# Arithmetic progressions and relative primes 

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#### Abstract

This article provides a formalization of the solution obtained by the author of the Problem "ARITHMETIC PROGRESSIONS" from the Putnam exam problems [1] of 2002. The statement of the problem is as follows: For which integers $n>1$ does the set of positive integers less than and relatively prime to $n$ constitute an arithmetic progression?


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1 Problem ARITHMETIC PROGRESSIONS (Put- nam exam problems 2002)
theory Arith-Prog-Rel-Primes imports

        Complex-Main
    
        HOL-Number-Theory.Number-Theory
    begin

Statement of the problem (from [1]): For which integers $n>1$ does the set of positive integers less than and relatively prime to $n$ constitute an arithmetic progression?

The solution of the above problem is theorem arith-prog-rel-primes-solution.
First, we will require some auxiliary material before we get started with the actual solution.

### 1.1 Auxiliary results

```
lemma even-and-odd-parts:
    fixes n::nat
```

```
    assumes <n\not=0>
    shows }\langle\existskq::nat.n=(2::nat)^ k*q\wedge odd q>
<proof>
lemma only-one-odd-div-power2:
    fixes n::nat
    assumes }\langlen\not=0\rangle\mathrm{ and «\ x. x dvd n > odd }x\Longrightarrowx=1
    shows }\langle\existsk.n=(2::nat)`k
    <proof\rangle
lemma coprime-power2:
    fixes n::nat
    assumes }\langlen\not=0\rangle\mathrm{ and «\ x. x< n (coprime x n }\longleftrightarrow\mathrm{ odd }x\mathrm{ )>
    shows <\exists k. n=(2::nat)}\mp@subsup{)}{}{\wedge}
<proof>
```


### 1.2 Main result

The solution to the problem ARITHMETIC PROGRESSIONS (Putnam exam problems 2002)
theorem arith-prog-rel-primes-solution:
fixes $n::$ nat
assumes $\langle n>1\rangle$
shows $\left\langle\left(\right.\right.$ prime $\left.n \vee\left(\exists k . n=2^{\wedge} k\right) \vee n=6\right) \longleftrightarrow$
$(\exists$ a b m. $m \neq 0 \wedge\{x \mid x . x<n \wedge$ coprime $x n\}=\{a+j * b \mid j::$ nat. $j<m\})$ >
$\langle p r o o f\rangle$
end

## References

[1] Problem "ARITHMETIC PROGRESSIONS", from Putnam exam problems 2002, https://www.ocf.berkeley.edu/ wwu/riddles/putnam.shtml.

