A Verified Code Generator from Isabelle/HOL to CakeML

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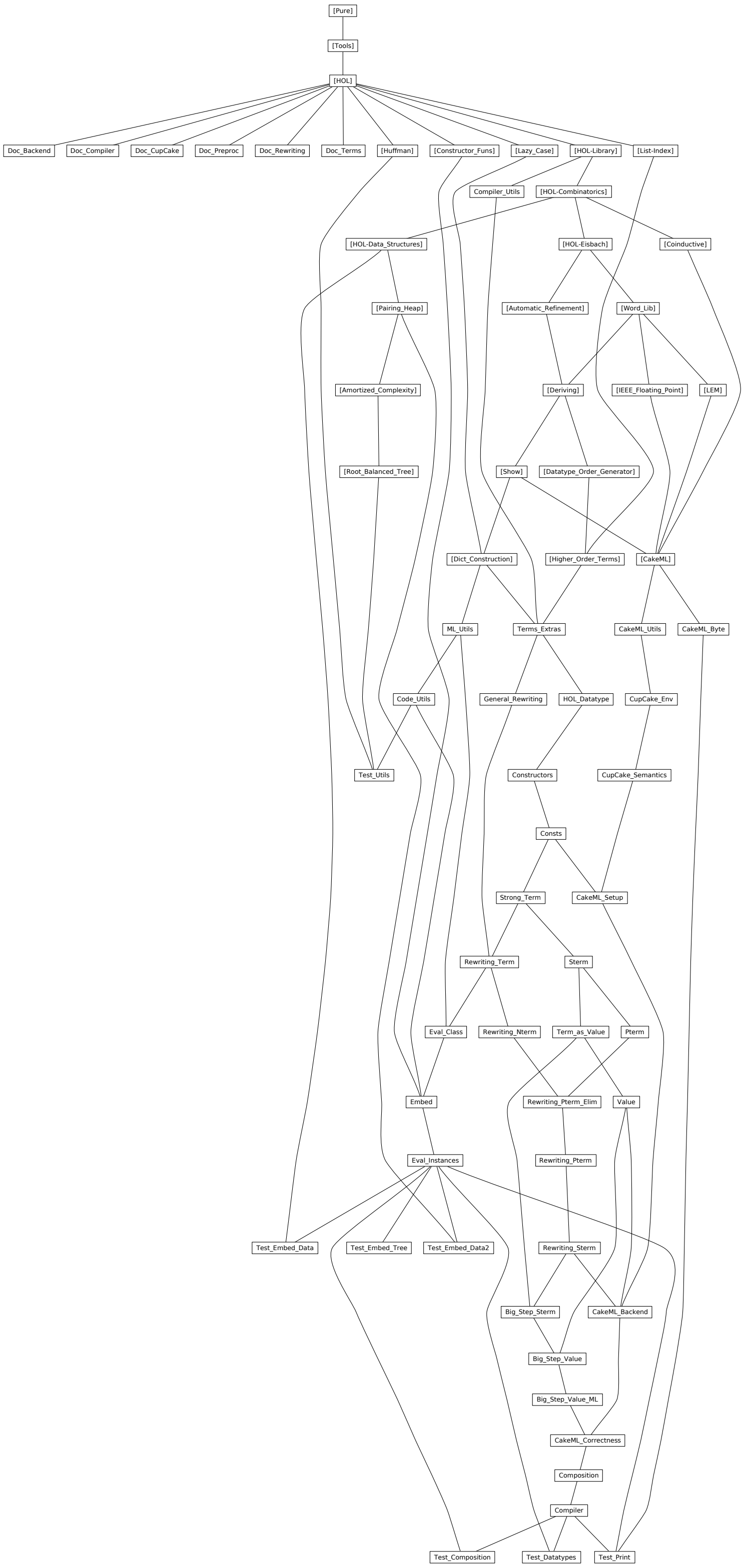
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Chapter 1

Terms

theory *Doc-Terms*
imports *Main*
begin

end

1.1 Additional material over the *Higher-Order-Terms* AFP entry

theory *Terms-Extras*
imports
  ../Utils/Compiler-Utils
  Higher-Order-Terms.Pats
  Dict-Construction.Dict-Construction
begin

no-notation *Mpat-Antiquot.mpaq-App* (**infixl** ‹$$› *900*)

ML-file *hol-term.ML*

primrec *basic-rule* :: - ⇒ *bool* **where**
basic-rule (*lhs*, *rhs*) ⟷
  linear lhs ∧
  is-const (*fst* (*strip-comb lhs*)) ∧
  ¬ *is-const lhs* ∧
  frees rhs |⊆| *frees lhs*

lemma *basic-ruleI*[*intro*]:
  assumes *linear lhs*
  assumes *is-const* (*fst* (*strip-comb lhs*))
  assumes ¬ *is-const lhs*
  assumes *frees rhs* |⊆| *frees lhs*
  shows *basic-rule* (*lhs*, *rhs*)

using *assms* **by** *simp*

primrec *split-rule* :: $(term \times {'a}) \Rightarrow (name \times (term\ list \times {'a}))$ **where**
split-rule (*lhs*, *rhs*) = (*let* (*name*, *args*) = *strip-comb lhs in* (*const-name name*, (*args*, *rhs*)))

fun *unsplit-rule* :: $(name \times (term\ list \times {'a})) \Rightarrow (term \times {'a})$ **where**
unsplit-rule (*name*, (*args*, *rhs*)) = (*name* $$ *args*, *rhs*)

lemma *split-unsplit*: *split-rule* (*unsplit-rule t*) = *t*
by (*induct t rule*: *unsplit-rule.induct*) (*simp add*: *strip-list-comb const-name-def*)

lemma *unsplit-split*:
  assumes *basic-rule r*
  shows *unsplit-rule* (*split-rule r*) = *r*
using *assms*
by (*cases r*) (*simp add*: *split-beta*)

datatype *pat* = *Patvar name* | *Patconstr name pat list*

fun *mk-pat* :: *term* ⇒ *pat* **where**
mk-pat pat = (*case strip-comb pat of* (*Const s*, *args*) ⇒ *Patconstr s* (*map mk-pat args*) | (*Free s*, []) ⇒ *Patvar s*)

declare *mk-pat.simps*[*simp del*]

lemma *mk-pat-simps*[*simp*]:
  mk-pat (*name* $$ *args*) = *Patconstr name* (*map mk-pat args*)
  mk-pat (*Free name*) = *Patvar name*
apply (*auto simp*: *mk-pat.simps strip-list-comb-const*)
apply (*simp add*: *const-term-def*)
done

primrec *patvars* :: *pat* ⇒ *name fset* **where**
patvars (*Patvar name*) = {| *name* |} |
patvars (*Patconstr - ps*) = *ffUnion* (*fset-of-list* (*map patvars ps*))

lemma *mk-pat-frees*:
  assumes *linear p*
  shows *patvars* (*mk-pat p*) = *frees p*
using *assms* **proof** (*induction p rule*: *linear-pat-induct*)
  case (*comb name args*)

  have *map* (*patvars* ∘ *mk-pat*) *args* = *map frees args*
    using *comb* **by** *force*
  hence *fset-of-list* (*map* (*patvars* ∘ *mk-pat*) *args*) = *fset-of-list* (*map frees args*)
    by *metis*
  thus *?case*
    by (*simp add*: *freess-def*)

qed *simp*

This definition might seem a little counter-intuitive. Assume we have two defining equations of a function, e.g. *map*: *map f* [] = [] *map f* (*x* # *xs*) = *f x* # *map f xs* The pattern "matrix" is compiled right-to-left. Equal patterns are grouped together. This definition is needed to avoid the following situation: *map f* [] = [] *map g* (*x* # *xs*) = *g x* # *map g xs* While this is logically the same as above, the problem is that *f* and *g* are overlapping but distinct patterns. Hence, instead of grouping them together, they stay separate. This leads to overlapping patterns in the target language which will produce wrong results. One way to deal with this is to rename problematic variables before invoking the compiler.

fun *pattern-compatible* :: *term* ⇒ *term* ⇒ *bool* **where**
pattern-compatible (t_1 \$ t_2) (u_1 \$ u_2) ⟷ *pattern-compatible* t_1 u_1 ∧ (t_1 = u_1 ⟶ *pattern-compatible* t_2 u_2) |
pattern-compatible t u ⟷ *t* = *u* ∨ *non-overlapping t u*

lemmas *pattern-compatible-simps*[*simp*] =
pattern-compatible.simps[*folded app-term-def*]

lemmas *pattern-compatible-induct* = *pattern-compatible.induct*[*case-names app-app*]

lemma *pattern-compatible-refl*[*intro?*]: *pattern-compatible t t*
by (*induct t*) *auto*

corollary *pattern-compatile-reflP*[*intro*!]: *reflp pattern-compatible*
by (*auto intro*: *pattern-compatible-refl reflpI*)

lemma *pattern-compatible-cases*[*consumes 1*]:
assumes *pattern-compatible t u*
obtains (*eq*) *t* = *u*
| (*non-overlapping*) *non-overlapping t u*
using *assms* **proof** (*induction arbitrary*: *thesis rule*: *pattern-compatible-induct*)
case (*app-app* t_1 t_2 u_1 u_2)

show *?case*
proof (*cases* t_1 = u_1 ∧ t_2 = u_2)
case *True*
with *app-app* **show** *thesis*
by *simp*
next
case *False*
from *app-app* **have** *pattern-compatible* t_1 u_1 t_1 = u_1 ⟹ *pattern-compatible* t_2 u_2
by *auto*
with *False* **have** *non-overlapping* (t_1 \$ t_2) (u_1 \$ u_2)
using *app-app* **by** (*metis non-overlapping-appI1 non-overlapping-appI2*)
thus *thesis*

by (*rule app-app.prems*(*2*))
qed
qed *auto*

inductive *rev-accum-rel* :: ($'a \Rightarrow 'a \Rightarrow$ *bool*) $\Rightarrow$ $'a$ *list* $\Rightarrow$ $'a$ *list* $\Rightarrow$ *bool* **for** *R*
where
nil: *rev-accum-rel R* [] [] |
snoc: *rev-accum-rel R xs ys* $\Longrightarrow$ (*xs* = *ys* $\Longrightarrow$ *R x y*) $\Longrightarrow$ *rev-accum-rel R* (*xs* @ [*x*]) (*ys* @ [*y*])

lemma *rev-accum-rel-refl*[*intro*]: *reflp R* $\Longrightarrow$ *rev-accum-rel R xs xs*
unfolding *reflp-def*
by (*induction xs rule*: *rev-induct*) (*auto intro*: *rev-accum-rel.intros*)

lemma *rev-accum-rel-length*:
assumes *rev-accum-rel R xs ys*
shows *length xs* = *length ys*
using *assms*
by *induct auto*

context begin

private inductive-cases *rev-accum-relE*[*consumes 1* , *case-names nil snoc*]: *rev-accum-rel P xs ys*

lemma *rev-accum-rel-butlast*[*intro*]:
assumes *rev-accum-rel P xs ys*
shows *rev-accum-rel P* (*butlast xs*) (*butlast ys*)
using *assms* **by** (*cases rule*: *rev-accum-relE*) (*auto intro*: *rev-accum-rel.intros*)

lemma *rev-accum-rel-snoc-eqE*: *rev-accum-rel P* (*xs* @ [*a*]) (*xs* @ [*b*]) $\Longrightarrow$ *P a b*
by (*auto elim*: *rev-accum-relE*)

end

abbreviation *patterns-compatible* :: *term list* $\Rightarrow$ *term list* $\Rightarrow$ *bool* **where**
patterns-compatible $\equiv$ *rev-accum-rel pattern-compatible*

abbreviation *patterns-compatibles* :: (*term list* $\times$ $'a$) *fset* $\Rightarrow$ *bool* **where**
patterns-compatibles $\equiv$ *fpairwise* (λ($pats_1$, -) ($pats_2$, -). *patterns-compatible* $pats_1$ $pats_2$)

lemma *pattern-compatible-combD*:
assumes *length xs* = *length ys pattern-compatible* (*list-comb f xs*) (*list-comb f ys*)
shows *patterns-compatible xs ys*
using *assms* **by** (*induction xs ys rule*: *rev-induct2*) (*auto intro*: *rev-accum-rel.intros*)

lemma *pattern-compatible-combI*[*intro*]:
assumes *patterns-compatible xs ys pattern-compatible f g*

shows *pattern-compatible* (*list-comb f xs*) (*list-comb g ys*)
using *assms*
proof (*induction rule*: *rev-accum-rel.induct*)
case (*snoc xs ys x y*)

then have *pattern-compatible* (*list-comb f xs*) (*list-comb g ys*)
by *auto*

moreover have *pattern-compatible x y* **if** *list-comb f xs* = *list-comb g ys*
proof (*rule snoc*, *rule list-comb-semi-inj*)
show *length xs* = *length ys*
using *snoc* **by** (*auto dest*: *rev-accum-rel-length*)
qed *fact*

ultimately show *?case*
by *auto*
qed (*auto intro*: *rev-accum-rel.intros*)

experiment begin

— The above example can be made concrete here. In general, the following identity does not hold:

lemma *pattern-compatible t u* ⟷ *t* = *u* ∨ *non-overlapping t u*
apply *rule*
apply (*erule pattern-compatible-cases*; *simp*)
apply (*erule disjE*)
apply (*metis pattern-compatible-refl*)
oops

— The counterexample:

definition *pats1* = [*Free* (*Name* ''*f*''), *Const* (*Name* ''*nil*'')]
definition *pats2* = [*Free* (*Name* ''*g*''), *Const* (*Name* ''*cons*'') $ *Free* (*Name* ''*x*'') $ *Free* (*Name* ''*xs*'')]

proposition *non-overlapping* (*list-comb c pats1*) (*list-comb c pats2*)
unfolding *pats1-def pats2-def*
apply (*simp add*: *app-term-def*)
apply (*rule non-overlapping-appI2*)
apply (*rule non-overlapping-const-appI*)
done

proposition ¬ *patterns-compatible pats1 pats2*
unfolding *pats1-def pats2-def*
apply *rule*
apply (*erule rev-accum-rel.cases*)
apply *simp*
apply *auto*

apply (*erule rev-accum-rel.cases*)
apply *auto*
apply (*erule rev-accum-rel.cases*)
apply *auto*
apply (*metis overlapping-var1I*)
done

end

abbreviation *pattern-compatibles* :: $(term \times {'a})\ fset \Rightarrow bool$ **where**
pattern-compatibles $\equiv$ *fpairwise* $(\lambda(lhs_1, \text{-})\ (lhs_2, \text{-}).\ pattern\text{-}compatible\ lhs_1\ lhs_2)$

corollary *match-compatible-pat-eq*:
assumes *pattern-compatible* t_1 t_2 *linear* t_1 *linear* t_2
assumes *match* t_1 u = *Some* env_1 *match* t_2 u = *Some* env_2
shows $t_1 = t_2$
using *assms* **by** (*metis pattern-compatible-cases match-overlapping*)

corollary *match-compatible-env-eq*:
assumes *pattern-compatible* t_1 t_2 *linear* t_1 *linear* t_2
assumes *match* t_1 u = *Some* env_1 *match* t_2 u = *Some* env_2
shows $env_1 = env_2$
using *assms* **by** (*metis match-compatible-pat-eq option.inject*)

corollary *matchs-compatible-eq*:
assumes *patterns-compatible* ts_1 ts_2 *linears* ts_1 *linears* ts_2
assumes *matchs* ts_1 *us* = *Some* env_1 *matchs* ts_2 *us* = *Some* env_2
shows $ts_1 = ts_2$ $env_1 = env_2$
proof −
fix *name*
have *match* (*name* $$ ts_1) (*name* $$ *us*) = *Some* env_1 *match* (*name* $$ ts_2) (*name* $$ *us*) = *Some* env_2
using *assms* **by** *auto*
moreover have *length* ts_1 = *length* ts_2
using *assms* **by** (*metis matchs-some-eq-length*)
ultimately have *pattern-compatible* (*name* $$ ts_1) (*name* $$ ts_2)
using *assms* **by** (*auto simp*: *const-term-def*)
moreover have *linear* (*name* $$ ts_1) *linear* (*name* $$ ts_2)
using *assms* **by** (*auto intro*: *linear-list-comb'*)
note ∗ = *calculation*

from ∗ **have** *name* $$ ts_1 = *name* $$ ts_2
by (*rule match-compatible-pat-eq*) *fact+*
thus $ts_1 = ts_2$
by (*meson list-comb-inj-second injD*)

from ∗ **show** $env_1 = env_2$
by (*rule match-compatible-env-eq*) *fact+*
qed

lemma *compatible-find-match*:
  assumes *pattern-compatibles* (*fset-of-list cs*) *list-all* (*linear* ∘ *fst*) *cs* *is-fmap* (*fset-of-list cs*)
  assumes *match pat t* = *Some env* (*pat*, *rhs*) ∈ *set cs*
  shows *find-match cs t* = *Some* (*env*, *pat*, *rhs*)
using *assms* **proof** (*induction cs arbitrary*: *pat rhs*)
 case (*Cons c cs*)
 then obtain *pat′ rhs′* **where** [*simp*]: *c* = (*pat′*, *rhs′*)
  by *force*
 have *find-match* ((*pat′*, *rhs′*) # *cs*) *t* = *Some* (*env*, *pat*, *rhs*)
  proof (*cases match pat′ t*)
   case *None*
   have *pattern-compatibles* (*fset-of-list cs*)
    using *Cons*
    by (*force simp*: *fpairwise-alt-def*)
   have *list-all* (*linear* ∘ *fst*) *cs*
    using *Cons*
    by (*auto simp*: *list-all-iff*)
   have *is-fmap* (*fset-of-list cs*)
    using *Cons*
    by (*meson fset-of-list-subset is-fmap-subset set-subset-Cons*)
   show *?thesis*
    apply (*simp add*: *None*)
    apply (*rule Cons*)
      apply *fact+*
    using *Cons None* **by** *force*
  next
   case (*Some env′*)
   have *linear pat linear pat′*
    using *Cons* **apply** (*metis Ball-set comp-apply fst-conv*)
    using *Cons* **by** *simp*
   moreover from *Cons* **have** *pattern-compatible pat pat′*
    apply (*cases pat* = *pat′*)
     apply (*simp add*: *pattern-compatible-refl*)
    unfolding *fpairwise-alt-def*
    by (*force simp*: *fset-of-list-elem*)
   ultimately have *env′* = *env pat′* = *pat*
    using *match-compatible-env-eq match-compatible-pat-eq*
    using *Cons Some*
    by *blast+*
   with *Cons* **have** *rhs′* = *rhs*
    using *is-fmapD*
    by (*metis* ‹*c* = (*pat′*, *rhs′*)› *fset-of-list-elem list.set-intros*(*1*))
   show *?thesis*
    apply (*simp add*: *Some*)
    apply (*intro conjI*)
    by *fact+*
  qed

thus *?case*
unfolding ‹c = -› **.**
qed *auto*

context *term* **begin**

definition *arity-compatible* :: $'a \Rightarrow 'a \Rightarrow$ *bool* **where**
arity-compatible t_1 t_2 = (
let
($head_1$, $pats_1$) = *strip-comb* t_1;
($head_2$, $pats_2$) = *strip-comb* t_2
in $head_1 = head_2 \longrightarrow$ *length* $pats_1$ = *length* $pats_2$
)

abbreviation *arity-compatibles* :: $('a \times 'b)$ *fset* $\Rightarrow$ *bool* **where**
arity-compatibles $\equiv$ *fpairwise* (λ(lhs_1, -) (lhs_2, -). *arity-compatible* lhs_1 lhs_2)

definition *head* :: $'a \Rightarrow$ *name* **where**
head $t \equiv$ *const-name* (*fst* (*strip-comb* t))

abbreviation *heads-of* :: (*term* $\times$ $'a$) *fset* $\Rightarrow$ *name fset* **where**
heads-of rs $\equiv$ (*head* $\circ$ *fst*) |‘| *rs*

end

definition *arity* :: ($'a$ *list* $\times$ $'b$) *fset* $\Rightarrow$ *nat* **where**
arity rs = *fthe-elem'* ((*length* $\circ$ *fst*) |‘| *rs*)

lemma *arityI*:
assumes *fBall rs* (λ(*pats*, -). *length pats* = n) *rs* $\neq$ {||}
shows *arity rs* = n
proof −
have (*length* $\circ$ *fst*) |‘| *rs* = {| n |}
using *assms* **by** *force*
thus *?thesis*
unfolding *arity-def fthe-elem'-eq* **by** *simp*
qed

end

1.2 Reflecting HOL datatype definitions

theory *HOL-Datatype*
imports
Terms-Extras
HOL−Library.Datatype-Records
HOL−Library.Finite-Map
Higher-Order-Terms.Name
begin

datatype *typ* =
  TVar name |
  TApp name typ list

datatype-compat *typ*

context begin

qualified definition *tapp-0* **where**
tapp-0 tc = *TApp tc* []

qualified definition *tapp-1* **where**
tapp-1 tc t1 = *TApp tc* [*t1*]

qualified definition *tapp-2* **where**
tapp-2 tc t1 t2 = *TApp tc* [*t1*, *t2*]

end

quickcheck-generator *typ*
  constructors:
    TVar,
    HOL-Datatype.tapp-0,
    HOL-Datatype.tapp-1,
    HOL-Datatype.tapp-2

datatype-record *dt-def* =
  tparams :: *name list*
  constructors :: (*name*, *typ list*) *fmap*

ML-file *hol-datatype.ML*

end

1.3 Constructor information

theory *Constructors*
imports
  Terms-Extras
  HOL-Datatype
begin

type-synonym *C-info* = (*name*, *dt-def*) *fmap*

locale *constructors* =
  fixes *C-info* :: *C-info*
begin

definition *flat-C-info* :: (*string* × *nat* × *string*) *list* **where**
flat-C-info = *do* {
  (*tname*, *Cs*) ← *sorted-list-of-fmap C-info*;
  (*C*, *params*) ← *sorted-list-of-fmap* (*constructors Cs*);
  [(*as-string C*, (*length params*, *as-string tname*))]
}

definition *all-tdefs* :: *name fset* **where**
all-tdefs = *fmdom C-info*

definition *C* :: *name fset* **where**
C = *ffUnion* (*fmdom* |'| *constructors* |'| *fmran C-info*)

definition *all-constructors* :: *name list* **where**
all-constructors =
  concat (*map* (λ(-, *Cs*). *map fst* (*sorted-list-of-fmap* (*constructors Cs*))) (*sorted-list-of-fmap C-info*))

end

declare *constructors.C-def*[*code*]
declare *constructors.flat-C-info-def*[*code*]
declare *constructors.all-constructors-def*[*code*]

export-code
  constructors.C constructors.flat-C-info constructors.all-constructors
  checking *Scala*

end

1.4 Special constants

theory *Consts*
imports
  Constructors
  Higher-Order-Terms.Nterm
begin

locale *special-constants* = *constructors*

locale *pre-constants* = *special-constants* +
  fixes *heads* :: *name fset*
begin

definition *all-consts* :: *name fset* **where**
all-consts = *heads* |∪| *C*

abbreviation *welldefined* :: *'a::term* ⇒ *bool* **where**
welldefined t ≡ *consts t* |⊆| *all-consts*

sublocale *welldefined*: *simple-syntactic-and welldefined*
by *standard auto*

end

declare *pre-constants.all-consts-def* [*code*]

locale *constants* = *pre-constants* +
  assumes *disjnt*: *fdisjnt heads C*
  — Conceptually the following assumptions should belong into *constructors*, but I prefer to keep that one assumption-free.
  assumes *distinct-ctr*: *distinct all-constructors*
begin

lemma *distinct-ctr'*: *distinct* (*map as-string all-constructors*)
unfolding *distinct-map*
using *distinct-ctr*
by (*auto intro*: *inj-onI dest*: *name.expand*)

end

end

1.5 Term algebra extended with wellformedness

theory *Strong-Term*
imports *Consts*
begin

class *pre-strong-term* = *term* +
  fixes *wellformed* :: *'a* ⇒ *bool*
  fixes *all-frees* :: *'a* ⇒ *name fset*
  assumes *wellformed-const*[*simp*]: *wellformed* (*const name*)
  assumes *wellformed-free*[*simp*]: *wellformed* (*free name*)
  assumes *wellformed-app*[*simp*]: *wellformed* (*app* u_1 u_2) ⟷ *wellformed* u_1 ∧ *wellformed* u_2
  assumes *all-frees-const*[*simp*]: *all-frees* (*const name*) = *fempty*
  assumes *all-frees-free*[*simp*]: *all-frees* (*free name*) = {| *name* |}
  assumes *all-frees-app*[*simp*]: *all-frees* (*app* u_1 u_2) = *all-frees* u_1 |∪| *all-frees* u_2
begin

abbreviation *wellformed-env* :: (*name*, *'a*) *fmap* ⇒ *bool* **where**
wellformed-env ≡ *fmpred* (λ-. *wellformed*)

end

context *pre-constants* **begin**

definition *shadows-consts* :: $'a$::*pre-strong-term* $\Rightarrow$ *bool* **where**
shadows-consts $t \longleftrightarrow \neg$ *fdisjnt all-consts* (*all-frees* t)

sublocale *shadows*: *simple-syntactic-or shadows-consts*
  by *standard* (*auto simp*: *shadows-consts-def fdisjnt-alt-def*)

abbreviation *not-shadows-consts-env* :: (*name*, $'a$::*pre-strong-term*) *fmap* $\Rightarrow$ *bool*
where
not-shadows-consts-env $\equiv$ *fmpred* (λ- s. $\neg$ *shadows-consts* s)

end

declare *pre-constants.shadows-consts-def*[*code*]

class *strong-term* = *pre-strong-term* +
  assumes *raw-frees-all-frees*: *abs-pred* (λt. *frees* t |⊆| *all-frees* t) t
  assumes *raw-subst-wellformed*: *abs-pred* (λt. *wellformed* $t \longrightarrow (\forall$ *env*. *wellformed-env* *env* $\longrightarrow$ *wellformed* (*subst* t *env*))) t
begin

lemma *frees-all-frees*: *frees* t |⊆| *all-frees* t
proof (*induction* t *rule*: *raw-induct*)
  case (*abs* t)
  show *?case*
    by (*rule raw-frees-all-frees*)
qed *auto*

lemma *subst-wellformed*: *wellformed* $t \Longrightarrow$ *wellformed-env env* $\Longrightarrow$ *wellformed* (*subst* t *env*)
proof (*induction* t *arbitrary*: *env rule*: *raw-induct*)
  case (*abs* t)
  show *?case*
    by (*rule raw-subst-wellformed*)
qed (*auto split*: *option.splits*)

end

global-interpretation *wellformed*: *subst-syntactic-and wellformed* :: $'a$::*strong-term* $\Rightarrow$ *bool*
by *standard* (*auto simp*: *subst-wellformed*)

instantiation *term* :: *strong-term* **begin**

fun *all-frees-term* :: *term* $\Rightarrow$ *name fset* **where**
all-frees-term (*Free* x) = {| x |} |
all-frees-term (t_1 \$ t_2) = *all-frees-term* t_1 |∪| *all-frees-term* t_2 |

all-frees-term (Λ *t*) = *all-frees-term t* |
all-frees-term - = {||}

lemma *frees-all-frees-term*[*simp*]: *all-frees t* = *frees* (*t*::*term*)
by (*induction t*) *auto*

definition *wellformed-term* :: *term* ⇒ *bool* **where**
[*simp*]: *wellformed-term t* ⟷ *Term.wellformed t*

instance proof (*standard*, *goal-cases*)
  case *8*

  have ∗: *abs-pred P t* **if** *P t* **for** *P* **and** *t* :: *term*
    unfolding *abs-pred-term-def* **using** *that*
    by *auto*

  show *?case*
    apply (*rule* ∗)
    unfolding *wellformed-term-def*
    by (*auto simp*: *Term.subst-wellformed*)
qed (*auto simp*: *const-term-def free-term-def app-term-def abs-pred-term-def*)

end

instantiation *nterm* :: *strong-term* **begin**

definition *wellformed-nterm* :: *nterm* ⇒ *bool* **where**
[*simp*]: *wellformed-nterm t* ⟷ *True*

fun *all-frees-nterm* :: *nterm* ⇒ *name fset* **where**
all-frees-nterm (*Nvar x*) = {| *x* |} |
all-frees-nterm (t_1 $\$_n$ t_2) = *all-frees-nterm* t_1 |∪| *all-frees-nterm* t_2 |
all-frees-nterm (Λ_n *x*. *t*) = *finsert x* (*all-frees-nterm t*) |
all-frees-nterm (*Nconst* -) = {||}

instance proof (*standard*, *goal-cases*)
  case (*7 t*)
  show *?case*
    unfolding *abs-pred-nterm-def*
    by *auto*
qed (*auto simp*: *const-nterm-def free-nterm-def app-nterm-def abs-pred-nterm-def*)

end

lemma (**in** *pre-constants*) *shadows-consts-frees*:
  fixes *t* :: ′*a*::*strong-term*
  shows ¬ *shadows-consts t* ⟹ *fdisjnt all-consts* (*frees t*)
unfolding *fdisjnt-alt-def shadows-consts-def*

using *frees-all-frees*
  by *auto*

abbreviation *wellformed-clauses* :: - ⇒ *bool* **where**
wellformed-clauses cs ≡ *list-all* (λ(*pat*, *t*). *linear pat* ∧ *wellformed t*) *cs* ∧ *distinct* (*map fst cs*) ∧ *cs* ≠ []

end

1.6 Terms with sequential pattern matching

theory *Sterm*
imports *Strong-Term*
begin

datatype *sterm* =
  Sconst name |
  Svar name |
  Sabs (*clauses*: (*term* × *sterm*) *list*) |
  Sapp sterm sterm (**infixl** ‹$\$_s$› *70*)

datatype-compat *sterm*

derive *linorder sterm*

abbreviation *Sabs-single* (‹Λ_s -. -› [*0*, *50*] *50*) **where**
Sabs-single x rhs ≡ *Sabs* [(*Free x*, *rhs*)]

type-synonym *sclauses* = (*term* × *sterm*) *list*

lemma *sterm-induct*[*case-names Sconst Svar Sabs Sapp*]:
  assumes $\bigwedge x.\ P\ (Sconst\ x)$
  assumes $\bigwedge x.\ P\ (Svar\ x)$
  assumes $\bigwedge cs.\ (\bigwedge pat\ t.\ (pat,\ t) \in set\ cs \Longrightarrow P\ t) \Longrightarrow P\ (Sabs\ cs)$
  assumes $\bigwedge t\ u.\ P\ t \Longrightarrow P\ u \Longrightarrow P\ (t\ \$_s\ u)$
  shows *P t*
using *assms*
  apply *induction-schema*
    apply *pat-completeness*
   apply *lexicographic-order*
  done

instantiation *sterm* :: *pre-term* **begin**

definition *app-sterm* **where**
app-sterm t u = *t* $\$_s$ *u*

fun *unapp-sterm* **where**
unapp-sterm (*t* $\$_s$ *u*) = *Some* (*t*, *u*) |

unapp-sterm - = *None*

definition *const-sterm* **where**
const-sterm = *Sconst*

fun *unconst-sterm* **where**
unconst-sterm (*Sconst name*) = *Some name* |
unconst-sterm - = *None*

fun *unfree-sterm* **where**
unfree-sterm (*Svar name*) = *Some name* |
unfree-sterm - = *None*

definition *free-sterm* **where**
free-sterm = *Svar*

fun *frees-sterm* **where**
frees-sterm (*Svar name*) = {|*name*|} |
frees-sterm (*Sconst* -) = {||} |
frees-sterm (*Sabs cs*) = *ffUnion* (*fset-of-list* (*map* (λ(*pat*, *rhs*). *frees-sterm rhs* − *frees pat*) *cs*)) |
frees-sterm (t $\$_s$ u) = *frees-sterm* t |∪| *frees-sterm* u

fun *subst-sterm* **where**
subst-sterm (*Svar s*) *env* = (*case fmlookup env s of Some t* ⇒ *t* | *None* ⇒ *Svar s*) |
subst-sterm (t_1 $\$_s$ t_2) *env* = *subst-sterm* t_1 *env* $\$_s$ *subst-sterm* t_2 *env* |
subst-sterm (*Sabs cs*) *env* = *Sabs* (*map* (λ(*pat*, *rhs*). (*pat*, *subst-sterm rhs* (*fmdrop-fset* (*frees pat*) *env*))) *cs*) |
subst-sterm t env = *t*

fun *consts-sterm* :: *sterm* ⇒ *name fset* **where**
consts-sterm (*Svar* -) = {||} |
consts-sterm (*Sconst name*) = {|*name*|} |
consts-sterm (*Sabs cs*) = *ffUnion* (*fset-of-list* (*map* (λ(-, *rhs*). *consts-sterm rhs*) *cs*)) |
consts-sterm (t $\$_s$ u) = *consts-sterm* t |∪| *consts-sterm* u

instance
by *standard*
  (*auto*
    simp: *app-sterm-def const-sterm-def free-sterm-def*
    elim: *unapp-sterm.elims unconst-sterm.elims unfree-sterm.elims*
    split: *option.splits*)

end

instantiation *sterm* :: *term* **begin**

definition *abs-pred-sterm* :: (*sterm* ⇒ *bool*) ⇒ *sterm* ⇒ *bool* **where**
[*code del*]: *abs-pred P t* ⟷ (∀ *cs*. *t* = *Sabs cs* ⟶ (∀ *pat t*. (*pat*, *t*) ∈ *set cs* ⟶ *P t*) ⟶ *P t*)

lemma *abs-pred-stermI*[*intro*]:
  assumes ⋀*cs*. (⋀*pat t*. (*pat*, *t*) ∈ *set cs* ⟹ *P t*) ⟹ *P* (*Sabs cs*)
  shows *abs-pred P t*
using *assms* **unfolding** *abs-pred-sterm-def* **by** *auto*

instance proof (*standard*, *goal-cases*)
  case (*1 P t*)
  then show *?case*
   by (*induction t*) (*auto simp*: *const-sterm-def free-sterm-def app-sterm-def abs-pred-sterm-def*)
next
  case (*2 t*)
  show *?case*
    apply *rule*
    apply *auto*
    apply (*subst* (*3*) *list.map-id*[*symmetric*])
    apply (*rule list.map-cong0*)
    apply *auto*
    by *blast*
next
  case (*3 x t*)
  show *?case*
    apply *rule*
    apply *clarsimp*
    subgoal for *cs env pat rhs*
      apply (*cases x* |∈| *frees pat*)
      subgoal
        apply (*rule arg-cong*[**where** *f* = *subst rhs*])
        by (*auto intro*: *fmap-ext*)
      subgoal premises *prems*[*rule-format*]
        apply (*subst* (*2*) *prems*(*1*)[*symmetric*, **where** *pat* = *pat*])
        subgoal by *fact*
        subgoal
          using *prems*
          unfolding *ffUnion-alt-def*
          by (*auto simp add*: *fset-of-list.rep-eq elim*!: *fBallE*)
        subgoal
          apply (*rule arg-cong*[**where** *f* = *subst rhs*])
          by (*auto intro*: *fmap-ext*)
        done
      done
    done
next
  case (*4 t*)
  show *?case*
    apply *rule*

```
    apply clarsimp
    subgoal premises prems[rule-format]
      apply (rule prems(1)[OF prems(4)])
      subgoal using prems by auto
      subgoal using prems unfolding fdisjnt-alt-def by auto
      done
    done
next
  case 5
  show ?case
    proof (intro abs-pred-stermI allI impI, goal-cases)
      case (1 cs env)
      show ?case
        proof safe
          fix name
          assume name |∈| frees (subst (Sabs cs) env)
          then obtain pat rhs
            where (pat, rhs) ∈ set cs
              and name |∈| frees (subst rhs (fmdrop-fset (frees pat) env))
              and name |∉| frees pat
           by (auto simp: fset-of-list-elem case-prod-twice comp-def ffUnion-alt-def)

          hence name |∈| frees rhs |-| fmdom (fmdrop-fset (frees pat) env)
            using 1 by (simp add: fmpred-drop-fset)

          hence name |∈| frees rhs |-| frees pat
            using ‹name |∉| frees pat› by blast

          show name |∈| frees (Sabs cs)
            apply (simp add: ffUnion-alt-def)
            apply (rule fBexI[where x = (pat, rhs)])
            unfolding prod.case
             apply (fact ‹name |∈| frees rhs |-| frees pat›)
            unfolding fset-of-list-elem
            by fact

          assume name |∈| fmdom env
          thus False
           using ‹name |∈| frees rhs |-| fmdom (fmdrop-fset (frees pat) env)› ‹name
|∉| frees pat›
            by fastforce
        next
          fix name
          assume name |∈| frees (Sabs cs) name |∉| fmdom env

          then obtain pat rhs
            where (pat, rhs) ∈ set cs name |∈| frees rhs name |∉| frees pat
            by (auto simp: fset-of-list-elem ffUnion-alt-def)
```

moreover hence *name* |∈| *frees rhs* |−| *fmdom* (*fmdrop-fset* (*frees pat*) *env*) |−| *frees pat*
using ‹*name* |∉| *fmdom env*› **by** *fastforce*

ultimately have *name* |∈| *frees* (*subst rhs* (*fmdrop-fset* (*frees pat*) *env*)) |−| *frees pat*
using *1* **by** (*simp add*: *fmpred-drop-fset*)

show *name* |∈| *frees* (*subst* (*Sabs cs*) *env*)
apply (*simp add*: *case-prod-twice comp-def*)
unfolding *ffUnion-alt-def*
apply (*rule fBexI*)
apply (*fact* ‹*name* |∈| *frees* (*subst rhs* (*fmdrop-fset* (*frees pat*) *env*)) |−| *frees pat*›)
apply (*subst fimage-iff*)
apply (*rule fBexI*[**where** *x* = (*pat*, *rhs*)])
apply *simp*
using ‹(*pat*, *rhs*) ∈ *set cs*›
by (*auto simp*: *fset-of-list-elem*)
qed
qed
next
case *6*
show *?case*
proof (*intro abs-pred-stermI allI impI*, *goal-cases*)
case (*1 cs env*)

— some property on various operations that is only useful in here
have ∗: *fbind* (*fmimage m* (*fbind A g*)) *f* = *fbind A* (λ*x*. *fbind* (*fmimage m* (*g x*)) *f*)
for *m A f g*
including *fset.lifting* **and** *fmap.lifting*
by *transfer′ force*

have *consts* (*subst* (*Sabs cs*) *env*) = *fbind* (*fset-of-list cs*) (λ(*pat*, *rhs*). *consts rhs* |∪| *ffUnion* (*consts* |‘| *fmimage* (*fmdrop-fset* (*frees pat*) *env*) (*frees rhs*)))
apply (*simp add*: *funion-image-bind-eq*)
apply (*rule fbind-cong*[*OF refl*])
apply (*clarsimp split*: *prod.splits*)
apply (*subst 1*)
apply (*subst* (*asm*) *fset-of-list-elem*, *assumption*)
apply *simp*
by (*simp add*: *funion-image-bind-eq*)
also have . . . = *fbind* (*fset-of-list cs*) (*consts* ∘ *snd*) |∪| *fbind* (*fset-of-list cs*) (λ(*pat*, *rhs*). *ffUnion* (*consts* |‘| *fmimage* (*fmdrop-fset* (*frees pat*) *env*) (*frees rhs*)))
apply (*subst fbind-fun-funion*[*symmetric*])
apply (*rule fbind-cong*[*OF refl*])
by *auto*
also have . . . = *consts* (*Sabs cs*) |∪| *fbind* (*fset-of-list cs*) (λ(*pat*, *rhs*). *ffUnion*

(*consts* |`| *fmimage* (*fmdrop-fset* (*frees pat*) *env*) (*frees rhs*)))
apply (*rule cong*[*OF cong*, *OF refl* - *refl*, **where** *f1* = *funion*])
apply (*subst funion-image-bind-eq*[*symmetric*])
unfolding *consts-sterm.simps*
apply (*rule arg-cong*[**where** *f* = *ffUnion*])
apply (*subst fset-of-list-map*)
apply (*rule fset.map-cong*[*OF refl*])
by *auto*
also have ... = *consts* (*Sabs cs*) |∪| *fbind* (*fmimage env* (*fbind* (*fset-of-list cs*) (λ(*pat*, *rhs*). *frees rhs* |−| *frees pat*))) *consts*
apply (*subst funion-image-bind-eq*)
apply (*subst fmimage-drop-fset*)
apply (*rule cong*[*OF cong*, *OF refl refl*, **where** *f1* = *funion*])
apply (*subst* ∗)
apply (*rule fbind-cong*[*OF refl*])
by *auto*
also have ... = *consts* (*Sabs cs*) |∪| *ffUnion* (*consts* |`| *fmimage env* (*frees* (*Sabs cs*)))
by (*simp only*: *frees-sterm.simps fset-of-list-map fmimage-Union funion-image-bind-eq*)

finally show *?case* **.**
qed
qed (*auto simp*: *abs-pred-sterm-def*)

end

lemma *no-abs-abs*[*simp*]: ¬ *no-abs* (*Sabs cs*)
by (*subst no-abs.simps*) (*auto simp*: *term-cases-def*)

lemma *closed-except-simps*:
closed-except (*Svar x*) *S* ⟷ *x* |∈| *S*
closed-except (t_1 $\$_s$ t_2) *S* ⟷ *closed-except* t_1 *S* ∧ *closed-except* t_2 *S*
closed-except (*Sabs cs*) *S* ⟷ *list-all* (λ(*pat*, *t*). *closed-except t* (*S* |∪| *frees pat*)) *cs*
closed-except (*Sconst name*) *S* ⟷ *True*
proof *goal-cases*
case *3*
show *?case*
proof (*standard*, *goal-cases*)
case *1*
then show *?case*
apply (*auto simp*: *list-all-iff ffUnion-alt-def fset-of-list-elem closed-except-def*)
apply (*drule ffUnion-least-rev*)
apply *auto*
by (*smt case-prod-conv fbspec fimageI fminusI fset-of-list-elem fset-rev-mp*)
next
case *2*
then show *?case*

by (*fastforce simp*: *list-all-iff ffUnion-alt-def fset-of-list-elem closed-except-def*)
qed
qed (*auto simp*: *ffUnion-alt-def closed-except-def*)

lemma *closed-except-sabs*:
assumes *closed* (*Sabs cs*) (*pat*, *rhs*) ∈ *set cs*
shows *closed-except rhs* (*frees pat*)
using *assms* **unfolding** *closed-except-def*
apply *auto*
by (*metis bot.extremum-uniqueI fempty-iff ffUnion-subset-elem fimageI fminusI fset-of-list-elem old.prod.case*)

instantiation *sterm* :: *strong-term* **begin**

fun *wellformed-sterm* :: *sterm* ⇒ *bool* **where**
wellformed-sterm (t_1 $\$_s$ t_2) ⟷ *wellformed-sterm* t_1 ∧ *wellformed-sterm* t_2 |
wellformed-sterm (*Sabs cs*) ⟷ *list-all* (λ(*pat*, *t*). *linear pat* ∧ *wellformed-sterm t*) *cs* ∧ *distinct* (*map fst cs*) ∧ *cs* ≠ [] |
wellformed-sterm - ⟷ *True*

primrec *all-frees-sterm* :: *sterm* ⇒ *name fset* **where**
all-frees-sterm (*Svar x*) = {|*x*|} |
all-frees-sterm (t_1 $\$_s$ t_2) = *all-frees-sterm* t_1 |∪| *all-frees-sterm* t_2 |
all-frees-sterm (*Sabs cs*) = *ffUnion* (*fset-of-list* (*map* (λ(*P*, *T*). *P* |∪| *T*) (*map* (*map-prod frees all-frees-sterm*) *cs*))) |
all-frees-sterm (*Sconst* -) = {||}

instance proof (*standard*, *goal-cases*)
case (*7 t*)
show *?case*
apply (*intro abs-pred-stermI allI impI*)
apply *simp*
apply (*rule ffUnion-least*)
apply (*rule fBallI*)
apply *auto*
apply (*subst ffUnion-alt-def*)
apply *simp*
apply (*rule-tac x* = (*a*, *b*) **in** *fBexI*)
by (*auto simp*: *fset-of-list-elem*)
next
case (*8 t*)
show *?case*
apply (*intro abs-pred-stermI allI impI*)
apply (*simp add*: *list.pred-map comp-def case-prod-twice*, *safe*)
subgoal
apply (*subst list-all-iff*)
apply (*rule ballI*)
apply *safe*[*1*]
apply (*fastforce simp*: *list-all-iff*)

subgoal premises *prems*[*rule-format*]
apply (*rule prems*)
apply (*fact prems*)
using *prems* **apply** (*fastforce simp*: *list-all-iff*)
using *prems* **by** *force*
done
subgoal
apply (*subst map-cong*[*OF refl*])
by *auto*
done
qed (*auto simp*: *const-sterm-def free-sterm-def app-sterm-def*)

end

lemma *match-sabs*[*simp*]: $\neg$ *is-free t* $\Longrightarrow$ *match t* (*Sabs cs*) = *None*
by (*cases t*) *auto*

context *pre-constants* **begin**

lemma *welldefined-sabs*: *welldefined* (*Sabs cs*) $\longleftrightarrow$ *list-all* (λ(-, *t*). *welldefined t*) *cs*
apply (*auto simp*: *list-all-iff ffUnion-alt-def dest*!: *ffUnion-least-rev*)
apply (*subst* (*asm*) *list-all-iff-fset*[*symmetric*])
apply (*auto simp*: *list-all-iff fset-of-list-elem*)
done

lemma *shadows-consts-sterm-simps*[*simp*]:
shadows-consts (t_1 $\$_s$ t_2) $\longleftrightarrow$ *shadows-consts* t_1 $\vee$ *shadows-consts* t_2
shadows-consts (*Svar name*) $\longleftrightarrow$ *name* $|\in|$ *all-consts*
shadows-consts (*Sabs cs*) $\longleftrightarrow$ *list-ex* (λ(*pat*, *t*). $\neg$ *fdisjnt all-consts* (*frees pat*) $\vee$ *shadows-consts t*) *cs*
shadows-consts (*Sconst name*) $\longleftrightarrow$ *False*
proof (*goal-cases*)
case *3*
show *?case*
unfolding *shadows-consts-def list-ex-iff*
apply *rule*
subgoal
by (*force simp*: *ffUnion-alt-def fset-of-list-elem fdisjnt-alt-def elim*!: *ballE*)
subgoal
apply (*auto simp*: *fset-of-list-elem fdisjnt-alt-def*)
by (*auto simp*: *fset-eq-empty-iff ffUnion-alt-def fset-of-list-elem elim*!: *allE fBallE*)
done
qed (*auto simp*: *shadows-consts-def fdisjnt-alt-def*)

lemma *subst-shadows*:
assumes $\neg$ *shadows-consts* (*t*::*sterm*) *not-shadows-consts-env* Γ

shows ¬ *shadows-consts* (*subst t* Γ)
using *assms* **proof** (*induction t arbitrary*: Γ *rule*: *sterm-induct*)
case (*Sabs cs*)
show *?case*
apply (*simp add*: *list-ex-iff case-prod-twice*)
apply (*rule ballI*)
subgoal for *c*
apply (*cases c*, *hypsubst-thin*, *simp*)
apply (*rule conjI*)
subgoal using *Sabs*(*2*) **by** (*fastforce simp*: *list-ex-iff*)
apply (*rule Sabs*(*1*))
apply *assumption*
subgoal using *Sabs*(*2*) **by** (*fastforce simp*: *list-ex-iff*)
subgoal using *Sabs*(*3*) **by** *force*
done
done
qed (*auto split*: *option.splits*)

end

end

1.7 Terms with explicit pattern matching

theory *Pterm*
imports
../Utils/Compiler-Utils
Consts
Sterm — Inclusion of this theory might seem a bit strange. Indeed, it is only for technical reasons: to allow for a **quickcheck** setup.
begin

datatype *pterm* =
Pconst name |
Pvar name |
Pabs (*term* × *pterm*) *fset* |
Papp pterm pterm (**infixl** ‹$\$_p$› *70*)

primrec *sterm-to-pterm* :: *sterm* ⇒ *pterm* **where**
sterm-to-pterm (*Sconst name*) = *Pconst name* |
sterm-to-pterm (*Svar name*) = *Pvar name* |
sterm-to-pterm (*t* $\$_s$ *u*) = *sterm-to-pterm t* $\$_p$ *sterm-to-pterm u* |
sterm-to-pterm (*Sabs cs*) = *Pabs* (*fset-of-list* (*map* (*map-prod id sterm-to-pterm*) *cs*))

quickcheck-generator *pterm*
— will print some fishy “constructor” names, but at least it works
constructors: *sterm-to-pterm*

```
lemma sterm-to-pterm-total:
  obtains t' where t = sterm-to-pterm t'
proof (induction t arbitrary: thesis)
  case (Pconst x)
  then show ?case
    by (metis sterm-to-pterm.simps)
next
  case (Pvar x)
  then show ?case
    by (metis sterm-to-pterm.simps)
next
  case (Pabs cs)
  from Pabs.IH obtain cs' where cs = fset-of-list (map (map-prod id sterm-to-pterm)
cs')
    apply atomize-elim
    proof (induction cs)
      case empty
      show ?case
        apply (rule exI[where x = []])
        by simp
    next
      case (insert c cs)
      obtain pat rhs where c = (pat, rhs) by (cases c) auto

      have ∃ cs'. cs = fset-of-list (map (map-prod id sterm-to-pterm) cs')
        apply (rule insert)
        using insert.prems unfolding finsert.rep-eq
        by blast
      then obtain cs' where cs = fset-of-list (map (map-prod id sterm-to-pterm)
cs')
        by blast

      obtain rhs' where rhs = sterm-to-pterm rhs'
        apply (rule insert.prems[of (pat, rhs) rhs])
        unfolding ‹c = -› by simp+

      show ?case
        apply (rule exI[where x = (pat, rhs') # cs'])
        unfolding ‹c = -› ‹cs = -› ‹rhs = -›
        by (simp add: id-def)
    qed
  hence Pabs cs = sterm-to-pterm (Sabs cs')
    by simp
  then show ?case
    using Pabs by metis
next
  case (Papp t1 t2)
  then obtain t1' t2' where t1 = sterm-to-pterm t1' t2 = sterm-to-pterm t2'
    by metis
```

then have $t1\ \$_p\ t2 = sterm\text{-}to\text{-}pterm\ (t1'\ \$_s\ t2')$
by *simp*
with *Papp* **show** *?case*
by *metis*
qed

lemma *pterm-induct*[*case-names Pconst Pvar Pabs Papp*]:
assumes $\bigwedge x.\ P\ (Pconst\ x)$
assumes $\bigwedge x.\ P\ (Pvar\ x)$
assumes $\bigwedge cs.\ (\bigwedge pat\ t.\ (pat,\ t)\ |\in|\ cs \Longrightarrow P\ t) \Longrightarrow P\ (Pabs\ cs)$
assumes $\bigwedge t\ u.\ P\ t \Longrightarrow P\ u \Longrightarrow P\ (t\ \$_p\ u)$
shows $P\ t$
proof (*rule pterm.induct*, *goal-cases*)
case (*3 cs*)
show *?case*
apply (*rule assms*)
using *3*
by *auto*
qed *fact+*

instantiation *pterm* :: *pre-term* **begin**

definition *app-pterm* **where**
$app\text{-}pterm\ t\ u = t\ \$_p\ u$

fun *unapp-pterm* **where**
$unapp\text{-}pterm\ (t\ \$_p\ u) = Some\ (t,\ u)\ |$
unapp-pterm - = None

definition *const-pterm* **where**
const-pterm = Pconst

fun *unconst-pterm* **where**
unconst-pterm (Pconst name) = Some name |
unconst-pterm - = None

definition *free-pterm* **where**
free-pterm = Pvar

fun *unfree-pterm* **where**
unfree-pterm (Pvar name) = Some name |
unfree-pterm - = None

function (*sequential*) *subst-pterm* **where**
$subst\text{-}pterm\ (Pvar\ s)\ env = (case\ fmlookup\ env\ s\ of\ Some\ t \Rightarrow t\ |\ None \Rightarrow Pvar\ s)\ |$
$subst\text{-}pterm\ (t_1\ \$_p\ t_2)\ env = subst\text{-}pterm\ t_1\ env\ \$_p\ subst\text{-}pterm\ t_2\ env\ |$
$subst\text{-}pterm\ (Pabs\ cs)\ env = Pabs\ ((\lambda(pat,\ rhs).\ (pat,\ subst\text{-}pterm\ rhs\ (fmdrop\text{-}fset\ (frees\ pat)\ env)))\ |`|\ cs)\ |$

subst-pterm t - = t
by *pat-completeness auto*

termination
proof (*relation measure* (*size* ∘ *fst*), *goal-cases*)
  case *4*
  then show *?case*
    apply *auto*
    including *fset.lifting* **apply** *transfer*
    apply (*rule le-imp-less-Suc*)
    apply (*rule sum-nat-le-single*[**where** *y* = (*a*, (*b*, *size b*)) **for** *a b*])
    by *auto*
qed *auto*

primrec *consts-pterm* :: *pterm* ⇒ *name fset* **where**
consts-pterm (*Pconst x*) = {|*x*|} |
consts-pterm (t_1 $\$_p$ t_2) = *consts-pterm* t_1 |∪| *consts-pterm* t_2 |
consts-pterm (*Pabs cs*) = *ffUnion* (*snd* |'| *map-prod id consts-pterm* |'| *cs*) |
consts-pterm (*Pvar* -) = {||}

primrec *frees-pterm* :: *pterm* ⇒ *name fset* **where**
frees-pterm (*Pvar x*) = {|*x*|} |
frees-pterm (t_1 $\$_p$ t_2) = *frees-pterm* t_1 |∪| *frees-pterm* t_2 |
frees-pterm (*Pabs cs*) = *ffUnion* ((λ(*pv*, *tv*). *tv* − *frees pv*) |'| *map-prod id frees-pterm* |'| *cs*) |
frees-pterm (*Pconst* -) = {||}

instance
by *standard*
   (*auto*
      simp: *app-pterm-def const-pterm-def free-pterm-def*
      elim: *unapp-pterm.elims unconst-pterm.elims unfree-pterm.elims*
      split: *option.splits*)

end

corollary *subst-pabs-id*:
  assumes ⋀*pat rhs*. (*pat*, *rhs*) |∈| *cs* ⟹ *subst rhs* (*fmdrop-fset* (*frees pat*) *env*) = *rhs*
  shows *subst* (*Pabs cs*) *env* = *Pabs cs*
apply (*subst subst-pterm.simps*)
apply (*rule arg-cong*[**where** *f* = *Pabs*])
apply (*rule fset-map-snd-id*)
apply (*rule assms*)
apply *assumption*
done

corollary *frees-pabs-alt-def*:
  frees (*Pabs cs*) = *ffUnion* ((λ(*pat*, *rhs*). *frees rhs* − *frees pat*) |'| *cs*)

apply *simp*
apply (*rule arg-cong*[**where** *f* = *ffUnion*])
apply (*rule fset.map-cong*[*OF refl*])
by *auto*

lemma *sterm-to-pterm-frees*[*simp*]: *frees* (*sterm-to-pterm t*) = *frees t*
proof (*induction t*)
  case (*Sabs cs*)
  show *?case*
    apply *simp*
    apply (*rule arg-cong*[**where** *f* = *ffUnion*])
    apply (*rule fimage-cong*[*OF refl*])
    apply *clarsimp*
    apply (*subst Sabs*)
    by (*auto simp*: *fset-of-list-elem snds.simps*)
qed *auto*

lemma *sterm-to-pterm-consts*[*simp*]: *consts* (*sterm-to-pterm t*) = *consts t*
proof (*induction t*)
  case (*Sabs cs*)
  show *?case*
    apply *simp*
    apply (*rule arg-cong*[**where** *f* = *ffUnion*])
    apply (*rule fimage-cong*[*OF refl*])
    apply *clarsimp*
    apply (*subst Sabs*)
    by (*auto simp*: *fset-of-list-elem snds.simps*)
qed *auto*

lemma *subst-sterm-to-pterm*:
  subst (*sterm-to-pterm t*) (*fmmap sterm-to-pterm env*) = *sterm-to-pterm* (*subst t env*)
proof (*induction t arbitrary*: *env rule*: *sterm-induct*)
  case (*Sabs cs*)
  show *?case*
    apply *simp*
    apply (*rule fset.map-cong0*)
    apply (*auto split*: *prod.splits*)
    apply (*rule Sabs*)
    by (*auto simp*: *fset-of-list.rep-eq*)
qed (*auto split*: *option.splits*)

instantiation *pterm* :: *term* **begin**

definition *abs-pred-pterm* :: (*pterm* ⇒ *bool*) ⇒ *pterm* ⇒ *bool* **where**
[*code del*]: *abs-pred P t* ⟷ (∀ *cs*. *t* = *Pabs cs* ⟶ (∀ *pat t*. (*pat*, *t*) |∈| *cs* ⟶ *P t*) ⟶ *P t*)

context begin

```
private lemma abs-pred-trivI0: P t ⟹ abs-pred P (t::pterm)
unfolding abs-pred-pterm-def by auto

instance proof (standard, goal-cases)
  case (1 P t)
  then show ?case
    by (induction t rule: pterm-induct)
      (auto simp: const-pterm-def free-pterm-def app-pterm-def abs-pred-pterm-def)
next
  case (2 t)
  show ?case
    unfolding abs-pred-pterm-def
    apply clarify
    apply (rule subst-pabs-id)
    by blast
next
  case (3 x t)
  show ?case
    unfolding abs-pred-pterm-def
    apply clarsimp
    apply (rule fset.map-cong0)
    apply (rename-tac c)
    apply (case-tac c; hypsubst-thin)
    apply simp
    subgoal for cs env pat rhs
      apply (cases x |∈| frees pat)
      subgoal
        apply (rule arg-cong[where f = subst rhs])
        by (auto intro: fmap-ext)
      subgoal premises prems[rule-format]
        apply (subst (2) prems(1)[symmetric, where pat = pat])
        subgoal
          by fact
        subgoal
          using prems unfolding ffUnion-alt-def
          by (auto simp: fset-of-list.rep-eq elim!: fBallE)
        subgoal
          apply (rule arg-cong[where f = subst rhs])
          by (auto intro: fmap-ext)
        done
      done
    done
next
```

```
  case (4 t)
  show ?case
    unfolding abs-pred-pterm-def
    apply clarsimp
    apply (rule fset.map-cong0)
    apply clarsimp
    subgoal premises prems[rule-format] for cs env₁ env₂ a b
      apply (rule prems(2)[OF prems(5)])
      using prems unfolding fdisjnt-alt-def by auto
    done
next
  case 5
  show ?case
    proof (rule abs-pred-trivI0, clarify)
      fix t :: pterm
      fix env :: (name, pterm) fmap

      obtain t' where t = sterm-to-pterm t'
        by (rule sterm-to-pterm-total)
      obtain env' where env = fmmap sterm-to-pterm env'
        by (metis fmmap-total sterm-to-pterm-total)

      show frees (subst t env) = frees t − fmdom env if fmpred (λ-. closed) env
        unfolding ‹t = -› ‹env = -›
        apply simp
        apply (subst subst-sterm-to-pterm)
        apply simp
        apply (rule subst-frees)
        using that unfolding ‹env = -›
        apply simp
        apply (rule fmpred-mono-strong; assumption?)
        unfolding closed-except-def by simp
    qed
next
  case 6
  show ?case
    proof (rule abs-pred-trivI0, clarify)
      fix t :: pterm
      fix env :: (name, pterm) fmap

      obtain t' where t = sterm-to-pterm t'
        by (rule sterm-to-pterm-total)
      obtain env' where env = fmmap sterm-to-pterm env'
        by (metis fmmap-total sterm-to-pterm-total)

     show consts (subst t env) = consts t |∪| ffUnion (consts |`| fmimage env (frees
t))
        unfolding ‹t = -› ‹env = -›
        apply simp
```

```
      apply (subst comp-def)
      apply simp
      apply (subst subst-sterm-to-pterm)
      apply simp
      apply (rule subst-consts')
      done
    qed
qed (rule abs-pred-trivI0)

end

end

lemma no-abs-abs[simp]: ¬ no-abs (Pabs cs)
by (subst no-abs.simps) (auto simp: term-cases-def)

lemma sterm-to-pterm:
  assumes no-abs t
  shows sterm-to-pterm t = convert-term t
using assms proof induction
  case (free name)
  show ?case
    apply simp
    apply (simp add: free-sterm-def free-pterm-def)
    done
next
  case (const name)
  show ?case
    apply simp
    apply (simp add: const-sterm-def const-pterm-def)
    done
next
  case (app t₁ t₂)
  then show ?case
    apply simp
    apply (simp add: app-sterm-def app-pterm-def)
    done
qed

abbreviation Pabs-single (‹Λp -. -› [0, 50] 50) where
Pabs-single x rhs ≡ Pabs {| (Free x, rhs) |}

lemma closed-except-simps:
  closed-except (Pvar x) S ⟷ x |∈| S
  closed-except (t₁ $p t₂) S ⟷ closed-except t₁ S ∧ closed-except t₂ S
  closed-except (Pabs cs) S ⟷ fBall cs (λ(pat, t). closed-except t (S |∪| frees pat))
  closed-except (Pconst name) S ⟷ True
proof goal-cases
  case 3
```

show *?case*
proof (*standard*, *goal-cases*)
case *1*
then show *?case*
apply (*auto simp*: *ffUnion-alt-def closed-except-def*)
apply (*drule ffUnion-least-rev*)
apply *auto*
by (*smt case-prod-conv fBall-alt-def fminus-iff fset-rev-mp id-apply map-prod-simp*)
next
case *2*
then show *?case*
by (*fastforce simp*: *ffUnion-alt-def closed-except-def*)
qed
qed (*auto simp*: *ffUnion-alt-def closed-except-def*)

instantiation *pterm* :: *pre-strong-term* **begin**

function (*sequential*) *wellformed-pterm* :: *pterm* ⇒ *bool* **where**
wellformed-pterm (t_1 $\$_p$ t_2) ⟷ *wellformed-pterm* t_1 ∧ *wellformed-pterm* t_2 |
wellformed-pterm (*Pabs cs*) ⟷ *fBall cs* (λ(*pat*, *t*). *linear pat* ∧ *wellformed-pterm* *t*) ∧ *is-fmap cs* ∧ *pattern-compatibles cs* ∧ *cs* ≠ {||} |
wellformed-pterm - ⟷ *True*
by *pat-completeness auto*

termination
proof (*relation measure size*, *goal-cases*)
case *4*
then show *?case*
apply *auto*
including *fset.lifting* **apply** *transfer*
apply (*rule le-imp-less-Suc*)
apply (*rule sum-nat-le-single*[**where** *y* = (*a*, (*b*, *size b*)) **for** *a b*])
by *auto*
qed *auto*

primrec *all-frees-pterm* :: *pterm* ⇒ *name fset* **where**
all-frees-pterm (*Pvar x*) = {|*x*|} |
all-frees-pterm (t_1 $\$_p$ t_2) = *all-frees-pterm* t_1 |∪| *all-frees-pterm* t_2 |
all-frees-pterm (*Pabs cs*) = *ffUnion* ((λ(*P*, *T*). *P* |∪| *T*) |`| *map-prod frees all-frees-pterm* |`| *cs*) |
all-frees-pterm (*Pconst* -) = {||}

instance
by *standard* (*auto simp*: *const-pterm-def free-pterm-def app-pterm-def*)

end

lemma *sterm-to-pterm-all-frees*[*simp*]: *all-frees* (*sterm-to-pterm t*) = *all-frees t*
proof (*induction t*)

case (*Sabs cs*)
show *?case*
apply *simp*
apply (*rule arg-cong*[**where** *f* = *ffUnion*])
apply (*rule fimage-cong*[*OF refl*])
apply *clarsimp*
apply (*subst Sabs*)
by (*auto simp*: *fset-of-list-elem snds.simps*)
qed *auto*

instance *pterm* :: *strong-term* **proof** (*standard*, *goal-cases*)
case (*1 t*)
obtain *t′* **where** *t* = *sterm-to-pterm t′*
by (*metis sterm-to-pterm-total*)
show *?case*
apply (*rule abs-pred-trivI*)
unfolding ‹*t* = -› *sterm-to-pterm-all-frees sterm-to-pterm-frees*
by (*rule frees-all-frees*)
next
case (*2 t*)
show *?case*
unfolding *abs-pred-pterm-def*
apply (*intro allI impI*)
apply (*simp add*: *case-prod-twice*, *intro conjI*)
subgoal by *blast*
subgoal by (*auto intro*: *is-fmap-image*)
subgoal
unfolding *fpairwise-image fpairwise-alt-def*
by (*auto elim*!: *fBallE*)
done
qed

lemma *wellformed-PabsI*:
assumes *is-fmap cs pattern-compatibles cs cs* ≠ {||}
assumes $\bigwedge$*pat t*. (*pat*, *t*) |∈| *cs* ⟹ *linear pat*
assumes $\bigwedge$*pat t*. (*pat*, *t*) |∈| *cs* ⟹ *wellformed t*
shows *wellformed* (*Pabs cs*)
using *assms* **by** *auto*

corollary *subst-closed-pabs*:
assumes (*pat*, *rhs*) |∈| *cs closed* (*Pabs cs*)
shows *subst rhs* (*fmdrop-fset* (*frees pat*) *env*) = *rhs*
using *assms* **by** (*subst subst-closed-except-id*) (*auto simp*: *fdisjnt-alt-def closed-except-simps*)

lemma (**in** *constants*) *shadows-consts-pterm-simps*[*simp*]:
shadows-consts (t_1 $\$_p$ t_2) ⟷ *shadows-consts* t_1 ∨ *shadows-consts* t_2
shadows-consts (*Pvar name*) ⟷ *name* |∈| *all-consts*
shadows-consts (*Pabs cs*) ⟷ *fBex cs* (λ(*pat*, *t*). *shadows-consts pat* ∨ *shadows-consts t*)

shadows-consts (*Pconst name*) ⟷ *False*
proof *goal-cases*
case *3*

show *?case*
unfolding *shadows-consts-def*
apply *rule*
subgoal
by (*force simp*: *ffUnion-alt-def fset-of-list-elem fdisjnt-alt-def elim*!: *ballE*)
subgoal
apply (*auto simp*: *fset-of-list-elem fdisjnt-alt-def*)
by (*auto simp*: *fset-eq-empty-iff ffUnion-alt-def fset-of-list-elem elim*!: *allE fBallE*)
done
qed (*auto simp*: *shadows-consts-def fdisjnt-alt-def*)

end

1.8 Irreducible terms (values)

theory *Term-as-Value*
imports *Sterm*
begin

1.9 Viewing *sterm* as values

declare *list.pred-mono*[*mono*]

context *constructors* **begin**

inductive *is-value* :: *sterm* ⇒ *bool* **where**
abs: *is-value* (*Sabs cs*) |
constr: *list-all is-value vs* ⟹ *name* |∈| *C* ⟹ *is-value* (*name* $$ *vs*)

lemma *value-distinct*:
Sabs cs ≠ *name* $$ *ts* (**is** *?P*)
name $$ *ts* ≠ *Sabs cs* (**is** *?Q*)
proof −
show *?P*
apply (*rule list-comb-cases'*[**where** *f* = *const name* **and** *xs* = *ts*])
apply (*auto simp*: *const-sterm-def is-app-def elim*: *unapp-sterm.elims*)
done
thus *?Q*
by *simp*
qed

abbreviation *value-env* :: (*name*, *sterm*) *fmap* ⇒ *bool* **where**

```
value-env ≡ fmpred (λ-. is-value)

lemma svar-value[simp]: ¬ is-value (Svar name)
proof
  assume is-value (Svar name)
  thus False
    apply (cases rule: is-value.cases)
    apply (fold free-sterm-def)
    by simp
qed

lemma value-cases:
  obtains (comb) name vs where list-all is-value vs t = name $$ vs name |∈| C
        | (abs) cs where t = Sabs cs
        | (nonvalue) ¬ is-value t
proof (cases t)
  case Svar
  thus thesis using nonvalue by simp
next
  case Sabs
  thus thesis using abs by (auto intro: is-value.abs)
next
  case (Sconst name)

  have list-all is-value [] by simp
  have t = name $$ [] unfolding Sconst by (simp add: const-sterm-def)
  show thesis
    using comb is-value.cases abs nonvalue by blast
next
  case Sapp

  show thesis
    proof (cases is-value t)
      case False
      thus thesis using nonvalue by simp
    next
      case True
      then obtain name vs where list-all is-value vs t = name $$ vs name |∈| C
        unfolding Sapp
        by cases auto
      thus thesis using comb by simp
    qed
qed

end

fun smatch' :: pat ⇒ sterm ⇒ (name, sterm) fmap option where
smatch' (Patvar name) t = Some (fmap-of-list [(name, t)]) |
smatch' (Patconstr name ps) t =
```

(*case strip-comb t of*
  (*Sconst name'*, *vs*) ⇒
    (*if name* = *name'* ∧ *length ps* = *length vs then*
      map-option (*foldl* ($++_f$) *fmempty*) (*those* (*map2 smatch' ps vs*))
    else
      None)
  | - ⇒ *None*)

lemmas *smatch'-induct* = *smatch'.induct*[*case-names var constr*]

context *constructors* **begin**

context begin

private lemma *smatch-list-comb-is-value*:
  assumes *is-value t*
  shows *match* (*name* $$ *ps*) *t* = (*case strip-comb t of*
  (*Sconst name'*, *vs*) ⇒
    (*if name* = *name'* ∧ *length ps* = *length vs then*
      map-option (*foldl* ($++_f$) *fmempty*) (*those* (*map2 match ps vs*))
    else
      None)
  | - ⇒ *None*)
using *assms*
apply *cases*
apply (*auto simp*: *strip-list-comb split*: *option.splits*)
apply (*subst* (*2*) *const-sterm-def*)
apply (*auto simp*: *matchs-alt-def*)
done

lemma *smatch-smatch'-eq*:
  assumes *linear pat is-value t*
  shows *match pat t* = *smatch'* (*mk-pat pat*) *t*
using *assms*
proof (*induction pat arbitrary*: *t rule*: *linear-pat-induct*)
  case (*comb name args*)

  show *?case*
    using ‹*is-value t*›
    proof (*cases rule*: *is-value.cases*)
      case (*abs cs*)
      thus *?thesis*
        by (*force simp*: *strip-list-comb-const*)
    next
      case (*constr args' name'*)

      have *map2 match args args'* = *map2 smatch'* (*map mk-pat args*) *args'* **if** *length args* = *length args'*
        using *that constr*(*2*) *comb*(*2*)

by (*induct args args′ rule*: *list-induct2*) *auto*

thus *?thesis*
using *constr*
apply (*auto*
simp: *smatch-list-comb-is-value strip-list-comb map-option-case strip-list-comb-const*
intro: *is-value.intros*)
apply (*subst* (*2*) *const-sterm-def*)
apply (*auto simp*: *matchs-alt-def*)
done
qed
qed *simp*

end

end

end

1.10 A dedicated value type

theory *Value*
imports *Term-as-Value*
begin

datatype *value* =
is-Vconstr: *Vconstr name value list* |
Vabs sclauses (*name*, *value*) *fmap* |
Vrecabs (*name*, *sclauses*) *fmap name* (*name*, *value*) *fmap*

type-synonym *vrule* = *name* × *value*

setup ‹*Sign.mandatory-path quickcheck*›

datatype *value* =
Vconstr name value list |
Vabs sclauses (*name* × *value*) *list* |
Vrecabs (*name* × *sclauses*) *list name* (*name* × *value*) *list*

primrec *Value* :: *quickcheck.value* ⇒ *value* **where**
Value (*quickcheck.Vconstr s vs*) = *Vconstr s* (*map Value vs*) |
Value (*quickcheck.Vabs cs* Γ) = *Vabs cs* (*fmap-of-list* (*map* (*map-prod id Value*) Γ)) |
Value (*quickcheck.Vrecabs css name* Γ) = *Vrecabs* (*fmap-of-list css*) *name* (*fmap-of-list* (*map* (*map-prod id Value*) Γ))

setup ‹*Sign.parent-path*›

quickcheck-generator *value*
constructors: *quickcheck.Value*

fun *vmatch* :: *pat* ⇒ *value* ⇒ (*name*, *value*) *fmap option* **where**
vmatch (*Patvar name*) *v* = *Some* (*fmap-of-list* [(*name*, *v*)]) |
vmatch (*Patconstr name ps*) (*Vconstr name′ vs*) =
(*if name* = *name′* ∧ *length ps* = *length vs then*
map-option (*foldl* ($++_f$) *fmempty*) (*those* (*map2 vmatch ps vs*))
else
None) |
vmatch - - = *None*

lemmas *vmatch-induct* = *vmatch.induct*[*case-names var constr*]

locale *value-pred* =
fixes *P* :: (*name*, *value*) *fmap* ⇒ *sclauses* ⇒ *bool*
fixes *Q* :: *name* ⇒ *bool*
fixes *R* :: *name fset* ⇒ *bool*
begin

primrec *pred* :: *value* ⇒ *bool* **where**
pred (*Vconstr name vs*) ⟷ *Q name* ∧ *list-all id* (*map pred vs*) |
pred (*Vabs cs* Γ) ⟷ *pred-fmap id* (*fmmap pred* Γ) ∧ *P* Γ *cs* |
pred (*Vrecabs css name* Γ) ⟷
pred-fmap id (*fmmap pred* Γ) ∧
pred-fmap (*P* Γ) *css* ∧
name |∈| *fmdom css* ∧
R (*fmdom css*)

declare *pred.simps*[*simp del*]

lemma *pred-alt-def*[*simp*, *code*]:
pred (*Vconstr name vs*) ⟷ *Q name* ∧ *list-all pred vs*
pred (*Vabs cs* Γ) ⟷ *fmpred* (λ-. *pred*) Γ ∧ *P* Γ *cs*
pred (*Vrecabs css name* Γ) ⟷ *fmpred* (λ-. *pred*) Γ ∧ *pred-fmap* (*P* Γ) *css* ∧ *name* |∈| *fmdom css* ∧ *R* (*fmdom css*)
by (*auto simp*: *list-all-iff pred.simps*)

For technical reasons, we don't introduce an abbreviation for *fmpred* (λ-. *pred*) *env* here. This locale is supposed to be interpreted with **global-interpretation** (or **sublocale** and a **defines** clause. However, this does not affect abbreviations: the abbreviation would still refer to the locale constant, not the constant introduced by the interpretation.

lemma *vmatch-env*:
assumes *vmatch pat v* = *Some env pred v*
shows *fmpred* (λ-. *pred*) *env*
using *assms* **proof** (*induction pat v arbitrary*: *env rule*: *vmatch-induct*)
case (*constr name ps name′ vs*)

hence
map-option (*foldl* ($++_f$) *fmempty*) (*those* (*map2 vmatch ps vs*)) = *Some env*
name = *name'* *length ps* = *length vs*
by (*auto split*: *if-splits*)
then obtain *envs* **where** *env* = *foldl* ($++_f$) *fmempty envs map2 vmatch ps vs* = *map Some envs*
by (*blast dest*: *those-someD*)

moreover have *fmpred* (λ-. *pred*) *env* **if** *env* ∈ *set envs* **for** *env*
proof −
from *that* **have** *Some env* ∈ *set* (*map2 vmatch ps vs*)
unfolding ‹*map2* - - - = -› **by** *simp*
then obtain *p v* **where** *p* ∈ *set ps v* ∈ *set vs vmatch p v* = *Some env*
apply (*rule map2-elemE*)
by *auto*
hence *pred v*
using *constr* **by** (*simp add*: *list-all-iff*)
show *?thesis*
by (*rule constr*; *safe?*) *fact+*
qed

ultimately show *?case*
by *auto*
qed *auto*

end

primrec *value-to-sterm* :: *value* ⇒ *sterm* **where**
value-to-sterm (*Vconstr name vs*) = *name* $$ *map value-to-sterm vs* |
value-to-sterm (*Vabs cs* Γ) = *Sabs* (*map* (λ(*pat*, *t*). (*pat*, *subst t* (*fmdrop-fset* (*frees pat*) (*fmmap value-to-sterm* Γ)))) *cs*) |
value-to-sterm (*Vrecabs css name* Γ) =
Sabs (*map* (λ(*pat*, *t*). (*pat*, *subst t* (*fmdrop-fset* (*frees pat*) (*fmmap value-to-sterm* Γ)))) (*the* (*fmlookup css name*)))

This locale establishes a connection between a predicate on *value*s with the corresponding predicate on *sterm*s, by means of *value-to-sterm*.

locale *pre-value-sterm-pred* = *value-pred* +
fixes *S*
assumes *value-to-sterm*: *pred v* ⟹ *S* (*value-to-sterm v*)
begin

corollary *value-to-sterm-env*:
assumes *fmpred* (λ-. *pred*) Γ
shows *fmpred* (λ-. *S*) (*fmmap value-to-sterm* Γ)
unfolding *fmpred-map* **proof**
fix *name v*
assume *fmlookup* Γ *name* = *Some v*
with *assms* **have** *pred v* **by** (*metis fmpredD*)

```
  thus S (value-to-sterm v) by (rule value-to-sterm)
qed

end

locale value-sterm-pred = value-pred + S: simple-syntactic-and S for S +
  assumes const: ⋀name. Q name ⟹ S (const name)
  assumes abs: ⋀Γ cs.
    (⋀n v. fmlookup Γ n = Some v ⟹ pred v ⟹ S (value-to-sterm v)) ⟹
    fmpred (λ-. pred) Γ ⟹
    P Γ cs ⟹
    S (Sabs (map (λ(pat, t). (pat, subst t (fmmap value-to-sterm (fmdrop-fset (frees
pat) Γ)))) cs))
begin

sublocale pre-value-sterm-pred
proof
  fix v
  assume pred v
  then show S (value-to-sterm v)
    proof (induction v)
      case (Vconstr x1 x2)
      show ?case
        apply simp
        unfolding S.list-comb
        apply rule
         apply (rule const)
        using Vconstr by (auto simp: list-all-iff)
    next
      case (Vabs x1 x2)
      show ?case
        apply auto
        apply (rule abs)
        using Vabs
        by (auto intro: fmran'I)
    next
      case (Vrecabs x1 x2 x3)
      show ?case
        apply auto
        apply (rule abs)
        using Vrecabs
        by (auto simp: fmlookup-dom-iff fmpred-iff intro: fmran'I)
    qed
qed

end

global-interpretation vwellformed:
  value-sterm-pred
```

```
    λ-. wellformed-clauses
    λ-. True
    λ-. True
    wellformed
  defines vwellformed = vwellformed.pred
proof (standard, goal-cases)
  case (2 Γ cs)
  hence cs ≠ []
    by simp

  moreover have wellformed (subst rhs (fmdrop-fset (frees pat) (fmmap value-to-sterm
Γ)))
    if (pat, rhs) ∈ set cs for pat rhs
    proof −
      show ?thesis
        apply (rule subst-wellformed)
        subgoal using 2 that by (force simp: list-all-iff)
        apply (rule fmpred-drop-fset)
        using 2 by auto
    qed

  moreover have distinct (map (fst ∘ (λ(pat, t). (pat, subst t (fmmap value-to-sterm
(fmdrop-fset (frees pat) Γ))))) cs)
    apply (subst map-cong[OF refl, where g = fst])
    using 2 by auto

  ultimately show ?case
    using 2 by (auto simp: list-all-iff)
qed (auto simp: const-sterm-def)

abbreviation wellformed-venv ≡ fmpred (λ-. vwellformed)

global-interpretation vclosed:
  value-sterm-pred
    λΓ cs. list-all (λ(pat, t). closed-except t (fmdom Γ |∪| frees pat)) cs
    λ-. True
    λ-. True
    closed
  defines vclosed = vclosed.pred
proof (standard, goal-cases)
  case (2 Γ cs)
  show ?case
    apply (simp add: list-all-iff case-prod-twice Sterm.closed-except-simps)
    apply safe
    apply (subst closed-except-def)
    apply (subst subst-frees)
     apply simp
    subgoal
      apply (rule fmpred-drop-fset)
```

```
    apply (rule fmpredI)
    apply (rule 2)
     apply assumption
    using 2 by auto
  subgoal
    using 2 by (auto simp: list-all-iff closed-except-def)
  done
qed simp

abbreviation closed-venv ≡ fmpred (λ-. vclosed)

context pre-constants begin

sublocale vwelldefined:
  value-sterm-pred
    λ- cs. list-all (λ(-, t). welldefined t) cs
    λname. name |∈| C
    λdom. dom |⊆| heads
    welldefined
  defines vwelldefined = vwelldefined.pred
proof (standard, goal-cases)
  case (2 Γ cs)
  note fset-of-list-map[simp del]

  show ?case
    apply simp
    apply (rule ffUnion-least)
    apply (rule fBallI)
    apply (subst (asm) fset-of-list-elem)
    apply simp
    apply (erule imageE)
    apply (simp add: case-prod-twice)
    subgoal for - x
      apply (cases x)
      apply simp
      apply (rule subst-consts)
      subgoal
        using 2 by (fastforce simp: list-all-iff)
      subgoal
        apply simp
        apply (rule fmpred-drop-fset)
        unfolding fmpred-map
        apply (rule fmpredI)
        using 2 by auto
      done
    done
qed (simp add: all-consts-def)

lemmas vwelldefined-alt-def = vwelldefined.pred-alt-def
```

end

declare *pre-constants.vwelldefined-alt-def*[*code*]

context *constructors* **begin**

sublocale *vconstructor-value*:
  pre-value-sterm-pred
    λ- -. True
    λname. name |∈| *C*
    λ-. True
    is-value
  defines *vconstructor-value* = *vconstructor-value.pred*
proof
  fix *v*
  assume *value-pred.pred* (*λ- -. True*) (*λname. name* |∈| *C*) (*λ-. True*) *v*
  then show *is-value* (*value-to-sterm v*)
    proof (*induction v*)
      case (*Vconstr name vs*)
      hence *list-all is-value* (*map value-to-sterm vs*)
        by (*fastforce simp*: *list-all-iff value-pred.pred-alt-def*)
      show *?case*
        unfolding *value-to-sterm.simps*
        apply (*rule is-value.constr*)
         apply *fact*
        using *Vconstr* **by** (*simp add*: *value-pred.pred-alt-def*)
    qed (*auto simp*: *disjnt-def intro*: *is-value.intros*)
qed

lemmas *vconstructor-value-alt-def* = *vconstructor-value.pred-alt-def*

abbreviation *vconstructor-value-env* ≡ *fmpred* (*λ-. vconstructor-value*)

definition *vconstructor-value-rs* :: *vrule list* ⇒ *bool* **where**
vconstructor-value-rs rs ⟷
  list-all (*λ*(-, *rhs*). *vconstructor-value rhs*) *rs* ∧
  fdisjnt (*fset-of-list* (*map fst rs*)) *C*

end

declare *constructors.vconstructor-value-alt-def*[*code*]
declare *constructors.vconstructor-value-rs-def*[*code*]

context *pre-constants* **begin**

sublocale *not-shadows-vconsts*:
  value-sterm-pred
    λ- cs. list-all (*λ*(*pat*, *t*). *fdisjnt all-consts* (*frees pat*) ∧ ¬ *shadows-consts t*) *cs*

```
    λ-. True
    λ-. True
    λt. ¬ shadows-consts t
  defines not-shadows-vconsts = not-shadows-vconsts.pred
proof (standard, goal-cases)
  case (2 Γ cs)
  show ?case
    apply (simp add: list-all-iff list-ex-iff case-prod-twice)
    apply (rule ballI)
    subgoal for x
      apply (cases x, simp)
      apply (rule conjI)
      subgoal
        using 2 by (force simp: list-all-iff)
      apply (rule subst-shadows)
      subgoal
        using 2 by (force simp: list-all-iff)
      apply simp
      apply (rule fmpred-drop-fset)
      apply (rule fmpredI)
      using 2 by auto
    done
qed (auto simp: const-sterm-def app-sterm-def)

lemmas not-shadows-vconsts-alt-def = not-shadows-vconsts.pred-alt-def

abbreviation not-shadows-vconsts-env ≡ fmpred (λ- s. not-shadows-vconsts s)

end

declare pre-constants.not-shadows-vconsts-alt-def[code]

fun term-to-value :: sterm ⇒ value where
term-to-value t =
  (case strip-comb t of
    (Sconst name, args) ⇒ Vconstr name (map term-to-value args)
  | (Sabs cs, []) ⇒ Vabs cs fmempty)

lemma (in constructors) term-to-value-to-sterm:
  assumes is-value t
  shows value-to-sterm (term-to-value t) = t
using assms proof induction
  case (constr vs name)

  have map (value-to-sterm ∘ term-to-value) vs = map id vs
    proof (rule list.map-cong0, unfold comp-apply id-apply)
      fix v
      assume v ∈ set vs
      with constr show value-to-sterm (term-to-value v) = v
```

by (*simp add*: *list-all-iff*)
qed
thus *?case*
apply (*simp add*: *strip-list-comb-const*)
apply (*subst const-sterm-def*)
by *simp*
qed *simp*

lemma *vmatch-dom*:
assumes *vmatch pat v* = *Some env*
shows *fmdom env* = *patvars pat*
using *assms* **proof** (*induction pat v arbitrary*: *env rule*: *vmatch-induct*)
case (*constr name ps name′ vs*)
hence
map-option (*foldl* ($++_f$) *fmempty*) (*those* (*map2 vmatch ps vs*)) = *Some env*
name = *name′ length ps* = *length vs*
by (*auto split*: *if-splits*)
then obtain *envs* **where** *env* = *foldl* ($++_f$) *fmempty envs map2 vmatch ps vs*
= *map Some envs*
by (*blast dest*: *those-someD*)

moreover have *fset-of-list* (*map fmdom envs*) = *fset-of-list* (*map patvars ps*)
proof *safe*
fix *names*
assume *names* |∈| *fset-of-list* (*map fmdom envs*)
hence *names* ∈ *set* (*map fmdom envs*)
unfolding *fset-of-list-elem* **.**
then obtain *env* **where** *env* ∈ *set envs names* = *fmdom env*
by *auto*
hence *Some env* ∈ *set* (*map2 vmatch ps vs*)
unfolding ‹*map2* - - - = -› **by** *simp*
then obtain *p v* **where** *p* ∈ *set ps v* ∈ *set vs vmatch p v* = *Some env*
by (*auto elim*: *map2-elemE*)
moreover hence *fmdom env* = *patvars p*
using *constr* **by** *fastforce*
ultimately have *names* ∈ *set* (*map patvars ps*)
unfolding ‹*names* = -› **by** *simp*
thus *names* |∈| *fset-of-list* (*map patvars ps*)
unfolding *fset-of-list-elem* **.**
next
fix *names*
assume *names* |∈| *fset-of-list* (*map patvars ps*)
hence *names* ∈ *set* (*map patvars ps*)
unfolding *fset-of-list-elem* **.**
then obtain *p* **where** *p* ∈ *set ps names* = *patvars p*
by *auto*
then obtain *v* **where** *v* ∈ *set vs vmatch p v* ∈ *set* (*map2 vmatch ps vs*)
using ‹*length ps* = *length vs*› **by** (*auto elim*!: *map2-elemE1*)
then obtain *env* **where** *env* ∈ *set envs vmatch p v* = *Some env*

unfolding ‹*map2 vmatch ps vs* = -› **by** *auto*
moreover hence *fmdom env* = *patvars p*
using *constr* ‹*name* = *name*′› ‹*length ps* = *length vs*› ‹*p* ∈ *set ps*› ‹*v* ∈ *set vs*›
by *fastforce*
ultimately have *names* ∈ *set* (*map fmdom envs*)
unfolding ‹*names* = -› **by** *auto*
thus *names* |∈| *fset-of-list* (*map fmdom envs*)
unfolding *fset-of-list-elem* **.**
qed

ultimately show *?case*
by (*auto simp*: *fmdom-foldl-add*)
qed *auto*

fun *vfind-match* :: *sclauses* ⇒ *value* ⇒ ((*name*, *value*) *fmap* × *term* × *sterm*) *option* **where**
vfind-match [] - = *None* |
vfind-match ((*pat*, *rhs*) # *cs*) *t* =
(*case vmatch* (*mk-pat pat*) *t of*
Some env ⇒ *Some* (*env*, *pat*, *rhs*)
| *None* ⇒ *vfind-match cs t*)

lemma *vfind-match-elem*:
assumes *vfind-match cs t* = *Some* (*env*, *pat*, *rhs*)
shows (*pat*, *rhs*) ∈ *set cs vmatch* (*mk-pat pat*) *t* = *Some env*
using *assms*
by (*induct cs*) (*auto split*: *option.splits*)

inductive *veq-structure* :: *value* ⇒ *value* ⇒ *bool* **where**
abs-abs: *veq-structure* (*Vabs* - -) (*Vabs* - -) |
recabs-recabs: *veq-structure* (*Vrecabs* - - -) (*Vrecabs* - - -) |
constr-constr: *list-all2 veq-structure ts us* ⟹ *veq-structure* (*Vconstr name ts*) (*Vconstr name us*)

lemma *veq-structure-simps*[*code*, *simp*]:
veq-structure (*Vabs* cs_1 Γ_1) (*Vabs* cs_2 Γ_2)
veq-structure (*Vrecabs* css_1 $name_1$ Γ_1) (*Vrecabs* css_2 $name_2$ Γ_2)
veq-structure (*Vconstr* $name_1$ *ts*) (*Vconstr* $name_2$ *us*) ⟷ $name_1$ = $name_2$ ∧ *list-all2 veq-structure ts us*
by (*auto intro*: *veq-structure.intros elim*: *veq-structure.cases*)

lemma *veq-structure-refl*[*simp*]: *veq-structure t t*
by (*induction t*) (*auto simp*: *list.rel-refl-strong*)

global-interpretation *vno-abs*: *value-pred* λ- -. *False* λ-. *True* λ-. *False*
defines *vno-abs* = *vno-abs.pred* **.**

lemma *veq-structure-eq-left*:

```
  assumes veq-structure t u vno-abs t
  shows t = u
using assms proof (induction rule: veq-structure.induct)
  case (constr-constr ts us name)
  have ts = us if list-all vno-abs ts
    using constr-constr.IH that
    by induction auto
  with constr-constr show ?case
    by auto
qed auto

lemma veq-structure-eq-right:
  assumes veq-structure t u vno-abs u
  shows t = u
using assms proof (induction rule: veq-structure.induct)
  case (constr-constr ts us name)
  have ts = us if list-all vno-abs us
    using constr-constr.IH that
    by induction auto
  with constr-constr show ?case
    by auto
qed auto

fun vmatch' :: pat ⇒ value ⇒ (name, value) fmap option where
vmatch' (Patvar name) v = Some (fmap-of-list [(name, v)]) |
vmatch' (Patconstr name ps) v =
  (case v of
    Vconstr name' vs ⇒
      (if name = name' ∧ length ps = length vs then
        map-option (foldl (++f) fmempty) (those (map2 vmatch' ps vs))
      else
        None)
  | - ⇒ None)

lemma vmatch-vmatch'-eq: vmatch p v = vmatch' p v
proof (induction rule: vmatch.induct)
  case (2 name ps name' vs)
  then show ?case
    apply auto
    apply (rule map-option-cong[OF - refl])
    apply (rule arg-cong[where f = those])
    apply (rule map2-cong[OF refl refl])
    apply blast
    done
qed auto

locale value-struct-rel =
  fixes Q :: value ⇒ value ⇒ bool
  assumes Q-impl-struct: Q t1 t2 ⟹ veq-structure t1 t2
```

assumes *Q-def*[*simp*]: *Q* (*Vconstr name ts*) (*Vconstr name′ us*) ⟷ *name* = *name′* ∧ *list-all2 Q ts us*
begin

lemma *eq-left*: *Q t u* ⟹ *vno-abs t* ⟹ *t* = *u*
using *Q-impl-struct* **by** (*metis veq-structure-eq-left*)

lemma *eq-right*: *Q t u* ⟹ *vno-abs u* ⟹ *t* = *u*
using *Q-impl-struct* **by** (*metis veq-structure-eq-right*)

context begin

private lemma *vmatch′-rel*:
  assumes *Q* t_1 t_2
  shows *rel-option* (*fmrel Q*) (*vmatch′ p* t_1) (*vmatch′ p* t_2)
using *assms*(*1*) **proof** (*induction p arbitrary*: t_1 t_2)
  case (*Patconstr name ps*)
  with *Q-impl-struct* **have** *veq-structure* t_1 t_2
    by *blast*

  thus *?case*
    proof (*cases rule*: *veq-structure.cases*)
      case (*constr-constr ts us name′*)

      {
        assume *length ps* = *length ts*

        have *list-all2* (*rel-option* (*fmrel Q*)) (*map2 vmatch′ ps ts*) (*map2 vmatch′ ps us*)
          using ‹*list-all2 veq-structure ts us*› *Patconstr* ‹*length ps* = *length ts*›
          unfolding ‹t_1 = -› ‹t_2 = -›
          proof (*induction arbitrary*: *ps*)
            case (*Cons t ts u us ps0*)
            then obtain *p ps* **where** *ps0* = *p* # *ps*
              by (*cases ps0*) *auto*

            have *length ts* = *length us*
              using *Cons* **by** (*auto dest*: *list-all2-lengthD*)

            hence *Q t u*
              using ‹*Q* (*Vconstr name′* (*t* # *ts*)) (*Vconstr name′* (*u* # *us*))›
              by (*simp add*: *list-all-iff*)
            hence *rel-option* (*fmrel Q*) (*vmatch′ p t*) (*vmatch′ p u*)
              using *Cons* **unfolding** ‹*ps0* = -› **by** *simp*

            moreover have *list-all2* (*rel-option* (*fmrel Q*)) (*map2 vmatch′ ps ts*) (*map2 vmatch′ ps us*)
              apply (*rule Cons*)
              subgoal

apply (*rule Cons*)
unfolding ‹*ps0* = -› **apply** *simp*
by *assumption*
subgoal
using ‹*Q* (*Vconstr name′* (*t* # *ts*)) (*Vconstr name′* (*u* # *us*))› ‹*length ts* = *length us*›
by (*simp add*: *list-all-iff*)
subgoal
using ‹*length ps0* = *length* (*t* # *ts*)›
unfolding ‹*ps0* = -› **by** *simp*
done

ultimately show *?case*
unfolding ‹*ps0* = -›
by *auto*
qed *auto*

hence *rel-option* (*list-all2* (*fmrel Q*)) (*those* (*map2 vmatch′ ps ts*)) (*those* (*map2 vmatch′ ps us*))
by (*rule rel-funD*[*OF those-transfer*])

have
rel-option (*fmrel Q*)
(*map-option* (*foldl* ($++_f$) *fmempty*) (*those* (*map2 vmatch′ ps ts*)))
(*map-option* (*foldl* ($++_f$) *fmempty*) (*those* (*map2 vmatch′ ps us*)))
apply (*rule rel-funD*[*OF rel-funD*[*OF option.map-transfer*]])
apply *transfer-prover*
by *fact*
}
note ∗ = *this*

have *length ts* = *length us*
using *constr-constr* **by** (*auto dest*: *list-all2-lengthD*)

thus *?thesis*
unfolding ‹t_1 = -› ‹t_2 = -›
apply *auto*
apply (*rule* ∗)
by *simp*
qed *auto*
qed *auto*

lemma *vmatch-rel*: *Q* t_1 t_2 ⟹ *rel-option* (*fmrel Q*) (*vmatch p* t_1) (*vmatch p* t_2)
unfolding *vmatch-vmatch′-eq* **by** (*rule vmatch′-rel*)

lemma *vfind-match-rel*:
assumes *list-all2* (*rel-prod* (=) *R*) cs_1 cs_2
assumes *Q* t_1 t_2
shows *rel-option* (*rel-prod* (*fmrel Q*) (*rel-prod* (=) *R*)) (*vfind-match* cs_1 t_1)

$(vfind\text{-}match\ cs_2\ t_2)$
using *assms(1)* **proof** *induction*
  case $(Cons\ c_1\ cs_1\ c_2\ cs_2)$
  moreover obtain $pat_1\ rhs_1$ **where** $c_1 = (pat_1, rhs_1)$ **by** *fastforce*
  moreover obtain $pat_2\ rhs_2$ **where** $c_2 = (pat_2, rhs_2)$ **by** *fastforce*

  ultimately have $pat_1 = pat_2\ R\ rhs_1\ rhs_2$
    by *auto*

  have $rel\text{-}option\ (fmrel\ Q)\ (vmatch\ (mk\text{-}pat\ pat_1)\ t_1)\ (vmatch\ (mk\text{-}pat\ pat_1)\ t_2)$
    by *(rule vmatch-rel) fact*
  thus *?case*
    proof *cases*
      case *None*
      thus *?thesis*
        unfolding ‹c_1 = -› ‹c_2 = -› ‹pat_1 = -›
        using *Cons* **by** *auto*
    next
      case $(Some\ \Gamma_1\ \Gamma_2)$
      thus *?thesis*
        unfolding ‹c_1 = -› ‹c_2 = -› ‹pat_1 = -›
        using ‹$R\ rhs_1\ rhs_2$›
        by *auto*
    qed
qed *simp*

lemmas *vfind-match-rel′* =
  vfind-match-rel[
    where $R = (=)$ **and** $cs_1 = cs$ **and** $cs_2 = cs$ **for** *cs*,
    unfolded prod.rel-eq,
    OF list.rel-refl, *OF refl*]

end end

hide-fact *vmatch-vmatch′-eq*
hide-const *vmatch′*

global-interpretation *veq-structure*: *value-struct-rel veq-structure*
by *standard auto*

abbreviation *env-eq* **where**
$env\text{-}eq \equiv fmrel\ (\lambda v\ t.\ t = value\text{-}to\text{-}sterm\ v)$

lemma *env-eq-eq*:
  assumes *env-eq venv senv*
  shows $senv = fmmap\ value\text{-}to\text{-}sterm\ venv$
proof *(rule fmap-ext, unfold fmlookup-map)*
  fix *name*
  from *assms* **have** $rel\text{-}option\ (\lambda v\ t.\ t = value\text{-}to\text{-}sterm\ v)\ (fmlookup\ venv\ name)$

(*fmlookup senv name*)
    by *auto*
  thus *fmlookup senv name* = *map-option value-to-sterm* (*fmlookup venv name*)
    by *cases auto*
qed

context *constructors* **begin**

context begin

private lemma *vmatch-eq0*: *rel-option env-eq* (*vmatch p v*) (*smatch' p* (*value-to-sterm v*))
proof (*induction p v rule*: *vmatch-induct*)
  case (*constr name ps name' vs*)

  have
    rel-option env-eq
      (*map-option* (*foldl* ($++_f$) Γ) (*those* (*map2 vmatch ps vs*)))
      (*map-option* (*foldl* ($++_f$) Γ') (*those* (*map2 smatch' ps* (*map value-to-sterm vs*))))
    if *length ps* = *length vs* **and** *name* = *name'* **and** *env-eq* Γ Γ' **for** Γ Γ'
    using *that constr*
    proof (*induction arbitrary*: Γ Γ' *rule*: *list-induct2*)
      case (*Cons p ps v vs*)
      hence *rel-option env-eq* (*vmatch p v*) (*smatch' p* (*value-to-sterm v*))
        by *auto*
      thus *?case*
        proof *cases*
          case (*Some* Γ_1 Γ_2)
          thus *?thesis*
            apply (*simp add*: *option.map-comp comp-def*)
            apply (*rule Cons*)
            using *Cons* **by** *auto*
        qed *simp*
    qed *fastforce*

  thus *?case*
    apply (*auto simp*: *strip-list-comb-const*)
    apply (*subst const-sterm-def*, *simp*)+
    done
qed *auto*

corollary *vmatch-eq*:
  assumes *linear p vconstructor-value v*
  shows *rel-option env-eq* (*vmatch* (*mk-pat p*) *v*) (*match p* (*value-to-sterm v*))
using *assms*
by (*metis smatch-smatch'-eq vmatch-eq0 vconstructor-value.value-to-sterm*)

end

end

abbreviation *match-related* **where**
match-related ≡ (λ(Γ_1, pat_1, rhs_1) (Γ_2, pat_2, rhs_2). rhs_1 = rhs_2 ∧ pat_1 = pat_2 ∧ *env-eq* Γ_1 Γ_2)

lemma (**in** *constructors*) *find-match-eq*:
  assumes *list-all* (*linear* ∘ *fst*) *cs vconstructor-value v*
  shows *rel-option match-related* (*vfind-match cs v*) (*find-match cs* (*value-to-sterm v*))
using *assms* **proof** (*induct cs*)
  case (*Cons c cs*)
  then obtain *p t* **where** *c* = (*p*, *t*) **by** *fastforce*
  hence *rel-option env-eq* (*vmatch* (*mk-pat p*) *v*) (*match p* (*value-to-sterm v*))
    using *Cons* **by** (*fastforce intro*: *vmatch-eq*)
  thus *?case*
    using *Cons* **unfolding** ‹*c* = -›
    by *cases auto*
qed *auto*

inductive *erelated* :: *value* ⇒ *value* ⇒ *bool* (‹- $\approx_e$ -›) **where**
constr: *list-all2 erelated ts us* ⟹ *Vconstr name ts* $\approx_e$ *Vconstr name us* |
abs: *fmrel-on-fset* (*ids* (*Sabs cs*)) *erelated* Γ_1 Γ_2 ⟹ *Vabs cs* Γ_1 $\approx_e$ *Vabs cs* Γ_2 |
rec-abs:
  pred-fmap (λ*cs*. *fmrel-on-fset* (*ids* (*Sabs cs*)) *erelated* Γ_1 Γ_2) *css* ⟹
    Vrecabs css name Γ_1 $\approx_e$ *Vrecabs css name* Γ_2

code-pred *erelated* **.**

global-interpretation *erelated*: *value-struct-rel erelated*
proof
  fix v_1 v_2
  assume $v_1 \approx_e v_2$
  thus *veq-structure* v_1 v_2
    by *induction* (*auto intro*: *list.rel-mono-strong*)
next
  fix *name name′* **and** *ts us* :: *value list*
  show *Vconstr name ts* $\approx_e$ *Vconstr name′ us* ⟷ (*name* = *name′* ∧ *list-all2 erelated ts us*)
    by (*auto intro*: *erelated.intros elim*: *erelated.cases*)
qed

lemma *erelated-refl*[*intro*]: $t \approx_e t$
proof (*induction t*)
  case *Vrecabs*
  thus *?case*
    apply (*auto intro*!: *erelated.intros fmpredI fmrel-on-fset-refl-strong*)
    apply (*auto intro*: *fmran′I*)

done
qed (*auto intro*!: *erelated.intros list.rel-refl-strong fmrel-on-fset-refl-strong fmran'I*)

export-code
value-to-sterm vmatch vwellformed vclosed erelated-i-i pre-constants.vwelldefined constructors.vconstructor-value-rs pre-constants.not-shadows-vconsts term-to-value vfind-match veq-structure vno-abs
checking *Scala*

end

Chapter 2

A smaller version of CakeML: *CupCakeML*

theory *Doc-CupCake*
imports *Main*
begin

end

2.1 CupCake environments

theory *CupCake-Env*
imports *../Utils/CakeML-Utils*
begin

fun *cake-no-abs* :: $v \Rightarrow$ *bool* **where**
cake-no-abs (*Conv* - *vs*) $\longleftrightarrow$ *list-all cake-no-abs vs* |
cake-no-abs - $\longleftrightarrow$ *False*

fun *is-cupcake-pat* :: *Ast.pat* $\Rightarrow$ *bool* **where**
is-cupcake-pat (*Ast.Pvar* -) $\longleftrightarrow$ *True* |
is-cupcake-pat (*Ast.Pcon* (*Some* (*Short* -)) *xs*) $\longleftrightarrow$ *list-all is-cupcake-pat xs* |
is-cupcake-pat - $\longleftrightarrow$ *False*

fun *is-cupcake-exp* :: *exp* $\Rightarrow$ *bool* **where**
is-cupcake-exp (*Ast.Var* (*Short* -)) $\longleftrightarrow$ *True* |
is-cupcake-exp (*Ast.App oper es*) $\longleftrightarrow$ *oper* = *Ast.Opapp* $\wedge$ *list-all is-cupcake-exp es* |
is-cupcake-exp (*Ast.Con* (*Some* (*Short* -)) *es*) $\longleftrightarrow$ *list-all is-cupcake-exp es* |
is-cupcake-exp (*Ast.Fun* - *e*) $\longleftrightarrow$ *is-cupcake-exp e* |
is-cupcake-exp (*Ast.Mat e cs*) $\longleftrightarrow$ *is-cupcake-exp e* $\wedge$ *list-all* ($\lambda(p, e)$. *is-cupcake-pat p* $\wedge$ *is-cupcake-exp e*) *cs* $\wedge$ *cake-linear-clauses cs* |
is-cupcake-exp - $\longleftrightarrow$ *False*

abbreviation *cupcake-clauses* :: (*Ast.pat* × *exp*) *list* ⇒ *bool* **where**
cupcake-clauses ≡ *list-all* (λ(*p*, *e*). *is-cupcake-pat p* ∧ *is-cupcake-exp e*)

fun *cupcake-c-ns* :: *c-ns* ⇒ *bool* **where**
cupcake-c-ns (*Bind cs mods*) ⟷
  mods = [] ∧ *list-all* (λ(-, -, *tid*). *case tid of TypeId* (*Short* -) ⇒ *True* | - ⇒ *False*)
cs

locale *cakeml-static-env* =
  fixes *static-cenv* :: *c-ns*
  assumes *static-cenv*: *cupcake-c-ns static-cenv*
begin

definition *empty-sem-env* :: *v sem-env* **where**
empty-sem-env = (| *sem-env.v* = *nsEmpty*, *sem-env.c* = *static-cenv* |)

lemma *v-of-empty-sem-env*[*simp*]: *sem-env.v empty-sem-env* = *nsEmpty*
unfolding *empty-sem-env-def* **by** *simp*

lemma *c-of-empty-sem-env*[*simp*]: *c empty-sem-env* = *static-cenv*
unfolding *empty-sem-env-def* **by** *simp*

fun *is-cupcake-value* :: *SemanticPrimitives.v* ⇒ *bool*
and *is-cupcake-all-env* :: *all-env* ⇒ *bool* **where**
is-cupcake-value (*Conv* (*Some* (-, *TypeId* (*Short* -))) *vs*) ⟷ *list-all is-cupcake-value vs* |
is-cupcake-value (*Closure env* - *e*) ⟷ *is-cupcake-exp e* ∧ *is-cupcake-all-env env* |
is-cupcake-value (*Recclosure env es* -) ⟷ *list-all* (λ(-, -, *e*). *is-cupcake-exp e*) *es* ∧ *is-cupcake-all-env env* |
is-cupcake-value - ⟷ *False* |
is-cupcake-all-env (| *sem-env.v* = *Bind v0* [], *sem-env.c* = *c0* |) ⟷ *c0* = *static-cenv* ∧ *list-all* (*is-cupcake-value* ∘ *snd*) *v0* |
is-cupcake-all-env - ⟷ *False*

lemma *is-cupcake-all-envE*:
  assumes *is-cupcake-all-env env*
  obtains *v c* **where** *env* = (| *sem-env.v* = *Bind v* [], *sem-env.c* = *c* |) *c* = *static-cenv list-all* (*is-cupcake-value* ∘ *snd*) *v*
using *assms*
by (*auto elim*!: *is-cupcake-all-env.elims*)

fun *is-cupcake-ns* :: *v-ns* ⇒ *bool* **where**
is-cupcake-ns (*Bind v0* []) ⟷ *list-all* (*is-cupcake-value* ∘ *snd*) *v0* |
is-cupcake-ns - ⟷ *False*

lemma *is-cupcake-nsE*:
  assumes *is-cupcake-ns ns*
  obtains *v* **where** *ns* = *Bind v* [] *list-all* (*is-cupcake-value* ∘ *snd*) *v*
using *assms* **by** (*rule is-cupcake-ns.elims*)

lemma *is-cupcake-all-envD*:
  assumes *is-cupcake-all-env env*
  shows *is-cupcake-ns* (*sem-env.v env*) *cupcake-c-ns* (*c env*)
using *assms static-cenv* **by** (*auto elim*!: *is-cupcake-all-envE*)

lemma *is-cupcake-all-envI*:
  assumes *is-cupcake-ns* (*sem-env.v env*) *sem-env.c env* = *static-cenv*
  shows *is-cupcake-all-env env*
using *assms static-cenv*
apply (*cases env*)
apply *simp*
subgoal for *v c*
  apply (*cases v*)
  apply *simp*
  subgoal for *x1 x2*
    by (*cases x2*) *auto*
  done
done

end

end

2.2 CupCake semantics

theory *CupCake-Semantics*
imports
  CupCake-Env
  CakeML.Matching
  CakeML.Big-Step-Unclocked-Single
begin

fun *cupcake-nsLookup* :: ($'m$,$'n$,$'v$)*namespace* ⇒ $'n$ ⇒ $'v$ *option* **where**
cupcake-nsLookup (*Bind v1* -) *n* = *map-of v1 n*

lemma *cupcake-nsLookup-eq*[*simp*]: *nsLookup ns* (*Short n*) = *cupcake-nsLookup ns* *n*
by (*cases ns*) *auto*

fun *cupcake-pmatch* :: ((*string*),(*string*),(*nat*∗*tid-or-exn*))*namespace* ⇒ *pat* ⇒ *v* ⇒(*string*∗*v*)*list* ⇒((*string*∗*v*)*list*)*match-result* **where**
cupcake-pmatch cenv (*Pvar x*) *v0 env* = *Match* ((*x*, *v0*)# *env*) |
cupcake-pmatch cenv (*Pcon* (*Some* (*Short n*)) *ps*) (*Conv* (*Some* (n', t')) *vs*) *env* =
  (*case cupcake-nsLookup cenv n of*
      Some (*l*, *t*)=>
        if same-tid t t' ∧ (*List.length ps* = *l*) *then*
          if same-ctor (*n*, *t*) (n',t') *then*
            Matching.fold2 (λ*p v m*. *case m of*

Match env ⇒ *cupcake-pmatch cenv p v env*
| *m* ⇒ *m*) *Match-type-error ps vs* (*Match env*)
else
No-match
else
Match-type-error
| - => *Match-type-error*) |
cupcake-pmatch cenv - - - = *Match-type-error*

fun *cupcake-match-result* :: - ⇒ *v* ⇒(*pat*∗*exp*)*list* ⇒ *v* ⇒ (*exp* × *pat* × (*char list* × *v*) *list*, *v*)*result* **where**
cupcake-match-result - - [] *err-v* = *Rerr* (*Rraise err-v*) |
cupcake-match-result cenv v0 ((*p*, *e*) # *pes*) *err-v* =
(*if distinct* (*pat-bindings p* []) *then*
(*case cupcake-pmatch cenv p v0* [] *of*
Match env′ ⇒ *Rval* (*e*, *p*, *env*′) |
No-match ⇒ *cupcake-match-result cenv v0 pes err-v* |
Match-type-error ⇒ *Rerr* (*Rabort Rtype-error*))
else
Rerr (*Rabort Rtype-error*))

lemma *cupcake-match-resultE*:
assumes *cupcake-match-result cenv v0 pes err-v* = *Rval* (*e*, *p*, *env*′)
obtains *init rest*
where *pes* = *init* @ (*p*, *e*) # *rest*
and *distinct* (*pat-bindings p* [])
and *list-all* (λ(*p*, *e*). *cupcake-pmatch cenv p v0* [] = *No-match* ∧ *distinct* (*pat-bindings p* [])) *init*
and *cupcake-pmatch cenv p v0* [] = *Match env*′
using *assms*
proof (*induction pes*)
case (*Cons pe pes*)
obtain *p0 e0* **where** *pe* = (*p0*, *e0*)
by *fastforce*

show *thesis*
proof (*cases distinct* (*pat-bindings p0* []))
case *True*
thus *?thesis*
proof (*cases cupcake-pmatch cenv p0 v0* [])
case *No-match*
show *?thesis*
proof (*rule Cons*)
fix *init rest*
assume *pes* = *init* @ (*p*, *e*) # *rest*
assume *list-all* (λ(*p*, *e*). *cupcake-pmatch cenv p v0* [] = *No-match* ∧ *distinct* (*pat-bindings p* [])) *init*
assume *distinct* (*pat-bindings p* [])
assume *cupcake-pmatch cenv p v0* [] = *Match env*′

```
        moreover have pe # pes = ((p0, e0) # init) @ (p, e) # rest
          unfolding ‹pes = -› ‹pe = -› by simp

          moreover have list-all (λ(p, e). cupcake-pmatch cenv p v0 [] =
No-match ∧ distinct (pat-bindings p [])) ((p0, e0) # init)
          apply auto
          subgoal by fact
          subgoal using True by simp
          subgoal using ‹list-all - -› by simp
          done

        moreover have distinct (pat-bindings p [])
          by fact

        ultimately show thesis
          using Cons by blast
      next
        show cupcake-match-result cenv v0 pes err-v = Rval (e, p, env')
          using Cons No-match True unfolding ‹pe = -› by auto
      qed
    next
      case Match
      with Cons show ?thesis
        using True unfolding ‹pe = -› by force
    next
      case Match-type-error
      with Cons show ?thesis
        using True unfolding ‹pe = -› by force
    qed
  next
    case False
    hence False
      using Cons unfolding ‹pe = -› by force
    thus ?thesis ..
  qed
qed simp

lemma cupcake-pmatch-eq:
  is-cupcake-pat pat ⟹ pmatch-single envC s pat v0 env = cupcake-pmatch envC
pat v0 env
proof (induct rule: pmatch-single.induct)
  case 4
  from is-cupcake-pat.elims(2)[OF 4(2)] show ?case
    proof cases
      case 2
      then show ?thesis
        using 4(1) apply −
        apply simp
```

```
        apply (auto split: option.splits match-result.splits)
        apply (rule Matching.fold2-cong)
          apply (auto simp: fun-eq-iff split: match-result.splits)
        apply (metis in-set-conv-decomp-last list.pred-inject(2) list-all-append)
        done
    qed simp
qed auto

lemma cupcake-match-result-eq:
  cupcake-clauses pes ⟹
    match-result env s v pes err-v =
      map-result (λ(e, -, env'). (e, env')) id (cupcake-match-result (c env) v pes
err-v)
by (induction pes) (auto split: match-result.splits simp: cupcake-pmatch-eq pmatch-single-equiv)

context cakeml-static-env begin

lemma cupcake-nsBind-preserve:
  is-cupcake-ns ns ⟹ is-cupcake-value v0 ⟹ is-cupcake-ns (nsBind k v0 ns)
by (cases ns) (auto elim: is-cupcake-ns.elims)

lemma cupcake-build-rec-preserve:
 assumes is-cupcake-all-env cl-env is-cupcake-ns env list-all (λ(-, -, e). is-cupcake-exp
e) fs
  shows is-cupcake-ns (build-rec-env fs cl-env env)
proof −
  have is-cupcake-ns (foldr (λ(f, -) env'. nsBind f (Recclosure cl-env fs0 f) env')
fs env)
    if list-all (λ(-, -, e). is-cupcake-exp e) fs0
    for fs0
    using ‹is-cupcake-ns env›
    proof (induction fs arbitrary: env)
      case (Cons f fs)
      show ?case
        apply (cases f, simp)
        apply (rule cupcake-nsBind-preserve)
         apply (rule Cons.IH)
         apply (rule Cons)
        using that assms by auto
    qed auto
  thus ?thesis
    unfolding build-rec-env-def
    using assms
    by (simp add: cond-case-prod-eta)
qed

lemma cupcake-v-update-preserve:
  assumes is-cupcake-all-env env is-cupcake-ns (f (sem-env.v env))
  shows is-cupcake-all-env (sem-env.update-v f env)
```

```
using assms
 by (metis is-cupcake-all-env.simps(1) is-cupcake-all-envE is-cupcake-nsE sem-env.collapse
sem-env.record-simps(1) sem-env.record-simps(2) sem-env.sel(2))

lemma cupcake-nsAppend-preserve: is-cupcake-ns ns1 ⟹ is-cupcake-ns ns2 ⟹
is-cupcake-ns (nsAppend ns1 ns2)
by (auto elim!: is-cupcake-ns.elims)

lemma cupcake-alist-to-ns-preserve: list-all (is-cupcake-value ∘ snd) env ⟹ is-cupcake-ns
(alist-to-ns env)
unfolding alist-to-ns-def
by simp

lemma cupcake-opapp-preserve:
  assumes do-opapp vs = Some (env, e) list-all is-cupcake-value vs
  shows is-cupcake-all-env env is-cupcake-exp e
proof –
  obtain cl v0 where vs = [cl, v0]
    using assms
    by (cases vs rule: do-opapp.cases) auto
  with assms have is-cupcake-value cl is-cupcake-value v0
    by auto

  have is-cupcake-all-env env ∧ is-cupcake-exp e
    using ‹do-opapp vs = -› proof (cases rule: do-opapp-cases)
      case (closure env' n arg)
      then show ?thesis
        using ‹is-cupcake-value cl› ‹is-cupcake-value v0› ‹vs = [cl, v0]›
      by (auto intro: cupcake-v-update-preserve cupcake-nsBind-preserve dest:is-cupcake-all-envD(1))
    next
      case (recclosure env' funs name n)
      hence is-cupcake-all-env env'
        using ‹is-cupcake-value cl› ‹vs = [cl, v0]› by simp
      have (name, n, e) ∈ set funs
        using recclosure by (auto dest: map-of-SomeD)
      hence is-cupcake-exp e
        using ‹is-cupcake-value cl› ‹vs = [cl, v0]› recclosure
        by (auto simp: list-all-iff)
      thus ?thesis
         using ‹is-cupcake-all-env env'› ‹is-cupcake-value cl› ‹is-cupcake-value v0›
‹vs = [cl, v0]› recclosure
        unfolding ‹env = -›
      using cupcake-build-rec-preserve cupcake-nsBind-preserve cupcake-v-update-preserve
is-cupcake-all-envD(1)
        by auto
    qed

  thus is-cupcake-all-env env is-cupcake-exp e
    by simp+
```

qed

context begin

lemma *cup-pmatch-list-length-neq*:
  $length\ vs \neq length\ ps \Longrightarrow Matching.fold2(\lambda p\ v\ m.\ case\ m\ of$
    $Match\ env \Rightarrow cupcake\text{-}pmatch\ cenv\ p\ v\ env$
    $\mid m \Rightarrow m)\ Match\text{-}type\text{-}error\ ps\ vs\ m = Match\text{-}type\text{-}error$
  by (*induction ps vs arbitrary*:*m rule*:*List.list-induct2'*) *auto*

lemma *cup-pmatch-list-nomatch*:
  $length\ vs = length\ ps \Longrightarrow Matching.fold2(\lambda p\ v\ m.\ case\ m\ of$
    $Match\ env \Rightarrow cupcake\text{-}pmatch\ cenv\ p\ v\ env$
    $\mid m \Rightarrow m)\ Match\text{-}type\text{-}error\ ps\ vs\ No\text{-}match = No\text{-}match$
  by (*induction ps vs rule*:*List.list-induct2'*) *auto*

lemma *cup-pmatch-list-typerr*:
  $length\ vs = length\ ps \Longrightarrow Matching.fold2(\lambda p\ v\ m.\ case\ m\ of$
    $Match\ env \Rightarrow cupcake\text{-}pmatch\ cenv\ p\ v\ env$
    $\mid m \Rightarrow m)\ Match\text{-}type\text{-}error\ ps\ vs\ Match\text{-}type\text{-}error = Match\text{-}type\text{-}error$
  by (*induction ps vs rule*:*List.list-induct2'*) *auto*

private lemma *cupcake-pmatch-list-preserve*:
  assumes $\bigwedge p\ v\ env.\ p \in set\ ps \wedge v \in set\ vs \longrightarrow list\text{-}all\ (is\text{-}cupcake\text{-}value \circ snd)$
$env \longrightarrow if\text{-}match\ (list\text{-}all\ (is\text{-}cupcake\text{-}value \circ snd))\ (cupcake\text{-}pmatch\ cenv\ p\ v\ env)$
$list\text{-}all\ (is\text{-}cupcake\text{-}value \circ snd)\ env$
  shows $if\text{-}match\ (list\text{-}all\ (\lambda a.\ is\text{-}cupcake\text{-}value\ (snd\ a)))\ (Matching.fold2$
    $(\lambda p\ v\ m.\ case\ m\ of$
      $Match\ env \Rightarrow cupcake\text{-}pmatch\ cenv\ p\ v\ env$
    $\mid m \Rightarrow m)$
    $Match\text{-}type\text{-}error\ ps\ vs\ (Match\ env))$
  using *assms* **proof** (*induction ps vs arbitrary*: *env rule*:*list-induct2'*)
  case (*4 p ps v vs*)
  show *?case*
  proof (*cases cupcake-pmatch cenv p v env*)
    case *No-match*
    then show *?thesis*
     by (*cases length ps = length vs*) (*auto simp*:*cup-pmatch-list-nomatch cup-pmatch-list-length-neq*)
  next
    case *Match-type-error*
    then show *?thesis*
     by (*cases length ps = length vs*) (*auto simp*:*cup-pmatch-list-typerr cup-pmatch-list-length-neq*)
  next
    case (*Match env'*)
    then have *env'*: $list\text{-}all\ (is\text{-}cupcake\text{-}value \circ snd)\ env'$
      using *4* **by** *fastforce*
      then show *?thesis*
      apply (*cases length ps = length vs*)
      using *4 Match* **by** *fastforce+*

```
  qed
qed (auto simp: comp-def)

private lemma cupcake-pmatch-preserve0:
  is-cupcake-pat pat ⟹
    is-cupcake-value v0 ⟹
     list-all (is-cupcake-value ∘ snd) env ⟹
      cupcake-c-ns envC ⟹
       if-match (list-all (is-cupcake-value ∘ snd)) (cupcake-pmatch envC pat v0 env)
proof (induction rule: cupcake-pmatch.induct)
  case (2 cenv n ps n' t' vs env)
  have p:p ∈ set ps ⟹ is-cupcake-pat p for p
    using 2 by (metis Ball-set is-cupcake-pat.simps(2))
  have v:v ∈ set vs ⟹ is-cupcake-value v for v
   using 2 by (metis Ball-set is-cupcake-value.elims(2) v.distinct(11) v.distinct(13)
v.inject(2))
  show ?case
    by (auto intro!: cupcake-pmatch-list-preserve split:if-splits option.splits) (metis
2 p v)+
qed (auto split: option.splits if-splits elim: is-cupcake-pat.elims is-cupcake-value.elims)

lemma cupcake-pmatch-preserve:
  is-cupcake-pat pat ⟹
    is-cupcake-value v0 ⟹
     list-all (is-cupcake-value ∘ snd) env ⟹
      cupcake-c-ns envC ⟹
       cupcake-pmatch envC pat v0 env = Match env' ⟹
        list-all (is-cupcake-value ∘ snd) env'
by (metis if-match.simps(1) cupcake-pmatch-preserve0)+

end

lemma cupcake-match-result-preserve:
  cupcake-c-ns envC ⟹
    cupcake-clauses pes ⟹
      is-cupcake-value v ⟹
      if-rval (λ(e, p, env'). is-cupcake-pat p ∧ is-cupcake-exp e ∧ list-all (is-cupcake-value
∘ snd) env')
          (cupcake-match-result envC v pes err-v)
  apply (induction pes)
   apply (auto split: match-result.splits)
  apply (rule cupcake-pmatch-preserve)
      apply auto
  done

lemma static-cenv-lookup:
  assumes cupcake-nsLookup static-cenv i = Some (len, b)
  obtains name where b = TypeId (Short name)
using assms static-cenv
```

apply (*cases static-cenv*; *cases b*)
apply (*auto simp*: *list-all-iff split*: *prod.splits tid-or-exn.splits id0.splits dest*!: *map-of-SomeD elim*!: *ballE allE*)
using *static-cenv*
apply (*auto simp*: *list-all-iff split*: *prod.splits tid-or-exn.splits id0.splits dest*!: *map-of-SomeD elim*!: *ballE allE*)
done

lemma *cupcake-build-conv-preserve*:
  fixes *v*
  assumes *list-all is-cupcake-value vs build-conv static-cenv* (*Some* (*Short i*)) *vs* = *Some v*
  shows *is-cupcake-value v*
using *assms*
by (*auto simp*: *build-conv.simps split*: *option.splits elim*: *static-cenv-lookup*)

lemma *cupcake-nsLookup-preserve*:
  assumes *is-cupcake-ns ns nsLookup ns n* = *Some v0*
  shows *is-cupcake-value v0*
proof −
  obtain *vs* **where** *list-all* (*is-cupcake-value* ∘ *snd*) *vs ns* = *Bind vs* []
    using *assms*
    by (*auto elim*: *is-cupcake-ns.elims*)
  show *?thesis*
    proof (*cases n*)
      case (*Short id*)
      hence (*id*, *v0*) ∈ *set vs*
        using *assms* **unfolding** ‹*ns* = -› **by** (*auto dest*: *map-of-SomeD*)
      thus *?thesis*
        using ‹*list-all* (*is-cupcake-value* ∘ *snd*) *vs*›
        by (*auto simp*: *list-all-iff*)
    next
      case *Long*
      hence *nsLookup ns n* = *None*
        unfolding ‹*ns* = -› **by** *simp*
      thus *?thesis*
        using *assms* **by** *auto*
    qed
qed

corollary *match-all-preserve*:
  assumes *cupcake-match-result cenv v0 pes err-v* = *Rval* (*e*, *p*, *env′*) *cupcake-c-ns cenv*
  assumes *is-cupcake-value v0 cupcake-clauses pes*
  shows *list-all* (*is-cupcake-value* ∘ *snd*) *env′ is-cupcake-exp e is-cupcake-pat p*
proof −
  from *assms* **obtain** *init rest*
    where *pes* = *init* @ (*p*, *e*) # *rest* **and** *cupcake-pmatch cenv p v0* [] = *Match env′*

by (*elim cupcake-match-resultE*)
hence $(p, e) \in$ *set pes*
by *simp*
thus *is-cupcake-exp e is-cupcake-pat p*
using *assms* **by** (*auto simp*: *list-all-iff*)

show *list-all* (*is-cupcake-value* $\circ$ *snd*) *env'*
by (*rule cupcake-pmatch-preserve*[**where** *env* = []]) (*fact* | *simp*)+
qed

end

fun *list-all2-shortcircuit* **where**
list-all2-shortcircuit P (*x* # *xs*) (*y* # *ys*) $\longleftrightarrow$ (*case y of Rval* - $\Rightarrow$ *P x y* $\wedge$ *list-all2-shortcircuit P xs ys* | *Rerr* - $\Rightarrow$ *P x y*) |
list-all2-shortcircuit P [] [] $\longleftrightarrow$ *True* |
list-all2-shortcircuit P - - $\longleftrightarrow$ *False*

lemma *list-all2-shortcircuit-induct*[*consumes 1*, *case-names nil cons-val cons-err*]:
assumes *list-all2-shortcircuit P xs ys*
assumes *R* [] []
assumes $\bigwedge$*x xs y ys*. *P x* (*Rval y*) $\Longrightarrow$ *list-all2-shortcircuit P xs ys* $\Longrightarrow$ *R xs ys* $\Longrightarrow$ *R* (*x* # *xs*) (*Rval y* # *ys*)
assumes $\bigwedge$*x xs y ys*. *P x* (*Rerr y*) $\Longrightarrow$ *R* (*x* # *xs*) (*Rerr y* # *ys*)
shows *R xs ys*
using *assms*
proof (*induction P xs ys rule*: *list-all2-shortcircuit.induct*)
case (*1 P x xs y ys*)
thus *?case*
by (*cases y*) *auto*
qed *auto*

lemma *list-all2-shortcircuit-mono*[*mono*]:
assumes $R \le Q$
shows *list-all2-shortcircuit R* $\le$ *list-all2-shortcircuit Q*
proof
fix *xs ys*
assume *list-all2-shortcircuit R xs ys*
thus *list-all2-shortcircuit Q xs ys*
using *assms* **by** (*induction xs ys rule*: *list-all2-shortcircuit-induct*) *auto*
qed

lemma *list-all2-shortcircuit-weaken*: *list-all2-shortcircuit P xs ys* $\Longrightarrow$ ($\bigwedge$*xs ys*. *P xs ys* $\Longrightarrow$ *Q xs ys*) $\Longrightarrow$ *list-all2-shortcircuit Q xs ys*
by (*metis list-all2-shortcircuit-mono predicate2I rev-predicate2D*)

lemma *list-all2-shortcircuit-rval*[*simp*]:
list-all2-shortcircuit P xs (*map Rval ys*) $\longleftrightarrow$ *list-all2* (λ*x y*. *P x* (*Rval y*)) *xs ys*
(**is** *?lhs* $\longleftrightarrow$ *?rhs*)

proof
  assume *?lhs* **thus** *?rhs*
   by (*induction map Rval ys*::(′*b*, ′*c*) *result list arbitrary*: *ys rule*: *list-all2-shortcircuit-induct*) *auto*
next
  assume *?rhs* **thus** *?lhs*
    by (*induction rule*: *list-all2-induct*) *auto*
qed

inductive *cupcake-evaluate-single* :: *all-env* ⇒ *exp* ⇒ (*v*, *v*) *result* ⇒ *bool* **where**
con1:
  do-con-check (*c env*) *cn* (*length es*) ⟹
   list-all2-shortcircuit (*cupcake-evaluate-single env*) (*rev es*) *rs* ⟹
   sequence-result rs = *Rval vs* ⟹
   build-conv (*c env*) *cn* (*rev vs*) = *Some v0* ⟹
   cupcake-evaluate-single env (*Con cn es*) (*Rval v0*) |
con2:
  ¬ *do-con-check* (*c env*) *cn* (*List.length es*) ⟹
   cupcake-evaluate-single env (*Con cn es*) (*Rerr* (*Rabort Rtype-error*)) |
con3:
  do-con-check (*c env*) *cn* (*List.length es*) ⟹
   list-all2-shortcircuit (*cupcake-evaluate-single env*) (*rev es*) *rs* ⟹
   sequence-result rs = *Rerr err* ⟹
   cupcake-evaluate-single env (*Con cn es*) (*Rerr err*) |
var1:
  nsLookup (*sem-env.v env*) *n* = *Some v0* ⟹ *cupcake-evaluate-single env* (*Var n*) (*Rval v0*) |
var2:
  nsLookup (*sem-env.v env*) *n* = *None* ⟹ *cupcake-evaluate-single env* (*Var n*) (*Rerr* (*Rabort Rtype-error*)) |
fn:
  cupcake-evaluate-single env (*Fun n e*) (*Rval* (*Closure env n e*)) |
app1:
  list-all2-shortcircuit (*cupcake-evaluate-single env*) (*rev es*) *rs* ⟹
   sequence-result rs = *Rval vs* ⟹
   do-opapp (*rev vs*) = *Some* (*env*′, *e*) ⟹
   cupcake-evaluate-single env′ *e bv* ⟹
   cupcake-evaluate-single env (*App Opapp es*) *bv* |
app3:
  list-all2-shortcircuit (*cupcake-evaluate-single env*) (*rev es*) *rs* ⟹
   sequence-result rs = *Rval vs* ⟹
   do-opapp (*rev vs*) = *None* ⟹
   cupcake-evaluate-single env (*App Opapp es*) (*Rerr* (*Rabort Rtype-error*)) |
app6:
  list-all2-shortcircuit (*cupcake-evaluate-single env*) (*rev es*) *rs* ⟹
   sequence-result rs = *Rerr err* ⟹
   cupcake-evaluate-single env (*App op0 es*) (*Rerr err*) |
mat1:
  cupcake-evaluate-single env e (*Rval v0*) ⟹

cupcake-match-result (*c env*) *v0 pes Bindv* = *Rval* (*e*′, -, *env*′) $\Longrightarrow$
cupcake-evaluate-single (*env* (| *sem-env.v* := *nsAppend* (*alist-to-ns env*′) (*sem-env.v env*) |)) *e*′ *bv* $\Longrightarrow$
cupcake-evaluate-single env (*Mat e pes*) *bv* |
mat1error:
cupcake-evaluate-single env e (*Rval v0*) $\Longrightarrow$
cupcake-match-result (*c env*) *v0 pes Bindv* = *Rerr err* $\Longrightarrow$
cupcake-evaluate-single env (*Mat e pes*) (*Rerr err*) |
mat2:
cupcake-evaluate-single env e (*Rerr err*) $\Longrightarrow$
cupcake-evaluate-single env (*Mat e pes*) (*Rerr err*)

context *cakeml-static-env* **begin**

context begin

private lemma *cupcake-list-preserve0*:
list-all2-shortcircuit
(λ*e r*. *cupcake-evaluate-single env e r* $\wedge$ (*is-cupcake-all-env env* $\longrightarrow$ *is-cupcake-exp e* $\longrightarrow$ *if-rval is-cupcake-value r*)) *es rs* $\Longrightarrow$
is-cupcake-all-env env $\Longrightarrow$ *list-all is-cupcake-exp es* $\Longrightarrow$ *sequence-result rs* = *Rval vs* $\Longrightarrow$ *list-all is-cupcake-value vs*
proof (*induction es rs arbitrary*: *vs rule*:*list-all2-shortcircuit-induct*)
case (*cons-val* - - - *rs*)
thus *?case*
by (*cases sequence-result rs*) *auto*
qed *auto*

private lemma *cupcake-single-preserve0*:
cupcake-evaluate-single env e res $\Longrightarrow$ *is-cupcake-all-env env* $\Longrightarrow$ *is-cupcake-exp e* $\Longrightarrow$ *if-rval is-cupcake-value res*
proof (*induction rule*:*cupcake-evaluate-single.induct*)
case (*con1 env cn es rs vs v0*)
then obtain *tid* **where** *cn*: *cn* = *Some* (*Short tid*) **and** *list-all is-cupcake-exp* (*rev es*)
by (*cases rule*: *is-cupcake-exp.cases*[**where** *x* = *Con cn es*]) *auto*
hence *list-all is-cupcake-value* (*rev vs*) **and** *c env* = *static-cenv*
using *cupcake-list-preserve0 con1*
by (*fastforce elim*:*is-cupcake-all-envE*)+

then show *?case*
using *cupcake-build-conv-preserve con1 cn*
by *fastforce*
next
case (*app1 env es rs vs env*′ *e bv*)
hence *list-all is-cupcake-exp* (*rev es*)
by *fastforce*
hence *list-all is-cupcake-value* (*rev vs*)
using *app1 cupcake-list-preserve0* **by** *force*

hence *is-cupcake-exp e* **and** *is-cupcake-all-env env′*
using *app1 cupcake-opapp-preserve* **by** *blast+*
then show *?case*
using *app1* **by** *blast*
next
case (*mat1 env e v0 pes e′ uu env′ bv*)
hence *cupcake-c-ns* (*c env*) *cupcake-clauses pes is-cupcake-value v0*
by (*auto dest*: *is-cupcake-all-envD*)
hence *list-all* (*is-cupcake-value* ∘ *snd*) *env′* **and** *e′*: *is-cupcake-exp e′*
using *cupcake-match-result-preserve*[**where** *envC* = *c env* **and** *v* = *v0* **and** *pes* = *pes* **and** *err-v* = *Bindv*, *unfolded mat1*, *simplified*]
by *auto*
have *is-cupcake-all-env* (*update-v* (λ-. *nsAppend* (*alist-to-ns env′*) (*sem-env.v env*)) *env*)
apply (*rule cupcake-v-update-preserve*)
apply *fact*
apply (*rule cupcake-nsAppend-preserve*)
apply (*rule cupcake-alist-to-ns-preserve*)
apply *fact*
apply (*rule is-cupcake-all-envD*)
apply *fact*
done
then show *?case*
using *mat1 e′* **by** *blast*
qed (*auto intro*: *cupcake-nsLookup-preserve dest*: *is-cupcake-all-envD*)

lemma *cupcake-single-preserve*:
cupcake-evaluate-single env e (*Rval res*) ⟹ *is-cupcake-all-env env* ⟹ *is-cupcake-exp e* ⟹ *is-cupcake-value res*
by (*fastforce dest*:*cupcake-single-preserve0*)

lemma *cupcake-list-preserve*:
list-all2-shortcircuit (*cupcake-evaluate-single env*) *es rs* ⟹
is-cupcake-all-env env ⟹ *list-all is-cupcake-exp es* ⟹ *sequence-result rs* = *Rval vs* ⟹ *list-all is-cupcake-value vs*
by (*induction es rs arbitrary*:*vs rule*:*list-all2-shortcircuit-induct*) (*fastforce dest*:*cupcake-single-preserve*)+

private lemma *cupcake-list-correct-rval*:
assumes *list-all2-shortcircuit*
(λ*e r*.
cupcake-evaluate-single env e r ∧
(*is-cupcake-all-env env* ⟶ *is-cupcake-exp e* ⟶ (∀ (*s*::′*a state*). ∃ *s′*. *evaluate env s e* (*s′*, *r*))))
es rs is-cupcake-all-env env list-all is-cupcake-exp es sequence-result rs = *Rval vs*
shows ∃ *s′*. *evaluate-list* (*evaluate env*) (*s*::′*a state*) *es* (*s′*,*Rval vs*)
using *assms* **proof** (*induction es rs arbitrary*: *s vs rule*:*list-all2-shortcircuit-induct*)
case (*cons-val e es y ys*)
have *e*: *is-cupcake-exp e list-all is-cupcake-exp es*

using *cons-val* **by** *fastforce*+
then obtain vs' **where** *ys*: *sequence-result ys* = *Rval* vs'
using *cons-val* **by** *fastforce*
hence *vs*: *Rval vs* = *Rval* (*y* # vs')
using *cons-val* **by** *fastforce*

from *e* **obtain** s' **where** *evaluate env s e* (s', *Rval y*)
using *cons-val* **by** *fastforce*
from *e ys* **obtain** s'' **where** *evaluate-list* (*evaluate env*) s' *es* (s'', *Rval* vs')
using *cons-val* **by** *fastforce*

show *?case*
unfolding *vs*
by (*rule*; *rule evaluate-list.cons1*) *fact*+
qed (*auto intro*:*evaluate-list.intros*)

private lemma *cupcake-list-correct-rerr*:
assumes *list-all2-shortcircuit*
(λ*e r*.
cupcake-evaluate-single env e r $\wedge$
(*is-cupcake-all-env env* $\longrightarrow$ *is-cupcake-exp e* $\longrightarrow$ ($\forall$ (*s*::'*a state*). $\exists$ s'. *evaluate env s e* (s', *r*))))
es rs is-cupcake-all-env env list-all is-cupcake-exp es sequence-result rs = *Rerr err*
shows $\exists$ s'. *evaluate-list* (*evaluate env*) (*s*::'*a state*) *es* (s',*Rerr err*)
using *assms* **proof** (*induction es rs arbitrary*: *s err rule*:*list-all2-shortcircuit-induct*)
case (*cons-val e es y ys*)
then have *is-cupcake-exp e list-all is-cupcake-exp es*
by *fastforce*+
moreover have *err*: *sequence-result ys* = *Rerr err*
using *cons-val*
by (*cases sequence-result ys*) (*auto simp*: *error-result.map-id*)

ultimately show *?case*
using *cons3 cons-val*
by *fast*
qed (*auto intro*:*evaluate-list.intros*)

private lemma *cupcake-list-correct0*:
assumes *list-all2-shortcircuit*
(λ*e r*.
cupcake-evaluate-single env e r $\wedge$
(*is-cupcake-all-env env* $\longrightarrow$ *is-cupcake-exp e* $\longrightarrow$ ($\forall$ (*s*::'*a state*). $\exists$ s'. *evaluate env s e* (s', *r*))))
es rs is-cupcake-all-env env list-all is-cupcake-exp es
shows $\exists$ s'. *evaluate-list* (*evaluate env*) (*s*::'*a state*) *es* (s',*sequence-result rs*)
using *assms* **by** (*cases sequence-result rs*) (*fastforce intro*: *cupcake-list-correct-rval cupcake-list-correct-rerr*)+

```
lemma cupcake-single-correct:
  assumes cupcake-evaluate-single env e res is-cupcake-all-env env is-cupcake-exp
e
  shows ∃ s'. Big-Step-Unclocked-Single.evaluate env s e (s',res)
using assms proof (induction arbitrary:s rule:cupcake-evaluate-single.induct)
  case (con1 env cn es rs vs v0)
  then have list-all is-cupcake-exp (rev es)
    by (cases rule: is-cupcake-exp.cases[where x = Con cn es]) auto
  then show ?case
    using cupcake-list-correct-rval evaluate.con1 con1
    by blast
next
  case (con3 env cn es rs err)
    then have list-all is-cupcake-exp (rev es)
    by (cases rule: is-cupcake-exp.cases[where x = Con cn es]) auto
  then show ?case
    using cupcake-list-correct-rerr con3 evaluate.con3
    by blast
next
  case (app1 env es rs vs env' e bv)
  hence es: list-all is-cupcake-exp (rev es)
    by fastforce
  hence list-all is-cupcake-value (rev vs)
    using app1 cupcake-list-preserve list-all2-shortcircuit-weaken
    by (metis (no-types, lifting) list-all-rev)
  hence is-cupcake-exp e and is-cupcake-all-env env'
    using app1 cupcake-opapp-preserve by blast+

  then show ?case
    using cupcake-list-correct-rval es app1 evaluate.app1
    by blast
next
  case (app3 env es rs vs)
  hence list-all is-cupcake-exp (rev es)
    by simp
  then show ?case
    using cupcake-list-correct-rval evaluate.app3 app3
    by blast
next
  case (app6 env es rs err op0)
  hence list-all is-cupcake-exp (rev es)
    by simp
  then show ?case
    using cupcake-list-correct-rerr app6 evaluate.app6
    by blast
next
  case (mat1 env e v0 pes e' uu env' bv)
  hence e: is-cupcake-exp e and cupcake-c-ns (c env) and pes: cupcake-clauses pes
and is-cupcake-value v0
```

by (*fastforce dest*: *is-cupcake-all-envD cupcake-single-preserve*)+
hence *list-all* (*is-cupcake-value* ∘ *snd*) *env′* **and** *e′*: *is-cupcake-exp e′*
using *cupcake-match-result-preserve*[**where** *envC* = *c env* **and** *v* = *v0* **and** *pes* = *pes* **and** *err-v* = *Bindv*, *unfolded mat1*, *simplified*]
by *blast*+
have *env′*: *is-cupcake-all-env* (*update-v* (λ-. *nsAppend* (*alist-to-ns env′*) (*sem-env.v env*)) *env*)
apply (*rule cupcake-v-update-preserve*)
apply *fact*
apply (*rule cupcake-nsAppend-preserve*)
apply (*rule cupcake-alist-to-ns-preserve*)
apply *fact*
apply (*rule is-cupcake-all-envD*)
apply *fact*
done

from *e* **obtain** *s′* **where** *evaluate env s e* (*s′*, *Rval v0*)
using *mat1* **by** *blast*
have *match-result env s′ v0 pes Bindv* = *Rval* (*e′*, *env′*)
using *mat1 cupcake-match-result-eq*[*OF pes*, **where** *env* = *env* **and** *v* = *v0* **and** *err-v* = *Bindv* **and** *s* = *s′*]
by *fastforce*
from *e′ env′* **obtain** *s″* **where** *evaluate* (*update-v* (λ-. *nsAppend* (*alist-to-ns env′*) (*sem-env.v env*)) *env*) *s′ e′* (*s″*, *bv*)
using *mat1* **by** *blast*

show *?case*
by *rule*+ *fact*+
next
case (*mat1error env e v0 pes err*)
hence *is-cupcake-exp e* **and** *pes*: *cupcake-clauses pes*
by (*auto dest*: *is-cupcake-all-envD*)

then obtain *s′* **where** *Big-Step-Unclocked-Single.evaluate env s e* (*s′*, *Rval v0*)
using *mat1error* **by** *blast*
hence *match-result env s′ v0 pes Bindv* = *Rerr err*
using *cupcake-match-result-eq*[*OF pes*, **where** *env* = *env* **and** *s* = *s′* **and** *v* = *v0* **and** *err-v* = *Bindv*] **unfolding** *mat1error*
by (*simp add*: *error-result.map-id*)

show *?case*
by (*rule*; *rule evaluate.mat1b*) *fact*+
next
case (*mat2* - *e*)
hence *is-cupcake-exp e*
by *simp*
then show *?case*
using *mat2 evaluate.mat2* **by** *blast*
qed (*blast intro*:*evaluate.intros*)+

lemma *cupcake-list-correct*:
assumes *list-all2-shortcircuit* (*cupcake-evaluate-single env*) *es rs is-cupcake-all-env env list-all is-cupcake-exp es*
shows $\exists s'$. *evaluate-list* (*evaluate env*) (*s*::*'a state*) *es* (*s'*,*sequence-result rs*)
using *assms* **by** (*fastforce intro*:*cupcake-list-correct0 list-all2-shortcircuit-weaken cupcake-single-correct*)+

private lemma *cupcake-list-complete0*:
evaluate-list
(λ*s e r*. *evaluate env s e r* $\wedge$ (*is-cupcake-all-env env* $\longrightarrow$ *is-cupcake-exp e* $\longrightarrow$ *cupcake-evaluate-single env e* (*snd r*))) *s1 es res* $\Longrightarrow$
is-cupcake-all-env env $\Longrightarrow$ *list-all is-cupcake-exp es* $\Longrightarrow$ $\exists$ *rs*. *list-all2-shortcircuit* (*cupcake-evaluate-single env*) *es rs* $\wedge$ *sequence-result rs* = (*snd res*)
proof (*induction rule*:*evaluate-list.induct*)
case *empty*
have *list-all2-shortcircuit* (*cupcake-evaluate-single env*) [] []
by *fastforce*
then show *?case*
by *fastforce*
next
case (*cons1 s1 e s2 v es s3 vs*)
then obtain *rs* **where** *list-all2-shortcircuit* (*cupcake-evaluate-single env*) *es rs*
and *sequence-result rs* = *Rval vs*
and *list-all2-shortcircuit* (*cupcake-evaluate-single env*) (*e* # *es*) (*Rval v* # *rs*)
by *fastforce*+
then show *?case*
by *fastforce*
next
case (*cons2 s1 e s2 err es*)
hence *list-all2-shortcircuit* (*cupcake-evaluate-single env*) (*e* # *es*) [*Rerr err*]
by *simp*
then show *?case*
by *fastforce*
next
case (*cons3 s1 e s2 v es s3 err*)
then obtain *rs* **where** *list-all2-shortcircuit* (*cupcake-evaluate-single env*) *es rs*
and *err*:*sequence-result rs* = *Rerr err*
and *list-all2-shortcircuit* (*cupcake-evaluate-single env*) (*e* # *es*) (*Rval v* # *rs*)
by *fastforce*
moreover have *sequence-result* (*Rval v* # *rs*) = *Rerr err*
by (*auto simp*: *error-result.map-id err*)
ultimately show *?case*
by *fastforce*
qed

private lemma *cupcake-single-complete0*:
evaluate env s e res $\Longrightarrow$ *is-cupcake-all-env env* $\Longrightarrow$ *is-cupcake-exp e* $\Longrightarrow$ *cupcake-evaluate-single env e* (*snd res*)

proof (*induction rule*:*evaluate.induct*)
  case (*con1 env cn es vs v s1 s2*)
  hence *list-all is-cupcake-exp* (*rev es*)
    by (*cases rule*: *is-cupcake-exp.cases*[**where** x = *Con cn es*]) *auto*
  hence *list-all2-shortcircuit* (*cupcake-evaluate-single env*) (*rev es*) (*map Rval vs*)
    using *cupcake-list-complete0 con1* **by** *fastforce*
  show *?case*
    by (*simp*|*rule*|*fact*)+
next
  case (*con3 env cn es s1 s2 err*)
  hence *list-all is-cupcake-exp* (*rev es*)
    by (*cases rule*: *is-cupcake-exp.cases*[**where** x = *Con cn es*]) *auto*
  then obtain *rs* **where** *list-all2-shortcircuit* (*cupcake-evaluate-single env*) (*rev es*) *rs sequence-result rs* = *Rerr err*
    using *con3* **by** (*fastforce dest*:*cupcake-list-complete0*)
  show *?case*
    by (*simp*;*rule cupcake-evaluate-single.con3*) *fact*+
next
  case (*app1 env s1 es s2 vs env′ e bv*)
  then obtain *rs* **where** *rs*: *list-all2-shortcircuit* (*cupcake-evaluate-single env*) (*rev es*) *rs sequence-result rs* = *Rval vs*
    by (*fastforce dest*:*cupcake-list-complete0*)
  hence *list-all is-cupcake-exp* (*rev es*)
    using *app1* **by** *fastforce*
  hence *list-all is-cupcake-value vs list-all is-cupcake-value* (*rev vs*)
    using *cupcake-list-preserve app1 rs* **by** *fastforce*+
  hence *is-cupcake-exp e is-cupcake-all-env env′*
    using *app1 cupcake-opapp-preserve* **by** *fastforce*+
  hence *cupcake-evaluate-single env′ e* (*snd bv*)
    using *app1* **by** *fastforce*
  show *?case*
    by *rule fact*+
next
  case (*app3 env s1 es s2 vs*)
  hence *list-all is-cupcake-exp* (*rev es*)
    by *simp*
  obtain *rs* **where** *list-all2-shortcircuit* (*cupcake-evaluate-single env*) (*rev es*) *rs sequence-result rs* = *Rval vs*
    using *app3 cupcake-list-complete0* **by** *fastforce*
  show *?case*
    by (*simp*|*rule*|*fact*)+
next
  case (*app6 env s1 es s2 err op0*)
  obtain *rs* **where** *list-all2-shortcircuit* (*cupcake-evaluate-single env*) (*rev es*) *rs sequence-result rs* = *Rerr err*
    using *cupcake-list-complete0 app6* **by** *fastforce*
  show *?case*
    by (*simp*|*rule*|*fact*)+
next

case (*mat1 env s1 e s2 v1 pes e′ env′ bv*)
hence *is-cupcake-exp e* **and** *cupcake-c-ns* (*c env*) **and** *pes*:*cupcake-clauses pes* **and** *is-cupcake-value v1*
by (*fastforce dest*: *is-cupcake-all-envD cupcake-single-preserve*)+

moreover obtain *uu* **where** *cupcake-match-result* (*c env*) *v1 pes Bindv* = *Rval* (*e′*, *uu*, *env′*)
using *cupcake-match-result-eq*[*OF pes*,**where** *env* = *env* **and** *s*= *s2* **and** *v* = *v1* **and** *err-v* = *Bindv*, *unfolded mat1*]
by (*cases cupcake-match-result* (*c env*) *v1 pes Bindv*) *auto*

ultimately have *list-all* (*is-cupcake-value* ∘ *snd*) *env′ is-cupcake-exp e′*
using *cupcake-match-result-preserve*[**where** *envC* = *c env* **and** *v* = *v1* **and** *pes* = *pes* **and** *err-v* = *Bindv*]
by *fastforce*+
moreover have *is-cupcake-all-env* (*update-v* (λ-. *nsAppend* (*alist-to-ns env′*) (*sem-env.v env*)) *env*)
apply (*rule cupcake-v-update-preserve*)
apply *fact*
apply (*rule cupcake-nsAppend-preserve*)
apply (*rule cupcake-alist-to-ns-preserve*)
apply *fact*
apply (*rule is-cupcake-all-envD*)
apply *fact*
done

ultimately have *cupcake-evaluate-single env e* (*Rval v1*)
and *cupcake-evaluate-single* (*update-v* (λ-. *nsAppend* (*alist-to-ns env′*) (*sem-env.v env*)) *env*) *e′* (*snd bv*)
using *mat1* **by** *fastforce*+

show *?case*
by (*rule cupcake-evaluate-single.mat1*) *fact*+
next
case (*mat1b env s1 e s2 v1 pes err*)
hence *is-cupcake-exp e* **and** *pes*: *cupcake-clauses pes*
by (*auto dest*: *is-cupcake-all-envD*)

have *cupcake-evaluate-single env e* (*Rval v1*)
using *mat1b* **by** *fastforce*
have *cupcake-match-result* (*c env*) *v1 pes Bindv* = *Rerr err*
using *cupcake-match-result-eq*[*OF pes*, **where** *env* = *env* **and** *s* = *s2* **and** *v* = *v1* **and** *err-v* = *Bindv*] **unfolding** *mat1b*
by (*cases* (*cupcake-match-result* (*c env*) *v1 pes Bindv*)) (*auto simp*:*error-result.map-id*)
show *?case*
by (*simp*; *rule cupcake-evaluate-single.mat1error*) *fact*+
qed (*fastforce intro*: *cupcake-evaluate-single.intros*)+

lemma *cupcake-single-complete*:

evaluate env s e $(s', res) \Longrightarrow$ *is-cupcake-all-env env* $\Longrightarrow$ *is-cupcake-exp e* $\Longrightarrow$ *cupcake-evaluate-single env e res*
by (*fastforce dest*:*cupcake-single-complete0*)

lemma *cupcake-list-complete*:
evaluate-list (*evaluate env*) *s1 es res* $\Longrightarrow$
is-cupcake-all-env env $\Longrightarrow$ *list-all is-cupcake-exp es* $\Longrightarrow$ $\exists$ *rs. list-all2-shortcircuit* (*cupcake-evaluate-single env*) *es rs* $\wedge$ *sequence-result rs* = (*snd res*)
by (*fastforce intro*:*cupcake-list-complete0 cupcake-single-complete evaluate-list-mono-strong*)

private lemma *cupcake-list-state-preserve0*:
assumes *evaluate-list* (λ*s e res. Big-Step-Unclocked-Single.evaluate env s e res* $\wedge$ (*is-cupcake-all-env env* $\longrightarrow$ *is-cupcake-exp e* $\longrightarrow$ *s* = *fst res*)) *s es res*
list-all is-cupcake-exp es is-cupcake-all-env env
shows *s* = (*fst res*)
using *assms* **by** (*induction rule*:*evaluate-list.induct*) *auto*

lemma *cupcake-state-preserve*:
assumes *Big-Step-Unclocked-Single.evaluate env s e res is-cupcake-all-env env is-cupcake-exp e*
shows *s* = (*fst res*)
using *assms* **proof** (*induction arbitrary*: *rule*: *evaluate.induct*)
case (*con1 env cn es vs v s1 s2*)
hence *list-all is-cupcake-exp es*
by (*cases rule*: *is-cupcake-exp.cases*[**where** *x* = *Con cn es*]) *auto*
then show *?case*
using *con1* **by** (*fastforce dest*:*cupcake-list-state-preserve0*)
next
case (*con3 env cn es s1 s2 err*)
hence *list-all is-cupcake-exp es*
by (*cases rule*: *is-cupcake-exp.cases*[**where** *x* = *Con cn es*]) *auto*
then show *?case*
using *con3* **by** (*fastforce dest*:*cupcake-list-state-preserve0*)
next
case (*app1 env s1 es s2 vs env' e bv*)
have *list-all is-cupcake-exp* (*rev es*)
using *app1* **by** *fastforce*
then obtain *rs* **where** *rs*: *list-all2-shortcircuit* (*cupcake-evaluate-single env*) (*rev es*) *rs sequence-result rs* = *Rval vs*
using *app1* **by** (*fastforce dest*:*evaluate-list-mono-strong*[*THEN cupcake-list-complete*])
hence *list-all is-cupcake-value vs list-all is-cupcake-value* (*rev vs*)
using *cupcake-list-preserve app1 rs* **by** *fastforce+*
hence *is-cupcake-exp e is-cupcake-all-env env'*
using *app1 cupcake-opapp-preserve* **by** *fastforce+*
then show *?case*
using *app1* **by** (*fastforce dest*:*cupcake-list-state-preserve0*)
next
case (*mat1 env s1 e s2 v1 pes e' env' bv*)
hence *is-cupcake-exp e* **and** *cupcake-c-ns* (*c env*) **and** *pes*:*cupcake-clauses pes*

and *is-cupcake-value v1*
  by (*fastforce dest*: *is-cupcake-all-envD cupcake-single-complete cupcake-single-preserve*)+

  moreover obtain *uu* **where** *cupcake-match-result* (*c env*) *v1 pes Bindv* = *Rval* (e', *uu*, env')
    using *cupcake-match-result-eq*[*OF pes*,**where** *env* = *env* **and** *s*= *s2* **and** *v* = *v1* **and** *err-v* = *Bindv*, *unfolded mat1*]
    by (*cases cupcake-match-result* (*c env*) *v1 pes Bindv*) *auto*

  ultimately have *list-all* (*is-cupcake-value* ∘ *snd*) env' *is-cupcake-exp* e'
    using *cupcake-match-result-preserve*[**where** *envC* = *c env* **and** *v* = *v1* **and** *pes* = *pes* **and** *err-v* = *Bindv*]
    by *fastforce*+
  moreover have *is-cupcake-all-env* (*update-v* (λ-. *nsAppend* (*alist-to-ns* env') (*sem-env.v env*)) *env*)
    apply (*rule cupcake-v-update-preserve*)
     apply *fact*
    apply (*rule cupcake-nsAppend-preserve*)
     apply (*rule cupcake-alist-to-ns-preserve*)
     apply *fact*
    apply (*rule is-cupcake-all-envD*)
    apply *fact*
    done
  ultimately show *?case*
    using *mat1* **by** *fastforce*
qed (*fastforce dest*:*cupcake-list-state-preserve0*)+

corollary *cupcake-single-correct-strong*:
  assumes *cupcake-evaluate-single env e res is-cupcake-all-env env is-cupcake-exp e*
  shows *Big-Step-Unclocked-Single.evaluate env s e* (*s*,*res*)
  using *assms cupcake-single-correct cupcake-state-preserve* **by** *fastforce*

corollary *cupcake-single-complete-weak*:
  evaluate env s e (*s*, *res*) ⟹ *is-cupcake-all-env env* ⟹ *is-cupcake-exp e* ⟹ *cupcake-evaluate-single env e res*
  using *cupcake-single-complete* **by** *fastforce*

end end

hide-const (**open**) *c*

end

Chapter 3

Term rewriting

theory *Doc-Rewriting*
imports *Main*
begin

end
theory *General-Rewriting*
imports *Terms-Extras*
begin

locale *rewriting* =
  fixes $R :: 'a::term \Rightarrow 'a \Rightarrow bool$
  assumes *R-fun*: $R\ t\ t' \Longrightarrow R\ (app\ t\ u)\ (app\ t'\ u)$
  assumes *R-arg*: $R\ u\ u' \Longrightarrow R\ (app\ t\ u)\ (app\ t\ u')$
begin

lemma *rt-fun*:
  $R^{**}\ t\ t' \Longrightarrow R^{**}\ (app\ t\ u)\ (app\ t'\ u)$
by (*induct rule*: *rtranclp.induct*) (*auto intro*: *rtranclp.rtrancl-into-rtrancl R-fun*)

lemma *rt-arg*:
  $R^{**}\ u\ u' \Longrightarrow R^{**}\ (app\ t\ u)\ (app\ t\ u')$
by (*induct rule*: *rtranclp.induct*) (*auto intro*: *rtranclp.rtrancl-into-rtrancl R-arg*)

lemma *rt-comb*:
  $R^{**}\ t_1\ u_1 \Longrightarrow R^{**}\ t_2\ u_2 \Longrightarrow R^{**}\ (app\ t_1\ t_2)\ (app\ u_1\ u_2)$
by (*metis rt-fun rt-arg rtranclp-trans*)

lemma *rt-list-comb*:
  assumes *list-all2* R^{**} *ts us* R^{**} *t u*
  shows R^{**} (*list-comb t ts*) (*list-comb u us*)
using *assms*
by (*induction ts us arbitrary*: *t u rule*: *list.rel-induct*) (*auto intro*: *rt-comb*)

end

end

3.1 Higher-order term rewriting using de-Bruijn indices

theory *Rewriting-Term*
imports
  ../Terms/General-Rewriting
  ../Terms/Strong-Term
begin

3.1.1 Matching and rewriting

type-synonym *rule* = *term* × *term*

inductive *rewrite* :: *rule fset* ⇒ *term* ⇒ *term* ⇒ *bool* (‹-/ ⊢/ - ⟶/ -› [*50*,*0*,*50*] *50*) **for** *rs* **where**
step: *r* |∈| *rs* ⟹ *r* ⊢ *t* → *u* ⟹ *rs* ⊢ *t* ⟶ *u* |
beta: *rs* ⊢ (Λ *t* $ *t'*) ⟶ *t* [*t'*]$_\beta$ |
fun: *rs* ⊢ *t* ⟶ *t'*⟹ *rs* ⊢ *t* $ *u* ⟶ *t'* $ *u* |
arg: *rs* ⊢ *u* ⟶ *u'*⟹ *rs* ⊢ *t* $ *u* ⟶ *t* $ *u'*

global-interpretation *rewrite*: *rewriting rewrite rs* **for** *rs*
by *standard* (*auto intro*: *rewrite.intros simp*: *app-term-def*)+

abbreviation *rewrite-rt* :: *rule fset* ⇒ *term* ⇒ *term* ⇒ *bool* (‹-/ ⊢/ - ⟶*/ -› [*50*,*0*,*50*] *50*) **where**
rewrite-rt rs ≡ (*rewrite rs*)**

lemma *rewrite-beta-alt*: *t* [*t'*]$_\beta$ = *u* ⟹ *wellformed t'* ⟹ *rs* ⊢ (Λ *t* $ *t'*) ⟶ *u*
by (*metis rewrite.beta*)

3.1.2 Wellformedness

primrec *rule* :: *rule* ⇒ *bool* **where**
rule (*lhs*, *rhs*) ⟷ *basic-rule* (*lhs*, *rhs*) ∧ *Term.wellformed rhs*

lemma *ruleI*[*intro*]:
  assumes *basic-rule* (*lhs*, *rhs*)
  assumes *Term.wellformed rhs*
  shows *rule* (*lhs*, *rhs*)
using *assms* **by** *simp*

lemma *split-rule-fst*: *fst* (*split-rule r*) = *head* (*fst r*)
unfolding *head-def* **by** (*smt prod.case-eq-if prod.collapse prod.inject split-rule.simps*)

locale *rules* = *constants C-info heads-of rs* **for** *C-info* **and** *rs* :: *rule fset* +
  assumes *all-rules*: *fBall rs rule*

assumes *arity*: *arity-compatibles rs*
assumes *fmap*: *is-fmap rs*
assumes *patterns*: *pattern-compatibles rs*
assumes *nonempty*: $rs \neq \{||\}$
assumes *not-shadows*: *fBall rs* (λ(*lhs*, -). $\neg$ *shadows-consts lhs*)
assumes *welldefined-rs*: *fBall rs* (λ(-, *rhs*). *welldefined rhs*)
begin

lemma *rewrite-wellformed*:
assumes $rs \vdash t \longrightarrow t'$ *wellformed t*
shows *wellformed* t'
using *assms*
proof (*induction rule*: *rewrite.induct*)
case (*step r t u*)
obtain *lhs rhs* **where** $r = (lhs, rhs)$
by *force*
hence *wellformed rhs*
using *all-rules step* **by** *force*
show *?case*
apply (*rule wellformed.rewrite-step*)
apply (*rule step*(*2*)[*unfolded* ‹r = -›])
apply *fact+*
done
next
case (*beta u t*)
show *?case*
unfolding *wellformed-term-def*
apply (*rule replace-bound-wellformed*)
using *beta* **by** *auto*
qed *auto*

lemma *rewrite-rt-wellformed*: $rs \vdash t \longrightarrow* t' \Longrightarrow$ *wellformed* $t \Longrightarrow$ *wellformed* t'
by (*induction rule*: *rtranclp.induct*) (*auto intro*: *rewrite-wellformed simp del*: *wellformed-term-def*)

lemma *rewrite-closed*: $rs \vdash t \longrightarrow t' \Longrightarrow$ *closed* $t \Longrightarrow$ *closed* t'
proof (*induction* t t' *rule*: *rewrite.induct*)
case (*step r t u*)
obtain *lhs rhs* **where** $r = (lhs, rhs)$
by *force*
with *step* **have** *rule* (*lhs*, *rhs*)
using *all-rules* **by** *blast*
hence *frees rhs* $|\subseteq|$ *frees lhs*
by *simp*
moreover have $(lhs, rhs) \vdash t \rightarrow u$
using *step* **unfolding** ‹r = -› **by** *simp*
show *?case*
apply (*rule rewrite-step-closed*)
apply *fact+*

done
next
case (*beta t t′*)
have *frees t* $[t']_\beta$ $|\subseteq|$ *frees t* $|\cup|$ *frees t′*
by (*rule replace-bound-frees*)
with *beta* **show** *?case*
unfolding *closed-except-def* **by** *auto*
qed (*auto simp*: *closed-except-def*)

lemma *rewrite-rt-closed*: $rs \vdash t \longrightarrow\!* t' \Longrightarrow closed\ t \Longrightarrow closed\ t'$
by (*induction rule*: *rtranclp.induct*) (*auto intro*: *rewrite-closed*)

end

end

3.2 Higher-order term rewriting using explicit bound variable names

theory *Rewriting-Nterm*
imports
Rewriting-Term
Higher-Order-Terms.Term-to-Nterm
../Terms/Strong-Term
begin

3.2.1 Definitions

type-synonym *nrule* = *term* × *nterm*

abbreviation *nrule* :: *nrule* ⇒ *bool* **where**
nrule ≡ *basic-rule*

fun (**in** *constants*) *not-shadowing* :: *nrule* ⇒ *bool* **where**
not-shadowing (*lhs*, *rhs*) ⟷ ¬ *shadows-consts lhs* ∧ ¬ *shadows-consts rhs*

locale *nrules* = *constants C-info heads-of rs* **for** *C-info* **and** *rs* :: *nrule fset* +
assumes *all-rules*: *fBall rs nrule*
assumes *arity*: *arity-compatibles rs*
assumes *fmap*: *is-fmap rs*
assumes *patterns*: *pattern-compatibles rs*
assumes *nonempty*: *rs* ≠ {||}
assumes *not-shadows*: *fBall rs not-shadowing*
assumes *welldefined-rs*: *fBall rs* (λ(-, *rhs*). *welldefined rhs*)

3.2.2 Matching and rewriting

inductive *nrewrite* :: *nrule fset* ⇒ *nterm* ⇒ *nterm* ⇒ *bool* (‹-/ $\vdash_n$/ - ⟶/ -›
[*50*,*0*,*50*] *50*) **for** *rs* **where**

step: r |∈| $rs \Longrightarrow r \vdash t \to u \Longrightarrow rs \vdash_n t \longrightarrow u$ |
beta: $rs \vdash_n ((\Lambda_n\ x.\ t)\ \$_n\ t') \longrightarrow$ *subst* t (*fmap-of-list* $[(x,\ t')]$) |
fun: $rs \vdash_n t \longrightarrow t' \Longrightarrow rs \vdash_n t\ \$_n\ u \longrightarrow t'\ \$_n\ u$ |
arg: $rs \vdash_n u \longrightarrow u' \Longrightarrow rs \vdash_n t\ \$_n\ u \longrightarrow t\ \$_n\ u'$

global-interpretation *nrewrite*: *rewriting nrewrite rs* **for** *rs*
by *standard* (*auto intro*: *nrewrite.intros simp*: *app-nterm-def*)+

abbreviation *nrewrite-rt* :: *nrule fset* ⇒ *nterm* ⇒ *nterm* ⇒ *bool* (‹-/ $\vdash_n$/ - ⟶∗/ -› [*50*,*0*,*50*] *50*) **where**
nrewrite-rt rs ≡ (*nrewrite rs*)**

lemma (**in** *nrules*) *nrewrite-closed*:
  assumes $rs \vdash_n t \longrightarrow t'$ *closed t*
  shows *closed t′*
using *assms* **proof** *induction*
  case (*step r t u*)
  obtain *lhs rhs* **where** *r* = (*lhs*, *rhs*)
    by *force*
  with *step* **have** *nrule* (*lhs*, *rhs*)
    using *all-rules* **by** *blast*
  hence *frees rhs* |⊆| *frees lhs*
    by *simp*
  have (*lhs*, *rhs*) $\vdash t \to u$
    using *step* **unfolding** ‹*r* = -› **by** *simp*

  show *?case*
    apply (*rule rewrite-step-closed*)
      apply *fact+*
    done
next
  case *beta*
  show *?case*
    apply *simp*
    apply (*subst closed-except-def*)
    apply (*subst subst-frees*)
    using *beta* **unfolding** *closed-except-def* **by** *auto*
qed (*auto simp*: *closed-except-def*)

corollary (**in** *nrules*) *nrewrite-rt-closed*:
  assumes $rs \vdash_n t \longrightarrow\!\ast\ t'$ *closed t*
  shows *closed t′*
using *assms*
by *induction* (*auto intro*: *nrewrite-closed*)

3.2.3 Translation from *Term-Class.term* to *nterm*

context begin

```
private lemma term-to-nterm-all-vars0:
  assumes wellformed' (length Γ) t
  shows ∃ T. all-frees (fst (run-state (term-to-nterm Γ t) x)) |⊆| fset-of-list Γ |∪|
frees t |∪| T ∧ fBall T (λy. y > x)
using assms proof (induction Γ t arbitrary: x rule: term-to-nterm-induct)
  case (bound Γ i)
  hence Γ ! i |∈| fset-of-list Γ
    by (simp add: fset-of-list-elem)
  with bound show ?case
    by (auto simp: State-Monad.return-def)
next
  case (abs Γ t)

  obtain T
     where all-frees (fst (run-state (term-to-nterm (next x # Γ) t) (next x))) |⊆|
fset-of-list (next x # Γ) |∪| frees t |∪| T
      and fBall T ((<) (next x))
    apply atomize-elim
    apply (rule abs(1))
    using abs by auto

  show ?case
    apply (simp add: split-beta create-alt-def)
    apply (rule exI[where x = finsert (next x) T])
    apply (intro conjI)
    subgoal by auto
    subgoal using ‹all-frees (fst (run-state - (next x))) |⊆| -› by simp
    subgoal
      apply simp
      apply (rule conjI)
       apply (rule next-ge)
      using ‹fBall T ((<) (next x))›
      by (metis fBallE fBallI fresh-class.next-ge order.strict-trans)
    done
next
  case (app Γ t1 t2 x1)
  obtain t1' x2 where run-state (term-to-nterm Γ t1) x1 = (t1', x2)
    by fastforce
  moreover obtain T1
     where all-frees (fst (run-state (term-to-nterm Γ t1) x1)) |⊆| fset-of-list Γ |∪|
frees t1 |∪| T1
      and fBall T1 ((<) x1)
    apply atomize-elim
    apply (rule app(1))
    using app by auto
  ultimately have all-frees t1' |⊆| fset-of-list Γ |∪| frees t1 |∪| T1
    by simp

  obtain T2
```

where *all-frees* (*fst* (*run-state* (*term-to-nterm* Γ t_2) x_2)) $|\subseteq|$ *fset-of-list* Γ $|\cup|$ *frees* t_2 $|\cup|$ T_2
and *fBall* T_2 (($<$) x_2)
apply *atomize-elim*
apply (*rule app*(*2*))
using *app* **by** *auto*
moreover obtain $t_2{}'$ x' **where** *run-state* (*term-to-nterm* Γ t_2) x_2 = ($t_2{}'$, x')
by *fastforce*
ultimately have *all-frees* $t_2{}'$ $|\subseteq|$ *fset-of-list* Γ $|\cup|$ *frees* t_2 $|\cup|$ T_2
by *simp*

have $x_1 \leq x_2$
apply (*rule state-io-relD*[*OF term-to-nterm-mono*])
apply *fact*
done

show *?case*
apply *simp*
unfolding ‹*run-state* (*term-to-nterm* Γ t_1) x_1 = -›
apply *simp*
unfolding ‹*run-state* (*term-to-nterm* Γ t_2) x_2 = -›
apply *simp*
apply (*rule exI*[**where** $x = T_1$ $|\cup|$ T_2])
apply (*intro conjI*)
subgoal using ‹*all-frees* $t_1{}'$ $|\subseteq|$ -› **by** *auto*
subgoal using ‹*all-frees* $t_2{}'$ $|\subseteq|$ -› **by** *auto*
subgoal
apply *auto*
using ‹*fBall* T_1 (($<$) x_1)› **apply** *auto*[]
using ‹*fBall* T_2 (($<$) x_2)› ‹$x_1 \leq x_2$›
by *auto*
done
qed *auto*

lemma *term-to-nterm-all-vars*:
assumes *wellformed t fdisjnt* (*frees t*) *S*
shows *fdisjnt* (*all-frees* (*fresh-frun* (*term-to-nterm* [] *t*) (*T* $|\cup|$ *S*))) *S*
proof −
let *?*Γ = []
let *?x* = *fresh-fNext* (*T* $|\cup|$ *S*)
from *assms* **have** *wellformed'* (*length ?*Γ) *t*
by *simp*
then obtain *F*
where *all-frees* (*fst* (*run-state* (*term-to-nterm ?*Γ *t*) *?x*)) $|\subseteq|$ *fset-of-list ?*Γ $|\cup|$ *frees t* $|\cup|$ *F*
and *fBall F* ($\lambda y.\ y >$ *?x*)
by (*metis term-to-nterm-all-vars0*)

have *fdisjnt F* (*T* $|\cup|$ *S*) **if** $S \neq \{||\}$

apply (*rule fdisjnt-ge-max*)
apply (*rule fBall-pred-weaken*[*OF - ‹fBall F* (*λy. y > ?x*)›])
apply (*rule less-trans*)
apply (*rule fNext-ge-max*)
using *that* **by** *auto*
show *?thesis*
apply (*rule fdisjnt-subset-left*)
apply (*subst fresh-frun-def*)
apply (*subst fresh-fNext-def*[*symmetric*])
apply *fact*
apply *simp*
apply (*rule fdisjnt-union-left*)
apply *fact*
using ‹- ⟹ *fdisjnt F* (*T* |∪| *S*)› **by** (*auto simp*: *fdisjnt-alt-def*)
qed

end

fun *translate-rule* :: *name fset* ⇒ *rule* ⇒ *nrule* **where**
translate-rule S (*lhs*, *rhs*) = (*lhs*, *fresh-frun* (*term-to-nterm* [] *rhs*) (*frees lhs* |∪| *S*))

lemma *translate-rule-alt-def*:
translate-rule S = (λ(*lhs*, *rhs*). (*lhs*, *fresh-frun* (*term-to-nterm* [] *rhs*) (*frees lhs* |∪| *S*)))
by *auto*

definition *compile′* **where**
compile′ C-info rs = *translate-rule* (*pre-constants.all-consts C-info* (*heads-of rs*)) |‘| *rs*

context *rules* **begin**

definition *compile* :: *nrule fset* **where**
compile = *translate-rule all-consts* |‘| *rs*

lemma *compile′-compile-eq*[*simp*]: *compile′ C-info rs* = *compile*
unfolding *compile′-def compile-def* **..**

lemma *compile-heads*: *heads-of compile* = *heads-of rs*
unfolding *compile-def translate-rule-alt-def head-def*[*abs-def*]
by *force*

lemma *compile-rules*: *nrules C-info compile*
proof
have *fBall compile* (λ(*lhs*, *rhs*). *nrule* (*lhs*, *rhs*))
proof *safe*
fix *lhs rhs*
assume (*lhs*, *rhs*) |∈| *compile*

then obtain *rhs'*
where (*lhs*, *rhs'*) |∈| *rs*
and *rhs*: *rhs* = *fresh-frun* (*term-to-nterm* [] *rhs'*) (*frees lhs* |∪| *all-consts*)
unfolding *compile-def* **by** *force*
then have *rule*: *rule* (*lhs*, *rhs'*)
using *all-rules* **by** *blast*

show *nrule* (*lhs*, *rhs*)
proof
from *rule* **show** *linear lhs is-const* (*fst* (*strip-comb lhs*)) ¬ *is-const lhs* **by** *auto*

have *frees rhs* |⊆| *frees rhs'*
unfolding *rhs* **using** *rule*
by (*metis rule.simps term-to-nterm-vars*)
also have *frees rhs'* |⊆| *frees lhs*
using *rule* **by** *auto*
finally show *frees rhs* |⊆| *frees lhs* **.**
qed
qed

thus *fBall compile nrule*
by *fast*
next
show *arity-compatibles compile*
proof *safe*
fix lhs_1 rhs_1 lhs_2 rhs_2
assume (lhs_1, rhs_1) |∈| *compile* (lhs_2, rhs_2) |∈| *compile*
then obtain rhs_1' rhs_2' **where** (lhs_1, rhs_1') |∈| *rs* (lhs_2, rhs_2') |∈| *rs*
unfolding *compile-def* **by** *force*
thus *arity-compatible* lhs_1 lhs_2
using *arity* **by** (*blast dest*: *fpairwiseD*)
qed
next
have *is-fmap rs*
using *fmap* **by** *simp*
thus *is-fmap compile*
unfolding *compile-def translate-rule-alt-def*
by (*rule is-fmap-image*)
next
have *pattern-compatibles rs*
using *patterns* **by** *simp*
thus *pattern-compatibles compile*
unfolding *compile-def translate-rule-alt-def*
by (*auto dest*: *fpairwiseD*)
next
show *fdisjnt* (*heads-of compile*) *C*
using *disjnt* **by** (*simp add*: *compile-heads*)
next

have *fBall compile not-shadowing*
proof *safe*
fix *lhs rhs*
assume (*lhs*, *rhs*) |∈| *compile*
then obtain *rhs'*
where *rhs* = *fresh-frun* (*term-to-nterm* [] *rhs'*) (*frees lhs* |∪| *all-consts*)
and (*lhs*, *rhs'*) |∈| *rs*
unfolding *compile-def translate-rule-alt-def* **by** *auto*
hence *rule* (*lhs*, *rhs'*) ¬ *shadows-consts lhs*
using *all-rules not-shadows* **by** *blast*+
moreover hence *wellformed rhs' frees rhs'* |⊆| *frees lhs fdisjnt all-consts* (*frees lhs*)
unfolding *shadows-consts-def* **by** *simp*+

moreover have ¬ *shadows-consts rhs*
apply (*subst shadows-consts-def*)
apply *simp*
unfolding ‹*rhs* = -›
apply (*rule fdisjnt-swap*)
apply (*rule term-to-nterm-all-vars*)
apply *fact*
apply (*rule fdisjnt-subset-left*)
apply *fact*
apply (*rule fdisjnt-swap*)
apply *fact*
done

ultimately show *not-shadowing* (*lhs*, *rhs*)
unfolding *compile-heads* **by** *simp*
qed
thus *fBall compile* (*constants.not-shadowing C-info* (*heads-of compile*))
unfolding *compile-heads* **.**

have *fBall compile* (λ(-, *rhs*). *welldefined rhs*)
unfolding *compile-heads*
unfolding *compile-def ball-simps*
apply (*rule fBall-pred-weaken*[*OF* - *welldefined-rs*])
subgoal for *x*
apply (*cases x*)
apply *simp*
apply (*subst fresh-frun-def*)
apply (*subst term-to-nterm-consts*[*THEN pred-state-run-state*])
by *auto*
done
thus *fBall compile* (λ(-, *rhs*). *consts rhs* |⊆| *pre-constants.all-consts C-info* (*heads-of compile*))
unfolding *compile-heads* **.**
next
show *compile* ≠ {||}

using *nonempty* **unfolding** *compile-def* **by** *auto*
next
show *distinct all-constructors*
by (*fact distinct-ctr*)
qed

sublocale *rules-as-nrules*: *nrules C-info compile*
by (*fact compile-rules*)

end

3.2.4 Correctness of translation

theorem (**in** *rules*) *compile-correct*:
assumes *compile* $\vdash_n$ *u* $\longrightarrow$ *u′ closed u*
shows *rs* $\vdash$ *nterm-to-term′ u* $\longrightarrow$ *nterm-to-term′ u′*
using *assms* **proof** (*induction u u′*)
case (*step r u u′*)
moreover obtain *pat rhs′* **where** *r* = (*pat*, *rhs′*)
by *force*
ultimately obtain *nenv* **where** *match pat u* = *Some nenv u′* = *subst rhs′ nenv*
by *auto*
then obtain *env* **where** *nrelated.P-env* [] *env nenv match pat* (*nterm-to-term* [] *u*) = *Some env*
by (*metis nrelated.match-rel option.exhaust option.rel-distinct*(*1*) *option.rel-inject*(*2*))

have *closed-env nenv*
using *step* ‹*match pat u* = *Some nenv*› **by** (*intro closed.match*)

from *step* **obtain** *rhs*
where *rhs′* = *fresh-frun* (*term-to-nterm* [] *rhs*) (*frees pat* |∪| *all-consts*) (*pat*, *rhs*) |∈| *rs*
unfolding ‹*r* = -› *compile-def* **by** *auto*
with *assms* **have** *rule* (*pat*, *rhs*)
using *all-rules* **by** *blast*
hence *rhs* = *nterm-to-term* [] *rhs′*
unfolding ‹*rhs′* = -›
by (*simp add*: *term-to-nterm-nterm-to-term fresh-frun-def*)

have *compile* $\vdash_n$ *u* $\longrightarrow$ *u′*
using *step* **by** (*auto intro*: *nrewrite.step*)
hence *closed u′*
by (*rule rules-as-nrules.nrewrite-closed*) *fact*

show *?case*
proof (*rule rewrite.step*)
show (*pat*, *rhs*) $\vdash$ *nterm-to-term* [] *u* $\rightarrow$ *nterm-to-term* [] *u′*
apply (*subst nterm-to-term-eq-closed*)
apply *fact*

apply *simp*
apply (*rule exI*[**where** $x = env$])
apply (*rule conjI*)
apply *fact*
unfolding ‹*rhs* = -›
apply (*subst nrelated-subst*)
apply *fact*
apply *fact*
unfolding *fdisjnt-alt-def* **apply** *simp*
unfolding ‹u' = *subst* rhs' *nenv*› **by** *simp*
qed *fact*
next
case *beta*
show *?case*
apply *simp*
apply (*subst subst-single-eq*[*symmetric*, *simplified*])
apply (*subst nterm-to-term-subst-replace-bound*[**where** $n = 0$])
subgoal using *beta* **by** (*simp add*: *closed-except-def*)
subgoal by *simp*
subgoal by *simp*
subgoal by *simp* (*rule rewrite.beta*)
done
qed (*auto intro*: *rewrite.intros simp*: *closed-except-def*)

3.2.5 Completeness of translation

context *rules* **begin**

context
notes [*simp*] = *closed-except-def fdisjnt-alt-def*
begin

private lemma *compile-complete0*:
assumes $rs \vdash t \longrightarrow t'$ *closed t wellformed t*
obtains u' **where** *compile* $\vdash_n$ *fst* (*run-state* (*term-to-nterm* [] *t*) *s*) $\longrightarrow u'$ $u' \approx_\alpha$ *fst* (*run-state* (*term-to-nterm* [] t') s')
apply *atomize-elim*
using *assms* **proof** (*induction* t t' *arbitrary*: s s')
case (*step* r t t')
let $?t_n$ = *fst* (*run-state* (*term-to-nterm* [] *t*) *s*)
let $?t_n{}'$ = *fst* (*run-state* (*term-to-nterm* [] t') s')
from *step* **have** *closed t closed* $?t_n$
using *term-to-nterm-vars0*[**where** $\Gamma = []$]
by *force+*
from *step* **have** *nterm-to-term'* $?t_n = t$
by (*auto intro*!: *term-to-nterm-nterm-to-term0*)

obtain *pat* rhs' **where** $r = (pat, rhs')$
by *fastforce*

with *step* **obtain** *env′* **where** *match pat t* = *Some env′ t′* = *subst rhs′ env′*
by *fastforce*
with ‹- = *t*› **have** *rel-option* (*nrelated.P-env* []) (*match pat t*) (*match pat* (*?t*$_n$))
by (*metis nrelated.match-rel*)
with *step* ‹- = *Some env′*›
obtain *env* **where** *nrelated.P-env* [] *env′ env match pat ?t*$_n$ = *Some env*
by (*metis* (*no-types*, *lifting*) *option-rel-Some1*)
with ‹*closed ?t*$_n$› **have** *closed-env env*
by (*blast intro*: *closed.match*)

from *step* **obtain** *rhs*
where *rhs* = *fresh-frun* (*term-to-nterm* [] *rhs′*) (*frees pat* |∪| *all-consts*) (*pat*, *rhs*) |∈| *compile*
unfolding ‹*r* = -› *compile-def*
by *force*
with *step* **have** *rule* (*pat*, *rhs′*)
unfolding ‹*r* = -›
using *all-rules* **by** *fast*
hence *nterm-to-term′ rhs* = *rhs′*
unfolding ‹*rhs* = -›
by (*simp add*: *fresh-frun-def term-to-nterm-nterm-to-term*)
obtain *u′* **where** *subst rhs env* = *u′*
by *simp*
have *t′* = *nterm-to-term′ u′*
unfolding ‹*t′* = -›
unfolding ‹- = *rhs′*›[*symmetric*]
apply (*subst nrelated-subst*)
apply *fact+*
using ‹- = *u′*›
by *simp+*

have *compile* $\vdash_n$ *?t*$_n$ ⟶ *u′*
apply (*rule nrewrite.step*)
apply *fact*
apply *simp*
apply (*intro conjI exI*)
apply *fact+*
done
with ‹*closed ?t*$_n$› **have** *closed u′*
by (*blast intro*:*rules-as-nrules.nrewrite-closed*)
with ‹*t′* = *nterm-to-term′* -› **have** *u′* $\approx_\alpha$ *?t*$_n$*′*
by (*force intro*: *nterm-to-term-term-to-nterm*[**where** Γ = [] **and** Γ′ = [],*simplified*])

show *?case*
apply (*intro conjI exI*)
apply (*rule nrewrite.step*)
apply *fact*
apply *simp*
apply (*intro conjI exI*)

apply *fact+*
done
next
case (*beta t t'*)
let *?name* = *next s*
let $?t_n$ = *fst* (*run-state* (*term-to-nterm* [*?name*] *t*) (*?name*))
let $?t_n{}'$ = *fst* (*run-state* (*term-to-nterm* [] *t'*) (*snd* (*run-state* (*term-to-nterm* [*?name*] *t*) (*?name*))))

from *beta* **have** *closed t closed* (t $[t']_\beta$) *closed* (Λ *t* \$ *t'*) *closed t'*
using *replace-bound-frees*
by *fastforce+*
moreover from *beta* **have** *wellformed'* (*Suc 0*) *t wellformed t'*
by *simp+*
ultimately have *t* = *nterm-to-term* [*?name*] $?t_n$
and *t'* = *nterm-to-term'* $?t_n{}'$
and $*$:*frees* $?t_n$ = {|*?name*|} ∨ *frees* $?t_n$ = *fempty*
and *closed* $?t_n{}'$
using *term-to-nterm-vars0*[**where** Γ = [*?name*]]
using *term-to-nterm-vars0*[**where** Γ = []]
by (*force simp*: *term-to-nterm-nterm-to-term0*)+

hence $**$:*t* $[t']_\beta$ = *nterm-to-term'* (*subst-single* $?t_n$ *?name* $?t_n{}'$)
by (*auto simp*: *nterm-to-term-subst-replace-bound*[**where** *n* = *0*])

from ‹*closed* $?t_n{}'$› **have** *closed-env* (*fmap-of-list* [(*?name*, $?t_n{}'$)])
by *auto*

show *?case*
apply (*rule exI*)
apply (*auto simp*: *split-beta create-alt-def*)
apply (*rule nrewrite.beta*)
apply (*subst subst-single-eq*[*symmetric*])
apply (*subst* $**$)
apply (*rule nterm-to-term-term-to-nterm*[**where** Γ = [] **and** Γ' = [], *simplified*])
apply (*subst subst-single-eq*)
apply (*subst subst-frees*[*OF* ‹*closed-env* -›])
using $*$ **by** *force*
next
case (*fun t t' u*)
hence *closed t closed u closed* (*t* \$ *u*)
and *wellform*:*wellformed t wellformed u*
by *fastforce+*
from *fun* **obtain** *u'*
where *compile* $\vdash_n$ *fst* (*run-state* (*term-to-nterm* [] *t*) *s*) ⟶ *u'*
u' $\approx_\alpha$ *fst* (*run-state* (*term-to-nterm* [] *t'*) *s'*)
by *force*
show *?case*
apply (*rule exI*)

apply (*auto simp*: *split-beta create-alt-def*)
 apply (*rule nrewrite.fun*)
 apply *fact*
apply *rule*
 apply *fact*
apply (*subst term-to-nterm-alpha-equiv*[*of* [] [], *simplified*])
using ‹*closed u*› ‹*wellformed u*› **by** *auto*

next
 case (*arg u u′ t*)
 hence *closed t closed u closed* (*t* $ *u*)
  and *wellform*:*wellformed t wellformed u*
 by *fastforce*+
 from *arg* **obtain** *t′*
 where *compile* $\vdash_n$ *fst* (*run-state* (*term-to-nterm* [] *u*) (*snd* (*run-state* (*term-to-nterm* [] *t*) *s*))) $\longrightarrow$ *t′*
  t′ $\approx_\alpha$ *fst* (*run-state* (*term-to-nterm* [] *u′*) (*snd* (*run-state* (*term-to-nterm* [] *t*) *s′*)))
 by *force*
 show *?case*
 apply (*rule exI*)
 apply (*auto simp*: *split-beta create-alt-def*)
  apply *rule*
  apply *fact*
 apply *rule*
  apply (*subst term-to-nterm-alpha-equiv*[*of* [] [], *simplified*])
 using ‹*closed t*› ‹*wellformed t*› **apply** *force*+
 by *fact*

qed

lemma *compile-complete*:
 assumes *rs* ⊢ *t* $\longrightarrow$ *t′ closed t wellformed t*
 obtains *u′* **where** *compile* $\vdash_n$ *term-to-nterm′ t* $\longrightarrow$ *u′ u′* $\approx_\alpha$ *term-to-nterm′ t′*
unfolding *term-to-nterm′-def*
using *assms* **by** (*metis compile-complete0*)

end

end

3.2.6 Splitting into constants

type-synonym *crules* = (*term list* × *nterm*) *fset*
type-synonym *crule-set* = (*name* × *crules*) *fset*

abbreviation *arity-compatibles* :: (*term list* × *′a*) *fset* ⇒ *bool* **where**
arity-compatibles ≡ *fpairwise* (λ($pats_1$, -) ($pats_2$, -). *length* $pats_1$ = *length* $pats_2$)

lemma *arity-compatible-length*:
 assumes *arity-compatibles rs* (*pats*, *rhs*) |∈| *rs*

shows *length pats* = *arity rs*
proof −
have *fBall rs* (λ($pats_1$, -). *fBall rs* (λ($pats_2$, -). *length* $pats_1$ = *length* $pats_2$))
using *assms* **unfolding** *fpairwise-alt-def* **by** *blast*
hence *fBall rs* (λx. *fBall rs* (λy. (*length* ∘ *fst*) *x* = (*length* ∘ *fst*) *y*))
by *force*
hence (*length* ∘ *fst*) (*pats*, *rhs*) = *arity rs*
using *assms*(*2*) **unfolding** *arity-def fthe-elem′-eq* **by** (*rule singleton-fset-holds*)
thus *?thesis*
by *simp*
qed

locale *pre-crules* = *constants C-info fst* |‘| *rs* **for** *C-info* **and** *rs* :: *crule-set*

locale *crules* = *pre-crules* +
assumes *fmap*: *is-fmap rs*
assumes *nonempty*: *rs* ≠ {||}
assumes *inner*:
fBall rs (λ(-, *crs*).
arity-compatibles crs ∧
is-fmap crs ∧
patterns-compatibles crs ∧
crs ≠ {||} ∧
fBall crs (λ(*pats*, *rhs*).
linears pats ∧
pats ≠ [] ∧
fdisjnt (*freess pats*) *all-consts* ∧
¬ *shadows-consts rhs* ∧
frees rhs |⊆| *freess pats* ∧
welldefined rhs))

lemma (**in** *pre-crules*) *crulesI*:
assumes ⋀*name crs*. (*name*, *crs*) |∈| *rs* ⟹ *arity-compatibles crs*
assumes ⋀*name crs*. (*name*, *crs*) |∈| *rs* ⟹ *is-fmap crs*
assumes ⋀*name crs*. (*name*, *crs*) |∈| *rs* ⟹ *patterns-compatibles crs*
assumes ⋀*name crs*. (*name*, *crs*) |∈| *rs* ⟹ *crs* ≠ {||}
assumes ⋀*name crs pats rhs*. (*name*, *crs*) |∈| *rs* ⟹ (*pats*, *rhs*) |∈| *crs* ⟹ *linears pats*
assumes ⋀*name crs pats rhs*. (*name*, *crs*) |∈| *rs* ⟹ (*pats*, *rhs*) |∈| *crs* ⟹ *pats* ≠ []
assumes ⋀*name crs pats rhs*. (*name*, *crs*) |∈| *rs* ⟹ (*pats*, *rhs*) |∈| *crs* ⟹ *fdisjnt* (*freess pats*) *all-consts*
assumes ⋀*name crs pats rhs*. (*name*, *crs*) |∈| *rs* ⟹ (*pats*, *rhs*) |∈| *crs* ⟹ ¬ *shadows-consts rhs*
assumes ⋀*name crs pats rhs*. (*name*, *crs*) |∈| *rs* ⟹ (*pats*, *rhs*) |∈| *crs* ⟹ *frees rhs* |⊆| *freess pats*
assumes ⋀*name crs pats rhs*. (*name*, *crs*) |∈| *rs* ⟹ (*pats*, *rhs*) |∈| *crs* ⟹ *welldefined rhs*
assumes *is-fmap rs rs* ≠ {||}

```
  shows crules C-info rs
proof unfold-locales
  show is-fmap rs
    using assms(11) .
next
  show rs ≠ {||}
    using assms(12) .
next
  show fBall rs (λ(-, crs). Rewriting-Nterm.arity-compatibles crs ∧ is-fmap crs ∧
    patterns-compatibles crs ∧ crs ≠ {||} ∧
    fBall crs (λ(pats, rhs). linears pats ∧ pats ≠ [] ∧ fdisjnt (freess pats) all-consts
∧
      ¬ shadows-consts rhs ∧ frees rhs |⊆| freess pats ∧ consts rhs |⊆| all-consts))
    using assms(1−10)
    by (intro prod-BallI conjI) metis+
qed

lemmas crulesI[intro!] = pre-crules.crulesI[unfolded pre-crules-def]

definition consts-of :: nrule fset ⇒ crule-set where
consts-of = fgroup-by split-rule

lemma consts-of-heads: fst |`| consts-of rs = heads-of rs
unfolding consts-of-def
by (simp add: split-rule-fst comp-def)

lemma (in nrules) consts-rules: crules C-info (consts-of rs)
proof
  have is-fmap rs
    using fmap by simp
  thus is-fmap (consts-of rs)
    unfolding consts-of-def by auto

  show consts-of rs ≠ {||}
    using nonempty unfolding consts-of-def
    by (meson fgroup-by-nonempty)

  show constants C-info (fst |`| consts-of rs)
    proof
      show fdisjnt (fst |`| consts-of rs) C
        using disjnt by (auto simp: consts-of-heads)
    next
      show distinct all-constructors
        by (fact distinct-ctr)
    qed

  fix name crs
  assume crs: (name, crs) |∈| consts-of rs
```

thus *crs* $\neq$ {||}
unfolding *consts-of-def*
by (*meson femptyE fgroup-by-nonempty-inner*)

show *arity-compatibles crs patterns-compatibles crs*
proof *safe*
fix $pats_1$ rhs_1 $pats_2$ rhs_2
assume ($pats_1$, rhs_1) |∈| *crs* ($pats_2$, rhs_2) |∈| *crs*
with *crs* **obtain** lhs_1 lhs_2
where *rs*: (lhs_1, rhs_1) |∈| *rs* (lhs_2, rhs_2) |∈| *rs* **and**
split: *split-rule* (lhs_1, rhs_1) = (*name*, ($pats_1$, rhs_1))
split-rule (lhs_2, rhs_2) = (*name*, ($pats_2$, rhs_2))
unfolding *consts-of-def* **by** (*force simp*: *split-beta*)

hence *arity*: *arity-compatible* lhs_1 lhs_2
using *arity* **by** (*force dest*: *fpairwiseD*)

from *rs* **have** *const*: *is-const* (*fst* (*strip-comb* lhs_1)) *is-const* (*fst* (*strip-comb* lhs_2))
using *all-rules* **by** *force+*

have *name* = *const-name* (*fst* (*strip-comb* lhs_1)) *name* = *const-name* (*fst* (*strip-comb* lhs_2))
using *split* **by** (*auto simp*: *split-beta*)
with *const* **have** *fst* (*strip-comb* lhs_1) = *Const name fst* (*strip-comb* lhs_2) = *Const name*
apply (*fold const-term-def*)
subgoal by *simp*
subgoal by *fastforce*
done
hence *fst*: *fst* (*strip-comb* lhs_1) = *fst* (*strip-comb* lhs_2)
by *simp*

with *arity* **have** *length* (*snd* (*strip-comb* lhs_1)) = *length* (*snd* (*strip-comb* lhs_2))
unfolding *arity-compatible-def*
by (*simp add*: *split-beta*)

with *split* **show** *length* $pats_1$ = *length* $pats_2$
by (*auto simp*: *split-beta*)

have *pattern-compatible* lhs_1 lhs_2
using *rs patterns* **by** (*auto dest*: *fpairwiseD*)
moreover have lhs_1 = *name* $$ $pats_1$
using *split*(*1*) *const*(*1*) **by** (*auto simp*: *split-beta*)
moreover have lhs_2 = *name* $$ $pats_2$
using *split*(*2*) *const*(*2*) **by** (*auto simp*: *split-beta*)
ultimately have *pattern-compatible* (*name* $$ $pats_1$) (*name* $$ $pats_2$)
by *simp*

    thus *patterns-compatible* $pats_1$ $pats_2$
      using ‹*length* $pats_1$ = -› **by** (*auto dest*: *pattern-compatible-combD*)
  qed

show *is-fmap crs*
  proof
    fix *pats* rhs_1 rhs_2
    assume (*pats*, rhs_1) |∈| *crs* (*pats*, rhs_2) |∈| *crs*
    with *crs* **obtain** lhs_1 lhs_2
      where *rs*: (lhs_1, rhs_1) |∈| *rs* (lhs_2, rhs_2) |∈| *rs* **and**
          split: *split-rule* (lhs_1, rhs_1) = (*name*, (*pats*, rhs_1))
                split-rule (lhs_2, rhs_2) = (*name*, (*pats*, rhs_2))
      unfolding *consts-of-def* **by** (*force simp*: *split-beta*)

    have $lhs_1 = lhs_2$
      proof (*rule ccontr*)
        assume $lhs_1 \neq lhs_2$
        then consider (*fst*) *fst* (*strip-comb* lhs_1) ≠ *fst* (*strip-comb* lhs_2)
                | (*snd*) *snd* (*strip-comb* lhs_1) ≠ *snd* (*strip-comb* lhs_2)
          by (*metis list-strip-comb*)
        thus *False*
          proof *cases*
            case *fst*
          moreover have *is-const* (*fst* (*strip-comb* lhs_1)) *is-const* (*fst* (*strip-comb* lhs_2))
              using *rs all-rules* **by** *force+*
            ultimately show *?thesis*
              using *split const-name-simps* **by** (*fastforce simp*: *split-beta*)
          next
            case *snd*
            with *split* **show** *?thesis*
              by (*auto simp*: *split-beta*)
          qed
      qed

    with *rs* **show** $rhs_1 = rhs_2$
      using ‹*is-fmap rs*› **by** (*auto dest*: *is-fmapD*)
  qed

fix *pats rhs*
assume (*pats*, *rhs*) |∈| *crs*
then obtain *lhs* **where** (*lhs*, *rhs*) |∈| *rs pats* = *snd* (*strip-comb lhs*)
  using *crs* **unfolding** *consts-of-def* **by** (*force simp*: *split-beta*)
hence *nrule* (*lhs*, *rhs*)
  using *all-rules* **by** *blast*
hence *linear lhs frees rhs* |⊆| *frees lhs*
  by *auto*
thus *linears pats*
  unfolding ‹*pats* = -› **by** (*intro linears-strip-comb*)

have ¬ *is-const lhs is-const* (*fst* (*strip-comb lhs*))
  using ‹*nrule* -› **by** *auto*
thus *pats* ≠ []
  unfolding ‹*pats* = -› **using** ‹*linear lhs*›
  apply (*cases lhs*)
    apply (*fold app-term-def*)
  by (*auto split*: *prod.splits*)

from ‹*nrule* (*lhs*, *rhs*)› **have** *frees* (*fst* (*strip-comb lhs*)) = {||}
  by (*cases fst* (*strip-comb lhs*)) (*auto simp*: *is-const-def*)
hence *frees lhs* = *freess* (*snd* (*strip-comb lhs*))
  by (*subst frees-strip-comb*) *auto*
thus *frees rhs* |⊆| *freess pats*
  unfolding ‹*pats* = -› **using** ‹*frees rhs* |⊆| *frees lhs*› **by** *simp*

have ¬ *shadows-consts rhs*
  using ‹(*lhs*, *rhs*) |∈| *rs*› *not-shadows*
  by *force*
thus ¬ *pre-constants.shadows-consts C-info* (*fst* |‘| *consts-of rs*) *rhs*
  by (*simp add*: *consts-of-heads*)

have *fdisjnt all-consts* (*frees lhs*)
  using ‹(*lhs*, *rhs*) |∈| *rs*› *not-shadows*
  by (*force simp*: *shadows-consts-def*)
moreover have *freess pats* |⊆| *frees lhs*
  unfolding ‹*pats* = -› ‹*frees lhs* = -›
  by *simp*
ultimately have *fdisjnt* (*freess pats*) *all-consts*
  by (*metis fdisjnt-subset-right fdisjnt-swap*)

thus *fdisjnt* (*freess pats*) (*pre-constants.all-consts C-info* (*fst* |‘| *consts-of rs*))
  by (*simp add*: *consts-of-heads*)

show *pre-constants.welldefined C-info* (*fst* |‘| *consts-of rs*) *rhs*
  using *welldefined-rs* ‹(*lhs*, *rhs*) |∈| *rs*›
  by (*force simp*: *consts-of-heads*)
qed

sublocale *nrules* ⊆ *nrules-as-crules?*: *crules C-info consts-of rs*
by (*fact consts-rules*)

3.2.7 Computability

export-code
  translate-rule consts-of arity nterm-to-term
  checking *Scala*

end

3.3 Higher-order term rewriting with explicit pattern matching

theory *Rewriting-Pterm-Elim*
imports
Rewriting-Nterm
../Terms/Pterm
begin

3.3.1 Intermediate rule sets

type-synonym *irules* = (*term list* × *pterm*) *fset*
type-synonym *irule-set* = (*name* × *irules*) *fset*

locale *pre-irules* = *constants C-info fst* |'| *rs* **for** *C-info* **and** *rs* :: *irule-set*

locale *irules* = *pre-irules* +
assumes *fmap*: *is-fmap rs*
assumes *nonempty*: $rs \neq \{||\}$
assumes *inner*:
fBall rs (λ(-, *irs*).
arity-compatibles irs ∧
is-fmap irs ∧
patterns-compatibles irs ∧
$irs \neq \{||\}$ ∧
fBall irs (λ(*pats*, *rhs*).
linears pats ∧
abs-ish pats rhs ∧
closed-except rhs (*freess pats*) ∧
fdisjnt (*freess pats*) *all-consts* ∧
wellformed rhs ∧
¬ *shadows-consts rhs* ∧
welldefined rhs))

lemma (**in** *pre-irules*) *irulesI*:
assumes ⋀*name irs*. (*name*, *irs*) |∈| *rs* ⟹ *arity-compatibles irs*
assumes ⋀*name irs*. (*name*, *irs*) |∈| *rs* ⟹ *is-fmap irs*
assumes ⋀*name irs*. (*name*, *irs*) |∈| *rs* ⟹ *patterns-compatibles irs*
assumes ⋀*name irs*. (*name*, *irs*) |∈| *rs* ⟹ $irs \neq \{||\}$
assumes ⋀*name irs pats rhs*. (*name*, *irs*) |∈| *rs* ⟹ (*pats*, *rhs*) |∈| *irs* ⟹ *linears pats*
assumes ⋀*name irs pats rhs*. (*name*, *irs*) |∈| *rs* ⟹ (*pats*, *rhs*) |∈| *irs* ⟹ *abs-ish pats rhs*
assumes ⋀*name irs pats rhs*. (*name*, *irs*) |∈| *rs* ⟹ (*pats*, *rhs*) |∈| *irs* ⟹ *fdisjnt* (*freess pats*) *all-consts*
assumes ⋀*name irs pats rhs*. (*name*, *irs*) |∈| *rs* ⟹ (*pats*, *rhs*) |∈| *irs* ⟹ *closed-except rhs* (*freess pats*)
assumes ⋀*name irs pats rhs*. (*name*, *irs*) |∈| *rs* ⟹ (*pats*, *rhs*) |∈| *irs* ⟹ *wellformed rhs*

assumes $\bigwedge$*name irs pats rhs*. (*name*, *irs*) |∈| *rs* $\Longrightarrow$ (*pats*, *rhs*) |∈| *irs* $\Longrightarrow$ ¬ *shadows-consts rhs*
assumes $\bigwedge$*name irs pats rhs*. (*name*, *irs*) |∈| *rs* $\Longrightarrow$ (*pats*, *rhs*) |∈| *irs* $\Longrightarrow$ *welldefined rhs*
assumes *is-fmap rs rs* ≠ {||}
shows *irules C-info rs*
proof *unfold-locales*
show *is-fmap rs*
using *assms*(*12*) **.**
next
show *rs* ≠ {||}
using *assms*(*13*) **.**
next
show *fBall rs* (λ(-, *irs*). *Rewriting-Nterm.arity-compatibles irs* ∧ *is-fmap irs* ∧ *patterns-compatibles irs* ∧ *irs* ≠ {||} ∧
fBall irs (λ(*pats*, *rhs*). *linears pats* ∧ *abs-ish pats rhs* ∧ *closed-except rhs* (*freess pats*) ∧
fdisjnt (*freess pats*) *all-consts* ∧ *pre-strong-term-class.wellformed rhs* ∧
¬ *shadows-consts rhs* ∧ *consts rhs* |⊆| *all-consts*))
using *assms*(*1*−*11*)
by (*intro prod-BallI conjI*) *metis+*
qed

lemmas *irulesI*[*intro*!] = *pre-irules.irulesI*[*unfolded pre-irules-def*]

Translation from *nterm* to *pterm*

fun *nterm-to-pterm* :: *nterm* ⇒ *pterm* **where**
nterm-to-pterm (*Nvar s*) = *Pvar s* |
nterm-to-pterm (*Nconst s*) = *Pconst s* |
nterm-to-pterm (t_1 $\$_n$ t_2) = *nterm-to-pterm* t_1 $\$_p$ *nterm-to-pterm* t_2 |
nterm-to-pterm (Λ_n *x*. *t*) = (Λ_p *x*. *nterm-to-pterm t*)

lemma *nterm-to-pterm-inj*: *nterm-to-pterm x* = *nterm-to-pterm y* $\Longrightarrow$ *x* = *y*
by (*induction y arbitrary*: *x*) (*auto elim*: *nterm-to-pterm.elims*)

lemma *nterm-to-pterm*:
assumes *no-abs t*
shows *nterm-to-pterm t* = *convert-term t*
using *assms*
apply *induction*
apply *auto*
by (*auto simp*: *free-nterm-def free-pterm-def const-nterm-def const-pterm-def app-nterm-def app-pterm-def*)

lemma *nterm-to-pterm-frees*[*simp*]: *frees* (*nterm-to-pterm t*) = *frees t*
by (*induct t*) *auto*

lemma *closed-nterm-to-pterm*[*intro*]: *closed-except* (*nterm-to-pterm t*) (*frees t*)

unfolding *closed-except-def* **by** *simp*

lemma (**in** *constants*) *shadows-nterm-to-pterm*[*simp*]: *shadows-consts* (*nterm-to-pterm* *t*) = *shadows-consts t*
by (*induct t*) (*auto simp*: *shadows-consts-def fdisjnt-alt-def*)

lemma *wellformed-nterm-to-pterm*[*intro*]: *wellformed* (*nterm-to-pterm t*)
by (*induct t*) *auto*

lemma *consts-nterm-to-pterm*[*simp*]: *consts* (*nterm-to-pterm t*) = *consts t*
by (*induct t*) *auto*

Translation from *crule-set* to *irule-set*

definition *translate-crules* :: *crules* ⇒ *irules* **where**
translate-crules = *fimage* (*map-prod id nterm-to-pterm*)

definition *compile* :: *crule-set* ⇒ *irule-set* **where**
compile = *fimage* (*map-prod id translate-crules*)

lemma *compile-heads*: *fst* |'| *compile rs* = *fst* |'| *rs*
unfolding *compile-def* **by** *simp*

lemma (**in** *crules*) *compile-rules*: *irules C-info* (*compile rs*)
proof
  have *is-fmap rs*
    using *fmap* **by** *simp*
  thus *is-fmap* (*compile rs*)
    unfolding *compile-def map-prod-def id-apply* **by** (*rule is-fmap-image*)

  show *compile rs* ≠ {||}
    using *nonempty* **unfolding** *compile-def* **by** *auto*

  show *constants C-info* (*fst* |'| *compile rs*)
    proof
      show *fdisjnt* (*fst* |'| *compile rs*) *C*
        using *disjnt* **unfolding** *compile-def*
        by *force*
    next
      show *distinct all-constructors*
        by (*fact distinct-ctr*)
    qed

  fix *name irs*
  assume *irs*: (*name*, *irs*) |∈| *compile rs*
  then obtain *irs'* **where** (*name*, *irs'*) |∈| *rs irs* = *translate-crules irs'*
    unfolding *compile-def* **by** *force*
  hence *arity-compatibles irs'*
    using *inner*

by (*metis* (*no-types*, *lifting*) *case-prodD*)
thus *arity-compatibles irs*
unfolding ‹*irs* = *translate-crules irs*′› *translate-crules-def*
by (*force dest*: *fpairwiseD*)

have *patterns-compatibles irs*′
using ‹(*name*, *irs*′) |∈| *rs*› *inner*
by (*blast dest*: *fpairwiseD*)
thus *patterns-compatibles irs*
unfolding ‹*irs* = -› *translate-crules-def*
by (*auto dest*: *fpairwiseD*)

have *is-fmap irs*′
using ‹(*name*, *irs*′) |∈| *rs*› *inner* **by** *auto*
thus *is-fmap irs*
unfolding ‹*irs* = *translate-crules irs*′› *translate-crules-def map-prod-def id-apply*
by (*rule is-fmap-image*)

have *irs*′ ≠ {||}
using ‹(*name*, *irs*′) |∈| *rs*› *inner* **by** *auto*
thus *irs* ≠ {||}
unfolding ‹*irs* = *translate-crules irs*′› *translate-crules-def* **by** *simp*

fix *pats rhs*
assume (*pats*, *rhs*) |∈| *irs*
then obtain *rhs*′ **where** (*pats*, *rhs*′) |∈| *irs*′ *rhs* = *nterm-to-pterm rhs*′
unfolding ‹*irs* = *translate-crules irs*′› *translate-crules-def* **by** *force*
hence *linears pats pats* ≠ [] *frees rhs*′ |⊆| *freess pats* ¬ *shadows-consts rhs*′
using *fbspec*[*OF inner* ‹(*name*, *irs*′) |∈| *rs*›]
by *blast*+

show *linears pats* **by** *fact*
show *closed-except rhs* (*freess pats*)
unfolding ‹*rhs* = *nterm-to-pterm rhs*′›
using ‹*frees rhs*′ |⊆| *freess pats*›
by (*metis dual-order.trans closed-nterm-to-pterm closed-except-def*)

show *wellformed rhs*
unfolding ‹*rhs* = *nterm-to-pterm rhs*′› **by** *auto*

have *fdisjnt* (*freess pats*) *all-consts*
using ‹(*pats*, *rhs*′) |∈| *irs*′› ‹(*name*, *irs*′) |∈| *rs*› *inner*
by *auto*
thus *fdisjnt* (*freess pats*) (*pre-constants.all-consts C-info* (*fst* |‘| *compile rs*))
unfolding *compile-def* **by** *simp*

have ¬ *shadows-consts rhs*
unfolding ‹*rhs* = -› **using** ‹¬ *shadows-consts* -› **by** *simp*
thus ¬ *pre-constants.shadows-consts C-info* (*fst* |‘| *compile rs*) *rhs*

  unfolding *compile-heads* **.**

 show *abs-ish pats rhs*
  using ‹*pats* ≠ []› **unfolding** *abs-ish-def* **by** *simp*

 have *welldefined rhs′*
  using *fbspec*[*OF inner* ‹(*name*, *irs′*) |∈| *rs*›, *simplified*]
  using ‹(*pats*, *rhs′*) |∈| *irs′*›
  by *blast*

 thus *pre-constants.welldefined C-info* (*fst* |‘| *compile rs*) *rhs*
  unfolding *compile-def* ‹*rhs* = -›
  by *simp*
qed

sublocale *crules* ⊆ *crules-as-irules*: *irules C-info compile rs*
by (*fact compile-rules*)

Transformation of *irule-set*

definition *transform-irules* :: *irules* ⇒ *irules* **where**
transform-irules rs = (
 if arity rs = *0 then rs*
 else map-prod id Pabs |‘| *fgroup-by* (λ(*pats*, *rhs*). (*butlast pats*, (*last pats*, *rhs*)))
rs)

lemma *arity-compatibles-transform-irules*:
 assumes *arity-compatibles rs*
 shows *arity-compatibles* (*transform-irules rs*)
proof (*cases arity rs* = *0*)
 case *True*
 thus *?thesis*
  unfolding *transform-irules-def* **using** *assms* **by** *simp*
next
 case *False*
 let *?rs′* = *transform-irules rs*
 let *?f* = λ(*pats*, *rhs*). (*butlast pats*, (*last pats*, *rhs*))
 let *?grp* = *fgroup-by ?f rs*
 have *rs′*: *?rs′* = *map-prod id Pabs* |‘| *?grp*
  using *False* **unfolding** *transform-irules-def* **by** *simp*
 show *?thesis*
  proof *safe*
   fix $pats_1$ rhs_1 $pats_2$ rhs_2
   assume ($pats_1$, rhs_1) |∈| *?rs′* ($pats_2$, rhs_2) |∈| *?rs′*
   then obtain rhs_1' rhs_2' **where** ($pats_1$, rhs_1') |∈| *?grp* ($pats_2$, rhs_2') |∈| *?grp*
    unfolding *rs′* **by** *auto*
   then obtain $pats_1'$ $pats_2'$ *x y* — dummies
    where *fst* (*?f* ($pats_1'$, *x*)) = $pats_1$ ($pats_1'$, *x*) |∈| *rs*
     and *fst* (*?f* ($pats_2'$, *y*)) = $pats_2$ ($pats_2'$, *y*) |∈| *rs*

```
      by (fastforce simp: split-beta elim: fgroup-byE2)
    hence pats₁ = butlast pats₁' pats₂ = butlast pats₂' length pats₁' = length pats₂'
      using assms by (force dest: fpairwiseD)+
    thus length pats₁ = length pats₂
      by auto
  qed
qed

lemma arity-transform-irules:
  assumes arity-compatibles rs rs ≠ {||}
  shows arity (transform-irules rs) = (if arity rs = 0 then 0 else arity rs − 1)
proof (cases arity rs = 0)
  case True
  thus ?thesis
    unfolding transform-irules-def by simp
next
  case False
  let ?f = λ(pats, rhs). (butlast pats, (last pats, rhs))
  let ?grp = fgroup-by ?f rs
  let ?rs' = map-prod id Pabs |`| ?grp

  have arity ?rs' = arity rs − 1
    proof (rule arityI)
      show fBall ?rs' (λ(pats, -). length pats = arity rs − 1)
        proof (rule prod-fBallI)
          fix pats rhs
          assume (pats, rhs) |∈| ?rs'
          then obtain cs where (pats, cs) |∈| ?grp rhs = Pabs cs
            by force
          then obtain pats' x — dummy
            where pats = butlast pats' (pats', x) |∈| rs
            by (fastforce simp: split-beta elim: fgroup-byE2)
          hence length pats' = arity rs
            using assms by (metis arity-compatible-length)
          thus length pats = arity rs − 1
            unfolding ‹pats = butlast pats'› using False by simp
        qed
    next
      show ?rs' ≠ {||}
        using assms by (simp add: fgroup-by-nonempty)
    qed

  with False show ?thesis
    unfolding transform-irules-def by simp
qed

definition transform-irule-set :: irule-set ⇒ irule-set where
transform-irule-set = fimage (map-prod id transform-irules)
```

lemma *transform-irule-set-heads*: *fst* |'| *transform-irule-set rs* = *fst* |'| *rs*
unfolding *transform-irule-set-def* **by** *simp*

lemma (**in** *irules*) *rules-transform*: *irules C-info* (*transform-irule-set rs*)
proof
  have *is-fmap rs*
    using *fmap* **by** *simp*
  thus *is-fmap* (*transform-irule-set rs*)
    unfolding *transform-irule-set-def map-prod-def id-apply* **by** (*rule is-fmap-image*)

  show *transform-irule-set rs* ≠ {||}
    using *nonempty* **unfolding** *transform-irule-set-def* **by** *auto*

  show *constants C-info* (*fst* |'| *transform-irule-set rs*)
    proof
      show *fdisjnt* (*fst* |'| *transform-irule-set rs*) *C*
        using *disjnt* **unfolding** *transform-irule-set-def*
        by *force*
    next
      show *distinct all-constructors*
        by (*fact distinct-ctr*)
    qed

  fix *name irs*
  assume *irs*: (*name*, *irs*) |∈| *transform-irule-set rs*
  then obtain *irs'* **where** (*name*, *irs'*) |∈| *rs irs* = *transform-irules irs'*
    unfolding *transform-irule-set-def* **by** *force*
  hence *arity-compatibles irs'*
    using *inner*
    by (*metis* (*no-types*, *lifting*) *case-prodD*)
  thus *arity-compatibles irs*
    unfolding ‹*irs* = *transform-irules irs'*› **by** (*rule arity-compatibles-transform-irules*)

  have *irs'* ≠ {||}
    using ‹(*name*, *irs'*) |∈| *rs*› *inner* **by** *blast*
  thus *irs* ≠ {||}
    unfolding ‹*irs* = *transform-irules irs'*› *transform-irules-def*
    by (*simp add*: *fgroup-by-nonempty*)

  let *?f* = λ(*pats*, *rhs*). (*butlast pats*, (*last pats*, *rhs*))
  let *?grp* = *fgroup-by ?f irs'*

  have *patterns-compatibles irs'*
    using ‹(*name*, *irs'*) |∈| *rs*› *inner*
    by (*blast dest*: *fpairwiseD*)
  show *patterns-compatibles irs*
    proof (*cases arity irs'* = *0*)
      case *True*
      thus *?thesis*

unfolding ‹irs = transform-irules irs'› transform-irules-def
    using ‹patterns-compatibles irs'› by simp
next
  case False
  hence irs': irs = map-prod id Pabs |`| ?grp
    unfolding ‹irs = transform-irules irs'› transform-irules-def by simp

  show ?thesis
    proof safe
      fix $pats_1$ rhs_1 $pats_2$ rhs_2
      assume ($pats_1$, rhs_1) |∈| irs ($pats_2$, rhs_2) |∈| irs
      with irs' obtain cs_1 cs_2 where ($pats_1$, cs_1) |∈| ?grp ($pats_2$, cs_2) |∈| ?grp
        by force
      then obtain $pats_1'$ $pats_2'$ and x y — dummies
        where ($pats_1'$, x) |∈| irs' ($pats_2'$, y) |∈| irs'
          and $pats_1$ = butlast $pats_1'$ $pats_2$ = butlast $pats_2'$
        unfolding irs'
        by (fastforce elim: fgroup-byE2)
      hence patterns-compatible $pats_1'$ $pats_2'$
        using ‹patterns-compatibles irs'› by (auto dest: fpairwiseD)
      thus patterns-compatible $pats_1$ $pats_2$
        unfolding ‹$pats_1$ = -› ‹$pats_2$ = -›
        by auto
    qed
qed

have is-fmap irs'
  using ‹(name, irs') |∈| rs› inner by blast
show is-fmap irs
  proof (cases arity irs' = 0)
    case True
    thus ?thesis
      unfolding ‹irs = transform-irules irs'› transform-irules-def
      using ‹is-fmap irs'› by simp
  next
    case False
    hence irs': irs = map-prod id Pabs |`| ?grp
      unfolding ‹irs = transform-irules irs'› transform-irules-def by simp

    show ?thesis
      proof
        fix pats rhs_1 rhs_2
        assume (pats, rhs_1) |∈| irs (pats, rhs_2) |∈| irs
        with irs' obtain cs_1 cs_2
          where (pats, cs_1) |∈| ?grp rhs_1 = Pabs cs_1
            and (pats, cs_2) |∈| ?grp rhs_2 = Pabs cs_2
          by force
        moreover have is-fmap ?grp
          by auto

```
        ultimately show rhs₁ = rhs₂
          by (auto dest: is-fmapD)
      qed
  qed

fix pats rhs
assume (pats, rhs) |∈| irs

show linears pats
  proof (cases arity irs' = 0)
    case True
    thus ?thesis
      using ‹(pats, rhs) |∈| irs› ‹(name, irs') |∈| rs› inner
      unfolding ‹irs = transform-irules irs'› transform-irules-def
      by auto
  next
    case False
    hence irs': irs = map-prod id Pabs |`| ?grp
      unfolding ‹irs = transform-irules irs'› transform-irules-def by simp
    then obtain cs where (pats, cs) |∈| ?grp
      using ‹(pats, rhs) |∈| irs› by force
    then obtain pats' x — dummy
      where fst (?f (pats', x)) = pats (pats', x) |∈| irs'
      by (fastforce simp: split-beta elim: fgroup-byE2)
    hence pats = butlast pats'
      by simp
    moreover have linears pats'
      using ‹(pats', x) |∈| irs'› ‹(name, irs') |∈| rs› inner
      by auto
    ultimately show ?thesis
      by auto
  qed

have fdisjnt (freess pats) all-consts
  proof (cases arity irs' = 0)
    case True
    thus ?thesis
      using ‹(pats, rhs) |∈| irs› ‹(name, irs') |∈| rs› inner
      unfolding ‹irs = transform-irules irs'› transform-irules-def
      by auto
  next
    case False
    hence irs': irs = map-prod id Pabs |`| ?grp
      unfolding ‹irs = transform-irules irs'› transform-irules-def by simp
    then obtain cs where (pats, cs) |∈| ?grp
      using ‹(pats, rhs) |∈| irs› by force
    then obtain pats' x — dummy
      where fst (?f (pats', x)) = pats (pats', x) |∈| irs'
      by (fastforce simp: split-beta elim: fgroup-byE2)
```

hence *pats* = *butlast pats'*
by *simp*
moreover have *fdisjnt* (*freess pats'*) *all-consts*
using ‹(*pats'*, *x*) |∈| *irs'*› ‹(*name*, *irs'*) |∈| *rs*› *inner* **by** *auto*
ultimately show *?thesis*
by (*metis subsetI in-set-butlastD freess-subset fdisjnt-subset-left*)
qed

thus *fdisjnt* (*freess pats*) (*pre-constants.all-consts C-info* (*fst* |'| *transform-irule-set rs*))
unfolding *transform-irule-set-def* **by** *simp*

show *closed-except rhs* (*freess pats*)
proof (*cases arity irs'* = *0*)
case *True*
thus *?thesis*
using ‹(*pats*, *rhs*) |∈| *irs*› ‹(*name*, *irs'*) |∈| *rs*› *inner*
unfolding ‹*irs* = *transform-irules irs'*› *transform-irules-def*
by *auto*
next
case *False*
hence *irs'*: *irs* = *map-prod id Pabs* |'| *?grp*
unfolding ‹*irs* = *transform-irules irs'*› *transform-irules-def* **by** *simp*
then obtain *cs* **where** (*pats*, *cs*) |∈| *?grp rhs* = *Pabs cs*
using ‹(*pats*, *rhs*) |∈| *irs*› **by** *force*

show *?thesis*
unfolding ‹*rhs* = *Pabs cs*› *closed-except-simps*
proof *safe*
fix *pat t*
assume (*pat*, *t*) |∈| *cs*
then obtain *pats'* **where** (*pats'*, *t*) |∈| *irs' ?f* (*pats'*, *t*) = (*pats*, (*pat*, *t*))
using ‹(*pats*, *cs*) |∈| *?grp*› **by** *auto*
hence *closed-except t* (*freess pats'*)
using ‹(*name*, *irs'*) |∈| *rs*› *inner* **by** *auto*

have *pats'* ≠ []
using ‹*arity-compatibles irs'*› ‹(*pats'*, *t*) |∈| *irs'*› *False*
by (*metis list.size*(*3*) *arity-compatible-length*)
hence *pats'* = *pats* @ [*pat*]
using ‹*?f* (*pats'*, *t*) = (*pats*, (*pat*, *t*))›
by (*fastforce simp*: *split-beta snoc-eq-iff-butlast*)
hence *freess pats* |∪| *frees pat* = *freess pats'*
unfolding *freess-def* **by** *auto*

thus *closed-except t* (*freess pats* |∪| *frees pat*)
using ‹*closed-except t* (*freess pats'*)› **by** *simp*
qed
qed

```
show wellformed rhs
  proof (cases arity irs' = 0)
    case True
    thus ?thesis
      using ‹(pats, rhs) |∈| irs› ‹(name, irs') |∈| rs› inner
      unfolding ‹irs = transform-irules irs'› transform-irules-def
      by auto
  next
    case False
    hence irs': irs = map-prod id Pabs |`| ?grp
      unfolding ‹irs = transform-irules irs'› transform-irules-def by simp
    then obtain cs where (pats, cs) |∈| ?grp rhs = Pabs cs
      using ‹(pats, rhs) |∈| irs› by force

    show ?thesis
      unfolding ‹rhs = Pabs cs›
      proof (rule wellformed-PabsI)
        show cs ≠ {||}
          using ‹(pats, cs) |∈| ?grp› ‹irs' ≠ {||}›
          by (meson femptyE fgroup-by-nonempty-inner)
      next
        show is-fmap cs
          proof
            fix pat t₁ t₂
            assume (pat, t₁) |∈| cs (pat, t₂) |∈| cs
            then obtain pats₁' pats₂'
              where (pats₁', t₁) |∈| irs' ?f (pats₁', t₁) = (pats, (pat, t₁))
                and (pats₂', t₂) |∈| irs' ?f (pats₂', t₂) = (pats, (pat, t₂))
              using ‹(pats, cs) |∈| ?grp› by force
            moreover hence pats₁' ≠ [] pats₂' ≠ []
              using ‹arity-compatibles irs'› False
              unfolding prod.case
              by (metis list.size(3) arity-compatible-length)+
            ultimately have pats₁' = pats @ [pat] pats₂' = pats @ [pat]
              unfolding split-beta fst-conv snd-conv
              by (metis prod.inject snoc-eq-iff-butlast)+
            with ‹is-fmap irs'› show t₁ = t₂
              using ‹(pats₁', t₁) |∈| irs'› ‹(pats₂', t₂) |∈| irs'›
              by (blast dest: is-fmapD)
          qed
      next
        show pattern-compatibles cs
          proof safe
            fix pat₁ rhs₁ pat₂ rhs₂
            assume (pat₁, rhs₁) |∈| cs (pat₂, rhs₂) |∈| cs
            then obtain pats₁' pats₂'
              where (pats₁', rhs₁) |∈| irs' ?f (pats₁', rhs₁) = (pats, (pat₁, rhs₁))
                and (pats₂', rhs₂) |∈| irs' ?f (pats₂', rhs₂) = (pats, (pat₂, rhs₂))
```

```
          using ‹(pats, cs) |∈| ?grp›
          by force
        moreover hence pats_1' ≠ [] pats_2' ≠ []
          using ‹arity-compatibles irs'› False
          unfolding prod.case
          by (metis list.size(3) arity-compatible-length)+
        ultimately have pats_1' = pats @ [pat_1] pats_2' = pats @ [pat_2]
          unfolding split-beta fst-conv snd-conv
          by (metis prod.inject snoc-eq-iff-butlast)+
        moreover have patterns-compatible pats_1' pats_2'
      using ‹(pats_1', rhs_1) |∈| irs'› ‹(pats_2', rhs_2) |∈| irs'› ‹patterns-compatibles
irs'›
          by (auto dest: fpairwiseD)
        ultimately show pattern-compatible pat_1 pat_2
          by (auto elim: rev-accum-rel-snoc-eqE)
      qed
    next
      fix pat t
      assume (pat, t) |∈| cs
      then obtain pats' where (pats', t) |∈| irs' pat = last pats'
        using ‹(pats, cs) |∈| ?grp› by auto
      moreover hence pats' ≠ []
        using ‹arity-compatibles irs'› False
        by (metis list.size(3) arity-compatible-length)
      ultimately have pat ∈ set pats'
        by auto

      moreover have linears pats'
        using ‹(pats', t) |∈| irs'› ‹(name, irs') |∈| rs› inner by auto
      ultimately show linear pat
        by (metis linears-linear)

      show wellformed t
        using ‹(pats', t) |∈| irs'› ‹(name, irs') |∈| rs› inner by auto
    qed
  qed

  have ¬ shadows-consts rhs
    proof (cases arity irs' = 0)
      case True
      thus ?thesis
        using ‹(pats, rhs) |∈| irs› ‹(name, irs') |∈| rs› inner
        unfolding ‹irs = transform-irules irs'› transform-irules-def
        by auto
    next
      case False
      hence irs': irs = map-prod id Pabs |`| ?grp
        unfolding ‹irs = transform-irules irs'› transform-irules-def by simp
      then obtain cs where (pats, cs) |∈| ?grp rhs = Pabs cs
```

```
      using ‹(pats, rhs) |∈| irs› by force

    show ?thesis
      unfolding ‹rhs = -›
      proof
        assume shadows-consts (Pabs cs)
        then obtain pat t where (pat, t) |∈| cs shadows-consts t ∨ shadows-consts
pat
          by force
        then obtain pats' where (pats', t) |∈| irs' pat = last pats'
          using ‹(pats, cs) |∈| ?grp› by auto
        moreover hence pats' ≠ []
          using ‹arity-compatibles irs'› False
          by (metis list.size(3) arity-compatible-length)
        ultimately have pat ∈ set pats'
          by auto

        show False
          using ‹shadows-consts t ∨ shadows-consts pat›
          proof
            assume shadows-consts t
            thus False
              using ‹(name, irs') |∈| rs› ‹(pats', t) |∈| irs'› inner by auto
          next
            assume shadows-consts pat

            have fdisjnt (freess pats') all-consts
              using ‹(name, irs') |∈| rs› ‹(pats', t) |∈| irs'› inner by auto
            have fdisjnt (frees pat) all-consts
              apply (rule fdisjnt-subset-left)
               apply (subst freess-single[symmetric])
               apply (rule freess-subset)
               apply simp
               apply fact+
              done

            thus False
              using ‹shadows-consts pat›
              unfolding shadows-consts-def fdisjnt-alt-def by auto
          qed
      qed
  qed
  thus ¬ pre-constants.shadows-consts C-info (fst |`| transform-irule-set rs) rhs
    by (simp add: transform-irule-set-heads)

  show abs-ish pats rhs
    proof (cases arity irs' = 0)
      case True
      thus ?thesis
```

```
      using ‹(pats, rhs) |∈| irs› ‹(name, irs') |∈| rs› inner
      unfolding ‹irs = transform-irules irs'› transform-irules-def
      by auto
  next
    case False
    hence irs': irs = map-prod id Pabs |`| ?grp
      unfolding ‹irs = transform-irules irs'› transform-irules-def by simp
    then obtain cs where (pats, cs) |∈| ?grp rhs = Pabs cs
      using ‹(pats, rhs) |∈| irs› by force
    thus ?thesis
      unfolding abs-ish-def by (simp add: is-abs-def term-cases-def)
  qed

have welldefined rhs
  proof (cases arity irs' = 0)
    case True
    hence ‹(pats, rhs) |∈| irs'›
      using ‹(pats, rhs) |∈| irs› ‹(name, irs') |∈| rs› inner
      unfolding ‹irs = transform-irules irs'› transform-irules-def
      by (smt fBallE split-conv)
    thus ?thesis
      unfolding transform-irule-set-def
      using fbspec[OF inner ‹(name, irs') |∈| rs›, simplified]
      by force
  next
    case False
    hence irs': irs = map-prod id Pabs |`| ?grp
      unfolding ‹irs = transform-irules irs'› transform-irules-def by simp
    then obtain cs where (pats, cs) |∈| ?grp rhs = Pabs cs
      using ‹(pats, rhs) |∈| irs› by force
    show ?thesis
      unfolding ‹rhs = -›
      apply simp
      apply (rule ffUnion-least)
      unfolding ball-simps
      apply rule
      apply (rename-tac x, case-tac x, hypsubst-thin)
      apply simp
      subgoal premises prems for pat t
        proof −
          from prems obtain pats' where (pats', t) |∈| irs'
            using ‹(pats, cs) |∈| ?grp› by auto
          hence welldefined t
            using fbspec[OF inner ‹(name, irs') |∈| rs›, simplified]
            by blast
          thus ?thesis
            unfolding transform-irule-set-def
            by simp
        qed
```

done
qed
thus *pre-constants.welldefined C-info* (*fst* |'| *transform-irule-set rs*) *rhs*
unfolding *transform-irule-set-heads* **.**
qed

Matching and rewriting

definition *irewrite-step* :: *name* ⇒ *term list* ⇒ *pterm* ⇒ *pterm* ⇒ *pterm option*
where
irewrite-step name pats rhs t = *map-option* (*subst rhs*) (*match* (*name* $$ *pats*) *t*)

abbreviation *irewrite-step'* :: *name* ⇒ *term list* ⇒ *pterm* ⇒ *pterm* ⇒ *pterm* ⇒ *bool* (‹-, -, - $\vdash_i$/ - →/ -› [*50*,*0*,*50*] *50*) **where**
name, *pats*, *rhs* $\vdash_i$ *t* → *u* ≡ *irewrite-step name pats rhs t* = *Some u*

lemma *irewrite-stepI*:
assumes *match* (*name* $$ *pats*) *t* = *Some env subst rhs env* = *u*
shows *name*, *pats*, *rhs* $\vdash_i$ *t* → *u*
using *assms* **unfolding** *irewrite-step-def* **by** *simp*

inductive *irewrite* :: *irule-set* ⇒ *pterm* ⇒ *pterm* ⇒ *bool* (‹-/ $\vdash_i$/ - ⟶/ -› [*50*,*0*,*50*] *50*) **for** *irs* **where**
step: ⟦ (*name*, *rs*) |∈| *irs*; (*pats*, *rhs*) |∈| *rs*; *name*, *pats*, *rhs* $\vdash_i$ *t* → *t'* ⟧ ⟹ *irs* $\vdash_i$ *t* ⟶ *t'* |
beta: ⟦ *c* |∈| *cs*; *c* ⊢ *t* → *t'* ⟧ ⟹ *irs* $\vdash_i$ *Pabs cs* $\$_p$ *t* ⟶ *t'* |
fun: *irs* $\vdash_i$ *t* ⟶ *t'* ⟹ *irs* $\vdash_i$ *t* $\$_p$ *u* ⟶ *t'* $\$_p$ *u* |
arg: *irs* $\vdash_i$ *u* ⟶ *u'* ⟹ *irs* $\vdash_i$ *t* $\$_p$ *u* ⟶ *t* $\$_p$ *u'*

global-interpretation *irewrite*: *rewriting irewrite rs* **for** *rs*
by *standard* (*auto intro*: *irewrite.intros simp*: *app-pterm-def*)+

abbreviation *irewrite-rt* :: *irule-set* ⇒ *pterm* ⇒ *pterm* ⇒ *bool* (‹-/ $\vdash_i$/ - ⟶∗/ -› [*50*,*0*,*50*] *50*) **where**
irewrite-rt rs ≡ (*irewrite rs*)**

lemma (**in** *irules*) *irewrite-closed*:
assumes *rs* $\vdash_i$ *t* ⟶ *u closed t*
shows *closed u*
using *assms* **proof** *induction*
case (*step name rs pats rhs t t'*)
then obtain *env* **where** *match* (*name* $$ *pats*) *t* = *Some env t'* = *subst rhs env*
unfolding *irewrite-step-def* **by** *auto*
hence *closed-env env*
using *step* **by** (*auto intro*: *closed.match*)

show *?case*
unfolding ‹*t'* = -›
apply (*subst closed-except-def*)

apply (*subst subst-frees*)
 apply *fact*
apply (*subst match-dom*)
 apply *fact*
apply (*subst frees-list-comb*)
apply *simp*
apply (*subst closed-except-def*[*symmetric*])
using *inner step* **by** *fastforce*

next
case (*beta c cs t t′*)
then obtain *pat rhs* **where** $c = (pat, rhs)$
by (*cases c*) *auto*
with *beta* **obtain** *env* **where** *match pat t* = *Some env t′* = *subst rhs env*
by *auto*
moreover have *closed t*
using *beta* **unfolding** *closed-except-def* **by** *simp*
ultimately have *closed-env env*
using *beta* **by** (*auto intro*: *closed.match*)

show *?case*
unfolding ‹*t′* = *subst rhs env*›
apply (*subst closed-except-def*)
apply (*subst subst-frees*)
 apply *fact*
apply (*subst match-dom*)
 apply *fact*
apply *simp*
apply (*subst closed-except-def*[*symmetric*])
using *inner beta* ‹*c* = -› **by** (*auto simp*: *closed-except-simps*)

qed (*auto simp*: *closed-except-def*)

corollary (**in** *irules*) *irewrite-rt-closed*:
assumes $rs \vdash_i t \longrightarrow* u$ *closed t*
shows *closed u*
using *assms* **by** *induction* (*auto intro*: *irewrite-closed*)

Correctness of translation

abbreviation *irelated* :: *nterm* ⇒ *pterm* ⇒ *bool* (‹- $\approx_i$ -› [*0*,*50*] *50*) **where**
$n \approx_i p \equiv$ *nterm-to-pterm n* = *p*

global-interpretation *irelated*: *term-struct-rel-strong irelated*
by *standard*
(*auto simp*: *app-pterm-def app-nterm-def const-pterm-def const-nterm-def elim*: *nterm-to-pterm.elims*)

lemma *irelated-vars*: $t \approx_i u \Longrightarrow$ *frees t* = *frees u*
by *auto*

lemma *irelated-no-abs*:
  assumes $t \approx_i u$
  shows *no-abs* $t \longleftrightarrow$ *no-abs* u
using *assms*
apply (*induction arbitrary*: *t*)
 apply (*auto elim*!: *nterm-to-pterm.elims*)
  apply (*fold const-nterm-def const-pterm-def free-nterm-def free-pterm-def app-pterm-def app-nterm-def*)
by *auto*

lemma *irelated-subst*:
  assumes $t \approx_i u$ *irelated.P-env nenv penv*
  shows *subst t nenv* $\approx_i$ *subst u penv*
using *assms* **proof** (*induction arbitrary*: *nenv penv u rule*: *nterm-to-pterm.induct*)
  case (*1 s*)
  then show *?case*
    by (*auto elim*!: *fmrel-cases*[**where** $x = s$])
next
  case *4*
  from *4*(*2*)[*symmetric*] **show** *?case*
    apply *simp*
    apply (*rule 4*)
     apply *simp*
    using *4*(*3*)
    by (*simp add*: *fmrel-drop*)
qed *auto*

lemma *related-irewrite-step*:
  assumes *name*, *pats*, *nterm-to-pterm rhs* $\vdash_i u \rightarrow u'$ $t \approx_i u$
  obtains t' **where** *unsplit-rule* (*name*, *pats*, *rhs*) $\vdash t \rightarrow t'$ $t' \approx_i u'$
proof −
  let *?rhs'* = *nterm-to-pterm rhs*
  let *?x* = *name* $$ *pats*

  from *assms* **obtain** *env* **where** *match ?x u* = *Some env* u' = *subst ?rhs' env*
    unfolding *irewrite-step-def* **by** *blast*
  then obtain *nenv* **where** *match ?x t* = *Some nenv irelated.P-env nenv env*
    using *assms*
   by (*metis Option.is-none-def not-None-eq option.rel-distinct*(*1*) *option.sel rel-option-unfold irelated.match-rel*)

  show *thesis*
    proof
      show *unsplit-rule* (*name*, *pats*, *rhs*) $\vdash t \rightarrow$ *subst rhs nenv*
        using ‹*match ?x t* = -› **by** *auto*
    next
      show *subst rhs nenv* $\approx_i u'$
        unfolding ‹u' = -› **using** ‹*irelated.P-env nenv env*›
        by (*auto intro*: *irelated-subst*)

    qed
qed

theorem (**in** *nrules*) *compile-correct*:
  assumes *compile* (*consts-of rs*) $\vdash_i$ *u* $\longrightarrow$ *u′ t* $\approx_i$ *u closed t*
  obtains *t′* **where** *rs* $\vdash_n$ *t* $\longrightarrow$ *t′ t′* $\approx_i$ *u′*
using *assms*(*1*−*3*) **proof** (*induction arbitrary*: *t thesis rule*: *irewrite.induct*)
  case (*step name irs pats rhs u u′*)
  then obtain *crs* **where** *irs* = *translate-crules crs* (*name*, *crs*) |∈| *consts-of rs*
    unfolding *compile-def* **by** *force*
  moreover with *step* **obtain** *rhs′* **where** *rhs* = *nterm-to-pterm rhs′* (*pats*, *rhs′*) |∈| *crs*
    unfolding *translate-crules-def* **by** *force*
  ultimately obtain *rule* **where** *split-rule rule* = (*name*, (*pats*, *rhs′*)) *rule* |∈| *rs*
    unfolding *consts-of-def* **by** *blast*
  hence *nrule rule*
    using *all-rules* **by** *blast*

  obtain *t′* **where** *unsplit-rule* (*name*, *pats*, *rhs′*) $\vdash$ *t* $\rightarrow$ *t′ t′* $\approx_i$ *u′*
    using ‹*name*, *pats*, *rhs* $\vdash_i$ *u* $\rightarrow$ *u′*› ‹*t* $\approx_i$ *u*› **unfolding** ‹*rhs* = *nterm-to-pterm rhs′*›
    by (*elim related-irewrite-step*)
  hence *rule* $\vdash$ *t* $\rightarrow$ *t′*
    using ‹*nrule rule*› ‹*split-rule rule* = (*name*, (*pats*, *rhs′*))›
    by (*metis unsplit-split*)

  show *?case*
    proof (*rule step.prems*)
      show *rs* $\vdash_n$ *t* $\longrightarrow$ *t′*
        apply (*rule nrewrite.step*)
         apply *fact*
        apply *fact*
        done
    next
      show *t′* $\approx_i$ *u′*
        by *fact*
    qed
next
  case (*beta c cs u u′*)
  then obtain *pat rhs* **where** *c* = (*pat*, *rhs*) (*pat*, *rhs*) |∈| *cs*
    by (*cases c*) *auto*
  obtain *v w* **where** *t* = *v* $\$_n$ *w v* $\approx_i$ *Pabs cs w* $\approx_i$ *u*
    using ‹*t* $\approx_i$ *Pabs cs* $\$_p$ *u*› **by** (*auto elim*: *nterm-to-pterm.elims*)
  obtain *x nrhs irhs* **where** *v* = (Λ_n *x*. *nrhs*) *cs* = {| (*Free x*, *irhs*) |} *nrhs* $\approx_i$ *irhs*
    using ‹*v* $\approx_i$ *Pabs cs*› **by** (*auto elim*: *nterm-to-pterm.elims*)
  hence *t* = (Λ_n *x*. *nrhs*) $\$_n$ *w* Λ_n *x*. *nrhs* $\approx_i$ Λ_p *x*. *irhs*
    unfolding ‹*t* = *v* $\$_n$ *w*› **using** ‹*v* $\approx_i$ *Pabs cs*› **by** *auto*

  have *pat* = *Free x rhs* = *irhs*

using ‹$cs = \{|\ (\mathit{Free}\ x,\ \mathit{irhs})\ |\}$› ‹$(\mathit{pat},\ \mathit{rhs})\ |\in|\ cs$› **by** *auto*
hence $(\mathit{Free}\ x,\ \mathit{irhs}) \vdash u \to u'$
using *beta* ‹$c = \text{-}$› **by** *simp*
hence $u' = \mathit{subst}\ \mathit{irhs}\ (\mathit{fmap\text{-}of\text{-}list}\ [(x,\ u)])$
by *simp*

show *?case*
proof (*rule beta.prems*)
show $rs \vdash_n t \longrightarrow \mathit{subst}\ \mathit{nrhs}\ (\mathit{fmap\text{-}of\text{-}list}\ [(x,\ w)])$
unfolding ‹$t = (\Lambda_n\ x.\ \mathit{nrhs})\ \$_n\ w$›
by (*rule nrewrite.beta*)
next
show $\mathit{subst}\ \mathit{nrhs}\ (\mathit{fmap\text{-}of\text{-}list}\ [(x,\ w)]) \approx_i u'$
unfolding ‹$u' = \mathit{subst}\ \mathit{irhs}\ \text{-}$›
apply (*rule irelated-subst*)
apply *fact*
apply *simp*
apply *rule*
apply *rule*
apply *fact*
done
qed
next
case $(\mathit{fun}\ v\ v'\ u)$
obtain $w\ x$ **where** $t = w\ \$_n\ x$ $w \approx_i v$ $x \approx_i u$
using ‹$t \approx_i v\ \$_p\ u$› **by** (*auto elim*: *nterm-to-pterm.elims*)
with *fun* **obtain** w' **where** $rs \vdash_n w \longrightarrow w'$ $w' \approx_i v'$
unfolding *closed-except-def* **by** *auto*

show *?case*
proof (*rule fun.prems*)
show $rs \vdash_n t \longrightarrow w'\ \$_n\ x$
unfolding ‹$t = w\ \$_n\ x$›
by (*rule nrewrite.fun*) *fact*
next
show $w'\ \$_n\ x \approx_i v'\ \$_p\ u$
by *auto fact+*
qed
next
case $(\mathit{arg}\ u\ u'\ v)$
obtain $w\ x$ **where** $t = w\ \$_n\ x$ $w \approx_i v$ $x \approx_i u$
using ‹ $t \approx_i v\ \$_p\ u$› **by** (*auto elim*: *nterm-to-pterm.elims*)
with *arg* **obtain** x' **where** $rs \vdash_n x \longrightarrow x'$ $x' \approx_i u'$
unfolding *closed-except-def* **by** *auto*

show *?case*
proof (*rule arg.prems*)
show $rs \vdash_n t \longrightarrow w\ \$_n\ x'$
unfolding ‹$t = w\ \$_n\ x$›

by (*rule nrewrite.arg*) *fact*
next
show $w \ \$_n \ x' \approx_i v \ \$_p \ u'$
by *auto fact+*
qed
qed

corollary (**in** *nrules*) *compile-correct-rt*:
assumes *compile* (*consts-of rs*) $\vdash_i u \longrightarrow* u'$ $t \approx_i u$ *closed t*
obtains t' **where** *rs* $\vdash_n t \longrightarrow* t'$ $t' \approx_i u'$
using *assms* **proof** (*induction arbitrary*: *thesis t*)
case (*step* u' u'')

obtain t' **where** *rs* $\vdash_n t \longrightarrow* t'$ $t' \approx_i u'$
using *step* **by** *blast*

obtain t'' **where** *rs* $\vdash_n t' \longrightarrow* t''$ $t'' \approx_i u''$
proof (*rule compile-correct*)
show *compile* (*consts-of rs*) $\vdash_i u' \longrightarrow u''$ $t' \approx_i u'$
by *fact+*
next
show *closed* t'
using ‹*rs* $\vdash_n t \longrightarrow* t'$› ‹*closed t*›
by (*rule nrewrite-rt-closed*)
qed *blast*

show *?case*
proof (*rule step.prems*)
show *rs* $\vdash_n t \longrightarrow* t''$
using ‹*rs* $\vdash_n t \longrightarrow* t'$› ‹*rs* $\vdash_n t' \longrightarrow* t''$› **by** *auto*
qed *fact*
qed *blast*

Completeness of translation

lemma (**in** *nrules*) *compile-complete*:
assumes *rs* $\vdash_n t \longrightarrow t'$ *closed t*
shows *compile* (*consts-of rs*) $\vdash_i$ *nterm-to-pterm t* $\longrightarrow$ *nterm-to-pterm* t'
using *assms* **proof** *induction*
case (*step r t* t')
then obtain *pat* rhs' **where** $r = (pat, rhs')$
by *force*
then have (pat, rhs') $|\in|$ *rs* $(pat, rhs') \vdash t \rightarrow t'$
using *step* **by** *blast+*
then have *nrule* (pat, rhs')
using *all-rules* **by** *blast*
then obtain *name pats* **where** $(name, (pats, rhs')) = split\text{-}rule\ r$ $pat = name$
$$ *pats*
unfolding *split-rule-def* ‹$r = -$›

```
    apply atomize-elim
    by (auto simp: split-beta)

  obtain crs where (name, crs) |∈| consts-of rs (pats, rhs') |∈| crs
    using step ‹- = split-rule r› ‹r = -›
    by (metis consts-of-def fgroup-by-complete fst-conv snd-conv)
  then obtain irs where irs = translate-crules crs
    by blast
  then have (name, irs) |∈| compile (consts-of rs)
    unfolding compile-def
    using ‹(name, -) |∈| -›
    by (metis fimageI id-def map-prod-simp)
  obtain rhs where rhs = nterm-to-pterm rhs' (pats, rhs) |∈| irs
    using ‹irs = -› ‹- |∈| crs›
    unfolding translate-crules-def
    by (metis fimageI id-def map-prod-simp)

  from step obtain env' where match pat t = Some env' t' = subst rhs' env'
    unfolding ‹r = -› using rewrite-step.simps
    by force
  then obtain env where match pat (nterm-to-pterm t) = Some env irelated.P-env
env' env
    by (metis irelated.match-rel option-rel-Some1)
  then have subst rhs env = nterm-to-pterm t'
    unfolding ‹t' = -›
    apply −
    apply (rule sym)
    apply (rule irelated-subst)
    unfolding ‹rhs = -›
    by auto

  have name, pats, rhs ⊢ᵢ nterm-to-pterm t → nterm-to-pterm t'
    apply (rule irewrite-stepI)
    using ‹match - - = Some env› unfolding ‹pat = -›
     apply assumption
    by fact

  show ?case
    by rule fact+
next
  case (beta x t t')
  obtain c where c = (Free x, nterm-to-pterm t)
    by blast
  from beta have closed (nterm-to-pterm t')
    using closed-nterm-to-pterm[where t = t']
    unfolding closed-except-def
    by auto
  show ?case
    apply simp
```

apply *rule*
using ‹c = -›
by (*fastforce intro*: *irelated-subst*[*THEN sym*])+
next
case (*fun t t′ u*)
show *?case*
apply *simp*
apply *rule*
apply (*rule fun*)
using *fun*
unfolding *closed-except-def*
apply *simp*
done
next
case (*arg u u′ t*)
show *?case*
apply *simp*
apply *rule*
apply (*rule arg*)
using *arg*
unfolding *closed-except-def*
by *simp*
qed

Correctness of transformation

abbreviation *irules-deferred-matches* :: *pterm list* ⇒ *irules* ⇒ (*term* × *pterm*) *fset* **where**
irules-deferred-matches args ≡ *fselect*
(λ(*pats*, *rhs*). *map-option* (λ*env*. (*last pats*, *subst rhs env*)) (*matchs* (*butlast pats*) *args*))

context *irules* **begin**

inductive *prelated* :: *pterm* ⇒ *pterm* ⇒ *bool* (‹- $\approx_p$ -› [*0*,*50*] *50*) **where**
const: *Pconst x* $\approx_p$ *Pconst x* |
var: *Pvar x* $\approx_p$ *Pvar x* |
app: $t_1 \approx_p u_1 \Longrightarrow t_2 \approx_p u_2 \Longrightarrow t_1$ \$$_p$ $t_2 \approx_p u_1$ \$$_p$ u_2 |
pat: *rel-fset* (*rel-prod* (=) *prelated*) cs_1 cs_2 ⟹ *Pabs* cs_1 $\approx_p$ *Pabs* cs_2 |
defer:
(*name*, *rsi*) |∈| *rs* ⟹ *0* < *arity rsi* ⟹
rel-fset (*rel-prod* (=) *prelated*) (*irules-deferred-matches args rsi*) *cs* ⟹
list-all closed args ⟹
name \$\$ *args* $\approx_p$ *Pabs cs*

inductive-cases *prelated-absE*[*consumes 1*, *case-names pat defer*]: *t* $\approx_p$ *Pabs cs*

lemma *prelated-refl*[*intro*!]: *t* $\approx_p$ *t*
proof (*induction t*)

```
  case Pabs
  thus ?case
    by (auto simp: snds.simps intro!: prelated.pat rel-fset-refl-strong rel-prod.intros)
qed (auto intro: prelated.intros)

sublocale prelated: term-struct-rel prelated
by standard (auto simp: const-pterm-def app-pterm-def intro: prelated.intros elim:
prelated.cases)

lemma prelated-pvars:
  assumes t ≈p u
  shows frees t = frees u
using assms proof (induction rule: prelated.induct)
  case (pat cs1 cs2)
  show ?case
    apply simp
    apply (rule arg-cong[where f = ffUnion])
    apply (rule rel-fset-image-eq)
     apply fact
    apply auto
    done
next
  case (defer name rsi args cs)

  {
    fix pat t
    assume (pat, t) |∈| cs
    with defer obtain t'
      where (pat, t') |∈| irules-deferred-matches args rsi frees t = frees t'
      by (auto elim: rel-fsetE2)
    then obtain pats rhs env
      where pat = last pats (pats, rhs) |∈| rsi
        and matchs (butlast pats) args = Some env t' = subst rhs env
      by auto

    have closed-except rhs (freess pats) linears pats
      using ‹(pats, rhs) |∈| rsi› ‹(name, rsi) |∈| rs› inner by auto

    have arity-compatibles rsi
      using defer inner
      by (metis (no-types, lifting) case-prodD)
    have length pats > 0
      by (subst arity-compatible-length) fact+
    hence pats = butlast pats @ [last pats]
      by simp

    note ‹frees t = frees t'›
    also have frees t' = frees rhs − fmdom env
      unfolding ‹t' = -›
```

```
    apply (rule subst-frees)
    apply (rule closed.matchs)
     apply fact+
    done
  also have ... = frees rhs − freess (butlast pats)
    using ‹matchs - - = -› by (metis matchs-dom)
  also have ... |⊆| freess pats − freess (butlast pats)
    using ‹closed-except - -›
    by (auto simp: closed-except-def)
  also have ... = frees (last pats) |−| freess (butlast pats)
    by (subst ‹pats = -›) (simp add: funion-fminus)
  also have ... = frees (last pats)
    proof (rule fminus-triv)
      have fdisjnt (freess (butlast pats)) (freess [last pats])
        using ‹linears pats› ‹pats = -›
        by (metis linears-appendD)
      thus frees (last pats) |∩| freess (butlast pats) = {||}
        by (fastforce simp: fdisjnt-alt-def)
    qed
  also have ... = frees pat unfolding ‹pat = -› ..
  finally have frees t |⊆| frees pat .
 }
 hence closed (Pabs cs)
   unfolding closed-except-simps
   by (auto simp: closed-except-def)
 moreover have closed (name $$ args)
   unfolding closed-list-comb by fact
 ultimately show ?case
   unfolding closed-except-def by simp
qed auto

corollary prelated-closed: t ≈p u ⟹ closed t ⟷ closed u
unfolding closed-except-def
by (auto simp: prelated-pvars)

lemma prelated-no-abs-right:
 assumes t ≈p u no-abs u
 shows t = u
using assms
apply (induction rule: prelated.induct)
    apply auto
 apply (fold app-pterm-def)
 apply auto
done

corollary env-prelated-refl[intro!]: prelated.P-env env env
by (auto intro: fmap.rel-refl)
```

The following, more general statement does not hold: $t \approx_p u \Longrightarrow$ *rel-option* *prelated.P-env* (*match* x t) (*match* x u) If t and u are related because of the *prelated.defer* rule, they have completely different shapes. Establishing *is-abs* t = *is-abs* u as a precondition would rule out this case, but at the same time be too restrictive.

Instead, we use $[\![$*match ?x ?u = Some ?env*; *?t* $\approx_p$ *?u*; $\bigwedge$*env'*. $[\![$*match ?x ?t = Some env'*; *prelated.P-env env' ?env*$]\!] \Longrightarrow$ *?thesis*$]\!] \Longrightarrow$ *?thesis*.

lemma *prelated-subst*:
  assumes $t_1 \approx_p t_2$ *prelated.P-env* env_1 env_2
  shows *subst* t_1 $env_1 \approx_p$ *subst* t_2 env_2
using *assms* **proof** (*induction arbitrary*: env_1 env_2 *rule*: *prelated.induct*)
  case (*var x*)
  thus *?case*
    proof (*cases rule*: *fmrel-cases*[**where** $x = x$])
      case *none*
      thus *?thesis*
        by (*auto intro*: *prelated.var*)
    next
      case (*some t u*)
      thus *?thesis*
        by *simp*
    qed
next
  case (*pat* cs_1 cs_2)
  let *?drop* = λ*env*. λ(*pat*::*term*). *fmdrop-fset* (*frees pat*) *env*
  from *pat* **have** *prelated.P-env* (*?drop* env_1 *pat*) (*?drop* env_2 *pat*) **for** *pat*
    by *blast*
  with *pat* **show** *?case*
    by (*auto intro*!: *prelated.pat rel-fset-image*)
next
  case (*defer name rsi args cs*)
  have *name* $$ *args* $\approx_p$ *Pabs cs*
    apply (*rule prelated.defer*)
       apply *fact+*
     apply (*rule fset.rel-mono-strong*)
      apply *fact*
     apply *force*
    apply *fact*
    done
  moreover have *closed* (*name* $$ *args*)
    unfolding *closed-list-comb* **by** *fact*
  ultimately have *closed* (*Pabs cs*)
    by (*metis prelated-closed*)

  let *?drop* = λ*env*. λ*pat*. *fmdrop-fset* (*frees pat*) *env*
  let *?f* = λ*env*. (λ(*pat*, *rhs*). (*pat*, *subst rhs* (*?drop env pat*)))

  have *name* $$ *args* $\approx_p$ *Pabs* (*?f* env_2 |'| *cs*)
    proof (*rule prelated.defer*)

show $(\mathit{name}, \mathit{rsi})$ $|\in|$ rs $0 < \mathit{arity}\ \mathit{rsi}$ $\mathit{list\text{-}all}\ \mathit{closed}\ \mathit{args}$
using *defer* **by** *auto*
next
{
fix pat_1 rhs_1
fix pat_2 rhs_2
assume $(\mathit{pat}_2, \mathit{rhs}_2)$ $|\in|$ cs
assume $\mathit{pat}_1 = \mathit{pat}_2$ $\mathit{rhs}_1 \approx_p \mathit{rhs}_2$
have $\mathit{rhs}_1 \approx_p \mathit{subst}\ \mathit{rhs}_2\ (\mathit{fmdrop\text{-}fset}\ (\mathit{frees}\ \mathit{pat}_2)\ \mathit{env}_2)$
by (*subst subst-closed-pabs*) *fact+*
}
hence *rel-fset* (*rel-prod* (=) *prelated*) (*id* |'| *irules-deferred-matches args rsi*) ($?f$ env_2 |'| *cs*)
by (*force intro*!: *rel-fset-image*[*OF* ‹*rel-fset* - - -›])

thus *rel-fset* (*rel-prod* (=) *prelated*) (*irules-deferred-matches args rsi*) ($?f$ env_2 |'| *cs*)
by *simp*
qed

moreover have $\mathit{map}\ (\lambda t.\ \mathit{subst}\ t\ \mathit{env}_1)\ \mathit{args} = \mathit{args}$
apply (*rule map-idI*)
apply (*rule subst-closed-id*)
using *defer* **by** (*simp add*: *list-all-iff*)

ultimately show *?case*
by (*simp add*: *subst-list-comb*)
qed (*auto intro*: *prelated.intros*)

lemma *prelated-step*:
assumes $\mathit{name}, \mathit{pats}, \mathit{rhs} \vdash_i u \to u'$ $t \approx_p u$
obtains t' **where** $\mathit{name}, \mathit{pats}, \mathit{rhs} \vdash_i t \to t'$ $t' \approx_p u'$
proof –
let $?\mathit{lhs} = \mathit{name}$ $$ *pats*
from *assms* **obtain** *env* **where** $\mathit{match}\ ?\mathit{lhs}\ u = \mathit{Some}\ \mathit{env}$ $u' = \mathit{subst}\ \mathit{rhs}\ \mathit{env}$
unfolding *irewrite-step-def* **by** *blast*
then obtain env' **where** $\mathit{match}\ ?\mathit{lhs}\ t = \mathit{Some}\ \mathit{env}'$ $\mathit{prelated.P\text{-}env}\ \mathit{env}'\ \mathit{env}$
using *assms* **by** (*auto elim*: *prelated.related-match*)
hence $\mathit{subst}\ \mathit{rhs}\ \mathit{env}' \approx_p \mathit{subst}\ \mathit{rhs}\ \mathit{env}$
using *assms* **by** (*auto intro*: *prelated-subst*)

show *thesis*
proof
show $\mathit{name}, \mathit{pats}, \mathit{rhs} \vdash_i t \to \mathit{subst}\ \mathit{rhs}\ \mathit{env}'$
unfolding *irewrite-step-def* **using** ‹$\mathit{match}\ ?\mathit{lhs}\ t = \mathit{Some}\ \mathit{env}'$›
by *simp*
next
show $\mathit{subst}\ \mathit{rhs}\ \mathit{env}' \approx_p u'$
unfolding ‹$u' = \mathit{subst}\ \mathit{rhs}\ \mathit{env}$›

by *fact*
qed
qed

lemma *prelated-beta*: — same problem as *prelated.related-match*
assumes $(pat,\ rhs_2) \vdash t_2 \rightarrow u_2$ $rhs_1 \approx_p rhs_2$ $t_1 \approx_p t_2$
obtains u_1 **where** $(pat,\ rhs_1) \vdash t_1 \rightarrow u_1$ $u_1 \approx_p u_2$
proof −
from *assms* **obtain** env_2 **where** $match\ pat\ t_2 = Some\ env_2$ $u_2 = subst\ rhs_2\ env_2$
by *auto*
with *assms* **obtain** env_1 **where** $match\ pat\ t_1 = Some\ env_1$ $prelated.P\text{-}env\ env_1$ env_2
by (*auto elim*: *prelated.related-match*)
with *assms* **have** $subst\ rhs_1\ env_1 \approx_p subst\ rhs_2\ env_2$
by (*auto intro*: *prelated-subst*)

show *thesis*
proof
show $(pat,\ rhs_1) \vdash t_1 \rightarrow subst\ rhs_1\ env_1$
using ‹$match\ pat\ t_1 = \text{-}$› **by** *simp*
next
show $subst\ rhs_1\ env_1 \approx_p u_2$
unfolding ‹$u_2 = \text{-}$› **by** *fact*
qed
qed

theorem *transform-correct*:
assumes $transform\text{-}irule\text{-}set\ rs \vdash_i u \longrightarrow u'$ $t \approx_p u$ $closed\ t$
obtains t' **where** $rs \vdash_i t \longrightarrow* t'$ — zero or one step **and** $t' \approx_p u'$
using *assms*(*1*−*3*) **proof** (*induction arbitrary*: *t thesis rule*: *irewrite.induct*)
case $(beta\ c\ cs_2\ u_2\ x_2')$
obtain $v\ u_1$ **where** $t = v\ \$_p\ u_1$ $v \approx_p Pabs\ cs_2$ $u_1 \approx_p u_2$
using ‹$t \approx_p Pabs\ cs_2\ \$_p\ u_2$› **by** *cases*
with *beta* **have** $closed\ u_1$
by (*simp add*: *closed-except-def*)

obtain $pat\ rhs_2$ **where** $c = (pat,\ rhs_2)$ **by** (*cases c*) *auto*

from ‹$v \approx_p Pabs\ cs_2$› **show** *?case*
proof (*cases rule*: *prelated-absE*)
case $(pat\ cs_1)$
with *beta* ‹$c = \text{-}$› **obtain** rhs_1 **where** $(pat,\ rhs_1)\ |{\in}|\ cs_1$ $rhs_1 \approx_p rhs_2$
by (*auto elim*: *rel-fsetE2*)
with *beta* **obtain** x_1' **where** $(pat,\ rhs_1) \vdash u_1 \rightarrow x_1'$ $x_1' \approx_p x_2'$
using ‹$u_1 \approx_p u_2$› *assms* ‹$c = \text{-}$›
by (*auto elim*: *prelated-beta simp del*: *rewrite-step.simps*)

show *?thesis*

proof (*rule beta.prems*)
show $rs \vdash_i t \longrightarrow* x_1'$
unfolding ‹t = -› ‹v = -›
by (*intro r-into-rtranclp irewrite.beta*) *fact*+
next
show $x_1' \approx_p x_2'$
by *fact*
qed
next
case (*defer name rsi args*)
with *beta* ‹c = -› **obtain** rhs_1' **where** (pat, rhs_1') |∈| *irules-deferred-matches args rsi* $rhs_1' \approx_p rhs_2$
by (*auto elim*: *rel-fsetE2*)
then obtain env_a rhs_1 *pats*
where *matchs* (*butlast pats*) *args* = *Some* env_a *pat* = *last pats* rhs_1' = *subst* rhs_1 env_a
and (*pats*, rhs_1) |∈| *rsi*
by *auto*
hence *linears pats*
using ‹(*name*, *rsi*) |∈| *rs*› *inner* **unfolding** *irules-def* **by** *auto*

have *arity-compatibles rsi*
using *defer inner*
by (*metis* (*no-types*, *lifting*) *case-prodD*)
have *length pats* > *0*
by (*subst arity-compatible-length*) *fact*+
hence *pats* = *butlast pats* @ [*pat*]
unfolding ‹*pat* = -› **by** *simp*

from *beta* ‹c = -› **obtain** env_b **where** *match pat* u_2 = *Some* env_b x_2' = *subst* rhs_2 env_b
by *fastforce*
with ‹$u_1 \approx_p u_2$› **obtain** env_b' **where** *match pat* u_1 = *Some* env_b' *prelated.P-env* env_b' env_b
by (*metis prelated.related-match*)

have *closed-env* env_a
by (*rule closed.matchs*) *fact*+
have *closed-env* env_b'
apply (*rule closed.matchs*[**where** *pats* = [*pat*] **and** *ts* = [u_1]])
apply *simp*
apply *fact*
apply *simp*
apply *fact*
done

have *fmdom* env_a = *freess* (*butlast pats*)
by (*rule matchs-dom*) *fact*
moreover have *fmdom* env_b' = *frees pat*

```
        by (rule match-dom) fact
      moreover have fdisjnt (freess (butlast pats)) (frees pat)
        using ‹pats = -› ‹linears pats›
        by (metis freess-single linears-appendD(3))
      ultimately have fdisjnt (fmdom env_a) (fmdom env_b')
        by simp

      show ?thesis
        proof (rule beta.prems)
          have rs ⊢_i name $$ args $_p u_1 ⟶ subst rhs_1' env_b'
            proof (rule irewrite.step)
              show (name, rsi) |∈| rs (pats, rhs_1) |∈| rsi
                by fact+
            next
              show name, pats, rhs_1 ⊢_i name $$ args $_p u_1 → subst rhs_1' env_b'
                apply (rule irewrite-stepI)
                 apply (fold app-pterm-def)
                 apply (subst list-comb-snoc)
                 apply (subst matchs-match-list-comb)
                 apply (subst ‹pats = -›)
                 apply (rule matchs-appI)
                  apply fact
                 apply simp
                 apply fact
                unfolding ‹rhs_1' = -›
                apply (rule subst-indep')
                 apply fact+
                done
            qed
          thus rs ⊢_i t ⟶* subst rhs_1' env_b'
            unfolding ‹t = -› ‹v = -›
            by (rule r-into-rtranclp)
        next
          show subst rhs_1' env_b' ≈_p x_2'
            unfolding ‹x_2' = -›
            by (rule prelated-subst) fact+
        qed
    qed
next
  case (step name rs_2 pats rhs u u')
  then obtain rs_1 where rs_2 = transform-irules rs_1 (name, rs_1) |∈| rs
    unfolding transform-irule-set-def by force
  hence arity-compatibles rs_1
    using inner
    by (metis (no-types, lifting) case-prodD)

  show ?case
    proof (cases arity rs_1 = 0)
      case True
```

hence $rs_2 = rs_1$
unfolding ‹rs_2 = -› *transform-irules-def* **by** *simp*
with *step* **have** $(pats, rhs)$ |∈| rs_1
by *simp*
from *step* **obtain** t' **where** *name*, *pats*, *rhs* $\vdash_i t \to t'$ $t' \approx_p u'$
using *assms*
by (*auto elim*: *prelated-step*)

show *?thesis*
proof (*rule step.prems*)
show *rs* $\vdash_i t \longrightarrow* t'$
by (*intro conjI exI r-into-rtranclp irewrite.step*) *fact+*
qed *fact*
next
let *?f* = λ(*pats*, *rhs*). (*butlast pats*, *last pats*, *rhs*)
let *?grp* = *fgroup-by ?f* rs_1

case *False*
hence rs_2 = *map-prod id Pabs* |'| *?grp*
unfolding ‹rs_2 = -› *transform-irules-def* **by** *simp*
with *step* **obtain** *cs* **where** *rhs* = *Pabs cs* (*pats*, *cs*) |∈| *?grp*
by *force*

from *step* **obtain** env_2 **where** *match* (*name* $$ *pats*) *u* = *Some* env_2 u' = *subst rhs* env_2
unfolding *irewrite-step-def* **by** *auto*
then obtain $args_2$ **where** *u* = *name* $$ $args_2$ *matchs pats* $args_2$ = *Some* env_2
by (*auto elim*: *match-list-combE*)
with *step* **obtain** $args_1$ **where** *t* = *name* $$ $args_1$ *list-all2 prelated* $args_1$ $args_2$
by (*auto elim*: *prelated.list-combE*)

then obtain env_1 **where** *matchs pats* $args_1$ = *Some* env_1 *prelated.P-env* env_1 env_2
using ‹*matchs pats* $args_2$ = -› **by** (*metis prelated.related-matchs*)
hence *fmdom* env_1 = *freess pats*
by (*auto simp*: *matchs-dom*)

obtain cs' **where** u' = *Pabs* cs'
unfolding ‹u' = -› ‹*rhs* = -› **by** *auto*

hence cs' = (λ(*pat*, *rhs*). (*pat*, *subst rhs* (*fmdrop-fset* (*frees pat*) env_2))) |'| *cs*
using ‹u' = *subst rhs* env_2› **unfolding** ‹*rhs* = -›
by *simp*

show *?thesis*
proof (*rule step.prems*)
show *rs* $\vdash_i t \longrightarrow* t$

by (*rule rtranclp.rtrancl-refl*)
next
show $t \approx_p u'$
unfolding ‹$u' = Pabs\ cs'$› ‹$t = -$›
proof (*intro prelated.defer rel-fsetI*; *safe?*)
show (*name*, rs_1) |∈| *rs*
by *fact*
next
show $0 < arity\ rs_1$
using *False* **by** *simp*
next
show *list-all closed* $args_1$
using ‹*closed t*› **unfolding** ‹$t = -$› *closed-list-comb* **.**
next
fix *pat* rhs'
assume (*pat*, rhs') |∈| *irules-deferred-matches* $args_1$ rs_1
then obtain $pats'$ *rhs env*
where ($pats'$, *rhs*) |∈| rs_1
and *matchs* (*butlast* $pats'$) $args_1$ = *Some env pat* = *last* $pats'$ rhs' = *subst rhs env*
by *auto*
with *False* **have** $pats' \neq []$
using ‹*arity-compatibles* rs_1›
by (*metis list.size(3) arity-compatible-length*)
hence *butlast* $pats'$ @ [*last* $pats'$] = $pats'$
by *simp*

from ‹(*pats*, *cs*) |∈| *?grp*› **obtain** $pats_e$ rhs_e
where ($pats_e$, rhs_e) |∈| rs_1 *pats* = *butlast* $pats_e$
by (*auto elim*: *fgroup-byE2*)

have *patterns-compatible* (*butlast* $pats'$) *pats*
unfolding ‹*pats* = -›
apply (*rule rev-accum-rel-butlast*)
using ‹($pats'$, *rhs*) |∈| rs_1› ‹($pats_e$, rhs_e) |∈| rs_1› ‹(*name*, rs_1) |∈| *rs*› *inner*
by (*blast dest*: *fpairwiseD*)

interpret *irules'*: *irules C-info transform-irule-set rs* **by** (*rule rules-transform*)

have *butlast* $pats'$ = *pats env* = env_1
apply (*rule matchs-compatible-eq*)
subgoal by *fact*
subgoal
apply (*rule linears-butlastI*)
using ‹($pats'$, *rhs*) |∈| rs_1› ‹(*name*, rs_1) |∈| *rs*› *inner* **by** *auto*
subgoal
using ‹(*pats*, -) |∈| rs_2› ‹(*name*, rs_2) |∈| *transform-irule-set rs*›

```
    using irules'.inner by auto
    apply fact+
  subgoal
    apply (rule matchs-compatible-eq)
        apply fact
      apply (rule linears-butlastI)
    using ‹(pats', rhs) |∈| rs1› ‹(name, rs1) |∈| rs› inner
    apply auto []
    using ‹(pats, -) |∈| rs2› ‹(name, rs2) |∈| transform-irule-set rs›
    using irules'.inner apply auto[]
    by fact+
  done

let ?rhs-subst = λenv. subst rhs (fmdrop-fset (frees pat) env)

have fmdom env2 = freess pats
  using ‹match (- $$ -) - = Some env2›
  by (simp add: match-dom)

show fBex cs' (rel-prod (=) prelated (pat, rhs'))
  unfolding ‹rhs' = -›
  proof (rule fBexI, rule rel-prod.intros)
    have fdisjnt (freess (butlast pats')) (frees (last pats'))
      apply (subst freess-single[symmetric])
      apply (rule linears-appendD)
      apply (subst ‹butlast pats' @ [last pats'] = pats'›)
      using ‹(pats', rhs) |∈| rs1› ‹(name, rs1) |∈| rs› inner
      by auto

    show subst rhs env ≈p ?rhs-subst env2
      apply (rule prelated-subst)
       apply (rule prelated-refl)
      unfolding fmfilter-alt-defs
      apply (subst fmfilter-true)
      subgoal premises prems for x y
        using fmdomI[OF prems]
        unfolding ‹pat = -› ‹fmdom env2 = -›
        apply (subst (asm) ‹butlast pats' = pats›[symmetric])
        using ‹fdisjnt (freess (butlast pats')) (frees (last pats'))›
        by (auto simp: fdisjnt-alt-def)
      subgoal
        unfolding ‹env = -›
        by fact
      done
  next
    have (pat, rhs) |∈| cs
      unfolding ‹pat = -›
      apply (rule fgroup-byD[where a = (x, y) for x y])
        apply fact
```

apply *simp*
apply (*intro conjI*)
apply *fact*
apply (*rule refl*)+
apply *fact*
done
thus (*pat*, *?rhs-subst* env_2) |∈| *cs′*
unfolding ‹*cs′* = -› **by** *force*
qed *simp*
next
fix *pat rhs′*
assume (*pat*, *rhs′*) |∈| *cs′*
then obtain *rhs*
where (*pat*, *rhs*) |∈| *cs*
and *rhs′* = *subst rhs* (*fmdrop-fset* (*frees pat*) env_2)
unfolding ‹*cs′* = -› **by** *auto*
with ‹(*pats*, *cs*) |∈| *?grp*› **obtain** *pats′*
where (*pats′*, *rhs*) |∈| rs_1 *pats* = *butlast pats′ pat* = *last pats′*
by *auto*
with *False* **have** *length pats′* ≠ *0*
using ‹*arity-compatibles* -› **by** (*metis arity-compatible-length*)
hence *pats′* = *pats* @ [*pat*]
unfolding ‹*pats* = -› ‹*pat* = -› **by** *auto*
moreover have *linears pats′*
using ‹(*pats′*, *rhs*) |∈| rs_1› ‹(*name*, rs_1) |∈| -› *inner* **by** *auto*
ultimately have *fdisjnt* (*fmdom* env_1) (*frees pat*)
unfolding ‹*fmdom* env_1 = -›
by (*auto dest*: *linears-appendD*)

let *?rhs-subst* = λ*env*. *subst rhs* (*fmdrop-fset* (*frees pat*) *env*)

show *fBex* (*irules-deferred-matches* $args_1$ rs_1) (λ*e*. *rel-prod* (=) *prelated e* (*pat*, *rhs′*))
unfolding ‹*rhs′* = -›
proof (*rule fBexI*, *rule rel-prod.intros*)
show *?rhs-subst* env_1 $\approx_p$ *?rhs-subst* env_2
using ‹*prelated.P-env* env_1 env_2› *inner*
by (*auto intro*: *prelated-subst*)
next
have *matchs* (*butlast pats′*) $args_1$ = *Some* env_1
using ‹*matchs pats* $args_1$ = -› ‹*pats* = -› **by** *simp*
moreover have *subst rhs* env_1 = *?rhs-subst* env_1
apply (*rule arg-cong*[**where** *f* = *subst rhs*])
unfolding *fmfilter-alt-defs*
apply (*rule fmfilter-true*[*symmetric*])
using ‹*fdisjnt* (*fmdom* env_1) -›
by (*auto simp*: *fdisjnt-alt-def intro*: *fmdomI*)
ultimately show (*pat*, *?rhs-subst* env_1) |∈| *irules-deferred-matches* $args_1$ rs_1

```
          using ‹(pats', rhs) |∈| rs₁› ‹pat = last pats'›
          by auto
        qed simp
      qed
    qed
  qed
next
  case (fun v v' u)
  obtain w x where t = w $ₚ x w ≈ₚ v x ≈ₚ u closed w
    using ‹t ≈ₚ v $ₚ u› ‹closed t› by cases (auto simp: closed-except-def)
  with fun obtain w' where rs ⊢ᵢ w ⟶* w' w' ≈ₚ v'
    by blast

  show ?case
    proof (rule fun.prems)
      show rs ⊢ᵢ t ⟶* w' $ₚ x
        unfolding ‹t = -›
        by (intro irewrite.rt-comb[unfolded app-pterm-def] rtranclp.rtrancl-refl) fact
    next
      show w' $ₚ x ≈ₚ v' $ₚ u
        by (rule prelated.app) fact+
    qed
next
  case (arg u u' v)
  obtain w x where t = w $ₚ x w ≈ₚ v x ≈ₚ u closed x
    using ‹t ≈ₚ v $ₚ u› ‹closed t› by cases (auto simp: closed-except-def)
  with arg obtain x' where rs ⊢ᵢ x ⟶* x' x' ≈ₚ u'
    by blast

  show ?case
    proof (rule arg.prems)
      show rs ⊢ᵢ t ⟶* w $ₚ x'
        unfolding ‹t = w $ₚ x›
        by (intro irewrite.rt-comb[unfolded app-pterm-def] rtranclp.rtrancl-refl) fact
    next
      show w $ₚ x' ≈ₚ v $ₚ u'
        by (rule prelated.app) fact+
    qed
qed

end
```

Completeness of transformation

```
lemma (in irules) transform-completeness:
  assumes rs ⊢ᵢ t ⟶ t' closed t
  shows transform-irule-set rs ⊢ᵢ t ⟶* t'
using assms proof induction
  case (step name irs' pats' rhs' t t')
```

then obtain *irs* **where** *irs* = *transform-irules irs′* (*name*, *irs*) |∈| *transform-irule-set rs*
unfolding *transform-irule-set-def*
by (*metis fimageI id-apply map-prod-simp*)
show *?case*
proof (*cases arity irs′* = *0*)
case *True*
hence *irs* = *irs′*
unfolding ‹*irs* = -›
unfolding *transform-irules-def*
by *simp*
with *step* **have** (*pats′*, *rhs′*) |∈| *irs name*, *pats′*, *rhs′* $\vdash_i$ *t* → *t′*
by *blast+*
have *transform-irule-set rs* $\vdash_i$ *t* ⟶∗ *t′*
apply (*rule r-into-rtranclp*)
apply *rule*
by *fact+*

show *?thesis* **by** *fact*
next
let *?f* = λ(*pats*, *rhs*). (*butlast pats*, *last pats*, *rhs*)
let *?grp* = *fgroup-by ?f irs′*
note *closed-except-def* [*simp add*]
case *False*
then have *irs* = *map-prod id Pabs* |‘| *?grp*
unfolding ‹*irs* = -›
unfolding *transform-irules-def*
by *simp*
with *False* **have** *irs* = *transform-irules irs′*
unfolding *transform-irules-def*
by *simp*
obtain *pat pats* **where** *pat* = *last pats′ pats* = *butlast pats′*
by *blast*
from *step False* **have** *length pats′* ≠ *0*
using *arity-compatible-length inner*
by (*smt* (*verit*, *ccfv-threshold*) *case-prodD*)
then have *pats′* = *pats* @ [*pat*]
unfolding ‹*pat* = -› ‹*pats* = -›
by *simp*
from *step* **have** *linears pats′*
using *inner* **by** *auto*
then have *fdisjnt* (*freess pats*) (*frees pat*)
unfolding ‹*pats′* = -›
using *linears-appendD*(*3*) *freess-single*
by *force*

from *step* **obtain** *cs* **where** (*pats*, *cs*) |∈| *?grp*
unfolding ‹*pats* = -›
by (*metis* (*no-types*, *lifting*) *fgroup-by-complete fst-conv prod.simps*(*2*))

with *step* **have** (*pat*, *rhs'*) |∈| *cs*
unfolding ‹*pat* = -› ‹*pats* = -›
by (*meson fgroup-byD old.prod.case*)
have (*pats*, *Pabs cs*) |∈| *irs*
using ‹*irs* = *map-prod id Pabs* |‘| *?grp*› ‹(*pats*, *cs*) |∈| -›
by (*metis* (*no-types*, *lifting*) *eq-snd-iff fst-conv fst-map-prod id-def rev-fimage-eqI snd-map-prod*)
from *step* **obtain** *env'* **where** *match* (*name* $$ *pats'*) *t* = *Some env'* *subst rhs' env'* = *t'*
using *irewrite-step-def* **by** *auto*
have *name* $$ *pats'* = (*name* $$ *pats*) $ *pat*
unfolding ‹*pats'* = -›
by (*simp add*: *app-term-def*)
then obtain t_0 t_1 env_0 env_1 **where** $t = t_0$ $\$_p$ t_1 *match* (*name* $$ *pats*) t_0 = *Some* env_0 *match pat* t_1 = *Some* env_1 $env' = env_0 ++_f env_1$
using *match-appE-split*[*OF* ‹*match* (*name* $$ *pats'*) - = -›[*unfolded* ‹*name* $$ *pats'* = -›], *unfolded app-pterm-def*]
by *blast*
with *step* **have** *closed* t_0 *closed* t_1
by *auto*
then have *closed-env* env_0 *closed-env* env_1
using *match-vars*[*OF* ‹*match* - t_0 = -›] *match-vars*[*OF* ‹*match* - t_1 = -›]
unfolding *closed-except-def*
by *auto*
obtain t_0' **where** *subst* (*Pabs cs*) env_0 = t_0'
by *blast*
then obtain *cs'* **where** t_0' = *Pabs cs'* *cs'* = ((λ(*pat*, *rhs*). (*pat*, *subst rhs* (*fmdrop-fset* (*frees pat*) env_0))) |‘| *cs*)
using *subst-pterm.simps*(*3*) **by** *blast*
obtain *rhs* **where** *subst rhs'* (*fmdrop-fset* (*frees pat*) env_0) = *rhs*
by *blast*
then have (*pat*, *rhs*) |∈| *cs'*
unfolding ‹*cs'* = -›
using ‹- |∈| *cs*›
by (*metis* (*mono-tags*, *lifting*) *old.prod.case rev-fimage-eqI*)
have $env_0 ++_f env_1$ = (*fmdrop-fset* (*frees pat*) env_0) $++_f$ env_1
apply (*subst fmadd-drop-left-dom*[*symmetric*])
using ‹*match pat* - = -› *match-dom*
by *metis*
have *fdisjnt* (*fmdom* env_0) (*fmdom* env_1)
using *match-dom*
using ‹*match pat* - = -› ‹*match* (*name* $$ *pats*) - = -›
using ‹*fdisjnt* - -›
unfolding *fdisjnt-alt-def*
by (*metis matchs-dom match-list-combE*)
have *subst rhs* env_1 = *t'*
unfolding ‹- = *rhs*›[*symmetric*]
unfolding ‹- = *t'*›[*symmetric*]
unfolding ‹*env'* = -›

unfolding ‹env_0 $++_f$ - = -›
apply (*subst subst-indep'*)
using ‹*closed-env* env_0›
apply *blast*
using ‹*fdisjnt* (*fmdom* -) -›
unfolding *fdisjnt-alt-def*
by *auto*

show *?thesis*
unfolding ‹*t* = -›
apply *rule*
apply (*rule r-into-rtranclp*)
apply (*rule irewrite.intros*(*3*))
apply *rule*
apply *fact+*
apply (*rule irewrite-stepI*)
apply *fact+*
unfolding ‹t_0' = -›
apply *rule*
apply *fact*
using ‹*match pat* t_1 = -› ‹*subst rhs* - = -›
by *force*
qed
qed (*auto intro*: *irewrite.rt-comb*[*unfolded app-pterm-def*] *intro*!: *irewrite.intros simp*: *closed-except-def*)

Computability

export-code
compile transform-irules
checking *Scala SML*

end

3.3.2 Pure pattern matching rule sets

theory *Rewriting-Pterm*
imports *Rewriting-Pterm-Elim*
begin

type-synonym *prule* = *name* × *pterm*

primrec *prule* :: *prule* ⇒ *bool* **where**
prule (-, *rhs*) ⟷ *wellformed rhs* ∧ *closed rhs* ∧ *is-abs rhs*

lemma *pruleI*[*intro*!]: *wellformed rhs* ⟹ *closed rhs* ⟹ *is-abs rhs* ⟹ *prule* (*name*, *rhs*)
by *simp*

locale *prules* = *constants C-info fst* |'| *rs* **for** *C-info* **and** *rs* :: *prule fset* +

assumes *all-rules*: *fBall rs prule*
assumes *fmap*: *is-fmap rs*
assumes *not-shadows*: *fBall rs* (λ(-, *rhs*). ¬ *shadows-consts rhs*)
assumes *welldefined-rs*: *fBall rs* (λ(-, *rhs*). *welldefined rhs*)

Rewriting

inductive *prewrite* :: *prule fset* ⇒ *pterm* ⇒ *pterm* ⇒ *bool* (‹-/ $\vdash_p$/ - ⟶/ -› [*50*,*0*,*50*] *50*) **for** *rs* **where**
step: (*name*, *rhs*) |∈| *rs* ⟹ *rs* $\vdash_p$ *Pconst name* ⟶ *rhs* |
beta: *c* |∈| *cs* ⟹ *c* ⊢ *t* → *t′* ⟹ *rs* $\vdash_p$ *Pabs cs* $\$_p$ *t* ⟶ *t′* |
fun: *rs* $\vdash_p$ *t* ⟶ *t′* ⟹ *rs* $\vdash_p$ *t* $\$_p$ *u* ⟶ *t′* $\$_p$ *u* |
arg: *rs* $\vdash_p$ *u* ⟶ *u′* ⟹ *rs* $\vdash_p$ *t* $\$_p$ *u* ⟶ *t* $\$_p$ *u′*

global-interpretation *prewrite*: *rewriting prewrite rs* **for** *rs*
by *standard* (*auto intro*: *prewrite.intros simp*: *app-pterm-def*)+

abbreviation *prewrite-rt* :: *prule fset* ⇒ *pterm* ⇒ *pterm* ⇒ *bool* (‹-/ $\vdash_p$/ - ⟶∗/ -› [*50*,*0*,*50*] *50*) **where**
prewrite-rt rs ≡ (*prewrite rs*)**

Translation from *irule-set* to *prule fset*

definition *finished* :: *irule-set* ⇒ *bool* **where**
finished rs = *fBall rs* (λ(-, *irs*). *arity irs* = *0*)

definition *translate-rhs* :: *irules* ⇒ *pterm* **where**
translate-rhs = *snd* ∘ *fthe-elem*

definition *compile* :: *irule-set* ⇒ *prule fset* **where**
compile = *fimage* (*map-prod id translate-rhs*)

lemma *compile-heads*: *fst* |‘| *compile rs* = *fst* |‘| *rs*
unfolding *compile-def* **by** *simp*

Correctness of translation

lemma *arity-zero-shape*:
assumes *arity-compatibles rs arity rs* = *0 is-fmap rs rs* ≠ {||}
obtains *t* **where** *rs* = {| ([], *t*) |}
proof −
from *assms* **obtain** *ppats prhs* **where** (*ppats*, *prhs*) |∈| *rs*
by *fast*

moreover {
fix *pats rhs*
assume (*pats*, *rhs*) |∈| *rs*
with *assms* **have** *length pats* = *0*
by (*metis arity-compatible-length*)
hence *pats* = []

```
    by simp
  }
  note all = this

  ultimately have proto: ([], prhs) |∈| rs by auto

  have fBall rs (λ(pats, rhs). pats = [] ∧ rhs = prhs)
    proof safe
      fix pats rhs
      assume cur: (pats, rhs) |∈| rs
      with all show pats = [] .
      with cur have ([], rhs) |∈| rs by auto

      with proto show rhs = prhs
        using assms by (auto dest: is-fmapD)
    qed
  hence fBall rs (λr. r = ([], prhs))
    by blast
  with assms have rs = {| ([], prhs) |}
    by (simp add: singleton-fset-is)
  thus thesis
    by (rule that)
qed

lemma (in irules) compile-rules:
  assumes finished rs
  shows prules C-info (compile rs)
proof
  show is-fmap (compile rs)
    using fmap
    unfolding compile-def map-prod-def id-apply
    by (rule is-fmap-image)
next
  show fdisjnt (fst |`| compile rs) C
    unfolding compile-def
    using disjnt by simp
next
  have
    fBall (compile rs) prule
    fBall (compile rs) (λ(-, rhs). ¬ shadows-consts rhs)
    fBall (compile rs) (λ(-, rhs). welldefined rhs)
    proof (safe del: fsubsetI)
      fix name rhs
      assume (name, rhs) |∈| compile rs
      then obtain irs where (name, irs) |∈| rs rhs = translate-rhs irs
        unfolding compile-def by force
      hence is-fmap irs irs ≠ {||} arity irs = 0
        using assms inner unfolding finished-def by blast+
      moreover have arity-compatibles irs
```

```
      using ‹(name, irs) |∈| rs› inner
      by (metis (no-types, lifting) case-prodD)
    ultimately obtain u where irs = {| ([], u) |}
      by (metis arity-zero-shape)
    hence rhs = u and u: ([], u) |∈| irs
      unfolding ‹rhs = -› translate-rhs-def by simp+
    hence abs-ish [] u
      using inner ‹(name, irs) |∈| rs› by fastforce
    thus is-abs rhs
      unfolding abs-ish-def ‹rhs = u› by simp

    show wellformed rhs
      using u ‹(name, irs) |∈| rs› inner unfolding ‹rhs = u›
      by fastforce

    have closed-except u {||}
      using u inner ‹(name, irs) |∈| rs›
      by fastforce
    thus closed rhs
      unfolding ‹rhs = u› .

    {
      assume shadows-consts rhs
      hence shadows-consts u
        unfolding compile-def ‹rhs = u› by simp
      moreover have ¬ shadows-consts u
        using inner ‹([], u) |∈| irs› ‹(name, irs) |∈| rs› by fastforce
      ultimately show False by blast
    }

    have welldefined u
      using fbspec[OF inner ‹(name, irs) |∈| rs›, simplified] ‹([], u) |∈| irs›
      by blast
    thus welldefined rhs
      unfolding ‹rhs = u› compile-def
      by simp
  qed
 thus
  fBall (compile rs) prule
   fBall (compile rs) (λ(-, rhs). ¬ pre-constants.shadows-consts C-info (fst |‘| compile rs) rhs)
  fBall (compile rs) (λ(-, rhs). pre-constants.welldefined C-info (fst |‘| compile rs) rhs)
  unfolding compile-heads by auto
next
 show distinct all-constructors
  by (fact distinct-ctr)
qed
```

theorem (**in** *irules*) *compile-correct*:
  assumes *compile rs* $\vdash_p$ *t* $\longrightarrow$ *t′ finished rs*
  shows *rs* $\vdash_i$ *t* $\longrightarrow$ *t′*
using *assms*(*1*) **proof** *induction*
  case (*step name rhs*)
  then obtain *irs* **where** *rhs* = *translate-rhs irs* (*name*, *irs*) |∈| *rs*
    unfolding *compile-def* **by** *force*
  hence *arity-compatibles irs*
    using *inner*
    by (*metis* (*no-types*, *lifting*) *case-prodD*)

  have *is-fmap irs irs* ≠ {||} *arity irs* = *0*
    using *assms inner* ‹(*name*, *irs*) |∈| *rs*› **unfolding** *finished-def* **by** *blast+*
  then obtain *u* **where** *irs* = {| ([], *u*) |}
    using ‹*arity-compatibles irs*›
    by (*metis arity-zero-shape*)

  show *?case*
    unfolding ‹*rhs* = -›
    apply (*rule irewrite.step*)
      apply *fact*
    unfolding ‹*irs* = -› *translate-rhs-def irewrite-step-def*
    by (*auto simp*: *const-term-def*)
qed (*auto intro*: *irewrite.intros*)

theorem (**in** *irules*) *compile-complete*:
  assumes *rs* $\vdash_i$ *t* $\longrightarrow$ *t′ finished rs*
  shows *compile rs* $\vdash_p$ *t* $\longrightarrow$ *t′*
using *assms*(*1*) **proof** *induction*
  case (*step name irs params rhs t t′*)
  hence *arity-compatibles irs*
    using *inner*
    by (*metis* (*no-types*, *lifting*) *case-prodD*)

  have *is-fmap irs irs* ≠ {||} *arity irs* = *0*
    using *assms inner step* **unfolding** *finished-def* **by** *blast+*
  then obtain *u* **where** *irs* = {| ([], *u*) |}
    using ‹*arity-compatibles irs*›
    by (*metis arity-zero-shape*)
  with *step* **have** *name*, [], *u* $\vdash_i$ *t* → *t′*
    by *simp*
  hence *t* = *Pconst name*
    unfolding *irewrite-step-def*
    by (*cases t*) (*auto split*: *if-splits simp*: *const-term-def*)
  hence *t′* = *u*
    using ‹*name*, [], *u* $\vdash_i$ *t* → *t′*›
    unfolding *irewrite-step-def*
    by (*cases t*) (*auto split*: *if-splits simp*: *const-term-def*)

have $(name,\ t')$ |∈| *compile rs*
unfolding *compile-def*
proof
show $(name,\ t') = $ *map-prod id translate-rhs* $(name,\ irs)$
using ‹*irs* = -› ‹$t' = u$›
by (*simp add*: *split-beta translate-rhs-def*)
qed *fact*
thus *?case*
unfolding ‹*t* = -›
by (*rule prewrite.step*)
qed (*auto intro*: *prewrite.intros*)

export-code
compile finished
checking *Scala*

end

3.4 Sequential pattern matching

theory *Rewriting-Sterm*
imports *Rewriting-Pterm*
begin

type-synonym *srule* = *name* × *sterm*

abbreviation *closed-srules* :: *srule list* ⇒ *bool* **where**
closed-srules ≡ *list-all* (*closed* ∘ *snd*)

primrec *srule* :: *srule* ⇒ *bool* **where**
srule (-, *rhs*) ⟷ *wellformed rhs* ∧ *closed rhs* ∧ *is-abs rhs*

lemma *sruleI*[*intro*!]: *wellformed rhs* ⟹ *closed rhs* ⟹ *is-abs rhs* ⟹ *srule* (*name*, *rhs*)
by *simp*

locale *srules* = *constants C-info fst* |'| *fset-of-list rs* **for** *C-info* **and** *rs* :: *srule list* +
assumes *all-rules*: *list-all srule rs*
assumes *distinct*: *distinct* (*map fst rs*)
assumes *not-shadows*: *list-all* (λ(-, *rhs*). ¬ *shadows-consts rhs*) *rs*
assumes *swelldefined-rs*: *list-all* (λ(-, *rhs*). *welldefined rhs*) *rs*
begin

lemma *map*: *is-map* (*set rs*)
using *distinct* **by** (*rule distinct-is-map*)

lemma *clausesE*:
assumes (*name*, *rhs*) ∈ *set rs*

obtains *cs* **where** *rhs* = *Sabs cs*
proof −
from *assms* **have** *is-abs rhs*
using *all-rules* **unfolding** *list-all-iff* **by** *auto*
then obtain *cs* **where** *rhs* = *Sabs cs*
by (*cases rhs*) (*auto simp*: *is-abs-def term-cases-def*)
with *that* **show** *thesis* **.**
qed

end

Rewriting

inductive *srewrite-step* **where**
cons-match: *srewrite-step* ((*name*, *rhs*) # *rest*) *name rhs* |
cons-nomatch: *name* ≠ *name'* ⟹ *srewrite-step rs name rhs* ⟹ *srewrite-step* ((*name'*, *rhs'*) # *rs*) *name rhs*

lemma *srewrite-stepI0*:
assumes (*name*, *rhs*) ∈ *set rs is-map* (*set rs*)
shows *srewrite-step rs name rhs*
using *assms* **proof** (*induction rs*)
case (*Cons r rs*)
then obtain *name' rhs'* **where** *r* = (*name'*, *rhs'*) **by** *force*
show *?case*
proof (*cases name* = *name'*)
case *False*
show *?thesis*
unfolding ‹*r* = -›
apply (*rule srewrite-step.cons-nomatch*)
subgoal by *fact*
apply (*rule Cons*)
using *False Cons*(*2*) ‹*r* = -› **apply** *force*
using *Cons*(*3*) **unfolding** *is-map-def* **by** *auto*
next
case *True*
have *rhs* = *rhs'*
apply (*rule is-mapD*)
apply *fact*
unfolding ‹*r* = -›
using *Cons*(*2*) ‹*r* = -› **apply** *simp*
using *True* **apply** *simp*
done
show *?thesis*
unfolding ‹*r* = -› ‹*name* = -› ‹*rhs* = -›
by (*rule srewrite-step.cons-match*)
qed
qed *auto*

lemma (**in** *srules*) *srewrite-stepI*: (*name*, *rhs*) ∈ *set rs* ⟹ *srewrite-step rs name rhs*
using *map*
by (*metis srewrite-stepI0*)

hide-fact *srewrite-stepI0*

inductive *srewrite* :: *srule list* ⇒ *sterm* ⇒ *sterm* ⇒ *bool* (‹-/ $\vdash_s$/ - ⟶/ -› [*50*,*0*,*50*] *50*) **for** *rs* **where**
step: *srewrite-step rs name rhs* ⟹ *rs* $\vdash_s$ *Sconst name* ⟶ *rhs* |
beta: *rewrite-first cs t t′* ⟹ *rs* $\vdash_s$ *Sabs cs* $\$_s$ *t* ⟶ *t′* |
fun: *rs* $\vdash_s$ *t* ⟶ *t′* ⟹ *rs* $\vdash_s$ *t* $\$_s$ *u* ⟶ *t′* $\$_s$ *u* |
arg: *rs* $\vdash_s$ *u* ⟶ *u′* ⟹ *rs* $\vdash_s$ *t* $\$_s$ *u* ⟶ *t* $\$_s$ *u′*

code-pred *srewrite* **.**

abbreviation *srewrite-rt* :: *srule list* ⇒ *sterm* ⇒ *sterm* ⇒ *bool* (‹-/ $\vdash_s$/ - ⟶∗/ -› [*50*,*0*,*50*] *50*) **where**
srewrite-rt rs ≡ (*srewrite rs*)**

global-interpretation *srewrite*: *rewriting srewrite rs* **for** *rs*
by *standard* (*auto intro*: *srewrite.intros simp*: *app-sterm-def*)+

code-pred (*modes*: *i* ⇒ *i* ⇒ *o* ⇒ *bool*) *srewrite-step* **.**
code-pred (*modes*: *i* ⇒ *i* ⇒ *o* ⇒ *bool*) *srewrite* **.**

Translation from *pterm* to *sterm*

In principle, any function of type (*′a* × *′b*) *fset* ⇒ (*′a* × *′b*) *list* that orders by keys would do here. However, For simplicity's sake, we choose a fixed one (*ordered-fmap*) here.

primrec *pterm-to-sterm* :: *pterm* ⇒ *sterm* **where**
pterm-to-sterm (*Pconst name*) = *Sconst name* |
pterm-to-sterm (*Pvar name*) = *Svar name* |
pterm-to-sterm (*t* $\$_p$ *u*) = *pterm-to-sterm t* $\$_s$ *pterm-to-sterm u* |
pterm-to-sterm (*Pabs cs*) = *Sabs* (*ordered-fmap* (*map-prod id pterm-to-sterm* |‘| *cs*))

lemma *pterm-to-sterm*:
  assumes *no-abs t*
  shows *pterm-to-sterm t* = *convert-term t*
using *assms* **proof** *induction*
  case (*free name*)
  show *?case*
    apply *simp*
    apply (*simp add*: *free-sterm-def free-pterm-def*)
    done
next
  case (*const name*)

  show *?case*
    apply *simp*
    apply (*simp add*: *const-sterm-def const-pterm-def*)
    done
next
  case (*app* t_1 t_2)
  then show *?case*
    apply *simp*
    apply (*simp add*: *app-sterm-def app-pterm-def*)
    done
qed

sterm-to-pterm has to be defined, for technical reasons, in *CakeML-Codegen.Pterm*.

lemma *pterm-to-sterm-wellformed*:
  assumes *wellformed t*
  shows *wellformed* (*pterm-to-sterm t*)
using *assms* **proof** (*induction t rule*: *pterm-induct*)
  case (*Pabs cs*)
  show *?case*
    apply *simp*
    unfolding *map-prod-def id-apply*
    apply (*intro conjI*)
    subgoal
      apply (*subst list-all-iff-fset*)
      apply (*subst ordered-fmap-set-eq*)
       apply (*rule is-fmap-image*)
      using *Pabs* **apply** *simp*
      apply (*rule fBallI*)
      apply (*erule fimageE*)
      apply *auto*[]
      using *Pabs*(*2*) **apply** *auto*[]
      apply (*rule Pabs*)
      using *Pabs*(*2*) **by** *auto*
    subgoal
      apply (*rule ordered-fmap-distinct*)
      apply (*rule is-fmap-image*)
      using *Pabs*(*2*) **by** *simp*
    subgoal
      apply (*subgoal-tac cs* $\neq$ {||})
      including *fset.lifting* **apply** *transfer*
      unfolding *ordered-map-def*
      using *Pabs*(*2*) **by** *auto*
    done
qed *auto*

lemma *pterm-to-sterm-sterm-to-pterm*:
  assumes *wellformed t*
  shows *sterm-to-pterm* (*pterm-to-sterm t*) = *t*
using *assms* **proof** (*induction t*)

case (*Pabs cs*)
note *fset-of-list-map*[*simp del*]
show *?case*
apply *simp*
unfolding *map-prod-def id-apply*
apply (*subst ordered-fmap-image*)
subgoal
apply (*rule is-fmap-image*)
using *Pabs* **by** *simp*
apply (*subst ordered-fmap-set-eq*)
subgoal
apply (*rule is-fmap-image*)
apply (*rule is-fmap-image*)
using *Pabs* **by** *simp*
subgoal
apply (*subst fset.map-comp*)
apply (*subst map-prod-def*[*symmetric*])+
unfolding *o-def*
apply (*subst prod.map-comp*)
apply (*subst id-def*[*symmetric*])+
apply *simp*
apply (*subst map-prod-def*)
unfolding *id-def*
apply (*rule fset-map-snd-id*)
apply *simp*
apply (*rule Pabs*)
using *Pabs*(*2*) **by** (*auto simp*: *snds.simps*)
done
qed *auto*

corollary *pterm-to-sterm-frees*: *wellformed t* $\Longrightarrow$ *frees* (*pterm-to-sterm t*) = *frees t*
by (*metis pterm-to-sterm-sterm-to-pterm sterm-to-pterm-frees*)

corollary *pterm-to-sterm-closed*:
closed-except t S $\Longrightarrow$ *wellformed t* $\Longrightarrow$ *closed-except* (*pterm-to-sterm t*) *S*
unfolding *closed-except-def*
by (*simp add*: *pterm-to-sterm-frees*)

corollary *pterm-to-sterm-consts*: *wellformed t* $\Longrightarrow$ *consts* (*pterm-to-sterm t*) = *consts t*
by (*metis pterm-to-sterm-sterm-to-pterm sterm-to-pterm-consts*)

corollary (**in** *constants*) *pterm-to-sterm-shadows*:
wellformed t $\Longrightarrow$ *shadows-consts t* $\longleftrightarrow$ *shadows-consts* (*pterm-to-sterm t*)
unfolding *shadows-consts-def*
by (*metis pterm-to-sterm-sterm-to-pterm sterm-to-pterm-all-frees*)

definition *compile* :: *prule fset* $\Rightarrow$ *srule list* **where**

compile rs = *ordered-fmap* (*map-prod id pterm-to-sterm* |\`| *rs*)

Correctness of translation

context *prules* **begin**

```
lemma compile-heads: fst |`| fset-of-list (compile rs) = fst |`| rs
  unfolding compile-def
  apply (subst ordered-fmap-set-eq)
   apply (subst map-prod-def, subst id-apply)
   apply (rule is-fmap-image)
   apply (rule fmap)
  apply simp
  done

lemma compile-rules: srules C-info (compile rs)
proof
  show list-all srule (compile rs)
    using fmap all-rules
    unfolding compile-def list-all-iff
    including fset.lifting
    apply transfer
    apply (subst ordered-map-set-eq)
    subgoal by simp
    subgoal
      unfolding map-prod-def id-def
      by (erule is-map-image)
    subgoal
      apply (rule ballI)
      apply safe
      subgoal
        apply (rule pterm-to-sterm-wellformed)
        apply fastforce
        done
      subgoal
        apply (rule pterm-to-sterm-closed)
         apply fastforce
        apply fastforce
        done
      subgoal for - - a b
        apply (erule ballE[where x = (a, b)])
         apply (cases b; auto)
            apply (auto simp: is-abs-def term-cases-def)
        done
      done
    done
next
  show distinct (map fst (compile rs))
    unfolding compile-def
```

```
    apply (rule ordered-fmap-distinct)
    unfolding map-prod-def id-def
    apply (rule is-fmap-image)
    apply (rule fmap)
    done
next
  have list-all (λ(-, rhs). welldefined rhs) (compile rs)
    unfolding compile-def
    apply (subst ordered-fmap-list-all)
    subgoal
      apply (subst map-prod-def)
      apply (subst id-apply)
      apply (rule is-fmap-image)
      by (fact fmap)
    apply simp
    apply (rule fBallI)
    subgoal for x
      apply (cases x, simp)
      apply (subst pterm-to-sterm-consts)
      using all-rules apply force
      using welldefined-rs by force
    done
   thus list-all (λ(-, rhs). consts rhs |⊆| pre-constants.all-consts C-info (fst |`|
fset-of-list (compile rs))) (compile rs)
    by (simp add: compile-heads)
next
  interpret c: constants - fset-of-list (map fst (compile rs))
    by (simp add: constants-axioms compile-heads)
  have all-consts: c.all-consts = all-consts
    by (simp add: compile-heads)

  note fset-of-list-map[simp del]
  have list-all (λ(-, rhs). ¬ shadows-consts rhs) (compile rs)
    unfolding compile-def
    apply (subst list-all-iff-fset)
    apply (subst ordered-fmap-set-eq)
     apply (subst map-prod-def)
    unfolding id-apply
     apply (rule is-fmap-image)
     apply (fact fmap)
    apply simp
    apply (rule fBall-pred-weaken[where P = λ(-, rhs). ¬ shadows-consts rhs])
    subgoal for x
      apply (cases x, simp)
      apply (subst (asm) pterm-to-sterm-shadows)
      using all-rules apply force
      by simp
    subgoal
      using not-shadows by force
```

done
thus *list-all* (λ(-, *rhs*). ¬ *pre-constants.shadows-consts C-info* (*fst* |'| *fset-of-list* (*compile rs*)) *rhs*) (*compile rs*)
unfolding *compile-heads all-consts* **.**
next
show *fdisjnt* (*fst* |'| *fset-of-list* (*compile rs*)) *C*
unfolding *compile-def*
apply (*subst fset-of-list-map*[*symmetric*])
apply (*subst ordered-fmap-keys*)
apply (*subst map-prod-def*)
apply (*subst id-apply*)
apply (*rule is-fmap-image*)
using *fmap disjnt* **by** *auto*
next
show *distinct all-constructors*
by (*fact distinct-ctr*)
qed

sublocale *prules-as-srules*: *srules C-info compile rs*
by (*fact compile-rules*)

end

global-interpretation *srelated*: *term-struct-rel-strong* ($\lambda p\ s.\ p = $ *sterm-to-pterm* *s*)
proof (*standard*, *goal-cases*)
case (*5 name t*)
then show *?case* **by** (*cases t*) (*auto simp*: *const-sterm-def const-pterm-def split*: *option.splits*)
next
case (*6* u_1 u_2 *t*)
then show *?case* **by** (*cases t*) (*auto simp*: *app-sterm-def app-pterm-def split*: *option.splits*)
qed (*auto simp*: *const-sterm-def const-pterm-def app-sterm-def app-pterm-def*)

lemma *srelated-subst*:
assumes *srelated.P-env penv senv*
shows *subst* (*sterm-to-pterm t*) *penv* = *sterm-to-pterm* (*subst t senv*)
using *assms*
proof (*induction t arbitrary*: *penv senv*)
case (*Svar name*)
thus *?case*
by (*cases rule*: *fmrel-cases*[**where** x = *name*]) *auto*
next
case (*Sabs cs*)
show *?case*
apply *simp*
including *fset.lifting*
apply (*transfer′ fixing*: *cs penv senv*)

unfolding *set-map image-comp*
apply (*rule image-cong*[*OF refl*])
unfolding *comp-apply*
apply (*case-tac x*)
apply *hypsubst-thin*
apply *simp*
apply (*rule Sabs*)
apply *assumption*
apply (*simp add*: *snds.simps*)
apply *rule*
apply (*rule Sabs*)
done
qed *auto*

context begin

private lemma *srewrite-step-non-empty*: *srewrite-step rs′ name rhs* $\Longrightarrow$ *rs′* $\neq$ []
by (*induct rule*: *srewrite-step.induct*) *auto*

private lemma *compile-consE*:
assumes (*name*, *rhs′*) # *rest* = *compile rs is-fmap rs*
obtains *rhs* **where** *rhs′* = *pterm-to-sterm rhs* (*name*, *rhs*) |∈| *rs rest* = *compile* (*rs* − {| (*name*, *rhs*) |})
proof −
from *assms* **have** *ordered-fmap* (*map-prod id pterm-to-sterm* |‘| *rs*) = (*name*, *rhs′*) # *rest*
unfolding *compile-def*
by *simp*
hence (*name*, *rhs′*) ∈ *set* (*ordered-fmap* (*map-prod id pterm-to-sterm* |‘| *rs*))
by *simp*

have (*name*, *rhs′*) |∈| *map-prod id pterm-to-sterm* |‘| *rs*
apply (*rule ordered-fmap-sound*)
subgoal
unfolding *map-prod-def id-apply*
apply (*rule is-fmap-image*)
apply *fact*
done
subgoal by *fact*
done
then obtain *rhs* **where** *rhs′* = *pterm-to-sterm rhs* (*name*, *rhs*) |∈| *rs*
by *auto*

have *rest* = *compile* (*rs* − {| (*name*, *rhs*) |})
unfolding *compile-def*
apply (*subst inj-on-fimage-set-diff*[**where** *C* = *rs*])
subgoal
apply (*rule inj-onI*)
apply *safe*

```
      apply auto
     using ‹is-fmap rs› by (blast dest: is-fmapD)
    subgoal by simp
    subgoal using ‹(name, rhs) |∈| rs› by simp
    subgoal
     apply simp
     apply (subst ordered-fmap-remove)
       apply (subst map-prod-def)
     unfolding id-apply
       apply (rule is-fmap-image)
       apply fact
     using ‹(name, rhs) |∈| rs› apply force
     apply (subst ‹rhs' = pterm-to-sterm rhs›[symmetric])
     apply (subst ‹ordered-fmap - = -›[unfolded id-def])
     by simp
    done

  show thesis
    by (rule that) fact+
qed

private lemma compile-correct-step:
  assumes srewrite-step (compile rs) name rhs is-fmap rs fBall rs prule
  shows (name, sterm-to-pterm rhs) |∈| rs
using assms proof (induction compile rs name rhs arbitrary: rs)
  case (cons-match name rhs' rest)
  then obtain rhs where rhs' = pterm-to-sterm rhs (name, rhs) |∈| rs
    by (auto elim: compile-consE)

  show ?case
    unfolding ‹rhs' = -›
    apply (subst pterm-to-sterm-sterm-to-pterm)
    using fbspec[OF ‹fBall rs prule› ‹(name, rhs) |∈| rs›] apply force
    by fact
next
  case (cons-nomatch name name₁ rest rhs rhs₁')
  then obtain rhs₁ where rhs₁' = pterm-to-sterm rhs₁ (name₁, rhs₁) |∈| rs rest
= compile (rs − {| (name₁, rhs₁) |})
    by (auto elim: compile-consE)

  let ?rs' = rs − {| (name₁, rhs₁) |}
  have (name, sterm-to-pterm rhs) |∈| ?rs'
    proof (intro cons-nomatch)
     show rest = compile ?rs'
       by fact

     show is-fmap (rs |−| {|(name₁, rhs₁)|})
       using ‹is-fmap rs›
       by (rule is-fmap-subset) auto
```

show *fBall ?rs′ prule*
using *cons-nomatch* **by** *blast*
qed
thus *?case*
by *simp*
qed

lemma *compile-correct0*:
assumes *compile rs* $\vdash_s$ *u* $\longrightarrow$ *u′ prules C rs*
shows *rs* $\vdash_p$ *sterm-to-pterm u* $\longrightarrow$ *sterm-to-pterm u′*
using *assms* **proof** *induction*
case (*beta cs t t′*)
then obtain *pat rhs env* **where** (*pat*, *rhs*) ∈ *set cs match pat t* = *Some env t′* = *subst rhs env*
by (*auto elim*: *rewrite-firstE*)

then obtain *env′* **where** *match pat* (*sterm-to-pterm t*) = *Some env′ srelated.P-env env′ env*
by (*metis option.distinct*(*1*) *option.inject option.rel-cases srelated.match-rel*)
hence *subst* (*sterm-to-pterm rhs*) *env′* = *sterm-to-pterm* (*subst rhs env*)
by (*simp add*: *srelated-subst*)

let *?rhs′* = *sterm-to-pterm rhs*

have (*pat*, *?rhs′*) |∈| *fset-of-list* (*map* (*map-prod id sterm-to-pterm*) *cs*)
using ‹(*pat*, *rhs*) ∈ *set cs*›
including *fset.lifting*
by *transfer′ force*

note *fset-of-list-map*[*simp del*]
show *?case*
apply *simp*
apply (*rule prewrite.intros*)
apply *fact*
unfolding *rewrite-step.simps*
apply (*subst map-option-eq-Some*)
apply (*intro exI conjI*)
apply *fact*
unfolding ‹*t′* = -›
by *fact*
next
case (*step name rhs*)
hence (*name*, *sterm-to-pterm rhs*) |∈| *rs*
unfolding *prules-def prules-axioms-def*
by (*metis compile-correct-step*)
thus *?case*
by (*auto intro*: *prewrite.intros*)
qed (*auto intro*: *prewrite.intros*)

end

lemma (**in** *prules*) *compile-correct*:
  assumes *compile rs* $\vdash_s$ *u* $\longrightarrow$ *u'*
  shows *rs* $\vdash_p$ *sterm-to-pterm u* $\longrightarrow$ *sterm-to-pterm u'*
by (*rule compile-correct0*) (*fact* | *standard*)+

hide-fact *compile-correct0*

Completeness of translation

global-interpretation *srelated'*: *term-struct-rel-strong* (λp *s*. *pterm-to-sterm p* = *s*)
proof (*standard*, *goal-cases*)
  case (*1 t name*)
  then show *?case* **by** (*cases t*) (*auto simp*: *const-sterm-def const-pterm-def split*: *option.splits*)
next
  case (*3 t* u_1 u_2)
   then show *?case* **by** (*cases t*) (*auto simp*: *app-sterm-def app-pterm-def split*: *option.splits*)
qed (*auto simp*: *const-sterm-def const-pterm-def app-sterm-def app-pterm-def*)

corollary *srelated-env-unique*:
  srelated'.P-env penv senv $\Longrightarrow$ *srelated'.P-env penv senv'* $\Longrightarrow$ *senv* = *senv'*
apply (*subst* (*asm*) *fmrel-iff*)+
apply (*subst* (*asm*) *option.rel-sel*)+
apply (*rule fmap-ext*)
by (*metis option.exhaust-sel*)

lemma *srelated-subst'*:
  assumes *srelated'.P-env penv senv wellformed t*
  shows *pterm-to-sterm* (*subst t penv*) = *subst* (*pterm-to-sterm t*) *senv*
using *assms* **proof** (*induction t arbitrary*: *penv senv*)
  case (*Pvar name*)
  thus *?case*
    by (*cases rule*: *fmrel-cases*[**where** *x* = *name*]) *auto*
next
  case (*Pabs cs*)
  hence *is-fmap cs*
    by *force*

  show *?case*
    apply *simp*
    unfolding *map-prod-def id-apply*
    apply (*subst ordered-fmap-image*[*symmetric*])
     apply *fact*
    apply (*subst fset.map-comp*[*symmetric*])

apply (*subst ordered-fmap-image*[*symmetric*])
subgoal by (*rule is-fmap-image*) *fact*
apply (*subst ordered-fmap-image*[*symmetric*])
apply *fact*
apply *auto*
apply (*drule ordered-fmap-sound*[*OF* ‹*is-fmap cs*›])
subgoal for *pat rhs*
apply (*rule Pabs*)
apply *assumption*
apply *auto*
using *Pabs* **by** *force+*
done
qed *auto*

lemma *srelated-find-match*:
assumes *find-match cs t* = *Some* (*penv*, *pat*, *rhs*) *srelated'.P-env penv senv*
shows *find-match* (*map* (*map-prod id pterm-to-sterm*) *cs*) (*pterm-to-sterm t*) = *Some* (*senv*, *pat*, *pterm-to-sterm rhs*)
proof −
let *?cs'* = *map* (*map-prod id pterm-to-sterm*) *cs*
let *?t'* = *pterm-to-sterm t*
have ∗: *list-all2* (*rel-prod* (=) (λ*p s*. *pterm-to-sterm p* = *s*)) *cs ?cs'*
unfolding *list.rel-map*
by (*auto intro*: *list.rel-refl*)

obtain *senv0*
where *find-match ?cs' ?t'* = *Some* (*senv0*, *pat*, *pterm-to-sterm rhs*) *srelated'.P-env penv senv0*
using *srelated'.find-match-rel*[*OF* ∗ *refl*, **where** *t* = *t*, *unfolded assms*]
unfolding *option-rel-Some1 rel-prod-conv*
by *auto*
with *assms* **have** *senv* = *senv0*
by (*metis srelated-env-unique*)
show *?thesis*
unfolding ‹*senv* = -› **by** *fact*
qed

lemma (**in** *prules*) *compile-complete*:
assumes *rs* $\vdash_p$ *t* ⟶ *t'* *wellformed t*
shows *compile rs* $\vdash_s$ *pterm-to-sterm t* ⟶ *pterm-to-sterm t'*
using *assms* **proof** *induction*
case (*step name rhs*)
then show *?case*
apply *simp*
apply *rule*
apply (*rule prules-as-srules.srewrite-stepI*)
unfolding *compile-def*
apply (*subst fset-of-list-elem*[*symmetric*])
apply (*subst ordered-fmap-set-eq*)

apply (*insert fmap*)
apply (*rule is-fmapI*)
apply (*force dest*: *is-fmapD*)
by (*metis fimage-eqI id-def map-prod-simp*)
next
case (*beta c cs t t′*)
from *beta* **obtain** *pat rhs penv* **where** $c = (pat, rhs)$ *match pat t* = *Some penv*
subst rhs penv = t'
by (*metis* (*no-types*, *lifting*) *map-option-eq-Some rewrite-step.simps surj-pair*)
then obtain *senv* **where** *match pat* (*pterm-to-sterm t*) = *Some senv srelated′.P-env*
penv senv
by (*metis option-rel-Some1 srelated′.match-rel*)
have *wellformed rhs*
using *beta* ‹*c* = -› *prules.all-rules prule.simps*
by *force*
then have *subst* (*pterm-to-sterm rhs*) *senv* = *pterm-to-sterm t′*
using *srelated-subst′* ‹- = *t′*› ‹*srelated′.P-env* - -›
by *metis*
have (*pat*, *pterm-to-sterm rhs*) |∈| *map-prod id pterm-to-sterm* |‘| *cs*
using *beta* ‹*c* = -›
by (*metis fimage-eqI id-def map-prod-simp*)
have *is-fmap cs*
using *beta*
by *auto*
have *find-match* (*ordered-fmap cs*) *t* = *Some* (*penv*, *pat*, *rhs*)
apply (*rule compatible-find-match*)
subgoal
apply (*subst ordered-fmap-set-eq*[*OF* ‹*is-fmap cs*›])+
using *beta* **by** *simp*
subgoal
unfolding *list-all-iff*
apply *rule*
apply (*rename-tac x*, *case-tac x*)
apply *simp*
apply (*drule ordered-fmap-sound*[*OF* ‹*is-fmap cs*›])
using *beta* **by** *auto*
subgoal
apply (*subst ordered-fmap-set-eq*)
by *fact*
subgoal
by *fact*
subgoal
using *beta*(*1*) ‹*c* = -› ‹*is-fmap cs*›
using *fset-of-list-elem ordered-fmap-set-eq* **by** *fast*
done

show *?case*
apply *simp*
apply *rule*

  apply (*subst* ‹- = *pterm-to-sterm t'*›[*symmetric*])
  apply (*rule find-match-rewrite-first*)
  unfolding *map-prod-def id-apply*
  apply (*subst ordered-fmap-image*[*symmetric*])
   apply *fact*
  apply (*subst map-prod-def*[*symmetric*])
  apply (*subst id-def*[*symmetric*])
  apply (*rule srelated-find-match*)
  by *fact*+
qed (*auto intro*: *srewrite.intros*)

Computability

export-code *compile*
 checking *Scala*

end

3.5 Big-step semantics

theory *Big-Step-Sterm*
imports
 Rewriting-Sterm
 ../Terms/Term-as-Value
begin

3.5.1 Big-step semantics evaluating to irreducible *sterm*s

inductive (**in** *constructors*) *seval* :: *srule list* ⇒ (*name*, *sterm*) *fmap* ⇒ *sterm* ⇒ *sterm* ⇒ *bool* (‹-, -/ $\vdash_s$/ - ↓/ -› [*50*,*0*,*50*] *50*) **for** *rs* **where**
const: (*name*, *rhs*) ∈ *set rs* ⟹ *rs*, Γ $\vdash_s$ *Sconst name* ↓ *rhs* |
var: *fmlookup* Γ *name* = *Some val* ⟹ *rs*, Γ $\vdash_s$ *Svar name* ↓ *val* |
abs: *rs*, Γ $\vdash_s$ *Sabs cs* ↓ *Sabs* (*map* (λ(*pat*, *t*). (*pat*, *subst t* (*fmdrop-fset* (*frees pat*) Γ))) *cs*) |
comb:
 rs, Γ $\vdash_s$ *t* ↓ *Sabs cs* ⟹ *rs*, Γ $\vdash_s$ *u* ↓ *u'* ⟹
 find-match cs u' = *Some* (*env*, -, *rhs*) ⟹
 rs, Γ $++_f$ *env* $\vdash_s$ *rhs* ↓ *val* ⟹
 rs, Γ $\vdash_s$ *t* $\$_s$ *u* ↓ *val* |
constr:
 name |∈| *C* ⟹
 list-all2 (*seval rs* Γ) *ts us* ⟹
 rs, Γ $\vdash_s$ *name* $$ *ts* ↓ *name* $$ *us*

lemma (**in** *constructors*) *seval-closed*:
 assumes *rs*, Γ $\vdash_s$ *t* ↓ *u closed-srules rs closed-env* Γ *closed-except t* (*fmdom* Γ)
 shows *closed u*
using *assms* **proof** *induction*
 case (*const name rhs* Γ)

```
  thus ?case
    by (auto simp: list-all-iff)
next
  case (comb Γ t cs u u' env pat rhs val)
  hence closed (Sabs cs) closed u'
    by (auto simp: closed-except-def)
  moreover have (pat, rhs) ∈ set cs match pat u' = Some env
    using comb by (auto simp: find-match-elem)
  ultimately have closed-except rhs (frees pat)
    by (auto dest: closed-except-sabs)

  show ?case
    proof (rule comb)
      have closed-env env
        by (rule closed.match) fact+
      thus closed-env (Γ ++_f env)
        using ‹closed-env Γ› by auto
    next
      have closed-except rhs (fmdom Γ |∪| frees pat)
        using ‹closed-except rhs -›
        unfolding closed-except-def by auto
      hence closed-except rhs (fmdom Γ |∪| fmdom env)
        using ‹match pat u' = Some env› by (metis match-dom)
      thus closed-except rhs (fmdom (Γ ++_f env))
        using comb by simp
    qed fact
next
  case (abs Γ cs)
  show ?case
    apply (subst subst-sterm.simps[symmetric])
    apply (subst closed-except-def)
    apply (subst subst-frees)
    apply fact+
    apply (subst fminus-fsubset-conv)
    apply (subst closed-except-def[symmetric])
    apply (subst funion-fempty-right)
    apply fact
    done
next
  case (constr name Γ ts us)
  have list-all closed us
    using ‹list-all2 - - -› ‹closed-except (list-comb - -) -›
    proof (induction ts us rule: list.rel-induct)
      case (Cons v vs u us)
      thus ?case
        using constr unfolding closed.list-comb
        by auto
    qed simp
  thus ?case
```

```
    unfolding closed.list-comb
    by (simp add: closed-except-def)
qed auto

lemma (in srules) seval-wellformed:
  assumes rs, Γ ⊢s t ↓ u wellformed t wellformed-env Γ
  shows wellformed u
using assms proof induction
  case (const name rhs Γ)
  thus ?case
    using all-rules
    by (auto simp: list-all-iff)
next
  case (comb Γ t cs u u' env pat rhs val)
  hence (pat, rhs) ∈ set cs match pat u' = Some env
    by (auto simp: find-match-elem)

  show ?case
    proof (rule comb)
      show wellformed rhs
        using ‹(pat, rhs) ∈ set cs› comb
        by (auto simp: list-all-iff)
    next
      have wellformed-env env
        apply (rule wellformed.match)
         apply fact
        apply (rule comb)
        using comb apply simp
        apply fact+
        done
      thus wellformed-env (Γ ++f env)
        using comb by auto
    qed
next
  case (abs Γ cs)
  thus ?case
    by (metis subst-sterm.simps subst-wellformed)
next
  case (constr name Γ ts us)
  have list-all wellformed us
    using ‹list-all2 - - -› ‹wellformed (list-comb - -)›
    proof (induction ts us rule: list.rel-induct)
      case (Cons v vs u us)
      thus ?case
        using constr by (auto simp: wellformed.list-comb app-sterm-def)
    qed simp
  thus ?case
    by (simp add: wellformed.list-comb const-sterm-def)
qed auto
```

```
lemma (in constants) seval-shadows:
  assumes rs, Γ ⊢s t ↓ u ¬ shadows-consts t
  assumes list-all (λ(-, rhs). ¬ shadows-consts rhs) rs
  assumes not-shadows-consts-env Γ
  shows ¬ shadows-consts u
using assms proof induction
  case (const name rhs Γ)
  thus ?case
    unfolding srules-def
    by (auto simp: list-all-iff)
next
  case (comb Γ t cs u u′ env pat rhs val)
  hence ¬ shadows-consts (Sabs cs) ¬ shadows-consts u′
    by auto
  moreover from comb have (pat, rhs) ∈ set cs match pat u′ = Some env
    by (auto simp: find-match-elem)
  ultimately have ¬ shadows-consts rhs
    by (auto simp: list-ex-iff)

  moreover have not-shadows-consts-env env
    using comb ‹match pat u′ = -› by (auto intro: shadows.match)

  ultimately show ?case
    using comb by blast
next
  case (abs Γ cs)
  show ?case
    apply (subst subst-sterm.simps[symmetric])
    apply (rule subst-shadows)
     apply fact+
    done
next
  case (constr name Γ ts us)
  have list-all (Not ∘ shadows-consts) us
    using ‹list-all2 - - -› ‹¬ shadows-consts (name $$ ts)›
    proof (induction ts us rule: list.rel-induct)
      case (Cons v vs u us)
      thus ?case
      using constr by (auto simp: shadows.list-comb const-sterm-def app-sterm-def)
    qed simp
  thus ?case
    by (auto simp: shadows.list-comb list-ex-iff list-all-iff const-sterm-def)
qed auto

lemma (in constructors) seval-list-comb-abs:
  assumes rs, Γ ⊢s name $$ args ↓ Sabs cs
  shows name ∈ dom (map-of rs)
using assms
```

```
proof (induction Γ name $$ args Sabs cs arbitrary: args cs)
  case (constr name' - - us)
  hence Sabs cs = name' $$ us by simp
  hence False
    by (cases rule: list-comb-cases) (auto simp: const-sterm-def app-sterm-def)
  thus ?case ..
next
  case (comb Γ t cs' u u' env pat rhs)

  hence strip-comb (t $_s u) = strip-comb (name $$ args)
    by simp
  hence strip-comb t = (Sconst name, butlast args) u = last args
     apply −
    subgoal
      apply (simp add: strip-list-comb-const)
      apply (fold app-sterm-def const-sterm-def)
      by (auto split: prod.splits)
    subgoal
      apply (simp add: strip-list-comb-const)
      apply (fold app-sterm-def const-sterm-def)
      by (auto split: prod.splits)
    done
  hence t = name $$ butlast args
    apply (fold const-sterm-def)
    by (metis list-strip-comb fst-conv snd-conv)

  thus ?case
    using comb by auto
qed (auto elim: list-comb-cases simp: const-sterm-def app-sterm-def intro: weak-map-of-SomeI)

lemma (in constructors) is-value-eval-id:
  assumes is-value t closed t
  shows rs, Γ ⊢_s t ↓ t
using assms proof induction
  case (abs cs)

   have rs, Γ ⊢_s Sabs cs ↓ Sabs (map (λ(pat, t). (pat, subst t (fmdrop-fset (frees
pat) Γ))) cs)
    by (rule seval.abs)
  moreover have subst (Sabs cs) Γ = Sabs cs
    using abs by (metis subst-closed-id)
  ultimately show ?case
    by simp
next
  case (constr vs name)
  have list-all2 (seval rs Γ) vs vs
    proof (rule list.rel-refl-strong)
      fix v
      assume v ∈ set vs
```

```
      moreover hence closed v
        using constr
        unfolding closed.list-comb
        by (auto simp: list-all-iff)
      ultimately show rs, Γ ⊢_s v ↓ v
        using ‹list-all - -›
        by (force simp: list-all-iff)
    qed
    with ‹name |∈| C› show ?case
      by (rule seval.constr)
qed

lemma (in constructors) ssubst-eval:
  assumes rs, Γ ⊢_s t ↓ t' Γ' ⊆_f Γ closed-env Γ value-env Γ
  shows rs, Γ ⊢_s subst t Γ' ↓ t'
using assms proof induction
  case (var Γ name val)
  show ?case
    proof (cases fmlookup Γ' name)
      case None
      thus ?thesis
        using var by (auto intro: seval.intros)
    next
      case (Some val')
      with var have val' = val
        unfolding fmsubset-alt-def by force
      show ?thesis
        apply simp
        apply (subst Some)
        apply (subst ‹val' = -›)
        apply simp
        apply (rule is-value-eval-id)
        using var by auto
    qed
next
  case (abs Γ cs)
  hence subst (subst (Sabs cs) Γ') Γ = subst (Sabs cs) Γ
    by (metis subst-twice fmsubset-pred)
  moreover have rs, Γ ⊢_s subst (Sabs cs) Γ' ↓ subst (subst (Sabs cs) Γ') Γ
    apply simp
    apply (subst map-map[symmetric])
    apply (rule seval.abs)
    done
  ultimately have rs, Γ ⊢_s subst (Sabs cs) Γ' ↓ subst (Sabs cs) Γ
    by metis
  thus ?case by simp
next
  case (constr name Γ ts us)
  hence list-all2 (λt. seval rs Γ (subst t Γ')) ts us
```

by (*blast intro*: *list.rel-mono-strong*)
with *constr* **show** *?case*
by (*auto simp*: *subst-list-comb list-all2-map1 intro*: *seval.constr*)
qed (*auto intro*: *seval.intros*)

lemma (**in** *constructors*) *seval-agree-eq*:
assumes *rs*, $\Gamma \vdash_s t \downarrow u$ *fmrestrict-fset* S Γ = *fmrestrict-fset* S Γ' *closed-except* t S
assumes S $|\subseteq|$ *fmdom* Γ *closed-srules rs closed-env* Γ
shows *rs*, $\Gamma' \vdash_s t \downarrow u$
using *assms* **proof** (*induction arbitrary*: Γ' S)
case (*var* Γ *name val*)
hence *name* $|\in|$ S
by (*simp add*: *closed-except-def*)
hence *fmlookup* Γ *name* = *fmlookup* Γ' *name*
using ‹*fmrestrict-fset* S Γ = -›
unfolding *fmfilter-alt-defs*
including *fmap.lifting*
by *transfer*′ (*auto simp*: *map-filter-def fun-eq-iff split*: *if-splits*)
with *var* **show** *?case*
by (*auto intro*: *seval.var*)
next
case (*abs* Γ *cs*)

— Intentionally local: not really a useful lemma outside of its scope
have ∗: *fmdrop-fset* S (*fmrestrict-fset* T m) = *fmrestrict-fset* (T $|\cup|$ S) (*fmdrop-fset* S m) **for** S T m
unfolding *fmfilter-alt-defs fmfilter-comp*
by (*rule fmfilter-cong*) *auto*

{
fix *pat* t
assume (*pat*, t) $\in$ *set cs*
with *abs* **have** *closed-except* t (S $|\cup|$ *frees pat*)
by (*auto simp*: *Sterm.closed-except-simps list-all-iff*)

have
subst t (*fmdrop-fset* (*frees pat*) (*fmrestrict-fset* S Γ)) = *subst* t (*fmdrop-fset* (*frees pat*) Γ)
apply (*subst* ∗)
apply (*rule subst-restrict-closed*)
apply *fact*
done

moreover have
subst t (*fmdrop-fset* (*frees pat*) (*fmrestrict-fset* S Γ')) = *subst* t (*fmdrop-fset* (*frees pat*) Γ')
apply (*subst* ∗)
apply (*rule subst-restrict-closed*)

```
      apply fact
      done

    ultimately have subst t (fmdrop-fset (frees pat) Γ) = subst t (fmdrop-fset
(frees pat) Γ′)
      using abs by metis
  }

  hence map (λ(pat, t). (pat, subst t (fmdrop-fset (frees pat) Γ))) cs =
        map (λ(pat, t). (pat, subst t (fmdrop-fset (frees pat) Γ′))) cs
    by auto

  thus ?case
    by (metis seval.abs)
next
  case (comb Γ t cs u u′ env pat rhs val)
  have fmdom env = frees pat
    apply (rule match-dom)
    apply (rule find-match-elem)
    apply fact
    done

  show ?case
    proof (rule seval.comb)
      show rs, Γ′ ⊢s t ↓ Sabs cs rs, Γ′ ⊢s u ↓ u′
        using comb by (auto simp: Sterm.closed-except-simps)
    next
      show rs, Γ′ ++f env ⊢s rhs ↓ val
        proof (rule comb)
          have fmrestrict-fset (S |∪| fmdom env) (Γ ++f env) = fmrestrict-fset (S
|∪| fmdom env) (Γ′ ++f env)
            using comb(8)
            unfolding fmfilter-alt-defs
            including fmap.lifting and fset.lifting
              by transfer′ (auto simp: map-filter-def fun-eq-iff map-add-def split:
option.splits if-splits)

          thus fmrestrict-fset (S |∪| frees pat) (Γ ++f env) = fmrestrict-fset (S |∪|
frees pat) (Γ′ ++f env)
            unfolding ‹fmdom env = -› .
        next
          have closed-except t S
            using comb by (simp add: Sterm.closed-except-simps)

          have closed (Sabs cs)
            apply (rule seval-closed)
            apply fact+
            using ‹closed-except t S› ‹S |⊆| fmdom Γ›
            unfolding closed-except-def apply simp
```

```
          done

        have (pat, rhs) ∈ set cs
          using ‹find-match - - = -› by (rule find-match-elem)
        hence closed-except rhs (frees pat)
          using ‹closed (Sabs cs)› by (auto dest: closed-except-sabs)
        thus closed-except rhs (S |∪| frees pat)
          unfolding closed-except-def by auto
      next
        show S |∪| frees pat |⊆| fmdom (Γ ++f env)
          apply simp
          apply (intro conjI)
          using comb(10) apply blast
          unfolding ‹fmdom env = -› by blast
      next
        have closed-except u S
          using comb by (auto simp: closed-except-def)

        show closed-env (Γ ++f env)
          apply rule
           apply fact
          apply (rule closed.match[where t = u' and pat = pat])
          subgoal
            by (rule find-match-elem) fact
          subgoal
            apply (rule seval-closed)
               apply fact+
           using ‹closed-except u S› ‹S |⊆| fmdom Γ› unfolding closed-except-def
by blast
          done
      qed fact
    qed fact
next
  case (constr name Γ ts us)
  show ?case
    apply (rule seval.constr)
     apply fact
    apply (rule list.rel-mono-strong)
     apply fact
    using constr
    unfolding closed.list-comb list-all-iff
    by auto
qed (auto intro: seval.intros)
```

Correctness wrt *srewrite*

context *srules* **begin context begin**

private lemma *seval-correct0*:

```
  assumes rs, Γ ⊢s t ↓ u closed-except t (fmdom Γ) closed-env Γ
  shows rs ⊢s subst t Γ ⟶* u
using assms proof induction
  case (const name rhs Γ)

  have srewrite-step rs name rhs
    by (rule srewrite-stepI) fact
  thus ?case
    by (auto intro: srewrite.intros)
next
  case (comb Γ t cs u u' env pat rhs val)
  hence closed-except t (fmdom Γ) closed-except u (fmdom Γ)
    by (simp add: Sterm.closed-except-simps)+
  moreover have closed-srules rs
    using all-rules
    unfolding list-all-iff by fastforce
  ultimately have closed (Sabs cs) closed u'
    using comb by (metis seval-closed)+

  from comb have (pat, rhs) ∈ set cs match pat u' = Some env
    by (auto simp: find-match-elem)
  hence closed-except rhs (frees pat)
    using ‹closed (Sabs cs)› by (auto dest: closed-except-sabs)
  hence frees rhs |⊆| frees pat
    by (simp add: closed-except-def)
  moreover have fmdom env = frees pat
    using ‹match pat u' = -› by (auto simp: match-dom)
  ultimately have frees rhs |⊆| fmdom env
    by simp
  hence subst rhs (Γ ++f env) = subst rhs env
    by (rule subst-add-shadowed-env)

  have rs ⊢s subst t Γ $s subst u Γ ⟶* Sabs cs $s u'
     using comb by (force intro: srewrite.rt-comb[unfolded app-sterm-def] simp:
Sterm.closed-except-simps)
  also have rs ⊢s Sabs cs $s u' ⟶* subst rhs env
    using comb ‹closed u'› by (force intro: srewrite.beta find-match-rewrite-first)
  also have rs ⊢s subst rhs env ⟶* subst rhs (Γ ++f env)
    unfolding ‹subst rhs (Γ ++f env) = -› by simp
  also have rs ⊢s subst rhs (Γ ++f env) ⟶* val
    proof (rule comb)
      show closed-except rhs (fmdom (Γ ++f env))
       using comb ‹match pat u' = Some env› ‹fmdom env = -› ‹frees rhs |⊆| frees
pat›
        by (auto simp: closed-except-def)
    next
      show closed-env (Γ ++f env)
        using comb ‹match pat u' = Some env› ‹closed u'›
        by (blast intro: closed.match)
```

qed

finally show *?case* **by** *simp*
next
case (*constr name* Γ *ts us*)
show *?case*
apply (*simp add*: *subst-list-comb*)
apply (*rule srewrite.rt-list-comb*)
subgoal
apply (*simp add*: *list.rel-map*)
apply (*rule list.rel-mono-strong*[*OF constr*(*2*)])
apply *clarify*
apply (*elim impE*)
using *constr*(*3*) **apply** (*erule closed.list-combE*)
apply (*rule constr*)+
apply (*auto simp*: *const-sterm-def*)
done
subgoal by *auto*
done
qed *auto*

corollary *seval-correct*:
assumes *rs*, *fmempty* $\vdash_s$ *t* ↓ *u closed t*
shows *rs* $\vdash_s$ *t* ⟶∗ *u*
proof −
have *closed-except t* (*fmdom fmempty*)
using *assms* **by** *simp*
with *assms* **have** *rs* $\vdash_s$ *subst t fmempty* ⟶∗ *u*
by (*fastforce intro*!: *seval-correct0*)
thus *?thesis*
by *simp*
qed

end end

end
theory *Big-Step-Value*
imports
Big-Step-Sterm
../Terms/Value
begin

3.5.2 Big-step semantics evaluating to *value*

primrec *vrule* :: *vrule* ⇒ *bool* **where**
vrule (-, *rhs*) ⟷ *vwellformed rhs* ∧ *vclosed rhs* ∧ ¬ *is-Vconstr rhs*

locale *vrules* = *constants C-info fst* |'| *fset-of-list rs* **for** *C-info* **and** *rs* :: *vrule list*
+

assumes *all-rules*: *list-all vrule rs*
assumes *distinct*: *distinct* (*map fst rs*)
assumes *not-shadows*: *list-all* (λ(-, *rhs*). *not-shadows-vconsts rhs*) *rs*
assumes *vconstructor-value-rs*: *vconstructor-value-rs rs*
assumes *vwelldefined-rs*: *list-all* (λ(-, *rhs*). *vwelldefined rhs*) *rs*
begin

lemma *map*: *is-map* (*set rs*)
using *distinct* **by** (*rule distinct-is-map*)

end

abbreviation *value-to-sterm-rules* :: *vrule list* ⇒ *srule list* **where**
value-to-sterm-rules ≡ *map* (*map-prod id value-to-sterm*)

inductive (**in** *special-constants*)
veval :: (*name* × *value*) *list* ⇒ (*name*, *value*) *fmap* ⇒ *sterm* ⇒ *value* ⇒ *bool* (‹-, -/ $\vdash_v$/ - ↓/ -› [*50*,*0*,*50*] *50*) **for** *rs* **where**
const: (*name*, *rhs*) ∈ *set rs* ⟹ *rs*, Γ $\vdash_v$ *Sconst name* ↓ *rhs* |
var: *fmlookup* Γ *name* = *Some val* ⟹ *rs*, Γ $\vdash_v$ *Svar name* ↓ *val* |
abs: *rs*, Γ $\vdash_v$ *Sabs cs* ↓ *Vabs cs* Γ |
comb:
rs, Γ $\vdash_v$ *t* ↓ *Vabs cs* Γ′ ⟹ *rs*, Γ $\vdash_v$ *u* ↓ *u*′ ⟹
vfind-match cs u′ = *Some* (*env*, -, *rhs*) ⟹
rs, Γ′ $++_f$ *env* $\vdash_v$ *rhs* ↓ *val* ⟹
rs, Γ $\vdash_v$ *t* $\$_s$ *u* ↓ *val* |
rec-comb:
rs, Γ $\vdash_v$ *t* ↓ *Vrecabs css name* Γ′ ⟹
fmlookup css name = *Some cs* ⟹
rs, Γ $\vdash_v$ *u* ↓ *u*′ ⟹
vfind-match cs u′ = *Some* (*env*, -, *rhs*) ⟹
rs, Γ′ $++_f$ *env* $\vdash_v$ *rhs* ↓ *val* ⟹
rs, Γ $\vdash_v$ *t* $\$_s$ *u* ↓ *val* |
constr:
name |∈| *C* ⟹
list-all2 (*veval rs* Γ) *ts us* ⟹
rs, Γ $\vdash_v$ *name* $$ *ts* ↓ *Vconstr name us*

lemma (**in** *vrules*) *veval-wellformed*:
assumes *rs*, Γ $\vdash_v$ *t* ↓ *v wellformed t wellformed-venv* Γ
shows *vwellformed v*
using *assms* **proof** *induction*
case *const*
thus *?case*
using *all-rules*
by (*auto simp*: *list-all-iff*)
next
case *comb*
show *?case*

```
    apply (rule comb)
      using comb by (auto simp: list-all-iff dest: vfind-match-elem intro: vwell-
formed.vmatch-env)
next
  case (rec-comb Γ t css name Γ' cs u u' env pat rhs val)
  hence (pat, rhs) ∈ set cs vmatch (mk-pat pat) u' = Some env
    by (metis vfind-match-elem)+

  show ?case
    proof (rule rec-comb)
      have wellformed t
        using rec-comb by simp
      have vwellformed (Vrecabs css name Γ')
        by (rule rec-comb) fact+
      thus wellformed rhs
        using rec-comb ‹(pat, rhs) ∈ set cs›
        by (auto simp: list-all-iff)

      have wellformed-venv Γ'
        using ‹vwellformed (Vrecabs css name Γ')› by simp
      moreover have wellformed-venv env
        using rec-comb ‹vmatch (mk-pat pat) u' = Some env›
        by (auto intro: vwellformed.vmatch-env)
      ultimately show wellformed-venv (Γ' ++_f env)
        by blast
    qed
next
  case (constr name Γ ts us)
  have list-all vwellformed us
    using ‹list-all2 - - -› ‹wellformed (list-comb - -)›
    proof (induction ts us rule: list.rel-induct)
      case (Cons v vs u us)
      with constr show ?case
        unfolding wellformed.list-comb by auto
    qed simp
  thus ?case
    by (simp add: list-all-iff)
qed auto

lemma (in vrules) veval-closed:
  assumes rs, Γ ⊢_v t ↓ v closed-except t (fmdom Γ) closed-venv Γ
  assumes wellformed t wellformed-venv Γ
  shows vclosed v
using assms proof induction
  case (const name rhs Γ)
  hence (name, rhs) ∈ set rs
    by (auto dest: map-of-SomeD)
  thus ?case
    using const all-rules
```

by (*auto simp*: *list-all-iff*)
next
  case (*comb* Γ *t cs* Γ' *u* u' *env pat rhs val*)
  hence *pat*: (*pat*, *rhs*) $\in$ *set cs vmatch* (*mk-pat pat*) u' = *Some env*
    by (*metis vfind-match-elem*)+

  show *?case*
    proof (*rule comb*)
      have *vclosed* u'
        using *comb* **by** (*auto simp*: *Sterm.closed-except-simps*)
      have *closed-venv env*
        by (*rule vclosed.vmatch-env*) *fact*+
      thus *closed-venv* (Γ' $++_f$ *env*)
        using *comb* **by** (*auto simp*: *Sterm.closed-except-simps*)
    next
      have *wellformed t*
        using *comb* **by** *simp*
      have *vwellformed* (*Vabs cs* Γ')
        by (*rule veval-wellformed*) *fact*+
      thus *wellformed rhs*
        using *pat* **by** (*auto simp*: *list-all-iff*)

      have *wellformed-venv* Γ'
        using ‹*vwellformed* (*Vabs cs* Γ')› **by** *simp*
      moreover have *wellformed-venv env*
        using *comb pat*
        by (*auto intro*: *vwellformed.vmatch-env veval-wellformed*)
      ultimately show *wellformed-venv* (Γ' $++_f$ *env*)
        by *blast*

      have *vclosed* (*Vabs cs* Γ')
        using *comb* **by** (*auto simp*: *Sterm.closed-except-simps*)
      hence *closed-except rhs* (*fmdom* Γ' $|\cup|$ *frees pat*)
        using *pat* **by** (*auto simp*: *list-all-iff*)

      moreover have *fmdom env* = *frees pat*
        using ‹*vwellformed* (*Vabs cs* Γ')› *pat*
        by (*auto simp*: *vmatch-dom mk-pat-frees list-all-iff*)

      ultimately show *closed-except rhs* (*fmdom* (Γ' $++_f$ *env*))
        using ‹*vclosed* (*Vabs cs* Γ')›
        by *simp*
    qed
next
  case (*rec-comb* Γ *t css name* Γ' *cs u* u' *env pat rhs val*)
  hence *pat*: (*pat*, *rhs*) $\in$ *set cs vmatch* (*mk-pat pat*) u' = *Some env*
    by (*metis vfind-match-elem*)+

  show *?case*

```
      proof (rule rec-comb)
        have vclosed u′
          using rec-comb by (auto simp: Sterm.closed-except-simps)
        have closed-venv env
          by (rule vclosed.vmatch-env) fact+
        thus closed-venv (Γ′ ++_f env)
          using rec-comb by (auto simp: Sterm.closed-except-simps)
      next
        have wellformed t
          using rec-comb by simp
        have vwellformed (Vrecabs css name Γ′)
          by (rule veval-wellformed) fact+
        thus wellformed rhs
          using pat rec-comb by (auto simp: list-all-iff)

        have wellformed-venv Γ′
          using ‹vwellformed (Vrecabs css name Γ′)› by simp
        moreover have wellformed-venv env
          using rec-comb pat
          by (auto intro: vwellformed.vmatch-env veval-wellformed)
        ultimately show wellformed-venv (Γ′ ++_f env)
          by blast

        have wellformed-clauses cs
         using ‹vwellformed (Vrecabs css name Γ′)› ‹fmlookup css name = Some cs›
          by auto

        have vclosed (Vrecabs css name Γ′)
          using rec-comb by (auto simp: Sterm.closed-except-simps)
        hence closed-except rhs (fmdom Γ′ |∪| frees pat)
          using rec-comb pat by (auto simp: list-all-iff)
        moreover have fmdom env = frees pat
          using ‹wellformed-clauses cs› pat
          by (auto simp: list-all-iff vmatch-dom mk-pat-frees)
        ultimately show closed-except rhs (fmdom (Γ′ ++_f env))
          using ‹vclosed (Vrecabs css name Γ′)›
          by simp
      qed
  next
    case (constr name Γ ts us)
    have list-all vclosed us
      using ‹list-all2 - - -› ‹closed-except (- $$ -) -› ‹wellformed (name $$ ts)›
      proof (induction ts us rule: list.rel-induct)
        case (Cons v vs u us)
        with constr show ?case
          unfolding closed.list-comb wellformed.list-comb
          by (auto simp: list-all-iff Sterm.closed-except-simps)
      qed simp
    thus ?case
```

by (*simp add*: *list-all-iff*)
qed (*auto simp*: *Sterm.closed-except-simps*)

lemma (**in** *vrules*) *veval-constructor-value*:
assumes *rs*, $\Gamma \vdash_v t \downarrow v$ *vconstructor-value-env* Γ
shows *vconstructor-value v*
using *assms* **proof** *induction*
case (*comb* Γ *t cs* Γ' *u* u' *env pat rhs val*)
hence (*pat*, *rhs*) $\in$ *set cs vmatch* (*mk-pat pat*) u' = *Some env*
by (*metis vfind-match-elem*)+
hence *vconstructor-value-env* (Γ' $++_f$ *env*)
using *comb* **by** (*auto intro*: *vconstructor-value.vmatch-env*)
thus *?case*
using *comb* **by** *auto*
next
case (*constr name* Γ *ts us*)
hence *list-all vconstructor-value us*
by (*auto elim*: *list-all2-rightE*)
with *constr* **show** *?case*
by *simp*
next
case *const*
thus *?case*
using *vconstructor-value-rs*
by (*auto simp*: *list-all-iff vconstructor-value-rs-def*)
next
case (*rec-comb* Γ *t css name* Γ' *cs u* u' *env pat rhs val*)
hence (*pat*, *rhs*) $\in$ *set cs vmatch* (*mk-pat pat*) u' = *Some env*
by (*metis vfind-match-elem*)+
hence *vconstructor-value-env* (Γ' $++_f$ *env*)
using *rec-comb* **by** (*auto intro*: *vconstructor-value.vmatch-env*)
thus *?case*
using *rec-comb* **by** *auto*
qed (*auto simp*: *list-all-iff vconstructor-value-rs-def*)

lemma (**in** *vrules*) *veval-welldefined*:
assumes *rs*, $\Gamma \vdash_v t \downarrow v$ *fmpred* (λ-. *vwelldefined*) Γ *welldefined t*
shows *vwelldefined v*
using *assms* **proof** *induction*
case *const*
thus *?case*
using *vwelldefined-rs assms*
unfolding *list-all-iff*
by (*auto simp*: *list-all-iff*)
next
case (*comb* Γ *t cs* Γ' *u* u' *env pat rhs val*)
hence *vwelldefined* (*Vabs cs* Γ')
by *auto*

```
  show ?case
    proof (rule comb)
      have fmpred (λ-. vwelldefined) Γ′
        using ‹vwelldefined (Vabs cs Γ′)›
        by simp
      moreover have fmpred (λ-. vwelldefined) env
        apply (rule vwelldefined.vmatch-env)
         apply (rule vfind-match-elem)
        using comb by auto
      ultimately show fmpred (λ-. vwelldefined) (Γ′ ++_f env)
        by auto
    next
      have (pat, rhs) ∈ set cs
        using comb by (metis vfind-match-elem)
      thus welldefined rhs
        using ‹vwelldefined (Vabs cs Γ′)›
        by (auto simp: list-all-iff)
    qed
next
  case (rec-comb Γ t css name Γ′ cs u u′ env pat rhs val)
  have (pat, rhs) ∈ set cs
    by (rule vfind-match-elem) fact
  show ?case
    proof (rule rec-comb)
      show fmpred (λ-. vwelldefined) (Γ′ ++_f env)
        proof
          show fmpred (λ-. vwelldefined) env
         using rec-comb by (auto dest: vfind-match-elem intro: vwelldefined.vmatch-env)
        next
          show fmpred (λ-. vwelldefined) Γ′
            using rec-comb by auto
        qed
    next
      have vwelldefined (Vrecabs css name Γ′)
        using rec-comb by auto

      thus welldefined rhs
        apply simp
        apply (elim conjE)
        apply (drule fmpredD[where m = css])
        using ‹(pat, rhs) ∈ set cs› rec-comb by (auto simp: list-all-iff)
    qed
next
  case (constr name Γ ts us)
  have list-all vwelldefined us
    using ‹list-all2 - - -› ‹welldefined (- $$ -)›
    proof (induction ts us rule: list.rel-induct)
      case (Cons v vs u us)
      with constr show ?case
```

```
      unfolding welldefined.list-comb
      by auto
    qed simp
  with constr show ?case
    by (simp add: list-all-iff all-consts-def)
next
  case abs
  thus ?case
    unfolding welldefined-sabs by auto
qed auto
```

Correctness wrt *constructors.seval*

```
context vrules begin

definition rs' :: srule list where
rs' = value-to-sterm-rules rs

lemma value-to-sterm-srules: srules C-info rs'
proof
  show distinct (map fst rs')
    unfolding rs'-def
    using distinct by auto
next
  show list-all srule rs'
    unfolding rs'-def list.pred-map
    apply (rule list.pred-mono-strong[OF all-rules])
    apply (auto intro: vclosed.value-to-sterm vwellformed.value-to-sterm)
    subgoal by (auto intro: vwellformed.value-to-sterm)
    subgoal by (auto intro: vclosed.value-to-sterm)
    subgoal for a b by (cases b) (auto simp: is-abs-def term-cases-def)
    done
next
  show fdisjnt (fst |`| fset-of-list rs') C
    using vconstructor-value-rs unfolding rs'-def vconstructor-value-rs-def
    by auto
  interpret c: constants - fst |`| fset-of-list rs'
    by standard (fact | fact distinct-ctr)+
  have all-consts: c.all-consts = all-consts
    unfolding c.all-consts-def all-consts-def
    by (simp add: rs'-def)
  have shadows-consts: c.shadows-consts rhs = shadows-consts rhs for rhs :: sterm
    by (induction rhs; fastforce simp: all-consts list-ex-iff)

  have list-all (λ(-, rhs). ¬ shadows-consts rhs) rs'
    unfolding rs'-def
    unfolding list.pred-map map-prod-def id-def case-prod-twice list-all-iff
    apply auto
    unfolding comp-def all-consts-def
```

using *not-shadows*
by (*fastforce simp*: *list-all-iff dest*: *not-shadows-vconsts.value-to-sterm*)
thus *list-all* (λ(-, *rhs*). $\neg$ *c.shadows-consts rhs*) *rs'*
unfolding *shadows-consts* **.**

have *list-all* (λ(-, *rhs*). *welldefined rhs*) *rs'*
unfolding *rs'-def list.pred-map*
apply (*rule list.pred-mono-strong*[*OF vwelldefined-rs*])
subgoal for *z*
apply (*cases z*; *hypsubst-thin*)
apply *simp*
apply (*erule vwelldefined.value-to-sterm*)
done
done
moreover have *map fst rs* = *map fst rs'*
unfolding *rs'-def* **by** *simp*
ultimately have *list-all* (λ(-, *rhs*). *welldefined rhs*) *rs'*
by *simp*
thus *list-all* (λ(-, *rhs*). *c.welldefined rhs*) *rs'*
unfolding *all-consts* **.**
next
show *distinct all-constructors*
by (*fact distinct-ctr*)
qed

end

When we evaluate *sterm*s using *veval*, the result is a *value* which possibly contains a closure (constructor *Vabs*). Such a closure is essentially a case-lambda (like *Sabs*), but with an additionally captured environment of type *string* $\rightharpoonup$ *value* (which is usually called Γ'). The contained case-lambda might not be closed.

The proof idea is that we can always substitute with Γ' and obtain a regular *sterm* value. The only interesting part of the proof is the case when a case-lambda gets applied to a value, because in that process, a hidden environment is *unveiled*. That environment may not bear any relation to the active environment Γ at all. But pattern matching and substitution proceeds only with that hidden environment.

context *vrules* **begin**

context begin

private lemma *veval-correct0*:
assumes *rs*, $\Gamma \vdash_v t \downarrow v$ *wellformed t wellformed-venv* Γ
assumes *closed-except t* (*fmdom* Γ) *closed-venv* Γ
assumes *vconstructor-value-env* Γ
shows *rs'*, *fmmap value-to-sterm* $\Gamma \vdash_s t \downarrow$ *value-to-sterm v*
using *assms* **proof** *induction*

```
  case (constr name Γ ts us)

  have list-all2 (seval rs' (fmmap value-to-sterm Γ)) ts (map value-to-sterm us)
    unfolding list-all2-map2
    proof (rule list.rel-mono-strong[OF ‹list-all2 - - -›], elim conjE impE)
      fix t u
      assume t ∈ set ts u ∈ set us
      assume rs, Γ ⊢v t ↓ u

      show wellformed t  closed-except t (fmdom Γ)
        using ‹t ∈ set ts› constr
        unfolding wellformed.list-comb closed.list-comb list-all-iff
        by auto
    qed (rule constr | assumption)+

  thus ?case
    using ‹name |∈| C›
    by (auto intro: seval.constr)
next
  case (comb Γ t cs Γ' u u' venv pat rhs val)

  — We first need to establish a ton of boring side-conditions.

  hence vmatch (mk-pat pat) u' = Some venv
    by (auto dest: vfind-match-elem)

  have wellformed t
    using comb by simp
  have vwellformed (Vabs cs Γ')
    by (rule veval-wellformed) fact+

  hence
    list-all (linear ∘ fst) cs
    wellformed-venv Γ'
    by (auto simp: list-all-iff split-beta)

  have rel-option match-related (vfind-match cs u') (find-match cs (value-to-sterm
u'))
    apply (rule find-match-eq)
     apply fact
    apply (rule veval-constructor-value)
     apply fact+
    done

  then obtain senv
    where find-match cs (value-to-sterm u') = Some (senv, pat, rhs)
      and env-eq venv senv
    using ‹vfind-match - - = -›
    by cases auto
```

hence $(pat, rhs) \in set\ cs\ match\ pat\ (value\text{-}to\text{-}sterm\ u') = Some\ senv$
  by (*auto dest*: *find-match-elem*)
hence *fmdom senv* = *frees pat*
  by (*simp add*: *match-dom*)

moreover have *senv* = *fmmap value-to-sterm venv*
  using ‹*env-eq venv senv*›
  by (*rule env-eq-eq*)

ultimately have *fmdom venv* = *frees pat*
  by *simp*

have *closed-except t* (*fmdom* Γ) *wellformed t*
  using *comb* **by** (*simp add*: *closed-except-def*)+
have *vclosed* (*Vabs cs* Γ')
  by (*rule veval-closed*) *fact*+

have *vconstructor-value* (*Vabs cs* Γ') *vconstructor-value* u'
  by (*rule veval-constructor-value*; *fact*)+
hence *vconstructor-value-env* Γ'
  by *simp*
have *vconstructor-value-env venv*
  by (*rule vconstructor-value.vmatch-env*) *fact*+

have *wellformed u*
  using *comb* **by** *simp*
have *vwellformed* u'
  by (*rule veval-wellformed*) *fact*+
have *wellformed-venv venv*
  by (*rule vwellformed.vmatch-env*) *fact*+

have *closed-except u* (*fmdom* Γ)
  using *comb* **by** (*simp add*: *closed-except-def*)
have *vclosed* u'
  by (*rule veval-closed*) *fact*+
have *closed-venv venv*
  by (*rule vclosed.vmatch-env*) *fact*+

have *closed-venv* Γ'
  using ‹*vclosed* (*Vabs cs* Γ')› **by** *simp*

let *?subst* = λ*pat t*. *subst t* (*fmdrop-fset* (*frees pat*) (*fmmap value-to-sterm* Γ'))

1. We know the following (induction hypothesis): *rs*$'$, *fmmap value-to-sterm* (Γ' $++_f$ *venv*) $\vdash_s$ *rhs* $\downarrow$ *value-to-sterm val*
2. ... first, we can deduce using *ssubst-eval* that this is equivalent to substituting *rhs* first: *rs*$'$, *fmmap value-to-sterm* (Γ' $++_f$ *venv*) $\vdash_s$ *subst rhs* (*fmdrop-fset* (*frees pat*) (*fmmap value-to-sterm* Γ')) $\downarrow$ *value-to-sterm val*

3. ... second, we can replace the hidden environment Γ' by the active environment Γ using *seval-agree-eq* because it does not contain useful information at this point: *rs'*, *fmmap value-to-sterm* (Γ $++_f$ *venv*) $\vdash_s$ *subst rhs* (*fmdrop-fset* (*frees pat*) (*fmmap value-to-sterm* Γ')) $\downarrow$ *value-to-sterm val*

4. ... finally we can apply a step in the original semantics and arrive at the conclusion: *rs'*, *fmmap value-to-sterm* Γ $\vdash_s$ *t* $\$_s$ *u* $\downarrow$ *value-to-sterm val*

have *rs'*, *fmmap value-to-sterm* (Γ' $++_f$ *venv*) $\vdash_s$ *?subst pat rhs* $\downarrow$ *value-to-sterm val*
  proof (*rule ssubst-eval*)
    show *rs'*, *fmmap value-to-sterm* (Γ' $++_f$ *venv*) $\vdash_s$ *rhs* $\downarrow$ *value-to-sterm val*
      proof (*rule comb*)
        have *linear pat closed-except rhs* (*fmdom* Γ' |∪| *frees pat*)
          using ‹(*pat*, *rhs*) ∈ *set cs*› ‹*vwellformed* (*Vabs cs* Γ')› ‹*vclosed* (*Vabs cs* Γ')›
          by (*auto simp*: *list-all-iff*)
        hence *closed-except rhs* (*fmdom* Γ' |∪| *fmdom venv*)
          using ‹*vmatch* (*mk-pat pat*) *u'* = *Some venv*›
          by (*auto simp*: *mk-pat-frees vmatch-dom*)
        thus *closed-except rhs* (*fmdom* (Γ' $++_f$ *venv*))
          by *simp*
      next
        show *wellformed-venv* (Γ' $++_f$ *venv*)
          using ‹*wellformed-venv* Γ'› ‹*wellformed-venv venv*›
          by *blast*
      next
        show *closed-venv* (Γ' $++_f$ *venv*)
          using ‹*closed-venv* Γ'› ‹*closed-venv venv*›
          by *blast*
      next
        show *vconstructor-value-env* (Γ' $++_f$ *venv*)
          using ‹*vconstructor-value-env* Γ'› ‹*vconstructor-value-env venv*›
          by *blast*
      next
        show *wellformed rhs*
          using ‹(*pat*, *rhs*) ∈ *set cs*› ‹*vwellformed* (*Vabs cs* Γ')›
          by (*fastforce simp*: *list-all-iff*)
      qed
  next
    have *fmdrop-fset* (*fmdom venv*) Γ' $\subseteq_f$ Γ' $++_f$ *venv*
      including *fmap.lifting* **and** *fset.lifting*
      by *transfer'*
        (*auto simp*: *map-drop-set-def map-filter-def map-le-def map-add-def split*: *if-splits*)
    thus *fmdrop-fset* (*frees pat*) (*fmmap value-to-sterm* Γ') $\subseteq_f$ *fmmap value-to-sterm* (Γ' $++_f$ *venv*)
      unfolding ‹*fmdom venv* = *frees pat*›
      by (*metis fmdrop-fset-fmmap fmmap-subset*)
  next

show *closed-env* (*fmmap value-to-sterm* (Γ' $++_f$ *venv*))
apply *auto*
apply *rule*
apply (*rule vclosed.value-to-sterm-env*, *fact*)+
done
next
show *value-env* (*fmmap value-to-sterm* (Γ' $++_f$ *venv*))
apply *auto*
apply *rule*
apply (*rule vconstructor-value.value-to-sterm-env*, *fact*)+
done
qed

show *?case*
proof (*rule seval.comb*)
have *rs'*, *fmmap value-to-sterm* $\Gamma \vdash_s$ *t* $\downarrow$ *value-to-sterm* (*Vabs cs* Γ')
using *comb* **by** (*auto simp*: *closed-except-def*)
thus *rs'*, *fmmap value-to-sterm* $\Gamma \vdash_s$ *t* $\downarrow$ *Sabs* (*map* (λ(*pat*, *t*). (*pat*, *?subst pat t*)) *cs*)
by *simp*
next
show *rs'*, *fmmap value-to-sterm* $\Gamma \vdash_s$ *u* $\downarrow$ *value-to-sterm u'*
using *comb* **by** (*simp add*: *closed-except-def*)
next
show *rs'*, *fmmap value-to-sterm* Γ $++_f$ *senv* $\vdash_s$ *?subst pat rhs* $\downarrow$ *value-to-sterm val*
proof (*rule seval-agree-eq*)
show *rs'*, *fmmap value-to-sterm* Γ' $++_f$ *fmmap value-to-sterm venv* $\vdash_s$ *?subst pat rhs* $\downarrow$ *value-to-sterm val*
using ‹*rs'*, *fmmap value-to-sterm* (Γ' $++_f$ *venv*) $\vdash_s$ *?subst pat rhs* $\downarrow$ *value-to-sterm val*› **by** *simp*
next
show *fmrestrict-fset* (*frees pat*) (*fmmap value-to-sterm* Γ' $++_f$ *fmmap value-to-sterm venv*) =
fmrestrict-fset (*frees pat*) (*fmmap value-to-sterm* Γ $++_f$ *senv*)
unfolding ‹*senv* = -›
apply (*subst* ‹*fmdom venv* = *frees pat*›[*symmetric*])+
apply (*subst fmdom-map*[*symmetric*])
apply (*subst fmadd-restrict-right-dom*)
apply (*subst fmdom-map*[*symmetric*])
apply (*subst fmadd-restrict-right-dom*)
apply *simp*
done
next
have *closed* (*value-to-sterm* (*Vabs cs* Γ'))
using ‹*vclosed* (*Vabs cs* Γ')›
by (*rule vclosed.value-to-sterm*)
thus *closed-except* (*subst rhs* (*fmdrop-fset* (*frees pat*) (*fmmap value-to-sterm* Γ'))) (*frees pat*)

using ‹$(pat, rhs) \in set\ cs$›
by (*auto simp*: *Sterm.closed-except-simps list-all-iff*)
next
show *closed-env* (*fmmap value-to-sterm* Γ' $++_f$ *fmmap value-to-sterm venv*)
using ‹*closed-venv* Γ'› ‹*closed-venv venv*›
by (*auto intro*: *vclosed.value-to-sterm-env*)
next
show *frees pat* $|\subseteq|$ *fmdom* (*fmmap value-to-sterm* Γ' $++_f$ *fmmap value-to-sterm venv*)
using ‹*fmdom venv* = *frees pat*›
by *fastforce*
next
show *closed-srules rs'*
using *all-rules*
unfolding *rs'-def list-all-iff*
by (*fastforce intro*: *vclosed.value-to-sterm*)
qed
next
show *find-match* (*map* (λ(*pat*, *t*). (*pat*, *?subst pat t*)) *cs*) (*value-to-sterm u'*) = *Some* (*senv*, *pat*, *?subst pat rhs*)
using ‹*find-match* - - = -›
by (*auto simp*: *find-match-map*)
qed
next
— Basically a verbatim copy from the *comb* case

case (*rec-comb* Γ *t css name* Γ' *cs u u' venv pat rhs val*)

hence *vmatch* (*mk-pat pat*) *u'* = *Some venv*
by (*auto dest*: *vfind-match-elem*)

have *cs* = *the* (*fmlookup css name*)
using *rec-comb* **by** *simp*

have *wellformed t*
using *rec-comb* **by** *simp*
have *vwellformed* (*Vrecabs css name* Γ')
by (*rule veval-wellformed*) *fact*+
hence *vwellformed* (*Vabs cs* Γ') — convenient hack: *cs* is not really part of a *Vabs*
using *rec-comb* **by** *auto*

hence
list-all (*linear* $\circ$ *fst*) *cs*
wellformed-venv Γ'
by (*auto simp*: *list-all-iff split-beta*)

have *rel-option match-related* (*vfind-match cs u'*) (*find-match cs* (*value-to-sterm*

u'))
  apply (*rule find-match-eq*)
   apply *fact*
  apply (*rule veval-constructor-value*)
   apply *fact*+
  done

 then obtain *senv*
  where *find-match cs* (*value-to-sterm* u') = *Some* (*senv*, *pat*, *rhs*)
   and *env-eq venv senv*
  using ‹*vfind-match* - - = -›
  by *cases auto*
 hence (*pat*, *rhs*) ∈ *set cs match pat* (*value-to-sterm* u') = *Some senv*
  by (*auto dest*: *find-match-elem*)
 hence *fmdom senv* = *frees pat*
  by (*simp add*: *match-dom*)

 moreover have *senv* = *fmmap value-to-sterm venv*
  using ‹*env-eq venv senv*›
  by (*rule env-eq-eq*)

 ultimately have *fmdom venv* = *frees pat*
  by *simp*

 have *closed-except t* (*fmdom* Γ) *wellformed t*
  using *rec-comb* **by** (*simp add*: *closed-except-def*)+
 have *vclosed* (*Vrecabs css name* Γ′)
  by (*rule veval-closed*) *fact*+
 hence *vclosed* (*Vabs cs* Γ′)
  using *rec-comb* **by** *auto*

 have *vconstructor-value* u'
  by (*rule veval-constructor-value*) *fact*+
 have *vconstructor-value* (*Vrecabs css name* Γ′)
  by (*rule veval-constructor-value*) *fact*+
 hence *vconstructor-value-env* Γ′
  by *simp*
 have *vconstructor-value-env venv*
  by (*rule vconstructor-value.vmatch-env*) *fact*+

 have *wellformed u*
  using *rec-comb* **by** *simp*
 have *vwellformed* u'
  by (*rule veval-wellformed*) *fact*+
 have *wellformed-venv venv*
  by (*rule vwellformed.vmatch-env*) *fact*+

 have *closed-except u* (*fmdom* Γ)
  using *rec-comb* **by** (*simp add*: *closed-except-def*)

have *vclosed* u'
by (*rule veval-closed*) *fact+*
have *closed-venv venv*
by (*rule vclosed.vmatch-env*) *fact+*

have *closed-venv* Γ'
using ‹*vclosed* (*Vabs cs* Γ')› **by** *simp*

let *?subst* = λ*pat t. subst t* (*fmdrop-fset* (*frees pat*) (*fmmap value-to-sterm* Γ'))
have *rs'*, *fmmap value-to-sterm* (Γ' $++_f$ *venv*) $\vdash_s$ *?subst pat rhs* $\downarrow$ *value-to-sterm val*
proof (*rule ssubst-eval*)
show *rs'*, *fmmap value-to-sterm* (Γ' $++_f$ *venv*) $\vdash_s$ *rhs* $\downarrow$ *value-to-sterm val*
proof (*rule rec-comb*)
have *linear pat closed-except rhs* (*fmdom* Γ' |∪| *frees pat*)
using ‹(*pat*, *rhs*) ∈ *set cs*› ‹*vwellformed* (*Vabs cs* Γ')› ‹*vclosed* (*Vabs cs* Γ')›
by (*auto simp*: *list-all-iff*)
hence *closed-except rhs* (*fmdom* Γ' |∪| *fmdom venv*)
using ‹*vmatch* (*mk-pat pat*) u' = *Some venv*›
by (*auto simp*: *mk-pat-frees vmatch-dom*)
thus *closed-except rhs* (*fmdom* (Γ' $++_f$ *venv*))
by *simp*
next
show *wellformed-venv* (Γ' $++_f$ *venv*)
using ‹*wellformed-venv* Γ'› ‹*wellformed-venv venv*›
by *blast*
next
show *closed-venv* (Γ' $++_f$ *venv*)
using ‹*closed-venv* Γ'› ‹*closed-venv venv*›
by *blast*
next
show *vconstructor-value-env* (Γ' $++_f$ *venv*)
using ‹*vconstructor-value-env* Γ'› ‹*vconstructor-value-env venv*›
by *blast*
next
show *wellformed rhs*
using ‹(*pat*, *rhs*) ∈ *set cs*› ‹*vwellformed* (*Vabs cs* Γ')›
by (*fastforce simp*: *list-all-iff*)
qed
next
have *fmdrop-fset* (*fmdom venv*) Γ' $\subseteq_f$ Γ' $++_f$ *venv*
including *fmap.lifting* **and** *fset.lifting*
by *transfer'*
(*auto simp*: *map-drop-set-def map-filter-def map-le-def map-add-def split*: *if-splits*)
thus *fmdrop-fset* (*frees pat*) (*fmmap value-to-sterm* Γ') $\subseteq_f$ *fmmap value-to-sterm* (Γ' $++_f$ *venv*)
unfolding ‹*fmdom venv* = *frees pat*›

by (*metis fmdrop-fset-fmmap fmmap-subset*)
next
show *closed-env* (*fmmap value-to-sterm* (Γ' $++_f$ *venv*))
apply *auto*
apply *rule*
apply (*rule vclosed.value-to-sterm-env*, *fact*)+
done
next
show *value-env* (*fmmap value-to-sterm* (Γ' $++_f$ *venv*))
apply *auto*
apply *rule*
apply (*rule vconstructor-value.value-to-sterm-env*, *fact*)+
done
qed

show *?case*
proof (*rule seval.comb*)
have *rs'*, *fmmap value-to-sterm* $\Gamma \vdash_s t \downarrow$ *value-to-sterm* (*Vrecabs css name* Γ')
using *rec-comb* **by** (*auto simp*: *closed-except-def*)
thus *rs'*, *fmmap value-to-sterm* $\Gamma \vdash_s t \downarrow$ *Sabs* (*map* (λ(*pat*, *t*). (*pat*, *?subst pat t*)) *cs*)
unfolding ‹*cs* = -› **by** *simp*
next
show *rs'*, *fmmap value-to-sterm* $\Gamma \vdash_s u \downarrow$ *value-to-sterm* u'
using *rec-comb* **by** (*simp add*: *closed-except-def*)
next
show *rs'*, *fmmap value-to-sterm* Γ $++_f$ *senv* $\vdash_s$ *?subst pat rhs* $\downarrow$ *value-to-sterm val*
proof (*rule seval-agree-eq*)
show *rs'*, *fmmap value-to-sterm* Γ' $++_f$ *fmmap value-to-sterm venv* $\vdash_s$ *?subst pat rhs* $\downarrow$ *value-to-sterm val*
using ‹*rs'*, *fmmap value-to-sterm* (Γ' $++_f$ *venv*) $\vdash_s$ *?subst pat rhs* $\downarrow$ *value-to-sterm val*› **by** *simp*
next
show *fmrestrict-fset* (*frees pat*) (*fmmap value-to-sterm* Γ' $++_f$ *fmmap value-to-sterm venv*) =
fmrestrict-fset (*frees pat*) (*fmmap value-to-sterm* Γ $++_f$ *senv*)
unfolding ‹*senv* = -›
apply (*subst* ‹*fmdom venv* = *frees pat*›[*symmetric*])+
apply (*subst fmdom-map*[*symmetric*])
apply (*subst fmadd-restrict-right-dom*)
apply (*subst fmdom-map*[*symmetric*])
apply (*subst fmadd-restrict-right-dom*)
apply *simp*
done
next
have *closed* (*value-to-sterm* (*Vabs cs* Γ'))
using ‹*vclosed* (*Vabs cs* Γ')›

by (*rule vclosed.value-to-sterm*)
thus *closed-except* (*subst rhs* (*fmdrop-fset* (*frees pat*) (*fmmap value-to-sterm* Γ'))) (*frees pat*)
using ‹(*pat*, *rhs*) ∈ *set cs*›
by (*auto simp*: *Sterm.closed-except-simps list-all-iff*)
next
show *closed-env* (*fmmap value-to-sterm* Γ' $++_f$ *fmmap value-to-sterm venv*)
using ‹*closed-venv* Γ'› ‹*closed-venv venv*›
by (*auto intro*: *vclosed.value-to-sterm-env*)
next
show *frees pat* $|\subseteq|$ *fmdom* (*fmmap value-to-sterm* Γ' $++_f$ *fmmap value-to-sterm venv*)
using ‹*fmdom venv* = *frees pat*›
by *fastforce*
next
show *closed-srules rs'*
using *all-rules*
unfolding *rs'-def list-all-iff*
by (*fastforce intro*: *vclosed.value-to-sterm*)
qed
next
show *find-match* (*map* (λ(*pat*, *t*). (*pat*, *?subst pat t*)) *cs*) (*value-to-sterm u'*) = *Some* (*senv*, *pat*, *?subst pat rhs*)
using ‹*find-match* - - = -›
by (*auto simp*: *find-match-map*)
qed
next
case (*const name rhs* Γ)
show *?case*
apply (*rule seval.const*)
unfolding *rs'-def*
using ‹(*name*, *rhs*) ∈ -› **by** *force*
next
case *abs*
show *?case*
by (*auto simp del*: *fmdrop-fset-fmmap intro*: *seval.abs*)
qed (*auto intro*: *seval.var seval.abs*)

lemma *veval-correct*:
assumes *rs*, *fmempty* $\vdash_v$ *t* $\downarrow$ *v wellformed t closed t*
shows *rs'*, *fmempty* $\vdash_s$ *t* $\downarrow$ *value-to-sterm v*
proof −
have *rs'*, *fmmap value-to-sterm fmempty* $\vdash_s$ *t* $\downarrow$ *value-to-sterm v*
using *assms*
by (*auto intro*: *veval-correct0 simp del*: *fmmap-empty*)
thus *?thesis*
by *simp*
qed

end end

end

3.5.3 Big-step semantics with conflation of constants and variables

theory *Big-Step-Value-ML*
imports *Big-Step-Value*
begin

definition *mk-rec-env* :: (*name*, *sclauses*) *fmap* ⇒ (*name*, *value*) *fmap* ⇒ (*name*, *value*) *fmap* **where**
mk-rec-env css Γ′ = *fmmap-keys* (λ*name cs. Vrecabs css name* Γ′) *css*

context *special-constants* **begin**

inductive *veval′* :: (*name*, *value*) *fmap* ⇒ *sterm* ⇒ *value* ⇒ *bool* (‹-/ $\vdash_v$/ - ↓/ -› [*50*,*0*,*50*] *50*) **where**
const: *name* |∉| *C* ⟹ *fmlookup* Γ *name* = *Some val* ⟹ Γ $\vdash_v$ *Sconst name* ↓ *val* |
var: *fmlookup* Γ *name* = *Some val* ⟹ Γ $\vdash_v$ *Svar name* ↓ *val* |
abs: Γ $\vdash_v$ *Sabs cs* ↓ *Vabs cs* Γ |
comb:
  Γ $\vdash_v$ *t* ↓ *Vabs cs* Γ′ ⟹ Γ $\vdash_v$ *u* ↓ *u′* ⟹
  vfind-match cs u′ = *Some* (*env*, -, *rhs*) ⟹
  Γ′ $++_f$ *env* $\vdash_v$ *rhs* ↓ *val* ⟹
  Γ $\vdash_v$ *t* $\$_s$ *u* ↓ *val* |
rec-comb:
  Γ $\vdash_v$ *t* ↓ *Vrecabs css name* Γ′ ⟹
  fmlookup css name = *Some cs* ⟹
  Γ $\vdash_v$ *u* ↓ *u′* ⟹
  vfind-match cs u′ = *Some* (*env*, -, *rhs*) ⟹
  Γ′ $++_f$ *mk-rec-env css* Γ′ $++_f$ *env* $\vdash_v$ *rhs* ↓ *val* ⟹
  Γ $\vdash_v$ *t* $\$_s$ *u* ↓ *val* |
constr: *name* |∈| *C* ⟹ *list-all2* (*veval′* Γ) *ts us* ⟹ Γ $\vdash_v$ *name* $$ *ts* ↓ *Vconstr name us*

lemma *veval′-sabs-svarE*:
  assumes Γ $\vdash_v$ *Sabs cs* $\$_s$ *Svar n* ↓ *v*
  obtains *u′ env pat rhs*
    where *fmlookup* Γ *n* = *Some u′*
      vfind-match cs u′ = *Some* (*env*, *pat*, *rhs*)
      Γ $++_f$ *env* $\vdash_v$ *rhs* ↓ *v*
using *assms* **proof** *cases*
  case (*constr name ts*)
  hence *strip-comb* (*Sabs cs* $\$_s$ *Svar n*) = *strip-comb* (*name* $$ *ts*)
    by *simp*

```
  hence False
    apply (fold app-sterm-def)
    apply (simp add: strip-list-comb-const)
    apply (simp add: const-sterm-def)
    done
  thus ?thesis by simp
next
  case rec-comb
  hence False by cases
  thus ?thesis by simp
next
  case (comb cs' Γ' u' env pat rhs)
  moreover have fmlookup Γ n = Some u'
    using ‹Γ ⊢_v Svar n ↓ u'›
    proof cases
      case (constr name ts)
      hence False
        by (fold free-sterm-def) simp
      thus ?thesis by simp
    qed auto
  moreover have cs = cs' Γ = Γ'
    using ‹Γ ⊢_v Sabs cs ↓ Vabs cs' Γ'›
    by (cases; auto)+

  ultimately show ?thesis
    using that by auto
qed

lemma veval'-wellformed:
  assumes Γ ⊢_v t ↓ v wellformed t wellformed-venv Γ
  shows vwellformed v
using assms proof induction
  case comb
  show ?case
    apply (rule comb)
      using comb by (auto simp: list-all-iff dest: vfind-match-elem intro: vwell-
formed.vmatch-env)
next
  case (rec-comb Γ t css name Γ' cs u u' env pat rhs val)
  have (pat, rhs) ∈ set cs
    by (rule vfind-match-elem) fact
  show ?case
    proof (rule rec-comb)
      show wellformed-venv (Γ' ++_f mk-rec-env css Γ' ++_f env)
        proof (intro fmpred-add)
          show wellformed-venv Γ'
            using rec-comb by auto
        next
          show wellformed-venv env
```

```
          using rec-comb by (auto dest: vfind-match-elem intro: vwellformed.vmatch-env)
        next
          show wellformed-venv (mk-rec-env css Γ′)
            unfolding mk-rec-env-def
            using rec-comb by (auto intro: fmdomI)
        qed
    next
      have vwellformed (Vrecabs css name Γ′)
        unfolding mk-rec-env-def
        using rec-comb by (auto intro: fmdom′I)
      thus wellformed rhs
        using ‹(pat, rhs) ∈ set cs› rec-comb by (auto simp: list-all-iff)
    qed
next
  case (constr name Γ ts us)
  have list-all vwellformed us
    using ‹list-all2 - - -› ‹wellformed (- $$ -)›
    proof (induction ts us rule: list.rel-induct)
      case (Cons v vs u us)
      thus ?case
        using constr by (auto simp: app-sterm-def wellformed.list-comb)
    qed simp
  thus ?case
    by (simp add: list-all-iff)
qed auto

lemma (in constants) veval′-shadows:
  assumes Γ ⊢_v t ↓ v not-shadows-vconsts-env Γ ¬ shadows-consts t
  shows not-shadows-vconsts v
using assms proof induction
  case comb
  show ?case
    apply (rule comb)
   using comb by (auto simp: list-all-iff dest: vfind-match-elem intro: not-shadows-vconsts.vmatch-env)
next
  case (rec-comb Γ t css name Γ′ cs u u′ env pat rhs val)
  have (pat, rhs) ∈ set cs
    by (rule vfind-match-elem) fact
  show ?case
    proof (rule rec-comb)
      show not-shadows-vconsts-env (Γ′ ++_f mk-rec-env css Γ′ ++_f env)
        proof (intro fmpred-add)
          show not-shadows-vconsts-env env
          using rec-comb by (auto dest: vfind-match-elem intro: not-shadows-vconsts.vmatch-env)
        next
          show not-shadows-vconsts-env (mk-rec-env css Γ′)
            unfolding mk-rec-env-def
            using rec-comb by (auto intro: fmdomI)
        next
```

```
        show not-shadows-vconsts-env Γ'
          using rec-comb by auto
      qed
  next
    have not-shadows-vconsts (Vrecabs css name Γ')
      using rec-comb by auto
    thus ¬ shadows-consts rhs
      using ‹(pat, rhs) ∈ set cs› rec-comb by (auto simp: list-all-iff)
  qed
next
  case (constr name Γ ts us)
  have list-all (not-shadows-vconsts) us
    using ‹list-all2 - - -› ‹¬ shadows-consts (name $$ ts)›
    proof (induction ts us rule: list.rel-induct)
      case (Cons v vs u us)
      thus ?case
        using constr by (auto simp: shadows.list-comb app-sterm-def)
    qed simp
  thus ?case
    by (simp add: list-all-iff)
qed (auto simp: list-all-iff list-ex-iff)

lemma veval'-closed:
  assumes Γ ⊢_v t ↓ v closed-except t (fmdom Γ) closed-venv Γ
  assumes wellformed t wellformed-venv Γ
  shows vclosed v
using assms proof induction
  case (comb Γ t cs Γ' u u' env pat rhs val)
  hence vclosed (Vabs cs Γ')
    by (auto simp: closed-except-def)

  have (pat, rhs) ∈ set cs vmatch (mk-pat pat) u' = Some env
    by (rule vfind-match-elem; fact)+
  hence fmdom env = patvars (mk-pat pat)
    by (simp add: vmatch-dom)

  have vwellformed (Vabs cs Γ')
    apply (rule veval'-wellformed)
    using comb by auto
  hence linear pat
    using ‹(pat, rhs) ∈ set cs›
    by (auto simp: list-all-iff)
  hence fmdom env = frees pat
    unfolding ‹fmdom env = -›
    by (simp add: mk-pat-frees)

  show ?case
    proof (rule comb)
      show wellformed rhs
```

```
        using ‹(pat, rhs) ∈ set cs› ‹vwellformed (Vabs cs Γ')›
        by (auto simp: list-all-iff)
    next
      show closed-venv (Γ' ++f env)
        apply rule
        using ‹vclosed (Vabs cs Γ')› apply auto[]
        apply (rule vclosed.vmatch-env)
         apply (rule vfind-match-elem)
        using comb by (auto simp: closed-except-def)
    next
      show closed-except rhs (fmdom (Γ' ++f env))
        using ‹vclosed (Vabs cs Γ')› ‹fmdom env = frees pat› ‹(pat, rhs) ∈ set cs›
        by (auto simp: list-all-iff)
    next
      show wellformed-venv (Γ' ++f env)
        apply rule
        using ‹vwellformed (Vabs cs Γ')› apply auto[]
        apply (rule vwellformed.vmatch-env)
         apply (rule vfind-match-elem)
         apply fact
        apply (rule veval'-wellformed)
        using comb by auto
    qed
next
  case (rec-comb Γ t css name Γ' cs u u' env pat rhs val)
  have (pat, rhs) ∈ set cs vmatch (mk-pat pat) u' = Some env
    by (rule vfind-match-elem; fact)+
  hence fmdom env = patvars (mk-pat pat)
    by (simp add: vmatch-dom)

  have vwellformed (Vrecabs css name Γ')
    apply (rule veval'-wellformed)
    using rec-comb by auto
  hence wellformed-clauses cs
    using rec-comb by auto
  hence linear pat
    using ‹(pat, rhs) ∈ set cs›
    by (auto simp: list-all-iff)
  hence fmdom env = frees pat
    unfolding ‹fmdom env = -›
    by (simp add: mk-pat-frees)
  show ?case
    proof (rule rec-comb)
      show closed-venv (Γ' ++f mk-rec-env css Γ' ++f env)
        proof (intro fmpred-add)
          show closed-venv Γ'
            using rec-comb by (auto simp: closed-except-def)
        next
          show closed-venv env
```

```
          using rec-comb by (auto simp: closed-except-def dest: vfind-match-elem
intro: vclosed.vmatch-env)
        next
          show closed-venv (mk-rec-env css Γ')
            unfolding mk-rec-env-def
            using rec-comb by (auto simp: closed-except-def intro: fmdomI)
        qed
    next
      have vclosed (Vrecabs css name Γ')
        using mk-rec-env-def
        using rec-comb by (auto simp: closed-except-def intro: fmdom'I)
      hence closed-except rhs (fmdom Γ' |∪| frees pat)
        apply simp
        apply (elim conjE)
        apply (drule fmpredD[where m = css])
         apply (rule rec-comb)
        using ‹(pat, rhs) ∈ set cs›
        unfolding list-all-iff by auto

      thus closed-except rhs (fmdom (Γ' ++_f mk-rec-env css Γ' ++_f env))
        unfolding closed-except-def
        using ‹fmdom env = frees pat›
        by auto
    next
      show wellformed rhs
        using ‹wellformed-clauses cs› ‹(pat, rhs) ∈ set cs›
        by (auto simp: list-all-iff)
    next
      show wellformed-venv (Γ' ++_f mk-rec-env css Γ' ++_f env)
        proof (intro fmpred-add)
          show wellformed-venv Γ'
            using ‹vwellformed (Vrecabs css name Γ')› by auto
        next
          show wellformed-venv env
            using rec-comb by (auto dest: vfind-match-elem intro: veval'-wellformed
vwellformed.vmatch-env)
        next
          show wellformed-venv (mk-rec-env css Γ')
            unfolding mk-rec-env-def
            using ‹vwellformed (Vrecabs css name Γ')› by (auto intro: fmdomI)
        qed
    qed
next
  case (constr name Γ ts us)
  have list-all vclosed us
    using ‹list-all2 - - -› ‹closed-except (- $$ -) -› ‹wellformed (- $$ -)›
    proof (induction ts us rule: list.rel-induct)
      case (Cons v vs u us)
      with constr show ?case
```

```
      unfolding closed.list-comb wellformed.list-comb
      by (auto simp: Sterm.closed-except-simps)
    qed simp
  thus ?case
    by (simp add: list-all-iff)
qed (auto simp: Sterm.closed-except-simps)

primrec vwelldefined' :: value ⇒ bool where
vwelldefined' (Vconstr name vs) ⟷ list-all vwelldefined' vs |
vwelldefined' (Vabs cs Γ) ⟷
  pred-fmap id (fmmap vwelldefined' Γ) ∧
  list-all (λ(pat, t). consts t |⊆| (fmdom Γ |∪| C)) cs ∧
  fdisjnt C (fmdom Γ) |
vwelldefined' (Vrecabs css name Γ) ⟷
  pred-fmap id (fmmap vwelldefined' Γ) ∧
  pred-fmap (λcs.
    list-all (λ(pat, t). consts t |⊆| fmdom Γ |∪| (C |∪| fmdom css)) cs ∧
    fdisjnt C (fmdom Γ)) css ∧
  name |∈| fmdom css ∧
  fdisjnt C (fmdom css)

lemma vmatch-welldefined':
  assumes vmatch pat v = Some env vwelldefined' v
  shows fmpred (λ-. vwelldefined') env
using assms proof (induction pat v arbitrary: env rule: vmatch-induct)
  case (constr name ps name' vs)
  hence
    map-option (foldl (++f) fmempty) (those (map2 vmatch ps vs)) = Some env
    name = name' length ps = length vs
    by (auto split: if-splits)
  then obtain envs where env = foldl (++f) fmempty envs map2 vmatch ps vs
= map Some envs
    by (blast dest: those-someD)

  moreover have fmpred (λ-. vwelldefined') env if env ∈ set envs for env
    proof −
      from that have Some env ∈ set (map2 vmatch ps vs)
        unfolding ‹map2 - - - = -› by simp
      then obtain p v where p ∈ set ps v ∈ set vs vmatch p v = Some env
        by (auto elim: map2-elemE)
      hence vwelldefined' v
        using constr by (simp add: list-all-iff)
      show ?thesis
        by (rule constr; safe?) fact+
    qed

    ultimately show ?case
      by auto
qed auto
```

lemma *sconsts-list-comb*:
  $consts\ (list\text{-}comb\ f\ xs)\ |\subseteq|\ S \longleftrightarrow consts\ f\ |\subseteq|\ S \wedge list\text{-}all\ (\lambda x.\ consts\ x\ |\subseteq|\ S)\ xs$
by (*induction xs arbitrary*: *f*) *auto*

lemma *sconsts-sabs*:
  $consts\ (Sabs\ cs)\ |\subseteq|\ S \longleftrightarrow list\text{-}all\ (\lambda(\text{-},\ t).\ consts\ t\ |\subseteq|\ S)\ cs$
  apply (*auto simp*: *list-all-iff ffUnion-alt-def dest*!: *ffUnion-least-rev*)
   apply (*subst* (*asm*) *list-all-iff-fset*[*symmetric*])
   apply (*auto simp*: *list-all-iff fset-of-list-elem*)
  done

lemma (**in** *constants*) *veval′-welldefined′*:
  assumes $\Gamma \vdash_v t \downarrow v$ *fdisjnt C* (*fmdom* Γ)
  assumes *consts t* $|\subseteq|$ *fmdom* Γ $|\cup|$ *C fmpred* (λ-. *vwelldefined′*) Γ
  assumes *wellformed t wellformed-venv* Γ
  assumes $\neg$ *shadows-consts t not-shadows-vconsts-env* Γ
  shows *vwelldefined′ v*
using *assms* **proof** *induction*
  case (*abs* Γ *cs*)
  thus *?case*
    unfolding *sconsts-sabs*
    by (*auto simp*: *list-all-iff list-ex-iff*)
next
  case (*comb* Γ *t cs* Γ' *u u′ env pat rhs val*)
  hence (*pat*, *rhs*) $\in$ *set cs*
    by (*auto dest*: *vfind-match-elem*)
  moreover have *vwelldefined′* (*Vabs cs* Γ')
    using *comb* **by** *auto*
  ultimately have *consts rhs* $|\subseteq|$ *fmdom* Γ' $|\cup|$ *C*
    by (*auto simp*: *list-all-iff*)

  have *vwellformed* (*Vabs cs* Γ')
    apply (*rule veval′-wellformed*)
    using *comb* **by** *auto*
  hence *linear pat*
    using ‹(*pat*, *rhs*) $\in$ *set cs*›
    by (*auto simp*: *list-all-iff*)
  hence *frees pat* = *patvars* (*mk-pat pat*)
    by (*simp add*: *mk-pat-frees*)
  hence *fmdom env* = *frees pat*
    apply *simp*
    apply (*rule vmatch-dom*)
    apply (*rule vfind-match-elem*)
    apply (*rule comb*)
    done

  have *not-shadows-vconsts* (*Vabs cs* Γ')

```
  apply (rule veval'-shadows)
  using comb by auto

have vwelldefined' (Vabs cs Γ')
  using comb by auto

show ?case
  proof (rule comb)
    show consts rhs |⊆| fmdom (Γ' ++_f env) |∪| C
      using ‹consts rhs |⊆| fmdom Γ' |∪| C›
      by auto
  next
    have vwelldefined' u'
      using comb by auto
    hence fmpred (λ-. vwelldefined') env
      using comb
      by (auto intro: vmatch-welldefined' dest: vfind-match-elem)
    thus fmpred (λ-. vwelldefined') (Γ' ++_f env)
      using ‹vwelldefined' (Vabs cs Γ')› by auto
  next
    have fdisjnt C (fmdom Γ')
      using ‹vwelldefined' (Vabs cs Γ')›
      by simp
    moreover have fdisjnt C (fmdom env)
      unfolding ‹fmdom env = frees pat›
      using ‹(pat, rhs) ∈ set cs› ‹not-shadows-vconsts (Vabs cs Γ')›
      by (auto simp: list-all-iff all-consts-def fdisjnt-alt-def)
    ultimately show fdisjnt C (fmdom (Γ' ++_f env))
      unfolding fdisjnt-alt-def by auto
  next
    show wellformed rhs
      using ‹(pat, rhs) ∈ set cs› ‹vwellformed (Vabs cs Γ')›
      by (auto simp: list-all-iff)
  next
    have wellformed-venv Γ'
      using ‹vwellformed (Vabs cs Γ')› by simp
    moreover have wellformed-venv env
      apply (rule vwellformed.vmatch-env)
       apply (rule vfind-match-elem)
       apply fact
      apply (rule veval'-wellformed)
      using comb by auto
    ultimately show wellformed-venv (Γ' ++_f env)
      by blast
  next
    have not-shadows-vconsts-env Γ'
      using ‹not-shadows-vconsts (Vabs cs Γ')› by simp
    moreover have not-shadows-vconsts-env env
      apply (rule not-shadows-vconsts.vmatch-env)
```

apply (*rule vfind-match-elem*)
apply *fact*
apply (*rule veval′-shadows*)
using *comb* **by** *auto*
ultimately show *not-shadows-vconsts-env* (Γ' $++_f$ *env*)
by *blast*
next
show ¬ *shadows-consts rhs*
using ‹*not-shadows-vconsts* (*Vabs cs* Γ')› ‹(*pat*, *rhs*) ∈ *set cs*›
by (*auto simp*: *list-all-iff*)
qed
next
case (*rec-comb* Γ *t css name* Γ' *cs u u′ env pat rhs val*)
hence (*pat*, *rhs*) ∈ *set cs*
by (*auto dest*: *vfind-match-elem*)
moreover have *vwelldefined′* (*Vrecabs css name* Γ')
using *rec-comb* **by** *auto*
ultimately have *consts rhs* |⊆| *fmdom* Γ' |∪| (*C* |∪| *fmdom css*)
using ‹*fmlookup css name* = *Some cs*›
by (*auto simp*: *list-all-iff dest*!: *fmpredD*[**where** *m* = *css*])

have *vwellformed* (*Vrecabs css name* Γ')
apply (*rule veval′-wellformed*)
using *rec-comb* **by** *auto*
hence *wellformed-clauses cs*
using *rec-comb* **by** *auto*
hence *linear pat*
using ‹(*pat*, *rhs*) ∈ *set cs*›
by (*auto simp*: *list-all-iff*)
hence *frees pat* = *patvars* (*mk-pat pat*)
by (*simp add*: *mk-pat-frees*)
hence *fmdom env* = *frees pat*
apply *simp*
apply (*rule vmatch-dom*)
apply (*rule vfind-match-elem*)
apply (*rule rec-comb*)
done

have *not-shadows-vconsts* (*Vrecabs css name* Γ')
apply (*rule veval′-shadows*)
using *rec-comb* **by** *auto*

have *vwelldefined′* (*Vrecabs css name* Γ')
using *rec-comb* **by** *auto*

show *?case*
proof (*rule rec-comb*)
show *consts rhs* |⊆| *fmdom* (Γ' $++_f$ *mk-rec-env css* Γ' $++_f$ *env*) |∪| *C*
using ‹*consts rhs* |⊆| -› **unfolding** *mk-rec-env-def*

by *auto*
next
have *fmpred* (λ-. *vwelldefined'*) Γ'
using ‹*vwelldefined'* (*Vrecabs css name* Γ')› **by** *auto*
moreover have *fmpred* (λ-. *vwelldefined'*) (*mk-rec-env css* Γ')
unfolding *mk-rec-env-def*
using *rec-comb* **by** (*auto intro*: *fmdomI*)
moreover have *fmpred* (λ-. *vwelldefined'*) *env*
using *rec-comb* **by** (*auto dest*: *vfind-match-elem intro*: *vmatch-welldefined'*)
ultimately show *fmpred* (λ-. *vwelldefined'*) (Γ' $++_f$ *mk-rec-env css* Γ' $++_f$ *env*)
by *blast*
next
have *fdisjnt C* (*fmdom* Γ')
using *rec-comb* **by** *auto*
moreover have *fdisjnt C* (*fmdom env*)
unfolding ‹*fmdom env* = *frees pat*›
using ‹*fmlookup css name* = *Some cs*› ‹(*pat*, *rhs*) $\in$ *set cs*› ‹*not-shadows-vconsts* (*Vrecabs css name* Γ')›
apply *auto*
apply (*drule fmpredD*[**where** *m* = *css*])
by (*auto simp*: *list-all-iff all-consts-def fdisjnt-alt-def*)
moreover have *fdisjnt C* (*fmdom* (*mk-rec-env css* Γ'))
unfolding *mk-rec-env-def*
using ‹*vwelldefined'* (*Vrecabs css name* Γ')›
by *simp*
ultimately show *fdisjnt C* (*fmdom* (Γ' $++_f$ *mk-rec-env css* Γ' $++_f$ *env*))
unfolding *fdisjnt-alt-def* **by** *auto*
next
show *wellformed rhs*
using ‹(*pat*, *rhs*) $\in$ *set cs*› ‹*wellformed-clauses cs*›
by (*auto simp*: *list-all-iff*)
next
have *wellformed-venv* Γ'
using ‹*vwellformed* (*Vrecabs css name* Γ')› **by** *simp*
moreover have *wellformed-venv* (*mk-rec-env css* Γ')
unfolding *mk-rec-env-def*
using ‹*vwellformed* (*Vrecabs css name* Γ')›
by (*auto intro*: *fmdomI*)
moreover have *wellformed-venv env*
apply (*rule vwellformed.vmatch-env*)
apply (*rule vfind-match-elem*)
apply *fact*
apply (*rule veval'-wellformed*)
using *rec-comb* **by** *auto*
ultimately show *wellformed-venv* (Γ' $++_f$ *mk-rec-env css* Γ' $++_f$ *env*)
by *blast*
next
have *not-shadows-vconsts-env* Γ'

using ‹*not-shadows-vconsts* (*Vrecabs css name* Γ′)› **by** *simp*
moreover have *not-shadows-vconsts-env* (*mk-rec-env css* Γ′)
unfolding *mk-rec-env-def*
using ‹*not-shadows-vconsts* (*Vrecabs css name* Γ′)›
by (*auto intro*: *fmdomI*)
moreover have *not-shadows-vconsts-env env*
apply (*rule not-shadows-vconsts.vmatch-env*)
apply (*rule vfind-match-elem*)
apply *fact*
apply (*rule veval′-shadows*)
using *rec-comb* **by** *auto*
ultimately show *not-shadows-vconsts-env* (Γ′ $++_f$ *mk-rec-env css* Γ′ $++_f$ *env*)
by *blast*
next
show ¬ *shadows-consts rhs*
using *rec-comb* ‹*not-shadows-vconsts* (*Vrecabs css name* Γ′)› ‹(*pat*, *rhs*) ∈ *set cs*›
by (*auto simp*: *list-all-iff*)
qed
next
case (*constr name* Γ *ts us*)
have *list-all vwelldefined′ us*
using ‹*list-all2* - - -› ‹*consts* (*name* $$ *ts*) |⊆| -›
using ‹*wellformed* (*name* $$ *ts*)› ‹¬ *shadows-consts* (*name* $$ *ts*)›
proof (*induction ts us rule*: *list.rel-induct*)
case (*Cons v vs u us*)
with *constr* **show** *?case*
unfolding *wellformed.list-comb shadows.list-comb*
by (*auto simp*: *consts-list-comb*)
qed *simp*
thus *?case*
by (*simp add*: *list-all-iff*)
qed *auto*

end

Correctness wrt *veval*

context *vrules* **begin**

The following relation can be characterized as follows:

- Values have to have the same structure. (We prove an interpretation of *value-struct-rel*.)
- For closures, the captured environments must agree on constants and variables occurring in the body. The first *value* argument is from *veval* (i.e. from *CakeML-Codegen.Big-Step-Value*), the second from *veval′*.

coinductive *vrelated* :: *value* ⇒ *value* ⇒ *bool* (‹$\vdash_v$/ - ≈ -› [*0*, *50*] *50*) **where**
constr: *list-all2 vrelated ts us* ⟹ $\vdash_v$ *Vconstr name ts* ≈ *Vconstr name us* |
abs:
  fmrel-on-fset (*frees* (*Sabs cs*)) *vrelated* Γ_1 Γ_2 ⟹
   fmrel-on-fset (*consts* (*Sabs cs*)) *vrelated* (*fmap-of-list rs*) Γ_2 ⟹
    $\vdash_v$ *Vabs cs* Γ_1 ≈ *Vabs cs* Γ_2 |
rec-abs:
  pred-fmap (λ*cs*.
    fmrel-on-fset (*frees* (*Sabs cs*)) *vrelated* Γ_1 Γ_2 ∧
    fmrel-on-fset (*consts* (*Sabs cs*)) *vrelated* (*fmap-of-list rs*) (Γ_2 $++_f$ *mk-rec-env css* Γ_2)) *css* ⟹
     name |∈| *fmdom css* ⟹
     $\vdash_v$ *Vrecabs css name* Γ_1 ≈ *Vrecabs css name* Γ_2

Perhaps unexpectedly, *vrelated* is not reflexive. The reason is that it does not just check syntactic equality including captured environments, but also adherence to the external rules.

sublocale *vrelated*: *value-struct-rel vrelated*
proof
  fix t_1 t_2
  assume $\vdash_v$ t_1 ≈ t_2
  thus *veq-structure* t_1 t_2
    apply (*induction* t_1 *arbitrary*: t_2)
      apply (*erule vrelated.cases*; *auto*)
      apply (*erule list.rel-mono-strong*)
      apply *simp*
     apply (*erule vrelated.cases*; *auto*)
    apply (*erule vrelated.cases*; *auto*)
    done
next
  fix *name name'* **and** *ts us* :: *value list*
  show $\vdash_v$ *Vconstr name ts* ≈ *Vconstr name' us* ⟷ (*name* = *name'* ∧ *list-all2 vrelated ts us*)
    proof *safe*
      assume $\vdash_v$ *Vconstr name ts* ≈ *Vconstr name' us*
      thus *name* = *name' list-all2 vrelated ts us*
        by (*cases*; *auto*)+
    qed (*auto intro*: *vrelated.intros*)
qed

The technically involved relation *vrelated* implies a weaker, but more intuitive property: If $\vdash_v$ $t \approx u$ then t and u are equal after termification (i.e. conversion with *value-to-sterm*). In fact, if both terms are ground terms, it collapses to equality. This follows directly from the interpretation of *value-struct-rel*.

lemma *veval'-correct*:
  assumes $\Gamma_2 \vdash_v t \downarrow v_2$ *wellformed t wellformed-venv* Γ_2
  assumes ¬ *shadows-consts t not-shadows-vconsts-env* Γ_2

assumes *welldefined t*
assumes *fmpred* (λ-. *vwelldefined*) Γ_1
assumes *fmrel-on-fset* (*frees t*) *vrelated* Γ_1 Γ_2
assumes *fmrel-on-fset* (*consts t*) *vrelated* (*fmap-of-list rs*) Γ_2
obtains v_1 **where** $rs, \Gamma_1 \vdash_v t \downarrow v_1 \vdash_v v_1 \approx v_2$
apply *atomize-elim*
using *assms* **proof** (*induction arbitrary*: Γ_1)
case (*const name* Γ_2 val_2)
hence *fmrel-on-fset* {|*name*|} *vrelated* (*fmap-of-list rs*) Γ_2
by *simp*
have *rel-option vrelated* (*fmlookup* (*fmap-of-list rs*) *name*) (*fmlookup* Γ_2 *name*)
apply (*rule fmrel-on-fsetD*[**where** S = {|*name*|}])
apply *simp*
apply *fact*
done
then obtain val_1 **where** *fmlookup* (*fmap-of-list rs*) *name* = *Some* $val_1 \vdash_v val_1$
$\approx val_2$
using *const* **by** *cases auto*
hence (*name*, val_1) $\in$ *set rs*
by (*auto dest*: *fmap-of-list-SomeD*)

show *?case*
apply (*intro conjI exI*)
apply (*rule veval.const*)
apply *fact+*
done
next
case (*var* Γ_2 *name* val_2)
hence *fmrel-on-fset* {|*name*|} *vrelated* Γ_1 Γ_2
by *simp*
have *rel-option vrelated* (*fmlookup* Γ_1 *name*) (*fmlookup* Γ_2 *name*)
apply (*rule fmrel-on-fsetD*[**where** S = {|*name*|}])
apply *simp*
apply *fact*
done
then obtain val_1 **where** *fmlookup* Γ_1 *name* = *Some* $val_1 \vdash_v val_1 \approx val_2$
using *var* **by** *cases auto*
show *?case*
apply (*intro conjI exI*)
apply (*rule veval.var*)
apply *fact+*
done
next
case *abs*
thus *?case*
by (*auto intro*!: *veval.abs vrelated.abs*)
next
case (*comb* Γ_2 *t cs* Γ'_2 *u* u'_2 env_2 *pat rhs* val_2)
hence $\exists v.\ rs, \Gamma_1 \vdash_v t \downarrow v \wedge \vdash_v v \approx Vabs\ cs\ \Gamma'_2$

by (*auto intro*: *fmrel-on-fsubset*)
then obtain v **where** $\vdash_v\ v \approx$ *Vabs cs* Γ'_2 *rs*, $\Gamma_1 \vdash_v\ t \downarrow v$
by *blast*
moreover then obtain Γ'_1
where $v =$ *Vabs cs* Γ'_1
and *fmrel-on-fset* (*frees* (*Sabs cs*)) *vrelated* Γ'_1 Γ'_2
and *fmrel-on-fset* (*consts* (*Sabs cs*)) *vrelated* (*fmap-of-list rs*) Γ'_2
by *cases auto*
ultimately have *rs*, $\Gamma_1 \vdash_v\ t \downarrow$ *Vabs cs* Γ'_1
by *simp*

have $\exists\, u_1'.$ *rs*, $\Gamma_1 \vdash_v\ u \downarrow u_1' \wedge \vdash_v\ u_1' \approx u'_2$
using *comb* **by** (*auto intro*: *fmrel-on-fsubset*)
then obtain u'_1 **where** $\vdash_v\ u'_1 \approx u'_2$ *rs*, $\Gamma_1 \vdash_v\ u \downarrow u'_1$
by *blast*

have *rel-option* (*rel-prod* (*fmrel vrelated*) (=)) (*vfind-match cs* u'_1) (*vfind-match cs* u'_2)
by (*rule vrelated.vfind-match-rel'*) *fact*
then obtain env_1 **where** *vfind-match cs* u'_1 = *Some* (env_1, *pat*, *rhs*) *fmrel vrelated* env_1 env_2
using ‹*vfind-match cs* u'_2 = -›
by *cases auto*

have (*pat*, *rhs*) $\in$ *set cs*
by (*rule vfind-match-elem*) *fact*

have *vwellformed* (*Vabs cs* Γ'_2)
apply (*rule veval'-wellformed*)
apply *fact*
using ‹*wellformed* (t $\$_s$ u)› **apply** *simp*
apply *fact+*
done
hence *wellformed-venv* Γ'_2
by *simp*

have *vwelldefined v*
apply (*rule veval-welldefined*)
apply *fact+*
using *comb* **by** *simp*
hence *vwelldefined* (*Vabs cs* Γ'_1)
unfolding ‹v = -› **.**

have *linear pat*
using ‹(*pat*, *rhs*) $\in$ *set cs*› ‹*vwellformed* (*Vabs cs* Γ'_2)›
by (*auto simp*: *list-all-iff*)

have *fmdom* env_1 = *patvars* (*mk-pat pat*)
apply (*rule vmatch-dom*)

```
    apply (rule vfind-match-elem)
    apply fact
    done
  with ‹linear pat› have fmdom env_1 = frees pat
    by (simp add: mk-pat-frees)

  have fmdom env_2 = patvars (mk-pat pat)
    apply (rule vmatch-dom)
    apply (rule vfind-match-elem)
    apply fact
    done
  with ‹linear pat› have fmdom env_2 = frees pat
    by (simp add: mk-pat-frees)

  note fset-of-list-map[simp del]
  have ∃ val_1. rs, Γ'_1 ++_f env_1 ⊢_v rhs ↓ val_1 ∧ ⊢_v val_1 ≈ val_2
    proof (rule comb)
      show fmrel-on-fset (frees rhs) vrelated (Γ'_1 ++_f env_1) (Γ'_2 ++_f env_2)
        proof
          fix name
          assume name |∈| frees rhs
          show rel-option vrelated (fmlookup (Γ'_1 ++_f env_1) name) (fmlookup (Γ'_2
++_f env_2) name)
            proof (cases name |∈| frees pat)
              case True
              hence name |∈| fmdom env_1 name |∈| fmdom env_2
                using ‹fmdom env_1 = frees pat› ‹fmdom env_2 = frees pat›
                by simp+
             hence fmlookup (Γ'_1 ++_f env_1) name = fmlookup env_1 name fmlookup
(Γ'_2 ++_f env_2) name = fmlookup env_2 name
                by auto
              thus ?thesis
                using ‹fmrel vrelated env_1 env_2›
                by auto
            next
              case False
              hence name |∉| fmdom env_1 name |∉| fmdom env_2
                using ‹fmdom env_1 = frees pat› ‹fmdom env_2 = frees pat›
                by simp+
              hence fmlookup (Γ'_1 ++_f env_1) name = fmlookup Γ'_1 name fmlookup
(Γ'_2 ++_f env_2) name = fmlookup Γ'_2 name
                by auto

              moreover have name |∈| frees (Sabs cs)
                using False ‹name |∈| frees rhs› ‹(pat, rhs) ∈ set cs›
                apply (auto simp: ffUnion-alt-def)
                apply (rule fBexI[where x = frees rhs |−| frees pat])
                 apply (auto simp: fset-of-list-elem)
                done
```

ultimately show *?thesis*
using ‹*fmrel-on-fset* (*frees* (*Sabs cs*)) *vrelated* Γ'_1 Γ'_2›
by (*auto dest*: *fmrel-on-fsetD*)
qed
qed
next
have *not-shadows-vconsts* (*Vabs cs* Γ'_2)
apply (*rule veval*′*-shadows*)
apply *fact*+
using *comb* **by** *auto*
thus ¬ *shadows-consts rhs*
using ‹(*pat*, *rhs*) ∈ *set cs*›
by (*auto simp*: *list-all-iff*)

show *not-shadows-vconsts-env* (Γ'_2 $++_f$ env_2)
apply *rule*
using ‹*not-shadows-vconsts* (*Vabs cs* Γ'_2)› **apply** *simp*
apply (*rule not-shadows-vconsts.vmatch-env*)
apply (*rule vfind-match-elem*)
apply *fact*
apply (*rule veval*′*-shadows*)
apply *fact*
apply *fact*
using *comb* **by** *auto*

show *fmrel-on-fset* (*consts rhs*) *vrelated* (*fmap-of-list rs*) (Γ'_2 $++_f$ env_2)
proof
fix *name*
assume *name* |∈| *consts rhs*
hence *name* |∈| *consts* (*Sabs cs*)
using ‹(*pat*, *rhs*) ∈ *set cs*›
by (*auto intro*!: *fBexI simp*: *fset-of-list-elem ffUnion-alt-def*)
hence *rel-option vrelated* (*fmlookup* (*fmap-of-list rs*) *name*) (*fmlookup* Γ'_2 *name*)
using ‹*fmrel-on-fset* (*consts* (*Sabs cs*)) *vrelated* (*fmap-of-list rs*) Γ'_2›
by (*auto dest*: *fmrel-on-fsetD*)
moreover have *name* |∉| *fmdom* env_2
proof
assume *name* |∈| *fmdom* env_2
hence *fmlookup* env_2 *name* ≠ *None*
by (*meson fmdom-notI*)
then obtain *v* **where** *fmlookup* env_2 *name* = *Some v*
by *blast*
hence *name* |∈| *fmdom* env_2
by (*auto intro*: *fmdomI*)
hence *name* |∈| *frees pat*
using ‹*fmdom* env_2 = *frees pat*›
by *simp*

```
          have welldefined rhs
            using ‹vwelldefined (Vabs cs Γ'_1)› ‹(pat, rhs) ∈ set cs›
            by (auto simp: list-all-iff)
          hence name |∈| fst |`| fset-of-list rs |∪| C
            using ‹name |∈| consts rhs›
            by (auto simp: all-consts-def)
          moreover have ¬ shadows-consts pat
            using ‹not-shadows-vconsts (Vabs cs Γ'_2)› ‹(pat, rhs) ∈ set cs›
            by (auto simp: list-all-iff shadows-consts-def all-consts-def)
          ultimately show False
            using ‹name |∈| frees pat›
            unfolding shadows-consts-def fdisjnt-alt-def all-consts-def
            by auto
        qed
        ultimately show rel-option vrelated (fmlookup (fmap-of-list rs) name)
(fmlookup (Γ'_2 ++_f env_2) name)
        by simp
      qed
    next
      show wellformed rhs
        using ‹(pat, rhs) ∈ set cs› ‹vwellformed (Vabs cs Γ'_2)›
        by (auto simp: list-all-iff)
    next
      have wellformed-venv Γ'_2
        by fact
      moreover have wellformed-venv env_2
        apply (rule vwellformed.vmatch-env)
         apply (rule vfind-match-elem)
         apply fact
        apply (rule veval'-wellformed)
          apply fact
        using ‹wellformed (t $_s u)› apply simp
        apply fact+
        done
      ultimately show wellformed-venv (Γ'_2 ++_f env_2)
        by blast
    next
      show welldefined rhs
        using ‹vwelldefined (Vabs cs Γ'_1)› ‹(pat, rhs) ∈ set cs›
        by (auto simp: list-all-iff)
    next
      have fmpred (λ-. vwelldefined) Γ'_1
        using ‹vwelldefined (Vabs cs Γ'_1)› by simp

      moreover have fmpred (λ-. vwelldefined) env_1
        apply (rule vwelldefined.vmatch-env)
         apply (rule vfind-match-elem)
         apply fact
```

apply (*rule veval-welldefined*)
apply *fact*
apply *fact*
using *comb* **apply** *simp*
done

ultimately show *fmpred* (λ-. *vwelldefined*) (Γ'_1 $++_f$ env_1)
by *blast*
qed

then obtain val_1 **where** *rs*, Γ'_1 $++_f$ env_1 $\vdash_v$ *rhs* $\downarrow$ val_1 $\vdash_v$ $val_1 \approx val_2$
by *blast*

show *?case*
apply (*intro conjI exI*)
apply (*rule veval.comb*)
apply *fact+*
done
next
— Almost verbatim copy from *comb* case.
case (*rec-comb* Γ_2 *t css name* Γ'_2 *cs u* u'_2 env_2 *pat rhs* val_2)
hence $\exists\, v.$ *rs*, $\Gamma_1 \vdash_v t \downarrow v \wedge \vdash_v v \approx$ *Vrecabs css name* Γ'_2
by (*auto intro*: *fmrel-on-fsubset*)
then obtain *v* **where** $\vdash_v v \approx$ *Vrecabs css name* Γ'_2 *rs*, $\Gamma_1 \vdash_v t \downarrow v$
by *blast*
moreover then obtain Γ'_1
where $v =$ *Vrecabs css name* Γ'_1
and *fmrel-on-fset* (*frees* (*Sabs cs*)) *vrelated* Γ'_1 Γ'_2
and *fmrel-on-fset* (*consts* (*Sabs cs*)) *vrelated* (*fmap-of-list rs*) (Γ'_2 $++_f$ *mk-rec-env css* Γ'_2)
using ‹*fmlookup css name* = *Some cs*›
by *cases auto*
ultimately have *rs*, $\Gamma_1 \vdash_v t \downarrow$ *Vrecabs css name* Γ'_1
by *simp*

have $\exists\, u_1'.$ *rs*, $\Gamma_1 \vdash_v u \downarrow u_1' \wedge \vdash_v u_1' \approx u'_2$
using *rec-comb* **by** (*auto intro*: *fmrel-on-fsubset*)
then obtain u'_1 **where** $\vdash_v u'_1 \approx u'_2$ *rs*, $\Gamma_1 \vdash_v u \downarrow u'_1$
by *blast*

have *rel-option* (*rel-prod* (*fmrel vrelated*) (=)) (*vfind-match cs* u'_1) (*vfind-match cs* u'_2)
by (*rule vrelated.vfind-match-rel'*) *fact*
then obtain env_1 **where** *vfind-match cs* u'_1 = *Some* (env_1, *pat*, *rhs*) *fmrel vrelated* env_1 env_2
using ‹*vfind-match cs* u'_2 = -›
by *cases auto*

have (*pat*, *rhs*) $\in$ *set cs*

  by (*rule vfind-match-elem*) *fact*

have *vwellformed* (*Vrecabs css name* Γ'_2)
  apply (*rule veval'-wellformed*)
    apply *fact*
  using ‹*wellformed* (t $\$_s$ u)› **apply** *simp*
  apply *fact+*
  done
hence *wellformed-venv* Γ'_2 *vwellformed* (*Vabs cs* Γ'_2)
  using *rec-comb* **by** *auto*

have *vwelldefined v*
  apply (*rule veval-welldefined*)
    apply *fact+*
  using *rec-comb* **by** *simp*
hence *vwelldefined* (*Vrecabs css name* Γ'_1)
  unfolding ‹v = -› **.**
hence *vwelldefined* (*Vabs cs* Γ'_1)
  using *rec-comb* **by** *auto*

have *linear pat*
  using ‹(*pat*, *rhs*) $\in$ *set cs*› ‹*vwellformed* (*Vabs cs* Γ'_2)›
  by (*auto simp*: *list-all-iff*)

have *fmdom* env_1 = *patvars* (*mk-pat pat*)
  apply (*rule vmatch-dom*)
  apply (*rule vfind-match-elem*)
  apply *fact*
  done
with ‹*linear pat*› **have** *fmdom* env_1 = *frees pat*
  by (*simp add*: *mk-pat-frees*)

have *fmdom* env_2 = *patvars* (*mk-pat pat*)
  apply (*rule vmatch-dom*)
  apply (*rule vfind-match-elem*)
  apply *fact*
  done
with ‹*linear pat*› **have** *fmdom* env_2 = *frees pat*
  by (*simp add*: *mk-pat-frees*)

note *fset-of-list-map*[*simp del*]
have $\exists\, val_1.\ rs,\ \Gamma'_1 \mathbin{++_f} env_1 \vdash_v rhs \downarrow val_1 \wedge \vdash_v val_1 \approx val_2$
  proof (*rule rec-comb*)
    have *not-shadows-vconsts* (*Vrecabs css name* Γ'_2)
      apply (*rule veval'-shadows*)
        apply *fact+*
      using *rec-comb* **by** *auto*
    thus $\neg$ *shadows-consts rhs*
      using ‹(*pat*, *rhs*) $\in$ *set cs*› *rec-comb*

by (*auto simp*: *list-all-iff*)
hence *fdisjnt all-consts* (*frees rhs*)
by (*rule shadows-consts-frees*)

have *not-shadows-vconsts-env* Γ'_2
using ‹*not-shadows-vconsts* (*Vrecabs css name* Γ'_2)›
by *simp*

moreover have *not-shadows-vconsts-env* (*mk-rec-env css* Γ'_2)
unfolding *mk-rec-env-def*
using ‹*not-shadows-vconsts* (*Vrecabs css name* Γ'_2)›
by (*auto intro*: *fmdomI*)

moreover have *not-shadows-vconsts-env* env_2
apply (*rule not-shadows-vconsts.vmatch-env*)
apply (*rule vfind-match-elem*)
apply *fact*
apply (*rule veval'-shadows*)
apply *fact*
apply *fact*
using *rec-comb* **by** *auto*

ultimately show *not-shadows-vconsts-env* (Γ'_2 $++_f$ *mk-rec-env css* Γ'_2 $++_f$ env_2)
by *auto*

have *not-shadows-vconsts* (*Vabs cs* Γ'_2)
using ‹*not-shadows-vconsts* (*Vrecabs* - - -)› *rec-comb*
by *auto*

show *fmrel-on-fset* (*frees rhs*) *vrelated* (Γ'_1 $++_f$ env_1) (Γ'_2 $++_f$ *mk-rec-env css* Γ'_2 $++_f$ env_2)
proof
fix *name*
assume *name* |∈| *frees rhs*
moreover have *fmdom css* |⊆| *all-consts*
using ‹*vwelldefined* (*Vrecabs* - - -)› **unfolding** *all-consts-def*
by *auto*
ultimately have *name* |∉| *fmdom css*
using ‹*fdisjnt* - (*frees rhs*)›
unfolding *fdisjnt-alt-def*
by (*metis* (*full-types*) *fempty-iff finterI fset-rev-mp*)

show *rel-option vrelated* (*fmlookup* (Γ'_1 $++_f$ env_1) *name*) (*fmlookup* (Γ'_2 $++_f$ *mk-rec-env css* Γ'_2 $++_f$ env_2) *name*)
proof (*cases name* |∈| *frees pat*)
case *True*
hence *name* |∈| *fmdom* env_1 *name* |∈| *fmdom* env_2
using ‹*fmdom* env_1 = *frees pat*› ‹*fmdom* env_2 = *frees pat*›

by *simp+*
hence
fmlookup $(\Gamma'_1 ++_f env_1)$ *name* = *fmlookup* env_1 *name*
fmlookup $(\Gamma'_2 ++_f$ *mk-rec-env css* $\Gamma'_2 ++_f env_2)$ *name* = *fmlookup* env_2 *name*
by *auto*
thus *?thesis*
using ‹*fmrel vrelated* env_1 env_2›
by *auto*
next
case *False*
hence *name* $|\notin|$ *fmdom* env_1 *name* $|\notin|$ *fmdom* env_2
using ‹*fmdom* env_1 = *frees pat*› ‹*fmdom* env_2 = *frees pat*›
by *simp+*
hence
fmlookup $(\Gamma'_1 ++_f env_1)$ *name* = *fmlookup* Γ'_1 *name*
fmlookup $(\Gamma'_2 ++_f$ *mk-rec-env css* $\Gamma'_2 ++_f env_2)$ *name* = *fmlookup* Γ'_2 *name*
unfolding *mk-rec-env-def* **using** ‹*name* $|\notin|$ *fmdom css*›
by *auto*

moreover have *name* $|\in|$ *frees* (*Sabs cs*)
using *False* ‹*name* $|\in|$ *frees rhs*› ‹(*pat*, *rhs*) ∈ *set cs*›
apply (*auto simp*: *ffUnion-alt-def*)
apply (*rule fBexI*[**where** x = *frees rhs* $|-|$ *frees pat*])
apply (*auto simp*: *fset-of-list-elem*)
done

ultimately show *?thesis*
using ‹*fmrel-on-fset* (*frees* (*Sabs cs*)) *vrelated* Γ'_1 Γ'_2›
by (*auto dest*: *fmrel-on-fsetD*)
qed
qed

show *fmrel-on-fset* (*consts rhs*) *vrelated* (*fmap-of-list rs*) $(\Gamma'_2 ++_f$ *mk-rec-env css* $\Gamma'_2 ++_f env_2)$
proof
fix *name*
assume *name* $|\in|$ *consts rhs*
hence *name* $|\in|$ *consts* (*Sabs cs*)
using ‹(*pat*, *rhs*) ∈ *set cs*›
by (*auto intro*!: *fBexI simp*: *fset-of-list-elem ffUnion-alt-def*)
hence *rel-option vrelated* (*fmlookup* (*fmap-of-list rs*) *name*) (*fmlookup* $(\Gamma'_2 ++_f$ *mk-rec-env css* $\Gamma'_2)$ *name*)
using ‹*fmrel-on-fset* (*consts* (*Sabs cs*)) *vrelated* (*fmap-of-list rs*) $(\Gamma'_2 ++_f$ *mk-rec-env css* $\Gamma'_2)$›
by (*auto dest*: *fmrel-on-fsetD*)
moreover have *name* $|\notin|$ *fmdom* env_2
proof

assume *name* |∈| *fmdom* env_2
hence *fmlookup* env_2 *name* ≠ *None*
by (*meson fmdom-notI*)
then obtain *v* **where** *fmlookup* env_2 *name* = *Some v*
by *blast*
hence *name* |∈| *fmdom* env_2
by (*auto intro*: *fmdomI*)
hence *name* |∈| *frees pat*
using ‹*fmdom* env_2 = *frees pat*›
by *simp*

have *vwelldefined* (*Vabs cs* Γ'_1)
using ‹*vwelldefined* (*Vrecabs css* - Γ'_1)›
using *rec-comb*
by *auto*

hence *welldefined rhs*
using ‹(*pat*, *rhs*) ∈ *set cs*› *rec-comb*
by (*auto simp*: *list-all-iff*)
hence *name* |∈| *fst* |‘| *fset-of-list rs* |∪| *C*
using ‹*name* |∈| *consts rhs*› *all-consts-def*
by *blast*
moreover have ¬ *shadows-consts pat*
using ‹*not-shadows-vconsts* (*Vabs cs* Γ'_2)› ‹(*pat*, *rhs*) ∈ *set cs*›
by (*auto simp*: *list-all-iff shadows-consts-def all-consts-def*)
ultimately show *False*
using ‹*name* |∈| *frees pat*›
unfolding *shadows-consts-def fdisjnt-alt-def all-consts-def*
by *auto*
qed
ultimately show *rel-option vrelated* (*fmlookup* (*fmap-of-list rs*) *name*) (*fmlookup* (Γ'_2 $++_f$ *mk-rec-env css* Γ'_2 $++_f$ env_2) *name*)
by *simp*
qed
next
show *wellformed rhs*
using ‹(*pat*, *rhs*) ∈ *set cs*› ‹*vwellformed* (*Vabs cs* Γ'_2)›
by (*auto simp*: *list-all-iff*)
next
have *wellformed-venv* Γ'_2
by *fact*
moreover have *wellformed-venv* env_2
apply (*rule vwellformed.vmatch-env*)
apply (*rule vfind-match-elem*)
apply *fact*
apply (*rule veval'-wellformed*)
apply *fact*
using ‹*wellformed* (*t* $\$_s$ *u*)› **apply** *simp*
apply *fact*+

```
        done
      moreover have wellformed-venv (mk-rec-env css Γ'₂)
        unfolding mk-rec-env-def
        using ‹vwellformed (Vrecabs - - -)›
        by (auto intro: fmdomI)
      ultimately show wellformed-venv (Γ'₂ ++_f mk-rec-env css Γ'₂ ++_f env₂)
        by blast
    next
      show welldefined rhs
        using ‹vwelldefined (Vabs cs Γ'₁)› ‹(pat, rhs) ∈ set cs›
        by (auto simp: list-all-iff)
    next
      have fmpred (λ-. vwelldefined) Γ'₁
        using ‹vwelldefined (Vabs cs Γ'₁)› by simp

      moreover have fmpred (λ-. vwelldefined) env₁
        apply (rule vwelldefined.vmatch-env)
         apply (rule vfind-match-elem)
         apply fact
        apply (rule veval-welldefined)
          apply fact
         apply fact
        using rec-comb apply simp
        done

      ultimately show fmpred (λ-. vwelldefined) (Γ'₁ ++_f env₁)
        by blast
    qed

  then obtain val₁ where rs, Γ'₁ ++_f env₁ ⊢_v rhs ↓ val₁ ⊢_v val₁ ≈ val₂
    by blast

  show ?case
    apply (intro conjI exI)
     apply (rule veval.rec-comb)
         apply fact+
    done
next
  case (constr name Γ₂ ts us₂)

  have list-all2 (λt u₂. (∃u₁. rs, Γ₁ ⊢_v t ↓ u₁ ∧ ⊢_v u₁ ≈ u₂)) ts us₂
    using ‹list-all2 - ts us₂›
    proof (rule list.rel-mono-strong, elim conjE impE allE exE)
      fix t u₂
      assume t ∈ set ts u₂ ∈ set us₂
      assume Γ₂ ⊢_v t ↓ u₂

      show wellformed t welldefined t ¬ shadows-consts t
        using constr ‹t ∈ set ts›
```

unfolding *welldefined.list-comb wellformed.list-comb shadows.list-comb*
by (*auto simp*: *list-all-iff list-ex-iff*)

show
wellformed-venv Γ_2
not-shadows-vconsts-env Γ_2
fmpred (λ-. *vwelldefined*) Γ_1
by *fact*+

have *consts t* |∈| *fset-of-list* (*map consts ts*)
using ‹*t* ∈ *set ts*› **by** (*simp add*: *fset-of-list-elem*)
hence *consts t* |⊆| *consts* (*name* $$ *ts*)
unfolding *consts-list-comb*
by (*metis ffUnion-subset-elem le-supI2*)
thus *fmrel-on-fset* (*consts t*) *vrelated* (*fmap-of-list rs*) Γ_2
using *constr* **by** (*blast intro*: *fmrel-on-fsubset*)

have *frees t* |∈| *fset-of-list* (*map frees ts*)
using ‹*t* ∈ *set ts*› **by** (*simp add*: *fset-of-list-elem*)
hence *frees t* |⊆| *frees* (*name* $$ *ts*)
unfolding *frees-list-comb const-sterm-def freess-def*
by (*auto intro*!: *ffUnion-subset-elem*)
thus *fmrel-on-fset* (*frees t*) *vrelated* Γ_1 Γ_2
using *constr* **by** (*blast intro*: *fmrel-on-fsubset*)
qed *auto*

then obtain us_1 **where** *list-all2* (*veval rs* Γ_1) *ts* us_1 *list-all2 vrelated* us_1 us_2
by *induction auto*

thus *?case*
using *constr*
by (*auto intro*: *veval.constr vrelated.constr*)
qed

lemma *veval′-correct′*:
assumes $\Gamma_2 \vdash_v t \downarrow v_2$ *wellformed t wellformed-venv* Γ_2
assumes ¬ *shadows-consts t not-shadows-vconsts-env* Γ_2
assumes *welldefined t*
assumes *closed t*
assumes *fmrel-on-fset* (*consts t*) *vrelated* (*fmap-of-list rs*) Γ_2
obtains v_1 **where** *rs*, *fmempty* $\vdash_v t \downarrow v_1 \vdash_v v_1 \approx v_2$
proof (*rule veval′-correct*[**where** Γ_1 = *fmempty*])
show *fmpred* (λ-. *vwelldefined*) *fmempty* **by** *simp*
next
show *fmrel-on-fset* (*frees t*) *vrelated fmempty* Γ_2
using ‹*closed t*› **unfolding** *closed-except-def* **by** *auto*
qed (*rule assms*)+

end

Preservation of extensional equality

lemma (**in** *constants*) *veval′-agree-eq*:
  assumes $\Gamma \vdash_v t \downarrow v$ *fmrel-on-fset* (*ids t*) *erelated* Γ' Γ
  assumes *closed-venv* Γ *closed-except t* (*fmdom* Γ)
  assumes *wellformed t wellformed-venv* Γ *fdisjnt C* (*fmdom* Γ)
  assumes *consts t* $|\subseteq|$ *fmdom* Γ $|\cup|$ *C fmpred* (λ-. *vwelldefined′*) Γ
  assumes $\neg$ *shadows-consts t not-shadows-vconsts-env* Γ
  obtains v' **where** $\Gamma' \vdash_v t \downarrow v'$ $v' \approx_e v$
using *assms* **proof** (*induction arbitrary*: Γ' *thesis*)
  case (*const name* Γ *val*)
  hence *name* $|\in|$ *ids* (*Sconst name*)
    unfolding *ids-def* **by** *simp*
  with *const* **have** *rel-option erelated* (*fmlookup* Γ' *name*) (*fmlookup* Γ *name*)
    by (*auto dest*: *fmrel-on-fsetD*)
  then obtain *val′* **where** *fmlookup* Γ' *name* = *Some val′ val′* $\approx_e$ *val*
    using ‹*fmlookup* Γ *name* = *Some val*›
    by *cases auto*
  thus *?case*
    using *const* **by** (*auto intro*: *veval′.const*)
next
  case (*var* Γ *name val*)
  hence *name* $|\in|$ *ids* (*Svar name*)
    unfolding *ids-def* **by** *simp*
  with *var* **have** *rel-option erelated* (*fmlookup* Γ' *name*) (*fmlookup* Γ *name*)
    by (*auto dest*: *fmrel-on-fsetD*)
  then obtain *val′* **where** *fmlookup* Γ' *name* = *Some val′ val′* $\approx_e$ *val*
    using ‹*fmlookup* Γ *name* = *Some val*›
    by *cases auto*
  thus *?case*
    using *var* **by** (*auto intro*: *veval′.var*)
next
  case (*abs* Γ *cs*)
  hence *Vabs cs* $\Gamma' \approx_e$ *Vabs cs* Γ
    by (*auto intro*: *erelated.abs*)
  thus *?case*
    using *abs* **by** (*auto intro*: *veval′.abs*)
next
  case (*comb* Γ *t cs* Γ_Λ *u* v_2 *env pat rhs val*)

  have *fmrel-on-fset* (*ids t*) *erelated* Γ' Γ
    apply (*rule fmrel-on-fsubset*)
     apply *fact*
    unfolding *ids-def* **by** *auto*
  then obtain v_1' **where** $\Gamma' \vdash_v t \downarrow v_1'$ $v_1' \approx_e$ *Vabs cs* Γ_Λ
    using *comb* **by** (*auto simp*: *closed-except-def*)
  then obtain Γ_Λ' **where** v_1' = *Vabs cs* Γ_Λ' *fmrel-on-fset* (*ids* (*Sabs cs*)) *erelated* Γ_Λ' Γ_Λ
    by (*auto elim*: *erelated.cases*)

have *fmrel-on-fset* (*ids u*) *erelated* Γ' Γ
apply (*rule fmrel-on-fsubset*)
apply *fact*
unfolding *ids-def* **by** *auto*
then obtain $v_2{}'$ **where** $\Gamma' \vdash_v u \downarrow v_2{}'$ $v_2{}' \approx_e v_2$
apply −
apply (*erule comb.IH*(*2*))
using *comb* **by** (*auto simp*: *closed-except-def*)

have *rel-option* (*rel-prod* (*fmrel erelated*) (=)) (*vfind-match cs* $v_2{}'$) (*vfind-match cs* v_2)
using ‹$v_2{}' \approx_e v_2$› **by** (*rule erelated.vfind-match-rel'*)
then obtain *env'* **where** *fmrel erelated env' env vfind-match cs* $v_2{}'$ = *Some* (*env'*, *pat*, *rhs*)
using *comb* **by** *cases auto*

have *vclosed* (*Vabs cs* Γ_Λ)
apply (*rule veval'-closed*)
using *comb* **by** (*auto simp*: *closed-except-def*)
have *vclosed* v_2
apply (*rule veval'-closed*)
using *comb* **by** (*auto simp*: *closed-except-def*)

have *closed-except* (*Sabs cs*) (*fmdom* Γ_Λ)
using ‹*vclosed* (*Vabs cs* Γ_Λ)› **by** (*auto simp*: *Sterm.closed-except-simps*)
hence *frees* (*Sabs cs*) $|\subseteq|$ *fmdom* Γ_Λ
unfolding *closed-except-def* **.**

have *vwellformed* (*Vabs cs* Γ_Λ)
apply (*rule veval'-wellformed*)
apply *fact*
using *comb* **by** *auto*

have (*pat*, *rhs*) ∈ *set cs*
by (*rule vfind-match-elem*) *fact*
hence *linear pat*
using ‹*vwellformed* (*Vabs cs* Γ_Λ)›
by (*auto simp*: *list-all-iff*)
hence *frees pat* = *patvars* (*mk-pat pat*)
by (*simp add*: *mk-pat-frees*)
hence *fmdom env* = *frees pat*
apply *simp*
apply (*rule vmatch-dom*)
apply (*rule vfind-match-elem*)
apply (*rule comb*)
done

have *vwelldefined'* (*Vabs cs* Γ_Λ)
apply (*rule veval'-welldefined'*)

```
        apply fact
  using comb by auto
hence consts rhs |⊆| fmdom Γ_Λ |∪| C fdisjnt C (fmdom Γ_Λ)
  using ‹(pat, rhs) ∈ set cs›
  by (auto simp: list-all-iff)

have not-shadows-vconsts (Vabs cs Γ_Λ)
  apply (rule veval'-shadows)
  using comb by auto

obtain val' where Γ_Λ' ++_f env' ⊢_v rhs ↓ val' val' ≈_e val
  proof (erule comb.IH)
    show closed-venv (Γ_Λ ++_f env)
      apply rule
      using ‹vclosed (Vabs cs Γ_Λ)› apply simp
      apply (rule vclosed.vmatch-env)
       apply (rule vfind-match-elem)
       apply fact
      apply fact
      done
  next
    show wellformed rhs
      using ‹(pat, rhs) ∈ set cs› ‹vwellformed (Vabs cs Γ_Λ)›
      by (auto simp: list-all-iff)
  next
    show wellformed-venv (Γ_Λ ++_f env)
      apply rule
      using ‹vwellformed (Vabs cs Γ_Λ)› apply simp
      apply (rule vwellformed.vmatch-env)
       apply (rule vfind-match-elem)
       apply (rule comb)
      apply (rule veval'-wellformed)
        apply fact
      using comb by auto
  next
    show closed-except rhs (fmdom (Γ_Λ ++_f env))
      using ‹(pat, rhs) ∈ set cs› ‹vclosed (Vabs cs Γ_Λ)› ‹fmdom env = frees pat›
      by (auto simp: list-all-iff)

    have fmdom env = fmdom env'
      using ‹fmrel erelated env' env›
      by (metis fmrel-fmdom-eq)

    show fmrel-on-fset (ids rhs) erelated (Γ_Λ' ++_f env') (Γ_Λ ++_f env)
      proof
        fix id
        assume id |∈| ids rhs

        thus rel-option erelated (fmlookup (Γ_Λ' ++_f env') id) (fmlookup (Γ_Λ ++_f
```

```
env) id)
        unfolding ids-def
        proof (cases rule: funion-strictE)
          case A
          hence id |∈| fmdom env |∪| fmdom ΓΛ
            using ‹closed-except rhs (fmdom (ΓΛ ++f env))›
            unfolding closed-except-def
            by auto

          thus ?thesis
            proof (cases rule: funion-strictE)
              case A
              hence id |∈| fmdom env'
                  using ‹fmdom env = frees pat› ‹fmdom env = fmdom env'› by
simp
              with A show ?thesis
                using ‹fmrel erelated env' env› by auto
            next
              case B
              hence id |∉| frees pat
                using ‹fmdom env = frees pat› by simp
              hence id |∈| frees (Sabs cs)
                apply auto
                unfolding ffUnion-alt-def
                apply simp
                apply (rule fBexI[where x = (pat, rhs)])
                using ‹id |∈| frees rhs› apply simp
                unfolding fset-of-list-elem
                apply (rule ‹(pat, rhs) ∈ set cs›)
                done
              hence id |∈| ids (Sabs cs)
                unfolding ids-def by simp

              have id |∉| fmdom env'
                using B unfolding ‹fmdom env = fmdom env'› by simp
              thus ?thesis
                using ‹id |∉| fmdom env›
                apply simp
                apply (rule fmrel-on-fsetD)
                 apply (rule ‹id |∈| ids (Sabs cs)›)
                apply (rule ‹fmrel-on-fset (ids (Sabs cs)) erelated ΓΛ' ΓΛ›)
                done
            qed
        next
          case B
          have id |∈| consts (Sabs cs)
            apply auto
            unfolding ffUnion-alt-def
            apply simp
```

```
        apply (rule fBexI[where x = (pat, rhs)])
         apply simp
         apply fact
        unfolding fset-of-list-elem
        apply (rule ‹(pat, rhs) ∈ set cs›)
        done
      hence id |∈| ids (Sabs cs)
        unfolding ids-def by auto

      show ?thesis
        using ‹fmdom env = fmdom env'›
        apply auto
         apply (rule fmrelD)
         apply (rule ‹fmrel erelated env' env›)
        apply (rule fmrel-on-fsetD)
         apply (rule ‹id |∈| ids (Sabs cs)›)
        apply (rule ‹fmrel-on-fset (ids (Sabs cs)) erelated ΓΛ' ΓΛ›)
        done
    qed
  qed
next
  show fmpred (λ-. vwelldefined') (ΓΛ ++f env)
  proof
    have vwelldefined' (Vabs cs ΓΛ)
      apply (rule veval'-welldefined')
          apply fact
      using comb by auto
    thus fmpred (λ-. vwelldefined') ΓΛ
      by simp
  next
    have vwelldefined' v2
      apply (rule veval'-welldefined')
          apply fact
      using comb by auto

    show fmpred (λ-. vwelldefined') env
      apply (rule vmatch-welldefined')
       apply (rule vfind-match-elem)
       apply fact+
      done
  qed
next
  have fdisjnt C (fmdom ΓΛ)
    using ‹vwelldefined' (Vabs cs ΓΛ)› by simp
  moreover have fdisjnt C (fmdom env)
    unfolding ‹fmdom env = -›
    using ‹(pat, rhs) ∈ set cs› ‹not-shadows-vconsts (Vabs cs ΓΛ)›
    by (auto simp: list-all-iff all-consts-def fdisjnt-alt-def)
  ultimately show fdisjnt C (fmdom (ΓΛ ++f env))
```

```
      unfolding fdisjnt-alt-def by auto
    next
      show ¬ shadows-consts rhs
        using ‹(pat, rhs) ∈ set cs› ‹not-shadows-vconsts (Vabs cs Γ_Λ)›
        by (auto simp: list-all-iff)
    next
      have not-shadows-vconsts-env Γ_Λ
        using ‹not-shadows-vconsts (Vabs cs Γ_Λ)› by auto
      moreover have not-shadows-vconsts-env env
        apply (rule not-shadows-vconsts.vmatch-env)
         apply (rule vfind-match-elem)
         apply fact
        apply (rule veval'-shadows)
        using comb by auto
      ultimately show not-shadows-vconsts-env (Γ_Λ ++_f env)
        by blast
    next
      show consts rhs |⊆| fmdom (Γ_Λ ++_f env) |∪| C
        using ‹consts rhs |⊆| -›
        by auto
    qed

  moreover have Γ' ⊢_v t $_s u ↓ val'
    proof (rule veval'.comb)
      show Γ' ⊢_v t ↓ Vabs cs Γ_Λ'
        using ‹Γ' ⊢_v t ↓ v_1'›
        unfolding ‹v_1' = -› .
    qed fact+

  ultimately show ?case
    using comb by metis
next
  case (rec-comb Γ t css name Γ_Λ cs u v_2 env pat rhs val)

  have fmrel-on-fset (ids t) erelated Γ' Γ
    apply (rule fmrel-on-fsubset)
    apply fact
    unfolding ids-def by auto
  then obtain v_1' where Γ' ⊢_v t ↓ v_1' v_1' ≈_e Vrecabs css name Γ_Λ
    using rec-comb by (auto simp: closed-except-def)
  then obtain Γ_Λ'
    where v_1' = Vrecabs css name Γ_Λ'
      and pred-fmap (λcs. fmrel-on-fset (ids (Sabs cs)) erelated Γ_Λ' Γ_Λ) css
    by (auto elim: erelated.cases)

  have fmrel-on-fset (ids u) erelated Γ' Γ
    apply (rule fmrel-on-fsubset)
     apply fact
    unfolding ids-def by auto
```

then obtain $v_2{}'$ **where** $\Gamma' \vdash_v u \downarrow v_2{}'$ $v_2{}' \approx_e v_2$
apply −
apply (*erule rec-comb.IH*(*2*))
using *rec-comb* **by** (*auto simp*: *closed-except-def*)

have *rel-option* (*rel-prod* (*fmrel erelated*) (=)) (*vfind-match cs* $v_2{}'$) (*vfind-match cs* v_2)
using ‹$v_2{}' \approx_e v_2$› **by** (*rule erelated.vfind-match-rel′*)
then obtain *env′* **where** *fmrel erelated env′ env vfind-match cs* $v_2{}'$ = *Some* (*env′*, *pat*, *rhs*)
using *rec-comb* **by** *cases auto*

have *vclosed* (*Vrecabs css name* Γ_Λ)
apply (*rule veval′-closed*)
using *rec-comb* **by** (*auto simp*: *closed-except-def*)
hence *vclosed* (*Vabs cs* Γ_Λ)
using *rec-comb* **by** (*auto simp*: *closed-except-def*)
have *vclosed* v_2
apply (*rule veval′-closed*)
using *rec-comb* **by** (*auto simp*: *closed-except-def*)

have *closed-except* (*Sabs cs*) (*fmdom* Γ_Λ)
using ‹*vclosed* (*Vabs cs* Γ_Λ)› **by** (*auto simp*: *Sterm.closed-except-simps*)
hence *frees* (*Sabs cs*) $|\subseteq|$ *fmdom* Γ_Λ
unfolding *closed-except-def* **.**

have *vwellformed* (*Vrecabs css name* Γ_Λ)
apply (*rule veval′-wellformed*)
apply *fact*
using *rec-comb* **by** *auto*
hence *vwellformed* (*Vabs cs* Γ_Λ)
using *rec-comb* **by** (*auto simp*: *closed-except-def*)

have (*pat*, *rhs*) ∈ *set cs*
by (*rule vfind-match-elem*) *fact*
hence *linear pat*
using ‹*vwellformed* (*Vabs cs* Γ_Λ)›
by (*auto simp*: *list-all-iff*)
hence *frees pat* = *patvars* (*mk-pat pat*)
by (*simp add*: *mk-pat-frees*)
hence *fmdom env* = *frees pat*
apply *simp*
apply (*rule vmatch-dom*)
apply (*rule vfind-match-elem*)
apply (*rule rec-comb*)
done

have *vwelldefined′* (*Vrecabs css name* Γ_Λ)
apply (*rule veval′-welldefined′*)

```
      apply fact
  using rec-comb by auto
 hence consts rhs |⊆| fmdom Γ_Λ |∪| (C |∪| fmdom css) fdisjnt C (fmdom Γ_Λ)
  using ‹(pat, rhs) ∈ set cs› ‹fmlookup css name = Some cs›
  by (auto simp: list-all-iff dest!: fmpredD[where m = css])

 have not-shadows-vconsts (Vrecabs css name Γ_Λ)
  apply (rule veval'-shadows)
  using rec-comb by auto
 hence not-shadows-vconsts (Vabs cs Γ_Λ)
  using rec-comb by auto

 obtain val' where Γ_Λ' ++_f mk-rec-env css Γ_Λ' ++_f env' ⊢_v rhs ↓ val' val' ≈_e
val
  proof (erule rec-comb.IH)
   show closed-venv (Γ_Λ ++_f mk-rec-env css Γ_Λ ++_f env)
    apply rule
     apply rule
    using ‹vclosed (Vabs cs Γ_Λ)› apply simp
    unfolding mk-rec-env-def
    using ‹vclosed (Vrecabs css name Γ_Λ)› apply (auto intro: fmdomI)[]
    apply (rule vclosed.vmatch-env)
     apply (rule vfind-match-elem)
     apply fact
    apply fact
    done
  next
   show wellformed rhs
    using ‹(pat, rhs) ∈ set cs› ‹vwellformed (Vabs cs Γ_Λ)›
    by (auto simp: list-all-iff)
  next
   show wellformed-venv (Γ_Λ ++_f mk-rec-env css Γ_Λ ++_f env)
    apply rule
     apply rule
    using ‹vwellformed (Vabs cs Γ_Λ)› apply simp
    unfolding mk-rec-env-def
    using ‹vwellformed (Vrecabs css name Γ_Λ)› apply (auto intro: fmdomI)[]
    apply (rule vwellformed.vmatch-env)
     apply (rule vfind-match-elem)
     apply fact
    apply (rule veval'-wellformed)
      apply fact
    using rec-comb by auto
  next
   have closed-except rhs (fmdom (Γ_Λ ++_f env))
    using ‹(pat, rhs) ∈ set cs› ‹vclosed (Vabs cs Γ_Λ)› ‹fmdom env = frees pat›
    by (auto simp: list-all-iff closed-except-def)
   thus closed-except rhs (fmdom (Γ_Λ ++_f mk-rec-env css Γ_Λ ++_f env))
    unfolding closed-except-def
```

by *auto*

have *fmdom env* = *fmdom env'*
using ‹*fmrel erelated env' env*›
by (*metis fmrel-fmdom-eq*)

have *fmrel-on-fset* (*ids rhs*) *erelated* (*mk-rec-env css* $\Gamma_\Lambda{}'$) (*mk-rec-env css* Γ_Λ)
unfolding *mk-rec-env-def*
apply *rule*
apply *simp*
unfolding *option.rel-map*
apply (*rule option.rel-refl*)
apply (*rule erelated.intros*)
apply (*rule* ‹*pred-fmap* (λ*cs. fmrel-on-fset* (*ids* (*Sabs cs*)) *erelated* $\Gamma_\Lambda{}'$ Γ_Λ) *css*›)
done

have *fmrel-on-fset* (*ids* (*Sabs cs*)) *erelated* $\Gamma_\Lambda{}'$ Γ_Λ
using ‹*pred-fmap* (λ*cs. fmrel-on-fset* (*ids* (*Sabs cs*)) *erelated* $\Gamma_\Lambda{}'$ Γ_Λ) *css*› *rec-comb*
by *auto*

have *fmdom* (*mk-rec-env css* Γ_Λ) = *fmdom* (*mk-rec-env css* $\Gamma_\Lambda{}'$)
unfolding *mk-rec-env-def* **by** *auto*

show *fmrel-on-fset* (*ids rhs*) *erelated* ($\Gamma_\Lambda{}'$ $++_f$ *mk-rec-env css* $\Gamma_\Lambda{}'$ $++_f$ *env'*) (Γ_Λ $++_f$ *mk-rec-env css* Γ_Λ $++_f$ *env*)
proof
fix *id*
assume *id* |∈| *ids rhs*

thus *rel-option erelated* (*fmlookup* ($\Gamma_\Lambda{}'$ $++_f$ *mk-rec-env css* $\Gamma_\Lambda{}'$ $++_f$ *env'*) *id*) (*fmlookup* (Γ_Λ $++_f$ *mk-rec-env css* Γ_Λ $++_f$ *env*) *id*)
unfolding *ids-def*
proof (*cases rule*: *funion-strictE*)
case *A*
hence *id* |∈| *fmdom env* |∪| *fmdom* Γ_Λ
using ‹*closed-except rhs* (*fmdom* (Γ_Λ $++_f$ *env*))›
unfolding *closed-except-def*
by *auto*

thus *?thesis*
proof (*cases rule*: *funion-strictE*)
case *A*
hence *id* |∈| *fmdom env'*
using ‹*fmdom env* = *frees pat*› ‹*fmdom env* = *fmdom env'*› **by** *simp*
with *A* **show** *?thesis*
using ‹*fmrel erelated env' env*› **by** *auto*

```
      next
        case B
        hence id |∉| frees pat
          using ‹fmdom env = frees pat› by simp
        hence id |∈| frees (Sabs cs)
          apply auto
          unfolding ffUnion-alt-def
          apply simp
          apply (rule fBexI[where x = (pat, rhs)])
          using ‹id |∈| frees rhs› apply simp
          unfolding fset-of-list-elem
          apply (rule ‹(pat, rhs) ∈ set cs›)
          done
        hence id |∈| ids (Sabs cs)
          unfolding ids-def by simp

        have id |∉| fmdom env'
          using B unfolding ‹fmdom env = fmdom env'› by simp
        thus ?thesis
          using ‹id |∉| fmdom env› ‹fmdom (mk-rec-env css ΓΛ) = fmdom
(mk-rec-env css ΓΛ')›
          apply auto
           apply (rule fmrel-on-fsetD)
            apply (rule ‹id |∈| ids rhs›)
           apply (rule ‹fmrel-on-fset (ids rhs) erelated (mk-rec-env css ΓΛ')
(mk-rec-env css ΓΛ)›)
          apply (rule fmrel-on-fsetD)
           apply (rule ‹id |∈| ids (Sabs cs)›)
          apply (rule ‹fmrel-on-fset (ids (Sabs cs)) erelated ΓΛ' ΓΛ›)
          done
      qed
    next
      case B
      have id |∈| consts (Sabs cs)
        apply auto
        unfolding ffUnion-alt-def
        apply simp
        apply (rule fBexI[where x = (pat, rhs)])
         apply simp
         apply fact
        unfolding fset-of-list-elem
        apply (rule ‹(pat, rhs) ∈ set cs›)
        done
      hence id |∈| ids (Sabs cs)
        unfolding ids-def by auto

      show ?thesis
          using ‹fmdom env = fmdom env'› ‹fmdom (mk-rec-env css ΓΛ) =
fmdom (mk-rec-env css ΓΛ')›
```

apply *auto*
apply (*rule fmrelD*)
apply (*rule* ‹*fmrel erelated env′ env*›)
apply (*rule fmrel-on-fsetD*)
apply (*rule* ‹*id* |∈| *ids rhs*›)
apply (*rule* ‹*fmrel-on-fset* (*ids rhs*) *erelated* (*mk-rec-env css* Γ_Λ′) (*mk-rec-env css* Γ_Λ)›)
apply (*rule fmrelD*)
apply (*rule* ‹*fmrel erelated env′ env*›)
apply (*rule fmrel-on-fsetD*)
apply (*rule* ‹*id* |∈| *ids* (*Sabs cs*)›)
apply (*rule* ‹*fmrel-on-fset* (*ids* (*Sabs cs*)) *erelated* Γ_Λ′ Γ_Λ›)
done
qed
qed
next
show *fmpred* (λ-. *vwelldefined′*) (Γ_Λ $++_f$ *mk-rec-env css* Γ_Λ $++_f$ *env*)
proof (*intro fmpred-add*)
have *vwelldefined′* (*Vrecabs css name* Γ_Λ)
apply (*rule veval′-welldefined′*)
apply *fact*
using *rec-comb* **by** *auto*
thus *fmpred* (λ-. *vwelldefined′*) Γ_Λ *fmpred* (λ-. *vwelldefined′*) (*mk-rec-env css* Γ_Λ)
unfolding *mk-rec-env-def*
by (*auto intro*: *fmdomI*)
next
have *vwelldefined′* v_2
apply (*rule veval′-welldefined′*)
apply *fact*
using *rec-comb* **by** *auto*

show *fmpred* (λ-. *vwelldefined′*) *env*
apply (*rule vmatch-welldefined′*)
apply (*rule vfind-match-elem*)
apply *fact*+
done
qed
next
have *fdisjnt C* (*fmdom env*)
unfolding ‹*fmdom env* = -›
using ‹(*pat*, *rhs*) ∈ *set cs*› ‹*not-shadows-vconsts* (*Vabs cs* Γ_Λ)›
by (*auto simp*: *list-all-iff all-consts-def fdisjnt-alt-def*)
moreover have *fdisjnt C* (*fmdom css*)
using ‹*vwelldefined′* (*Vrecabs css name* Γ_Λ)› **by** *simp*
ultimately show *fdisjnt C* (*fmdom* (Γ_Λ $++_f$ *mk-rec-env css* Γ_Λ $++_f$ *env*))
using ‹*fdisjnt C* (*fmdom* Γ_Λ)›
unfolding *fdisjnt-alt-def mk-rec-env-def* **by** *auto*
next

```
      show ¬ shadows-consts rhs
        using ‹(pat, rhs) ∈ set cs› ‹not-shadows-vconsts (Vabs cs Γ_Λ)›
        by (auto simp: list-all-iff)
    next
      have not-shadows-vconsts-env Γ_Λ
        using ‹not-shadows-vconsts (Vabs cs Γ_Λ)› by auto
      moreover have not-shadows-vconsts-env env
        apply (rule not-shadows-vconsts.vmatch-env)
         apply (rule vfind-match-elem)
         apply fact
        apply (rule veval'-shadows)
        using rec-comb by auto
      moreover have not-shadows-vconsts-env (mk-rec-env css Γ_Λ)
        unfolding mk-rec-env-def
        using ‹not-shadows-vconsts (Vrecabs css name Γ_Λ)›
        by (auto intro: fmdomI)
      ultimately show not-shadows-vconsts-env (Γ_Λ ++_f mk-rec-env css Γ_Λ ++_f
env)
        by blast
    next
      show consts rhs |⊆| fmdom (Γ_Λ ++_f mk-rec-env css Γ_Λ ++_f env) |∪| C
        using ‹consts rhs |⊆| -› unfolding mk-rec-env-def
        by auto
    qed

  moreover have Γ' ⊢_v t $_s u ↓ val'
    proof (rule veval'.rec-comb)
      show Γ' ⊢_v t ↓ Vrecabs css name Γ_Λ'
        using ‹Γ' ⊢_v t ↓ v_1'›
        unfolding ‹v_1' = -› .
    qed fact+

  ultimately show ?case
    using rec-comb by metis
next
  case (constr name Γ ts us)

  have list-all (λt. fmrel-on-fset (ids t) erelated Γ' Γ ∧ closed-except t (fmdom Γ)
∧ wellformed t ∧ consts t |⊆| fmdom Γ |∪| C ∧ ¬ shadows-consts t) ts
    apply (rule list-allI)
    apply rule
     apply (rule fmrel-on-fsubset)
      apply (rule constr)
    subgoal
      unfolding ids-list-comb
      by (induct ts; auto)
    subgoal
      apply (intro conjI)
      subgoal
```

```
      using ‹closed-except (name $$ ts) (fmdom Γ)›
      unfolding closed.list-comb by (auto simp: list-all-iff)
    subgoal
      using ‹wellformed (name $$ ts)›
      unfolding wellformed.list-comb by (auto simp: list-all-iff)
    subgoal
      using ‹consts (name $$ ts) |⊆| fmdom Γ |∪| C›
      unfolding consts-list-comb
      by (metis Ball-set constr.prems(8) special-constants.sconsts-list-comb)
    subgoal
      using ‹¬ shadows-consts (name $$ ts)›
      unfolding shadows.list-comb by (auto simp: list-ex-iff)
    done
  done

obtain us' where list-all3 (λt u u'. Γ' ⊢v t ↓ u' ∧ u' ≈e u) ts us us'
  using ‹list-all2 - - -› ‹list-all - ts›
  proof (induction arbitrary: thesis rule: list.rel-induct)
    case (Cons t ts u us)
    then obtain us' where list-all3 (λt u u'. Γ' ⊢v t ↓ u' ∧ u' ≈e u) ts us us'
      by auto
    have
      fmrel-on-fset (ids t) erelated Γ' Γ closed-except t (fmdom Γ)
      wellformed t consts t |⊆| fmdom Γ |∪| C ¬ shadows-consts t
      using Cons by auto

    then obtain u' where Γ' ⊢v t ↓ u' u' ≈e u
       using ‹closed-venv Γ› ‹wellformed-venv Γ› ‹fdisjnt C (fmdom Γ)› ‹fmpred
(λ-. vwelldefined') Γ›
      using ‹not-shadows-vconsts-env Γ› Cons.hyps
      by blast

    show ?case
      apply (rule Cons.prems)
      apply (rule list-all3-cons)
       apply fact
      apply (rule conjI)
       apply fact+
      done
  qed auto

show ?case
  apply (rule constr.prems)
   apply (rule veval'.constr[where us = us'])
    apply fact
  using ‹list-all3 - ts us us'›
   apply (induct; auto)
  apply (rule erelated.intros)
  using ‹list-all3 - ts us us'›
```

apply (*induct*; *auto*)
done
qed

end

Chapter 4

Preprocessing of code equations

theory *Doc-Preproc*
imports *Main*
begin

end

4.1 A type class for correspondence between HOL expressions and terms

theory *Eval-Class*
imports
  ../Rewriting/Rewriting-Term
  ../Utils/ML-Utils
  Deriving.Derive-Manager
  Dict-Construction.Dict-Construction
begin

no-notation *Mpat-Antiquot.mpaq-App* (**infixl** ‹$$› *900*)
hide-const (**open**) *Strong-Term.wellformed*
declare *Strong-Term.wellformed-term-def*[*simp del*]

class *evaluate* =
  fixes *eval* :: *rule fset* ⇒ *term* ⇒ *'a* ⇒ *bool* (‹-/ ⊢/ (- ≈/ -)› [*50*,*0*,*50*] *50*)
  assumes *eval-wellformed*: $rs \vdash t \approx a \Longrightarrow wellformed\ t$
begin

definition *eval'* :: *rule fset* ⇒ *term* ⇒ *'a* ⇒ *bool* (‹-/ ⊢/ (- ↓/ -)› [*50*,*0*,*50*] *50*)
where
$rs \vdash t \downarrow a \longleftrightarrow wellformed\ t \wedge (\exists\, t'.\ rs \vdash t \longrightarrow\!*\ t' \wedge rs \vdash t' \approx a)$

lemma *eval'I*[*intro*]:

assumes *wellformed t* $rs \vdash t \longrightarrow* t'$ $rs \vdash t' \approx a$
shows $rs \vdash t \downarrow a$
using *assms* **unfolding** *eval′-def* **by** *auto*

lemma *eval′E*[*elim*]:
assumes $rs \vdash t \downarrow a$
obtains t' **where** *wellformed t* $rs \vdash t \longrightarrow* t'$ $rs \vdash t' \approx a$
using *assms* **unfolding** *eval′-def* **by** *auto*

lemma *eval-trivI*: $rs \vdash t \approx a \Longrightarrow rs \vdash t \downarrow a$
by (*auto dest*: *eval-wellformed*)

lemma *eval-compose*:
assumes *wellformed t* $rs \vdash t \longrightarrow* t'$ $rs \vdash t' \downarrow a$
shows $rs \vdash t \downarrow a$
proof −
from ‹$rs \vdash t' \downarrow a$› **obtain** t'' **where** $rs \vdash t' \longrightarrow* t''$ $rs \vdash t'' \approx a$
by *blast*
moreover hence $rs \vdash t \longrightarrow* t''$
using *assms* **by** *auto*
ultimately show $rs \vdash t \downarrow a$
using *assms* **by** *auto*
qed

end

instantiation *fun* :: (*evaluate*, *evaluate*) *evaluate* **begin**

definition *eval-fun* **where**
eval-fun rs t a $\longleftrightarrow$ *wellformed t* $\wedge$ $(\forall x\ t_x.\ rs \vdash t_x \downarrow x \longrightarrow rs \vdash t\ \$\ t_x \downarrow a\ x)$

instance
by *standard* (*simp add*: *eval-fun-def*)

end

corollary *eval-funD*:
assumes $rs \vdash t \approx f$ $rs \vdash t_x \downarrow x$
shows $rs \vdash t\ \$\ t_x \downarrow f\ x$
using *assms* **unfolding** *eval-fun-def* **by** *blast*

corollary *eval′-funD*:
assumes $rs \vdash t \downarrow f$ $rs \vdash t_x \downarrow x$
shows $rs \vdash t\ \$\ t_x \downarrow f\ x$
proof −
from *assms* **obtain** t' **where** $rs \vdash t \longrightarrow* t'$ $rs \vdash t' \approx f$
by *auto*
have *wellformed* $(t\ \$\ t_x)$
using *assms* **by** *auto*

moreover have $rs \vdash t \$ t_x \longrightarrow* t' \$ t_x$
  using ‹$rs \vdash t \longrightarrow* t'$› **by** (*rule rewrite.rt-fun*[*unfolded app-term-def*])
moreover have $rs \vdash t' \$ t_x \downarrow f\ x$
  using ‹$rs \vdash t' \approx f$› *assms*(*2*) **by** (*rule eval-funD*)
ultimately show $rs \vdash t \$ t_x \downarrow f\ x$
  by (*rule eval-compose*)
qed

lemma *eval-ext*:
  assumes *wellformed* f $\bigwedge x\ t.\ rs \vdash t \downarrow x \Longrightarrow rs \vdash f \$ t \downarrow a\ x$
  shows $rs \vdash f \approx a$
using *assms* **unfolding** *eval-fun-def* **by** *blast*

lemma *eval'-ext*:
  assumes *wellformed* f $\bigwedge x\ t.\ rs \vdash t \downarrow x \Longrightarrow rs \vdash f \$ t \downarrow a\ x$
  shows $rs \vdash f \downarrow a$
apply (*rule eval'I*[*OF* ‹*wellformed f*›])
apply (*rule rtranclp.rtrancl-refl*)
apply (*rule eval-ext*)
using *assms* **by** *auto*

lemma *eval'-ext-alt*:
  fixes f :: *'a*::*evaluate* ⇒ *'b*::*evaluate*
  assumes *wellformed' 1 t* $\bigwedge u\ x.\ rs \vdash u \downarrow x \Longrightarrow rs \vdash t\ [u]_\beta \downarrow f\ x$
  shows $rs \vdash \Lambda\ t \downarrow f$
proof (*rule eval'-ext*)
  show *wellformed* (Λ *t*)
    using *assms* **by** *simp*
next
  fix *x* :: *'a* **and** *u*
  assume $rs \vdash u \downarrow x$
  show $rs \vdash \Lambda\ t \$ u \downarrow f\ x$
    proof (*rule eval-compose*)
      show *wellformed* (Λ $t \$ u$)
        using *assms* ‹$rs \vdash u \downarrow x$› **by** *auto*
    next
      show $rs \vdash \Lambda\ t \$ u \longrightarrow* t\ [u]_\beta$
        using ‹$rs \vdash u \downarrow x$› **by** (*auto intro*: *rewrite.beta*)
    next
      show $rs \vdash t\ [u]_\beta \downarrow f\ x$
        using *assms* ‹$rs \vdash u \downarrow x$› **by** *auto*
    qed
qed

lemma *eval-impl-wellformed*[*dest*]: $rs \vdash t \approx a \Longrightarrow$ *wellformed' n t*
by (*auto dest*: *wellformed-inc eval-wellformed*[*unfolded wellformed-term-def*])

lemma *eval'-impl-wellformed*[*dest*]: $rs \vdash t \downarrow a \Longrightarrow$ *wellformed' n t*
unfolding *eval'-def* **by** (*auto dest*: *wellformed-inc*)

lemma *wellformed-unpack*:
  $wellformed'\ n\ (t\ \$\ u) \Longrightarrow wellformed'\ n\ t$
  $wellformed'\ n\ (t\ \$\ u) \Longrightarrow wellformed'\ n\ u$
  $wellformed'\ n\ (\Lambda\ t) \Longrightarrow wellformed'\ (Suc\ n)\ t$
by *auto*

lemma *replace-bound-aux*:
  $n < 0 \longleftrightarrow False$
  $Suc\ n < Suc\ m \longleftrightarrow n < m$
  $0 < Suc\ n \longleftrightarrow True$
  $((0::nat) = 0) \longleftrightarrow True$
  $(0 = Suc\ m) \longleftrightarrow False$
  $(Suc\ m = Suc\ n) \longleftrightarrow n = m$
  $(Suc\ m = 0) \longleftrightarrow False$
  $(if\ True\ then\ P\ else\ Q) = P$
  $(if\ False\ then\ P\ else\ Q) = Q$
  $int\ (0::nat) = 0$
by *auto*

named-theorems *eval-data-intros*
named-theorems *eval-data-elims*

context begin

private definition *rewrite-step-term* :: $term \times term \Rightarrow term \Rightarrow term\ option$
where
rewrite-step-term = *rewrite-step*

private lemmas *rewrite-rt-fun* = *rewrite.rt-fun*[*unfolded app-term-def*]
private lemmas *rewrite-rt-arg* = *rewrite.rt-arg*[*unfolded app-term-def*]

ML-file *tactics.ML*

end

method-setup *wellformed* = ‹*Scan.succeed* (*SIMPLE-METHOD′ o Tactics.wellformed-tac*)›

end

4.2 Deep embedding of Pure terms into term-rewriting logic

theory *Embed*
imports
  Constructor-Funs.Constructor-Funs
  ../Utils/Code-Utils

Eval-Class
keywords *embed* :: *thy-goal*
begin

fun *non-overlapping′* :: *term* ⇒ *term* ⇒ *bool* **where**
non-overlapping′ (*Const x*) (*Const y*) ⟷ $x \neq y$ |
non-overlapping′ (*Const* -) (- $ -) ⟷ *True* |
non-overlapping′ (- $ -) (*Const* -) ⟷ *True* |
non-overlapping′ (t_1 $ t_2) (u_1 $ u_2) ⟷ *non-overlapping′* t_1 u_1 ∨ *non-overlapping′* t_2 u_2 |
non-overlapping′ - - ⟷ *False*

lemma *non-overlapping-approx*:
assumes *non-overlapping′ t u*
shows *non-overlapping t u*
using *assms*
by (*induct t u rule*: *non-overlapping′.induct*) *fastforce+*

fun *pattern-compatible′* :: *term* ⇒ *term* ⇒ *bool* **where**
pattern-compatible′ (t_1 $ t_2) (u_1 $ u_2) ⟷ *pattern-compatible′* t_1 u_1 ∧ ($t_1 = u_1$ ⟶ *pattern-compatible′* t_2 u_2) |
pattern-compatible′ t u ⟷ $t = u$ ∨ *non-overlapping′ t u*

lemma *pattern-compatible-approx*:
assumes *pattern-compatible′ t u*
shows *pattern-compatible t u*
using *assms*
proof (*induction t u rule*: *pattern-compatible.induct*)
case *2-1*
thus *?case*
by (*force simp*: *non-overlapping-approx*)
next
case *2-5*
thus *?case*
by (*force simp*: *non-overlapping-approx*)
qed *auto*

abbreviation *pattern-compatibles′* :: (*term* × *′a*) *fset* ⇒ *bool* **where**
pattern-compatibles′ ≡ *fpairwise* (λ(lhs_1, -) (lhs_2, -). *pattern-compatible′* lhs_1 lhs_2)

definition *rules′* :: *C-info* ⇒ *rule fset* ⇒ *bool* **where**
rules′ C-info rs ⟷
fBall rs rule ∧
arity-compatibles rs ∧
is-fmap rs ∧
pattern-compatibles′ rs ∧
rs ≠ {||} ∧
fBall rs (λ(*lhs*, -). ¬ *pre-constants.shadows-consts C-info* (*heads-of rs*) *lhs*) ∧
fdisjnt (*heads-of rs*) (*constructors.C C-info*) ∧

fBall rs (λ(-, *rhs*). *pre-constants.welldefined C-info* (*heads-of rs*) *rhs*) ∧
distinct (*constructors.all-constructors C-info*)

lemma *rules-approx*:
assumes *rules' C-info rs*
shows *rules C-info rs*
proof
show *fBall rs rule arity-compatibles rs is-fmap rs rs* ≠ {||}
and *fBall rs* (λ(*lhs*, -). ¬ *pre-constants.shadows-consts C-info* (*heads-of rs*) *lhs*)
and *fBall rs* (λ(-, *rhs*). *pre-constants.welldefined C-info* (*heads-of rs*) *rhs*)
and *fdisjnt* (*heads-of rs*) (*constructors.C C-info*)
and *distinct* (*constructors.all-constructors C-info*)
using *assms* **unfolding** *rules'-def* **by** *simp+*
next
have *pattern-compatibles' rs*
using *assms* **unfolding** *rules'-def* **by** *simp*
thus *pattern-compatibles rs*
by (*rule fpairwise-weaken*) (*blast intro*: *pattern-compatible-approx*)
qed

lemma *embed-ext*: $f \equiv g \Longrightarrow f\ x \equiv g\ x$
by *auto*

ML-file *embed.ML*

consts *lift-term* :: *'a* ⇒ *term* (‹⟨-⟩›)

setup‹
let
fun embed ((*Const* (@{*const-name lift-term*}, -)) $ *t*) = *HOL-Term.mk-term false t*
| *embed* (*t* $ *u*) = *embed t* $ *embed u*
| *embed t* = *t*
in Context.theory-map (*Syntax-Phases.term-check 99 lift* (*K* (*map embed*))) *end*
›

end

4.3 Default instances

theory *Eval-Instances*
imports *Embed*
begin

ML-file *eval-instances.ML*

setup ‹*Eval-Instances.setup*›

derive *evaluate nat bool list unit prod sum option char num name term*

end

Chapter 5

Final stage: Translation to CakeML

theory *Doc-Backend*
imports *Main*
begin

end

5.1 Basic CakeML setup

theory *CakeML-Setup*
imports
  ../CupCakeML/CupCake-Semantics
  CakeML.CakeML-Code
  ../Terms/Consts
begin

global-interpretation *name*: *rekey Name*
  rewrites *inv Name* = *as-string*
proof −
  have *bij Name*
    by (*metis bijI′ name.exhaust name.inject*)
  show *rekey Name*
    by *standard fact*

  show *inv Name* = *as-string*
    by (*metis inv-equality name.exhaust-sel name.sel*)
qed

global-interpretation *name-as-string*: *rekey as-string*
  by (*rule name.inv*)

hide-const (**open**) *Lem-string.concat*

hide-const (**open**) *sem-env.c*
hide-const (**open**) *sem-env.v*

definition *empty-locn* :: *locn* **where**
empty-locn = (| *row* = *0*, *col* = *0*, *offset* = *0* |)

definition *empty-locs* :: *locs* **where**
empty-locs = (*empty-locn*, *empty-locn*)

definition *empty-state* :: *unit SemanticPrimitives.state* **where**
empty-state = (| *clock* = *0*, *refs* = [], *ffi* = *empty-ffi-state*, *defined-types* = {}, *defined-mods* = {} |)

fun *fmap-of-ns* :: (*'b*, *string*, *'a*) *namespace* ⇒ (*name*, *'a*) *fmap* **where**
fmap-of-ns (*Bind xs -*) = *fmap-of-list* (*map* (*map-prod Name id*) *xs*)

lemma *fmlookup-ns*[*simp*]: *fmlookup* (*fmap-of-ns ns*) *k* = *cupcake-nsLookup ns* (*as-string k*)
by (*cases ns*) (*simp add*: *fmlookup-of-list map-prod-def name.map-of-rekey option.map-ident*)

lemma *fmap-of-nsBind*[*simp*]: *fmap-of-ns* (*nsBind* (*as-string k*) *v0 ns*) = *fmupd k v0* (*fmap-of-ns ns*)
by (*cases ns*) *auto*

lemma *fmap-of-nsAppend*[*simp*]: *fmap-of-ns* (*nsAppend ns1 ns2*) = *fmap-of-ns ns2* $++_f$ *fmap-of-ns ns1*
by (*cases ns1*; *cases ns2*) *simp*

lemma *fmap-of-alist-to-ns*[*simp*]: *fmap-of-ns* (*alist-to-ns xs*) = *fmap-of-list* (*map* (*map-prod Name id*) *xs*)
unfolding *alist-to-ns-def* **by** *simp*

lemma *fmap-of-nsEmpty*[*simp*]: *fmap-of-ns nsEmpty* = *fmempty*
unfolding *nsEmpty-def* **by** *simp*

context begin

private lemma *build-rec-env-fmap0*:
fmap-of-ns (*foldr* (λ(*f*, *x*, *e*). *nsBind f* (*Recclosure* env_Λ *funs' f*)) *funs env*) =
fmap-of-ns env $++_f$ *fmap-of-list* (*map* (λ(*f*, -). (*Name f*, *Recclosure* env_Λ *funs' f*)) *funs*)
apply (*induction funs arbitrary*: *env*)
apply *auto*
by (*metis* (*no-types*, *lifting*) *fmap-of-nsBind name.sel*)

definition *cake-mk-rec-env* **where**
cake-mk-rec-env funs env = *fmap-of-list* (*map* (λ(*f*, -). (*Name f*, *Recclosure env funs f*)) *funs*)

lemma *build-rec-env-fmap*:
fmap-of-ns (*build-rec-env funs* env_Λ *env*) = *fmap-of-ns env* $++_f$ *cake-mk-rec-env funs* env_Λ
unfolding *build-rec-env-def cake-mk-rec-env-def*
by (*rule build-rec-env-fmap0*)

end

5.2 Constructors according to CakeML

definition *cake-tctor* :: *string* ⇒ *tctor* **where**
cake-tctor name = (*if name* = *''fun'' then Ast.TC-fn else Ast.TC-name* (*Short name*))

primrec *typ-to-t* :: *typ* ⇒ *Ast.t* **where**
typ-to-t (*TVar name*) = *Ast.Tvar* (*as-string name*) |
typ-to-t (*TApp name args*) = *Ast.Tapp* (*map typ-to-t args*) (*cake-tctor* (*as-string name*))

context *constructors* **begin**

definition *as-static-cenv* :: *c-ns* **where**
as-static-cenv = *Bind* (*rev* (*map* (*map-prod id* (*map-prod id* (*TypeId* ∘ *Short*))) *flat-C-info*)) []

lemma *as-static-cenv-cakeml-static-env*: *cakeml-static-env as-static-cenv*
unfolding *cakeml-static-env-def as-static-cenv-def*
by (*auto simp*: *list.pred-map split*: *prod.splits*)

sublocale *cake-static-env?*: *cakeml-static-env as-static-cenv*
by (*rule as-static-cenv-cakeml-static-env*)

definition *as-cake-type-def* :: *Ast.type-def* **where**
as-cake-type-def =
map (λ(*name*, *dt-def*). (*map as-string* (*tparams dt-def*), *as-string name*,
map (λ(*C*, *params*). (*as-string C*, *map typ-to-t params*))
(*sorted-list-of-fmap* (*constructors dt-def*))))
(*sorted-list-of-fmap C-info*)

definition *cake-dt-prelude* :: *Ast.dec* **where**
cake-dt-prelude = *Ast.Dtype empty-locs as-cake-type-def*

definition *cake-all-types* :: *tid-or-exn set* **where**
cake-all-types = (*TypeId* ∘ *Short* ∘ *as-string*) ‘ *fset all-tdefs*

definition *empty-state-with-types* :: *unit SemanticPrimitives.state* **where**
empty-state-with-types =
⦇ *clock* = *0*, *refs* = [], *ffi* = *empty-ffi-state*, *defined-types* = *cake-all-types*, *defined-mods* = {} ⦈

lemma *empty-state-with-types-alt-def*:
empty-state-with-types = *empty-state* ⦇ *defined-types* := *cake-all-types* ⦈
unfolding *empty-state-with-types-def empty-state-def*

by (*auto simp*: *datatype-record-update*)

end

5.2.1 Running the generated type declarations through the semantics

context *constants* **begin**

context begin

private lemma *state-types-update*:
update-defined-types (λ-. *cake-all-types* ∪ *defined-types empty-state*) *empty-state* =
empty-state-with-types
unfolding *empty-state-with-types-def empty-state-def*

by (*simp add*: *datatype-record-update*)

private lemma *env-types-update*: *build-tdefs* [] *as-cake-type-def* = *as-static-cenv*
unfolding *as-cake-type-def-def as-static-cenv-def build-tdefs-def alist-to-ns-def flat-C-info-def*
apply (*auto simp*: *List.bind-def map-concat*)
apply (*rule arg-cong*[**where** *f* = *concat*])
by (*auto simp*: *map-concat comp-def split-beta*)

private lemmas *evaluate-type* =
evaluate-dec.dtype1[
where *new-tdecs* = *cake-all-types* **and** *s* = *empty-state* **and** *mn* = [] **and** *tds* = *as-cake-type-def*,
unfolded state-types-update env-types-update,
folded empty-sem-env-def]

private lemma *type-defs-to-new-tdecs*:
type-defs-to-new-tdecs [] *as-cake-type-def* =
set (*map* (λ*name*. *TypeId* (*Short* (*as-string name*))) (*sorted-list-of-fset* (*fmdom C-info*)))
unfolding *cake-all-types-def type-defs-to-new-tdecs-def as-cake-type-def-def all-tdefs-def*
by (*simp add*: *case-prod-twice sorted-list-of-fmap-def*)

private lemma *cakeml-convoluted1*: *foldr* (λ(*n*, *ts*). (#) *n*) *ys xs* = *map fst ys* @ *xs*
by (*induction ys arbitrary*: *xs*) *auto*

private lemma *cakeml-convoluted2*: *foldr* ($\lambda x\ y.\ f\ x$ @ y) *xs ys* = *concat* (*map f xs*) @ *ys*
by (*induction xs arbitrary*: *ys*) *auto*

private lemma *check-dup-ctors-alt-def*: *check-dup-ctors tds* $\longleftrightarrow$ *distinct* (*tds* $\gg=$ (λ(-, -, *cons*). *map fst cons*))
unfolding *check-dup-ctors-def*
apply *simp*
apply (*rule arg-cong*[**where** *f* = *distinct*])
apply (*subst foldr-cong*[*OF refl refl*, **where** *g* = $\lambda x\ a.$ *map fst* (*snd* (*snd x*)) @ *a*])
subgoal
  apply (*subst split-beta*)
  apply (*subst split-beta*)
  by (*rule cakeml-convoluted1*)
subgoal
  apply (*subst cakeml-convoluted2*)
  apply *auto*
  unfolding *List.bind-def*
  apply (*rule arg-cong*[**where** *f* = *concat*])
  by *auto*
done

lemma *evaluate-dec-prelude*:
  evaluate-dec t [] *env empty-state cake-dt-prelude* (*empty-state-with-types*, *Rval empty-sem-env*)
unfolding *cake-dt-prelude-def*
proof (*rule evaluate-type*, *intro conjI*)
  show *check-dup-ctors as-cake-type-def*
    using *distinct-ctr′*
    unfolding *check-dup-ctors-alt-def List.bind-def as-cake-type-def-def all-constructors-def*
    by (*auto simp*: *comp-def split-beta map-concat*)
next
  show *allDistinct* (*map* ($\lambda x.$ *case x of* (*tvs*, *tn*, *ctors*) $\Rightarrow$ *tn*) *as-cake-type-def*)
    unfolding *all-distinct-alt-def as-cake-type-def-def*
    apply (*auto simp*: *comp-def case-prod-twice*)
    apply (*rule name-as-string.fst-distinct*)
    unfolding *sorted-list-of-fmap-def*
    by (*auto simp*: *comp-def*)
next
  show *cake-all-types* = *type-defs-to-new-tdecs* [] *as-cake-type-def*
    unfolding *cake-all-types-def type-defs-to-new-tdecs all-tdefs-def*
    by *simp*
next
  show *disjnt cake-all-types* (*defined-types empty-state*)
    unfolding *empty-state-def disjnt-def* **by** *simp*
qed

end

end

Computability

declare *constructors.as-static-cenv-def*[*code*]
declare *constructors.as-cake-type-def-def*[*code*]
declare *constructors.cake-dt-prelude-def*[*code*]

export-code *constructors.as-static-cenv constructors.cake-dt-prelude*
  checking *Scala*

end

5.3 CakeML backend

theory *CakeML-Backend*
imports
  CakeML-Setup
  ../Terms/Value
  ../Rewriting/Rewriting-Sterm
begin

5.3.1 Compilation

fun *mk-ml-pat* :: *pat* ⇒ *Ast.pat* **where**
mk-ml-pat (*Patvar s*) = *Ast.Pvar* (*as-string s*) |
mk-ml-pat (*Patconstr s args*) = *Ast.Pcon* (*Some* (*Short* (*as-string s*))) (*map mk-ml-pat args*)

lemma *mk-pat-cupcake*[*intro*]: *is-cupcake-pat* (*mk-ml-pat pat*)
by (*induct pat*) (*auto simp*: *list-all-iff*)

context begin

private fun *frees′* :: *term* ⇒ *name list* **where**
frees′ (*Free x*) = [*x*] |
frees′ (t_1 \$ t_2) = *frees′* t_2 @ *frees′* t_1 |
frees′ (Λ *t*) = *frees′ t* |
frees′ - = []

private lemma *frees′-eq*[*simp*]: *fset-of-list* (*frees′ t*) = *frees t*
by (*induction t*) *auto*

private lemma *frees′-list-comb*: *frees′* (*list-comb f xs*) = *concat* (*rev* (*map frees′ xs*)) @ *frees′ f*
by (*induction xs arbitrary*: *f*) (*auto simp*: *app-term-def*)

private lemma *frees′-distinct*: *linear pat* ⟹ *distinct* (*frees′ pat*)
proof (*induction pat*)
  case (*App t u*)

hence *distinct* (*frees′ u* @ *frees′ t*)
  by (*fastforce intro*: *distinct-append-fset fdisjnt-swap*)
thus *?case*
  by *simp*
qed *auto*

private fun *pat-bindings′* :: *Ast.pat* ⇒ *name list* **where**
pat-bindings′ (*Ast.Pvar n*) = [*Name n*] |
pat-bindings′ (*Ast.Pcon - ps*) = *concat* (*rev* (*map pat-bindings′ ps*)) |
pat-bindings′ (*Ast.Pref p*) = *pat-bindings′ p* |
pat-bindings′ (*Ast.Ptannot p -*) = *pat-bindings′ p* |
pat-bindings′ - = []

private lemma *pat-bindings′-eq*:
  map Name (*pats-bindings ps xs*) = *concat* (*rev* (*map pat-bindings′ ps*)) @ *map Name xs*
  map Name (*pat-bindings p xs*) = *pat-bindings′ p* @ *map Name xs*
by (*induction ps xs* **and** *p xs rule*: *pats-bindings-pat-bindings.induct*) (*auto simp*: *ac-simps*)

private lemma *pat-bindings′-empty-eq*: *map Name* (*pat-bindings p* []) = *pat-bindings′ p*
by (*simp add*: *pat-bindings′-eq*)

private lemma *pat-bindings′-eq-frees*: *linear p* ⟹ *pat-bindings′* (*mk-ml-pat* (*mk-pat p*)) = *frees′ p*
proof (*induction rule*: *mk-pat.induct*)
  case (*1 t*)

  show *?case*
    using ‹*linear t*› **proof** (*cases rule*: *linear-strip-comb-cases*)
      case (*comb s args*)
      have *map* (*pat-bindings′* ∘ *mk-ml-pat* ∘ *mk-pat*) *args* = *map frees′ args*
        proof (*rule list.map-cong0*, *unfold comp-apply*)
          fix *x*
          assume *x* ∈ *set args*
          moreover hence *linear x*
            using *1 comb* **by** (*metis linears-linear linears-strip-comb snd-conv*)
          ultimately show *pat-bindings′* (*mk-ml-pat* (*mk-pat x*)) = *frees′ x*
            using *1 comb* **by** *auto*
        qed

      hence *concat* (*rev* (*map* (*pat-bindings′* ∘ *mk-ml-pat* ∘ *mk-pat*) *args*)) = *concat* (*rev* (*map frees′ args*))
        by *metis*
      with *comb* **show** *?thesis*
        apply (*fold const-term-def*)
        apply (*auto simp*: *strip-list-comb-const frees′-list-comb comp-assoc*)
        apply (*unfold const-term-def*)

apply *simp*
done
qed *auto*
qed

lemma *mk-pat-distinct*: *linear pat* $\Longrightarrow$ *distinct* (*pat-bindings* (*mk-ml-pat* (*mk-pat pat*)) [])
by (*metis pat-bindings′-eq-frees pat-bindings′-empty-eq frees′-distinct distinct-map*)

end

locale *cakeml* = *pre-constants*
begin

fun
mk-exp :: *name fset* ⇒ *sterm* ⇒ *exp* **and**
mk-clauses :: *name fset* ⇒ (*term* × *sterm*) *list* ⇒ (*Ast.pat* × *exp*) *list* **and**
mk-con :: *name fset* ⇒ *sterm* ⇒ *exp* **where**
mk-exp - (*Svar s*) = *Ast.Var* (*Short* (*as-string s*)) |
mk-exp - (*Sconst s*) = (*if s* |∈| *C then Ast.Con* (*Some* (*Short* (*as-string s*))) [] *else Ast.Var* (*Short* (*as-string s*))) |
mk-exp S (t_1 $\$_s$ t_2) = *Ast.App Ast.Opapp* [*mk-con S* t_1, *mk-con S* t_2] |
mk-exp S (*Sabs cs*) = (
let n = *fresh-fNext S in*
Ast.Fun (*as-string n*) (*Ast.Mat* (*Ast.Var* (*Short* (*as-string n*))) (*mk-clauses S cs*))) |
mk-con S t =
(*case strip-comb t of*
(*Sconst c*, *args*) ⇒
if c |∈| *C then Ast.Con* (*Some* (*Short* (*as-string c*))) (*map* (*mk-con S*) *args*) *else mk-exp S t*
| - ⇒ *mk-exp S t*) |
mk-clauses S cs = *map* (λ(*pat*, *t*). (*mk-ml-pat* (*mk-pat pat*), *mk-con* (*frees pat* |∪| *S*) *t*)) *cs*

context begin

private lemma *mk-exp-cupcake0*:
wellformed t $\Longrightarrow$ *is-cupcake-exp* (*mk-exp S t*)
wellformed-clauses cs $\Longrightarrow$ *cupcake-clauses* (*mk-clauses S cs*) ∧ *cake-linear-clauses* (*mk-clauses S cs*)
wellformed t $\Longrightarrow$ *is-cupcake-exp* (*mk-con S t*)
proof (*induction rule*: *mk-exp-mk-clauses-mk-con.induct*)
case (*5 S t*)
show *?case*
apply (*simp split*!: *prod.splits sterm.splits if-splits*)
subgoal premises *prems* **for** *args c*
proof −
from *prems* **have** *t* = *c* $$ *args*

```
        apply (fold const-sterm-def)
        by (metis fst-conv list-strip-comb snd-conv)
      show ?thesis
        apply (auto simp: list-all-iff simp del: mk-con.simps)
        apply (rule 5(1))
            apply (rule prems(1)[symmetric])
           apply (rule refl)
          apply (rule prems)
         apply assumption
        using ‹wellformed t› ‹t = -›
        apply (auto simp: wellformed.list-comb list-all-iff)
        done
    qed
  using 5 by (auto split: prod.splits sterm.splits)
qed (auto simp: Let-def list-all-iff intro: mk-pat-distinct)

declare mk-con.simps[simp del]

lemma mk-exp-cupcake:
  wellformed t ⟹ is-cupcake-exp (mk-exp S t)
  wellformed t ⟹ is-cupcake-exp (mk-con S t)
by (metis mk-exp-cupcake0)+

end

definition mk-letrec-body where
mk-letrec-body S rs = (
  map (λ(name, rhs).
    (as-string name, (
      let n = fresh-fNext S in
      (as-string n, Ast.Mat (Ast.Var (Short (as-string n))) (mk-clauses S (sterm.clauses
rhs)))))) rs
)

definition compile-group :: name fset ⇒ srule list ⇒ Ast.dec where
compile-group S rs = Ast.Dletrec empty-locs (mk-letrec-body S rs)

definition compile :: srule list ⇒ Ast.prog where
compile rs = [Ast.Tdec (compile-group all-consts rs)]

end

declare cakeml.mk-con.simps[code]
declare cakeml.mk-exp.simps[code]
declare cakeml.mk-clauses.simps[code]
declare cakeml.mk-letrec-body-def[code]
declare cakeml.compile-group-def[code]
declare cakeml.compile-def[code]
```

locale *cakeml′* = *cakeml* + *constants*

context *srules* **begin**

sublocale *srules-as-cake?*: *cakeml′ C-info fst* |'| *fset-of-list rs* **by** *standard*

lemma *mk-letrec-cupcake*:
  list-all (λ(-, -, *exp*). *is-cupcake-exp exp*) (*mk-letrec-body S rs*)
unfolding *mk-letrec-body-def*
using *all-rules*
apply (*auto simp*: *Let-def list-all-iff intro*!: *mk-pat-cupcake mk-exp-cupcake mk-pat-distinct*)
subgoal for *a b*
  apply (*erule ballE*[**where** *x* = (*a*, *b*)]; *cases b*)
      apply (*auto simp*: *list-all-iff is-abs-def term-cases-def*)
  done
subgoal for *a b*
  apply (*erule ballE*[**where** *x* = (*a*, *b*)]; *cases b*)
      apply (*auto simp*: *list-all-iff is-abs-def term-cases-def*)
  done
done

end

definition *compile′* **where**
compile′ C-info rs = *cakeml.compile C-info* (*fst* |'| *fset-of-list rs*) *rs*

lemma (**in** *srules*) *compile′-compile-eq*: *compile′ C-info rs* = *compile rs*
unfolding *compile′-def* **..**

5.3.2 Computability

export-code *cakeml.compile*
  checking *Scala*

5.3.3 Correctness of semantic functions

abbreviation *related-pat* :: *term* ⇒ *Ast.pat* ⇒ *bool* **where**
related-pat t p ≡ (*p* = *mk-ml-pat* (*mk-pat t*))

context *cakeml′* **begin**

inductive *related-exp* :: *sterm* ⇒ *exp* ⇒ *bool* **where**
var: *related-exp* (*Svar name*) (*Ast.Var* (*Short* (*as-string name*))) |
const: *name* |∉| *C* ⟹ *related-exp* (*Sconst name*) (*Ast.Var* (*Short* (*as-string name*)))
|
constr: *name* |∈| *C* ⟹ *list-all2 related-exp ts es* ⟹
      related-exp (*name* $$ *ts*) (*Ast.Con* (*Some* (*Short* (*as-string name*))) *es*) |
app: *related-exp* t_1 u_1 ⟹ *related-exp* t_2 u_2 ⟹ *related-exp* (t_1 $\$_s$ t_2) (*Ast.App Ast.Opapp* [u_1, u_2]) |
fun: *list-all2* (*rel-prod related-pat related-exp*) *cs ml-cs* ⟹

```
    n |∉| ids (Sabs cs) ⟹ n |∉| all-consts ⟹
      related-exp (Sabs cs) (Ast.Fun (as-string n) (Ast.Mat (Ast.Var (Short
(as-string n))) ml-cs)) |
mat: list-all2 (rel-prod related-pat related-exp) cs ml-cs ⟹
    related-exp scr ml-scr ⟹
      related-exp (Sabs cs $_s scr) (Ast.Mat ml-scr ml-cs)

lemma related-exp-is-cupcake:
  assumes related-exp t e wellformed t
  shows is-cupcake-exp e
using assms proof induction
  case (fun cs ml-cs n)
  hence list-all (λ(pat, t). linear pat ∧ wellformed t) cs by simp
  moreover have cupcake-clauses ml-cs ∧ cake-linear-clauses ml-cs
    using ‹list-all2 - cs ml-cs› ‹list-all - cs›
    proof induction
      case (Cons c cs ml-c ml-cs)
      obtain ml-p ml-e where ml-c = (ml-p, ml-e) by fastforce
      obtain p t where c = (p, t) by fastforce

      have ml-p = mk-ml-pat (mk-pat p)
        using Cons unfolding ‹ml-c = -› ‹c = -› by simp
      thus ?case
        using Cons unfolding ‹ml-c = -› ‹c = -› by (auto intro: mk-pat-distinct)
    qed simp

  ultimately show ?case
    by auto
next
  case (mat cs ml-cs scr ml-scr)
  hence list-all (λ(pat, t). linear pat ∧ wellformed t) cs by simp
  moreover have cupcake-clauses ml-cs ∧ cake-linear-clauses ml-cs
    using ‹list-all2 - cs ml-cs› ‹list-all - cs›
    proof induction
      case (Cons c cs ml-c ml-cs)
      obtain ml-p ml-e where ml-c = (ml-p, ml-e) by fastforce
      obtain p t where c = (p, t) by fastforce

      have ml-p = mk-ml-pat (mk-pat p)
        using Cons unfolding ‹ml-c = -› ‹c = -› by simp
      thus ?case
        using Cons unfolding ‹ml-c = -› ‹c = -› by (auto intro: mk-pat-distinct)
    qed simp

  ultimately show ?case
    using mat by auto
next
  case (constr name ts es)
  hence list-all wellformed ts
```

by (*simp add*: *wellformed.list-comb*)
with ‹*list-all2* - *ts es*› **have** *list-all is-cupcake-exp es*
by *induction auto*
thus *?case*
by *simp*
qed *auto*

definition *related-fun* :: (*term* × *sterm*) *list* ⇒ *name* ⇒ *exp* ⇒ *bool* **where**
related-fun cs n e ⟷
n |∉| *ids* (*Sabs cs*) ∧ *n* |∉| *all-consts* ∧ (*case e of*
(*Ast.Mat* (*Ast.Var* (*Short n'*)) *ml-cs*) ⇒
n = *Name n'* ∧ *list-all2* (*rel-prod related-pat related-exp*) *cs ml-cs*
| - ⇒ *False*)

lemma *related-fun-alt-def*:
related-fun cs n (*Ast.Mat* (*Ast.Var* (*Short* (*as-string n*))) *ml-cs*) ⟷
list-all2 (*rel-prod related-pat related-exp*) *cs ml-cs* ∧
n |∉| *ids* (*Sabs cs*) ∧ *n* |∉| *all-consts*
unfolding *related-fun-def*
by *auto*

lemma *related-funE*:
assumes *related-fun cs n e*
obtains *ml-cs*
where *e* = *Ast.Mat* (*Ast.Var* (*Short* (*as-string n*))) *ml-cs n* |∉| *ids* (*Sabs cs*)
n |∉| *all-consts*
and *list-all2* (*rel-prod related-pat related-exp*) *cs ml-cs*
using *assms* **unfolding** *related-fun-def*
by (*simp split*: *exp0.splits id0.splits*)

lemma *related-exp-fun*:
related-fun cs n e ⟷ *related-exp* (*Sabs cs*) (*Ast.Fun* (*as-string n*) *e*) ∧ *n* |∉| *ids*
(*Sabs cs*) ∧ *n* |∉| *all-consts*
(**is** *?lhs* ⟷ *?rhs*)
proof
assume *?rhs*
hence *related-exp* (*Sabs cs*) (*Ast.Fun* (*as-string n*) *e*) **by** *simp*
thus *?lhs*
by *cases* (*auto simp*: *related-fun-def dest*: *name.expand*)
next
assume *?lhs*
thus *?rhs*
by (*auto intro*: *related-exp.fun elim*: *related-funE*)
qed

inductive *related-v* :: *value* ⇒ *v* ⇒ *bool* **where**
conv:
list-all2 related-v us vs ⟹
related-v (*Vconstr name us*) (*Conv* (*Some* (*as-string name*, -)) *vs*) |

closure:
  related-fun cs n e $\Longrightarrow$
   fmrel-on-fset (*ids* (*Sabs cs*)) *related-v* Γ (*fmap-of-ns* (*sem-env.v env*)) $\Longrightarrow$
    related-v (*Vabs cs* Γ) (*Closure env* (*as-string n*) *e*) |
rec-closure:
  fmrel-on-fset (*fbind* (*fmran css*) (*ids* $\circ$ *Sabs*)) *related-v* Γ (*fmap-of-ns* (*sem-env.v env*)) $\Longrightarrow$
   fmrel (λ*cs.* λ(*n*, *e*). *related-fun cs n e*) *css* (*fmap-of-list* (*map* (*map-prod Name* (*map-prod Name id*)) *exps*)) $\Longrightarrow$
    related-v (*Vrecabs css name* Γ) (*Recclosure env exps* (*as-string name*))

abbreviation *var-env* :: (*name*, *value*) *fmap* $\Rightarrow$ (*string* $\times$ *v*) *list* $\Rightarrow$ *bool* **where**
var-env Γ *ns* $\equiv$ *fmrel related-v* Γ (*fmap-of-list* (*map* (*map-prod Name id*) *ns*))

lemma *related-v-ext*:
  assumes *related-v v ml-v*
  assumes $v' \approx_e v$
  shows *related-v* v' *ml-v*
using *assms* **proof** (*induction arbitrary*: v')
  case (*conv us ml-us name*)
  obtain *ts* **where** v' = *Vconstr name ts list-all2 erelated ts us*
    using ‹$v' \approx_e$ *Vconstr name us*›
    by *cases auto*

  have *list-all2 related-v ts ml-us*
    by (*rule list-all2-trans*[*OF* - ‹*list-all2 erelated ts us*› *conv*(*1*)]) *auto*

  thus *?case*
    using *conv* **unfolding** ‹v' = -›
    by (*auto intro*: *related-v.conv*)
next
  case (*closure cs n e* Γ_2 *env*)
  obtain Γ_1 **where** v' = *Vabs cs* Γ_1 *fmrel-on-fset* (*ids* (*Sabs cs*)) *erelated* Γ_1 Γ_2
    using ‹$v' \approx_e$ -›
    by *cases auto*

  have *fmrel-on-fset* (*ids* (*Sabs cs*)) *related-v* Γ_1 (*fmap-of-ns* (*sem-env.v env*))
    apply *rule*
    subgoal premises *prems* **for** *x*
      apply (*insert prems*)
      apply (*drule fmrel-on-fsetD*)
       apply (*rule closure*)
      subgoal
        apply (*insert prems*)
        apply (*drule fmrel-on-fsetD*)
         apply (*rule* ‹*fmrel-on-fset* (*ids* (*Sabs cs*)) *erelated* Γ_1 Γ_2›)
        apply (*metis* (*mono-tags*, *lifting*) *option.rel-sel*)
        done
      done

done

show *?case*
unfolding ‹$v' = -$›
by (*rule related-v.closure*) *fact+*
next
case (*rec-closure css* Γ_2 *env exps name*)
obtain Γ_1 **where** $v' =$ *Vrecabs css name* Γ_1 *pred-fmap* (λ*cs. fmrel-on-fset* (*ids* (*Sabs cs*)) *erelated* Γ_1 Γ_2) *css*
using ‹$v' \approx_e$ -›
by *cases auto*

have *fmrel-on-fset* (*fbind* (*fmran css*) (*ids* ∘ *Sabs*)) *related-v* Γ_1 (*fmap-of-ns* (*sem-env.v env*))
apply (*rule fmrel-on-fsetI*)
subgoal premises *prems* **for** *x*
apply (*insert prems*)
apply (*drule fmrel-on-fsetD*)
apply (*rule rec-closure*)
subgoal
apply (*insert prems*)
apply (*erule fbindE*)
apply (*drule pred-fmapD*[*OF* ‹*pred-fmap - css*›])
unfolding *comp-apply*
apply (*drule fmrel-on-fsetD*)
apply *assumption*
apply (*metis* (*mono-tags*, *lifting*) *option.rel-sel*)
done
done
done

show *?case*
unfolding ‹$v' = -$›
by (*rule related-v.rec-closure*) *fact+*
qed

context begin

private inductive *match-result-related* :: (*string* × *v*) *list* ⇒ (*string* × *v*) *list match-result* ⇒ (*name*, *value*) *fmap option* ⇒ *bool* **for** *eenv* **where**
no-match: *match-result-related eenv No-match None* |
error: *match-result-related eenv Match-type-error -* |
match: *var-env* Γ *eenv-m* ⟹ *match-result-related eenv* (*Match* (*eenv-m* @ *eenv*)) (*Some* Γ)

private corollary *match-result-related-empty*: *match-result-related eenv* (*Match eenv*) (*Some fmempty*)
proof −
have *match-result-related eenv* (*Match* ([] @ *eenv*)) (*Some fmempty*)

by (*rule match-result-related.match*) *auto*
thus *?thesis*
by *simp*
qed

private fun *is-Match* :: *'a match-result* ⇒ *bool* **where**
is-Match (*Match* -) ⟷ *True* |
is-Match - ⟷ *False*

lemma *cupcake-pmatch-related*:
assumes *related-v v ml-v*
shows *match-result-related eenv* (*cupcake-pmatch as-static-cenv* (*mk-ml-pat pat*) *ml-v eenv*) (*vmatch pat v*)
using *assms* **proof** (*induction pat arbitrary*: *v ml-v eenv*)
case (*Patvar name*)
have *var-env* (*fmap-of-list* [(*name*, *v*)]) [(*as-string name*, *ml-v*)]
using *Patvar* **by** *auto*
hence *match-result-related eenv* (*Match* ([(*as-string name*, *ml-v*)] @ *eenv*)) (*Some* (*fmap-of-list* [(*name*, *v*)]))
by (*rule match-result-related.match*)
thus *?case*
by *simp*
next
case (*Patconstr name ps v0*)

show *?case*
using *Patconstr.prems*
proof (*cases rule*: *related-v.cases*)
case (*conv us vs name'* -)

define *f* **where**
f p v m =
(*case m of*
Match env ⇒ *cupcake-pmatch as-static-cenv p v env*
| *m* ⇒ *m*) **for** *p v m*

{
assume *name* = *name'*

assume *length ps* = *length us*
hence *list-all2* (λ- -. *True*) *us ps*
by (*induct rule*: *list-induct2*) *auto*
hence *list-all3* (λ*t p v*. *related-v t v*) *us ps vs*
using ‹*list-all2 related-v us vs*›
by (*rule list-all3-from-list-all2s*) *auto*

hence ∗: *match-result-related eenv*
(*Matching.fold2 f Match-type-error* (*map mk-ml-pat ps*) *vs* (*Match* (*eenv-m* @ *eenv*)))

($map\text{-}option$ ($foldl$ ($++_f$) Γ) ($those$ ($map2$ $vmatch$ ps us))) (**is** *?rel*)
if *var-env* Γ *eenv-m*
for *eenv-m* Γ
using *Patconstr.IH* ‹*related-v v0 ml-v*› *that*
proof (*induction us ps vs arbitrary*: Γ *eenv-m rule*: *list-all3-induct*)
case (*Cons t us p ps v vs*)

have *match-result-related* (*eenv-m* @ *eenv*) (*cupcake-pmatch as-static-cenv* (*mk-ml-pat p*) *v* (*eenv-m* @ *eenv*)) (*vmatch p t*)
using *Cons* **by** *simp*

thus *?case*
proof *cases*
case *no-match*
thus *?thesis*
unfolding *f-def*
apply (*cases length* (*map mk-ml-pat ps*) = *length vs*)
by (*fastforce intro*: *match-result-related.intros simp*:*cup-pmatch-list-nomatch* *cup-pmatch-list-length-neq*)+
next
case *error*
thus *?thesis*
unfolding *f-def*
apply (*cases length* (*map mk-ml-pat ps*) = *length vs*)
by (*fastforce intro*: *match-result-related.intros simp*:*cup-pmatch-list-typerr* *cup-pmatch-list-length-neq*)+
next
case (*match* Γ' *eenv-m'*)

have *match-result-related eenv*
(*Matching.fold2 f Match-type-error* (*map mk-ml-pat ps*) *vs* (*Match* ((*eenv-m'* @ *eenv-m*) @ *eenv*)))
($map\text{-}option$ ($foldl$ ($++_f$) (Γ $++_f$ Γ')) ($those$ ($map2$ $vmatch$ ps us)))
proof (*rule Cons*, *rule Cons*)
show *var-env* (Γ $++_f$ Γ') (*eenv-m'* @ *eenv-m*)
using ‹*var-env* Γ *eenv-m*› *match*
by *force*
qed (*simp* | *fact*)+

thus *?thesis*
using *match*
unfolding *f-def*
by (*auto simp*: *map-option.compositionality comp-def*)
qed
qed (*auto intro*: *match-result-related.match*)

moreover have *var-env fmempty* []
by *force*

ultimately have *?rel* [] *fmempty*
by *fastforce*

hence *?thesis*
using *conv* ‹*length ps* = *length us*›
unfolding *f-def* ‹*name* = *name'*›
by (*auto intro*: *match-result-related.intros split*: *option.splits elim*: *static-cenv-lookup*)
}
moreover
{
assume *name* ≠ *name'*
with *conv* **have** *?thesis*
by (*auto intro*: *match-result-related.intros split*: *option.splits elim*: *same-ctor.elims simp*: *name.expand*)
}
moreover
{
let *?fold* = *Matching.fold2 f Match-type-error* (*map mk-ml-pat ps*) *vs* (*Match eenv*)

assume ∗: *length ps* ≠ *length us*
moreover have *length us* = *length vs*
using ‹*list-all2 related-v us vs*› **by** (*rule list-all2-lengthD*)
ultimately have *length ps* ≠ *length vs*
by *simp*

moreover have ¬ *is-Match* (*Matching.fold2 f err xs ys init*)
if ¬ *is-Match err* **and** *length xs* ≠ *length ys* **for** *init err xs ys*
using *that f-def*
by (*induct f err xs ys init rule*: *fold2.induct*) (*auto split*: *match-result.splits*)
ultimately have ¬ *is-Match ?fold*
by *simp*
hence *?fold* = *Match-type-error* ∨ *?fold* = *No-match*
by (*cases ?fold*) *auto*

with ∗ **have** *?thesis*
unfolding ‹*ml-v* = -› ‹*v0* = -› *f-def*
by (*auto intro*: *match-result-related.intros split*: *option.splits*)
}

ultimately show *?thesis*
by *auto*
qed (*auto intro*: *match-result-related.intros*)
qed

lemma *match-all-related*:
assumes *list-all2* (*rel-prod related-pat related-exp*) *cs ml-cs*
assumes *list-all* (λ(*pat*, -). *linear pat*) *cs*

assumes *related-v v ml-v*
assumes *cupcake-match-result as-static-cenv ml-v ml-cs Bindv* = *Rval* (*ml-rhs*, *ml-pat*, *eenv*)
obtains *rhs pat* Γ **where**
ml-pat = *mk-ml-pat* (*mk-pat pat*)
related-exp rhs ml-rhs
vfind-match cs v = *Some* (Γ, *pat*, *rhs*)
var-env Γ *eenv*
using *assms*
proof (*induction cs ml-cs arbitrary*: *thesis ml-pat ml-rhs rule*: *list-all2-induct*)
case (*Cons c cs ml-c ml-cs*)
moreover obtain *pat0 rhs0* **where** *c* = (*pat0*, *rhs0*) **by** *fastforce*
moreover obtain *ml-pat0 ml-rhs0* **where** *ml-c* = (*ml-pat0*, *ml-rhs0*) **by** *fastforce*
ultimately have *ml-pat0* = *mk-ml-pat* (*mk-pat pat0*) *related-exp rhs0 ml-rhs0* **by** *auto*

have *linear pat0*
using *Cons*(*5*) **unfolding** ‹*c* = -› **by** *simp*+

have *rel*: *match-result-related* [] (*cupcake-pmatch as-static-cenv* (*mk-ml-pat* (*mk-pat pat0*)) *ml-v* []) (*vmatch* (*mk-pat pat0*) *v*)
by (*rule cupcake-pmatch-related*) *fact*+

show *?case*
proof (*cases cupcake-pmatch as-static-cenv ml-pat0 ml-v* [])
case *Match-type-error*
hence *False*
using *Cons*(*7*) **unfolding** ‹*ml-c* = -›
by (*simp split*: *if-splits*)
thus *thesis* **..**
next
case *No-match*

show *thesis*
proof (*rule Cons*(*3*))
show *cupcake-match-result as-static-cenv ml-v ml-cs Bindv* = *Rval* (*ml-rhs*, *ml-pat*, *eenv*)
using *Cons*(*7*) *No-match* **unfolding** ‹*ml-c* = -›
by (*simp split*: *if-splits*)
next
fix *pat rhs* Γ
assume *ml-pat* = *mk-ml-pat* (*mk-pat pat*)
assume *related-exp rhs ml-rhs*
assume *vfind-match cs v* = *Some* (Γ, *pat*, *rhs*)
assume *var-env* Γ *eenv*

from *rel* **have** *match-result-related* [] *No-match* (*vmatch* (*mk-pat pat0*) *v*)
using *No-match* **unfolding** ‹*ml-pat0* = -›
by *simp*

```
        hence vmatch (mk-pat pat0) v = None
          by (cases rule: match-result-related.cases)

        show thesis
          proof (rule Cons(4))
            show vfind-match (c # cs) v = Some (Γ, pat, rhs)
              unfolding ‹c = -›
              using ‹vfind-match cs v = -› ‹vmatch (mk-pat pat0) v = -›
              by simp
          qed fact+
      next
        show list-all (λ(pat, -). linear pat) cs
          using Cons(5) by simp
      qed fact+
  next
    case (Match eenv')
    hence ml-rhs = ml-rhs0 ml-pat = ml-pat0 eenv = eenv'
      using Cons(7) unfolding ‹ml-c = -›
      by (simp split: if-splits)+

    from rel have match-result-related [] (Match eenv') (vmatch (mk-pat pat0) v)

      using Match unfolding ‹ml-pat0 = -› by simp
    then obtain Γ where vmatch (mk-pat pat0) v = Some Γ var-env Γ eenv'
      by (cases rule: match-result-related.cases) auto

    show thesis
      proof (rule Cons(4))
        show ml-pat = mk-ml-pat (mk-pat pat0)
          unfolding ‹ml-pat = -› by fact
      next
        show related-exp rhs0 ml-rhs
          unfolding ‹ml-rhs = -› by fact
      next
        show var-env Γ eenv
          unfolding ‹eenv = -› by fact
      next
        show vfind-match (c # cs) v = Some (Γ, pat0, rhs0)
          unfolding ‹c = -›
          using ‹vmatch (mk-pat pat0) v = Some Γ›
          by simp
      qed
  qed
qed simp

end end

end
```

5.3.4 Correctness of compilation

theory *CakeML-Correctness*
imports
CakeML-Backend
../Rewriting/Big-Step-Value-ML
begin

context *cakeml'* **begin**

lemma *mk-rec-env-related*:
assumes *fmrel* (λ*cs* (*n*, *e*). *related-fun cs n e*) *css* (*fmap-of-list* (*map* (*map-prod Name* (*map-prod Name id*)) *funs*))
assumes *fmrel-on-fset* (*fbind* (*fmran css*) (*ids* $\circ$ *Sabs*)) *related-v* Γ_Λ (*fmap-of-ns* (*sem-env.v env*$_\Lambda$))
shows *fmrel related-v* (*mk-rec-env css* Γ_Λ) (*cake-mk-rec-env funs env*$_\Lambda$)
proof (*rule fmrelI*)
fix *name*
have *rel-option* (λ*cs* (*n*, *e*). *related-fun cs n e*) (*fmlookup css name*) (*map-of* (*map* (*map-prod Name* (*map-prod Name id*)) *funs*) *name*)
using *assms* **by** (*auto simp*: *fmap-of-list.rep-eq*)

then have *rel-option* (λ*cs* (*n*, *e*). *related-fun cs* (*Name n*) *e*) (*fmlookup css name*) (*map-of funs* (*as-string name*))
unfolding *name.map-of-rekey'*
by *cases auto*

have $*$: *related-v* (*Vrecabs css name* Γ_Λ) (*Recclosure env*$_\Lambda$ *funs* (*as-string name*))
using *assms* **by** (*auto intro*: *related-v.rec-closure*)

show *rel-option related-v* (*fmlookup* (*mk-rec-env css* Γ_Λ) *name*) (*fmlookup* (*cake-mk-rec-env funs env*$_\Lambda$) *name*)
unfolding *mk-rec-env-def cake-mk-rec-env-def fmap-of-list.rep-eq*
apply (*simp add*: *map-of-map-keyed name.map-of-rekey option.rel-map*)
apply (*rule option.rel-mono-strong*)
apply *fact*
apply (*rule* $*$)
done
qed

lemma *mk-exp-correctness*:
ids t $|\subseteq|$ *S* $\Longrightarrow$ *all-consts* $|\subseteq|$ *S* $\Longrightarrow$ $\neg$ *shadows-consts t* $\Longrightarrow$ *related-exp t* (*mk-exp S t*)
ids (*Sabs cs*) $|\subseteq|$ *S* $\Longrightarrow$ *all-consts* $|\subseteq|$ *S* $\Longrightarrow$ $\neg$ *shadows-consts* (*Sabs cs*) $\Longrightarrow$ *list-all2* (*rel-prod related-pat related-exp*) *cs* (*mk-clauses S cs*)
ids t $|\subseteq|$ *S* $\Longrightarrow$ *all-consts* $|\subseteq|$ *S* $\Longrightarrow$ $\neg$ *shadows-consts t* $\Longrightarrow$ *related-exp t* (*mk-con S t*)
proof (*induction rule*: *mk-exp-mk-clauses-mk-con.induct*)
case (*2 S name*)
show *?case*

```
    proof (cases name |∈| C)
      case True
      hence related-exp (name $$ []) (mk-exp S (Sconst name))
        by (auto intro: related-exp.intros simp del: list-comb.simps)
      thus ?thesis
        by (simp add: const-sterm-def)
    qed (auto intro: related-exp.intros)
next
  case (4 S cs)

  have fresh-fNext (S |∪| all-consts) |∉| S |∪| all-consts
    by (rule fNext-not-member)
  hence fresh-fNext S |∉| S |∪| all-consts
    using ‹all-consts |⊆| S› by (simp add: sup-absorb1)
  hence fresh-fNext S |∉| ids (Sabs cs) |∪| all-consts
    using 4 by auto

  show ?case
    apply (simp add: Let-def)
    apply (rule related-exp.fun)
      apply (rule 4.IH[unfolded mk-clauses.simps])
         apply (rule refl)
        apply fact+
    using ‹fresh-fNext S |∉| ids (Sabs cs) |∪| all-consts› by auto
next
  case (5 S t)
  show ?case
    apply (simp add: mk-con.simps split!: prod.splits sterm.splits if-splits)
    subgoal premises prems for args c
      proof −
        from prems have t = c $$ args
          apply (fold const-sterm-def)
          by (metis fst-conv list-strip-comb snd-conv)
        show ?thesis
          unfolding ‹t = -›
          apply (rule related-exp.constr)
           apply fact
          apply (simp add: list.rel-map)
          apply (rule list.rel-refl-strong)
          apply (rule 5(1))
                apply (rule prems(1)[symmetric])
               apply (rule refl)
          subgoal by (rule prems)
          subgoal by assumption
          subgoal
            using ‹ids t |⊆| S› unfolding ‹t = -›
            apply (auto simp: ids-list-comb)
            by (meson ffUnion-subset-elem fimage-eqI fset-of-list-elem fset-rev-mp)
          subgoal by (rule 5)
```

```
        subgoal
          using ‹¬ shadows-consts t› unfolding ‹t = -›
          unfolding shadows.list-comb
          by (auto simp: list-ex-iff)
        done
      qed
    using 5 by (auto split: prod.splits sterm.splits)
next
  case (6 S cs)

  have list-all2 (λx y. rel-prod related-pat related-exp x (case y of (pat, t) ⇒
(mk-ml-pat (mk-pat pat), mk-con (frees pat |∪| S) t))) cs cs
    proof (rule list.rel-refl-strong, safe intro!: rel-prod.intros)
      fix pat rhs
      assume (pat, rhs) ∈ set cs

      hence consts rhs |⊆| S
        using ‹ids (Sabs cs) |⊆| S›
        unfolding ids-def
        apply auto
        apply (drule ffUnion-least-rev)+
        apply (auto simp: fset-of-list-elem elim!: fBallE)
        done

      have frees rhs |⊆| frees pat |∪| S
        using ‹ids (Sabs cs) |⊆| S› ‹(pat, rhs) ∈ set cs›
        unfolding ids-def
        apply auto
        apply (drule ffUnion-least-rev)+
        apply (auto simp: fset-of-list-elem elim!: fBallE)
        done

      have ¬ shadows-consts rhs
        using ‹(pat, rhs) ∈ set cs› 6
        by (auto simp: list-ex-iff)

      show related-exp rhs (mk-con (frees pat |∪| S) rhs)
        apply (rule 6)
            apply fact
        subgoal by simp
        subgoal
          unfolding ids-def
          using ‹consts rhs |⊆| S› ‹frees rhs |⊆| frees pat |∪| S›
          by auto
        subgoal using 6(3) by auto
        subgoal by fact
        done
    qed
```

thus *?case*
  by (*simp add*: *list.rel-map*)
qed (*auto intro*: *related-exp.intros simp*: *ids-def fdisjnt-alt-def*)

context begin

private lemma *semantic-correctness0*:
  fixes *exp*
  assumes *cupcake-evaluate-single env exp r is-cupcake-all-env env*
  assumes *fmrel-on-fset* (*ids t*) *related-v* Γ (*fmap-of-ns* (*sem-env.v env*))
  assumes *related-exp t exp*
  assumes *wellformed t wellformed-venv* Γ
  assumes *closed-venv* Γ *closed-except t* (*fmdom* Γ)
  assumes *fmpred* (λ-. *vwelldefined′*) Γ *consts t* |⊆| *fmdom* Γ |∪| *C*
  assumes *fdisjnt C* (*fmdom* Γ)
  assumes ¬ *shadows-consts t not-shadows-vconsts-env* Γ
  shows *if-rval* (λ*ml-v*. ∃ *v*. $\Gamma \vdash_v$ *t* ↓ *v* ∧ *related-v v ml-v*) *r*
using *assms* **proof** (*induction arbitrary*: Γ *t*)
  case (*con1 env cn es ress ml-vs ml-v′*)
  obtain *name ts* **where** *cn* = *Some* (*Short* (*as-string name*)) *name* |∈| *C t* = *name* $$ *ts list-all2 related-exp ts es*
    using ‹*related-exp t* (*Con cn es*)›
    by *cases auto*
  with *con1* **obtain** *tid* **where** *ml-v′* = *Conv* (*Some* (*id-to-n* (*Short* (*as-string name*)), *tid*)) (*rev ml-vs*)
    by (*auto split*: *option.splits*)

  have *ress* = *map Rval ml-vs*
    using *con1* **by** *auto*

  define *ml-vs′* **where** *ml-vs′* = *rev ml-vs*

  note *IH* = ‹*list-all2-shortcircuit* - - -›[
      unfolded ‹*ress* = -› *list-all2-shortcircuit-rval list-all2-rev1*,
      folded ml-vs′-def]
  moreover have
    list-all wellformed ts list-all (λ*t*. ¬ *shadows-consts t*) *ts*
    list-all (λ*t*. *consts t* |⊆| *fmdom* Γ |∪| *C*) *ts list-all* (λ*t*. *closed-except t* (*fmdom* Γ)) *ts*
    subgoal
      using ‹*wellformed t*› **unfolding** ‹*t* = -› *wellformed.list-comb* **by** *simp*
    subgoal
      using ‹¬ *shadows-consts t*› **unfolding** ‹*t* = -› *shadows.list-comb*
      by (*simp add*: *list-all-iff list-ex-iff*)
    subgoal
      using ‹*consts t* |⊆| *fmdom* Γ |∪| *C*›
      unfolding *list-all-iff*
    by (*metis Ball-set* ‹*t* = *name* $$ *ts*› *con1.prems*(*9*) *special-constants.sconsts-list-comb*)
    subgoal

using ‹*closed-except t* (*fmdom* Γ)› **unfolding** ‹*t* = -› *closed.list-comb* **by** *simp*
done
moreover have
list-all (λ*t*′. *fmrel-on-fset* (*ids t*′) *related-v* Γ (*fmap-of-ns* (*sem-env.v env*))) *ts*
proof (*standard*, *rule fmrel-on-fsubset*)
fix *t*′
assume *t*′ ∈ *set ts*
thus *ids t*′ |⊆| *ids t*
unfolding ‹*t* = -›
apply (*simp add*: *ids-list-comb*)
apply (*subst* (*2*) *ids-def*)
apply *simp*
apply (*rule fsubset-finsertI2*)
apply (*auto simp*: *fset-of-list-elem intro*!: *ffUnion-subset-elem*)
done
show *fmrel-on-fset* (*ids t*) *related-v* Γ (*fmap-of-ns* (*sem-env.v env*))
by *fact*
qed

ultimately obtain *us* **where** *list-all2* (*veval*′ Γ) *ts us list-all2 related-v us ml-vs*′
using ‹*list-all2 related-exp ts es*›
proof (*induction es ml-vs*′ *arbitrary*: *ts thesis rule*: *list.rel-induct*)
case (*Cons e es ml-v ml-vs ts0*)
then obtain *t ts* **where** *ts0* = *t* # *ts related-exp t e* **by** (*cases ts0*) *auto*
with *Cons* **have** *list-all2 related-exp ts es* **by** *simp*
with *Cons* **obtain** *us* **where** *list-all2* (*veval*′ Γ) *ts us list-all2 related-v us ml-vs*
unfolding ‹*ts0* = -›
by *auto*

from *Cons.hyps*[*simplified*, *THEN conjunct2*, *rule-format*, *of t* Γ]
obtain *u* **where** Γ $\vdash_v$ *t* ↓ *u* *related-v u ml-v*
proof
show
is-cupcake-all-env env related-exp t e wellformed-venv Γ *closed-venv* Γ
fmpred (λ-. *vwelldefined*′) Γ *fdisjnt C* (*fmdom* Γ)
not-shadows-vconsts-env Γ
by *fact*+
next
show
wellformed t ¬ *shadows-consts t closed-except t* (*fmdom* Γ)
consts t |⊆| *fmdom* Γ |∪| *C fmrel-on-fset* (*ids t*) *related-v* Γ (*fmap-of-ns* (*sem-env.v env*))
using *Cons* **unfolding** ‹*ts0* = -›
by *auto*
qed *blast*

show *?case*
apply (*rule Cons*(*3*)[*of u* # *us*])

```
      unfolding ‹ts0 = -›
       apply auto
         apply fact+
      done
  qed auto

  show ?case
    apply simp
    apply (intro exI conjI)
    unfolding ‹t = -›
     apply (rule veval'.constr)
      apply fact+
    unfolding ‹ml-v' = -›
    apply (subst ml-vs'-def[symmetric])
    apply simp
    apply (rule related-v.conv)
    apply fact
    done
next
  case (var1 env id ml-v)

  from ‹related-exp t (Var id)› obtain name where id = Short (as-string name)
    by cases auto
   with var1 have cupcake-nsLookup (sem-env.v env) (as-string name) = Some
ml-v
    by auto

  from ‹related-exp t (Var id)› consider
      (var) t = Svar name
    | (const) t = Sconst name name |∉| C
    unfolding ‹id = -›
    apply (cases t)
    using name.expand by blast+
  thus ?case
    proof cases
      case var
      hence name |∈| ids t
        unfolding ids-def by simp

       have rel-option related-v (fmlookup Γ name) (cupcake-nsLookup (sem-env.v
env) (as-string name))
        using ‹fmrel-on-fset (ids t) - - -›
        apply −
        apply (drule fmrel-on-fsetD[OF ‹name |∈| ids t›])
        apply simp
        done
      then obtain v where related-v v ml-v fmlookup Γ name = Some v
        using ‹cupcake-nsLookup (sem-env.v env) - = -›
        by cases auto
```

```
      show ?thesis
        unfolding ‹t = -›
        apply simp
        apply (rule exI)
        apply (rule conjI)
         apply (rule veval'.var)
         apply fact+
        done
    next
      case const
      hence name |∈| ids t
        unfolding ids-def by simp

      have rel-option related-v (fmlookup Γ name) (cupcake-nsLookup (sem-env.v env) (as-string name))
        using ‹fmrel-on-fset (ids t) - - -›
        apply –
        apply (drule fmrel-on-fsetD[OF ‹name |∈| ids t›])
        apply simp
        done
      then obtain v where related-v v ml-v fmlookup Γ name = Some v
        using ‹cupcake-nsLookup (sem-env.v env) - = -›
        by cases auto

      show ?thesis
        unfolding ‹t = -›
        apply simp
        apply (rule exI)
        apply (rule conjI)
         apply (rule veval'.const)
          apply fact+
        done
    qed
next
  case (fn env n u)
  obtain n' where n = as-string n'
    by (metis name.sel)
  obtain cs ml-cs
    where t = Sabs cs u = Mat (Var (Short (as-string n'))) ml-cs n' |∉| ids (Sabs cs) n' |∉| all-consts
      and list-all2 (rel-prod related-pat related-exp) cs ml-cs
    using ‹related-exp t (Fun n u)› unfolding ‹n = -›
    by cases (auto dest: name.expand)

  obtain ns where fmap-of-ns (sem-env.v env) = fmap-of-list ns
    apply (cases env)
    apply simp
    subgoal for v by (cases v) simp
```

done

show *?case*
apply *simp*
apply (*rule exI*)
apply (*rule conjI*)
unfolding ‹t = -›
apply (*rule veval'.abs*)
unfolding ‹n = -›
apply (*rule related-v.closure*)
unfolding ‹u = -›
apply (*subst related-fun-alt-def*; *rule conjI*)
apply *fact*
apply (*rule conjI*; *fact*)
using ‹*fmrel-on-fset* (*ids t*) - - -›
unfolding ‹t = -› ‹*fmap-of-ns* (*sem-env.v env*) = -›
by *simp*
next
case (*app1 env exps ress ml-vs env' exp' bv*)
from ‹*related-exp t* -› **obtain** exp_1 exp_2 t_1 t_2
where *rev exps* = [exp_2, exp_1] *exps* = [exp_1, exp_2] $t = t_1$ $\$_s$ t_2
and *related-exp* t_1 exp_1 *related-exp* t_2 exp_2
by *cases auto*
moreover from *app1* **have** *ress* = *map Rval ml-vs*
by *auto*
ultimately obtain *ml*-v_1 *ml*-v_2 **where** *ml-vs* = [*ml*-v_2, *ml*-v_1]
using *app1*(*1*)
by (*smt list-all2-shortcircuit-rval list-all2-Cons1 list-all2-Nil*)

have *is-cupcake-exp* exp_1 *is-cupcake-exp* exp_2
using *app1* **unfolding** ‹*exps* = -› **by** (*auto dest*: *related-exp-is-cupcake*)
moreover have *fmrel-on-fset* (*ids* t_1) *related-v* Γ (*fmap-of-ns* (*sem-env.v env*))
using *app1* **unfolding** *ids-def* ‹t = -›
by (*auto intro*: *fmrel-on-fsubset*)
moreover have *fmrel-on-fset* (*ids* t_2) *related-v* Γ (*fmap-of-ns* (*sem-env.v env*))
using *app1* **unfolding** *ids-def* ‹t = -›
by (*auto intro*: *fmrel-on-fsubset*)
ultimately have
cupcake-evaluate-single env exp_1 (*Rval ml*-v_1) *cupcake-evaluate-single env* exp_2 (*Rval ml*-v_2) **and**
$\exists\, t_1'.\ \Gamma \vdash_v t_1 \downarrow t_1' \wedge$ *related-v* t_1' *ml*-v_1 $\exists\, t_2'.\ \Gamma \vdash_v t_2 \downarrow t_2' \wedge$ *related-v* t_2' *ml*-v_2
using *app1* ‹*related-exp* t_1 exp_1› ‹*related-exp* t_2 exp_2›
unfolding ‹*ress* = -› ‹*exps* = -› ‹*ml-vs* = -› ‹t = -›
by (*auto simp*: *closed-except-def*)

then obtain v_1 v_2
where $\Gamma \vdash_v t_1 \downarrow v_1$ *related-v* v_1 *ml*-v_1
and $\Gamma \vdash_v t_2 \downarrow v_2$ *related-v* v_2 *ml*-v_2
by *blast*

```
have is-cupcake-value ml-v₁
  by (rule cupcake-single-preserve) fact+
moreover have is-cupcake-value ml-v₂
  by (rule cupcake-single-preserve) fact+
ultimately have list-all is-cupcake-value (rev ml-vs)
  unfolding ‹ml-vs = -› by simp

hence is-cupcake-exp exp′ is-cupcake-all-env env′
  using ‹do-opapp - = -› by (metis cupcake-opapp-preserve)+

have vclosed v₁
  proof (rule veval′-closed)
    show closed-except t₁ (fmdom Γ)
      using ‹closed-except - (fmdom Γ)›
      unfolding ‹t = -›
      by (simp add: closed-except-def)
  next
    show wellformed t₁
      using ‹wellformed t› unfolding ‹t = -›
      by simp
  qed fact+
have vclosed v₂
  apply (rule veval′-closed)
      apply fact
  using app1 unfolding ‹t = -› by (auto simp: closed-except-def)

have vwellformed v₁
  apply (rule veval′-wellformed)
    apply fact
  using app1 unfolding ‹t = -› by auto
have vwellformed v₂
  apply (rule veval′-wellformed)
    apply fact
  using app1 unfolding ‹t = -› by auto

have vwelldefined′ v₁
  apply (rule veval′-welldefined′)
        apply fact
  using app1 unfolding ‹t = -› by auto
have vwelldefined′ v₂
  apply (rule veval′-welldefined′)
        apply fact
  using app1 unfolding ‹t = -› by auto

have not-shadows-vconsts v₁
  apply (rule veval′-shadows)
    apply fact
  using app1 unfolding ‹t = -› by auto
```

have *not-shadows-vconsts* v_2
apply (*rule veval′-shadows*)
apply *fact*
using *app1* **unfolding** ‹t = -› **by** *auto*

show *?case*
proof (*rule if-rvalI*)
fix *ml-v*
assume *bv* = *Rval ml-v*
show $\exists\, v.\ \Gamma \vdash_v t \downarrow v \wedge$ *related-v v ml-v*
using ‹*do-opapp* - = -›
proof (*cases rule*: *do-opapp-cases*)
case (*closure* env_Λ *n*)
then have *closure′*:
$ml\text{-}v_1$ = *Closure* env_Λ (*as-string* (*Name n*)) *exp′*
env′ = *update-v* (λ-. *nsBind* (*as-string* (*Name n*)) $ml\text{-}v_2$ (*sem-env.v* env_Λ)) env_Λ
unfolding ‹*ml-vs* = -› **by** *auto*
obtain Γ_Λ *cs*
where v_1 = *Vabs cs* Γ_Λ *related-fun cs* (*Name n*) *exp′*
and *fmrel-on-fset* (*ids* (*Sabs cs*)) *related-v* Γ_Λ (*fmap-of-ns* (*sem-env.v* env_Λ))
using ‹*related-v* v_1 $ml\text{-}v_1$› **unfolding** ‹$ml\text{-}v_1$ = -›
by *cases auto*

then obtain *ml-cs*
where *exp′* = *Mat* (*Var* (*Short* (*as-string* (*Name n*)))) *ml-cs Name n* |∉| *ids* (*Sabs cs*) *Name n* |∉| *all-consts*
and *list-all2* (*rel-prod related-pat related-exp*) *cs ml-cs*
by (*auto elim*: *related-funE*)

hence *cupcake-evaluate-single env′* (*Mat* (*Var* (*Short* (*as-string* (*Name n*)))) *ml-cs*) (*Rval ml-v*)
using ‹*cupcake-evaluate-single env′ exp′ bv*›
unfolding ‹*bv* = -›
by *simp*

then obtain *m-env v′ ml-rhs ml-pat*
where *cupcake-evaluate-single env′* (*Var* (*Short* (*as-string* (*Name n*)))) (*Rval v′*)
and *cupcake-match-result* (*sem-env.c env′*) *v′ ml-cs Bindv* = *Rval* (*ml-rhs*, *ml-pat*, *m-env*)
and *cupcake-evaluate-single* (*env′* ⦇ *sem-env.v* := *nsAppend* (*alist-to-ns m-env*) (*sem-env.v env′*) ⦈) *ml-rhs* (*Rval ml-v*)
by *cases auto*

have
closed-venv (*fmupd* (*Name n*) v_2 Γ_Λ) *wellformed-venv* (*fmupd* (*Name n*) v_2 Γ_Λ)

not-shadows-vconsts-env (*fmupd* (*Name n*) v_2 Γ_Λ) *fmpred* (λ-. *vwelldefined'*) (*fmupd* (*Name n*) v_2 Γ_Λ)
using ‹*vclosed* v_1› ‹*vclosed* v_2›
using ‹*vwellformed* v_1› ‹*vwellformed* v_2›
using ‹*not-shadows-vconsts* v_1› ‹*not-shadows-vconsts* v_2›
using ‹*vwelldefined'* v_1› ‹*vwelldefined'* v_2›
unfolding ‹v_1 = -›
by *auto*

have *closed-except* (*Sabs cs*) (*fmdom* (*fmupd* (*Name n*) v_2 Γ_Λ))
using ‹*vclosed* v_1› **unfolding** ‹v_1 = -›
apply (*auto simp*: *Sterm.closed-except-simps list-all-iff*)
apply (*auto simp*: *closed-except-def*)
done

have *consts* (*Sabs cs*) |⊆| *fmdom* (*fmupd* (*Name n*) v_2 Γ_Λ) |∪| *C*
using ‹*vwelldefined'* v_1› **unfolding** ‹v_1 = -›
unfolding *sconsts-sabs*
by (*auto simp*: *list-all-iff*)

have ¬ *shadows-consts* (*Sabs cs*)
using ‹*not-shadows-vconsts* v_1› **unfolding** ‹v_1 = -›
by (*auto simp*: *list-all-iff list-ex-iff*)

have *fdisjnt C* (*fmdom* Γ_Λ)
using ‹*vwelldefined'* v_1› **unfolding** ‹v_1 = -›
by *simp*

have *if-rval* (λ*ml-v*. ∃ *v*. *fmupd* (*Name n*) v_2 Γ_Λ $\vdash_v$ *Sabs cs* $\$_s$ *Svar* (*Name n*) ↓ *v* ∧ *related-v v ml-v*) *bv*
proof (*rule app1*(*2*))
show *fmrel-on-fset* (*ids* (*Sabs cs* $\$_s$ *Svar* (*Name n*))) *related-v* (*fmupd* (*Name n*) v_2 Γ_Λ) (*fmap-of-ns* (*sem-env.v env'*))
unfolding *closure'*
apply (*simp del*: *frees-sterm.simps*(*3*) *consts-sterm.simps*(*3*) *name.sel add*: *ids-def split*!: *sem-env.splits*)
apply (*rule fmrel-on-fset-updateI*)
apply (*fold ids-def*)
using ‹*fmrel-on-fset* (*ids* (*Sabs cs*)) *related-v* Γ_Λ -› **apply** *simp*
apply (*rule* ‹*related-v* v_2 *ml-*v_2›)
done
next
show *wellformed* (*Sabs cs* $\$_s$ *Svar* (*Name n*))
using ‹*vwellformed* v_1› **unfolding** ‹v_1 = -›
by *simp*
next
show *related-exp* (*Sabs cs* $\$_s$ *Svar* (*Name n*)) *exp'*
unfolding ‹*exp'* = -›
using ‹*list-all2* (*rel-prod related-pat related-exp*) *cs ml-cs*›

by (*auto intro*:*related-exp.intros simp del*: *name.sel*)
next
show *closed-except* (*Sabs cs* $\$_s$ *Svar* (*Name n*)) (*fmdom* (*fmupd* (*Name n*) v_2 Γ_Λ))
using ‹*closed-except* (*Sabs cs*) (*fmdom* (*fmupd* (*Name n*) v_2 Γ_Λ))› **by** (*simp add*: *closed-except-def*)
next
show ¬ *shadows-consts* (*Sabs cs* $\$_s$ *Svar* (*Name n*))
using ‹¬ *shadows-consts* (*Sabs cs*)› ‹*Name n* |∉| *all-consts*› **by** *simp*
next
show *consts* (*Sabs cs* $\$_s$ *Svar* (*Name n*)) |⊆| *fmdom* (*fmupd* (*Name n*) v_2 Γ_Λ) |∪| *C*
using ‹*consts* (*Sabs cs*) |⊆| *fmdom* (*fmupd* (*Name n*) v_2 Γ_Λ) |∪| *C*› **by** *simp*
next
show *fdisjnt C* (*fmdom* (*fmupd* (*Name n*) v_2 Γ_Λ))
using ‹*Name n* |∉| *all-consts*› ‹*fdisjnt C* (*fmdom* Γ_Λ)›
unfolding *fdisjnt-alt-def all-consts-def* **by** *auto*
qed *fact+*
then obtain *v* **where** *fmupd* (*Name n*) v_2 Γ_Λ $\vdash_v$ *Sabs cs* $\$_s$ *Svar* (*Name n*) ↓ *v related-v v ml-v*
unfolding ‹*bv* = -›
by *auto*

then obtain *env pat rhs*
where *vfind-match cs* v_2 = *Some* (*env*, *pat*, *rhs*)
and *fmupd* (*Name n*) v_2 Γ_Λ $++_f$ *env* $\vdash_v$ *rhs* ↓ *v*
by (*auto elim*: *veval′-sabs-svarE*)
hence (*pat*, *rhs*) ∈ *set cs vmatch* (*mk-pat pat*) v_2 = *Some env*
by (*metis vfind-match-elem*)+
hence *linear pat wellformed rhs*
using ‹*vwellformed* v_1› **unfolding** ‹v_1 = -›
by (*auto simp*: *list-all-iff*)
hence *frees pat* = *patvars* (*mk-pat pat*)
by (*simp add*: *mk-pat-frees*)
hence *fmdom env* = *frees pat*
apply *simp*
apply (*rule vmatch-dom*)
apply (*rule* ‹*vmatch* (*mk-pat pat*) v_2 = *Some env*›)
done

obtain v' **where** Γ_Λ $++_f$ *env* $\vdash_v$ *rhs* ↓ v' $v' \approx_e v$
proof (*rule veval′-agree-eq*)
show *fmupd* (*Name n*) v_2 Γ_Λ $++_f$ *env* $\vdash_v$ *rhs* ↓ *v* **by** *fact*
next
have ∗: *Name n* |∉| *ids rhs* **if** *Name n* |∉| *fmdom env*
proof
assume *Name n* |∈| *ids rhs*
thus *False*

```
          unfolding ids-def
          proof (cases rule: funion-strictE)
            case A
            moreover have Name n |∉| frees pat
              using that unfolding ‹fmdom env = frees pat› .
            ultimately have Name n |∈| frees (Sabs cs)
              apply auto
              unfolding ffUnion-alt-def
              apply simp
              apply (rule fBexI[where x = (pat, rhs)])
               apply (auto simp: fset-of-list-elem)
              apply (rule ‹(pat, rhs) ∈ set cs›)
              done
            thus ?thesis
              using ‹Name n |∉| ids (Sabs cs)› unfolding ids-def
              by blast
          next
            case B
            hence Name n |∈| consts (Sabs cs)
              apply auto
              unfolding ffUnion-alt-def
              apply simp
              apply (rule fBexI[where x = (pat, rhs)])
               apply (auto simp: fset-of-list-elem)
              apply (rule ‹(pat, rhs) ∈ set cs›)
              done
            thus ?thesis
              using ‹Name n |∉| ids (Sabs cs)› unfolding ids-def
              by blast
          qed
        qed

      show fmrel-on-fset (ids rhs) erelated (Γ_Λ ++_f env) (fmupd (Name n)
v2 Γ_Λ ++_f env)
        apply rule
        apply auto
           apply (rule option.rel-refl; rule erelated-refl)
        using * apply auto[]
         apply (rule option.rel-refl; rule erelated-refl)+
        done
    next
      show closed-venv (fmupd (Name n) v2 Γ_Λ ++_f env)
        apply rule
         apply fact
        apply (rule vclosed.vmatch-env)
         apply fact
        apply fact
        done
    next
```

show *wellformed-venv* (*fmupd* (*Name n*) v_2 Γ_Λ $++_f$ *env*)
apply *rule*
apply *fact*
apply (*rule vwellformed.vmatch-env*)
apply *fact*
apply *fact*
done
next
show *closed-except rhs* (*fmdom* (*fmupd* (*Name n*) v_2 Γ_Λ $++_f$ *env*))
using ‹*fmdom env* = *frees pat*› ‹(*pat*, *rhs*) ∈ *set cs*›
using ‹*closed-except* (*Sabs cs*) (*fmdom* (*fmupd* (*Name n*) v_2 Γ_Λ))›
by (*auto simp*: *Sterm.closed-except-simps list-all-iff*)
next
show *wellformed rhs* **by** *fact*
next
show *consts rhs* |⊆| *fmdom* (*fmupd* (*Name n*) v_2 Γ_Λ $++_f$ *env*) |∪| *C*
using ‹*consts* (*Sabs cs*) |⊆| *fmdom* (*fmupd* (*Name n*) v_2 Γ_Λ) |∪| *C*›
‹(*pat*, *rhs*) ∈ *set cs*›
unfolding *sconsts-sabs*
by (*auto simp*: *list-all-iff*)
next
have *fdisjnt C* (*fmdom env*)
using ‹(*pat*, *rhs*) ∈ *set cs*› ‹¬ *shadows-consts* (*Sabs cs*)›
unfolding ‹*fmdom env* = *frees pat*›
by (*auto simp*: *list-ex-iff fdisjnt-alt-def all-consts-def*)
thus *fdisjnt C* (*fmdom* (*fmupd* (*Name n*) v_2 Γ_Λ $++_f$ *env*))
using ‹*Name n* |∉| *all-consts*› ‹*fdisjnt C* (*fmdom* Γ_Λ)›
unfolding *fdisjnt-alt-def*
by (*auto simp*: *all-consts-def*)
next
show ¬ *shadows-consts rhs*
using ‹(*pat*, *rhs*) ∈ *set cs*› ‹¬ *shadows-consts* (*Sabs cs*)›
by (*auto simp*: *list-ex-iff*)
next
have *not-shadows-vconsts-env env*
by (*rule not-shadows-vconsts.vmatch-env*) *fact+*
thus *not-shadows-vconsts-env* (*fmupd* (*Name n*) v_2 Γ_Λ $++_f$ *env*)
using ‹*not-shadows-vconsts-env* (*fmupd* (*Name n*) v_2 Γ_Λ)› **by** *blast*
next
have *fmpred* (λ-. *vwelldefined′*) *env*
by (*rule vmatch-welldefined′*) *fact+*
thus *fmpred* (λ-. *vwelldefined′*) (*fmupd* (*Name n*) v_2 Γ_Λ $++_f$ *env*)
using ‹*fmpred* (λ-. *vwelldefined′*) (*fmupd* (*Name n*) v_2 Γ_Λ)› **by** *blast*
qed *blast*

show *?thesis*
apply (*intro exI conjI*)
unfolding ‹*t* = -›
apply (*rule veval′.comb*)

using ‹$\Gamma \vdash_v t_1 \downarrow v_1$› **unfolding** ‹$v_1$ = -›
apply *blast*
apply *fact*
apply *fact+*
apply (*rule related-v-ext*)
apply *fact+*
done
next
case (*recclosure env*$_\Lambda$ *funs name n*)
with *recclosure* **have** *recclosure′*:
ml-v$_1$ = *Recclosure env*$_\Lambda$ *funs name*
env′ = *update-v* (λ-. *nsBind* (*as-string* (*Name n*)) *ml-v*$_2$ (*build-rec-env funs env*$_\Lambda$ (*sem-env.v env*$_\Lambda$))) *env*$_\Lambda$
unfolding ‹*ml-vs* = -› **by** *auto*
obtain Γ_Λ *css*
where v_1 = *Vrecabs css* (*Name name*) Γ_Λ
and *fmrel-on-fset* (*fbind* (*fmran css*) (*ids* ∘ *Sabs*)) *related-v* Γ_Λ (*fmap-of-ns* (*sem-env.v env*$_\Lambda$))
and *fmrel* (λ*cs* (*n*, *e*). *related-fun cs n e*) *css* (*fmap-of-list* (*map* (*map-prod Name* (*map-prod Name id*)) *funs*))
using ‹*related-v* v_1 *ml-v*$_1$› **unfolding** ‹*ml-v*$_1$ = -›
by *cases auto*
from ‹*fmrel* - - -› **have** *rel-option* (λ*cs* (*n*, *e*). *related-fun cs* (*Name n*) *e*) (*fmlookup css* (*Name name*)) (*find-recfun name funs*)
apply −
apply (*subst option.rel-sel*)
apply *auto*
apply (*drule fmrel-fmdom-eq*)
apply (*drule fmdom-notI*)
using ‹v_1 = *Vrecabs css* (*Name name*) Γ_Λ› ‹*vwellformed* v_1› **apply** *auto*[*1*]
using *recclosure*(*3*) **apply** *auto*[*1*]
apply (*erule fmrel-cases*[**where** *x* = *Name name*])
apply *simp*
apply (*subst* (*asm*) *fmlookup-of-list*)
apply (*simp add*: *name.map-of-rekey′*)
by *blast*

then obtain *cs* **where** *fmlookup css* (*Name name*) = *Some cs related-fun cs* (*Name n*) *exp′*
using ‹*find-recfun* - - = -›
by *cases auto*

then obtain *ml-cs*
where *exp′* = *Mat* (*Var* (*Short* (*as-string* (*Name n*)))) *ml-cs Name n* |∉| *ids* (*Sabs cs*) *Name n* |∉| *all-consts*
and *list-all2* (*rel-prod related-pat related-exp*) *cs ml-cs*
by (*auto elim*: *related-funE*)

hence *cupcake-evaluate-single env′* (*Mat* (*Var* (*Short n*)) *ml-cs*) (*Rval ml-v*)
using ‹*cupcake-evaluate-single env′ exp′ bv*›
unfolding ‹*bv* = -›
by *simp*

then obtain *m-env v′ ml-rhs ml-pat*
where *cupcake-evaluate-single env′* (*Var* (*Short n*)) (*Rval v′*)
and *cupcake-match-result* (*sem-env.c env′*) *v′ ml-cs Bindv* = *Rval* (*ml-rhs*, *ml-pat*, *m-env*)
and *cupcake-evaluate-single* (*env′* ⦇ *sem-env.v* := *nsAppend* (*alist-to-ns m-env*) (*sem-env.v env′*) ⦈) *ml-rhs* (*Rval ml-v*)
by *cases auto*

have *closed-venv* (*fmupd* (*Name n*) v_2 (Γ_Λ $++_f$ *mk-rec-env css* Γ_Λ))
using ‹*vclosed* v_1› ‹*vclosed* v_2›
using ‹*fmlookup css* (*Name name*) = *Some cs*›
unfolding ‹v_1 = -› *mk-rec-env-def*
apply *auto*
apply *rule*
apply *rule*
apply (*auto intro*: *fmdomI*)
done
have *wellformed-venv* (*fmupd* (*Name n*) v_2 (Γ_Λ $++_f$ *mk-rec-env css* Γ_Λ))
using ‹*vwellformed* v_1› ‹*vwellformed* v_2›
using ‹*fmlookup css* (*Name name*) = *Some cs*›
unfolding ‹v_1 = -› *mk-rec-env-def*
apply *auto*
apply *rule*
apply *rule*
apply (*auto intro*: *fmdomI*)
done
have *not-shadows-vconsts-env* (*fmupd* (*Name n*) v_2 (Γ_Λ $++_f$ *mk-rec-env css* Γ_Λ))
using ‹*not-shadows-vconsts* v_1› ‹*not-shadows-vconsts* v_2›
using ‹*fmlookup css* (*Name name*) = *Some cs*›
unfolding ‹v_1 = -› *mk-rec-env-def*
apply *auto*
apply *rule*
apply *rule*
apply (*auto intro*: *fmdomI*)
done
have *fmpred* (λ-. *vwelldefined′*) (*fmupd* (*Name n*) v_2 (Γ_Λ $++_f$ *mk-rec-env css* Γ_Λ))
using ‹*vwelldefined′* v_1› ‹*vwelldefined′* v_2›
using ‹*fmlookup css* (*Name name*) = *Some cs*›
unfolding ‹v_1 = -› *mk-rec-env-def*
apply *auto*
apply *rule*

apply *rule*
apply (*auto intro*: *fmdomI*)
done

have *closed-except* (*Sabs cs*) (*fmdom* (*fmupd* (*Name n*) v_2 Γ_Λ))
using ‹*vclosed* v_1› **unfolding** ‹v_1 = -›
apply (*auto simp*: *Sterm.closed-except-simps list-all-iff*)
using ‹*fmlookup css* (*Name name*) = *Some cs*›
apply (*auto simp*: *closed-except-def dest*!: *fmpredD*[**where** *m* = *css*])
done

have *consts* (*Sabs cs*) |⊆| *fmdom* (*fmupd* (*Name n*) v_2 Γ_Λ) |∪| (*C* |∪| *fmdom css*)
using ‹*vwelldefined'* v_1› **unfolding** ‹v_1 = -›
unfolding *sconsts-sabs*
using ‹*fmlookup css* (*Name name*) = *Some cs*›
by (*auto simp*: *list-all-iff dest*!: *fmpredD*[**where** *m* = *css*])

have ¬ *shadows-consts* (*Sabs cs*)
using ‹*not-shadows-vconsts* v_1› **unfolding** ‹v_1 = -›
using ‹*fmlookup css* (*Name name*) = *Some cs*›
by (*auto simp*: *list-all-iff list-ex-iff*)

have *fdisjnt C* (*fmdom* Γ_Λ)
using ‹*vwelldefined'* v_1› **unfolding** ‹v_1 = -›
using ‹*fmlookup css* (*Name name*) = *Some cs*›
by *auto*

have *if-rval* (*λml-v*. ∃ *v*. *fmupd* (*Name n*) v_2 (Γ_Λ $++_f$ *mk-rec-env css* Γ_Λ) $\vdash_v$ *Sabs cs* $\$_s$ *Svar* (*Name n*) ↓ *v* ∧ *related-v v ml-v*) *bv*
proof (*rule app1*(*2*))
have *fmrel-on-fset* (*ids* (*Sabs cs*)) *related-v* Γ_Λ (*fmap-of-ns* (*sem-env.v* env_Λ))
apply (*rule fmrel-on-fsubset*)
apply *fact*
apply (*subst funion-image-bind-eq*[*symmetric*])
apply (*rule ffUnion-subset-elem*)
apply (*subst fimage-iff*)
apply (*rule fBexI*)
apply *simp*
apply (*rule fmranI*)
apply *fact*
done

have *fmrel-on-fset* (*ids* (*Sabs cs*)) *related-v* (*mk-rec-env css* Γ_Λ) (*cake-mk-rec-env funs* env_Λ)
apply *rule*
apply (*rule mk-rec-env-related*[*THEN fmrelD*])
apply (*rule* ‹*fmrel - css -*›)

apply (*rule* ‹*fmrel-on-fset* (*fbind* - -) *related-v* Γ_Λ -›)
done

show *fmrel-on-fset* (*ids* (*Sabs cs* $\$_s$ *Svar* (*Name n*))) *related-v* (*fmupd* (*Name n*) v_2 (Γ_Λ $++_f$ *mk-rec-env css* Γ_Λ)) (*fmap-of-ns* (*sem-env.v env*′))
unfolding *recclosure*′
apply (*simp del*: *frees-sterm.simps*(*3*) *consts-sterm.simps*(*3*) *name.sel add*: *ids-def split*!: *sem-env.splits*)
apply (*rule fmrel-on-fset-updateI*)
unfolding *build-rec-env-fmap*
apply (*rule fmrel-on-fset-addI*)
apply (*fold ids-def*)
subgoal
using ‹*fmrel-on-fset* (*ids* (*Sabs cs*)) *related-v* Γ_Λ -› **by** *simp*
subgoal
using ‹*fmrel-on-fset* (*ids* (*Sabs cs*)) *related-v* (*mk-rec-env css* Γ_Λ) -› **by** *simp*
apply (*rule* ‹*related-v* v_2 *ml-*v_2›)
done
next
show *wellformed* (*Sabs cs* $\$_s$ *Svar* (*Name n*))
using ‹*vwellformed* v_1› **unfolding** ‹v_1 = -›
using ‹*fmlookup css* (*Name name*) = *Some cs*›
by *auto*
next
show *related-exp* (*Sabs cs* $\$_s$ *Svar* (*Name n*)) *exp*′
unfolding ‹*exp*′ = -›
apply (*rule related-exp.intros*)
apply *fact*
apply (*rule related-exp.intros*)
done
next
show *closed-except* (*Sabs cs* $\$_s$ *Svar* (*Name n*)) (*fmdom* (*fmupd* (*Name n*) v_2 (Γ_Λ $++_f$ *mk-rec-env css* Γ_Λ)))
using ‹*closed-except* (*Sabs cs*) (*fmdom* (*fmupd* (*Name n*) v_2 Γ_Λ))›
by (*auto simp*: *list-all-iff closed-except-def*)
next
show ¬ *shadows-consts* (*Sabs cs* $\$_s$ *Svar* (*Name n*))
using ‹¬ *shadows-consts* (*Sabs cs*)› ‹*Name n* |∉| *all-consts*› **by** *simp*
next
show *consts* (*Sabs cs* $\$_s$ *Svar* (*Name n*)) |⊆| *fmdom* (*fmupd* (*Name n*) v_2 (Γ_Λ $++_f$ *mk-rec-env css* Γ_Λ)) |∪| *C*
using ‹*consts* (*Sabs cs*) |⊆| -› **unfolding** *mk-rec-env-def*
by *auto*
next
show *fdisjnt C* (*fmdom* (*fmupd* (*Name n*) v_2 (Γ_Λ $++_f$ *mk-rec-env css* Γ_Λ)))
using ‹*Name n* |∉| *all-consts*› ‹*fdisjnt C* (*fmdom* Γ_Λ)› ‹*vwelldefined*′ v_1›

unfolding *mk-rec-env-def* ‹v_1 = -›
by (*auto simp*: *fdisjnt-alt-def all-consts-def*)
qed *fact*+
then obtain v
where *fmupd* (*Name n*) v_2 (Γ_Λ $++_f$ *mk-rec-env css* Γ_Λ) $\vdash_v$ *Sabs cs* $\$_s$ *Svar* (*Name n*) $\downarrow$ *v related-v v ml-v*
unfolding ‹*bv* = -›
by *auto*

then obtain *env pat rhs*
where *vfind-match cs* v_2 = *Some* (*env*, *pat*, *rhs*)
and *fmupd* (*Name n*) v_2 (Γ_Λ $++_f$ *mk-rec-env css* Γ_Λ) $++_f$ *env* $\vdash_v$ *rhs* $\downarrow$ *v*
by (*auto elim*: *veval′-sabs-svarE*)
hence (*pat*, *rhs*) $\in$ *set cs vmatch* (*mk-pat pat*) v_2 = *Some env*
by (*metis vfind-match-elem*)+
hence *linear pat wellformed rhs*
using ‹*vwellformed* v_1› **unfolding** ‹v_1 = -›
using ‹*fmlookup css* (*Name name*) = *Some cs*›
by (*auto simp*: *list-all-iff*)
hence *frees pat* = *patvars* (*mk-pat pat*)
by (*simp add*: *mk-pat-frees*)
hence *fmdom env* = *frees pat*
apply *simp*
apply (*rule vmatch-dom*)
apply (*rule* ‹*vmatch* (*mk-pat pat*) v_2 = *Some env*›)
done

obtain v' **where** Γ_Λ $++_f$ *mk-rec-env css* Γ_Λ $++_f$ *env* $\vdash_v$ *rhs* $\downarrow$ v' $v' \approx_e v$
proof (*rule veval′-agree-eq*)
show *fmupd* (*Name n*) v_2 (Γ_Λ $++_f$ *mk-rec-env css* Γ_Λ) $++_f$ *env* $\vdash_v$ *rhs* $\downarrow$ *v* **by** *fact*
next
have $*$: *Name n* $|\notin|$ *ids rhs* **if** *Name n* $|\notin|$ *fmdom env*
proof
assume *Name n* $|\in|$ *ids rhs*
thus *False*
unfolding *ids-def*
proof (*cases rule*: *funion-strictE*)
case *A*
moreover have *Name n* $|\notin|$ *frees pat*
using *that* **unfolding** ‹*fmdom env* = *frees pat*› **.**
ultimately have *Name n* $|\in|$ *frees* (*Sabs cs*)
apply *auto*
unfolding *ffUnion-alt-def*
apply *simp*
apply (*rule fBexI*[**where** *x* = (*pat*, *rhs*)])
apply (*auto simp*: *fset-of-list-elem*)
apply (*rule* ‹(*pat*, *rhs*) $\in$ *set cs*›)

done
thus *?thesis*
using ‹*Name n* |∉| *ids* (*Sabs cs*)› **unfolding** *ids-def*
by *blast*
next
case *B*
hence *Name n* |∈| *consts* (*Sabs cs*)
apply *auto*
unfolding *ffUnion-alt-def*
apply *simp*
apply (*rule fBexI*[**where** x = (*pat*, *rhs*)])
apply (*auto simp*: *fset-of-list-elem*)
apply (*rule* ‹(*pat*, *rhs*) ∈ *set cs*›)
done
thus *?thesis*
using ‹*Name n* |∉| *ids* (*Sabs cs*)› **unfolding** *ids-def*
by *blast*
qed
qed

show *fmrel-on-fset* (*ids rhs*) *erelated* (Γ_Λ $++_f$ *mk-rec-env css* Γ_Λ $++_f$ *env*) (*fmupd* (*Name n*) v_2 (Γ_Λ $++_f$ *mk-rec-env css* Γ_Λ) $++_f$ *env*)
apply *rule*
apply *auto*
apply (*rule option.rel-refl*; *rule erelated-refl*)
using ∗ **apply** *auto*[]
apply (*rule option.rel-refl*; *rule erelated-refl*)+
using ∗ **apply** *auto*[]
apply (*rule option.rel-refl*; *rule erelated-refl*)+
done
next
show *closed-venv* (*fmupd* (*Name n*) v_2 (Γ_Λ $++_f$ *mk-rec-env css* Γ_Λ) $++_f$ *env*)
apply *rule*
apply *fact*
apply (*rule vclosed.vmatch-env*)
apply *fact*
apply *fact*
done
next
show *wellformed-venv* (*fmupd* (*Name n*) v_2 (Γ_Λ $++_f$ *mk-rec-env css* Γ_Λ) $++_f$ *env*)
apply *rule*
apply *fact*
apply (*rule vwellformed.vmatch-env*)
apply *fact*
apply *fact*
done
next

show *closed-except rhs* (*fmdom* (*fmupd* (*Name n*) v_2 (Γ_Λ $++_f$ *mk-rec-env css* Γ_Λ) $++_f$ *env*))
using ‹*fmdom env* = *frees pat*› ‹(*pat*, *rhs*) ∈ *set cs*›
using ‹*closed-except* (*Sabs cs*) (*fmdom* (*fmupd* (*Name n*) v_2 Γ_Λ))›
apply (*auto simp*: *Sterm.closed-except-simps list-all-iff*)
apply (*erule ballE*[**where** x = (*pat*, *rhs*)])
apply (*auto simp*: *closed-except-def*)
done
next
show *wellformed rhs* **by** *fact*
next
show *consts rhs* |⊆| *fmdom* (*fmupd* (*Name n*) v_2 (Γ_Λ $++_f$ *mk-rec-env css* Γ_Λ) $++_f$ *env*) |∪| *C*
using ‹*consts* (*Sabs cs*) |⊆| -› ‹(*pat*, *rhs*) ∈ *set cs*›
unfolding *sconsts-sabs mk-rec-env-def*
by (*auto simp*: *list-all-iff*)
next
have *fdisjnt C* (*fmdom env*)
using ‹(*pat*, *rhs*) ∈ *set cs*› ‹¬ *shadows-consts* (*Sabs cs*)›
unfolding ‹*fmdom env* = *frees pat*›
by (*auto simp*: *list-ex-iff all-consts-def fdisjnt-alt-def*)
moreover have *fdisjnt C* (*fmdom css*)
using ‹*vwelldefined′* v_1› **unfolding** ‹v_1 = -›
by *simp*
ultimately show *fdisjnt C* (*fmdom* (*fmupd* (*Name n*) v_2 (Γ_Λ $++_f$ *mk-rec-env css* Γ_Λ) $++_f$ *env*))
using ‹*Name n* |∉| *all-consts*› ‹*fdisjnt C* (*fmdom* Γ_Λ)›
unfolding *fdisjnt-alt-def mk-rec-env-def*
by (*auto simp*: *all-consts-def*)
next
show ¬ *shadows-consts rhs*
using ‹(*pat*, *rhs*) ∈ *set cs*› ‹¬ *shadows-consts* (*Sabs cs*)›
by (*auto simp*: *list-ex-iff*)
next
have *not-shadows-vconsts-env env*
by (*rule not-shadows-vconsts.vmatch-env*) *fact+*
thus *not-shadows-vconsts-env* (*fmupd* (*Name n*) v_2 (Γ_Λ $++_f$ *mk-rec-env css* Γ_Λ) $++_f$ *env*)
using ‹*not-shadows-vconsts-env* (*fmupd* (*Name n*) v_2 (Γ_Λ $++_f$ *mk-rec-env css* Γ_Λ))› **by** *blast*
next
have *fmpred* (λ-. *vwelldefined′*) *env*
by (*rule vmatch-welldefined′*) *fact+*
thus *fmpred* (λ-. *vwelldefined′*) (*fmupd* (*Name n*) v_2 (Γ_Λ $++_f$ *mk-rec-env css* Γ_Λ) $++_f$ *env*)
using ‹*fmpred* (λ-. *vwelldefined′*) (*fmupd* (*Name n*) v_2 (Γ_Λ $++_f$ *mk-rec-env css* Γ_Λ))› **by** *blast*
qed *simp*

show *?thesis*
apply (*intro exI conjI*)
unfolding ‹*t* = -›
apply (*rule veval′.rec-comb*)
using ‹$\Gamma \vdash_v t_1 \downarrow v_1$› **unfolding** ‹$v_1$ = -› **apply** *blast*
apply *fact+*
apply (*rule related-v-ext*)
apply *fact+*
done
qed
qed
next
case (*mat1 env ml-scr ml-scr′ ml-cs ml-rhs ml-pat env′ ml-res*)

obtain *scr cs*
where *t* = *Sabs cs* $\$_s$ *scr related-exp scr ml-scr*
and *list-all2* (*rel-prod related-pat related-exp*) *cs ml-cs*
using ‹*related-exp t* (*Mat ml-scr ml-cs*)›
by *cases*

have *sem-env.c env* = *as-static-cenv*
using ‹*is-cupcake-all-env env*›
by (*auto elim*: *is-cupcake-all-envE*)

obtain *scr′* **where** $\Gamma \vdash_v$ *scr* ↓ *scr′ related-v scr′ ml-scr′*
using *mat1*(*4*) **unfolding** *if-rval.simps*
proof
show
is-cupcake-all-env env related-exp scr ml-scr wellformed-venv Γ
closed-venv Γ *fmpred* (λ-. *vwelldefined′*) Γ *fdisjnt C* (*fmdom* Γ)
not-shadows-vconsts-env Γ
by *fact+*
next
show *fmrel-on-fset* (*ids scr*) *related-v* Γ (*fmap-of-ns* (*sem-env.v env*))
apply (*rule fmrel-on-fsubset*)
apply *fact*
unfolding ‹*t* = -› *ids-def*
apply *auto*
done
next
show
wellformed scr consts scr |⊆| *fmdom* Γ |∪| *C*
closed-except scr (*fmdom* Γ) ¬ *shadows-consts scr*
using *mat1* **unfolding** ‹*t* = -› **by** (*auto simp*: *closed-except-def*)
qed *blast*

have *list-all* (λ(*pat*, -). *linear pat*) *cs*
using *mat1* **unfolding** ‹*t* = -› **by** (*auto simp*: *list-all-iff*)

obtain *rhs pat* Γ'
  where *ml-pat* = *mk-ml-pat* (*mk-pat pat*) *related-exp rhs ml-rhs*
    and *vfind-match cs scr'* = *Some* (Γ', *pat*, *rhs*)
    and *var-env* Γ' *env'*
  using ‹*list-all2 - cs ml-cs*› ‹*list-all - cs*› ‹*related-v scr' ml-scr'*› *mat1*(*2*)
  unfolding ‹*sem-env.c env* = *as-static-cenv*›
  by (*auto elim*: *match-all-related*)

hence *vmatch* (*mk-pat pat*) *scr'* = *Some* Γ'
  by (*auto dest*: *vfind-match-elem*)
hence *patvars* (*mk-pat pat*) = *fmdom* Γ'
  by (*auto simp*: *vmatch-dom*)

have (*pat*, *rhs*) $\in$ *set cs*
  by (*rule vfind-match-elem*) *fact*

have *linear pat*
  using ‹(*pat*, *rhs*) $\in$ *set cs*› ‹*wellformed t*› **unfolding** ‹*t* = -›
  by (*auto simp*: *list-all-iff*)
hence *fmdom* Γ' = *frees pat*
  using ‹*patvars* (*mk-pat pat*) = *fmdom* Γ'›
  by (*simp add*: *mk-pat-frees*)

show *?case*
  proof (*rule if-rvalI*)
    fix *ml-rhs'*
    assume *ml-res* = *Rval ml-rhs'*

    obtain *rhs'* **where** $\Gamma ++_f \Gamma' \vdash_v$ *rhs* $\downarrow$ *rhs' related-v rhs' ml-rhs'*
      using *mat1*(*5*) **unfolding** ‹*ml-res* = -› *if-rval.simps*
      proof
        show *is-cupcake-all-env* (*env* ⦇ *sem-env.v* := *nsAppend* (*alist-to-ns env'*)
(*sem-env.v env*) ⦈)
          proof (*rule cupcake-v-update-preserve*)
            have *list-all* (*is-cupcake-value* $\circ$ *snd*) *env'*
              proof (*rule match-all-preserve*)
                show *cupcake-c-ns* (*sem-env.c env*)
                  unfolding ‹*sem-env.c env* = -› **by** (*rule static-cenv*)
              next
                have *is-cupcake-exp* (*Mat ml-scr ml-cs*)
                  apply (*rule related-exp-is-cupcake*)
                  using *mat1* **by** *auto*
                thus *cupcake-clauses ml-cs*
                  by *simp*

                show *is-cupcake-value ml-scr'*
                  apply (*rule cupcake-single-preserve*)
                    apply (*rule mat1*)
                   apply (*rule mat1*)

```
            using ‹is-cupcake-exp (Mat ml-scr ml-cs)› by simp
          qed fact+
         hence is-cupcake-ns (alist-to-ns env')
          by (rule cupcake-alist-to-ns-preserve)
         moreover have is-cupcake-ns (sem-env.v env)
          by (rule is-cupcake-all-envD) fact
        ultimately show is-cupcake-ns (nsAppend (alist-to-ns env') (sem-env.v
env))
          by (rule cupcake-nsAppend-preserve)
      qed fact
    next
      show related-exp rhs ml-rhs
        by fact
    next
      have *: fmdom (fmap-of-list (map (map-prod Name id) env')) = fmdom
Γ'
        using ‹var-env Γ' env'›
        by (metis fmrel-fmdom-eq)

      have **: id |∈| ids t if id |∈| ids rhs id |∉| fmdom Γ' for id
        using ‹id |∈| ids rhs› unfolding ids-def
        proof (cases rule: funion-strictE)
          case A
          from that have id |∉| frees pat
            unfolding ‹fmdom Γ' = frees pat› by simp
          hence id |∈| frees t
            using ‹(pat, rhs) ∈ set cs›
            unfolding ‹t = -›
            apply auto
            apply (subst ffUnion-alt-def)
            apply simp
            apply (rule fBexI[where x = (pat, rhs)])
            using A apply (auto simp: fset-of-list-elem)
            done
          thus id |∈| frees t |∪| consts t by simp
        next
          case B
          hence id |∈| consts t
            using ‹(pat, rhs) ∈ set cs›
            unfolding ‹t = -›
            apply auto
            apply (subst ffUnion-alt-def)
            apply simp
            apply (rule fBexI[where x = (pat, rhs)])
             apply (auto simp: fset-of-list-elem)
            done
          thus id |∈| frees t |∪| consts t by simp
        qed
```

have *fmrel-on-fset* (*ids rhs*) *related-v* (Γ $++_f$ Γ') (*fmap-of-ns* (*sem-env.v env*) $++_f$ *fmap-of-list* (*map* (*map-prod Name id*) *env'*))
apply *rule*
apply *simp*
apply *safe*
subgoal
apply (*rule fmrelD*)
apply (*rule* ‹*var-env* Γ' *env'*›)
done
subgoal
unfolding ∗[*symmetric*]
using *fmdom-of-list fset-of-list-map fset.set-map image-image fst-map-prod*
by *simp*
subgoal using ∗
by (*metis* (*no-types*, *opaque-lifting*) *comp-def fimageI fmdom-fmap-of-list fset-of-list-map fst-comp-map-prod*)
subgoal using ∗∗
by (*metis fmlookup-ns fmrel-on-fsetD mat1.prems*(*2*))
done

thus *fmrel-on-fset* (*ids rhs*) *related-v* (Γ $++_f$ Γ') (*fmap-of-ns* (*sem-env.v* (*env* ⦇ *sem-env.v* := *nsAppend* (*alist-to-ns env'*) (*sem-env.v env*) ⦈)))
by (*auto split*: *sem-env.splits*)
next
show *wellformed-venv* (Γ $++_f$ Γ')
apply *rule*
apply *fact*
apply (*rule vwellformed.vmatch-env*)
apply *fact*
apply (*rule veval'-wellformed*)
apply *fact*
using *mat1* **unfolding** ‹*t* = -› **by** *auto*
next
show *closed-venv* (Γ $++_f$ Γ')
apply *rule*
apply *fact*
apply (*rule vclosed.vmatch-env*)
apply *fact*
apply (*rule veval'-closed*)
apply *fact*
using *mat1* **unfolding** ‹*t* = -› **by** (*auto simp*: *closed-except-def*)
next
show *fmpred* (λ-. *vwelldefined'*) (Γ $++_f$ Γ')
apply *rule*
apply *fact*
apply (*rule vmatch-welldefined'*)
apply *fact*
apply (*rule veval'-welldefined'*)
apply *fact*

```
        using mat1 unfolding ‹t = -› by auto
    next
      show not-shadows-vconsts-env (Γ ++f Γ')
        apply rule
         apply fact
        apply (rule not-shadows-vconsts.vmatch-env)
         apply fact
        apply (rule veval'-shadows)
          apply fact
        using mat1 unfolding ‹t = -› by auto
    next
      show wellformed rhs
        using ‹(pat, rhs) ∈ set cs› ‹wellformed t› unfolding ‹t = -›
        by (auto simp: list-all-iff)
    next
      show closed-except rhs (fmdom (Γ ++f Γ'))
        apply simp
        unfolding ‹fmdom Γ' = frees pat›
        using ‹(pat, rhs) ∈ set cs› ‹closed-except t (fmdom Γ)› unfolding ‹t =
-›
        by (auto simp: Sterm.closed-except-simps list-all-iff)
    next
      have consts (Sabs cs) |⊆| fmdom Γ |∪| C
        using mat1 unfolding ‹t = -› by auto
      show consts rhs |⊆| fmdom (Γ ++f Γ') |∪| C
        apply simp
        unfolding ‹fmdom Γ' = frees pat›
        using ‹(pat, rhs) ∈ set cs› ‹consts (Sabs cs) |⊆| -›
        unfolding sconsts-sabs
        by (auto simp: list-all-iff)
    next
      have fdisjnt C (fmdom Γ')
        unfolding ‹fmdom Γ' = frees pat›
        using ‹¬ shadows-consts t› ‹(pat, rhs) ∈ set cs›
        unfolding ‹t = -›
        by (auto simp: list-ex-iff fdisjnt-alt-def all-consts-def)

      thus fdisjnt C (fmdom (Γ ++f Γ'))
        using ‹fdisjnt C (fmdom Γ)›
        unfolding fdisjnt-alt-def by auto
    next
      show ¬ shadows-consts rhs
        using ‹(pat, rhs) ∈ set cs› ‹¬ shadows-consts t› unfolding ‹t = -›
        by (auto simp: list-ex-iff)
    qed blast

  show ∃ t'. Γ ⊢v t ↓ t' ∧ related-v t' ml-rhs'
    unfolding ‹t = -›
    apply (intro exI conjI seval.intros)
```

apply (*rule veval′.intros*)
apply (*rule veval′.intros*)
apply *fact+*
done
qed
qed *auto*

theorem *semantic-correctness*:
fixes *exp*
assumes *cupcake-evaluate-single env exp* (*Rval ml-v*) *is-cupcake-all-env env*
assumes *fmrel-on-fset* (*ids t*) *related-v* Γ (*fmap-of-ns* (*sem-env.v env*))
assumes *related-exp t exp*
assumes *wellformed t wellformed-venv* Γ
assumes *closed-venv* Γ *closed-except t* (*fmdom* Γ)
assumes *fmpred* (λ-. *vwelldefined′*) Γ *consts t* |⊆| *fmdom* Γ |∪| *C*
assumes *fdisjnt C* (*fmdom* Γ)
assumes ¬ *shadows-consts t not-shadows-vconsts-env* Γ
obtains *v* **where** $\Gamma \vdash_v t \downarrow v$ *related-v v ml-v*
using *semantic-correctness0*[*OF assms*]
by *auto*

end end

end

5.4 Converting bytes to integers

theory *CakeML-Byte*
imports
CakeML.Evaluate-Single
Show.Show-Instances
begin

definition *pat* :: *Ast.pat* **where**
pat = *Ast.Pcon* (*Some* (*Short* ″*String-char-Char*″)) (*map* (λ*n*. *Ast.Pvar* (″*b*″ @ *show n*)) [*0*..<*8*])

definition *cake-plus* :: *exp* ⇒ *exp* ⇒ *exp* **where**
cake-plus t u = *Ast.App* (*Ast.Opn Ast.Plus*) [*t*, *u*]

lemma *cake-plus-correct*:
assumes *evaluate env s u* = (*s′*, *Rval* (*Litv* (*IntLit y*)))
assumes *evaluate env s′ t* = (*s″*, *Rval* (*Litv* (*IntLit x*)))
shows *evaluate env s* (*cake-plus t u*) = (*s″*, *Rval* (*Litv* (*IntLit* (*x* + *y*))))
unfolding *cake-plus-def* **using** *assms* **by** *simp*

definition *cake-times* :: *exp* ⇒ *exp* ⇒ *exp* **where**
cake-times t u = *Ast.App* (*Ast.Opn Ast.Times*) [*t*, *u*]

lemma *cake-times-correct*:
  assumes *evaluate env s u* = (*s*′, *Rval* (*Litv* (*IntLit y*)))
  assumes *evaluate env s*′ *t* = (*s*′′, *Rval* (*Litv* (*IntLit x*)))
  shows *evaluate env s* (*cake-times t u*) = (*s*′′, *Rval* (*Litv* (*IntLit* (*x* ∗ *y*))))
unfolding *cake-times-def* **using** *assms* **by** *simp*

definition *cake-int-of-bool* :: *exp* ⇒ *exp* **where**
cake-int-of-bool e = *Ast.Mat e*
  [(*Ast.Pcon* (*Some* (*Short* ′′*HOL-False*′′)) [], *Lit* (*IntLit 0*)),
   (*Ast.Pcon* (*Some* (*Short* ′′*HOL-True*′′)) [], *Lit* (*IntLit 1*))]

definition *summands* :: *exp list* **where**
summands = *map* (λ*n*. *cake-times* (*Lit* (*IntLit* (*2* ^ *n*))) (*cake-int-of-bool* (*Ast.Var* (*Short* (′′*b*′′ @ *show n*))))) [*0*..<*8*]

definition *cake-int-of-byte* :: *exp* **where**
cake-int-of-byte =
  Ast.Fun ′′*x*′′ (*Ast.Mat* (*Ast.Var* (*Short* ′′*x*′′)) [(*pat*, *foldl cake-plus* (*Lit* (*IntLit* *0*)) *summands*)])

end

Chapter 6

Composition of phases and full compilation pipeline

theory *Doc-Compiler*
imports *Main*
begin

end

6.1 Composition of correctness results

theory *Composition*
imports *../Backend/CakeML-Correctness*
begin

hide-const (**open**) *sem-env.v*

$\mathit{Term\text{-}Class.term} \longrightarrow \mathit{nterm} \longrightarrow \mathit{pterm} \longrightarrow \mathit{sterm}$

6.1.1 Reflexive-transitive closure of *irules.compile-correct*.

lemma (**in** *prules*) *prewrite-closed*:
  assumes $rs \vdash_p t \longrightarrow t'$ *closed t*
  shows *closed* t'
using *assms* **proof** *induction*
  case (*step name rhs*)
  thus *?case*
    using *all-rules* **by** *force*
next
  case (*beta c*)
  obtain *pat rhs* **where** $c = (pat, rhs)$ **by** (*cases c*) *auto*
  with *beta* **have** *closed-except rhs* (*frees pat*)
    by (*auto simp*: *closed-except-simps*)
  show *?case*

apply (*rule rewrite-step-closed*[*OF* - *beta*(*2*)[*unfolded* ‹*c* = -›]])
using ‹*closed-except rhs* (*frees pat*)› *beta* **by** (*auto simp*: *closed-except-def*)
qed (*auto simp*: *closed-except-def*)

corollary (**in** *prules*) *prewrite-rt-closed*:
assumes $rs \vdash_p t \longrightarrow{*} t'$ *closed t*
shows *closed* t'
using *assms*
by *induction* (*auto intro*: *prewrite-closed*)

corollary (**in** *irules*) *compile-correct-rt*:
assumes *Rewriting-Pterm.compile* $rs \vdash_p t \longrightarrow{*} t'$ *finished rs*
shows $rs \vdash_i t \longrightarrow{*} t'$
using *assms* **proof** (*induction rule*: *rtranclp-induct*)
case *step*
thus *?case*
by (*meson compile-correct rtranclp.simps*)
qed *auto*

6.1.2 Reflexive-transitive closure of *prules.compile-correct*.

lemma (**in** *prules*) *compile-correct-rt*:
assumes *Rewriting-Sterm.compile* $rs \vdash_s u \longrightarrow{*} u'$
shows $rs \vdash_p$ *sterm-to-pterm u* $\longrightarrow{*}$ *sterm-to-pterm* u'
using *assms* **proof** *induction*
case *step*
thus *?case*
by (*meson compile-correct rtranclp.simps*)
qed *auto*

lemma *srewrite-stepD*:
assumes *srewrite-step rs name t*
shows (*name*, *t*) ∈ *set rs*
using *assms* **by** *induct auto*

lemma (**in** *srules*) *srewrite-wellformed*:
assumes $rs \vdash_s t \longrightarrow t'$ *wellformed t*
shows *wellformed* t'
using *assms* **proof** *induction*
case (*step name rhs*)
hence (*name*, *rhs*) ∈ *set rs*
by (*auto dest*: *srewrite-stepD*)
thus *?case*
using *all-rules* **by** (*auto simp*: *list-all-iff*)
next
case (*beta cs t* t')
then obtain *pat rhs env* **where** (*pat*, *rhs*) ∈ *set cs match pat t* = *Some env* t' = *subst rhs env*
by (*elim rewrite-firstE*)

show *?case*
unfolding ‹$t' = -$›
proof (*rule subst-wellformed*)
show *wellformed rhs*
using ‹(*pat*, *rhs*) $\in$ *set cs*› *beta* **by** (*auto simp*: *list-all-iff*)
next
show *wellformed-env env*
using ‹*match pat t* = *Some env*› *beta*
by (*auto intro*: *wellformed.match*)
qed
qed *auto*

lemma (**in** *srules*) *srewrite-wellformed-rt*:
assumes *rs* $\vdash_s t \longrightarrow\!\!* \ t'$ *wellformed t*
shows *wellformed* t'
using *assms*
by *induction* (*auto intro*: *srewrite-wellformed*)

lemma *vno-abs-value-to-sterm*: *no-abs* (*value-to-sterm v*) $\longleftrightarrow$ *vno-abs v* **for** *v*
by (*induction v*) (*auto simp*: *no-abs.list-comb list-all-iff*)

6.1.3 Reflexive-transitive closure of *rules.compile-correct*.

corollary (**in** *rules*) *compile-correct-rt*:
assumes *compile* $\vdash_n u \longrightarrow\!\!* \ u'$ *closed u*
shows *rs* $\vdash$ *nterm-to-term*$'$ $u \longrightarrow\!\!*$ *nterm-to-term*$'$ u'
using *assms*
proof *induction*
case (*step* u' u'')
hence *rs* $\vdash$ *nterm-to-term*$'$ $u \longrightarrow\!\!*$ *nterm-to-term*$'$ u'
by *auto*
also have *rs* $\vdash$ *nterm-to-term*$'$ $u' \longrightarrow$ *nterm-to-term*$'$ u''
using *step* **by** (*auto dest*: *rewrite-rt-closed intro*!: *compile-correct simp*: *closed-except-def*)
finally show *?case* **.**
qed *auto*

6.1.4 Reflexive-transitive closure of *irules.transform-correct*.

corollary (**in** *irules*) *transform-correct-rt*:
assumes *transform-irule-set rs* $\vdash_i u \longrightarrow\!\!* \ u''$ $t \approx_p u$ *closed t*
obtains t'' **where** *rs* $\vdash_i t \longrightarrow\!\!* \ t''$ $t'' \approx_p u''$
using *assms* **proof** (*induction arbitrary*: *thesis t*)
case (*step* u' u'')

obtain t' **where** *rs* $\vdash_i t \longrightarrow\!\!* \ t'$ $t' \approx_p u'$
using *step* **by** *blast*

obtain t'' **where** *rs* $\vdash_i t' \longrightarrow\!\!* \ t''$ $t'' \approx_p u''$
apply (*rule transform-correct*)
apply (*rule* ‹*transform-irule-set rs* $\vdash_i u' \longrightarrow u''$›)

apply (*rule* ‹$t' \approx_p u'$›)
apply (*rule irewrite-rt-closed*)
apply (*rule* ‹*rs* $\vdash_i$ $t \longrightarrow* t'$›)
apply (*rule* ‹*closed t*›)
apply *blast*
done

show *?case*
apply (*rule step.prems*)
apply (*rule rtranclp-trans*)
apply *fact+*
done
qed *blast*

corollary (**in** *irules*) *transform-correct-rt-no-abs*:
assumes *transform-irule-set rs* $\vdash_i$ $t \longrightarrow* u$ *closed t no-abs u*
shows *rs* $\vdash_i$ $t \longrightarrow* u$
proof −
have $t \approx_p t$ **by** (*rule prelated-refl*)
obtain t' **where** *rs* $\vdash_i$ $t \longrightarrow* t'$ $t' \approx_p u$
apply (*rule transform-correct-rt*)
apply (*rule assms*)
apply (*rule* ‹$t \approx_p t$›)
apply (*rule assms*)
apply *blast*
done
thus *?thesis*
using *assms* **by** (*metis prelated-no-abs-right*)
qed

corollary *transform-correct-rt-n-no-abs0*:
assumes *irules C rs* (*transform-irule-set* ^^ *n*) *rs* $\vdash_i$ $t \longrightarrow* u$ *closed t no-abs u*
shows *rs* $\vdash_i$ $t \longrightarrow* u$
using *assms*(*1*,*2*) **proof** (*induction n arbitrary*: *rs*)
case (*Suc n*)
interpret *irules C rs* **by** *fact*
show *?case*
apply (*rule transform-correct-rt-no-abs*)
apply (*rule Suc.IH*)
apply (*rule rules-transform*)
using *Suc*(*3*) **apply** (*simp add*: *funpow-swap1*)
apply *fact+*
done
qed *auto*

corollary (**in** *irules*) *transform-correct-rt-n-no-abs*:
assumes (*transform-irule-set* ^^ *n*) *rs* $\vdash_i$ $t \longrightarrow* u$ *closed t no-abs u*
shows *rs* $\vdash_i$ $t \longrightarrow* u$
by (*rule transform-correct-rt-n-no-abs0*) (*rule irules-axioms assms*)+

hide-fact *transform-correct-rt-n-no-abs0*

6.1.5 Iterated application of *transform-irule-set*.

definition *max-arity* :: *irule-set* ⇒ *nat* **where**
max-arity rs = *fMax* ((*arity* ∘ *snd*) |‘| *rs*)

lemma *rules-transform-iter0*:
  assumes *irules C-info rs*
  shows *irules C-info* ((*transform-irule-set* ^^ *n*) *rs*)
using *assms*
by (*induction n*) (*auto intro*: *irules.rules-transform del*: *irulesI*)

lemma (**in** *irules*) *rules-transform-iter*: *irules C-info* ((*transform-irule-set* ^^ *n*) *rs*)
by (*rule rules-transform-iter0*) (*rule irules-axioms*)

lemma *transform-irule-set-n-heads*: *fst* |‘| ((*transform-irule-set* ^^ *n*) *rs*) = *fst* |‘| *rs*
by (*induction n*) (*auto simp*: *transform-irule-set-heads*)

hide-fact *rules-transform-iter0*

definition *transform-irule-set-iter* :: *irule-set* ⇒ *irule-set* **where**
transform-irule-set-iter rs = (*transform-irule-set* ^^ *max-arity rs*) *rs*

lemma *transform-irule-set-iter-heads*: *fst* |‘| *transform-irule-set-iter rs* = *fst* |‘| *rs*
unfolding *transform-irule-set-iter-def* **by** (*simp add*: *transform-irule-set-n-heads*)

lemma (**in** *irules*) *finished-alt-def*: *finished rs* ⟷ *max-arity rs* = *0*
proof
  assume *max-arity rs* = *0*
  hence ¬ *fBex* ((*arity* ∘ *snd*) |‘| *rs*) (λ*x*. *0* < *x*)
    using *nonempty*
    unfolding *max-arity-def*
    by (*metis fBex-fempty fmax-ex-gr not-less0*)
  thus *finished rs*
    unfolding *finished-def*
    by *force*
next
  assume *finished rs*
  have *fMax* ((*arity* ∘ *snd*) |‘| *rs*) ≤ *0*
    proof (*rule fMax-le*)
      show *fBall* ((*arity* ∘ *snd*) |‘| *rs*) (λ*x*. *x* ≤ *0*)
        using ‹*finished rs*› **unfolding** *finished-def* **by** *force*
    next
      show (*arity* ∘ *snd*) |‘| *rs* ≠ {||}
        using *nonempty* **by** *force*

    qed
  thus *max-arity rs* = *0*
    unfolding *max-arity-def* by *simp*
qed

lemma (in *irules*) *transform-finished-id*: *finished rs* ⟹ *transform-irule-set rs* = *rs*
unfolding *transform-irule-set-def finished-def transform-irules-def map-prod-def id-apply*
by (*rule fset-map-snd-id*) (*auto elim*!: *fBallE*)

lemma (in *irules*) *max-arity-decr*: *max-arity* (*transform-irule-set rs*) = *max-arity rs* − *1*
proof (*cases finished rs*)
  case *True*
  thus *?thesis*
    by (*auto simp*: *transform-finished-id finished-alt-def*)
next
  case *False*
  have (*arity* ∘ *snd*) |'| *transform-irule-set rs* = (λ*x*. *x* − *1*) |'| (*arity* ∘ *snd*) |'| *rs*
    unfolding *transform-irule-set-def fset.map-comp*
   proof (*rule fset.map-cong0*, *safe*, *unfold o-apply map-prod-simp id-apply snd-conv*)
      fix *name irs*
      assume (*name*, *irs*) |∈| *rs*
      hence *arity-compatibles irs irs* ≠ {||}
        using *nonempty inner*
        unfolding *atomize-conj*
        by (*smt* (*verit*, *del-insts*) ‹(*name*, *irs*) |∈| *rs*› *case-prodD fBallE inner*)
      thus *arity* (*transform-irules irs*) = *arity irs* − *1*
        by (*simp add*: *arity-transform-irules*)
    qed
  hence *max-arity* (*transform-irule-set rs*) = *fMax* ((λ*x*. *x* − *1*) |'| (*arity* ∘ *snd*) |'| *rs*)
    unfolding *max-arity-def* by *simp*
  also have ... = *fMax* ((*arity* ∘ *snd*) |'| *rs*) − *1*
    proof (*rule fmax-decr*)
      show *fBex* ((*arity* ∘ *snd*) |'| *rs*) ((≤) *1*)
        using *False* unfolding *finished-def* by *force*
    qed
  finally show *?thesis*
    unfolding *max-arity-def*
    by *simp*
qed

lemma *max-arity-decr'0*:
  assumes *irules C rs*
  shows *max-arity* ((*transform-irule-set* ^^ *n*) *rs*) = *max-arity rs* − *n*
proof (*induction n*)
  case (*Suc n*)
  show *?case*

apply *simp*
apply (*subst irules.max-arity-decr*)
using *Suc assms* **by** (*auto intro*: *irules.rules-transform-iter del*: *irulesI*)
qed *auto*

lemma (**in** *irules*) *max-arity-decr'*: *max-arity* ((*transform-irule-set* ^^ *n*) *rs*) = *max-arity rs* − *n*
by (*rule max-arity-decr'0*) (*rule irules-axioms*)

hide-fact *max-arity-decr'0*

lemma (**in** *irules*) *transform-finished*: *finished* (*transform-irule-set-iter rs*)
unfolding *transform-irule-set-iter-def*
by (*subst irules.finished-alt-def*)
(*auto simp*: *max-arity-decr' intro*: *rules-transform-iter del*: *Rewriting-Pterm-Elim.irulesI*)

Trick as described in §*7.1* in the locale manual.

locale *irules'* = *irules*

sublocale *irules'* ⊆ *irules'-as-irules*: *irules C-info transform-irule-set-iter rs*
unfolding *transform-irule-set-iter-def* **by** (*rule rules-transform-iter*)

sublocale *crules* ⊆ *crules-as-irules'*: *irules' C-info Rewriting-Pterm-Elim.compile rs*
unfolding *irules'-def* **by** (*fact compile-rules*)

sublocale *irules'* ⊆ *irules'-as-prules*: *prules C-info Rewriting-Pterm.compile* (*transform-irule-set-iter rs*)
by (*rule irules'-as-irules.compile-rules*) (*rule transform-finished*)

6.1.6 Big-step semantics

context *srules* **begin**

definition *global-css* :: (*name*, *sclauses*) *fmap* **where**
global-css = *fmap-of-list* (*map* (*map-prod id clauses*) *rs*)

lemma *fmdom-global-css*: *fmdom global-css* = *fst* |`| *fset-of-list rs*
unfolding *global-css-def* **by** *simp*

definition *as-vrules* :: *vrule list* **where**
as-vrules = *map* (λ(*name*, -). (*name*, *Vrecabs global-css name fmempty*)) *rs*

lemma *as-vrules-fst*[*simp*]: *fst* |`| *fset-of-list as-vrules* = *fst* |`| *fset-of-list rs*
unfolding *as-vrules-def*
apply *simp*
apply (*rule fset.map-cong*)
apply (*rule refl*)
by *auto*

lemma *as-vrules-fst'*[*simp*]: *map fst as-vrules* = *map fst rs*
unfolding *as-vrules-def*
  by *auto*

lemma *list-all-as-vrulesI*:
  assumes *list-all* (λ(-, *t*). *P fmempty* (*clauses t*)) *rs*
  assumes *R* (*fst* |'| *fset-of-list rs*)
  shows *list-all* (λ(-, *t*). *value-pred.pred P Q R t*) *as-vrules*
proof (*rule list-allI*, *safe*)
  fix *name rhs*
  assume (*name*, *rhs*) ∈ *set as-vrules*
  hence *rhs* = *Vrecabs global-css name fmempty*
    unfolding *as-vrules-def* **by** *auto*

  moreover have *fmpred* (λ-. *P fmempty*) *global-css*
    unfolding *global-css-def list.pred-map*
    using *assms* **by** (*auto simp*: *list-all-iff intro*!: *fmpred-of-list*)

  moreover have *name* |∈| *fmdom global-css*
    unfolding *global-css-def*
    apply *auto*
    using ‹(*name*, *rhs*) ∈ *set as-vrules*› **unfolding** *as-vrules-def*
    including *fset.lifting* **apply** *transfer'*
    by *force*

  moreover have *R* (*fmdom global-css*)
    using *assms* **unfolding** *global-css-def*
    by *auto*

  ultimately show *value-pred.pred P Q R rhs*
    by (*simp add*: *value-pred.pred-alt-def*)
qed

lemma *srules-as-vrules*: *vrules C-info as-vrules*
proof (*standard*, *unfold as-vrules-fst*)
  have *list-all* (λ(-, *t*). *vwellformed t*) *as-vrules*
    unfolding *vwellformed-def*
    apply (*rule list-all-as-vrulesI*)
     apply (*rule list.pred-mono-strong*)
      apply (*rule all-rules*)
     apply (*auto elim*: *clausesE*)
    done

  moreover have *list-all* (λ(-, *t*). *vclosed t*) *as-vrules*
    unfolding *vclosed-def*
    apply (*rule list-all-as-vrulesI*)
     apply *auto*
    apply (*rule list.pred-mono-strong*)

```
     apply (rule all-rules)
    apply (auto elim: clausesE simp: Sterm.closed-except-simps)
    done

  moreover have list-all (λ(-, t). ¬ is-Vconstr t) as-vrules
    unfolding as-vrules-def
    by (auto simp: list-all-iff)

  ultimately show list-all vrule as-vrules
    unfolding list-all-iff by fastforce
next
  show distinct (map fst as-vrules)
    using distinct by auto
next
  show fdisjnt (fst |`| fset-of-list rs) C
    using disjnt by simp
next
  show list-all (λ(-, rhs). not-shadows-vconsts rhs) as-vrules
    unfolding not-shadows-vconsts-def
    apply (rule list-all-as-vrulesI)
     apply auto
    apply (rule list.pred-mono-strong)
     apply (rule not-shadows)
    by (auto simp: list-all-iff list-ex-iff all-consts-def elim!: clausesE)
next
  show vconstructor-value-rs as-vrules
    unfolding vconstructor-value-rs-def
    apply (rule conjI)
    unfolding vconstructor-value-def
     apply (rule list-all-as-vrulesI)
      apply (simp add: list-all-iff)
     apply simp
    apply simp
    using disjnt by simp
next
  show list-all (λ(-, rhs). vwelldefined rhs) as-vrules
    unfolding vwelldefined-def
    apply (rule list-all-as-vrulesI)
     apply auto
    apply (rule list.pred-mono-strong)
     apply (rule swelldefined-rs)
    apply auto
    apply (erule clausesE)
    apply hypsubst-thin
    apply (subst (asm) welldefined-sabs)
    by simp
next
  show distinct all-constructors
    by (fact distinct-ctr)
```

qed

sublocale *srules-as-vrules*: *vrules C-info as-vrules*
by (*fact srules-as-vrules*)

lemma *rs′-rs-eq*: *srules-as-vrules.rs′* = *rs*
unfolding *srules-as-vrules.rs′-def*
unfolding *as-vrules-def*
apply (*subst map-prod-def*)
apply *simp*
unfolding *comp-def*
apply (*subst case-prod-twice*)
apply (*rule list-map-snd-id*)
unfolding *global-css-def*
using *all-rules map*
apply (*auto simp*: *list-all-iff map-of-is-map map-of-map map-prod-def fmap-of-list.rep-eq*)
subgoal for *a b*
  by (*erule ballE*[**where** $x = (a, b)$], *cases b*, *auto simp*: *is-abs-def term-cases-def*)
done

lemma *veval-correct*:
  fixes *v*
  assumes *as-vrules*, *fmempty* $\vdash_v$ *t* ↓ *v wellformed t closed t*
  shows *rs*, *fmempty* $\vdash_s$ *t* ↓ *value-to-sterm v*
using *assms*
by (*rule srules-as-vrules.veval-correct*[*unfolded rs′-rs-eq*])

end

6.1.7 ML-style semantics

context *srules* **begin**

lemma *as-vrules-mk-rec-env*: *fmap-of-list as-vrules* = *mk-rec-env global-css fmempty*
apply (*subst global-css-def*)
apply (*subst as-vrules-def*)
apply (*subst mk-rec-env-def*)
apply (*rule fmap-ext*)
apply (*subst fmlookup-fmmap-keys*)
apply (*subst fmap-of-list.rep-eq*)
apply (*subst fmap-of-list.rep-eq*)
apply (*subst map-of-map-keyed*)
apply (*subst* (*2*) *map-prod-def*)
apply (*subst id-apply*)
apply (*subst map-of-map*)
apply *simp*
apply (*subst option.map-comp*)
apply (*rule option.map-cong*)
 apply (*rule refl*)

apply *simp*
apply (*subst global-css-def*)
apply (*rule refl*)
done

abbreviation (*input*) *vrelated* ≡ *srules-as-vrules.vrelated*
notation *srules-as-vrules.vrelated* (‹$\vdash_v$/ - ≈ -› [*0*, *50*] *50*)

lemma *vrecabs-global-css-refl*:
  assumes *name* |∈| *fmdom global-css*
  shows $\vdash_v$ *Vrecabs global-css name fmempty* ≈ *Vrecabs global-css name fmempty*
using *assms*
proof (*coinduction arbitrary*: *name*)
  case *vrelated*
  have *rel-option* (λv_1 v_2. (∃ *name*. v_1 = *Vrecabs global-css name fmempty* ∧ v_2 = *Vrecabs global-css name fmempty* ∧ *name* |∈| *fmdom global-css*) ∨ $\vdash_v$ v_1 ≈ v_2) (*fmlookup* (*fmap-of-list as-vrules*) *y*) (*fmlookup* (*mk-rec-env global-css fmempty*) *y*)
for *y*
    apply (*subst as-vrules-mk-rec-env*)
    apply (*rule option.rel-refl-strong*)
    apply (*rule disjI1*)
    apply (*simp add*: *mk-rec-env-def*)
    apply (*elim conjE exE*)
    by (*auto intro*: *fmdomI*)
  with *vrelated* **show** *?case*
    by *fastforce*
qed

lemma *as-vrules-refl-rs*: *fmrel-on-fset* (*fst* |'| *fset-of-list as-vrules*) *vrelated* (*fmap-of-list as-vrules*) (*fmap-of-list as-vrules*)
apply *rule*
apply (*subst* (*2*) *as-vrules-def*)
apply (*subst* (*2*) *as-vrules-def*)
apply (*simp add*: *fmap-of-list.rep-eq*)
apply (*rule rel-option-reflI*)
apply *simp*
apply (*drule map-of-SomeD*)
apply *auto*
apply (*rule vrecabs-global-css-refl*)
unfolding *global-css-def*
by (*auto simp*: *fset-of-list-elem intro*: *rev-image-eqI*)

lemma *as-vrules-refl-C*: *fmrel-on-fset C vrelated* (*fmap-of-list as-vrules*) (*fmap-of-list as-vrules*)
proof
  fix *c*
  assume *c* |∈| *C*
  hence *c* |∉| *fset-of-list* (*map fst as-vrules*)
    using *srules-as-vrules.vconstructor-value-rs*

unfolding *vconstructor-value-rs-def fdisjnt-alt-def*
by *auto*
hence c |∉| *fmdom* (*fmap-of-list as-vrules*)
by *simp*
hence *fmlookup* (*fmap-of-list as-vrules*) c = *None*
by (*metis fmdom-notD*)
thus *rel-option vrelated* (*fmlookup* (*fmap-of-list as-vrules*) *c*) (*fmlookup* (*fmap-of-list as-vrules*) *c*)
by *simp*
qed

lemma *veval′-correct″*:
fixes *t v*
assumes *fmap-of-list as-vrules* $\vdash_v t \downarrow v$
assumes *wellformed t*
assumes ¬ *shadows-consts t*
assumes *welldefined t*
assumes *closed t*
assumes *vno-abs v*
shows *as-vrules*, *fmempty* $\vdash_v t \downarrow v$
proof −
obtain v_1 **where** *as-vrules*, *fmempty* $\vdash_v t \downarrow v_1 \vdash_v v_1 \approx v$
using ‹*fmap-of-list as-vrules* $\vdash_v t \downarrow v$›
proof (*rule srules-as-vrules.veval′-correct′*, *unfold as-vrules-fst*)
show *wellformed t* ¬ *shadows-consts t closed t consts t* |⊆| *all-consts*
by *fact*+
next
show *wellformed-venv* (*fmap-of-list as-vrules*)
apply *rule*
using *srules-as-vrules.all-rules*
apply (*auto simp*: *list-all-iff*)
done
next
show *not-shadows-vconsts-env* (*fmap-of-list as-vrules*)
apply *rule*
using *srules-as-vrules.not-shadows*
apply (*auto simp*: *list-all-iff*)
done
next
have *fmrel-on-fset* (*fst* |‘| *fset-of-list as-vrules* |∪| *C*) *vrelated* (*fmap-of-list as-vrules*) (*fmap-of-list as-vrules*)
apply (*rule fmrel-on-fset-unionI*)
apply (*rule as-vrules-refl-rs*)
apply (*rule as-vrules-refl-C*)
done

show *fmrel-on-fset* (*consts t*) *vrelated* (*fmap-of-list as-vrules*) (*fmap-of-list as-vrules*)
apply (*rule fmrel-on-fsubset*)

apply *fact+*
using *assms* **by** (*auto simp*: *all-consts-def*)
qed

thus *?thesis*
using *assms* **by** (*metis srules-as-vrules.vrelated.eq-right*)
qed

end

6.1.8 CakeML

context *srules* **begin**

definition *as-sem-env* :: *v sem-env* ⇒ *v sem-env* **where**
as-sem-env env = (| *sem-env.v* = *build-rec-env* (*mk-letrec-body all-consts rs*) *env nsEmpty*, *sem-env.c* = *nsEmpty* |)

lemma *compile-sem-env*:
evaluate-dec ck mn env state (*compile-group all-consts rs*) (*state*, *Rval* (*as-sem-env env*))
unfolding *compile-group-def as-sem-env-def*
apply (*rule evaluate-dec.dletrec1*)
unfolding *mk-letrec-body-def Let-def*
apply (*simp add*:*comp-def case-prod-twice*)
using *name-as-string.fst-distinct*[*OF distinct*]
by *auto*

lemma *compile-sem-env′*:
fun-evaluate-decs mn state env [(*compile-group all-consts rs*)] = (*state*, *Rval* (*as-sem-env env*))
unfolding *compile-group-def as-sem-env-def mk-letrec-body-def Let-def*
apply (*simp add*: *comp-def case-prod-twice*)
using *name-as-string.fst-distinct*[*OF distinct*]
by *auto*

lemma *compile-prog*[*unfolded combine-dec-result.simps*, *simplified*]:
evaluate-prog ck env state (*compile rs*) (*state*, *combine-dec-result* (*as-sem-env env*) (*Rval* (| *sem-env.v* = *nsEmpty*, *sem-env.c* = *nsEmpty* |)))
unfolding *compile-def*
apply (*rule evaluate-prog.cons1*)
apply *rule*
apply (*rule evaluate-top.tdec1*)
apply (*rule compile-sem-env*)
apply (*rule evaluate-prog.empty*)
done

lemma *compile-prog′*[*unfolded combine-dec-result.simps*, *simplified*]:
fun-evaluate-prog state env (*compile rs*) = (*state*, *combine-dec-result* (*as-sem-env*

env) (*Rval* ⦇ *sem-env.v* = *nsEmpty*, *sem-env.c* = *nsEmpty* ⦈))
unfolding *compile-def fun-evaluate-prog-def no-dup-mods-def no-dup-top-types-def prog-to-mods-def prog-to-top-types-def decs-to-types-def*
using *compile-sem-env′ compile-group-def* **by** *simp*

definition *sem-env* :: *v sem-env* **where**
sem-env ≡ *extend-dec-env* (*as-sem-env empty-sem-env*) *empty-sem-env*

lemma *cupcake-sem-env*: *is-cupcake-all-env sem-env*
unfolding *as-sem-env-def sem-env-def*
apply (*rule is-cupcake-all-envI*)
apply (*simp add*: *extend-dec-env-def empty-sem-env-def nsEmpty-def*)
apply (*rule cupcake-nsAppend-preserve*)
apply (*rule cupcake-build-rec-preserve*)
  apply (*simp add*: *empty-sem-env-def*)
 apply (*simp add*: *nsEmpty-def*)
apply (*rule mk-letrec-cupcake*)
apply *simp*
apply (*simp add*: *empty-sem-env-def*)
done

lemma *sem-env-refl*: *fmrel related-v* (*fmap-of-list as-vrules*) (*fmap-of-ns* (*sem-env.v sem-env*))
proof
  fix *name*
   show *rel-option related-v* (*fmlookup* (*fmap-of-list as-vrules*) *name*) (*fmlookup* (*fmap-of-ns* (*sem-env.v sem-env*)) *name*)
    apply (*simp add*:
        as-sem-env-def build-rec-env-fmap cake-mk-rec-env-def sem-env-def
        fmap-of-list.rep-eq map-of-map-keyed option.rel-map
        as-vrules-def mk-letrec-body-def comp-def case-prod-twice)
    apply (*rule option.rel-refl-strong*)
    apply (*rule related-v.rec-closure*)
     apply *auto*[]
    apply (*simp add*:
          fmmap-of-list[*symmetric*, *unfolded apsnd-def map-prod-def id-def*] *fmap.rel-map*
        global-css-def Let-def map-prod-def comp-def case-prod-twice)
    apply (*thin-tac map-of rs name* = -)
    apply (*rule fmap.rel-refl-strong*)
    apply *simp*
    subgoal premises *prems* **for** *rhs*
      proof −
        obtain *name* **where** (*name*, *rhs*) ∈ *set rs*
          using *prems*
          including *fmap.lifting*
          by *transfer′* (*auto dest*: *map-of-SomeD*)

hence *is-abs rhs closed rhs welldefined rhs*
using *all-rules swelldefined-rs* **by** (*auto simp add*: *list-all-iff*)
then obtain *cs* **where** *clauses rhs* = *cs rhs* = *Sabs cs wellformed-clauses cs*
using ‹(*name*, *rhs*) ∈ *set rs*› *all-rules*
by (*cases rhs*) (*auto simp*: *list-all-iff is-abs-def term-cases-def*)
show *?thesis*
unfolding *related-fun-alt-def* ‹*clauses rhs* = *cs*›
proof (*intro conjI*)
show *list-all2* (*rel-prod related-pat related-exp*) *cs* (*map* (λ(*pat*, *t*). (*mk-ml-pat* (*mk-pat pat*), *mk-con* (*frees pat* |∪| *all-consts*) *t*)) *cs*)
unfolding *list.rel-map*
apply (*rule list.rel-refl-strong*)
apply (*rename-tac z*, *case-tac z*, *hypsubst-thin*)
apply *simp*
subgoal premises *prems* **for** *pat t*
proof (*rule mk-exp-correctness*)
have ¬ *shadows-consts rhs*
using ‹(*name*, *rhs*) ∈ *set rs*› *not-shadows*
by (*auto simp*: *list-all-iff all-consts-def*)
thus ¬ *shadows-consts t*
unfolding ‹*rhs* = *Sabs cs*› **using** *prems*
by (*auto simp*: *list-all-iff list-ex-iff*)
next
have *frees t* |⊆| *frees pat*
using ‹*closed rhs*› *prems* **unfolding** ‹*rhs* = -›
apply (*auto simp*: *list-all-iff Sterm.closed-except-simps*)
apply (*erule ballE*[**where** *x* = (*pat*, *t*)])
apply (*auto simp*: *closed-except-def*)
done
moreover have *consts t* |⊆| *all-consts*
using ‹*welldefined rhs*› *prems* **unfolding** ‹*rhs* = -› *welldefined-sabs*
by (*auto simp*: *list-all-iff all-consts-def*)
ultimately show *ids t* |⊆| *frees pat* |∪| *all-consts*
unfolding *ids-def* **by** *auto*
qed (*auto simp*: *all-consts-def*)
done
next
have *1*: *frees* (*Sabs cs*) = {||}
using ‹*closed rhs*› **unfolding** ‹*rhs* = *Sabs cs*›
by (*auto simp*: *closed-except-def*)

have *2*: *welldefined rhs*
using *swelldefined-rs* ‹(*name*, *rhs*) ∈ *set rs*›
by (*auto simp*: *list-all-iff*)

show *fresh-fNext all-consts* |∉| *ids* (*Sabs cs*)
apply (*rule fNext-not-member-subset*)
unfolding *ids-def 1*

```
          using 2 ‹rhs = -› by (simp add: all-consts-def del: consts-sterm.simps)
        next
          show fresh-fNext all-consts |∉| all-consts
            by (rule fNext-not-member)
        qed
      qed
    done
qed

lemma semantic-correctness':
  assumes cupcake-evaluate-single sem-env (mk-con all-consts t) (Rval ml-v)
  assumes welldefined t closed t ¬ shadows-consts t wellformed t
  obtains v where fmap-of-list as-vrules ⊢v t ↓ v related-v v ml-v
using assms(1) proof (rule semantic-correctness)
  show is-cupcake-all-env sem-env
    by (fact cupcake-sem-env)
next
  show related-exp t (mk-con all-consts t)
    apply (rule mk-exp-correctness)
    using assms
    unfolding ids-def closed-except-def by (auto simp: all-consts-def)
next
  show wellformed t ¬ shadows-consts t by fact+
next
  show closed-except t (fmdom (fmap-of-list as-vrules))
    using ‹closed t› by (auto simp: closed-except-def)
next
  show closed-venv (fmap-of-list as-vrules)
    apply (rule fmpred-of-list)
    using srules-as-vrules.all-rules
    by (auto simp: list-all-iff)

  show wellformed-venv (fmap-of-list as-vrules)
    apply (rule fmpred-of-list)
    using srules-as-vrules.all-rules
    by (auto simp: list-all-iff)
next
   have 1: fmpred (λ-. list-all (λ(pat, t). consts t |⊆| C |∪| fmdom global-css))
global-css
    apply (subst (2) global-css-def)
    apply (rule fmpred-of-list)
    apply (auto simp: map-prod-def)
    subgoal premises prems for pat t
      proof −
        from prems obtain cs where t = Sabs cs
          by (elim clausesE)
        have welldefined t
          using swelldefined-rs prems
          by (auto simp: list-all-iff fmdom-global-css)
```

```
      show ?thesis
        using ‹welldefined t›
        unfolding ‹t = -› welldefined-sabs
        by (auto simp: all-consts-def list-all-iff fmdom-global-css)
    qed
  done

 show fmpred (λ-. vwelldefined′) (fmap-of-list as-vrules)
  apply (rule fmpred-of-list)
  unfolding as-vrules-def
  apply simp
  apply (erule imageE)
  apply (auto split: prod.splits)
    apply (subst fdisjnt-alt-def)
    apply simp
    apply (rule 1)
   apply (subst global-css-def)
   apply simp
  subgoal for x1 x2
    apply (rule image-eqI[where x = (x1, x2)])
    by (auto simp: fset-of-list-elem)
  subgoal
    using disjnt by (auto simp: fdisjnt-alt-def fmdom-global-css)
  done
next
 show not-shadows-vconsts-env (fmap-of-list as-vrules)
  apply (rule fmpred-of-list)
  using srules-as-vrules.not-shadows
  unfolding list-all-iff
  by auto
next
 show fdisjnt C (fmdom (fmap-of-list as-vrules))
  using disjnt by (auto simp: fdisjnt-alt-def)
next
 show fmrel-on-fset (ids t) related-v (fmap-of-list as-vrules) (fmap-of-ns (sem-env.v
sem-env))
  unfolding fmrel-on-fset-fmrel-restrict
  apply (rule fmrel-restrict-fset)
  apply (rule sem-env-refl)
  done
next
 show consts t |⊆| fmdom (fmap-of-list as-vrules) |∪| C
  apply (subst fmdom-fmap-of-list)
  apply (subst as-vrules-fst′)
  apply simp
  using assms by (auto simp: all-consts-def)
qed blast
```

end

fun *cake-to-value* :: *v* ⇒ *value* **where**
cake-to-value (*Conv* (*Some* (*name*, -)) *vs*) = *Vconstr* (*Name name*) (*map cake-to-value vs*)

context *cakeml′* **begin**

lemma *cake-to-value-abs-free*:
  assumes *is-cupcake-value v cake-no-abs v*
  shows *vno-abs* (*cake-to-value v*)
  using *assms* **by** (*induction v*) (*auto elim*: *is-cupcake-value.elims simp*: *list-all-iff*)

lemma *cake-to-value-related*:
  assumes *cake-no-abs v is-cupcake-value v*
  shows *related-v* (*cake-to-value v*) *v*
using *assms* **proof** (*induction v*)
  case (*Conv c vs*)
  then obtain *name tid* **where** *c* = *Some* ((*as-string name*), *TypeId* (*Short tid*))
    apply (*elim is-cupcake-value.elims*)
    subgoal
      by (*metis name.sel v.simps*(*2*))
    by *auto*
  show *?case*
    unfolding ‹*c* = -›
    apply *simp*
    apply (*rule related-v.conv*)
    apply (*simp add*: *list.rel-map*)
    apply (*rule list.rel-refl-strong*)
    apply (*rule Conv*)
    using *Conv* **unfolding** ‹*c* = -›
    by (*auto simp*: *list-all-iff*)
qed *auto*

lemma *related-v-abs-free-uniq*:
  assumes *related-v* v_1 *ml-v related-v* v_2 *ml-v cake-no-abs ml-v*
  shows $v_1 = v_2$
using *assms* **proof** (*induction arbitrary*: v_2)
  case (*conv* vs_1 *ml-vs name*)
  then obtain vs_2 **where** v_2 = *Vconstr name* vs_2 *list-all2 related-v* vs_2 *ml-vs*
    by (*auto elim*: *related-v.cases simp*: *name.expand*)
  moreover have *list-all cake-no-abs ml-vs*
    using *conv* **by** *simp*
  have *list-all2* (=) vs_1 vs_2
    using ‹*list-all2* - vs_1 -› ‹*list-all2* - vs_2 -› ‹*list-all cake-no-abs ml-vs*›
    by (*induction arbitrary*: vs_2 *rule*: *list.rel-induct*) (*auto simp*: *list-all2-Cons2*)
  thus *?case*
    unfolding ‹v_2 = -›
    by (*simp add*: *list.rel-eq*)

qed *auto*

corollary *related-v-abs-free-cake-to-value*:
  assumes *related-v v ml-v cake-no-abs ml-v is-cupcake-value ml-v*
  shows v = *cake-to-value ml-v*
using *assms* **by** (*metis cake-to-value-related related-v-abs-free-uniq*)

end

context *srules* **begin**

lemma *cupcake-sem-env-preserve*:
  assumes *cupcake-evaluate-single sem-env* (*mk-con S t*) (*Rval ml-v*) *wellformed t*
  shows *is-cupcake-value ml-v*
apply (*rule cupcake-single-preserve*[*OF assms*(*1*)])
 apply (*rule cupcake-sem-env*)
apply (*rule mk-exp-cupcake*)
apply *fact*
done

lemma *semantic-correctness''*:
  assumes *cupcake-evaluate-single sem-env* (*mk-con all-consts t*) (*Rval ml-v*)
  assumes *welldefined t closed t* ¬ *shadows-consts t wellformed t*
  assumes *cake-no-abs ml-v*
  shows *fmap-of-list as-vrules* $\vdash_v$ $t \downarrow$ *cake-to-value ml-v*
using *assms*
by (*metis cupcake-sem-env-preserve semantic-correctness' related-v-abs-free-cake-to-value*)

end

6.1.9 Composition

context *rules* **begin**

abbreviation *term-to-nterm* **where**
term-to-nterm t ≡ *fresh-frun* (*Term-to-Nterm.term-to-nterm* [] *t*) *all-consts*

abbreviation *sterm-to-cake* **where**
sterm-to-cake ≡ *rules-as-nrules.crules-as-irules'.irules'-as-prules.prules-as-srules.mk-con all-consts*

abbreviation *term-to-cake t* ≡ *sterm-to-cake* (*pterm-to-sterm* (*nterm-to-pterm* (*term-to-nterm t*)))
abbreviation *cake-to-term t* ≡ (*convert-term* (*value-to-sterm* (*cake-to-value t*)) :: *term*)

abbreviation *cake-sem-env* ≡ *rules-as-nrules.crules-as-irules'.irules'-as-prules.prules-as-srules.sem-env*

definition *compiled* ≡ *rules-as-nrules.crules-as-irules'.irules'-as-prules.prules-as-srules.as-vrules*

lemma *fmdom-compiled*: *fmdom* (*fmap-of-list compiled*) = *heads-of rs*
unfolding *compiled-def*
by (*simp add*:
rules-as-nrules.crules-as-irules'.irules'-as-prules.compile-heads
Rewriting-Pterm.compile-heads transform-irule-set-iter-heads
Rewriting-Pterm-Elim.compile-heads
compile-heads consts-of-heads)

lemma *cake-semantic-correctness*:
assumes *cupcake-evaluate-single cake-sem-env* (*sterm-to-cake t*) (*Rval ml-v*)
assumes *welldefined t closed t* ¬ *shadows-consts t wellformed t*
assumes *cake-no-abs ml-v*
shows *fmap-of-list compiled* $\vdash_v$ *t* ↓ *cake-to-value ml-v*
unfolding *compiled-def*
apply (*rule rules-as-nrules.crules-as-irules'.irules'-as-prules.prules-as-srules.semantic-correctness''*)
using *assms*
by (*simp-all add*:
rules-as-nrules.crules-as-irules'.irules'-as-prules.compile-heads
Rewriting-Pterm.compile-heads transform-irule-set-iter-heads
Rewriting-Pterm-Elim.compile-heads
compile-heads consts-of-heads all-consts-def)

Lo and behold, this is the final correctness theorem!

theorem *compiled-correct*:
— If CakeML evaluation of a term succeeds ...
assumes ∃ *k*. *Evaluate-Single.evaluate cake-sem-env* (*s* ⦇ *clock* := *k* ⦈) (*term-to-cake t*) = (*s'*, *Rval ml-v*)
— ... producing a constructor term without closures ...
assumes *cake-no-abs ml-v*
— ... and some syntactic properties of the involved terms hold ...
assumes *closed t* ¬ *shadows-consts t welldefined t wellformed t*
— ... then this evaluation can be reproduced in the term-rewriting semantics
shows *rs* ⊢ *t* ⟶* *cake-to-term ml-v*
proof −
let *?heads* = *fst* |'| *fset-of-list rules-as-nrules.crules-as-irules'.irules'-as-prules.prules-as-srules.as-vrules*
have *?heads* = *heads-of rs*
using *fmdom-compiled* **unfolding** *compiled-def* **by** *simp*

have *wellformed* (*nterm-to-pterm* (*term-to-nterm t*))
by *auto*
hence *wellformed* (*pterm-to-sterm* (*nterm-to-pterm* (*term-to-nterm t*)))
by (*auto intro*: *pterm-to-sterm-wellformed*)

have *is-cupcake-all-env cake-sem-env*
by (*rule rules-as-nrules.nrules-as-crules.crules-as-irules'.irules'-as-prules.prules-as-srules.cupcake-sem-env*)
have *is-cupcake-exp* (*term-to-cake t*)
by (*rule rules-as-nrules.nrules-as-crules.crules-as-irules'.irules'-as-prules.prules-as-srules.srules-as-cake.mk-fact*

```
 obtain k where Evaluate-Single.evaluate cake-sem-env (s ⦇ clock := k ⦈) (term-to-cake
t) = (s', Rval ml-v)
    using assms by blast
  then have Big-Step-Unclocked-Single.evaluate cake-sem-env (s ⦇ clock := (clock
s') ⦈) (term-to-cake t) (s', Rval ml-v)
    using unclocked-single-fun-eq by fastforce
 have cupcake-evaluate-single cake-sem-env (sterm-to-cake (pterm-to-sterm (nterm-to-pterm
(term-to-nterm t)))) (Rval ml-v)
    apply (rule cupcake-single-complete)
      apply fact+
    done
  hence is-cupcake-value ml-v
   apply (rule rules-as-nrules.crules-as-irules'.irules'-as-prules.prules-as-srules.cupcake-sem-env-preserve)
    by (auto intro: pterm-to-sterm-wellformed)
  hence vno-abs (cake-to-value ml-v)
    using ‹cake-no-abs -›
   by (metis rules-as-nrules.nrules-as-crules.crules-as-irules'.irules'-as-prules.prules-as-srules.srules-as-cake.ca

  hence no-abs (value-to-sterm (cake-to-value ml-v))
    by (metis vno-abs-value-to-sterm)

  hence no-abs (sterm-to-pterm (value-to-sterm (cake-to-value ml-v)))
    by (metis sterm-to-pterm convert-term-no-abs)

  have welldefined (term-to-nterm t)
    unfolding term-to-nterm'-def
    apply (subst fresh-frun-def)
    apply (subst pred-stateD[OF term-to-nterm-consts])
     apply (subst surjective-pairing)
     apply (rule refl)
    apply fact
    done
  have welldefined (pterm-to-sterm (nterm-to-pterm (term-to-nterm t)))
    apply (subst pterm-to-sterm-consts)
     apply fact
    apply (subst consts-nterm-to-pterm)
    apply fact+
    done

  have ¬ shadows-consts t
    using assms unfolding shadows-consts-def fdisjnt-alt-def
    by auto
  hence ¬ shadows-consts (term-to-nterm t)
    unfolding shadows-consts-def shadows-consts-def
    apply auto
    using term-to-nterm-all-vars[folded wellformed-term-def]
    by (metis assms(6) fdisjnt-swap sup-idem)

  have ¬ shadows-consts (pterm-to-sterm (nterm-to-pterm (term-to-nterm t)))
```

apply (*subst pterm-to-sterm-shadows*[*symmetric*])
apply *fact*
apply (*subst shadows-nterm-to-pterm*)
unfolding *shadows-consts-def*
apply *simp*
apply (*rule term-to-nterm-all-vars*[**where** *T* = *fempty*, *simplified*, *THEN fdisjnt-swap*])
apply (*fold wellformed-term-def*)
apply *fact*
using ‹*closed t*› **unfolding** *closed-except-def* **by** (*auto simp*: *fdisjnt-alt-def*)

have *closed* (*term-to-nterm t*)
using *assms* **unfolding** *closed-except-def*
using *term-to-nterm-vars* **unfolding** *wellformed-term-def* **by** *blast*
hence *closed* (*nterm-to-pterm* (*term-to-nterm t*))
using *closed-nterm-to-pterm* **unfolding** *closed-except-def*
by *auto*
have *closed* (*pterm-to-sterm* (*nterm-to-pterm* (*term-to-nterm t*)))
unfolding *closed-except-def*
apply (*subst pterm-to-sterm-frees*)
apply *fact*
using ‹*closed* (*term-to-nterm t*)› *closed-nterm-to-pterm* **unfolding** *closed-except-def*
by *auto*

have *fmap-of-list compiled* $\vdash_v$ *pterm-to-sterm* (*nterm-to-pterm* (*term-to-nterm t*)) ↓ *cake-to-value ml-v*
by (*rule cake-semantic-correctness*) *fact*+
hence *fmap-of-list rules-as-nrules.crules-as-irules′.irules′-as-prules.prules-as-srules.as-vrules* $\vdash_v$ *pterm-to-sterm* (*nterm-to-pterm* (*term-to-nterm t*)) ↓ *cake-to-value ml-v*
using *assms* **unfolding** *compiled-def* **by** *simp*
hence *rules-as-nrules.crules-as-irules′.irules′-as-prules.prules-as-srules.as-vrules*, *fmempty* $\vdash_v$ *pterm-to-sterm* (*nterm-to-pterm* (*term-to-nterm t*)) ↓ *cake-to-value ml-v*
proof (*rule rules-as-nrules.crules-as-irules′.irules′-as-prules.prules-as-srules.veval′-correct″*)
show ¬ *rules-as-nrules.crules-as-irules′.irules′-as-prules.prules-as-srules.shadows-consts* (*pterm-to-sterm* (*nterm-to-pterm* (*term-to-nterm t*)))
using ‹¬ *shadows-consts* (-::*sterm*)› ‹*?heads* = *heads-of rs*› **by** *auto*
next
show *consts* (*pterm-to-sterm* (*nterm-to-pterm* (*term-to-nterm t*))) |⊆| *rules-as-nrules.crules-as-irules′.irule*
using ‹*welldefined* (*pterm-to-sterm* -)› ‹*?heads* = -› **by** *auto*
qed *fact*+
hence *Rewriting-Sterm.compile* (*Rewriting-Pterm.compile* (*transform-irule-set-iter* (*Rewriting-Pterm-Elim.compile* (*consts-of compile*)))), *fmempty* $\vdash_s$ *pterm-to-sterm* (*nterm-to-pterm* (*term-to-nterm t*)) ↓ *value-to-sterm* (*cake-to-value ml-v*)
by (*rule rules-as-nrules.crules-as-irules′.irules′-as-prules.prules-as-srules.veval-correct*) *fact*+
hence *Rewriting-Sterm.compile* (*Rewriting-Pterm.compile* (*transform-irule-set-iter* (*Rewriting-Pterm-Elim.compile* (*consts-of compile*)))) $\vdash_s$ *pterm-to-sterm* (*nterm-to-pterm* (*term-to-nterm t*)) ⟶∗ *value-to-sterm* (*cake-to-value ml-v*)

by (*rule rules-as-nrules.crules-as-irules'.irules'-as-prules.prules-as-srules.seval-correct*) *fact*

hence *Rewriting-Pterm.compile* (*transform-irule-set-iter* (*Rewriting-Pterm-Elim.compile* (*consts-of compile*))) $\vdash_p$ *sterm-to-pterm* (*pterm-to-sterm* (*nterm-to-pterm* (*term-to-nterm t*))) $\longrightarrow*$ *sterm-to-pterm* (*value-to-sterm* (*cake-to-value ml-v*))

by (*rule rules-as-nrules.crules-as-irules'.irules'-as-prules.compile-correct-rt*)

hence *Rewriting-Pterm.compile* (*transform-irule-set-iter* (*Rewriting-Pterm-Elim.compile* (*consts-of compile*))) $\vdash_p$ *nterm-to-pterm* (*term-to-nterm t*) $\longrightarrow*$ *sterm-to-pterm* (*value-to-sterm* (*cake-to-value ml-v*))

by (*subst* (*asm*) *pterm-to-sterm-sterm-to-pterm*) *fact*

hence *transform-irule-set-iter* (*Rewriting-Pterm-Elim.compile* (*consts-of compile*)) $\vdash_i$ *nterm-to-pterm* (*term-to-nterm t*) $\longrightarrow*$ *sterm-to-pterm* (*value-to-sterm* (*cake-to-value ml-v*))

by (*rule rules-as-nrules.crules-as-irules'.irules'-as-irules.compile-correct-rt*)
(*rule rules-as-nrules.crules-as-irules.transform-finished*)

have *Rewriting-Pterm-Elim.compile* (*consts-of compile*) $\vdash_i$ *nterm-to-pterm* (*term-to-nterm t*) $\longrightarrow*$ *sterm-to-pterm* (*value-to-sterm* (*cake-to-value ml-v*))

apply (*rule rules-as-nrules.crules-as-irules.transform-correct-rt-n-no-abs*)
using ‹*transform-irule-set-iter* - $\vdash_i$ - $\longrightarrow*$ -› **unfolding** *transform-irule-set-iter-def*
apply *simp*
apply *fact+*
done

then obtain t' **where** *compile* $\vdash_n$ *term-to-nterm t* $\longrightarrow*$ t' $t' \approx_i$ *sterm-to-pterm* (*value-to-sterm* (*cake-to-value ml-v*))

using ‹*closed* (*term-to-nterm t*)›
by (*metis rules-as-nrules.compile-correct-rt*)
hence *no-abs* t'
using ‹*no-abs* (*sterm-to-pterm* -)›
by (*metis irelated-no-abs*)

have *rs* $\vdash$ *nterm-to-term'* (*term-to-nterm t*) $\longrightarrow*$ *nterm-to-term'* t'
by (*rule compile-correct-rt*) *fact+*
hence *rs* $\vdash$ *t* $\longrightarrow*$ *nterm-to-term'* t'
apply (*subst* (*asm*) *fresh-frun-def*)
apply (*subst* (*asm*) *term-to-nterm-nterm-to-term*[**where** S = *fempty* **and** t = t, *simplified*])
apply (*fold wellformed-term-def*)
apply *fact*
using *assms* **unfolding** *closed-except-def* **by** *auto*

have *nterm-to-pterm* t' = *sterm-to-pterm* (*value-to-sterm* (*cake-to-value ml-v*))
using ‹$t' \approx_i$ -›
by *auto*
hence (*convert-term* t' :: *pterm*) = *convert-term* (*value-to-sterm* (*cake-to-value ml-v*))
apply (*subst* (*asm*) *nterm-to-pterm*)
apply *fact*
apply (*subst* (*asm*) *sterm-to-pterm*)

apply *fact*
apply *assumption*
done
hence *nterm-to-term′ t′* = *convert-term* (*value-to-sterm* (*cake-to-value ml-v*))
apply (*subst nterm-to-term′*)
apply (*rule* ‹*no-abs t′*›)
apply (*rule convert-term-inj*)
subgoal premises
apply (*rule convert-term-no-abs*)
apply *fact*
done
subgoal premises
apply (*rule convert-term-no-abs*)
apply *fact*
done
apply (*subst convert-term-idem*)
apply (*rule* ‹*no-abs t′*›)
apply (*subst convert-term-idem*)
apply (*rule* ‹*no-abs* (*value-to-sterm* (*cake-to-value ml-v*))›)
apply *assumption*
done

thus *?thesis*
using ‹*rs* ⊢ *t* ⟶∗ *nterm-to-term′ t′*› **by** *simp*
qed

end

end

6.2 Executable compilation chain

theory *Compiler*
imports *Composition*
begin

definition *term-to-exp* :: *C-info* ⇒ *rule fset* ⇒ *term* ⇒ *exp* **where**
term-to-exp C-info rs t =
cakeml.mk-con C-info (*heads-of rs* |∪| *constructors.C C-info*)
(*pterm-to-sterm* (*nterm-to-pterm* (*fresh-frun* (*term-to-nterm* [] *t*) (*heads-of rs* |∪| *constructors.C C-info*))))

lemma (**in** *rules*) *Compiler.term-to-exp C-info rs* = *term-to-cake*
unfolding *term-to-exp-def* **by** (*simp add*: *all-consts-def*)

primrec *compress-pterm* :: *pterm* ⇒ *pterm* **where**
compress-pterm (*Pabs cs*) = *Pabs* (*fcompress* (*map-prod id compress-pterm* |‘| *cs*))
|
compress-pterm (*Pconst name*) = *Pconst name* |

compress-pterm (*Pvar name*) = *Pvar name* |
compress-pterm (t $\$_p$ u) = *compress-pterm* t $\$_p$ *compress-pterm* u

lemma *compress-pterm-eq*[*simp*]: *compress-pterm* $t = t$
by (*induction* t) (*auto simp*: *subst-pabs-id fset-map-snd-id map-prod-def*)

definition *compress-crule-set* :: *crule-set* ⇒ *crule-set* **where**
compress-crule-set = *fcompress* ∘ *fimage* (*map-prod id fcompress*)

definition *compress-irule-set* :: *irule-set* ⇒ *irule-set* **where**
compress-irule-set = *fcompress* ∘ *fimage* (*map-prod id* (*fcompress* ∘ *fimage* (*map-prod id compress-pterm*)))

definition *compress-prule-set* :: *prule fset* ⇒ *prule fset* **where**
compress-prule-set = *fcompress* ∘ *fimage* (*map-prod id compress-pterm*)

lemma *compress-crule-set-eq*[*simp*]: *compress-crule-set rs* = *rs*
unfolding *compress-crule-set-def* **by** *force*

lemma *compress-irule-set-eq*[*simp*]: *compress-irule-set rs* = *rs*
unfolding *compress-irule-set-def map-prod-def* **by** *simp*

lemma *compress-prule-set*[*simp*]: *compress-prule-set rs* = *rs*
unfolding *compress-prule-set-def* **by** *force*

definition *transform-irule-set-iter* :: *irule-set* ⇒ *irule-set* **where**
transform-irule-set-iter rs = ((*transform-irule-set* ∘ *compress-irule-set*) ^^ *max-arity rs*) *rs*

definition *as-sem-env* :: *C-info* ⇒ *srule list* ⇒ *v sem-env* ⇒ *v sem-env* **where**
as-sem-env C-info rs env =
  ⦇ *sem-env.v* =
      build-rec-env (*cakeml.mk-letrec-body C-info* (*fset-of-list* (*map fst rs*) |∪| *constructors.C C-info*) *rs*) *env nsEmpty*,
    sem-env.c =
      nsEmpty ⦈

definition *empty-sem-env* :: *C-info* ⇒ *v sem-env* **where**
empty-sem-env C-info = ⦇ *sem-env.v* = *nsEmpty*, *sem-env.c* = *constructors.as-static-cenv C-info* ⦈

definition *sem-env* :: *C-info* ⇒ *srule list* ⇒ *v sem-env* **where**
sem-env C-info rs = *extend-dec-env* (*as-sem-env C-info rs* (*empty-sem-env C-info*)) (*empty-sem-env C-info*)

definition *compile* :: *C-info* ⇒ *rule fset* ⇒ *Ast.prog* **where**
compile C-info =
  CakeML-Backend.compile′ C-info ∘
  Rewriting-Sterm.compile ∘

compress-prule-set ∘
Rewriting-Pterm.compile ∘
transform-irule-set-iter ∘
compress-irule-set ∘
Rewriting-Pterm-Elim.compile ∘
compress-crule-set ∘
Rewriting-Nterm.consts-of ∘
fcompress ∘
Rewriting-Nterm.compile′ C-info ∘
fcompress

definition *compile-to-env* :: *C-info* ⇒ *rule fset* ⇒ *v sem-env* **where**
compile-to-env C-info =
sem-env C-info ∘
Rewriting-Sterm.compile ∘
compress-prule-set ∘
Rewriting-Pterm.compile ∘
transform-irule-set-iter ∘
compress-irule-set ∘
Rewriting-Pterm-Elim.compile ∘
compress-crule-set ∘
Rewriting-Nterm.consts-of ∘
fcompress ∘
Rewriting-Nterm.compile′ C-info ∘
fcompress

lemma (**in** *rules*) *Compiler.compile-to-env C-info rs* = *rules.cake-sem-env C-info rs*
unfolding *Compiler.compile-to-env-def Compiler.sem-env-def Compiler.as-sem-env-def Compiler.empty-sem-env-def*
unfolding *rules-as-nrules.crules-as-irules′.irules′-as-prules.prules-as-srules.sem-env-def*
unfolding *rules-as-nrules.crules-as-irules′.irules′-as-prules.prules-as-srules.as-sem-env-def*
unfolding *empty-sem-env-def*
by (*auto simp*:
Compiler.compress-irule-set-eq[*abs-def*]
Composition.transform-irule-set-iter-def[*abs-def*]
Compiler.transform-irule-set-iter-def[*abs-def*] *comp-def pre-constants.all-consts-def*)

export-code
term-to-exp compile compile-to-env
checking *Scala*

end