A Theory of Architectural Design Patterns

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Abstract

The following document formalizes and verifies several architectural design patterns [1]. Each pattern specification is formalized in terms of a locale where the locale assumptions correspond to the assumptions which a pattern poses on an architecture. Thus, pattern specifications may build on top of each other by interpreting the corresponding locale. A pattern is verified using the framework provided by the AFP entry *Dynamic Architectures* [3].

Currently, the document consists of formalizations of 4 different patterns: the singleton, the publisher subscriber, the blackboard pattern, and the blockchain pattern. Thereby, the publisher component of the publisher subscriber pattern is modeled as an instance of the singleton pattern and the blackboard pattern is modeled as an instance of the publisher subscriber pattern.

In general, this entry provides the first steps towards an overall theory of architectural design patterns [2].

Contents

1 A Theory of Singletons

In the following, we formalize the specification of the singleton pattern as described in [4].

theory *Singleton*
imports *DynamicArchitectures.Dynamic-Architecture-Calculus*
begin

1.1 Singletons

In the following we formalize a variant of the Singleton pattern.

locale *singleton* = *dynamic-component cmp active*
  for *active* :: *'id* ⇒ *cnf* ⇒ *bool* (‹‖-‖_› [*0*,*110*]*60*)
  and *cmp* :: *'id* ⇒ *cnf* ⇒ *'cmp* (‹σ_(-)› [*0*,*110*]*60*) +
assumes *alwaysActive*: ⋀*k*. ∃ *id*. ‖*id*‖$_k$
  and *unique*: ∃ *id*. ∀ *k*. ∀ *id'*. (‖*id'*‖$_k$ ⟶ *id* = *id'*)
begin

1.1.1 Calculus Interpretation

baIA: ⟦∃ *i*≥*n*. ‖*c*‖$_{t\ i}$; *φ* (σ$_c$*t* ⟨*c* → *t*⟩$_n$)⟧ ⟹ *eval c t t' n* [*φ*]$_b$

baIN1: ⟦∃ *i*. ‖*c*‖$_{t\ i}$; ¬ (∃ *i*≥*n*. ‖*c*‖$_{t\ i}$); *φ* (*t'* (*n* − ⟨*c* ∧ *t*⟩ − *1*))⟧ ⟹ *eval c t t' n* [*φ*]$_b$

baIN2: ⟦∄ *i*. ‖*c*‖$_{t\ i}$; *φ* (*t' n*)⟧ ⟹ *eval c t t' n* [*φ*]$_b$

1.1.2 Architectural Guarantees

definition *the-singleton* ≡ *THE id*. ∀ *k*. ∀ *id'*. ‖*id'*‖$_k$ ⟶ *id'* = *id*

theorem *ts-prop*:
  fixes *k*::*cnf*
  shows ⋀*id*. ‖*id*‖$_k$ ⟹ *id* = *the-singleton*
    and ‖*the-singleton*‖$_k$
⟨*proof*⟩
declare *ts-prop*(*2*)[*simp*]

lemma *lNact-active*[*simp*]:
  fixes *cid t n*
  shows ⟨*the-singleton* ⇐ *t*⟩$_n$ = *n*
  ⟨*proof*⟩

lemma *lNxt-active*[*simp*]:
  fixes *cid t n*
  shows ⟨*the-singleton* → *t*⟩$_n$ = *n*

⟨*proof*⟩

lemma *baI*[*intro*]:
fixes t n a
assumes φ $(\sigma_{\textit{the-singleton}}(t\ n))$
shows *eval the-singleton* t t' n $[\varphi]_b$ ⟨*proof*⟩

lemma *baE*[*elim*]:
fixes t n a
assumes *eval the-singleton* t t' n $[\varphi]_b$
shows φ $(\sigma_{\textit{the-singleton}}(t\ n))$ ⟨*proof*⟩

lemma *evtE*[*elim*]:
fixes t *id* n a
assumes *eval the-singleton* t t' n $(\Diamond_b\ \gamma)$
shows $\exists\, n'{\geq}n$. *eval the-singleton* t t' n' γ
⟨*proof*⟩

lemma *globE*[*elim*]:
fixes t *id* n a
assumes *eval the-singleton* t t' n $(\Box_b\ \gamma)$
shows $\forall\, n'{\geq}n$. *eval the-singleton* t t' n' γ
⟨*proof*⟩

lemma *untilI*[*intro*]:
fixes t::*nat* ⇒ *cnf*
and t'::*nat* ⇒ *'cmp*
and n::*nat*
and n'::*nat*
assumes $n'{\geq}n$
and *eval the-singleton* t t' n' γ
and $\bigwedge n''$. $[\![n{\leq}n'';\ n''{<}n']\!] \Longrightarrow$ *eval the-singleton* t t' n'' γ'
shows *eval the-singleton* t t' n $(\gamma'\ \mathfrak{U}_b\ \gamma)$
⟨*proof*⟩

lemma *untilE*[*elim*]:
fixes t *id* n γ' γ
assumes *eval the-singleton* t t' n $(\gamma'\ \mathfrak{U}_b\ \gamma)$
shows $\exists\, n'{\geq}n$. *eval the-singleton* t t' n' $\gamma \wedge (\forall\, n''{\geq}n.\ n'' < n' \longrightarrow$ *eval the-singleton* t t' n'' $\gamma')$
⟨*proof*⟩
end

end

2 A Theory of Publisher-Subscriber Architectures

In the following, we formalize the specification of the publisher subscriber pattern as described in [4].

theory *Publisher-Subscriber*
imports *Singleton*
begin

2.1 Subscriptions

datatype *'evt subscription = sub 'evt | unsub 'evt*

2.2 Publisher-Subscriber Architectures

locale *publisher-subscriber* =
  pb: *singleton pbactive pbcmp* +
  sb: *dynamic-component sbcmp sbactive*
    for *pbactive* :: *'pid* ⇒ *cnf* ⇒ *bool* (‹‖-‖_› [0,110]60)
    and *pbcmp* :: *'pid* ⇒ *cnf* ⇒ *'PB* (‹σ_(-)› [0,110]60)
    and *sbactive* :: *'sid* ⇒ *cnf* ⇒ *bool* (‹‖-‖_› [0,110]60)
    and *sbcmp* :: *'sid* ⇒ *cnf* ⇒ *'SB* (‹σ_(-)› [0,110]60) +
  fixes *pbsb* :: *'PB* ⇒ (*'evt set*) *subscription set*
    and *pbnt* :: *'PB* ⇒ (*'evt* × *'msg*)
    and *sbnt* :: *'SB* ⇒ (*'evt* × *'msg*) *set*
    and *sbsb* :: *'SB* ⇒ (*'evt set*) *subscription*
  assumes *conn1*: ⋀*k pid*. $\|pid\|_k$
      ⟹ *pbsb* ($\sigma_{pid}(k)$) = (⋃ *sid*∈{*sid*. $\|sid\|_k$}. {*sbsb* ($\sigma_{sid}(k)$)})
    and *conn2*: ⋀*t n n'' sid pid E e m*.
      ⟦*t* ∈ *arch*; $\|pid\|_{t\ n}$; $\|sid\|_{t\ n}$; *sub E* = *sbsb* ($\sigma_{sid}(t\ n)$); *n''*≥ *n*; *e* ∈ *E*;
      ∄*n' E'*. *n'* ≥ *n* ∧ *n'* ≤ *n''* ∧ $\|sid\|_{t\ n'}$ ∧
        unsub E' = *sbsb* ($\sigma_{sid}(t\ n')$) ∧ *e* ∈ *E'*;
      (*e*, *m*) = *pbnt* ($\sigma_{pid}(t\ n'')$); $\|sid\|_{t\ n''}$⟧
      ⟹ *pbnt* ($\sigma_{pid}(t\ n'')$) ∈ *sbnt* ($\sigma_{sid}(t\ n'')$)
begin

2.2.1 Calculus Interpretation

pb.baIA: ⟦∃ *i*≥*n*. $\|c\|_{t\ i}$; *φ* ($\sigma_c t$ (*pb.nxtAct c t n*))⟧ ⟹ *pb.eval c t t' n* $[\varphi]_b$

sb.baIA: ⟦∃ *i*≥*n*. $\|c\|_{t\ i}$; *φ* ($\sigma_c t$ (*sb.nxtAct c t n*))⟧ ⟹ *sb.eval c t t' n* $[\varphi]_b$

2.2.2 Results from Singleton

abbreviation *the-pb* :: *'pid* **where**
the-pb ≡ *pb.the-singleton*

pb.ts-prop (*1*): $\|id\|_k$ ⟹ *id* = *the-pb*

pb.ts-prop (*2*): $\|the\text{-}pb\|_k$

2.2.3 Architectural Guarantees

The following theorem ensures that a subscriber indeed receives all messages associated with an event for which he is subscribed.

theorem *msgDelivery*:
  fixes *t n n''* **and** *sid*::*'sid* **and** *E e m*
  assumes *t* ∈ *arch*
    and $\|sid\|_{t\ n}$
    and *sub E* = *sbsb* ($\sigma_{sid}(t\ n)$)
    and *n''* ≥ *n*
    and ∄*n' E'*. *n'* ≥ *n* ∧ *n'* ≤ *n''* ∧ $\|sid\|_{t\ n'}$ ∧ *unsub E'* = *sbsb*($\sigma_{sid}(t\ n')$)
        ∧ *e* ∈ *E'*
    and *e* ∈ *E*

and $(e,m) = pbnt\ (\sigma_{the\text{-}pb}(t\ n''))$
and $\|sid\|_{t\ n''}$
shows $(e,m) \in sbnt\ (\sigma_{sid}(t\ n''))$
⟨*proof*⟩

Since a publisher is actually a singleton, we can provide an alternative version of constraint *conn1*.

lemma *conn1A*:
fixes k
shows $pbsb\ (\sigma_{the\text{-}pb}(k)) = (\bigcup sid \in \{sid.\ \|sid\|_k\}.\ \{sbsb\ (\sigma_{sid}(k))\})$
⟨*proof*⟩
end

end

3 A Theory of Blackboard Architectures

In the following, we formalize the specification of the blackboard pattern as described in [4].

theory *Blackboard*
imports *Publisher-Subscriber*
begin

3.1 Problems and Solutions

Blackboards work with problems and solutions for them.

typedecl *PROB*
consts $sb :: (PROB \times PROB)\ set$
axiomatization where *sbWF*: *wf sb*
typedecl *SOL*
consts *solve*:: $PROB \Rightarrow SOL$

3.2 Blackboard Architectures

In the following, we describe the locale for the blackboard pattern.

locale *blackboard = publisher-subscriber bbactive bbcmp ksactive kscmp bbrp bbcs kscs ksrp*
for *bbactive* :: $'bid \Rightarrow cnf \Rightarrow bool\ (\langle\|\text{-}\|_{\text{-}}\rangle\ [0,110]60)$
and *bbcmp* :: $'bid \Rightarrow cnf \Rightarrow 'BB\ (\langle\sigma_{\text{-}}(\text{-})\rangle\ [0,110]60)$
and *ksactive* :: $'kid \Rightarrow cnf \Rightarrow bool\ (\langle\|\text{-}\|_{\text{-}}\rangle\ [0,110]60)$
and *kscmp* :: $'kid \Rightarrow cnf \Rightarrow 'KS\ (\langle\sigma_{\text{-}}(\text{-})\rangle\ [0,110]60)$
and *bbrp* :: $'BB \Rightarrow (PROB\ set)\ subscription\ set$
and *bbcs* :: $'BB \Rightarrow (PROB \times SOL)$
and *kscs* :: $'KS \Rightarrow (PROB \times SOL)\ set$
and *ksrp* :: $'KS \Rightarrow (PROB\ set)\ subscription +$
fixes *bbns* :: $'BB \Rightarrow (PROB \times SOL)\ set$
and *ksns* :: $'KS \Rightarrow (PROB \times SOL)$
and *bbop* :: $'BB \Rightarrow PROB$
and *ksop* :: $'KS \Rightarrow PROB\ set$
and *prob* :: $'kid \Rightarrow PROB$
assumes
ks1: $\forall p.\ \exists ks.\ p{=}prob\ ks$ — Component Parameter
— Assertions about component behavior.
and *bhvbb1*: $\bigwedge t\ t'\ bId\ p\ s.\ [\![t \in arch]\!] \Longrightarrow pb.eval\ bId\ t\ t'\ 0$

$(\Box_b\ ([\lambda bb.\ (p,s) \in bbns\ bb]_b$
$\longrightarrow^b\ (\Diamond_b\ [\lambda bb.\ (p,s) = bbcs\ bb]_b)))$
and *bhvbb2*: $\bigwedge t\ t'\ bId\ P\ q.\ [\![t \in arch]\!] \Longrightarrow pb.eval\ bId\ t\ t'\ 0$
$(\Box_b\ ([\lambda bb.\ sub\ P \in bbrp\ bb \wedge q \in P]_b \longrightarrow^b$
$(\Diamond_b\ [\lambda bb.\ q = bbop\ bb]_b)))$
and *bhvbb3*: $\bigwedge t\ t'\ bId\ p\ .\ [\![t \in arch]\!] \Longrightarrow pb.eval\ bId\ t\ t'\ 0$
$(\Box_b\ ([\lambda bb.\ p = bbop(bb)]_b \longrightarrow^b$
$([\lambda bb.\ p{=}bbop(bb)]_b\ \mathfrak{W}_b\ [\lambda bb.\ (p,solve(p)) = bbcs(bb)]_b)))$
and *bhvks1*: $\bigwedge t\ t'\ kId\ p\ P.\ [\![t \in arch;\ p = prob\ kId]\!] \Longrightarrow sb.eval\ kId\ t\ t'\ 0$
$(\Box_b\ ([\lambda ks.\ sub\ P = ksrp\ ks]_b \wedge^b$
$(\forall_b\ q.\ ((sb.pred\ (q \in P)) \longrightarrow^b (\Diamond_b\ ([\lambda ks.\ (q,solve(q)) \in kscs\ ks]_b))))$
$\longrightarrow^b (\Diamond_b\ [\lambda ks.\ (p,\ solve\ p) = ksns\ ks]_b)))$
and *bhvks2*: $\bigwedge t\ t'\ kId\ p\ P\ q.\ [\![t \in arch; p = prob\ kId]\!] \Longrightarrow sb.eval\ kId\ t\ t'\ 0$
$(\Box_b\ [\lambda ks.\ sub\ P = ksrp\ ks \wedge q \in P \longrightarrow (q,p) \in sb]_b)$
and *bhvks3*: $\bigwedge t\ t'\ kId\ p.\ [\![t \in arch; p = prob\ kId]\!] \Longrightarrow sb.eval\ kId\ t\ t'\ 0$
$(\Box_b\ ([\lambda ks.\ p \in ksop\ ks]_b \longrightarrow^b (\Diamond_b\ [\lambda ks.\ (\exists P.\ sub\ P = ksrp\ ks)]_b)))$
and *bhvks4*: $\bigwedge t\ t'\ kId\ p\ P.\ [\![t \in arch;\ p \in P]\!] \Longrightarrow sb.eval\ kId\ t\ t'\ 0$
$(\Box_b\ ([\lambda ks.\ sub\ P = ksrp\ ks]_b \longrightarrow^b$
$((\neg^b\ (\exists_b\ P'.\ (sb.pred\ (p \in P') \wedge^b [\lambda ks.\ unsub\ P' = ksrp\ ks]_b)))\ \mathfrak{W}_b$
$[\lambda ks.\ (p,solve\ p) \in kscs\ ks]_b)))$

— Assertions about component activation.
and *actks*:
$\bigwedge t\ n\ kid\ p.\ [\![t \in arch;\ \|kid\|_{t\ n};\ p{=}prob\ kid;\ p \in ksop\ (\sigma_{kid}(t\ n))]\!]$
$\Longrightarrow (\exists\, n' \geq n.\ \|kid\|_{t\ n'} \wedge (p,\ solve\ p) = ksns\ (\sigma_{kid}(t\ n')) \wedge$
$(\forall\, n'' \geq n.\ n'' < n' \longrightarrow \|kid\|_{t\ n''}))$
$\vee\ (\forall\, n' \geq n.\ (\|kid\|_{t\ n'} \wedge (\neg(p,\ solve\ p) = ksns\ (\sigma_{kid}(t\ n')))))$

— Assertions about connections.
and *conn1*: $\bigwedge k\ bid.\ \|bid\|_k$
$\Longrightarrow bbns\ (\sigma_{bid}(k)) = (\bigcup kid \in \{kid.\ \|kid\|_k\}.\ \{ksns\ (\sigma_{kid}(k))\})$
and *conn2*: $\bigwedge k\ kid.\ \|kid\|_k$
$\Longrightarrow ksop\ (\sigma_{kid}(k)) = (\bigcup bid \in \{bid.\ \|bid\|_k\}.\ \{bbop\ (\sigma_{bid}(k))\})$
begin
notation *sb.lNAct* $(\langle\langle\text{-} \Leftarrow \text{-}\rangle_{\text{-}}\rangle)$
notation *sb.nxtAct* $(\langle\langle\text{-} \rightarrow \text{-}\rangle_{\text{-}}\rangle)$
notation *pb.lNAct* $(\langle\langle\text{-} \Leftarrow \text{-}\rangle_{\text{-}}\rangle)$
notation *pb.nxtAct* $(\langle\langle\text{-} \rightarrow \text{-}\rangle_{\text{-}}\rangle)$

3.2.1 Calculus Interpretation

pb.baIA: $[\![\exists\, i \geq n.\ \|c\|_{t\ i};\ \varphi\ (\sigma_c t\ \langle c \rightarrow t\rangle_n)]\!] \Longrightarrow pb.eval\ c\ t\ t'\ n\ [\varphi]_b$

sb.baIA: $[\![\exists\, i \geq n.\ \|c\|_{t\ i};\ \varphi\ (\sigma_c t\ \langle c \rightarrow t\rangle_n)]\!] \Longrightarrow sb.eval\ c\ t\ t'\ n\ [\varphi]_b$

3.2.2 Results from Singleton

abbreviation *the-bb* $\equiv$ *the-pb*

pb.ts-prop (*1*): $\|id\|_k \Longrightarrow id = the\text{-}bb$

pb.ts-prop (*2*): $\|the\text{-}bb\|_k$

3.2.3 Results from Publisher Subscriber

msgDelivery: $[\![t \in arch;\ \|sid\|_{t\ n};\ sub\ E = ksrp\ (\sigma_{sid} t\ n);\ n \leq n'';\ \nexists n'\ E'.\ n \leq n' \wedge n' \leq n'' \wedge \|sid\|_{t\ n'} \wedge unsub\ E' = ksrp\ (\sigma_{sid} t\ n') \wedge e \in E';\ e \in E;\ (e,\ m) = bbcs\ (\sigma_{the\text{-}bb} t\ n'');\ \|sid\|_{t\ n''}]\!] \Longrightarrow (e,\ m) \in kscs\ (\sigma_{sid} t\ n'')$

lemma *conn2-bb*:
fixes k **and** kid::$'kid$
assumes $\|kid\|_k$
shows $bbop\ (\sigma_{the\text{-}bb}(k)) \in ksop\ (\sigma_{kid}(k))$
⟨*proof*⟩

3.2.4 Knowledge Sources

In the following we introduce an abbreviation for knowledge sources which are able to solve a specific problem.

definition *sKs*:: $PROB \Rightarrow 'kid$ **where**
$sKs\ p \equiv (SOME\ kid.\ p = prob\ kid)$

lemma *sks-prob*:
$p = prob\ (sKs\ p)$
⟨*proof*⟩

3.2.5 Architectural Guarantees

The following theorem verifies that a problem is eventually solved by the pattern even if no knowledge source exist which can solve the problem on its own. It assumes, however, that for every open sub problem, a corresponding knowledge source able to solve the problem will be eventually activated.

lemma *pSolved-Ind*:
fixes t **and** t'::$nat \Rightarrow 'BB$ **and** p **and** t''::$nat \Rightarrow 'KS$
assumes $t \in arch$ **and**
$\forall n.\ (\exists n' \geq n.\ \|sKs\ (bbop(\sigma_{the\text{-}bb}(t\ n)))\|_{t\ n'})$
shows
$\forall n.\ (\exists P.\ sub\ P \in bbrp(\sigma_{the\text{-}bb}(t\ n)) \wedge p \in P) \longrightarrow$
$(\exists m \geq n.\ (p, solve(p)) = bbcs\ (\sigma_{the\text{-}bb}(t\ m)))$
— The proof is by well-founded induction over the subproblem relation *sb*
⟨*proof*⟩

theorem *pSolved*:
fixes t **and** t'::$nat \Rightarrow 'BB$ **and** t''::$nat \Rightarrow 'KS$
assumes $t \in arch$ **and**
$\forall n.\ (\exists n' \geq n.\ \|sKs\ (bbop(\sigma_{the\text{-}bb}(t\ n)))\|_{t\ n'})$
shows
$\forall n.\ (\forall P.\ (sub\ P \in bbrp(\sigma_{the\text{-}bb}(t\ n))$
$\longrightarrow (\forall p \in P.\ (\exists m \geq n.\ (p, solve(p)) = bbcs\ (\sigma_{the\text{-}bb}(t\ m))))))$
⟨*proof*⟩
end

end

4 Some Auxiliary Results

theory *Auxiliary* **imports** *Main*

begin

lemma *disjE3*: $P \vee Q \vee R \Longrightarrow (P \Longrightarrow S) \Longrightarrow (Q \Longrightarrow S) \Longrightarrow (R \Longrightarrow S) \Longrightarrow S$ ⟨*proof*⟩

lemma *ge-induct*[*consumes 1*, *case-names step*]:
fixes *i*::*nat* **and** *j*::*nat* **and** *P*::*nat* ⇒ *bool*
shows $i \leq j \Longrightarrow (\bigwedge n.\ i \leq n \Longrightarrow ((\forall m \geq i.\ \ m<n \longrightarrow P\ m) \Longrightarrow P\ n)) \Longrightarrow P\ j$
⟨*proof*⟩

lemma *my-induct*[*consumes 1*, *case-names base step*]:
fixes *P*::*nat*⇒*bool*
assumes *less*: $i \leq j$
and *base*: $P\ j$
and *step*: $\bigwedge n.\ i \leq n \Longrightarrow n < j \Longrightarrow (\forall n'>n.\ n' \leq j \longrightarrow P\ n') \Longrightarrow P\ n$
shows $P\ i$
⟨*proof*⟩

lemma *Greatest-ex-le-nat*: **assumes** $\exists k.\ P\ k \wedge (\forall k'.\ P\ k' \longrightarrow k' \leq k)$ **shows** $\neg(\exists n'>Greatest\ P.\ P\ n')$
⟨*proof*⟩

lemma *cardEx*: **assumes** *finite A* **and** *finite B* **and** *card A* > *card B* **shows** $\exists x \in A.\ \neg\ x \in B$
⟨*proof*⟩

lemma *cardshift*: *card* $\{i$::*nat*. $i>n \wedge i \leq n' \wedge p\ (n'' + i)\} =$ *card* $\{i.\ i>(n + n'') \wedge i \leq (n' + n'') \wedge p\ i\}$
⟨*proof*⟩

end

5 Relative Frequency LTL

theory *RF-LTL*
imports *Main HOL−Library.Sublist Auxiliary DynamicArchitectures.Dynamic-Architecture-Calculus*
begin

type-synonym *'s seq* = *nat* ⇒ *'s*

abbreviation *ccard n n' p* ≡ *card* $\{i.\ i>n \wedge i \leq n' \wedge p\ i\}$

lemma *ccard-same*:
assumes ¬ *p* (*Suc n'*)
shows *ccard n n' p* = *ccard n* (*Suc n'*) *p*
⟨*proof*⟩

lemma *ccard-zero*[*simp*]:
fixes *n*::*nat*
shows *ccard n n p* = *0*
⟨*proof*⟩

lemma *ccard-inc*:
assumes *p* (*Suc n'*)
and $n' \geq n$
shows *ccard n* (*Suc n'*) *p* = *Suc* (*ccard n n' p*)
⟨*proof*⟩

lemma *ccard-mono*:
assumes $n'{\geq}n$
shows $n''{\geq}n' \Longrightarrow ccard\ n\ (n''{::}nat)\ p \geq ccard\ n\ n'\ p$
⟨*proof*⟩

lemma *ccard-ub*[*simp*]:
$ccard\ n\ n'\ p \leq Suc\ n' - n$
⟨*proof*⟩

lemma *ccard-sum*:
fixes $n{::}nat$
assumes $n'{\geq}n''$
and $n''{\geq}n$
shows $ccard\ n\ n'\ P = ccard\ n\ n''\ P + ccard\ n''\ n'\ P$
⟨*proof*⟩

lemma *ccard-ex*:
fixes $n{::}nat$
shows $c{\geq}1 \Longrightarrow c < ccard\ n\ n''\ P \Longrightarrow \exists\, n'{<}n''.\ n'{>}n \wedge ccard\ n\ n'\ P = c$
⟨*proof*⟩

lemma *ccard-freq*:
assumes $(n'{::}nat){\geq}n$
and $ccard\ n\ n'\ P > ccard\ n\ n'\ Q + cnf$
shows $\exists\, n'\ n''.\ ccard\ n'\ n''\ P > cnf \wedge ccard\ n'\ n''\ Q \leq cnf$
⟨*proof*⟩

locale *honest* =
fixes $bc{::}\ ('a\ list)\ seq$
and $n{::}nat$
assumes *growth*: $n'{\neq}0 \Longrightarrow n'{\leq}n \Longrightarrow bc\ n' = bc\ (n'{-}1) \vee (\exists\, b.\ bc\ n' = bc\ (n' - 1)\ @\ b)$
begin
end

locale *dishonest* =
fixes $bc{::}\ ('a\ list)\ seq$
and $mining{::}bool\ seq$
assumes *growth*: $\bigwedge n{::}nat.\ prefix\ (bc\ (Suc\ n))\ (bc\ n) \vee (\exists\, b{::}'a.\ bc\ (Suc\ n) = bc\ n\ @\ [b]) \wedge mining\ (Suc\ n)$
begin

lemma *prefix-save*:
assumes $prefix\ sbc\ (bc\ n')$
and $\forall\, n'''{>}n'.\ n'''{\leq}n'' \longrightarrow length\ (bc\ n''') \geq length\ sbc$
shows $n''{\geq}n' \Longrightarrow prefix\ sbc\ (bc\ n'')$
⟨*proof*⟩

theorem *prefix-length*:
assumes $prefix\ sbc\ (bc\ n')$ **and** $\neg\ prefix\ sbc\ (bc\ n'')$ **and** $n'{\leq}n''$
shows $\exists\, n'''{>}n'.\ n'''{\leq}n'' \wedge length\ (bc\ n''') < length\ sbc$
⟨*proof*⟩

theorem *grow-mining*:
assumes $length\ (bc\ n) < length\ (bc\ (Suc\ n))$
shows $mining\ (Suc\ n)$

⟨*proof*⟩

lemma *length-suc-length*:
length (*bc* (*Suc n*)) ≤ *Suc* (*length* (*bc n*))
⟨*proof*⟩

end

locale *dishonest-growth* =
fixes *bc*:: *nat seq*
and *mining*:: *nat* ⇒ *bool*
assumes *as1*: ⋀*n*::*nat*. *bc* (*Suc n*) ≤ *Suc* (*bc n*)
and *as2*: ⋀*n*::*nat*. *bc* (*Suc n*) > *bc n* ⟹ *mining* (*Suc n*)
begin

end

sublocale *dishonest* ⊆ *dishonest-growth* λ*n*. *length* (*bc n*) ⟨*proof*⟩

context *dishonest-growth*
begin
theorem *ccard-diff-lgth*:
n′≥*n* ⟹ *ccard n n*′ (λ*n*. *mining n*) ≥ (*bc n*′ − *bc n*)
⟨*proof*⟩
end

locale *honest-growth* =
fixes *bc*:: *nat seq*
and *mining*:: *nat* ⇒ *bool*
and *init*:: *nat*
assumes *as1*: ⋀*n*::*nat*. *bc* (*Suc n*) ≥ *bc n*
and *as2*: ⋀*n*::*nat*. *mining* (*Suc n*) ⟹ *bc* (*Suc n*) > *bc n*
begin
lemma *grow-mono*: *n*′≥*n*⟹*bc n*′≥*bc n*
⟨*proof*⟩

theorem *ccard-diff-lgth*:
shows *n*′≥*n* ⟹ *bc n*′ − *bc n* ≥ *ccard n n*′ (λ*n*. *mining n*)
⟨*proof*⟩
end

locale *bounded-growth* = *hg*: *honest-growth hbc hmining* + *dg*: *dishonest-growth dbc dmining*
for *hbc*:: *nat seq*
and *dbc*:: *nat seq*
and *hmining*:: *nat* ⇒ *bool*
and *dmining*:: *nat* ⇒ *bool*
and *sbc*::*nat*
and *cnf*::*nat* +
assumes *fair*: ⋀*n n*′. *ccard n n*′ (λ*n*. *dmining n*) > *cnf* ⟹ *ccard n n*′ (λ*n*. *hmining n*) > *cnf*
and *a2*: *hbc 0* ≥ *sbc*+*cnf*
and *a3*: *dbc 0* < *sbc*
begin

theorem *hn-upper-bound*: **shows** *dbc n* < *hbc n*
⟨*proof*⟩

end

end

6 Blockchain Architectures

theory *Blockchain* **imports** *Auxiliary DynamicArchitectures.Dynamic-Architecture-Calculus RF-LTL*
begin

6.1 Blockchains

A blockchain itself is modeled as a simple list.

type-synonym *'a BC = 'a list*

abbreviation *max-cond*:: *('a BC) set ⇒ 'a BC ⇒ bool*
  where *max-cond B b ≡ b ∈ B ∧ (∀ b'∈B. length b' ≤ length b)*

no-syntax
  -MAX1 :: *pttrns ⇒ 'b ⇒ 'b* (‹(‹*indent=3 notation=‹binder MAX››MAX -./ -*)› [*0, 10*] *10*)
  -MAX :: *pttrn ⇒ 'a set ⇒ 'b ⇒ 'b* (‹(‹*indent=3 notation=‹binder MAX››MAX -∈-./ -*)› [*0, 0, 10*] *10*)

definition *MAX*:: *('a BC) set ⇒ 'a BC*
  where *MAX B = (SOME b. max-cond B b)*

lemma *max-ex*:
  fixes *XS*::*('a BC) set*
  assumes *XS ≠ {}*
    and *finite XS*
  shows *∃ xs∈XS. (∀ ys∈XS. length ys ≤ length xs)*
⟨*proof*⟩

lemma *max-prop*:
  fixes *XS*::*('a BC) set*
  assumes *XS ≠ {}*
    and *finite XS*
  shows *MAX XS ∈ XS*
    and *∀ b'∈XS. length b' ≤ length (MAX XS)*
⟨*proof*⟩

lemma *max-less*:
  fixes *b*::*'a BC* **and** *b'*::*'a BC* **and** *B*::*('a BC) set*
  assumes *b∈B*
    and *finite B*
    and *length b > length b'*
  shows *length (MAX B) > length b'*
⟨*proof*⟩

6.2 Blockchain Architectures

In the following we describe the locale for blockchain architectures.

locale *Blockchain = dynamic-component cmp active*
  for *active* :: *'nid ⇒ cnf ⇒ bool* (‹‖-‖-› [*0,110*]*60*)

and *cmp* :: *'nid* ⇒ *cnf* ⇒ *'ND* (‹σ_(-)› [0,110]60) +
fixes *pin* :: *'ND* ⇒ (*'nid BC*) *set*
and *pout* :: *'ND* ⇒ *'nid BC*
and *bc* :: *'ND* ⇒ *'nid BC*
and *mining* :: *'ND* ⇒ *bool*
and *honest* :: *'nid* ⇒ *bool*
and *actHn* :: *cnf* ⇒ *'nid set*
and *actDn* :: *cnf* ⇒ *'nid set*
and *PoW*:: *trace* ⇒ *nat* ⇒ *nat*
and *hmining*:: *trace* ⇒ *nat* ⇒ *bool*
and *dmining*:: *trace* ⇒ *nat* ⇒ *bool*
and *cb*:: *nat*
defines *actHn k* ≡ {*nid*. $\|nid\|_k$ ∧ *honest nid*}
and *actDn k* ≡ {*nid*. $\|nid\|_k$ ∧ ¬ *honest nid*}
and *PoW t n* ≡ (*LEAST x*. ∀ *nid*∈*actHn* (*t n*). *length* (*bc* (σ_{nid}(*t n*))) ≤ *x*)
and *hmining t* ≡ (λ*n*. ∃ *nid*∈*actHn* (*t n*). *mining* (σ_{nid}(*t n*)))
and *dmining t* ≡ (λ*n*. ∃ *nid*∈*actDn* (*t n*). *mining* (σ_{nid}(*t n*)))
assumes *consensus*: ⋀*nid t t' bc'*::(*'nid BC*). ⟦*honest nid*⟧ ⟹ *eval nid t t' 0*
(□$_b$ ([λ*nd*. *bc'* = (*if* (∃ *b*∈*pin nd*. *length b* > *length* (*bc nd*)) *then* (*MAX* (*pin nd*)) *else* (*bc nd*))]$_b$
⟶^{b} ○$_b$ [λ*nd*.(¬ *mining nd* ∧ *bc nd* = *bc'* ∨ *mining nd* ∧ (∃ *b*. *bc nd* = *bc'* @ [*b*]))]$_b$))
and *attacker*: ⋀*nid t t' bc'*. ⟦¬ *honest nid*⟧ ⟹ *eval nid t t' 0*
(□$_b$ ([λ*nd*. *bc'* = (*SOME b*. *b* ∈ (*pin nd* ∪ {*bc nd*}))]$_b$ ⟶^{b}
○$_b$ [λ*nd*.(¬ *mining nd* ∧ *prefix* (*bc nd*) *bc'* ∨ *mining nd* ∧ (∃ *b*. *bc nd* = *bc'* @ [*b*]))]$_b$))
and *forward*: ⋀*nid t t'*. *eval nid t t' 0* (□$_b$ [λ*nd*. *pout nd* = *bc nd*]$_b$)
— At each time point a node will forward its blockchain to the network
and *init*: ⋀*nid t t'*. *eval nid t t' 0* [λ*nd*. *bc nd*=[]]$_b$
and *conn*: ⋀*k nid*. ⟦$\|nid\|_k$; *honest nid*⟧
⟹ *pin* (*cmp nid k*) = (⋃ *nid'*∈*actHn k*. {*pout* (*cmp nid' k*)})
and *act*: ⋀*t n*::*nat*. *finite* {*nid*::*'nid*. $\|nid\|_{t\ n}$}
and *actHn*: ⋀*t n*::*nat*. ∃ *nid*. *honest nid* ∧ $\|nid\|_{t\ n}$ ∧ $\|nid\|_{t\ (Suc\ n)}$
and *fair*: ⋀*n n'*. *ccard n n'* (*dmining t*) > *cb* ⟹ *ccard n n'* (*hmining t*) > *cb*
and *closed*: ⋀*t nid b n*::*nat*. ⟦$\|nid\|_{t\ n}$; *b* ∈ *pin* (σ_{nid}(*t n*))⟧ ⟹ ∃ *nid'*. $\|nid'\|_{t\ n}$ ∧ *bc* ($\sigma_{nid'}$(*t n*))
= *b*
and *mine*: ⋀*t nid n*::*nat*. ⟦*honest nid*; $\|nid\|_{t\ (Suc\ n)}$; *mining* (σ_{nid}(*t* (*Suc n*)))⟧ ⟹ $\|nid\|_{t\ n}$
begin

lemma *init-model*:
assumes ¬ (∃ *n'*. *latestAct-cond nid t n n'*)
and $\|nid\|_{t\ n}$
shows *bc* (σ_{nid}*t n*) = []
⟨*proof*⟩

lemma *fwd-bc*:
fixes *nid* **and** *t*::*nat* ⇒ *cnf* **and** *t'*::*nat* ⇒ *'ND*
assumes $\|nid\|_{t\ n}$
shows *pout* (σ_{nid}*t n*) = *bc* (σ_{nid}*t n*)
⟨*proof*⟩

lemma *finite-input*:
fixes *t n nid*
assumes $\|nid\|_{t\ n}$
defines *dep nid'* ≡ *pout* ($\sigma_{nid'}$(*t n*))
shows *finite* (*pin* (*cmp nid* (*t n*)))
⟨*proof*⟩

lemma *nempty-input*:
fixes t n *nid*
assumes $\|nid\|_{t\ n}$
and *honest nid*
shows *pin* (*cmp nid* (t n))≠{} ⟨*proof*⟩

lemma *onlyone*:
assumes ∃ n'≥n. $\|tid\|_{t\ n'}$
and ∃ n'<n. $\|tid\|_{t\ n'}$
shows ∃!i. $\langle tid \leftarrow t\rangle_n \leq i \wedge i < \langle tid \rightarrow t\rangle_n \wedge \|tid\|_{t\ i}$
⟨*proof*⟩

6.2.1 Component Behavior

lemma *bhv-hn-ex*:
fixes t **and** t'::*nat* ⇒ *'ND* **and** *tid*
assumes *honest tid*
and ∃ n'≥n. $\|tid\|_{t\ n'}$
and ∃ n'<n. $\|tid\|_{t\ n'}$
and ∃ b∈*pin* ($\sigma_{tid} t\ \langle tid \leftarrow t\rangle_n$). *length* b > *length* (*bc* ($\sigma_{tid} t\ \langle tid \leftarrow t\rangle_n$))
shows ¬ *mining* ($\sigma_{tid} t\ \langle tid \rightarrow t\rangle_n$) ∧ *bc* ($\sigma_{tid} t\ \langle tid \rightarrow t\rangle_n$) =
Blockchain.MAX (*pin* ($\sigma_{tid} t\ \langle tid \leftarrow t\rangle_n$)) ∨ *mining* ($\sigma_{tid} t\ \langle tid \rightarrow t\rangle_n$) ∧
(∃ b. *bc* ($\sigma_{tid} t\ \langle tid \rightarrow t\rangle_n$) = *Blockchain.MAX* (*pin* ($\sigma_{tid} t\ \langle tid \leftarrow t\rangle_n$)) @ [$b$])
⟨*proof*⟩

lemma *bhv-hn-in*:
fixes t **and** t'::*nat* ⇒ *'ND* **and** *tid*
assumes *honest tid*
and ∃ n'≥n. $\|tid\|_{t\ n'}$
and ∃ n'<n. $\|tid\|_{t\ n'}$
and ¬ (∃ b∈*pin* ($\sigma_{tid} t\ \langle tid \leftarrow t\rangle_n$). *length* b > *length* (*bc* ($\sigma_{tid} t\ \langle tid \leftarrow t\rangle_n$)))
shows ¬ *mining* ($\sigma_{tid} t\ \langle tid \rightarrow t\rangle_n$) ∧ *bc* ($\sigma_{tid} t\ \langle tid \rightarrow t\rangle_n$) = *bc* ($\sigma_{tid} t\ \langle tid \leftarrow t\rangle_n$) ∨
mining ($\sigma_{tid} t\ \langle tid \rightarrow t\rangle_n$) ∧ (∃ b. *bc* ($\sigma_{tid} t\ \langle tid \rightarrow t\rangle_n$) = *bc* ($\sigma_{tid} t\ \langle tid \leftarrow t\rangle_n$) @ [$b$])
⟨*proof*⟩

lemma *bhv-hn-context*:
assumes *honest tid*
and $\|tid\|_{t\ n}$
and ∃ n'<n. $\|tid\|_{t\ n'}$
shows ∃ nid'. $\|nid'\|_{t\ \langle tid \leftarrow t\rangle_n}$ ∧ (*mining* ($\sigma_{tid} t\ n$) ∧ (∃ b. *bc* ($\sigma_{tid} t\ n$) = *bc* ($\sigma_{nid'} t\ \langle tid \leftarrow t\rangle_n$) @ [$b$]) ∨
¬ *mining* ($\sigma_{tid} t\ n$) ∧ *bc* ($\sigma_{tid} t\ n$) = *bc* ($\sigma_{nid'} t\ \langle tid \leftarrow t\rangle_n$))
⟨*proof*⟩

lemma *bhv-dn*:
fixes t **and** t'::*nat* ⇒ *'ND* **and** *uid*
assumes ¬ *honest uid*
and ∃ n'≥n. $\|uid\|_{t\ n'}$
and ∃ n'<n. $\|uid\|_{t\ n'}$
shows ¬ *mining* ($\sigma_{uid} t\ \langle uid \rightarrow t\rangle_n$) ∧ *prefix* (*bc* ($\sigma_{uid} t\ \langle uid \rightarrow t\rangle_n$)) (*SOME* b. b ∈ *pin* ($\sigma_{uid} t\ \langle uid \leftarrow t\rangle_n$) ∪ {*bc* ($\sigma_{uid} t\ \langle uid \leftarrow t\rangle_n$)})
∨ *mining* ($\sigma_{uid} t\ \langle uid \rightarrow t\rangle_n$) ∧ (∃ b. *bc* ($\sigma_{uid} t\ \langle uid \rightarrow t\rangle_n$) = (*SOME* b. b ∈ *pin* ($\sigma_{uid} t\ \langle uid \leftarrow t\rangle_n$) ∪ {*bc* ($\sigma_{uid} t\ \langle uid \leftarrow t\rangle_n$)}) @ [$b$])
⟨*proof*⟩

lemma *bhv-dn-context*:

assumes ¬ *honest uid*
and $\|uid\|_{t\ n}$
and ∃ *n'*<*n*. $\|uid\|_{t\ n'}$
shows ∃ *nid'*. $\|nid'\|_{t\ \langle uid \leftarrow t\rangle n}$ ∧ (*mining* (σ_{uid}*t n*) ∧ (∃ *b*. *prefix* (*bc* (σ_{uid}*t n*)) (*bc* ($\sigma_{nid'}$*t* ⟨*uid* ← *t*⟩*n*) @ [*b*]))
∨ ¬ *mining* (σ_{uid}*t n*) ∧ *prefix* (*bc* (σ_{uid}*t n*)) (*bc* ($\sigma_{nid'}$*t* ⟨*uid* ← *t*⟩*n*)))
⟨*proof*⟩

6.2.2 Maximal Honest Blockchains

abbreviation *mbc-cond*:: *trace* ⇒ *nat* ⇒ *'nid* ⇒ *bool*
where *mbc-cond t n nid* ≡ *nid*∈*actHn* (*t n*) ∧ (∀ *nid'*∈*actHn* (*t n*). *length* (*bc* ($\sigma_{nid'}$(*t n*))) ≤ *length* (*bc* (σ_{nid}(*t n*))))

lemma *mbc-ex*:
fixes *t n*
shows ∃ *x*. *mbc-cond t n x*
⟨*proof*⟩

definition *MBC*:: *trace* ⇒ *nat* ⇒ *'nid*
where *MBC t n* = (*SOME b*. *mbc-cond t n b*)

lemma *mbc-prop*[*simp*]:
shows *mbc-cond t n* (*MBC t n*)
⟨*proof*⟩

6.2.3 Honest Proof of Work

An important construction is the maximal proof of work available in the honest community. The construction was already introduces in the locale itself since it was used to express some of the locale assumptions.

abbreviation *pow-cond*:: *trace* ⇒ *nat* ⇒ *nat* ⇒ *bool*
where *pow-cond t n n'* ≡ ∀ *nid*∈*actHn* (*t n*). *length* (*bc* (σ_{nid}(*t n*))) ≤ *n'*

lemma *pow-ex*:
fixes *t n*
shows *pow-cond t n* (*length* (*bc* ($\sigma_{MBC\ t\ n}$(*t n*))))
and ∀ *x'*. *pow-cond t n x'* ⟶ *x'*≥*length* (*bc* ($\sigma_{MBC\ t\ n}$(*t n*)))
⟨*proof*⟩

lemma *pow-prop*:
pow-cond t n (*PoW t n*)
⟨*proof*⟩

lemma *pow-eq*:
fixes *n*
assumes ∃ *tid*∈*actHn* (*t n*). *length* (*bc* (σ_{tid}(*t n*))) = *x*
and ∀ *tid*∈*actHn* (*t n*). *length* (*bc* (σ_{tid}(*t n*))) ≤ *x*
shows *PoW t n* = *x*
⟨*proof*⟩

lemma *pow-mbc*:
shows *length* (*bc* ($\sigma_{MBC\ t\ n}$*t n*)) = *PoW t n*
⟨*proof*⟩

lemma *pow-less*:
fixes t n nid
assumes *pow-cond* t n x
shows $PoW\ t\ n \leq x$
⟨*proof*⟩

lemma *pow-le-max*:
assumes *honest tid*
and $\|tid\|_{t\ n}$
shows $PoW\ t\ n \leq length\ (MAX\ (pin\ (\sigma_{tid} t\ n)))$
⟨*proof*⟩

lemma *pow-ge-lgth*:
assumes *honest tid*
and $\|tid\|_{t\ n}$
shows $length\ (bc\ (\sigma_{tid} t\ n)) \leq PoW\ t\ n$
⟨*proof*⟩

lemma *pow-le-lgth*:
assumes *honest tid*
and $\|tid\|_{t\ n}$
and $\neg(\exists\, b{\in}pin\ (\sigma_{tid} t\ n).\ length\ b > length\ (bc\ (\sigma_{tid} t\ n)))$
shows $length\ (bc\ (\sigma_{tid} t\ n)) \geq PoW\ t\ n$
⟨*proof*⟩

lemma *pow-mono*:
shows $n'{\geq}n \Longrightarrow PoW\ t\ n' \geq PoW\ t\ n$
⟨*proof*⟩

lemma *pow-equals*:
assumes $PoW\ t\ n = PoW\ t\ n'$
and $n'{\geq}n$
and $n''{\geq}n$
and $n''{\leq}n'$
shows $PoW\ t\ n = PoW\ t\ n''$ ⟨*proof*⟩

lemma *pow-mining-suc*:
assumes *hmining* t $(Suc\ n)$
shows $PoW\ t\ n < PoW\ t\ (Suc\ n)$
⟨*proof*⟩

6.2.4 History

In the following we introduce an operator which extracts the development of a blockchain up to a time point n.

abbreviation *his-prop* t n nid n' nid' $x \equiv$
$(\exists\, n.\ latestAct\text{-}cond\ nid'\ t\ n')\ n) \wedge \|snd\ x\|_{t\ (fst\ x)} \wedge fst\ x = \langle nid' \leftarrow t\rangle_{n'} \wedge$
$(prefix\ (bc\ (\sigma_{nid'}(t\ n')))\ (bc\ (\sigma_{snd\ x}(t\ (fst\ x))))) \vee$
$(\exists\, b.\ bc\ (\sigma_{nid'}(t\ n')) = (bc\ (\sigma_{snd\ x}(t\ (fst\ x)))) \mathbin{@} [b] \wedge mining\ (\sigma_{nid'}(t\ n'))))$

inductive-set
his:: $trace \Rightarrow nat \Rightarrow 'nid \Rightarrow (nat \times 'nid)\ set$
for t::*trace* **and** n::*nat* **and** *nid*::$'nid$
where $[\![\|nid\|_{t\ n}]\!] \Longrightarrow (n,nid) \in his\ t\ n\ nid$
| $[\![(n',nid') \in his\ t\ n\ nid;\ \exists\, x.\ his\text{-}prop\ t\ n\ nid\ n'\ nid'\ x]\!] \Longrightarrow (SOME\ x.\ his\text{-}prop\ t\ n\ nid\ n'\ nid'\ x) \in$

his t n nid

lemma *his-act*:
assumes $(n',nid') \in his\ t\ n\ nid$
shows $\|nid'\|_{t\ n'}$
⟨*proof*⟩

In addition we also introduce an operator to obtain the predecessor of a blockchains development.

definition *hisPred*
where $hisPred\ t\ n\ nid\ n' \equiv (GREATEST\ n''.\ \exists nid'.\ (n'',nid') \in his\ t\ n\ nid \wedge n'' < n')$

lemma *hisPrev-prop*:
assumes $\exists n''<n'.\ \exists nid'.\ (n'',nid') \in his\ t\ n\ nid$
shows $hisPred\ t\ n\ nid\ n' < n'$ **and** $\exists nid'.\ (hisPred\ t\ n\ nid\ n',nid') \in his\ t\ n\ nid$
⟨*proof*⟩

lemma *hisPrev-nex-less*:
assumes $\exists n''<n'.\ \exists nid'.\ (n'',nid') \in his\ t\ n\ nid$
shows $\neg(\exists x \in his\ t\ n\ nid.\ fst\ x < n' \wedge fst\ x > hisPred\ t\ n\ nid\ n')$
⟨*proof*⟩

lemma *his-le*:
assumes $x \in his\ t\ n\ nid$
shows $fst\ x \leq n$
⟨*proof*⟩

lemma *his-determ-base*:
shows $(n,\ nid') \in his\ t\ n\ nid \Longrightarrow nid'=nid$
⟨*proof*⟩

lemma *hisPrev-same*:
assumes $\exists n'<n''.\ \exists nid'.\ (n',nid') \in his\ t\ n\ nid$
and $\exists n''<n'.\ \exists nid'.\ (n'',nid') \in his\ t\ n\ nid$
and $(n',nid') \in his\ t\ n\ nid$
and $(n'',nid'') \in his\ t\ n\ nid$
and $hisPred\ t\ n\ nid\ n'=hisPred\ t\ n\ nid\ n''$
shows $n'=n''$
⟨*proof*⟩

lemma *his-determ-ext*:
shows $n' \leq n \Longrightarrow (\exists nid'.\ (n',nid') \in his\ t\ n\ nid) \Longrightarrow (\exists! nid'.\ (n',nid') \in his\ t\ n\ nid) \wedge$
$((\exists n''<n'.\ \exists nid'.\ (n'',nid') \in his\ t\ n\ nid) \longrightarrow (\exists x.\ his\text{-}prop\ t\ n\ nid\ n'\ (THE\ nid'.\ (n',nid') \in his\ t\ n\ nid)\ x) \wedge$
$(hisPred\ t\ n\ nid\ n',\ (SOME\ nid'.\ (hisPred\ t\ n\ nid\ n',\ nid') \in his\ t\ n\ nid)) = (SOME\ x.\ his\text{-}prop\ t\ n\ nid\ n'\ (THE\ nid'.\ (n',nid') \in his\ t\ n\ nid)\ x))$
⟨*proof*⟩

corollary *his-determ-ex*:
assumes $(n',nid') \in his\ t\ n\ nid$
shows $\exists! nid'.\ (n',nid') \in his\ t\ n\ nid$
⟨*proof*⟩

corollary *his-determ*:
assumes $(n',nid') \in his\ t\ n\ nid$

and $(n',nid'') \in his\ t\ n\ nid$
shows $nid'=nid''$ ⟨*proof*⟩

corollary *his-determ-the*:
assumes $(n',nid') \in his\ t\ n\ nid$
shows $(THE\ nid'.\ (n',\ nid') \in his\ t\ n\ nid) = nid'$
⟨*proof*⟩

6.2.5 Blockchain Development

definition $devBC :: trace \Rightarrow nat \Rightarrow 'nid \Rightarrow nat \Rightarrow 'nid\ option$
where $devBC\ t\ n\ nid\ n' \equiv$
$(if\ (\exists\, nid'.\ (n',\ nid') \in his\ t\ n\ nid)\ then\ (Some\ (THE\ nid'.\ (n',\ nid') \in his\ t\ n\ nid))$
$else\ Option.None)$

lemma *devBC-some*[*simp*]: **assumes** $\|nid\|_{t\ n}$ **shows** $devBC\ t\ n\ nid\ n = Some\ nid$
⟨*proof*⟩

lemma *devBC-act*: **assumes** $\neg\ Option.is\text{-}none\ (devBC\ t\ n\ nid\ n')$ **shows** $\|the\ (devBC\ t\ n\ nid\ n')\|_{t\ n'}$
⟨*proof*⟩

lemma *his-ex*:
assumes $\neg Option.is\text{-}none\ (devBC\ t\ n\ nid\ n')$
shows $\exists\, nid'.\ (n',nid') \in his\ t\ n\ nid$
⟨*proof*⟩

lemma *devExt-nopt-leq*:
assumes $\neg Option.is\text{-}none\ (devBC\ t\ n\ nid\ n')$
shows $n' \leq n$
⟨*proof*⟩

An extended version of the development in which deactivations are filled with the last value.

function $devExt :: trace \Rightarrow nat \Rightarrow 'nid \Rightarrow nat \Rightarrow nat \Rightarrow 'nid\ BC$
where $[\![\exists\, n'<n_s.\ \neg Option.is\text{-}none\ (devBC\ t\ n\ nid\ n');\ Option.is\text{-}none\ (devBC\ t\ n\ nid\ n_s)]\!] \Longrightarrow devExt\ t\ n\ nid\ n_s\ 0 = bc\ (\sigma_{the\ (devBC\ t\ n\ nid\ (GREATEST\ n'.\ n'<n_s \wedge \neg Option.is\text{-}none\ (devBC\ t\ n\ nid\ n')))}(t\ (GREATEST\ n'.\ n'<n_s \wedge \neg Option.is\text{-}none\ (devBC\ t\ n\ nid\ n'))))$
| $[\![\neg\ (\exists\, n'<n_s.\ \neg Option.is\text{-}none\ (devBC\ t\ n\ nid\ n'));\ Option.is\text{-}none\ (devBC\ t\ n\ nid\ n_s)]\!] \Longrightarrow devExt\ t\ n\ nid\ n_s\ 0 = []$
| $\neg\ Option.is\text{-}none\ (devBC\ t\ n\ nid\ n_s) \Longrightarrow devExt\ t\ n\ nid\ n_s\ 0 = bc\ (\sigma_{the\ (devBC\ t\ n\ nid\ n_s)}(t\ n_s))$
| $\neg\ Option.is\text{-}none\ (devBC\ t\ n\ nid\ (n_s + Suc\ n')) \Longrightarrow devExt\ t\ n\ nid\ n_s\ (Suc\ n') = bc\ (\sigma_{the\ (devBC\ t\ n\ nid\ (n_s + Suc\ n'))}($
$(n_s + Suc\ n')))$
| $Option.is\text{-}none\ (devBC\ t\ n\ nid\ (n_s + Suc\ n')) \Longrightarrow devExt\ t\ n\ nid\ n_s\ (Suc\ n') = devExt\ t\ n\ nid\ n_s\ n'$
⟨*proof*⟩
termination ⟨*proof*⟩

lemma *devExt-same*:
assumes $\forall\, n'''>n'.\ n'''\leq n'' \longrightarrow Option.is\text{-}none\ (devBC\ t\ n\ nid\ n''')$
and $n' \geq n_s$
and $n''' \leq n''$
shows $n''' \geq n' \Longrightarrow devExt\ t\ n\ nid\ n_s\ (n'''-n_s) = devExt\ t\ n\ nid\ n_s\ (n'-n_s)$
⟨*proof*⟩

lemma *devExt-bc*[*simp*]:
assumes $\neg\ Option.is\text{-}none\ (devBC\ t\ n\ nid\ (n'+n''))$
shows $devExt\ t\ n\ nid\ n'\ n'' = bc\ (\sigma_{the\ (devBC\ t\ n\ nid\ (n'+n''))}(t\ (n'+n'')))$

⟨*proof*⟩

lemma *devExt-greatest*:
assumes $\exists\, n'''{<}n'{+}n''.\ \neg$ *Option.is-none* (*devBC t n nid n'''*)
and *Option.is-none* (*devBC t n nid* (*n'*+*n''*)) **and** $\neg$ *n''*=*0*
shows *devExt t n nid n' n''* = *bc* ($\sigma_{the\ (devBC\ t\ n\ nid\ (GREATEST\ n'''.\ n'''<(n'+n'')\ \wedge\ \neg Option.is\text{-}none\ (devBC\ t\ n\ nid\ n'}$
(*GREATEST n'''. n'''*<(*n'*+*n''*) ∧ ¬*Option.is-none* (*devBC t n nid n'''*))))
⟨*proof*⟩

lemma *devExt-shift*: *devExt t n nid* (*n'*+*n''*) *0* = *devExt t n nid n' n''*
⟨*proof*⟩

lemma *devExt-bc-geq*:
assumes ¬ *Option.is-none* (*devBC t n nid n'*) **and** *n'*≥n_s
shows *devExt t n nid* n_s (*n'*−n_s) = *bc* ($\sigma_{the\ (devBC\ t\ n\ nid\ n')}(t\ n')$) (**is** *?LHS* = *?RHS*)
⟨*proof*⟩

lemma *his-bc-empty*:
assumes (*n'*,*nid'*)∈ *his t n nid* **and** ¬(∃ *n''*<*n'*. ∃ *nid''*. (*n''*,*nid''*)∈ *his t n nid*)
shows *bc* ($\sigma_{nid'}(t\ n')$) = []
⟨*proof*⟩

lemma *devExt-devop*:
prefix (*devExt t n nid* n_s (*Suc n'*)) (*devExt t n nid* n_s *n'*) ∨ (∃ *b*. *devExt t n nid* n_s (*Suc n'*) = *devExt t n nid* n_s *n'* @ [*b*]) ∧ ¬ *Option.is-none* (*devBC t n nid* (n_s + *Suc n'*)) ∧ ‖*the* (*devBC t n nid* (n_s + *Suc n'*))‖$_{t\ (n_s\ +\ Suc\ n')}$ ∧ n_s + *Suc n'* ≤ *n* ∧ *mining* ($\sigma_{the\ (devBC\ t\ n\ nid\ (n_s\ +\ Suc\ n'))}(t\ (n_s\ +\ Suc\ n'))$)
⟨*proof*⟩

abbreviation *devLgthBC* **where** *devLgthBC t n nid* n_s ≡ (λ*n'*. *length* (*devExt t n nid* n_s *n'*))

theorem *blockchain-save*:
fixes *t*::*nat*⇒*cnf* **and** n_s **and** *sbc* **and** *n*
assumes ∀ *nid*. *honest nid* ⟶ *prefix sbc* (*bc* (σ_{nid}(*t* (⟨*nid* → *t*⟩$_{n_s}$))))
and ∀ *nid*∈*actDn* (*t* n_s). *length* (*bc* (σ_{nid}(*t* n_s))) < *length sbc*
and *PoW t* n_s≥*length sbc* + *cb*
and ∀ *n'*<n_s. ∀ *nid*. ‖*nid*‖$_{t\ n'}$ ⟶ *length* (*bc* (σ_{nid}*t n'*)) < *length sbc* ∨ *prefix sbc* (*bc* (σ_{nid}(*t n'*)))
and *n*≥n_s
shows ∀ *nid* ∈ *actHn* (*t n*). *prefix sbc* (*bc* (σ_{nid}(*t n*)))
⟨*proof*⟩

end

end

References

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