

Bondy's Theorem

Jeremy Avigad and Stefan Hetzl

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Abstract

A proof of Bondy's Theorem following Bollobás [1].

theory *Bondy*
imports *Main*
begin

lemma *card-less-if-surj-not-inj*:

$\llbracket \text{finite } A; f \text{ ' } A = B; \neg \text{inj-on } f \text{ } A \rrbracket \implies \text{card } B < \text{card } A$
<proof>

theorem *Bondy* :

assumes $\forall A \in F. A \subseteq X$ **and** $\text{card } X \geq 1$ **and** $\text{card } F = \text{card } X$
shows $\exists D. D \subseteq X \ \& \ \text{card } D < \text{card } X \ \& \ \text{card } (\text{inter } D \text{ ' } F) = \text{card } F$
<proof>

end

References

- [1] B. Bollobás. *Combinatorics: set systems, hypergraphs, families of vectors and combinatorial probability*. Cambridge University Press, 1986.