A bytecode logic for JML and types (Isabelle/HOL sources)

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Abstract

This document contains the Isabelle/HOL sources underlying our paper A bytecode logic for JML and types [2], updated to Isabelle 2008. We present a program logic for a subset of sequential Java bytecode that is suitable for representing both, features found in high-level specification language JML as well as interpretations of high-level type systems. To this end, we introduce a fine-grained collection of assertions, including strong invariants, local annotations and VDM-reminiscent partial-correctness specifications. Thanks to a goal-oriented structure and interpretation of judgements, verification may proceed without recourse to an additional control flow analysis. The suitability for interpreting intensional type systems is illustrated by the proof-carryingcode style encoding of a type system for a first-order functional language which guarantees a constant upper bound on the number of objects allocated throughout an execution, be the execution terminating or non-terminating.

Like the published paper, the formal development is restricted to a comparatively small subset of the JVML, lacking (among other features) exceptions, arrays, virtual methods, and static fields. This shortcoming has been overcome meanwhile, as our paper has formed the basis of the MOBIUS base logic [9], a program logic for the full sequential fragment of the JVML. Indeed, the present formalisation formed the basis of a subsequent formalisation of the MOBIUS base logic in the proof assistant Coq, which includes a proof of soundness with respect to the Bicolano operational semantics [10].

Contents

1	Pre	limina	ries:	ass	ocia	tio	n	list	s									2
2	Lan	guage																3
	2.1	Syntax	x															3
	2.2	Dynan	nic s	emar	tics													5
		2.2.1	Sen	nanti	c co	mp	on	ents	5									5

		2.2.2 Operational judgements	6										
	2.3	Basic properties	8										
3	Axi	iomatic semantics	8										
	3.1	Assertion forms	8										
	3.2	Proof system	9										
4	Aux	xiliary operational judgements	14										
	4.1	Multistep execution	14										
	4.2	Reachability relation	15										
5	Soundness												
	5.1	Validity	16										
	5.2	Soundness under valid contexts	18										
	5.3	Soundness of verified programs	20										
6			20										
0	A d	lerived logic for a strong type system	20										
0	A d 6.1	Syntax and semantics of judgements	20 21										
0	A d 6.1 6.2	Syntax and semantics of judgements	20 21 22										

1 Preliminaries: association lists

Finite maps are used frequently, both in the representation of syntax and in the program logics. Instead of restricting Isabelle's partial map type $\alpha \rightarrow \beta$ to finite domains, we found it easier for the present development to use the following adhoc data type of association lists.

type-synonym ('a, 'b) AssList = ('a \times 'b) list

primrec lookup::('a, 'b) AssList \Rightarrow 'a \Rightarrow ('b option) ($\langle -\downarrow -\rangle$ [90,0] 90) where lookup [] $l = None \mid$ lookup (h # t) l = (if fst h = l then Some(snd h) else lookup t l)

The statement following the type declaration of *lookup* indicates that we may use the infix notation $L \downarrow a$ for the lookup operation, and asserts some precedence for bracketing. In a similar way, shorthands are introduced for various operations throughout this document.

primrec delete::('a, 'b) AssList $\Rightarrow 'a \Rightarrow ('a, 'b)$ AssList **where** delete [] a = [] |delete (h # t) a = (if (fst h) = a then delete t a else (h # (delete t a)))

definition $upd::('a, 'b) AssList \Rightarrow 'a \Rightarrow 'b \Rightarrow ('a, 'b) AssList$

 $(\langle -[\rightarrow -] \rangle \ [1000, 0, 0] \ 1000)$ where $upd \ L \ a \ b = (a, b) \ \# \ (delete \ L \ a)$

The empty map is represented by the empty list.

definition emp::('a, 'b)AssList **where** emp = [] **definition** contained::('a,'b) AssList \Rightarrow ('a,'b) AssList \Rightarrow bool **where** contained L M = (\forall a b . L \downarrow a = Some b \longrightarrow M \downarrow a = Some b)

The following operation defined the cardinality of a map.

fun AL-Size :: ('a, 'b) AssList \Rightarrow nat (<|-|> [1000] 1000) where AL-Size [] = 0 | AL-Size (h # t) = Suc (AL-Size (delete t (fst h)))

Some obvious basic properties of association lists and their operations are easily proven, but have been suppressed during the document preparation.

2 Language

2.1 Syntax

We have syntactic classes of (local) variables, class names, field names, and method names. Naming restrictions, namespaces, long Java names etc. are not modelled.

typedecl Var typedecl Class typedecl Field typedecl Method

Since arithmetic operations are modelled as unimplemented functions, we introduce the type of values in this section. The domain of heap locations is arbitrary.

typedecl Addr

A reference is either null or an address.

datatype $Ref = Nullref \mid Loc \ Addr$

Values are either integer numbers or references.

datatype $Val = RVal Ref \mid IVal int$

The type of (instruction) labels is fixed, since the operational semantics increments the program counter after each instruction.

type-synonym Label = int

Regarding the instructions, we support basic operand-stack manipulations, object creation, field modifications, casts, static method invocations, conditional and unconditional jumps, and a return instruction.

For every (Isabelle) function $f : Val \Rightarrow Val \Rightarrow Val$ we have an instruction *binop* f whose semantics is to invoke f on the two topmost values on the operand stack and replace them with the result. Similarly for *unop* f.

datatype Instr =

const Valduppopswap load Var store Var $binop Val \Rightarrow Val \Rightarrow Val$ $unop Val \Rightarrow Val$ new Class getfield Class Field putfield Class Field checkcast Class invokeS Class Method *qoto* Label iftrue Label vreturn

Method body declarations contain a list of formal parameters, a mapping from instruction labels to instructions, and a start label. The operational semantics assumes that instructions are labelled consecutively¹.

type-synonym $Mbody = Var \ list \times (Label, \ Instr) \ AssList \times Label$

A class definition associates method bodies to method names.

type-synonym Classdef = (Method, Mbody) AssList

Finally, a program consists of classes.

type-synonym Prog = (Class, Classdef) AssList

Taken together, the three types *Prog*, *Classdef*, and *Mbody* represent an abstract model of the virtual machine environment. In our opinion, it would be desirable to avoid modelling this environment at a finer level, at least for the purpose of the program logic. For example, we prefer not to consider in detail the representation of the constant pool.

¹In the paper, we slightly abstract from this by including a successor functions on labels

2.2 Dynamic semantics

2.2.1 Semantic components

An object consists of the identifier of its dynamic class and a map from field names to values. Currently, we do not model type-correctness, nor do we require that all (or indeed any) of the fields stem from the static definition of the class, or a super-class. Note, however, that type correctness can be expressed in the logic.

type-synonym $Object = Class \times (Field, Val) AssList$

The heap is represented as a map from addresses to values. The JVM specification does not prescribe any particular object layout. The proposed type reflects this indeterminacy, but allows one to calculate the byte-correct size of a heap only after a layout scheme has been supplied. Alternative heap models would be the store-less semantics in the sense of Jonkers [8] and Deutsch [5], (where the heap is modelled as a partial equivalence relation on access paths), or object-based semantics in the sense of Reddy [11], where the heap is represented as a history of update operations. Hähnle et al. use a variant of the latter in their dynamic logic for a JAVACARD [6].

type-synonym Heap = (Addr, Object) AssList

Later, one might extend heaps by a component for static fields.

The types of the (register) store and the operand stack are as expected.

type-synonym Store = (Var, Val) AssList**type-synonym** $OpStack = Val \ list$

States contain an operand stack, a store, and a heap.

type-synonym $State = OpStack \times Store \times Heap$

definition $heap::State \Rightarrow Heap$ where $heap \ s = snd(snd \ s)$

The operational semantics and the program logic are defined relative to a fixed program P. Alternatively, the type of the operational semantics (and proof judgements) could be extended by a program component. We also define the constant value TRUE, the representation of which does not matter for the current formalisation.

axiomatization P::Prog and TRUE::Val

In order to obtain more readable rules, we define operations for extracting method bodies and instructions from the program.

definition *mbody-is*::*Class* \Rightarrow *Method* \Rightarrow *Mbody* \Rightarrow *bool* **where** *mbody-is* $C \ m \ M = (\exists \ CD \ . \ P \downarrow C = Some \ CD \land \ CD \downarrow m = Some \ M)$ **definition** get-ins:: $Mbody \Rightarrow Label \Rightarrow Instroption$ where get-ins $M \ l = (fst(snd \ M)) \downarrow l$

definition *ins-is::Class* \Rightarrow *Method* \Rightarrow *Label* \Rightarrow *Instr* \Rightarrow *bool* **where** *ins-is* $C \ m \ l \ ins = (\exists M \ . \ mbody-is \ C \ m \ M \ \land get-ins \ M \ l = Some \ ins)$

The transfer of method arguments from the caller's operand stack to the formal parameters of an invoked method is modelled by the predicate

 $\begin{array}{l} \textbf{inductive-set} \ Frame::(OpStack \times (Var \ list) \times Store \times OpStack) \ set \\ \textbf{where} \\ FrameNil: [[oo=ops]] \Longrightarrow (ops, [], emp, oo) : Frame \\ | \\ Frame-cons: [[(oo, par, S, ops) : Frame; R = S[x \mapsto v]]] \\ \implies (v \ \# \ oo, \ x \ \# \ par, R, ops):Frame \end{array}$

In order to obtain a deterministic semantics, we assume the existence of a function, with the obvious freshness axiom for this construction.

axiomatization $nextLoc::Heap \Rightarrow Addr$ **where** nextLoc-fresh: $h\downarrow(nextLoc h) = None$

2.2.2 Operational judgements

Similar to Bannwart-Müller [1], we define two operational judgements: a one-step relation and a relation that represents the transitive closure of the former until the end of the current method invocation. These relations are mutually recursive, since the method invocation rule contracts the execution of the invoked method to a single step. The one-step relation associates a state to its immediate successor state, where the program counter is interpreted with respect to the current method body. The transitive closure ignores the bottom part of the operand stack and the store of the final configuration. It simply returns the heap and the result of the method invocation, where the latter is given by the topmost value on the operand stack. In contrast to [1], we do not use an explicit *return* variable. Both relations take an additional index of type *nat* that monitors the derivation height. This is useful in the proof of soundness of the program logic.

Intuitively, (M,l,s,n,l',s'):Step means that method (body) M evolves in one step from state s to state s', while statement (M,s,n,h,v):Exec indicates that executing from s in method M leads eventually to a state whose final value is h, where precisely the last step in this sequence is a *vreturn* instruction and the return value is v.

Like Bannwart and Müller, we define a "frame-less" semantics. i.e. the execution of a method body is modelled by a transitive closure of the basic step-relation, which results in a one-step reduction at the invocation site. Arguably, an operational semantics with an explicit frame stack is closer to the real JVM. It should not be difficult to verify the operational soundness of the present system w.r.t. such a finer model, or to modify the semantics.

inductive-set

 $Step::(Mbody \times Label \times State \times nat \times Label \times State) \ set$ and *Exec::*($Mbody \times Label \times State \times nat \times Heap \times Val$) set where Const: [get-ins $M \ l = Some \ (const \ v); \ NEXT = (v \ \# \ os, s, h); \ ll = l+1$] $\implies (M, l, (os, s, h), 1, ll, NEXT) : Step$ Dup: $[get-ins \ M \ l = Some \ dup; \ NEXT = (v \ \# \ v \ \# \ os, s, h); \ ll = l+1]$ $\implies (M, l, (v \# os, s, h), 1, ll, NEXT) : Step$ Pop: $[get-ins M \ l = Some \ pop; NEXT = (os,s,h); \ ll = l+1]$ $\implies (M, l, (v \# os, s, h), 1, ll, NEXT) : Step$ Swap: $[get-ins \ M \ l = Some \ swap; \ NEXT = (w \ \# \ (v \ \# \ os), s, h); \ ll = l+1]$ $\implies (M,l,(v \# (w \# os),s,h), 1, ll, NEXT) : Step$ Load: [get-ins $M \ l = Some \ (load \ x); \ s \downarrow x = Some \ v;$ NEXT = (v # os, s, h); ll = l+1 $\implies (M, l, (os, s, h), 1, ll, NEXT) : Step$ Store: $[get-ins \ M \ l = Some \ (store \ x); \ NEXT = (os, s[x \mapsto v], h); \ ll = l+1]$ $\implies (M, l, (v \# os, s, h), 1, ll, NEXT) : Step$ Binop: $[get-ins \ M \ l = Some \ (binop \ f); \ NEXT = ((f \ v \ w) \ \# \ os, s, h); \ ll = l+1]$ $\implies (M, l, (v \# (w \# os), s, h), 1, ll, NEXT) : Step$ Unop: $[qet-ins M \ l = Some \ (unop \ f); NEXT = ((f \ v) \ \# \ os, s, h); ll = l+1]$ $\implies (M, l, (v \# os, s, h), 1, ll, NEXT) : Step$ New: $[get-ins \ M \ l = Some \ (new \ d); \ newobj = (d, \ emp); \ a=nextLoc \ h;$ $NEXT = ((RVal \ (Loc \ a)) \ \# \ os, s, h[a \mapsto newobj]); \ ll = l+1]$ $\implies (M, l, (os, s, h), 1, ll, NEXT) : Step$ Get: $[[get-ins \ M \ l = Some \ (getfield \ d \ F); \ h \downarrow a = Some \ (d, \ Flds);$ $Flds \downarrow F = Some \ v; \ NEXT = (v \ \# \ os, s, h); \ ll = l+1$ $\implies (M, l, ((RVal \ (Loc \ a)) \ \# \ os, s, h), \ 1, \ ll, NEXT) : Step$ Put: $[get-ins \ M \ l = Some \ (put field \ d \ F); \ h \downarrow a = Some \ (d, \ Flds);$ $newobj = (d, Flds[F \mapsto v]); NEXT = (os, s, h[a \mapsto newobj]); ll = l+1]$ $\implies (M,l,(v \ \# ((RVal \ (Loc \ a)) \ \# \ os),s,h), \ 1, \ ll, \ NEXT) : Step$ Cast: [get-ins $M \ l = Some \ (checkcast \ d); \ h \downarrow a = Some \ (d, \ Flds);$ $NEXT = ((RVal \ (Loc \ a)) \ \# \ os, s, h); \ ll = l+1$ $\implies (M,l,((RVal \ (Loc \ a)) \ \# \ os,s,h), \ 1, \ ll,NEXT) : Step$

 $\begin{array}{l} \textit{Run: } \llbracket (M,l,s,n,ll,t) : \textit{Step; } (M,ll,t,m,h,v) : \textit{Exec; } k = (max \ n \ m) \ +1 \ \rrbracket \\ \implies (M,l,s,k,h,v) : \textit{Exec} \end{array}$

A big-step operational judgement that abstracts from the derivation height is easily defined.

definition $Opsem::Mbody \Rightarrow Label \Rightarrow State \Rightarrow Heap \Rightarrow Val \Rightarrow bool$ where $Opsem M \ l \ s \ h \ v = (\exists \ n \ . \ (M, l, s, n, h, v):Exec)$

2.3 Basic properties

We provide elimination lemmas for the inductively defined relations

inductive-cases eval-cases: (M,l,s,n,ll,t) : Step(M,l,s,n,h,v) : Exec

and observe that no derivations of height 0 exist.

lemma no-zero-height-Step-derivs: (M,l,s,0,ll,t):Step \implies False **lemma** no-zero-height-Exec-derivs: (M,l,s,0,h,v):Exec \implies False

By induction on the derivation system one can show determinism.

3 Axiomatic semantics

3.1 Assertion forms

We introduce two further kinds of states. Initial states do not contain operand stacks, terminal states lack operand stacks and local variables, but include return values. **type-synonym** $InitState = Store \times Heap$ **type-synonym** $TermState = Heap \times Val$

A judgements relating to a specific program point C.m.l consists of pre- and post-conditions, an invariant, and – optionally – a local annotation. Local pre-conditions and annotations relate initial states to states, i.e. are of type

type-synonym $Assn = InitState \Rightarrow State \Rightarrow bool$

Post-conditions additionally depend on a terminal state

type-synonym $Post = InitState \Rightarrow State \Rightarrow TermState \Rightarrow bool$

Invariants hold for the heap components of all future (reachable) states in the current frame as well as its subframes. They relate these heaps to the current state and the initial state of the current frame.

type-synonym $Inv = InitState \Rightarrow State \Rightarrow Heap \Rightarrow bool$

Local annotations of a method implementation are collected in a table of type

type-synonym ANNO = (Label, Assn) AssList

Implicitly, the labels are always interpreted with respect to the current method. In addition to such a table, the behaviour of methods is specified by a partial-correctness assertion of type

type-synonym $MethSpec = InitState \Rightarrow TermState \Rightarrow bool$

and a method invariant of type

type-synonym $MethInv = InitState \Rightarrow Heap \Rightarrow bool$

A method invariant is expected to be satisfied by the heap components of all states during the execution of the method, including states in subframes, irrespectively of the termination behaviour.

All method specifications are collected in a table of type

type-synonym $MSPEC = (Class \times Method, MethSpec \times MethInv \times ANNO)$ AssList

A table of this type assigns to each method a partial-correctness specification, a method invariant, and a table of local annotations for the instructions in this method.

3.2 Proof system

The proof system derives judgements of the form $G \triangleright \{A\}C, m, l\{B\}I$ where A is a pre-condition, B is a post-condition, I is a strong invariant, C.m.l represents a program point, and G is a proof context (see below). The proof

system consists of syntax-directed rules and structural rules. These rules are formulated in such a way that assertions in the conclusions are unconstrained, i.e. a rule can be directly applied to derive a judgement. In the case of the syntax-directed rules, the hypotheses require the derivation of related statements for the control flow successor instructions. Judgements occurring as hypotheses involve assertions that are notationally constrained, and relate to the conclusions' assertions via uniform constructions that resemble strongest postconditions in Hoare-style logics.

On pre-conditions, the operator

definition SP-pre::Mbody \Rightarrow Label \Rightarrow Assn \Rightarrow Assn where SP-pre M l A = (λ s0 r . (\exists s l1 n. A s0 s \wedge (M,l,s,n,l1,r):Step))

constructs an assertion that holds of a state r precisely if the argument assertion A held at the predecessor state of r. Similar readings explain the constructions on post-conditions

definition SP-post::Mbody \Rightarrow Label \Rightarrow Post \Rightarrow Post where SP-post M l B = (λ s0 r t . (\forall s l1 n. (M,l,s,n,l1,r):Step \longrightarrow B s0 s t))

and invariants

definition SP-inv::Mbody \Rightarrow Label \Rightarrow Inv \Rightarrow Inv where SP-inv M l I = (λ s0 r h . \forall s l1 n. (M,l,s,n,l1,r):Step \longrightarrow I s0 s h)

For the basic instructions, the appearance of the single-step execution relation in these constructions makes the strongest-postcondition interpretation apparent, but could easily be eliminated by unfolding the definition of *Step*. In the proof rule for static method invocations, such a direct reference to the operational semantics is clearly undesirable. Instead, the proof rule extracts the invoked method's specification from the specification table. In order to simplify the formulation of the proof rule, we introduce three operators which manipulate the extracted assertions in a similar way as the above *SP*-operators.

definition SINV-pre:: Var list \Rightarrow MethSpec \Rightarrow Assn \Rightarrow Assn where SINV-pre par T A =

 $\begin{array}{l} (\lambda \ s0 \ s \ . \ (\exists \ ops1 \ ops2 \ S \ R \ h \ k \ w. \\ (ops1, par, R, ops2) : \ Frame \ \land \ T \ (R, \ k) \ (h, w) \ \land \\ A \ s0 \ (ops1, S, k) \ \land \ s = (w \# ops2, S, \ h))) \end{array}$

definition SINV-post:: Var list \Rightarrow MethSpec \Rightarrow Post \Rightarrow Post where SINV-post par T B =

 $\begin{array}{l} (\lambda \ s0 \ s \ t \ . \ \forall \ ops1 \ ops2 \ S \ R \ h \ k \ w \ r. \\ (ops1, par, R, ops2) : \ Frame \longrightarrow T \ (R, \ k) \ (h, w) \longrightarrow \\ s = (w \# ops2, S, h) \longrightarrow r = (ops1, S, k) \longrightarrow B \ s0 \ r \ t) \end{array}$

definition SINV-inv:: Var list \Rightarrow MethSpec \Rightarrow Inv \Rightarrow Inv where SINV-inv par T I = $\begin{array}{ccc} (\lambda \ s0 \ s \ h' \ . \ \forall \ ops1 \ ops2 \ S \ R \ h \ k \ w. \\ (ops1, par, R, ops2) \ : \ Frame \longrightarrow T \ (R, k) \ (h, w) \longrightarrow \\ s = (w \# ops2, S, h) \longrightarrow I \ s0 \ (ops1, S, k) \ h') \end{array}$

The derivation system is formulated using contexts G of proof-theoretic assumptions representing local judgements. The type of contexts is

type-synonym $CTXT = (Class \times Method \times Label, Assn \times Post \times Inv) AssList$

The existence of the proof context also motivates that the hypotheses in the syntax-directed rules are formulated using an auxiliary judgement form, $G \triangleright \langle A \rangle C, m, l \langle B \rangle$ I. Statements for this auxiliary form can be derived by essentially two rules. The first rule, AX, allows us to extract assumptions from the context, while the second rule, *INJECT*, converts an ordinary judgement $G \triangleright \{A\}C, m, l\{B\}$ I into $G \triangleright \langle A \rangle C, m, l\langle B \rangle$ I. No rule is provided for a direct embedding in the opposite direction. As a consequence of this formulation, contextual assumptions cannot be used directly to justify a statement $G \triangleright \{A\}C, m, l\{B\}$ I. Instead, the extraction of such an assumption has to be followed by at least one "proper" (syntax-driven) rule. In particular, attempts to verify jumps by assuming a judgement and immediately using it to prove the rule's hypothesis are ruled out. More specifically, the purpose of this technical device will become obvious when we discharge the context in the proof of soundness (Section 5.3). Here, each entry ((C', m', l'), (A', B', I')) of the context will need to be justified by a derivation $G \triangleright \{A'\}C', m', l'\{B'\}$ I'. This justification should in principle be allowed to rely on other entries of the context. However, an axiom rule for judgements $G \triangleright \{A\}C, m, l\{B\}$ I would allow a trivial discharge of any context entry. The introduction of the auxiliary judgement form $G \triangleright \langle A \rangle C, m, l \langle B \rangle$ I thus ensures that the discharge of a contextual assumption involves at least one application of a "proper" (i.e. syntax-directed) rule. Rule *INJECT* is used to chain together syntax-directed rules. Indeed, it allows us to discharge a hypothesis $G \triangleright \langle A \rangle C, m, l \langle B \rangle$ I of a syntax-directed rule using a derivation of $G \triangleright \{A\}C, m, l\{B\} I$.

The proof rules are defined relative to a fixed method specification table MST.

axiomatization MST::MSPEC

In Isabelle, the distinction between the two judgement forms may for example be achieved by introducing a single judgement form that includes a boolean flag, and defining two pretty-printing notions.

 $(\langle - \triangleright \{ - \} - , -, - \{ - \} - \rangle [100, 100, 100, 100, 100, 100, 100] 50)$ and SP-Assum :: CTXT => Assn => Class => Method => Label => $Post \implies Inv \implies bool$ $(\langle - \triangleright \langle - \rangle , -, -, - \langle - \rangle) \rightarrow [100, 100, 100, 100, 100, 100, 100] 50)$ where $G \vartriangleright \{\!\!\{ A \\!\!\} \ C,m,l \ \!\!\{ B \\!\!\} \ I == (False, \ G, \ C, \ m, \ l, \ A, \ B, \ I): SP-Judgement$ $G \triangleright \langle A \rangle C, m, l \langle B \rangle I == (True, G, C, m, l, A, B, I): SP-Judgement$ INSTR: $\llbracket mbody$ -is C m M; get-ins M l = Some ins; lookup MST (C,m) = Some (Mspec, Minv, Anno); $\forall Q \text{ . lookup Anno } l = Some Q \longrightarrow (\forall s0 s \text{ . A s0 } s \longrightarrow Q s0 s);$ $\forall s0 s . A s0 s \longrightarrow I s0 s (heap s);$ $ins \in \{ const c, dup, pop, swap, load x, store x, binop f, unop q, \}$ new d, getfield d F, putfield d F, checkcast d}; $G \triangleright \langle (SP\text{-}pre \ M \ l \ A) \rangle C, m, (l+1) \langle (SP\text{-}post \ M \ l \ B) \rangle (SP\text{-}inv \ M \ l \ I) \rangle$ \implies $G \triangleright \{ A \} C, m, l \{ B \} I$ GOTO: $\llbracket mbody$ -is C m M; get-ins M l = Some (goto pc); $MST\downarrow(C,m) = Some \ (Mspec, Minv, Anno);$ $\forall Q . Anno \downarrow (l) = Some Q \longrightarrow (\forall s0 s . A s0 s \longrightarrow Q s0 s);$ $\forall s0 s . A s0 s \longrightarrow I s0 s (heap s);$ $G \vartriangleright \langle SP\text{-}pre \ M \ l \ A \rangle \ C, m, pc \ \langle SP\text{-}post \ M \ l \ B \rangle \ (SP\text{-}inv \ M \ l \ I)]$ \implies $G \triangleright \{ A \} C, m, l \{ B \} I$ IF: \llbracket mbody-is C m M; get-ins M l = Some (iftrue pc); $MST\downarrow(C,m) = Some (Mspec, Minv, Anno);$ $\forall Q . Anno \downarrow (l) = Some Q \longrightarrow (\forall s0 s . A s0 s \longrightarrow Q s0 s);$ $\forall s0 s . A s0 s \longrightarrow I s0 s (heap s);$ $G \rhd \langle SP\text{-}pre \ M \ l \ (\lambda \ s0 \ s \ . \ (\forall \ ops \ S \ k \ . \ s=(TRUE \# ops, S, k) \longrightarrow A \ s0 \ s)) \rangle$ C,m,pc $\langle SP\text{-}post \ M \ l \ (\lambda \ s0 \ s \ t \ .$ $(\forall ops \ S \ k \ . \ s = (TRUE \# ops, S, k) \longrightarrow B \ s0 \ s \ t)) \rangle$ $(SP-inv \ M \ l \ (\lambda \ s0 \ s \ t \ .$ $(\forall ops \ S \ k \ . \ s = (TRUE \# ops, S, k) \longrightarrow I \ s0 \ s \ t)));$ $G \vartriangleright \langle SP\text{-}pre \ M \ l \ (\lambda \ s0 \ s \ .$ $(\forall ops \ S \ k \ v \ . \ s = (v \# ops, S, k) \longrightarrow v \neq TRUE \longrightarrow A \ s0 \ s)) \rangle$ C, m, (l+1) $\langle SP\text{-}post \ M \ l \ (\lambda \ s0 \ s \ t.$ $(\forall ops \ S \ k \ v \ . \ s = (v \# ops, S, k) \longrightarrow v \neq TRUE \longrightarrow B \ s0 \ s \ t))\rangle$ (SP-inv M l (λ s0 s t. $(\forall ops \ S \ k \ v \ . \ s = (v \# ops, S, k) \longrightarrow v \neq TRUE \longrightarrow I \ s0 \ s \ t)) \)]$ $\implies G \rhd \{ A \} C, m, l \{ B \} I$

VRET: \llbracket mbody-is C m M; get-ins M l = Some vreturn; $MST\downarrow(C,m) = Some \ (Mspec, Minv, Anno);$ $\forall Q . Anno \downarrow (l) = Some Q \longrightarrow (\forall s0 s . A s0 s \longrightarrow Q s0 s);$ $\forall s0 s . A s0 s \longrightarrow I s0 s (heap s);$ $\forall s0 s . A s0 s \longrightarrow (\forall v ops S h . s = (v \# ops, S, h) \longrightarrow B s0 s (h, v))$ $\implies G \rhd \{ A \} C, m, l \{ B \} I$ INVS: $\llbracket mbody$ -is C m M; get-ins M l = Some (invokeS D m'); $MST\downarrow(C,m) = Some (Mspec,Minv,Anno);$ $MST\downarrow(D,m') = Some (T,MI,Anno2); mbody-is D m' (par,code,l0);$ $\forall Q . Anno \downarrow (l) = Some Q \longrightarrow (\forall s0 s . A s0 s \longrightarrow Q s0 s);$ $\forall s0 s . A s0 s \longrightarrow I s0 s (heap s);$ $\forall s0 \ ops1 \ ops2 \ S \ R \ k \ t \ . \ (ops1, par, R, ops2) : Frame \longrightarrow$ A so $(ops1,S,k) \longrightarrow MI(R,k) t \longrightarrow I so (ops1,S,k) t;$ $G \triangleright \langle SINV\text{-}pre \ par \ T \ A \rangle \ C, m, (l+1) \langle SINV\text{-}post \ par \ T \ B \rangle$ $(SINV-inv \ par \ T \ I)$ \implies $G \triangleright \{ A \} C, m, l \{ B \} I$ CONSEQ: $[(b,G,C,m,l,AA,BB,II) \in SP$ -Judgement; $\forall s0 s . A s0 s \longrightarrow AA s0 s; \forall s0 s t . BB s0 s t \longrightarrow B s0 s t;$ $\forall s0 \ s \ k. \ II \ s0 \ s \ k \longrightarrow I \ s0 \ s \ k$ $\implies (b, G, C, m, l, A, B, I) \in SP$ -Judgement $INJECT: \llbracket G \triangleright \{ A \} C, m, l \{ B \} I \implies G \triangleright \langle A \rangle C, m, l \langle B \rangle I$ AX: $\llbracket G \downarrow (C,m,l) = Some (A,B,I); MST \downarrow (C,m) = Some (Mspec,Minv,Anno);$ $\forall Q . Anno \downarrow (l) = Some Q \longrightarrow (\forall s0 s . A s0 s \longrightarrow Q s0 s);$ $\forall s0 s . A s0 s \longrightarrow I s0 s (heap s)]$ $\implies G \rhd \langle A \rangle C, m, l \langle B \rangle I$

As a first consequence, we can prove by induction on the proof system that a derivable judgement entails its strong invariant and an annotation that may be attached to the instruction.

For *verified programs*, all preconditions can be justified by proof derivations, and initial labels of all methods (again provably) satisfy the method preconditions.

definition $mkState::InitState \Rightarrow State$ where $mkState \ s0 = ([], fst \ s0, snd \ s0)$ **definition** $mkPost::MethSpec \Rightarrow Post$ where $mkPost T = (\lambda \ s0 \ s \ t \ . \ s=mkState \ s0 \longrightarrow T \ s0 \ t)$

definition $mkInv::MethInv \Rightarrow Inv$ where $mkInv MI = (\lambda \ s0 \ s \ t \ . \ s=mkState \ s0 \longrightarrow MI \ s0 \ t)$

 $\begin{array}{l} \textbf{definition} \ VP\text{-}G::CTXT \Rightarrow bool \ \textbf{where} \\ VP\text{-}G \ G = \\ ((\forall \ C \ m \ l \ A \ B \ I. \ G \downarrow (C,m,l) = Some \ (A,B,I) \longrightarrow G \rhd \ \{\!\!\{A\}\!\!\} \ C,m,l \ \{\!\!\{B\}\!\!\} \ I) \land \\ (\forall \ C \ m \ par \ code \ l0 \ T \ MI \ Anno. \\ mbody\text{-}is \ C \ m \ (par,code,l0) \longrightarrow MST \downarrow (C,m) = Some(T,MI,Anno) \longrightarrow \\ G \rhd \ \{\!\!(\lambda \ s0 \ s. \ s = \ mkState \ s0)\}\!\!\} \ C,m,l0 \ \{\!\!\{mkPost \ T\}\!\!\} \ (mkInv \ MI))) \end{array}$

definition VP::bool where $VP = (\exists G . VP - G G)$

4 Auxiliary operational judgements

Beside the basic operational judgements *Step* and *Exec*, the interpretation of judgements refers to two multi-step relations which we now define.

4.1 Multistep execution

The first additional operational judgement is the reflexive and transitive closure of *Step*. It relates states s and t if the latter can be reached from the former by a chain if single steps, all in the same frame. Note that t does not need to be a terminal state. As was the case in the definition of *Step*, we first define a relation with an explicit derivation height index (*MStep*).

 $\begin{array}{l} \textbf{inductive-set} \\ MStep::(Mbody \times Label \times State \times nat \times Label \times State) \ set \\ \textbf{where} \\ MS\text{-zero: } \llbracket k=0; \ t=s; \ ll=l \rrbracket \Longrightarrow (M,l,s,k,ll,t):MStep \\ | \\ MS\text{-step: } \llbracket (M,l,s,n,l1,r):Step; \ (M,l1,r,k,l2,t):MStep; \ m=Suc \ k+n \rrbracket \\ \implies (M,l,s,m,l2,t) \ : \ MStep \\ \end{array}$

The following properties of *MStep* are useful to notice.

 $\begin{array}{l} \textbf{lemma} \ ZeroHeightMultiElim: \ (M,l,s,0,ll,r) \in MStep \implies r=s \land ll=l\\ \textbf{lemma} \ MultiSplit: \\ \llbracket (M, \ l, \ s, \ k, \ ll, \ t) \in MStep; \ 1 \leq k \rrbracket \Longrightarrow \\ \exists \ n \ m \ r \ l1. \ (M,l,s,n,l1,r):Step \land (M,l1,r,m,ll,t):MStep \land Suc \ m+n=k\\ \textbf{lemma} \ MStep-returnElim: \\ \llbracket (M,l,s,k,ll,t) \in MStep; \ get-ins \ M \ l= Some \ vreturn \rrbracket \Longrightarrow \ t=s \land ll=l\\ \textbf{lemma} \ MultiApp: \\ \llbracket (M,l,s,k,l1,r):MStep; \ (M,l1,r,n,l2,t):Step \rrbracket \Longrightarrow (M,l,s,Suc \ k+n,l2,t):MStep \end{array}$

Here are two simple lemmas relating the operational judgements.

Finally, the definition of the non-height-indexed relation.

definition $MS::Mbody \Rightarrow Label \Rightarrow State \Rightarrow Label \Rightarrow State \Rightarrow bool$ where $MS \ M \ l \ s \ ll \ t = (\exists \ k \ . \ (M,l,s,k,ll,t):MStep)$

4.2 Reachability relation

The second auxiliary operational judgement is required for the interpretation of invariants and method invariants. Invariants are expected to be satisfied in all heap components of (future) states that occur either in the same frame as the current state or a subframe thereof. Likewise, method invariants are expected to be satisfied by all heap components of states observed during the execution of a method, including subframes. None of the previous three operational judgements allows us to express these interpretations, as *Step* injects the execution of an invoked method as a single step. Thus, states occurring in subframes cannot be related to states occurring in the parent frame using these judgements. This motivates the introduction of predicates relating states s and t whenever the latter can be reach from the former, i.e. whenever t occurs as a successor of s in the same frame as s or one of its subframes. Again, we first define a relation that includes an explicit derivation height index.

 $\begin{array}{l} \textbf{inductive-set} \\ Reachable::(Mbody \times Label \times State \times nat \times State) \; set \\ \textbf{where} \\ Reachable-zero: [[k=0; t=s]] \Longrightarrow (M,l,s,k,t):Reachable \\ | \\ Reachable-step: \\ [[(M,l,s,n,ll,r):Step; (M,ll,r,m,t):Reachable; k=Suc m+n]] \\ \Longrightarrow (M,l,s,k,t) : Reachable \\ | \\ Reachable-invS: \\ [[mbody-is \; C \; m \; (par,code,l0); \; get-ins \; M \; l = \; Some \; (invokeS \; C \; m); \\ s = (ops,S,h); \; (ops,par,R,ops1):Frame; \\ ((par,code,l0), \; l0, \; ([],R,h), \; n, \; t):Reachable; \; k=Suc \; n]] \end{array}$

 $\implies (M, l, s, k, t) : Reachable$

The following properties of are useful to notice.

Similar to the operational semantics, we define a variation of the reachability relation that hides the index.

definition Reach:: $Mbody \Rightarrow Label \Rightarrow State \Rightarrow bool$ where Reach $M \ l \ s \ t = (\exists \ k \ . \ (M, l, s, k, t): Reachable)$

5 Soundness

This section contains the soundness proof of the program logic. In the first subsection, we define our notion of validity, thus formalising our intuitive explanation of the terms preconditions, specifications, and invariants. The following two subsections contain the details of the proof and can easily be skipped during a first pass through the document.

5.1 Validity

A judgement is valid at the program point C.m.l (i.e. at label l in method m of class C), written valid C m l A B I or, in symbols,

$$\models \{A\} C, m, l\{B\} I,$$

if A is a precondition for B and for all local annotations following l in an execution of m, and all reachable states in the current frame or yet-to-be created subframes satisfy I. More precisely, whenever an execution of the method starting in an initial state s_0 reaches the label l with state s, the following properties are implied by $A(s_0, s)$.

- 1. If the continued execution from s reaches a final state t (i.e. the method terminates), then that final state t satisfies $B(s_0, s, t)$.
- 2. Any state s' visited in the current frame during the remaining program execution whose label carries an annotation Q will satisfy $Q(s_0, s')$, even if the execution of the frame does not terminate.

3. Any state s' visited in the current frame or a subframe of the current frame will satisfy $I(s_0, s, heap(s'))$, again even if the execution does not terminate.

Formally, this interpretation is expressed as follows.

definition valid:: Class \Rightarrow Method \Rightarrow Label \Rightarrow Assn \Rightarrow Post \Rightarrow Inv \Rightarrow bool where valid C m l A B I =

 $\begin{array}{l} (\forall \ M. \ mbody-is \ C \ m \ M \longrightarrow \\ (\forall \ Mspec \ Minv \ Anno \ . \ MST \downarrow (C,m) = \ Some(Mspec, Minv, Anno) \longrightarrow \\ (\forall \ par \ code \ lo \ . \ M = (par, code, lo) \longrightarrow \\ (\forall \ so \ s \ . \ MS \ M \ lo \ (mkState \ so) \ l \ s \longrightarrow A \ so \ s \longrightarrow \\ ((\forall \ h \ v. \ Opsem \ M \ l \ s \ h \ v \longrightarrow B \ so \ s \ (h, v)) \land \\ (\forall \ ll \ r \ . \ (MS \ M \ l \ s \ ll \ r \longrightarrow (\forall \ Q \ . \ Anno \downarrow (ll) = \ Some \ Q \longrightarrow Q \ so \ r)) \land \\ (Reach \ M \ l \ s \ r \longrightarrow I \ so \ s \ (heap \ r)))))))) \end{array}$

abbreviation valid-syntax :: $Assn \Rightarrow Class \Rightarrow Method \Rightarrow$ $Label \Rightarrow Post \Rightarrow Inv \Rightarrow bool$ $(\langle \models \{ - \} - , - , - \{ - \} - \rangle [200, 200, 200, 200, 200] 200)$

where valid-syntax $A \ C \ m \ l \ B \ I == valid \ C \ m \ l \ A \ B \ I$

This notion of validity extends that of Bannwart-Müller by allowing the postcondition to differ from method specification and to refer to the initial state, and by including invariants. In the logic of Bannwart and Müller, the validity of a method specification is given by a partial correctness (Hoare-style) interpretation, while the validity of preconditions of individual instructions is such that a precondition at l implies the preconditions of its immediate control flow successors.

Validity us lifted to contexts and the method specification table. In the case of the former, we simply require that all entries be valid.

definition *G*-valid:: $CTXT \Rightarrow bool$ where *G*-valid $G = (\forall C m \ l \ A \ B \ I. \ G \downarrow (C,m,l) = Some \ (A,B,I) \longrightarrow$ $\models \{ A \} \ C, \ m, \ l \ \{ B \} \ I)$

Regarding the specification table, we require that the initial label of each method satisfies an assertion that ties the method precondition to the current state.

 $\begin{array}{l} \textbf{definition} \ MST\text{-valid} ::bool \ \textbf{where} \\ MST\text{-valid} = (\forall \ C \ m \ par \ code \ l0 \ T \ MI \ Anno. \\ mbody\text{-}is \ C \ m \ (par, code, l0) \longrightarrow MST \downarrow (C, \ m) = Some \ (T, MI, Anno) \longrightarrow \\ \models \{ (\lambda \ s0 \ s. \ s = \ mkState \ s0) \} \ C, \ m, \ l0 \ \{ (mkPost \ T) \} \ (mkInv \ MI)) \end{array}$

definition *Prog-valid::bool* **where** $Prog-valid = (\exists G . G-valid G \land MST-valid)$

The remainder of this section contains a proof of soundness, i.e. of the property

 $VP \Longrightarrow Prog-valid$,

and is structured into two parts. The first step (Section 5.2) establishes a soundness result where the VP property is replaced by validity assumptions regarding the method specification table and the context. In the second step (Section 5.3), we show that these validity assumptions are satisfied by verified programs, which implies the overall soundness theorem.

5.2 Soundness under valid contexts

The soundness proof proceeds by induction on the axiomatic semantics, based on an auxiliary lemma for method invocations that is proven by induction on the derivation height of the operational semantics. For the latter induction, relativised notions of validity are employed that restrict the derivation height of the program continuations affected by an assertion. The appropriate definitions of relativised validity for judgements, for the precondition table, and for the method specification table are as follows.

definition *validn*::

 $\begin{array}{l} nat \Rightarrow Class \Rightarrow Method \Rightarrow Label \Rightarrow Assn \Rightarrow Post \Rightarrow Inv \Rightarrow bool \ \mbox{where}\\ validn \ K \ C \ m \ l \ A \ B \ I = \\ (\forall \ M. \ mbody-is \ C \ m \ M \longrightarrow \\ (\forall \ Mspec \ Minv \ Anno \ . \ MST \downarrow (C,m) = Some(Mspec, Minv, Anno) \longrightarrow \\ (\forall \ par \ code \ l0 \ . \ M = (par, code, l0) \longrightarrow \\ (\forall \ so \ s \ . \ MS \ M \ l0 \ (mkState \ so) \ l \ s \longrightarrow A \ so \ s \longrightarrow \\ (\forall \ k \ . \ k \leq K \longrightarrow \\ ((\forall \ h \ v. \ (M, l, s, k, h, v): Exec \ \longrightarrow B \ so \ s \ (h, v)) \land \\ (\forall \ ll \ r \ . \ ((M, l, s, k, ll, r): MStep \ \longrightarrow \\ ((M, l, s, k, r): Reachable \ \longrightarrow \ I \ so \ s \ (heap \ r)))))))))) \end{array}$

abbreviation validn-syntax :: nat \Rightarrow Assn \Rightarrow Class \Rightarrow Method \Rightarrow Label \Rightarrow Post \Rightarrow Inv \Rightarrow bool ($\langle \models$ - { - , - , - { - } - } [200,200,200,200,200,200] 200)

where validn-syntax K A C m l B I == validn K C m l A B I

 $\begin{array}{l} \text{definition } G\text{-validn::}nat \Rightarrow CTXT \Rightarrow bool \text{ where} \\ G\text{-validn } K \ G = (\forall \ C \ m \ l \ A \ B \ I. \ G \downarrow (C,m,l) = Some \ (A,B,I) \longrightarrow \\ \models_K \ \{\!\!\{A\}\!\!\} \ C, \ m, \ l \ \{\!\!\{B\}\!\!\} \ I) \end{array}$

 $\begin{array}{l} \textbf{definition} \ MST\text{-validn}::nat \Rightarrow bool \ \textbf{where} \\ MST\text{-validn} \ K = (\forall \ C \ m \ par \ code \ l0 \ T \ MI \ Anno. \\ mbody\text{-}is \ C \ m \ (par, code, l0) \longrightarrow MST\downarrow(C, \ m) = Some \ (T, MI, Anno) \longrightarrow \\ \models_K \ \{(\lambda \ s0 \ s. \ s = \ mkState \ s0)\} \ C, \ m, \ l0 \ \{(mkPost \ T)\} \ (mkInv \ MI)) \end{array}$

definition Prog-validn:: $nat \Rightarrow bool$ where Prog-validn $K = (\exists G . G$ -validn $K G \land MST$ -validn K)

The relativised notions are related to each other, and to the native notions of validity as follows.

lemma valid-validn: \models {A} C, m, l {B} $I \Longrightarrow \models_K$ {A} C, m, l {B} I **lemma** validn-valid: [$\forall K . \models_K$ {A} C, m, l {B} I] $\Longrightarrow \models$ {A} C, m, l {B} I**lemma** validn-lower:

 $\llbracket\models_{K} \{\!\{A\}\!\} C, m, l \{\!\{B\}\!\} I; L \leq K \rrbracket \Longrightarrow \models_{L} \{\!\{A\}\!\} C, m, l \{\!\{B\}\!\} I \\ \texttt{lemma } G\text{-valid-valid}: \llbracket \forall K . G\text{-valid} K G \rrbracket \Longrightarrow G\text{-valid} K G \\ \texttt{lemma } G\text{-validn-valid}: \llbracket \forall K . G\text{-valid} K G \rrbracket \Longrightarrow G\text{-valid} G \\ \texttt{lemma } G\text{-validn-lower}: \llbracket G\text{-valid} K G; L \leq K \rrbracket \Longrightarrow G\text{-valid} L G \\ \texttt{lemma } MST\text{-valid-valid}: \llbracket \forall K. MST\text{-valid} K \rrbracket \Longrightarrow MST\text{-valid} \\ \texttt{lemma } MST\text{-validn-valid}: \llbracket \forall K. MST\text{-valid} K \rrbracket \Longrightarrow MST\text{-valid} \\ \texttt{lemma } MST\text{-validn-lower}: \llbracket MST\text{-valid} K ; L \leq K \rrbracket \Longrightarrow MST\text{-valid} L \\ \texttt{lemma } MST\text{-validn-lower}: \llbracket MST\text{-valid} K ; L \leq K \rrbracket \Longrightarrow MST\text{-valid} L \\ \texttt{lemma } MST\text{-validn-lower}: \llbracket MST\text{-valid} K ; L \leq K \rrbracket \Longrightarrow MST\text{-valid} L \\ \texttt{lemma } MST\text{-validn-lower}: \llbracket MST\text{-valid} K ; L \leq K \rrbracket \Longrightarrow MST\text{-valid} L \\ \texttt{lemma } MST\text{-validn-lower}: \llbracket MST\text{-valid} K ; L \leq K \rrbracket \Longrightarrow MST\text{-valid} L \\ \texttt{lemma } MST\text{-valid} R \\ \texttt{lemma } MST\text{-valid} R ; L \leq K \rrbracket \Longrightarrow MST\text{-valid} R \\ \texttt{lemma } MST\text{-valid} R ; L \leq K \rrbracket \Longrightarrow MST\text{-valid} R \\ \texttt{lemma } MST\text{-valid} R ; L \leq K \rrbracket \Longrightarrow MST\text{-valid} R \\ \texttt{lemma } RST\text{-valid} R ; L \leq K \rrbracket \Longrightarrow MST\text{-valid} R \\ \texttt{lemma } RST\text{-valid} R ; L \leq K \rrbracket \Longrightarrow MST\text{-valid} R \\ \texttt{lemma } RST\text{-valid} R ; L \leq K \rrbracket \Longrightarrow MST\text{-valid} R \\ \texttt{lemma } RST\text{-valid} R ; L \leq K \rrbracket \Longrightarrow MST\text{-valid} R \\ \texttt{lemma } RST\text{-valid} R ; L \leq K \rrbracket \Longrightarrow MST\text{-valid} R \\ \texttt{lemma } RST\text{-valid} R ; L \leq K \rrbracket \Longrightarrow MST\text{-valid} R \\ \texttt{lemma } RST\text{-valid} R ; L \leq K \rrbracket \Longrightarrow MST\text{-valid} R \\ \texttt{lemma } RST\text{-valid} R ; L \leq K \rrbracket \Longrightarrow MST\text{-valid} R \\ \texttt{lemma } RST\text{-valid} R ; L \leq K \rrbracket \Longrightarrow MST\text{-valid} R \\ \texttt{lemma } RST\text{-valid} R ; L \leq K \rrbracket \Longrightarrow MST\text{-valid} R \\ \texttt{lemma } RST\text{-valid} R ; L \leq K \rrbracket \boxtimes R ; L \leq K \rrbracket \Longrightarrow R$

We define an abbreviation for the side conditions of the rule for static method invocations...

definition INVS-SC::

 $\begin{array}{l} Class \Rightarrow Method \Rightarrow Label \Rightarrow Class \Rightarrow Method \Rightarrow MethSpec \Rightarrow MethInv \Rightarrow \\ ANNO \Rightarrow ANNO \Rightarrow Mbody \Rightarrow Assn \Rightarrow Inv \Rightarrow bool ~ ~ ~ ~ ~ ~ ~ ~ \\ INVS-SC C m l D m' T MI Anno Anno2 M' A I = (\exists M par code l0 T1 MI1. \\ mbody-is C m M \land get-ins M l = Some (invokeS D m') \land \\ MST\downarrow(C,m) = Some (T1,MI1,Anno) \land \\ MST\downarrow(D,m') = Some (T,MI,Anno2) \land \\ mbody-is D m' M' \land M'=(par,code,l0) \land \\ (\forall Q . Anno\downarrow(l) = Some Q \longrightarrow (\forall s0 s . A s0 s \longrightarrow Q s0 s)) \land \\ (\forall s0 ops1 ops2 S R h t . (ops1,par,R,ops2) : Frame \longrightarrow \\ A s0 (ops1,S,h) \longrightarrow MI (R,h) t \longrightarrow I s0 (ops1,S,h) t)) \end{array}$

... and another abbreviation for the soundness property of the same rule.

definition *INVS-soundK*::

 $\begin{array}{l} nat \Rightarrow CTXT \Rightarrow Class \Rightarrow Method \Rightarrow Label \Rightarrow Class \Rightarrow Method \Rightarrow\\ MethSpec \Rightarrow MethInv \Rightarrow ANNO \Rightarrow ANNO \Rightarrow Mbody \Rightarrow Assn \Rightarrow\\ Post \Rightarrow Inv \Rightarrow bool ~ where\\ INVS-soundK K G C m l D m' T MI Anno Anno2 M' A B I =\\ (INVS-SC C m l D m' T MI Anno Anno2 M' A I \longrightarrow\\ G-validn K G \longrightarrow MST-validn K \longrightarrow\\ \models_K \{ (SINV-pre ~ (fst M') T A) \} C,m,(l+1) \\ ~ \{ (SINV-post ~ (fst M') T B) \} (SINV-inv ~ (fst M') T I) \\ \longrightarrow \models_(K+1) \{ A \} C,m,l \{ B \} I) \end{array}$

The proof that this property holds for all K proceeds by induction on K.

lemma INVS-soundK-all: INVS-soundK K G C m l D m' T MI Anno Anno2 M' A B I

The heart of the soundness proof - the induction on the axiomatic semantics.

 $\begin{array}{l} \textbf{lemma SOUND-Aux[rule-format]:} \\ (b,G,C,m,l,A,B,I):SP-Judgement \implies G\text{-validn } K \ G \longrightarrow MST\text{-validn } K \longrightarrow \\ ((b \longrightarrow \models_{K} \{\!\!\{A\}\!\!\} \ C, \ m, \ l \ \{\!\!\{B\}\!\!\} \ I) \land \\ ((\neg \ b) \longrightarrow \models_{l} Suc \ K) \ \{\!\!\{A\}\!\!\} \ C, \ m, \ l \ \{\!\!\{B\}\!\!\} \ I)) \end{array}$

The statement of this lemma gives a semantic interpretation of the two judgement forms, as *SP-Assum*-judgements enjoy validity up to execution height K, while *SP-Deriv*-judgements are valid up to level K + 1.

From this, we obtain a soundness result that still involves context validity.

theorem SOUND-in-CTXT: $\llbracket G \triangleright \{\!\!\{A\}\!\!\} C, m, l \{\!\!\{B\}\!\!\} I; G\text{-valid } G; MST\text{-valid} \rrbracket \Longrightarrow \models \{\!\!\{A\}\!\!\} C, m, l \{\!\!\{B\}\!\!\} I$

We will now show that the two semantic assumptions can be replaced by the verified-program property.

5.3 Soundness of verified programs

In order to obtain a soundness result that does not require validity assumptions of the context or the specification table, we show that the VP property implies context validity. First, the elimination of contexts. By induction on k we prove

lemma VPG-MSTn-Gn[rule-format]: VP-G $G \longrightarrow MST$ -validn $k \longrightarrow G$ -validn k G

which implies

lemma VPG-MST-G: $[\![VP$ -G G; MST- $valid]\!] \implies G$ -valid G

Next, the elimination of MST-valid. Again by induction on k, we prove

lemma VPG-MSTn[rule-format]: VP- $G \ G \longrightarrow MST$ -validn k

which yields

lemma VPG-MST: VP- $G \ G \implies MST$ -valid

Combining these two results, and unfolding the definition of program validity yields the final soundness result.

theorem *VP-VALID*: *VP* \implies *Prog-valid*

6 A derived logic for a strong type system

In this section we consider a system of derived assertions, for a type system for bounded heap consumption. The type system arises by reformulating the analysis of Cachera, Jensen, Pichardie, and Schneider [4] for a highlevel functional language. The original approach of Cachera et al. consists of formalising the correctness proof of a certain analysis technique in Coq. Consequently, the verification of a program requires the execution of the analysis algorithm inside the theorem prover, which involves the computation of the (method) call graph and fixed point iterations. In contrast, our approach follows the proof-carrying code paradigm more closely: the analysis amounts to a type inference which is left unformalised and can thus be carried out outside the trusted code base. Only the result of the analysis is communicated to the code recipient. The recipient verifies the validity of the certificate by a largely syntax-directed single-pass traversal of the (low-level) code using a domain-specific program logic. This approach to proof-carrying code was already explored in the MRG project, with respect to program logics of partial correctness [3] and a type system for memory consumption by Hofmann and Jost [7]. In order to obtain syntax-directedness of the proof rules, these had to be formulated at the granularity of typing judgements. In contrast, the present proof system admits proof rules for individual JVM instructions.

Having derived proof rules for individual JVM instructions, we introduce a type system for a small functional language, and a compilation into bytecode. The type system associates a natural number n to an expression e, in a typing context Σ . Informally, the interpretation of a typing judgement $\Sigma \triangleright e : n$ is that the evaluation of e (which may include the invocation of functions whose resource behaviour is specified in Σ) does not perform more than n allocations. The type system is then formally proven sound, using the derived logic for bytecode. By virtue of the invariants, the guarantee given by the present system is stronger than the one given by our encoding of the Hofmann-Jost system, as even non-terminating programs can be verified in a meaningful way.

6.1 Syntax and semantics of judgements

The formal interpretation at JVM level of a type n is given by a triple

$$Cachera(n) = (A, B, I)$$

consisting of a (trivial) precondition, a post-condition, and a strong invariant.

 $\begin{array}{l} \textbf{definition} \ Cachera::nat \Rightarrow (Assn \times Post \times Inv) \ \textbf{where} \\ Cachera \ n = (\lambda \ s0 \ s \ . \ True, \\ \lambda \ s0 \ (ops,s,h) \ (k,v) \ . \ |k| \leq |h| + n, \\ \lambda \ s0 \ (ops,s,h) \ k. \ \ |k| \leq |h| + n) \end{array}$

This definition is motivated by the expectation that $\triangleright \{A\} \ulcorner e \urcorner \{B\} I$ should be derivable whenever the type judgement $\Sigma \triangleright e : n$ holds, where $\ulcorner e \urcorner$ is the translation of compiling the expression e into JVML, and the specification table MST contains the interpretations of the entries in Σ .

We abbreviate the above construction of judgements by a predicate *deriv*.

 $deriv \ G \ C \ m \ l \ (ABI) = (let \ (A,B,I) = ABI \ in \ (G \rhd \{ A \} C,m,l \ \{ B \} I))$

Thus, the intended interpretation of a typing judgement $\Sigma \triangleright e : n$ is

deriv C m l (Cachera n)

if e translates to a code block whose first instruction is at C.m.l.

We also define a judgement of the auxiliary form of sequents.

definition $derivAssum::CTXT \Rightarrow Class \Rightarrow Method \Rightarrow Label \Rightarrow$ $(Assn \times Post \times Inv) \Rightarrow bool$ where $derivAssum \ G \ C \ m \ l \ (ABI) = (let \ (A,B,I) = ABI \ in \ G \rhd \ \langle A \rangle \ C,m,l \ \langle B \rangle \ I)$

The following operation converts a derived judgement into the syntactical form of method specifications.

 $\begin{array}{l} \textbf{definition} \ mkSPEC::(Assn \times Post \times Inv) \Rightarrow ANNO \Rightarrow \\ (MethSpec \times MethInv \times ANNO) \ \textbf{where} \\ mkSPEC \ (ABI) \ Anno = (let \ (A,B,I) = ABI \ in \\ (\lambda \ s0 \ t \ . \ B \ s0 \ (mkState \ s0) \ t, \ \lambda \ s0 \ h \ . \ I \ s0 \ (mkState \ s0) \ h, \ Anno)) \end{array}$

This enables the interpretation of typing contexts Σ as a set of constraints on the specification table *MST*.

6.2 Derived proof rules

We are now ready to prove derived rules, i.e. proof rules where assumptions as well as conclusions are of the restricted assertion form. While their justification unfolds the definition of the predicate *deriv*, their application will not. We first give syntax-directed proof rules for all JVM instructions:

```
lemma CACH-NEW:
[ ins-is C \ m \ l \ (new \ c); MST \downarrow (C,m) = Some(Mspec,Minv,Anno);
   Anno\downarrow(l) = None; n = k + 1; derivAssum G C m (l+1) (Cachera k)
 \implies deriv G C m l (Cachera n)
lemma CACH-INSTR:
 [] ins-is C m l I;
   I \in \{ \text{ const } c, dup, pop, swap, load x, store x, binop f, \}
         unop g, getfield d F, putfield d F, checkcast d;
   MST \downarrow (C,m) = Some(Mspec,Minv,Anno); Anno \downarrow (l) = None;
   derivAssum G C m (l+1) (Cachera n)
\implies deriv G C m l (Cachera n)
lemma CACH-RET:
 [ ins-is C \ m \ l \ vreturn; MST \downarrow (C,m) = Some(Mspec,Minv,Anno);
   Anno\downarrow(l) = None
  \implies deriv G C m l (Cachera 0)
lemma CACH-GOTO:
 [ ins-is C \ m \ l \ (goto \ pc); MST \downarrow (C,m) = Some(Mspec,Minv,Anno);
   Anno\downarrow(l) = None; derivAssum \ G \ C \ m \ pc \ (Cachera \ n)
  \Rightarrow deriv G C m l (Cachera n)
```

lemma CACH-IF:

 $\begin{bmatrix} ins-is \ C \ m \ l \ (iftrue \ pc); \ MST \downarrow (C,m) = Some(Mspec,Minv,Anno); \\ Anno \downarrow (l) = None; \ derivAssum \ G \ C \ m \ pc \ (Cachera \ n); \\ derivAssum \ G \ C \ m \ (l+1) \ (Cachera \ n) \ \end{bmatrix} \\ \Longrightarrow deriv \ G \ C \ m \ l \ (Cachera \ n) \\ \textbf{lemma} \ CACH-INVS: \\ \begin{bmatrix} ins-is \ C \ m \ l \ (invokeS \ D \ m'); \ mbody-is \ D \ m' \ (par,code,l0); \\ MST \downarrow (C,m) = Some(Mspec,Minv,Anno); \ Anno \downarrow (l) = None; \\ MST \downarrow (D, \ m') = Some(mkSPEC \ (Cachera \ k) \ Anno2); \\ nk = \ n+k; \ derivAssum \ G \ C \ m \ (l+1) \ (Cachera \ n) \\ \end{bmatrix}$

 \implies deriv G C m l (Cachera nk)

In addition, we have two rules for subtyping

 $\begin{array}{l} \textbf{lemma } CACH-SUB: \\ \llbracket \ deriv \ G \ C \ m \ l \ (Cachera \ n); \ n \leq k \rrbracket \Longrightarrow deriv \ G \ C \ m \ l \ (Cachera \ k) \\ \textbf{lemma } CACHAssum-SUB: \\ \llbracket \ derivAssum \ G \ C \ m \ l \ (Cachera \ n); \ n \leq k \rrbracket \\ \Longrightarrow \ derivAssum \ G \ C \ m \ l \ (Cachera \ k) \end{array}$

and specialised forms of the axiom rule and the injection rule.

lemma CACH-AX: $\begin{bmatrix} G \downarrow (C,m,l) = Some \ (Cachera \ n); \ MST \downarrow (C,m) = Some(Mspec,Minv,Anno); \\ Anno \downarrow (l) = None \end{bmatrix}$ $\implies derivAssum \ G \ C \ m \ l \ (Cachera \ n)$ **lemma** CACH-INJECT: deriv \ G \ C \ m \ l \ (Cachera \ n) \implies derivAssum \ G \ C \ m \ l \ (Cachera \ n)

Finally, a verified-program rule relates specifications to judgements for the method bodies. Thus, even the method specifications may be given as derived assertions (modulo the mkSPEC-conversion).

 $\begin{array}{l} \textbf{lemma } CACH-VP: \\ \llbracket \forall \ c \ m \ par \ code \ l0. \ mbody-is \ c \ m \ (par, \ code, \ l0) \longrightarrow \\ (\exists \ n \ Anno \ . \ MST \downarrow (c,m) = \ Some(mkSPEC(Cachera \ n) \ Anno) \land \\ deriv \ G \ c \ m \ l0 \ (Cachera \ n)); \\ \forall \ c \ m \ l \ A \ B \ I. \ G \downarrow (c,m,l) = \ Some(A,B,I) \longrightarrow \\ (\exists \ n \ . \ (A,B,I) = \ Cachera \ n \ \land \ deriv \ G \ c \ m \ l \ (Cachera \ n)) \rrbracket \\ \Longrightarrow \ VP \end{array}$

6.3 Soundness of high-level type system

We define a first-order functional language where expressions are stratified into primitive expressions and general expressions. The language supports the construction of lists using constructors NilPrim and ConsPrim h t, and includes a corresponding pattern match operation. In order to simplify the compilation, function identifiers are taken to be pairs of class names and method names.

type-synonym $Fun = Class \times Method$

datatype Prim =
 IntPrim int
 UnPrim Val ⇒ Val Var
 BinPrim Val ⇒ Val ⇒ Val Var Var
 NilPrim
 ConsPrim Var Var
 CallPrim Fun Var list

datatype Expr = PrimE Prim | LetE Var Prim Expr | CondE Var Expr Expr | MatchE Var Expr Var Var Expr

type-synonym $FunProg = (Fun, Var \ list \times Expr)$ AssList

The type system uses contexts that associate a type (natural number) to function identifiers.

type-synonym TP-Sig = (Fun, nat) AssList

We first give the rules for primitive expressions.

inductive-set TP-prim:: $(TP-Sig \times Prim \times nat)set$ where TP-int: $(\Sigma, IntPrim \ i, 0)$: TP-prim | TP-un: $(\Sigma, UnPrim \ f \ x, 0)$: TP-prim

TP-bin: $(\Sigma, BinPrim f x y, 0)$: TP-prim

TP-nil: $(\Sigma, NilPrim, \theta)$: TP-prim

TP-cons: $(\Sigma, ConsPrim \ x \ y, 1)$: TP-prim

 $\textit{TP-Call: } \llbracket \Sigma {\downarrow} f = \textit{Some n} \rrbracket \Longrightarrow (\Sigma, \textit{CallPrim f args}, n) : \textit{TP-prim}$

Next, the rules for general expressions.

 $\begin{array}{l} \text{inductive-set } TP\text{-}expr::(TP\text{-}Sig \times Expr \times nat) \ set \\ \text{where} \\ TP\text{-}sub: \ \llbracket (\Sigma, e, m): TP\text{-}expr; \ m \leq n \rrbracket \Longrightarrow (\Sigma, e, n): TP\text{-}expr \\ | \\ TP\text{-}prim: \llbracket (\Sigma, p, n): TP\text{-}prim \rrbracket \Longrightarrow (\Sigma, PrimE \ p, n): \ TP\text{-}expr \\ | \\ TP\text{-}let: \ \llbracket (\Sigma, p, k): TP\text{-}prim; \ (\Sigma, e, m): TP\text{-}expr; \ n = k+m \rrbracket \\ \Longrightarrow (\Sigma, LetE \ x \ p \ e, n): \ TP\text{-}expr \\ | \\ TP\text{-}Cond: \llbracket (\Sigma, e1, n): TP\text{-}expr; \ (\Sigma, e2, n): TP\text{-}expr \rrbracket \\ \Longrightarrow (\Sigma, CondE \ x \ e1 \ e2, n): \ TP\text{-}expr \\ | \\ \end{array}$

 $TP-Match: \llbracket (\Sigma, e1, n): TP-expr; \ (\Sigma, e2, n): TP-expr \\ \implies (\Sigma, MatchE \ x \ e1 \ h \ t \ e2, n): TP-expr \\ \end{bmatrix}$

A functional program is well-typed if its domain agrees with that of some context such that each function body validates the context entry.

 $\begin{array}{l} \textbf{definition} \ TP::TP-Sig \Rightarrow FunProg \Rightarrow bool \ \textbf{where} \\ TP \ \Sigma \ F = ((\forall \ f \ . \ (\Sigma \downarrow f = None)) = (F \downarrow f = None)) \land \\ (\forall \ f \ n \ par \ e \ . \ \Sigma \downarrow f = Some \ n \longrightarrow F \downarrow f = Some \ (par, e) \longrightarrow (\Sigma, e, n):TP-expr)) \end{array}$

For the translation into bytecode, we introduce identifiers for a class of lists, the expected field names, and a temporary (reserved) variable name.

axiomatization

LIST::Class and HD::Field and TL::Field and tmp::Var

The compilation of primitive expressions extends a code block by a sequence of JVM instructions that leave a value on the top of the operand stack.

inductive-set compilePrim::

```
(Label \times (Label, Instr) AssList \times Prim \times ((Label, Instr) AssList \times Label)) set
where
compileInt: (l, code, IntPrim i, (code[l \mapsto (const (IVal i))], l+1)) : compilePrim
compileUn:
 (l, code, UnPrim f x, (code[l \mapsto (load x)][(l+1) \mapsto (unop f)], l+2)) : compilePrim
compileBin:
(l, code, BinPrim f x y,
    (code[l\mapsto (load x)][(l+1)\mapsto (load y)][(l+2)\mapsto (binop f)], l+3)): compilePrim
compileNil:
  (l, code, NilPrim, (code[l \mapsto (const (RVal Nullref))], l+1)) : compilePrim
compileCons:
 (l, code, ConsPrim x y,
     (code[l\mapsto (load y)][(l+1)\mapsto (load x)]
          [(l+2)\mapsto (new \ LIST)][(l+3)\mapsto store \ tmp]
          [(l+4)\mapsto load tmp][(l+5)\mapsto (put field LIST HD)]
          [(l+6)\mapsto load tmp][(l+7)\mapsto (put field LIST TL)]
          [(l+8)\mapsto(load\ tmp)],\ l+9)):\ compilePrim
compileCall-Nil:
 (l, code, CallPrim f [], (code[l \mapsto invokeS (fst f) (snd f)], l+1)): compilePrim
compileCall-Cons:
  [(l+1, code[l \mapsto load x], CallPrim f args, OUT) : compilePrim]
  \implies (l, code, CallPrim f (x#args), OUT): compilePrim
```

The following lemma shows that the resulting code is an extension of the code submitted as an argument, and that the new instructions define a contiguous block.

A signature corresponds to a method specification table if all context entries are represented as MST entries and method names that are defined in the global program P.

 $\begin{array}{l} \textbf{definition } Sig-good::TP-Sig \Rightarrow bool \textbf{ where} \\ Sig-good \ \Sigma = \\ (\forall \ C \ m \ n. \ \Sigma \downarrow (C,m) = Some \ n \longrightarrow \\ (MST \downarrow (C, \ m) = Some \ (mkSPEC \ (Cachera \ n) \ emp) \land \\ (\exists \ par \ code \ l0 \ . \ mbody-is \ C \ m \ (par, code, l0)))) \end{array}$

This definition requires MST to associate the specification

mkSPEC (Cachera n) emp

to each method to which the type signature associates the type n. In particular, this requires the annotation table of such a method to be empty. Additionally, the global program P is required to contain a method definition for each method (i.e. function) name occurring in the domain of the signature.

An auxiliary abbreviation that captures when a block of code has trivial annotations and only comprises defined program labels.

 $\begin{array}{l} \textbf{definition Segment::} \\ Class \Rightarrow Method \Rightarrow Label \Rightarrow Label \Rightarrow (Label, Instr) AssList \Rightarrow bool \\ \textbf{where} \\ Segment \ C \ m \ l \ l1 \ code = \\ (\exists \ Mspec \ Minv \ Anno \ . \ MST \downarrow (C,m) = Some(Mspec, Minv, Anno) \land \\ (\forall \ ll. \ l \leq ll \longrightarrow ll < ll \longrightarrow \\ Anno \downarrow (ll) = None \land (\exists \ ins. \ ins-is \ C \ m \ ll \ ins \land \ code \downarrow ll = Some \ ins))) \end{array}$

The soundness of (the translation of) a function call is proven by induction on the list of arguments.

$$\begin{split} MST \downarrow (C, \ m) &= Some \ (T, \ MI, Anno) \longrightarrow Segment \ C \ m \ l \ l1 \ code1 \longrightarrow \\ derivAssum \ G \ C \ m \ l1 \ (Cachera \ k) \longrightarrow \\ deriv \ G \ C \ m \ l \ (Cachera \ (n+k))) \end{split}$$
lemma Call-Sound: [[Sig-good $\Sigma; \ \Sigma \downarrow f = Some \ n;$ (l, code, CallPrim f args, code1, l1) \in compilePrim; $MST \downarrow (C,m) = Some \ (T, \ MI, Anno); \ Segment \ C \ m \ l \ l1 \ code1; \\ derivAssum \ G \ C \ m \ l1 \ (Cachera \ nn); \ k = n+nn]$] $\implies deriv \ G \ C \ m \ l \ (Cachera \ k)$

The definition of basic instructions.

 $\begin{array}{l} \textbf{definition} \ basic::Instr \Rightarrow bool \ \textbf{where} \\ basic \ ins = ((\exists \ c \ . \ ins = \ const \ c) \lor \ ins = \ dup \lor \\ ins = \ pop \lor \ ins = \ swap \lor (\exists \ x. \ ins = \ load \ x) \lor \\ (\exists \ y. \ ins = \ store \ y) \lor (\exists \ f. \ ins = \ load \ x) \lor \\ (\exists \ g. \ ins = \ unop \ g) \lor (\exists \ c1 \ F1. \ ins = \ getfield \ c1 \ F1) \lor \\ (\exists \ c2 \ F2. \ ins = \ putfield \ c2 \ F2) \lor \\ (\exists \ c3. \ ins = \ checkcast \ c3)) \end{array}$

Next, we prove the soundness of basic instructions. The hypothesis refers to instructions located at the program continuation.

lemma Basic-Sound:

 $\begin{bmatrix} Segment \ C \ m \ l \ ll \ code; \ MST \downarrow (C,m) = Some \ (T, \ MI, Anno); \ l \le l1; \ l1 < ll; \\ l2 = l1 + 1; \ code \downarrow l1 = Some \ ins; \ basic \ ins; \ derivAssum \ G \ C \ m \ l2 \ (Cachera \ n) \end{bmatrix} \implies deriv \ G \ C \ m \ l1 \ (Cachera \ n)$

Following this, the soundness of the type system for primitive expressions. The proof proceeds by induction on the typing judgement.

lemma *TP-prim-Sound*[*rule-format*]:

 $\begin{array}{l} (\Sigma,p,n)\colon TP\text{-}prim \Longrightarrow\\ Sig-good \ \Sigma \longrightarrow\\ (\forall \ l \ code \ code1 \ l1 \ G \ C \ m \ T \ MI \ Anno \ nn \ k.\\ (l, \ code, \ p, \ (code1,l1)) \ : \ compilePrim \longrightarrow\\ MST\downarrow(C,m) = Some \ (T,MI,Anno) \longrightarrow\\ Segment \ C \ m \ l \ l1 \ code1 \longrightarrow derivAssum \ G \ C \ m \ l1 \ (Cachera \ nn) \longrightarrow\\ k = \ n+nn \longrightarrow deriv \ G \ C \ m \ l \ (Cachera \ k)) \end{array}$

The translation of general expressions is defined similarly, but no code continuation is required.

inductive-set compileExpr:: (Label × (Label,Instr) AssList × Expr × ((Label,Instr) AssList × Label)) set where compilePrimE: $[[(l, code, p, (code1,l1)) : compilePrim; OUT = (code1[l1 \mapsto vreturn],l1+1)]]$ $\implies (l, code, PrimE p, OUT):compileExpr$ | compileLetE: $[[(l, code, p, (code1,l1)) : compilePrim; (code2,l2) = (code1[l1 \mapsto (store x)],l1+1);$

(l2, code2, e, OUT) : compileExpr \implies (l, code, LetE x p e, OUT) : compileExpr *compileCondE*: [(l+2, code, e2, (codeElse, XXX)) : compileExpr;(XXX, codeElse, e1, (codeThen, YYY)) : compileExpr; $OUT = (codeThen[l \mapsto load x][(l+1) \mapsto (iftrue XXX)], YYY)]$ \implies (l, code, CondE x e1 e2, OUT): compileExpr *compileMatchE*: [(l+9, code, e2, (codeCons, lNil)) : compileExpr;(lNil, codeCons, e1, (codeNil,lRes)) : compileExpr; $OUT = (codeNil[l \mapsto (load x)]$ $[(l+1)\mapsto (unop \ (\lambda \ v \ . \ if \ v = RVal \ Nullref$ then TRUE else FALSE))] $[(l+2)\mapsto (iftrue\ lNil)]$ $[(l+3)\mapsto(load x)]$ $[(l+4)\mapsto (getfield\ LIST\ HD)]$ $[(l+5)\mapsto(store\ h)]$ $[(l+6)\mapsto(load x)]$ $[(l+7)\mapsto (get field \ LIST \ TL)]$ $[(l+8)\mapsto(store\ t)],\ lRes)$ \implies (l, code, MatchE x e1 h t e2, OUT): compileExpr

Again, we prove an auxiliary result on the emitted code, by induction on the compilation judgement.

Then, soundness of the epxression type system is proven by induction on the typing judgement.

The full translation of a functional program into a bytecode program is defined as follows.

definition compileProg::FunProg \Rightarrow bool where compileProg F = $((\forall C m par e. F\downarrow(C,m) = Some(par,e) \longrightarrow$ $(\exists code l0 l. mbody-is C m (rev par,code,l0) \land$ $(l0, [], e, (code, l)): compileExpr)) \land$ $(\forall C m. (\exists M. mbody-is C m M) = (\exists fdecl . F \downarrow (C, m) = Some fdecl)))$

The final condition relating a typing context to a method specification table.

 $\begin{array}{l} \textbf{definition} \ TP\text{-}MST:: TP\text{-}Sig \Rightarrow bool \ \textbf{where} \\ TP\text{-}MST \ \Sigma = \\ (\forall \ C \ m \ . \ case \ (MST \downarrow (C,m)) \ of \\ None \Rightarrow \Sigma \downarrow (C,m) = None \\ \mid Some(T,MI,Anno) \Rightarrow Anno = emp \land \\ (\exists \ n \ . \Sigma \downarrow (C,m) = Some \ n \land \\ (T,MI,Anno) = mkSPEC \ (Cachera \ n) \ emp)) \end{array}$

For well-typed programs, this property implies the earlier condition on signatures.

lemma translation-good: $[compileProg F; TP-MST \Sigma; TP \Sigma F] \implies Sig-good \Sigma$

We can thus prove that well-typed function bodies satisfy the specifications asserted by the typing context.

lemma CACH-BodiesDerivable[rule-format]: [[mbody-is C m (par, code, l); compileProg F; TP-MST Σ ; TP Σ F]] $\implies \exists n . MST \downarrow (C,m) = Some(mkSPEC(Cachera n) emp) \land$ deriv [] C m l (Cachera n)

From this, the overall soundness result follows easily.

theorem CACH-VERIFIED: $[TP \Sigma F; TP-MST \Sigma; compileProg F] \implies VP$

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