# Broadcast Psi-calculi

Palle Raabjerg  Johannes Åman Pohjola  Tjark Weber

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**Abstract**

We provide an Isabelle/HOL-Nominal formalisation of the definitions, theorems and proofs in the paper *Broadcast Psi-calculi with an Application to Wireless Protocols* by Borgström et al., which extends the Psi-calculi framework with primitives for broadcast communication in order to model wireless protocols.

## 1 Introduction

We provide an Isabelle/HOL-Nominal formalisation of the definitions, theorems and proofs in the paper *Broadcast Psi-calculi with an Application to Wireless Protocols* [4, 5], which extends the Psi-calculi framework [2, 3, 1] with primitives for broadcast communication in order to model wireless protocols.

The file `Broadcast_Thms.thy` contains a collection of the relevant definitions and theorems, with comments relating them directly to the paper.

## 2 Formalisation

**theory** *Broadcast-Chain*
  **imports** *Psi-Calculi.Chain*
**begin**

**lemma** *pair-perm-fresh-contr*:
  **fixes** *a*::*'a* **and** *b*::*'a*
  **assumes**
    *at*: *at TYPE*(*'a*)
  **and**
    *prems*: *b* ♯ *pi* (*a*, *b*) ∈ *set pi*
  **shows** *False*
  ⟨*proof*⟩

**lemma** *pair-perm-fresh-contr'*:
  **fixes** *a*::*'a* **and** *b*::*'a*
  **assumes**

*at*: *at TYPE*('*a*)
**and**
*prems*: *a* ♯ *pi* (*a*, *b*) ∈ *set pi*
**shows** *False*
⟨*proof*⟩

**lemma** *list-set-supp*:
**fixes** *l* :: ('*d*::*fs-name*) *list*
**shows** *supp* (*set l*) = (*supp l* :: *name set*)
⟨*proof*⟩

**lemma** *name-set-supp*:
**assumes** *finite a*
**shows** *supp a* = (*a*::*name set*)
⟨*proof*⟩

**lemma** *supp-idem*:
**fixes** *l* :: ('*d*::*fs-name*)
**shows** *supp*((*supp l*)::*name set*) = (*supp*(*l*)::*name set*)
⟨*proof*⟩

**lemma** *fresh-supp*:
**fixes** *a* :: *name*
**and** *X* :: ('*d*::*fs-name*)
**shows** *a* ♯ ((*supp X*)::*name set*) = *a* ♯ *X*
⟨*proof*⟩

**lemma** *fresh-chain-supp*:
**fixes** *A* :: *name list*
**and** *X* :: ('*d*::*fs-name*)
**shows** *A* ♯∗ ((*supp X*)::*name set*) = *A* ♯∗ *X*
⟨*proof*⟩

**lemma** *fresh-chain-fin-union*:
**fixes** *X*::('*d*::*fs-name set*)
**and** *Y*::('*d*::*fs-name set*)
**and** *A*::*name list*
**assumes** *f1*: *finite X*
**and** *f2*: *finite Y*
**shows** *A*♯∗(*X*∪*Y*) = (*A*♯∗*X* ∧ *A*♯∗ *Y*)
⟨*proof*⟩

**lemma** *fresh-subset*:
**fixes** *S* :: *name set*
**and** *S'* :: *name set*
**and** *a* :: *name*
**assumes** *a* ♯ *S*
**and** *S'* ⊆ *S*
**and** *finite S*

**shows** $a \sharp S'$
⟨*proof*⟩

**lemma** *fresh-subset'*:
**fixes** $S :: 'd::fs\text{-}name\ set$
**and** $S' :: 'd::fs\text{-}name\ set$
**and** $a :: name$
**assumes** $a \sharp S$
**and** $S' \subseteq S$
**and** *finite* $S$

**shows** $a \sharp S'$
⟨*proof*⟩

**lemma** *fresh-star-subset'*:
**fixes** $S :: 'd::fs\text{-}name\ set$
**and** $S' :: 'd::fs\text{-}name\ set$
**and** $A :: name\ list$
**assumes** $A \sharp* S$
**and** $S' \subseteq S$
**and** *finite* $S$

**shows** $A \sharp* S'$
⟨*proof*⟩

**lemma** *fresh-star-subset*:
**fixes** $S :: name\ set$
**and** $S' :: name\ set$
**and** $A :: name\ list$
**assumes** $A \sharp* S$
**and** $S' \subseteq S$
**and** *finite* $S$

**shows** $A \sharp* S'$
⟨*proof*⟩

**lemma** *times-set-fresh*:
**fixes** $a :: name$
**and** $S :: name\ list$
**and** $S' :: name\ list$
**assumes** $a \sharp set\ S$
**and** $a \sharp set\ S'$
**shows** $a \sharp set\ S \times set\ S'$
⟨*proof*⟩

**lemma** *times-set-fresh-star*:
**fixes** $A :: name\ list$
**and** $S :: name\ list$

**and** $S'$ :: *name list*
**assumes** $A \sharp* set\ S$
**and** $A \sharp* set\ S'$
**shows** $A \sharp* (set\ S \times set\ S')$
⟨*proof*⟩

**lemma** *supp-list-set*:
**fixes** $M$::$'d$::*fs-name list*
**shows** $(supp\ M) = ((supp(set\ M))$::*name set*$)$
⟨*proof*⟩

**lemma** *fresh-list-set*:
**fixes** $M$::$'d$::*fs-name list*
**and** $A$::*name list*
**shows** $A \sharp* set\ M = A \sharp* M$
⟨*proof*⟩

**lemma** *permSupp*:
**fixes** $\Psi$ :: *name prm*
**and** $\Psi'$ :: $'d$::*fs-name*

**shows** $(supp(\Psi \cdot \Psi')$::*name set*$) \subseteq ((supp\ \Psi) \cup (supp\ \Psi'))$
⟨*proof*⟩

**end**
**theory** *Broadcast-Frame*
**imports** *Psi-Calculi.Frame*
**begin**

**locale** *assertionAux* = *Frame.assertionAux SCompose SImp SBottom SChanEq*
**for** *SCompose* :: $'b$::*fs-name* ⇒ $'b$ ⇒ $'b$ (**infixr** ⊗ *80*)
**and** *SImp* :: $'b$ ⇒ $'c$::*fs-name* ⇒ *bool* (- ⊢ - [*70*, *70*] *70*)
**and** *SBottom* :: $'b$ (⊥ *90*)
**and** *SChanEq* :: ($'a$::*fs-name* ⇒ $'a$ ⇒ $'c$) (- ↔ - [*80*, *80*] *80*)
+
**fixes** *SOutCon* :: $'a$::*fs-name* ⇒ $'a$ ⇒ $'c$ (- ≺ - [*80*, *80*] *80*)
**and** *SInCon* :: $'a$::*fs-name* ⇒ $'a$ ⇒ $'c$ (- ≻ - [*80*, *80*] *80*)

**assumes** *statEqvt'''*[*eqvt*]: ⋀*p*::*name prm*. $p \cdot (M \prec N) = (p \cdot M) \prec (p \cdot N)$
**and** *statEqvt''''*[*eqvt*]: ⋀*p*::*name prm*. $p \cdot (M \succ N) = (p \cdot M) \succ (p \cdot N)$

**begin**

**lemma** *chanInConSupp*:
**fixes** $M$ :: $'a$
**and** $N$ :: $'a$

**shows** $(supp(M \succ N)$::*name set*$) \subseteq ((supp\ M) \cup (supp\ N))$
⟨*proof*⟩

**lemma** *chanOutConSupp*:
**fixes** $M :: 'a$
**and** $N :: 'a$

**shows** $(supp(M \preceq N)::name\ set) \subseteq ((supp\ M) \cup (supp\ N))$
⟨*proof*⟩

**lemma** *freshInCon*[*intro*]:
**fixes** $x :: name$
**and** $M :: 'a$
**and** $N :: 'a$

**assumes** $x \sharp M$
**and** $x \sharp N$

**shows** $x \sharp M \succeq N$
⟨*proof*⟩

**lemma** *freshInConChain*[*intro*]:
**fixes** $xvec :: name\ list$
**and** $Xs :: name\ set$
**and** $M :: 'a$
**and** $N :: 'a$

**shows** $[\![xvec \sharp* M; xvec \sharp* N]\!] \Longrightarrow xvec \sharp* (M \succeq N)$
**and** $[\![Xs \sharp* M; Xs \sharp* N]\!] \Longrightarrow Xs \sharp* (M \succeq N)$
⟨*proof*⟩

**lemma** *freshOutCon*[*intro*]:
**fixes** $x :: name$
**and** $M :: 'a$
**and** $N :: 'a$

**assumes** $x \sharp M$
**and** $x \sharp N$

**shows** $x \sharp M \preceq N$
⟨*proof*⟩

**lemma** *freshOutConChain*[*intro*]:
**fixes** $xvec :: name\ list$
**and** $Xs :: name\ set$
**and** $M :: 'a$
**and** $N :: 'a$

**shows** $[\![xvec \sharp* M; xvec \sharp* N]\!] \Longrightarrow xvec \sharp* (M \preceq N)$
**and** $[\![Xs \sharp* M; Xs \sharp* N]\!] \Longrightarrow Xs \sharp* (M \preceq N)$
⟨*proof*⟩

**lemma** *chanOutConClosed*:
**fixes** $\Psi :: 'b$
**and** $M :: 'a$
**and** $N :: 'a$
**and** $p :: name\ prm$

**assumes** $\Psi \vdash M \preceq N$

**shows** $(p \cdot \Psi) \vdash (p \cdot M) \preceq (p \cdot N)$
⟨*proof*⟩

**lemma** *chanInConClosed*:
**fixes** $\Psi :: 'b$
**and** $M :: 'a$
**and** $N :: 'a$
**and** $p :: name\ prm$

**assumes** $\Psi \vdash M \succeq N$

**shows** $(p \cdot \Psi) \vdash (p \cdot M) \succeq (p \cdot N)$
⟨*proof*⟩

**end**

**locale** *assertion* = *assertionAux SCompose SImp SBottom SChanEq SOutCon SInCon* + *assertion SCompose SImp SBottom SChanEq*
**for** *SCompose* :: *'b*::*fs-name* ⇒ *'b* ⇒ *'b*
**and** *SImp* :: *'b* ⇒ *'c*::*fs-name* ⇒ *bool*
**and** *SBottom* :: *'b*
**and** *SChanEq* :: *'a*::*fs-name* ⇒ *'a* ⇒ *'c*
**and** *SOutCon* :: *'a*::*fs-name* ⇒ *'a* ⇒ *'c*
**and** *SInCon* :: *'a*::*fs-name* ⇒ *'a* ⇒ *'c* +

**assumes** *chanOutConSupp*: *SImp* Ψ (*SOutCon M N*) ⟹ (((*supp N*)::*name set*) ⊆ ((*supp M*)::*name set*))
**and** *chanInConSupp*: *SImp* Ψ (*SInCon N M*) ⟹ (((*supp N*)::*name set*) ⊆ ((*supp M*)::*name set*))

**begin**

**notation** *SOutCon* (- ≼ - [*90*, *90*] *90*)
**notation** *SInCon* (- ≽ - [*90*, *90*] *90*)

**end**

**end**
**theory** *Semantics*
**imports** *Broadcast-Chain Broadcast-Frame*

**begin**

This file is a (heavily modified) variant of the theory *Psi_Calculi.Semantics* from [1]. The nominal datatypes *('a,'b,'c) residual* and *'a action* have been extended with constructors for broadcast input and output. This leads to a different semantics.

**nominal-datatype** $('a, 'b, 'c)$ *boundOutput* =
  *BOut* $'a$::*fs-name* $('a, 'b$::*fs-name*, $'c$::*fs-name*) *psi* (- $\prec''$ - [*110*, *110*] *110*)
  | *BStep* «*name*» $('a, 'b, 'c)$ *boundOutput* (⦇$\nu$-⦈- [*110*, *110*] *110*)

**primrec** *BOresChain* :: *name list* ⇒ ($'a$::*fs-name*, $'b$::*fs-name*, $'c$::*fs-name*) *boundOutput* ⇒
  $('a, 'b, 'c)$ *boundOutput*
  **where**
    *Base*: *BOresChain* [] $B = B$
  | *Step*: *BOresChain* $(x\#xs)$ $B$ = ⦇$\nu x$⦈(*BOresChain* $xs$ $B$)

**abbreviation**
  *BOresChainJudge* (⦇$\nu$∗-⦈- [*80*, *80*] *80*) **where** ⦇$\nu$∗*xvec*⦈$B \equiv$ *BOresChain* *xvec* $B$

**lemma** *BOresChainEqvt*[*eqvt*]:
  **fixes** *perm* :: *name prm*
    **and** *lst* :: *name list*
    **and** $B$ :: ($'a$::*fs-name*, $'b$::*fs-name*, $'c$::*fs-name*) *boundOutput*

**shows** *perm* ∙ (⦇$\nu$∗*xvec*⦈$B$) = ⦇$\nu$∗(*perm* ∙ *xvec*)⦈(*perm* ∙ $B$)
  ⟨*proof*⟩

**lemma** *BOresChainSimps*[*simp*]:
  **fixes** *xvec* :: *name list*
    **and** $N$ :: $'a$::*fs-name*
    **and** $P$ :: ($'a$, $'b$::*fs-name*, $'c$::*fs-name*) *psi*
    **and** $N'$ :: $'a$
    **and** $P'$ :: $('a, 'b, 'c)$ *psi*
    **and** $B$ :: $('a, 'b, 'c)$ *boundOutput*
    **and** $B'$ :: $('a, 'b, 'c)$ *boundOutput*

**shows** (⦇$\nu$∗*xvec*⦈$N \prec' P = N' \prec' P') = (xvec = [] \wedge N = N' \wedge P = P')$
  **and** $(N' \prec' P' =$ ⦇$\nu$∗*xvec*⦈$N \prec' P) = (xvec = [] \wedge N = N' \wedge P = P')$
  **and** $(N' \prec' P' = N \prec' P) = (N = N' \wedge P = P')$
  **and** (⦇$\nu$∗*xvec*⦈$B =$ ⦇$\nu$∗*xvec*⦈$B') = (B = B')$
  ⟨*proof*⟩

**lemma** *outputFresh*[*simp*]:
  **fixes** *Xs* :: *name set*
    **and** *xvec* :: *name list*
    **and** $N$ :: $'a$::*fs-name*
    **and** $P$ :: ($'a$, $'b$::*fs-name*, $'c$::*fs-name*) *psi*

**shows** $(Xs \sharp* (N \prec' P)) = ((Xs \sharp* N) \wedge (Xs \sharp* P))$
**and** $(xvec \sharp* (N \prec' P)) = ((xvec \sharp* N) \wedge (xvec \sharp* P))$
⟨*proof*⟩

**lemma** *boundOutputFresh*:
**fixes** $x$ :: *name*
**and** $xvec$ :: *name list*
**and** $B$ :: $('a::fs\text{-}name, 'b::fs\text{-}name, 'c::fs\text{-}name)$ *boundOutput*

**shows** $(x \sharp (\!|\nu*xvec|\!) B)) = (x \in set\ xvec \vee x \sharp B)$
⟨*proof*⟩

**lemma** *boundOutputFreshSet*:
**fixes** $Xs$ :: *name set*
**and** $xvec$ :: *name list*
**and** $B$ :: $('a::fs\text{-}name, 'b::fs\text{-}name, 'c::fs\text{-}name)$ *boundOutput*
**and** $yvec$ :: *name list*
**and** $x$ :: *name*

**shows** $Xs \sharp* ((\!|\nu*xvec|\!) B) = (\forall x \in Xs.\ x \in set\ xvec \vee x \sharp B)$
**and** $yvec \sharp* ((\!|\nu*xvec|\!) B) = (\forall x \in (set\ yvec).\ x \in set\ xvec \vee x \sharp B)$
**and** $Xs \sharp* ((\!|\nu x|\!) B) = Xs \sharp* [x].B$
**and** $xvec \sharp* ((\!|\nu x|\!) B) = xvec \sharp* [x].B$
⟨*proof*⟩

**lemma** *BOresChainSupp*:
**fixes** $xvec$ :: *name list*
**and** $B$ :: $('a::fs\text{-}name, 'b::fs\text{-}name, 'c::fs\text{-}name)$ *boundOutput*

**shows** $(supp((\!|\nu*xvec|\!) B)::name\ set) = (supp\ B) - (supp\ xvec)$
⟨*proof*⟩

**lemma** *boundOutputFreshSimps*[*simp*]:
**fixes** $Xs$ :: *name set*
**and** $xvec$ :: *name list*
**and** $B$ :: $('a::fs\text{-}name, 'b::fs\text{-}name, 'c::fs\text{-}name)$ *boundOutput*
**and** $yvec$ :: *name list*
**and** $x$ :: *name*

**shows** $Xs \sharp* xvec \Longrightarrow (Xs \sharp* ((\!|\nu*xvec|\!) B)) = (Xs \sharp* B)$
**and** $yvec \sharp* xvec \Longrightarrow yvec \sharp* ((\!|\nu*xvec|\!) B) = yvec \sharp* B$
**and** $xvec \sharp* ((\!|\nu*xvec|\!) B)$
**and** $x \sharp xvec \Longrightarrow x \sharp (\!|\nu*xvec|\!) B = x \sharp B$
⟨*proof*⟩

**lemma** *boundOutputChainAlpha*:
**fixes** $p$ :: *name prm*
**and** $xvec$ :: *name list*
**and** $B$ :: $('a::fs\text{-}name, 'b::fs\text{-}name, 'c::fs\text{-}name)$ *boundOutput*

**and** $yvec :: name\ list$

**assumes** $xvecFreshB$: $(p \cdot xvec) \sharp* B$
**and** $S$: $set\ p \subseteq set\ xvec \times set\ (p \cdot xvec)$
**and** $(set\ xvec) \subseteq (set\ yvec)$

**shows** $(\langle\!|\nu* yvec|\!\rangle B) = (\langle\!|\nu*(p \cdot yvec)|\!\rangle(p \cdot B))$
⟨*proof*⟩

**lemma** *boundOutputChainAlpha′*:
**fixes** $p$ :: *name prm*
**and** $xvec$ :: *name list*
**and** $B$ :: $('a::fs\text{-}name, 'b::fs\text{-}name, 'c::fs\text{-}name)\ boundOutput$
**and** $yvec$ :: *name list*
**and** $zvec$ :: *name list*

**assumes** $xvecFreshB$: $xvec \sharp* B$
**and** $S$: $set\ p \subseteq set\ xvec \times set\ yvec$
**and** $yvec \sharp* (\langle\!|\nu* zvec|\!\rangle B)$

**shows** $(\langle\!|\nu* zvec|\!\rangle B) = (\langle\!|\nu*(p \cdot zvec)|\!\rangle(p \cdot B))$
⟨*proof*⟩

**lemma** *boundOutputChainAlpha″*:
**fixes** $p$ :: *name prm*
**and** $xvec$ :: *name list*
**and** $M$ :: $'a::fs\text{-}name$
**and** $P$ :: $('a::fs\text{-}name, 'b::fs\text{-}name, 'c::fs\text{-}name)\ psi$
**and** $yvec$ :: *name list*

**assumes** $(p \cdot xvec) \sharp* M$
**and** $(p \cdot xvec) \sharp* P$
**and** $set\ p \subseteq set\ xvec \times set\ (p \cdot xvec)$
**and** $(set\ xvec) \subseteq (set\ yvec)$

**shows** $(\langle\!|\nu* yvec|\!\rangle M \prec' P) = (\langle\!|\nu*(p \cdot yvec)|\!\rangle(p \cdot M) \prec' (p \cdot P))$
⟨*proof*⟩

**lemma** *boundOutputChainSwap*:
**fixes** $x$ :: *name*
**and** $y$ :: *name*
**and** $N$ :: $'a::fs\text{-}name$
**and** $P$ :: $('a, 'b::fs\text{-}name, 'c::fs\text{-}name)\ psi$
**and** $xvec$ :: *name list*

**assumes** $y \sharp N$
**and** $y \sharp P$
**and** $x \in (set\ xvec)$

**shows** $(\!|\nu * xvec|\!)N \prec' P = (\!|\nu * ([(x,\ y)] \cdot xvec)|\!)([(x\ ,y)] \cdot N) \prec' ([(x,\ y)] \cdot P)$
⟨*proof*⟩

**lemma** *alphaBoundOutput*:
**fixes** $x$ :: *name*
**and** $y$ :: *name*
**and** $B$ :: ('*a*::*fs-name*, '*b*::*fs-name*, '*c*::*fs-name*) *boundOutput*

**assumes** $y \sharp B$

**shows** $(\!|\nu x|\!)B = (\!|\nu y|\!)([(x,\ y)] \cdot B)$
⟨*proof*⟩

**lemma** *boundOutputEqFresh*:
**fixes** $B$ :: ('*a*::*fs-name*, '*b*::*fs-name*, '*c*::*fs-name*) *boundOutput*
**and** $C$ :: ('*a*, '*b*, '*c*) *boundOutput*
**and** $x$ :: *name*
**and** $y$ :: *name*

**assumes** $(\!|\nu x|\!)B = (\!|\nu y|\!)C$
**and** $x \sharp B$

**shows** $y \sharp C$
⟨*proof*⟩

**lemma** *boundOutputEqSupp*:
**fixes** $B$ :: ('*a*::*fs-name*, '*b*::*fs-name*, '*c*::*fs-name*) *boundOutput*
**and** $C$ :: ('*a*, '*b*, '*c*) *boundOutput*
**and** $x$ :: *name*
**and** $y$ :: *name*

**assumes** $(\!|\nu x|\!)B = (\!|\nu y|\!)C$
**and** $x \in supp\ B$

**shows** $y \in supp\ C$
⟨*proof*⟩

**lemma** *boundOutputChainEq*:
**fixes** *xvec* :: *name list*
**and** $B$ :: ('*a*::*fs-name*, '*b*::*fs-name*, '*c*::*fs-name*) *boundOutput*
**and** *yvec* :: *name list*
**and** $B'$ :: ('*a*, '*b*, '*c*) *boundOutput*

**assumes** $(\!|\nu * xvec|\!)B = (\!|\nu * yvec|\!)B'$
**and** $xvec \sharp * yvec$
**and** $length\ xvec = length\ yvec$

**shows** $\exists p.\ (set\ p) \subseteq (set\ xvec) \times set\ (yvec) \wedge distinctPerm\ p \wedge\ B = p \cdot B' \wedge (set\ (map\ fst\ p)) \subseteq (supp\ B) \wedge xvec \sharp * B' \wedge yvec \sharp * B$

⟨*proof*⟩

**lemma** *boundOutputChainEqLength*:
**fixes** *xvec* :: *name list*
**and** *M* :: *'a*::*fs-name*
**and** *P* :: (*'a*, *'b*::*fs-name*, *'c*::*fs-name*) *psi*
**and** *yvec* :: *name list*
**and** *N* :: *'a*::*fs-name*
**and** *Q* :: (*'a*, *'b*::*fs-name*, *'c*::*fs-name*) *psi*

**assumes** (|*ν*∗*xvec*|)*M* ≺′ *P* = (|*ν*∗*yvec*|)*N* ≺′ *Q*

**shows** *length xvec* = *length yvec*
⟨*proof*⟩

**lemma** *boundOutputChainEq′*:
**fixes** *xvec* :: *name list*
**and** *M* :: *'a*::*fs-name*
**and** *P* :: (*'a*, *'b*::*fs-name*, *'c*::*fs-name*) *psi*
**and** *yvec* :: *name list*
**and** *N* :: *'a*
**and** *Q* :: (*'a*::*fs-name*, *'b*::*fs-name*, *'c*::*fs-name*) *psi*

**assumes** (|*ν*∗*xvec*|)*M* ≺′ *P* = (|*ν*∗*yvec*|)*N* ≺′ *Q*
**and** *xvec* ♯∗ *yvec*

**shows** ∃ *p*. (*set p*) ⊆ (*set xvec*) × *set* (*yvec*) ∧ *distinctPerm p* ∧ *M* = *p* ∙ *N* ∧ *P* = *p* ∙ *Q* ∧ *xvec* ♯∗ *N* ∧ *xvec* ♯∗ *Q* ∧ *yvec* ♯∗ *M* ∧ *yvec* ♯∗ *P*
⟨*proof*⟩

**lemma** *boundOutputChainEq′′*:
**fixes** *xvec* :: *name list*
**and** *M* :: *'a*::*fs-name*
**and** *P* :: (*'a*, *'b*::*fs-name*, *'c*::*fs-name*) *psi*
**and** *yvec* :: *name list*
**and** *N* :: *'a*
**and** *Q* :: (*'a*::*fs-name*, *'b*::*fs-name*, *'c*::*fs-name*) *psi*

**assumes** (|*ν*∗*xvec*|)*M* ≺′ *P* = (|*ν*∗*yvec*|)*N* ≺′ *Q*
**and** *xvec* ♯∗ *yvec*
**and** *distinct xvec*
**and** *distinct yvec*

**obtains** *p* **where** (*set p*) ⊆ (*set xvec*) × *set* (*p* ∙ *xvec*) **and** *distinctPerm p* **and** *yvec* = *p* ∙ *xvec* **and** *N* = *p* ∙ *M* **and** *Q* = *p* ∙ *P* **and** *xvec* ♯∗ *N* **and** *xvec* ♯∗ *Q* **and** (*p* ∙ *xvec*) ♯∗ *M* **and** (*p* ∙ *xvec*) ♯∗ *P*
⟨*proof*⟩

**lemma** *boundOutputEqSupp′*:

**fixes** $x$ :: *name*
**and** *xvec* :: *name list*
**and** $M$ :: $'a$::*fs-name*
**and** $P$ :: ($'a$, $'b$::*fs-name*, $'c$::*fs-name*) *psi*
**and** $y$ :: *name*
**and** *yvec* :: *name list*
**and** $N$ :: $'a$
**and** $Q$ :: ($'a$, $'b$, $'c$) *psi*

**assumes** *Eq*: $(\!|\nu x|\!)((\!|\nu{*}xvec|\!)M \prec' P) = (\!|\nu y|\!)((\!|\nu{*}yvec|\!)N \prec' Q)$
**and** $x \neq y$
**and** $x \sharp yvec$
**and** $x \sharp xvec$
**and** $y \sharp xvec$
**and** $y \sharp yvec$
**and** $xvec \sharp* yvec$
**and** $x \in supp\ M$

**shows** $y \in supp\ N$
⟨*proof*⟩

**lemma** *boundOutputChainOpenIH*:
**fixes** *xvec* :: *name list*
**and** $x$ :: *name*
**and** $B$ :: ($'a$::*fs-name*, $'b$::*fs-name*, $'c$::*fs-name*) *boundOutput*
**and** *yvec* :: *name list*
**and** $y$ :: *name*
**and** $B'$ :: ($'a$, $'b$, $'c$) *boundOutput*

**assumes** *Eq*: $(\!|\nu{*}xvec|\!)((\!|\nu x|\!)B) = (\!|\nu{*}yvec|\!)((\!|\nu y|\!)B')$
**and** *L*: *length xvec* = *length yvec*
**and** *xFreshB'*: $x \sharp B'$
**and** *xFreshxvec*: $x \sharp xvec$
**and** *xFreshyvec*: $x \sharp yvec$

**shows** $(\!|\nu{*}xvec|\!)B = (\!|\nu{*}yvec|\!)([(x, y)] \cdot B')$
⟨*proof*⟩

**lemma** *boundOutputPar1Dest*:
**fixes** *xvec* :: *name list*
**and** $M$ :: $'a$::*fs-name*
**and** $P$ :: ($'a$, $'b$::*fs-name*, $'c$::*fs-name*) *psi*
**and** *yvec* :: *name list*
**and** $N$ :: $'a$
**and** $Q$ :: ($'a$, $'b$, $'c$) *psi*
**and** $R$ :: ($'a$, $'b$, $'c$) *psi*

**assumes** $(\!|\nu{*}xvec|\!)M \prec' P = (\!|\nu{*}yvec|\!)N \prec' (Q \parallel R)$
**and** $xvec \sharp* R$

**and** $yvec \sharp* R$

**obtains** $T$ **where** $P = T \parallel R$ **and** $(\!|\nu * xvec|\!) M \prec' T = (\!|\nu * yvec|\!) N \prec' Q$
⟨*proof*⟩

**lemma** *boundOutputPar1Dest'*:
**fixes** $xvec :: name\ list$
**and** $M :: 'a::fs\text{-}name$
**and** $P :: ('a, 'b::fs\text{-}name, 'c::fs\text{-}name)\ psi$
**and** $yvec :: name\ list$
**and** $N :: 'a$
**and** $Q :: ('a, 'b, 'c)\ psi$
**and** $R :: ('a, 'b, 'c)\ psi$

**assumes** $(\!|\nu * xvec|\!) M \prec' P = (\!|\nu * yvec|\!) N \prec' (Q \parallel R)$
**and** $xvec \sharp* yvec$

**obtains** $T\ p$ **where** $set\ p \subseteq set\ xvec \times set\ yvec$ **and** $P = T \parallel (p \cdot R)$ **and** $(\!|\nu * xvec|\!) M \prec' T = (\!|\nu * yvec|\!) N \prec' Q$
⟨*proof*⟩

**lemma** *boundOutputPar2Dest*:
**fixes** $xvec :: name\ list$
**and** $M :: 'a::fs\text{-}name$
**and** $P :: ('a, 'b::fs\text{-}name, 'c::fs\text{-}name)\ psi$
**and** $yvec :: name\ list$
**and** $N :: 'a$
**and** $Q :: ('a, 'b, 'c)\ psi$
**and** $R :: ('a, 'b, 'c)\ psi$

**assumes** $(\!|\nu * xvec|\!) M \prec' P = (\!|\nu * yvec|\!) N \prec' (Q \parallel R)$
**and** $xvec \sharp* Q$
**and** $yvec \sharp* Q$

**obtains** $T$ **where** $P = Q \parallel T$ **and** $(\!|\nu * xvec|\!) M \prec' T = (\!|\nu * yvec|\!) N \prec' R$
⟨*proof*⟩

**lemma** *boundOutputPar2Dest'*:
**fixes** $xvec :: name\ list$
**and** $M :: 'a::fs\text{-}name$
**and** $P :: ('a, 'b::fs\text{-}name, 'c::fs\text{-}name)\ psi$
**and** $yvec :: name\ list$
**and** $N :: 'a$
**and** $Q :: ('a, 'b, 'c)\ psi$
**and** $R :: ('a, 'b, 'c)\ psi$

**assumes** $(\!|\nu * xvec|\!) M \prec' P = (\!|\nu * yvec|\!) N \prec' (Q \parallel R)$
**and** $xvec \sharp* yvec$

**obtains** $T$ $p$ **where** $set\ p \subseteq set\ xvec \times set\ yvec$ **and** $P = (p \cdot Q) \parallel T$ **and** $(\!|\nu{*}xvec|\!)M \prec' T = (\!|\nu{*}yvec|\!)N \prec' R$
⟨*proof*⟩

**lemma** *boundOutputApp*:
**fixes** *xvec* :: *name list*
**and** *yvec* :: *name list*
**and** $B$ :: ('*a*::*fs-name*, '*b*::*fs-name*, '*c*::*fs-name*) *boundOutput*

**shows** $(\!|\nu{*}(xvec@yvec)|\!)B = (\!|\nu{*}xvec|\!)((\!|\nu{*}yvec|\!)B)$
⟨*proof*⟩

**lemma** *openInjectAux*:
**fixes** *xvec1* :: *name list*
**and** $x$ :: *name*
**and** *xvec2* :: *name list*
**and** *yvec* :: *name list*

**assumes** $length(xvec1@x\#xvec2) = length\ yvec$

**shows** $\exists\, yvec1\ y\ yvec2.\ yvec = yvec1@y\#yvec2 \wedge length\ xvec1 = length\ yvec1 \wedge length\ xvec2 = length\ yvec2$
⟨*proof*⟩

**lemma** *boundOutputOpenDest*:
**fixes** *yvec* :: *name list*
**and** $M$ :: '*a*::*fs-name*
**and** $P$ :: ('*a*, '*b*::*fs-name*, '*c*::*fs-name*) *psi*
**and** *xvec1* :: *name list*
**and** $x$ :: *name*
**and** *xvec2* :: *name list*
**and** $N$ :: '*a*
**and** $Q$ :: ('*a*, '*b*, '*c*) *psi*

**assumes** *Eq*: $(\!|\nu{*}(xvec1@x\#xvec2)|\!)M \prec' P = (\!|\nu{*}yvec|\!)N \prec' Q$
**and** $x \sharp xvec1$
**and** $x \sharp yvec$
**and** $x \sharp N$
**and** $x \sharp Q$
**and** *distinct yvec*

**obtains** *yvec1 y yvec2* **where** *yvec*=*yvec1*@*y*#*yvec2* **and** *length xvec1* = *length yvec1* **and** *length xvec2* = *length yvec2*
**and** $(\!|\nu{*}(xvec1@xvec2)|\!)M \prec' P = (\!|\nu{*}(yvec1@yvec2)|\!)([(x, y)] \cdot N) \prec' ([(x, y)] \cdot Q)$
⟨*proof*⟩

**lemma** *boundOutputOpenDest'*:

**fixes** *yvec* :: *name list*
**and** *M* :: *'a::fs-name*
**and** *P* :: *('a, 'b::fs-name, 'c::fs-name) psi*
**and** *xvec1* :: *name list*
**and** *x* :: *name*
**and** *xvec2* :: *name list*
**and** *N* :: *'a*
**and** *Q* :: *('a, 'b, 'c) psi*

**assumes** *Eq*: $(\!|\nu*(xvec1@x\#xvec2)|\!)M \prec' P = (\!|\nu*yvec|\!)N \prec' Q$
**and** $x \sharp xvec1$
**and** $x \sharp yvec$
**and** $x \sharp N$
**and** $x \sharp Q$

**obtains** *yvec1 y yvec2* **where** *yvec=yvec1@y#yvec2* **and** *length xvec1 = length yvec1* **and** *length xvec2 = length yvec2*
**and** $(\!|\nu*(xvec1@xvec2)|\!)M \prec' P = (\!|\nu*(yvec1@[(x, y)] \cdot yvec2)|\!)([(x, y)] \cdot N) \prec' ([(x, y)] \cdot Q)$
⟨*proof*⟩

**lemma** *boundOutputScopeDest*:
**fixes** *xvec* :: *name list*
**and** *M* :: *'a::fs-name*
**and** *P* :: *('a, 'b::fs-name, 'c::fs-name) psi*
**and** *yvec* :: *name list*
**and** *N* :: *'a*
**and** *x* :: *name*
**and** *Q* :: *('a, 'b, 'c) psi*

**assumes** $(\!|\nu*xvec|\!)M \prec' P = (\!|\nu*yvec|\!)N \prec' (\!|\nu z|\!)Q$
**and** $z \sharp xvec$
**and** $z \sharp yvec$

**obtains** *R* **where** $P = (\!|\nu z|\!)R$ **and** $(\!|\nu*xvec|\!)M \prec' R = (\!|\nu*yvec|\!)N \prec' Q$
⟨*proof*⟩

**nominal-datatype** *('a, 'b, 'c) residual* =
*RIn 'a::fs-name 'a ('a, 'b::fs-name, 'c::fs-name) psi*
| *RBrIn 'a::fs-name 'a ('a, 'b::fs-name, 'c::fs-name) psi*
| *ROut 'a ('a, 'b, 'c) boundOutput*
| *RBrOut 'a ('a, 'b, 'c) boundOutput*
| *RTau ('a, 'b, 'c) psi*

**nominal-datatype** *'a action* = *In 'a::fs-name 'a* (-(|-|) [*90*, *90*] *90*)
| *BrIn 'a::fs-name 'a* (¿-(|-|) [*90*, *90*] *90*)
| *Out 'a::fs-name name list 'a* (-(|ν*-|)⟨-⟩ [*90*, *90*, *90*] *90*)
| *BrOut 'a::fs-name name list 'a* (¡-(|ν*-|)⟨-⟩ [*90*, *90*, *90*] *90*)

```
  | Tau                                    (τ 90)

nominal-primrec bn :: ('a::fs-name) action ⇒ name list
  where
    bn (M(⦇N⦈)) = []
  | bn (¿M(⦇N⦈)) = []
  | bn (M(⦇ν*xvec⦈)⟨N⟩) = xvec
  | bn (¡M(⦇ν*xvec⦈)⟨N⟩) = xvec
  | bn (τ) = []
  ⟨proof⟩

lemma bnEqvt[eqvt]:
  fixes p :: name prm
    and α :: ('a::fs-name) action

shows (p ∙ bn α) = bn(p ∙ α)
  ⟨proof⟩

nominal-primrec create-residual :: ('a::fs-name) action ⇒ ('a, 'b::fs-name, 'c::fs-name)
psi ⇒ ('a, 'b, 'c) residual (- ≺ - [80, 80] 80)
  where
    (M(⦇N⦈)) ≺ P = RIn M N P
  | (¿M(⦇N⦈)) ≺ P = RBrIn M N P
  | M(⦇ν*xvec⦈)⟨N⟩ ≺ P = ROut M (⦇ν*xvec⦈(N ≺' P))
  | (¡M(⦇ν*xvec⦈)⟨N⟩) ≺ P = RBrOut M (⦇ν*xvec⦈(N ≺' P))
  | τ ≺ P = (RTau P)
  ⟨proof⟩

nominal-primrec subject :: ('a::fs-name) action ⇒ 'a option
  where
    subject (M(⦇N⦈)) = Some M
  | subject (¿M(⦇N⦈)) = Some M
  | subject (M(⦇ν*xvec⦈)⟨N⟩) = Some M
  | subject (¡M(⦇ν*xvec⦈)⟨N⟩) = Some M
  | subject (τ) = None
  ⟨proof⟩

nominal-primrec object :: ('a::fs-name) action ⇒ 'a option
  where
    object (M(⦇N⦈)) = Some N
  | object (¿M(⦇N⦈)) = Some N
  | object (M(⦇ν*xvec⦈)⟨N⟩) = Some N
  | object (¡M(⦇ν*xvec⦈)⟨N⟩) = Some N
  | object (τ) = None
  ⟨proof⟩

lemma optionFreshChain[simp]:
  fixes xvec :: name list
    and X    :: name set
```

**shows** $xvec \sharp* (Some\ x) = xvec \sharp* x$
**and** $X \sharp* (Some\ x) = X \sharp* x$
**and** $xvec \sharp* None$
**and** $X \sharp* None$
⟨*proof*⟩

**lemmas** [*simp*] = *fresh-some fresh-none*

**lemma** *actionFresh*[*simp*]:
**fixes** $x :: name$
**and** $\alpha :: ('a::fs\text{-}name)\ action$

**shows** $(x \sharp \alpha) = (x \sharp (subject\ \alpha) \wedge x \sharp (bn\ \alpha) \wedge x \sharp (object\ \alpha))$
⟨*proof*⟩

**lemma** *actionFreshChain*[*simp*]:
**fixes** $X :: name\ set$
**and** $\alpha :: ('a::fs\text{-}name)\ action$
**and** $xvec :: name\ list$

**shows** $(X \sharp* \alpha) = (X \sharp* (subject\ \alpha) \wedge X \sharp* (bn\ \alpha) \wedge X \sharp* (object\ \alpha))$
**and** $(xvec \sharp* \alpha) = (xvec \sharp* (subject\ \alpha) \wedge xvec \sharp* (bn\ \alpha) \wedge xvec \sharp* (object\ \alpha))$
⟨*proof*⟩

**lemma** *subjectEqvt*[*eqvt*]:
**fixes** $p :: name\ prm$
**and** $\alpha :: ('a::fs\text{-}name)\ action$

**shows** $(p \cdot subject\ \alpha) = subject(p \cdot \alpha)$
⟨*proof*⟩

**lemma** *okjectEqvt*[*eqvt*]:
**fixes** $p :: name\ prm$
**and** $\alpha :: ('a::fs\text{-}name)\ action$

**shows** $(p \cdot object\ \alpha) = object(p \cdot \alpha)$
⟨*proof*⟩

**lemma** *create-residualEqvt*[*eqvt*]:
**fixes** $p :: name\ prm$
**and** $\alpha :: ('a::fs\text{-}name)\ action$
**and** $P :: ('a, 'b::fs\text{-}name, 'c::fs\text{-}name)\ psi$

**shows** $(p \cdot (\alpha \prec P)) = (p \cdot \alpha) \prec (p \cdot P)$
⟨*proof*⟩

**lemma** *residualFresh*:
**fixes** $x :: name$

**and** $\alpha$ :: *'a::fs-name action*
**and** $P$ :: (*'a*, *'b::fs-name*, *'c::fs-name*) *psi*

**shows** $(x \sharp (\alpha \prec P)) = (x \sharp (subject\ \alpha) \wedge (x \in (set(bn(\alpha))) \vee (x \sharp object(\alpha) \wedge x \sharp P)))$
⟨*proof*⟩

**lemma** *residualFresh2*[*simp*]:
**fixes** $x$ :: *name*
**and** $\alpha$ :: (*'a::fs-name*) *action*
**and** $P$ :: (*'a*, *'b::fs-name*, *'c::fs-name*) *psi*

**assumes** $x \sharp \alpha$
**and** $x \sharp P$

**shows** $x \sharp \alpha \prec P$
⟨*proof*⟩

**lemma** *residualFreshChain2*[*simp*]:
**fixes** *xvec* :: *name list*
**and** $X$ :: *name set*
**and** $\alpha$ :: (*'a::fs-name*) *action*
**and** $P$ :: (*'a*, *'b::fs-name*, *'c::fs-name*) *psi*

**shows** $[\![xvec \sharp* \alpha; xvec \sharp* P]\!] \Longrightarrow xvec \sharp* (\alpha \prec P)$
**and** $[\![X \sharp* \alpha; X \sharp* P]\!] \Longrightarrow X \sharp* (\alpha \prec P)$
⟨*proof*⟩

**lemma** *residualFreshSimp*[*simp*]:
**fixes** $x$ :: *name*
**and** $M$ :: *'a::fs-name*
**and** $N$ :: *'a*
**and** $P$ :: (*'a*, *'b::fs-name*, *'c::fs-name*) *psi*

**shows** $x \sharp (M(\!|N|\!) \prec P) = (x \sharp M \wedge x \sharp N \wedge x \sharp P)$
**and** $x \sharp (¿\ M(\!|N|\!) \prec P) = (x \sharp M \wedge x \sharp N \wedge x \sharp P)$
**and** $x \sharp (M(\!|\nu*xvec|\!)\langle N\rangle \prec P) = (x \sharp M \wedge x \sharp ((\!|\nu*xvec|\!)(N \prec' P)))$
**and** $x \sharp (¡M(\!|\nu*xvec|\!)\langle N\rangle \prec P) = (x \sharp M \wedge x \sharp ((\!|\nu*xvec|\!)(N \prec' P)))$
**and** $x \sharp (\tau \prec P) = (x \sharp P)$
⟨*proof*⟩

**lemma** *residualInject'*:

**shows** $(\alpha \prec P = RIn\ M\ N\ Q) = (P = Q \wedge \alpha = M(\!|N|\!))$
**and** $(\alpha \prec P = RBrIn\ M\ N\ Q) = (P = Q \wedge \alpha = ¿M(\!|N|\!))$
**and** $(\alpha \prec P = ROut\ M\ B) = (\exists\, xvec\ N.\ \alpha = M(\!|\nu*xvec|\!)\langle N\rangle \wedge B = (\!|\nu*xvec|\!)(N \prec' P))$
**and** $(\alpha \prec P = RBrOut\ M\ B) = (\exists\, xvec\ N.\ \alpha = ¡M(\!|\nu*xvec|\!)\langle N\rangle \wedge B = (\!|\nu*xvec|\!)(N$

$\prec' P))$
**and** $(\alpha \prec P = RTau\ Q) = (\alpha = \tau \wedge P = Q)$
**and** $(RIn\ M\ N\ Q = \alpha \prec P) = (P = Q \wedge \alpha = M(\!|N|\!))$
**and** $(RBrIn\ M\ N\ Q = \alpha \prec P) = (P = Q \wedge \alpha = ¿M(\!|N|\!))$
**and** $(ROut\ M\ B = \alpha \prec P) = (\exists\, xvec\ N.\ \alpha = M(\!|\nu{*}xvec|\!)\langle N\rangle \wedge B = (\!|\nu{*}xvec|\!)(N \prec' P))$
**and** $(RBrOut\ M\ B = \alpha \prec P) = (\exists\, xvec\ N.\ \alpha = ¡M(\!|\nu{*}xvec|\!)\langle N\rangle \wedge B = (\!|\nu{*}xvec|\!)(N \prec' P))$
**and** $(RTau\ Q = \alpha \prec P) = (\alpha = \tau \wedge P = Q)$
⟨*proof*⟩

**lemma** *residualFreshChainSimp*[*simp*]:
**fixes** *xvec* :: *name list*
**and** $X$ :: *name set*
**and** $M$ :: $'a$::*fs-name*
**and** $N$ :: $'a$
**and** *yvec* :: *name list*
**and** $P$ :: ($'a$, $'b$::*fs-name*, $'c$::*fs-name*) *psi*

**shows** $xvec\ \sharp*\ (M(\!|N|\!) \prec P) = (xvec\ \sharp*\ M \wedge xvec\ \sharp*\ N \wedge xvec\ \sharp*\ P)$
**and** $xvec\ \sharp*\ (¿M(\!|N|\!) \prec P) = (xvec\ \sharp*\ M \wedge xvec\ \sharp*\ N \wedge xvec\ \sharp*\ P)$
**and** $xvec\ \sharp*\ (M(\!|\nu{*}yvec|\!)\langle N\rangle \prec P) = (xvec\ \sharp*\ M \wedge xvec\ \sharp*\ ((\!|\nu{*}yvec|\!)(N \prec' P)))$
**and** $xvec\ \sharp*\ (¡M(\!|\nu{*}yvec|\!)\langle N\rangle \prec P) = (xvec\ \sharp*\ M \wedge xvec\ \sharp*\ ((\!|\nu{*}yvec|\!)(N \prec' P)))$
**and** $xvec\ \sharp*\ (\tau \prec P) = (xvec\ \sharp*\ P)$
**and** $X\ \sharp*\ (M(\!|N|\!) \prec P) = (X\ \sharp*\ M \wedge X\ \sharp*\ N \wedge X\ \sharp*\ P)$
**and** $X\ \sharp*\ (¿M(\!|N|\!) \prec P) = (X\ \sharp*\ M \wedge X\ \sharp*\ N \wedge X\ \sharp*\ P)$
**and** $X\ \sharp*\ (M(\!|\nu{*}yvec|\!)\langle N\rangle \prec P) = (X\ \sharp*\ M \wedge X\ \sharp*\ ((\!|\nu{*}yvec|\!)(N \prec' P)))$
**and** $X\ \sharp*\ (¡M(\!|\nu{*}yvec|\!)\langle N\rangle \prec P) = (X\ \sharp*\ M \wedge X\ \sharp*\ ((\!|\nu{*}yvec|\!)(N \prec' P)))$
**and** $X\ \sharp*\ (\tau \prec P) = (X\ \sharp*\ P)$
⟨*proof*⟩

**lemma** *residualFreshChainSimp2*[*simp*]:
**fixes** *xvec* :: *name list*
**and** $X$ :: *name set*
**and** $M$ :: $'a$::*fs-name*
**and** $N$ :: $'a$
**and** *yvec* :: *name list*
**and** $P$ :: ($'a$, $'b$::*fs-name*, $'c$::*fs-name*) *psi*

**shows** $xvec\ \sharp*\ (RIn\ M\ N\ P) = (xvec\ \sharp*\ M \wedge xvec\ \sharp*\ N \wedge xvec\ \sharp*\ P)$
**and** $xvec\ \sharp*\ (RBrIn\ M\ N\ P) = (xvec\ \sharp*\ M \wedge xvec\ \sharp*\ N \wedge xvec\ \sharp*\ P)$
**and** $xvec\ \sharp*\ (ROut\ M\ B) = (xvec\ \sharp*\ M \wedge xvec\ \sharp*\ B)$
**and** $xvec\ \sharp*\ (RBrOut\ M\ B) = (xvec\ \sharp*\ M \wedge xvec\ \sharp*\ B)$
**and** $xvec\ \sharp*\ (RTau\ P) = (xvec\ \sharp*\ P)$
**and** $X\ \sharp*\ (RIn\ M\ N\ P) = (X\ \sharp*\ M \wedge X\ \sharp*\ N \wedge X\ \sharp*\ P)$
**and** $X\ \sharp*\ (RBrIn\ M\ N\ P) = (X\ \sharp*\ M \wedge X\ \sharp*\ N \wedge X\ \sharp*\ P)$
**and** $X\ \sharp*\ (ROut\ M\ B) = (X\ \sharp*\ M \wedge X\ \sharp*\ B)$
**and** $X\ \sharp*\ (RBrOut\ M\ B) = (X\ \sharp*\ M \wedge X\ \sharp*\ B)$
**and** $X\ \sharp*\ (RTau\ P) = (X\ \sharp*\ P)$

⟨*proof*⟩

**lemma** *freshResidual3*[*dest*]:
**fixes** $x$ :: *name*
**and** $\alpha$ :: ('*a*::*fs-name*) *action*
**and** $P$ :: ('*a*, '*b*::*fs-name*, '*c*::*fs-name*) *psi*

**assumes** $x \sharp bn\ \alpha$
**and** $x \sharp \alpha \prec P$

**shows** $x \sharp \alpha$ **and** $x \sharp P$
⟨*proof*⟩

**lemma** *freshResidualChain3*[*dest*]:
**fixes** *xvec* :: *name list*
**and** $\alpha$ :: ('*a*::*fs-name*) *action*
**and** $P$ :: ('*a*, '*b*::*fs-name*, '*c*::*fs-name*) *psi*

**assumes** *xvec* $\sharp*$ $(\alpha \prec P)$
**and** *xvec* $\sharp*$ $bn\ \alpha$

**shows** *xvec* $\sharp*$ $\alpha$ **and** *xvec* $\sharp*$ $P$
⟨*proof*⟩

**lemma** *freshResidual4*[*dest*]:
**fixes** $x$ :: *name*
**and** $\alpha$ :: ('*a*::*fs-name*) *action*
**and** $P$ :: ('*a*, '*b*::*fs-name*, '*c*::*fs-name*) *psi*

**assumes** $x \sharp \alpha \prec P$

**shows** $x \sharp subject\ \alpha$
⟨*proof*⟩

**lemma** *freshResidualChain4*[*dest*]:
**fixes** *xvec* :: *name list*
**and** $\alpha$ :: ('*a*::*fs-name*) *action*
**and** $P$ :: ('*a*, '*b*::*fs-name*, '*c*::*fs-name*) *psi*

**assumes** *xvec* $\sharp*$ $(\alpha \prec P)$

**shows** *xvec* $\sharp*$ *subject* $\alpha$
⟨*proof*⟩

**lemma** *alphaOutputResidual*:
**fixes** $M$ :: '*a*::*fs-name*
**and** *xvec* :: *name list*
**and** $N$ :: '*a*
**and** $P$ :: ('*a*, '*b*::*fs-name*, '*c*::*fs-name*) *psi*

**and** $p$ :: *name prm*

**assumes** $(p \cdot xvec) \sharp* N$
**and** $(p \cdot xvec) \sharp* P$
**and** $set\ p \subseteq set\ xvec \times set(p \cdot xvec)$
**and** $set\ xvec \subseteq set\ yvec$

**shows** $M(\!|\nu* yvec|\!)\langle N\rangle \prec P = M(\!|\nu*(p \cdot yvec)|\!)\langle(p \cdot N)\rangle \prec (p \cdot P)$
**and** $¡M(\!|\nu* yvec|\!)\langle N\rangle \prec P = ¡M(\!|\nu*(p \cdot yvec)|\!)\langle(p \cdot N)\rangle \prec (p \cdot P)$
⟨*proof*⟩

**lemmas**[*simp del*] = *create-residual.simps*

**lemma** *residualInject''*:

**assumes** $bn\ \alpha = bn\ \beta$

**shows** $(\alpha \prec P = \beta \prec Q) = (\alpha = \beta \wedge P = Q)$
⟨*proof*⟩

**lemmas** *residualInject* = *residual.inject create-residual.simps residualInject' residualInject''*

**lemma** *bnFreshResidual*[*simp*]:
**fixes** $\alpha$ :: (*'a*::*fs-name*) *action*

**shows** $(bn\ \alpha) \sharp* (\alpha \prec P) = bn\ \alpha \sharp* (subject\ \alpha)$
⟨*proof*⟩

**lemma** *actionCases*[*case-names cInput cBrInput cOutput cBrOutput cTau*]:
**fixes** $\alpha$ :: (*'a*::*fs-name*) *action*

**assumes** $\bigwedge M\ N.\ \alpha = M(\!|N|\!) \Longrightarrow Prop$
**and** $\bigwedge M\ N.\ \alpha = ¿M(\!|N|\!) \Longrightarrow Prop$
**and** $\bigwedge M\ xvec\ N.\ \alpha = M(\!|\nu* xvec|\!)\langle N\rangle \Longrightarrow Prop$
**and** $\bigwedge M\ xvec\ N.\ \alpha = ¡M(\!|\nu* xvec|\!)\langle N\rangle \Longrightarrow Prop$
**and** $\alpha = \tau \Longrightarrow Prop$

**shows** *Prop*
⟨*proof*⟩

**lemma** *actionPar1Dest*:
**fixes** $\alpha$ :: (*'a*::*fs-name*) *action*
**and** $P$ :: (*'a*, *'b*::*fs-name*, *'c*::*fs-name*) *psi*
**and** $\beta$ :: (*'a*::*fs-name*) *action*
**and** $Q$ :: (*'a*, *'b*, *'c*) *psi*
**and** $R$ :: (*'a*, *'b*, *'c*) *psi*

**assumes** $\alpha \prec P = \beta \prec (Q \parallel R)$

**and** $bn\ \alpha \sharp* bn\ \beta$

**obtains** $T\ p$ **where** $set\ p \subseteq set(bn\ \alpha) \times set(bn\ \beta)$ **and** $P = T \parallel (p \cdot R)$ **and** $\alpha \prec T = \beta \prec Q$
⟨*proof*⟩

**lemma** *actionPar2Dest*:
**fixes** $\alpha :: ('a::fs\text{-}name)\ action$
**and** $P :: ('a, 'b::fs\text{-}name, 'c::fs\text{-}name)\ psi$
**and** $\beta :: ('a::fs\text{-}name)\ action$
**and** $Q :: ('a, 'b, 'c)\ psi$
**and** $R :: ('a, 'b, 'c)\ psi$

**assumes** $\alpha \prec P = \beta \prec (Q \parallel R)$
**and** $bn\ \alpha \sharp* bn\ \beta$

**obtains** $T\ p$ **where** $set\ p \subseteq set(bn\ \alpha) \times set(bn\ \beta)$ **and** $P = (p \cdot Q) \parallel T$ **and** $\alpha \prec T = \beta \prec R$
⟨*proof*⟩

**lemma** *actionScopeDest*:
**fixes** $\alpha :: ('a::fs\text{-}name)\ action$
**and** $P :: ('a, 'b::fs\text{-}name, 'c::fs\text{-}name)\ psi$
**fixes** $\beta :: ('a::fs\text{-}name)\ action$
**and** $x :: name$
**and** $Q :: ('a, 'b, 'c)\ psi$

**assumes** $\alpha \prec P = \beta \prec (\!|\nu x|\!)\, Q$
**and** $x \sharp bn\ \alpha$
**and** $x \sharp bn\ \beta$

**obtains** $R$ **where** $P = (\!|\nu x|\!)\, R$ **and** $\alpha \prec R = \beta \prec Q$
⟨*proof*⟩

**lemma** *emptyFreshName*:
**fixes** $x :: name$
**and** $M :: 'a::fs\text{-}name$

**assumes** $supp\ M = (\{\}::name\ set)$

**shows** $x \sharp M$
⟨*proof*⟩

**lemma** *emptyFresh*:
**fixes** $xvec :: name\ list$
**and** $M \quad :: 'a::fs\text{-}name$

**assumes** $supp\ M = (\{\}::name\ set)$

**shows** *xvec* $\sharp*$ *M*
⟨*proof*⟩

**lemma** *permEmptyEq*:
**fixes** *p* :: *name prm*
**and** *M* :: *'a*::*fs-name*

**assumes** *suppE*: *supp M* = ({}::*name set*)

**shows** (*p* ∙ *M*) = *M*
⟨*proof*⟩

**abbreviation**
*outputJudge* (-⟨-⟩ [*110*, *110*] *110*) **where** $M\langle N\rangle \equiv M(\!|\nu*([])|\!)\langle N\rangle$

**abbreviation**
*brOutputJudge* (¡-⟨-⟩ [*110*, *110*] *110*) **where** $¡M\langle N\rangle \equiv ¡M(\!|\nu*([])|\!)\langle N\rangle$

**declare** [[*unify-trace-bound*=*100*]]

**locale** *env* = *substPsi substTerm substAssert substCond* +
*assertion SCompose' SImp' SBottom' SChanEq' SOutCon' SInCon'*
**for** *substTerm* :: (*'a*::*fs-name*) ⇒ *name list* ⇒ *'a*::*fs-name list* ⇒ *'a*
**and** *substAssert* :: (*'b*::*fs-name*) ⇒ *name list* ⇒ *'a*::*fs-name list* ⇒ *'b*
**and** *substCond* :: (*'c*::*fs-name*) ⇒ *name list* ⇒ *'a*::*fs-name list* ⇒ *'c*
**and** *SCompose'* :: *'b* ⇒ *'b* ⇒ *'b*
**and** *SImp'* :: *'b* ⇒ *'c* ⇒ *bool*
**and** *SBottom'* :: *'b*
**and** *SChanEq'* :: *'a* ⇒ *'a* ⇒ *'c*
**and** *SOutCon'* :: *'a* ⇒ *'a* ⇒ *'c*
**and** *SInCon'* :: *'a* ⇒ *'a* ⇒ *'c*
**begin**
**notation** *SCompose'* (**infixr** ⊗ *90*)
**notation** *SImp'* (- ⊢ - [*85*, *85*] *85*)
**notation** *FrameImp* (- $\vdash_F$ - [*85*, *85*] *85*)
**abbreviation**
*FBottomJudge* ($\bot_F$ *90*) **where** $\bot_F \equiv$ (*FAssert SBottom'*)
**notation** *SChanEq'* (- ↔ - [*90*, *90*] *90*)
**notation** *SOutCon'* (- ≼ - [*90*, *90*] *90*)
**notation** *SInCon'* (- ≽ - [*90*, *90*] *90*)
**notation** *substTerm* (-[-::=-] [*100*, *100*, *100*] *100*)
**notation** *subs* (-[-::=-] [*100*, *100*, *100*] *100*)
**notation** *AssertionStatEq* (- ≃ - [*80*, *80*] *80*)
**notation** *FrameStatEq* (- $\simeq_F$ - [*80*, *80*] *80*)
**notation** *SBottom'* (**1** *190*)
**abbreviation** *insertAssertion'* (*insertAssertion*) **where** *insertAssertion'* ≡ *assertionAux.insertAssertion* (⊗)

**inductive** *semantics* :: *'b* ⇒ (*'a*, *'b*, *'c*) *psi* ⇒ (*'a*, *'b*, *'c*) *residual* ⇒ *bool*

$(\text{-} \rhd \text{-} \longmapsto \text{-}\ [50,\ 50,\ 50]\ 50)$
**where**
*cInput*: $[\![\Psi \vdash M \leftrightarrow K;\ \mathit{distinct}\ \mathit{xvec};\ \mathit{set}\ \mathit{xvec} \subseteq \mathit{supp}\ N;\ \mathit{xvec} \sharp* \mathit{Tvec};$
$\mathit{length}\ \mathit{xvec} = \mathit{length}\ \mathit{Tvec};$
$\mathit{xvec} \sharp* \Psi;\ \mathit{xvec} \sharp* M;\ \mathit{xvec} \sharp* K]\!] \Longrightarrow \Psi \rhd M(\!|\lambda * \mathit{xvec}\ N|\!).P \longmapsto K(\!|(N[\mathit{xvec}::=\mathit{Tvec}])|\!) \prec P[\mathit{xvec}::=\mathit{Tvec}]$
| *cBrInput*:$[\![\Psi \vdash K \succeq M;\ \mathit{distinct}\ \mathit{xvec};\ \mathit{set}\ \mathit{xvec} \subseteq \mathit{supp}\ N;\ \mathit{xvec} \sharp* \mathit{Tvec};$
$\mathit{length}\ \mathit{xvec} = \mathit{length}\ \mathit{Tvec};$
$\mathit{xvec} \sharp* \Psi;\ \mathit{xvec} \sharp* M;\ \mathit{xvec} \sharp* K]\!] \Longrightarrow \Psi \rhd M(\!|\lambda * \mathit{xvec}\ N|\!).P \longmapsto ¿K(\!|(N[\mathit{xvec}::=\mathit{Tvec}])|\!) \prec P[\mathit{xvec}::=\mathit{Tvec}]$
| *Output*: $[\![\Psi \vdash M \leftrightarrow K]\!] \Longrightarrow \Psi \rhd M\langle N\rangle.P \longmapsto K\langle N\rangle \prec P$
| *BrOutput*: $[\![\Psi \vdash M \preceq K]\!] \Longrightarrow \Psi \rhd M\langle N\rangle.P \longmapsto ¡K\langle N\rangle \prec P$
| *Case*: $[\![\Psi \rhd P \longmapsto Rs;\ (\varphi,\ P) \in \mathit{set}\ Cs;\ \Psi \vdash \varphi;\ \mathit{guarded}\ P]\!] \Longrightarrow \Psi \rhd \mathit{Cases}\ Cs \longmapsto Rs$
| *cPar1*: $[\![(\Psi \otimes \Psi_Q) \rhd P \longmapsto \alpha \prec P';\ \mathit{extractFrame}\ Q = \langle A_Q,\ \Psi_Q\rangle;\ \mathit{distinct}\ A_Q;$
$A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* \Psi;\ A_Q \sharp* \alpha;\ A_Q \sharp* P';\ \mathit{distinct}(\mathit{bn}\ \alpha);$
$\mathit{bn}\ \alpha \sharp* \Psi;\ \mathit{bn}\ \alpha \sharp* \Psi_Q;\ \mathit{bn}\ \alpha \sharp* Q;\ \mathit{bn}\ \alpha \sharp* P;\ \mathit{bn}\ \alpha \sharp* (\mathit{subject}\ \alpha)]\!] \Longrightarrow$
$\Psi \rhd P \parallel Q \longmapsto \alpha \prec (P' \parallel Q)$
| *cPar2*: $[\![(\Psi \otimes \Psi_P) \rhd Q \longmapsto \alpha \prec Q';\ \mathit{extractFrame}\ P = \langle A_P,\ \Psi_P\rangle;\ \mathit{distinct}\ A_P;$
$A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* \Psi;\ A_P \sharp* \alpha;\ A_P \sharp* Q';\ \mathit{distinct}(\mathit{bn}\ \alpha);$
$\mathit{bn}\ \alpha \sharp* \Psi;\ \mathit{bn}\ \alpha \sharp* \Psi_P;\ \mathit{bn}\ \alpha \sharp* P;\ \mathit{bn}\ \alpha \sharp* Q;\ \mathit{bn}\ \alpha \sharp* (\mathit{subject}\ \alpha)]\!] \Longrightarrow$
$\Psi \rhd P \parallel Q \longmapsto \alpha \prec (P \parallel Q')$
| *cComm1*: $[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P';\ \mathit{extractFrame}\ P = \langle A_P,\ \Psi_P\rangle;\ \mathit{distinct}\ A_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu * \mathit{xvec}|\!)\langle N\rangle \prec Q';\ \mathit{extractFrame}\ Q = \langle A_Q,\ \Psi_Q\rangle;\ \mathit{distinct}\ A_Q;$
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K;$
$A_P \sharp* \Psi;\ A_P \sharp* \Psi_Q;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';$
$A_P \sharp* Q;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* \mathit{xvec};$
$A_Q \sharp* \Psi;\ A_Q \sharp* \Psi_P;\ A_Q \sharp* P;\ A_Q \sharp* N;\ A_Q \sharp* P';$
$A_Q \sharp* Q;\ A_Q \sharp* K;\ A_Q \sharp* Q';\ A_Q \sharp* \mathit{xvec};\ \mathit{distinct}\ \mathit{xvec};$
$\mathit{xvec} \sharp* \Psi;\ \mathit{xvec} \sharp* \Psi_P;\ \mathit{xvec} \sharp* \Psi_Q;\ \mathit{xvec} \sharp* P;\ \mathit{xvec} \sharp* M;$
$\mathit{xvec} \sharp* Q;\ \mathit{xvec} \sharp* K]\!] \Longrightarrow$
$\Psi \rhd P \parallel Q \longmapsto \tau \prec (\!|\nu * \mathit{xvec}|\!)(P' \parallel Q')$
| *cComm2*: $[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu * \mathit{xvec}|\!)\langle N\rangle \prec P';\ \mathit{extractFrame}\ P = \langle A_P,\ \Psi_P\rangle;\ \mathit{distinct}\ A_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q';\ \mathit{extractFrame}\ Q = \langle A_Q,\ \Psi_Q\rangle;\ \mathit{distinct}\ A_Q;$
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K;$
$A_P \sharp* \Psi;\ A_P \sharp* \Psi_Q;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';$
$A_P \sharp* Q;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* \mathit{xvec};$
$A_Q \sharp* \Psi;\ A_Q \sharp* \Psi_P;\ A_Q \sharp* P;\ A_Q \sharp* N;\ A_Q \sharp* P';$
$A_Q \sharp* Q;\ A_Q \sharp* K;\ A_Q \sharp* Q';\ A_Q \sharp* \mathit{xvec};\ \mathit{distinct}\ \mathit{xvec};$
$\mathit{xvec} \sharp* \Psi;\ \mathit{xvec} \sharp* \Psi_P;\ \mathit{xvec} \sharp* \Psi_Q;\ \mathit{xvec} \sharp* P;\ \mathit{xvec} \sharp* M;$
$\mathit{xvec} \sharp* Q;\ \mathit{xvec} \sharp* K]\!] \Longrightarrow$
$\Psi \rhd P \parallel Q \longmapsto \tau \prec (\!|\nu * \mathit{xvec}|\!)(P' \parallel Q')$
| *cBrMerge*: $[\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¿M(\!|N|\!) \prec P';\ \mathit{extractFrame}\ P = \langle A_P,\ \Psi_P\rangle;\ \mathit{distinct}\ A_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto ¿M(\!|N|\!) \prec Q';\ \mathit{extractFrame}\ Q = \langle A_Q,\ \Psi_Q\rangle;\ \mathit{distinct}$

$A_Q$;
$A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* N$; $A_P \sharp* P'$;
$A_P \sharp* Q$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$;
$A_P \sharp* M$; $A_Q \sharp* M$;
$A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$; $A_Q \sharp* P$; $A_Q \sharp* N$; $A_Q \sharp* P'$;
$A_Q \sharp* Q$; $A_Q \sharp* Q'\rrbracket \Longrightarrow$
$\Psi \rhd P \parallel Q \longmapsto ¿M(\!|N|\!) \prec (P' \parallel Q')$

| *cBrComm1*: $[\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¿M(\!|N|\!) \prec P'$; *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* $A_P$;
$\Psi \otimes \Psi_P \rhd Q \longmapsto ¡M(\!|\nu*xvec|\!)\langle N\rangle \prec Q'$; *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* $A_Q$;
$A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* N$; $A_P \sharp* P'$;
$A_P \sharp* Q$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* xvec$;
$A_P \sharp* M$; $A_Q \sharp* M$;
$A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$; $A_Q \sharp* P$; $A_Q \sharp* N$; $A_Q \sharp* P'$;
$A_Q \sharp* Q$; $A_Q \sharp* Q'$; $A_Q \sharp* xvec$; *distinct* $xvec$;
$xvec \sharp* \Psi$; $xvec \sharp* \Psi_P$; $xvec \sharp* \Psi_Q$; $xvec \sharp* P$;
$xvec \sharp* Q$; $xvec \sharp* M\rrbracket \Longrightarrow$
$\Psi \rhd P \parallel Q \longmapsto ¡M(\!|\nu*xvec|\!)\langle N\rangle \prec (P' \parallel Q')$
| *cBrComm2*: $[\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¡M(\!|\nu*xvec|\!)\langle N\rangle \prec P'$; *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* $A_P$;
$\Psi \otimes \Psi_P \rhd Q \longmapsto ¿M(\!|N|\!) \prec Q'$; *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* $A_Q$;
$A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* N$; $A_P \sharp* P'$;
$A_P \sharp* Q$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* xvec$;
$A_P \sharp* M$; $A_Q \sharp* M$;
$A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$; $A_Q \sharp* P$; $A_Q \sharp* N$; $A_Q \sharp* P'$;
$A_Q \sharp* Q$; $A_Q \sharp* Q'$; $A_Q \sharp* xvec$; *distinct* $xvec$;
$xvec \sharp* \Psi$; $xvec \sharp* \Psi_P$; $xvec \sharp* \Psi_Q$; $xvec \sharp* P$;
$xvec \sharp* Q$; $xvec \sharp* M\rrbracket \Longrightarrow$
$\Psi \rhd P \parallel Q \longmapsto ¡M(\!|\nu*xvec|\!)\langle N\rangle \prec (P' \parallel Q')$
| *cBrClose*: $[\![\Psi \rhd P \longmapsto ¡M(\!|\nu*xvec|\!)\langle N\rangle \prec P'$;
$x \in supp\ M$;
*distinct* $xvec$; $xvec \sharp* \Psi$; $xvec \sharp* P$;
$xvec \sharp* M$;
$x \sharp \Psi$; $x \sharp xvec\rrbracket \Longrightarrow$
$\Psi \rhd (\!|\nu x|\!)P \longmapsto \tau \prec (\!|\nu x|\!)(\!|\nu*xvec|\!)P')$
| *cOpen*: $[\![\Psi \rhd P \longmapsto M(\!|\nu*(xvec@yvec)|\!)\langle N\rangle \prec P'$; $x \in supp\ N$; $x \sharp xvec$; $x \sharp yvec$; $x \sharp M$; $x \sharp \Psi$;
*distinct* $xvec$; *distinct* $yvec$;
$xvec \sharp* \Psi$; $xvec \sharp* P$; $xvec \sharp* M$; $xvec \sharp* yvec$; $yvec \sharp* \Psi$; $yvec \sharp* P$; $yvec \sharp* M\rrbracket \Longrightarrow$
$\Psi \rhd (\!|\nu x|\!)P \longmapsto M(\!|\nu*(xvec@x\#yvec)|\!)\langle N\rangle \prec P'$
| *cBrOpen*: $[\![\Psi \rhd P \longmapsto ¡M(\!|\nu*(xvec@yvec)|\!)\langle N\rangle \prec P'$; $x \in supp\ N$; $x \sharp xvec$; $x \sharp yvec$; $x \sharp M$; $x \sharp \Psi$;
*distinct* $xvec$; *distinct* $yvec$;
$xvec \sharp* \Psi$; $xvec \sharp* P$; $xvec \sharp* M$; $xvec \sharp* yvec$; $yvec \sharp* \Psi$; $yvec \sharp* P$; $yvec \sharp* M\rrbracket \Longrightarrow$

$\Psi \rhd (\!|\nu x|\!) P \longmapsto ¡M(\!|\nu*(xvec@x\#yvec)|\!)\langle N\rangle \prec P'$
| *cScope*: $[\![\Psi \rhd P \longmapsto\!\alpha \prec P'; x \sharp \Psi; x \sharp \alpha; bn\ \alpha \sharp* \Psi; bn\ \alpha \sharp* P; bn\ \alpha \sharp* (subject\ \alpha); distinct(bn\ \alpha)]\!] \Longrightarrow \Psi \rhd (\!|\nu x|\!) P \longmapsto\!\alpha \prec ((\!|\nu x|\!) P')$
| *Bang*: $[\![\Psi \rhd P \parallel !P \longmapsto Rs; guarded\ P]\!] \Longrightarrow \Psi \rhd !P \longmapsto Rs$

**abbreviation**
*semanticsBottomJudge* (- $\longmapsto$ - [50, 50] 50) **where** $P \longmapsto Rs \equiv \mathbf{1} \rhd P \longmapsto Rs$

**equivariance** *env.semantics*

**nominal-inductive2** *env.semantics*
**avoids** *cInput*: *set xvec*
| *cBrInput*: *set xvec*
| *cPar1*: *set* $A_Q \cup$ *set*(*bn* $\alpha$)
| *cPar2*: *set* $A_P \cup$ *set*(*bn* $\alpha$)
| *cComm1*: *set* $A_P \cup$ *set* $A_Q \cup$ *set xvec*
| *cComm2*: *set* $A_P \cup$ *set* $A_Q \cup$ *set xvec*
| *cBrMerge*: *set* $A_P \cup$ *set* $A_Q$
| *cBrComm1*: *set* $A_P \cup$ *set* $A_Q \cup$ *set xvec*
| *cBrComm2*: *set* $A_P \cup$ *set* $A_Q \cup$ *set xvec*
| *cBrClose*: $\{x\} \cup$ *set xvec*
| *cOpen*: $\{x\} \cup$ *set xvec* $\cup$ *set yvec*
| *cBrOpen*: $\{x\} \cup$ *set xvec* $\cup$ *set yvec*
| *cScope*: $\{x\} \cup$ *set*(*bn* $\alpha$)
⟨*proof*⟩

**lemma** *nilTrans1*:
**fixes** $\Psi$ :: $'b$
**and** $M$ :: $'a$
**and** *xvec* :: *name list*
**and** $N$ :: $'a$
**and** $P$ :: ($'a$, $'b$, $'c$) *psi*

**assumes** $\Psi \rhd \mathbf{0} \longmapsto M(\!|\nu*xvec|\!)\langle N\rangle \prec P$

**shows** *False*
⟨*proof*⟩

**lemma** *nilTrans1'*:
**fixes** $\Psi$ :: $'b$
**and** $M$ :: $'a$
**and** *xvec* :: *name list*
**and** $N$ :: $'a$
**and** $P$ :: ($'a$, $'b$, $'c$) *psi*

**assumes** $\Psi \rhd \mathbf{0} \longmapsto ¡M(\!|\nu*xvec|\!)\langle N\rangle \prec P$

**shows** *False*
⟨*proof*⟩

**lemma** *nilTrans2*:
**fixes** $\Psi$ :: $'b$
**and** $Rs$ :: $('a, 'b, 'c)$ *residual*

**assumes** $\Psi \rhd \mathbf{0} \longmapsto Rs$

**shows** *False*
⟨*proof*⟩

**lemma** *nilTrans3*:
**fixes** $\Psi$ :: $'b$
**and** $M$ :: $'a$
**and** $M'$ :: $'a$
**and** *xvec* :: *name list*
**and** *yvec* :: *name list*
**and** $N$ :: $'a$
**and** $N'$ :: $'a$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $P'$ :: $('a, 'b, 'c)$ *psi*

**assumes** $\Psi \rhd M(\!|\lambda * xvec\ N|\!).P \longmapsto M'(\!|\nu * yvec|\!)\langle N'\rangle \prec P'$

**shows** *False*
⟨*proof*⟩

**lemma** *nilTrans3'*:
**fixes** $\Psi$ :: $'b$
**and** $M$ :: $'a$
**and** $M'$ :: $'a$
**and** *xvec* :: *name list*
**and** *yvec* :: *name list*
**and** $N$ :: $'a$
**and** $N'$ :: $'a$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $P'$ :: $('a, 'b, 'c)$ *psi*

**assumes** $\Psi \rhd M(\!|\lambda * xvec\ N|\!).P \longmapsto ¡M'(\!|\nu * yvec|\!)\langle N'\rangle \prec P'$

**shows** *False*
⟨*proof*⟩

**lemma** *nilTrans4*:
**fixes** $\Psi$ :: $'b$
**and** $Rs$ :: $('a, 'b, 'c)$ *residual*

**assumes** $\Psi \rhd M(\!|\lambda * xvec\ N|\!).P \longmapsto \tau \prec P'$

**shows** *False*

⟨*proof*⟩

**lemma** *nilTrans5*:
**fixes** $\Psi$ :: $'b$
**fixes** $\Psi'$ :: $'b$
**and** $M$ :: $'a$
**and** *xvec* :: *name list*
**and** $N$ :: $'a$
**and** $P$ :: $('a, 'b, 'c)$ *psi*

**assumes** $\Psi \rhd \{\!|\Psi'|\!\} \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P$

**shows** *False*
⟨*proof*⟩

**lemma** *nilTrans5'*:
**fixes** $\Psi$ :: $'b$
**fixes** $\Psi'$ :: $'b$
**and** $M$ :: $'a$
**and** *xvec* :: *name list*
**and** $N$ :: $'a$
**and** $P$ :: $('a, 'b, 'c)$ *psi*

**assumes** $\Psi \rhd \{\!|\Psi'|\!\} \longmapsto ¡M(\!|\nu * xvec|\!)\langle N\rangle \prec P$

**shows** *False*
⟨*proof*⟩

**lemma** *nilTrans6*:
**fixes** $\Psi$ :: $'b$
**and** *Rs* :: $('a, 'b, 'c)$ *residual*

**assumes** $\Psi \rhd \{\!|\Psi'|\!\} \longmapsto Rs$

**shows** *False*
⟨*proof*⟩

**lemma** *nilTrans*[*dest*]:
**fixes** $\Psi$ :: $'b$
**and** *Rs* :: $('a, 'b, 'c)$ *residual*
**and** $M$ :: $'a$
**and** *xvec* :: *name list*
**and** $N$ :: $'a$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $K$ :: $'a$
**and** *yvec* :: *name list*
**and** $N'$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** *CsP* :: $('c \times ('a, 'b, 'c)$ *psi*$)$ *list*

**and** $\Psi'$ :: $'b$

**shows** $\Psi \rhd \mathbf{0} \longmapsto Rs \Longrightarrow False$
**and** $\Psi \rhd M(\!|\lambda * xvec\ N|\!).P \longmapsto K(\!|\nu * yvec|\!)\langle N'\rangle \prec P' \Longrightarrow False$
**and** $\Psi \rhd M(\!|\lambda * xvec\ N|\!).P \longmapsto ¡K(\!|\nu * yvec|\!)\langle N'\rangle \prec P' \Longrightarrow False$
**and** $\Psi \rhd M(\!|\lambda * xvec\ N|\!).P \longmapsto \tau \prec P' \Longrightarrow False$
**and** $\Psi \rhd M\langle N\rangle.P \longmapsto K(\!|N'|\!) \prec P' \Longrightarrow False$
**and** $\Psi \rhd M\langle N\rangle.P \longmapsto ¿K(\!|N'|\!) \prec P' \Longrightarrow False$
**and** $\Psi \rhd M\langle N\rangle.P \longmapsto \tau \prec P' \Longrightarrow False$
**and** $\Psi \rhd \{\!|\Psi'|\!\} \longmapsto Rs \Longrightarrow False$
⟨*proof*⟩

**lemma** *residualEq*:
**fixes** $\alpha$ :: $'a$ *action*
**and** $P$ :: ($'a$, $'b$, $'c$) *psi*
**and** $\beta$ :: $'a$ *action*
**and** $Q$ :: ($'a$, $'b$, $'c$) *psi*

**assumes** $\alpha \prec P = \beta \prec Q$
**and** $bn\ \alpha \sharp * (bn\ \beta)$
**and** $distinct(bn\ \alpha)$
**and** $distinct(bn\ \beta)$
**and** $bn\ \alpha \sharp * (\alpha \prec P)$
**and** $bn\ \beta \sharp * (\beta \prec Q)$

**obtains** $p$ **where** $set\ p \subseteq set(bn\ \alpha) \times set(bn(p \cdot \alpha))$ **and** $distinctPerm\ p$ **and** $\beta = p \cdot \alpha$ **and** $Q = p \cdot P$ **and** $bn\ \alpha \sharp * \beta$ **and** $bn\ \alpha \sharp * Q$ **and** $bn(p \cdot \alpha) \sharp * \alpha$ **and** $bn(p \cdot \alpha) \sharp * P$
⟨*proof*⟩

**lemma** *semanticsInduct*[*consumes 3*, *case-names cAlpha cInput cBrInput cOutput cBrOutput cCase cPar1 cPar2 cComm1 cComm2 cBrMerge cBrComm1 cBrComm2 cBrClose cOpen cBrOpen cScope cBang*]:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: ($'a$, $'b$, $'c$) *psi*
**and** $\alpha$ :: $'a$ *action*
**and** $P'$ :: ($'a$, $'b$, $'c$) *psi*
**and** *Prop* :: $'f$::*fs-name* ⇒ $'b$ ⇒ ($'a$, $'b$, $'c$) *psi* ⇒ $'a$ *action* ⇒ ($'a$, $'b$, $'c$) *psi* ⇒ *bool*
**and** $C$ :: $'f$::*fs-name*

**assumes** $\Psi \rhd P \longmapsto \alpha \prec P'$
**and** $bn\ \alpha \sharp * (subject\ \alpha)$
**and** $distinct(bn\ \alpha)$
**and** *rAlpha*: $\bigwedge \Psi\ P\ \alpha\ P'\ p\ C.\ [\![bn\ \alpha \sharp * \Psi;\ bn\ \alpha \sharp * P;\ bn\ \alpha \sharp * (subject\ \alpha);$
$bn\ \alpha \sharp * C;\ bn\ \alpha \sharp * (bn(p \cdot \alpha));$
$set\ p \subseteq set(bn\ \alpha) \times set(bn(p \cdot \alpha));\ distinctPerm\ p;$
$(bn(p \cdot \alpha)) \sharp * \alpha;\ (bn(p \cdot \alpha)) \sharp * P';\ Prop\ C\ \Psi\ P\ \alpha\ P'\!]\!] \Longrightarrow$

$$Prop\ C\ \Psi\ P\ (p \cdot \alpha)\ (p \cdot P')$$

**and** *rInput*: $\bigwedge \Psi\ M\ K\ xvec\ N\ Tvec\ P\ C.$
$[\![\Psi \vdash M \leftrightarrow K;\ distinct\ xvec;\ set\ xvec \subseteq supp\ N;$
$length\ xvec = length\ Tvec;\ xvec \sharp* \Psi;$
$xvec \sharp* M;\ xvec \sharp* K;\ xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (M(\!|\lambda* xvec\ N|\!).P)$
$(K(\!|(N[xvec::=Tvec])|\!))\ (P[xvec::=Tvec])$

**and** *rBrInput*:$\bigwedge \Psi\ K\ M\ xvec\ N\ Tvec\ P\ C.$
$[\![\Psi \vdash K \succeq M;\ distinct\ xvec;\ set\ xvec \subseteq supp\ N;$
$length\ xvec = length\ Tvec;\ xvec \sharp* \Psi;$
$xvec \sharp* M;\ xvec \sharp* K;\ xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (M(\!|\lambda* xvec\ N|\!).P)$
$(¿K(\!|(N[xvec::=Tvec])|\!))\ (P[xvec::=Tvec])$

**and** *rOutput*: $\bigwedge \Psi\ M\ K\ N\ P\ C.\ [\![\Psi \vdash M \leftrightarrow K]\!] \Longrightarrow Prop\ C\ \Psi\ (M\langle N\rangle.P)$ $(K\langle N\rangle)\ P$

**and** *rBrOutput*:$\bigwedge \Psi\ M\ K\ N\ P\ C.\ [\![\Psi \vdash M \preceq K]\!] \Longrightarrow Prop\ C\ \Psi\ (M\langle N\rangle.P)$ $(¡K\langle N\rangle)\ P$

**and** *rCase*: $\bigwedge \Psi\ P\ \alpha\ P'\ \varphi\ Cs\ C.\ [\![\Psi \rhd P \longmapsto \alpha \prec P';\ \bigwedge C.\ Prop\ C\ \Psi\ P\ \alpha\ P';$ $(\varphi,\ P) \in set\ Cs;\ \Psi \vdash \varphi;\ guarded\ P]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (Cases\ Cs)\ \alpha\ P'$

**and** *rPar1*: $\bigwedge \Psi\ \Psi_Q\ P\ \alpha\ P'\ A_Q\ Q\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto \alpha \prec P';\ extractFrame\ Q = \langle A_Q,\ \Psi_Q\rangle;\ distinct$ $A_Q;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)\ P\ \alpha\ P';$
$A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* \Psi;\ A_Q \sharp* \alpha;\ A_Q \sharp* P';\ A_Q \sharp* C;$ $distinct(bn\ \alpha);\ bn\ \alpha \sharp* Q;$
$bn\ \alpha \sharp* \Psi;\ bn\ \alpha \sharp* \Psi_Q;\ bn\ \alpha \sharp* P;\ bn\ \alpha \sharp* subject\ \alpha;\ bn\ \alpha \sharp* C]\!]$ $\Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ \alpha\ (P' \parallel Q)$

**and** *rPar2*: $\bigwedge \Psi\ \Psi_P\ Q\ \alpha\ Q'\ A_P\ P\ C.$
$[\![\Psi \otimes \Psi_P \rhd Q \longmapsto \alpha \prec Q';\ extractFrame\ P = \langle A_P,\ \Psi_P\rangle;\ distinct$ $A_P;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_P)\ Q\ \alpha\ Q';$
$A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* \Psi;\ A_P \sharp* \alpha;\ A_P \sharp* Q';\ A_P \sharp* C;$ $distinct(bn\ \alpha);\ bn\ \alpha \sharp* Q;$
$bn\ \alpha \sharp* \Psi;\ bn\ \alpha \sharp* \Psi_P;\ bn\ \alpha \sharp* P;\ bn\ \alpha \sharp* subject\ \alpha;\ bn\ \alpha \sharp* C]\!]$ $\Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ \alpha\ (P \parallel Q')$

**and** *rComm1*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ N\ P'\ A_P\ \Psi_P\ Q\ K\ xvec\ Q'\ A_Q\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P';\ \bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)\ P\ (M(\!|N|\!))$ $P';$
$extractFrame\ P = \langle A_P,\ \Psi_P\rangle;\ distinct\ A_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu* xvec|\!)\langle N\rangle \prec Q';\ \bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_P)\ Q$ $(K(\!|\nu* xvec|\!)\langle N\rangle)\ Q';$
$extractFrame\ Q = \langle A_Q,\ \Psi_Q\rangle;\ distinct\ A_Q;$
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K;$
$A_P \sharp* \Psi;\ A_P \sharp* \Psi_Q;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';$
$A_P \sharp* Q;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* xvec;\ A_Q \sharp* \Psi;\ A_Q \sharp* \Psi_P;$
$A_Q \sharp* P;\ A_Q \sharp* N;\ A_Q \sharp* P';\ A_Q \sharp* Q;\ A_Q \sharp* K;\ A_Q \sharp* Q';$

$$\textit{distinct } xvec;$$
$$A_Q \sharp* xvec;\ xvec \sharp* \Psi;\ xvec \sharp* \Psi_P;\ xvec \sharp* \Psi_Q;\ xvec \sharp* P;\ xvec \sharp* M;$$
$$xvec \sharp* Q;\ xvec \sharp* K;\ A_P \sharp* C;\ A_Q \sharp* C;\ xvec \sharp* C[\![ \Longrightarrow$$
$$Prop\ C\ \Psi\ (P \parallel Q)\ (\tau)\ ((\!|\nu* xvec|\!)(P' \parallel Q'))$$

**and** *rComm2*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ xvec\ N\ P'\ A_P\ \Psi_P\ Q\ K\ Q'\ A_Q\ C.$
$$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu* xvec|\!)\langle N\rangle \prec P';\ \bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)\ P\ (M(\!|\nu* xvec|\!)\langle N\rangle)\ P';$$
$$extractFrame\ P = \langle A_P,\ \Psi_P\rangle;\ \textit{distinct } A_P;$$
$$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q';\ \bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_P)\ Q\ (K(\!|N|\!))\ Q';$$
$$extractFrame\ Q = \langle A_Q,\ \Psi_Q\rangle;\ \textit{distinct } A_Q;$$
$$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K;$$
$$A_P \sharp* \Psi;\ A_P \sharp* \Psi_Q;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';$$
$$A_P \sharp* Q;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* xvec;\ A_Q \sharp* \Psi;\ A_Q \sharp* \Psi_P;$$
$$A_Q \sharp* P;\ A_Q \sharp* N;\ A_Q \sharp* P';\ A_Q \sharp* Q;\ A_Q \sharp* K;\ A_Q \sharp* Q';$$
$$\textit{distinct } xvec;$$
$$A_Q \sharp* xvec;\ xvec \sharp* \Psi;\ xvec \sharp* \Psi_P;\ xvec \sharp* \Psi_Q;\ xvec \sharp* P;\ xvec \sharp* M;$$
$$xvec \sharp* Q;\ xvec \sharp* K;\ A_P \sharp* C;\ A_Q \sharp* C;\ xvec \sharp* C[\![ \Longrightarrow$$
$$Prop\ C\ \Psi\ (P \parallel Q)\ (\tau)\ ((\!|\nu* xvec|\!)(P' \parallel Q'))$$

**and** *rBrMerge*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ N\ P'\ A_P\ \Psi_P\ Q\ Q'\ A_Q\ C.$
$$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¿M(\!|N|\!) \prec P';\ \bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)\ P\ (¿M(\!|N|\!))\ P';$$
$$extractFrame\ P = \langle A_P,\ \Psi_P\rangle;\ \textit{distinct } A_P;$$
$$\Psi \otimes \Psi_P \rhd Q \longmapsto ¿M(\!|N|\!) \prec Q';\ \bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_P)\ Q\ (¿M(\!|N|\!))\ Q';$$
$$extractFrame\ Q = \langle A_Q,\ \Psi_Q\rangle;\ \textit{distinct } A_Q;$$
$$A_P \sharp* \Psi;\ A_P \sharp* \Psi_Q;\ A_P \sharp* P;\ A_P \sharp* N;\ A_P \sharp* P';$$
$$A_P \sharp* Q;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* M;\ A_Q \sharp* M;$$
$$A_Q \sharp* \Psi;\ A_Q \sharp* \Psi_P;\ A_Q \sharp* P;\ A_Q \sharp* N;\ A_Q \sharp* P';$$
$$A_Q \sharp* Q;\ A_Q \sharp* Q';\ A_P \sharp* C;\ A_Q \sharp* C[\![ \Longrightarrow$$
$$Prop\ C\ \Psi\ (P \parallel Q)\ (¿M(\!|N|\!))\ (P' \parallel Q')$$

**and** *rBrComm1*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ N\ P'\ A_P\ \Psi_P\ Q\ xvec\ Q'\ A_Q\ C.$
$$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¿M(\!|N|\!) \prec P';\ \bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)\ P\ (¿M(\!|N|\!))\ P';$$
$$extractFrame\ P = \langle A_P,\ \Psi_P\rangle;\ \textit{distinct } A_P;$$
$$\Psi \otimes \Psi_P \rhd Q \longmapsto ¡M(\!|\nu* xvec|\!)\langle N\rangle \prec Q';\ \bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_P)\ Q\ (¡M(\!|\nu* xvec|\!)\langle N\rangle)\ Q';$$
$$extractFrame\ Q = \langle A_Q,\ \Psi_Q\rangle;\ \textit{distinct } A_Q;$$
$$A_P \sharp* \Psi;\ A_P \sharp* \Psi_Q;\ A_P \sharp* P;\ A_P \sharp* N;\ A_P \sharp* P';$$
$$A_P \sharp* Q;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* xvec;\ A_Q \sharp* \Psi;\ A_Q \sharp* \Psi_P;$$
$$A_Q \sharp* P;\ A_Q \sharp* N;\ A_Q \sharp* P';\ A_Q \sharp* Q;\ A_Q \sharp* Q';\ \textit{distinct } xvec;$$
$$A_P \sharp* M;\ A_Q \sharp* M;\ xvec \sharp* M;$$
$$A_Q \sharp* xvec;\ xvec \sharp* \Psi;\ xvec \sharp* \Psi_P;\ xvec \sharp* \Psi_Q;\ xvec \sharp* P;$$
$$xvec \sharp* Q;\ A_P \sharp* C;\ A_Q \sharp* C;\ xvec \sharp* C[\![ \Longrightarrow$$
$$Prop\ C\ \Psi\ (P \parallel Q)\ (¡M(\!|\nu* xvec|\!)\langle N\rangle)\ (P' \parallel Q')$$

**and** *rBrComm2*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ xvec\ N\ P'\ A_P\ \Psi_P\ Q\ Q'\ A_Q\ C.$
$$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¡M(\!|\nu* xvec|\!)\langle N\rangle \prec P';\ \bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)$$

$P\ (¡M(\!|\nu{*}xvec|\!)\langle N\rangle)\ P'$;
*extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* $A_P$;
$\Psi \otimes \Psi_P \rhd Q \longmapsto ¿M(\!|N|\!) \prec Q'$; $\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_P)\ Q\ (¿M(\!|N|\!))\ Q'$;
*extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* $A_Q$;
$A_P \sharp{*}\ \Psi$; $A_P \sharp{*}\ \Psi_Q$; $A_P \sharp{*}\ P$; $A_P \sharp{*}\ N$; $A_P \sharp{*}\ P'$;
$A_P \sharp{*}\ Q$; $A_P \sharp{*}\ Q'$; $A_P \sharp{*}\ A_Q$; $A_P \sharp{*}\ xvec$; $A_Q \sharp{*}\ \Psi$; $A_Q \sharp{*}\ \Psi_P$;
$A_Q \sharp{*}\ P$; $A_Q \sharp{*}\ N$; $A_Q \sharp{*}\ P'$; $A_Q \sharp{*}\ Q$; $A_Q \sharp{*}\ Q'$; *distinct* $xvec$;
$A_P \sharp{*}\ M$; $A_Q \sharp{*}\ M$; $xvec \sharp{*}\ M$;
$A_Q \sharp{*}\ xvec$; $xvec \sharp{*}\ \Psi$; $xvec \sharp{*}\ \Psi_P$; $xvec \sharp{*}\ \Psi_Q$; $xvec \sharp{*}\ P$;
$xvec \sharp{*}\ Q$; $A_P \sharp{*}\ C$; $A_Q \sharp{*}\ C$; $xvec \sharp{*}\ C\rrbracket \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ (¡M(\!|\nu{*}xvec|\!)\langle N\rangle)\ (P' \parallel Q')$

**and** *rBrClose*: $\bigwedge \Psi\ P\ M\ xvec\ N\ P'\ x\ C.$
$\llbracket \Psi \rhd P \longmapsto ¡M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P'$;
$\bigwedge C.\ Prop\ C\ \Psi\ P\ (¡M(\!|\nu{*}xvec|\!)\langle N\rangle)\ P'$;
$x \in supp\ M$;
*distinct* $xvec$; $xvec \sharp{*}\ \Psi$; $xvec \sharp{*}\ P$;
$xvec \sharp{*}\ M$;
$x \sharp \Psi$; $x \sharp xvec\rrbracket \Longrightarrow$
$Prop\ C\ \Psi\ ((\!|\nu x|\!)P)\ (\tau)\ ((\!|\nu x|\!)((\!|\nu{*}xvec|\!)P'))$

**and** *rOpen*: $\bigwedge \Psi\ P\ M\ xvec\ yvec\ N\ P'\ x\ C.$
$\llbracket \Psi \rhd P \longmapsto M(\!|\nu{*}(xvec@yvec)|\!)\langle N\rangle \prec P'$; $x \in supp\ N$; $\bigwedge C.\ Prop\ C\ \Psi\ P\ (M(\!|\nu{*}(xvec@yvec)|\!)\langle N\rangle)\ P'$;
$x \sharp \Psi$; $x \sharp M$; $x \sharp xvec$; $x \sharp yvec$; $xvec \sharp{*}\ \Psi$; $xvec \sharp{*}\ P$; $xvec \sharp{*}\ M$; *distinct* $xvec$; *distinct* $yvec$;
$yvec \sharp{*}\ \Psi$; $yvec \sharp{*}\ P$; $yvec \sharp{*}\ M$; $yvec \sharp{*}\ C$; $x \sharp C$; $xvec \sharp{*}\ C\rrbracket \Longrightarrow$
$Prop\ C\ \Psi\ ((\!|\nu x|\!)P)\ (M(\!|\nu{*}(xvec@x\#yvec)|\!)\langle N\rangle)\ P'$

**and** *rBrOpen*: $\bigwedge \Psi\ P\ M\ xvec\ yvec\ N\ P'\ x\ C.$
$\llbracket \Psi \rhd P \longmapsto ¡M(\!|\nu{*}(xvec@yvec)|\!)\langle N\rangle \prec P'$; $x \in supp\ N$; $\bigwedge C.\ Prop\ C\ \Psi\ P\ (¡M(\!|\nu{*}(xvec@yvec)|\!)\langle N\rangle)\ P'$;
$x \sharp \Psi$; $x \sharp M$; $x \sharp xvec$; $x \sharp yvec$; $xvec \sharp{*}\ \Psi$; $xvec \sharp{*}\ P$; $xvec \sharp{*}\ M$; *distinct* $xvec$; *distinct* $yvec$;
$yvec \sharp{*}\ \Psi$; $yvec \sharp{*}\ P$; $yvec \sharp{*}\ M$; $yvec \sharp{*}\ C$; $x \sharp C$; $xvec \sharp{*}\ C\rrbracket \Longrightarrow$
$Prop\ C\ \Psi\ ((\!|\nu x|\!)P)\ (¡M(\!|\nu{*}(xvec@x\#yvec)|\!)\langle N\rangle)\ P'$

**and** *rScope*: $\bigwedge \Psi\ P\ \alpha\ P'\ x\ C.$
$\llbracket \Psi \rhd P \longmapsto \alpha \prec P'$; $\bigwedge C.\ Prop\ C\ \Psi\ P\ \alpha\ P'$;
$x \sharp \Psi$; $x \sharp \alpha$; $bn\ \alpha \sharp{*}\ \Psi$;
$bn\ \alpha \sharp{*}\ P$; $bn\ \alpha \sharp{*}\ (subject\ \alpha)$; $x \sharp C$; $bn\ \alpha \sharp{*}\ C$; $distinct(bn\ \alpha)\rrbracket \Longrightarrow$
$Prop\ C\ \Psi\ ((\!|\nu x|\!)P)\ \alpha\ ((\!|\nu x|\!)P')$

**and** *rBang*: $\bigwedge \Psi\ P\ \alpha\ P'\ C.$
$\llbracket \Psi \rhd P \parallel !P \longmapsto \alpha \prec P'$; *guarded* $P$; $\bigwedge C.\ Prop\ C\ \Psi\ (P \parallel !P)\ \alpha\ P'\rrbracket \Longrightarrow$
$Prop\ C\ \Psi\ (!P)\ \alpha\ P'$

**shows** $Prop\ C\ \Psi\ P\ \alpha\ P'$
⟨*proof*⟩

**lemma** *outputInduct*[*consumes 1*, *case-names cOutput cCase cPar1 cPar2 cOpen*

*cScope cBang*]:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** $B$ :: $('a, 'b, 'c)$ *boundOutput*
**and** *Prop* :: $'f$::*fs-name* $\Rightarrow$ $'b \Rightarrow ('a, 'b, 'c)$ *psi* $\Rightarrow$
$'a \Rightarrow ('a, 'b, 'c)$ *boundOutput* $\Rightarrow$ *bool*
**and** $C$ :: $'f$::*fs-name*

**assumes** $\Psi \rhd P \longmapsto ROut\ M\ B$
**and** *rOutput*: $\bigwedge \Psi\ M\ K\ N\ P\ C.\ [\![\Psi \vdash M \leftrightarrow K]\!] \Longrightarrow Prop\ C\ \Psi\ (M\langle N\rangle.P)\ K\ (N \prec' P)$
**and** *rCase*: $\bigwedge \Psi\ P\ M\ B\ \varphi\ Cs\ C.$
$[\![\Psi \rhd P \longmapsto (ROut\ M\ B);\ \bigwedge C.\ Prop\ C\ \Psi\ P\ M\ B;\ (\varphi, P) \in set\ Cs;\ \Psi \vdash \varphi;\ guarded\ P]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (Cases\ Cs)\ M\ B$
**and** *rPar1*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ xvec\ N\ \ P'\ A_Q\ Q\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P';\ extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)\ P\ M\ ((\!|\nu * xvec|\!)N \prec' P');$
$A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* \Psi;\ A_Q \sharp* M;$
$A_Q \sharp* xvec;\ A_Q \sharp* N;\ A_Q \sharp* P';\ A_Q \sharp* C;\ xvec \sharp* Q;$
$xvec \sharp* \Psi;\ xvec \sharp* \Psi_Q;\ xvec \sharp* P;\ xvec \sharp* M;\ xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ M\ ((\!|\nu * xvec|\!)N \prec' (P' \parallel Q))$
**and** *rPar2*: $\bigwedge \Psi\ \Psi_P\ Q\ M\ xvec\ N\ \ Q'\ A_P\ P\ C.$
$[\![\Psi \otimes \Psi_P \rhd Q \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec Q';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_P)\ Q\ M\ ((\!|\nu * xvec|\!)N \prec' Q');$
$A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* \Psi;\ A_P \sharp* M;$
$A_P \sharp* xvec;\ A_P \sharp* N;\ A_P \sharp* Q';\ A_P \sharp* C;\ xvec \sharp* P;$
$xvec \sharp* \Psi;\ xvec \sharp* \Psi_P;\ xvec \sharp* Q;\ xvec \sharp* M;\ xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ M\ ((\!|\nu * xvec|\!)N \prec' (P \parallel Q'))$
**and** *rOpen*: $\bigwedge \Psi\ P\ M\ xvec\ yvec\ N\ P'\ x\ C.$
$[\![\Psi \rhd P \longmapsto M(\!|\nu * (xvec@yvec)|\!)\langle N\rangle \prec P';\ x \in supp\ N;\ \bigwedge C.\ Prop\ C\ \Psi\ P\ M\ ((\!|\nu * (xvec@yvec)|\!)N \prec' P');$
$x \sharp \Psi;\ x \sharp M;\ x \sharp xvec;\ x \sharp yvec;\ xvec \sharp* \Psi;\ xvec \sharp* P;\ xvec \sharp* M;$
$xvec \sharp* yvec;\ yvec \sharp* \Psi;\ yvec \sharp* P;\ yvec \sharp* M;\ yvec \sharp* C;\ x \sharp C;\ xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ ((\!|\nu x|\!)P)\ M\ ((\!|\nu * (xvec@x\#yvec)|\!)N \prec' P')$
**and** *rScope*: $\bigwedge \Psi\ P\ M\ xvec\ N\ P'\ x\ C.$
$[\![\Psi \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P';\ \bigwedge C.\ Prop\ C\ \Psi\ P\ M\ ((\!|\nu * xvec|\!)N \prec' P');$
$x \sharp \Psi;\ x \sharp M;\ x \sharp xvec;\ x \sharp N;\ xvec \sharp* \Psi;\ xvec \sharp* P;\ xvec \sharp* M;$
$x \sharp C;\ xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ ((\!|\nu x|\!)P)\ M\ ((\!|\nu * xvec|\!)N \prec' (\!|\nu x|\!)P')$
**and** *rBang*: $\bigwedge \Psi\ P\ M\ B\ C.$
$[\![\Psi \rhd P \parallel !P \longmapsto (ROut\ M\ B);\ guarded\ P;\ \bigwedge C.\ Prop\ C\ \Psi\ (P \parallel !P)\ M\ B]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (!P)\ M\ B$

**shows** *Prop C Ψ P M B*
⟨*proof*⟩

**lemma** *brOutputInduct*[*consumes 1*, *case-names cBrOutput cCase cPar1 cPar2 cBrComm1 cBrComm2 cBrOpen cScope cBang*]:
**fixes** Ψ :: *'b*
**and** *P* :: (*'a*, *'b*, *'c*) *psi*
**and** *M* :: *'a*
**and** *B* :: (*'a*, *'b*, *'c*) *boundOutput*
**and** *Prop* :: *'f*::*fs-name* ⇒ *'b* ⇒ (*'a*, *'b*, *'c*) *psi* ⇒
*'a* ⇒ (*'a*, *'b*, *'c*) *boundOutput* ⇒ *bool*
**and** *C* :: *'f*::*fs-name*

**assumes** Ψ ▷ *P* ⟼*RBrOut M B*
**and** *rBrOutput*: ⋀Ψ *M K N P C*. ⟦Ψ ⊢ *M* ≤ *K*⟧ ⟹ *Prop C* Ψ (*M*⟨*N*⟩.*P*) *K* (*N* ≺′ *P*)
**and** *rCase*: ⋀Ψ *P M B φ Cs C*.
⟦Ψ ▷ *P* ⟼(*RBrOut M B*); ⋀*C*. *Prop C* Ψ *P M B*; (*φ*, *P*) ∈ *set Cs*; Ψ ⊢ *φ*; *guarded P*⟧ ⟹
*Prop C* Ψ (*Cases Cs*) *M B*
**and** *rPar1*: ⋀Ψ Ψ$_Q$ *P M xvec N P′ A$_Q$ Q C*.
⟦Ψ ⊗ Ψ$_Q$ ▷ *P* ⟼¡*M*(⦇*ν*∗*xvec*⦈)⟨*N*⟩ ≺ *P′*; *extractFrame Q* = ⟨*A$_Q$*, Ψ$_Q$⟩; *distinct A$_Q$*;
⋀*C*. *Prop C* (Ψ ⊗ Ψ$_Q$) *P M* ((⦇*ν*∗*xvec*⦈)*N* ≺′ *P′*);
*A$_Q$* ♯∗ *P*; *A$_Q$* ♯∗ *Q*; *A$_Q$* ♯∗ Ψ; *A$_Q$* ♯∗ *M*;
*A$_Q$* ♯∗ *xvec*; *A$_Q$* ♯∗ *N*; *A$_Q$* ♯∗ *P′*; *A$_Q$* ♯∗ *C*; *xvec* ♯∗ *Q*;
*xvec* ♯∗ Ψ; *xvec* ♯∗ Ψ$_Q$; *xvec* ♯∗ *P*; *xvec* ♯∗ *M*; *xvec* ♯∗ *C*⟧ ⟹
*Prop C* Ψ (*P* ∥ *Q*) *M* ((⦇*ν*∗*xvec*⦈)*N* ≺′ (*P′* ∥ *Q*))
**and** *rPar2*: ⋀Ψ Ψ$_P$ *Q M xvec N Q′ A$_P$ P C*.
⟦Ψ ⊗ Ψ$_P$ ▷ *Q* ⟼¡*M*(⦇*ν*∗*xvec*⦈)⟨*N*⟩ ≺ *Q′*; *extractFrame P* = ⟨*A$_P$*, Ψ$_P$⟩; *distinct A$_P$*;
⋀*C*. *Prop C* (Ψ ⊗ Ψ$_P$) *Q M* ((⦇*ν*∗*xvec*⦈)*N* ≺′ *Q′*);
*A$_P$* ♯∗ *P*; *A$_P$* ♯∗ *Q*; *A$_P$* ♯∗ Ψ; *A$_P$* ♯∗ *M*;
*A$_P$* ♯∗ *xvec*; *A$_P$* ♯∗ *N*; *A$_P$* ♯∗ *Q′*; *A$_P$* ♯∗ *C*; *xvec* ♯∗ *P*;
*xvec* ♯∗ Ψ; *xvec* ♯∗ Ψ$_P$; *xvec* ♯∗ *Q*; *xvec* ♯∗ *M*; *xvec* ♯∗ *C*⟧ ⟹
*Prop C* Ψ (*P* ∥ *Q*) *M* ((⦇*ν*∗*xvec*⦈)*N* ≺′ (*P* ∥ *Q′*))
**and** *rBrComm1*: ⋀Ψ Ψ$_Q$ *P M N P′ A$_P$* Ψ$_P$ *Q xvec Q′ A$_Q$ C*.
⟦Ψ ⊗ Ψ$_Q$ ▷ *P* ⟼¿*M*(⦇*N*⦈) ≺ *P′*;
*extractFrame P* = ⟨*A$_P$*, Ψ$_P$⟩; *distinct A$_P$*;
Ψ ⊗ Ψ$_P$ ▷ *Q* ⟼¡*M*(⦇*ν*∗*xvec*⦈)⟨*N*⟩ ≺ *Q′*; ⋀*C*. *Prop C* (Ψ ⊗ Ψ$_P$) *Q M* ((⦇*ν*∗*xvec*⦈)*N* ≺′ *Q′*);
*extractFrame Q* = ⟨*A$_Q$*, Ψ$_Q$⟩; *distinct A$_Q$*;
*A$_P$* ♯∗ Ψ; *A$_P$* ♯∗ Ψ$_Q$; *A$_P$* ♯∗ *P*; *A$_P$* ♯∗ *N*; *A$_P$* ♯∗ *P′*;
*A$_P$* ♯∗ *Q*; *A$_P$* ♯∗ *Q′*; *A$_P$* ♯∗ *A$_Q$*; *A$_P$* ♯∗ *xvec*; *A$_Q$* ♯∗ Ψ; *A$_Q$* ♯∗ Ψ$_P$;
*A$_Q$* ♯∗ *P*; *A$_Q$* ♯∗ *N*; *A$_Q$* ♯∗ *P′*; *A$_Q$* ♯∗ *Q*; *A$_Q$* ♯∗ *Q′*; *distinct xvec*;
*A$_P$* ♯∗ *M*; *A$_Q$* ♯∗ *M*; *xvec* ♯∗ *M*;
*A$_Q$* ♯∗ *xvec*; *xvec* ♯∗ Ψ; *xvec* ♯∗ Ψ$_P$; *xvec* ♯∗ Ψ$_Q$; *xvec* ♯∗ *P*;
*xvec* ♯∗ *Q*; *A$_P$* ♯∗ *C*; *A$_Q$* ♯∗ *C*; *xvec* ♯∗ *C*⟧ ⟹
*Prop C* Ψ (*P* ∥ *Q*) *M* ((⦇*ν*∗*xvec*⦈)*N* ≺′ (*P′* ∥ *Q′*))

**and** *rBrComm2*: $\bigwedge\Psi\ \Psi_Q\ P\ M\ xvec\ N\ P'\ A_P\ \Psi_P\ Q\ Q'\ A_Q\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¡M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P';\ \bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)\ P\ M\ ((\!|\nu{*}xvec|\!)N \prec' P');$
$extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto ¿M(\!|N|\!) \prec Q';$
$extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$A_P \sharp* \Psi;\ A_P \sharp* \Psi_Q;\ A_P \sharp* P;\ A_P \sharp* N;\ A_P \sharp* P';$
$A_P \sharp* Q;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* xvec;\ A_Q \sharp* \Psi;\ A_Q \sharp* \Psi_P;$
$A_Q \sharp* P;\ A_Q \sharp* N;\ A_Q \sharp* P';\ A_Q \sharp* Q;\ A_Q \sharp* Q';\ distinct\ xvec;$
$A_P \sharp* M;\ A_Q \sharp* M;\ xvec \sharp* M;$
$A_Q \sharp* xvec;\ xvec \sharp* \Psi;\ xvec \sharp* \Psi_P;\ xvec \sharp* \Psi_Q;\ xvec \sharp* P;$
$xvec \sharp* Q;\ A_P \sharp* C;\ A_Q \sharp* C;\ xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ M\ ((\!|\nu{*}xvec|\!)N \prec' (P' \parallel Q'))$

**and** *rBrOpen*: $\bigwedge\Psi\ P\ M\ xvec\ yvec\ N\ P'\ x\ C.$
$[\![\Psi \rhd P \longmapsto ¡M(\!|\nu{*}(xvec@yvec)|\!)\langle N\rangle \prec P';\ x \in supp\ N;\ \bigwedge C.\ Prop\ C\ \Psi\ P\ M\ ((\!|\nu{*}(xvec@yvec)|\!)N \prec' P');$
$x \sharp \Psi;\ x \sharp M;\ x \sharp xvec;\ x \sharp yvec;\ xvec \sharp* \Psi;\ xvec \sharp* P;\ xvec \sharp* M;$
$xvec \sharp* yvec;\ yvec \sharp* \Psi;\ yvec \sharp* P;\ yvec \sharp* M;\ yvec \sharp* C;\ x \sharp C;\ xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ ((\!|\nu x|\!)P)\ M\ ((\!|\nu{*}(xvec@x\#yvec)|\!)N \prec' P')$

**and** *rScope*: $\bigwedge\Psi\ P\ M\ xvec\ N\ P'\ x\ C.$
$[\![\Psi \rhd P \longmapsto ¡M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P';\ \bigwedge C.\ Prop\ C\ \Psi\ P\ M\ ((\!|\nu{*}xvec|\!)N \prec' P');$
$x \sharp \Psi;\ x \sharp M;\ x \sharp xvec;\ x \sharp N;\ xvec \sharp* \Psi;\ xvec \sharp* P;\ xvec \sharp* M;$
$x \sharp C;\ xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ ((\!|\nu x|\!)P)\ M\ ((\!|\nu{*}xvec|\!)N \prec' (\!|\nu x|\!)P')$

**and** *rBang*: $\bigwedge\Psi\ P\ M\ B\ C.$
$[\![\Psi \rhd P \parallel !P \longmapsto (RBrOut\ M\ B);\ guarded\ P;\ \bigwedge C.\ Prop\ C\ \Psi\ (P \parallel !P)\ M\ B]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (!P)\ M\ B$

**shows** $Prop\ C\ \Psi\ P\ M\ B$
⟨*proof*⟩

**lemma** *boundOutputBindObject*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $M$ :: $'a$
**and** $yvec$ :: $name\ list$
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)\ psi$
**and** $y$ :: $name$

**assumes** $\Psi \rhd P \longmapsto \alpha \prec P'$
**and** $bn\ \alpha \sharp* subject\ \alpha$
**and** $distinct(bn\ \alpha)$
**and** $y \in set(bn\ \alpha)$

**shows** $y \in supp(object\ \alpha)$
⟨*proof*⟩

**lemma** *alphaBoundOutputChain'*:
  **fixes** *yvec* :: *name list*
    **and** *xvec* :: *name list*
    **and** *B* :: (*'a*, *'b*, *'c*) *boundOutput*

**assumes** *length xvec* = *length yvec*
  **and** *yvec* $\sharp*$ *B*
  **and** *yvec* $\sharp*$ *xvec*
  **and** *distinct yvec*

**shows** $(\!|\nu{*}xvec|\!)B = (\!|\nu{*}yvec|\!)([xvec\ yvec] \cdot_v B)$
  ⟨*proof*⟩

**lemma** *alphaBoundOutputChain''*:
  **fixes** *yvec* :: *name list*
    **and** *xvec* :: *name list*
    **and** *N* :: *'a*
    **and** *P* :: (*'a*, *'b*, *'c*) *psi*

**assumes** *length xvec* = *length yvec*
  **and** *yvec* $\sharp*$ *N*
  **and** *yvec* $\sharp*$ *P*
  **and** *yvec* $\sharp*$ *xvec*
  **and** *distinct yvec*

**shows** $(\!|\nu{*}xvec|\!)(N \prec' P) = (\!|\nu{*}yvec|\!)(([xvec\ yvec] \cdot_v N) \prec' ([xvec\ yvec] \cdot_v P))$
⟨*proof*⟩

**lemma** *alphaDistinct*:
  **fixes** *xvec* :: *name list*
    **and** *N* :: *'a*
    **and** *P* :: (*'a*, *'b*, *'c*) *psi*
    **and** *yvec* :: *name list*
    **and** *M* :: *'a*
    **and** *Q* :: (*'a*, *'b*, *'c*) *psi*

**assumes** $\alpha \prec P = \beta \prec Q$
  **and** *distinct*(*bn* $\alpha$)
  **and** $\bigwedge x.\ x \in set(bn\ \alpha) \Longrightarrow x \in supp(object\ \alpha)$
  **and** *bn* $\alpha$ $\sharp*$ *bn* $\beta$
  **and** *bn* $\alpha$ $\sharp*$ (*object* $\beta$)
  **and** *bn* $\alpha$ $\sharp*$ *Q*

**shows** *distinct*(*bn* $\beta$)
  ⟨*proof*⟩

**lemma** *boundOutputDistinct*:
  **fixes** $\Psi$ :: *'b*

**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $\alpha$ :: $'a$ *action*
**and** $P'$ :: $('a, 'b, 'c)$ *psi*

**assumes** $\Psi \rhd P \longmapsto \alpha \prec P'$

**shows** *distinct*(*bn* $\alpha$)
⟨*proof*⟩

**lemma** *inputDistinct*:
**fixes** $\Psi$ :: $'b$
**and** $M$ :: $'a$
**and** *xvec* :: *name list*
**and** $N$ :: $'a$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** *Rs* :: $('a, 'b, 'c)$ *residual*

**assumes** $\Psi \rhd M(\!|\lambda * xvec\ N|\!).P \longmapsto Rs$

**shows** *distinct xvec*
⟨*proof*⟩

**lemma** *outputInduct'*[*consumes 2*, *case-names cAlpha cOutput cCase cPar1 cPar2 cOpen cScope cBang*]:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** *yvec* :: *name list*
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** *Prop* :: $'f$::*fs-name* ⇒ $'b$ ⇒ $('a, 'b, 'c)$ *psi* ⇒ $'a$ ⇒ *name list* ⇒ $'a$ ⇒ $('a, 'b, 'c)$ *psi* ⇒ *bool*
**and** $C$ :: $'f$::*fs-name*

**assumes** $\Psi \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
**and** *xvec* $\sharp*$ $M$
**and** *rAlpha*: $\bigwedge \Psi\ P\ M\ xvec\ N\ P'\ p\ C.$ ⟦*xvec* $\sharp*$ $\Psi$; *xvec* $\sharp*$ $P$; *xvec* $\sharp*$ $M$; *xvec* $\sharp*$ $C$; *xvec* $\sharp*$ $(p \cdot xvec)$; *set* $p \subseteq$ *set xvec* × *set*$(p \cdot xvec)$; *distinctPerm* $p$; $(p \cdot xvec)$ $\sharp*$ $N$; $(p \cdot xvec)$ $\sharp*$ $P'$; *Prop* $C$ $\Psi$ $P$ $M$ *xvec* $N$ $P'$⟧ ⟹ *Prop* $C$ $\Psi$ $P$ $M$ $(p \cdot xvec)$ $(p \cdot N)$ $(p \cdot P')$
**and** *rOutput*: $\bigwedge \Psi\ M\ K\ N\ P\ C.$ ⟦$\Psi \vdash M \leftrightarrow K$⟧ ⟹ *Prop* $C$ $\Psi$ $(M\langle N\rangle.P)$ $K$ $([])$ $N$ $P$
**and** *rCase*: $\bigwedge \Psi\ P\ M\ xvec\ N\ P'\ \varphi\ Cs\ C.$ ⟦$\Psi \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$; $\bigwedge C.$ *Prop* $C$ $\Psi$ $P$ $M$ *xvec* $N$ $P'$; $(\varphi, P) \in$ *set Cs*; $\Psi \vdash \varphi$; *guarded* $P$⟧ ⟹ *Prop* $C$ $\Psi$ (*Cases Cs*) $M$ *xvec* $N$ $P'$
**and** *rPar1*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ xvec\ N\ P'\ A_Q\ Q\ C.$ ⟦$\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$; *extractFrame* $Q = \langle A_Q,$

$\Psi_Q\rangle$; *distinct* $A_Q$;
$\bigwedge C$. *Prop* $C$ $(\Psi \otimes \Psi_Q)$ $P$ $M$ *xvec* $N$ $P'$;
$A_Q \sharp^* P$; $A_Q \sharp^* Q$; $A_Q \sharp^* \Psi$; $A_Q \sharp^* M$;
$A_Q \sharp^* xvec$; $A_Q \sharp^* N$; $A_Q \sharp^* P'$; $A_Q \sharp^* C$; $xvec \sharp^* Q$;
$xvec \sharp^* \Psi$; $xvec \sharp^* \Psi_Q$; $xvec \sharp^* P$; $xvec \sharp^* M$; $xvec \sharp^* C\rrbracket \Longrightarrow$
*Prop* $C$ $\Psi$ $(P \parallel Q)$ $M$ *xvec* $N$ $(P' \parallel Q)$

**and** *rPar2*: $\bigwedge \Psi\ \Psi_P\ Q\ M$ *xvec* $N\ \ Q'\ A_P\ P\ C$.
$\llbracket \Psi \otimes \Psi_P \rhd Q \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec Q'$; *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* $A_P$;
$\bigwedge C$. *Prop* $C$ $(\Psi \otimes \Psi_P)$ $Q$ $M$ *xvec* $N$ $Q'$;
$A_P \sharp^* P$; $A_P \sharp^* Q$; $A_P \sharp^* \Psi$; $A_P \sharp^* M$;
$A_P \sharp^* xvec$; $A_P \sharp^* N$; $A_P \sharp^* Q'$; $A_P \sharp^* C$; $xvec \sharp^* Q$;
$xvec \sharp^* \Psi$; $xvec \sharp^* \Psi_P$; $xvec \sharp^* P$; $xvec \sharp^* M$; $xvec \sharp^* C\rrbracket \Longrightarrow$
*Prop* $C$ $\Psi$ $(P \parallel Q)$ $M$ *xvec* $N$ $(P \parallel Q')$

**and** *rOpen*: $\bigwedge \Psi\ P\ M$ *xvec yvec* $N\ P'\ x\ C$.
$\llbracket \Psi \rhd P \longmapsto M(\!|\nu * (xvec @ yvec)|\!)\langle N\rangle \prec P'$; $x \in$ *supp* $N$; $\bigwedge C$. *Prop* $C$ $\Psi$ $P$ $M$ $(xvec @ yvec)$ $N$ $P'$;
$x \sharp \Psi$; $x \sharp M$; $x \sharp xvec$; $x \sharp yvec$; $xvec \sharp^* \Psi$; $xvec \sharp^* P$; $xvec \sharp^* M$;
$yvec \sharp^* \Psi$; $yvec \sharp^* P$; $yvec \sharp^* M$; $yvec \sharp^* C$; $x \sharp C$; $xvec \sharp^* C\rrbracket \Longrightarrow$
*Prop* $C$ $\Psi$ $((\!|\nu x|\!)P)$ $M$ $(xvec @ x \# yvec)$ $N$ $P'$

**and** *rScope*: $\bigwedge \Psi\ P\ M$ *xvec* $N\ P'\ x\ C$.
$\llbracket \Psi \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$; $\bigwedge C$. *Prop* $C$ $\Psi$ $P$ $M$ *xvec* $N$ $P'$;
$x \sharp \Psi$; $x \sharp M$; $x \sharp xvec$; $x \sharp N$; $xvec \sharp^* \Psi$;
$xvec \sharp^* P$; $xvec \sharp^* M$; $x \sharp C$; $xvec \sharp^* C\rrbracket \Longrightarrow$
*Prop* $C$ $\Psi$ $((\!|\nu x|\!)P)$ $M$ *xvec* $N$ $((\!|\nu x|\!)P')$

**and** *rBang*: $\bigwedge \Psi\ P\ M$ *xvec* $N\ P'\ C$.
$\llbracket \Psi \rhd P \parallel\ !P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$; *guarded* $P$; $\bigwedge C$. *Prop* $C$ $\Psi$ $(P \parallel\ !P)$ $M$ *xvec* $N$ $P'\rrbracket \Longrightarrow$
*Prop* $C$ $\Psi$ $(!P)$ $M$ *xvec* $N$ $P'$

**shows** *Prop* $C$ $\Psi$ $P$ $M$ *xvec* $N$ $P'$
⟨*proof*⟩

**lemma** *brOutputInduct'*[*consumes 2*, *case-names cAlpha cBrOutput cCase cPar1 cPar2 cBrComm1 cBrComm2 cBrOpen cScope cBang*]:
**fixes** $\Psi$ :: *'b*
**and** $P$ :: *('a, 'b, 'c) psi*
**and** $M$ :: *'a*
**and** *yvec* :: *name list*
**and** $N$ :: *'a*
**and** $P'$ :: *('a, 'b, 'c) psi*
**and** *Prop* :: *'f::fs-name* ⇒ *'b* ⇒ *('a, 'b, 'c) psi* ⇒
*'a* ⇒ *name list* ⇒ *'a* ⇒ *('a, 'b, 'c) psi* ⇒ *bool*
**and** $C$ :: *'f::fs-name*

**assumes** $\Psi \rhd P \longmapsto ¡M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
**and** *xvec* $\sharp^*$ $M$
**and** *rAlpha*: $\bigwedge \Psi\ P\ M$ *xvec* $N\ P'\ p\ C$. $\llbracket xvec \sharp^* \Psi$; $xvec \sharp^* P$; $xvec \sharp^* M$; $xvec \sharp^* C$; $xvec \sharp^* (p \cdot xvec)$;
*set* $p \subseteq$ *set xvec* $\times$ *set*$(p \cdot xvec)$; *distinctPerm* $p$;

$(p \cdot xvec) \sharp\!* N$; $(p \cdot xvec) \sharp\!* P'$; *Prop* $C\ \Psi\ P\ M$ $xvec\ N\ P'\rrbracket \Longrightarrow$

*Prop* $C\ \Psi\ P\ M\ (p \cdot xvec)\ (p \cdot N)\ (p \cdot P')$

**and** *rBrOutput*: $\bigwedge \Psi\ M\ K\ N\ P\ C.\ [\![\Psi \vdash M \preceq K]\!] \Longrightarrow$ *Prop* $C\ \Psi\ (M\langle N\rangle.P)\ K\ ([])\ N\ P$

**and** *rCase*: $\bigwedge \Psi\ P\ M\ xvec\ N\ P'\ \varphi\ Cs\ C.\ [\![\Psi \rhd P \longmapsto\!\!¡ M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$; $\bigwedge C.$ *Prop* $C\ \Psi\ P\ M\ xvec\ N\ P'$; $(\varphi, P) \in$ *set* $Cs$; $\Psi \vdash \varphi$; *guarded* $P]\!] \Longrightarrow$

*Prop* $C\ \Psi$ (*Cases* $Cs$) $M\ xvec\ N\ P'$

**and** *rPar1*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ xvec\ N\ \ P'\ A_Q\ Q\ C.$

$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto\!\!¡ M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$; *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* $A_Q$;

$\bigwedge C.$ *Prop* $C\ (\Psi \otimes \Psi_Q)\ P\ M\ xvec\ N\ P'$;

$A_Q \sharp\!* P$; $A_Q \sharp\!* Q$; $A_Q \sharp\!* \Psi$; $A_Q \sharp\!* M$;

$A_Q \sharp\!* xvec$; $A_Q \sharp\!* N$; $A_Q \sharp\!* P'$; $A_Q \sharp\!* C$; $xvec \sharp\!* Q$;

$xvec \sharp\!* \Psi$; $xvec \sharp\!* \Psi_Q$; $xvec \sharp\!* P$; $xvec \sharp\!* M$; $xvec \sharp\!* C]\!] \Longrightarrow$

*Prop* $C\ \Psi\ (P \parallel Q)\ M\ xvec\ N\ (P' \parallel Q)$

**and** *rPar2*: $\bigwedge \Psi\ \Psi_P\ Q\ M\ xvec\ N\ \ Q'\ A_P\ P\ C.$

$[\![\Psi \otimes \Psi_P \rhd Q \longmapsto\!\!¡ M(\!|\nu * xvec|\!)\langle N\rangle \prec Q'$; *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* $A_P$;

$\bigwedge C.$ *Prop* $C\ (\Psi \otimes \Psi_P)\ Q\ M\ xvec\ N\ Q'$;

$A_P \sharp\!* P$; $A_P \sharp\!* Q$; $A_P \sharp\!* \Psi$; $A_P \sharp\!* M$;

$A_P \sharp\!* xvec$; $A_P \sharp\!* N$; $A_P \sharp\!* Q'$; $A_P \sharp\!* C$; $xvec \sharp\!* Q$;

$xvec \sharp\!* \Psi$; $xvec \sharp\!* \Psi_P$; $xvec \sharp\!* P$; $xvec \sharp\!* M$; $xvec \sharp\!* C]\!] \Longrightarrow$

*Prop* $C\ \Psi\ (P \parallel Q)\ M\ xvec\ N\ (P \parallel Q')$

**and** *rBrComm1*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ N\ P'\ A_P\ \Psi_P\ Q\ xvec\ Q'\ A_Q\ C.$

$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto\!\!¿ M(\!|N|\!) \prec P'$;

*extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* $A_P$;

$\Psi \otimes \Psi_P \rhd Q \longmapsto\!\!¡ M(\!|\nu * xvec|\!)\langle N\rangle \prec Q'$; $\bigwedge C.$ *Prop* $C\ (\Psi \otimes \Psi_P)\ Q\ M\ xvec\ N\ Q'$;

*extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* $A_Q$;

$A_P \sharp\!* \Psi$; $A_P \sharp\!* \Psi_Q$; $A_P \sharp\!* P$; $A_P \sharp\!* N$; $A_P \sharp\!* P'$;

$A_P \sharp\!* Q$; $A_P \sharp\!* Q'$; $A_P \sharp\!* A_Q$; $A_P \sharp\!* xvec$; $A_Q \sharp\!* \Psi$; $A_Q \sharp\!* \Psi_P$;

$A_Q \sharp\!* P$; $A_Q \sharp\!* N$; $A_Q \sharp\!* P'$; $A_Q \sharp\!* Q$; $A_Q \sharp\!* Q'$; *distinct* $xvec$;

$A_P \sharp\!* M$; $A_Q \sharp\!* M$; $xvec \sharp\!* M$;

$A_Q \sharp\!* xvec$; $xvec \sharp\!* \Psi$; $xvec \sharp\!* \Psi_P$; $xvec \sharp\!* \Psi_Q$; $xvec \sharp\!* P$;

$xvec \sharp\!* Q$; $A_P \sharp\!* C$; $A_Q \sharp\!* C$; $xvec \sharp\!* C]\!] \Longrightarrow$

*Prop* $C\ \Psi\ (P \parallel Q)\ M\ xvec\ N\ (P' \parallel Q')$

**and** *rBrComm2*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ xvec\ N\ P'\ A_P\ \Psi_P\ Q\ Q'\ A_Q\ C.$

$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto\!\!¡ M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$; $\bigwedge C.$ *Prop* $C\ (\Psi \otimes \Psi_Q)\ P\ M\ xvec\ N\ P'$;

*extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* $A_P$;

$\Psi \otimes \Psi_P \rhd Q \longmapsto\!\!¿ M(\!|N|\!) \prec Q'$;

*extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* $A_Q$;

$A_P \sharp\!* \Psi$; $A_P \sharp\!* \Psi_Q$; $A_P \sharp\!* P$; $A_P \sharp\!* N$; $A_P \sharp\!* P'$;

$A_P \sharp\!* Q$; $A_P \sharp\!* Q'$; $A_P \sharp\!* A_Q$; $A_P \sharp\!* xvec$; $A_Q \sharp\!* \Psi$; $A_Q \sharp\!* \Psi_P$;

$A_Q \sharp\!* P$; $A_Q \sharp\!* N$; $A_Q \sharp\!* P'$; $A_Q \sharp\!* Q$; $A_Q \sharp\!* Q'$; *distinct* $xvec$;

$A_P \sharp\!* M$; $A_Q \sharp\!* M$; $xvec \sharp\!* M$;

$A_Q \sharp\!* xvec$; $xvec \sharp\!* \Psi$; $xvec \sharp\!* \Psi_P$; $xvec \sharp\!* \Psi_Q$; $xvec \sharp\!* P$;

$xvec \sharp\!* Q$; $A_P \sharp\!* C$; $A_Q \sharp\!* C$; $xvec \sharp\!* C]\!] \Longrightarrow$

$Prop\ C\ \Psi\ (P \parallel Q)\ M\ xvec\ N\ (P' \parallel Q')$
**and** *rOpen*: $\bigwedge \Psi\ P\ M\ xvec\ yvec\ N\ P'\ x\ C.$
$[\![\Psi \rhd P \longmapsto ¡M(\!|\nu*(xvec@yvec)|\!)\langle N\rangle \prec P';\ x \in supp\ N;\ \bigwedge C.\ Prop\ C\ \Psi\ P\ M\ (xvec@yvec)\ N\ P';$
$x \sharp \Psi;\ x \sharp M;\ x \sharp xvec;\ x \sharp yvec;\ xvec \sharp* \Psi;\ xvec \sharp* P;\ xvec \sharp* M;$
$yvec \sharp* \Psi;\ yvec \sharp* P;\ yvec \sharp* M;\ yvec \sharp* C;\ x \sharp C;\ xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ ((\!|\nu x|\!)P)\ M\ (xvec@x\#yvec)\ N\ P'$
**and** *rScope*: $\bigwedge \Psi\ P\ M\ xvec\ N\ P'\ x\ C.$
$[\![\Psi \rhd P \longmapsto ¡M(\!|\nu*xvec|\!)\langle N\rangle \prec P';\ \bigwedge C.\ Prop\ C\ \Psi\ P\ M\ xvec\ N\ P';$
$x \sharp \Psi;\ x \sharp M;\ x \sharp xvec;\ x \sharp N;\ xvec \sharp* \Psi;$
$xvec \sharp* P;\ xvec \sharp* M;\ x \sharp C;\ xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ ((\!|\nu x|\!)P)\ M\ xvec\ N\ ((\!|\nu x|\!)P')$
**and** *rBang*: $\bigwedge \Psi\ P\ M\ xvec\ N\ P'\ C.$
$[\![\Psi \rhd P \parallel !P \longmapsto ¡M(\!|\nu*xvec|\!)\langle N\rangle \prec P';\ guarded\ P;\ \bigwedge C.\ Prop\ C\ \Psi\ (P \parallel !P)\ M\ xvec\ N\ P']\!] \Longrightarrow$
$Prop\ C\ \Psi\ (!P)\ M\ xvec\ N\ P'$
**shows** $Prop\ C\ \Psi\ P\ M\ xvec\ N\ P'$
⟨*proof*⟩

**lemma** *inputInduct*[*consumes 1*, *case-names cInput cCase cPar1 cPar2 cScope cBang*]:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $M$ :: $'a$
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)\ psi$
**and** *Prop* :: $'f$::*fs-name* ⇒ $'b$ ⇒ $('a, 'b, 'c)\ psi$ ⇒ $'a$ ⇒ $'a$ ⇒ $('a, 'b, 'c)\ psi$ ⇒ *bool*
**and** $C$ :: $'f$::*fs-name*

**assumes** *Trans*: $\Psi \rhd P \longmapsto M(\!|N|\!) \prec P'$
**and** *rInput*: $\bigwedge \Psi\ M\ K\ xvec\ N\ Tvec\ P\ C.$
$[\![\Psi \vdash M \leftrightarrow K;\ distinct\ xvec;\ set\ xvec \subseteq supp\ N;$
$length\ xvec = length\ Tvec;\ xvec \sharp* \Psi;$
$xvec \sharp* M;\ xvec \sharp* K;\ xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (M(\!|\lambda*xvec\ N|\!).P)$
$K\ (N[xvec::=Tvec])\ (P[xvec::=Tvec])$
**and** *rCase*: $\bigwedge \Psi\ P\ M\ N\ P'\ \varphi\ Cs\ C.\ [\![\Psi \rhd P \longmapsto M(\!|N|\!) \prec P';\ \bigwedge C.\ Prop\ C\ \Psi\ P\ M\ N\ P';\ (\varphi, P) \in set\ Cs;\ \Psi \vdash \varphi;\ guarded\ P]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (Cases\ Cs)\ M\ N\ P'$
**and** *rPar1*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ N\ P'\ A_Q\ Q\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P';\ extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)\ P\ M\ N\ P';\ distinct\ A_Q;$
$A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* \Psi;\ A_Q \sharp* M;\ A_Q \sharp* N;$
$A_Q \sharp* P';\ A_Q \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ M\ N\ (P' \parallel Q)$
**and** *rPar2*: $\bigwedge \Psi\ \Psi_P\ Q\ M\ N\ Q'\ A_P\ P\ C.$
$[\![\Psi \otimes \Psi_P \rhd Q \longmapsto M(\!|N|\!) \prec Q';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;$

*distinct* $A_P$;
$\bigwedge C.$ *Prop* $C$ $(\Psi \otimes \Psi_P)$ $Q$ $M$ $N$ $Q'$; *distinct* $A_P$;
$A_P \sharp* P$; $A_P \sharp* Q$; $A_P \sharp* \Psi$; $A_P \sharp* M$; $A_P \sharp* N$;
$A_P \sharp* Q'$; $A_P \sharp* C]\!] \Longrightarrow$
*Prop* $C$ $\Psi$ $(P \parallel Q)$ $M$ $N$ $(P \parallel Q')$

**and** *rScope*: $\bigwedge \Psi$ $P$ $M$ $N$ $P'$ $x$ $C$.
$[\![\Psi \rhd P \longmapsto M(\!|N|\!) \prec P'$; $\bigwedge C.$ *Prop* $C$ $\Psi$ $P$ $M$ $N$ $P'$; $x \sharp \Psi$; $x \sharp M$; $x \sharp N$; $x \sharp C]\!] \Longrightarrow$
*Prop* $C$ $\Psi$ $((\!|\nu x|\!)P)$ $M$ $N$ $((\!|\nu x|\!)P')$

**and** *rBang*: $\bigwedge \Psi$ $P$ $M$ $N$ $P'$ $C$.
$[\![\Psi \rhd P \parallel !P \longmapsto M(\!|N|\!) \prec P'$; *guarded* $P$; $\bigwedge C.$ *Prop* $C$ $\Psi$ $(P \parallel !P)$ $M$ $N$ $P]\!] \Longrightarrow$ *Prop* $C$ $\Psi$ $(!P)$ $M$ $N$ $P'$

**shows** *Prop* $C$ $\Psi$ $P$ $M$ $N$ $P'$
⟨*proof*⟩

**lemma** *brInputInduct*[*consumes 1*, *case-names cBrInput cCase cPar1 cPar2 cBrMerge cScope cBang*]:
**fixes** $\Psi$ :: *'b*
**and** $P$ :: *('a, 'b, 'c) psi*
**and** $M$ :: *'a*
**and** $N$ :: *'a*
**and** $P'$ :: *('a, 'b, 'c) psi*
**and** *Prop* :: *'f::fs-name* ⇒ *'b* ⇒ *('a, 'b, 'c) psi* ⇒ *'a* ⇒ *'a* ⇒ *('a, 'b, 'c) psi* ⇒ *bool*
**and** $C$ :: *'f::fs-name*

**assumes** *Trans*: $\Psi \rhd P \longmapsto ¿ M(\!|N|\!) \prec P'$
**and** *rBrInput*: $\bigwedge \Psi$ $K$ $M$ *xvec* $N$ *Tvec* $P$ $C$.
$[\![\Psi \vdash K \succeq M$; *distinct xvec*; *set xvec* $\subseteq$ *supp* $N$;
*length xvec* = *length Tvec*; *xvec* $\sharp* \Psi$;
*xvec* $\sharp* M$; *xvec* $\sharp* K$; *xvec* $\sharp* C]\!] \Longrightarrow$
*Prop* $C$ $\Psi$ $(M(\!|\lambda* xvec\ N|\!).P)$
$K$ $(N[xvec ::= Tvec])$ $(P[xvec ::= Tvec])$

**and** *rCase*: $\bigwedge \Psi$ $P$ $M$ $N$ $P'$ $\varphi$ *Cs* $C$. $[\![\Psi \rhd P \longmapsto ¿ M(\!|N|\!) \prec P'$; $\bigwedge C.$ *Prop* $C$ $\Psi$ $P$ $M$ $N$ $P'$; $(\varphi, P) \in$ *set Cs*; $\Psi \vdash \varphi$; *guarded* $P]\!] \Longrightarrow$
*Prop* $C$ $\Psi$ $(Cases\ Cs)$ $M$ $N$ $P'$

**and** *rPar1*: $\bigwedge \Psi$ $\Psi_Q$ $P$ $M$ $N$ $P'$ $A_Q$ $Q$ $C$.
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¿ M(\!|N|\!) \prec P'$; *extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$; *distinct* $A_Q$;
$\bigwedge C.$ *Prop* $C$ $(\Psi \otimes \Psi_Q)$ $P$ $M$ $N$ $P'$; *distinct* $A_Q$;
$A_Q \sharp* P$; $A_Q \sharp* Q$; $A_Q \sharp* \Psi$; $A_Q \sharp* M$; $A_Q \sharp* N$;
$A_Q \sharp* P'$; $A_Q \sharp* C]\!] \Longrightarrow$
*Prop* $C$ $\Psi$ $(P \parallel Q)$ $M$ $N$ $(P' \parallel Q)$

**and** *rPar2*: $\bigwedge \Psi$ $\Psi_P$ $Q$ $M$ $N$ $Q'$ $A_P$ $P$ $C$.
$[\![\Psi \otimes \Psi_P \rhd Q \longmapsto ¿ M(\!|N|\!) \prec Q'$; *extractFrame* $P = \langle A_P, \Psi_P \rangle$; *distinct* $A_P$;
$\bigwedge C.$ *Prop* $C$ $(\Psi \otimes \Psi_P)$ $Q$ $M$ $N$ $Q'$; *distinct* $A_P$;
$A_P \sharp* P$; $A_P \sharp* Q$; $A_P \sharp* \Psi$; $A_P \sharp* M$; $A_P \sharp* N$;
$A_P \sharp* Q'$; $A_P \sharp* C]\!] \Longrightarrow$

$Prop\ C\ \Psi\ (P \parallel Q)\ M\ N\ (P \parallel Q')$
**and** *rBrMerge*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ N\ P'\ A_P\ \Psi_P\ Q\ Q'\ A_Q\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¿M(\!|N|\!) \prec P';\ \bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)\ P\ M\ N\ P';$
$extractFrame\ P = \langle A_P,\ \Psi_P\rangle;\ distinct\ A_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto ¿M(\!|N|\!) \prec Q';\ \bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_P)\ Q\ M\ N\ Q';$
$extractFrame\ Q = \langle A_Q,\ \Psi_Q\rangle;\ distinct\ A_Q;$
$A_P \sharp* \Psi;\ A_P \sharp* \Psi_Q;\ A_P \sharp* P;\ A_P \sharp* N;\ A_P \sharp* P';$
$A_P \sharp* Q;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* M;\ A_Q \sharp* M;$
$A_Q \sharp* \Psi;\ A_Q \sharp* \Psi_P;\ A_Q \sharp* P;\ A_Q \sharp* N;\ A_Q \sharp* P';$
$A_Q \sharp* Q;\ A_Q \sharp* Q';\ A_P \sharp* C;\ A_Q \sharp* C;$
$A_P \sharp* M;\ A_Q \sharp* M]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ M\ N\ (P' \parallel Q')$
**and** *rScope*: $\bigwedge \Psi\ P\ M\ N\ P'\ x\ C.$
$[\![\Psi \rhd P \longmapsto ¿M(\!|N|\!) \prec P';\ \bigwedge C.\ Prop\ C\ \Psi\ P\ M\ N\ P';\ x \sharp \Psi;\ x \sharp M;\ x \sharp N;\ x \sharp C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ ((\!|\nu x|\!)P)\ M\ N\ ((\!|\nu x|\!)P')$
**and** *rBang*: $\bigwedge \Psi\ P\ M\ N\ P'\ C.$
$[\![\Psi \rhd P \parallel\ !P \longmapsto ¿M(\!|N|\!) \prec P';\ guarded\ P;\ \bigwedge C.\ Prop\ C\ \Psi\ (P \parallel\ !P)\ M\ N\ P']\!] \Longrightarrow Prop\ C\ \Psi\ (!P)\ M\ N\ P'$
**shows** $Prop\ C\ \Psi\ P\ M\ N\ P'$
⟨*proof*⟩

**lemma** *tauInduct*[*consumes 1*, *case-names cCase cPar1 cPar2 cComm1 cComm2 cBrClose cScope cBang*]:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $Rs$ :: $('a, 'b, 'c)\ residual$
**and** $Prop$ :: $'f$::*fs-name* ⇒ $'b$ ⇒ $('a, 'b, 'c)\ psi$ ⇒ $('a, 'b, 'c)\ psi$ ⇒ *bool*
**and** $C$ :: $'f$::*fs-name*

**assumes** *Trans*: $\Psi \rhd P \longmapsto \tau \prec P'$
**and** *rCase*: $\bigwedge \Psi\ P\ P'\ \varphi\ Cs\ C.\ [\![\Psi \rhd P \longmapsto \tau \prec P';\ \bigwedge C.\ Prop\ C\ \Psi\ P\ P';\ (\varphi, P) \in set\ Cs;\ \Psi \vdash \varphi;\ guarded\ P]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (Cases\ Cs)\ P'$
**and** *rPar1*: $\bigwedge \Psi\ \Psi_Q\ P\ P'\ A_Q\ Q\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto \tau \prec P';\ extractFrame\ Q = \langle A_Q,\ \Psi_Q\rangle;\ distinct\ A_Q;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)\ P\ P';$
$A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* \Psi;$
$A_Q \sharp* P';\ A_Q \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ (P' \parallel Q)$
**and** *rPar2*: $\bigwedge \Psi\ \Psi_P\ Q\ Q'\ A_P\ P\ C.$
$[\![\Psi \otimes \Psi_P \rhd Q \longmapsto \tau \prec Q';\ extractFrame\ P = \langle A_P,\ \Psi_P\rangle;\ distinct\ A_P;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_P)\ Q\ Q';$
$A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* \Psi;$

$A_P \sharp* Q'$; $A_P \sharp* C$$\rrbracket \Longrightarrow$
*Prop* $C$ $\Psi$ $(P \parallel Q)$ $(P \parallel Q')$

**and** *rComm1*: $\bigwedge \Psi$ $\Psi_Q$ $P$ $M$ $N$ $P'$ $A_P$ $\Psi_P$ $Q$ $K$ *xvec* $Q'$ $A_Q$ $C$.
$\llbracket \Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P'$; *extractFrame* $P = \langle A_P, \Psi_P \rangle$; *distinct* $A_P$;
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu * xvec|\!)\langle N \rangle \prec Q'$; *extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$; *distinct* $A_Q$;
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$;
$A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* M$; $A_P \sharp* N$; $A_P \sharp* P'$;
$A_P \sharp* Q$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* xvec$; $A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$;
$A_Q \sharp* P$; $A_Q \sharp* N$; $A_Q \sharp* P'$; $A_Q \sharp* Q$; $A_Q \sharp* K$; $A_Q \sharp* Q'$;
$A_Q \sharp* xvec$; *xvec* $\sharp* \Psi$; *xvec* $\sharp* \Psi_P$; *xvec* $\sharp* \Psi_Q$; *xvec* $\sharp* P$; *xvec* $\sharp* M$;
*xvec* $\sharp* Q$; *xvec* $\sharp* K$; $A_P \sharp* C$; $A_Q \sharp* C$; *xvec* $\sharp* C$$\rrbracket \Longrightarrow$
*Prop* $C$ $\Psi$ $(P \parallel Q)$ $((\!|\nu * xvec|\!)(P' \parallel Q'))$

**and** *rComm2*: $\bigwedge \Psi$ $\Psi_Q$ $P$ $M$ *xvec* $N$ $P'$ $A_P$ $\Psi_P$ $Q$ $K$ $Q'$ $A_Q$ $C$.
$\llbracket \Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N \rangle \prec P'$; *extractFrame* $P = \langle A_P, \Psi_P \rangle$; *distinct* $A_P$;
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q'$; *extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$; *distinct* $A_Q$;
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$;
$A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* M$; $A_P \sharp* N$; $A_P \sharp* P'$;
$A_P \sharp* Q$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* xvec$; $A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$;
$A_Q \sharp* P$; $A_Q \sharp* N$; $A_Q \sharp* P'$; $A_Q \sharp* Q$; $A_Q \sharp* K$; $A_Q \sharp* Q'$;
$A_Q \sharp* xvec$; *xvec* $\sharp* \Psi$; *xvec* $\sharp* \Psi_P$; *xvec* $\sharp* \Psi_Q$; *xvec* $\sharp* P$; *xvec* $\sharp* M$;
*xvec* $\sharp* Q$; *xvec* $\sharp* K$; $A_P \sharp* C$; $A_Q \sharp* C$; *xvec* $\sharp* C$$\rrbracket \Longrightarrow$
*Prop* $C$ $\Psi$ $(P \parallel Q)$ $((\!|\nu * xvec|\!)(P' \parallel Q'))$

**and** *rBrClose*: $\bigwedge \Psi$ $P$ $M$ *xvec* $N$ $P'$ $A_P$ $\Psi_P$ $x$ $C$.
$\llbracket \Psi \rhd P \longmapsto ¡M(\!|\nu * xvec|\!)\langle N \rangle \prec P'$;
$x \in supp\ M$;
*distinct xvec*; *xvec* $\sharp* \Psi$; *xvec* $\sharp* P$;
*xvec* $\sharp* M$;
$x \sharp \Psi$; $x \sharp$ *xvec*$\rrbracket \Longrightarrow$
*Prop* $C$ $\Psi$ $((\!|\nu x|\!)P)$ $((\!|\nu x|\!)((\!|\nu * xvec|\!)P'))$

**and** *rScope*: $\bigwedge \Psi$ $P$ $P'$ $x$ $C$.
$\llbracket \Psi \rhd P \longmapsto \tau \prec P'$; $\bigwedge C$. *Prop* $C$ $\Psi$ $P$ $P'$; $x \sharp \Psi$; $x \sharp C$$\rrbracket \Longrightarrow$
*Prop* $C$ $\Psi$ $((\!|\nu x|\!)P)$ $((\!|\nu x|\!)P')$

**and** *rBang*: $\bigwedge \Psi$ $P$ $P'$ $C$.
$\llbracket \Psi \rhd P \parallel !P \longmapsto \tau \prec P'$; *guarded* $P$; $\bigwedge C$. *Prop* $C$ $\Psi$ $(P \parallel !P)$ $P'$$\rrbracket$
$\Longrightarrow$ *Prop* $C$ $\Psi$ $(!P)$ $P'$

**shows** *Prop* $C$ $\Psi$ $P$ $P'$
⟨*proof*⟩

**lemma** *semanticsFrameInduct*[*consumes 3*, *case-names cAlpha cInput cBrInput cOutput cBrOutput cCase cPar1 cPar2 cComm1 cComm2 cBrMerge cBrComm1 cBrComm2 cBrClose cOpen cBrOpen cScope cBang*]:
**fixes** $\Psi$ :: *'b*
**and** $P$ :: (*'a*, *'b*, *'c*) *psi*

**and** $Rs$ :: $('a, 'b, 'c)$ *residual*
**and** $A_P$ :: *name list*
**and** $\Psi_P$ :: $'b$
**and** *Prop* :: $'f$::*fs-name* $\Rightarrow$ $'b \Rightarrow ('a, 'b, 'c)$ *psi* $\Rightarrow$
$('a, 'b, 'c)$ *residual* $\Rightarrow$ *name list* $\Rightarrow$ $'b \Rightarrow$ *bool*
**and** $C$ :: $'f$::*fs-name*

**assumes** *Trans*: $\Psi \rhd P \longmapsto Rs$
**and** *FrP*: *extractFrame* $P = \langle A_P, \Psi_P \rangle$
**and** *distinct* $A_P$
**and** *rAlpha*: $\bigwedge \Psi\ P\ A_P\ \Psi_P\ p\ Rs\ C.\ [\![A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* (p \cdot A_P);\ A_P \sharp* Rs;\ A_P \sharp* C;$
$set\ p \subseteq set\ A_P \times set(p \cdot A_P);\ distinctPerm\ p;$
$Prop\ C\ \Psi\ P\ Rs\ A_P\ \Psi_P]\!] \Longrightarrow Prop\ C\ \Psi\ P\ Rs\ (p \cdot A_P)\ (p \cdot \Psi_P)$
**and** *rInput*: $\bigwedge \Psi\ M\ K\ xvec\ N\ Tvec\ P\ C.$
$[\![\Psi \vdash M \leftrightarrow K;\ distinct\ xvec;\ set\ xvec \subseteq supp\ N;$
$length\ xvec = length\ Tvec;\ xvec \sharp* \Psi;$
$xvec \sharp* M;\ xvec \sharp* K;\ xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (M(\!|\lambda* xvec\ N|\!).P)$
$(K(\!|(N[xvec::=Tvec])|\!) \prec (P[xvec::=Tvec]))\ ([])\ (\mathbf{1})$
**and** *rBrInput*: $\bigwedge \Psi\ M\ K\ xvec\ N\ Tvec\ P\ C.$
$[\![\Psi \vdash K \succeq M;\ distinct\ xvec;\ set\ xvec \subseteq supp\ N;$
$length\ xvec = length\ Tvec;\ xvec \sharp* \Psi;$
$xvec \sharp* M;\ xvec \sharp* K;\ xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (M(\!|\lambda* xvec\ N|\!).P)$
$(¿K(\!|(N[xvec::=Tvec])|\!) \prec (P[xvec::=Tvec]))\ ([])\ (\mathbf{1})$
**and** *rOutput*: $\bigwedge \Psi\ M\ K\ N\ P\ C.\ \Psi \vdash M \leftrightarrow K \Longrightarrow Prop\ C\ \Psi\ (M\langle N \rangle.P)\ (K\langle N \rangle \prec P)\ ([])\ (\mathbf{1})$
**and** *rBrOutput*: $\bigwedge \Psi\ M\ K\ N\ P\ C.\ \Psi \vdash M \preceq K \Longrightarrow Prop\ C\ \Psi\ (M\langle N \rangle.P)\ (¡K\langle N \rangle \prec P)\ ([])\ (\mathbf{1})$
**and** *rCase*: $\bigwedge \Psi\ P\ Rs\ \varphi\ Cs\ A_P\ \Psi_P\ C.\ [\![\Psi \rhd P \longmapsto Rs;\ extractFrame\ P = \langle A_P, \Psi_P \rangle;\ distinct\ A_P;\ \bigwedge C.\ Prop\ C\ \Psi\ P\ Rs\ A_P\ \Psi_P;$
$(\varphi, P) \in set\ Cs;\ \Psi \vdash \varphi;\ guarded\ P;\ \Psi_P \simeq \mathbf{1};$
$(supp\ \Psi_P) = (\{\}::name\ set);$
$A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* Rs;\ A_P \sharp* C]\!] \Longrightarrow Prop\ C\ \Psi\ (Cases\ Cs)\ Rs\ ([])\ (\mathbf{1})$
**and** *rPar1*: $\bigwedge \Psi\ \Psi_Q\ P\ \alpha\ P'\ A_Q\ Q\ A_P\ \Psi_P\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto \alpha \prec P';$
$extractFrame\ P = \langle A_P, \Psi_P \rangle;\ distinct\ A_P;$
$extractFrame\ Q = \langle A_Q, \Psi_Q \rangle;\ distinct\ A_Q;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)\ P\ (\alpha \prec P')\ A_P\ \Psi_P;\ distinct(bn\ \alpha);$
$A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* \Psi;\ A_P \sharp* \alpha;\ A_P \sharp* P';\ A_P \sharp* A_Q;\ A_P \sharp* \Psi_Q;$
$A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* \Psi;\ A_Q \sharp* \alpha;\ A_Q \sharp* P';\ A_Q \sharp* \Psi_P;$
$bn\ \alpha \sharp* \Psi;\ bn\ \alpha \sharp* P;\ bn\ \alpha \sharp* Q;\ bn\ \alpha \sharp* subject\ \alpha;\ bn\ \alpha \sharp* \Psi_P;$
$bn\ \alpha \sharp* \Psi_Q;$
$A_P \sharp* C;\ A_Q \sharp* C;\ bn\ \alpha \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ (\alpha \prec (P' \parallel Q))\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

**and** *rPar2*: $\bigwedge \Psi\ \Psi_P\ Q\ \alpha\ Q'\ A_P\ P\ A_Q\ \Psi_Q\ C.$
$[\![\Psi \otimes \Psi_P \rhd Q \longmapsto \alpha \prec Q';$
$\mathit{extractFrame}\ P = \langle A_P, \Psi_P\rangle;\ \mathit{distinct}\ A_P;$
$\mathit{extractFrame}\ Q = \langle A_Q, \Psi_Q\rangle;\ \mathit{distinct}\ A_Q;$
$\bigwedge C.\ \mathit{Prop}\ C\ (\Psi \otimes \Psi_P)\ Q\ (\alpha \prec Q')\ A_Q\ \Psi_Q;\ \mathit{distinct}(\mathit{bn}\ \alpha);$
$A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* \Psi;\ A_P \sharp* \alpha;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* \Psi_Q;$
$A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* \Psi;\ A_Q \sharp* \alpha;\ A_Q \sharp* Q';\ A_Q \sharp* \Psi_P;$
$\mathit{bn}\ \alpha \sharp* \Psi;\ \mathit{bn}\ \alpha \sharp* P;\ \mathit{bn}\ \alpha \sharp* Q;\ \mathit{bn}\ \alpha \sharp* \mathit{subject}\ \alpha;\ \mathit{bn}\ \alpha \sharp* \Psi_P;\ \mathit{bn}\ \alpha \sharp* \Psi_Q;$
$A_P \sharp* C;\ A_Q \sharp* C;\ \mathit{bn}\ \alpha \sharp* C]\!] \Longrightarrow$
$\mathit{Prop}\ C\ \Psi\ (P \parallel Q)\ (\alpha \prec (P \parallel Q'))\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

**and** *rComm1*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ N\ P'\ A_P\ \Psi_P\ Q\ K\ \mathit{xvec}\ Q'\ A_Q\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P';\ \mathit{extractFrame}\ P = \langle A_P, \Psi_P\rangle;\ \mathit{distinct}\ A_P;$
$\bigwedge C.\ \mathit{Prop}\ C\ (\Psi \otimes \Psi_Q)\ P\ ((M(\!|N|\!)) \prec P')\ A_P\ \Psi_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu * \mathit{xvec}|\!)\langle N\rangle \prec Q';\ \mathit{extractFrame}\ Q = \langle A_Q, \Psi_Q\rangle;\ \mathit{distinct}\ A_Q;$
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K;$
$\bigwedge C.\ \mathit{Prop}\ C\ (\Psi \otimes \Psi_P)\ Q\ (K(\!|\nu * \mathit{xvec}|\!)\langle N\rangle \prec Q')\ A_Q\ \Psi_Q;\ \mathit{distinct}\ \mathit{xvec};$
$A_P \sharp* \Psi;\ A_P \sharp* \Psi_Q;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';$
$A_P \sharp* Q;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* \mathit{xvec};\ A_Q \sharp* \Psi;\ A_Q \sharp* \Psi_P;$
$A_Q \sharp* P;\ A_Q \sharp* N;\ A_Q \sharp* P';\ A_Q \sharp* Q;\ A_Q \sharp* K;\ A_Q \sharp* Q';$
$A_Q \sharp* \mathit{xvec};\ \mathit{xvec} \sharp* \Psi;\ \mathit{xvec} \sharp* \Psi_P;\ \mathit{xvec} \sharp* \Psi_Q;\ \mathit{xvec} \sharp* P;\ \mathit{xvec} \sharp* M;$
$\mathit{xvec} \sharp* Q;\ \mathit{xvec} \sharp* K;\ A_P \sharp* C;\ A_Q \sharp* C;\ \mathit{xvec} \sharp* C]\!] \Longrightarrow$
$\mathit{Prop}\ C\ \Psi\ (P \parallel Q)\ (\tau \prec (\!|\nu * \mathit{xvec}|\!)(P' \parallel Q'))\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

**and** *rComm2*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ \mathit{xvec}\ N\ P'\ A_P\ \Psi_P\ Q\ K\ Q'\ A_Q\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu * \mathit{xvec}|\!)\langle N\rangle \prec P';\ \mathit{extractFrame}\ P = \langle A_P, \Psi_P\rangle;\ \mathit{distinct}\ A_P;$
$\bigwedge C.\ \mathit{Prop}\ C\ (\Psi \otimes \Psi_Q)\ P\ (M(\!|\nu * \mathit{xvec}|\!)\langle N\rangle \prec P')\ A_P\ \Psi_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q';\ \mathit{extractFrame}\ Q = \langle A_Q, \Psi_Q\rangle;\ \mathit{distinct}\ A_Q;$
$\bigwedge C.\ \mathit{Prop}\ C\ (\Psi \otimes \Psi_P)\ Q\ (K(\!|N|\!) \prec Q')\ A_Q\ \Psi_Q;$
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K;\ \mathit{distinct}\ \mathit{xvec};$
$A_P \sharp* \Psi;\ A_P \sharp* \Psi_Q;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';$
$A_P \sharp* Q;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* \mathit{xvec};\ A_Q \sharp* \Psi;\ A_Q \sharp* \Psi_P;$
$A_Q \sharp* P;\ A_Q \sharp* N;\ A_Q \sharp* P';\ A_Q \sharp* Q;\ A_Q \sharp* K;\ A_Q \sharp* Q';$
$A_Q \sharp* \mathit{xvec};\ \mathit{xvec} \sharp* \Psi;\ \mathit{xvec} \sharp* \Psi_P;\ \mathit{xvec} \sharp* \Psi_Q;\ \mathit{xvec} \sharp* P;\ \mathit{xvec} \sharp* M;$
$\mathit{xvec} \sharp* Q;\ \mathit{xvec} \sharp* K;\ A_P \sharp* C;\ A_Q \sharp* C;\ \mathit{xvec} \sharp* C]\!] \Longrightarrow$
$\mathit{Prop}\ C\ \Psi\ (P \parallel Q)\ (\tau \prec (\!|\nu * \mathit{xvec}|\!)(P' \parallel Q'))\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

**and** *rBrMerge*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ N\ P'\ A_P\ \Psi_P\ Q\ Q'\ A_Q\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¿M(\!|N|\!) \prec P';\ \bigwedge C.\ \mathit{Prop}\ C\ (\Psi \otimes \Psi_Q)\ P\ (¿M(\!|N|\!) \prec P')\ A_P\ \Psi_P;$
$\mathit{extractFrame}\ P = \langle A_P, \Psi_P\rangle;\ \mathit{distinct}\ A_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto ¿M(\!|N|\!) \prec Q';\ \bigwedge C.\ \mathit{Prop}\ C\ (\Psi \otimes \Psi_P)\ Q\ (¿M(\!|N|\!) \prec Q')\ A_Q\ \Psi_Q;$

$$
\begin{array}{l}
\quad\quad\quad\quad \mathit{extractFrame}\ Q = \langle A_Q,\ \Psi_Q\rangle;\ \mathit{distinct}\ A_Q;\\
\quad\quad\quad\quad A_P \sharp\!* \Psi;\ A_P \sharp\!* \Psi_Q;\ A_P \sharp\!* P;\ A_P \sharp\!* N;\ A_P \sharp\!* P';\\
\quad\quad\quad\quad A_P \sharp\!* Q;\ A_P \sharp\!* Q';\ A_P \sharp\!* A_Q;\ A_P \sharp\!* M;\ A_Q \sharp\!* M;\\
\quad\quad\quad\quad A_Q \sharp\!* \Psi;\ A_Q \sharp\!* \Psi_P;\ A_Q \sharp\!* P;\ A_Q \sharp\!* N;\ A_Q \sharp\!* P';\\
\quad\quad\quad\quad A_Q \sharp\!* Q;\ A_Q \sharp\!* Q';\ A_P \sharp\!* C;\ A_Q \sharp\!* C;\\
\quad\quad\quad\quad A_P \sharp\!* M;\ A_Q \sharp\!* M[\!] \Longrightarrow\\
\quad\quad\quad\quad \mathit{Prop}\ C\ \Psi\ (P \parallel Q)\ (¿M(\!|N|\!) \prec (P' \parallel Q'))\ (A_P@A_Q)\ (\Psi_P \otimes \Psi_Q)\\
\textbf{and}\quad \mathit{rBrComm1}{:}\bigwedge \Psi\ \Psi_Q\ P\ M\ N\ P'\ A_P\ \Psi_P\ Q\ \mathit{xvec}\ Q'\ A_Q\ C.\\
\quad\quad\quad\quad [\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¿M(\!|N|\!) \prec P';\ \mathit{extractFrame}\ P = \langle A_P,\ \Psi_P\rangle;\\
\mathit{distinct}\ A_P;\\
\quad\quad\quad\quad \bigwedge C.\ \mathit{Prop}\ C\ (\Psi \otimes \Psi_Q)\ P\ ((¿M(\!|N|\!)) \prec P')\ A_P\ \Psi_P;\\
\quad\quad\quad\quad \Psi \otimes \Psi_P \rhd Q \longmapsto ¡M(\!|\nu{*}\mathit{xvec}|\!)\langle N\rangle \prec Q';\ \mathit{extractFrame}\ Q = \langle A_Q,\\
\Psi_Q\rangle;\ \mathit{distinct}\ A_Q;\\
\quad\quad\quad\quad \bigwedge C.\ \mathit{Prop}\ C\ (\Psi \otimes \Psi_P)\ Q\ (¡M(\!|\nu{*}\mathit{xvec}|\!)\langle N\rangle \prec Q')\ A_Q\ \Psi_Q;\ \mathit{distinct}\\
\mathit{xvec};\\
\quad\quad\quad\quad A_P \sharp\!* \Psi;\ A_P \sharp\!* \Psi_Q;\ A_P \sharp\!* P;\ A_P \sharp\!* N;\ A_P \sharp\!* P';\\
\quad\quad\quad\quad A_P \sharp\!* Q;\ A_P \sharp\!* Q';\ A_P \sharp\!* A_Q;\ A_P \sharp\!* \mathit{xvec};\ A_Q \sharp\!* \Psi;\ A_Q \sharp\!* \Psi_P;\\
\quad\quad\quad\quad A_Q \sharp\!* P;\ A_Q \sharp\!* N;\ A_Q \sharp\!* P';\ A_Q \sharp\!* Q;\ A_Q \sharp\!* Q';\\
\quad\quad\quad\quad A_Q \sharp\!* \mathit{xvec};\ \mathit{xvec} \sharp\!* \Psi;\ \mathit{xvec} \sharp\!* \Psi_P;\ \mathit{xvec} \sharp\!* \Psi_Q;\ \mathit{xvec} \sharp\!* P;\\
\quad\quad\quad\quad \mathit{xvec} \sharp\!* Q;\ A_P \sharp\!* C;\ A_Q \sharp\!* C;\ \mathit{xvec} \sharp\!* C;\\
\quad\quad\quad\quad A_P \sharp\!* M;\ A_Q \sharp\!* M;\ \mathit{xvec} \sharp\!* M[\!] \Longrightarrow\\
\quad\quad\quad\quad \mathit{Prop}\ C\ \Psi\ (P \parallel Q)\ (¡M(\!|\nu{*}\mathit{xvec}|\!)\langle N\rangle \prec (P' \parallel Q'))\ (A_P@A_Q)\ (\Psi_P\\
\otimes \Psi_Q)\\
\textbf{and}\quad \mathit{rBrComm2}{:}\bigwedge \Psi\ \Psi_Q\ P\ M\ \mathit{xvec}\ N\ P'\ A_P\ \Psi_P\ Q\ Q'\ A_Q\ C.\\
\quad\quad\quad\quad [\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¡M(\!|\nu{*}\mathit{xvec}|\!)\langle N\rangle \prec P';\ \mathit{extractFrame}\ P = \langle A_P,\\
\Psi_P\rangle;\ \mathit{distinct}\ A_P;\\
\quad\quad\quad\quad \bigwedge C.\ \mathit{Prop}\ C\ (\Psi \otimes \Psi_Q)\ P\ (¡M(\!|\nu{*}\mathit{xvec}|\!)\langle N\rangle \prec P')\ A_P\ \Psi_P;\\
\quad\quad\quad\quad \Psi \otimes \Psi_P \rhd Q \longmapsto ¿M(\!|N|\!) \prec Q';\ \mathit{extractFrame}\ Q = \langle A_Q,\ \Psi_Q\rangle;\\
\mathit{distinct}\ A_Q;\\
\quad\quad\quad\quad \bigwedge C.\ \mathit{Prop}\ C\ (\Psi \otimes \Psi_P)\ Q\ (¿M(\!|N|\!) \prec Q')\ A_Q\ \Psi_Q;\ \mathit{distinct}\ \mathit{xvec};\\
\quad\quad\quad\quad A_P \sharp\!* \Psi;\ A_P \sharp\!* \Psi_Q;\ A_P \sharp\!* P;\ A_P \sharp\!* N;\ A_P \sharp\!* P';\\
\quad\quad\quad\quad A_P \sharp\!* Q;\ A_P \sharp\!* Q';\ A_P \sharp\!* A_Q;\ A_P \sharp\!* \mathit{xvec};\ A_Q \sharp\!* \Psi;\ A_Q \sharp\!* \Psi_P;\\
\quad\quad\quad\quad A_Q \sharp\!* P;\ A_Q \sharp\!* N;\ A_Q \sharp\!* P';\ A_Q \sharp\!* Q;\ A_Q \sharp\!* Q';\\
\quad\quad\quad\quad A_Q \sharp\!* \mathit{xvec};\ \mathit{xvec} \sharp\!* \Psi;\ \mathit{xvec} \sharp\!* \Psi_P;\ \mathit{xvec} \sharp\!* \Psi_Q;\ \mathit{xvec} \sharp\!* P;\\
\quad\quad\quad\quad \mathit{xvec} \sharp\!* Q;\ A_P \sharp\!* C;\ A_Q \sharp\!* C;\ \mathit{xvec} \sharp\!* C;\\
\quad\quad\quad\quad A_P \sharp\!* M;\ A_Q \sharp\!* M;\ \mathit{xvec} \sharp\!* M[\!] \Longrightarrow\\
\quad\quad\quad\quad \mathit{Prop}\ C\ \Psi\ (P \parallel Q)\ (¡M(\!|\nu{*}\mathit{xvec}|\!)\langle N\rangle \prec (P' \parallel Q'))\ (A_P@A_Q)\ (\Psi_P\\
\otimes \Psi_Q)\\
\textbf{and}\quad \mathit{rBrClose}{:}\ \bigwedge \Psi\ P\ M\ \mathit{xvec}\ N\ P'\ A_P\ \Psi_P\ x\ C.\\
\quad\quad\quad\quad [\![\Psi \rhd P \longmapsto ¡M(\!|\nu{*}\mathit{xvec}|\!)\langle N\rangle \prec P';\\
\quad\quad\quad\quad x \in \mathit{supp}\ M;\\
\quad\quad\quad\quad \bigwedge C.\ \mathit{Prop}\ C\ \Psi\ P\ (¡M(\!|\nu{*}\mathit{xvec}|\!)\langle N\rangle \prec P')\ A_P\ \Psi_P;\\
\quad\quad\quad\quad \mathit{extractFrame}\ P = \langle A_P,\ \Psi_P\rangle;\ \mathit{distinct}\ A_P;\\
\quad\quad\quad\quad A_P \sharp\!* \Psi;\ A_P \sharp\!* P;\ A_P \sharp\!* M;\ A_P \sharp\!* N;\ A_P \sharp\!* P';\ A_P \sharp\!* \mathit{xvec};\\
\quad\quad\quad\quad \mathit{distinct}\ \mathit{xvec};\ \mathit{xvec} \sharp\!* \Psi;\ \mathit{xvec} \sharp\!* \Psi_P;\ \mathit{xvec} \sharp\!* P;\\
\quad\quad\quad\quad \mathit{xvec} \sharp\!* M;\\
\quad\quad\quad\quad x \sharp \Psi;\ x \sharp \mathit{xvec};\ x \sharp A_P;\\
\quad\quad\quad\quad A_P \sharp\!* C;\ \mathit{xvec} \sharp\!* C;\ x \sharp C[\!] \Longrightarrow\\
\quad\quad\quad\quad \mathit{Prop}\ C\ \Psi\ ((\!|\nu x|\!)P)\ (\tau \prec ((\!|\nu x|\!)((\!|\nu{*}\mathit{xvec}|\!)P')))\ (x\#A_P)\ \Psi_P
\end{array}
$$

**and** *rOpen*: $\bigwedge\Psi\ P\ M\ xvec\ yvec\ N\ P'\ x\ A_P\ \Psi_P\ C.$
$[\![\Psi \rhd P \longmapsto M(\!|\nu*(xvec@yvec)|\!)\langle N\rangle \prec P';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$\bigwedge C.\ Prop\ C\ \Psi\ P\ (M(\!|\nu*(xvec@yvec)|\!)\langle N\rangle \prec P')\ A_P\ \Psi_P;\ x \in supp\ N;\ x \sharp \Psi;\ x \sharp M;$
$x \sharp A_P;\ x \sharp xvec;\ x \sharp yvec;\ A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';$
$A_P \sharp* xvec;\ A_P \sharp* yvec;\ xvec \sharp* yvec;\ distinct\ xvec;\ distinct\ yvec;$
$xvec \sharp* \Psi;\ xvec \sharp* P;\ xvec \sharp* M;\ xvec \sharp* \Psi_P;\ yvec \sharp* \Psi_P;$
$yvec \sharp* \Psi;\ yvec \sharp* P;\ yvec \sharp* M;\ A_P \sharp* C;\ x \sharp C;\ xvec \sharp* C;\ yvec \sharp* C] \Longrightarrow$
$Prop\ C\ \Psi\ ((\!|\nu x|\!)P)\ (M(\!|\nu*(xvec@x\#yvec)|\!)\langle N\rangle \prec P')\ (x\#A_P)\ \Psi_P$

**and** *rBrOpen*: $\bigwedge\Psi\ P\ M\ xvec\ yvec\ N\ P'\ x\ A_P\ \Psi_P\ C.$
$[\![\Psi \rhd P \longmapsto ¡M(\!|\nu*(xvec@yvec)|\!)\langle N\rangle \prec P';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$\bigwedge C.\ Prop\ C\ \Psi\ P\ (¡M(\!|\nu*(xvec@yvec)|\!)\langle N\rangle \prec P')\ A_P\ \Psi_P;\ x \in supp\ N;\ x \sharp \Psi;\ x \sharp M;$
$x \sharp A_P;\ x \sharp xvec;\ x \sharp yvec;\ A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';$
$A_P \sharp* xvec;\ A_P \sharp* yvec;\ xvec \sharp* yvec;\ distinct\ xvec;\ distinct\ yvec;$
$xvec \sharp* \Psi;\ xvec \sharp* P;\ xvec \sharp* M;\ xvec \sharp* \Psi_P;\ yvec \sharp* \Psi_P;$
$yvec \sharp* \Psi;\ yvec \sharp* P;\ yvec \sharp* M;\ A_P \sharp* C;\ x \sharp C;\ xvec \sharp* C;\ yvec \sharp* C] \Longrightarrow$
$Prop\ C\ \Psi\ ((\!|\nu x|\!)P)\ (¡M(\!|\nu*(xvec@x\#yvec)|\!)\langle N\rangle \prec P')\ (x\#A_P)\ \Psi_P$

**and** *rScope*: $\bigwedge\Psi\ P\ \alpha\ P'\ x\ A_P\ \Psi_P\ C.$
$[\![\Psi \rhd P \longmapsto \alpha \prec P';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$\bigwedge C.\ Prop\ C\ \Psi\ P\ (\alpha \prec P')\ A_P\ \Psi_P;$
$x \sharp \Psi;\ x \sharp \alpha;\ x \sharp A_P;\ A_P \sharp* \Psi;\ A_P \sharp* P;$
$A_P \sharp* \alpha;\ A_P \sharp* P';\ distinct(bn\ \alpha);$
$bn\ \alpha \sharp* \Psi;\ bn\ \alpha \sharp* P;\ bn\ \alpha \sharp* subject\ \alpha;\ bn\ \alpha \sharp* \Psi_P;$
$A_P \sharp* C;\ x \sharp C;\ bn\ \alpha \sharp* C] \Longrightarrow$
$Prop\ C\ \Psi\ ((\!|\nu x|\!)P)\ (\alpha \prec ((\!|\nu x|\!)P'))\ (x\#A_P)\ \Psi_P$

**and** *rBang*: $\bigwedge\Psi\ P\ Rs\ A_P\ \Psi_P\ C.$
$[\![\Psi \rhd P \parallel\ !P \longmapsto Rs;\ guarded\ P;\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$\bigwedge C.\ Prop\ C\ \Psi\ (P \parallel\ !P)\ Rs\ A_P\ (\Psi_P \otimes \mathbf{1});\ \Psi_P \simeq \mathbf{1};\ supp\ \Psi_P = (\{\}::name\ set);$
$A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* Rs;\ A_P \sharp* C] \Longrightarrow Prop\ C\ \Psi\ (!P)\ Rs\ ([])\ (\mathbf{1})$

**shows** $Prop\ C\ \Psi\ P\ Rs\ A_P\ \Psi_P$

⟨*proof*⟩

**lemma** *semanticsFrameInduct'*[*consumes 5*, *case-names cAlpha cFrameAlpha cInput cBrInput cOutput cBrOutput cCase cPar1 cPar2 cComm1 cComm2 cBrMerge cBrComm1 cBrComm2 cBrClose cOpen cBrOpen cScope cBang*]:

**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $Rs$ :: $('a, 'b, 'c)\ residual$
**and** $A_P$ :: *name list*

**and** $\Psi_P$ :: $'b$
**and** *Prop* :: $'f$::*fs-name* $\Rightarrow$ $'b$ $\Rightarrow$ ($'a$, $'b$, $'c$) *psi* $\Rightarrow$ $'a$ *action* $\Rightarrow$
($'a$, $'b$, $'c$) *psi* $\Rightarrow$ *name list* $\Rightarrow$ $'b$ $\Rightarrow$ *bool*
**and** $C$ :: $'f$::*fs-name*

**assumes** *Trans*: $\Psi \rhd P \longmapsto \alpha \prec P'$
**and** *FrP*: *extractFrame* $P = \langle A_P, \Psi_P \rangle$
**and** *distinct* $A_P$
**and** *bn* $\alpha \sharp* \mathit{subject}\ \alpha$
**and** *distinct*(*bn* $\alpha$)
**and** *rAlpha*: $\bigwedge \Psi\ P\ \alpha\ P'\ p\ A_P\ \Psi_P\ C.$ $[\![ bn\ \alpha \sharp* \Psi;\ bn\ \alpha \sharp* P;\ bn\ \alpha \sharp* \mathit{subject}\ \alpha;\ bn\ \alpha \sharp* \Psi_P;$
$bn\ \alpha \sharp* C;\ bn\ \alpha \sharp* (p \cdot \alpha);\ A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* \alpha;\ A_P \sharp* P';\ A_P \sharp* C;$
$\mathit{set}\ p \subseteq \mathit{set}(bn\ \alpha) \times \mathit{set}(bn(p \cdot \alpha));\ \mathit{distinctPerm}\ p;$
$bn(p \cdot \alpha) \sharp* \alpha;\ (bn(p \cdot \alpha)) \sharp* P';\ \mathit{Prop}\ C\ \Psi\ P\ \alpha\ P'\ A_P\ \Psi_P ]\!] \Longrightarrow$
$\mathit{Prop}\ C\ \Psi\ P\ (p \cdot \alpha)\ (p \cdot P')\ A_P\ \Psi_P$
**and** *rFrameAlpha*: $\bigwedge \Psi\ P\ A_P\ \Psi_P\ p\ \alpha\ P'\ C.$ $[\![ A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* (p \cdot A_P);\ A_P \sharp* \alpha;\ A_P \sharp* P';\ A_P \sharp* C;$
$\mathit{set}\ p \subseteq \mathit{set}\ A_P \times \mathit{set}(p \cdot A_P);\ \mathit{distinctPerm}\ p;\ A_P \sharp* \mathit{subject}\ \alpha;$
$\mathit{Prop}\ C\ \Psi\ P\ \alpha\ P'\ A_P\ \Psi_P ]\!] \Longrightarrow \mathit{Prop}\ C\ \Psi\ P\ \alpha\ P'\ (p \cdot A_P)\ (p \cdot \Psi_P)$
**and** *rInput*: $\bigwedge \Psi\ M\ K\ \mathit{xvec}\ N\ \mathit{Tvec}\ P\ C.$
$[\![ \Psi \vdash M \leftrightarrow K;\ \mathit{distinct}\ \mathit{xvec};\ \mathit{set}\ \mathit{xvec} \subseteq \mathit{supp}\ N;$
$\mathit{length}\ \mathit{xvec} = \mathit{length}\ \mathit{Tvec};\ \mathit{xvec} \sharp* \Psi;$
$\mathit{xvec} \sharp* M;\ \mathit{xvec} \sharp* K;\ \mathit{xvec} \sharp* C ]\!] \Longrightarrow$
$\mathit{Prop}\ C\ \Psi\ (M(\!|\lambda* \mathit{xvec}\ N|\!).P)$
$(K(\!|(N[\mathit{xvec}::=\mathit{Tvec}])|\!))\ (P[\mathit{xvec}::=\mathit{Tvec}])\ ([])\ (\mathbf{1})$
**and** *rBrInput*: $\bigwedge \Psi\ M\ K\ \mathit{xvec}\ N\ \mathit{Tvec}\ P\ C.$
$[\![ \Psi \vdash K \succeq M;\ \mathit{distinct}\ \mathit{xvec};\ \mathit{set}\ \mathit{xvec} \subseteq \mathit{supp}\ N;$
$\mathit{length}\ \mathit{xvec} = \mathit{length}\ \mathit{Tvec};\ \mathit{xvec} \sharp* \Psi;$
$\mathit{xvec} \sharp* M;\ \mathit{xvec} \sharp* K;\ \mathit{xvec} \sharp* C ]\!] \Longrightarrow$
$\mathit{Prop}\ C\ \Psi\ (M(\!|\lambda* \mathit{xvec}\ N|\!).P)$
$(¿K(\!|(N[\mathit{xvec}::=\mathit{Tvec}])|\!))\ (P[\mathit{xvec}::=\mathit{Tvec}])\ ([])\ (\mathbf{1})$
**and** *rOutput*: $\bigwedge \Psi\ M\ K\ N\ P\ C.\ \Psi \vdash M \leftrightarrow K \Longrightarrow \mathit{Prop}\ C\ \Psi\ (M\langle N \rangle.P)\ (K\langle N \rangle)\ P\ ([])\ (\mathbf{1})$
**and** *rBrOutput*: $\bigwedge \Psi\ M\ K\ N\ P\ C.\ \Psi \vdash M \preceq K \Longrightarrow \mathit{Prop}\ C\ \Psi\ (M\langle N \rangle.P)\ (¡K\langle N \rangle)\ P\ ([])\ (\mathbf{1})$
**and** *rCase*: $\bigwedge \Psi\ P\ \alpha\ P'\ \varphi\ \mathit{Cs}\ A_P\ \Psi_P\ C.$ $[\![ \Psi \rhd P \longmapsto \alpha \prec P';\ \mathit{extractFrame}\ P = \langle A_P, \Psi_P \rangle;\ \mathit{distinct}\ A_P;\ \bigwedge C.\ \mathit{Prop}\ C\ \Psi\ P\ \alpha\ P'\ A_P\ \Psi_P;$
$(\varphi, P) \in \mathit{set}\ \mathit{Cs};\ \Psi \vdash \varphi;\ \mathit{guarded}\ P;\ \Psi_P \simeq \mathbf{1};$
$(\mathit{supp}\ \Psi_P) = (\{\}::\mathit{name\ set});$
$A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* \alpha;\ A_P \sharp* P';\ A_P \sharp* C ]\!] \Longrightarrow \mathit{Prop}\ C\ \Psi\ (\mathit{Cases}\ \mathit{Cs})\ \alpha\ P'\ ([])\ (\mathbf{1})$
**and** *rPar1*: $\bigwedge \Psi\ \Psi_Q\ P\ \alpha\ P'\ A_Q\ Q\ A_P\ \Psi_P\ C.$
$[\![ \Psi \otimes \Psi_Q \rhd P \longmapsto \alpha \prec P';$

$$
\begin{array}{l}
\quad\quad\quad\quad \mathit{extractFrame}\ P = \langle A_P,\ \Psi_P\rangle;\ \mathit{distinct}\ A_P;\\
\quad\quad\quad\quad \mathit{extractFrame}\ Q = \langle A_Q,\ \Psi_Q\rangle;\ \mathit{distinct}\ A_Q;\\
\quad\quad\quad\quad \bigwedge C.\ \mathit{Prop}\ C\ (\Psi \otimes \Psi_Q)\ P\ \alpha\ P'\ A_P\ \Psi_P;\\
\quad\quad\quad\quad A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* \Psi;\ A_P \sharp* \alpha;\ A_P \sharp* P';\ A_P \sharp* A_Q;\ A_P \sharp* \Psi_Q;\\
\quad\quad\quad\quad A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* \Psi;\ A_Q \sharp* \alpha;\ A_Q \sharp* P';\ A_Q \sharp* \Psi_P;\\
\quad\quad\quad\quad \mathit{bn}\ \alpha \sharp* \Psi;\ \mathit{bn}\ \alpha \sharp* P;\ \mathit{bn}\ \alpha \sharp* Q;\ \mathit{bn}\ \alpha \sharp* \mathit{subject}\ \alpha;\ \mathit{bn}\ \alpha \sharp* \Psi_P;\ \mathit{bn}\ \alpha \sharp* \Psi_Q;\\
\quad\quad\quad\quad A_P \sharp* C;\ A_Q \sharp* C;\ \mathit{bn}\ \alpha \sharp* C[\!] \Longrightarrow\\
\quad\quad\quad\quad \mathit{Prop}\ C\ \Psi\ (P \parallel Q)\ \alpha\ (P' \parallel Q)\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)\\
\textbf{and}\quad \mathit{rPar2}\colon \bigwedge \Psi\ \Psi_P\ Q\ \alpha\ Q'\ A_P\ P\ A_Q\ \Psi_Q\ C.\\
\quad\quad\quad\quad [\![\Psi \otimes \Psi_P \rhd Q \longmapsto \alpha \prec Q';\\
\quad\quad\quad\quad \mathit{extractFrame}\ P = \langle A_P,\ \Psi_P\rangle;\ \mathit{distinct}\ A_P;\\
\quad\quad\quad\quad \mathit{extractFrame}\ Q = \langle A_Q,\ \Psi_Q\rangle;\ \mathit{distinct}\ A_Q;\\
\quad\quad\quad\quad \bigwedge C.\ \mathit{Prop}\ C\ (\Psi \otimes \Psi_P)\ Q\ \alpha\ Q'\ A_Q\ \Psi_Q;\\
\quad\quad\quad\quad A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* \Psi;\ A_P \sharp* \alpha;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* \Psi_Q;\\
\quad\quad\quad\quad A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* \Psi;\ A_Q \sharp* \alpha;\ A_Q \sharp* Q';\ A_Q \sharp* \Psi_P;\\
\quad\quad\quad\quad \mathit{bn}\ \alpha \sharp* \Psi;\ \mathit{bn}\ \alpha \sharp* P;\ \mathit{bn}\ \alpha \sharp* Q;\ \mathit{bn}\ \alpha \sharp* \mathit{subject}\ \alpha;\ \mathit{bn}\ \alpha \sharp* \Psi_P;\ \mathit{bn}\ \alpha \sharp* \Psi_Q;\\
\quad\quad\quad\quad A_P \sharp* C;\ A_Q \sharp* C;\ \mathit{bn}\ \alpha \sharp* C[\!] \Longrightarrow\\
\quad\quad\quad\quad \mathit{Prop}\ C\ \Psi\ (P \parallel Q)\ \alpha\ (P \parallel Q')\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)\\
\textbf{and}\quad \mathit{rComm1}\colon \bigwedge \Psi\ \Psi_Q\ P\ M\ N\ P'\ A_P\ \Psi_P\ Q\ K\ \mathit{xvec}\ Q'\ A_Q\ C.\\
\quad\quad\quad\quad\quad [\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P';\ \mathit{extractFrame}\ P = \langle A_P,\ \Psi_P\rangle;\\
\mathit{distinct}\ A_P;\\
\quad\quad\quad\quad \bigwedge C.\ \mathit{Prop}\ C\ (\Psi \otimes \Psi_Q)\ P\ (M(\!|N|\!))\ P'\ A_P\ \Psi_P;\\
\quad\quad\quad\quad\quad \Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu{*}\mathit{xvec}|\!)\langle N\rangle \prec Q';\ \mathit{extractFrame}\ Q = \langle A_Q,\\
\Psi_Q\rangle;\ \mathit{distinct}\ A_Q;\\
\quad\quad\quad\quad \Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K;\ \mathit{distinct}\ \mathit{xvec};\\
\quad\quad\quad\quad \bigwedge C.\ \mathit{Prop}\ C\ (\Psi \otimes \Psi_P)\ Q\ (K(\!|\nu{*}\mathit{xvec}|\!)\langle N\rangle)\ Q'\ A_Q\ \Psi_Q;\\
\quad\quad\quad\quad A_P \sharp* \Psi;\ A_P \sharp* \Psi_Q;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';\\
\quad\quad\quad\quad A_P \sharp* Q;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* \mathit{xvec};\ A_Q \sharp* \Psi;\ A_Q \sharp* \Psi_P;\\
\quad\quad\quad\quad A_Q \sharp* P;\ A_Q \sharp* N;\ A_Q \sharp* P';\ A_Q \sharp* Q;\ A_Q \sharp* K;\ A_Q \sharp* Q';\\
\quad\quad\quad\quad A_Q \sharp* \mathit{xvec};\ \mathit{xvec} \sharp* \Psi;\ \mathit{xvec} \sharp* \Psi_P;\ \mathit{xvec} \sharp* \Psi_Q;\ \mathit{xvec} \sharp* P;\ \mathit{xvec} \sharp*\\
M;\\
\quad\quad\quad\quad\quad \mathit{xvec} \sharp* Q;\ \mathit{xvec} \sharp* K;\ A_P \sharp* C;\ A_Q \sharp* C;\ \mathit{xvec} \sharp* C[\!] \Longrightarrow\\
\quad\quad\quad\quad \mathit{Prop}\ C\ \Psi\ (P \parallel Q)\ (\tau)\ ((\!|\nu{*}\mathit{xvec}|\!)(P' \parallel Q'))\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)\\
\textbf{and}\quad \mathit{rComm2}\colon \bigwedge \Psi\ \Psi_Q\ P\ M\ \mathit{xvec}\ N\ P'\ A_P\ \Psi_P\ Q\ K\ Q'\ A_Q\ C.\\
\quad\quad\quad\quad\quad [\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu{*}\mathit{xvec}|\!)\langle N\rangle \prec P';\ \mathit{extractFrame}\ P = \langle A_P,\\
\Psi_P\rangle;\ \mathit{distinct}\ A_P;\\
\quad\quad\quad\quad \bigwedge C.\ \mathit{Prop}\ C\ (\Psi \otimes \Psi_Q)\ P\ (M(\!|\nu{*}\mathit{xvec}|\!)\langle N\rangle)\ P'\ A_P\ \Psi_P;\\
\quad\quad\quad\quad\quad \Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q';\ \mathit{extractFrame}\ Q = \langle A_Q,\ \Psi_Q\rangle;\\
\mathit{distinct}\ A_Q;\\
\quad\quad\quad\quad \bigwedge C.\ \mathit{Prop}\ C\ (\Psi \otimes \Psi_P)\ Q\ (K(\!|N|\!))\ Q'\ A_Q\ \Psi_Q;\\
\quad\quad\quad\quad \Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K;\ \mathit{distinct}\ \mathit{xvec};\\
\quad\quad\quad\quad A_P \sharp* \Psi;\ A_P \sharp* \Psi_Q;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';\\
\quad\quad\quad\quad A_P \sharp* Q;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* \mathit{xvec};\ A_Q \sharp* \Psi;\ A_Q \sharp* \Psi_P;\\
\quad\quad\quad\quad A_Q \sharp* P;\ A_Q \sharp* N;\ A_Q \sharp* P';\ A_Q \sharp* Q;\ A_Q \sharp* K;\ A_Q \sharp* Q';\\
\quad\quad\quad\quad A_Q \sharp* \mathit{xvec};\ \mathit{xvec} \sharp* \Psi;\ \mathit{xvec} \sharp* \Psi_P;\ \mathit{xvec} \sharp* \Psi_Q;\ \mathit{xvec} \sharp* P;\ \mathit{xvec} \sharp*
\end{array}
$$

$M$;
$xvec \sharp* Q$; $xvec \sharp* K$; $A_P \sharp* C$; $A_Q \sharp* C$; $xvec \sharp* C[\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ (\tau)\ ((\!|\nu{*}xvec|\!)(P' \parallel Q'))\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

**and** *rBrMerge*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ N\ P'\ A_P\ \Psi_P\ Q\ Q'\ A_Q\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¿M(\!|N|\!) \prec P';\ \bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)\ P\ (¿M(\!|N|\!))\ P'\ A_P\ \Psi_P;$
$extractFrame\ P = \langle A_P, \Psi_P \rangle$; $distinct\ A_P$;
$\Psi \otimes \Psi_P \rhd Q \longmapsto ¿M(\!|N|\!) \prec Q';\ \bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_P)\ Q\ (¿M(\!|N|\!))\ Q'\ A_Q\ \Psi_Q;$
$extractFrame\ Q = \langle A_Q, \Psi_Q \rangle$; $distinct\ A_Q$;
$A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* N$; $A_P \sharp* P'$;
$A_P \sharp* Q$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* M$; $A_Q \sharp* M$;
$A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$; $A_Q \sharp* P$; $A_Q \sharp* N$; $A_Q \sharp* P'$;
$A_Q \sharp* Q$; $A_Q \sharp* Q'$; $A_P \sharp* C$; $A_Q \sharp* C[\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ (¿M(\!|N|\!))\ (P' \parallel Q')\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

**and** *rBrComm1*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ N\ P'\ A_P\ \Psi_P\ Q\ xvec\ Q'\ A_Q\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¿M(\!|N|\!) \prec P';\ extractFrame\ P = \langle A_P, \Psi_P \rangle;$
$distinct\ A_P;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)\ P\ (¿M(\!|N|\!))\ P'\ A_P\ \Psi_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto ¡M(\!|\nu{*}xvec|\!)\langle N \rangle \prec Q';\ extractFrame\ Q = \langle A_Q, \Psi_Q \rangle;$ $distinct\ A_Q$;
$distinct\ xvec$;
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_P)\ Q\ (¡M(\!|\nu{*}xvec|\!)\langle N \rangle)\ Q'\ A_Q\ \Psi_Q;$
$A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* N$; $A_P \sharp* P'$;
$A_P \sharp* Q$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* xvec$; $A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$;
$A_Q \sharp* P$; $A_Q \sharp* N$; $A_Q \sharp* P'$; $A_Q \sharp* Q$; $A_Q \sharp* Q'$;
$A_Q \sharp* xvec$; $xvec \sharp* \Psi$; $xvec \sharp* \Psi_P$; $xvec \sharp* \Psi_Q$; $xvec \sharp* P$;
$xvec \sharp* Q$; $A_P \sharp* C$; $A_Q \sharp* C$; $xvec \sharp* C$;
$A_P \sharp* M$; $A_Q \sharp* M$; $xvec \sharp* M[\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ (¡M(\!|\nu{*}xvec|\!)\langle N \rangle)\ (P' \parallel Q')\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

**and** *rBrComm2*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ xvec\ N\ P'\ A_P\ \Psi_P\ Q\ Q'\ A_Q\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¡M(\!|\nu{*}xvec|\!)\langle N \rangle \prec P';\ extractFrame\ P = \langle A_P, \Psi_P \rangle;\ distinct\ A_P;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)\ P\ (¡M(\!|\nu{*}xvec|\!)\langle N \rangle)\ P'\ A_P\ \Psi_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto ¿M(\!|N|\!) \prec Q';\ extractFrame\ Q = \langle A_Q, \Psi_Q \rangle;$ $distinct\ A_Q$;
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_P)\ Q\ (¿M(\!|N|\!))\ Q'\ A_Q\ \Psi_Q;$
$distinct\ xvec$;
$A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* N$; $A_P \sharp* P'$;
$A_P \sharp* Q$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* xvec$; $A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$;
$A_Q \sharp* P$; $A_Q \sharp* N$; $A_Q \sharp* P'$; $A_Q \sharp* Q$; $A_Q \sharp* Q'$;
$A_Q \sharp* xvec$; $xvec \sharp* \Psi$; $xvec \sharp* \Psi_P$; $xvec \sharp* \Psi_Q$; $xvec \sharp* P$;
$xvec \sharp* Q$; $A_P \sharp* C$; $A_Q \sharp* C$; $xvec \sharp* C$;
$A_P \sharp* M$; $A_Q \sharp* M$; $xvec \sharp* M[\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ (¡M(\!|\nu{*}xvec|\!)\langle N \rangle)\ (P' \parallel Q')\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

**and** *rBrClose*: $\bigwedge \Psi\ P\ M\ xvec\ N\ P'\ A_P\ \Psi_P\ x\ C.$
$[\![\Psi \rhd P \longmapsto ¡M(\!|\nu{*}xvec|\!)\langle N \rangle \prec P';$

$x \in supp\ M$;
$\bigwedge C.\ Prop\ C\ \Psi\ P\ (¡M(\!|\nu{*}xvec|\!)\langle N\rangle)\ P'\ A_P\ \Psi_P$;
$extractFrame\ P = \langle A_P, \Psi_P\rangle$; $distinct\ A_P$;
$A_P \sharp* \Psi$; $A_P \sharp* P$; $A_P \sharp* M$; $A_P \sharp* N$; $A_P \sharp* P'$; $A_P \sharp* xvec$;
$distinct\ xvec$; $xvec \sharp* \Psi$; $xvec \sharp* \Psi_P$; $xvec \sharp* P$;
$xvec \sharp* M$;
$x \sharp \Psi$; $x \sharp xvec$; $x \sharp A_P$;
$A_P \sharp* C$; $xvec \sharp* C$; $x \sharp C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ ((\!|\nu x|\!)P)\ (\tau)\ ((\!|\nu x|\!)(\!|\nu{*}xvec|\!)P'))\ (x\#A_P)\ \Psi_P$

**and** *rOpen*: $\bigwedge \Psi\ P\ M\ xvec\ yvec\ N\ P'\ x\ A_P\ \Psi_P\ y\ C.$
$[\![\Psi \rhd P \longmapsto M(\!|\nu{*}(xvec@yvec)|\!)\langle N\rangle \prec P'$; $extractFrame\ P = \langle A_P, \Psi_P\rangle$; $distinct\ A_P$;
$\bigwedge C.\ Prop\ C\ \Psi\ P\ (M(\!|\nu{*}(xvec@yvec)|\!)\langle N\rangle)\ P'\ A_P\ \Psi_P$; $x \in supp\ N$; $x \sharp \Psi$; $x \sharp M$;
$x \sharp A_P$; $x \sharp xvec$; $x \sharp yvec$; $A_P \sharp* \Psi$; $A_P \sharp* P$; $A_P \sharp* M$; $A_P \sharp* N$; $A_P \sharp* P'$;
$A_P \sharp* xvec$; $A_P \sharp* yvec$; $xvec \sharp* yvec$; $distinct\ xvec$; $distinct\ yvec$;
$xvec \sharp* \Psi$; $xvec \sharp* P$; $xvec \sharp* M$; $xvec \sharp* \Psi_P$;
$yvec \sharp* \Psi$; $yvec \sharp* P$; $yvec \sharp* M$; $A_P \sharp* C$; $x \sharp C$; $xvec \sharp* C$; $yvec \sharp* C$;
$y \neq x$; $y \sharp \Psi$; $y \sharp P$; $y \sharp M$; $y \sharp xvec$; $y \sharp yvec$; $y \sharp N$; $y \sharp P'$; $y \sharp A_P$; $y \sharp \Psi_P$; $y \sharp C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ ((\!|\nu x|\!)P)\ (M(\!|\nu{*}(xvec@y\#yvec)|\!)\langle([(x,\ y)] \cdot N)\rangle)\ ([(x,\ y)] \cdot P')\ (x\#A_P)\ \Psi_P$

**and** *rBrOpen*: $\bigwedge \Psi\ P\ M\ xvec\ yvec\ N\ P'\ x\ A_P\ \Psi_P\ y\ C.$
$[\![\Psi \rhd P \longmapsto ¡M(\!|\nu{*}(xvec@yvec)|\!)\langle N\rangle \prec P'$; $extractFrame\ P = \langle A_P, \Psi_P\rangle$; $distinct\ A_P$;
$\bigwedge C.\ Prop\ C\ \Psi\ P\ (¡M(\!|\nu{*}(xvec@yvec)|\!)\langle N\rangle)\ P'\ A_P\ \Psi_P$; $x \in supp\ N$; $x \sharp \Psi$; $x \sharp M$;
$x \sharp A_P$; $x \sharp xvec$; $x \sharp yvec$; $A_P \sharp* \Psi$; $A_P \sharp* P$; $A_P \sharp* M$; $A_P \sharp* N$; $A_P \sharp* P'$;
$A_P \sharp* xvec$; $A_P \sharp* yvec$; $xvec \sharp* yvec$; $distinct\ xvec$; $distinct\ yvec$;
$xvec \sharp* \Psi$; $xvec \sharp* P$; $xvec \sharp* M$; $xvec \sharp* \Psi_P$;
$yvec \sharp* \Psi$; $yvec \sharp* P$; $yvec \sharp* M$; $A_P \sharp* C$; $x \sharp C$; $xvec \sharp* C$; $yvec \sharp* C$;
$y \neq x$; $y \sharp \Psi$; $y \sharp P$; $y \sharp M$; $y \sharp xvec$; $y \sharp yvec$; $y \sharp N$; $y \sharp P'$; $y \sharp A_P$; $y \sharp \Psi_P$; $y \sharp C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ ((\!|\nu x|\!)P)\ (¡M(\!|\nu{*}(xvec@y\#yvec)|\!)\langle([(x,\ y)] \cdot N)\rangle)\ ([(x,\ y)] \cdot P')\ (x\#A_P)\ \Psi_P$

**and** *rScope*: $\bigwedge \Psi\ P\ \alpha\ P'\ x\ A_P\ \Psi_P\ C.$
$[\![\Psi \rhd P \longmapsto \alpha \prec P'$; $extractFrame\ P = \langle A_P, \Psi_P\rangle$; $distinct\ A_P$;
$\bigwedge C.\ Prop\ C\ \Psi\ P\ \alpha\ P'\ A_P\ \Psi_P$;
$x \sharp \Psi$; $x \sharp \alpha$; $x \sharp A_P$; $A_P \sharp* \Psi$; $A_P \sharp* P$;
$A_P \sharp* \alpha$; $A_P \sharp* P'$;
$bn\ \alpha \sharp* \Psi$; $bn\ \alpha \sharp* P$; $bn\ \alpha \sharp* subject\ \alpha$; $bn\ \alpha \sharp* \Psi_P$;
$A_P \sharp* C$; $x \sharp C$; $bn\ \alpha \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ ((\!|\nu x|\!)P)\ \alpha\ ((\!|\nu x|\!)P')\ (x\#A_P)\ \Psi_P$

**and** *rBang*: $\bigwedge \Psi\ P\ \alpha\ P'\ A_P\ \Psi_P\ C.$
$[\![\Psi \rhd P \parallel\ !P \longmapsto \alpha \prec P'$; $guarded\ P$; $extractFrame\ P = \langle A_P, \Psi_P\rangle$;

*distinct* $A_P$;
$\bigwedge C.\ Prop\ C\ \Psi\ (P \parallel\ !P)\ \alpha\ P'\ A_P\ (\Psi_P \otimes \mathbf{1});\ \Psi_P \simeq \mathbf{1};\ supp\ \Psi_P = (\{\}::name\ set);$
$A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* \alpha;\ A_P \sharp* P';\ A_P \sharp* C] \Longrightarrow Prop\ C\ \Psi\ (!P)\ \alpha\ P'\ ([])\ (\mathbf{1})$
**shows** *Prop* $C\ \Psi\ P\ \alpha\ P'\ A_P\ \Psi_P$
⟨*proof*⟩

**lemma** *inputFrameInduct*[*consumes 3*, *case-names cAlpha cInput cCase cPar1 cPar2 cScope cBang*]:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $M$ :: $'a$
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)\ psi$
**and** *Prop* :: $'f::fs\text{-}name \Rightarrow 'b \Rightarrow ('a, 'b, 'c)\ psi \Rightarrow 'a \Rightarrow 'a \Rightarrow ('a, 'b, 'c)\ psi \Rightarrow name\ list \Rightarrow 'b \Rightarrow bool$
**and** $C$ :: $'f::fs\text{-}name$

**assumes** *Trans*: $\Psi \rhd P \longmapsto M(\!|N|\!) \prec P'$
**and** *FrP*: $extractFrame\ P = \langle A_P, \Psi_P\rangle$
**and** $distinct\ A_P$
**and** *rAlpha*: $\bigwedge \Psi\ P\ M\ N\ P'\ A_P\ \Psi_P\ p\ C.\ [\![A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';\ A_P \sharp* (p \cdot A_P);\ A_P \sharp* C;$
$set\ p \subseteq set\ A_P \times set(p \cdot A_P);\ distinctPerm\ p;$
$Prop\ C\ \Psi\ P\ M\ N\ P'\ A_P\ \Psi_P] \Longrightarrow Prop\ C\ \Psi\ P\ M\ N\ P'\ (p \cdot A_P)\ (p \cdot \Psi_P)$
**and** *rInput*: $\bigwedge \Psi\ M\ K\ xvec\ N\ Tvec\ P\ C.$
$[\![\Psi \vdash M \leftrightarrow K;\ distinct\ xvec;\ set\ xvec \subseteq supp\ N;$
$length\ xvec = length\ Tvec;\ xvec \sharp* \Psi;$
$xvec \sharp* M;\ xvec \sharp* K;\ xvec \sharp* C] \Longrightarrow$
$Prop\ C\ \Psi\ (M(\!|\lambda* xvec\ N|\!).P)$
$K\ (N[xvec::=Tvec])\ (P[xvec::=Tvec])\ ([])\ (\mathbf{1})$
**and** *rCase*: $\bigwedge \Psi\ P\ M\ N\ P'\ \varphi\ Cs\ A_P\ \Psi_P\ C.\ [\![\Psi \rhd P \longmapsto M(\!|N|\!) \prec P';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;\ \bigwedge C.\ Prop\ C\ \Psi\ P\ M\ N\ P'\ A_P\ \Psi_P;$
$(\varphi, P) \in set\ Cs;\ \Psi \vdash \varphi;\ guarded\ P;\ \Psi_P \simeq \mathbf{1};\ (supp\ \Psi_P) = (\{\}::name\ set);$
$A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';\ A_P \sharp* C] \Longrightarrow Prop\ C\ \Psi\ (Cases\ Cs)\ M\ N\ P'\ ([])\ (\mathbf{1})$
**and** *rPar1*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ N\ P'\ A_Q\ Q\ A_P\ \Psi_P\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P';$
$extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)\ P\ M\ N\ P'\ A_P\ \Psi_P;$
$A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* \Psi;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';\ A_P \sharp* A_Q;\ A_P \sharp* \Psi_Q;$
$A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* \Psi;\ A_Q \sharp* M;\ A_Q \sharp* N;\ A_Q \sharp* P';\ A_Q \sharp* \Psi_P;$
$A_P \sharp* C;\ A_Q \sharp* C] \Longrightarrow$

$Prop\ C\ \Psi\ (P \parallel Q)\ M\ N\ (P' \parallel Q)\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$
**and** *rPar2*: $\bigwedge \Psi\ \Psi_P\ Q\ M\ N\ Q'\ A_P\ P\ A_Q\ \Psi_Q\ C.$
$[\![\Psi \otimes \Psi_P \rhd Q \longmapsto M(\!|N|\!) \prec Q';$
$extractFrame\ P = \langle A_P, \Psi_P \rangle;\ distinct\ A_P;$
$extractFrame\ Q = \langle A_Q, \Psi_Q \rangle;\ distinct\ A_Q;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_P)\ Q\ M\ N\ Q'\ A_Q\ \Psi_Q;$
$A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* \Psi;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* \Psi_Q;$
$A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* \Psi;\ A_Q \sharp* M;\ A_Q \sharp* N;\ A_Q \sharp* Q';\ A_Q \sharp* \Psi_P;$
$A_P \sharp* C;\ A_Q \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ M\ N\ (P \parallel Q')\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$
**and** *rScope*: $\bigwedge \Psi\ P\ M\ N\ P'\ x\ A_P\ \Psi_P\ C.$
$[\![\Psi \rhd P \longmapsto M(\!|N|\!) \prec P';\ extractFrame\ P = \langle A_P, \Psi_P \rangle;\ distinct\ A_P;$
$\bigwedge C.\ Prop\ C\ \Psi\ P\ M\ N\ P'\ A_P\ \Psi_P;\ x \sharp \Psi;\ x \sharp M;\ x \sharp N;$
$x \sharp A_P;\ A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';$
$A_P \sharp* C;\ x \sharp C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ ((\!|\nu x|\!)P)\ M\ N\ ((\!|\nu x|\!)P')\ (x \# A_P)\ \Psi_P$
**and** *rBang*: $\bigwedge \Psi\ P\ M\ N\ P'\ A_P\ \Psi_P\ C.$
$[\![\Psi \rhd P \parallel !P \longmapsto M(\!|N|\!) \prec P';\ guarded\ P;\ extractFrame\ P = \langle A_P, \Psi_P \rangle;\ distinct\ A_P;$
$\bigwedge C.\ Prop\ C\ \Psi\ (P \parallel !P)\ M\ N\ P'\ A_P\ (\Psi_P \otimes \mathbf{1});\ \Psi_P \simeq \mathbf{1};\ (supp\ \Psi_P) = (\{\}::name\ set);$
$A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';\ A_P \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (!P)\ M\ N\ P'\ ([])\ (\mathbf{1})$
**shows** $Prop\ C\ \Psi\ P\ M\ N\ P'\ A_P\ \Psi_P$
⟨*proof*⟩

**lemma** *brinputFrameInduct*[*consumes 3*, *case-names cAlpha cBrInput cCase cPar1 cPar2 cBrMerge cScope cBang*]:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $M$ :: $'a$
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)\ psi$
**and** $Prop$ :: $'f::fs\text{-}name \Rightarrow 'b \Rightarrow ('a, 'b, 'c)\ psi \Rightarrow$
$'a \Rightarrow 'a \Rightarrow ('a, 'b, 'c)\ psi \Rightarrow name\ list \Rightarrow 'b \Rightarrow bool$
**and** $C$ :: $'f::fs\text{-}name$

**assumes** *Trans*: $\Psi \rhd P \longmapsto ¿ M(\!|N|\!) \prec P'$
**and** *FrP*: $extractFrame\ P = \langle A_P, \Psi_P \rangle$
**and** $distinct\ A_P$
**and** *rAlpha*: $\bigwedge \Psi\ P\ M\ N\ P'\ A_P\ \Psi_P\ p\ C.\ [\![A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';\ A_P \sharp* (p \cdot A_P);\ A_P \sharp* C;$
$set\ p \subseteq set\ A_P \times set(p \cdot A_P);\ distinctPerm\ p;$
$Prop\ C\ \Psi\ P\ M\ N\ P'\ A_P\ \Psi_P]\!] \Longrightarrow Prop\ C\ \Psi\ P\ M\ N\ P'\ (p \cdot A_P)\ (p \cdot \Psi_P)$
**and** *rBrInput*: $\bigwedge \Psi\ M\ K\ xvec\ N\ Tvec\ P\ C.$
$[\![\Psi \vdash K \succeq M;\ distinct\ xvec;\ set\ xvec \subseteq supp\ N;$

$$\begin{array}{l}
\quad \mathit{length}\ \mathit{xvec} = \mathit{length}\ \mathit{Tvec};\ \mathit{xvec} \sharp* \Psi;\\
\quad \mathit{xvec} \sharp* M;\ \mathit{xvec} \sharp* K;\ \mathit{xvec} \sharp* C\rrbracket \Longrightarrow\\
\quad \mathit{Prop}\ C\ \Psi\ (M\langle\!|\lambda*\mathit{xvec}\ N|\!\rangle.P)\\
\qquad\qquad K\ (N[\mathit{xvec}::=\mathit{Tvec}])\ (P[\mathit{xvec}::=\mathit{Tvec}])\ ([])\ (\mathbf{1})
\end{array}$$

**and** *rCase*: $\bigwedge \Psi\ P\ M\ N\ P'\ \varphi\ \mathit{Cs}\ A_P\ \Psi_P\ C.\ [\![\Psi \rhd P \longmapsto ¿M\langle\!|N|\!\rangle \prec P';\ \mathit{extractFrame}\ P = \langle A_P,\ \Psi_P\rangle;\ \mathit{distinct}\ A_P;\ \bigwedge C.\ \mathit{Prop}\ C\ \Psi\ P\ M\ N\ P'\ A_P\ \Psi_P;$
$(\varphi,\ P) \in \mathit{set}\ \mathit{Cs};\ \Psi \vdash \varphi;\ \mathit{guarded}\ P;\ \Psi_P \simeq \mathbf{1};\ (\mathit{supp}\ \Psi_P) = (\{\}::\mathit{name\ set});$
$A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';\ A_P \sharp* C]\!] \Longrightarrow \mathit{Prop}\ C\ \Psi\ (\mathit{Cases}\ \mathit{Cs})\ M\ N\ P'\ ([])\ (\mathbf{1})$

**and** *rPar1*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ N\ P'\ A_Q\ Q\ A_P\ \Psi_P\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¿M\langle\!|N|\!\rangle \prec P';$
$\mathit{extractFrame}\ P = \langle A_P,\ \Psi_P\rangle;\ \mathit{distinct}\ A_P;$
$\mathit{extractFrame}\ Q = \langle A_Q,\ \Psi_Q\rangle;\ \mathit{distinct}\ A_Q;$
$\bigwedge C.\ \mathit{Prop}\ C\ (\Psi \otimes \Psi_Q)\ P\ M\ N\ P'\ A_P\ \Psi_P;$
$A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* \Psi;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';\ A_P \sharp* A_Q;\ A_P \sharp* \Psi_Q;$
$A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* \Psi;\ A_Q \sharp* M;\ A_Q \sharp* N;\ A_Q \sharp* P';\ A_Q \sharp* \Psi_P;$
$A_P \sharp* C;\ A_Q \sharp* C]\!] \Longrightarrow$
$\mathit{Prop}\ C\ \Psi\ (P \parallel Q)\ M\ N\ (P' \parallel Q)\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

**and** *rPar2*: $\bigwedge \Psi\ \Psi_P\ Q\ M\ N\ Q'\ A_P\ P\ A_Q\ \Psi_Q\ C.$
$[\![\Psi \otimes \Psi_P \rhd Q \longmapsto ¿M\langle\!|N|\!\rangle \prec Q';$
$\mathit{extractFrame}\ P = \langle A_P,\ \Psi_P\rangle;\ \mathit{distinct}\ A_P;$
$\mathit{extractFrame}\ Q = \langle A_Q,\ \Psi_Q\rangle;\ \mathit{distinct}\ A_Q;$
$\bigwedge C.\ \mathit{Prop}\ C\ (\Psi \otimes \Psi_P)\ Q\ M\ N\ Q'\ A_Q\ \Psi_Q;$
$A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* \Psi;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* \Psi_Q;$
$A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* \Psi;\ A_Q \sharp* M;\ A_Q \sharp* N;\ A_Q \sharp* Q';\ A_Q \sharp* \Psi_P;$
$A_P \sharp* C;\ A_Q \sharp* C]\!] \Longrightarrow$
$\mathit{Prop}\ C\ \Psi\ (P \parallel Q)\ M\ N\ (P \parallel Q')\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

**and** *rBrMerge*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ N\ P'\ A_P\ \Psi_P\ Q\ Q'\ A_Q\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¿M\langle\!|N|\!\rangle \prec P';\ \bigwedge C.\ \mathit{Prop}\ C\ (\Psi \otimes \Psi_Q)\ P\ M\ N\ P'\ A_P\ \Psi_P;$
$\mathit{extractFrame}\ P = \langle A_P,\ \Psi_P\rangle;\ \mathit{distinct}\ A_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto ¿M\langle\!|N|\!\rangle \prec Q';\ \bigwedge C.\ \mathit{Prop}\ C\ (\Psi \otimes \Psi_P)\ Q\ M\ N\ Q'\ A_Q\ \Psi_Q;$
$\mathit{extractFrame}\ Q = \langle A_Q,\ \Psi_Q\rangle;\ \mathit{distinct}\ A_Q;$
$A_P \sharp* \Psi;\ A_P \sharp* \Psi_Q;\ A_P \sharp* P;\ A_P \sharp* N;\ A_P \sharp* P';$
$A_P \sharp* Q;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* M;\ A_Q \sharp* M;$
$A_Q \sharp* \Psi;\ A_Q \sharp* \Psi_P;\ A_Q \sharp* P;\ A_Q \sharp* N;\ A_Q \sharp* P';$
$A_Q \sharp* Q;\ A_Q \sharp* Q';\ A_P \sharp* C;\ A_Q \sharp* C]\!] \Longrightarrow$
$\mathit{Prop}\ C\ \Psi\ (P \parallel Q)\ M\ N\ (P' \parallel Q')\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

**and** *rScope*: $\bigwedge \Psi\ P\ M\ N\ P'\ x\ A_P\ \Psi_P\ C.$
$[\![\Psi \rhd P \longmapsto ¿M\langle\!|N|\!\rangle \prec P';\ \mathit{extractFrame}\ P = \langle A_P,\ \Psi_P\rangle;\ \mathit{distinct}\ A_P;$
$\bigwedge C.\ \mathit{Prop}\ C\ \Psi\ P\ M\ N\ P'\ A_P\ \Psi_P;\ x \sharp \Psi;\ x \sharp M;\ x \sharp N;$
$x \sharp A_P;\ A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';$

$A_P \sharp* C$; $x \sharp C]\!] \Longrightarrow$
*Prop* $C\ \Psi\ (([\!|\nu x|\!])P)\ M\ N\ (([\!|\nu x|\!])P')\ (x\#A_P)\ \Psi_P$

**and** *rBang*: $\bigwedge \Psi\ P\ M\ N\ P'\ A_P\ \Psi_P\ C.$
$[\![\Psi \rhd P \parallel !P \longmapsto ¿M([\!|N|\!]) \prec P'$; *guarded* $P$; *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* $A_P$;
$\bigwedge C.$ *Prop* $C\ \Psi\ (P \parallel !P)\ M\ N\ P'\ A_P\ (\Psi_P \otimes \mathbf{1})$; $\Psi_P \simeq \mathbf{1}$; (*supp* $\Psi_P$) = ({}::*name set*);
$A_P \sharp* \Psi$; $A_P \sharp* P$; $A_P \sharp* M$; $A_P \sharp* N$; $A_P \sharp* P'$; $A_P \sharp* C]\!] \Longrightarrow$ *Prop* $C\ \Psi\ (!P)\ M\ N\ P'\ ([])\ (\mathbf{1})$

**shows** *Prop* $C\ \Psi\ P\ M\ N\ P'\ A_P\ \Psi_P$

⟨*proof*⟩

**lemma** *outputFrameInduct*[*consumes 3*, *case-names cAlpha cOutput cCase cPar1 cPar2 cOpen cScope cBang*]:

**fixes** $\Psi$ :: *'b*
**and** $P$ :: (*'a*, *'b*, *'c*) *psi*
**and** $M$ :: *'a*
**and** $B$ :: (*'a*, *'b*, *'c*) *boundOutput*
**and** $A_P$ :: *name list*
**and** $\Psi_P$ :: *'b*
**and** *Prop* :: *'f*::*fs-name* ⇒ *'b* ⇒ (*'a*, *'b*, *'c*) *psi* ⇒
*'a* ⇒ (*'a*, *'b*, *'c*) *boundOutput* ⇒ *name list* ⇒ *'b* ⇒ *bool*
**and** $C$ :: *'f*::*fs-name*

**assumes** *Trans*: $\Psi \rhd P \longmapsto ROut\ M\ B$
**and** *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$
**and** *distinct* $A_P$
**and** *rAlpha*: $\bigwedge \Psi\ P\ M\ A_P\ \Psi_P\ p\ B\ C.\ [\![A_P \sharp* \Psi$; $A_P \sharp* P$; $A_P \sharp* M$; $A_P \sharp* (p \cdot A_P)$; $A_P \sharp* B$; $A_P \sharp* C$;
*set* $p \subseteq$ *set* $A_P \times$ *set*$(p \cdot A_P)$; *distinctPerm* $p$;
*Prop* $C\ \Psi\ P\ M\ B\ A_P\ \Psi_P]\!] \Longrightarrow$ *Prop* $C\ \Psi\ P\ M\ B\ (p \cdot A_P)\ (p \cdot \Psi_P)$
**and** *rOutput*: $\bigwedge \Psi\ M\ K\ N\ P\ C.\ \Psi \vdash M \leftrightarrow K \Longrightarrow$ *Prop* $C\ \Psi\ (M\langle N\rangle.P)\ K\ (N \prec' P)\ ([])\ (\mathbf{1})$
**and** *rCase*: $\bigwedge \Psi\ P\ M\ B\ \varphi\ Cs\ A_P\ \Psi_P\ C.\ [\![\Psi \rhd P \longmapsto (ROut\ M\ B)$; *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* $A_P$; $\bigwedge C.$ *Prop* $C\ \Psi\ P\ M\ B\ A_P\ \Psi_P$;
$(\varphi, P) \in$ *set* $Cs$; $\Psi \vdash \varphi$; *guarded* $P$; $\Psi_P \simeq \mathbf{1}$; (*supp* $\Psi_P$) = ({}::*name set*);
$A_P \sharp* \Psi$; $A_P \sharp* P$; $A_P \sharp* M$; $A_P \sharp* B$; $A_P \sharp* C]\!] \Longrightarrow$ *Prop* $C\ \Psi$ (*Cases* $Cs$) $M\ B\ ([])\ (\mathbf{1})$
**and** *rPar1*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ xvec\ N\ P'\ A_Q\ Q\ A_P\ \Psi_P\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M([\!|\nu * xvec|\!])\langle N\rangle \prec P'$;
*extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* $A_P$;
*extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* $A_Q$;
$\bigwedge C.$ *Prop* $C\ (\Psi \otimes \Psi_Q)\ P\ M\ (([\!|\nu * xvec|\!])N \prec' P')\ A_P\ \Psi_P$;
$A_P \sharp* P$; $A_P \sharp* Q$; $A_P \sharp* \Psi$; $A_P \sharp* M$; $A_P \sharp* xvec$; $A_P \sharp* N$; $A_P \sharp* P'$; $A_P \sharp* A_Q$; $A_P \sharp* \Psi_Q$;
$A_Q \sharp* P$; $A_Q \sharp* Q$; $A_Q \sharp* \Psi$; $A_Q \sharp* M$; $A_Q \sharp* xvec$; $A_Q \sharp* N$; $A_Q \sharp* P'$; $A_Q \sharp* \Psi_P$;

$xvec \sharp* \Psi$; $xvec \sharp* P$; $xvec \sharp* Q$; $xvec \sharp* M$; $xvec \sharp* \Psi_P$; $xvec \sharp* \Psi_Q$;
$A_P \sharp* C$; $A_Q \sharp* C$; $xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ M\ ((\!|\nu * xvec|\!) N \prec' (P' \parallel Q))\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

**and** *rPar2*: $\bigwedge \Psi\ \Psi_P\ Q\ M\ xvec\ N\ Q'\ A_P\ P\ A_Q\ \Psi_Q\ C.$
$[\![\Psi \otimes \Psi_P \rhd Q \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec Q'$;
$extractFrame\ P = \langle A_P, \Psi_P\rangle$; $distinct\ A_P$;
$extractFrame\ Q = \langle A_Q, \Psi_Q\rangle$; $distinct\ A_Q$;
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_P)\ Q\ M\ ((\!|\nu * xvec|\!) N \prec' Q')\ A_Q\ \Psi_Q$;
$A_P \sharp* P$; $A_P \sharp* Q$; $A_P \sharp* \Psi$; $A_P \sharp* M$; $A_P \sharp* xvec$; $A_P \sharp* N$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* \Psi_Q$;
$A_Q \sharp* P$; $A_Q \sharp* Q$; $A_Q \sharp* \Psi$; $A_Q \sharp* M$; $A_Q \sharp* xvec$; $A_Q \sharp* N$; $A_Q \sharp* Q'$; $A_Q \sharp* \Psi_P$;
$xvec \sharp* \Psi$; $xvec \sharp* P$; $xvec \sharp* Q$; $xvec \sharp* M$; $xvec \sharp* \Psi_P$; $xvec \sharp* \Psi_Q$;
$A_P \sharp* C$; $A_Q \sharp* C$; $xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ M\ ((\!|\nu * xvec|\!) N \prec' (P \parallel Q'))\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

**and** *rOpen*: $\bigwedge \Psi\ P\ M\ xvec\ yvec\ N\ P'\ x\ A_P\ \Psi_P\ C.$
$[\![\Psi \rhd P \longmapsto M(\!|\nu * (xvec @ yvec)|\!)\langle N\rangle \prec P'$; $extractFrame\ P = \langle A_P, \Psi_P\rangle$; $distinct\ A_P$;
$\bigwedge C.\ Prop\ C\ \Psi\ P\ M\ ((\!|\nu * (xvec @ yvec)|\!) N \prec' P')\ A_P\ \Psi_P$; $x \in supp\ N$; $x \sharp \Psi$; $x \sharp M$;
$x \sharp A_P$; $x \sharp xvec$; $x \sharp yvec$; $A_P \sharp* \Psi$; $A_P \sharp* P$; $A_P \sharp* M$; $A_P \sharp* N$; $A_P \sharp* P'$;
$A_P \sharp* xvec$; $A_P \sharp* yvec$;
$xvec \sharp* \Psi$; $xvec \sharp* P$; $xvec \sharp* M$; $xvec \sharp* \Psi_P$;
$yvec \sharp* \Psi$; $yvec \sharp* P$; $yvec \sharp* M$; $A_P \sharp* C$; $x \sharp C$; $xvec \sharp* C$; $yvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ ((\!|\nu x|\!) P)\ M\ ((\!|\nu * (xvec @ x \# yvec)|\!) N \prec' P')\ (x \# A_P)\ \Psi_P$

**and** *rScope*: $\bigwedge \Psi\ P\ M\ xvec\ N\ P'\ x\ A_P\ \Psi_P\ C.$
$[\![\Psi \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$; $extractFrame\ P = \langle A_P, \Psi_P\rangle$; $distinct\ A_P$;
$\bigwedge C.\ Prop\ C\ \Psi\ P\ M\ ((\!|\nu * xvec|\!) N \prec' P')\ A_P\ \Psi_P$;
$x \sharp \Psi$; $x \sharp M$; $x \sharp xvec$; $x \sharp N$; $x \sharp A_P$; $A_P \sharp* \Psi$; $A_P \sharp* P$;
$A_P \sharp* M$; $A_P \sharp* N$; $A_P \sharp* P'$; $A_P \sharp* xvec$;
$xvec \sharp* \Psi$; $xvec \sharp* P$; $xvec \sharp* M$; $xvec \sharp* \Psi_P$;
$A_P \sharp* C$; $x \sharp C$; $xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ ((\!|\nu x|\!) P)\ M\ ((\!|\nu * xvec|\!) N \prec' ((\!|\nu x|\!) P'))\ (x \# A_P)\ \Psi_P$

**and** *rBang*: $\bigwedge \Psi\ P\ M\ B\ A_P\ \Psi_P\ C.$
$[\![\Psi \rhd P \parallel !P \longmapsto ROut\ M\ B$; $guarded\ P$; $extractFrame\ P = \langle A_P, \Psi_P\rangle$; $distinct\ A_P$;
$\bigwedge C.\ Prop\ C\ \Psi\ (P \parallel !P)\ M\ B\ A_P\ (\Psi_P \otimes \mathbf{1})$; $\Psi_P \simeq \mathbf{1}$; $supp\ \Psi_P = (\{\}::name\ set)$;
$A_P \sharp* \Psi$; $A_P \sharp* P$; $A_P \sharp* M$; $A_P \sharp* C]\!] \Longrightarrow Prop\ C\ \Psi\ (!P)\ M\ B\ ([])\ (\mathbf{1})$

**shows** $Prop\ C\ \Psi\ P\ M\ B\ A_P\ \Psi_P$
⟨*proof*⟩

**lemma** *broutputFrameInduct*[*consumes 3*, *case-names cAlpha cBrOutput cCase*

*cPar1 cPar2 cBrComm1 cBrComm2 cBrOpen cScope cBang*]:
**fixes** $\Psi$ :: *'b*
**and** $P$ :: (*'a*, *'b*, *'c*) *psi*
**and** $M$ :: *'a*
**and** $B$ :: (*'a*, *'b*, *'c*) *boundOutput*
**and** $A_P$ :: *name list*
**and** $\Psi_P$ :: *'b*
**and** *Prop* :: *'f*::*fs-name* $\Rightarrow$ *'b* $\Rightarrow$ (*'a*, *'b*, *'c*) *psi* $\Rightarrow$
*'a* $\Rightarrow$ (*'a*, *'b*, *'c*) *boundOutput* $\Rightarrow$ *name list* $\Rightarrow$ *'b* $\Rightarrow$ *bool*
**and** $C$ :: *'f*::*fs-name*

**assumes** *Trans*: $\Psi \rhd P \longmapsto RBrOut\ M\ B$
**and** *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$
**and** *distinct* $A_P$
**and** *rAlpha*: $\bigwedge \Psi\ P\ M\ A_P\ \Psi_P\ p\ B\ C.\ [\![A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* (p \cdot A_P);\ A_P \sharp* B;\ A_P \sharp* C;$
$set\ p \subseteq set\ A_P \times set(p \cdot A_P);\ distinctPerm\ p;$
$Prop\ C\ \Psi\ P\ M\ B\ A_P\ \Psi_P]\!] \Longrightarrow Prop\ C\ \Psi\ P\ M\ B\ (p \cdot A_P)\ (p \cdot \Psi_P)$
**and** *rBrOutput*: $\bigwedge \Psi\ M\ K\ N\ P\ C.\ \Psi \vdash M \preceq K \Longrightarrow Prop\ C\ \Psi\ (M\langle N\rangle.P)\ K\ (N \prec' P)\ ([])\ (\mathbf{1})$
**and** *rCase*: $\bigwedge \Psi\ P\ M\ B\ \varphi\ Cs\ A_P\ \Psi_P\ C.\ [\![\Psi \rhd P \longmapsto (RBrOut\ M\ B);\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;\ \bigwedge C.\ Prop\ C\ \Psi\ P\ M\ B\ A_P\ \Psi_P;$
$(\varphi, P) \in set\ Cs;\ \Psi \vdash \varphi;\ guarded\ P;\ \Psi_P \simeq \mathbf{1};\ (supp\ \Psi_P) = (\{\}::name\ set);$
$A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* B;\ A_P \sharp* C]\!] \Longrightarrow Prop\ C\ \Psi\ (Cases\ Cs)\ M\ B\ ([])\ (\mathbf{1})$
**and** *rPar1*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ xvec\ N\ P'\ A_Q\ Q\ A_P\ \Psi_P\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¡M(\!|\nu * xvec|\!)\langle N\rangle \prec P';$
$extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)\ P\ M\ ((\!|\nu * xvec|\!)N \prec' P')\ A_P\ \Psi_P;$
$A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* \Psi;\ A_P \sharp* M;\ A_P \sharp* xvec;\ A_P \sharp* N;\ A_P \sharp* P';\ A_P \sharp* A_Q;\ A_P \sharp* \Psi_Q;$
$A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* \Psi;\ A_Q \sharp* M;\ A_Q \sharp* xvec;\ A_Q \sharp* N;\ A_Q \sharp* P';\ A_Q \sharp* \Psi_P;$
$xvec \sharp* \Psi;\ xvec \sharp* P;\ xvec \sharp* Q;\ xvec \sharp* M;\ xvec \sharp* \Psi_P;\ xvec \sharp* \Psi_Q;$
$A_P \sharp* C;\ A_Q \sharp* C;\ xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ M\ ((\!|\nu * xvec|\!)N \prec' (P' \parallel Q))\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$
**and** *rPar2*: $\bigwedge \Psi\ \Psi_P\ Q\ M\ xvec\ N\ Q'\ A_P\ P\ A_Q\ \Psi_Q\ C.$
$[\![\Psi \otimes \Psi_P \rhd Q \longmapsto ¡M(\!|\nu * xvec|\!)\langle N\rangle \prec Q';$
$extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_P)\ Q\ M\ ((\!|\nu * xvec|\!)N \prec' Q')\ A_Q\ \Psi_Q;$
$A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* \Psi;\ A_P \sharp* M;\ A_P \sharp* xvec;\ A_P \sharp* N;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* \Psi_Q;$
$A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* \Psi;\ A_Q \sharp* M;\ A_Q \sharp* xvec;\ A_Q \sharp* N;\ A_Q \sharp* Q';\ A_Q \sharp* \Psi_P;$

$xvec \sharp* \Psi$; $xvec \sharp* P$; $xvec \sharp* Q$; $xvec \sharp* M$; $xvec \sharp* \Psi_P$; $xvec \sharp* \Psi_Q$;
$A_P \sharp* C$; $A_Q \sharp* C$; $xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ M\ ((\!|\nu* xvec|\!)N \prec' (P \parallel Q'))\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

**and** *rBrComm1*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ N\ P'\ A_P\ \Psi_P\ Q\ xvec\ Q'\ A_Q\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¡M(\!|N|\!) \prec P'$; $extractFrame\ P = \langle A_P, \Psi_P \rangle$; $distinct\ A_P$;
$\Psi \otimes \Psi_P \rhd Q \longmapsto ¡M(\!|\nu* xvec|\!)\langle N \rangle \prec Q'$; $extractFrame\ Q = \langle A_Q, \Psi_Q \rangle$; $distinct\ A_Q$;
$distinct\ xvec$;
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_P)\ Q\ M\ ((\!|\nu* xvec|\!)N \prec' Q')\ A_Q\ \Psi_Q$;
$A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* N$; $A_P \sharp* P'$;
$A_P \sharp* Q$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* xvec$; $A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$;
$A_Q \sharp* P$; $A_Q \sharp* N$; $A_Q \sharp* P'$; $A_Q \sharp* Q$; $A_Q \sharp* Q'$;
$A_Q \sharp* xvec$; $xvec \sharp* \Psi$; $xvec \sharp* \Psi_P$; $xvec \sharp* \Psi_Q$; $xvec \sharp* P$;
$xvec \sharp* Q$; $A_P \sharp* C$; $A_Q \sharp* C$; $xvec \sharp* C$;
$A_P \sharp* M$; $A_Q \sharp* M$; $xvec \sharp* M]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ M\ ((\!|\nu* xvec|\!)N \prec' (P' \parallel Q'))\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

**and** *rBrComm2*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ xvec\ N\ P'\ A_P\ \Psi_P\ Q\ Q'\ A_Q\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¡M(\!|\nu* xvec|\!)\langle N \rangle \prec P'$; $extractFrame\ P = \langle A_P, \Psi_P \rangle$; $distinct\ A_P$;
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)\ P\ M\ ((\!|\nu* xvec|\!)N \prec' P')\ A_P\ \Psi_P$;
$\Psi \otimes \Psi_P \rhd Q \longmapsto ¡M(\!|N|\!) \prec Q'$; $extractFrame\ Q = \langle A_Q, \Psi_Q \rangle$; $distinct\ A_Q$;
$distinct\ xvec$;
$A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* N$; $A_P \sharp* P'$;
$A_P \sharp* Q$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* xvec$; $A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$;
$A_Q \sharp* P$; $A_Q \sharp* N$; $A_Q \sharp* P'$; $A_Q \sharp* Q$; $A_Q \sharp* Q'$;
$A_Q \sharp* xvec$; $xvec \sharp* \Psi$; $xvec \sharp* \Psi_P$; $xvec \sharp* \Psi_Q$; $xvec \sharp* P$;
$xvec \sharp* Q$; $A_P \sharp* C$; $A_Q \sharp* C$; $xvec \sharp* C$;
$A_P \sharp* M$; $A_Q \sharp* M$; $xvec \sharp* M]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ M\ ((\!|\nu* xvec|\!)N \prec' (P' \parallel Q'))\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

**and** *rBrOpen*: $\bigwedge \Psi\ P\ M\ xvec\ yvec\ N\ P'\ x\ A_P\ \Psi_P\ C.$
$[\![\Psi \rhd P \longmapsto ¡M(\!|\nu* (xvec @ yvec)|\!)\langle N \rangle \prec P'$; $extractFrame\ P = \langle A_P, \Psi_P \rangle$; $distinct\ A_P$;
$\bigwedge C.\ Prop\ C\ \Psi\ P\ M\ ((\!|\nu* (xvec @ yvec)|\!)N \prec' P')\ A_P\ \Psi_P$; $x \in supp\ N$; $x \sharp \Psi$; $x \sharp M$;
$x \sharp A_P$; $x \sharp xvec$; $x \sharp yvec$; $A_P \sharp* \Psi$; $A_P \sharp* P$; $A_P \sharp* M$; $A_P \sharp* N$; $A_P \sharp* P'$;
$A_P \sharp* xvec$; $A_P \sharp* yvec$;
$xvec \sharp* \Psi$; $xvec \sharp* P$; $xvec \sharp* M$; $xvec \sharp* \Psi_P$;
$yvec \sharp* \Psi$; $yvec \sharp* P$; $yvec \sharp* M$; $A_P \sharp* C$; $x \sharp C$; $xvec \sharp* C$; $yvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ ((\!|\nu x|\!)P)\ M\ ((\!|\nu* (xvec @ x \# yvec)|\!)N \prec' P')\ (x \# A_P)\ \Psi_P$

**and** *rScope*: $\bigwedge \Psi\ P\ M\ xvec\ N\ P'\ x\ A_P\ \Psi_P\ C.$
$[\![\Psi \rhd P \longmapsto ¡M(\!|\nu* xvec|\!)\langle N \rangle \prec P'$; $extractFrame\ P = \langle A_P, \Psi_P \rangle$; $distinct\ A_P$;

$\bigwedge C.\ Prop\ C\ \Psi\ P\ M\ ((\!|\nu{*}xvec|\!)N \prec' P')\ A_P\ \Psi_P;$
$x \sharp \Psi;\ x \sharp M;\ x \sharp xvec;\ x \sharp N;\ x \sharp A_P;\ A_P \sharp{*} \Psi;\ A_P \sharp{*} P;$
$A_P \sharp{*} M;\ A_P \sharp{*} N;\ A_P \sharp{*} P';\ A_P \sharp{*} xvec;$
$xvec \sharp{*} \Psi;\ xvec \sharp{*} P;\ xvec \sharp{*} M;\ xvec \sharp{*} \Psi_P;$
$A_P \sharp{*} C;\ x \sharp C;\ xvec \sharp{*} C[\!] \Longrightarrow$
$Prop\ C\ \Psi\ ((\!|\nu x|\!)P)\ M\ ((\!|\nu{*}xvec|\!)N \prec' ((\!|\nu x|\!)P'))\ (x\#A_P)\ \Psi_P$

**and** *rBang*: $\bigwedge \Psi\ P\ M\ B\ A_P\ \Psi_P\ C.$
$[\![\Psi \rhd P \parallel\ !P \longmapsto RBrOut\ M\ B;\ guarded\ P;\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$\bigwedge C.\ Prop\ C\ \Psi\ (P \parallel\ !P)\ M\ B\ A_P\ (\Psi_P \otimes \mathbf{1});\ \Psi_P \simeq \mathbf{1};\ supp\ \Psi_P = (\{\}::name\ set);$
$A_P \sharp{*} \Psi;\ A_P \sharp{*} P;\ A_P \sharp{*} M;\ A_P \sharp{*} C[\!] \Longrightarrow Prop\ C\ \Psi\ (!P)\ M\ B\ ([\!])\ (\mathbf{1})$

**shows** $Prop\ C\ \Psi\ P\ M\ B\ A_P\ \Psi_P$

⟨*proof*⟩

**lemma** *tauFrameInduct*[*consumes 3*, *case-names cAlpha cCase cPar1 cPar2 cComm1 cComm2 cBrClose cScope cBang*]:

**fixes** $\Psi$ :: *'b*
**and** $P$ :: *('a, 'b, 'c) psi*
**and** $P'$ :: *('a, 'b, 'c) psi*
**and** *Prop* :: *'f::fs-name* ⇒ *'b* ⇒ *('a, 'b, 'c) psi* ⇒ *('a, 'b, 'c) psi* ⇒ *name list* ⇒ *'b* ⇒ *bool*
**and** $C$ :: *'f::fs-name*

**assumes** *Trans*: $\Psi \rhd P \longmapsto \tau \prec P'$
**and** *FrP*: $extractFrame\ P = \langle A_P, \Psi_P\rangle$
**and** $distinct\ A_P$
**and** *rAlpha*: $\bigwedge \Psi\ P\ P'\ A_P\ \Psi_P\ p\ C.\ [\![A_P \sharp{*} \Psi;\ A_P \sharp{*} P;\ A_P \sharp{*} P';\ A_P \sharp{*} (p \cdot A_P);\ A_P \sharp{*} C;$
$set\ p \subseteq set\ A_P \times set\ (p \cdot A_P);\ distinctPerm\ p;$
$Prop\ C\ \Psi\ P\ P'\ A_P\ \Psi_P[\!] \Longrightarrow Prop\ C\ \Psi\ P\ P'\ (p \cdot A_P)\ (p \cdot \Psi_P)$
**and** *rCase*: $\bigwedge \Psi\ P\ P'\ \varphi\ Cs\ A_P\ \Psi_P\ C.\ [\![\Psi \rhd P \longmapsto \tau \prec P';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;\ \bigwedge C.\ Prop\ C\ \Psi\ P\ P'\ A_P\ \Psi_P;$
$(\varphi, P) \in set\ Cs;\ \Psi \vdash \varphi;\ guarded\ P;\ \Psi_P \simeq \mathbf{1};\ (supp\ \Psi_P) = (\{\}::name\ set);$
$A_P \sharp{*} \Psi;\ A_P \sharp{*} P;\ A_P \sharp{*} P';\ A_P \sharp{*} C[\!] \Longrightarrow Prop\ C\ \Psi\ (Cases\ Cs)\ P'\ ([\!])\ (\mathbf{1})$
**and** *rPar1*: $\bigwedge \Psi\ \Psi_Q\ P\ P'\ A_Q\ Q\ A_P\ \Psi_P\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto \tau \prec P';$
$extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)\ P\ P'\ A_P\ \Psi_P;$
$A_P \sharp{*} P;\ A_P \sharp{*} Q;\ A_P \sharp{*} \Psi;\ A_P \sharp{*} P';\ A_P \sharp{*} A_Q;\ A_P \sharp{*} \Psi_Q;$
$A_Q \sharp{*} P;\ A_Q \sharp{*} Q;\ A_Q \sharp{*} \Psi;\ A_Q \sharp{*} P';\ A_Q \sharp{*} \Psi_P;$
$A_P \sharp{*} C;\ A_Q \sharp{*} C[\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ (P' \parallel Q)\ (A_P@A_Q)\ (\Psi_P \otimes \Psi_Q)$
**and** *rPar2*: $\bigwedge \Psi\ \Psi_P\ Q\ Q'\ A_P\ P\ A_Q\ \Psi_Q\ C.$

$[\![\Psi \otimes \Psi_P \rhd Q \longmapsto\!\!\!\!\rightarrow \tau \prec Q';$
$\mathit{extractFrame}\ P = \langle A_P, \Psi_P\rangle;\ \mathit{distinct}\ A_P;$
$\mathit{extractFrame}\ Q = \langle A_Q, \Psi_Q\rangle;\ \mathit{distinct}\ A_Q;$
$\bigwedge C.\ \mathit{Prop}\ C\ (\Psi \otimes \Psi_P)\ Q\ Q'\ A_Q\ \Psi_Q;$
$A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* \Psi;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* \Psi_Q;$
$A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* \Psi;\ A_Q \sharp* Q';\ A_Q \sharp* \Psi_P;$
$A_P \sharp* C;\ A_Q \sharp* C]\!] \Longrightarrow$
$\mathit{Prop}\ C\ \Psi\ (P \parallel Q)\ (P \parallel Q')\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

**and** *rComm1*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ N\ P'\ A_P\ \Psi_P\ Q\ K\ \mathit{xvec}\ Q'\ A_Q\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P';\ \mathit{extractFrame}\ P = \langle A_P, \Psi_P\rangle;$
$\mathit{distinct}\ A_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu * \mathit{xvec}|\!)\langle N\rangle \prec Q';\ \mathit{extractFrame}\ Q = \langle A_Q, \Psi_Q\rangle;\ \mathit{distinct}\ A_Q;$
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K;\ \mathit{distinct}\ \mathit{xvec};$
$A_P \sharp* \Psi;\ A_P \sharp* \Psi_Q;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';$
$A_P \sharp* Q;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* \mathit{xvec};\ A_Q \sharp* \Psi;\ A_Q \sharp* \Psi_P;$
$A_Q \sharp* P;\ A_Q \sharp* N;\ A_Q \sharp* P';\ A_Q \sharp* Q;\ A_Q \sharp* K;\ A_Q \sharp* Q';$
$A_Q \sharp* \mathit{xvec};\ \mathit{xvec} \sharp* \Psi;\ \mathit{xvec} \sharp* \Psi_P;\ \mathit{xvec} \sharp* \Psi_Q;\ \mathit{xvec} \sharp* P;\ \mathit{xvec} \sharp* M;$
$\mathit{xvec} \sharp* Q;\ \mathit{xvec} \sharp* K;\ A_P \sharp* C;\ A_Q \sharp* C;\ \mathit{xvec} \sharp* C]\!] \Longrightarrow$
$\mathit{Prop}\ C\ \Psi\ (P \parallel Q)\ ((\!|\nu * \mathit{xvec}|\!)(P' \parallel Q'))\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

**and** *rComm2*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ \mathit{xvec}\ N\ P'\ A_P\ \Psi_P\ Q\ K\ Q'\ A_Q\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu * \mathit{xvec}|\!)\langle N\rangle \prec P';\ \mathit{extractFrame}\ P = \langle A_P, \Psi_P\rangle;\ \mathit{distinct}\ A_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q';\ \mathit{extractFrame}\ Q = \langle A_Q, \Psi_Q\rangle;\ \mathit{distinct}\ A_Q;$
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K;\ \mathit{distinct}\ \mathit{xvec};$
$A_P \sharp* \Psi;\ A_P \sharp* \Psi_Q;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';$
$A_P \sharp* Q;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* \mathit{xvec};\ A_Q \sharp* \Psi;\ A_Q \sharp* \Psi_P;$
$A_Q \sharp* P;\ A_Q \sharp* N;\ A_Q \sharp* P';\ A_Q \sharp* Q;\ A_Q \sharp* K;\ A_Q \sharp* Q';$
$A_Q \sharp* \mathit{xvec};\ \mathit{xvec} \sharp* \Psi;\ \mathit{xvec} \sharp* \Psi_P;\ \mathit{xvec} \sharp* \Psi_Q;\ \mathit{xvec} \sharp* P;\ \mathit{xvec} \sharp* M;$
$\mathit{xvec} \sharp* Q;\ \mathit{xvec} \sharp* K;\ A_P \sharp* C;\ A_Q \sharp* C;\ \mathit{xvec} \sharp* C]\!] \Longrightarrow$
$\mathit{Prop}\ C\ \Psi\ (P \parallel Q)\ ((\!|\nu * \mathit{xvec}|\!)(P' \parallel Q'))\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

**and** *rBrClose*: $\bigwedge \Psi\ P\ M\ \mathit{xvec}\ N\ P'\ A_P\ \Psi_P\ x\ C.$
$[\![\Psi \rhd P \longmapsto \text{¡}M(\!|\nu * \mathit{xvec}|\!)\langle N\rangle \prec P';$
$x \in \mathit{supp}\ M;$
$\mathit{extractFrame}\ P = \langle A_P, \Psi_P\rangle;\ \mathit{distinct}\ A_P;$
$A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';\ A_P \sharp* \mathit{xvec};$
$\mathit{distinct}\ \mathit{xvec};\ \mathit{xvec} \sharp* \Psi;\ \mathit{xvec} \sharp* \Psi_P;\ \mathit{xvec} \sharp* P;$
$\mathit{xvec} \sharp* M;$
$x \sharp \Psi;\ x \sharp \mathit{xvec};\ x \sharp A_P;$
$A_P \sharp* C;\ \mathit{xvec} \sharp* C;\ x \sharp C]\!] \Longrightarrow$
$\mathit{Prop}\ C\ \Psi\ ((\!|\nu x|\!)P)\ ((\!|\nu x|\!)((\!|\nu * \mathit{xvec}|\!)P'))\ (x \# A_P)\ \Psi_P$

**and** *rScope*: $\bigwedge \Psi\ P\ P'\ x\ A_P\ \Psi_P\ C.$
$[\![\Psi \rhd P \longmapsto\!\!\!\!\rightarrow \tau \prec P';\ \mathit{extractFrame}\ P = \langle A_P, \Psi_P\rangle;\ \mathit{distinct}\ A_P;$
$\bigwedge C.\ \mathit{Prop}\ C\ \Psi\ P\ P'\ A_P\ \Psi_P;\ x \sharp \Psi;$
$x \sharp A_P;\ A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* P';$
$A_P \sharp* C;\ x \sharp C]\!] \Longrightarrow$

$$\mathit{Prop}\ C\ \Psi\ (\langle\!|\nu x|\!\rangle P)\ (\langle\!|\nu x|\!\rangle P')\ (x\#A_P)\ \Psi_P$$

**and** *rBang*: $\bigwedge \Psi\ P\ P'\ A_P\ \Psi_P\ C.$

$$[\![\Psi \rhd P \parallel\ !P \longmapsto \tau \prec P';\ \mathit{guarded}\ P;\ \mathit{extractFrame}\ P = \langle A_P, \Psi_P\rangle;$$

$\mathit{distinct}\ A_P;$

$$\bigwedge C.\ \mathit{Prop}\ C\ \Psi\ (P \parallel\ !P)\ P'\ A_P\ (\Psi_P \otimes \mathbf{1});\ \Psi_P \simeq \mathbf{1};\ \mathit{supp}\ \Psi_P =$$

$(\{\}::\mathit{name\ set});$

$$A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* P';\ A_P \sharp* C]\!] \Longrightarrow \mathit{Prop}\ C\ \Psi\ (!P)\ P'$$

$([])\ (\mathbf{1})$

**shows** $\mathit{Prop}\ C\ \Psi\ P\ P'\ A_P\ \Psi_P$

⟨*proof*⟩

**lemma** *inputFreshDerivative*:

**fixes** $\Psi :: 'b$

**and** $P :: ('a, 'b, 'c)\ psi$

**and** $M :: 'a$

**and** $N :: 'a$

**and** $P' :: ('a, 'b, 'c)\ psi$

**and** $x :: name$

**assumes** $\Psi \rhd P \longmapsto M(\!|N|\!) \prec P'$

**and** $x \sharp P$

**and** $x \sharp N$

**shows** $x \sharp P'$

⟨*proof*⟩

**lemma** *brinputFreshDerivative*:

**fixes** $\Psi :: 'b$

**and** $P :: ('a, 'b, 'c)\ psi$

**and** $M :: 'a$

**and** $N :: 'a$

**and** $P' :: ('a, 'b, 'c)\ psi$

**and** $x :: name$

**assumes** $\Psi \rhd P \longmapsto ¿M(\!|N|\!) \prec P'$

**and** $x \sharp P$

**and** $x \sharp N$

**shows** $x \sharp P'$

⟨*proof*⟩

**lemma** *inputFreshChainDerivative*:

**fixes** $\Psi :: 'b$

**and** $P :: ('a, 'b, 'c)\ psi$

**and** $M :: 'a$

**and** $N :: 'a$

**and** $P' :: ('a, 'b, 'c)\ psi$

**and** $xvec :: name\ list$

**assumes** $\Psi \rhd P \longmapsto M(\!|N|\!) \prec P'$
**and** $xvec \sharp* P$
**and** $xvec \sharp* N$

**shows** $xvec \sharp* P'$
⟨*proof*⟩

**lemma** *brinputFreshChainDerivative*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** *xvec* :: *name list*

**assumes** $\Psi \rhd P \longmapsto ¿M(\!|N|\!) \prec P'$
**and** $xvec \sharp* P$
**and** $xvec \sharp* N$

**shows** $xvec \sharp* P'$
⟨*proof*⟩

**lemma** *outputFreshDerivativeN*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** *xvec* :: *name list*
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $x$ :: *name*

**assumes** $\Psi \rhd P \longmapsto M(\!|\nu*xvec|\!)\langle N\rangle \prec P'$
**and** $xvec \sharp* M$
**and** *distinct xvec*
**and** $x \sharp P$
**and** $x \sharp xvec$

**shows** $x \sharp N$
⟨*proof*⟩

**lemma** *broutputFreshDerivativeN*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** *xvec* :: *name list*
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $x$ :: *name*

**assumes** $\Psi \rhd P \longmapsto ¡M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P'$
**and** $xvec \sharp* M$
**and** *distinct xvec*
**and** $x \sharp P$
**and** $x \sharp xvec$

**shows** $x \sharp N$
⟨*proof*⟩

**lemma** *outputFreshDerivativeP*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** *xvec* :: *name list*
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $x$ :: *name*

**assumes** $\Psi \rhd P \longmapsto M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P'$
**and** $xvec \sharp* M$
**and** *distinct xvec*
**and** $x \sharp P$
**and** $x \sharp xvec$

**shows** $x \sharp P'$
⟨*proof*⟩

**lemma** *broutputFreshDerivativeP*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** *xvec* :: *name list*
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $x$ :: *name*

**assumes** $\Psi \rhd P \longmapsto ¡M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P'$
**and** $xvec \sharp* M$
**and** *distinct xvec*
**and** $x \sharp P$
**and** $x \sharp xvec$

**shows** $x \sharp P'$
⟨*proof*⟩

**lemma** *outputFreshDerivative*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$

**and** $xvec :: name\ list$
**and** $N :: 'a$
**and** $P' :: ('a, 'b, 'c)\ psi$
**and** $x :: name$

**assumes** $\Psi \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
**and** $xvec \sharp* M$
**and** $distinct\ xvec$
**and** $x \sharp P$
**and** $x \sharp xvec$

**shows** $x \sharp N$
**and** $x \sharp P'$
⟨*proof*⟩

**lemma** *broutputFreshDerivative*:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $M :: 'a$
**and** $xvec :: name\ list$
**and** $N :: 'a$
**and** $P' :: ('a, 'b, 'c)\ psi$
**and** $x :: name$

**assumes** $\Psi \rhd P \longmapsto ¡M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
**and** $xvec \sharp* M$
**and** $distinct\ xvec$
**and** $x \sharp P$
**and** $x \sharp xvec$

**shows** $x \sharp N$
**and** $x \sharp P'$
⟨*proof*⟩

**lemma** *outputFreshChainDerivative*:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $M :: 'a$
**and** $xvec :: name\ list$
**and** $N :: 'a$
**and** $P' :: ('a, 'b, 'c)\ psi$
**and** $yvec :: name\ list$

**assumes** $\Psi \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
**and** $xvec \sharp* M$
**and** $distinct\ xvec$
**and** $yvec \sharp* P$
**and** $yvec \sharp* xvec$

**shows** *yvec* $\sharp*$ *N*
**and** *yvec* $\sharp*$ *P'*
⟨*proof*⟩

**lemma** *broutputFreshChainDerivative*:
**fixes** Ψ :: *'b*
**and** *P* :: (*'a*, *'b*, *'c*) *psi*
**and** *M* :: *'a*
**and** *xvec* :: *name list*
**and** *N* :: *'a*
**and** *P'* :: (*'a*, *'b*, *'c*) *psi*
**and** *yvec* :: *name list*

**assumes** Ψ ▷ *P* ⟼¡*M*⦇ν∗*xvec*⦈⟨*N*⟩ ≺ *P'*
**and** *xvec* $\sharp*$ *M*
**and** *distinct xvec*
**and** *yvec* $\sharp*$ *P*
**and** *yvec* $\sharp*$ *xvec*

**shows** *yvec* $\sharp*$ *N*
**and** *yvec* $\sharp*$ *P'*
⟨*proof*⟩

**lemma** *tauFreshDerivative*:
**fixes** Ψ :: *'b*
**and** *P* :: (*'a*, *'b*, *'c*) *psi*
**and** *P'* :: (*'a*, *'b*, *'c*) *psi*
**and** *x* :: *name*

**assumes** Ψ ▷ *P* ⟼τ ≺ *P'*
**and** *x* ♯ *P*

**shows** *x* ♯ *P'*
⟨*proof*⟩

**lemma** *tauFreshChainDerivative*:
**fixes** Ψ :: *'b*
**and** *P* :: (*'a*, *'b*, *'c*) *psi*
**and** *M* :: *'a*
**and** *N* :: *'a*
**and** *P'* :: (*'a*, *'b*, *'c*) *psi*
**and** *xvec* :: *name list*

**assumes** Ψ ▷ *P* ⟼τ ≺ *P'*
**and** *xvec* $\sharp*$ *P*

**shows** *xvec* $\sharp*$ *P'*
⟨*proof*⟩

**lemma** *freeFreshDerivative*:
  **fixes** $\Psi$ :: $'b$
    **and** $P$ :: $('a, 'b, 'c)$ *psi*
    **and** $\alpha$ :: $'a$ *action*
    **and** $P'$ :: $('a, 'b, 'c)$ *psi*
    **and** $x$ :: *name*

**assumes** $\Psi \rhd P \longmapsto \alpha \prec P'$
  **and** *bn* $\alpha \sharp* subject\ \alpha$
  **and** *distinct*(*bn* $\alpha$)
  **and** $x \sharp \alpha$
  **and** $x \sharp P$

**shows** $x \sharp P'$
  ⟨*proof*⟩

**lemma** *freeFreshChainDerivative*:
  **fixes** $\Psi$ :: $'b$
    **and** $P$ :: $('a, 'b, 'c)$ *psi*
    **and** $\alpha$ :: $'a$ *action*
    **and** $P'$ :: $('a, 'b, 'c)$ *psi*
    **and** *xvec* :: *name list*

**assumes** $\Psi \rhd P \longmapsto \alpha \prec P'$
  **and** *bn* $\alpha \sharp* subject\ \alpha$
  **and** *distinct*(*bn* $\alpha$)
  **and** *xvec* $\sharp* P$
  **and** *xvec* $\sharp* \alpha$

**shows** *xvec* $\sharp* P'$
  ⟨*proof*⟩

**lemma** *Input*:
  **fixes** $\Psi$ :: $'b$
    **and** $M$ :: $'a$
    **and** $K$ :: $'a$
    **and** *xvec* :: *name list*
    **and** $N$ :: $'a$
    **and** *Tvec* :: $'a$ *list*

**assumes** $\Psi \vdash M \leftrightarrow K$
  **and** *distinct xvec*
  **and** *set xvec* $\subseteq$ *supp N*
  **and** *length xvec* = *length Tvec*

**shows** $\Psi \rhd M(\!|\lambda* xvec\ N|\!).P \longmapsto K(\!|N[xvec::=Tvec]|\!) \prec P[xvec::=Tvec]$
⟨*proof*⟩

**lemma** *BrInput*:

**fixes** $\Psi$ :: $'b$
**and** $M$ :: $'a$
**and** $K$ :: $'a$
**and** $xvec$ :: $name\ list$
**and** $N$ :: $'a$
**and** $Tvec$ :: $'a\ list$

**assumes** $\Psi \vdash K \succeq M$
**and** $distinct\ xvec$
**and** $set\ xvec \subseteq supp\ N$
**and** $length\ xvec = length\ Tvec$

**shows** $\Psi \rhd M(\!|\lambda * xvec\ N|\!).P \longmapsto ¿K(\!|N[xvec::=Tvec]|\!) \prec P[xvec::=Tvec]$
⟨*proof*⟩

**lemma** *residualAlpha*:
**fixes** $p$ :: $name\ prm$
**and** $\alpha$ :: $'a\ action$
**and** $P$ :: $('a, 'b, 'c)\ psi$

**assumes** $bn(p \cdot \alpha) \sharp * object\ \alpha$
**and** $bn(p \cdot \alpha) \sharp * P$
**and** $bn\ \alpha \sharp * subject\ \alpha$
**and** $bn(p \cdot \alpha) \sharp * subject\ \alpha$
**and** $set\ p \subseteq set(bn\ \alpha) \times set(bn(p \cdot \alpha))$

**shows** $\alpha \prec P = (p \cdot \alpha) \prec (p \cdot P)$
⟨*proof*⟩

**lemma** *Par1*:
**fixes** $\Psi$ :: $'b$
**and** $\Psi_Q$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $\alpha$ :: $'a\ action$
**and** $P'$ :: $('a, 'b, 'c)\ psi$
**and** $A_Q$ :: $name\ list$
**and** $Q$ :: $('a, 'b, 'c)\ psi$

**assumes** *Trans*: $\Psi \otimes \Psi_Q \rhd P \longmapsto \alpha \prec P'$
**and** $extractFrame\ Q = \langle A_Q, \Psi_Q \rangle$
**and** $bn\ \alpha \sharp * Q$
**and** $A_Q \sharp * \Psi$
**and** $A_Q \sharp * P$
**and** $A_Q \sharp * \alpha$

**shows** $\Psi \rhd P \parallel Q \longmapsto \alpha \prec (P' \parallel Q)$
⟨*proof*⟩

**lemma** *Par2*:

**fixes** $\Psi$ :: $'b$
**and** $\Psi_P$ :: $'b$
**and** $Q$ :: $('a, 'b, 'c)$ *psi*
**and** $\alpha$ :: $'a$ *action*
**and** $Q'$ :: $('a, 'b, 'c)$ *psi*
**and** $A_P$ :: *name list*
**and** $P$ :: $('a, 'b, 'c)$ *psi*

**assumes** *Trans*: $\Psi \otimes \Psi_P \rhd Q \longmapsto \alpha \prec Q'$
**and** *extractFrame* $P = \langle A_P, \Psi_P \rangle$
**and** *bn* $\alpha \sharp* P$
**and** $A_P \sharp* \Psi$
**and** $A_P \sharp* Q$
**and** $A_P \sharp* \alpha$

**shows** $\Psi \rhd P \parallel Q \longmapsto \alpha \prec (P \parallel Q')$
⟨*proof*⟩

**lemma** *Open*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** *xvec* :: *name list*
**and** *yvec* :: *name list*
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $x$ :: *name*

**assumes** *Trans*: $\Psi \rhd P \longmapsto M(\!|\nu*(xvec@yvec)|\!)\langle N \rangle \prec P'$
**and** $x \in supp\ N$
**and** $x \sharp \Psi$
**and** $x \sharp M$
**and** $x \sharp xvec$
**and** $x \sharp yvec$

**shows** $\Psi \rhd (\!|\nu x|\!)P \longmapsto M(\!|\nu*(xvec@x\#yvec)|\!)\langle N \rangle \prec P'$
⟨*proof*⟩

**lemma** *BrOpen*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** *xvec* :: *name list*
**and** *yvec* :: *name list*
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $x$ :: *name*

**assumes** *Trans*: $\Psi \rhd P \longmapsto ¡M(\!|\nu*(xvec@yvec)|\!)\langle N \rangle \prec P'$

**and** $x \in supp\ N$
**and** $x \sharp \Psi$
**and** $x \sharp M$
**and** $x \sharp xvec$
**and** $x \sharp yvec$

**shows** $\Psi \rhd (\!|\nu x|\!)P \longmapsto ¡M(\!|\nu*(xvec@x\#yvec)|\!)\langle N\rangle \prec P'$
⟨*proof*⟩

**lemma** *Scope*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $\alpha$ :: $'a\ action$
**and** $P'$ :: $('a, 'b, 'c)\ psi$
**and** $x$ :: *name*

**assumes** $\Psi \rhd P \longmapsto \alpha \prec P'$
**and** $x \sharp \Psi$
**and** $x \sharp \alpha$

**shows** $\Psi \rhd (\!|\nu x|\!)P \longmapsto \alpha \prec (\!|\nu x|\!)P'$
⟨*proof*⟩

**lemma** *inputSwapFrameSubject*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $M$ :: $'a$
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)\ psi$
**and** $x$ :: *name*
**and** $y$ :: *name*

**assumes** $\Psi \rhd P \longmapsto M(\!|N|\!) \prec P'$
**and** $x \sharp P$
**and** $y \sharp P$

**shows** $([(x, y)] \cdot \Psi) \rhd P \longmapsto ([(x, y)] \cdot M)(\!|N|\!) \prec P'$
⟨*proof*⟩

**lemma** *brinputSwapFrameSubject*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $M$ :: $'a$
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)\ psi$
**and** $x$ :: *name*
**and** $y$ :: *name*

**assumes** $\Psi \rhd P \longmapsto ¿M(\!|N|\!) \prec P'$

**and** $x \sharp P$
**and** $y \sharp P$

**shows** $([(x, y)] \cdot \Psi) \rhd P \longmapsto ¿([(x, y)] \cdot M)(\!|N|\!) \prec P'$
⟨*proof*⟩

**lemma** *inputPermFrameSubject*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $p$ :: *name prm*
**and** $Xs$ :: *name set*
**and** $Ys$ :: *name set*

**assumes** $\Psi \rhd P \longmapsto M(\!|N|\!) \prec P'$
**and** $S$: *set* $p \subseteq Xs \times Ys$
**and** $Xs \sharp* P$
**and** $Ys \sharp* P$

**shows** $(p \cdot \Psi) \rhd P \longmapsto (p \cdot M)(\!|N|\!) \prec P'$
⟨*proof*⟩

**lemma** *brinputPermFrameSubject*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $p$ :: *name prm*
**and** $Xs$ :: *name set*
**and** $Ys$ :: *name set*

**assumes** $\Psi \rhd P \longmapsto ¿M(\!|N|\!) \prec P'$
**and** $S$: *set* $p \subseteq Xs \times Ys$
**and** $Xs \sharp* P$
**and** $Ys \sharp* P$

**shows** $(p \cdot \Psi) \rhd P \longmapsto ¿(p \cdot M)(\!|N|\!) \prec P'$
⟨*proof*⟩

**lemma** *inputSwapSubject*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $x$ :: *name*

**and** $y$ :: *name*

**assumes** $\Psi \rhd P \longmapsto M(\!|N|\!) \prec P'$
**and** $x \sharp P$
**and** $y \sharp P$
**and** $x \sharp \Psi$
**and** $y \sharp \Psi$

**shows** $\Psi \rhd P \longmapsto ([(x, y)] \cdot M)(\!|N|\!) \prec P'$
⟨*proof*⟩

**lemma** *brinputSwapSubject*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $x$ :: *name*
**and** $y$ :: *name*

**assumes** $\Psi \rhd P \longmapsto ¿M(\!|N|\!) \prec P'$
**and** $x \sharp P$
**and** $y \sharp P$
**and** $x \sharp \Psi$
**and** $y \sharp \Psi$

**shows** $\Psi \rhd P \longmapsto ¿([(x, y)] \cdot M)(\!|N|\!) \prec P'$
⟨*proof*⟩

**lemma** *inputPermSubject*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $p$ :: *name prm*
**and** $Xs$ :: *name set*
**and** $Ys$ :: *name set*

**assumes** $\Psi \rhd P \longmapsto M(\!|N|\!) \prec P'$
**and** $S$: *set* $p \subseteq Xs \times Ys$
**and** $Xs \sharp* P$
**and** $Ys \sharp* P$
**and** $Xs \sharp* \Psi$
**and** $Ys \sharp* \Psi$

**shows** $\Psi \rhd P \longmapsto (p \cdot M)(\!|N|\!) \prec P'$
⟨*proof*⟩

**lemma** *brinputPermSubject*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $p$ :: *name prm*
**and** $Xs$ :: *name set*
**and** $Ys$ :: *name set*

**assumes** $\Psi \rhd P \longmapsto ¿M(\!|N|\!) \prec P'$
**and** $S$: *set* $p \subseteq Xs \times Ys$
**and** $Xs \sharp* P$
**and** $Ys \sharp* P$
**and** $Xs \sharp* \Psi$
**and** $Ys \sharp* \Psi$

**shows** $\Psi \rhd P \longmapsto ¿(p \cdot M)(\!|N|\!) \prec P'$
⟨*proof*⟩

**lemma** *inputSwapFrame*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $x$ :: *name*
**and** $y$ :: *name*

**assumes** $\Psi \rhd P \longmapsto M(\!|N|\!) \prec P'$
**and** $x \sharp P$
**and** $y \sharp P$
**and** $x \sharp M$
**and** $y \sharp M$

**shows** $([(x, y)] \cdot \Psi) \rhd P \longmapsto M(\!|N|\!) \prec P'$
⟨*proof*⟩

**lemma** *brinputSwapFrame*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $x$ :: *name*
**and** $y$ :: *name*

**assumes** $\Psi \rhd P \longmapsto ¿M(\!|N|\!) \prec P'$
**and** $x \sharp P$

**and** $y \sharp P$
**and** $x \sharp M$
**and** $y \sharp M$

**shows** $([(x, y)] \cdot \Psi) \rhd P \longmapsto ¿M(\!|N|\!) \prec P'$
⟨*proof*⟩

**lemma** *inputPermFrame*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $p$ :: *name prm*
**and** *Xs* :: *name set*
**and** *Ys* :: *name set*

**assumes** $\Psi \rhd P \longmapsto M(\!|N|\!) \prec P'$
**and** *S*: *set* $p \subseteq Xs \times Ys$
**and** $Xs \sharp* P$
**and** $Ys \sharp* P$
**and** $Xs \sharp* M$
**and** $Ys \sharp* M$

**shows** $(p \cdot \Psi) \rhd P \longmapsto M(\!|N|\!) \prec P'$
⟨*proof*⟩

**lemma** *brinputPermFrame*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $p$ :: *name prm*
**and** *Xs* :: *name set*
**and** *Ys* :: *name set*

**assumes** $\Psi \rhd P \longmapsto ¿M(\!|N|\!) \prec P'$
**and** *S*: *set* $p \subseteq Xs \times Ys$
**and** $Xs \sharp* P$
**and** $Ys \sharp* P$
**and** $Xs \sharp* M$
**and** $Ys \sharp* M$

**shows** $(p \cdot \Psi) \rhd P \longmapsto ¿M(\!|N|\!) \prec P'$
⟨*proof*⟩

**lemma** *inputAlpha*:
**fixes** $\Psi$ :: $'b$

**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $p$ :: *name prm*
**and** *xvec* :: *name list*

**assumes** $\Psi \rhd P \longmapsto M(\!|N|\!) \prec P'$
**and** *set* $p \subseteq$ (*set xvec*) $\times$ (*set* $(p \cdot xvec)$)
**and** *distinctPerm* $p$
**and** *xvec* $\sharp*$ $P$
**and** $(p \cdot xvec) \sharp* P$

**shows** $\Psi \rhd P \longmapsto M(\!|(p \cdot N)|\!) \prec (p \cdot P')$
⟨*proof*⟩

**lemma** *brinputAlpha*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $p$ :: *name prm*
**and** *xvec* :: *name list*

**assumes** $\Psi \rhd P \longmapsto ¿M(\!|N|\!) \prec P'$
**and** *set* $p \subseteq$ (*set xvec*) $\times$ (*set* $(p \cdot xvec)$)
**and** *distinctPerm* $p$
**and** *xvec* $\sharp*$ $P$
**and** $(p \cdot xvec) \sharp* P$

**shows** $\Psi \rhd P \longmapsto ¿M(\!|(p \cdot N)|\!) \prec (p \cdot P')$
⟨*proof*⟩

**lemma** *frameFresh*[*dest*]:
**fixes** $x$ :: *name*
**and** $A_F$ :: *name list*
**and** $\Psi_F$ :: $'b$

**assumes** $x \sharp A_F$
**and** $x \sharp \langle A_F, \Psi_F \rangle$

**shows** $x \sharp \Psi_F$
⟨*proof*⟩

**lemma** *outputSwapFrameSubject*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$

**and** $xvec :: name\ list$
**and** $N :: 'a$
**and** $x :: name$
**and** $y :: name$

**assumes** $\Psi \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
**and** $xvec \sharp* M$
**and** $x \sharp P$
**and** $y \sharp P$

**shows** $([(x, y)] \cdot \Psi) \rhd P \longmapsto([(x, y)] \cdot M)(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
$\langle proof\rangle$

**lemma** *broutputSwapFrameSubject*:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $M :: 'a$
**and** $xvec :: name\ list$
**and** $N :: 'a$
**and** $x :: name$
**and** $y :: name$

**assumes** $\Psi \rhd P \longmapsto ¡M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
**and** $xvec \sharp* M$
**and** $x \sharp P$
**and** $y \sharp P$

**shows** $([(x, y)] \cdot \Psi) \rhd P \longmapsto ¡([(x, y)] \cdot M)(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
$\langle proof\rangle$

**lemma** *outputPermFrameSubject*:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $M :: 'a$
**and** $xvec :: name\ list$
**and** $N :: 'a$
**and** $P' :: ('a, 'b, 'c)\ psi$
**and** $p :: name\ prm$
**and** $yvec :: name\ list$
**and** $zvec :: name\ list$

**assumes** $\Psi \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
**and** $S$: $set\ p \subseteq set\ yvec \times set\ zvec$
**and** $yvec \sharp* P$
**and** $zvec \sharp* P$

**shows** $(p \cdot \Psi) \rhd P \longmapsto(p \cdot M)(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
$\langle proof\rangle$

**lemma** *broutputPermFrameSubject*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** *xvec* :: *name list*
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $p$ :: *name prm*
**and** *yvec* :: *name list*
**and** *zvec* :: *name list*

**assumes** $\Psi \rhd P \longmapsto ¡ M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
**and** $S$: *set* $p \subseteq$ *set yvec* $\times$ *set zvec*
**and** *yvec* $\sharp*$ $P$
**and** *zvec* $\sharp*$ $P$

**shows** $(p \cdot \Psi) \rhd P \longmapsto ¡ (p \cdot M)(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
⟨*proof*⟩

**lemma** *outputSwapSubject*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** $B$ :: $('a, 'b, 'c)$ *boundOutput*
**and** $x$ :: *name*
**and** $y$ :: *name*

**assumes** $\Psi \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
**and** *xvec* $\sharp*$ $M$
**and** $x \sharp P$
**and** $y \sharp P$
**and** $x \sharp \Psi$
**and** $y \sharp \Psi$

**shows** $\Psi \rhd P \longmapsto ([(x, y)] \cdot M)(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
⟨*proof*⟩

**lemma** *broutputSwapSubject*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** $B$ :: $('a, 'b, 'c)$ *boundOutput*
**and** $x$ :: *name*
**and** $y$ :: *name*

**assumes** $\Psi \rhd P \longmapsto ¡ M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
**and** *xvec* $\sharp*$ $M$
**and** $x \sharp P$
**and** $y \sharp P$

**and** $x \sharp \Psi$
**and** $y \sharp \Psi$

**shows** $\Psi \rhd P \longmapsto ¡([(x,\ y)] \cdot M)(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
⟨*proof*⟩

**lemma** *outputPermSubject*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a,\ 'b,\ 'c)$ *psi*
**and** $M$ :: $'a$
**and** $B$ :: $('a,\ 'b,\ 'c)$ *boundOutput*
**and** $p$ :: *name prm*
**and** *yvec* :: *name list*
**and** *zvec* :: *name list*

**assumes** $\Psi \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
**and** $S$: *set p* $\subseteq$ *set yvec* $\times$ *set zvec*
**and** *yvec* $\sharp*$ $P$
**and** *zvec* $\sharp*$ $P$
**and** *yvec* $\sharp*$ $\Psi$
**and** *zvec* $\sharp*$ $\Psi$

**shows** $\Psi \rhd P \longmapsto (p \cdot M)(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
⟨*proof*⟩

**lemma** *broutputPermSubject*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a,\ 'b,\ 'c)$ *psi*
**and** $M$ :: $'a$
**and** $B$ :: $('a,\ 'b,\ 'c)$ *boundOutput*
**and** $p$ :: *name prm*
**and** *yvec* :: *name list*
**and** *zvec* :: *name list*

**assumes** $\Psi \rhd P \longmapsto ¡M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
**and** $S$: *set p* $\subseteq$ *set yvec* $\times$ *set zvec*
**and** *yvec* $\sharp*$ $P$
**and** *zvec* $\sharp*$ $P$
**and** *yvec* $\sharp*$ $\Psi$
**and** *zvec* $\sharp*$ $\Psi$

**shows** $\Psi \rhd P \longmapsto ¡(p \cdot M)(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
⟨*proof*⟩

**lemma** *outputSwapFrame*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a,\ 'b,\ 'c)$ *psi*
**and** $M$ :: $'a$
**and** $B$ :: $('a,\ 'b,\ 'c)$ *boundOutput*

**and** $x$ :: *name*
**and** $y$ :: *name*

**assumes** $\Psi \rhd P \longmapsto M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P'$
**and** $xvec \sharp* M$
**and** $x \sharp P$
**and** $y \sharp P$
**and** $x \sharp M$
**and** $y \sharp M$

**shows** $([(x, y)] \cdot \Psi) \rhd P \longmapsto M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P'$
⟨*proof*⟩

**lemma** *broutputSwapFrame*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** $B$ :: $('a, 'b, 'c)$ *boundOutput*
**and** $x$ :: *name*
**and** $y$ :: *name*

**assumes** $\Psi \rhd P \longmapsto ¡M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P'$
**and** $xvec \sharp* M$
**and** $x \sharp P$
**and** $y \sharp P$
**and** $x \sharp M$
**and** $y \sharp M$

**shows** $([(x, y)] \cdot \Psi) \rhd P \longmapsto ¡M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P'$
⟨*proof*⟩

**lemma** *outputPermFrame*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** $B$ :: $('a, 'b, 'c)$ *boundOutput*
**and** $p$ :: *name prm*
**and** *yvec* :: *name list*
**and** *zvec* :: *name list*

**assumes** $\Psi \rhd P \longmapsto M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P'$
**and** $S$: *set* $p \subseteq$ *set yvec* $\times$ *set zvec*
**and** *yvec* $\sharp*$ $P$
**and** *zvec* $\sharp*$ $P$
**and** *yvec* $\sharp*$ $M$
**and** *zvec* $\sharp*$ $M$

**shows** $(p \cdot \Psi) \rhd P \longmapsto M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P'$
⟨*proof*⟩

**lemma** *broutputPermFrame*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** $B$ :: $('a, 'b, 'c)$ *boundOutput*
**and** $p$ :: *name prm*
**and** *yvec* :: *name list*
**and** *zvec* :: *name list*

**assumes** $\Psi \rhd P \longmapsto ¡ M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
**and** $S$: *set p* $\subseteq$ *set yvec* $\times$ *set zvec*
**and** *yvec* $\sharp *$ $P$
**and** *zvec* $\sharp *$ $P$
**and** *yvec* $\sharp *$ $M$
**and** *zvec* $\sharp *$ $M$

**shows** $(p \cdot \Psi) \rhd P \longmapsto ¡ M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
⟨*proof*⟩

**lemma** *Comm1*:
**fixes** $\Psi$ :: $'b$
**and** $\Psi_Q$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $A_P$ :: *name list*
**and** $\Psi_P$ :: $'b$
**and** $Q$ :: $('a, 'b, 'c)$ *psi*
**and** $K$ :: $'a$
**and** *xvec* :: *name list*
**and** $Q'$ :: $('a, 'b, 'c)$ *psi*
**and** $A_Q$ :: *name list*

**assumes** $\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P'$
**and** *extractFrame* $P = \langle A_P, \Psi_P\rangle$
**and** $\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec Q'$
**and** *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$
**and** $\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$
**and** $A_P \sharp * \Psi$
**and** $A_P \sharp * P$
**and** $A_P \sharp * Q$
**and** $A_P \sharp * M$
**and** $A_P \sharp * A_Q$
**and** $A_Q \sharp * \Psi$
**and** $A_Q \sharp * P$
**and** $A_Q \sharp * Q$
**and** $A_Q \sharp * K$

**and** $xvec \sharp* P$

**shows** $\Psi \rhd P \parallel Q \longmapsto \tau \prec (\!|\nu * xvec|\!)(P' \parallel Q')$
⟨*proof*⟩

**lemma** *Comm2*:
**fixes** $\Psi$ :: $'b$
**and** $\Psi_Q$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** *xvec* :: *name list*
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $A_P$ :: *name list*
**and** $\Psi_P$ :: $'b$
**and** $Q$ :: $('a, 'b, 'c)$ *psi*
**and** $K$ :: $'a$
**and** $Q'$ :: $('a, 'b, 'c)$ *psi*
**and** $A_Q$ :: *name list*

**assumes** $\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
**and** *extractFrame* $P = \langle A_P, \Psi_P\rangle$
**and** $\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q'$
**and** *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$
**and** $\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$
**and** $A_P \sharp* \Psi$
**and** $A_P \sharp* P$
**and** $A_P \sharp* Q$
**and** $A_P \sharp* M$
**and** $A_P \sharp* A_Q$
**and** $A_Q \sharp* \Psi$
**and** $A_Q \sharp* P$
**and** $A_Q \sharp* Q$
**and** $A_Q \sharp* K$
**and** $xvec \sharp* Q$

**shows** $\Psi \rhd P \parallel Q \longmapsto \tau \prec (\!|\nu * xvec|\!)(P' \parallel Q')$
⟨*proof*⟩

**lemma** *BrMerge*:
**fixes** $\Psi$ :: $'b$
**and** $\Psi_Q$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $A_P$ :: *name list*
**and** $\Psi_P$ :: $'b$
**and** $Q$ :: $('a, 'b, 'c)$ *psi*

**and** $Q'$ :: $('a, 'b, 'c)$ *psi*
**and** $A_Q$ :: *name list*

**assumes** $\Psi \otimes \Psi_Q \rhd P \longmapsto ¿M(\!|N|\!) \prec P'$
**and** *extractFrame* $P = \langle A_P, \Psi_P \rangle$
**and** $\Psi \otimes \Psi_P \rhd Q \longmapsto ¿M(\!|N|\!) \prec Q'$
**and** *extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$
**and** $A_P \sharp* \Psi$
**and** $A_P \sharp* P$
**and** $A_P \sharp* Q$
**and** $A_P \sharp* M$
**and** $A_P \sharp* A_Q$
**and** $A_Q \sharp* \Psi$
**and** $A_Q \sharp* P$
**and** $A_Q \sharp* Q$
**and** $A_Q \sharp* M$

**shows** $\Psi \rhd P \parallel Q \longmapsto ¿M(\!|N|\!) \prec (P' \parallel Q')$
⟨*proof*⟩

**lemma** *BrComm1*:
**fixes** $\Psi$ :: $'b$
**and** $\Psi_Q$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $A_P$ :: *name list*
**and** $\Psi_P$ :: $'b$
**and** $Q$ :: $('a, 'b, 'c)$ *psi*
**and** *xvec* :: *name list*
**and** $Q'$ :: $('a, 'b, 'c)$ *psi*
**and** $A_Q$ :: *name list*

**assumes** $\Psi \otimes \Psi_Q \rhd P \longmapsto ¿M(\!|N|\!) \prec P'$
**and** *extractFrame* $P = \langle A_P, \Psi_P \rangle$
**and** $\Psi \otimes \Psi_P \rhd Q \longmapsto ¡M(\!|\nu*xvec|\!)\langle N \rangle \prec Q'$
**and** *extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$
**and** $A_P \sharp* \Psi$
**and** $A_P \sharp* P$
**and** $A_P \sharp* Q$
**and** $A_P \sharp* M$
**and** $A_P \sharp* A_Q$
**and** $A_Q \sharp* \Psi$
**and** $A_Q \sharp* P$
**and** $A_Q \sharp* Q$
**and** $A_Q \sharp* M$
**and** *xvec* $\sharp* P$

**shows** $\Psi \rhd P \parallel Q \longmapsto ¡M(\!|\nu{*}xvec|\!)\langle N\rangle \prec (P' \parallel Q')$
⟨*proof*⟩

**lemma** *BrComm2*:
**fixes** $\Psi$ :: $'b$
**and** $\Psi_Q$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** *xvec* :: *name list*
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $A_P$ :: *name list*
**and** $\Psi_P$ :: $'b$
**and** $Q$ :: $('a, 'b, 'c)$ *psi*
**and** $Q'$ :: $('a, 'b, 'c)$ *psi*
**and** $A_Q$ :: *name list*

**assumes** $\Psi \otimes \Psi_Q \rhd P \longmapsto ¡M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P'$
**and** *extractFrame* $P = \langle A_P, \Psi_P\rangle$
**and** $\Psi \otimes \Psi_P \rhd Q \longmapsto ¿M(\!|N|\!) \prec Q'$
**and** *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$
**and** $A_P \sharp* \Psi$
**and** $A_P \sharp* P$
**and** $A_P \sharp* Q$
**and** $A_P \sharp* M$
**and** $A_P \sharp* A_Q$
**and** $A_Q \sharp* \Psi$
**and** $A_Q \sharp* P$
**and** $A_Q \sharp* Q$
**and** $A_Q \sharp* M$
**and** *xvec* $\sharp*$ $Q$

**shows** $\Psi \rhd P \parallel Q \longmapsto ¡M(\!|\nu{*}xvec|\!)\langle N\rangle \prec (P' \parallel Q')$
⟨*proof*⟩

**lemma** *BrClose*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** *xvec* :: *name list*
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $x$ :: *name*

**assumes** $\Psi \rhd P \longmapsto ¡M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P'$
**and** $x \in$ *supp* $M$
**and** $x \sharp \Psi$

**shows** $\Psi \rhd (\!|\nu x|\!)P \longmapsto \tau \prec (\!|\nu x|\!)((\!|\nu{*}xvec|\!)P')$

⟨*proof*⟩

**lemma** *semanticsCasesAux*[*consumes 1*, *case-names cInput cBrInput cOutput cBrOutput cCase cPar1 cPar2 cComm1 cComm2 cBrMerge cBrComm1 cBrComm2 cBrClose cOpen cBrOpen cScope cBang*]:
**fixes** $cP$ :: ('$a$, '$b$, '$c$) *psi*
**and** $cRs$ :: ('$a$, '$b$, '$c$) *residual*
**and** $C$ :: '$f$::*fs-name*
**and** $x$ :: *name*

**assumes** $\Psi \rhd cP \longmapsto cRs$
**and** *rInput*: $\bigwedge M\ K\ xvec\ N\ Tvec\ P.\ [\![cP = M(\!|\lambda * xvec\ N|\!).P;\ cRs = K(\!|(N[xvec ::= Tvec])|\!) \prec P[xvec ::= Tvec];$
$\Psi \vdash M \leftrightarrow K$; *distinct xvec*; *set xvec* $\subseteq$ *supp* $N$; *length xvec*=*length Tvec*;
*xvec* $\sharp*$ $Tvec$; *xvec* $\sharp*$ $\Psi$; *xvec* $\sharp*$ $M$; *xvec* $\sharp*$ $K$; *xvec* $\sharp*$ $C$$]\!] \Longrightarrow$ *Prop*
**and** *rBrInput*: $\bigwedge M\ K\ xvec\ N\ Tvec\ P.\ [\![cP = M(\!|\lambda * xvec\ N|\!).P;\ cRs = ¿K(\!|(N[xvec ::= Tvec])|\!) \prec P[xvec ::= Tvec];$
$\Psi \vdash K \succeq M$; *distinct xvec*; *set xvec* $\subseteq$ *supp* $N$; *length xvec*=*length Tvec*;
*xvec* $\sharp*$ $Tvec$; *xvec* $\sharp*$ $\Psi$; *xvec* $\sharp*$ $M$; *xvec* $\sharp*$ $K$; *xvec* $\sharp*$ $C$$]\!] \Longrightarrow$ *Prop*
**and** *rOutput*: $\bigwedge M\ K\ N\ P.\ [\![cP = M\langle N\rangle.P;\ cRs = K\langle N\rangle \prec P;\ \Psi \vdash M \leftrightarrow K]\!] \Longrightarrow$ *Prop*
**and** *rBrOutput*: $\bigwedge M\ K\ N\ P.\ [\![cP = M\langle N\rangle.P;\ cRs = ¡K\langle N\rangle \prec P;\ \Psi \vdash M \preceq K]\!] \Longrightarrow$ *Prop*
**and** *rCase*: $\bigwedge Cs\ P\ \varphi.\ [\![cP = Cases\ Cs;\ \Psi \rhd P \longmapsto cRs;\ (\varphi, P) \in set\ Cs;\ \Psi \vdash \varphi;\ guarded\ P]\!] \Longrightarrow$ *Prop*

**and** *rPar1*: $\bigwedge \Psi_Q\ P\ \alpha\ P'\ Q\ A_Q.\ [\![cP = P \parallel Q;\ cRs = \alpha \prec (P' \parallel Q);$
$(\Psi \otimes \Psi_Q) \rhd P \longmapsto (\alpha \prec P')$; *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* $A_Q$;
$A_Q \sharp* P$; $A_Q \sharp* Q$; $A_Q \sharp* \Psi$; $A_Q \sharp* \alpha$; $A_Q \sharp* C$; $A_Q \sharp* P'$; *bn* $\alpha \sharp* \Psi$; *bn* $\alpha$ $\sharp* \Psi_Q$;
*bn* $\alpha$ $\sharp* Q$; *bn* $\alpha$ $\sharp* P$; *bn* $\alpha \sharp*$ *subject* $\alpha$; *bn* $\alpha \sharp* C$; *distinct*(*bn* $\alpha$)$]\!] \Longrightarrow$
*Prop*
**and** *rPar2*: $\bigwedge \Psi_P\ Q\ \alpha\ Q'\ P\ A_P.\ [\![cP = P \parallel Q;\ cRs = \alpha \prec (P \parallel Q');$
$(\Psi \otimes \Psi_P) \rhd Q \longmapsto \alpha \prec Q'$; *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* $A_P$;
$A_P \sharp* P$; $A_P \sharp* Q$; $A_P \sharp* \Psi$; $A_P \sharp* \alpha$; $A_P \sharp* C$;
$A_P \sharp* Q'$; *bn* $\alpha \sharp* \Psi$; *bn* $\alpha \sharp* \Psi_P$; *bn* $\alpha \sharp* P$; *bn* $\alpha \sharp* Q$; *bn* $\alpha \sharp*$ *subject* $\alpha$; *bn* $\alpha \sharp* C$; *distinct*(*bn* $\alpha$)$]\!] \Longrightarrow$ *Prop*
**and** *rComm1*: $\bigwedge \Psi_Q\ P\ M\ N\ P'\ A_P\ \Psi_P\ Q\ K\ xvec\ Q'\ A_Q.$
$[\![cP = P \parallel Q;\ cRs = \tau \prec (\!|\nu * xvec|\!)P' \parallel Q';$
$\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P'$; *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* $A_P$;
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec Q'$; *extractFrame* $Q = \langle A_Q,$

$\Psi_Q\rangle$; *distinct* $A_Q$;
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$; $A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* M$; $A_P \sharp* N$;
$A_P \sharp* P'$; $A_P \sharp* Q$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* xvec$; $A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$;
$A_Q \sharp* P$; $A_Q \sharp* K$; $A_Q \sharp* N$; $A_Q \sharp* P'$; $A_Q \sharp* Q$; $A_Q \sharp* Q'$; $A_Q \sharp* xvec$;
$xvec \sharp* \Psi$; $xvec \sharp* \Psi_P$; $xvec \sharp* \Psi_Q$; $xvec \sharp* P$; $xvec \sharp* M$; $xvec \sharp* Q$;
$xvec \sharp* K$; $A_P \sharp* C$; $A_Q \sharp* C$; $xvec \sharp* C$; *distinct* $xvec$]] $\Longrightarrow$ *Prop*

**and** *rComm2*: $\bigwedge \Psi_Q\ P\ M\ xvec\ N\ P'\ A_P\ \Psi_P\ Q\ K\ Q'\ A_Q$.
[[$cP = P \parallel Q$; $cRs = \tau \prec (\!|\nu* xvec|\!) P' \parallel Q'$;
$\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu* xvec|\!)\langle N\rangle \prec P'$; *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* $A_P$;
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q'$; *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* $A_Q$;
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$; $A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* M$; $A_P \sharp* N$;
$A_P \sharp* P'$; $A_P \sharp* Q$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* xvec$; $A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$;
$A_Q \sharp* P$; $A_Q \sharp* K$; $A_Q \sharp* N$; $A_Q \sharp* P'$; $A_Q \sharp* Q$; $A_Q \sharp* Q'$; $A_Q \sharp* xvec$;
$xvec \sharp* \Psi$; $xvec \sharp* \Psi_P$; $xvec \sharp* \Psi_Q$; $xvec \sharp* P$; $xvec \sharp* M$; $xvec \sharp* Q$;
$xvec \sharp* K$; $A_P \sharp* C$; $A_Q \sharp* C$; $xvec \sharp* C$; *distinct* $xvec$]] $\Longrightarrow$ *Prop*

**and** *rBrMerge*: $\bigwedge \Psi_Q\ P\ M\ N\ P'\ A_P\ \Psi_P\ Q\ Q'\ A_Q$.
[[$cP = (P \parallel Q)$; $cRs = ¿M(\!|N|\!) \prec (P' \parallel Q')$;
$\Psi \otimes \Psi_Q \rhd P \longmapsto ¿M(\!|N|\!) \prec P'$; *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* $A_P$;
$\Psi \otimes \Psi_P \rhd Q \longmapsto ¿M(\!|N|\!) \prec Q'$; *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* $A_Q$;
$A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* N$; $A_P \sharp* P'$;
$A_P \sharp* Q$; $A_P \sharp* Q'$; $A_P \sharp* M$; $A_P \sharp* A_Q$;
$A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$; $A_Q \sharp* P$; $A_Q \sharp* N$; $A_Q \sharp* P'$;
$A_Q \sharp* Q$; $A_Q \sharp* Q'$; $A_Q \sharp* M$; $A_P \sharp* C$; $A_Q \sharp* C$]] $\Longrightarrow$ *Prop*

**and** *rBrComm1*: $\bigwedge \Psi_Q\ P\ M\ N\ P'\ A_P\ \Psi_P\ Q\ xvec\ Q'\ A_Q$.
[[$cP = P \parallel Q$; $cRs = ¡M(\!|\nu* xvec|\!)\langle N\rangle \prec (P' \parallel Q')$;
$\Psi \otimes \Psi_Q \rhd P \longmapsto ¿M(\!|N|\!) \prec P'$; *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* $A_P$;
$\Psi \otimes \Psi_P \rhd Q \longmapsto ¡M(\!|\nu* xvec|\!)\langle N\rangle \prec Q'$; *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* $A_Q$;
$A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* N$;
$A_P \sharp* P'$; $A_P \sharp* Q$; $A_P \sharp* Q'$; $A_P \sharp* M$; $A_P \sharp* A_Q$; $A_P \sharp* xvec$; $A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$;
$A_Q \sharp* P$; $A_Q \sharp* N$; $A_Q \sharp* P'$; $A_Q \sharp* M$; $A_Q \sharp* Q$; $A_Q \sharp* Q'$; $A_Q \sharp* xvec$;
$xvec \sharp* \Psi$; $xvec \sharp* \Psi_P$; $xvec \sharp* \Psi_Q$; $xvec \sharp* P$; $xvec \sharp* Q$; $xvec \sharp* M$;
$A_P \sharp* C$; $A_Q \sharp* C$; $xvec \sharp* C$; *distinct* $xvec$]] $\Longrightarrow$ *Prop*

**and** *rBrComm2*: $\bigwedge \Psi_Q\ P\ M\ xvec\ N\ P'\ A_P\ \Psi_P\ Q\ Q'\ A_Q$.
$[\![ cP = P \parallel Q;\ cRs = ¡M(\!|\nu{*}xvec|\!)\langle N\rangle \prec (P' \parallel Q');$
$\Psi \otimes \Psi_Q \rhd P \longmapsto ¡M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto ¿M(\!|N|\!) \prec Q';\ extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$A_P \sharp* \Psi;\ A_P \sharp* \Psi_Q;\ A_P \sharp* P;\ A_P \sharp* N;$
$A_P \sharp* P';\ A_P \sharp* Q;\ A_P \sharp* Q';\ A_P \sharp* M;\ A_P \sharp* A_Q;\ A_P \sharp* xvec;\ A_Q \sharp* \Psi;\ A_Q \sharp* \Psi_P;$
$A_Q \sharp* P;\ A_Q \sharp* N;\ A_Q \sharp* P';\ A_Q \sharp* M;\ A_Q \sharp* Q;\ A_Q \sharp* Q';\ A_Q \sharp* xvec;$
$xvec \sharp* \Psi;\ xvec \sharp* \Psi_P;\ xvec \sharp* \Psi_Q;\ xvec \sharp* P;\ xvec \sharp* Q;\ xvec \sharp* M;$
$A_P \sharp* C;\ A_Q \sharp* C;\ xvec \sharp* C;\ distinct\ xvec]\!] \Longrightarrow Prop$

**and** *rBrClose*: $\bigwedge P\ M\ xvec\ N\ P'\ x$.
$[\![ cP = (\!|\nu x|\!)P);\ cRs = \tau \prec (\!|\nu x|\!)((\!|\nu{*}xvec|\!)P');$
$x \in supp\ M;$
$\Psi \rhd P \longmapsto ¡M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P';$
$distinct\ xvec;\ xvec \sharp* \Psi;\ xvec \sharp* P;$
$xvec \sharp* M;$
$x \sharp \Psi;\ x \sharp xvec;$
$xvec \sharp* C;\ x \sharp C]\!] \Longrightarrow Prop$

**and** *rOpen*: $\bigwedge P\ M\ xvec\ yvec\ N\ P'\ x$.
$[\![ cP = (\!|\nu x|\!)P;\ cRs = M(\!|\nu{*}(xvec@x\#yvec)|\!)\langle N\rangle \prec P';$
$\Psi \rhd P \longmapsto M(\!|\nu{*}(xvec@yvec)|\!)\langle N\rangle \prec P';\ x \in supp\ N;\ x \sharp xvec;\ x \sharp yvec;\ x \sharp M;\ x \sharp \Psi;\ distinct\ xvec;\ distinct\ yvec;$
$xvec \sharp* \Psi;\ xvec \sharp* P;\ xvec \sharp* M;\ xvec \sharp* yvec;\ yvec \sharp* \Psi;\ yvec \sharp* P;\ yvec \sharp* M;\ xvec \sharp* C;\ x \sharp C;\ yvec \sharp* C]\!] \Longrightarrow$
$Prop$

**and** *rBrOpen*: $\bigwedge P\ M\ xvec\ yvec\ N\ P'\ x$.
$[\![ cP = (\!|\nu x|\!)P;\ cRs = ¡M(\!|\nu{*}(xvec@x\#yvec)|\!)\langle N\rangle \prec P';$
$\Psi \rhd P \longmapsto ¡M(\!|\nu{*}(xvec@yvec)|\!)\langle N\rangle \prec P';\ x \in supp\ N;\ x \sharp xvec;\ x \sharp yvec;\ x \sharp M;\ x \sharp \Psi;\ distinct\ xvec;\ distinct\ yvec;$
$xvec \sharp* \Psi;\ xvec \sharp* P;\ xvec \sharp* M;\ xvec \sharp* yvec;\ yvec \sharp* \Psi;\ yvec \sharp* P;\ yvec \sharp* M;\ xvec \sharp* C;\ x \sharp C;\ yvec \sharp* C]\!] \Longrightarrow$
$Prop$

**and** *rScope*: $\bigwedge P\ \alpha\ P'\ x$. $[\![ cP = (\!|\nu x|\!)P;\ cRs = \alpha \prec (\!|\nu x|\!)P';$
$\Psi \rhd P \longmapsto \alpha \prec P';\ x \sharp \Psi;\ x \sharp \alpha;\ x \sharp C;\ bn\ \alpha \sharp* \Psi;\ bn\ \alpha \sharp* P;\ bn\ \alpha \sharp* subject\ \alpha;\ bn\ \alpha \sharp* C;\ distinct(bn\ \alpha)]\!] \Longrightarrow Prop$

**and** *rBang*: $\bigwedge P$. $[\![ cP = !P;\ \Psi \rhd P \parallel !P \longmapsto cRs;\ guarded\ P]\!] \Longrightarrow Prop$

**shows** *Prop*

⟨*proof*⟩

**nominal-primrec**
*inputLength* :: ('*a*::*fs-name*, '*b*::*fs-name*, '*c*::*fs-name*) *psi* ⇒ *nat*
**and** *inputLength'* :: ('*a*::*fs-name*, '*b*::*fs-name*, '*c*::*fs-name*) *input* ⇒ *nat*
**and** *inputLength''* :: ('*a*::*fs-name*, '*b*::*fs-name*, '*c*::*fs-name*) *psiCase* ⇒ *nat*

**where**

$inputLength\ (\mathbf{0}) = 0$
| $inputLength\ (M\langle N\rangle.P) = 0$
| $inputLength\ (M(\!|I) = inputLength'\ I$
| $inputLength\ (Case\ C) = 0$
| $inputLength\ (P \parallel Q) = 0$
| $inputLength\ ((\!|\nu x|\!)P) = 0$
| $inputLength\ (\{\!|\Psi|\!\}) = 0$
| $inputLength\ (!P) = 0$

| $inputLength'\ (Trm\ M\ P) = 0$
| $inputLength'\ (\nu\ y\ I) = 1 + (inputLength'\ I)$

| $inputLength''\ (\bot_c) = 0$
| $inputLength''\ (\Box\Phi \Rightarrow P\ C) = 0$
⟨*proof*⟩

**nominal-primrec** *boundOutputLength* :: $('a, 'b, 'c)\ boundOutput \Rightarrow nat$
**where**
$boundOutputLength\ (BOut\ M\ P) = 0$
| $boundOutputLength\ (BStep\ x\ B) = (boundOutputLength\ B) + 1$
⟨*proof*⟩

**nominal-primrec** *residualLength* :: $('a, 'b, 'c)\ residual \Rightarrow nat$
**where**
$residualLength\ (RIn\ M\ N\ P) = 0$
| $residualLength\ (RBrIn\ M\ N\ P) = 0$
| $residualLength\ (ROut\ M\ B) = boundOutputLength\ B$
| $residualLength\ (RBrOut\ M\ B) = boundOutputLength\ B$
| $residualLength\ (RTau\ P) = 0$
⟨*proof*⟩

**lemma** *inputLengthProc*[*simp*]:
**shows** $inputLength(M(\!|\lambda*xvec\ N|\!).P) = length\ xvec$
⟨*proof*⟩

**lemma** *boundOutputLengthSimp*[*simp*]:
**shows** $residualLength(M(\!|\nu*xvec|\!)\langle N\rangle \prec P) = length\ xvec$
**and** $residualLength(¡M(\!|\nu*xvec|\!)\langle N\rangle \prec P) = length\ xvec$
⟨*proof*⟩

**lemma** *boundOuputLengthSimp2*[*simp*]:
**shows** $residualLength(\alpha \prec P) = length(bn\ \alpha)$
⟨*proof*⟩

**lemmas** [*simp del*] = *inputLength-inputLength'-inputLength''.simps residualLength.simps boundOutputLength.simps*

**lemma** *constructPerm*:
**fixes** *xvec* :: *name list*

**and** *yvec* :: *name list*

**assumes** *length xvec* = *length yvec*
**and** *xvec* ♯* *yvec*
**and** *distinct xvec*
**and** *distinct yvec*

**obtains** *p* **where** *set p* ⊆ *set xvec* × *set*(*p* ∙ *xvec*) **and** *distinctPerm p* **and** *yvec* = *p* ∙ *xvec*
⟨*proof*⟩

**lemma** *distinctApend*[*simp*]:
**fixes** *xvec* :: *name list*
**and** *yvec* :: *name list*

**shows** (*set xvec* ∩ *set yvec* = {}) = *xvec* ♯* *yvec*
⟨*proof*⟩

**lemma** *lengthAux*:
**fixes** *xvec* :: *name list*
**and** *y* :: *name*
**and** *yvec* :: *name list*

**assumes** *length xvec* = *length*(*y*#*yvec*)

**obtains** *z zvec* **where** *xvec* = *z*#*zvec* **and** *length zvec* = *length yvec*
⟨*proof*⟩

**lemma** *lengthAux2*:
**fixes** *xvec* :: *name list*
**and** *yvec* :: *name list*
**and** *zvec* :: *name list*

**assumes** *length xvec* = *length*(*yvec*@*y*#*zvec*)

**obtains** *xvec1 x xvec2* **where** *xvec*=*xvec1*@*x*#*xvec2* **and** *length xvec1* = *length yvec* **and** *length xvec2* = *length zvec*
⟨*proof*⟩

**lemma** *semanticsCases*[*consumes 19*, *case-names cInput cBrInput cOutput cBrOutput cCase cPar1 cPar2 cComm1 cComm2 cBrMerge cBrComm1 cBrComm2 cBrClose cOpen cBrOpen cScope cBang*]:
**fixes** Ψ :: *'b*
**and** *cP* :: (*'a*, *'b*, *'c*) *psi*
**and** *cRs* :: (*'a*, *'b*, *'c*) *residual*
**and** *C* :: *'f*::*fs-name*
**and** *x1* :: *name*
**and** *x2* :: *name*
**and** *x3* :: *name*

**and** *x4* :: *name*
**and** *xvec1* :: *name list*
**and** *xvec2* :: *name list*
**and** *xvec3* :: *name list*
**and** *xvec4* :: *name list*
**and** *xvec5* :: *name list*
**and** *xvec6* :: *name list*
**and** *xvec7* :: *name list*
**and** *xvec8* :: *name list*
**and** *xvec9* :: *name list*

**assumes** $\Psi \rhd cP \longmapsto cRs$
**and** *length xvec1* = *inputLength cP* **and** *distinct xvec1*
**and** *length xvec6* = *inputLength cP* **and** *distinct xvec6*
**and** *length xvec2* = *residualLength cRs* **and** *distinct xvec2*
**and** *length xvec3* = *residualLength cRs* **and** *distinct xvec3*
**and** *length xvec4* = *residualLength cRs* **and** *distinct xvec4*
**and** *length xvec5* = *residualLength cRs* **and** *distinct xvec5*
**and** *length xvec7* = *residualLength cRs* **and** *distinct xvec7*
**and** *length xvec8* = *residualLength cRs* **and** *distinct xvec8*
**and** *length xvec9* = *residualLength cRs* **and** *distinct xvec9*
**and** *rInput*: $\bigwedge M\ K\ N\ Tvec\ P.\ ([\![xvec1 \sharp* \Psi;\ xvec1 \sharp* cP;\ xvec1 \sharp* cRs]\!] \Longrightarrow cP = M(\!|\lambda* xvec1\ N|\!).P \wedge\ \ cRs = K(\!|(N[xvec1::=Tvec])|\!) \prec P[xvec1::=Tvec] \wedge \Psi \vdash M \leftrightarrow K \wedge distinct\ xvec1 \wedge set\ xvec1 \subseteq supp\ N \wedge length\ xvec1 = length\ Tvec \wedge xvec1 \sharp* Tvec \wedge xvec1 \sharp* \Psi \wedge xvec1 \sharp* M \wedge xvec1 \sharp* K) \Longrightarrow Prop$
**and** *rBrInput*: $\bigwedge M\ K\ N\ Tvec\ P.\ ([\![xvec6 \sharp* \Psi;\ xvec6 \sharp* cP;\ xvec6 \sharp* cRs]\!] \Longrightarrow cP = M(\!|\lambda* xvec6\ N|\!).P \wedge\ \ cRs = ¿K(\!|(N[xvec6::=Tvec])|\!) \prec P[xvec6::=Tvec] \wedge \Psi \vdash K \succeq M \wedge distinct\ xvec6 \wedge set\ xvec6 \subseteq supp\ N \wedge length\ xvec6 = length\ Tvec \wedge xvec6 \sharp* Tvec \wedge xvec6 \sharp* \Psi \wedge xvec6 \sharp* M \wedge xvec6 \sharp* K) \Longrightarrow Prop$
**and** *rOutput*: $\bigwedge M\ K\ N\ P.\ [\![cP = M\langle N\rangle.P;\ cRs = K\langle N\rangle \prec P;\ \Psi \vdash M \leftrightarrow K]\!] \Longrightarrow Prop$
**and** *rBrOutput*: $\bigwedge M\ K\ N\ P.\ [\![cP = M\langle N\rangle.P;\ cRs = ¡K\langle N\rangle \prec P;\ \Psi \vdash M \preceq K]\!] \Longrightarrow Prop$
**and** *rCase*: $\bigwedge Cs\ P\ \varphi.\ [\![cP = Cases\ Cs;\ \Psi \rhd P \longmapsto cRs;\ (\varphi, P) \in set\ Cs;\ \Psi \vdash \varphi;\ guarded\ P]\!] \Longrightarrow Prop$
**and** *rPar1*: $\bigwedge \Psi_Q\ P\ \alpha\ P'\ Q\ A_Q.\ ([\![xvec2 \sharp* \Psi;\ xvec2 \sharp* cP;\ xvec2 \sharp* cRs]\!] \Longrightarrow cP = P \parallel Q \wedge cRs = \alpha \prec (P' \parallel Q) \wedge xvec2 = bn\ \alpha \wedge \Psi \otimes \Psi_Q \rhd P \longmapsto \alpha \prec P' \wedge extractFrame\ Q = \langle A_Q, \Psi_Q\rangle \wedge distinct\ A_Q \wedge A_Q \sharp* P \wedge A_Q \sharp* Q \wedge A_Q \sharp* \Psi \wedge A_Q \sharp* \alpha \wedge A_Q \sharp* P' \wedge A_Q \sharp* C) \Longrightarrow Prop$
**and** *rPar2*: $\bigwedge \Psi_P\ Q\ \alpha\ Q'\ P\ A_P.\ ([\![xvec3 \sharp* \Psi;\ xvec3 \sharp* cP;\ xvec3 \sharp* cRs]\!] \Longrightarrow cP = P \parallel Q \wedge cRs = \alpha \prec (P \parallel Q') \wedge xvec3 = bn\ \alpha \wedge$

$\Psi \otimes \Psi_P \rhd Q \longmapsto \alpha \prec Q' \wedge$ *extractFrame* $P = \langle A_P, \Psi_P \rangle \wedge$ *distinct* $A_P \wedge$
$A_P \sharp^* P \wedge A_P \sharp^* Q \wedge A_P \sharp^* \Psi \wedge A_P \sharp^* \alpha \wedge A_P \sharp^* Q' \wedge A_P \sharp^* C) \Longrightarrow$ *Prop*

**and** *rComm1*: $\bigwedge \Psi_Q\ P\ M\ N\ P'\ A_P\ \Psi_P\ Q\ K\ xvec\ Q'\ A_Q.$
$[\![cP = P \parallel Q;\ cRs = \tau \prec (\!|\nu * xvec|\!) P' \parallel Q';$
$\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P';$ *extractFrame* $P = \langle A_P, \Psi_P \rangle;$ *distinct* $A_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu * xvec|\!)\langle N \rangle \prec Q';$ *extractFrame* $Q = \langle A_Q, \Psi_Q \rangle;$ *distinct* $A_Q;$
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K;\ A_P \sharp^* \Psi;\ A_P \sharp^* \Psi_Q;\ A_P \sharp^* P;\ A_P \sharp^* M;\ A_P \sharp^* N;$
$A_P \sharp^* P';\ A_P \sharp^* Q;\ A_P \sharp^* Q';\ A_P \sharp^* A_Q;\ A_P \sharp^* xvec;\ A_Q \sharp^* \Psi;\ A_Q \sharp^* \Psi_P;$
$A_Q \sharp^* P;\ A_Q \sharp^* K;\ A_Q \sharp^* N;\ A_Q \sharp^* P';\ A_Q \sharp^* Q;\ A_Q \sharp^* Q';\ A_Q \sharp^* xvec;$
$xvec \sharp^* \Psi;\ xvec \sharp^* \Psi_P;\ xvec \sharp^* \Psi_Q;\ xvec \sharp^* P;\ xvec \sharp^* M;\ xvec \sharp^* Q;$
$xvec \sharp^* K;\ A_P \sharp^* C;\ A_Q \sharp^* C;\ xvec \sharp^* C;$ *distinct* $xvec]\!] \Longrightarrow$ *Prop*

**and** *rComm2*: $\bigwedge \Psi_Q\ P\ M\ xvec\ N\ P'\ A_P\ \Psi_P\ Q\ K\ Q'\ A_Q.$
$[\![cP = P \parallel Q;\ cRs = \tau \prec (\!|\nu * xvec|\!) P' \parallel Q';$
$\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N \rangle \prec P';$ *extractFrame* $P = \langle A_P, \Psi_P \rangle;$ *distinct* $A_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q';$ *extractFrame* $Q = \langle A_Q, \Psi_Q \rangle;$ *distinct* $A_Q;$
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K;\ A_P \sharp^* \Psi;\ A_P \sharp^* \Psi_Q;\ A_P \sharp^* P;\ A_P \sharp^* M;\ A_P \sharp^* N;$
$A_P \sharp^* P';\ A_P \sharp^* Q;\ A_P \sharp^* Q';\ A_P \sharp^* A_Q;\ A_P \sharp^* xvec;\ A_Q \sharp^* \Psi;\ A_Q \sharp^* \Psi_P;$
$A_Q \sharp^* P;\ A_Q \sharp^* K;\ A_Q \sharp^* N;\ A_Q \sharp^* P';\ A_Q \sharp^* Q;\ A_Q \sharp^* Q';\ A_Q \sharp^* xvec;$
$xvec \sharp^* \Psi;\ xvec \sharp^* \Psi_P;\ xvec \sharp^* \Psi_Q;\ xvec \sharp^* P;\ xvec \sharp^* M;\ xvec \sharp^* Q;$
$xvec \sharp^* K;\ A_P \sharp^* C;\ A_Q \sharp^* C;\ xvec \sharp^* C;$ *distinct* $xvec]\!] \Longrightarrow$ *Prop*

**and** *rBrMerge*: $\bigwedge \Psi_Q\ P\ M\ N\ P'\ A_P\ \Psi_P\ Q\ Q'\ A_Q.$
$[\![cP = (P \parallel Q);\ cRs = ¿M(\!|N|\!) \prec (P' \parallel Q');$
$\Psi \otimes \Psi_Q \rhd P \longmapsto ¿M(\!|N|\!) \prec P';$ *extractFrame* $P = \langle A_P, \Psi_P \rangle;$ *distinct* $A_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto ¿M(\!|N|\!) \prec Q';$ *extractFrame* $Q = \langle A_Q, \Psi_Q \rangle;$ *distinct* $A_Q;$
$A_P \sharp^* \Psi;\ A_P \sharp^* \Psi_Q;\ A_P \sharp^* P;\ A_P \sharp^* N;\ A_P \sharp^* P';$
$A_P \sharp^* Q;\ A_P \sharp^* Q';\ A_P \sharp^* A_Q;\ A_P \sharp^* M;\ A_Q \sharp^* M;$
$A_Q \sharp^* \Psi;\ A_Q \sharp^* \Psi_P;\ A_Q \sharp^* P;\ A_Q \sharp^* N;\ A_Q \sharp^* P';$
$A_Q \sharp^* Q;\ A_Q \sharp^* Q';\ A_P \sharp^* C;\ A_Q \sharp^* C]\!] \Longrightarrow$ *Prop*

**and** *rBrComm1*: $\bigwedge \Psi_Q\ P\ M\ N\ P'\ A_P\ \Psi_P\ Q\ Q'\ A_Q.$
$([\![xvec7 \sharp^* \Psi;\ xvec7 \sharp^* cP;\ xvec7 \sharp^* cRs]\!] \Longrightarrow$
$cP = P \parallel Q \wedge cRs = ¡M(\!|\nu * xvec7|\!)\langle N \rangle \prec (P' \parallel Q') \wedge$
$\Psi \otimes \Psi_Q \rhd P \longmapsto ¿M(\!|N|\!) \prec P' \wedge$ *extractFrame* $P = \langle A_P, \Psi_P \rangle \wedge$ *distinct* $A_P \wedge$

$\Psi \otimes \Psi_P \rhd Q \longmapsto ¡M(\!|\nu{*}xvec7|\!)\langle N\rangle \prec Q' \wedge extractFrame\ Q = \langle A_Q, \Psi_Q\rangle \wedge distinct\ A_Q \wedge$
$A_P \sharp{*} \Psi \wedge A_P \sharp{*} \Psi_Q \wedge A_P \sharp{*} P \wedge A_P \sharp{*} N \wedge$
$A_P \sharp{*} P' \wedge A_P \sharp{*} Q \wedge A_P \sharp{*} Q' \wedge A_P \sharp{*} A_Q \wedge A_P \sharp{*} xvec7 \wedge A_Q \sharp{*} \Psi \wedge A_Q \sharp{*} \Psi_P \wedge$
$A_Q \sharp{*} P \wedge A_Q \sharp{*} N \wedge A_Q \sharp{*} P' \wedge A_Q \sharp{*} Q \wedge A_Q \sharp{*} Q' \wedge A_Q \sharp{*} xvec7 \wedge$
$xvec7 \sharp{*} \Psi \wedge xvec7 \sharp{*} \Psi_P \wedge xvec7 \sharp{*} \Psi_Q \wedge xvec7 \sharp{*} P \wedge xvec7 \sharp{*} Q \wedge$
$A_P \sharp{*} M \wedge A_Q \sharp{*} M \wedge xvec7 \sharp{*} M \wedge$
$A_P \sharp{*} C \wedge A_Q \sharp{*} C \wedge distinct\ xvec7) \Longrightarrow Prop$

**and** *rBrComm2*: $\bigwedge \Psi_Q\ P\ M\ N\ P'\ A_P\ \Psi_P\ Q\ Q'\ A_Q.$
$(\llbracket xvec8 \sharp{*} \Psi;\ xvec8 \sharp{*} cP;\ xvec8 \sharp{*} cRs \rrbracket \Longrightarrow$
$cP = P \parallel Q \wedge cRs = ¡M(\!|\nu{*}xvec8|\!)\langle N\rangle \prec (P' \parallel Q') \wedge$
$\Psi \otimes \Psi_Q \rhd P \longmapsto ¡M(\!|\nu{*}xvec8|\!)\langle N\rangle \prec P' \wedge extractFrame\ P = \langle A_P, \Psi_P\rangle \wedge distinct\ A_P \wedge$
$\Psi \otimes \Psi_P \rhd Q \longmapsto ¿M(\!|N|\!) \prec Q' \wedge extractFrame\ Q = \langle A_Q, \Psi_Q\rangle \wedge distinct\ A_Q \wedge$
$A_P \sharp{*} \Psi \wedge A_P \sharp{*} \Psi_Q \wedge A_P \sharp{*} P \wedge A_P \sharp{*} N \wedge$
$A_P \sharp{*} P' \wedge A_P \sharp{*} Q \wedge A_P \sharp{*} Q' \wedge A_P \sharp{*} A_Q \wedge A_P \sharp{*} xvec8 \wedge A_Q \sharp{*} \Psi \wedge A_Q \sharp{*} \Psi_P \wedge$
$A_Q \sharp{*} P \wedge A_Q \sharp{*} N \wedge A_Q \sharp{*} P' \wedge A_Q \sharp{*} Q \wedge A_Q \sharp{*} Q' \wedge A_Q \sharp{*} xvec8 \wedge$
$xvec8 \sharp{*} \Psi \wedge xvec8 \sharp{*} \Psi_P \wedge xvec8 \sharp{*} \Psi_Q \wedge xvec8 \sharp{*} P \wedge xvec8 \sharp{*} Q \wedge$
$A_P \sharp{*} M \wedge A_Q \sharp{*} M \wedge xvec8 \sharp{*} M \wedge$
$A_P \sharp{*} C \wedge A_Q \sharp{*} C \wedge distinct\ xvec8) \Longrightarrow Prop$

**and** *rBrClose*: $\bigwedge P\ M\ N\ xvec\ P'.$
$(\llbracket x3 \sharp \Psi;\ x3 \sharp cP;\ x3 \sharp cRs \rrbracket \Longrightarrow$
$cP = (\!|\nu x3|\!)P \wedge cRs = \tau \prec (\!|\nu x3|\!)((\!|\nu{*}xvec|\!)P') \wedge$
$x3 \in supp\ M \wedge$
$\Psi \rhd P \longmapsto ¡M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P' \wedge$
$distinct\ xvec \wedge xvec \sharp{*} \Psi \wedge xvec \sharp{*} P \wedge$
$xvec \sharp{*} M \wedge xvec \sharp{*} C \wedge$
$x3 \sharp \Psi \wedge x3 \sharp xvec) \Longrightarrow Prop$

**and** *rOpen*: $\bigwedge P\ M\ xvec\ y\ yvec\ N\ P'.$
$(\llbracket xvec4 \sharp{*} \Psi;\ xvec4 \sharp{*} cP;\ xvec4 \sharp{*} cRs;\ x1 \sharp \Psi;\ x1 \sharp cP;\ x1 \sharp cRs;\ x1 \sharp xvec4 \rrbracket \Longrightarrow$
$cP = (\!|\nu x1|\!)P \wedge cRs = M(\!|\nu{*}(xvec@x1\#yvec)|\!)\langle N\rangle \prec P' \wedge xvec4{=}xvec@y\#yvec \wedge$
$\Psi \rhd P \longmapsto M(\!|\nu{*}(xvec@yvec)|\!)\langle N\rangle \prec P' \wedge x1 \in supp\ N \wedge x1 \sharp xvec \wedge x1 \sharp yvec \wedge$
$distinct\ xvec \wedge distinct\ yvec \wedge xvec \sharp{*} \Psi \wedge xvec \sharp{*} P \wedge xvec \sharp{*} M \wedge xvec \sharp{*} yvec \wedge$
$yvec \sharp{*} \Psi \wedge yvec \sharp{*} P \wedge yvec \sharp{*} M) \Longrightarrow Prop$

**and** *rBrOpen*: $\bigwedge P\ M\ xvec\ y\ yvec\ N\ P'.$
$(\llbracket xvec9 \sharp{*} \Psi;\ xvec9 \sharp{*} cP;\ xvec9 \sharp{*} cRs;\ x4 \sharp \Psi;\ x4 \sharp cP;\ x4 \sharp cRs;\ x4 \sharp xvec9 \rrbracket \Longrightarrow$
$cP = (\!|\nu x4|\!)P \wedge cRs = ¡M(\!|\nu{*}(xvec@x4\#yvec)|\!)\langle N\rangle \prec P' \wedge$

*xvec9*=*xvec*@*y*#*yvec* ∧
Ψ ▷ *P* ⟼¡*M*(⦇*ν*∗(*xvec*@*yvec*)⦈⟨*N*⟩ ≺ *P*′ ∧ *x4* ∈ *supp N* ∧ *x4* ♯ *xvec* ∧ *x4* ♯ *yvec* ∧
*distinct xvec* ∧ *distinct yvec* ∧ *xvec* ♯∗ Ψ ∧ *xvec* ♯∗ *P* ∧ *xvec* ♯∗ *M* ∧ *xvec* ♯∗ *yvec* ∧
*yvec* ♯∗ Ψ ∧ *yvec* ♯∗ *P* ∧ *yvec* ♯∗ *M*) ⟹ *Prop*
**and** *rScope*: ⋀*P α P*′. (⟦*xvec5* ♯∗ Ψ; *xvec5* ♯∗ *cP*; *xvec5* ♯∗ *cRs*; *x2* ♯ Ψ; *x2* ♯ *cP*; *x2* ♯ *cRs*; *x2* ♯ *xvec5*⟧ ⟹
*cP* = ⦇*νx2*⦈*P* ∧ *cRs* = *α* ≺ ⦇*νx2*⦈*P*′ ∧ *xvec5* = *bn α* ∧
Ψ ▷ *P* ⟼*α* ≺ *P*′ ∧ *x2* ♯ Ψ ∧ *x2* ♯ *α* ∧ *bn α* ♯∗ *subject α* ∧ *distinct*(*bn α*)) ⟹ *Prop*
**and** *rBang*: ⋀*P*. ⟦*cP* = !*P*;
Ψ ▷ *P* ∥ !*P* ⟼*cRs*; *guarded P*⟧ ⟹ *Prop*
**shows** *Prop*
⟨*proof*⟩

**lemma** *resResidEq*:
**fixes** *xvec* :: *name list*
**and** *P* :: (′*a*, ′*b*, ′*c*) *psi*
**and** *Q* :: (′*a*, ′*b*, ′*c*) *psi*
**and** *M* :: ′*a*
**and** *N* :: ′*a*
**and** *N*′ :: ′*a*

**assumes** ¡*M*(⦇*ν*∗*xvec*⦈⟨*N*⟩ ≺ *P* = ¡*M*(⦇*ν*∗*xvec*⦈⟨*N*′⟩ ≺ *Q*
**and** *xvec* ♯∗ *M*

**shows** ¡*M*(⦇*ν*∗*xvec*⦈⟨*N*⟩ = ¡*M*(⦇*ν*∗*xvec*⦈⟨*N*′⟩
⟨*proof*⟩

**lemma** *parCases*[*consumes 5*, *case-names cPar1 cPar2 cComm1 cComm2 cBrMerge cBrComm1 cBrComm2*]:
**fixes** Ψ :: ′*b*
**and** *P* :: (′*a*, ′*b*, ′*c*) *psi*
**and** *Q* :: (′*a*, ′*b*, ′*c*) *psi*
**and** *α* :: ′*a action*
**and** *T* :: (′*a*, ′*b*, ′*c*) *psi*
**and** *C* :: ′*f*::*fs-name*
**and** *M* :: ′*a*
**and** *N* :: ′*a*

**assumes** *Trans*: Ψ ▷ *P* ∥ *Q* ⟼*α* ≺ *T*
**and** *bn α* ♯∗ Ψ
**and** *bn α* ♯∗ *P*
**and** *bn α* ♯∗ *Q*
**and** *bn α* ♯∗ *subject α*
**and** *rPar1*: ⋀*P*′ *A*$_Q$ Ψ$_Q$. ⟦Ψ ⊗ Ψ$_Q$ ▷ *P* ⟼*α* ≺ *P*′; *extractFrame Q* = ⟨*A*$_Q$, Ψ$_Q$⟩; *distinct A*$_Q$;
*A*$_Q$ ♯∗ Ψ; *A*$_Q$ ♯∗ *P*; *A*$_Q$ ♯∗ *Q*; *A*$_Q$ ♯∗ *α*; *A*$_Q$ ♯∗ *P*′; *A*$_Q$

$\sharp*\ C\rrbracket \Longrightarrow \mathit{Prop}\ \alpha\ (P' \parallel Q)$
**and** *rPar2*: $\bigwedge Q'\ A_P\ \Psi_P.\ \llbracket \Psi \otimes \Psi_P \rhd Q \longmapsto \alpha \prec Q';\ \mathit{extractFrame}\ P = \langle A_P, \Psi_P\rangle;\ \mathit{distinct}\ A_P;$
$A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* \alpha;\ A_P \sharp* Q';\ A_P \sharp* C\rrbracket \Longrightarrow \mathit{Prop}\ \alpha\ (P \parallel Q')$
**and** *rComm1*: $\bigwedge \Psi_Q\ M\ N\ P'\ A_P\ \Psi_P\ K\ \mathit{xvec}\ Q'\ A_Q.$
$\llbracket \Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P';\ \mathit{extractFrame}\ P = \langle A_P, \Psi_P\rangle;\ \mathit{distinct}\ A_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu*\mathit{xvec}|\!)\langle N\rangle \prec Q';\ \mathit{extractFrame}\ Q = \langle A_Q, \Psi_Q\rangle;\ \mathit{distinct}\ A_Q;$
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K;\ \mathit{distinct}\ \mathit{xvec};\ \alpha = \tau;$
$A_P \sharp* \Psi;\ A_P \sharp* \Psi_Q;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';\ A_P \sharp* Q;\ A_P \sharp* \mathit{xvec};\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* C;$
$A_Q \sharp* \Psi;\ A_Q \sharp* \Psi_P;\ A_Q \sharp* P;\ A_Q \sharp* K;\ A_Q \sharp* N;\ A_Q \sharp* P';\ A_Q \sharp* Q;\ A_Q \sharp* \mathit{xvec};\ A_Q \sharp* Q';\ A_Q \sharp* C;$
$\mathit{xvec} \sharp* \Psi;\ \mathit{xvec} \sharp* \Psi_P;\ \mathit{xvec} \sharp* P;\ \mathit{xvec} \sharp* M;\ \mathit{xvec} \sharp* K;\ \mathit{xvec} \sharp* Q;\ \mathit{xvec} \sharp* \Psi_Q;\ \mathit{xvec} \sharp* C\rrbracket \Longrightarrow$
$\mathit{Prop}\ (\tau)\ ((\!|\nu*\mathit{xvec}|\!)(P' \parallel Q'))$
**and** *rComm2*: $\bigwedge \Psi_Q\ M\ \mathit{xvec}\ N\ P'\ A_P\ \Psi_P\ K\ Q'\ A_Q.$
$\llbracket \Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu*\mathit{xvec}|\!)\langle N\rangle \prec P';\ \mathit{extractFrame}\ P = \langle A_P, \Psi_P\rangle;\ \mathit{distinct}\ A_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q';\ \mathit{extractFrame}\ Q = \langle A_Q, \Psi_Q\rangle;\ \mathit{distinct}\ A_Q;$
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K;\ \mathit{distinct}\ \mathit{xvec};\ \alpha = \tau;$
$A_P \sharp* \Psi;\ A_P \sharp* \Psi_Q;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';\ A_P \sharp* Q;\ A_P \sharp* \mathit{xvec};\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* C;$
$A_Q \sharp* \Psi;\ A_Q \sharp* \Psi_P;\ A_Q \sharp* P;\ A_Q \sharp* K;\ A_Q \sharp* N;\ A_Q \sharp* P';\ A_Q \sharp* Q;\ A_Q \sharp* \mathit{xvec};\ A_Q \sharp* Q';\ A_Q \sharp* C;$
$\mathit{xvec} \sharp* \Psi;\ \mathit{xvec} \sharp* \Psi_P;\ \mathit{xvec} \sharp* P;\ \mathit{xvec} \sharp* M;\ \mathit{xvec} \sharp* K;\ \mathit{xvec} \sharp* Q;\ \mathit{xvec} \sharp* \Psi_Q;\ \mathit{xvec} \sharp* C\rrbracket \Longrightarrow$
$\mathit{Prop}\ (\tau)\ ((\!|\nu*\mathit{xvec}|\!)(P' \parallel Q'))$
**and** *rBrMerge*: $\bigwedge \Psi_Q\ M\ N\ P'\ A_P\ \Psi_P\ Q'\ A_Q.$
$\llbracket \Psi \otimes \Psi_Q \rhd P \longmapsto ¿M(\!|N|\!) \prec P';\ \mathit{extractFrame}\ P = \langle A_P, \Psi_P\rangle;\ \mathit{distinct}\ A_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto ¿M(\!|N|\!) \prec Q';\ \mathit{extractFrame}\ Q = \langle A_Q, \Psi_Q\rangle;\ \mathit{distinct}\ A_Q;$
$A_P \sharp* \Psi;\ A_P \sharp* \Psi_Q;\ A_P \sharp* P;\ A_P \sharp* N;\ A_P \sharp* P';$
$A_P \sharp* Q;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* M;\ A_Q \sharp* M;$
$A_Q \sharp* \Psi;\ A_Q \sharp* \Psi_P;\ A_Q \sharp* P;\ A_Q \sharp* N;\ A_Q \sharp* P';$
$A_Q \sharp* Q;\ A_Q \sharp* Q';\ A_P \sharp* C;\ A_Q \sharp* C;\ \alpha = ¿M(\!|N|\!)\rrbracket \Longrightarrow$
$\mathit{Prop}\ (¿M(\!|N|\!))\ (P' \parallel Q')$
**and** *rBrComm1*: $\bigwedge \Psi_Q\ M\ N\ P'\ A_P\ \Psi_P\ \mathit{xvec}\ Q'\ A_Q.$
$\llbracket \Psi \otimes \Psi_Q \rhd P \longmapsto ¿M(\!|N|\!) \prec P';\ \mathit{extractFrame}\ P = \langle A_P, \Psi_P\rangle;\ \mathit{distinct}\ A_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto ¡M(\!|\nu*\mathit{xvec}|\!)\langle N\rangle \prec Q';\ \mathit{extractFrame}\ Q = \langle A_Q, \Psi_Q\rangle;\ \mathit{distinct}\ A_Q;$
$\mathit{distinct}\ \mathit{xvec};$
$A_P \sharp* \Psi;\ A_P \sharp* \Psi_Q;\ A_P \sharp* P;\ A_P \sharp* N;\ A_P \sharp* P';\ A_P \sharp* Q;\ A_P \sharp* \mathit{xvec};\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* C;$
$A_Q \sharp* \Psi;\ A_Q \sharp* \Psi_P;\ A_Q \sharp* P;\ A_Q \sharp* N;\ A_Q \sharp* P';\ A_Q \sharp* Q;\ A_Q \sharp* \mathit{xvec};\ A_Q \sharp* Q';\ A_Q \sharp* C;$

$A_P \sharp* M$; $A_Q \sharp* M$; $xvec \sharp* M$; $¡M(\!|\nu{*}xvec|\!)\langle N\rangle = \alpha$;
$xvec \sharp* \Psi$; $xvec \sharp* \Psi_P$; $xvec \sharp* P$; $xvec \sharp* Q$; $xvec \sharp* \Psi_Q\rrbracket \Longrightarrow$
*Prop* $(¡M(\!|\nu{*}xvec|\!)\langle N\rangle)$ $(P' \parallel Q')$

**and** *rBrComm2*: $\bigwedge \Psi_Q\ M\ xvec\ N\ P'\ A_P\ \Psi_P\ Q'\ A_Q$.
$\llbracket \Psi \otimes \Psi_Q \rhd P \longmapsto ¡M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P'$; *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* $A_P$;
$\Psi \otimes \Psi_P \rhd Q \longmapsto ¿M(\!|N|\!) \prec Q'$; *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* $A_Q$;
*distinct xvec*;
$A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* N$; $A_P \sharp* P'$; $A_P \sharp* Q$; $A_P \sharp* xvec$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* C$;
$A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$; $A_Q \sharp* P$; $A_Q \sharp* N$; $A_Q \sharp* P'$; $A_Q \sharp* Q$; $A_Q \sharp* xvec$; $A_Q \sharp* Q'$; $A_Q \sharp* C$;
$A_P \sharp* M$; $A_Q \sharp* M$; $xvec \sharp* M$; $¡M(\!|\nu{*}xvec|\!)\langle N\rangle = \alpha$;
$xvec \sharp* \Psi$; $xvec \sharp* \Psi_P$; $xvec \sharp* P$; $xvec \sharp* Q$; $xvec \sharp* \Psi_Q\rrbracket \Longrightarrow$
*Prop* $(¡M(\!|\nu{*}xvec|\!)\langle N\rangle)$ $(P' \parallel Q')$

**shows** *Prop* $\alpha$ $T$
⟨*proof*⟩

**lemma** *parInputCases*[*consumes 1*, *case-names cPar1 cPar2*]:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)$ *psi*
**and** $Q :: ('a, 'b, 'c)$ *psi*
**and** $M :: 'a$
**and** $N :: 'a$
**and** $R :: ('a, 'b, 'c)$ *psi*
**and** $C :: 'f$::*fs-name*

**assumes** *Trans*: $\Psi \rhd P \parallel Q \longmapsto M(\!|N|\!) \prec R$
**and** *rPar1*: $\bigwedge P'\ A_Q\ \Psi_Q$. $\llbracket \Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P'$; *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* $A_Q$;
$A_Q \sharp* \Psi$; $A_Q \sharp* P$; $A_Q \sharp* Q$; $A_Q \sharp* M$; $A_Q \sharp* N$; $A_Q \sharp* C\rrbracket$
$\Longrightarrow$ *Prop* $(P' \parallel Q)$
**and** *rPar2*: $\bigwedge Q'\ A_P\ \Psi_P$. $\llbracket \Psi \otimes \Psi_P \rhd Q \longmapsto M(\!|N|\!) \prec Q'$; *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* $A_P$;
$A_P \sharp* \Psi$; $A_P \sharp* P$; $A_P \sharp* Q$; $A_P \sharp* M$; $A_P \sharp* N$; $A_P \sharp* C\rrbracket$
$\Longrightarrow$ *Prop* $(P \parallel Q')$
**shows** *Prop* $R$
⟨*proof*⟩

**lemma** *parBrInputCases*[*consumes 1*, *case-names cPar1 cPar2 cBrMerge*]:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)$ *psi*
**and** $Q :: ('a, 'b, 'c)$ *psi*
**and** $M :: 'a$
**and** $N :: 'a$
**and** $R :: ('a, 'b, 'c)$ *psi*
**and** $C :: 'f$::*fs-name*

**assumes** *Trans*: $\Psi \rhd P \parallel Q \longmapsto ¿M(\!|N|\!) \prec R$
**and** *rPar1*: $\bigwedge P'\ A_Q\ \Psi_Q.\ [\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¿M(\!|N|\!) \prec P';\ extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$A_Q \sharp* \Psi;\ A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* M;\ A_Q \sharp* N;\ A_Q \sharp* C]\!] \Longrightarrow Prop\ (P' \parallel Q)$
**and** *rPar2*: $\bigwedge Q'\ A_P\ \Psi_P.\ [\![\Psi \otimes \Psi_P \rhd Q \longmapsto ¿M(\!|N|\!) \prec Q';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* C]\!] \Longrightarrow Prop\ (P \parallel Q')$
**and** *rBrMerge*: $\bigwedge \Psi_Q\ P'\ A_P\ \Psi_P\ Q'\ A_Q.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¿M(\!|N|\!) \prec P';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto ¿M(\!|N|\!) \prec Q';\ extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$A_P \sharp* \Psi;\ A_P \sharp* \Psi_Q;\ A_P \sharp* P;\ A_P \sharp* N;\ A_P \sharp* P';$
$A_P \sharp* Q;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* M;\ A_Q \sharp* M;$
$A_Q \sharp* \Psi;\ A_Q \sharp* \Psi_P;\ A_Q \sharp* P;\ A_Q \sharp* N;\ A_Q \sharp* P';$
$A_Q \sharp* Q;\ A_Q \sharp* Q';\ A_P \sharp* C;\ A_Q \sharp* C]\!] \Longrightarrow$
$Prop\ (P' \parallel Q')$
**shows** *Prop R*
⟨*proof*⟩

**lemma** *parOutputCases*[*consumes 5*, *case-names cPar1 cPar2*]:
**fixes** $\Psi$ :: *'b*
**and** $P$ :: *('a, 'b, 'c) psi*
**and** $Q$ :: *('a, 'b, 'c) psi*
**and** $M$ :: *'a*
**and** *xvec* :: *name list*
**and** $N$ :: *'a*
**and** $R$ :: *('a, 'b, 'c) psi*
**and** $C$ :: *'f::fs-name*

**assumes** *Trans*: $\Psi \rhd P \parallel Q \longmapsto M(\!|\nu*xvec|\!)\langle N\rangle \prec R$
**and** $xvec \sharp* \Psi$
**and** $xvec \sharp* P$
**and** $xvec \sharp* Q$
**and** $xvec \sharp* M$
**and** *rPar1*: $\bigwedge P'\ A_Q\ \Psi_Q.\ [\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu*xvec|\!)\langle N\rangle \prec P';\ extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$A_Q \sharp* \Psi;\ A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* M;\ A_Q \sharp* xvec;\ A_Q \sharp* N;\ A_Q \sharp* C;\ A_Q \sharp* xvec;\ distinct\ xvec]\!] \Longrightarrow Prop\ (P' \parallel Q)$
**and** *rPar2*: $\bigwedge Q'\ A_P\ \Psi_P.\ [\![\Psi \otimes \Psi_P \rhd Q \longmapsto M(\!|\nu*xvec|\!)\langle N\rangle \prec Q';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* M;\ A_P \sharp* xvec;\ A_P \sharp* N;\ A_P \sharp* C;\ A_P \sharp* xvec;\ distinct\ xvec]\!] \Longrightarrow Prop\ (P \parallel Q')$
**shows** *Prop R*
⟨*proof*⟩

**lemma** *parBrOutputCases*[*consumes 5*, *case-names cPar1 cPar2 cBrComm1 cBrComm2*]:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $Q :: ('a, 'b, 'c)\ psi$
**and** $M :: 'a$
**and** $xvec :: name\ list$
**and** $N :: 'a$
**and** $R :: ('a, 'b, 'c)\ psi$
**and** $C :: 'f::fs\text{-}name$

**assumes** *Trans*: $\Psi \rhd P \parallel Q \longmapsto ¡M(\!|\nu * xvec|\!)\langle N\rangle \prec R$
**and** $xvec \sharp* \Psi$
**and** $xvec \sharp* P$
**and** $xvec \sharp* Q$
**and** $xvec \sharp* M$
**and** *rPar1*: $\bigwedge P'\ A_Q\ \Psi_Q.\ [\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¡M(\!|\nu * xvec|\!)\langle N\rangle \prec P';\ extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$A_Q \sharp* \Psi;\ A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* M;\ A_Q \sharp* xvec;\ A_Q \sharp* N;\ A_Q \sharp* C;\ A_Q \sharp* xvec;\ distinct\ xvec]\!] \Longrightarrow Prop\ (P' \parallel Q)$
**and** *rPar2*: $\bigwedge Q'\ A_P\ \Psi_P.\ [\![\Psi \otimes \Psi_P \rhd Q \longmapsto ¡M(\!|\nu * xvec|\!)\langle N\rangle \prec Q';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* M;\ A_P \sharp* xvec;\ A_P \sharp* N;\ A_P \sharp* C;\ A_P \sharp* xvec;\ distinct\ xvec]\!] \Longrightarrow Prop\ (P \parallel Q')$
**and** *rBrComm1*: $\bigwedge \Psi_Q\ P'\ A_P\ \Psi_P\ Q'\ A_Q.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¿M(\!|N|\!) \prec P';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto ¡M(\!|\nu * xvec|\!)\langle N\rangle \prec Q';\ extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$distinct\ xvec;$
$A_P \sharp* \Psi;\ A_P \sharp* \Psi_Q;\ A_P \sharp* P;\ A_P \sharp* N;\ A_P \sharp* P';\ A_P \sharp* Q;\ A_P \sharp* xvec;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* C;$
$A_Q \sharp* \Psi;\ A_Q \sharp* \Psi_P;\ A_Q \sharp* P;\ A_Q \sharp* N;\ A_Q \sharp* P';\ A_Q \sharp* Q;\ A_Q \sharp* xvec;\ A_Q \sharp* Q';\ A_Q \sharp* C;$
$A_P \sharp* M;\ A_Q \sharp* M;\ xvec \sharp* M;$
$xvec \sharp* \Psi;\ xvec \sharp* \Psi_P;\ xvec \sharp* P;\ xvec \sharp* Q;\ xvec \sharp* \Psi_Q]\!] \Longrightarrow$
$Prop\ (P' \parallel Q')$
**and** *rBrComm2*: $\bigwedge \Psi_Q\ P'\ A_P\ \Psi_P\ Q'\ A_Q.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¡M(\!|\nu * xvec|\!)\langle N\rangle \prec P';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto ¿M(\!|N|\!) \prec Q';\ extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$distinct\ xvec;$
$A_P \sharp* \Psi;\ A_P \sharp* \Psi_Q;\ A_P \sharp* P;\ A_P \sharp* N;\ A_P \sharp* P';\ A_P \sharp* Q;\ A_P \sharp* xvec;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* C;$
$A_Q \sharp* \Psi;\ A_Q \sharp* \Psi_P;\ A_Q \sharp* P;\ A_Q \sharp* N;\ A_Q \sharp* P';\ A_Q \sharp* Q;\ A_Q \sharp* xvec;\ A_Q \sharp* Q';\ A_Q \sharp* C;$
$A_P \sharp* M;\ A_Q \sharp* M;\ xvec \sharp* M;$
$xvec \sharp* \Psi;\ xvec \sharp* \Psi_P;\ xvec \sharp* P;\ xvec \sharp* Q;\ xvec \sharp* \Psi_Q]\!] \Longrightarrow$

*Prop* $(P' \parallel Q')$
**shows** *Prop R*
⟨*proof*⟩

**lemma** *theEqvt*[*eqvt-force*]:
**fixes** $p$ :: *name prm*
**and** $\alpha$ :: $'a$ *action*

**assumes** $\alpha \neq \tau$

**shows** $(p \cdot the(subject\ \alpha)) = the(p \cdot (subject\ \alpha))$
⟨*proof*⟩

**lemma** *theSubjectFresh*[*simp*]:
**fixes** $\alpha$ :: $'a$ *action*
**and** $x$ :: *name*

**assumes** $\alpha \neq \tau$

**shows** $x \sharp the(subject\ \alpha) = x \sharp subject\ \alpha$
⟨*proof*⟩

**lemma** *theSubjectFreshChain*[*simp*]:
**fixes** $\alpha$ :: $'a$ *action*
**and** *xvec* :: *name list*

**assumes** $\alpha \neq \tau$

**shows** *xvec* $\sharp*$ $the(subject\ \alpha) =$ *xvec* $\sharp*$ *subject* $\alpha$
⟨*proof*⟩

**lemma** *inputObtainPrefix*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $A_P$ :: *name list*
**and** $\Psi_P$ :: $'b$
**and** $N$ :: $'a$
**and** $K$ :: $'a$
**and** $B$ :: *name list*

**assumes** $\Psi \rhd P \longmapsto K(\!|N|\!) \prec P'$
**and** *extractFrame* $P = \langle A_P, \Psi_P \rangle$
**and** *distinct* $A_P$
**and** $B \sharp* P$
**and** $A_P \sharp* \Psi$
**and** $A_P \sharp* B$
**and** $A_P \sharp* P$
**and** $A_P \sharp* K$

**obtains** $M$ **where** $\Psi \otimes \Psi_P \vdash K \leftrightarrow M$ **and** $B \sharp* M$
⟨*proof*⟩

**lemma** *outputObtainPrefix*:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $P' :: ('a, 'b, 'c)\ psi$
**and** $A_P :: name\ list$
**and** $\Psi_P :: 'b$
**and** $N :: 'a$
**and** $K :: 'a$
**and** $xvec :: name\ list$
**and** $B :: name\ list$

**assumes** $\Psi \rhd P \longmapsto ROut\ K\ ((\!|\nu * xvec|\!)N \prec' P')$
**and** $extractFrame\ P = \langle A_P, \Psi_P\rangle$
**and** $distinct\ A_P$
**and** $xvec \sharp* K$
**and** $distinct\ xvec$
**and** $B \sharp* P$
**and** $A_P \sharp* \Psi$
**and** $A_P \sharp* B$
**and** $A_P \sharp* P$
**and** $A_P \sharp* K$

**obtains** $M$ **where** $\Psi \otimes \Psi_P \vdash K \leftrightarrow M$ **and** $B \sharp* M$
⟨*proof*⟩

**lemma** *inputRenameSubject*:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $M :: 'a$
**and** $N :: 'a$
**and** $P' :: ('a, 'b, 'c)\ psi$
**and** $A_P :: name\ list$
**and** $\Psi_P :: 'b$

**assumes** $\Psi \rhd P \longmapsto M(\!|N|\!) \prec P'$
**and** $extractFrame\ P = \langle A_P, \Psi_P\rangle$
**and** $distinct\ A_P$
**and** $\Psi \otimes \Psi_P \vdash M \leftrightarrow K$
**and** $A_P \sharp* \Psi$
**and** $A_P \sharp* P$
**and** $A_P \sharp* M$
**and** $A_P \sharp* K$

**shows** $\Psi \rhd P \longmapsto K(\!|N|\!) \prec P'$
⟨*proof*⟩

**lemma** *outputRenameSubject*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** *xvec* :: *name list*
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $A_P$ :: *name list*
**and** $\Psi_P$ :: $'b$

**assumes** $\Psi \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
**and** *extractFrame* $P = \langle A_P, \Psi_P\rangle$
**and** *distinct* $A_P$
**and** $\Psi \otimes \Psi_P \vdash M \leftrightarrow K$
**and** $A_P \sharp* \Psi$
**and** $A_P \sharp* P$
**and** $A_P \sharp* M$
**and** $A_P \sharp* K$

**shows** $\Psi \rhd P \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
⟨*proof*⟩

**lemma** *parCasesSubject*[*consumes* 7, *case-names cPar1 cPar2 cComm1 cComm2 cBrMerge cBrComm1 cBrComm2*]:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $Q$ :: $('a, 'b, 'c)$ *psi*
**and** $\alpha$ :: $'a$ *action*
**and** $R$ :: $('a, 'b, 'c)$ *psi*
**and** $C$ :: $'f$::*fs-name*
**and** *yvec* :: *name list*

**assumes** *Trans*: $\Psi \rhd P \parallel Q \longmapsto \alpha \prec R$
**and** *bn* $\alpha \sharp* \Psi$
**and** *bn* $\alpha \sharp* P$
**and** *bn* $\alpha \sharp* Q$
**and** *bn* $\alpha \sharp* $ *subject* $\alpha$
**and** *yvec* $\sharp* P$
**and** *yvec* $\sharp* Q$
**and** *rPar1*: $\bigwedge P'\ A_Q\ \Psi_Q.\ [\![\Psi \otimes \Psi_Q \rhd P \longmapsto \alpha \prec P';$ *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* $A_Q$;
$A_Q \sharp* \Psi$; $A_Q \sharp* P$; $A_Q \sharp* \alpha$; $A_Q \sharp* C]\!] \Longrightarrow$ *Prop* $\alpha$ $(P' \parallel Q)$
**and** *rPar2*: $\bigwedge Q'\ A_P\ \Psi_P.\ [\![\Psi \otimes \Psi_P \rhd Q \longmapsto \alpha \prec Q';$ *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* $A_P$;
$A_P \sharp* \Psi$; $A_P \sharp* Q$; $A_P \sharp* \alpha$; $A_P \sharp* C]\!] \Longrightarrow$ *Prop* $\alpha$ $(P \parallel Q')$
**and** *rComm1*: $\bigwedge \Psi_Q\ M\ N\ P'\ A_P\ \Psi_P\ K\ xvec\ Q'\ A_Q.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P';$ *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* $A_P$;
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec Q';$ *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$;

*distinct* $A_Q$;
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$; *yvec* $\sharp*$ $M$; *yvec* $\sharp*$ $K$; *distinct xvec*; $\alpha = \tau$;
$A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* M$; $A_P \sharp* N$; $A_P \sharp* P'$; $A_P \sharp* Q$; $A_P \sharp*$ *xvec*; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* C$;
$A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$; $A_Q \sharp* P$; $A_Q \sharp* K$; $A_Q \sharp* N$; $A_Q \sharp* P'$; $A_Q \sharp* Q$; $A_Q \sharp*$ *xvec*; $A_Q \sharp* Q'$; $A_Q \sharp* C$;
*xvec* $\sharp*$ $\Psi$; *xvec* $\sharp*$ $\Psi_P$; *xvec* $\sharp*$ $P$; *xvec* $\sharp*$ $M$; *xvec* $\sharp*$ $K$; *xvec* $\sharp*$ $Q$; *xvec* $\sharp*$ $\Psi_Q$; *xvec* $\sharp*$ $C$$[\![ \Longrightarrow$
*Prop* ($\tau$) ($(\!|\nu*xvec|\!)(P' \parallel Q')$)
**and** *rComm2*: $\bigwedge \Psi_Q\ M\ xvec\ N\ P'\ A_P\ \Psi_P\ K\ Q'\ A_Q$.
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu*xvec|\!)\langle N\rangle \prec P'$; *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* $A_P$;
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q'$; *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* $A_Q$;
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$; *yvec* $\sharp*$ $M$; *yvec* $\sharp*$ $K$; *distinct xvec*; $\alpha = \tau$;
$A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* M$; $A_P \sharp* N$; $A_P \sharp* P'$; $A_P \sharp* Q$; $A_P \sharp*$ *xvec*; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* C$;
$A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$; $A_Q \sharp* P$; $A_Q \sharp* K$; $A_Q \sharp* N$; $A_Q \sharp* P'$; $A_Q \sharp* Q$; $A_Q \sharp*$ *xvec*; $A_Q \sharp* Q'$; $A_Q \sharp* C$;
*xvec* $\sharp*$ $\Psi$; *xvec* $\sharp*$ $\Psi_P$; *xvec* $\sharp*$ $P$; *xvec* $\sharp*$ $M$; *xvec* $\sharp*$ $K$; *xvec* $\sharp*$ $Q$; *xvec* $\sharp*$ $\Psi_Q$; *xvec* $\sharp*$ $C$$[\![ \Longrightarrow$
*Prop* ($\tau$) ($(\!|\nu*xvec|\!)(P' \parallel Q')$)
**and** *rBrMerge*: $\bigwedge \Psi_Q\ M\ N\ P'\ A_P\ \Psi_P\ Q'\ A_Q$.
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¿M(\!|N|\!) \prec P'$; *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* $A_P$;
$\Psi \otimes \Psi_P \rhd Q \longmapsto ¿M(\!|N|\!) \prec Q'$; *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* $A_Q$;
$A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* N$; $A_P \sharp* P'$;
$A_P \sharp* Q$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* M$; $A_Q \sharp* M$;
$A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$; $A_Q \sharp* P$; $A_Q \sharp* N$; $A_Q \sharp* P'$;
$A_Q \sharp* Q$; $A_Q \sharp* Q'$; $A_P \sharp* C$; $A_Q \sharp* C$; $\alpha = ¿M(\!|N|\!)$$[\![ \Longrightarrow$
*Prop* ($¿M(\!|N|\!)$) ($P' \parallel Q'$)
**and** *rBrComm1*: $\bigwedge \Psi_Q\ M\ N\ P'\ A_P\ \Psi_P\ xvec\ Q'\ A_Q$.
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¿M(\!|N|\!) \prec P'$; *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* $A_P$;
$\Psi \otimes \Psi_P \rhd Q \longmapsto ¡M(\!|\nu*xvec|\!)\langle N\rangle \prec Q'$; *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* $A_Q$;
*distinct xvec*;
$A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* N$; $A_P \sharp* P'$; $A_P \sharp* Q$; $A_P \sharp* xvec$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* C$;
$A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$; $A_Q \sharp* P$; $A_Q \sharp* N$; $A_Q \sharp* P'$; $A_Q \sharp* Q$; $A_Q \sharp* xvec$; $A_Q \sharp* Q'$; $A_Q \sharp* C$;
$A_P \sharp* M$; $A_Q \sharp* M$; *xvec* $\sharp*$ $M$; $¡M(\!|\nu*xvec|\!)\langle N\rangle = \alpha$;
*xvec* $\sharp*$ $\Psi$; *xvec* $\sharp*$ $\Psi_P$; *xvec* $\sharp*$ $P$; *xvec* $\sharp*$ $Q$; *xvec* $\sharp*$ $\Psi_Q$$[\![ \Longrightarrow$
*Prop* ($¡M(\!|\nu*xvec|\!)\langle N\rangle$) ($P' \parallel Q'$)
**and** *rBrComm2*: $\bigwedge \Psi_Q\ M\ xvec\ N\ P'\ A_P\ \Psi_P\ Q'\ A_Q$.
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¡M(\!|\nu*xvec|\!)\langle N\rangle \prec P'$; *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* $A_P$;
$\Psi \otimes \Psi_P \rhd Q \longmapsto ¿M(\!|N|\!) \prec Q'$; *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* $A_Q$;

*distinct xvec*;
$A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* N$; $A_P \sharp* P'$; $A_P \sharp* Q$; $A_P \sharp* xvec$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* C$;
$A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$; $A_Q \sharp* P$; $A_Q \sharp* N$; $A_Q \sharp* P'$; $A_Q \sharp* Q$; $A_Q \sharp* xvec$; $A_Q \sharp* Q'$; $A_Q \sharp* C$;
$A_P \sharp* M$; $A_Q \sharp* M$; $xvec \sharp* M$; ¡$M(\!|\nu*xvec|\!)\langle N\rangle = \alpha$;
$xvec \sharp* \Psi$; $xvec \sharp* \Psi_P$; $xvec \sharp* P$; $xvec \sharp* Q$; $xvec \sharp* \Psi_Q$]] ⟹
*Prop* (¡$M(\!|\nu*xvec|\!)\langle N\rangle$) ($P' \parallel Q'$)

**shows** *Prop* $\alpha$ $R$
⟨*proof*⟩

**lemma** *inputCases*[*consumes 1*, *case-names cInput cBrInput*]:
**fixes** $\Psi$ :: *'b*
**and** $M$ :: *'a*
**and** *xvec* :: *name list*
**and** $N$ :: *'a*
**and** $P$ :: (*'a*, *'b*, *'c*) *psi*
**and** $\alpha$ :: *'a action*
**and** $P'$ :: (*'a*, *'b*, *'c*) *psi*

**assumes** *Trans*: $\Psi \rhd M(\!|\lambda*xvec\ N|\!).P \longmapsto \alpha \prec P'$
**and** *rInput*: ⋀$K$ *Tvec*. [[$\Psi \vdash M \leftrightarrow K$; *set xvec* ⊆ *supp N*; *length xvec* = *length Tvec*; *distinct xvec*]] ⟹ *Prop* ($K(\!|N[xvec::=Tvec]|\!)$) ($P[xvec::=Tvec]$)
**and** *rBrInput*: ⋀$K$ *Tvec*. [[$\Psi \vdash K \succeq M$; *set xvec* ⊆ *supp N*; *length xvec* = *length Tvec*; *distinct xvec*]] ⟹ *Prop* (¿$K(\!|N[xvec::=Tvec]|\!)$) ($P[xvec::=Tvec]$)

**shows** *Prop* $\alpha$ $P'$
⟨*proof*⟩

**lemma** *outputCases*[*consumes 1*, *case-names cOutput cBrOutput*]:
**fixes** $\Psi$ :: *'b*
**and** $M$ :: *'a*
**and** $N$ :: *'a*
**and** $P$ :: (*'a*, *'b*, *'c*) *psi*
**and** $\alpha$ :: *'a action*
**and** $P'$ :: (*'a*, *'b*, *'c*) *psi*

**assumes** $\Psi \rhd M\langle N\rangle.P \longmapsto \alpha \prec P'$
**and** ⋀$K$. $\Psi \vdash M \leftrightarrow K$ ⟹ *Prop* ($K\langle N\rangle$) $P$
**and** ⋀$K$. $\Psi \vdash M \preceq K$ ⟹ *Prop* (¡$K\langle N\rangle$) $P$

**shows** *Prop* $\alpha$ $P'$
⟨*proof*⟩

**lemma** *caseCases*[*consumes 1*, *case-names cCase*]:
**fixes** $\Psi$ :: *'b*
**and** *Cs* :: (*'c* × (*'a*, *'b*, *'c*) *psi*) *list*
**and** $\alpha$ :: *'a action*

**and** $P' :: ('a, 'b, 'c)\ psi$

**assumes** *Trans*: $\Psi \rhd (Cases\ Cs) \longmapsto Rs$
**and** *rCase*: $\bigwedge \varphi\ P.\ [\![\Psi \rhd P \longmapsto Rs;\ (\varphi, P) \in set\ Cs;\ \Psi \vdash \varphi;\ guarded\ P]\!] \Longrightarrow Prop$

**shows** *Prop*
⟨*proof*⟩

**lemma** *resCases*[*consumes* 7, *case-names cOpen cBrOpen cRes cBrClose*]:
**fixes** $\Psi :: 'b$
**and** $x :: name$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $\alpha :: 'a\ action$
**and** $P' :: ('a, 'b, 'c)\ psi$
**and** $C :: 'f::fs\text{-}name$

**assumes** *Trans*: $\Psi \rhd (\!|\nu x|\!)P \longmapsto \alpha \prec P'$
**and** $x \sharp \Psi$
**and** $x \sharp \alpha$
**and** $x \sharp P'$
**and** $bn\ \alpha \sharp* \Psi$
**and** $bn\ \alpha \sharp* P$
**and** $bn\ \alpha \sharp* subject\ \alpha$
**and** *rOpen*: $\bigwedge M\ xvec\ yvec\ y\ N\ P'.\ [\![\Psi \rhd P \longmapsto M(\!|\nu*(xvec@yvec)|\!)\langle([(x, y)] \cdot N)\rangle \prec ([(x, y)] \cdot P');\ y \in supp\ N;$
$x \sharp N;\ x \sharp P';\ x \neq y;\ y \sharp xvec;\ y \sharp yvec;\ y \sharp M;$ *distinct xvec*; *distinct yvec*;
$xvec \sharp* \Psi;\ y \sharp \Psi;\ yvec \sharp* \Psi;\ xvec \sharp* P;\ y \sharp P;$ $yvec \sharp* P;\ xvec \sharp* M;\ y \sharp M;$
$yvec \sharp* M;\ xvec \sharp* yvec]\!] \Longrightarrow$
$Prop\ (M(\!|\nu*(xvec@y\#yvec)|\!)\langle N\rangle)\ P'$
**and** *rBrOpen*: $\bigwedge M\ xvec\ yvec\ y\ N\ P'.\ [\![\Psi \rhd P \longmapsto ¡M(\!|\nu*(xvec@yvec)|\!)\langle([(x, y)] \cdot N)\rangle \prec ([(x, y)] \cdot P');\ y \in supp\ N;$
$x \sharp N;\ x \sharp P';\ x \neq y;\ y \sharp xvec;\ y \sharp yvec;\ y \sharp M;$ *distinct xvec*; *distinct yvec*;
$xvec \sharp* \Psi;\ y \sharp \Psi;\ yvec \sharp* \Psi;\ xvec \sharp* P;\ y \sharp P;$ $yvec \sharp* P;\ xvec \sharp* M;\ y \sharp M;$
$yvec \sharp* M;\ xvec \sharp* yvec]\!] \Longrightarrow$
$Prop\ (¡M(\!|\nu*(xvec@y\#yvec)|\!)\langle N\rangle)\ P'$
**and** *rScope*: $\bigwedge P'.\ [\![\Psi \rhd P \longmapsto \alpha \prec P']\!] \Longrightarrow Prop\ \alpha\ ((\!|\nu x|\!)P')$
**and** *rBrClose*: $\bigwedge M\ xvec\ N\ P'.$
$[\![\Psi \rhd P \longmapsto ¡M(\!|\nu*xvec|\!)\langle N\rangle \prec P';$
$x \in supp\ M;$
$distinct\ xvec;\ xvec \sharp* \Psi;\ xvec \sharp* P;$
$xvec \sharp* M;$
$x \sharp \Psi;\ x \sharp xvec;$
$xvec \sharp* C]\!] \Longrightarrow Prop\ (\tau)\ ((\!|\nu x|\!)((\!|\nu*xvec|\!)P'))$
**shows** $Prop\ \alpha\ P'$

⟨*proof*⟩

**lemma** *resCases'*[*consumes 7*, *case-names cOpen cBrOpen cRes cBrClose*]:
**fixes** $\Psi$ :: *'b*
**and** $x$ :: *name*
**and** $P$ :: (*'a*, *'b*, *'c*) *psi*
**and** $\alpha$ :: *'a action*
**and** $P'$ :: (*'a*, *'b*, *'c*) *psi*
**and** $C$ :: *'f*::*fs-name*

**assumes** *Trans*: $\Psi \rhd (\!|\nu x|\!)P \longmapsto \alpha \prec P'$
**and** $x \sharp \Psi$
**and** $x \sharp \alpha$
**and** $x \sharp P'$
**and** $bn\ \alpha \sharp* \Psi$
**and** $bn\ \alpha \sharp* P$
**and** $bn\ \alpha \sharp* subject\ \alpha$
**and** *rOpen*: $\bigwedge M\ xvec\ yvec\ y\ N\ P'.\ [\![\Psi \rhd ([(x, y)] \cdot P) \longmapsto M(\!|\nu*(xvec@yvec)|\!)\langle N\rangle \prec P';\ y \in supp\ N;$
$x \sharp N;\ x \sharp P';\ x \neq y;\ y \sharp xvec;\ y \sharp yvec;\ y \sharp M;$
*distinct xvec*; *distinct yvec*;
$xvec \sharp* \Psi;\ y \sharp \Psi;\ yvec \sharp* \Psi;\ xvec \sharp* P;\ y \sharp P;$
$yvec \sharp* P;\ xvec \sharp* M;\ y \sharp M;$
$yvec \sharp* M;\ xvec \sharp* yvec]\!] \Longrightarrow$
$Prop\ (M(\!|\nu*(xvec@y\#yvec)|\!)\langle N\rangle)\ P'$
**and** *rBrOpen*: $\bigwedge M\ xvec\ yvec\ y\ N\ P'.\ [\![\Psi \rhd ([(x, y)] \cdot P) \longmapsto ¡M(\!|\nu*(xvec@yvec)|\!)\langle N\rangle \prec P';\ y \in supp\ N;$
$x \sharp N;\ x \sharp P';\ x \neq y;\ y \sharp xvec;\ y \sharp yvec;\ y \sharp M;$
*distinct xvec*; *distinct yvec*;
$xvec \sharp* \Psi;\ y \sharp \Psi;\ yvec \sharp* \Psi;\ xvec \sharp* P;\ y \sharp P;$
$yvec \sharp* P;\ xvec \sharp* M;\ y \sharp M;$
$yvec \sharp* M;\ xvec \sharp* yvec]\!] \Longrightarrow$
$Prop\ (¡M(\!|\nu*(xvec@y\#yvec)|\!)\langle N\rangle)\ P'$
**and** *rScope*: $\bigwedge P'.\ [\![\Psi \rhd P \longmapsto \alpha \prec P']\!] \Longrightarrow Prop\ \alpha\ ((\!|\nu x|\!)P')$
**and** *rBrClose*: $\bigwedge M\ xvec\ N\ P'.$
$[\![\Psi \rhd P \longmapsto ¡M(\!|\nu*xvec|\!)\langle N\rangle \prec P';$
$x \in supp\ M;$
*distinct xvec*; $xvec \sharp* \Psi;\ xvec \sharp* P;$
$xvec \sharp* M;$
$x \sharp \Psi;\ x \sharp xvec;$
$xvec \sharp* C]\!] \Longrightarrow Prop\ (\tau)\ ((\!|\nu x|\!)((\!|\nu*xvec|\!)P'))$

**shows** $Prop\ \alpha\ P'$
⟨*proof*⟩

**lemma** *resInputCases'*[*consumes 4*, *case-names cRes*]:
**fixes** $\Psi$ :: *'b*
**and** $x$ :: *name*
**and** $M$ :: *'a*

**and** $N$ :: $'a$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $R$ :: $('a, 'b, 'c)$ *psi*
**and** $C$ :: $'f$::*fs-name*

**assumes** *Trans*: $\Psi \rhd (\!|\nu x|\!)P \longmapsto M(\!|N|\!) \prec R$
**and** *2*: $x \sharp \Psi$
**and** $x \sharp (M(\!|N|\!))$
**and** *4*: $x \sharp R$
**and** *rScope*: $\bigwedge P'.\ [\![\Psi \rhd P \longmapsto M(\!|N|\!) \prec P']\!] \Longrightarrow Prop\ ((\!|\nu x|\!)P')$

**shows** *Prop R*
⟨*proof*⟩

**lemma** *resBrInputCases'*[*consumes 4*, *case-names cRes*]:
**fixes** $\Psi$ :: $'b$
**and** $x$ :: *name*
**and** $M$ :: $'a$
**and** $N$ :: $'a$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $R$ :: $('a, 'b, 'c)$ *psi*
**and** $C$ :: $'f$::*fs-name*

**assumes** *Trans*: $\Psi \rhd (\!|\nu x|\!)P \longmapsto ¿M(\!|N|\!) \prec R$
**and** *2*: $x \sharp \Psi$
**and** $x \sharp (¿M(\!|N|\!))$
**and** *4*: $x \sharp R$
**and** *rScope*: $\bigwedge P'.\ [\![\Psi \rhd P \longmapsto ¿M(\!|N|\!) \prec P']\!] \Longrightarrow Prop\ ((\!|\nu x|\!)P')$

**shows** *Prop R*
⟨*proof*⟩

**lemma** *resOutputCases'''*[*consumes 7*, *case-names cOpen cRes*]:
**fixes** $\Psi$ :: $'b$
**and** $x$ :: *name*
**and** $M$ :: $'a$
**and** $N$ :: $'a$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $R$ :: $('a, 'b, 'c)$ *psi*
**and** $C$ :: $'f$::*fs-name*

**assumes** *Trans*: $\Psi \rhd (\!|\nu x|\!)P \longmapsto M(\!|\nu*(zvec1\ @\ zvec2)|\!)\langle N\rangle \prec R$
**and** *1*: $x \sharp \Psi$
**and** $x \sharp (M(\!|\nu*(zvec1\ @\ zvec2)|\!)\langle N\rangle)$
**and** *3*: $x \sharp R$
**and** $(zvec1\ @\ zvec2) \sharp* \Psi$
**and** $(zvec1\ @\ zvec2) \sharp* P$
**and** $(zvec1\ @\ zvec2) \sharp* M$
**and** *rOpen*: $\bigwedge M\ xvec\ yvec\ y\ N\ P'.\ [\![\Psi \rhd ([(x, y)] \cdot P) \longmapsto M(\!|\nu*(xvec@yvec)|\!)\langle N\rangle$

$\prec P'$; $y \in supp\ N$;
$x \sharp N$; $x \sharp P'$; $x \neq y$; $y \sharp xvec$; $y \sharp yvec$; $y \sharp M$;
*distinct xvec*; *distinct yvec*;
$xvec \sharp* \Psi$; $y \sharp \Psi$; $yvec \sharp* \Psi$; $xvec \sharp* P$; $y \sharp P$;
$yvec \sharp* P$; $xvec \sharp* M$; $y \sharp M$;
$yvec \sharp* M$; $xvec \sharp* yvec$$\rrbracket \Longrightarrow$
*Prop* $P'$
**and** *rScope*: $\bigwedge P'.\ \llbracket \Psi \rhd P \longmapsto M(\!|\nu*(zvec1\ @\ zvec2)|\!)\langle N\rangle \prec P' \rrbracket \Longrightarrow$ *Prop* $((\!|\nu x|\!)P')$

**shows** *Prop* $R$
⟨*proof*⟩

**lemma** *resOutputCases''*[*consumes* 7, *case-names cOpen cRes*]:
**fixes** $\Psi$ :: $'b$
**and** $x$ :: *name*
**and** $z$ :: *name*
**and** $M$ :: $'a$
**and** $N$ :: $'a$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $R$ :: $('a, 'b, 'c)$ *psi*
**and** $C$ :: $'f$::*fs-name*

**assumes** *Trans*: $\Psi \rhd (\!|\nu x|\!)P \longmapsto M(\!|\nu*(zvec1@y\#zvec2)|\!)\langle N\rangle \prec R$
**and** *1*: $x \sharp \Psi$
**and** $x \sharp (M(\!|\nu*(zvec1@y\#zvec2)|\!)\langle N\rangle)$
**and** *3*: $x \sharp R$
**and** $(zvec1@y\#zvec2) \sharp* \Psi$
**and** $(zvec1@y\#zvec2) \sharp* P$
**and** $(zvec1@y\#zvec2) \sharp* M$
**and** *rOpen*: $\bigwedge M\ xvec\ yvec\ y\ N\ P'.\ \llbracket \Psi \rhd ([(x, y)] \cdot P) \longmapsto M(\!|\nu*(xvec@yvec)|\!)\langle N\rangle \prec P'$; $y \in supp\ N$;
$x \sharp N$; $x \sharp P'$; $x \neq y$; $y \sharp xvec$; $y \sharp yvec$; $y \sharp M$;
*distinct xvec*; *distinct yvec*;
$xvec \sharp* \Psi$; $y \sharp \Psi$; $yvec \sharp* \Psi$; $xvec \sharp* P$; $y \sharp P$;
$yvec \sharp* P$; $xvec \sharp* M$; $y \sharp M$;
$yvec \sharp* M$; $xvec \sharp* yvec$$\rrbracket \Longrightarrow$
*Prop* $P'$
**and** *rScope*: $\bigwedge P'.\ \llbracket \Psi \rhd P \longmapsto M(\!|\nu*(zvec1@y\#zvec2)|\!)\langle N\rangle \prec P' \rrbracket \Longrightarrow$ *Prop* $((\!|\nu x|\!)P')$

**shows** *Prop* $R$
⟨*proof*⟩

**abbreviation**
*statImpJudge* (- ↪ - [80, 80] 80)
**where** $\Psi \hookrightarrow \Psi' \equiv AssertionStatImp\ \Psi\ \Psi'$

**lemma** *statEqTransition*:

**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $Rs$ :: $('a, 'b, 'c)$ *residual*
**and** $\Psi'$ :: $'b$

**assumes** $\Psi \rhd P \longmapsto Rs$
**and** $\Psi \simeq \Psi'$

**shows** $\Psi' \rhd P \longmapsto Rs$
⟨*proof*⟩

**lemma** *brInputTermSupp*:
**fixes** $\Psi$ :: $'b$::*fs-name*
**and** $P$ :: $('a, 'b, ('c$::*fs-name*$))$ *psi*
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $N$ :: $'a$::*fs-name*
**and** $K$ :: $'a$::*fs-name*

**assumes** $\Psi \rhd P \longmapsto ¿K(\!|N|\!) \prec P'$

**shows** (*supp* $K$) $\subseteq$ ((*supp* $P$)::*name set*)
⟨*proof*⟩

**lemma** *brOutputTermSupp*:
**fixes** $\Psi$ :: $'b$::*fs-name*
**and** $P$ :: $('a, 'b, ('c$::*fs-name*$))$ *psi*
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $N$ :: $'a$::*fs-name*
**and** $K$ :: $'a$::*fs-name*
**and** *xvec* :: *name list*

**assumes** $\Psi \rhd P \longmapsto RBrOut\ K\ (([\!|\nu * xvec|\!])N \prec' P')$

**shows** (*supp* $K$) $\subseteq$ ((*supp* $P$)::*name set*)
⟨*proof*⟩

**lemma** *actionPar1Dest'*:
**fixes** $\alpha$ :: $('a$::*fs-name*$)$ *action*
**and** $P$ :: $('a, 'b$::*fs-name*$, 'c$::*fs-name*$)$ *psi*
**and** $\beta$ :: $('a$::*fs-name*$)$ *action*
**and** $Q$ :: $('a, 'b, 'c)$ *psi*
**and** $R$ :: $('a, 'b, 'c)$ *psi*

**assumes** $\alpha \prec P = \beta \prec (Q \parallel R)$
**and** *bn* $\alpha \sharp* R$
**and** *bn* $\beta \sharp* R$

**obtains** $T$ **where** $P = T \parallel R$ **and** $\alpha \prec T = \beta \prec Q$
⟨*proof*⟩

**lemma** *actionPar2Dest'*:
**fixes** $\alpha$ :: ('*a*::*fs-name*) *action*
**and** $P$ :: ('*a*, '*b*::*fs-name*, '*c*::*fs-name*) *psi*
**and** $\beta$ :: ('*a*::*fs-name*) *action*
**and** $Q$ :: ('*a*, '*b*, '*c*) *psi*
**and** $R$ :: ('*a*, '*b*, '*c*) *psi*

**assumes** $\alpha \prec P = \beta \prec (Q \parallel R)$
**and** *bn* $\alpha \sharp* Q$
**and** *bn* $\beta \sharp* Q$

**obtains** $T$ **where** $P = Q \parallel T$ **and** $\alpha \prec T = \beta \prec R$
⟨*proof*⟩

**lemma** *expandNonTauFrame*:
**fixes** $\Psi$ :: '*b*
**and** $P$ :: ('*a*, '*b*, '*c*) *psi*
**and** $\alpha$ :: '*a action*
**and** $P'$ :: ('*a*, '*b*, '*c*) *psi*
**and** $A_P$ :: *name list*
**and** $\Psi_P$ :: '*b*
**and** $C$ :: '*f*::*fs-name*
**and** $C'$ :: '*g*::*fs-name*

**assumes** *Trans*: $\Psi \rhd P \longmapsto \alpha \prec P'$
**and** *extractFrame* $P = \langle A_P, \Psi_P \rangle$
**and** *distinct* $A_P$
**and** *bn* $\alpha \sharp* subject\ \alpha$
**and** $A_P \sharp* P$
**and** $A_P \sharp* \alpha$
**and** $A_P \sharp* C$
**and** $A_P \sharp* C'$
**and** *bn* $\alpha \sharp* P$
**and** *bn* $\alpha \sharp* C'$
**and** $\alpha \neq \tau$

**obtains** $p$ $\Psi'$ $A_P'$ $\Psi_P'$ **where** *set* $p \subseteq set(bn\ \alpha) \times set(bn(p \cdot \alpha))$ **and** $(p \cdot \Psi_P) \otimes \Psi' \simeq \Psi_P'$ **and** *distinctPerm* $p$ **and**
*extractFrame* $P' = \langle A_P', \Psi_P' \rangle$ **and** $A_P' \sharp* P'$ **and** $A_P' \sharp* \alpha$ **and** $A_P' \sharp* (p \cdot \alpha)$ **and**
$A_P' \sharp* C$ **and** $(bn(p \cdot \alpha)) \sharp* C'$ **and** $(bn(p \cdot \alpha)) \sharp* \alpha$ **and** $(bn(p \cdot \alpha)) \sharp* P'$ **and** *distinct* $A_P'$
⟨*proof*⟩

**lemma** *expandTauFrame*:
**fixes** $\Psi$ :: '*b*
**and** $P$ :: ('*a*, '*b*,'*c*) *psi*
**and** $P'$ :: ('*a*, '*b*, '*c*) *psi*

**and** $A_P$ :: *name list*
**and** $\Psi_P$ :: $'b$
**and** $C$ :: $'f$::*fs-name*

**assumes** $\Psi \rhd P \longmapsto \tau \prec P'$
**and** *extractFrame* $P = \langle A_P, \Psi_P \rangle$
**and** *distinct* $A_P$
**and** $A_P \sharp* P$
**and** $A_P \sharp* C$

**obtains** $\Psi'$ $A_P'$ $\Psi_P'$ **where** *extractFrame* $P' = \langle A_P', \Psi_P' \rangle$ **and** $\Psi_P \otimes \Psi' \simeq \Psi_P'$ **and** $A_P' \sharp* C$ **and** $A_P' \sharp* P'$ **and** *distinct* $A_P'$
⟨*proof*⟩

**lemma** *expandFrame*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $\alpha$ :: $'a$ *action*
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $A_P$ :: *name list*
**and** $\Psi_P$ :: $'b$
**and** $C$ :: $'f$::*fs-name*
**and** $C'$ :: $'g$::*fs-name*

**assumes** *Trans*: $\Psi \rhd P \longmapsto \alpha \prec P'$
**and** *extractFrame* $P = \langle A_P, \Psi_P \rangle$
**and** *distinct* $A_P$
**and** *bn* $\alpha \sharp* subject\ \alpha$
**and** *distinct*(*bn* $\alpha$)
**and** $A_P \sharp* \alpha$
**and** $A_P \sharp* P$
**and** $A_P \sharp* C$
**and** $A_P \sharp* C'$
**and** *bn* $\alpha \sharp* P$
**and** *bn* $\alpha \sharp* C'$

**obtains** $p$ $\Psi'$ $A_P'$ $\Psi_P'$ **where** *set* $p \subseteq$ *set*(*bn* $\alpha$) $\times$ *set*(*bn*$(p \cdot \alpha)$) **and** $(p \cdot \Psi_P) \otimes \Psi' \simeq \Psi_P'$ **and** *distinctPerm* $p$ **and**
*extractFrame* $P' = \langle A_P', \Psi_P' \rangle$ **and** $A_P' \sharp* P'$ **and** $A_P' \sharp* \alpha$ **and** $A_P' \sharp* (p \cdot \alpha)$ **and**
$A_P' \sharp* C$ **and** $(bn(p \cdot \alpha)) \sharp* C'$ **and** $(bn(p \cdot \alpha)) \sharp* \alpha$ **and** $(bn(p \cdot \alpha)) \sharp* P'$ **and** *distinct* $A_P'$
⟨*proof*⟩

**abbreviation**
*frameImpJudge* (- $\hookrightarrow_F$ - [80, 80] 80)
**where** $F \hookrightarrow_F G \equiv$ *FrameStatImp* $F$ $G$

**lemma** *FrameStatEqImpCompose*:

**fixes** $F :: 'b\ frame$
**and** $G :: 'b\ frame$
**and** $H :: 'b\ frame$
**and** $I :: 'b\ frame$

**assumes** $F \simeq_F G$
**and** $G \hookrightarrow_F H$
**and** $H \simeq_F I$

**shows** $F \hookrightarrow_F I$
⟨*proof*⟩

**lemma** *transferNonTauFrame*:
**fixes** $\Psi_F :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $\alpha :: 'a\ action$
**and** $P' :: ('a, 'b, 'c)\ psi$
**and** $A_F :: name\ list$
**and** $A_G :: name\ list$
**and** $\Psi_G :: 'b$

**assumes** $\Psi_F \rhd P \longmapsto \alpha \prec P'$
**and** $extractFrame\ P = \langle A_P, \Psi_P \rangle$
**and** $distinct\ A_P$
**and** $distinct(bn\ \alpha)$
**and** $\langle A_F, \Psi_F \otimes \Psi_P \rangle \hookrightarrow_F \langle A_G, \Psi_G \otimes \Psi_P \rangle$
**and** $A_F \sharp* P$
**and** $A_G \sharp* P$
**and** $A_F \sharp* subject\ \alpha$
**and** $A_G \sharp* subject\ \alpha$
**and** $A_P \sharp* A_F$
**and** $A_P \sharp* A_G$
**and** $A_P \sharp* \Psi_F$
**and** $A_P \sharp* \Psi_G$
**and** $\alpha \neq \tau$

**shows** $\Psi_G \rhd P \longmapsto \alpha \prec P'$
⟨*proof*⟩

**lemma** *transferTauFrame*:
**fixes** $\Psi_F :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $P' :: ('a, 'b, 'c)\ psi$
**and** $A_F :: name\ list$
**and** $A_G :: name\ list$
**and** $\Psi_G :: 'b$

**assumes** $\Psi_F \rhd P \longmapsto \tau \prec P'$
**and** $extractFrame\ P = \langle A_P, \Psi_P \rangle$

**and** $\mathit{distinct}\ A_P$
**and** $\langle A_F, \Psi_F \otimes \Psi_P\rangle \hookrightarrow_F \langle A_G, \Psi_G \otimes \Psi_P\rangle$
**and** $A_F \sharp* P$
**and** $A_G \sharp* P$
**and** $A_P \sharp* A_F$
**and** $A_P \sharp* A_G$
**and** $A_P \sharp* \Psi_F$
**and** $A_P \sharp* \Psi_G$

**shows** $\Psi_G \rhd P \longmapsto\tau \prec P'$
⟨*proof*⟩

**lemma** *transferFrame*:
**fixes** $\Psi_F$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $\alpha$ :: $'a\ action$
**and** $P'$ :: $('a, 'b, 'c)\ psi$
**and** $A_F$ :: *name list*
**and** $A_G$ :: *name list*
**and** $\Psi_G$ :: $'b$

**assumes** $\Psi_F \rhd P \longmapsto\alpha \prec P'$
**and** $\mathit{extractFrame}\ P = \langle A_P, \Psi_P\rangle$
**and** $\mathit{distinct}\ A_P$
**and** $\langle A_F, \Psi_F \otimes \Psi_P\rangle \hookrightarrow_F \langle A_G, \Psi_G \otimes \Psi_P\rangle$
**and** $A_F \sharp* P$
**and** $A_G \sharp* P$
**and** $A_F \sharp* \mathit{subject}\ \alpha$
**and** $A_G \sharp* \mathit{subject}\ \alpha$
**and** $A_P \sharp* A_F$
**and** $A_P \sharp* A_G$
**and** $A_P \sharp* \Psi_F$
**and** $A_P \sharp* \Psi_G$

**shows** $\Psi_G \rhd P \longmapsto\alpha \prec P'$
⟨*proof*⟩

**lemma** *parCasesInputFrame*[*consumes 7*, *case-names cPar1 cPar2*]:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $Q$ :: $('a, 'b, 'c)\ psi$
**and** $M$ :: $'a$
**and** *xvec* :: *name list*
**and** $N$ :: $'a$
**and** $T$ :: $('a, 'b, 'c)\ psi$
**and** $C$ :: $'d$::*fs-name*

**assumes** *Trans*: $\Psi \rhd P \parallel Q \longmapsto M(\!\lfloor N \rfloor\!) \prec T$
**and** $\mathit{extractFrame}(P \parallel Q) = \langle A_{PQ}, \Psi_{PQ}\rangle$

**and** *distinct* $A_{PQ}$
**and** $A_{PQ} \sharp* \Psi$
**and** $A_{PQ} \sharp* P$
**and** $A_{PQ} \sharp* Q$
**and** $A_{PQ} \sharp* M$
**and** *rPar1*: $\bigwedge P'\ A_P\ \Psi_P\ A_Q\ \Psi_Q.\ [\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P'; \mathit{extractFrame}\ P = \langle A_P, \Psi_P\rangle; \mathit{extractFrame}\ Q = \langle A_Q, \Psi_Q\rangle;$
$\mathit{distinct}\ A_P; \mathit{distinct}\ A_Q; A_P \sharp* \Psi; A_P \sharp* P; A_P \sharp* Q; A_P \sharp* M;\ A_Q \sharp* \Psi; A_Q \sharp* P; A_Q \sharp* Q; A_Q \sharp* M;$
$A_P \sharp* \Psi_Q; A_Q \sharp* \Psi_P; A_P \sharp* A_Q; A_{PQ} = A_P @ A_Q; \Psi_{PQ} = \Psi_P \otimes \Psi_Q]\!] \Longrightarrow \mathit{Prop}\ (P' \parallel Q)$
**and** *rPar2*: $\bigwedge Q'\ A_P\ \Psi_P\ A_Q\ \Psi_Q.\ [\![\Psi \otimes \Psi_P \rhd Q \longmapsto M(\!|N|\!) \prec Q'; \mathit{extractFrame}\ P = \langle A_P, \Psi_P\rangle; \mathit{extractFrame}\ Q = \langle A_Q, \Psi_Q\rangle;$
$\mathit{distinct}\ A_P; \mathit{distinct}\ A_Q; A_P \sharp* \Psi; A_P \sharp* P; A_P \sharp* Q; A_P \sharp* M;\ A_Q \sharp* \Psi; A_Q \sharp* P; A_Q \sharp* Q; A_Q \sharp* M;$
$A_P \sharp* \Psi_Q; A_Q \sharp* \Psi_P; A_P \sharp* A_Q; A_{PQ} = A_P @ A_Q; \Psi_{PQ} = \Psi_P \otimes \Psi_Q]\!] \Longrightarrow \mathit{Prop}\ (P \parallel Q')$
**shows** *Prop T*
⟨*proof*⟩

**lemma** *parCasesBrInputFrame*[*consumes* 7, *case-names cPar1 cPar2 cBrMerge*]:
**fixes** $\Psi$ :: *'b*
**and** $P$ :: *('a, 'b, 'c) psi*
**and** $Q$ :: *('a, 'b, 'c) psi*
**and** $M$ :: *'a*
**and** *xvec* :: *name list*
**and** $N$ :: *'a*
**and** $T$ :: *('a, 'b, 'c) psi*
**and** $C$ :: *'d*::*fs-name*

**assumes** *Trans*: $\Psi \rhd P \parallel Q \longmapsto ¿M(\!|N|\!) \prec T$
**and** $\mathit{extractFrame}(P \parallel Q) = \langle A_{PQ}, \Psi_{PQ}\rangle$
**and** $\mathit{distinct}\ A_{PQ}$
**and** $A_{PQ} \sharp* \Psi$
**and** $A_{PQ} \sharp* P$
**and** $A_{PQ} \sharp* Q$
**and** $A_{PQ} \sharp* M$
**and** *rPar1*: $\bigwedge P'\ A_P\ \Psi_P\ A_Q\ \Psi_Q.\ [\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¿M(\!|N|\!) \prec P'; \mathit{extractFrame}\ P = \langle A_P, \Psi_P\rangle; \mathit{extractFrame}\ Q = \langle A_Q, \Psi_Q\rangle;$
$\mathit{distinct}\ A_P; \mathit{distinct}\ A_Q; A_P \sharp* \Psi; A_P \sharp* P; A_P \sharp* Q; A_P \sharp* M;\ A_Q \sharp* \Psi; A_Q \sharp* P; A_Q \sharp* Q; A_Q \sharp* M;$
$A_P \sharp* \Psi_Q; A_Q \sharp* \Psi_P; A_P \sharp* A_Q; A_{PQ} = A_P @ A_Q; \Psi_{PQ} = \Psi_P \otimes \Psi_Q]\!] \Longrightarrow \mathit{Prop}\ (P' \parallel Q)$
**and** *rPar2*: $\bigwedge Q'\ A_P\ \Psi_P\ A_Q\ \Psi_Q.\ [\![\Psi \otimes \Psi_P \rhd Q \longmapsto ¿M(\!|N|\!) \prec Q'; \mathit{extractFrame}\ P = \langle A_P, \Psi_P\rangle; \mathit{extractFrame}\ Q = \langle A_Q, \Psi_Q\rangle;$
$\mathit{distinct}\ A_P; \mathit{distinct}\ A_Q; A_P \sharp* \Psi; A_P \sharp* P; A_P \sharp* Q; A_P \sharp* M;\ A_Q \sharp* \Psi; A_Q \sharp* P; A_Q \sharp* Q; A_Q \sharp* M;$
$A_P \sharp* \Psi_Q; A_Q \sharp* \Psi_P; A_P \sharp* A_Q; A_{PQ} = A_P @ A_Q; \Psi_{PQ} = \Psi_P \otimes \Psi_Q]\!] \Longrightarrow \mathit{Prop}\ (P \parallel Q')$

**and** *rBrMerge*: $\bigwedge \Psi_Q\ P'\ A_P\ \Psi_P\ Q'\ A_Q.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¿M(\!|N|\!) \prec P';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;$
$distinct\ A_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto ¿M(\!|N|\!) \prec Q';\ extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;$
$distinct\ A_Q;$
$A_P \sharp* \Psi;\ A_P \sharp* \Psi_Q;\ A_P \sharp* P;$
$A_P \sharp* Q;\ A_P \sharp* A_Q;$
$A_Q \sharp* \Psi;\ A_Q \sharp* \Psi_P;\ A_Q \sharp* P;$
$A_Q \sharp* Q;$
$A_{PQ} = A_P@A_Q;\ \Psi_{PQ} = \Psi_P \otimes \Psi_Q]\!] \Longrightarrow$
$Prop\ (P' \parallel Q')$
**shows** *Prop T*
⟨*proof*⟩

**lemma** *parCasesOutputFrame*[*consumes 11*, *case-names cPar1 cPar2*]:
**fixes** $\Psi$ :: *'b*
**and** $P$ :: *('a, 'b, 'c) psi*
**and** $Q$ :: *('a, 'b, 'c) psi*
**and** $M$ :: *'a*
**and** *xvec* :: *name list*
**and** $N$ :: *'a*
**and** $T$ :: *('a, 'b, 'c) psi*
**and** $C$ :: *'d::fs-name*

**assumes** *Trans*: $\Psi \rhd P \parallel Q \longmapsto M(\!|\nu*xvec|\!)\langle N\rangle \prec T$
**and** $xvec \sharp* \Psi$
**and** $xvec \sharp* P$
**and** $xvec \sharp* Q$
**and** $xvec \sharp* M$
**and** $extractFrame(P \parallel Q) = \langle A_{PQ}, \Psi_{PQ}\rangle$
**and** $distinct\ A_{PQ}$
**and** $A_{PQ} \sharp* \Psi$
**and** $A_{PQ} \sharp* P$
**and** $A_{PQ} \sharp* Q$
**and** $A_{PQ} \sharp* M$
**and** *rPar1*: $\bigwedge P'\ A_P\ \Psi_P\ A_Q\ \Psi_Q.\ [\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu*xvec|\!)\langle N\rangle \prec P';$
$extractFrame\ P = \langle A_P, \Psi_P\rangle;\ extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;$
$distinct\ A_P;\ distinct\ A_Q;\ A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* M;\ A_Q \sharp* \Psi;\ A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* M;$
$A_P \sharp* \Psi_Q;\ A_Q \sharp* \Psi_P;\ A_P \sharp* A_Q;\ A_{PQ} = A_P@A_Q;$
$\Psi_{PQ} = \Psi_P \otimes \Psi_Q]\!] \Longrightarrow Prop\ (P' \parallel Q)$
**and** *rPar2*: $\bigwedge Q'\ A_P\ \Psi_P\ A_Q\ \Psi_Q.\ [\![\Psi \otimes \Psi_P \rhd Q \longmapsto M(\!|\nu*xvec|\!)\langle N\rangle \prec Q';$
$extractFrame\ P = \langle A_P, \Psi_P\rangle;\ extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;$
$distinct\ A_P;\ distinct\ A_Q;\ A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* M;\ A_Q \sharp* \Psi;\ A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* M;$
$A_P \sharp* \Psi_Q;\ A_Q \sharp* \Psi_P;\ A_P \sharp* A_Q;\ A_{PQ} = A_P@A_Q;$
$\Psi_{PQ} = \Psi_P \otimes \Psi_Q]\!] \Longrightarrow Prop\ (P \parallel Q')$
**shows** *Prop T*
⟨*proof*⟩

**lemma** *parCasesBrOutputFrame*[*consumes 11*, *case-names cPar1 cPar2 cBrComm1 cBrComm2*]:
**fixes** $\Psi$ :: *'b*
**and** $P$ :: (*'a*, *'b*, *'c*) *psi*
**and** $Q$ :: (*'a*, *'b*, *'c*) *psi*
**and** $M$ :: *'a*
**and** *xvec* :: *name list*
**and** $N$ :: *'a*
**and** $T$ :: (*'a*, *'b*, *'c*) *psi*
**and** $C$ :: *'d*::*fs-name*

**assumes** *Trans*: $\Psi \rhd P \parallel Q \longmapsto ¡M(\!|\nu * xvec|\!)\langle N\rangle \prec T$
**and** *xvec* $\sharp*$ $\Psi$
**and** *xvec* $\sharp*$ $P$
**and** *xvec* $\sharp*$ $Q$
**and** *xvec* $\sharp*$ $M$
**and** *extractFrame*$(P \parallel Q) = \langle A_{PQ}, \Psi_{PQ}\rangle$
**and** *distinct* $A_{PQ}$
**and** $A_{PQ}$ $\sharp*$ $\Psi$
**and** $A_{PQ}$ $\sharp*$ $P$
**and** $A_{PQ}$ $\sharp*$ $Q$
**and** $A_{PQ}$ $\sharp*$ $M$
**and** *rPar1*: $\bigwedge P'\ A_P\ \Psi_P\ A_Q\ \Psi_Q.$ $[\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¡M(\!|\nu * xvec|\!)\langle N\rangle \prec P';$ *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$;
*distinct* $A_P$; *distinct* $A_Q$; $A_P$ $\sharp*$ $\Psi$; $A_P$ $\sharp*$ $P$; $A_P$ $\sharp*$ $Q$; $A_P$ $\sharp*$ $M$; $A_Q$ $\sharp*$ $\Psi$; $A_Q$ $\sharp*$ $P$; $A_Q$ $\sharp*$ $Q$; $A_Q$ $\sharp*$ $M$;
$A_P$ $\sharp*$ $\Psi_Q$; $A_Q$ $\sharp*$ $\Psi_P$; $A_P$ $\sharp*$ $A_Q$; $A_{PQ} = A_P@A_Q$; $\Psi_{PQ} = \Psi_P \otimes \Psi_Q]\!] \Longrightarrow$ *Prop* $(P' \parallel Q)$
**and** *rPar2*: $\bigwedge Q'\ A_P\ \Psi_P\ A_Q\ \Psi_Q.$ $[\![\Psi \otimes \Psi_P \rhd Q \longmapsto ¡M(\!|\nu * xvec|\!)\langle N\rangle \prec Q';$ *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$;
*distinct* $A_P$; *distinct* $A_Q$; $A_P$ $\sharp*$ $\Psi$; $A_P$ $\sharp*$ $P$; $A_P$ $\sharp*$ $Q$; $A_P$ $\sharp*$ $M$; $A_Q$ $\sharp*$ $\Psi$; $A_Q$ $\sharp*$ $P$; $A_Q$ $\sharp*$ $Q$; $A_Q$ $\sharp*$ $M$;
$A_P$ $\sharp*$ $\Psi_Q$; $A_Q$ $\sharp*$ $\Psi_P$; $A_P$ $\sharp*$ $A_Q$; $A_{PQ} = A_P@A_Q$; $\Psi_{PQ} = \Psi_P \otimes \Psi_Q]\!] \Longrightarrow$ *Prop* $(P \parallel Q')$
**and** *rBrComm1*: $\bigwedge \Psi_Q\ P'\ A_P\ \Psi_P\ Q'\ A_Q.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¿M(\!|N|\!) \prec P';$ *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* $A_P$;
$\Psi \otimes \Psi_P \rhd Q \longmapsto ¡M(\!|\nu * xvec|\!)\langle N\rangle \prec Q';$ *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* $A_Q$;
*distinct xvec*;
$A_P$ $\sharp*$ $\Psi$; $A_P$ $\sharp*$ $\Psi_Q$; $A_P$ $\sharp*$ $P$; $A_P$ $\sharp*$ $Q$; $A_P$ $\sharp*$ $A_Q$;
$A_Q$ $\sharp*$ $\Psi$; $A_Q$ $\sharp*$ $\Psi_P$; $A_Q$ $\sharp*$ $P$; $A_Q$ $\sharp*$ $Q$;
$A_P$ $\sharp*$ $M$; $A_Q$ $\sharp*$ $M$; *xvec* $\sharp*$ $M$;
*xvec* $\sharp*$ $\Psi$; *xvec* $\sharp*$ $P$; *xvec* $\sharp*$ $Q$;
$A_{PQ} = A_P@A_Q$; $\Psi_{PQ} = \Psi_P \otimes \Psi_Q]\!] \Longrightarrow$
*Prop* $(P' \parallel Q')$
**and** *rBrComm2*: $\bigwedge \Psi_Q\ P'\ A_P\ \Psi_P\ Q'\ A_Q.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¡M(\!|\nu * xvec|\!)\langle N\rangle \prec P';$ *extractFrame* $P = \langle A_P, \Psi_P\rangle$;

*distinct* $A_P$;
$\Psi \otimes \Psi_P \rhd Q \longmapsto ¿M(\!|N|\!) \prec Q'$; *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* $A_Q$;
*distinct xvec*;
$A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* Q$; $A_P \sharp* A_Q$;
$A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$; $A_Q \sharp* P$; $A_Q \sharp* Q$;
$A_P \sharp* M$; $A_Q \sharp* M$; *xvec* $\sharp* M$;
*xvec* $\sharp* \Psi$; *xvec* $\sharp* P$; *xvec* $\sharp* Q$;
$A_{PQ} = A_P @ A_Q$; $\Psi_{PQ} = \Psi_P \otimes \Psi_Q]\!] \Longrightarrow$
*Prop* $(P' \parallel Q')$
**shows** *Prop* $T$
⟨*proof*⟩

**inductive** *bangPred* :: $('a, 'b, 'c)$ *psi* ⇒ $('a, 'b, 'c)$ *psi* ⇒ *bool*
**where**
*aux1*: *bangPred* $P$ $(!P)$
| *aux2*: *bangPred* $P$ $(P \parallel !P)$

**lemma** *bangInduct*[*consumes 1*, *case-names cPar1 cPar2 cComm1 cComm2 cBrMerge cBrComm1 cBrComm2 cBang*]:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** *Rs* :: $('a, 'b, 'c)$ *residual*
**and** *Prop* :: $'d$::*fs-name* ⇒ $'b$ ⇒ $('a, 'b, 'c)$ *psi* ⇒ $('a, 'b, 'c)$ *residual* ⇒ *bool*
**and** $C$ :: $'d$

**assumes** $\Psi \rhd !P \longmapsto Rs$
**and** *rPar1*: $\bigwedge \alpha\ P'\ C.\ [\![\Psi \rhd P \longmapsto \alpha \prec P'$; *bn* $\alpha \sharp* \Psi$; *bn* $\alpha \sharp* P$; *bn* $\alpha \sharp*$ *subject* $\alpha$; *bn* $\alpha \sharp* C$; *distinct*(*bn* $\alpha$)$]\!] \Longrightarrow$ *Prop* $C\ \Psi\ (P \parallel !P)\ (\alpha \prec (P' \parallel !P))$
**and** *rPar2*: $\bigwedge \alpha\ P'\ C.\ [\![\Psi \rhd !P \longmapsto \alpha \prec P'$; *bn* $\alpha \sharp* \Psi$; *bn* $\alpha \sharp* P$; *bn* $\alpha \sharp*$ *subject* $\alpha$; *bn* $\alpha \sharp* C$; *distinct*(*bn* $\alpha$)$;$
$\bigwedge C.$ *Prop* $C\ \Psi\ (!P)\ (\alpha \prec P')]\!] \Longrightarrow$ *Prop* $C\ \Psi\ (P \parallel !P)\ (\alpha \prec (P \parallel P'))$
**and** *rComm1*: $\bigwedge M\ N\ P'\ K$ *xvec* $P''\ C.\ [\![\Psi \rhd P \longmapsto M(\!|N|\!) \prec P'$; $\Psi \rhd !P \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec P''$; $\bigwedge C.$ *Prop* $C\ \Psi\ (!P)\ (K(\!|\nu * xvec|\!)\langle N\rangle \prec P'')$; $\Psi \vdash M \leftrightarrow K$;
*xvec* $\sharp* \Psi$; *xvec* $\sharp* P$; *xvec* $\sharp* M$; *xvec* $\sharp* K$; *xvec* $\sharp* C$; *distinct xvec*$]\!] \Longrightarrow$ *Prop* $C\ \Psi\ (P \parallel !P)\ (\tau \prec (\!|\nu * xvec|\!)(P' \parallel P''))$
**and** *rComm2*: $\bigwedge M$ *xvec* $N\ P'\ K\ P''\ C.\ [\![\Psi \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$; $\Psi \rhd !P \longmapsto K(\!|N|\!) \prec P''$; $\bigwedge C.$ *Prop* $C\ \Psi\ (!P)\ (K(\!|N|\!) \prec P'')$; $\Psi \vdash M \leftrightarrow K$;
*xvec* $\sharp* \Psi$; *xvec* $\sharp* P$; *xvec* $\sharp* M$; *xvec* $\sharp* K$; *xvec* $\sharp* C$; *distinct xvec*$]\!] \Longrightarrow$ *Prop* $C\ \Psi\ (P \parallel !P)\ (\tau \prec (\!|\nu * xvec|\!)(P' \parallel P''))$
**and** *rBrMerge*: $\bigwedge M\ N\ P'\ P''\ C.\ [\![\Psi \rhd P \longmapsto ¿M(\!|N|\!) \prec P'$; $\Psi \rhd !P \longmapsto ¿M(\!|N|\!) \prec P''$; $\bigwedge C.$ *Prop* $C\ \Psi\ (!P)\ (¿M(\!|N|\!) \prec P'')]\!] \Longrightarrow$
*Prop* $C\ \Psi\ (P \parallel !P)\ (¿M(\!|N|\!) \prec (P' \parallel P''))$
**and** *rBrComm1*: $\bigwedge M\ N\ P'$ *xvec* $P''\ C.\ [\![\Psi \rhd P \longmapsto ¿M(\!|N|\!) \prec P'$; $\Psi \rhd !P \longmapsto ¡M(\!|\nu * xvec|\!)\langle N\rangle \prec P''$; $\bigwedge C.$ *Prop* $C\ \Psi\ (!P)\ (¡M(\!|\nu * xvec|\!)\langle N\rangle \prec P'')$;
*xvec* $\sharp* \Psi$; *xvec* $\sharp* P$; *xvec* $\sharp* M$; *xvec* $\sharp* C$; *distinct xvec*$]\!] \Longrightarrow$ *Prop* $C\ \Psi\ (P \parallel !P)\ (¡M(\!|\nu * xvec|\!)\langle N\rangle \prec (P' \parallel P''))$

**and** *rBrComm2*: $\bigwedge M\ N\ P'\ xvec\ P''\ C.\ [\![\Psi \rhd P \longmapsto ¡M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P';\ \Psi \rhd\ !P \longmapsto ¿M(\!|N|\!) \prec P'';\ \bigwedge C.\ Prop\ C\ \Psi\ (!P)\ (¿M(\!|N|\!) \prec P'');\ xvec \sharp{*} \Psi;\ xvec \sharp{*} P;\ xvec \sharp{*} M;\ xvec \sharp{*} C;\ distinct\ xvec]\!] \Longrightarrow Prop\ C\ \Psi\ (P \parallel !P)\ (¡M(\!|\nu{*}xvec|\!)\langle N\rangle \prec (P' \parallel P''))$
**and** *rBang*: $\bigwedge Rs\ C.\ [\![\Psi \rhd P \parallel !P \longmapsto Rs;\ \bigwedge C.\ Prop\ C\ \Psi\ (P \parallel !P)\ Rs;\ guarded\ P]\!] \Longrightarrow Prop\ C\ \Psi\ (!P)\ Rs$
**shows** $Prop\ C\ \Psi\ (!P)\ Rs$
⟨*proof*⟩

**lemma** *bangInputInduct*[*consumes 1*, *case-names cPar1 cPar2 cBang*]:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $M$ :: $'a$
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)\ psi$
**and** $Prop$ :: $'b \Rightarrow ('a, 'b, 'c)\ psi \Rightarrow 'a \Rightarrow 'a \Rightarrow ('a, 'b, 'c)\ psi \Rightarrow bool$

**assumes** $\Psi \rhd\ !P \longmapsto M(\!|N|\!) \prec P'$
**and** *rPar1*: $\bigwedge P'.\ \Psi \rhd P \longmapsto M(\!|N|\!) \prec P' \Longrightarrow Prop\ \Psi\ (P \parallel !P)\ M\ N\ (P' \parallel !P)$
**and** *rPar2*: $\bigwedge P'.\ [\![\Psi \rhd\ !P \longmapsto M(\!|N|\!) \prec P';\ Prop\ \Psi\ (!P)\ M\ N\ P']\!] \Longrightarrow Prop\ \Psi\ (P \parallel !P)\ M\ N\ (P \parallel P')$
**and** *rBang*: $\bigwedge P'.\ [\![\Psi \rhd P \parallel !P \longmapsto M(\!|N|\!) \prec P';\ Prop\ \Psi\ (P \parallel !P)\ M\ N\ P';\ guarded\ P]\!] \Longrightarrow Prop\ \Psi\ (!P)\ M\ N\ P'$
**shows** $Prop\ \Psi\ (!P)\ M\ N\ P'$
⟨*proof*⟩

**lemma** *brbangInputInduct*[*consumes 1*, *case-names cPar1 cPar2 cBrMerge cBang*]:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $M$ :: $'a$
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)\ psi$
**and** $Prop$ :: $'b \Rightarrow ('a, 'b, 'c)\ psi \Rightarrow 'a \Rightarrow 'a \Rightarrow ('a, 'b, 'c)\ psi \Rightarrow bool$

**assumes** $\Psi \rhd\ !P \longmapsto ¿M(\!|N|\!) \prec P'$
**and** *rPar1*: $\bigwedge P'.\ \Psi \rhd P \longmapsto ¿M(\!|N|\!) \prec P' \Longrightarrow Prop\ \Psi\ (P \parallel !P)\ M\ N\ (P' \parallel !P)$
**and** *rPar2*: $\bigwedge P'.\ [\![\Psi \rhd\ !P \longmapsto ¿M(\!|N|\!) \prec P';\ Prop\ \Psi\ (!P)\ M\ N\ P']\!] \Longrightarrow Prop\ \Psi\ (P \parallel !P)\ M\ N\ (P \parallel P')$
**and** *rBrMerge*: $\bigwedge P'\ P''\ C.\ [\![\Psi \rhd P \longmapsto ¿M(\!|N|\!) \prec P';\ \Psi \rhd\ !P \longmapsto ¿M(\!|N|\!) \prec P'';\ \bigwedge C.\ Prop\ \Psi\ (!P)\ M\ N\ P']\!] \Longrightarrow Prop\ \Psi\ (P \parallel !P)\ M\ N\ (P' \parallel P'')$
**and** *rBang*: $\bigwedge P'.\ [\![\Psi \rhd P \parallel !P \longmapsto ¿M(\!|N|\!) \prec P';\ Prop\ \Psi\ (P \parallel !P)\ M\ N\ P';\ guarded\ P]\!] \Longrightarrow Prop\ \Psi\ (!P)\ M\ N\ P'$
**shows** $Prop\ \Psi\ (!P)\ M\ N\ P'$
⟨*proof*⟩

**lemma** *bangOutputInduct*[*consumes 1*, *case-names cPar1 cPar2 cBang*]:
**fixes** $\Psi$ :: $'b$

**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** *xvec* :: *name list*
**and** $N$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** *Prop* :: $'d$::*fs-name* $\Rightarrow$ $'b$ $\Rightarrow$ $('a, 'b, 'c)$ *psi* $\Rightarrow$ $'a$ $\Rightarrow$ $('a, 'b, 'c)$ *boundOutput* $\Rightarrow$ *bool*
**and** $C$ :: $'d$

**assumes** $\Psi \rhd !P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
**and** *rPar1*: $\bigwedge xvec\ N\ P'\ C.\ [\![\Psi \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P';\ xvec\ \sharp*\ \Psi;\ xvec\ \sharp*\ P;\ xvec\ \sharp*\ M;\ xvec\ \sharp*\ C;\ distinct\ xvec]\!] \Longrightarrow Prop\ C\ \Psi\ (P \parallel !P)\ M\ ((\!|\nu * xvec|\!)N \prec' (P' \parallel !P))$
**and** *rPar2*: $\bigwedge xvec\ N\ P'\ C.\ [\![\Psi \rhd !P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P';\ \bigwedge C.\ Prop\ C\ \Psi\ (!P)\ M\ ((\!|\nu * xvec|\!)N \prec' P');\ xvec\ \sharp*\ \Psi;\ xvec\ \sharp*\ P;\ xvec\ \sharp*\ M;\ xvec\ \sharp*\ C;\ distinct\ xvec]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel !P)\ M\ ((\!|\nu * xvec|\!)N \prec' (P \parallel P'))$
**and** *rBang*: $\bigwedge B\ C.\ [\![\Psi \rhd P \parallel !P \longmapsto (ROut\ M\ B);\ \bigwedge C.\ Prop\ C\ \Psi\ (P \parallel !P)\ M\ B;\ guarded\ P]\!] \Longrightarrow Prop\ C\ \Psi\ (!P)\ M\ B$

**shows** $Prop\ C\ \Psi\ (!P)\ M\ ((\!|\nu * xvec|\!)N \prec' P')$
⟨*proof*⟩

**lemma** *bangTauInduct*[*consumes 1*, *case-names cPar1 cPar2 cComm1 cComm2 cBang*]:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** *Prop* :: $'d$::*fs-name* $\Rightarrow$ $'b$ $\Rightarrow$ $('a, 'b, 'c)$ *psi* $\Rightarrow$ $('a, 'b, 'c)$ *psi* $\Rightarrow$ *bool*
**and** $C$ :: $'d$

**assumes** $\Psi \rhd !P \longmapsto \tau \prec P'$
**and** *rPar1*: $\bigwedge P'\ C.\ \Psi \rhd P \longmapsto \tau \prec P' \Longrightarrow Prop\ C\ \Psi\ (P \parallel !P)\ (P' \parallel !P)$
**and** *rPar2*: $\bigwedge P'\ C.\ [\![\Psi \rhd !P \longmapsto \tau \prec P';\ \bigwedge C.\ Prop\ C\ \Psi\ (!P)\ P']\!] \Longrightarrow Prop\ C\ \Psi\ (P \parallel !P)\ (P \parallel P')$
**and** *rComm1*: $\bigwedge M\ N\ P'\ K\ xvec\ P''\ C.\ [\![\Psi \rhd P \longmapsto M(\!|N|\!) \prec P';\ \Psi \rhd !P \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec P'';\ \Psi \vdash M \leftrightarrow K;$
$xvec\ \sharp*\ \Psi;\ xvec\ \sharp*\ P;\ xvec\ \sharp*\ M;\ xvec\ \sharp*\ K;\ xvec\ \sharp*\ C]\!] \Longrightarrow Prop\ C\ \Psi\ (P \parallel !P)\ ((\!|\nu * xvec|\!)(P' \parallel P''))$
**and** *rComm2*: $\bigwedge M\ N\ P'\ K\ xvec\ P''\ C.\ [\![\Psi \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P';\ \Psi \rhd !P \longmapsto K(\!|N|\!) \prec P'';\ \Psi \vdash M \leftrightarrow K;$
$xvec\ \sharp*\ \Psi;\ xvec\ \sharp*\ P;\ xvec\ \sharp*\ M;\ xvec\ \sharp*\ K;\ xvec\ \sharp*\ C]\!] \Longrightarrow Prop\ C\ \Psi\ (P \parallel !P)\ ((\!|\nu * xvec|\!)(P' \parallel P''))$
**and** *rBang*: $\bigwedge P'\ C.\ [\![\Psi \rhd P \parallel !P \longmapsto \tau \prec P';\ \bigwedge C.\ Prop\ C\ \Psi\ (P \parallel !P)\ P';\ guarded\ P]\!] \Longrightarrow Prop\ C\ \Psi\ (!P)\ P'$

**shows** $Prop\ C\ \Psi\ (!P)\ P'$
⟨*proof*⟩

**lemma** *bangInduct'*[*consumes 2*, *case-names cAlpha cPar1 cPar2 cComm1 cComm2 cBrMerge cBrComm1 cBrComm2 cBang*]:
**fixes** $\Psi$ :: *'b*
**and** $P$ :: (*'a*, *'b*, *'c*) *psi*
**and** $\alpha$ :: *'a action*
**and** $P'$ :: (*'a*, *'b*, *'c*) *psi*
**and** *Prop* :: *'d*::*fs-name* ⇒ *'b* ⇒ (*'a*, *'b*, *'c*) *psi* ⇒ *'a action* ⇒ (*'a*, *'b*, *'c*) *psi* ⇒ *bool*
**and** $C$ :: *'d*::*fs-name*

**assumes** $\Psi \rhd !P \longmapsto \alpha \prec P'$
**and** *bn* $\alpha$ $\sharp*$ *subject* $\alpha$
**and** *rAlpha*: $\bigwedge \alpha\ P'\ p\ C.$ ⟦*bn* $\alpha$ $\sharp*$ $\Psi$; *bn* $\alpha$ $\sharp*$ $P$; *bn* $\alpha$ $\sharp*$ *subject* $\alpha$; *bn* $\alpha$ $\sharp*$ $C$; *set* $p \subseteq$ *set*(*bn* $\alpha$) × *set*(*bn*($p \cdot \alpha$)); *distinctPerm* $p$; *bn*($p \cdot \alpha$) $\sharp*$ $\alpha$; *bn*($p \cdot \alpha$) $\sharp*$ $P'$; *Prop* $C$ $\Psi$ ($P \parallel !P$) $\alpha$ $P'$⟧ ⟹ *Prop* $C$ $\Psi$ ($P \parallel !P$) ($p \cdot \alpha$) ($p \cdot P'$)
**and** *rPar1*: $\bigwedge \alpha\ P'\ C.$ ⟦$\Psi \rhd P \longmapsto \alpha \prec P'$; *bn* $\alpha$ $\sharp*$ $\Psi$; *bn* $\alpha$ $\sharp*$ $P$; *bn* $\alpha$ $\sharp*$ *subject* $\alpha$; *bn* $\alpha$ $\sharp*$ $C$; *distinct*(*bn* $\alpha$)⟧ ⟹ *Prop* $C$ $\Psi$ ($P \parallel !P$) $\alpha$ ($P' \parallel !P$)
**and** *rPar2*: $\bigwedge \alpha\ P'\ C.$ ⟦$\Psi \rhd !P \longmapsto \alpha \prec P'$; $\bigwedge C.$ *Prop* $C$ $\Psi$ ($!P$) $\alpha$ $P'$; *bn* $\alpha$ $\sharp*$ $\Psi$; *bn* $\alpha$ $\sharp*$ $P$; *bn* $\alpha$ $\sharp*$ *subject* $\alpha$; *bn* $\alpha$ $\sharp*$ $C$; *distinct*(*bn* $\alpha$)⟧ ⟹ *Prop* $C$ $\Psi$ ($P \parallel !P$) $\alpha$ ($P \parallel P'$)
**and** *rComm1*: $\bigwedge M\ N\ P'\ K\ xvec\ P''\ C.$ ⟦$\Psi \rhd P \longmapsto M(\!|N|\!) \prec P'$; $\Psi \rhd !P \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec P''$; $\bigwedge C.$ *Prop* $C$ $\Psi$ ($!P$) ($K(\!|\nu * xvec|\!)\langle N\rangle$) $P''$; $\Psi \vdash M \leftrightarrow K$; *xvec* $\sharp*$ $\Psi$; *xvec* $\sharp*$ $P$; *xvec* $\sharp*$ $M$; *xvec* $\sharp*$ $K$; *xvec* $\sharp*$ $C$; *distinct xvec*⟧ ⟹ *Prop* $C$ $\Psi$ ($P \parallel !P$) ($\tau$) ($(\!|\nu * xvec|\!)(P' \parallel P'')$)
**and** *rComm2*: $\bigwedge M\ xvec\ N\ P'\ K\ P''\ C.$ ⟦$\Psi \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$; $\Psi \rhd !P \longmapsto K(\!|N|\!) \prec P''$; $\bigwedge C.$ *Prop* $C$ $\Psi$ ($!P$) ($K(\!|N|\!)$) $P''$; $\Psi \vdash M \leftrightarrow K$; *xvec* $\sharp*$ $\Psi$; *xvec* $\sharp*$ $P$; *xvec* $\sharp*$ $M$; *xvec* $\sharp*$ $K$; *xvec* $\sharp*$ $C$; *distinct xvec*⟧ ⟹ *Prop* $C$ $\Psi$ ($P \parallel !P$) ($\tau$) ($(\!|\nu * xvec|\!)(P' \parallel P'')$)
**and** *rBrMerge*: $\bigwedge M\ N\ P'\ P''\ C.$ ⟦$\Psi \rhd P \longmapsto ¿M(\!|N|\!) \prec P'$; $\Psi \rhd !P \longmapsto ¿M(\!|N|\!) \prec P''$; $\bigwedge C.$ *Prop* $C$ $\Psi$ ($!P$) ($¿M(\!|N|\!)$) $P'$⟧ ⟹ *Prop* $C$ $\Psi$ ($P \parallel !P$) ($¿M(\!|N|\!)$) ($P' \parallel P''$)
**and** *rBrComm1*: $\bigwedge M\ N\ P'\ xvec\ P''\ C.$ ⟦$\Psi \rhd P \longmapsto ¿M(\!|N|\!) \prec P'$; $\Psi \rhd !P \longmapsto ¡M(\!|\nu * xvec|\!)\langle N\rangle \prec P''$; $\bigwedge C.$ *Prop* $C$ $\Psi$ ($!P$) ($¡M(\!|\nu * xvec|\!)\langle N\rangle$) $P''$; *xvec* $\sharp*$ $\Psi$; *xvec* $\sharp*$ $P$; *xvec* $\sharp*$ $M$; *xvec* $\sharp*$ $C$; *distinct xvec*⟧ ⟹ *Prop* $C$ $\Psi$ ($P \parallel !P$) ($¡M(\!|\nu * xvec|\!)\langle N\rangle$) ($P' \parallel P''$)
**and** *rBrComm2*: $\bigwedge M\ N\ P'\ xvec\ P''\ C.$ ⟦$\Psi \rhd P \longmapsto ¡M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$; $\Psi \rhd !P \longmapsto ¿M(\!|N|\!) \prec P''$; $\bigwedge C.$ *Prop* $C$ $\Psi$ ($!P$) ($¿M(\!|N|\!)$) $P''$; *xvec* $\sharp*$ $\Psi$; *xvec* $\sharp*$ $P$; *xvec* $\sharp*$ $M$; *xvec* $\sharp*$ $C$; *distinct xvec*⟧ ⟹ *Prop* $C$ $\Psi$ ($P \parallel !P$) ($¡M(\!|\nu * xvec|\!)\langle N\rangle$) ($P' \parallel P''$)
**and** *rBang*: $\bigwedge \alpha\ P'\ C.$ ⟦$\Psi \rhd P \parallel !P \longmapsto \alpha \prec P'$; *guarded* $P$; $\bigwedge C.$ *Prop* $C$ $\Psi$ ($P \parallel !P$) $\alpha$ $P'$; *guarded* $P$; *distinct*(*bn* $\alpha$)⟧ ⟹ *Prop* $C$ $\Psi$ ($!P$) $\alpha$ $P'$

**shows** *Prop C* $\Psi$ $(!P)$ $\alpha$ $P'$
⟨*proof*⟩

**lemma** *brCommInAuxTooMuch*:
**fixes** $\Psi$ :: *'b*
**and** $\Psi_Q$ :: *'b*
**and** $R$ :: *('a, 'b, 'c) psi*
**and** $M$ :: *'a*
**and** $N$ :: *'a*
**and** $R'$ :: *('a, 'b, 'c) psi*
**and** $A_R$ :: *name list*
**and** $\Psi_R$ :: *'b*
**and** $A_P$ :: *name list*
**and** $\Psi_P$ :: *'b*
**and** $A_Q$ :: *name list*

**assumes** *RTrans*: $\Psi \otimes \Psi_Q \rhd R \longmapsto ¿ M(\!|N|\!) \prec R'$
**and** *FrR*: *extractFrame* $R = \langle A_R, \Psi_R \rangle$
**and** *distinct* $A_R$
**and** *QimpP*: $\langle A_Q, (\Psi \otimes \Psi_Q) \otimes \Psi_R \rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \Psi_R \rangle$
**and** $A_R \sharp* A_P$
**and** $A_R \sharp* A_Q$
**and** $A_R \sharp* \Psi$
**and** $A_R \sharp* \Psi_P$
**and** $A_R \sharp* \Psi_Q$
**and** $A_R \sharp* (\Psi \otimes \Psi_Q)$
**and** $A_R \sharp* R$
**and** $A_R \sharp* M$
**and** $A_P \sharp* R$
**and** $A_P \sharp* M$
**and** $A_Q \sharp* R$
**and** $A_Q \sharp* M$

**shows** $\Psi \otimes \Psi_P \rhd R \longmapsto ¿ M(\!|N|\!) \prec R'$
⟨*proof*⟩

**lemma** *brCommInAux*:
**fixes** $\Psi$ :: *'b*
**and** $\Psi_Q$ :: *'b*
**and** $R$ :: *('a, 'b, 'c) psi*
**and** $M$ :: *'a*
**and** $N$ :: *'a*
**and** $R'$ :: *('a, 'b, 'c) psi*
**and** $A_R$ :: *name list*
**and** $\Psi_R$ :: *'b*
**and** $A_P$ :: *name list*
**and** $\Psi_P$ :: *'b*
**and** $A_Q$ :: *name list*

**assumes** *RTrans*: $\Psi \otimes \Psi_Q \rhd R \longmapsto ¿M(\!|N|\!) \prec R'$
**and** *FrR*: *extractFrame* $R = \langle A_R, \Psi_R \rangle$
**and** *distinct* $A_R$
**and** *QimpP*: $\langle A_Q, (\Psi \otimes \Psi_Q) \otimes \Psi_R \rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \Psi_R \rangle$
**and** $A_R \sharp* A_P$
**and** $A_R \sharp* A_Q$
**and** $A_R \sharp* \Psi$
**and** $A_R \sharp* \Psi_P$
**and** $A_R \sharp* \Psi_Q$
**and** $A_R \sharp* R$
**and** $A_R \sharp* M$
**and** $A_P \sharp* R$
**and** $A_P \sharp* M$
**and** $A_Q \sharp* R$
**and** $A_Q \sharp* M$

**shows** $\Psi \otimes \Psi_P \rhd R \longmapsto ¿M(\!|N|\!) \prec R'$
⟨*proof*⟩

**lemma** *brCommOutAuxTooMuch*:
**fixes** $\Psi$ :: $'b$
**and** $\Psi_Q$ :: $'b$
**and** $R$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** *xvec* :: *name list*
**and** $N$ :: $'a$
**and** $R'$ :: $('a, 'b, 'c)$ *psi*
**and** $A_R$ :: *name list*
**and** $\Psi_R$ :: $'b$
**and** $A_P$ :: *name list*
**and** $\Psi_P$ :: $'b$
**and** $A_Q$ :: *name list*

**assumes** *RTrans*: $\Psi \otimes \Psi_Q \rhd R \longmapsto RBrOut\ M\ (\!|\nu * xvec|\!)N \prec' R'$
**and** *FrR*: *extractFrame* $R = \langle A_R, \Psi_R \rangle$
**and** *distinct* $A_R$
**and** *QimpP*: $\langle A_Q, (\Psi \otimes \Psi_Q) \otimes \Psi_R \rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \Psi_R \rangle$
**and** $A_R \sharp* A_P$
**and** $A_R \sharp* A_Q$
**and** $A_R \sharp* \Psi$
**and** $A_R \sharp* \Psi_P$
**and** $A_R \sharp* \Psi_Q$
**and** $A_R \sharp* (\Psi \otimes \Psi_Q)$
**and** $A_R \sharp* R$
**and** $A_R \sharp* M$
**and** $A_P \sharp* R$
**and** $A_P \sharp* M$
**and** $A_Q \sharp* R$
**and** $A_Q \sharp* M$

**shows** $\Psi \otimes \Psi_P \rhd R \longmapsto ¡M(\!|\nu{*}xvec|\!)\langle N\rangle \prec R'$
$\langle proof\rangle$

**lemma** *brCommOutAux*:
**fixes** $\Psi$ :: $'b$
**and** $\Psi_Q$ :: $'b$
**and** $R$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** *xvec* :: *name list*
**and** $N$ :: $'a$
**and** $R'$ :: $('a, 'b, 'c)$ *psi*
**and** $A_R$ :: *name list*
**and** $\Psi_R$ :: $'b$
**and** $A_P$ :: *name list*
**and** $\Psi_P$ :: $'b$
**and** $A_Q$ :: *name list*

**assumes** *RTrans*: $\Psi \otimes \Psi_Q \rhd R \longmapsto ¡M(\!|\nu{*}xvec|\!)\langle N\rangle \prec R'$
**and** *FrR*: *extractFrame* $R = \langle A_R, \Psi_R\rangle$
**and** *distinct* $A_R$
**and** *QimpP*: $\langle A_Q, (\Psi \otimes \Psi_Q) \otimes \Psi_R\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \Psi_R\rangle$
**and** $A_R \sharp* A_P$
**and** $A_R \sharp* A_Q$
**and** $A_R \sharp* \Psi$
**and** $A_R \sharp* \Psi_P$
**and** $A_R \sharp* \Psi_Q$
**and** $A_R \sharp* R$
**and** $A_R \sharp* M$
**and** $A_P \sharp* R$
**and** $A_P \sharp* M$
**and** $A_Q \sharp* R$
**and** $A_Q \sharp* M$

**shows** $\Psi \otimes \Psi_P \rhd R \longmapsto ¡M(\!|\nu{*}xvec|\!)\langle N\rangle \prec R'$
$\langle proof\rangle$

**lemma** *comm1Aux*:
**fixes** $\Psi$ :: $'b$
**and** $\Psi_Q$ :: $'b$
**and** $R$ :: $('a, 'b, 'c)$ *psi*
**and** $K$ :: $'a$
**and** *xvec* :: *name list*
**and** $N$ :: $'a$
**and** $R'$ :: $('a, 'b, 'c)$ *psi*
**and** $A_R$ :: *name list*
**and** $\Psi_R$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$

**and** $L$ :: $'a$
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $A_P$ :: *name list*
**and** $\Psi_P$ :: $'b$
**and** $A_Q$ :: *name list*

**assumes** *RTrans*: $\Psi \otimes \Psi_Q \rhd R \longmapsto K\langle\!|\nu * xvec|\!\rangle\langle N\rangle \prec R'$
**and** *FrR*: *extractFrame* $R = \langle A_R, \Psi_R\rangle$
**and** *PTrans*: $\Psi \otimes \Psi_R \rhd P \longmapsto M\langle\!|L|\!\rangle \prec P'$
**and** *MeqK*: $\Psi \otimes \Psi_Q \otimes \Psi_R \vdash M \leftrightarrow K$
**and** *PeqQ*: $\langle A_Q, (\Psi \otimes \Psi_Q) \otimes \Psi_R\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \Psi_R\rangle$
**and** *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$
**and** *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$
**and** *distinct* $A_P$
**and** *distinct* $A_R$
**and** $A_R \sharp* A_P$
**and** $A_R \sharp* A_Q$
**and** $A_R \sharp* \Psi$
**and** $A_R \sharp* P$
**and** $A_R \sharp* Q$
**and** $A_R \sharp* R$
**and** $A_R \sharp* K$
**and** $A_P \sharp* \Psi$
**and** $A_P \sharp* R$
**and** $A_P \sharp* P$
**and** $A_P \sharp* M$
**and** $A_Q \sharp* R$
**and** $A_Q \sharp* M$

**obtains** $K'$ **where** $\Psi \otimes \Psi_P \rhd R \longmapsto K'\langle\!|\nu * xvec|\!\rangle\langle N\rangle \prec R'$ **and** $\Psi \otimes \Psi_P \otimes \Psi_R \vdash M \leftrightarrow K'$ **and** $A_R \sharp* K'$
⟨*proof*⟩

**lemma** *comm2Aux*:
**fixes** $\Psi$ :: $'b$
**and** $\Psi_Q$ :: $'b$
**and** $R$ :: $('a, 'b, 'c)$ *psi*
**and** $K$ :: $'a$
**and** $N$ :: $'a$
**and** $R'$ :: $('a, 'b, 'c)$ *psi*
**and** $A_R$ :: *name list*
**and** $\Psi_R$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** *xvec* :: *name list*
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $A_P$ :: *name list*
**and** $\Psi_P$ :: $'b$
**and** $A_Q$ :: *name list*

**assumes** *RTrans*: $\Psi \otimes \Psi_Q \rhd R \longmapsto K\langle\!|N|\!\rangle \prec R'$
**and** *FrR*: *extractFrame* $R = \langle A_R, \Psi_R\rangle$
**and** *PTrans*: $\Psi \otimes \Psi_R \rhd P \longmapsto M\langle\!|\nu * xvec|\!\rangle\langle N\rangle \prec P'$
**and** *MeqK*: $\Psi \otimes \Psi_Q \otimes \Psi_R \vdash M \leftrightarrow K$
**and** *QimpP*: $\langle A_Q, (\Psi \otimes \Psi_Q) \otimes \Psi_R\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \Psi_R\rangle$
**and** *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$
**and** *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$
**and** *distinct* $A_P$
**and** *distinct* $A_R$
**and** $A_R \sharp* A_P$
**and** $A_R \sharp* A_Q$
**and** $A_R \sharp* \Psi$
**and** $A_R \sharp* P$
**and** $A_R \sharp* Q$
**and** $A_R \sharp* R$
**and** $A_R \sharp* K$
**and** $A_P \sharp* \Psi$
**and** $A_P \sharp* R$
**and** $A_P \sharp* P$
**and** $A_P \sharp* M$
**and** $A_Q \sharp* R$
**and** $A_Q \sharp* M$
**and** $A_R \sharp* xvec$
**and** $xvec \sharp* M$

**obtains** $K'$ **where** $\Psi \otimes \Psi_P \rhd R \longmapsto K'\langle\!|N|\!\rangle \prec R'$ **and** $\Psi \otimes \Psi_P \otimes \Psi_R \vdash M \leftrightarrow K'$ **and** $A_R \sharp* K'$
⟨*proof*⟩

**end**

**end**
**theory** *Simulation*
**imports** *Semantics*
**begin**

This file is a (heavily modified) variant of the theory *Psi_Calculi.Simulation* from [1].

**context** *env* **begin**

**definition**
*simulation* :: $'b \Rightarrow ('a, 'b, 'c)\ psi \Rightarrow$
$('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set \Rightarrow$
$('a, 'b, 'c)\ psi \Rightarrow bool$ $(- \rhd - \rightsquigarrow[-] - [80, 80, 80, 80]\ 80)$
**where**
$\Psi \rhd P \rightsquigarrow[Rel]\ Q \equiv \forall \alpha\ Q'.\ \Psi \rhd Q \longmapsto \alpha \prec Q' \longrightarrow bn\ \alpha \sharp* \Psi \longrightarrow bn\ \alpha \sharp* P \longrightarrow (\exists P'.\ \Psi \rhd P \longmapsto \alpha \prec P' \wedge (\Psi, P', Q') \in Rel)$

**abbreviation**
*simulationNilJudge* (- $\rightsquigarrow$[-] - [*80*, *80*, *80*] *80*) **where** $P \rightsquigarrow[Rel]\ Q \equiv SBottom' \rhd P \rightsquigarrow[Rel]\ Q$

**lemma** *simI*[*consumes 1*, *case-names cSim*]:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $Rel$ :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
**and** $Q$ :: $('a, 'b, 'c)\ psi$
**and** $C$ :: $'d$::*fs-name*

**assumes** *Eqvt*: *eqvt Rel*
**and** *Sim*: $\bigwedge\alpha\ Q'.\ [\![\Psi \rhd Q \longmapsto\alpha \prec Q';\ bn\ \alpha \sharp* P;\ bn\ \alpha \sharp* Q;\ bn\ \alpha \sharp* \Psi;\ distinct(bn\ \alpha);\ bn\ \alpha \sharp* (subject\ \alpha);\ bn\ \alpha \sharp* C]\!] \Longrightarrow \exists P'.\ \Psi \rhd P \longmapsto\alpha \prec P' \wedge (\Psi, P', Q') \in Rel$

**shows** $\Psi \rhd P \rightsquigarrow[Rel]\ Q$
⟨*proof*⟩

**lemma** *simI2*[*case-names cSim*]:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $Rel$ :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
**and** $Q$ :: $('a, 'b, 'c)\ psi$
**and** $C$ :: $'d$::*fs-name*

**assumes** *Sim*: $\bigwedge\alpha\ Q'.\ [\![\Psi \rhd Q \longmapsto\alpha \prec Q';\ bn\ \alpha \sharp* P;\ bn\ \alpha \sharp* \Psi;\ distinct(bn\ \alpha)]\!] \Longrightarrow \exists P'.\ \Psi \rhd P \longmapsto\alpha \prec P' \wedge (\Psi, P', Q') \in Rel$

**shows** $\Psi \rhd P \rightsquigarrow[Rel]\ Q$
⟨*proof*⟩

**lemma** *simIChainFresh*[*consumes 4*, *case-names cSim*]:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $Rel$ :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
**and** $Q$ :: $('a, 'b, 'c)\ psi$
**and** $xvec$ :: *name list*
**and** $C$ :: $'d$::*fs-name*

**assumes** *Eqvt*: *eqvt Rel*
**and** $xvec \sharp* \Psi$
**and** $xvec \sharp* P$
**and** $xvec \sharp* Q$
**and** *Sim*: $\bigwedge\alpha\ Q'.\ [\![\Psi \rhd Q \longmapsto\alpha \prec Q';\ bn\ \alpha \sharp* P;\ bn\ \alpha \sharp* Q;\ bn\ \alpha \sharp* \Psi;\ bn\ \alpha \sharp* subject\ \alpha;\ distinct(bn\ \alpha);\ bn\ \alpha \sharp* C;\ xvec \sharp* \alpha;\ xvec \sharp* Q']\!] \Longrightarrow \exists P'.\ \Psi \rhd P \longmapsto\alpha \prec P' \wedge (\Psi, P', Q') \in Rel$

**shows** $\Psi \rhd P \rightsquigarrow[Rel]\ Q$
⟨*proof*⟩

**lemma** *simIFresh*[*consumes 4*, *case-names cSim*]:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $Rel$ :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
**and** $Q$ :: $('a, 'b, 'c)\ psi$
**and** $x$ :: *name*
**and** $C$ :: $'d$::*fs-name*

**assumes** *Eqvt*: *eqvt Rel*
**and** $x \sharp \Psi$
**and** $x \sharp P$
**and** $x \sharp Q$
**and** $\bigwedge\alpha\ Q'.\ [\![\Psi \rhd Q \longmapsto\alpha \prec Q';\ bn\ \alpha \sharp* P;\ bn\ \alpha \sharp* Q;\ bn\ \alpha \sharp* \Psi;$
$bn\ \alpha \sharp* subject\ \alpha;\ distinct(bn\ \alpha);\ bn\ \alpha \sharp* C;\ x \sharp \alpha;\ x \sharp Q']\!] \Longrightarrow$
$\exists P'.\ \Psi \rhd P \longmapsto\alpha \prec P' \wedge (\Psi, P', Q') \in Rel$

**shows** $\Psi \rhd P \rightsquigarrow[Rel]\ Q$
⟨*proof*⟩

**lemma** *simE*:
**fixes** $F$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $Rel$ :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
**and** $Q$ :: $('a, 'b, 'c)\ psi$

**assumes** $\Psi \rhd P \rightsquigarrow[Rel]\ Q$

**shows** $\bigwedge\alpha\ Q'.\ [\![\Psi \rhd Q \longmapsto\alpha \prec Q';\ bn\ \alpha \sharp* \Psi;\ bn\ \alpha \sharp* P]\!] \Longrightarrow \exists P'.\ \Psi \rhd P \longmapsto\alpha \prec P' \wedge (\Psi, P', Q') \in Rel$
⟨*proof*⟩

**lemma** *simClosedAux*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $Rel$ :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
**and** $Q$ :: $('a, 'b, 'c)\ psi$
**and** $p$ :: *name prm*

**assumes** *EqvtRel*: *eqvt Rel*
**and** *PSimQ*: $\Psi \rhd P \rightsquigarrow[Rel]\ Q$

**shows** $(p \cdot \Psi) \rhd (p \cdot P) \rightsquigarrow[Rel]\ (p \cdot Q)$
⟨*proof*⟩

**lemma** *simClosed*:
**fixes** $\Psi$ :: $'b$

**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $Rel$ :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
**and** $Q$ :: $('a, 'b, 'c)\ psi$
**and** $p$ :: $name\ prm$

**assumes** *EqvtRel*: $eqvt\ Rel$

**shows** $\Psi \rhd P \rightsquigarrow[Rel]\ Q \Longrightarrow (p \cdot \Psi) \rhd (p \cdot P) \rightsquigarrow[Rel]\ (p \cdot Q)$
**and** $P \rightsquigarrow[Rel]\ Q \Longrightarrow (p \cdot P) \rightsquigarrow[Rel]\ (p \cdot Q)$
⟨*proof*⟩

**lemma** *reflexive*:
**fixes** $Rel$ :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
**and** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$

**assumes** $\{(\Psi, P, P) \mid \Psi\ P.\ True\} \subseteq Rel$

**shows** $\Psi \rhd P \rightsquigarrow[Rel]\ P$
⟨*proof*⟩

**lemma** *transitive*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $Rel$ :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
**and** $Q$ :: $('a, 'b, 'c)\ psi$
**and** $Rel'$ :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
**and** $R$ :: $('a, 'b, 'c)\ psi$
**and** $Rel''$ :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

**assumes** *PSimQ*: $\Psi \rhd P \rightsquigarrow[Rel]\ Q$
**and** *QSimR*: $\Psi \rhd Q \rightsquigarrow[Rel']\ R$
**and** *Eqvt*: $eqvt\ Rel''$
**and** *Set*: $\{(\Psi, P, R) \mid \Psi\ P\ R.\ \exists Q.\ (\Psi, P, Q) \in Rel \wedge (\Psi, Q, R) \in Rel'\} \subseteq Rel''$

**shows** $\Psi \rhd P \rightsquigarrow[Rel'']\ R$
⟨*proof*⟩

**lemma** *statEqSim*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $Rel$ :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
**and** $Q$ :: $('a, 'b, 'c)\ psi$
**and** $\Psi'$ :: $'b$

**assumes** *PSimQ*: $\Psi \rhd P \rightsquigarrow[Rel]\ Q$
**and** $eqvt\ Rel'$
**and** $\Psi \simeq \Psi'$

**and** *C1*: $\bigwedge \Psi'' \; R \; S \; \Psi'''. \; [\![(\Psi'', R, S) \in Rel; \Psi'' \simeq \Psi'']\!] \Longrightarrow (\Psi''', R, S) \in Rel'$

**shows** $\Psi' \rhd P \rightsquigarrow [Rel'] \; Q$
⟨*proof*⟩

**lemma** *monotonic*:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c) \; psi$
**and** $A :: ('b \times ('a, 'b, 'c) \; psi \times ('a, 'b, 'c) \; psi) \; set$
**and** $Q :: ('a, 'b, 'c) \; psi$
**and** $B :: ('b \times ('a, 'b, 'c) \; psi \times ('a, 'b, 'c) \; psi) \; set$

**assumes** $\Psi \rhd P \rightsquigarrow [A] \; Q$
**and** $A \subseteq B$

**shows** $\Psi \rhd P \rightsquigarrow [B] \; Q$
⟨*proof*⟩

**end**

**end**
**theory** *Bisimulation*
**imports** *Simulation*
**begin**

This file is a (heavily modified) variant of the theory *Psi_Calculi.Bisimulation* from [1].

**context** *env* **begin**

**lemma** *monoCoinduct*: $\bigwedge x \; y \; xa \; xb \; xc \; P \; Q \; \Psi.$
$x \leq y \Longrightarrow$
$(\Psi \rhd Q \rightsquigarrow [\{(xc, xb, xa).\ x \; xc \; xb \; xa\}] \; P) \longrightarrow$
$(\Psi \rhd Q \rightsquigarrow [\{(xb, xa, xc).\ y \; xb \; xa \; xc\}] \; P)$
⟨*proof*⟩

**coinductive-set** *bisim* :: $('b \times ('a, 'b, 'c) \; psi \times ('a, 'b, 'c) \; psi) \; set$
**where**
*step*: $[\![(insertAssertion \; (extractFrame \; P)) \; \Psi \simeq_F (insertAssertion \; (extractFrame \; Q) \; \Psi);$
$\Psi \rhd P \rightsquigarrow [bisim] \; Q;$
$\forall \Psi'. \; (\Psi \otimes \Psi', \; P, \; Q) \in bisim; \; (\Psi, \; Q, \; P) \in bisim]\!] \Longrightarrow (\Psi, \; P, \; Q) \in bisim$
**monos** *monoCoinduct*

**abbreviation**
*bisimJudge* $(- \rhd - \sim - \; [70, \; 70, \; 70] \; 65)$ **where** $\Psi \rhd P \sim Q \equiv (\Psi, \; P, \; Q) \in bisim$
**abbreviation**
*bisimNilJudge* $(- \sim - \; [70, \; 70] \; 65)$ **where** $P \sim Q \equiv SBottom' \rhd P \sim Q$

**lemma** *bisimCoinductAux*[*consumes 1*]:

**fixes** $F :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $Q :: ('a, 'b, 'c)\ psi$
**and** $X :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

**assumes** $(\Psi, P, Q) \in X$
**and** $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow insertAssertion\ (extractFrame\ P)\ \Psi \simeq_F insertAssertion\ (extractFrame\ Q)\ \Psi \wedge$
$(\Psi \rhd P \rightsquigarrow [(X \cup bisim)]\ Q) \wedge$
$(\forall \Psi'.\ (\Psi \otimes \Psi', P, Q) \in X \vee (\Psi \otimes \Psi', P, Q) \in bisim) \wedge$
$((\Psi, Q, P) \in X \vee (\Psi, Q, P) \in bisim)$

**shows** $(\Psi, P, Q) \in bisim$
⟨*proof*⟩

**lemma** *bisimCoinduct*[*consumes 1*, *case-names cStatEq cSim cExt cSym*]:
**fixes** $F :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $Q :: ('a, 'b, 'c)\ psi$
**and** $X :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

**assumes** $(\Psi, P, Q) \in X$
**and** $\bigwedge \Psi'\ R\ S.\ (\Psi', R, S) \in X \Longrightarrow insertAssertion\ (extractFrame\ R)\ \Psi' \simeq_F insertAssertion\ (extractFrame\ S)\ \Psi'$
**and** $\bigwedge \Psi'\ R\ S.\ (\Psi', R, S) \in X \Longrightarrow \Psi' \rhd R \rightsquigarrow [(X \cup bisim)]\ S$
**and** $\bigwedge \Psi'\ R\ S\ \Psi''.\ (\Psi', R, S) \in X \Longrightarrow (\Psi' \otimes \Psi'', R, S) \in X \vee (\Psi' \otimes \Psi'', R, S) \in bisim$
**and** $\bigwedge \Psi'\ R\ S.\ (\Psi', R, S) \in X \Longrightarrow (\Psi', S, R) \in X \vee (\Psi', S, R) \in bisim$

**shows** $(\Psi, P, Q) \in bisim$
⟨*proof*⟩

**lemma** *bisimWeakCoinductAux*[*consumes 1*]:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $Q :: ('a, 'b, 'c)\ psi$
**and** $X :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

**assumes** $(\Psi, P, Q) \in X$
**and** $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow insertAssertion\ (extractFrame\ P)\ \Psi \simeq_F insertAssertion\ (extractFrame\ Q)\ \Psi \wedge$
$\Psi \rhd P \rightsquigarrow [X]\ Q \wedge$
$(\forall \Psi'.\ (\Psi \otimes \Psi', P, Q) \in X) \wedge (\Psi, Q, P) \in X$

**shows** $(\Psi, P, Q) \in bisim$
⟨*proof*⟩

**lemma** *bisimWeakCoinduct*[*consumes 1*, *case-names cStatEq cSim cExt cSym*]:

**fixes** $F :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $Q :: ('a, 'b, 'c)\ psi$
**and** $X :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

**assumes** $(\Psi, P, Q) \in X$
**and** $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow insertAssertion\ (extractFrame\ P)\ \Psi \simeq_F insertAssertion\ (extractFrame\ Q)\ \Psi$
**and** $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow \Psi \rhd P \rightsquigarrow [X]\ Q$
**and** $\bigwedge \Psi\ P\ Q\ \Psi'.\ (\Psi, P, Q) \in X \Longrightarrow (\Psi \otimes \Psi', P, Q) \in X$
**and** $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow (\Psi, Q, P) \in X$

**shows** $(\Psi, P, Q) \in bisim$
⟨*proof*⟩

**lemma** *bisimE*:
**fixes** $P :: ('a, 'b, 'c)\ psi$
**and** $Q :: ('a, 'b, 'c)\ psi$
**and** $\Psi :: 'b$
**and** $\Psi' :: 'b$

**assumes** $(\Psi, P, Q) \in bisim$

**shows** $insertAssertion\ (extractFrame\ P)\ \Psi \simeq_F insertAssertion\ (extractFrame\ Q)\ \Psi$
**and** $\Psi \rhd P \rightsquigarrow [bisim]\ Q$
**and** $(\Psi \otimes \Psi', P, Q) \in bisim$
**and** $(\Psi, Q, P) \in bisim$
⟨*proof*⟩

**lemma** *bisimI*:
**fixes** $P :: ('a, 'b, 'c)\ psi$
**and** $Q :: ('a, 'b, 'c)\ psi$
**and** $\Psi :: 'b$

**assumes** $insertAssertion\ (extractFrame\ P)\ \Psi \simeq_F insertAssertion\ (extractFrame\ Q)\ \Psi$
**and** $\Psi \rhd P \rightsquigarrow [bisim]\ Q$
**and** $\forall \Psi'.\ (\Psi \otimes \Psi', P, Q) \in bisim$
**and** $(\Psi, Q, P) \in bisim$

**shows** $(\Psi, P, Q) \in bisim$
⟨*proof*⟩

**lemma** *bisimReflexive*:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$

**shows** $\Psi \rhd P \sim P$
⟨*proof*⟩

**lemma** *bisimClosed*:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $Q :: ('a, 'b, 'c)\ psi$
**and** $p :: name\ prm$

**assumes** *PBisimQ*: $\Psi \rhd P \sim Q$

**shows** $(p \cdot \Psi) \rhd (p \cdot P) \sim (p \cdot Q)$
⟨*proof*⟩

**lemma** *bisimEqvt*[*simp*]:
**shows** *eqvt bisim*
⟨*proof*⟩

**lemma** *statEqBisim*:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $Q :: ('a, 'b, 'c)\ psi$
**and** $\Psi' :: 'b$

**assumes** $\Psi \rhd P \sim Q$
**and** $\Psi \simeq \Psi'$

**shows** $\Psi' \rhd P \sim Q$
⟨*proof*⟩

**lemma** *bisimTransitive*:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $Q :: ('a, 'b, 'c)\ psi$
**and** $R :: ('a, 'b, 'c)\ psi$

**assumes** *PQ*: $\Psi \rhd P \sim Q$
**and** *QR*: $\Psi \rhd Q \sim R$

**shows** $\Psi \rhd P \sim R$
⟨*proof*⟩

**lemma** *weakTransitiveCoinduct*[*case-names cStatEq cSim cExt cSym*, *case-conclusion bisim step*, *consumes 2*]:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $Q :: ('a, 'b, 'c)\ psi$
**and** $X :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

**assumes** $p$: $(\Psi,\ P,\ Q) \in X$
**and** *Eqvt*: *eqvt* $X$
**and** *rStatEq*: $\bigwedge \Psi\ P\ Q.\ (\Psi,\ P,\ Q) \in X \Longrightarrow$ *insertAssertion* (*extractFrame* $P$) $\Psi$ $\simeq_F$ *insertAssertion* (*extractFrame* $Q$) $\Psi$
**and** *rSim*: $\bigwedge \Psi\ P\ Q.\ (\Psi,\ P,\ Q) \in X \Longrightarrow \Psi \rhd P \rightsquigarrow[(\{(\Psi,\ P,\ Q) \mid \Psi\ P\ P'\ Q'\ Q.\ \Psi \rhd P \sim P' \wedge (\Psi,\ P',\ Q') \in X \wedge \Psi \rhd Q' \sim Q\})]\ Q$
**and** *rExt*: $\bigwedge \Psi\ P\ Q\ \Psi'.\ (\Psi,\ P,\ Q) \in X \Longrightarrow (\Psi \otimes \Psi',\ P,\ Q) \in X$
**and** *rSym*: $\bigwedge \Psi\ P\ Q.\ (\Psi,\ P,\ Q) \in X \Longrightarrow (\Psi,\ Q,\ P) \in X$

**shows** $\Psi \rhd P \sim Q$
⟨*proof*⟩

**lemma** *weakTransitiveCoinduct'*[*case-names cStatEq cSim cExt cSym*, *case-conclusion bisim step*, *consumes 2*]:
**fixes** $\Psi$ :: *'b*
**and** $P$ :: (*'a*, *'b*, *'c*) *psi*
**and** $Q$ :: (*'a*, *'b*, *'c*) *psi*
**and** $X$ :: (*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set*

**assumes** $p$: $(\Psi,\ P,\ Q) \in X$
**and** *Eqvt*: *eqvt* $X$
**and** *rStatEq*: $\bigwedge \Psi\ P\ Q.\ (\Psi,\ P,\ Q) \in X \Longrightarrow$ *insertAssertion* (*extractFrame* $P$) $\Psi$ $\simeq_F$ *insertAssertion* (*extractFrame* $Q$) $\Psi$
**and** *rSim*: $\bigwedge \Psi\ P\ Q.\ (\Psi,\ P,\ Q) \in X \Longrightarrow \Psi \rhd P \rightsquigarrow[(\{(\Psi,\ P,\ Q) \mid \Psi\ P\ P'\ Q'\ Q.\ \Psi \rhd P \sim P' \wedge (\Psi,\ P',\ Q') \in X \wedge \Psi \rhd Q' \sim Q\})]\ Q$
**and** *rExt*: $\bigwedge \Psi\ P\ Q\ \Psi'.\ (\Psi,\ P,\ Q) \in X \Longrightarrow (\Psi \otimes \Psi',\ P,\ Q) \in X$
**and** *rSym*: $\bigwedge \Psi\ P\ Q.\ (\Psi,\ P,\ Q) \in X \Longrightarrow (\Psi,\ Q,\ P) \in \{(\Psi,\ P,\ Q) \mid \Psi\ P\ P'\ Q'\ Q.\ \Psi \rhd P \sim P' \wedge (\Psi,\ P',\ Q') \in X \wedge \Psi \rhd Q' \sim Q\}$

**shows** $\Psi \rhd P \sim Q$
⟨*proof*⟩

**lemma** *weakTransitiveCoinduct''*[*case-names cStatEq cSim cExt cSym*, *case-conclusion bisim step*, *consumes 2*]:
**fixes** $\Psi$ :: *'b*
**and** $P$ :: (*'a*, *'b*, *'c*) *psi*
**and** $Q$ :: (*'a*, *'b*, *'c*) *psi*
**and** $X$ :: (*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set*

**assumes** $p$: $(\Psi,\ P,\ Q) \in X$
**and** *Eqvt*: *eqvt* $X$
**and** *rStatEq*: $\bigwedge \Psi\ P\ Q.\ (\Psi,\ P,\ Q) \in X \Longrightarrow$ *insertAssertion* (*extractFrame* $P$) $\Psi$ $\simeq_F$ *insertAssertion* (*extractFrame* $Q$) $\Psi$
**and** *rSim*: $\bigwedge \Psi\ P\ Q.\ (\Psi,\ P,\ Q) \in X \Longrightarrow \Psi \rhd P \rightsquigarrow[(\{(\Psi,\ P,\ Q) \mid \Psi\ P\ P'\ Q'\ Q.$

$\Psi \rhd P \sim P' \wedge$
$(\Psi, P', Q') \in X \wedge$
$\Psi \rhd Q' \sim Q\})]\ Q$
**and** *rExt*: $\bigwedge \Psi\ P\ Q\ \Psi'.\ (\Psi, P, Q) \in \{(\Psi, P, Q) \mid \Psi\ P\ P'\ Q'\ Q.\ \Psi \rhd P \sim P' \wedge (\Psi, P', Q') \in X \wedge \Psi \rhd Q' \sim Q\} \Longrightarrow (\Psi \otimes \Psi', P, Q) \in \{(\Psi, P, Q) \mid \Psi\ P\ P'\ Q'\ Q.\ \Psi \rhd P \sim P' \wedge (\Psi, P', Q') \in X \wedge \Psi \rhd Q' \sim Q\}$
**and** *rSym*: $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in \{(\Psi, P, Q) \mid \Psi\ P\ P'\ Q'\ Q.\ \Psi \rhd P \sim P' \wedge (\Psi, P', Q') \in X \wedge \Psi \rhd Q' \sim Q\} \Longrightarrow (\Psi, Q, P) \in \{(\Psi, P, Q) \mid \Psi\ P\ P'\ Q'\ Q.\ \Psi \rhd P \sim P' \wedge (\Psi, P', Q') \in X \wedge \Psi \rhd Q' \sim Q\}$

**shows** $\Psi \rhd P \sim Q$
⟨*proof*⟩

**lemma** *transitiveCoinduct*[*case-names cStatEq cSim cExt cSym*, *case-conclusion bisim step*, *consumes 2*]:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $Q :: ('a, 'b, 'c)\ psi$
**and** $X :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

**assumes** *p*: $(\Psi, P, Q) \in X$
**and** *Eqvt*: *eqvt X*
**and** *rStatEq*: $\bigwedge \Psi'\ R\ S.\ (\Psi', R, S) \in X \Longrightarrow insertAssertion\ (extractFrame\ R)\ \Psi' \simeq_F insertAssertion\ (extractFrame\ S)\ \Psi'$
**and** *rSim*: $\bigwedge \Psi'\ R\ S.\ (\Psi', R, S) \in X \Longrightarrow \Psi' \rhd R \rightsquigarrow[(\{(\Psi', R, S) \mid \Psi'\ R\ R'\ S'\ S.\ \Psi' \rhd R \sim R' \wedge ((\Psi', R', S') \in X \vee \Psi' \rhd R' \sim S') \wedge \Psi' \rhd S' \sim S\})]\ S$
**and** *rExt*: $\bigwedge \Psi'\ R\ S\ \Psi''.\ (\Psi', R, S) \in X \Longrightarrow (\Psi' \otimes \Psi'', R, S) \in X \vee \Psi' \otimes \Psi'' \rhd R \sim S$
**and** *rSym*: $\bigwedge \Psi'\ R\ S.\ (\Psi', R, S) \in X \Longrightarrow (\Psi', S, R) \in X \vee \Psi' \rhd S \sim R$

**shows** $\Psi \rhd P \sim Q$
⟨*proof*⟩

**lemma** *transitiveCoinduct'*[*case-names cStatEq cSim cExt cSym*, *case-conclusion bisim step*, *consumes 2*]:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $Q :: ('a, 'b, 'c)\ psi$
**and** $X :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

**assumes** *p*: $(\Psi, P, Q) \in X$
**and** *Eqvt*: *eqvt X*
**and** *rStatEq*: $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow insertAssertion\ (extractFrame\ P)\ \Psi$

$\simeq_F$ *insertAssertion* (*extractFrame* $Q$) $\Psi$
**and** *rSim*: $\bigwedge \Psi\ P\ Q.\ (\Psi,\ P,\ Q) \in X \Longrightarrow \Psi \rhd P \rightsquigarrow [(\{(\Psi,\ P,\ Q) \mid \Psi\ P\ P'\ Q'\ Q.\ \Psi \rhd P \sim P' \wedge (\Psi,\ P',\ Q') \in (X \cup bisim) \wedge \Psi \rhd Q' \sim Q\})]\ Q$
**and** *rExt*: $\bigwedge \Psi\ P\ Q\ \Psi'.\ (\Psi,\ P,\ Q) \in X \Longrightarrow (\Psi \otimes \Psi',\ P,\ Q) \in X \vee \Psi \otimes \Psi' \rhd P \sim Q$
**and** *rSym*: $\bigwedge \Psi\ P\ Q.\ (\Psi,\ P,\ Q) \in X \Longrightarrow (\Psi,\ Q,\ P) \in \{(\Psi,\ P,\ Q) \mid \Psi\ P\ P'\ Q'\ Q.\ \Psi \rhd P \sim P' \wedge ((\Psi,\ P',\ Q') \in (X \cup bisim)) \wedge \Psi \rhd Q' \sim Q\}$

**shows** $\Psi \rhd P \sim Q$
⟨*proof*⟩

**lemma** *bisimSymmetric*:
**fixes** $\Psi :: 'b$
**and** $P :: ('a,\ 'b,\ 'c)\ psi$
**and** $Q :: ('a,\ 'b,\ 'c)\ psi$

**assumes** $\Psi \rhd P \sim Q$

**shows** $\Psi \rhd Q \sim P$
⟨*proof*⟩

**lemma** *eqvtTrans*[*intro*]:
**assumes** *eqvt* $X$

**shows** *eqvt* $\{(\Psi,\ P,\ Q) \mid \Psi\ P\ P'\ Q'\ Q.\ \Psi \rhd P \sim P' \wedge ((\Psi,\ P',\ Q') \in X \vee \Psi \rhd P' \sim Q') \wedge \Psi \rhd Q' \sim Q\}$
⟨*proof*⟩

**lemma** *eqvtWeakTrans*[*intro*]:
**assumes** *eqvt* $X$

**shows** *eqvt* $\{(\Psi,\ P,\ Q) \mid \Psi\ P\ P'\ Q'\ Q.\ \Psi \rhd P \sim P' \wedge (\Psi,\ P',\ Q') \in X \wedge \Psi \rhd Q' \sim Q\}$
⟨*proof*⟩

**inductive-set**
*rel-trancl* :: ('b × ('a,'b,'c) *psi* × ('a,'b,'c) *psi*) *set* ⇒ ('b × ('a,'b,'c) *psi* × ('a,'b,'c) *psi*) *set* ((-⋆) [*1000*] *999*)
**for** $r$ :: ('b × ('a,'b,'c) *psi* × ('a,'b,'c) *psi*) *set*
**where**
*r-into-rel-trancl* [*intro*, *Pure.intro*]: $(\Psi,\ P,Q) : r \Longrightarrow (\Psi,P,Q) : r^\star$
| *rel-trancl-into-rel-trancl* [*Pure.intro*]: $(\Psi,P,Q) : r^\star \Longrightarrow (\Psi,Q,R) : r \Longrightarrow (\Psi,P,R) : r^\star$

**lemma** *rel-trancl-transitive*:
**assumes** $(\Psi,P,Q) \in Rel^\star$

**and** $(\Psi,Q,R) \in Rel^\star$
**shows** $(\Psi,P,R) \in Rel^\star$
⟨*proof*⟩

**lemma** *rel-trancl-eqvt*:
**assumes** *eqvt X*
**shows** *eqvt*$(X^\star)$
⟨*proof*⟩

**lemma** *bisimStarWeakCoinduct*[*consumes 2*, *case-names cStatEq cSim cExt cSym*]:
**fixes** $F :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $Q :: ('a, 'b, 'c)\ psi$
**and** $X :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

**assumes** $(\Psi, P, Q) \in X$
**and** *eqvt X*
**and** *rStatEq*: $\bigwedge \Psi'\ R\ S.\ (\Psi', R, S) \in X \Longrightarrow insertAssertion\ (extractFrame\ R)\ \Psi' \simeq_F insertAssertion\ (extractFrame\ S)\ \Psi'$
**and** *rSim*: $\bigwedge \Psi'\ R\ S.\ (\Psi', R, S) \in X \Longrightarrow \Psi' \rhd R \rightsquigarrow[X^\star]\ S$
**and** *rExt*: $\bigwedge \Psi'\ R\ S\ \Psi''.\ (\Psi', R, S) \in X \Longrightarrow (\Psi' \otimes \Psi'', R, S) \in X$
**and** *rSym*: $\bigwedge \Psi'\ R\ S.\ (\Psi', R, S) \in X \Longrightarrow (\Psi', S, R) \in X$

**shows** $(\Psi, P, Q) \in bisim$
⟨*proof*⟩

**lemma** *weakTransitiveStarCoinduct*[*case-names cStatEq cSim cExt cSym*, *case-conclusion bisim step*, *consumes 2*]:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $Q :: ('a, 'b, 'c)\ psi$
**and** $X :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

**assumes** *p*: $(\Psi, P, Q) \in X$
**and** *Eqvt*: *eqvt X*
**and** *rStatEq*: $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow insertAssertion\ (extractFrame\ P)\ \Psi \simeq_F insertAssertion\ (extractFrame\ Q)\ \Psi$
**and** *rSim*: $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow \Psi \rhd P \rightsquigarrow[(\{(\Psi, P, Q) \mid \Psi\ P\ P'\ Q'\ Q.\ \Psi \rhd P \sim P' \wedge (\Psi, P', Q') \in X \wedge \Psi \rhd Q' \sim Q\})^\star]\ Q$
**and** *rExt*: $\bigwedge \Psi\ P\ Q\ \Psi'.\ (\Psi, P, Q) \in X \Longrightarrow (\Psi \otimes \Psi', P, Q) \in X$
**and** *rSym*: $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow (\Psi, Q, P) \in X$

**shows** $\Psi \rhd P \sim Q$
⟨*proof*⟩

**lemma** *weakTransitiveStarCoinduct'*[*case-names cStatEq cSim cExt cSym*, *case-conclusion bisim step*, *consumes 2*]:

**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $Q :: ('a, 'b, 'c)\ psi$
**and** $X :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

**assumes** $p$: $(\Psi, P, Q) \in X$
**and** *Eqvt*: $eqvt\ X$
**and** *rStatEq*: $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow insertAssertion\ (extractFrame\ P)\ \Psi \simeq_F insertAssertion\ (extractFrame\ Q)\ \Psi$
**and** *rSim*: $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow \Psi \rhd P \rightsquigarrow[(\{(\Psi, P, Q) \mid \Psi\ P\ P'\ Q'\ Q.\ \Psi \rhd P \sim P' \wedge (\Psi, P', Q') \in X \wedge \Psi \rhd Q' \sim Q\})^\star]\ Q$
**and** *rExt*: $\bigwedge \Psi\ P\ Q\ \Psi'.\ (\Psi, P, Q) \in X \Longrightarrow (\Psi \otimes \Psi', P, Q) \in X$
**and** *rSym*: $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow (\Psi, Q, P) \in \{(\Psi, P, Q) \mid \Psi\ P\ P'\ Q'\ Q.\ \Psi \rhd P \sim P' \wedge (\Psi, P', Q') \in X \wedge \Psi \rhd Q' \sim Q\}$

**shows** $\Psi \rhd P \sim Q$
⟨*proof*⟩

**lemma** *transitiveStarCoinduct*[*case-names cStatEq cSim cExt cSym*, *case-conclusion bisim step*, *consumes 2*]:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $Q :: ('a, 'b, 'c)\ psi$
**and** $X :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

**assumes** $p$: $(\Psi, P, Q) \in X$
**and** *Eqvt*: $eqvt\ X$
**and** *rStatEq*: $\bigwedge \Psi'\ R\ S.\ (\Psi', R, S) \in X \Longrightarrow insertAssertion\ (extractFrame\ R)\ \Psi' \simeq_F insertAssertion\ (extractFrame\ S)\ \Psi'$
**and** *rSim*: $\bigwedge \Psi'\ R\ S.\ (\Psi', R, S) \in X \Longrightarrow \Psi' \rhd R \rightsquigarrow[(\{(\Psi', R, S) \mid \Psi'\ R\ R'\ S'\ S.\ \Psi' \rhd R \sim R' \wedge ((\Psi', R', S') \in X \vee \Psi' \rhd R' \sim S') \wedge \Psi' \rhd S' \sim S\})^\star]\ S$
**and** *rExt*: $\bigwedge \Psi'\ R\ S\ \Psi''.\ (\Psi', R, S) \in X \Longrightarrow (\Psi' \otimes \Psi'', R, S) \in X \vee \Psi' \otimes \Psi'' \rhd R \sim S$
**and** *rSym*: $\bigwedge \Psi'\ R\ S.\ (\Psi', R, S) \in X \Longrightarrow (\Psi', S, R) \in X \vee \Psi' \rhd S \sim R$

**shows** $\Psi \rhd P \sim Q$
⟨*proof*⟩

**end**

**end**
**theory** *Sim-Pres*

**imports** *Simulation*
**begin**

This file is a (heavily modified) variant of the theory *Psi_Calculi.Sim_Pres* from [1].

**context** *env* **begin**

**lemma** *inputPres*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** *Rel* :: $('b \times ('a, 'b, 'c)$ *psi* $\times ('a, 'b, 'c)$ *psi*$)$ *set*
**and** $Q$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** *xvec* :: *name list*
**and** $N$ :: $'a$

**assumes** *PRelQ*: $\bigwedge$*Tvec*. *length xvec* = *length Tvec* $\Longrightarrow$ $(\Psi, P[xvec::=Tvec], Q[xvec::=Tvec])$ $\in$ *Rel*

**shows** $\Psi \rhd M(\!|\lambda * xvec\ N|\!).P \rightsquigarrow[Rel]\ M(\!|\lambda * xvec\ N|\!).Q$
⟨*proof*⟩

**lemma** *outputPres*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** *Rel* :: $('b \times ('a, 'b, 'c)$ *psi* $\times ('a, 'b, 'c)$ *psi*$)$ *set*
**and** $Q$ :: $('a, 'b, 'c)$ *psi*
**and** $M$ :: $'a$
**and** $N$ :: $'a$

**assumes** *PRelQ*: $(\Psi, P, Q) \in$ *Rel*

**shows** $\Psi \rhd M\langle N\rangle.P \rightsquigarrow[Rel]\ M\langle N\rangle.Q$
⟨*proof*⟩

**lemma** *casePres*:
**fixes** $\Psi$ :: $'b$
**and** *CsP* :: $('c \times ('a, 'b, 'c)$ *psi*$)$ *list*
**and** *Rel* :: $('b \times ('a, 'b, 'c)$ *psi* $\times ('a, 'b, 'c)$ *psi*$)$ *set*
**and** *CsQ* :: $('c \times ('a, 'b, 'c)$ *psi*$)$ *list*
**and** $M$ :: $'a$
**and** $N$ :: $'a$

**assumes** *PRelQ*: $\bigwedge\varphi\ Q.\ (\varphi, Q) \in set\ CsQ \Longrightarrow \exists P.\ (\varphi, P) \in set\ CsP \wedge guarded\ P \wedge (\Psi, P, Q) \in Rel$
**and** *Sim*: $\bigwedge\Psi'\ R\ S.\ (\Psi', R, S) \in Rel \Longrightarrow \Psi' \rhd R \rightsquigarrow[Rel]\ S$
**and** $Rel \subseteq Rel'$

**shows** $\Psi \rhd Cases\ CsP \rightsquigarrow[Rel']\ Cases\ CsQ$

⟨*proof*⟩

**lemma** *resPres*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $Rel$ :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
**and** $Q$ :: $('a, 'b, 'c)\ psi$
**and** $x$ :: $name$
**and** $Rel'$ :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

**assumes** *PSimQ*: $\Psi \rhd P \rightsquigarrow[Rel]\ Q$
**and** $eqvt\ Rel'$
**and** $x \sharp \Psi$
**and** $Rel \subseteq Rel'$
**and** *C1*: $\bigwedge \Psi'\ R\ S\ yvec.\ [\![(\Psi', R, S) \in Rel; yvec \sharp* \Psi']\!] \Longrightarrow (\Psi', (\!|\nu * yvec|\!)R, (\!|\nu * yvec|\!)S) \in Rel'$

**shows** $\Psi \rhd (\!|\nu x|\!)P \rightsquigarrow[Rel'] (\!|\nu x|\!)Q$
⟨*proof*⟩

**lemma** *resChainPres*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $Rel$ :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
**and** $Q$ :: $('a, 'b, 'c)\ psi$
**and** $xvec$ :: $name\ list$

**assumes** *PSimQ*: $\Psi \rhd P \rightsquigarrow[Rel]\ Q$
**and** $eqvt\ Rel$
**and** $xvec \sharp* \Psi$
**and** *C1*: $\bigwedge \Psi'\ R\ S\ yvec.\ [\![(\Psi', R, S) \in Rel; yvec \sharp* \Psi']\!] \Longrightarrow (\Psi', (\!|\nu * yvec|\!)R, (\!|\nu * yvec|\!)S) \in Rel$

**shows** $\Psi \rhd (\!|\nu * xvec|\!)P \rightsquigarrow[Rel] (\!|\nu * xvec|\!)Q$
⟨*proof*⟩

**lemma** *parPres*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $Rel$ :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
**and** $Q$ :: $('a, 'b, 'c)\ psi$
**and** $R$ :: $('a, 'b, 'c)\ psi$
**and** $Rel'$ :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

**assumes** *PRelQ*: $\bigwedge A_R\ \Psi_R.\ [\![extractFrame\ R = \langle A_R, \Psi_R\rangle; A_R \sharp* \Psi; A_R \sharp* P; A_R \sharp* Q]\!] \Longrightarrow (\Psi \otimes \Psi_R, P, Q) \in Rel$
**and** *Eqvt*: $eqvt\ Rel$
**and** *Eqvt'*: $eqvt\ Rel'$

**and** *StatImp*: $\bigwedge \Psi'\ S\ T.\ (\Psi', S, T) \in Rel \Longrightarrow insertAssertion\ (extractFrame\ T)\ \Psi' \hookrightarrow_F insertAssertion\ (extractFrame\ S)\ \Psi'$
**and** *Sim*: $\bigwedge \Psi'\ S\ T.\ (\Psi', S, T) \in Rel \Longrightarrow \Psi' \rhd S \rightsquigarrow [Rel]\ T$
**and** *Ext*: $\bigwedge \Psi'\ S\ T\ \Psi''.\ [\![(\Psi', S, T) \in Rel]\!] \Longrightarrow (\Psi' \otimes \Psi'', S, T) \in Rel$

**and** *C1*: $\bigwedge \Psi'\ S\ T\ A_U\ \Psi_U\ U.\ [\![(\Psi' \otimes \Psi_U, S, T) \in Rel; extractFrame\ U = \langle A_U, \Psi_U\rangle; A_U \sharp* \Psi'; A_U \sharp* S; A_U \sharp* T]\!] \Longrightarrow (\Psi', S \parallel U, T \parallel U) \in Rel'$
**and** *C2*: $\bigwedge \Psi'\ S\ T\ xvec.\ [\![(\Psi', S, T) \in Rel'; xvec \sharp* \Psi']\!] \Longrightarrow (\Psi', (\!|\nu* xvec|\!) S, (\!|\nu* xvec|\!)\ T) \in Rel'$
**and** *C3*: $\bigwedge \Psi'\ S\ T\ \Psi''.\ [\![(\Psi', S, T) \in Rel; \Psi' \simeq \Psi']\!] \Longrightarrow (\Psi'', S, T) \in Rel$

**shows** $\Psi \rhd P \parallel R \rightsquigarrow [Rel']\ Q \parallel R$
⟨*proof*⟩

**no-notation** *relcomp* (**infixr** *O* 75)

**lemma** *bangPres*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $Q$ :: $('a, 'b, 'c)\ psi$
**and** $R$ :: $('a, 'b, 'c)\ psi$
**and** $Rel$ :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
**and** $Rel'$ :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

**assumes** $(\Psi, P, Q) \in Rel$
**and** *eqvt Rel′*
**and** *guarded P*
**and** *guarded Q*
**and** *cSim*: $\bigwedge \Psi'\ S\ T.\ (\Psi', S, T) \in Rel \Longrightarrow \Psi' \rhd S \rightsquigarrow [Rel]\ T$
**and** *cExt*: $\bigwedge \Psi'\ S\ T\ \Psi''.\ (\Psi', S, T) \in Rel \Longrightarrow (\Psi' \otimes \Psi'', S, T) \in Rel$
**and** *cSym*: $\bigwedge \Psi'\ S\ T.\ (\Psi', S, T) \in Rel \Longrightarrow (\Psi', T, S) \in Rel$
**and** *StatEq*: $\bigwedge \Psi'\ S\ T\ \Psi''.\ [\![(\Psi', S, T) \in Rel; \Psi' \simeq \Psi']\!] \Longrightarrow (\Psi'', S, T) \in Rel$
**and** *Closed*: $\bigwedge \Psi'\ S\ T\ p.\ (\Psi', S, T) \in Rel \Longrightarrow ((p::name\ prm) \cdot \Psi', p \cdot S, p \cdot T) \in Rel$
**and** *Assoc*: $\bigwedge \Psi'\ S\ T\ U.\ (\Psi', S \parallel (T \parallel U), (S \parallel T) \parallel U) \in Rel$
**and** *ParPres*: $\bigwedge \Psi'\ S\ T\ U.\ (\Psi', S, T) \in Rel \Longrightarrow (\Psi', S \parallel U, T \parallel U) \in Rel$
**and** *FrameParPres*: $\bigwedge \Psi'\ \Psi_U\ S\ T\ U\ A_U.\ [\![(\Psi' \otimes \Psi_U, S, T) \in Rel; extractFrame\ U = \langle A_U, \Psi_U\rangle; A_U \sharp* \Psi'; A_U \sharp* S; A_U \sharp* T]\!] \Longrightarrow (\Psi', U \parallel S, U \parallel T) \in Rel$
**and** *ResPres*: $\bigwedge \Psi'\ S\ T\ xvec.\ [\![(\Psi', S, T) \in Rel; xvec \sharp* \Psi']\!] \Longrightarrow (\Psi', (\!|\nu* xvec|\!) S, (\!|\nu* xvec|\!)\ T) \in Rel$
**and** *ScopeExt*: $\bigwedge xvec\ \Psi'\ S\ T.\ [\![xvec \sharp* \Psi'; xvec \sharp* T]\!] \Longrightarrow (\Psi', (\!|\nu* xvec|\!)(S \parallel T), ((\!|\nu* xvec|\!) S) \parallel T) \in Rel$
**and** *Trans*: $\bigwedge \Psi'\ S\ T\ U.\ [\![(\Psi', S, T) \in Rel; (\Psi', T, U) \in Rel]\!] \Longrightarrow (\Psi', S, U) \in Rel$
**and** *Compose*: $\bigwedge \Psi'\ S\ T\ U\ O.\ [\![(\Psi', S, T) \in Rel; (\Psi', T, U) \in Rel'; (\Psi', U, O) \in Rel]\!] \Longrightarrow (\Psi', S, O) \in Rel'$
**and** *C1*: $\bigwedge \Psi\ S\ T\ U.\ [\![(\Psi, S, T) \in Rel; guarded\ S; guarded\ T]\!] \Longrightarrow (\Psi, U \parallel !S,$

$U \parallel !T) \in Rel'$
**and** *Der*: $\bigwedge \Psi' S \alpha S' T. [\![\Psi' \rhd !S \longmapsto \alpha \prec S'; (\Psi', S, T) \in Rel; bn\ \alpha \sharp* \Psi'; bn\ \alpha \sharp* S; bn\ \alpha \sharp* T; guarded\ T; bn\ \alpha \sharp* subject\ \alpha]\!] \Longrightarrow \exists T' U O. \Psi' \rhd !T \longmapsto \alpha \prec T' \wedge (\Psi', S', U \parallel !S) \in Rel \wedge (\Psi', T', O \parallel !T) \in Rel \wedge (\Psi', U, O) \in Rel \wedge ((supp\ U)::name\ set) \subseteq supp\ S' \wedge ((supp\ O)::name\ set) \subseteq supp\ T'$

**shows** $\Psi \rhd R \parallel !P \rightsquigarrow [Rel'] R \parallel !Q$
⟨*proof*⟩

**notation** *relcomp* (**infixr** *O* 75)

**end**

**end**
**theory** *Sim-Struct-Cong*
**imports** *Simulation HOL−Library.Multiset*
**begin**

This file is a (heavily modified) variant of the theory *Psi_Calculi.Sim_Struct_Cong* from [1].

**lemma** *partitionListLeft*:
**assumes** $xs@ys{=}xs'@y\#ys'$
**and** $y \in set\ xs$
**and** $distinct(xs@ys)$

**obtains** *zs* **where** $xs = xs'@y\#zs$ **and** $ys'{=}zs@ys$
⟨*proof*⟩

**lemma** *partitionListRight*:

**assumes** $xs@ys{=}xs'@y\#ys'$
**and** $y \in set\ ys$
**and** $distinct(xs@ys)$

**obtains** *zs* **where** $xs' = xs@zs$ **and** $ys{=}zs@y\#ys'$
⟨*proof*⟩

**context** *env* **begin**

**lemma** *resOutputCases''''*[*consumes* 8, *case-names cOpen cRes*]:
**fixes** $\Psi$ :: $'b$
**and** $x$ :: *name*
**and** *zvec* :: *name list*
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $\alpha$ :: $'a$ *action*
**and** $P'$ :: $('a, 'b, 'c)$ *psi*

**and** $C$ :: *'f::fs-name*

**assumes** *Trans*: $\Psi \rhd (\!|\nu x|\!)P \longmapsto \alpha \prec P'$
**and** *1*: $x \sharp \Psi$
**and** *2*: $x \sharp \alpha$
**and** *3*: $x \sharp P'$
**and** *4*: $bn\ \alpha \sharp* \Psi$
**and** *5*: $bn\ \alpha \sharp* P$
**and** *6*: $bn\ \alpha \sharp* subject\ \alpha$
**and** $\alpha = M(\!|\nu* zvec|\!)\langle N\rangle$
**and** *rOpen*: $\bigwedge M\ xvec\ yvec\ y\ N\ P'.\ [\![\Psi \rhd ([(x, y)] \cdot P) \longmapsto M(\!|\nu*(xvec@yvec)|\!)\langle N\rangle \prec P';\ y \in supp\ N;$
$x \sharp N;\ x \sharp P';\ x \neq y;\ y \sharp xvec;\ y \sharp yvec;\ y \sharp M;$
$distinct\ xvec;\ distinct\ yvec;$
$xvec \sharp* \Psi;\ y \sharp \Psi;\ yvec \sharp* \Psi;\ xvec \sharp* P;\ y \sharp P;$
$yvec \sharp* P;\ xvec \sharp* M;\ y \sharp M;$
$yvec \sharp* M;\ xvec \sharp* yvec]\!] \Longrightarrow$
$Prop\ (M(\!|\nu*(xvec@y\#yvec)|\!)\langle N\rangle)\ P'$
**and** *rScope*: $\bigwedge P'.\ [\![\Psi \rhd P \longmapsto \alpha \prec P']\!] \Longrightarrow Prop\ \alpha\ ((\!|\nu x|\!)P')$

**shows** $Prop\ \alpha\ P'$
⟨*proof*⟩

**lemma** *resOutputCases''''''*[*consumes 7*, *case-names cOpen cRes*]:
**fixes** $\Psi$ :: *'b*
**and** $x$ :: *name*
**and** *zvec* :: *name list*
**and** $P$ :: *('a, 'b, 'c) psi*
**and** $P'$ :: *('a, 'b, 'c) psi*
**and** $C$ :: *'f::fs-name*

**assumes** *Trans*: $\Psi \rhd (\!|\nu x|\!)P \longmapsto M(\!|\nu* zvec|\!)\langle N\rangle \prec P'$
**and** *1*: $x \sharp \Psi$
**and** $x \sharp M(\!|\nu* zvec|\!)\langle N\rangle$
**and** *3*: $x \sharp P'$
**and** $zvec \sharp* \Psi$
**and** $zvec \sharp* P$
**and** $zvec \sharp* M$
**and** *rOpen*: $\bigwedge M'\ xvec\ yvec\ y\ N'\ P'.\ [\![\Psi \rhd ([(x, y)] \cdot P) \longmapsto M'(\!|\nu*(xvec@yvec)|\!)\langle N'\rangle \prec P';\ y \in supp\ N';$
$x \sharp N';\ x \sharp P';\ x \neq y;\ y \sharp xvec;\ y \sharp yvec;\ y \sharp M';$
$distinct\ xvec;\ distinct\ yvec;$
$xvec \sharp* \Psi;\ y \sharp \Psi;\ yvec \sharp* \Psi;\ xvec \sharp* P;\ y \sharp P;$
$yvec \sharp* P;\ xvec \sharp* M';\ y \sharp M';$
$yvec \sharp* M';\ xvec \sharp* yvec;\ M'(\!|\nu*(xvec@y\#yvec)|\!)\langle N'\rangle = M(\!|\nu*zvec|\!)\langle N\rangle]\!] \Longrightarrow$
$Prop\ P'$
**and** *rScope*: $\bigwedge P'.\ [\![\Psi \rhd P \longmapsto M(\!|\nu*zvec|\!)\langle N\rangle \prec P']\!] \Longrightarrow Prop\ ((\!|\nu x|\!)P')$

**shows** *Prop* $P'$
⟨*proof*⟩

**lemma** *resBrOutputCases'*[*consumes* 7, *case-names cBrOpen cRes*]:
**fixes** $\Psi$ :: $'b$
**and** $x$ :: *name*
**and** *zvec* :: *name list*
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $P'$ :: $('a, 'b, 'c)$ *psi*
**and** $C$ :: $'f$::*fs-name*

**assumes** *Trans*: $\Psi \rhd (\!|\nu x|\!)P \longmapsto ¡M(\!|\nu{*}zvec|\!)\langle N\rangle \prec P'$
**and** *1*: $x \sharp \Psi$
**and** $x \sharp ¡M(\!|\nu{*}zvec|\!)\langle N\rangle$
**and** *3*: $x \sharp P'$
**and** $zvec \sharp{*} \Psi$
**and** $zvec \sharp{*} P$
**and** $zvec \sharp{*} M$
**and** *rBrOpen*: $\bigwedge M'\ xvec\ yvec\ y\ N'\ P'.\ [\![\Psi \rhd ([(x, y)] \cdot P) \longmapsto ¡M'(\!|\nu{*}(xvec@yvec)|\!)\langle N'\rangle \prec P';\ y \in supp\ N';$
$x \sharp N';\ x \sharp P';\ x \neq y;\ y \sharp xvec;\ y \sharp yvec;\ y \sharp M';$
*distinct xvec*; *distinct yvec*;
$xvec \sharp{*} \Psi;\ y \sharp \Psi;\ yvec \sharp{*} \Psi;\ xvec \sharp{*} P;\ y \sharp P;$
$yvec \sharp{*} P;\ xvec \sharp{*} M';\ y \sharp M';$
$yvec \sharp{*} M';\ xvec \sharp{*} yvec;\ ¡M'(\!|\nu{*}(xvec@y\#yvec)|\!)\langle N'\rangle = ¡M(\!|\nu{*}zvec|\!)\langle N\rangle]\!] \Longrightarrow$
$Prop\ P'$
**and** *rScope*: $\bigwedge P'.\ [\![\Psi \rhd P \longmapsto ¡M(\!|\nu{*}zvec|\!)\langle N\rangle \prec P']\!] \Longrightarrow Prop\ ((\!|\nu x|\!)P')$

**shows** *Prop* $P'$
⟨*proof*⟩

**lemma** *brOutputFreshSubject*:
**fixes** $x$::*name*
**assumes** $\Psi \rhd P \longmapsto ¡M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P'$
**and** $xvec \sharp{*} M$
**and** $x \sharp P$
**shows** $x \sharp M$
⟨*proof*⟩

**lemma** *brInputFreshSubject*:
**fixes** $x$::*name*
**assumes** $\Psi \rhd P \longmapsto ¿M(\!|N|\!) \prec P'$
**and** $x \sharp P$
**shows** $x \sharp M$
⟨*proof*⟩

**lemma** *resComm*:
**fixes** $\Psi$ :: $'b$

**and** $x$ :: *name*
**and** $y$ :: *name*
**and** *Rel* :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
**and** $P$ :: $('a, 'b, 'c)\ psi$

**assumes** $x \sharp \Psi$
**and** $y \sharp \Psi$
**and** *eqvt Rel*
**and** *R1*: $\bigwedge \Psi'\ Q.\ (\Psi', Q, Q) \in Rel$
**and** *R2*: $\bigwedge \Psi'\ a\ b\ Q.\ [\![a \sharp \Psi'; b \sharp \Psi']\!] \Longrightarrow (\Psi', (\!|\nu a|\!)((\!|\nu b|\!)\, Q), (\!|\nu b|\!)((\!|\nu a|\!)\, Q)) \in Rel$
**and** *R3*: $\bigwedge \Psi'\ xvec\ yvec\ Q.\ [\![xvec \sharp* \Psi'; mset\ xvec = mset\ yvec]\!] \Longrightarrow (\Psi', (\!|\nu* xvec|\!)\, Q, (\!|\nu* yvec|\!)\, Q) \in Rel$

**shows** $\Psi \rhd (\!|\nu x|\!)((\!|\nu y|\!)P) \rightsquigarrow [Rel]\ (\!|\nu y|\!)((\!|\nu x|\!)P)$
⟨*proof*⟩

**lemma** *parAssocLeft*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $Q$ :: $('a, 'b, 'c)\ psi$
**and** $R$ :: $('a, 'b, 'c)\ psi$
**and** *Rel* :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

**assumes** *eqvt Rel*
**and** *C1*: $\bigwedge \Psi'\ S\ T\ U.\ (\Psi, (S \parallel T) \parallel U, S \parallel (T \parallel U)) \in Rel$
**and** *C2*: $\bigwedge xvec\ \Psi'\ S\ T\ U.\ [\![xvec \sharp* \Psi'; xvec \sharp* S]\!] \Longrightarrow (\Psi', (\!|\nu* xvec|\!)((S \parallel T) \parallel U), S \parallel ((\!|\nu* xvec|\!)(T \parallel U))) \in Rel$
**and** *C3*: $\bigwedge xvec\ \Psi'\ S\ T\ U.\ [\![xvec \sharp* \Psi'; xvec \sharp* U]\!] \Longrightarrow (\Psi', ((\!|\nu* xvec|\!)(S \parallel T)) \parallel U, (\!|\nu* xvec|\!)(S \parallel (T \parallel U))) \in Rel$
**and** *C4*: $\bigwedge \Psi'\ S\ T\ xvec.\ [\![(\Psi', S, T) \in Rel; xvec \sharp* \Psi']\!] \Longrightarrow (\Psi', (\!|\nu* xvec|\!)S, (\!|\nu* xvec|\!)\, T) \in Rel$

**shows** $\Psi \rhd (P \parallel Q) \parallel R \rightsquigarrow [Rel]\ P \parallel (Q \parallel R)$
⟨*proof*⟩

**lemma** *parNilLeft*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** *Rel* :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

**assumes** *eqvt Rel*
**and** *C1*: $\bigwedge Q.\ (\Psi, Q \parallel \mathbf{0}, Q) \in Rel$

**shows** $\Psi \rhd (P \parallel \mathbf{0}) \rightsquigarrow [Rel]\ P$
⟨*proof*⟩

**lemma** *parNilRight*:
**fixes** $\Psi$ :: $'b$

**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $Rel$ :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

**assumes** *eqvt Rel*
**and** *C1*: $\bigwedge Q.\ (\Psi, Q, (Q \parallel \mathbf{0})) \in Rel$

**shows** $\Psi \rhd P \rightsquigarrow [Rel]\ (P \parallel \mathbf{0})$
⟨*proof*⟩

**lemma** *resNilLeft*:
**fixes** $x$ :: *name*
**and** $\Psi$ :: $'b$
**and** $Rel$ :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

**shows** $\Psi \rhd (\!|\nu x|\!)\mathbf{0} \rightsquigarrow [Rel]\ \mathbf{0}$
⟨*proof*⟩

**lemma** *resNilRight*:
**fixes** $x$ :: *name*
**and** $\Psi$ :: $'b$
**and** $Rel$ :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

**shows** $\Psi \rhd \mathbf{0} \rightsquigarrow [Rel]\ (\!|\nu x|\!)\mathbf{0}$
⟨*proof*⟩

**lemma** *inputPushResLeft*:
**fixes** $\Psi$ :: $'b$
**and** $x$ :: *name*
**and** $M$ :: $'a$
**and** *xvec* :: *name list*
**and** $N$ :: $'a$
**and** $P$ :: $('a, 'b, 'c)\ psi$

**assumes** *eqvt Rel*
**and** $x \sharp \Psi$
**and** $x \sharp M$
**and** $x \sharp xvec$
**and** $x \sharp N$
**and** *C1*: $\bigwedge Q.\ (\Psi, Q, Q) \in Rel$

**shows** $\Psi \rhd (\!|\nu x|\!)(M(\!|\lambda * xvec\ N|\!).P) \rightsquigarrow [Rel]\ M(\!|\lambda * xvec\ N|\!).(\!|\nu x|\!)P$
⟨*proof*⟩

**lemma** *inputPushResRight*:
**fixes** $\Psi$ :: $'b$
**and** $x$ :: *name*
**and** $M$ :: $'a$
**and** *xvec* :: *name list*
**and** $N$ :: $'a$

**and** $P$ :: $('a, 'b, 'c)$ *psi*

**assumes** *eqvt Rel*
**and** $x \sharp \Psi$
**and** $x \sharp M$
**and** $x \sharp xvec$
**and** $x \sharp N$
**and** *C1*: $\bigwedge Q.\ (\Psi, Q, Q) \in Rel$

**shows** $\Psi \rhd M(\!|\lambda * xvec\ N|\!).(\!|\nu x|\!)P \rightsquigarrow [Rel]\ (\!|\nu x|\!)(M(\!|\lambda * xvec\ N|\!).P)$
⟨*proof*⟩

**lemma** *outputPushResLeft*:
**fixes** $\Psi$ :: $'b$
**and** $x$ :: *name*
**and** $M$ :: $'a$
**and** $N$ :: $'a$
**and** $P$ :: $('a, 'b, 'c)$ *psi*

**assumes** *eqvt Rel*
**and** $x \sharp \Psi$
**and** $x \sharp M$
**and** $x \sharp N$
**and** *C1*: $\bigwedge Q.\ (\Psi, Q, Q) \in Rel$

**shows** $\Psi \rhd (\!|\nu x|\!)(M\langle N\rangle.P) \rightsquigarrow [Rel]\ M\langle N\rangle.(\!|\nu x|\!)P$
⟨*proof*⟩

**lemma** *broutputNoBind*:
**fixes** $\Psi$ :: $'b$
**and** $M$ :: $'a$
**and** $N$ :: $'a$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $\alpha$ :: $'a$ *action*
**and** $P'$ :: $('a, 'b, 'c)$ *psi*

**assumes** $\Psi \rhd M\langle N\rangle.P \longmapsto (¡K(\!|\nu * xvec|\!)\langle N'\rangle) \prec P'$
**shows** $xvec = []$
⟨*proof*⟩

**lemma** *broutputObjectEq*:
**fixes** $\Psi$ :: $'b$
**and** $M$ :: $'a$
**and** $N$ :: $'a$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $\alpha$ :: $'a$ *action*
**and** $P'$ :: $('a, 'b, 'c)$ *psi*

**assumes** $\Psi \rhd M\langle N\rangle.P \longmapsto (¡K(\!|\nu * xvec|\!)\langle N'\rangle) \prec P'$

**shows** $N = N'$
⟨*proof*⟩

**lemma** *brOutputOutputCases*[*consumes 1*, *case-names cBrOutput*]:
**fixes** $\Psi :: 'b$
**and** $M :: 'a$
**and** $N :: 'a$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $\alpha :: 'a\ action$
**and** $P' :: ('a, 'b, 'c)\ psi$

**assumes** *Trans*: $\Psi \rhd M\langle N\rangle.P \longmapsto (¡K(\!|\nu * xvec|\!)\langle N'\rangle) \prec P'$
**and** *rBrOutput*: $\bigwedge K.\ \Psi \vdash M \preceq K \Longrightarrow Prop\ (¡K\langle N\rangle)\ P$

**shows** $Prop\ (¡K(\!|\nu * xvec|\!)\langle N'\rangle)\ P'$
⟨*proof*⟩

**lemma** *outputPushResRight*:
**fixes** $\Psi :: 'b$
**and** $x :: name$
**and** $M :: 'a$
**and** $N :: 'a$
**and** $P :: ('a, 'b, 'c)\ psi$

**assumes** *eqvt Rel*
**and** $x \sharp \Psi$
**and** $x \sharp M$
**and** $x \sharp N$
**and** *C1*: $\bigwedge Q.\ (\Psi, Q, Q) \in Rel$

**shows** $\Psi \rhd M\langle N\rangle.(\!|\nu x|\!)P \rightsquigarrow [Rel]\ (\!|\nu x|\!)(M\langle N\rangle.P)$
⟨*proof*⟩

**lemma** *casePushResLeft*:
**fixes** $\Psi :: 'b$
**and** $x :: name$
**and** $Cs :: ('c \times ('a, 'b, 'c)\ psi)\ list$

**assumes** *eqvt Rel*
**and** $x \sharp \Psi$
**and** $x \sharp map\ fst\ Cs$
**and** *C1*: $\bigwedge Q.\ (\Psi, Q, Q) \in Rel$

**shows** $\Psi \rhd (\!|\nu x|\!)(Cases\ Cs) \rightsquigarrow [Rel]\ Cases\ (map\ (\lambda(\varphi, P).\ (\varphi, (\!|\nu x|\!)P))\ Cs)$
⟨*proof*⟩

**lemma** *casePushResRight*:
**fixes** $\Psi :: 'b$
**and** $x :: name$

**and** $Cs :: ('c \times ('a, 'b, 'c)\ psi)\ list$

**assumes** *eqvt Rel*
**and** $x \sharp \Psi$
**and** $x \sharp map\ fst\ Cs$
**and** $C1: \bigwedge Q.\ (\Psi, Q, Q) \in Rel$

**shows** $\Psi \rhd Cases\ (map\ (\lambda(\varphi, P).\ (\varphi, (\!|\nu x|\!)P))\ Cs) \rightsquigarrow [Rel]\ (\!|\nu x|\!)(Cases\ Cs)$
⟨*proof*⟩

**lemma** *resInputCases*[*consumes 5*, *case-names cRes*]:
**fixes** $\Psi :: 'b$
**and** $x :: name$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $M :: 'a$
**and** $N :: 'a$
**and** $P' :: ('a, 'b, 'c)\ psi$
**and** $C :: 'd::fs\text{-}name$

**assumes** *Trans*: $\Psi \rhd (\!|\nu x|\!)P \longmapsto M(\!|N|\!) \prec P'$
**and** $x \sharp \Psi$
**and** $x \sharp M$
**and** $x \sharp N$
**and** $x \sharp P'$
**and** *rScope*: $\bigwedge P'.\ [\![\Psi \rhd P \longmapsto M(\!|N|\!) \prec P']\!] \Longrightarrow Prop\ ((\!|\nu x|\!)P')$

**shows** *Prop* $P'$
⟨*proof*⟩

**lemma** *resBrInputCases*[*consumes 5*, *case-names cRes*]:
**fixes** $\Psi :: 'b$
**and** $x :: name$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $M :: 'a$
**and** $N :: 'a$
**and** $P' :: ('a, 'b, 'c)\ psi$
**and** $C :: 'd::fs\text{-}name$

**assumes** *Trans*: $\Psi \rhd (\!|\nu x|\!)P \longmapsto ¿ M(\!|N|\!) \prec P'$
**and** $x \sharp \Psi$
**and** $x \sharp M$
**and** $x \sharp N$
**and** $x \sharp P'$
**and** *rScope*: $\bigwedge P'.\ [\![\Psi \rhd P \longmapsto ¿ M(\!|N|\!) \prec P']\!] \Longrightarrow Prop\ ((\!|\nu x|\!)P')$

**shows** *Prop* $P'$
⟨*proof*⟩

**lemma** *swap-supp*:

**fixes** $x$ :: *name*
**and** $y$ :: *name*
**and** $N$ :: $'a$

**assumes** $y \in supp\ N$

**shows** $x \in supp\ ([(x,\ y)] \cdot N)$
⟨*proof*⟩

**lemma** *swap-supp'*:
**fixes** $x$ :: *name*
**and** $y$ :: *name*
**and** $N$ :: $'a$

**assumes** $x \in supp\ N$

**shows** $y \in supp\ ([(x,\ y)] \cdot N)$
⟨*proof*⟩

**lemma** *brOutputFreshSubjectChain*:
**fixes** $\Psi$ :: $'b$
**and** $Q$ :: $('a,\ 'b,\ 'c)\ psi$
**and** $M$ :: $'a$
**and** *xvec* :: *name list*
**and** $N$ :: $'a$
**and** $Q'$ :: $('a,\ 'b,\ 'c)\ psi$
**and** *yvec* :: *name list*

**assumes** $\Psi \rhd Q \longmapsto ¡M(\!|\nu{*}xvec|\!)\langle N\rangle \prec Q'$
**and** $xvec \sharp* M$
**and** $yvec \sharp* Q$

**shows** $yvec \sharp* M$
⟨*proof*⟩

**lemma** *scopeExtLeft*:
**fixes** $x$ :: *name*
**and** $P$ :: $('a,\ 'b,\ 'c)\ psi$
**and** $\Psi$ :: $'b$
**and** $Q$ :: $('a,\ 'b,\ 'c)\ psi$
**and** *Rel* :: $('b \times ('a,\ 'b,\ 'c)\ psi \times ('a,\ 'b,\ 'c)\ psi)\ set$

**assumes** $x \sharp P$
**and** $x \sharp \Psi$
**and** *eqvt Rel*
**and** *C1*: $\bigwedge \Psi'\ R.\ (\Psi',\ R,\ R) \in Rel$
**and** *C2*: $\bigwedge y\ \Psi'\ R\ S\ zvec.\ [\![y \sharp \Psi';\ y \sharp R;\ zvec \sharp* \Psi']\!] \Longrightarrow (\Psi',\ (\!|\nu y|\!)((\!|\nu{*}zvec|\!)(R \parallel S)),\ (\!|\nu{*}zvec|\!)(R \parallel (\!|\nu y|\!)S)) \in Rel$
**and** *C3*: $\bigwedge \Psi'\ zvec\ R\ y.\ [\![y \sharp \Psi';\ zvec \sharp* \Psi']\!] \Longrightarrow (\Psi',\ (\!|\nu y|\!)((\!|\nu{*}zvec|\!)R),$

$(\!|\nu{*}zvec|\!)((\!|\nu y|\!)R)) \in Rel$
— Addition for Broadcast
**and** *C4*: $\bigwedge \Psi'\ R\ S\ zvec.\ [\![zvec\ \sharp{*}\ R;\ zvec\ \sharp{*}\ \Psi']\!] \Longrightarrow (\Psi', ((\!|\nu{*}zvec|\!)(R \parallel S)), (R \parallel ((\!|\nu{*}zvec|\!)S))) \in Rel$

**shows** $\Psi \rhd (\!|\nu x|\!)(P \parallel Q) \rightsquigarrow[Rel]\ P \parallel (\!|\nu x|\!)Q$
⟨*proof*⟩

**lemma** *scopeExtRight*:
**fixes** $x$ :: *name*
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $\Psi$ :: $'b$
**and** $Q$ :: $('a, 'b, 'c)\ psi$
**and** $Rel$ :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

**assumes** $x \sharp P$
**and** $x \sharp \Psi$
**and** *eqvt Rel*
**and** *C1*: $\bigwedge \Psi'\ R.\ (\Psi, R, R) \in Rel$
**and** *C2*: $\bigwedge y\ \Psi'\ R\ S\ zvec.\ [\![y \sharp \Psi'; y \sharp R; zvec \sharp{*} \Psi']\!] \Longrightarrow (\Psi', (\!|\nu{*}zvec|\!)(R \parallel (\!|\nu y|\!)S), (\!|\nu y|\!)((\!|\nu{*}zvec|\!)(R \parallel S))) \in Rel$
— Addition for Broadcast
**and** *C3*: $\bigwedge \Psi'\ R\ S\ zvec.\ [\![zvec \sharp{*} R; zvec \sharp{*} \Psi']\!] \Longrightarrow (\Psi', (R \parallel ((\!|\nu{*}zvec|\!)S)), ((\!|\nu{*}zvec|\!)(R \parallel S))) \in Rel$

**shows** $\Psi \rhd P \parallel (\!|\nu x|\!)Q \rightsquigarrow[Rel]\ (\!|\nu x|\!)(P \parallel Q)$
⟨*proof*⟩

**lemma** *simParComm*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $Q$ :: $('a, 'b, 'c)\ psi$
**and** $Rel$ :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

**assumes** *eqvt Rel*
**and** *C1*: $\bigwedge \Psi'\ R\ S.\ (\Psi', R \parallel S, S \parallel R) \in Rel$
**and** *C2*: $\bigwedge \Psi'\ R\ S\ xvec.\ [\![(\Psi', R, S) \in Rel; xvec \sharp{*} \Psi']\!] \Longrightarrow (\Psi', (\!|\nu{*}xvec|\!)R, (\!|\nu{*}xvec|\!)S) \in Rel$

**shows** $\Psi \rhd P \parallel Q \rightsquigarrow[Rel]\ Q \parallel P$
⟨*proof*⟩

**lemma** *bangExtLeft*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$

**assumes** *guarded P*
**and** $\bigwedge \Psi'\ Q.\ (\Psi', Q, Q) \in Rel$

**shows** $\Psi \rhd !P \rightsquigarrow[Rel]\ P \parallel !P$
⟨*proof*⟩

**lemma** *bangExtRight*:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$

**assumes** *C1*: $\bigwedge \Psi'\ Q.\ (\Psi', Q, Q) \in Rel$

**shows** $\Psi \rhd P \parallel !P \rightsquigarrow[Rel]\ !P$
⟨*proof*⟩

**end**

**end**
**theory** *Bisim-Pres*
**imports** *Bisimulation Sim-Pres*
**begin**

This file is a (heavily modified) variant of the theory *Psi_Calculi.Bisim_Pres* from [1].

**context** *env* **begin**

**lemma** *bisimInputPres*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $Q$ :: $('a, 'b, 'c)\ psi$
**and** $M$ :: $'a$
**and** *xvec* :: *name list*
**and** $N$ :: $'a$

**assumes** $\bigwedge Tvec.\ length\ xvec = length\ Tvec \Longrightarrow \Psi \rhd P[xvec::=Tvec] \sim Q[xvec::=Tvec]$

**shows** $\Psi \rhd M(\!|\lambda * xvec\ N|\!).P \sim M(\!|\lambda * xvec\ N|\!).Q$
⟨*proof*⟩

**lemma** *bisimOutputPres*:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $Q :: ('a, 'b, 'c)\ psi$
**and** $M :: 'a$
**and** $N :: 'a$

**assumes** $\Psi \rhd P \sim Q$

**shows** $\Psi \rhd M\langle N\rangle.P \sim M\langle N\rangle.Q$
⟨*proof*⟩

**lemma** *bisimCasePres*:

**fixes** $\Psi$ :: $'b$
**and** $CsP$ :: $('c \times ('a, 'b, 'c)\ psi)\ list$
**and** $CsQ$ :: $('c \times ('a, 'b, 'c)\ psi)\ list$

**assumes** $\bigwedge\varphi\ P.\ (\varphi, P) \in set\ CsP \Longrightarrow \exists\, Q.\ (\varphi, Q) \in set\ CsQ \wedge guarded\ Q \wedge \Psi \rhd P \sim Q$
**and** $\bigwedge\varphi\ Q.\ (\varphi, Q) \in set\ CsQ \Longrightarrow \exists\, P.\ (\varphi, P) \in set\ CsP \wedge guarded\ P \wedge \Psi \rhd P \sim Q$

**shows** $\Psi \rhd Cases\ CsP \sim Cases\ CsQ$
⟨*proof*⟩

**lemma** *bisimResPres*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $Q$ :: $('a, 'b, 'c)\ psi$
**and** $x$ :: *name*

**assumes** $\Psi \rhd P \sim Q$
**and** $x \sharp \Psi$

**shows** $\Psi \rhd (\!|\nu x|\!)P \sim (\!|\nu x|\!)Q$
⟨*proof*⟩

**lemma** *bisimResChainPres*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $Q$ :: $('a, 'b, 'c)\ psi$
**and** *xvec* :: *name list*

**assumes** $\Psi \rhd P \sim Q$
**and** $xvec \sharp* \Psi$

**shows** $\Psi \rhd (\!|\nu * xvec|\!)P \sim (\!|\nu * xvec|\!)Q$
⟨*proof*⟩

**lemma** *bisimParPresAux*:
**fixes** $\Psi$ :: $'b$
**and** $\Psi_R$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $Q$ :: $('a, 'b, 'c)\ psi$
**and** $R$ :: $('a, 'b, 'c)\ psi$
**and** $A_R$ :: *name list*

**assumes** $\Psi \otimes \Psi_R \rhd P \sim Q$
**and** *FrR*: $extractFrame\ R = \langle A_R, \Psi_R\rangle$
**and** $A_R \sharp* \Psi$
**and** $A_R \sharp* P$
**and** $A_R \sharp* Q$

**shows** $\Psi \rhd P \parallel R \sim Q \parallel R$
⟨*proof*⟩

**lemma** *bisimParPres*:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $Q :: ('a, 'b, 'c)\ psi$
**and** $R :: ('a, 'b, 'c)\ psi$

**assumes** $\Psi \rhd P \sim Q$

**shows** $\Psi \rhd P \parallel R \sim Q \parallel R$
⟨*proof*⟩

**end**

**end**
**theory** *Bisim-Struct-Cong*
**imports** *Bisim-Pres Sim-Struct-Cong Psi-Calculi.Structural-Congruence*
**begin**

This file is a (heavily modified) variant of the theory *Psi_Calculi.Bisim_Struct_Cong* from [1].

**context** *env* **begin**

**lemma** *bisimParComm*:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $Q :: ('a, 'b, 'c)\ psi$

**shows** $\Psi \rhd P \parallel Q \sim Q \parallel P$
⟨*proof*⟩

**inductive-set** *resCommRel* :: $('b \times ('a,'b,'c)\ psi \times ('a,'b,'c)\ psi)\ set$
**where**
*resCommRel-refl* : $(\Psi,P,P) \in resCommRel$
| *resCommRel-swap* : $[\![a \sharp \Psi;\ b \sharp \Psi]\!] \Longrightarrow (\Psi,(\!|\nu a|\!)((\!|\nu b|\!)P),(\!|\nu b|\!)((\!|\nu a|\!)P)) \in resCommRel$
| *resCommRel-res* : $[\![(\Psi,P,Q) \in resCommRel;\ a \sharp \Psi]\!] \Longrightarrow (\Psi,(\!|\nu a|\!)P,(\!|\nu a|\!)Q) \in resCommRel$

**lemma** *eqvtResCommRel*: *eqvt resCommRel*
⟨*proof*⟩

**lemma** *resCommRelStarRes*:
**assumes** $(\Psi,P,Q) \in resCommRel^\star$
**and** $a \sharp \Psi$
**shows** $(\Psi,(\!|\nu a|\!)P,(\!|\nu a|\!)Q) \in resCommRel^\star$

⟨*proof*⟩

**lemma** *resCommRelStarResChain*:
**assumes** $(\Psi, P, Q) \in resCommRel^\star$
**and** $xvec \sharp^* \Psi$
**shows** $(\Psi, (\!|\nu * xvec|\!)P, (\!|\nu * xvec|\!)Q) \in resCommRel^\star$
⟨*proof*⟩

**lemma** *length-induct1*[*consumes 0*, *case-names Nil Cons*]:
**assumes** *b*: $P\ []$
**and** *s*: $\bigwedge x\ xvec.\ [\![\bigwedge yvec.\ length\ xvec = length\ yvec \Longrightarrow P\ yvec]\!] \Longrightarrow P(x\#xvec)$
**shows** $P\ xvec$
⟨*proof*⟩

**lemma** *oneStepPerm-in-rel*:
**assumes** $xvec \sharp^* \Psi$
**shows** $(\Psi, (\!|\nu * xvec|\!)P, (\!|\nu * (rotate1\ xvec)|\!)P) \in resCommRel^\star$
⟨*proof*⟩

**lemma** *fresh-same-multiset*:
**fixes** *xvec*::*name list*
**and** *yvec*::*name list*
**assumes** $mset\ xvec = mset\ yvec$
**and** $xvec \sharp^* X$
**shows** $yvec \sharp^* X$
⟨*proof*⟩

**lemma** *nStepPerm-in-rel*:
**assumes** $xvec \sharp^* \Psi$
**shows** $(\Psi, (\!|\nu * xvec|\!)P, (\!|\nu * (rotate\ n\ xvec)|\!)P) \in resCommRel^\star$
⟨*proof*⟩

**lemma** *any-perm-in-rel*:
**assumes** $xvec \sharp^* \Psi$
**and** $mset\ xvec = mset\ yvec$
**shows** $(\Psi, (\!|\nu * xvec|\!)P, (\!|\nu * yvec|\!)P) \in resCommRel^\star$
⟨*proof*⟩

**lemma** *bisimResComm*:
**fixes** $x :: name$
**and** $\Psi :: 'b$
**and** $y :: name$
**and** $P :: ('a, 'b, 'c)\ psi$

**shows** $\Psi \rhd (\!|\nu x|\!)((\!|\nu y|\!)P) \sim (\!|\nu y|\!)((\!|\nu x|\!)P)$
⟨*proof*⟩

**lemma** *bisimResComm'*:
**fixes** $x :: name$

**and** $\Psi$ :: $'b$
**and** $xvec$ :: $name\ list$
**and** $P$ :: $('a, 'b, 'c)\ psi$

**assumes** $x \sharp \Psi$
**and** $xvec \sharp* \Psi$

**shows** $\Psi \rhd (\!|\nu x|\!)((\!|\nu * xvec|\!)P) \sim (\!|\nu * xvec|\!)((\!|\nu x|\!)P)$
⟨*proof*⟩

**lemma** *bisimParPresSym*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $Q$ :: $('a, 'b, 'c)\ psi$
**and** $R$ :: $('a, 'b, 'c)\ psi$

**assumes** $\Psi \rhd P \sim Q$

**shows** $\Psi \rhd R \parallel P \sim R \parallel Q$
⟨*proof*⟩

**lemma** *bisimScopeExt*:
**fixes** $x$ :: $name$
**and** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $Q$ :: $('a, 'b, 'c)\ psi$

**assumes** $x \sharp P$

**shows** $\Psi \rhd (\!|\nu x|\!)(P \parallel Q) \sim P \parallel (\!|\nu x|\!)Q$
⟨*proof*⟩

**lemma** *bisimScopeExtChain*:
**fixes** $xvec$ :: $name\ list$
**and** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $Q$ :: $('a, 'b, 'c)\ psi$

**assumes** $xvec \sharp* \Psi$
**and** $xvec \sharp* P$

**shows** $\Psi \rhd (\!|\nu * xvec|\!)(P \parallel Q) \sim P \parallel ((\!|\nu * xvec|\!)Q)$
⟨*proof*⟩

**lemma** *bisimParAssoc*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $Q$ :: $('a, 'b, 'c)\ psi$
**and** $R$ :: $('a, 'b, 'c)\ psi$

**shows** $\Psi \rhd (P \parallel Q) \parallel R \sim P \parallel (Q \parallel R)$
⟨*proof*⟩

**lemma** *bisimParNil*:
**fixes** $P :: ('a, 'b, 'c)\ psi$

**shows** $\Psi \rhd P \parallel \mathbf{0} \sim P$
⟨*proof*⟩

**lemma** *bisimResNil*:
**fixes** $x :: name$
**and** $\Psi :: 'b$

**shows** $\Psi \rhd (\!|\nu x|\!)\mathbf{0} \sim \mathbf{0}$
⟨*proof*⟩

**lemma** *bisimOutputPushRes*:
**fixes** $x :: name$
**and** $\Psi :: 'b$
**and** $M :: 'a$
**and** $N :: 'a$
**and** $P :: ('a, 'b, 'c)\ psi$

**assumes** $x \sharp M$
**and** $x \sharp N$

**shows** $\Psi \rhd (\!|\nu x|\!)(M\langle N\rangle.P) \sim M\langle N\rangle.(\!|\nu x|\!)P$
⟨*proof*⟩

**lemma** *bisimInputPushRes*:
**fixes** $x :: name$
**and** $\Psi :: 'b$
**and** $M :: 'a$
**and** $xvec :: name\ list$
**and** $N :: 'a$
**and** $P :: ('a, 'b, 'c)\ psi$

**assumes** $x \sharp M$
**and** $x \sharp xvec$
**and** $x \sharp N$

**shows** $\Psi \rhd (\!|\nu x|\!)(M(\!|\lambda * xvec\ N|\!).P) \sim M(\!|\lambda * xvec\ N|\!).(\!|\nu x|\!)P$
⟨*proof*⟩

**lemma** *bisimCasePushRes*:
**fixes** $x :: name$
**and** $\Psi :: 'b$
**and** $Cs :: ('c \times ('a, 'b, 'c)\ psi)\ list$

**assumes** $x \sharp (map\ fst\ Cs)$

**shows** $\Psi \rhd (\!|\nu x|\!)(Cases\ Cs) \sim Cases(map\ (\lambda(\varphi, P).\ (\varphi, (\!|\nu x|\!)P))\ Cs)$
⟨*proof*⟩

**lemma** *bangExt*:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$

**assumes** *guarded* $P$

**shows** $\Psi \rhd !P \sim P \parallel !P$
⟨*proof*⟩

**lemma** *bisimScopeExtSym*:
**fixes** $x :: name$
**and** $Q :: ('a, 'b, 'c)\ psi$
**and** $P :: ('a, 'b, 'c)\ psi$

**assumes** $x \sharp \Psi$
**and** $x \sharp Q$

**shows** $\Psi \rhd (\!|\nu x|\!)(P \parallel Q) \sim ((\!|\nu x|\!)P) \parallel Q$
⟨*proof*⟩

**lemma** *bisimScopeExtChainSym*:
**fixes** $xvec :: name\ list$
**and** $Q :: ('a, 'b, 'c)\ psi$
**and** $P :: ('a, 'b, 'c)\ psi$

**assumes** $xvec \sharp* \Psi$
**and** $xvec \sharp* Q$

**shows** $\Psi \rhd (\!|\nu* xvec|\!)(P \parallel Q) \sim ((\!|\nu* xvec|\!)P) \parallel Q$
⟨*proof*⟩

**lemma** *bisimParPresAuxSym*:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $Q :: ('a, 'b, 'c)\ psi$
**and** $R :: ('a, 'b, 'c)\ psi$

**assumes** $\Psi \otimes \Psi_R \rhd P \sim Q$
**and** $extractFrame\ R = \langle A_R, \Psi_R \rangle$
**and** $A_R \sharp* \Psi$
**and** $A_R \sharp* P$
**and** $A_R \sharp* Q$

**shows** $\Psi \rhd R \parallel P \sim R \parallel Q$
⟨*proof*⟩

**lemma** *bangDerivative*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $\alpha$ :: $'a$ *action*
**and** $P'$ :: $('a, 'b, 'c)$ *psi*

**assumes** $\Psi \rhd !P \longmapsto \alpha \prec P'$
**and** $\Psi \rhd P \sim Q$
**and** *bn* $\alpha \sharp* \Psi$
**and** *bn* $\alpha \sharp* P$
**and** *bn* $\alpha \sharp* Q$
**and** *bn* $\alpha \sharp* subject\ \alpha$
**and** *guarded* $Q$

**obtains** $Q'$ $R$ $T$ **where** $\Psi \rhd !Q \longmapsto \alpha \prec Q'$ **and** $\Psi \rhd P' \sim R \parallel !P$ **and** $\Psi \rhd Q' \sim T \parallel !Q$ **and** $\Psi \rhd R \sim T$
**and** ((*supp* $R$)::*name set*) ⊆ *supp* $P'$ **and** ((*supp* $T$)::*name set*) ⊆ *supp* $Q'$
⟨*proof*⟩

**lemma** *structCongBisim*:
**fixes** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $Q$ :: $('a, 'b, 'c)$ *psi*

**assumes** $P \equiv_s Q$

**shows** $P \sim Q$
⟨*proof*⟩

**lemma** *bisimBangPres*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)$ *psi*
**and** $Q$ :: $('a, 'b, 'c)$ *psi*

**assumes** $\Psi \rhd P \sim Q$
**and** *guarded* $P$
**and** *guarded* $Q$

**shows** $\Psi \rhd !P \sim !Q$
⟨*proof*⟩

**end**

**end**
**theory** *Bisim-Subst*
**imports** *Bisim-Struct-Cong Psi-Calculi.Close-Subst*
**begin**

This file is a (heavily modified) variant of the theory *Psi_Calculi.Bisim_Subst* from [1].

**context** *env* **begin**

**abbreviation**
*bisimSubstJudge* (- ⊳ - ∼$_s$ - [70, 70, 70] 65) **where** Ψ ⊳ *P* ∼$_s$ *Q* ≡ (Ψ, *P*, *Q*) ∈ *closeSubst bisim*
**abbreviation**
*bisimSubstNilJudge* (- ∼$_s$ - [70, 70] 65) **where** *P* ∼$_s$ *Q* ≡ *SBottom*′ ⊳ *P* ∼$_s$ *Q*

**lemmas** *bisimSubstClosed*[*eqvt*] = *closeSubstClosed*[*OF bisimEqvt*]
**lemmas** *bisimSubstEqvt*[*simp*] = *closeSubstEqvt*[*OF bisimEqvt*]

**lemma** *bisimSubstOutputPres*:
**fixes** Ψ :: 'b
**and** *P* :: ('a, 'b, 'c) *psi*
**and** *Q* :: ('a, 'b, 'c) *psi*
**and** *M* :: 'a
**and** *N* :: 'a

**assumes** Ψ ⊳ *P* ∼$_s$ *Q*

**shows** Ψ ⊳ *M*⟨*N*⟩.*P* ∼$_s$ *M*⟨*N*⟩.*Q*
⟨*proof*⟩

**lemma** *seqSubstInputChain*[*simp*]:
**fixes** *xvec* :: *name list*
**and** *N* :: 'a
**and** *P* :: ('a, 'b, 'c) *psi*
**and** σ :: (*name list* × 'a *list*) *list*

**assumes** *xvec* ♯* σ

**shows** *seqSubs*′ (*inputChain xvec N P*) σ = *inputChain xvec* (*substTerm.seqSubst N* σ) (*seqSubs P* σ)
⟨*proof*⟩

**lemma** *bisimSubstInputPres*:
**fixes** Ψ :: 'b
**and** *P* :: ('a, 'b, 'c) *psi*
**and** *Q* :: ('a, 'b, 'c) *psi*
**and** *M* :: 'a
**and** *xvec* :: *name list*
**and** *N* :: 'a

**assumes** Ψ ⊳ *P* ∼$_s$ *Q*
**and** *xvec* ♯* Ψ
**and** *distinct xvec*

**shows** $\Psi \rhd M(\!|\lambda * xvec\ N|\!).P \sim_s M(\!|\lambda * xvec\ N|\!).Q$
⟨*proof*⟩

**lemma** *bisimSubstCasePresAux*:
**fixes** $\Psi$ :: $'b$
**and** *CsP* :: $('c \times ('a, 'b, 'c)\ psi)\ list$
**and** *CsQ* :: $('c \times ('a, 'b, 'c)\ psi)\ list$

**assumes** *C1*: $\bigwedge \varphi\ P.\ (\varphi, P) \in set\ CsP \Longrightarrow \exists\, Q.\ (\varphi, Q) \in set\ CsQ \wedge guarded\ Q \wedge \Psi \rhd P \sim_s Q$
**and** *C2*: $\bigwedge \varphi\ Q.\ (\varphi, Q) \in set\ CsQ \Longrightarrow \exists\, P.\ (\varphi, P) \in set\ CsP \wedge guarded\ P \wedge \Psi \rhd P \sim_s Q$

**shows** $\Psi \rhd Cases\ CsP \sim_s Cases\ CsQ$
⟨*proof*⟩

**lemma** *bisimSubstReflexive*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$

**shows** $\Psi \rhd P \sim_s P$
⟨*proof*⟩

**lemma** *bisimSubstTransitive*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $Q$ :: $('a, 'b, 'c)\ psi$
**and** $R$ :: $('a, 'b, 'c)\ psi$

**assumes** $\Psi \rhd P \sim_s Q$
**and** $\Psi \rhd Q \sim_s R$

**shows** $\Psi \rhd P \sim_s R$
⟨*proof*⟩

**lemma** *bisimSubstSymmetric*:
**fixes** $\Psi$ :: $'b$
**and** $P$ :: $('a, 'b, 'c)\ psi$
**and** $Q$ :: $('a, 'b, 'c)\ psi$

**assumes** $\Psi \rhd P \sim_s Q$

**shows** $\Psi \rhd Q \sim_s P$
⟨*proof*⟩

**lemma** *bisimSubstCasePres*:
**fixes** $\Psi$ :: $'b$
**and** *CsP* :: $('c \times ('a, 'b, 'c)\ psi)\ list$

**and** $CsQ :: ('c \times ('a, 'b, 'c)\ psi)\ list$

**assumes** $length\ CsP = length\ CsQ$
**and** $C$: $\bigwedge(i::nat)\ \varphi\ P\ \varphi'\ Q.\ [\![i <= length\ CsP; (\varphi, P) = nth\ CsP\ i; (\varphi', Q) = nth\ CsQ\ i]\!] \Longrightarrow \varphi = \varphi' \wedge \Psi \rhd P \sim_s Q \wedge guarded\ P \wedge guarded\ Q$

**shows** $\Psi \rhd Cases\ CsP \sim_s Cases\ CsQ$
⟨*proof*⟩

**lemma** *bisimSubstParPres*:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $Q :: ('a, 'b, 'c)\ psi$
**and** $R :: ('a, 'b, 'c)\ psi$

**assumes** $\Psi \rhd P \sim_s Q$

**shows** $\Psi \rhd P \parallel R \sim_s Q \parallel R$
⟨*proof*⟩

**lemma** *bisimSubstResPres*:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $Q :: ('a, 'b, 'c)\ psi$
**and** $x :: name$

**assumes** $\Psi \rhd P \sim_s Q$
**and** $x \sharp \Psi$

**shows** $\Psi \rhd (\!|\nu x|\!) P \sim_s (\!|\nu x|\!) Q$
⟨*proof*⟩

**lemma** *bisimSubstBangPres*:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $Q :: ('a, 'b, 'c)\ psi$

**assumes** $\Psi \rhd P \sim_s Q$
**and** $guarded\ P$
**and** $guarded\ Q$

**shows** $\Psi \rhd\ !P \sim_s\ !Q$
⟨*proof*⟩

**lemma** *substNil*[*simp*]:
**fixes** $xvec :: name\ list$
**and** $Tvec :: 'a\ list$

**assumes** $wellFormedSubst\ \sigma$

**and** *distinct xvec*

**shows** $(\mathbf{0}[<\sigma>]) = \mathbf{0}$
⟨*proof*⟩

**lemma** *bisimSubstParNil*:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$

**shows** $\Psi \rhd P \parallel \mathbf{0} \sim_s P$
⟨*proof*⟩

**lemma** *bisimSubstParComm*:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $Q :: ('a, 'b, 'c)\ psi$

**shows** $\Psi \rhd P \parallel Q \sim_s Q \parallel P$
⟨*proof*⟩

**lemma** *bisimSubstParAssoc*:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $Q :: ('a, 'b, 'c)\ psi$
**and** $R :: ('a, 'b, 'c)\ psi$

**shows** $\Psi \rhd (P \parallel Q) \parallel R \sim_s P \parallel (Q \parallel R)$
⟨*proof*⟩

**lemma** *bisimSubstResNil*:
**fixes** $\Psi :: 'b$
**and** $x :: name$

**shows** $\Psi \rhd (\!|\nu x|\!)\mathbf{0} \sim_s \mathbf{0}$
⟨*proof*⟩

**lemma** *seqSubst2*:
**fixes** $x :: name$
**and** $P :: ('a, 'b, 'c)\ psi$

**assumes** *wellFormedSubst* $\sigma$
**and** $x \sharp \sigma$
**and** $x \sharp P$

**shows** $x \sharp P[<\sigma>]$
⟨*proof*⟩

**notation** *substTerm.seqSubst* (-[<->] [*100*, *100*] *100*)

**lemma** *bisimSubstScopeExt*:
**fixes** $\Psi :: 'b$
**and** $x :: name$
**and** $P :: ('a, 'b, 'c)\ psi$
**and** $Q :: ('a, 'b, 'c)\ psi$

**assumes** $x \sharp P$

**shows** $\Psi \rhd (\!|\nu x|\!)(P \parallel Q) \sim_s P \parallel (\!|\nu x|\!) Q$
⟨*proof*⟩

**lemma** *bisimSubstCasePushRes*:
**fixes** $x :: name$
**and** $\Psi :: 'b$
**and** $Cs :: ('c \times ('a, 'b, 'c)\ psi)\ list$

**assumes** $x \sharp map\ fst\ Cs$

**shows** $\Psi \rhd (\!|\nu x|\!)(Cases\ Cs) \sim_s Cases\ map\ (\lambda(\varphi, P).\ (\varphi, (\!|\nu x|\!)P))\ Cs$
⟨*proof*⟩

**lemma** *bisimSubstOutputPushRes*:
**fixes** $x :: name$
**and** $\Psi :: 'b$
**and** $M :: 'a$
**and** $N :: 'a$
**and** $P :: ('a, 'b, 'c)\ psi$

**assumes** $x \sharp M$
**and** $x \sharp N$

**shows** $\Psi \rhd (\!|\nu x|\!)(M\langle N\rangle.P) \sim_s M\langle N\rangle.(\!|\nu x|\!)P$
⟨*proof*⟩

**lemma** *bisimSubstInputPushRes*:
**fixes** $x :: name$
**and** $\Psi :: 'b$
**and** $M :: 'a$
**and** $xvec :: name\ list$
**and** $N :: 'a$

**assumes** $x \sharp M$
**and** $x \sharp xvec$
**and** $x \sharp N$

**shows** $\Psi \rhd (\!|\nu x|\!)(M(\!|\lambda * xvec\ N|\!).P) \sim_s M(\!|\lambda * xvec\ N|\!).(\!|\nu x|\!)P$
⟨*proof*⟩

**lemma** *bisimSubstResComm*:

**fixes** $x :: name$
**and** $y :: name$

**shows** $\Psi \rhd (\!|\nu x|\!)((\!|\nu y|\!)P) \sim_s (\!|\nu y|\!)((\!|\nu x|\!)P)$
⟨*proof*⟩

**lemma** *bisimSubstExtBang*:
**fixes** $\Psi :: 'b$
**and** $P :: ('a, 'b, 'c)\ psi$

**assumes** *guarded* $P$

**shows** $\Psi \rhd !P \sim_s P \parallel !P$
⟨*proof*⟩

**lemma** *structCongBisimSubst*:
**fixes** $P :: ('a, 'b, 'c)\ psi$
**and** $Q :: ('a, 'b, 'c)\ psi$

**assumes** $P \equiv_s Q$

**shows** $P \sim_s Q$
⟨*proof*⟩

**end**

**end**
**theory** *Broadcast-Thms*
**imports** *Broadcast-Chain Broadcast-Frame Semantics Simulation Bisimulation Sim-Pres*
*Sim-Struct-Cong Bisim-Pres Bisim-Struct-Cong Bisim-Subst*
**begin**

**context** *env*
**begin**

## 2.1 Syntax

### 2.1.1 Psi calculus agents – Definition 2

$M$, $N$ range over message terms. $P$, $Q$ range over processes. $C$ ranges over cases.

- Output: $M\langle N\rangle.P$
- Input: $M(\!|\lambda * xvec\ N|\!).P$
- Case: $Case\ C$
- Par: $P \parallel Q$

- Res: $(\!|\nu x|\!)P$
- Assert: $\{\!|\Psi|\!\}$
- Bang: $!P$

- Cases: $\Box\ \varphi \Rightarrow P\ C$

### 2.1.2 Parameters – Definition 1

- Channel equivalence: $M \leftrightarrow N$
- Composition: $\Psi_P \otimes \Psi_Q$
- Unit: $\mathbf{1}$
- Entailment: $\Psi \vdash \varphi$

### 2.1.3 Extra predicates for broadcast – Definition 5

- Output connectivity: $M \preceq N$
- Input connectivity: $M \succeq N$

### 2.1.4 Transitions – Definition 3

- $\Psi \rhd P \longmapsto \alpha\ P'$

### 2.1.5 Actions ($\alpha$) – Definition 7

- Input: $M(\!|N|\!)$
- Output: $M(\!|\nu * xvec|\!)\langle N \rangle$
- Broadcast input: ¿$M(\!|N|\!)$
- Broadcast output: ¡$M(\!|\nu * xvec|\!)\langle N \rangle$
- Silent action: $\tau$

## 2.2 Semantics

### 2.2.1 Basic Psi semantics – Table 1

- Theorem *Input*:

  $[\![\Psi \vdash M \leftrightarrow K;\ \mathit{distinct\ xvec};\ \mathit{set\ xvec} \subseteq \mathit{supp}\ N;\ \mathit{length\ xvec} = \mathit{length\ Tvec}]\!]$
  $\Longrightarrow \Psi \rhd M(\!|\lambda{*}\mathit{xvec}\ N|\!).P \longmapsto K(\!|N[\mathit{xvec}::=\mathit{Tvec}]|\!) \prec P[\mathit{xvec}::=\mathit{Tvec}]$

- Theorem *Output*:

  $\Psi \vdash M \leftrightarrow K \Longrightarrow \Psi \rhd M\langle N\rangle.P \longmapsto K\langle N\rangle \prec P$

- Theorem *Case*:

  $[\![\Psi \rhd P \longmapsto \mathit{Rs};\ (\varphi,\ P)\ \mathit{mem\ Cs};\ \Psi \vdash \varphi;\ \mathit{guarded}\ P]\!] \Longrightarrow \Psi \rhd \mathit{Cases\ Cs} \longmapsto \mathit{Rs}$

- Theorems *Par1* and *Par2*:

  $[\![\Psi \otimes \Psi_Q \rhd P \longmapsto \alpha \prec P';\ \mathit{extractFrame}\ Q = \langle A_Q,\ \Psi_Q\rangle;\ \mathit{bn}\ \alpha\ \sharp{*}\ Q;\ A_Q\ \sharp{*}\ \Psi;$
  $A_Q\ \sharp{*}\ P;\ A_Q\ \sharp{*}\ \alpha]\!]$
  $\Longrightarrow \Psi \rhd P \parallel Q \longmapsto \alpha \prec P' \parallel Q$

  $[\![\Psi \otimes \Psi_P \rhd Q \longmapsto \alpha \prec Q';\ \mathit{extractFrame}\ P = \langle A_P,\ \Psi_P\rangle;\ \mathit{bn}\ \alpha\ \sharp{*}\ P;\ A_P\ \sharp{*}\ \Psi;$
  $A_P\ \sharp{*}\ Q;\ A_P\ \sharp{*}\ \alpha]\!]$
  $\Longrightarrow \Psi \rhd P \parallel Q \longmapsto \alpha \prec P \parallel Q'$

- Theorems *Comm1* and *Comm2*:

  $[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P';\ \mathit{extractFrame}\ P = \langle A_P,\ \Psi_P\rangle;$
  $\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu{*}\mathit{xvec}|\!)\langle N\rangle \prec Q';\ \mathit{extractFrame}\ Q = \langle A_Q,\ \Psi_Q\rangle;$
  $\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K;\ A_P\ \sharp{*}\ \Psi;\ A_P\ \sharp{*}\ P;\ A_P\ \sharp{*}\ Q;\ A_P\ \sharp{*}\ M;\ A_P\ \sharp{*}\ A_Q;\ A_Q\ \sharp{*}\ \Psi;$
  $A_Q\ \sharp{*}\ P;\ A_Q\ \sharp{*}\ Q;\ A_Q\ \sharp{*}\ K;\ \mathit{xvec}\ \sharp{*}\ P]\!]$
  $\Longrightarrow \Psi \rhd P \parallel Q \longmapsto \tau \prec (\!|\nu{*}\mathit{xvec}|\!)P' \parallel Q'$

  $[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu{*}\mathit{xvec}|\!)\langle N\rangle \prec P';\ \mathit{extractFrame}\ P = \langle A_P,\ \Psi_P\rangle;$
  $\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q';\ \mathit{extractFrame}\ Q = \langle A_Q,\ \Psi_Q\rangle;\ \Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K;$
  $A_P\ \sharp{*}\ \Psi;\ A_P\ \sharp{*}\ P;\ A_P\ \sharp{*}\ Q;\ A_P\ \sharp{*}\ M;\ A_P\ \sharp{*}\ A_Q;\ A_Q\ \sharp{*}\ \Psi;\ A_Q\ \sharp{*}\ P;\ A_Q\ \sharp{*}\ Q;$
  $A_Q\ \sharp{*}\ K;\ \mathit{xvec}\ \sharp{*}\ Q]\!]$
  $\Longrightarrow \Psi \rhd P \parallel Q \longmapsto \tau \prec (\!|\nu{*}\mathit{xvec}|\!)P' \parallel Q'$

- Theorem *Open*:

$[\![\Psi \rhd P \longmapsto M(\!|\nu*(xvec \mathbin{@} yvec)|\!)\langle N\rangle \prec P';\ x \in supp\ N;\ x \sharp \Psi;\ x \sharp M;\ x \sharp xvec;$
$x \sharp yvec]\!]$
$\Longrightarrow \Psi \rhd (\!|\nu x|\!)P \longmapsto M(\!|\nu*(xvec \mathbin{@} x \mathbin{\#} yvec)|\!)\langle N\rangle \prec P'$

- Theorem *Scope*:

  $[\![\Psi \rhd P \longmapsto \alpha \prec P';\ x \sharp \Psi;\ x \sharp \alpha]\!] \Longrightarrow \Psi \rhd (\!|\nu x|\!)P \longmapsto \alpha \prec (\!|\nu x|\!)P'$

- Theorem *Bang*:

  $[\![\Psi \rhd P \parallel !P \longmapsto Rs;\ guarded\ P]\!] \Longrightarrow \Psi \rhd !P \longmapsto Rs$

### 2.2.2 Broadcast rules – Table 2

- Theorem *BrInput*:

  $[\![\Psi \vdash K \succeq M;\ distinct\ xvec;\ set\ xvec \subseteq supp\ N;\ length\ xvec = length\ Tvec]\!]$
  $\Longrightarrow \Psi \rhd M(\!|\lambda*xvec\ N|\!).P \longmapsto ¿K(\!|N[xvec::=Tvec]|\!) \prec P[xvec::=Tvec]$

- Theorem *BrOutput*:

  $\Psi \vdash M \preceq K \Longrightarrow \Psi \rhd M\langle N\rangle.P \longmapsto ¡K\langle N\rangle \prec P$

- Theorem *BrMerge*:

  $[\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¿M(\!|N|\!) \prec P';\ extractFrame\ P = \langle A_P,\ \Psi_P\rangle;$
  $\Psi \otimes \Psi_P \rhd Q \longmapsto ¿M(\!|N|\!) \prec Q';\ extractFrame\ Q = \langle A_Q,\ \Psi_Q\rangle;\ A_P \sharp* \Psi;$
  $A_P \sharp* P;$
  $A_P \sharp* Q;\ A_P \sharp* M;\ A_P \sharp* A_Q;\ A_Q \sharp* \Psi;\ A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* M]\!]$
  $\Longrightarrow \Psi \rhd P \parallel Q \longmapsto ¿M(\!|N|\!) \prec P' \parallel Q'$

- Theorems *BrComm1* and *BrComm2*:

  $[\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¿M(\!|N|\!) \prec P';\ extractFrame\ P = \langle A_P,\ \Psi_P\rangle;$
  $\Psi \otimes \Psi_P \rhd Q \longmapsto ¡M(\!|\nu*xvec|\!)\langle N\rangle \prec Q';\ extractFrame\ Q = \langle A_Q,\ \Psi_Q\rangle;\ A_P \sharp* \Psi;$
  $A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* M;\ A_P \sharp* A_Q;\ A_Q \sharp* \Psi;\ A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* M;$
  $xvec \sharp* P]\!]$
  $\Longrightarrow \Psi \rhd P \parallel Q \longmapsto ¡M(\!|\nu*xvec|\!)\langle N\rangle \prec P' \parallel Q'$

  $[\![\Psi \otimes \Psi_Q \rhd P \longmapsto ¡M(\!|\nu*xvec|\!)\langle N\rangle \prec P';\ extractFrame\ P = \langle A_P,\ \Psi_P\rangle;$
  $\Psi \otimes \Psi_P \rhd Q \longmapsto ¿M(\!|N|\!) \prec Q';\ extractFrame\ Q = \langle A_Q,\ \Psi_Q\rangle;\ A_P \sharp* \Psi;$
  $A_P \sharp* P;$
  $A_P \sharp* Q;\ A_P \sharp* M;\ A_P \sharp* A_Q;\ A_Q \sharp* \Psi;\ A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* M;$
  $xvec \sharp* Q]\!]$
  $\Longrightarrow \Psi \rhd P \parallel Q \longmapsto ¡M(\!|\nu*xvec|\!)\langle N\rangle \prec P' \parallel Q'$

- Theorem *BrClose*:

$$[\![\Psi \rhd P \longmapsto ¡M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P';\ x \in supp\ M;\ x \sharp \Psi]\!]$$
$$\Longrightarrow \Psi \rhd (\!|\nu x|\!)P \longmapsto \tau \prec (\!|\nu x|\!)((\!|\nu{*}xvec|\!)P')$$

- Theorem *BrOpen*:

$$[\![\Psi \rhd P \longmapsto ¡M(\!|\nu{*}(xvec\ @\ yvec)|\!)\langle N\rangle \prec P';\ x \in supp\ N;\ x \sharp \Psi;\ x \sharp M;\ x \sharp xvec;$$
$$x \sharp yvec]\!]$$
$$\Longrightarrow \Psi \rhd (\!|\nu x|\!)P \longmapsto ¡M(\!|\nu{*}(xvec\ @\ x\ \#\ yvec)|\!)\langle N\rangle \prec P'$$

### 2.2.3 Requirements for broadcast – Definition 6

- Theorem *chanOutConSupp*:

$$\Psi \vdash M \preceq N \Longrightarrow supp\ N \subseteq supp\ M$$

- Theorem *chanInConSupp*:

$$\Psi \vdash N \succeq M \Longrightarrow supp\ N \subseteq supp\ M$$

### 2.2.4 Strong bisimulation – Definition 4

- Theorem *bisim.step*:

$$[\![insertAssertion\ (extractFrame\ P)\ \Psi \simeq_F insertAssertion\ (extractFrame\ Q)\ \Psi;$$
$$\Psi \rhd P \rightsquigarrow[bisim]\ Q;\ \forall \Psi'.\ \Psi \otimes \Psi' \rhd P \sim Q;\ \Psi \rhd Q \sim P]\!]$$
$$\Longrightarrow \Psi \rhd P \sim Q$$

## 2.3 Theorems

### 2.3.1 Congruence properties of strong bisimulation – Theorem 8

- Theorem *bisimOutputPres*:

$$\Psi \rhd P \sim Q \Longrightarrow \Psi \rhd M\langle N\rangle.P \sim M\langle N\rangle.Q$$

- Theorem *bisimInputPres*:

$$(\bigwedge Tvec.\ length\ xvec = length\ Tvec \Longrightarrow \Psi \rhd P[xvec::=Tvec] \sim Q[xvec::=Tvec])$$
$$\Longrightarrow$$
$$\Psi \rhd M(\!|\lambda{*}xvec\ N|\!).P \sim M(\!|\lambda{*}xvec\ N|\!).Q$$

- Theorem *bisimCasePres*:

  $[\![\bigwedge\varphi\ P.\ (\varphi,\ P)\ mem\ CsP \Longrightarrow \exists\, Q.\ (\varphi,\ Q)\ mem\ CsQ \wedge guarded\ Q \wedge \Psi \rhd P \sim Q;$
  $\bigwedge\varphi\ Q.\ (\varphi,\ Q)\ mem\ CsQ \Longrightarrow \exists\, P.\ (\varphi,\ P)\ mem\ CsP \wedge guarded\ P \wedge \Psi \rhd P \sim Q]\!]$
  $\Longrightarrow \Psi \rhd Cases\ CsP \sim Cases\ CsQ$

- Theorems *bisimParPres* and *bisimParPresSym*:

  $\Psi \rhd P \sim Q \Longrightarrow \Psi \rhd P \parallel R \sim Q \parallel R$

  $\Psi \rhd P \sim Q \Longrightarrow \Psi \rhd R \parallel P \sim R \parallel Q$

- Theorem *bisimResPres*:

  $[\![\Psi \rhd P \sim Q;\ x \sharp \Psi]\!] \Longrightarrow \Psi \rhd (\!|\nu x|\!)P \sim (\!|\nu x|\!)Q$

- Theorem *bisimBangPres*:

  $[\![\Psi \rhd P \sim Q;\ guarded\ P;\ guarded\ Q]\!] \Longrightarrow \Psi \rhd\ !P \sim\ !Q$

### 2.3.2 Strong congruence, bisimulation closed under substitution – Definition 9

- Theorem *closeSubst_def*:

  $closeSubst\ Rel \equiv$
  $\{(\Psi,\ P,\ Q)\ |\Psi\ P\ Q.\ \forall\sigma.\ wellFormedSubst\ \sigma \longrightarrow (\Psi,\ P[<\sigma>],\ Q[<\sigma>]) \in Rel\}$

- $\Psi \rhd P \sim_s Q$

### 2.3.3 Strong congruence is a process congruence for all $\Psi$ – Theorem 10

- Theorem *bisimSubstOutputPres*:

  $\Psi \rhd P \sim_s Q \Longrightarrow \Psi \rhd M\langle N\rangle.P \sim_s M\langle N\rangle.Q$

- Theorem *bisimSubstInputPres*:

  $[\![\Psi \rhd P \sim_s Q;\ xvec \sharp* \Psi;\ distinct\ xvec]\!] \Longrightarrow \Psi \rhd M(\!|\lambda* xvec\ N|\!).P \sim_s M(\!|\lambda* xvec\ N|\!).Q$

- Theorem *bisimSubstCasePres*:

  $[\![ length\ CsP = length\ CsQ;$
  $\bigwedge i\ \varphi\ P\ \varphi'\ Q.$
  $\quad [\![ i \leq length\ CsP;\ (\varphi,\ P) = CsP\ !\ i;\ (\varphi',\ Q) = CsQ\ !\ i ]\!]$
  $\quad \Longrightarrow \varphi = \varphi' \wedge \Psi \rhd P \sim_s Q \wedge guarded\ P \wedge guarded\ Q ]\!]$
  $\Longrightarrow \Psi \rhd Cases\ CsP \sim_s Cases\ CsQ$

- Theorem *bisimSubstParPres*:

  $\Psi \rhd P \sim_s Q \Longrightarrow \Psi \rhd P \parallel R \sim_s Q \parallel R$

- Theorem *bisimSubstResPres*:

  $[\![ \Psi \rhd P \sim_s Q;\ x \sharp \Psi ]\!] \Longrightarrow \Psi \rhd (\!|\nu x|\!) P \sim_s (\!|\nu x|\!) Q$

- Theorem *bisimSubstBangPres*:

  $[\![ \Psi \rhd P \sim_s Q;\ guarded\ P;\ guarded\ Q ]\!] \Longrightarrow \Psi \rhd\ !P \sim_s\ !Q$

### 2.3.4 Structural equivalence – Theorem 11

- Theorem *bisimCasePushRes*:

  $x \sharp map\ fst\ Cs \Longrightarrow \Psi \rhd (\!|\nu x|\!)(Cases\ Cs) \sim Cases\ map\ (\lambda(\varphi,\ P).\ (\varphi,\ (\!|\nu x|\!) P))\ Cs$

- Theorem *bisimInputPushRes*:

  $[\![ x \sharp M;\ x \sharp xvec;\ x \sharp N ]\!] \Longrightarrow \Psi \rhd (\!|\nu x|\!)(M(\!|\lambda * xvec\ N|\!).P) \sim M(\!|\lambda * xvec\ N|\!).(\!|\nu x|\!) P$

- Theorem *bisimOutputPushRes*:

  $[\![ x \sharp M;\ x \sharp N ]\!] \Longrightarrow \Psi \rhd (\!|\nu x|\!)(M\langle N\rangle.P) \sim M\langle N\rangle.(\!|\nu x|\!) P$

- Theorem *bisimParAssoc*:

  $\Psi \rhd P \parallel Q \parallel R \sim P \parallel (Q \parallel R)$

- Theorem *bisimParComm*:

  $\Psi \rhd P \parallel Q \sim Q \parallel P$

- Theorem *bisimResNil*:

$\Psi \rhd (\!|\nu x|\!)\mathbf{0} \sim \mathbf{0}$

- Theorem *bisimScopeExt*:

  $x \sharp P \Longrightarrow \Psi \rhd (\!|\nu x|\!)(P \parallel Q) \sim P \parallel (\!|\nu x|\!)Q$

- Theorem *bisimResComm*:

  $\Psi \rhd (\!|\nu x|\!)((\!|\nu y|\!)P) \sim (\!|\nu y|\!)((\!|\nu x|\!)P)$

- Theorem *bangExt*:

  $guarded\ P \Longrightarrow \Psi \rhd !P \sim P \parallel !P$

- Theorem *bisimParNil*:

  $\Psi \rhd P \parallel \mathbf{0} \sim P$

### 2.3.5 Support of processes in broadcast transitions – Lemma 14

- Theorem *brInputTermSupp*:

  $\Psi \rhd P \longmapsto ¿K(\!|N|\!) \prec P' \Longrightarrow supp\ K \subseteq supp\ P$

- Theorem *brOutputTermSupp*:

  $\Psi \rhd P \longmapsto RBrOut\ K\ ((\!|\nu{*}xvec|\!)N \prec' P') \Longrightarrow supp\ K \subseteq supp\ P$

**end**

**end**

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