \text{None} \\
& \longrightarrow \text{filter } ((\neq) \text{Marker}) (\text{msgs } c \ \text{cid}) \\
& = \text{map } \text{Msg } (\text{fst } (\text{channel-snapshot } c' \ \text{cid}))
\end{aligned}$$

definition *state-equal-to-snapshot* where

$$\begin{aligned}
\text{state-equal-to-snapshot } c \ c' & \equiv \\
& \text{ps-equal-to-snapshot } c \ c' \wedge \text{cs-equal-to-snapshot } c \ c'
\end{aligned}$$

lemma *init-is-s-t-0*:

assumes

$$\text{trace init } t \ \text{final}$$

shows

$$\text{init} = (S \ t \ 0)$$

$\langle \text{proof} \rangle$

lemma *final-is-s-t-len-t*:

assumes

$$\text{trace init } t \ \text{final}$$

shows

$$\text{final} = S \ t \ (\text{length } t)$$

$\langle \text{proof} \rangle$

lemma *snapshot-event*:

assumes

$$\text{trace init } t \ \text{final} \ \mathbf{and}$$

$$\sim \text{has-snapshotted } (S \ t \ i) \ p \ \mathbf{and}$$

$$\text{has-snapshotted } (S \ t \ (i+1)) \ p$$

shows

$$\text{isSnapshot } (t ! i) \vee \text{isRecvMarker } (t ! i)$$

$\langle \text{proof} \rangle$

lemma *snapshot-state*:

assumes

trace init t final and
states (S t i) p = u and
~ has-snapshot (S t i) p and
has-snapshot (S t (i+1)) p

shows

ps (S t (i+1)) p = Some u

<proof>

lemma *snapshot-state-unchanged-trace-2*:

shows

\llbracket *trace init t final; i ≤ j; j ≤ length t;*
ps (S t i) p = Some u
 $\rrbracket \implies$ *ps (S t j) p = Some u*

<proof>

lemma *no-recording-cs-if-not-snapshot*:

shows

\llbracket *trace init t final; ~ has-snapshot (S t i) p;*
channel cid = Some (q, p) \rrbracket \implies *cs (S t i) cid = cs init cid*

<proof>

lemma *cs-done-implies-has-snapshot*:

assumes

trace init t final and
snd (cs (S t i) cid) = Done and
channel cid = Some (p, q)

shows

has-snapshot (S t i) q

<proof>

lemma *exactly-one-snapshot*:

assumes

trace init t final

shows

$\exists! i. \sim$ *has-snapshot (S t i) p* \wedge *has-snapshot (S t (i+1)) p* (**is ?P**)

<proof>

lemma *initial-cs-changes-implies-nonregular-event*:

assumes

trace init t final and
snd (cs (S t i) cid) = NotStarted and
snd (cs (S t (i+1)) cid) ≠ NotStarted and
channel cid = Some (p, q)

shows

\sim *regular-event (t ! i)*

<proof>

lemma *cs-in-initial-state-implies-not-snapshotted*:

assumes

trace init t final and

snd (cs (S t i) cid) = NotStarted and

channel cid = Some (p, q)

shows

~ has-snapshotted (S t i) q

<proof>

lemma *nonregular-event-in-initial-state-implies-cs-changed*:

assumes

trace init t final and

snd (cs (S t i) cid) = NotStarted and

~ regular-event (t ! i) and

occurs-on (t ! i) = q and

channel cid = Some (p, q) and

i < length t

shows

snd (cs (S t (i+1)) cid) ≠ NotStarted

<proof>

lemma *cs-recording-implies-snapshot*:

assumes

trace init t final and

snd (cs (S t i) cid) = Recording and

channel cid = Some (p, q)

shows

has-snapshotted (S t i) q

<proof>

lemma *cs-done-implies-both-snapshotted*:

assumes

trace init t final and

snd (cs (S t i) cid) = Done and

i < length t and

channel cid = Some (p, q)

shows

has-snapshotted (S t i) p

has-snapshotted (S t i) q

<proof>

lemma *cs-done-implies-same-snapshots*:

assumes *trace init t final i ≤ j j ≤ length t*

shows *snd (cs (S t i) cid) = Done ⇒ channel cid = Some (p, q) ⇒ cs (S t i) cid = cs (S t j) cid*

<proof>

lemma *snapshotted-and-not-done-implies-marker-in-channel*:

assumes

trace init t final and
has-snapshotted (S t i) p and
snd (cs (S t i) cid) ≠ Done and
i ≤ length t and
channel cid = Some (p, q)
shows
Marker : set (msgs (S t i) cid)
 ⟨proof⟩

lemma *no-marker-left-in-final-state:*

assumes
trace init t final
shows
Marker ∉ set (msgs final cid) (is ?P)
 ⟨proof⟩

lemma *all-channels-done-in-final-state:*

assumes
trace init t final and
channel cid = Some (p, q)
shows
snd (cs final cid) = Done
 ⟨proof⟩

lemma *cs-NotStarted-implies-empty-cs:*

shows
 $\llbracket \text{trace init } t \text{ final; channel } cid = \text{Some } (p, q); i < \text{length } t; \sim \text{has-snapshotted } (S \ t \ i) \ q \rrbracket$
 $\implies cs \ (S \ t \ i) \ cid = (\llbracket, \text{NotStarted}\rrbracket)$
 ⟨proof⟩

lemma *fst-changed-by-recv-recording-trace:*

assumes
i < j and
j ≤ length t and
trace init t final and
fst (cs (S t i) cid) ≠ fst (cs (S t j) cid) and
channel cid = Some (p, q)
shows
 $\exists k. i \leq k \wedge k < j \wedge (\exists p \ q \ u \ u' \ m. t \ ! \ k = \text{Recv } cid \ q \ p \ u \ u' \ m) \wedge (\text{snd } (cs \ (S \ t \ k) \ cid) = \text{Recording}) \ (\text{is } ?P)$
 ⟨proof⟩

lemma *cs-not-nil-implies-postrecording-event:*

assumes
trace init t final and
fst (cs (S t i) cid) ≠ \llbracket and
i ≤ length t and
channel cid = Some (p, q)

shows
 $\exists j. j < i \wedge \text{postrecording-event } t \ j$
 $\langle \text{proof} \rangle$

5.3.4 Relating process states

lemma *snapshot-state-must-have-been-reached*:

assumes
 $\text{trace init } t \ \text{final}$ **and**
 $\text{ps final } p = \text{Some } u$ **and**
 $\sim \text{has-snapshotted } (S \ t \ i) \ p$ **and**
 $\text{has-snapshotted } (S \ t \ (i+1)) \ p$ **and**
 $i < \text{length } t$
shows
 $\text{states } (S \ t \ i) \ p = u$
 $\langle \text{proof} \rangle$

lemma *ps-after-all-prerecording-events*:

assumes
 $\text{trace init } t \ \text{final}$ **and**
 $\forall i'. i' \geq i \longrightarrow \sim \text{prerecording-event } t \ i'$ **and**
 $\forall j'. j' < i \longrightarrow \sim \text{postrecording-event } t \ j'$
shows
 $\text{ps-equal-to-snapshot } (S \ t \ i) \ \text{final}$
 $\langle \text{proof} \rangle$

5.3.5 Relating channel states

lemma *cs-when-recording*:

shows
 $\llbracket i < j; j \leq \text{length } t; \text{trace init } t \ \text{final};$
 $\text{has-snapshotted } (S \ t \ i) \ p;$
 $\text{snd } (cs \ (S \ t \ i) \ cid) = \text{Recording};$
 $\text{snd } (cs \ (S \ t \ j) \ cid) = \text{Done};$
 $\text{channel } cid = \text{Some } (p, q) \rrbracket$
 $\implies \text{map } \text{Msg } (\text{fst } (cs \ (S \ t \ j) \ cid))$
 $= \text{map } \text{Msg } (\text{fst } (cs \ (S \ t \ i) \ cid)) @ \text{takeWhile } ((\neq) \ \text{Marker}) \ (\text{msgs } (S \ t \ i)$
 $\text{cid})$
 $\langle \text{proof} \rangle$

lemma *cs-when-recording-2*:

shows
 $\llbracket i \leq j; \text{trace init } t \ \text{final};$
 $\sim \text{has-snapshotted } (S \ t \ i) \ p;$
 $\forall k. i \leq k \wedge k < j \longrightarrow \sim \text{occurs-on } (t \ ! \ k) = p;$
 $\text{snd } (cs \ (S \ t \ i) \ cid) = \text{Recording};$
 $\text{channel } cid = \text{Some } (p, q) \rrbracket$
 $\implies \text{map } \text{Msg } (\text{fst } (cs \ (S \ t \ j) \ cid)) @ \text{takeWhile } ((\neq) \ \text{Marker}) \ (\text{msgs } (S \ t \ j)$
 $\text{cid})$

$$= \text{map } \text{Msg } (\text{fst } (\text{cs } (S \ t \ i) \ \text{cid})) \ @ \ \text{takeWhile } ((\neq) \ \text{Marker}) \ (\text{msgs } (S \ t \ i) \ \text{cid})$$

$$\wedge \ \text{snd } (\text{cs } (S \ t \ j) \ \text{cid}) = \text{Recording}$$
 $\langle \text{proof} \rangle$

lemma *cs-when-recording-3*:

shows

$$\llbracket i \leq j; \text{trace init } t \ \text{final};$$

$$\sim \text{has-snapshotted } (S \ t \ i) \ q;$$

$$\forall k. i \leq k \wedge k < j \longrightarrow \sim \text{occurs-on } (t \ ! \ k) = q;$$

$$\text{snd } (\text{cs } (S \ t \ i) \ \text{cid}) = \text{NotStarted};$$

$$\text{has-snapshotted } (S \ t \ i) \ p;$$

$$\text{Marker} : \text{set } (\text{msgs } (S \ t \ i) \ \text{cid});$$

$$\text{channel } \text{cid} = \text{Some } (p, q) \rrbracket$$

$$\implies \text{map } \text{Msg } (\text{fst } (\text{cs } (S \ t \ j) \ \text{cid})) \ @ \ \text{takeWhile } ((\neq) \ \text{Marker}) \ (\text{msgs } (S \ t \ j) \ \text{cid})$$

$$= \text{map } \text{Msg } (\text{fst } (\text{cs } (S \ t \ i) \ \text{cid})) \ @ \ \text{takeWhile } ((\neq) \ \text{Marker}) \ (\text{msgs } (S \ t \ i) \ \text{cid})$$

$$\wedge \ \text{snd } (\text{cs } (S \ t \ j) \ \text{cid}) = \text{NotStarted}$$
 $\langle \text{proof} \rangle$

lemma *at-most-one-marker*:

shows

$$\llbracket \text{trace init } t \ \text{final}; \text{channel } \text{cid} = \text{Some } (p, q) \rrbracket$$

$$\implies \text{Marker} \notin \text{set } (\text{msgs } (S \ t \ i) \ \text{cid})$$

$$\vee (\exists ! j. j < \text{length } (\text{msgs } (S \ t \ i) \ \text{cid}) \wedge \text{msgs } (S \ t \ i) \ \text{cid} \ ! \ j = \text{Marker})$$
 $\langle \text{proof} \rangle$

lemma *last-changes-implies-send-when-msgs-nonempty*:

assumes

$$\text{trace init } t \ \text{final} \ \mathbf{and}$$

$$\text{msgs } (S \ t \ i) \ \text{cid} \neq [] \ \mathbf{and}$$

$$\text{msgs } (S \ t \ (i+1)) \ \text{cid} \neq [] \ \mathbf{and}$$

$$\text{last } (\text{msgs } (S \ t \ i) \ \text{cid}) = \text{Marker} \ \mathbf{and}$$

$$\text{last } (\text{msgs } (S \ t \ (i+1)) \ \text{cid}) \neq \text{Marker} \ \mathbf{and}$$

$$\text{channel } \text{cid} = \text{Some } (p, q)$$

shows

$$(\exists u \ u' \ m. t \ ! \ i = \text{Send } \text{cid } p \ q \ u \ u' \ m)$$
 $\langle \text{proof} \rangle$

lemma *no-marker-after-RecvMarker*:

assumes

$$\text{trace init } t \ \text{final} \ \mathbf{and}$$

$$(S \ t \ i) \vdash \text{RecvMarker } \text{cid } p \ q \mapsto (S \ t \ (i+1)) \ \mathbf{and}$$

$$\text{channel } \text{cid} = \text{Some } (q, p)$$

shows

$$\text{Marker} \notin \text{set } (\text{msgs } (S \ t \ (i+1)) \ \text{cid})$$
 $\langle \text{proof} \rangle$

lemma *no-marker-and-snapshotted-implies-no-more-markers-trace:*

shows

\llbracket *trace* *init* *t* *final*; $i \leq j$; $j \leq \text{length } t$;
has-snapshotted (*S t i*) *p*;
Marker $\notin \text{set } (\text{msgs } (S t i) \text{ cid})$;
channel *cid* = *Some* (*p*, *q*) \rrbracket
 $\implies \text{Marker} \notin \text{set } (\text{msgs } (S t j) \text{ cid})$

<proof>

lemma *marker-not-vanishing-means-always-present:*

shows

\llbracket *trace* *init* *t* *final*; $i \leq j$; $j \leq \text{length } t$;
Marker : *set* (*msgs* (*S t i*) *cid*);
Marker : *set* (*msgs* (*S t j*) *cid*);
channel *cid* = *Some* (*p*, *q*)
 $\rrbracket \implies \forall k. i \leq k \wedge k \leq j \longrightarrow \text{Marker} : \text{set } (\text{msgs } (S t k) \text{ cid})$

<proof>

lemma *last-stays-if-no-recv-marker-and-no-send:*

shows

\llbracket *trace* *init* *t* *final*; $i < j$; $j \leq \text{length } t$;
last (*msgs* (*S t i*) *cid*) = *Marker*;
Marker : *set* (*msgs* (*S t i*) *cid*);
Marker : *set* (*msgs* (*S t j*) *cid*);
 $\forall k. i \leq k \wedge k < j \longrightarrow \sim (\exists u u' m. t ! k = \text{Send } \text{cid } p \ q \ u \ u' \ m)$;
channel *cid* = *Some* (*p*, *q*) \rrbracket
 $\implies \text{last } (\text{msgs } (S t j) \text{ cid}) = \text{Marker}$

<proof>

lemma *last-changes-implies-send-when-msgs-nonempty-trace:*

assumes

trace *init* *t* *final*
 $i < j$
 $j \leq \text{length } t$
Marker : *set* (*msgs* (*S t i*) *cid*)
Marker : *set* (*msgs* (*S t j*) *cid*)
last (*msgs* (*S t i*) *cid*) = *Marker*
last (*msgs* (*S t j*) *cid*) \neq *Marker*
channel *cid* = *Some* (*p*, *q*)

shows

$\exists k u u' m. i \leq k \wedge k < j \wedge t ! k = \text{Send } \text{cid } p \ q \ u \ u' \ m$

<proof>

lemma *msg-after-marker-and-nonempty-implies-postrecording-event:*

assumes

trace *init* *t* *final* **and**
Marker : *set* (*msgs* (*S t i*) *cid*) **and**
Marker $\neq \text{last } (\text{msgs } (S t i) \text{ cid})$ **and**
 $i \leq \text{length } t$ **and**
channel *cid* = *Some* (*p*, *q*)

shows
 $\exists j. j < i \wedge \text{postrecording-event } t \ j \ (\text{is } ?P)$
 $\langle \text{proof} \rangle$

lemma *same-messages-if-no-occurrence-trace*:

shows
 $\llbracket \text{trace init } t \ \text{final}; i \leq j; j \leq \text{length } t;$
 $(\forall k. i \leq k \wedge k < j \longrightarrow \text{occurs-on } (t \ ! \ k) \neq p \wedge \text{occurs-on } (t \ ! \ k) \neq q);$
 $\text{channel } cid = \text{Some } (p, q) \rrbracket$
 $\implies \text{msgs } (S \ t \ i) \ cid = \text{msgs } (S \ t \ j) \ cid \wedge \text{cs } (S \ t \ i) \ cid = \text{cs } (S \ t \ j) \ cid$
 $\langle \text{proof} \rangle$

lemma *snapshot-step-cs-preservation-p*:

assumes
 $c \vdash ev \mapsto c'$ **and**
 $\sim \text{regular-event } ev$ **and**
 $\text{occurs-on } ev = p$ **and**
 $\text{channel } cid = \text{Some } (p, q)$
shows
 $\text{map } \text{Msg } (\text{fst } (\text{cs } c \ cid)) \ @ \ \text{takeWhile } ((\neq) \ \text{Marker}) \ (\text{msgs } c \ cid)$
 $= \text{map } \text{Msg } (\text{fst } (\text{cs } c' \ cid)) \ @ \ \text{takeWhile } ((\neq) \ \text{Marker}) \ (\text{msgs } c' \ cid)$
 $\wedge \text{snd } (\text{cs } c \ cid) = \text{snd } (\text{cs } c' \ cid)$
 $\langle \text{proof} \rangle$

lemma *snapshot-step-cs-preservation-q*:

assumes
 $c \vdash ev \mapsto c'$ **and**
 $\sim \text{regular-event } ev$ **and**
 $\text{occurs-on } ev = q$ **and**
 $\text{channel } cid = \text{Some } (p, q)$ **and**
 $\text{Marker} \notin \text{set } (\text{msgs } c \ cid)$ **and**
 $\sim \text{has-snapshotted } c \ q$
shows
 $\text{map } \text{Msg } (\text{fst } (\text{cs } c \ cid)) \ @ \ \text{takeWhile } ((\neq) \ \text{Marker}) \ (\text{msgs } c \ cid)$
 $= \text{map } \text{Msg } (\text{fst } (\text{cs } c' \ cid)) \ @ \ \text{takeWhile } ((\neq) \ \text{Marker}) \ (\text{msgs } c' \ cid)$
 $\wedge \text{snd } (\text{cs } c' \ cid) = \text{Recording}$
 $\langle \text{proof} \rangle$

lemma *Marker-in-channel-implies-not-done*:

assumes
 $\text{trace init } t \ \text{final}$ **and**
 $\text{Marker} : \text{set } (\text{msgs } (S \ t \ i) \ cid)$ **and**
 $\text{channel } cid = \text{Some } (p, q)$ **and**
 $i \leq \text{length } t$
shows
 $\text{snd } (\text{cs } (S \ t \ i) \ cid) \neq \text{Done}$
 $\langle \text{proof} \rangle$

lemma *keep-empty-if-no-events*:

shows

\llbracket trace init t final; $i \leq j$; $j \leq \text{length } t$;
msgs ($S \ t \ i$) cid = \llbracket ;
has-snapshotted ($S \ t \ i$) p ;
channel cid = Some (p, q);
 $\forall k. i \leq k \wedge k < j \wedge \text{regular-event } (t \ ! \ k) \longrightarrow \sim \text{occurs-on } (t \ ! \ k) = p \rrbracket$
 $\implies \text{msgs } (S \ t \ j) \text{ cid} = \llbracket$

$\langle \text{proof} \rangle$

lemma last-unchanged-or-empty-if-no-events:

shows

\llbracket trace init t final; $i \leq j$; $j \leq \text{length } t$;
msgs ($S \ t \ i$) cid $\neq \llbracket$;
last (msgs ($S \ t \ i$) cid) = Marker;
has-snapshotted ($S \ t \ i$) p ;
channel cid = Some (p, q);
 $\forall k. i \leq k \wedge k < j \wedge \text{regular-event } (t \ ! \ k) \longrightarrow \sim \text{occurs-on } (t \ ! \ k) = p \rrbracket$
 $\implies \text{msgs } (S \ t \ j) \text{ cid} = \llbracket \vee (\text{msgs } (S \ t \ j) \text{ cid} \neq \llbracket \wedge \text{last } (\text{msgs } (S \ t \ j) \text{ cid}) =$

Marker)

$\langle \text{proof} \rangle$

lemma cs-after-all-prerecording-events:

assumes

trace init t final and
 $\forall i'. i' \geq i \longrightarrow \sim \text{prerecording-event } t \ i'$ and
 $\forall j'. j' < i \longrightarrow \sim \text{postrecording-event } t \ j'$ and
 $i \leq \text{length } t$

shows

cs-equal-to-snapshot ($S \ t \ i$) final

$\langle \text{proof} \rangle$

lemma snapshot-after-all-prerecording-events:

assumes

trace init t final and
 $\forall i'. i' \geq i \longrightarrow \sim \text{prerecording-event } t \ i'$ and
 $\forall j'. j' < i \longrightarrow \sim \text{postrecording-event } t \ j'$ and
 $i \leq \text{length } t$

shows

state-equal-to-snapshot ($S \ t \ i$) final

$\langle \text{proof} \rangle$

5.4 Obtaining the desired traces

abbreviation all-prerecording-before-postrecording where

all-prerecording-before-postrecording $t \equiv \exists i. (\forall j. j < i \longrightarrow \sim \text{postrecording-event } t \ j)$

$\wedge (\forall j. j \geq i \longrightarrow \sim \text{prerecording-event } t \ j)$
 $\wedge i \leq \text{length } t$
 $\wedge \text{trace init } t \text{ final}$

definition *count-violations* :: ('a, 'b, 'c) trace \Rightarrow nat **where**
count-violations t = sum (λi . if postrecording-event t i
then card {j \in {i+1..*length* t}. prerecording-event t j}
else 0)
{0..*length* t}

lemma *violations-ge-0*:

shows

(if postrecording-event t i
then card {j \in {i+1..*length* t}. prerecording-event t j}
else 0) \geq 0

<proof>

lemma *count-violations-ge-0*:

shows

count-violations t \geq 0

<proof>

lemma *violations-0-implies-all-subterms-0*:

assumes

count-violations t = 0

shows

$\forall i \in \{0..*length* t\}$. (if postrecording-event t i
then card {j \in {i+1..*length* t}. prerecording-event t j}
else 0) = 0

<proof>

lemma *exists-postrecording-violation-if-count-greater-0*:

assumes

count-violations t $>$ 0

shows

$\exists i$. postrecording-event t i \wedge card {j \in {i+1..*length* t}. prerecording-event t j} $>$ 0 (**is** ?P)

<proof>

lemma *exists-prerecording-violation-when-card-greater-0*:

assumes

card {j \in {i+1..*length* t}. prerecording-event t j} $>$ 0

shows

$\exists j \in \{i+1..*length* t\}$. prerecording-event t j

<proof>

lemma *card-greater-0-if-post-after-pre*:

assumes

i < j **and**

postrecording-event t i **and**

prerecording-event t j

shows

$\text{card } \{j \in \{i+1..<\text{length } t\}. \text{ prerecording-event } t j\} > 0$
 ⟨proof⟩

lemma exists-neighboring-violation-pair:

assumes

trace init t final **and**
count-violations t > 0

shows

$\exists i j. i < j \wedge \text{postrecording-event } t i \wedge \text{prerecording-event } t j$
 $\wedge (\forall k. (i < k \wedge k < j) \longrightarrow \sim \text{regular-event } (t ! k)) \wedge j < \text{length } t$

⟨proof⟩

lemma same-cardinality-post-swap-1:

assumes

prerecording-event t j **and**
postrecording-event t i **and**
i < j **and**
j < length t **and**
count-violations t = Suc n **and**
 $\forall k. (i < k \wedge k < j) \longrightarrow \sim \text{regular-event } (t ! k)$ **and**
trace init t final

shows

$\{k \in \{0..<i\}. \text{prerecording-event } t k\}$
 $= \{k \in \{0..<i\}. \text{prerecording-event } (\text{swap-events } i j t) k\}$

⟨proof⟩

lemma same-cardinality-post-swap-2:

assumes

prerecording-event t j **and**
postrecording-event t i **and**
i < j **and**
j < length t **and**
count-violations t = Suc n **and**
 $\forall k. (i < k \wedge k < j) \longrightarrow \sim \text{regular-event } (t ! k)$ **and**
trace init t final

shows

$\text{card } \{k \in \{i..<j+1\}. \text{prerecording-event } t k\}$
 $= \text{card } \{k \in \{i..<j+1\}. \text{prerecording-event } (\text{swap-events } i j t) k\}$

⟨proof⟩

lemma same-cardinality-post-swap-3:

assumes

prerecording-event t j **and**
postrecording-event t i **and**
i < j **and**
j < length t **and**
count-violations t = Suc n **and**
 $\forall k. (i < k \wedge k < j) \longrightarrow \sim \text{regular-event } (t ! k)$ **and**
trace init t final

shows

$\{k \in \{j+1..<length\ t\}. \text{prerecording-event } t\ k\}$
 $= \{k \in \{j+1..<length\ (\text{swap-events } i\ j\ t)\}. \text{prerecording-event } (\text{swap-events } i\ j\ t)\ k\}$
(proof)

lemma *card-ip1-to-j-is-1-in-normal-events:*

assumes

prerecording-event $t\ j$ **and**
postrecording-event $t\ i$ **and**
 $i < j$ **and**
 $j < length\ t$ **and**
count-violations $t = Suc\ n$ **and**
 $\forall k. (i < k \wedge k < j) \longrightarrow \sim \text{regular-event } (t\ !\ k)$ **and**
count-violations $t = Suc\ n$ **and**
trace *init* t *final*

shows

$card\ \{k \in \{i+1..<j+1\}. \text{prerecording-event } t\ k\} = 1$
(proof)

lemma *card-ip1-to-j-is-0-in-swapped-events:*

assumes

prerecording-event $t\ j$ **and**
postrecording-event $t\ i$ **and**
 $i < j$ **and**
 $j < length\ t$ **and**
count-violations $t = Suc\ n$ **and**
 $\forall k. (i < k \wedge k < j) \longrightarrow \sim \text{regular-event } (t\ !\ k)$ **and**
count-violations $t = Suc\ n$ **and**
trace *init* t *final*

shows

$card\ \{k \in \{i+1..<j+1\}. \text{prerecording-event } (\text{swap-events } i\ j\ t)\ k\} = 0$
(proof)

lemma *count-violations-swap:*

assumes

prerecording-event $t\ j$ **and**
postrecording-event $t\ i$ **and**
 $i < j$ **and**
 $j < length\ t$ **and**
count-violations $t = Suc\ n$ **and**
 $\forall k. (i < k \wedge k < j) \longrightarrow \sim \text{regular-event } (t\ !\ k)$ **and**
count-violations $t = Suc\ n$ **and**
trace *init* t *final*

shows

count-violations $(\text{swap-events } i\ j\ t) = n$
(proof)

lemma *desired-trace-always-exists:*

assumes
trace init t final
shows
 $\exists t'. \text{mset } t' = \text{mset } t$
 $\wedge \text{all-prerecording-before-postrecording } t'$
⟨*proof*⟩

theorem *snapshot-algorithm-is-correct:*

assumes
trace init t final
shows
 $\exists t' i. \text{trace init } t' \text{ final} \wedge \text{mset } t' = \text{mset } t$
 $\wedge \text{state-equal-to-snapshot } (S \ t' \ i) \text{ final} \wedge i \leq \text{length } t'$
⟨*proof*⟩

5.5 Stable property detection

Finally, we show that the computed snapshot is indeed suitable for stable property detection, as claimed in [1].

definition *stable where*

stable p $\equiv (\forall c. p \ c \longrightarrow (\forall t \ c'. \text{trace } c \ t \ c' \longrightarrow p \ c'))$

lemma *has-snapshot-stable:*

assumes
trace init t final
shows
stable $(\lambda c. \text{has-snapshotted } c \ p)$
⟨*proof*⟩

definition *some-snapshot-state where*

some-snapshot-state t \equiv
 $SOME \ (t', i). \text{trace init } t \text{ final}$
 $\wedge \text{trace init } t' \text{ final} \wedge \text{mset } t' = \text{mset } t$
 $\wedge \text{state-equal-to-snapshot } (S \ t' \ i) \text{ final}$

lemma *split-S:*

assumes
trace init t final
shows
trace $(S \ t \ i) \ (\text{drop } i \ t) \ \text{final}$
⟨*proof*⟩

theorem *Stable-Property-Detection:*

assumes
stable p and
trace init t final and
 $(t', i) = \text{some-snapshot-state } t \ \text{and}$
 $p \ (S \ t' \ i)$
shows

```

    p final
  <proof>

end

end
theory Co-Snapshot
  imports
    Snapshot
    Ordered-Resolution-Prover.Lazy-List-Chain
begin

```

6 Extension to infinite traces

The computation locale assumes that there already exists a known final configuration c' to the given initial c and trace t . However, we can show that the snapshot algorithm must terminate correctly even if the underlying computation itself does not terminate. We relax the trace relation to allow for a potentially infinite number of “intermediate” events, and show that the algorithm’s correctness still holds when imposing the same constraints as in the computation locale.

We use a preexisting theory of lazy list chains by Schlichtkrull, Blanchette, Traytel and Waldmann [2] to construct infinite traces.

primrec *ltake* **where**

```

  ltake 0 t = []
| ltake (Suc i) t = (case t of LNil  $\Rightarrow$  [] | LCons x t'  $\Rightarrow$  x # ltake i t')
```

primrec *ldrop* **where**

```

  ldrop 0 t = t
| ldrop (Suc i) t = (case t of LNil  $\Rightarrow$  LNil | LCons x t'  $\Rightarrow$  ldrop i t')
```

lemma *ltake-LNil[simp]*: $ltake\ i\ LNil = []$

<proof>

lemma *ltake-LCons*: $0 < i \Longrightarrow ltake\ i\ (LCons\ x\ t) = x\ \#\ ltake\ (i - 1)\ t$

<proof>

lemma *take-ltake*: $i \leq j \Longrightarrow take\ i\ (ltake\ j\ xs) = ltake\ i\ xs$

<proof>

lemma *nth-ltake [simp]*: $i < \min\ n\ (\text{length}\ xs) \Longrightarrow (ltake\ n\ xs)\ !\ i = \text{nth}\ xs\ i$

<proof>

lemma *length-ltake[simp]*: $\text{length}\ (ltake\ i\ xs) = (\text{case}\ \text{length}\ xs\ \text{of}\ \infty \Rightarrow i\ | \text{enat}\ m \Rightarrow \min\ i\ m)$

<proof>

lemma *ltake-prepend*:

$ltake\ i\ (prepend\ xs\ t) = (if\ i \leq length\ xs\ then\ take\ i\ xs\ else\ xs\ @\ ltake\ (i - length\ xs)\ t)$
<proof>

lemma *prepend-ltake-ldrop-id*: $prepend\ (ltake\ i\ t)\ (ldrop\ i\ t) = t$
<proof>

context *distributed-system*
begin

coinductive *cotrace* **where**

cotr-init: $cotrace\ c\ LNil$
cotr-step: $\llbracket c \vdash ev \mapsto c'; cotrace\ c' t \rrbracket \implies cotrace\ c\ (LCons\ ev\ t)$

lemma *cotrace-trace*: $cotrace\ c\ t \implies \exists!c'. trace\ c\ (ltake\ i\ t)\ c'$
<proof>

lemma *cotrace-trace'*: $cotrace\ c\ t \implies \exists c'. trace\ c\ (ltake\ i\ t)\ c'$
<proof>

definition *cos* **where** $cos\ c\ t\ i = s\ c\ (ltake\ i\ t)\ i$

lemma *cotrace-trace-cos*: $cotrace\ c\ t \implies trace\ c\ (ltake\ i\ t)\ (cos\ c\ t\ i)$
<proof>

lemma *s-0[simp]*: $s\ c\ t\ 0 = c$
<proof>

lemma *s-chop*: $i \leq length\ t \implies s\ c\ t\ i = s\ c\ (take\ i\ t)\ i$
<proof>

lemma *cotrace-prepend*: $trace\ c\ t\ c' \implies cotrace\ c'\ u \implies cotrace\ c\ (prepend\ t\ u)$
<proof>

lemma *s-Cons*: $\exists c''. trace\ c'\ xs\ c'' \implies c \vdash ev \mapsto c' \implies s\ c\ (ev\ \# xs)\ (Suc\ i) = s\ c'\ xs\ i$
<proof>

lemma *cotrace-ldrop*: $cotrace\ c\ t \implies i \leq llength\ t \implies cotrace\ (cos\ c\ t\ i)\ (ldrop\ i\ t)$
<proof>

end

locale *cocomputation* = *distributed-system* +
fixes
init :: ('a, 'b, 'c) configuration
assumes

finite-channels:

finite $\{i. \exists p q. \text{channel } i = \text{Some } (p, q)\}$ **and**

strongly-connected-raw:

$\forall p q. (p \neq q) \longrightarrow$

$(\text{trancpl } (\lambda p q. (\exists i. \text{channel } i = \text{Some } (p, q)))) p q$ **and**

at-least-two-processes:

card $(UNIV :: 'a \text{ set}) > 1$ **and**

finite-processes:

finite $(UNIV :: 'a \text{ set})$ **and**

no-initial-Marker:

$\forall i. (\exists p q. \text{channel } i = \text{Some } (p, q))$

$\longrightarrow \text{Marker} \notin \text{set } (\text{msgs } \text{init } i)$ **and**

no-msgs-if-no-channel:

$\forall i. \text{channel } i = \text{None} \longrightarrow \text{msgs } \text{init } i = []$ **and**

no-initial-process-snapshot:

$\forall p. \neg \text{has-snapshotted } \text{init } p$ **and**

no-initial-channel-snapshot:

$\forall i. \text{channel-snapshot } \text{init } i = ([], \text{NotStarted})$ **and**

valid: $\exists t. \text{cotrace } \text{init } t$ **and**

l1: $\forall t i \text{ cid}. \text{cotrace } \text{init } t$

$\wedge \text{Marker} \in \text{set } (\text{msgs } (\text{cos } \text{init } t i) \text{ cid})$

$\longrightarrow (\exists j \leq \text{llength } t. j \geq i \wedge \text{Marker} \notin \text{set } (\text{msgs } (\text{cos } \text{init } t j) \text{ cid}))$ **and**

l2: $\forall t p. \text{cotrace } \text{init } t$

$\longrightarrow (\exists i \leq \text{llength } t. \text{has-snapshotted } (\text{cos } \text{init } t i) p)$

begin

abbreviation *coS* **where** $\text{coS} \equiv \text{cos } \text{init}$

definition *some-snapshot* $t p = (\text{SOME } i. \text{has-snapshotted } (\text{coS } t i) p \wedge i \leq \text{llength } t)$

lemma *has-snapshotted:*

$\text{cotrace } \text{init } t \Longrightarrow \text{has-snapshotted } (\text{coS } t (\text{some-snapshot } t p)) p \wedge \text{some-snapshot } t p \leq \text{llength } t$

<proof>

lemma *cotrace-cos:* $\text{cotrace } \text{init } t \Longrightarrow j < \text{llength } t \Longrightarrow$

$(\text{coS } t j) \vdash \text{lth } t j \mapsto (\text{coS } t (\text{Suc } j))$

<proof>

lemma *snapshot-stable:*

$\text{cotrace } \text{init } t \Longrightarrow i \leq j \Longrightarrow \text{has-snapshotted } (\text{coS } t i) p \Longrightarrow \text{has-snapshotted } (\text{coS } t j) p$

<proof>

lemma *no-markers-if-all-snapshotted:*

$\text{cotrace init } t \implies i \leq j \implies \forall p. \text{has-snapshotted } (\text{coS } t \ i) \ p \implies$
 $\text{Marker} \notin \text{set } (\text{msgs } (\text{coS } t \ i) \ c) \implies \text{Marker} \notin \text{set } (\text{msgs } (\text{coS } t \ j) \ c)$
 <proof>

lemma *cotrace-all-have-snapshotted*:

assumes *cotrace init t*

shows $\exists i \leq \text{llength } t. \forall p. \text{has-snapshotted } (\text{coS } t \ i) \ p$

<proof>

lemma *no-messages-if-no-channel*:

assumes *cotrace init t*

shows $\text{channel } \text{cid} = \text{None} \implies \text{msgs } (\text{coS } t \ i) \ \text{cid} = []$

<proof>

lemma *cotrace-all-have-snapshotted-and-no-markers*:

assumes *cotrace init t*

shows $\exists i \leq \text{llength } t. (\forall p. \text{has-snapshotted } (\text{coS } t \ i) \ p) \wedge$
 $(\forall c. \text{Marker} \notin \text{set } (\text{msgs } (\text{coS } t \ i) \ c))$

<proof>

context

fixes *t*

assumes *cotrace: cotrace init t*

begin

definition *final-i* \equiv

$(\text{SOME } i. i \leq \text{llength } t \wedge (\forall p. \text{has-snapshotted } (\text{coS } t \ i) \ p) \wedge (\forall c. \text{Marker} \notin \text{set } (\text{msgs } (\text{coS } t \ i) \ c)))$

definition *final where*

$\text{final} = \text{coS } t \ \text{final-i}$

lemma *final-i*: $\text{final-i} \leq \text{llength } t \ (\forall p. \text{has-snapshotted } (\text{coS } t \ \text{final-i}) \ p) \ (\forall c. \text{Marker} \notin \text{set } (\text{msgs } (\text{coS } t \ \text{final-i}) \ c))$

<proof>

lemma *final*: $\exists t. \text{trace init } t \ \text{final} \ (\forall p. \text{has-snapshotted } \text{final} \ p) \ (\forall c. \text{Marker} \notin \text{set } (\text{msgs } \text{final} \ c))$

<proof>

interpretation *computation channel trans send recv init final*

<proof>

definition *coperm where*

$\text{coperm } l \ r = (\exists xs \ ys \ z. \text{mset } xs = \text{mset } ys \wedge l = \text{prepend } xs \ z \wedge r = \text{prepend } ys \ z)$

lemma *copermIL*: $\text{mset } ys = \text{mset } xs \implies t = \text{prepend } xs \ z \implies \text{coperm } (\text{prepend } ys \ z) \ t$

<proof>

lemma *snapshot-algorithm-is-cocorrect*:

$\exists t' i. \text{cotrace init } t' \wedge \text{coperm } t' t \wedge \text{state-equal-to-snapshot } (\text{coS } t' i) \text{ final} \wedge i \leq \text{final-}i$
<proof>

end

print-statement *snapshot-algorithm-is-cocorrect*

end

end

7 Example

We provide an example in order to prove that our locale is non-vacuous. This example corresponds to the computation and associated snapshot described in Section 4 of [1].

theory *Example*

imports

Snapshot

begin

datatype *PType* = *P* | *Q*

datatype *MType* = *M* | *M'*

datatype *SType* = *S-Wait* | *S-Send* | *T-Wait* | *T-Send*

fun *trans* :: *PType* \Rightarrow *SType* \Rightarrow *SType* \Rightarrow *bool* **where**
trans *p s s'* = *False*

fun *send* :: *channel-id* \Rightarrow *PType* \Rightarrow *PType* \Rightarrow *SType*
 \Rightarrow *SType* \Rightarrow *MType* \Rightarrow *bool* **where**
send *c p q s s' m* = ((*c* = 0 \wedge *p* = *P* \wedge *q* = *Q*
 \wedge *s* = *S-Send* \wedge *s'* = *S-Wait* \wedge *m* = *M*)
 \vee (*c* = 1 \wedge *p* = *Q* \wedge *q* = *P*
 \wedge *s* = *T-Send* \wedge *s'* = *T-Wait* \wedge *m* = *M'*))

fun *recv* :: *channel-id* \Rightarrow *PType* \Rightarrow *PType* \Rightarrow *SType*
 \Rightarrow *SType* \Rightarrow *MType* \Rightarrow *bool* **where**
recv *c p q s s' m* = ((*c* = 1 \wedge *p* = *P* \wedge *q* = *Q*
 \wedge *s* = *S-Wait* \wedge *s'* = *S-Send* \wedge *m* = *M'*)
 \vee (*c* = 0 \wedge *p* = *Q* \wedge *q* = *P*
 \wedge *s* = *T-Wait* \wedge *s'* = *T-Send* \wedge *m* = *M*))

fun *chan* :: *nat* \Rightarrow (*PType* * *PType*) *option* **where**

*chan n = (if n = 0 then Some (P, Q)
else if n = 1 then Some (Q, P)
else None)*

abbreviation *init* :: (PType, SType, MType) configuration **where**

init \equiv (
states = (%p. if p = P then S-Send else T-Send),
msgs = (%d. []),
process-snapshot = (%p. None),
channel-snapshot = (%d. ([], NotStarted))
)

abbreviation *t0* **where** *t0* \equiv Snapshot P

abbreviation *s1* :: (PType, SType, MType) configuration **where**

s1 \equiv (
states = (%p. if p = P then S-Send else T-Send),
msgs = (%d. if d = 0 then [Marker] else []),
process-snapshot = (%p. if p = P then Some S-Send else None),
channel-snapshot = (%d. if d = 1 then ([], Recording) else ([], NotStarted))
)

abbreviation *t1* **where** *t1* \equiv Send 0 P Q S-Send S-Wait M

abbreviation *s2* :: (PType, SType, MType) configuration **where**

s2 \equiv (
states = (%p. if p = P then S-Wait else T-Send),
msgs = (%d. if d = 0 then [Marker, Msg M] else []),
process-snapshot = (%p. if p = P then Some S-Send else None),
channel-snapshot = (%d. if d = 1 then ([], Recording) else ([], NotStarted))
)

abbreviation *t2* **where** *t2* \equiv Send 1 Q P T-Send T-Wait M'

abbreviation *s3* :: (PType, SType, MType) configuration **where**

s3 \equiv (
states = (%p. if p = P then S-Wait else T-Wait),
msgs = (%d. if d = 0 then [Marker, Msg M] else if d = 1 then [Msg M'] else
[]),
process-snapshot = (%p. if p = P then Some S-Send else None),
channel-snapshot = (%d. if d = 1 then ([], Recording) else ([], NotStarted))
)

abbreviation *t3* **where** *t3* \equiv Snapshot Q

abbreviation *s4* :: (PType, SType, MType) configuration **where**

s4 \equiv (
states = (%p. if p = P then S-Wait else T-Wait),
msgs = (%d. if d = 0 then [Marker, Msg M] else if d = 1 then [Msg M'],
)

Marker] else []),
 process-snapshot = (%p. if p = P then Some S-Send else Some T-Wait),
 channel-snapshot = (%d. if d = 1 then ([], Recording) else if d = 0 then ([],
 Recording) else ([], NotStarted))
)

abbreviation $t4$ where $t4 \equiv \text{RecvMarker } 0 \ Q \ P$

abbreviation $s5$:: (PType, SType, MType) configuration where

$s5 \equiv$ (
 states = (%p. if p = P then S-Wait else T-Wait),
 msgs = (%d. if d = 0 then [Msg M] else if d = 1 then [Msg M', Marker] else
 []),
 process-snapshot = (%p. if p = P then Some S-Send else Some T-Wait),
 channel-snapshot = (%d. if d = 0 then ([], Done) else if d = 1 then ([],
 Recording) else ([], NotStarted))
)

abbreviation $t5$ where $t5 \equiv \text{Recv } 1 \ P \ Q \ S\text{-Wait } S\text{-Send } M'$

abbreviation $s6$:: (PType, SType, MType) configuration where

$s6 \equiv$ (
 states = (%p. if p = P then S-Send else T-Wait),
 msgs = (%d. if d = 0 then [Msg M] else if d = 1 then [Marker] else []),
 process-snapshot = (%p. if p = P then Some S-Send else Some T-Wait),
 channel-snapshot = (%d. if d = 0 then ([], Done) else if d = 1 then ([M'],
 Recording) else ([], NotStarted))
)

abbreviation $t6$ where $t6 \equiv \text{RecvMarker } 1 \ P \ Q$

abbreviation $s7$:: (PType, SType, MType) configuration where

$s7 \equiv$ (
 states = (%p. if p = P then S-Send else T-Wait),
 msgs = (%d. if d = 0 then [Msg M] else if d = 1 then [] else []),
 process-snapshot = (%p. if p = P then Some S-Send else Some T-Wait),
 channel-snapshot = (%d. if d = 0 then ([], Done) else if d = 1 then ([M'],
 Done) else ([], NotStarted))
)

lemma $s7\text{-no-marker}$:

shows

$\forall cid. \text{Marker} \notin \text{set } (\text{msgs } s7 \ cid)$

$\langle \text{proof} \rangle$

interpretation computation chan trans send recv init $s7$

$\langle \text{proof} \rangle$

end

References

- [1] K. M. Chandy and L. Lamport. Distributed snapshots: Determining global states of distributed systems. *ACM Trans. Comput. Syst.*, 3(1):63–75, 1985.
- [2] A. Schlichtkrull, J. C. Blanchette, D. Traytel, and U. Waldmann. Formalization of bachmair and ganzinger’s ordered resolution prover. *Archive of Formal Proofs*, Jan. 2018. http://isa-afp.org/entries/Ordered_Resolution_Prover.html, Formal proof development.