

Bondy's Theorem

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Abstract

A proof of Bondy's Theorem following Bollobás [1].

```
theory Bondy  
imports Main  
begin
```

```
lemma card-less-if-surj-not-inj:
```

```
   $\llbracket \text{finite } A; f \text{ ' } A = B; \neg \text{inj-on } f \text{ } A \rrbracket \implies \text{card } B < \text{card } A$   
  <proof>
```

```
theorem Bondy :
```

```
  assumes  $\forall A \in F. A \subseteq X$  and  $\text{card } X \geq 1$  and  $\text{card } F = \text{card } X$   
  shows  $\exists D. D \subseteq X \ \& \ \text{card } D < \text{card } X \ \& \ \text{card } (\text{inter } D \text{ ' } F) = \text{card } F$   
  <proof>
```

```
end
```

References

- [1] B. Bollobás. *Combinatorics: set systems, hypergraphs, families of vectors and combinatorial probability*. Cambridge University Press, 1986.