A Machine-Checked Model for a Java-like Language, Virtual Machine and Compiler

Gerwin Klein Tobias Nipkow

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Contents

Chapter 1

Preface

This document contains the automatically generated listings of the Isabelle sources for the theories defining and analysing Jinja (a Java-like programming language), the Jinja Virtual Machine, and the compiler. To shorten the document, all proofs have been hidden. For a detailed exposition of these theories see the paper by Klein and Nipkow [1, 2].

1.1 Theory Dependencies

Figure 1.1 shows the dependencies between the Isabelle theories in the following sections.

A tabled implementation of the reflexive transitive closure **theory** *Transitive-Closure-Table*
imports *Main*
begin

inductive *rtrancl-path* :: $('a \Rightarrow 'a \Rightarrow bool) \Rightarrow 'a \Rightarrow 'a\ list \Rightarrow 'a \Rightarrow bool$
  for r :: $'a \Rightarrow 'a \Rightarrow bool$
where
  base: *rtrancl-path* $r\ x\ []\ x$
| *step*: $r\ x\ y \Longrightarrow$ *rtrancl-path* $r\ y\ ys\ z \Longrightarrow$ *rtrancl-path* $r\ x\ (y\ \#\ ys)\ z$

lemma *rtranclp-eq-rtrancl-path*: $r^{**}\ x\ y = (\exists xs.$ *rtrancl-path* $r\ x\ xs\ y)$
⟨*proof*⟩

lemma *rtrancl-path-trans*:
  assumes *xy*: *rtrancl-path* $r\ x\ xs\ y$
  and *yz*: *rtrancl-path* $r\ y\ ys\ z$
  shows *rtrancl-path* $r\ x\ (xs\ @\ ys)\ z$ ⟨*proof*⟩

lemma *rtrancl-path-appendE*:
  assumes *xz*: *rtrancl-path* $r\ x\ (xs\ @\ y\ \#\ ys)\ z$
  obtains *rtrancl-path* $r\ x\ (xs\ @\ [y])\ y$ **and** *rtrancl-path* $r\ y\ ys\ z$ ⟨*proof*⟩

lemma *rtrancl-path-distinct*:
  assumes *xy*: *rtrancl-path* $r\ x\ xs\ y$
  obtains xs' **where** *rtrancl-path* $r\ x\ xs'\ y$ **and** *distinct* $(x\ \#\ xs')$ ⟨*proof*⟩

inductive *rtrancl-tab* :: $('a \Rightarrow 'a \Rightarrow bool) \Rightarrow 'a\ list \Rightarrow 'a \Rightarrow 'a \Rightarrow bool$
  for r :: $'a \Rightarrow 'a \Rightarrow bool$
where
  base: *rtrancl-tab* $r\ xs\ x\ x$
| *step*: $x \notin set\ xs \Longrightarrow r\ x\ y \Longrightarrow$ *rtrancl-tab* $r\ (x\ \#\ xs)\ y\ z \Longrightarrow$ *rtrancl-tab* $r\ xs\ x\ z$

lemma *rtrancl-path-imp-rtrancl-tab*:
  assumes *path*: *rtrancl-path* $r\ x\ xs\ y$
  and x: *distinct* $(x\ \#\ xs)$
  and *ys*: $(\{x\} \cup set\ xs) \cap set\ ys = \{\}$
  shows *rtrancl-tab* $r\ ys\ x\ y$ ⟨*proof*⟩

lemma *rtrancl-tab-imp-rtrancl-path*:
  assumes *tab*: *rtrancl-tab* $r\ ys\ x\ y$
  obtains *xs* **where** *rtrancl-path* $r\ x\ xs\ y$ ⟨*proof*⟩

lemma *rtranclp-eq-rtrancl-tab-nil*: $r^{**}\ x\ y =$ *rtrancl-tab* $r\ []\ x\ y$
⟨*proof*⟩

declare *rtranclp-rtrancl-eq*[*code del*]
declare *rtranclp-eq-rtrancl-tab-nil*[*THEN iffD2*, *code-pred-intro*]

code-pred *rtranclp* ⟨*proof*⟩

1.1.1 A simple example

datatype $ty = A \mid B \mid C$

inductive *test* :: *ty* ⇒ *ty* ⇒ *bool*
where
test A B
| *test B A*
| *test B C*

Invoking with the predicate compiler and the generic code generator

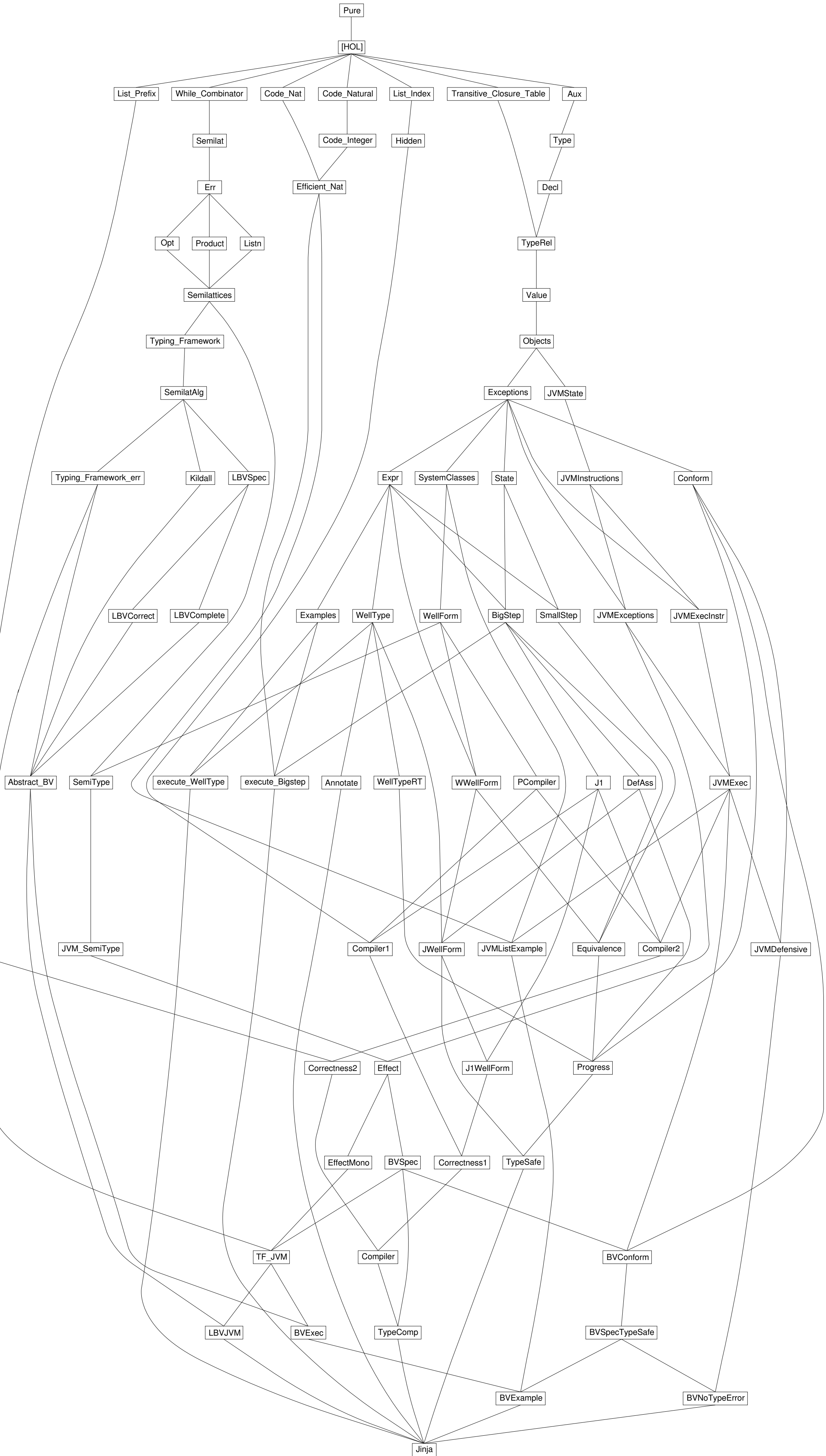
code-pred *test* ⟨*proof*⟩

values $\{x.\ test^{**}\ A\ C\}$
values $\{x.\ test^{**}\ C\ A\}$
values $\{x.\ test^{**}\ A\ x\}$
values $\{x.\ test^{**}\ x\ C\}$

value $test^{**}\ A\ C$
value $test^{**}\ C\ A$

hide-type *ty*
hide-const *test A B C*

end


Pure
[HOL]
List_Prefix
While_Combinator
Code_Nat
Code_Natural
List_Index
Transitive_Closure_Table
Aux
Semilat
Code_Integer
Hidden
Type
Err
Efficient_Nat
Decl
Opt
Product
Listn
TypeRel
Semilattices
Value
Typing_Framework
Objects
SemilatAlg
Exceptions
JVMState
Typing_Framework_err
Kildall
LBVSpec
Expr
SystemClasses
State
JVMInstructions
Conform
LBVCorrect
LBVComplete
Examples
WellType
WellForm
BigStep
SmallStep
JVMExceptions
JVMExecInstr
Abstract_BV
SemiType
execute_WellType
execute_Bigstep
Annotate
WellTypeRT
WWellForm
PCompiler
J1
DefAss
JVMExec
JVM_SemiType
Compiler1
JWellForm
JVMListExample
Equivalence
Compiler2
JVMDefensive
Correctness2
Effect
J1WellForm
Progress
EffectMono
BVSpec
Correctness1
TypeSafe
TF_JVM
Compiler
BVConform
LBVJVM
BVExec
TypeComp
BVSpecTypeSafe
BVExample
BVNoTypeError
Jinja

Chapter 2

Jinja Source Language

2.1 Auxiliary Definitions

theory *Aux* **imports** *Main* **begin**

lemma *nat-add-max-le*[*simp*]:
$((n::nat) + max\ i\ j \le m) = (n + i \le m \wedge n + j \le m)$
⟨*proof*⟩
lemma *Suc-add-max-le*[*simp*]:
$(Suc(n + max\ i\ j) \le m) = (Suc(n + i) \le m \wedge Suc(n + j) \le m)$⟨*proof*⟩

notation *Some* ((⌊-⌋))

2.1.1 *distinct-fst*

definition *distinct-fst* $::$ $('a \times 'b)\ list \Rightarrow bool$
where
distinct-fst $\equiv$ *distinct* $\circ$ *map fst*

lemma *distinct-fst-Nil* [*simp*]:
distinct-fst []
⟨*proof*⟩
lemma *distinct-fst-Cons* [*simp*]:
distinct-fst $((k,x)\#kxs) = (distinct\text{-}fst\ kxs \wedge (\forall y.\ (k,y) \notin set\ kxs))$⟨*proof*⟩

lemma *map-of-SomeI*:
$[\![$ *distinct-fst kxs*; $(k,x) \in set\ kxs$ $]\!] \Longrightarrow$ *map-of kxs k* $=$ *Some x*⟨*proof*⟩

2.1.2 Using *list-all2* for relations

definition *fun-of* $::$ $('a \times 'b)\ set \Rightarrow 'a \Rightarrow 'b \Rightarrow bool$
where
fun-of $S \equiv \lambda x\ y.\ (x,y) \in S$

Convenience lemmas

lemma *rel-list-all2-Cons* [*iff*]:
list-all2 (*fun-of* S) $(x\#xs)$ $(y\#ys)$ $=$
$((x,y) \in S \wedge$ *list-all2* (*fun-of* S) *xs ys*)
⟨*proof*⟩
lemma *rel-list-all2-Cons1*:
list-all2 (*fun-of* S) $(x\#xs)$ *ys* $=$
$(\exists z\ zs.\ ys = z\#zs \wedge (x,z) \in S \wedge$ *list-all2* (*fun-of* S) *xs zs*)
⟨*proof*⟩
lemma *rel-list-all2-Cons2*:
list-all2 (*fun-of* S) *xs* $(y\#ys)$ $=$
$(\exists z\ zs.\ xs = z\#zs \wedge (z,y) \in S \wedge$ *list-all2* (*fun-of* S) *zs ys*)
⟨*proof*⟩
lemma *rel-list-all2-refl*:
$(\bigwedge x.\ (x,x) \in S) \Longrightarrow$ *list-all2* (*fun-of* S) *xs xs*
⟨*proof*⟩
lemma *rel-list-all2-antisym*:
$[\![$ $(\bigwedge x\ y.\ [\![(x,y) \in S;\ (y,x) \in T]\!] \Longrightarrow x = y)$;
list-all2 (*fun-of* S) *xs ys*; *list-all2* (*fun-of* T) *ys xs* $]\!] \Longrightarrow xs = ys$
⟨*proof*⟩
lemma *rel-list-all2-trans*:

$\llbracket \bigwedge a\ b\ c.\ \llbracket(a,b) \in R;\ (b,c) \in S\rrbracket \Longrightarrow (a,c) \in T;$
$\quad \textit{list-all2}\ (\textit{fun-of}\ R)\ \textit{as}\ \textit{bs};\ \textit{list-all2}\ (\textit{fun-of}\ S)\ \textit{bs}\ \textit{cs}\rrbracket$
$\Longrightarrow \textit{list-all2}\ (\textit{fun-of}\ T)\ \textit{as}\ \textit{cs}$
⟨*proof*⟩
lemma *rel-list-all2-update-cong*:
$\llbracket\ i{<}\textit{size}\ \textit{xs};\ \textit{list-all2}\ (\textit{fun-of}\ S)\ \textit{xs}\ \textit{ys};\ (x,y) \in S\ \rrbracket$
$\Longrightarrow \textit{list-all2}\ (\textit{fun-of}\ S)\ (\textit{xs}[i{:=}x])\ (\textit{ys}[i{:=}y])$
⟨*proof*⟩
lemma *rel-list-all2-nthD*:
$\llbracket\ \textit{list-all2}\ (\textit{fun-of}\ S)\ \textit{xs}\ \textit{ys};\ p < \textit{size}\ \textit{xs}\ \rrbracket \Longrightarrow (\textit{xs}!p,\textit{ys}!p) \in S$
⟨*proof*⟩
lemma *rel-list-all2I*:
$\llbracket\ \textit{length}\ a = \textit{length}\ b;\ \bigwedge n.\ n < \textit{length}\ a \Longrightarrow (a!n,b!n) \in S\ \rrbracket \Longrightarrow \textit{list-all2}\ (\textit{fun-of}\ S)\ a\ b$
⟨*proof*⟩
end

2.2 Jinja types

theory *Type* **imports** *Aux* **begin**

type-synonym *cname* = *string* — class names
type-synonym *mname* = *string* — method name
type-synonym *vname* = *string* — names for local/field variables

definition *Object* :: *cname*
where
  Object ≡ ''*Object*''

definition *this* :: *vname*
where
  this ≡ ''*this*''

— types
datatype *ty*
  = *Void* — type of statements
  | *Boolean*
  | *Integer*
  | *NT* — null type
  | *Class cname* — class type

definition *is-refT* :: *ty* ⇒ *bool*
where
  $\textit{is-refT}\ T \ \equiv\ T = NT \lor (\exists\, C.\ T = \textit{Class}\ C)$

lemma [*iff*]: *is-refT NT*⟨*proof*⟩
lemma [*iff*]: *is-refT*(*Class C*)⟨*proof*⟩
lemma *refTE*:
  $[\![\textit{is-refT}\ T;\ T = NT \Longrightarrow P;\ \bigwedge C.\ T = \textit{Class}\ C \Longrightarrow P\ ]\!] \Longrightarrow P\langle \textit{proof}\rangle$
lemma *not-refTE*:
  $[\![\ \neg\textit{is-refT}\ T;\ T = \textit{Void} \lor T = \textit{Boolean} \lor T = \textit{Integer} \Longrightarrow P\ ]\!] \Longrightarrow P\langle \textit{proof}\rangle$
end

2.3 Class Declarations and Programs

theory *Decl* **imports** *Type* **begin**

type-synonym
$fdecl \ = vname \times ty$ — field declaration
type-synonym
$'m\ mdecl = mname \times ty\ list \times ty \times 'm$ — method = name, arg. types, return type, body
type-synonym
$'m\ class = cname \times fdecl\ list \times 'm\ mdecl\ list$ — class = superclass, fields, methods
type-synonym
$'m\ cdecl = cname \times 'm\ class$ — class declaration
type-synonym
$'m\ prog \ = 'm\ cdecl\ list$ — program

definition $class :: 'm\ prog \Rightarrow cname \rightharpoonup 'm\ class$
where
$class \ \equiv \ map\text{-}of$

definition $is\text{-}class :: 'm\ prog \Rightarrow cname \Rightarrow bool$
where
$is\text{-}class\ P\ C \ \equiv \ class\ P\ C \neq None$

lemma *finite-is-class*: $finite\ \{C.\ is\text{-}class\ P\ C\}$
$\langle proof \rangle$
definition $is\text{-}type :: 'm\ prog \Rightarrow ty \Rightarrow bool$
where
$is\text{-}type\ P\ T \ \equiv$
$(case\ T\ of\ Void \Rightarrow True \mid Boolean \Rightarrow True \mid Integer \Rightarrow True \mid NT \Rightarrow True$
$\mid Class\ C \Rightarrow is\text{-}class\ P\ C)$

lemma *is-type-simps* [*simp*]:
$is\text{-}type\ P\ Void \wedge is\text{-}type\ P\ Boolean \wedge is\text{-}type\ P\ Integer \wedge$
$is\text{-}type\ P\ NT \wedge is\text{-}type\ P\ (Class\ C) = is\text{-}class\ P\ C \langle proof \rangle$

abbreviation
$types\ P == Collect\ (is\text{-}type\ P)$

end

2.4 Relations between Jinja Types

theory *TypeRel* **imports**
  ~~/src/HOL/Library/Transitive-Closure-Table
  Decl
begin

2.4.1 The subclass relations

inductive-set
  $subcls1 :: {'m}\ prog \Rightarrow (cname \times cname)\ set$
  and $subcls1' :: {'m}\ prog \Rightarrow [cname, cname] \Rightarrow bool$ ($\text{-} \vdash \text{-} \prec^1 \text{-}$ $[71,71,71]$ 70)
  for $P :: {'m}\ prog$
where
  $P \vdash C \prec^1 D \equiv (C,D) \in subcls1\ P$
| $subcls1I$: $[\![class\ P\ C = Some\ (D,rest);\ C \neq Object]\!] \Longrightarrow P \vdash C \prec^1 D$

abbreviation
  $subcls :: {'m}\ prog \Rightarrow [cname, cname] \Rightarrow bool$ ($\text{-} \vdash \text{-} \preceq^* \text{-}$ $[71,71,71]$ 70)
  where $P \vdash C \preceq^* D \equiv (C,D) \in (subcls1\ P)^*$

lemma $subcls1D$: $P \vdash C \prec^1 D \Longrightarrow C \neq Object \wedge (\exists fs\ ms.\ class\ P\ C = Some\ (D,fs,ms))\langle proof\rangle$
lemma $[iff]$: $\neg\ P \vdash Object \prec^1 C\langle proof\rangle$
lemma $[iff]$: $(P \vdash Object \preceq^* C) = (C = Object)\langle proof\rangle$
lemma $subcls1\text{-}def2$:
  $subcls1\ P =$
    $(SIGMA\ C{:}\{C.\ is\text{-}class\ P\ C\}.\ \{D.\ C{\neq}Object \wedge fst\ (the\ (class\ P\ C)){=}D\})\langle proof\rangle$
lemma $finite\text{-}subcls1$: $finite\ (subcls1\ P)\langle proof\rangle$

2.4.2 The subtype relations

inductive
  $widen :: {'m}\ prog \Rightarrow ty \Rightarrow ty \Rightarrow bool$ ($\text{-} \vdash \text{-} \leq \text{-}$ $[71,71,71]$ 70)
  for $P :: {'m}\ prog$
where
  $widen\text{-}refl[iff]$: $P \vdash T \leq T$
| $widen\text{-}subcls$: $P \vdash C \preceq^* D \Longrightarrow P \vdash Class\ C \leq Class\ D$
| $widen\text{-}null[iff]$: $P \vdash NT \leq Class\ C$

abbreviation (*xsymbols*)
  $widens :: {'m}\ prog \Rightarrow ty\ list \Rightarrow ty\ list \Rightarrow bool$
    ($\text{-} \vdash \text{-} [\leq] \text{-}$ $[71,71,71]$ 70) **where**
  $widens\ P\ Ts\ Ts' \equiv list\text{-}all2\ (widen\ P)\ Ts\ Ts'$

lemma $[iff]$: $(P \vdash T \leq Void) = (T = Void)\langle proof\rangle$
lemma $[iff]$: $(P \vdash T \leq Boolean) = (T = Boolean)\langle proof\rangle$
lemma $[iff]$: $(P \vdash T \leq Integer) = (T = Integer)\langle proof\rangle$
lemma $[iff]$: $(P \vdash Void \leq T) = (T = Void)\langle proof\rangle$
lemma $[iff]$: $(P \vdash Boolean \leq T) = (T = Boolean)\langle proof\rangle$
lemma $[iff]$: $(P \vdash Integer \leq T) = (T = Integer)\langle proof\rangle$

lemma $Class\text{-}widen$: $P \vdash Class\ C \leq T \Longrightarrow \exists D.\ T = Class\ D\langle proof\rangle$
lemma $[iff]$: $(P \vdash T \leq NT) = (T = NT)\langle proof\rangle$
lemma $Class\text{-}widen\text{-}Class\ [iff]$: $(P \vdash Class\ C \leq Class\ D) = (P \vdash C \preceq^* D)\langle proof\rangle$
lemma $widen\text{-}Class$: $(P \vdash T \leq Class\ C) = (T = NT \vee (\exists D.\ T = Class\ D \wedge P \vdash D \preceq^* C))\langle proof\rangle$

lemma *widen-trans*[*trans*]: $[\![P \vdash S \leq U;\ P \vdash U \leq T]\!] \Longrightarrow P \vdash S \leq T\langle proof\rangle$
lemma *widens-trans* [*trans*]: $[\![P \vdash Ss\ [\leq]\ Ts;\ P \vdash Ts\ [\leq]\ Us]\!] \Longrightarrow P \vdash Ss\ [\leq]\ Us\langle proof\rangle$

2.4.3 Method lookup

inductive
$Methods :: [{}'m\ prog,\ cname,\ mname \rightharpoonup (ty\ list \times ty \times {}'m) \times cname] \Rightarrow bool$
$(\text{-} \vdash \text{-}\ sees'\text{-}methods\ \text{-}\ [51,51,51]\ 50)$
for $P :: {}'m\ prog$
where
sees-methods-Object:
$[\![\ class\ P\ Object = Some(D,fs,ms);\ Mm = Option.map\ (\lambda m.\ (m,Object)) \circ map\text{-}of\ ms\ ]\!]$
$\Longrightarrow P \vdash Object\ sees\text{-}methods\ Mm$
| *sees-methods-rec*:
$[\![\ class\ P\ C = Some(D,fs,ms);\ C \neq Object;\ P \vdash D\ sees\text{-}methods\ Mm;$
$Mm' = Mm\ {+}{+}\ (Option.map\ (\lambda m.\ (m,C)) \circ map\text{-}of\ ms)\ ]\!]$
$\Longrightarrow P \vdash C\ sees\text{-}methods\ Mm'$

lemma *sees-methods-fun*:
assumes *1*: $P \vdash C\ sees\text{-}methods\ Mm$
shows $\bigwedge Mm'.\ P \vdash C\ sees\text{-}methods\ Mm' \Longrightarrow Mm' = Mm$
$\langle proof\rangle$
lemma *visible-methods-exist*:
$P \vdash C\ sees\text{-}methods\ Mm \Longrightarrow Mm\ M = Some(m,D) \Longrightarrow$
$(\exists D'\ fs\ ms.\ class\ P\ D = Some(D',fs,ms) \wedge map\text{-}of\ ms\ M = Some\ m)$
$\langle proof\rangle$
lemma *sees-methods-decl-above*:
assumes *Csees*: $P \vdash C\ sees\text{-}methods\ Mm$
shows $Mm\ M = Some(m,D) \Longrightarrow P \vdash C \preceq^* D$
$\langle proof\rangle$
lemma *sees-methods-idemp*:
assumes *Cmethods*: $P \vdash C\ sees\text{-}methods\ Mm$
shows $\bigwedge m\ D.\ Mm\ M = Some(m,D) \Longrightarrow$
$\exists Mm'.\ (P \vdash D\ sees\text{-}methods\ Mm') \wedge Mm'\ M = Some(m,D)\langle proof\rangle$

lemma *sees-methods-decl-mono*:
assumes *sub*: $P \vdash C' \preceq^* C$
shows $P \vdash C\ sees\text{-}methods\ Mm \Longrightarrow$
$\exists Mm'\ Mm_2.\ P \vdash C'\ sees\text{-}methods\ Mm' \wedge Mm' = Mm\ {+}{+}\ Mm_2 \wedge$
$(\forall M\ m\ D.\ Mm_2\ M = Some(m,D) \longrightarrow P \vdash D \preceq^* C)\langle proof\rangle$

definition $Method :: {}'m\ prog \Rightarrow cname \Rightarrow mname \Rightarrow ty\ list \Rightarrow ty \Rightarrow {}'m \Rightarrow cname \Rightarrow bool$
$(\text{-} \vdash \text{-}\ sees\ \text{-}:\ \text{-}{\rightarrow}\text{-} = \text{-}\ in\ \text{-}\ [51,51,51,51,51,51,51]\ 50)$
where
$P \vdash C\ sees\ M:\ Ts{\rightarrow}T = m\ in\ D\ \equiv$
$\exists Mm.\ P \vdash C\ sees\text{-}methods\ Mm \wedge Mm\ M = Some((Ts,T,m),D)$

definition $has\text{-}method :: {}'m\ prog \Rightarrow cname \Rightarrow mname \Rightarrow bool\ (\text{-} \vdash \text{-}\ has\ \text{-}\ [51,0,51]\ 50)$
where
$P \vdash C\ has\ M \equiv \exists Ts\ T\ m\ D.\ P \vdash C\ sees\ M{:}Ts{\rightarrow}T = m\ in\ D$

lemma *sees-method-fun*:

$[\![P \vdash C \textit{ sees } M{:}TS{\rightarrow}T = m \textit{ in } D;\ P \vdash C \textit{ sees } M{:}TS'{\rightarrow}T' = m' \textit{ in } D']\!]$
$\Longrightarrow TS' = TS \wedge T' = T \wedge m' = m \wedge D' = D$
⟨*proof*⟩
lemma *sees-method-decl-above*:
$P \vdash C \textit{ sees } M{:}Ts{\rightarrow}T = m \textit{ in } D \Longrightarrow P \vdash C \preceq^* D$
⟨*proof*⟩
lemma *visible-method-exists*:
$P \vdash C \textit{ sees } M{:}Ts{\rightarrow}T = m \textit{ in } D \Longrightarrow$
$\exists D'\ fs\ ms.\ \textit{class } P\ D = \textit{Some}(D',fs,ms) \wedge \textit{map-of } ms\ M = \textit{Some}(Ts,T,m)$⟨*proof*⟩

lemma *sees-method-idemp*:
$P \vdash C \textit{ sees } M{:}Ts{\rightarrow}T{=}m \textit{ in } D \Longrightarrow P \vdash D \textit{ sees } M{:}Ts{\rightarrow}T{=}m \textit{ in } D$
⟨*proof*⟩
lemma *sees-method-decl-mono*:
$[\![P \vdash C' \preceq^* C;\ P \vdash C \textit{ sees } M{:}Ts{\rightarrow}T = m \textit{ in } D;$
$P \vdash C' \textit{ sees } M{:}Ts'{\rightarrow}T' = m' \textit{ in } D']\!] \Longrightarrow P \vdash D' \preceq^* D$
⟨*proof*⟩
lemma *sees-method-is-class*:
$[\![P \vdash C \textit{ sees } M{:}Ts{\rightarrow}T = m \textit{ in } D]\!] \Longrightarrow \textit{is-class } P\ C$⟨*proof*⟩

2.4.4 Field lookup

inductive
$\textit{Fields} :: [{}'m\ \textit{prog},\ \textit{cname},\ ((\textit{vname} \times \textit{cname}) \times \textit{ty})\ \textit{list}] \Rightarrow \textit{bool}$
$(\text{-} \vdash \text{-}\ \textit{has'-fields}\ \text{-}\ [51,51,51]\ 50)$
for $P :: {}'m\ \textit{prog}$
where
has-fields-rec:
$[\![\textit{class } P\ C = \textit{Some}(D,fs,ms);\ C \neq \textit{Object};\ P \vdash D \textit{ has-fields } FDTs;$
$FDTs' = \textit{map } (\lambda(F,T).\ ((F,C),T))\ fs\ @\ FDTs]\!]$
$\Longrightarrow P \vdash C \textit{ has-fields } FDTs'$
| *has-fields-Object*:
$[\![\textit{class } P\ \textit{Object} = \textit{Some}(D,fs,ms);\ FDTs = \textit{map } (\lambda(F,T).\ ((F,\textit{Object}),T))\ fs]\!]$
$\Longrightarrow P \vdash \textit{Object has-fields } FDTs$

lemma *has-fields-fun*:
assumes *1*: $P \vdash C \textit{ has-fields } FDTs$
shows $\bigwedge FDTs'.\ P \vdash C \textit{ has-fields } FDTs' \Longrightarrow FDTs' = FDTs$
⟨*proof*⟩
lemma *all-fields-in-has-fields*:
assumes *sub*: $P \vdash C \textit{ has-fields } FDTs$
shows $[\![P \vdash C \preceq^* D;\ \textit{class } P\ D = \textit{Some}(D',fs,ms);\ (F,T) \in \textit{set } fs]\!]$
$\Longrightarrow ((F,D),T) \in \textit{set } FDTs$⟨*proof*⟩

lemma *has-fields-decl-above*:
assumes *fields*: $P \vdash C \textit{ has-fields } FDTs$
shows $((F,D),T) \in \textit{set } FDTs \Longrightarrow P \vdash C \preceq^* D$⟨*proof*⟩

lemma *subcls-notin-has-fields*:
assumes *fields*: $P \vdash C \textit{ has-fields } FDTs$
shows $((F,D),T) \in \textit{set } FDTs \Longrightarrow (D,C) \notin (\textit{subcls1 } P)^+$⟨*proof*⟩

lemma *has-fields-mono-lem*:
assumes *sub*: $P \vdash D \preceq^* C$

shows $P \vdash C$ *has-fields FDTs*
$\Longrightarrow \exists\, pre.\ P \vdash D$ *has-fields pre@FDTs* $\wedge$ *dom*(*map-of pre*) $\cap$ *dom*(*map-of FDTs*) $= \{\}\langle proof\rangle$

definition *has-field* :: $'m$ *prog* $\Rightarrow$ *cname* $\Rightarrow$ *vname* $\Rightarrow$ *ty* $\Rightarrow$ *cname* $\Rightarrow$ *bool*
(- $\vdash$ - *has* -:- *in* - [51,51,51,51,51] 50)
where
$P \vdash C$ *has* $F{:}T$ *in* $D \equiv$
$\exists\, FDTs.\ P \vdash C$ *has-fields FDTs* $\wedge$ *map-of FDTs* $(F,D) =$ *Some* T

lemma *has-field-mono*:
$[\![\ P \vdash C$ *has* $F{:}T$ *in* $D;\ P \vdash C' \preceq^* C\ ]\!] \Longrightarrow P \vdash C'$ *has* $F{:}T$ *in* $D\langle proof\rangle$

definition *sees-field* :: $'m$ *prog* $\Rightarrow$ *cname* $\Rightarrow$ *vname* $\Rightarrow$ *ty* $\Rightarrow$ *cname* $\Rightarrow$ *bool*
(- $\vdash$ - *sees* -:- *in* - [51,51,51,51,51] 50)
where
$P \vdash C$ *sees* $F{:}T$ *in* $D \equiv$
$\exists\, FDTs.\ P \vdash C$ *has-fields FDTs* $\wedge$
map-of (*map* $(\lambda((F,D),T).\ (F,(D,T)))$ *FDTs*) $F =$ *Some*(D,T)

lemma *map-of-remap-SomeD*:
map-of (*map* $(\lambda((k,k'),x).\ (k,(k',x)))$ t) $k =$ *Some* $(k',x) \Longrightarrow$ *map-of* t $(k,\ k') =$ *Some* $x\langle proof\rangle$

lemma *has-visible-field*:
$P \vdash C$ *sees* $F{:}T$ *in* $D \Longrightarrow P \vdash C$ *has* $F{:}T$ *in* $D\langle proof\rangle$

lemma *sees-field-fun*:
$[\![P \vdash C$ *sees* $F{:}T$ *in* $D;\ P \vdash C$ *sees* $F{:}T'$ *in* $D']\!] \Longrightarrow T' = T \wedge D' = D\langle proof\rangle$

lemma *sees-field-decl-above*:
$P \vdash C$ *sees* $F{:}T$ *in* $D \Longrightarrow P \vdash C \preceq^* D\langle proof\rangle$

lemma *sees-field-idemp*:
$P \vdash C$ *sees* $F{:}T$ *in* $D \Longrightarrow P \vdash D$ *sees* $F{:}T$ *in* $D\langle proof\rangle$

2.4.5 Functional lookup

definition *method* :: $'m$ *prog* $\Rightarrow$ *cname* $\Rightarrow$ *mname* $\Rightarrow$ *cname* $\times$ *ty list* $\times$ *ty* $\times$ $'m$
where
method $P\ C\ M \equiv$ *THE* $(D,Ts,T,m).\ P \vdash C$ *sees* $M{:}Ts \rightarrow T = m$ *in* D

definition *field* :: $'m$ *prog* $\Rightarrow$ *cname* $\Rightarrow$ *vname* $\Rightarrow$ *cname* $\times$ *ty*
where
field $P\ C\ F \equiv$ *THE* $(D,T).\ P \vdash C$ *sees* $F{:}T$ *in* D

definition *fields* :: $'m$ *prog* $\Rightarrow$ *cname* $\Rightarrow$ ((*vname* $\times$ *cname*) $\times$ *ty*) *list*
where
fields $P\ C \equiv$ *THE FDTs*. $P \vdash C$ *has-fields FDTs*

lemma *fields-def2* [*simp*]: $P \vdash C$ *has-fields FDTs* $\Longrightarrow$ *fields* $P\ C = FDTs\langle proof\rangle$
lemma *field-def2* [*simp*]: $P \vdash C$ *sees* $F{:}T$ *in* $D \Longrightarrow$ *field* $P\ C\ F = (D,T)\langle proof\rangle$
lemma *method-def2* [*simp*]: $P \vdash C$ *sees* $M{:}\ Ts \rightarrow T = m$ *in* $D \Longrightarrow$ *method* $P\ C\ M = (D,Ts,T,m)\langle proof\rangle$

2.4.6 Code generator setup

code-pred
  ($modes$: $i \Rightarrow i \Rightarrow i \Rightarrow bool$, $i \Rightarrow i \Rightarrow o \Rightarrow bool$)
  subcls1p
⟨*proof*⟩
declare *subcls1-def* [*code-pred-def*]

code-pred
  ($modes$: $i \Rightarrow i \times o \Rightarrow bool$, $i \Rightarrow i \times i \Rightarrow bool$)
  [*inductify*]
  subcls1
⟨*proof*⟩
definition *subcls'* **where** $subcls'\ G = (subcls1p\ G)\hat{}{**}$
code-pred
  ($modes$: $i \Rightarrow i \Rightarrow i \Rightarrow bool$, $i \Rightarrow i \Rightarrow o \Rightarrow bool$)
  [*inductify*]
  subcls'
⟨*proof*⟩

lemma *subcls-conv-subcls'* [*code-unfold*]:
  $(subcls1\ G)\hat{}{*} = \{(C, D).\ subcls'\ G\ C\ D\}$
  ⟨*proof*⟩

code-pred
  ($modes$: $i \Rightarrow i \Rightarrow i \Rightarrow bool$)
  widen
⟨*proof*⟩

code-pred
  ($modes$: $i \Rightarrow i \Rightarrow o \Rightarrow bool$)
  Fields
⟨*proof*⟩

lemma *has-field-code* [*code-pred-intro*]:
  $[\![\ P \vdash C\ has\text{-}fields\ FDTs;\ map\text{-}of\ FDTs\ (F, D) = \lfloor T \rfloor\ ]\!]$
  $\Longrightarrow P \vdash C\ has\ F{:}T\ in\ D$
⟨*proof*⟩

code-pred
  ($modes$: $i \Rightarrow i \Rightarrow i \Rightarrow o \Rightarrow i \Rightarrow bool$, $i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow bool$)
  has-field
⟨*proof*⟩

lemma *sees-field-code* [*code-pred-intro*]:
  $[\![\ P \vdash C\ has\text{-}fields\ FDTs;\ map\text{-}of\ (map\ (\lambda((F, D), T).\ (F, D, T))\ FDTs)\ F = \lfloor(D, T)\rfloor\ ]\!]$
  $\Longrightarrow P \vdash C\ sees\ F{:}T\ in\ D$
⟨*proof*⟩

code-pred
  ($modes$: $i \Rightarrow i \Rightarrow i \Rightarrow o \Rightarrow o \Rightarrow bool$, $i \Rightarrow i \Rightarrow i \Rightarrow o \Rightarrow i \Rightarrow bool$,
      $i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow o \Rightarrow bool$, $i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow bool$)
  sees-field
⟨*proof*⟩

code-pred
$(modes: i \Rightarrow i \Rightarrow o \Rightarrow bool)$
Methods
$\langle proof \rangle$

lemma *Method-code* [*code-pred-intro*]:
$[\![P \vdash C \text{ sees-methods } Mm; Mm\ M = \lfloor((Ts, T, m), D)\rfloor]\!]$
$\Longrightarrow P \vdash C \text{ sees } M\text{: } Ts \rightarrow T = m \text{ in } D$
$\langle proof \rangle$

code-pred
$(modes: i \Rightarrow i \Rightarrow i \Rightarrow o \Rightarrow o \Rightarrow o \Rightarrow o \Rightarrow bool,$
$i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow bool)$
Method
$\langle proof \rangle$

lemma *eval-Method-i-i-i-o-o-o-o-conv*:
$Predicate.eval\ (Method\text{-}i\text{-}i\text{-}i\text{-}o\text{-}o\text{-}o\text{-}o\ P\ C\ M) = (\lambda(Ts, T, m, D).\ P \vdash C \text{ sees } M{:}Ts \rightarrow T{=}m \text{ in } D)$
$\langle proof \rangle$

lemma *method-code* [*code*]:
$method\ P\ C\ M =$
$Predicate.the\ (Predicate.bind\ (Method\text{-}i\text{-}i\text{-}i\text{-}o\text{-}o\text{-}o\text{-}o\ P\ C\ M)\ (\lambda(Ts, T, m, D).\ Predicate.single\ (D, Ts, T, m)))$
$\langle proof \rangle$

lemma *eval-Fields-conv*:
$Predicate.eval\ (Fields\text{-}i\text{-}i\text{-}o\ P\ C) = (\lambda FDTs.\ P \vdash C \text{ has-fields } FDTs)$
$\langle proof \rangle$

lemma *fields-code* [*code*]:
$fields\ P\ C = Predicate.the\ (Fields\text{-}i\text{-}i\text{-}o\ P\ C)$
$\langle proof \rangle$

lemma *eval-sees-field-i-i-i-o-o-conv*:
$Predicate.eval\ (sees\text{-}field\text{-}i\text{-}i\text{-}i\text{-}o\text{-}o\ P\ C\ F) = (\lambda(T, D).\ P \vdash C \text{ sees } F{:}T \text{ in } D)$
$\langle proof \rangle$

lemma *eval-sees-field-i-i-i-o-i-conv*:
$Predicate.eval\ (sees\text{-}field\text{-}i\text{-}i\text{-}i\text{-}o\text{-}i\ P\ C\ F\ D) = (\lambda T.\ P \vdash C \text{ sees } F{:}T \text{ in } D)$
$\langle proof \rangle$

lemma *field-code* [*code*]:
$field\ P\ C\ F = Predicate.the\ (Predicate.bind\ (sees\text{-}field\text{-}i\text{-}i\text{-}i\text{-}o\text{-}o\ P\ C\ F)\ (\lambda(T, D).\ Predicate.single\ (D, T)))$
$\langle proof \rangle$

2.5 Jinja Values

theory *Value* **imports** *TypeRel* **begin**

type-synonym $addr = nat$

datatype *val*
  $=$ *Unit* — dummy result value of void expressions
  $|$ *Null* — null reference
  $|$ *Bool bool* — Boolean value
  $|$ *Intg int* — integer value
  $|$ *Addr addr* — addresses of objects in the heap

primrec *the-Intg* $::$ *val* $\Rightarrow$ *int* **where**
  the-Intg (*Intg i*) $=$ *i*

primrec *the-Addr* $::$ *val* $\Rightarrow$ *addr* **where**
  the-Addr (*Addr a*) $=$ *a*

primrec *default-val* $::$ *ty* $\Rightarrow$ *val* — default value for all types **where**
  default-val Void $=$ *Unit*
$|$ *default-val Boolean* $=$ *Bool False*
$|$ *default-val Integer* $=$ *Intg 0*
$|$ *default-val NT* $=$ *Null*
$|$ *default-val* (*Class C*) $=$ *Null*

end

2.6 Objects and the Heap

theory *Objects* **imports** *TypeRel Value* **begin**

2.6.1 Objects

type-synonym
  $fields = vname \times cname \rightharpoonup val$ — field name, defining class, value
type-synonym
  $obj = cname \times fields$ — class instance with class name and fields

definition $obj\text{-}ty :: obj \Rightarrow ty$
where
  $obj\text{-}ty\ obj \equiv Class\ (fst\ obj)$

definition $init\text{-}fields :: ((vname \times cname) \times ty)\ list \Rightarrow fields$
where
  $init\text{-}fields \equiv map\text{-}of \circ map\ (\lambda(F,T).\ (F,default\text{-}val\ T))$

— a new, blank object with default values in all fields:
definition $blank :: 'm\ prog \Rightarrow cname \Rightarrow obj$
where
  $blank\ P\ C \equiv (C,init\text{-}fields\ (fields\ P\ C))$

lemma [*simp*]: $obj\text{-}ty\ (C,fs) = Class\ C\langle proof\rangle$

2.6.2 Heap

type-synonym $heap = addr \rightharpoonup obj$

abbreviation
  $cname\text{-}of :: heap \Rightarrow addr \Rightarrow cname$ **where**
  $cname\text{-}of\ hp\ a == fst\ (the\ (hp\ a))$

definition $new\text{-}Addr :: heap \Rightarrow addr\ option$
where
  $new\text{-}Addr\ h \equiv if\ \exists\, a.\ h\ a = None\ then\ Some(LEAST\ a.\ h\ a = None)\ else\ None$

definition $cast\text{-}ok :: 'm\ prog \Rightarrow cname \Rightarrow heap \Rightarrow val \Rightarrow bool$
where
  $cast\text{-}ok\ P\ C\ h\ v \equiv v = Null \lor P \vdash cname\text{-}of\ h\ (the\text{-}Addr\ v) \preceq^* C$

definition $hext :: heap \Rightarrow heap \Rightarrow bool$ (- $\trianglelefteq$ - [51,51] 50)
where
  $h \trianglelefteq h' \equiv \forall\, a\ C\ fs.\ h\ a = Some(C,fs) \longrightarrow (\exists\, fs'.\ h'\ a = Some(C,fs'))$

primrec $typeof\text{-}h :: heap \Rightarrow val \Rightarrow ty\ option$ ($typeof_{\_}$)
where
  $typeof_h\ Unit = Some\ Void$
| $typeof_h\ Null = Some\ NT$
| $typeof_h\ (Bool\ b) = Some\ Boolean$
| $typeof_h\ (Intg\ i) = Some\ Integer$
| $typeof_h\ (Addr\ a) = (case\ h\ a\ of\ None \Rightarrow None \mid Some(C,fs) \Rightarrow Some(Class\ C))$

lemma *new-Addr-SomeD*:

$new\text{-}Addr\ h = Some\ a \Longrightarrow h\ a = None$
$\langle proof \rangle$
lemma $[simp]$: $(typeof_h\ v = Some\ Boolean) = (\exists\, b.\ v = Bool\ b)$
$\langle proof \rangle$
lemma $[simp]$: $(typeof_h\ v = Some\ Integer) = (\exists\, i.\ v = Intg\ i)\langle proof \rangle$
lemma $[simp]$: $(typeof_h\ v = Some\ NT) = (v = Null)$
$\langle proof \rangle$
lemma $[simp]$: $(typeof_h\ v = Some(Class\ C)) = (\exists\, a\ fs.\ v = Addr\ a \wedge h\ a = Some(C,fs))$
$\langle proof \rangle$
lemma $[simp]$: $h\ a = Some(C,fs) \Longrightarrow typeof_{(h(a \mapsto (C,fs')))}\ v = typeof_h\ v$
$\langle proof \rangle$

For literal values the first parameter of *typeof* can be set to $Map.empty$ because they do not contain addresses:

abbreviation
$typeof :: val \Rightarrow ty\ option$ **where**
$typeof\ v == typeof\text{-}h\ empty\ v$

lemma *typeof-lit-typeof*:
$typeof\ v = Some\ T \Longrightarrow typeof_h\ v = Some\ T$
$\langle proof \rangle$
lemma *typeof-lit-is-type*:
$typeof\ v = Some\ T \Longrightarrow is\text{-}type\ P\ T$
$\langle proof \rangle$

2.6.3 Heap extension $\trianglelefteq$

lemma *hextI*: $\forall\, a\ C\ fs.\ h\ a = Some(C,fs) \longrightarrow (\exists\, fs'.\ h'\ a = Some(C,fs')) \Longrightarrow h \trianglelefteq h'\langle proof \rangle$
lemma *hext-objD*: $[\![\ h \trianglelefteq h';\ h\ a = Some(C,fs)\ ]\!] \Longrightarrow \exists\, fs'.\ h'\ a = Some(C,fs')\langle proof \rangle$
lemma *hext-refl* $[iff]$: $h \trianglelefteq h\langle proof \rangle$
lemma *hext-new* $[simp]$: $h\ a = None \Longrightarrow h \trianglelefteq h(a \mapsto x)\langle proof \rangle$
lemma *hext-trans*: $[\![\ h \trianglelefteq h';\ h' \trianglelefteq h''\ ]\!] \Longrightarrow h \trianglelefteq h''\langle proof \rangle$
lemma *hext-upd-obj*: $h\ a = Some\ (C,fs) \Longrightarrow h \trianglelefteq h(a \mapsto (C,fs'))\langle proof \rangle$
lemma *hext-typeof-mono*: $[\![\ h \trianglelefteq h';\ typeof_h\ v = Some\ T\ ]\!] \Longrightarrow typeof_{h'}\ v = Some\ T\langle proof \rangle$

Code generator setup for *new-Addr*

definition $gen\text{-}new\text{-}Addr :: heap \Rightarrow addr \Rightarrow addr\ option$
where $gen\text{-}new\text{-}Addr\ h\ n \equiv if\ \exists\, a.\ a \geq n \wedge h\ a = None\ then\ Some(LEAST\ a.\ a \geq n \wedge h\ a = None)$ $else\ None$

lemma *new-Addr-code-code* $[code]$:
$new\text{-}Addr\ h = gen\text{-}new\text{-}Addr\ h\ 0$
$\langle proof \rangle$

lemma *gen-new-Addr-code* $[code]$:
$gen\text{-}new\text{-}Addr\ h\ n = (if\ h\ n = None\ then\ Some\ n\ else\ gen\text{-}new\text{-}Addr\ h\ (Suc\ n))$
$\langle proof \rangle$

end

2.7 Exceptions

theory *Exceptions* **imports** *Objects* **begin**

definition *NullPointer* :: *cname*
where
$NullPointer \equiv {''NullPointer''}$

definition *ClassCast* :: *cname*
where
$ClassCast \equiv {''ClassCast''}$

definition *OutOfMemory* :: *cname*
where
$OutOfMemory \equiv {''OutOfMemory''}$

definition *sys-xcpts* :: *cname set*
where
$sys\text{-}xcpts \equiv \{NullPointer,\ ClassCast,\ OutOfMemory\}$

definition *addr-of-sys-xcpt* :: $cname \Rightarrow addr$
where
$addr\text{-}of\text{-}sys\text{-}xcpt\ s \equiv if\ s = NullPointer\ then\ 0\ else$
$if\ s = ClassCast\ then\ 1\ else$
$if\ s = OutOfMemory\ then\ 2\ else\ undefined$

definition *start-heap* :: $'c\ prog \Rightarrow heap$
where
$start\text{-}heap\ G \equiv empty\ (addr\text{-}of\text{-}sys\text{-}xcpt\ NullPointer \mapsto blank\ G\ NullPointer)$
$(addr\text{-}of\text{-}sys\text{-}xcpt\ ClassCast \mapsto blank\ G\ ClassCast)$
$(addr\text{-}of\text{-}sys\text{-}xcpt\ OutOfMemory \mapsto blank\ G\ OutOfMemory)$

definition *preallocated* :: $heap \Rightarrow bool$
where
$preallocated\ h \equiv \forall C \in sys\text{-}xcpts.\ \exists fs.\ h(addr\text{-}of\text{-}sys\text{-}xcpt\ C) = Some\ (C,fs)$

2.7.1 System exceptions

lemma [*simp*]: $NullPointer \in sys\text{-}xcpts \land OutOfMemory \in sys\text{-}xcpts \land ClassCast \in sys\text{-}xcpts\langle proof\rangle$

lemma *sys-xcpts-cases* [*consumes 1*, *cases set*]:
$[\![\ C \in sys\text{-}xcpts;\ P\ NullPointer;\ P\ OutOfMemory;\ P\ ClassCast]\!] \Longrightarrow P\ C\langle proof\rangle$

2.7.2 *preallocated*

lemma *preallocated-dom* [*simp*]:
$[\![\ preallocated\ h;\ C \in sys\text{-}xcpts\ ]\!] \Longrightarrow addr\text{-}of\text{-}sys\text{-}xcpt\ C \in dom\ h\langle proof\rangle$

lemma *preallocatedD*:
$[\![\ preallocated\ h;\ C \in sys\text{-}xcpts\ ]\!] \Longrightarrow \exists fs.\ h(addr\text{-}of\text{-}sys\text{-}xcpt\ C) = Some\ (C,\ fs)\langle proof\rangle$

lemma *preallocatedE* [*elim?*]:
$[\![\ preallocated\ h;\ \ C \in sys\text{-}xcpts;\ \bigwedge fs.\ h(addr\text{-}of\text{-}sys\text{-}xcpt\ C) = Some(C,fs) \Longrightarrow P\ h\ C]\!]$
$\Longrightarrow P\ h\ C\langle proof\rangle$

lemma *cname-of-xcp* [*simp*]:
$[\![$ *preallocated h*; $C \in$ *sys-xcpts* $]\!] \Longrightarrow$ *cname-of h* (*addr-of-sys-xcpt C*) $= C\langle proof\rangle$

lemma *typeof-ClassCast* [*simp*]:
preallocated h $\Longrightarrow$ $typeof_h$ (*Addr*(*addr-of-sys-xcpt ClassCast*)) = *Some*(*Class ClassCast*)$\langle proof\rangle$

lemma *typeof-OutOfMemory* [*simp*]:
preallocated h $\Longrightarrow$ $typeof_h$ (*Addr*(*addr-of-sys-xcpt OutOfMemory*)) = *Some*(*Class OutOfMemory*)$\langle proof\rangle$

lemma *typeof-NullPointer* [*simp*]:
preallocated h $\Longrightarrow$ $typeof_h$ (*Addr*(*addr-of-sys-xcpt NullPointer*)) = *Some*(*Class NullPointer*)$\langle proof\rangle$

lemma *preallocated-hext*:
$[\![$ *preallocated h*; $h \trianglelefteq h'$ $]\!] \Longrightarrow$ *preallocated* $h'\langle proof\rangle$

lemma *preallocated-start*:
preallocated (*start-heap P*)$\langle proof\rangle$

end

2.8 Expressions

theory *Expr*
imports *../Common/Exceptions*
begin

datatype *bop* = *Eq* | *Add* — names of binary operations

datatype *'a exp*
= *new cname* — class instance creation
| *Cast cname* (*'a exp*) — type cast
| *Val val* — value
| *BinOp* (*'a exp*) *bop* (*'a exp*) (- «-» - [*80*,*0*,*81*] *80*) — binary operation
| *Var 'a* — local variable (incl. parameter)
| *LAss 'a* (*'a exp*) (-:=- [*90*,*90*]*90*) — local assignment
| *FAcc* (*'a exp*) *vname cname* (-•-{-} [*10*,*90*,*99*]*90*) — field access
| *FAss* (*'a exp*) *vname cname* (*'a exp*) (-•-{-} := - [*10*,*90*,*99*,*90*]*90*) — field assignment
| *Call* (*'a exp*) *mname* (*'a exp list*) (-•-'(-') [*90*,*99*,*0*] *90*) — method call
| *Block 'a ty* (*'a exp*) ('{-:-; -})
| *Seq* (*'a exp*) (*'a exp*) (-;;/ - [*61*,*60*]*60*)
| *Cond* (*'a exp*) (*'a exp*) (*'a exp*) (*if* '(-') -/ *else* - [*80*,*79*,*79*]*70*)
| *While* (*'a exp*) (*'a exp*) (*while* '(-') - [*80*,*79*]*70*)
| *throw* (*'a exp*)
| *TryCatch* (*'a exp*) *cname 'a* (*'a exp*) (*try* -/ *catch*'(- -') - [*0*,*99*,*80*,*79*] *70*)

type-synonym
expr = *vname exp* — Jinja expression
type-synonym
J-mb = *vname list* × *expr* — Jinja method body: parameter names and expression
type-synonym
J-prog = *J-mb prog* — Jinja program

The semantics of binary operators:

fun *binop* :: *bop* × *val* × *val* ⇒ *val option* **where**
$binop(Eq,v_1,v_2) = Some(Bool\ (v_1 = v_2))$
| $binop(Add,Intg\ i_1,Intg\ i_2) = Some(Intg(i_1+i_2))$
| $binop(bop,v_1,v_2) = None$

lemma [*simp*]:
$(binop(Add,v_1,v_2) = Some\ v) = (\exists i_1\ i_2.\ v_1 = Intg\ i_1 \wedge v_2 = Intg\ i_2 \wedge v = Intg(i_1+i_2))$⟨*proof*⟩

2.8.1 Syntactic sugar

abbreviation (*input*)
InitBlock:: *'a* ⇒ *ty* ⇒ *'a exp* ⇒ *'a exp* ⇒ *'a exp* ((*1*'{-:- := -;/ -})) **where**
InitBlock V T e1 e2 == {*V*:*T*; *V* := *e1*;; *e2*}

abbreviation *unit* **where** *unit* == *Val Unit*
abbreviation *null* **where** *null* == *Val Null*
abbreviation *addr a* == *Val*(*Addr a*)
abbreviation *true* == *Val*(*Bool True*)
abbreviation *false* == *Val*(*Bool False*)

abbreviation

$Throw :: addr \Rightarrow {}'a\ exp$ **where**
$Throw\ a == throw(Val(Addr\ a))$

abbreviation
$THROW :: cname \Rightarrow {}'a\ exp$ **where**
$THROW\ xc == Throw(addr\text{-}of\text{-}sys\text{-}xcpt\ xc)$

2.8.2 Free Variables

primrec $fv :: expr \Rightarrow vname\ set$ **and** $fvs :: expr\ list \Rightarrow vname\ set$ **where**
$fv(new\ C) = \{\}$
$|\ fv(Cast\ C\ e) = fv\ e$
$|\ fv(Val\ v) = \{\}$
$|\ fv(e_1 \ll bop \gg e_2) = fv\ e_1 \cup fv\ e_2$
$|\ fv(Var\ V) = \{V\}$
$|\ fv(LAss\ V\ e) = \{V\} \cup fv\ e$
$|\ fv(e{\cdot}F\{D\}) = fv\ e$
$|\ fv(e_1{\cdot}F\{D\}{:=}e_2) = fv\ e_1 \cup fv\ e_2$
$|\ fv(e{\cdot}M(es)) = fv\ e \cup fvs\ es$
$|\ fv(\{V{:}T;\ e\}) = fv\ e - \{V\}$
$|\ fv(e_1;;e_2) = fv\ e_1 \cup fv\ e_2$
$|\ fv(if\ (b)\ e_1\ else\ e_2) = fv\ b \cup fv\ e_1 \cup fv\ e_2$
$|\ fv(while\ (b)\ e) = fv\ b \cup fv\ e$
$|\ fv(throw\ e) = fv\ e$
$|\ fv(try\ e_1\ catch(C\ V)\ e_2) = fv\ e_1 \cup (fv\ e_2 - \{V\})$
$|\ fvs([]) = \{\}$
$|\ fvs(e\#es) = fv\ e \cup fvs\ es$

lemma $[simp]$: $fvs(es_1\ @\ es_2) = fvs\ es_1 \cup fvs\ es_2 \langle proof \rangle$
lemma $[simp]$: $fvs(map\ Val\ vs) = \{\} \langle proof \rangle$
end

2.9 Program State

theory *State* **imports** *../Common/Exceptions* **begin**

type-synonym
$locals = vname \rightharpoonup val$ — local vars, incl. params and "this"
type-synonym
$state = heap \times locals$

definition $hp :: state \Rightarrow heap$
where
$hp \equiv fst$
definition $lcl :: state \Rightarrow locals$
where
$lcl \equiv snd$
end

2.10 Big Step Semantics

theory *BigStep* **imports** *Expr State* **begin**

inductive
$eval :: J\text{-}prog \Rightarrow expr \Rightarrow state \Rightarrow expr \Rightarrow state \Rightarrow bool$
$(\text{-} \vdash ((1\langle\text{-},/\text{-}\rangle) \Rightarrow/ (1\langle\text{-},/\text{-}\rangle)) [51,0,0,0,0] 81)$
and $evals :: J\text{-}prog \Rightarrow expr\ list \Rightarrow state \Rightarrow expr\ list \Rightarrow state \Rightarrow bool$
$(\text{-} \vdash ((1\langle\text{-},/\text{-}\rangle) [\Rightarrow]/ (1\langle\text{-},/\text{-}\rangle)) [51,0,0,0,0] 81)$
for $P :: J\text{-}prog$
where

New:
$[\![new\text{-}Addr\ h = Some\ a;\ P \vdash C\ has\text{-}fields\ FDTs;\ h' = h(a \mapsto (C, init\text{-}fields\ FDTs))]\!]$
$\Longrightarrow P \vdash \langle new\ C,(h,l)\rangle \Rightarrow \langle addr\ a,(h',l)\rangle$

| *NewFail*:
$new\text{-}Addr\ h = None \Longrightarrow$
$P \vdash \langle new\ C,\ (h,l)\rangle \Rightarrow \langle THROW\ OutOfMemory,(h,l)\rangle$

| *Cast*:
$[\![P \vdash \langle e,s_0\rangle \Rightarrow \langle addr\ a,(h,l)\rangle;\ h\ a = Some(D,fs);\ P \vdash D \preceq^* C]\!]$
$\Longrightarrow P \vdash \langle Cast\ C\ e,s_0\rangle \Rightarrow \langle addr\ a,(h,l)\rangle$

| *CastNull*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle null,s_1\rangle \Longrightarrow$
$P \vdash \langle Cast\ C\ e,s_0\rangle \Rightarrow \langle null,s_1\rangle$

| *CastFail*:
$[\![P \vdash \langle e,s_0\rangle \Rightarrow \langle addr\ a,(h,l)\rangle;\ h\ a = Some(D,fs);\ \neg\ P \vdash D \preceq^* C]\!]$
$\Longrightarrow P \vdash \langle Cast\ C\ e,s_0\rangle \Rightarrow \langle THROW\ ClassCast,(h,l)\rangle$

| *CastThrow*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash \langle Cast\ C\ e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$

| *Val*:
$P \vdash \langle Val\ v,s\rangle \Rightarrow \langle Val\ v,s\rangle$

| *BinOp*:
$[\![P \vdash \langle e_1,s_0\rangle \Rightarrow \langle Val\ v_1,s_1\rangle;\ P \vdash \langle e_2,s_1\rangle \Rightarrow \langle Val\ v_2,s_2\rangle;\ binop(bop,v_1,v_2) = Some\ v]\!]$
$\Longrightarrow P \vdash \langle e_1 \ll bop \gg e_2,s_0\rangle \Rightarrow \langle Val\ v,s_2\rangle$

| *BinOpThrow1*:
$P \vdash \langle e_1,s_0\rangle \Rightarrow \langle throw\ e,s_1\rangle \Longrightarrow$
$P \vdash \langle e_1 \ll bop \gg e_2,\ s_0\rangle \Rightarrow \langle throw\ e,s_1\rangle$

| *BinOpThrow2*:
$[\![P \vdash \langle e_1,s_0\rangle \Rightarrow \langle Val\ v_1,s_1\rangle;\ P \vdash \langle e_2,s_1\rangle \Rightarrow \langle throw\ e,s_2\rangle]\!]$
$\Longrightarrow P \vdash \langle e_1 \ll bop \gg e_2,s_0\rangle \Rightarrow \langle throw\ e,s_2\rangle$

| *Var*:
$l\ V = Some\ v \Longrightarrow$
$P \vdash \langle Var\ V,(h,l)\rangle \Rightarrow \langle Val\ v,(h,l)\rangle$

| *LAss*:
$[\![P \vdash \langle e,s_0\rangle \Rightarrow \langle Val\ v,(h,l)\rangle;\ l' = l(V\mapsto v)\ ]\!]$
$\Longrightarrow P \vdash \langle V{:=}e,s_0\rangle \Rightarrow \langle unit,(h,l')\rangle$

| *LAssThrow*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash \langle V{:=}e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$

| *FAcc*:
$[\![P \vdash \langle e,s_0\rangle \Rightarrow \langle addr\ a,(h,l)\rangle;\ h\ a = Some(C,fs);\ fs(F,D) = Some\ v\ ]\!]$
$\Longrightarrow P \vdash \langle e{\cdot}F\{D\},s_0\rangle \Rightarrow \langle Val\ v,(h,l)\rangle$

| *FAccNull*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle null,s_1\rangle \Longrightarrow$
$P \vdash \langle e{\cdot}F\{D\},s_0\rangle \Rightarrow \langle THROW\ NullPointer,s_1\rangle$

| *FAccThrow*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash \langle e{\cdot}F\{D\},s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$

| *FAss*:
$[\![P \vdash \langle e_1,s_0\rangle \Rightarrow \langle addr\ a,s_1\rangle;\ P \vdash \langle e_2,s_1\rangle \Rightarrow \langle Val\ v,(h_2,l_2)\rangle;$
$h_2\ a = Some(C,fs);\ fs' = fs((F,D)\mapsto v);\ h_2' = h_2(a\mapsto(C,fs'))\ ]\!]$
$\Longrightarrow P \vdash \langle e_1{\cdot}F\{D\}{:=}e_2,s_0\rangle \Rightarrow \langle unit,(h_2',l_2)\rangle$

| *FAssNull*:
$[\![P \vdash \langle e_1,s_0\rangle \Rightarrow \langle null,s_1\rangle;\ \ P \vdash \langle e_2,s_1\rangle \Rightarrow \langle Val\ v,s_2\rangle\ ]\!] \Longrightarrow$
$P \vdash \langle e_1{\cdot}F\{D\}{:=}e_2,s_0\rangle \Rightarrow \langle THROW\ NullPointer,s_2\rangle$

| *FAssThrow1*:
$P \vdash \langle e_1,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash \langle e_1{\cdot}F\{D\}{:=}e_2,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$

| *FAssThrow2*:
$[\![P \vdash \langle e_1,s_0\rangle \Rightarrow \langle Val\ v,s_1\rangle;\ P \vdash \langle e_2,s_1\rangle \Rightarrow \langle throw\ e',s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle e_1{\cdot}F\{D\}{:=}e_2,s_0\rangle \Rightarrow \langle throw\ e',s_2\rangle$

| *CallObjThrow*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash \langle e{\cdot}M(ps),s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$

| *CallParamsThrow*:
$[\![P \vdash \langle e,s_0\rangle \Rightarrow \langle Val\ v,s_1\rangle;\ P \vdash \langle es,s_1\rangle\ [\Rightarrow]\ \langle map\ Val\ vs\ @\ throw\ ex\ \#\ es',s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle e{\cdot}M(es),s_0\rangle \Rightarrow \langle throw\ ex,s_2\rangle$

| *CallNull*:
$[\![P \vdash \langle e,s_0\rangle \Rightarrow \langle null,s_1\rangle;\ \ P \vdash \langle ps,s_1\rangle\ [\Rightarrow]\ \langle map\ Val\ vs,s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle e{\cdot}M(ps),s_0\rangle \Rightarrow \langle THROW\ NullPointer,s_2\rangle$

| *Call*:
$[\![P \vdash \langle e,s_0\rangle \Rightarrow \langle addr\ a,s_1\rangle;\ \ P \vdash \langle ps,s_1\rangle\ [\Rightarrow]\ \langle map\ Val\ vs,(h_2,l_2)\rangle;$
$h_2\ a = Some(C,fs);\ \ P \vdash C\ sees\ M{:}Ts{\rightarrow}T = (pns,body)\ in\ D;$
$length\ vs = length\ pns;\ \ l_2' = [this\mapsto Addr\ a,\ pns[\mapsto]vs];$

$P \vdash \langle body,(h_2,l_2{}') \rangle \Rightarrow \langle e',(h_3,l_3) \rangle$ $]\!]$
$\Longrightarrow P \vdash \langle e{\cdot}M(ps),s_0 \rangle \Rightarrow \langle e',(h_3,l_2) \rangle$

| *Block*:
$P \vdash \langle e_0,(h_0,l_0(V{:=}None)) \rangle \Rightarrow \langle e_1,(h_1,l_1) \rangle \Longrightarrow$
$P \vdash \langle \{V{:}T;\ e_0\},(h_0,l_0) \rangle \Rightarrow \langle e_1,(h_1,l_1(V{:=}l_0\ V)) \rangle$

| *Seq*:
$[\![\ P \vdash \langle e_0,s_0 \rangle \Rightarrow \langle Val\ v,s_1 \rangle;\ P \vdash \langle e_1,s_1 \rangle \Rightarrow \langle e_2,s_2 \rangle\ ]\!]$
$\Longrightarrow P \vdash \langle e_0;;e_1,s_0 \rangle \Rightarrow \langle e_2,s_2 \rangle$

| *SeqThrow*:
$P \vdash \langle e_0,s_0 \rangle \Rightarrow \langle throw\ e,s_1 \rangle \Longrightarrow$
$P \vdash \langle e_0;;e_1,s_0 \rangle {\Rightarrow} \langle throw\ e,s_1 \rangle$

| *CondT*:
$[\![\ P \vdash \langle e,s_0 \rangle \Rightarrow \langle true,s_1 \rangle;\ P \vdash \langle e_1,s_1 \rangle \Rightarrow \langle e',s_2 \rangle\ ]\!]$
$\Longrightarrow P \vdash \langle if\ (e)\ e_1\ else\ e_2,s_0 \rangle \Rightarrow \langle e',s_2 \rangle$

| *CondF*:
$[\![\ P \vdash \langle e,s_0 \rangle \Rightarrow \langle false,s_1 \rangle;\ P \vdash \langle e_2,s_1 \rangle \Rightarrow \langle e',s_2 \rangle\ ]\!]$
$\Longrightarrow P \vdash \langle if\ (e)\ e_1\ else\ e_2,s_0 \rangle \Rightarrow \langle e',s_2 \rangle$

| *CondThrow*:
$P \vdash \langle e,s_0 \rangle \Rightarrow \langle throw\ e',s_1 \rangle \Longrightarrow$
$P \vdash \langle if\ (e)\ e_1\ else\ e_2,\ s_0 \rangle \Rightarrow \langle throw\ e',s_1 \rangle$

| *WhileF*:
$P \vdash \langle e,s_0 \rangle \Rightarrow \langle false,s_1 \rangle \Longrightarrow$
$P \vdash \langle while\ (e)\ c,s_0 \rangle \Rightarrow \langle unit,s_1 \rangle$

| *WhileT*:
$[\![\ P \vdash \langle e,s_0 \rangle \Rightarrow \langle true,s_1 \rangle;\ P \vdash \langle c,s_1 \rangle \Rightarrow \langle Val\ v_1,s_2 \rangle;\ P \vdash \langle while\ (e)\ c,s_2 \rangle \Rightarrow \langle e_3,s_3 \rangle\ ]\!]$
$\Longrightarrow P \vdash \langle while\ (e)\ c,s_0 \rangle \Rightarrow \langle e_3,s_3 \rangle$

| *WhileCondThrow*:
$P \vdash \langle e,s_0 \rangle \Rightarrow \langle\ throw\ e',s_1 \rangle \Longrightarrow$
$P \vdash \langle while\ (e)\ c,s_0 \rangle \Rightarrow \langle throw\ e',s_1 \rangle$

| *WhileBodyThrow*:
$[\![\ P \vdash \langle e,s_0 \rangle \Rightarrow \langle true,s_1 \rangle;\ P \vdash \langle c,s_1 \rangle \Rightarrow \langle throw\ e',s_2 \rangle]\!]$
$\Longrightarrow P \vdash \langle while\ (e)\ c,s_0 \rangle \Rightarrow \langle throw\ e',s_2 \rangle$

| *Throw*:
$P \vdash \langle e,s_0 \rangle \Rightarrow \langle addr\ a,s_1 \rangle \Longrightarrow$
$P \vdash \langle throw\ e,s_0 \rangle \Rightarrow \langle Throw\ a,s_1 \rangle$

| *ThrowNull*:
$P \vdash \langle e,s_0 \rangle \Rightarrow \langle null,s_1 \rangle \Longrightarrow$
$P \vdash \langle throw\ e,s_0 \rangle \Rightarrow \langle THROW\ NullPointer,s_1 \rangle$

| *ThrowThrow*:
$P \vdash \langle e,s_0 \rangle \Rightarrow \langle throw\ e',s_1 \rangle \Longrightarrow$
$P \vdash \langle throw\ e,s_0 \rangle \Rightarrow \langle throw\ e',s_1 \rangle$

| *Try*:
$P \vdash \langle e_1,s_0\rangle \Rightarrow \langle Val\ v_1,s_1\rangle \Longrightarrow$
$P \vdash \langle try\ e_1\ catch(C\ V)\ e_2,s_0\rangle \Rightarrow \langle Val\ v_1,s_1\rangle$

| *TryCatch*:
$[\![\ P \vdash \langle e_1,s_0\rangle \Rightarrow \langle Throw\ a,(h_1,l_1)\rangle;\ \ h_1\ a = Some(D,fs);\ \ P \vdash D \preceq^* C;$
$P \vdash \langle e_2,(h_1,l_1(V \mapsto Addr\ a))\rangle \Rightarrow \langle e_2{}',(h_2,l_2)\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle try\ e_1\ catch(C\ V)\ e_2,s_0\rangle \Rightarrow \langle e_2{}',(h_2,l_2(V{:=}l_1\ V))\rangle$

| *TryThrow*:
$[\![\ P \vdash \langle e_1,s_0\rangle \Rightarrow \langle Throw\ a,(h_1,l_1)\rangle;\ \ h_1\ a = Some(D,fs);\ \ \neg\ P \vdash D \preceq^* C\ ]\!]$
$\Longrightarrow P \vdash \langle try\ e_1\ catch(C\ V)\ e_2,s_0\rangle \Rightarrow \langle Throw\ a,(h_1,l_1)\rangle$

| *Nil*:
$P \vdash \langle [],s\rangle\ [\Rightarrow]\ \langle [],s\rangle$

| *Cons*:
$[\![\ P \vdash \langle e,s_0\rangle \Rightarrow \langle Val\ v,s_1\rangle;\ P \vdash \langle es,s_1\rangle\ [\Rightarrow]\ \langle es',s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle e\#es,s_0\rangle\ [\Rightarrow]\ \langle Val\ v\ \#\ es',s_2\rangle$

| *ConsThrow*:
$P \vdash \langle e,\ s_0\rangle \Rightarrow \langle throw\ e',\ s_1\rangle \Longrightarrow$
$P \vdash \langle e\#es,\ s_0\rangle\ [\Rightarrow]\ \langle throw\ e'\ \#\ es,\ s_1\rangle$

2.10.1 Final expressions

definition *final* :: $'a\ exp \Rightarrow bool$
where
$final\ e\ \equiv\ (\exists\, v.\ e = Val\ v) \vee (\exists\, a.\ e = Throw\ a)$

definition *finals*:: $'a\ exp\ list \Rightarrow bool$
where
$finals\ es\ \equiv\ (\exists\, vs.\ es = map\ Val\ vs) \vee (\exists\, vs\ a\ es'.\ es = map\ Val\ vs\ @\ Throw\ a\ \#\ es')$

lemma [*simp*]: $final(Val\ v)\langle proof\rangle$
lemma [*simp*]: $final(throw\ e) = (\exists\, a.\ e = addr\ a)\langle proof\rangle$
lemma *finalE*: $[\![\ final\ e;\ \ \bigwedge v.\ e = Val\ v \Longrightarrow R;\ \ \bigwedge a.\ e = Throw\ a \Longrightarrow R\ ]\!] \Longrightarrow R\langle proof\rangle$
lemma [*iff*]: $finals\ []\langle proof\rangle$
lemma [*iff*]: $finals\ (Val\ v\ \#\ es) = finals\ es\langle proof\rangle$
lemma *finals-app-map*[*iff*]: $finals\ (map\ Val\ vs\ @\ es) = finals\ es\langle proof\rangle$
lemma [*iff*]: $finals\ (map\ Val\ vs)\langle proof\rangle$
lemma [*iff*]: $finals\ (throw\ e\ \#\ es) = (\exists\, a.\ e = addr\ a)\langle proof\rangle$
lemma *not-finals-ConsI*: $\neg\ final\ e \Longrightarrow \neg\ finals(e\#es)$
$\langle proof\rangle$

lemma *eval-final*: $P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle \Longrightarrow final\ e'$
and *evals-final*: $P \vdash \langle es,s\rangle\ [\Rightarrow]\ \langle es',s'\rangle \Longrightarrow finals\ es'\langle proof\rangle$

lemma *eval-lcl-incr*: $P \vdash \langle e,(h_0,l_0)\rangle \Rightarrow \langle e',(h_1,l_1)\rangle \Longrightarrow dom\ l_0 \subseteq dom\ l_1$
and *evals-lcl-incr*: $P \vdash \langle es,(h_0,l_0)\rangle\ [\Rightarrow]\ \langle es',(h_1,l_1)\rangle \Longrightarrow dom\ l_0 \subseteq dom\ l_1\langle proof\rangle$

Only used later, in the small to big translation, but is already a good sanity check:

lemma *eval-finalId*: *final* $e \Longrightarrow P \vdash \langle e,s \rangle \Rightarrow \langle e,s \rangle\langle proof \rangle$

lemma *eval-finalsId*:
assumes *finals*: *finals es* **shows** $P \vdash \langle es,s \rangle \ [\Rightarrow] \ \langle es,s \rangle\langle proof \rangle$

theorem *eval-hext*: $P \vdash \langle e,(h,l) \rangle \Rightarrow \langle e',(h',l') \rangle \Longrightarrow h \trianglelefteq h'$
and *evals-hext*: $P \vdash \langle es,(h,l) \rangle \ [\Rightarrow] \ \langle es',(h',l') \rangle \Longrightarrow h \trianglelefteq h'\langle proof \rangle$

end

2.11 Small Step Semantics

theory *SmallStep*
imports *Expr State*
begin

fun *blocks* :: *vname list* ∗ *ty list* ∗ *val list* ∗ *expr* ⇒ *expr*
where
 blocks(*V*#*Vs*, *T*#*Ts*, *v*#*vs*, *e*) = {*V*:*T* := *Val v*; *blocks*(*Vs*,*Ts*,*vs*,*e*)}
|*blocks*([],[],[],*e*) = *e*

lemmas *blocks-induct* = *blocks.induct*[*split-format* (*complete*)]

lemma [*simp*]:
 ⟦ *size vs* = *size Vs*; *size Ts* = *size Vs* ⟧ ⟹ *fv*(*blocks*(*Vs*,*Ts*,*vs*,*e*)) = *fv e* − *set Vs*⟨*proof*⟩

definition *assigned* :: *vname* ⇒ *expr* ⇒ *bool*
where
 assigned V e ≡ ∃ *v e*′. *e* = (*V* := *Val v*;; *e*′)

inductive-set
 red :: *J-prog* ⇒ ((*expr* × *state*) × (*expr* × *state*)) *set*
 and *reds* :: *J-prog* ⇒ ((*expr list* × *state*) × (*expr list* × *state*)) *set*
 and *red*′ :: *J-prog* ⇒ *expr* ⇒ *state* ⇒ *expr* ⇒ *state* ⇒ *bool*
  (- ⊢ ((*1*⟨-,/-⟩) →/ (*1*⟨-,/-⟩)) [*51*,*0*,*0*,*0*,*0*] *81*)
 and *reds*′ :: *J-prog* ⇒ *expr list* ⇒ *state* ⇒ *expr list* ⇒ *state* ⇒ *bool*
  (- ⊢ ((*1*⟨-,/-⟩) [→]/ (*1*⟨-,/-⟩)) [*51*,*0*,*0*,*0*,*0*] *81*)
 for *P* :: *J-prog*
where

 P ⊢ ⟨*e*,*s*⟩ → ⟨*e*′,*s*′⟩ ≡ ((*e*,*s*), *e*′,*s*′) ∈ *red P*
| *P* ⊢ ⟨*es*,*s*⟩ [→] ⟨*es*′,*s*′⟩ ≡ ((*es*,*s*), *es*′,*s*′) ∈ *reds P*

| *RedNew*:
 ⟦ *new-Addr h* = *Some a*; *P* ⊢ *C has-fields FDTs*; *h*′ = *h*(*a*↦(*C*,*init-fields FDTs*)) ⟧
 ⟹ *P* ⊢ ⟨*new C*, (*h*,*l*)⟩ → ⟨*addr a*, (*h*′,*l*)⟩

| *RedNewFail*:
 new-Addr h = *None* ⟹
 P ⊢ ⟨*new C*, (*h*,*l*)⟩ → ⟨*THROW OutOfMemory*, (*h*,*l*)⟩

| *CastRed*:
 P ⊢ ⟨*e*,*s*⟩ → ⟨*e*′,*s*′⟩ ⟹
 P ⊢ ⟨*Cast C e*, *s*⟩ → ⟨*Cast C e*′, *s*′⟩

| *RedCastNull*:
 P ⊢ ⟨*Cast C null*, *s*⟩ → ⟨*null*,*s*⟩

| *RedCast*:
⟦ *hp s a* = *Some*(*D*,*fs*); *P* ⊢ *D* ⪯∗ *C* ⟧
 ⟹ *P* ⊢ ⟨*Cast C* (*addr a*), *s*⟩ → ⟨*addr a*, *s*⟩

| *RedCastFail*:
 ⟦ *hp s a* = *Some*(*D*,*fs*); ¬ *P* ⊢ *D* ⪯∗ *C* ⟧

$\Longrightarrow P \vdash \langle Cast\ C\ (addr\ a),\ s\rangle \rightarrow \langle THROW\ ClassCast,\ s\rangle$

| *BinOpRed1*:
$P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle e \ll bop \gg e_2,\ s\rangle \rightarrow \langle e' \ll bop \gg e_2,\ s'\rangle$

| *BinOpRed2*:
$P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle (Val\ v_1) \ll bop \gg e,\ s\rangle \rightarrow \langle (Val\ v_1) \ll bop \gg e',\ s'\rangle$

| *RedBinOp*:
$binop(bop,v_1,v_2) = Some\ v \Longrightarrow$
$P \vdash \langle (Val\ v_1) \ll bop \gg (Val\ v_2),\ s\rangle \rightarrow \langle Val\ v,s\rangle$

| *RedVar*:
$lcl\ s\ V = Some\ v \Longrightarrow$
$P \vdash \langle Var\ V,s\rangle \rightarrow \langle Val\ v,s\rangle$

| *LAssRed*:
$P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle V{:=}e,s\rangle \rightarrow \langle V{:=}e',s'\rangle$

| *RedLAss*:
$P \vdash \langle V{:=}(Val\ v),\ (h,l)\rangle \rightarrow \langle unit,\ (h,l(V \mapsto v))\rangle$

| *FAccRed*:
$P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle e{\cdot}F\{D\},\ s\rangle \rightarrow \langle e'{\cdot}F\{D\},\ s'\rangle$

| *RedFAcc*:
$[\![\ hp\ s\ a = Some(C,fs);\ fs(F,D) = Some\ v\ ]\!]$
$\Longrightarrow P \vdash \langle (addr\ a){\cdot}F\{D\},\ s\rangle \rightarrow \langle Val\ v,s\rangle$

| *RedFAccNull*:
$P \vdash \langle null{\cdot}F\{D\},\ s\rangle \rightarrow \langle THROW\ NullPointer,\ s\rangle$

| *FAssRed1*:
$P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle e{\cdot}F\{D\}{:=}e_2,\ s\rangle \rightarrow \langle e'{\cdot}F\{D\}{:=}e_2,\ s'\rangle$

| *FAssRed2*:
$P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle Val\ v{\cdot}F\{D\}{:=}e,\ s\rangle \rightarrow \langle Val\ v{\cdot}F\{D\}{:=}e',\ s'\rangle$

| *RedFAss*:
$h\ a = Some(C,fs) \Longrightarrow$
$P \vdash \langle (addr\ a){\cdot}F\{D\}{:=}(Val\ v),\ (h,l)\rangle \rightarrow \langle unit,\ (h(a \mapsto (C,fs((F,D) \mapsto v))),l)\rangle$

| *RedFAssNull*:
$P \vdash \langle null{\cdot}F\{D\}{:=}Val\ v,\ s\rangle \rightarrow \langle THROW\ NullPointer,\ s\rangle$

| *CallObj*:
$P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle e{\cdot}M(es),s\rangle \rightarrow \langle e'{\cdot}M(es),s'\rangle$

| *CallParams*:
$P \vdash \langle es,s\rangle\ [\rightarrow]\ \langle es',s'\rangle \Longrightarrow$
$P \vdash \langle (Val\ v)\cdot M(es),s\rangle \rightarrow \langle (Val\ v)\cdot M(es'),s'\rangle$

| *RedCall*:
$[\![\ hp\ s\ a = Some(C,fs);\ P \vdash C\ sees\ M{:}Ts{\rightarrow}T = (pns,body)\ in\ D;\ size\ vs = size\ pns;\ size\ Ts = size\ pns\ ]\!]$
$\Longrightarrow P \vdash \langle (addr\ a)\cdot M(map\ Val\ vs),\ s\rangle \rightarrow \langle blocks(this\#pns,\ Class\ D\#Ts,\ Addr\ a\#vs,\ body),\ s\rangle$

| *RedCallNull*:
$P \vdash \langle null\cdot M(map\ Val\ vs),s\rangle \rightarrow \langle THROW\ NullPointer,s\rangle$

| *BlockRedNone*:
$[\![\ P \vdash \langle e,\ (h,l(V{:=}None))\rangle \rightarrow \langle e',\ (h',l')\rangle;\ l'\ V = None;\ \neg\ assigned\ V\ e\ ]\!]$
$\Longrightarrow P \vdash \langle \{V{:}T;\ e\},\ (h,l)\rangle \rightarrow \langle \{V{:}T;\ e'\},\ (h',l'(V := l\ V))\rangle$

| *BlockRedSome*:
$[\![\ P \vdash \langle e,\ (h,l(V{:=}None))\rangle \rightarrow \langle e',\ (h',l')\rangle;\ l'\ V = Some\ v;\neg\ assigned\ V\ e\ ]\!]$
$\Longrightarrow P \vdash \langle \{V{:}T;\ e\},\ (h,l)\rangle \rightarrow \langle \{V{:}T := Val\ v;\ e'\},\ (h',l'(V := l\ V))\rangle$

| *InitBlockRed*:
$[\![\ P \vdash \langle e,\ (h,l(V{\mapsto}v))\rangle \rightarrow \langle e',\ (h',l')\rangle;\ l'\ V = Some\ v'\ ]\!]$
$\Longrightarrow P \vdash \langle \{V{:}T := Val\ v;\ e\},\ (h,l)\rangle \rightarrow \langle \{V{:}T := Val\ v';\ e'\},\ (h',l'(V := l\ V))\rangle$

| *RedBlock*:
$P \vdash \langle \{V{:}T;\ Val\ u\},\ s\rangle \rightarrow \langle Val\ u,\ s\rangle$

| *RedInitBlock*:
$P \vdash \langle \{V{:}T := Val\ v;\ Val\ u\},\ s\rangle \rightarrow \langle Val\ u,\ s\rangle$

| *SeqRed*:
$P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle e;;e_2,\ s\rangle \rightarrow \langle e';;e_2,\ s'\rangle$

| *RedSeq*:
$P \vdash \langle (Val\ v);;e_2,\ s\rangle \rightarrow \langle e_2,\ s\rangle$

| *CondRed*:
$P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle if\ (e)\ e_1\ else\ e_2,\ s\rangle \rightarrow \langle if\ (e')\ e_1\ else\ e_2,\ s'\rangle$

| *RedCondT*:
$P \vdash \langle if\ (true)\ e_1\ else\ e_2,\ s\rangle \rightarrow \langle e_1,\ s\rangle$

| *RedCondF*:
$P \vdash \langle if\ (false)\ e_1\ else\ e_2,\ s\rangle \rightarrow \langle e_2,\ s\rangle$

| *RedWhile*:
$P \vdash \langle while(b)\ c,\ s\rangle \rightarrow \langle if(b)\ (c;;while(b)\ c)\ else\ unit,\ s\rangle$

| *ThrowRed*:
$P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle throw\ e,\ s\rangle \rightarrow \langle throw\ e',\ s'\rangle$

| *RedThrowNull*:
  $P \vdash \langle \mathit{throw\ null},\ s\rangle \rightarrow \langle \mathit{THROW\ NullPointer},\ s\rangle$

| *TryRed*:
  $P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle \Longrightarrow$
  $P \vdash \langle \mathit{try}\ e\ \mathit{catch}(C\ V)\ e_2,\ s\rangle \rightarrow \langle \mathit{try}\ e'\ \mathit{catch}(C\ V)\ e_2,\ s'\rangle$

| *RedTry*:
  $P \vdash \langle \mathit{try}\ (\mathit{Val}\ v)\ \mathit{catch}(C\ V)\ e_2,\ s\rangle \rightarrow \langle \mathit{Val}\ v,\ s\rangle$

| *RedTryCatch*:
  $[\![\ \mathit{hp}\ s\ a = \mathit{Some}(D,\mathit{fs});\ P \vdash D \preceq^* C\ ]\!]$
  $\Longrightarrow P \vdash \langle \mathit{try}\ (\mathit{Throw}\ a)\ \mathit{catch}(C\ V)\ e_2,\ s\rangle \rightarrow \langle \{V{:}\mathit{Class}\ C := \mathit{addr}\ a;\ e_2\},\ s\rangle$

| *RedTryFail*:
  $[\![\ \mathit{hp}\ s\ a = \mathit{Some}(D,\mathit{fs});\ \neg\ P \vdash D \preceq^* C\ ]\!]$
  $\Longrightarrow P \vdash \langle \mathit{try}\ (\mathit{Throw}\ a)\ \mathit{catch}(C\ V)\ e_2,\ s\rangle \rightarrow \langle \mathit{Throw}\ a,\ s\rangle$

| *ListRed1*:
  $P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle \Longrightarrow$
  $P \vdash \langle e\#es,s\rangle\ [\rightarrow]\ \langle e'\#es,s'\rangle$

| *ListRed2*:
  $P \vdash \langle es,s\rangle\ [\rightarrow]\ \langle es',s'\rangle \Longrightarrow$
  $P \vdash \langle \mathit{Val}\ v\ \#\ es,s\rangle\ [\rightarrow]\ \langle \mathit{Val}\ v\ \#\ es',s'\rangle$

— Exception propagation

| *CastThrow*: $P \vdash \langle \mathit{Cast}\ C\ (\mathit{throw}\ e),\ s\rangle \rightarrow \langle \mathit{throw}\ e,\ s\rangle$
| *BinOpThrow1*: $P \vdash \langle (\mathit{throw}\ e)\ \ll\!\mathit{bop}\!\gg\ e_2,\ s\rangle \rightarrow \langle \mathit{throw}\ e,\ s\rangle$
| *BinOpThrow2*: $P \vdash \langle (\mathit{Val}\ v_1)\ \ll\!\mathit{bop}\!\gg\ (\mathit{throw}\ e),\ s\rangle \rightarrow \langle \mathit{throw}\ e,\ s\rangle$
| *LAssThrow*: $P \vdash \langle V{:=}(\mathit{throw}\ e),\ s\rangle \rightarrow \langle \mathit{throw}\ e,\ s\rangle$
| *FAccThrow*: $P \vdash \langle (\mathit{throw}\ e){\cdot}F\{D\},\ s\rangle \rightarrow \langle \mathit{throw}\ e,\ s\rangle$
| *FAssThrow1*: $P \vdash \langle (\mathit{throw}\ e){\cdot}F\{D\}{:=}e_2,\ s\rangle \rightarrow \langle \mathit{throw}\ e,s\rangle$
| *FAssThrow2*: $P \vdash \langle \mathit{Val}\ v{\cdot}F\{D\}{:=}(\mathit{throw}\ e),\ s\rangle \rightarrow \langle \mathit{throw}\ e,\ s\rangle$
| *CallThrowObj*: $P \vdash \langle (\mathit{throw}\ e){\cdot}M(es),\ s\rangle \rightarrow \langle \mathit{throw}\ e,\ s\rangle$
| *CallThrowParams*: $[\![\ es = \mathit{map}\ \mathit{Val}\ vs\ @\ \mathit{throw}\ e\ \#\ es'\ ]\!] \Longrightarrow P \vdash \langle (\mathit{Val}\ v){\cdot}M(es),\ s\rangle \rightarrow \langle \mathit{throw}\ e,\ s\rangle$
| *BlockThrow*: $P \vdash \langle \{V{:}T;\ \mathit{Throw}\ a\},\ s\rangle \rightarrow \langle \mathit{Throw}\ a,\ s\rangle$
| *InitBlockThrow*: $P \vdash \langle \{V{:}T := \mathit{Val}\ v;\ \mathit{Throw}\ a\},\ s\rangle \rightarrow \langle \mathit{Throw}\ a,\ s\rangle$
| *SeqThrow*: $P \vdash \langle (\mathit{throw}\ e);;e_2,\ s\rangle \rightarrow \langle \mathit{throw}\ e,\ s\rangle$
| *CondThrow*: $P \vdash \langle \mathit{if}\ (\mathit{throw}\ e)\ e_1\ \mathit{else}\ e_2,\ s\rangle \rightarrow \langle \mathit{throw}\ e,\ s\rangle$
| *ThrowThrow*: $P \vdash \langle \mathit{throw}(\mathit{throw}\ e),\ s\rangle \rightarrow \langle \mathit{throw}\ e,\ s\rangle$

2.11.1 The reflexive transitive closure

abbreviation
  Step :: $\mathit{J\text{-}prog} \Rightarrow \mathit{expr} \Rightarrow \mathit{state} \Rightarrow \mathit{expr} \Rightarrow \mathit{state} \Rightarrow \mathit{bool}$
    $(\text{-} \vdash ((1\langle\text{-},/\text{-}\rangle) \rightarrow\!*/\ (1\langle\text{-},/\text{-}\rangle))\ [51,0,0,0,0]\ 81)$
  where $P \vdash \langle e,s\rangle \rightarrow\!*\ \langle e',s'\rangle \equiv ((e,s),\ e',s') \in (\mathit{red}\ P)^*$

abbreviation
  Steps :: $\mathit{J\text{-}prog} \Rightarrow \mathit{expr\ list} \Rightarrow \mathit{state} \Rightarrow \mathit{expr\ list} \Rightarrow \mathit{state} \Rightarrow \mathit{bool}$
    $(\text{-} \vdash ((1\langle\text{-},/\text{-}\rangle)\ [\rightarrow]\!*/\ (1\langle\text{-},/\text{-}\rangle))\ [51,0,0,0,0]\ 81)$

where $P \vdash \langle es,s\rangle\ [\to]{*}\ \langle es',s'\rangle \equiv ((es,s),\ es',s') \in (reds\ P)^*$

lemma *converse-rtrancl-induct-red*[*consumes 1*]:
assumes $P \vdash \langle e,(h,l)\rangle \to{*}\ \langle e',(h',l')\rangle$
and $\bigwedge e\ h\ l.\ R\ e\ h\ l\ e\ h\ l$
and $\bigwedge e_0\ h_0\ l_0\ e_1\ h_1\ l_1\ e'\ h'\ l'.$
$[\![\ P \vdash \langle e_0,(h_0,l_0)\rangle \to \langle e_1,(h_1,l_1)\rangle;\ R\ e_1\ h_1\ l_1\ e'\ h'\ l'\ ]\!] \Longrightarrow R\ e_0\ h_0\ l_0\ e'\ h'\ l'$
shows $R\ e\ h\ l\ e'\ h'\ l'$⟨*proof*⟩

2.11.2 Some easy lemmas

lemma [*iff*]: $\neg\ P \vdash \langle [],s\rangle\ [\to]\ \langle es',s'\rangle$⟨*proof*⟩
lemma [*iff*]: $\neg\ P \vdash \langle Val\ v,s\rangle \to \langle e',s'\rangle$⟨*proof*⟩
lemma [*iff*]: $\neg\ P \vdash \langle Throw\ a,s\rangle \to \langle e',s'\rangle$⟨*proof*⟩

lemma *red-hext-incr*: $P \vdash \langle e,(h,l)\rangle \to \langle e',(h',l')\rangle \Longrightarrow h \trianglelefteq h'$
and *reds-hext-incr*: $P \vdash \langle es,(h,l)\rangle\ [\to]\ \langle es',(h',l')\rangle \Longrightarrow h \trianglelefteq h'$⟨*proof*⟩

lemma *red-lcl-incr*: $P \vdash \langle e,(h_0,l_0)\rangle \to \langle e',(h_1,l_1)\rangle \Longrightarrow dom\ l_0 \subseteq dom\ l_1$
and $P \vdash \langle es,(h_0,l_0)\rangle\ [\to]\ \langle es',(h_1,l_1)\rangle \Longrightarrow dom\ l_0 \subseteq dom\ l_1$⟨*proof*⟩

lemma *red-lcl-add*: $P \vdash \langle e,(h,l)\rangle \to \langle e',(h',l')\rangle \Longrightarrow (\bigwedge l_0.\ P \vdash \langle e,(h,l_0{+}{+}l)\rangle \to \langle e',(h',l_0{+}{+}l')\rangle)$
and $P \vdash \langle es,(h,l)\rangle\ [\to]\ \langle es',(h',l')\rangle \Longrightarrow (\bigwedge l_0.\ P \vdash \langle es,(h,l_0{+}{+}l)\rangle\ [\to]\ \langle es',(h',l_0{+}{+}l')\rangle)$⟨*proof*⟩

lemma *Red-lcl-add*:
assumes $P \vdash \langle e,(h,l)\rangle \to{*}\ \langle e',(h',l')\rangle$ **shows** $P \vdash \langle e,(h,l_0{+}{+}l)\rangle \to{*}\ \langle e',(h',l_0{+}{+}l')\rangle$⟨*proof*⟩

end

2.12 System Classes

theory *SystemClasses*
imports *Decl Exceptions*
begin

This theory provides definitions for the *Object* class, and the system exceptions.

definition *ObjectC* :: *'m cdecl*
where
$ObjectC \equiv (Object, (undefined,[],[]))$

definition *NullPointerC* :: *'m cdecl*
where
$NullPointerC \equiv (NullPointer, (Object,[],[]))$

definition *ClassCastC* :: *'m cdecl*
where
$ClassCastC \equiv (ClassCast, (Object,[],[]))$

definition *OutOfMemoryC* :: *'m cdecl*
where
$OutOfMemoryC \equiv (OutOfMemory, (Object,[],[]))$

definition *SystemClasses* :: *'m cdecl list*
where
$SystemClasses \equiv [ObjectC, NullPointerC, ClassCastC, OutOfMemoryC]$

end

2.13 Generic Well-formedness of programs

theory *WellForm* **imports** *TypeRel SystemClasses* **begin**

This theory defines global well-formedness conditions for programs but does not look inside method bodies. Hence it works for both Jinja and JVM programs. Well-typing of expressions is defined elsewhere (in theory *WellType*).

Because Jinja does not have method overloading, its policy for method overriding is the classical one: *covariant in the result type but contravariant in the argument types.* This means the result type of the overriding method becomes more specific, the argument types become more general.

type-synonym $'m$ *wf-mdecl-test* = $'m$ *prog* ⇒ *cname* ⇒ $'m$ *mdecl* ⇒ *bool*

definition *wf-fdecl* :: $'m$ *prog* ⇒ *fdecl* ⇒ *bool*
where
wf-fdecl P ≡ $\lambda(F,T)$. *is-type* P T

definition *wf-mdecl* :: $'m$ *wf-mdecl-test* ⇒ $'m$ *wf-mdecl-test*
where
wf-mdecl wf-md P C ≡ $\lambda(M,Ts,T,mb)$.
(∀ T∈*set* Ts. *is-type* P T) ∧ *is-type* P T ∧ *wf-md* P C (M,Ts,T,mb)

definition *wf-cdecl* :: $'m$ *wf-mdecl-test* ⇒ $'m$ *prog* ⇒ $'m$ *cdecl* ⇒ *bool*
where
wf-cdecl wf-md P ≡ $\lambda(C,(D,fs,ms))$.
(∀ f∈*set fs*. *wf-fdecl* P f) ∧ *distinct-fst fs* ∧
(∀ m∈*set ms*. *wf-mdecl wf-md* P C m) ∧ *distinct-fst ms* ∧
(C ≠ *Object* ⟶
is-class P D ∧ ¬ P ⊢ D $\preceq^*$ C ∧
(∀ (M,Ts,T,m)∈*set ms*.
∀ D' Ts' T' m'. P ⊢ D *sees* M:Ts' → T' = m' *in* D' ⟶
P ⊢ Ts' [≤] Ts ∧ P ⊢ T ≤ T'))

definition *wf-syscls* :: $'m$ *prog* ⇒ *bool*
where
wf-syscls P ≡ {*Object*} ∪ *sys-xcpts* ⊆ *set*(*map fst* P)

definition *wf-prog* :: $'m$ *wf-mdecl-test* ⇒ $'m$ *prog* ⇒ *bool*
where
wf-prog wf-md P ≡ *wf-syscls* P ∧ (∀ c ∈ *set* P. *wf-cdecl wf-md* P c) ∧ *distinct-fst* P

2.13.1 Well-formedness lemmas

lemma *class-wf*:
⟦*class* P C = *Some* c; *wf-prog wf-md* P⟧ ⟹ *wf-cdecl wf-md* P (C,c)⟨*proof*⟩

lemma *class-Object* [*simp*]:
wf-prog wf-md P ⟹ ∃ C *fs ms*. *class* P *Object* = *Some* (C,fs,ms)⟨*proof*⟩

lemma *is-class-Object* [*simp*]:
wf-prog wf-md P ⟹ *is-class* P *Object*⟨*proof*⟩

lemma *is-class-xcpt*:
⟦ C ∈ *sys-xcpts*; *wf-prog wf-md* P ⟧ ⟹ *is-class* P C⟨*proof*⟩

lemma *subcls1-wfD*:
$[\![\ P \vdash C \prec^1 D;\ \textit{wf-prog wf-md P}\ ]\!] \Longrightarrow D \neq C \wedge (D,C) \notin (\textit{subcls1 P})^+ \langle \textit{proof} \rangle$

lemma *wf-cdecl-supD*:
$[\![\textit{wf-cdecl wf-md P}\ (C,D,r);\ C \neq \textit{Object}]\!] \Longrightarrow \textit{is-class P D} \langle \textit{proof} \rangle$

lemma *subcls-asym*:
$[\![\ \textit{wf-prog wf-md P};\ (C,D) \in (\textit{subcls1 P})^+\ ]\!] \Longrightarrow (D,C) \notin (\textit{subcls1 P})^+ \langle \textit{proof} \rangle$

lemma *subcls-irrefl*:
$[\![\ \textit{wf-prog wf-md P};\ (C,D) \in (\textit{subcls1 P})^+\ ]\!] \Longrightarrow C \neq D \langle \textit{proof} \rangle$

lemma *acyclic-subcls1*:
$\textit{wf-prog wf-md P} \Longrightarrow \textit{acyclic}\ (\textit{subcls1 P}) \langle \textit{proof} \rangle$

lemma *wf-subcls1*:
$\textit{wf-prog wf-md P} \Longrightarrow \textit{wf}\ ((\textit{subcls1 P})^{-1}) \langle \textit{proof} \rangle$

lemma *single-valued-subcls1*:
$\textit{wf-prog wf-md G} \Longrightarrow \textit{single-valued}\ (\textit{subcls1 G}) \langle \textit{proof} \rangle$

lemma *subcls-induct*:
$[\![\ \textit{wf-prog wf-md P};\ \bigwedge C.\ \forall D.\ (C,D) \in (\textit{subcls1 P})^+ \longrightarrow Q\ D \Longrightarrow Q\ C\ ]\!] \Longrightarrow Q\ C \langle \textit{proof} \rangle$

lemma *subcls1-induct-aux*:
$[\![\ \textit{is-class P C};\ \textit{wf-prog wf-md P};\ Q\ \textit{Object};$
$\bigwedge C\ D\ \textit{fs ms}.$
$[\![\ C \neq \textit{Object};\ \textit{is-class P C};\ \textit{class P C} = \textit{Some}\ (D,\textit{fs},\textit{ms}) \wedge$
$\textit{wf-cdecl wf-md P}\ (C,D,\textit{fs},\textit{ms}) \wedge P \vdash C \prec^1 D \wedge \textit{is-class P D} \wedge Q\ D]\!] \Longrightarrow Q\ C\ ]\!]$
$\Longrightarrow Q\ C \langle \textit{proof} \rangle$

lemma *subcls1-induct* [*consumes 2*, *case-names Object Subcls*]:
$[\![\ \textit{wf-prog wf-md P};\ \textit{is-class P C};\ Q\ \textit{Object};$
$\bigwedge C\ D.\ [\![C \neq \textit{Object};\ P \vdash C \prec^1 D;\ \textit{is-class P D};\ Q\ D]\!] \Longrightarrow Q\ C\ ]\!]$
$\Longrightarrow Q\ C \langle \textit{proof} \rangle$

lemma *subcls-C-Object*:
$[\![\ \textit{is-class P C};\ \textit{wf-prog wf-md P}\ ]\!] \Longrightarrow P \vdash C \preceq^* \textit{Object} \langle \textit{proof} \rangle$

lemma *is-type-pTs*:
assumes *wf-prog wf-md P* **and** $(C,S,\textit{fs},\textit{ms}) \in \textit{set P}$ **and** $(M,\textit{Ts},T,m) \in \textit{set ms}$
shows $\textit{set Ts} \subseteq \textit{types P} \langle \textit{proof} \rangle$

2.13.2 Well-formedness and method lookup

lemma *sees-wf-mdecl*:
$[\![\ \textit{wf-prog wf-md P};\ P \vdash C\ \textit{sees}\ M{:}\textit{Ts}{\rightarrow}T = m\ \textit{in}\ D\ ]\!] \Longrightarrow \textit{wf-mdecl wf-md P D}\ (M,\textit{Ts},T,m) \langle \textit{proof} \rangle$

lemma *sees-method-mono* [*rule-format* (*no-asm*)]:
$[\![\ P \vdash C' \preceq^* C;\ \textit{wf-prog wf-md P}\ ]\!] \Longrightarrow$
$\forall D\ \textit{Ts}\ T\ m.\ P \vdash C\ \textit{sees}\ M{:}\textit{Ts}{\rightarrow}T = m\ \textit{in}\ D \longrightarrow$
$(\exists D'\ \textit{Ts}'\ T'\ m'.\ P \vdash C'\ \textit{sees}\ M{:}\textit{Ts}'{\rightarrow}T' = m'\ \textit{in}\ D' \wedge P \vdash \textit{Ts}\ [\le]\ \textit{Ts}' \wedge P \vdash T' \le T) \langle \textit{proof} \rangle$

lemma *sees-method-mono2*:
$[\![$ $P \vdash C' \preceq^* C$; *wf-prog wf-md P*;
$P \vdash C$ *sees* $M{:}Ts{\rightarrow}T = m$ *in* D; $P \vdash C'$ *sees* $M{:}Ts'{\rightarrow}T' = m'$ *in* D' $]\!]$
$\Longrightarrow P \vdash Ts\ [\le]\ Ts' \wedge P \vdash T' \le T\langle proof\rangle$

lemma *mdecls-visible*:
assumes *wf*: *wf-prog wf-md P* **and** *class*: *is-class P C*
shows $\bigwedge D\ fs\ ms.$ *class P C* $=$ *Some*(D,fs,ms)
$\Longrightarrow \exists\, Mm.\ P \vdash C$ *sees-methods* $Mm \wedge (\forall (M,Ts,T,m) \in set\ ms.\ Mm\ M = Some((Ts,T,m),C))\langle proof\rangle$

lemma *mdecl-visible*:
assumes *wf*: *wf-prog wf-md P* **and** *C*: $(C,S,fs,ms) \in set\ P$ **and** *m*: $(M,Ts,T,m) \in set\ ms$
shows $P \vdash C$ *sees* $M{:}Ts{\rightarrow}T = m$ *in* $C\langle proof\rangle$

lemma *Call-lemma*:
$[\![$ $P \vdash C$ *sees* $M{:}Ts{\rightarrow}T = m$ *in* D; $P \vdash C' \preceq^* C$; *wf-prog wf-md P* $]\!]$
$\Longrightarrow \exists\, D'\ Ts'\ T'\ m'.$
$P \vdash C'$ *sees* $M{:}Ts'{\rightarrow}T' = m'$ *in* $D' \wedge P \vdash Ts\ [\le]\ Ts' \wedge P \vdash T' \le T \wedge P \vdash C' \preceq^* D'$
$\wedge$ *is-type* $P\ T' \wedge (\forall\, T{\in}set\ Ts'.\ is\text{-}type\ P\ T) \wedge$ *wf-md* $P\ D'\ (M,Ts',T',m')\langle proof\rangle$

lemma *wf-prog-lift*:
assumes *wf*: *wf-prog* $(\lambda P\ C\ bd.\ A\ P\ C\ bd)\ P$
and *rule*:
$\bigwedge$*wf-md* $C\ M\ Ts\ C\ T\ m\ bd.$
wf-prog wf-md $P \Longrightarrow$
$P \vdash C$ *sees* $M{:}Ts{\rightarrow}T = m$ *in* $C \Longrightarrow$
set $Ts \subseteq$ *types* $P \Longrightarrow$
$bd = (M,Ts,T,m) \Longrightarrow$
$A\ P\ C\ bd \Longrightarrow$
$B\ P\ C\ bd$
shows *wf-prog* $(\lambda P\ C\ bd.\ B\ P\ C\ bd)\ P\langle proof\rangle$

2.13.3 Well-formedness and field lookup

lemma *wf-Fields-Ex*:
$[\![$ *wf-prog wf-md P*; *is-class P C* $]\!] \Longrightarrow \exists\, FDTs.\ P \vdash C$ *has-fields* $FDTs\langle proof\rangle$

lemma *has-fields-types*:
$[\![$ $P \vdash C$ *has-fields* $FDTs$; $(FD,T) \in set\ FDTs$; *wf-prog wf-md P* $]\!] \Longrightarrow$ *is-type* $P\ T\langle proof\rangle$

lemma *sees-field-is-type*:
$[\![$ $P \vdash C$ *sees* $F{:}T$ *in* D; *wf-prog wf-md P* $]\!] \Longrightarrow$ *is-type* $P\ T\langle proof\rangle$
lemma *wf-syscls*:
set SystemClasses $\subseteq$ *set* $P \Longrightarrow$ *wf-syscls* $P\langle proof\rangle$
end

2.14 Weak well-formedness of Jinja programs

theory *WWellForm* **imports** ../*Common*/*WellForm* *Expr* **begin**

definition *wwf-J-mdecl* :: *J-prog* ⇒ *cname* ⇒ *J-mb mdecl* ⇒ *bool*
where
wwf-J-mdecl P C ≡ λ(*M*,*Ts*,*T*,(*pns*,*body*)).
length Ts = *length pns* ∧ *distinct pns* ∧ *this* ∉ *set pns* ∧ *fv body* ⊆ {*this*} ∪ *set pns*

lemma *wwf-J-mdecl*[*simp*]:
wwf-J-mdecl P C (*M*,*Ts*,*T*,*pns*,*body*) =
(*length Ts* = *length pns* ∧ *distinct pns* ∧ *this* ∉ *set pns* ∧ *fv body* ⊆ {*this*} ∪ *set pns*)⟨*proof*⟩
abbreviation
wwf-J-prog :: *J-prog* ⇒ *bool* **where**
wwf-J-prog == *wf-prog wwf-J-mdecl*

end

2.15 Equivalence of Big Step and Small Step Semantics

theory *Equivalence* **imports** *BigStep SmallStep WWellForm* **begin**

2.15.1 Small steps simulate big step

Cast

lemma *CastReds*:
$P \vdash \langle e,s\rangle \rightarrow* \langle e',s'\rangle \Longrightarrow P \vdash \langle Cast\ C\ e,s\rangle \rightarrow* \langle Cast\ C\ e',s'\rangle\langle proof\rangle$
lemma *CastRedsNull*:
$P \vdash \langle e,s\rangle \rightarrow* \langle null,s'\rangle \Longrightarrow P \vdash \langle Cast\ C\ e,s\rangle \rightarrow* \langle null,s'\rangle\langle proof\rangle$
lemma *CastRedsAddr*:
$[\![\ P \vdash \langle e,s\rangle \rightarrow* \langle addr\ a,s'\rangle;\ hp\ s'\ a = Some(D,fs);\ P \vdash D \preceq^* C\ ]\!] \Longrightarrow$
$P \vdash \langle Cast\ C\ e,s\rangle \rightarrow* \langle addr\ a,s'\rangle\langle proof\rangle$
lemma *CastRedsFail*:
$[\![\ P \vdash \langle e,s\rangle \rightarrow* \langle addr\ a,s'\rangle;\ hp\ s'\ a = Some(D,fs);\ \neg\ P \vdash D \preceq^* C\ ]\!] \Longrightarrow$
$P \vdash \langle Cast\ C\ e,s\rangle \rightarrow* \langle THROW\ ClassCast,s'\rangle\langle proof\rangle$
lemma *CastRedsThrow*:
$[\![\ P \vdash \langle e,s\rangle \rightarrow* \langle throw\ a,s'\rangle\ ]\!] \Longrightarrow P \vdash \langle Cast\ C\ e,s\rangle \rightarrow* \langle throw\ a,s'\rangle\langle proof\rangle$

LAss

lemma *LAssReds*:
$P \vdash \langle e,s\rangle \rightarrow* \langle e',s'\rangle \Longrightarrow P \vdash \langle\ V{:=}e,s\rangle \rightarrow* \langle\ V{:=}e',s'\rangle\langle proof\rangle$
lemma *LAssRedsVal*:
$[\![\ P \vdash \langle e,s\rangle \rightarrow* \langle Val\ v,(h',l')\rangle\ ]\!] \Longrightarrow P \vdash \langle\ V{:=}e,s\rangle \rightarrow* \langle unit,(h',l'(V\mapsto v))\rangle\langle proof\rangle$
lemma *LAssRedsThrow*:
$[\![\ P \vdash \langle e,s\rangle \rightarrow* \langle throw\ a,s'\rangle\ ]\!] \Longrightarrow P \vdash \langle\ V{:=}e,s\rangle \rightarrow* \langle throw\ a,s'\rangle\langle proof\rangle$

BinOp

lemma *BinOp1Reds*:
$P \vdash \langle e,s\rangle \rightarrow* \langle e',s'\rangle \Longrightarrow P \vdash \langle\ e \ll bop \gg e_2,\ s\rangle \rightarrow* \langle e' \ll bop \gg e_2,\ s'\rangle\langle proof\rangle$
lemma *BinOp2Reds*:
$P \vdash \langle e,s\rangle \rightarrow* \langle e',s'\rangle \Longrightarrow P \vdash \langle(Val\ v) \ll bop \gg e,\ s\rangle \rightarrow* \langle(Val\ v) \ll bop \gg e',\ s'\rangle\langle proof\rangle$
lemma *BinOpRedsVal*:
$[\![\ P \vdash \langle e_1,s_0\rangle \rightarrow* \langle Val\ v_1,s_1\rangle;\ P \vdash \langle e_2,s_1\rangle \rightarrow* \langle Val\ v_2,s_2\rangle;\ binop(bop,v_1,v_2) = Some\ v\ ]\!]$
$\Longrightarrow P \vdash \langle e_1 \ll bop \gg e_2,\ s_0\rangle \rightarrow* \langle Val\ v,s_2\rangle\langle proof\rangle$
lemma *BinOpRedsThrow1*:
$P \vdash \langle e,s\rangle \rightarrow* \langle throw\ e',s'\rangle \Longrightarrow P \vdash \langle e \ll bop \gg e_2,\ s\rangle \rightarrow* \langle throw\ e',\ s'\rangle\langle proof\rangle$
lemma *BinOpRedsThrow2*:
$[\![\ P \vdash \langle e_1,s_0\rangle \rightarrow* \langle Val\ v_1,s_1\rangle;\ P \vdash \langle e_2,s_1\rangle \rightarrow* \langle throw\ e,s_2\rangle]\!]$
$\Longrightarrow P \vdash \langle e_1 \ll bop \gg e_2,\ s_0\rangle \rightarrow* \langle throw\ e,s_2\rangle\langle proof\rangle$

FAcc

lemma *FAccReds*:
$P \vdash \langle e,s\rangle \rightarrow* \langle e',s'\rangle \Longrightarrow P \vdash \langle e{\cdot}F\{D\},\ s\rangle \rightarrow* \langle e'{\cdot}F\{D\},\ s'\rangle\langle proof\rangle$
lemma *FAccRedsVal*:
$[\![P \vdash \langle e,s\rangle \rightarrow* \langle addr\ a,s'\rangle;\ hp\ s'\ a = Some(C,fs);\ fs(F,D) = Some\ v\ ]\!]$
$\Longrightarrow P \vdash \langle e{\cdot}F\{D\},s\rangle \rightarrow* \langle Val\ v,s'\rangle\langle proof\rangle$
lemma *FAccRedsNull*:
$P \vdash \langle e,s\rangle \rightarrow* \langle null,s'\rangle \Longrightarrow P \vdash \langle e{\cdot}F\{D\},s\rangle \rightarrow* \langle THROW\ NullPointer,s'\rangle\langle proof\rangle$
lemma *FAccRedsThrow*:

$P \vdash \langle e,s\rangle \to* \langle throw\ a,s'\rangle \Longrightarrow P \vdash \langle e{\cdot}F\{D\},s\rangle \to* \langle throw\ a,s'\rangle\langle proof\rangle$

FAss

lemma *FAssReds1*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle e{\cdot}F\{D\}{:=}e_2,\ s\rangle \to* \langle e'{\cdot}F\{D\}{:=}e_2,\ s'\rangle\langle proof\rangle$
lemma *FAssReds2*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle Val\ v{\cdot}F\{D\}{:=}e,\ s\rangle \to* \langle Val\ v{\cdot}F\{D\}{:=}e',\ s'\rangle\langle proof\rangle$
lemma *FAssRedsVal*:
$[\![\ P \vdash \langle e_1,s_0\rangle \to* \langle addr\ a,s_1\rangle;\ P \vdash \langle e_2,s_1\rangle \to* \langle Val\ v,(h_2,l_2)\rangle;\ Some(C,fs) = h_2\ a\ ]\!] \Longrightarrow$
$P \vdash \langle e_1{\cdot}F\{D\}{:=}e_2,\ s_0\rangle \to* \langle unit,\ (h_2(a\mapsto(C,fs((F,D)\mapsto v))),l_2)\rangle\langle proof\rangle$
lemma *FAssRedsNull*:
$[\![\ P \vdash \langle e_1,s_0\rangle \to* \langle null,s_1\rangle;\ P \vdash \langle e_2,s_1\rangle \to* \langle Val\ v,s_2\rangle\ ]\!] \Longrightarrow$
$P \vdash \langle e_1{\cdot}F\{D\}{:=}e_2,\ s_0\rangle \to* \langle THROW\ NullPointer,\ s_2\rangle\langle proof\rangle$
lemma *FAssRedsThrow1*:
$P \vdash \langle e,s\rangle \to* \langle throw\ e',s'\rangle \Longrightarrow P \vdash \langle e{\cdot}F\{D\}{:=}e_2,\ s\rangle \to* \langle throw\ e',\ s'\rangle\langle proof\rangle$
lemma *FAssRedsThrow2*:
$[\![\ P \vdash \langle e_1,s_0\rangle \to* \langle Val\ v,s_1\rangle;\ P \vdash \langle e_2,s_1\rangle \to* \langle throw\ e,s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle e_1{\cdot}F\{D\}{:=}e_2,s_0\rangle \to* \langle throw\ e,s_2\rangle\langle proof\rangle$

;;

lemma *SeqReds*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle e;;e_2,\ s\rangle \to* \langle e';;e_2,\ s'\rangle\langle proof\rangle$
lemma *SeqRedsThrow*:
$P \vdash \langle e,s\rangle \to* \langle throw\ e',s'\rangle \Longrightarrow P \vdash \langle e;;e_2,\ s\rangle \to* \langle throw\ e',\ s'\rangle\langle proof\rangle$
lemma *SeqReds2*:
$[\![\ P \vdash \langle e_1,s_0\rangle \to* \langle Val\ v_1,s_1\rangle;\ P \vdash \langle e_2,s_1\rangle \to* \langle e_2',s_2\rangle\ ]\!] \Longrightarrow P \vdash \langle e_1;;e_2,\ s_0\rangle \to* \langle e_2',s_2\rangle\langle proof\rangle$

If

lemma *CondReds*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle if\ (e)\ e_1\ else\ e_2,s\rangle \to* \langle if\ (e')\ e_1\ else\ e_2,s'\rangle\langle proof\rangle$
lemma *CondRedsThrow*:
$P \vdash \langle e,s\rangle \to* \langle throw\ a,s'\rangle \Longrightarrow P \vdash \langle if\ (e)\ e_1\ else\ e_2,\ s\rangle \to* \langle throw\ a,s'\rangle\langle proof\rangle$
lemma *CondReds2T*:
$[\![P \vdash \langle e,s_0\rangle \to* \langle true,s_1\rangle;\ P \vdash \langle e_1,\ s_1\rangle \to* \langle e',s_2\rangle\ ]\!] \Longrightarrow P \vdash \langle if\ (e)\ e_1\ else\ e_2,\ s_0\rangle \to* \langle e',s_2\rangle\langle proof\rangle$
lemma *CondReds2F*:
$[\![P \vdash \langle e,s_0\rangle \to* \langle false,s_1\rangle;\ P \vdash \langle e_2,\ s_1\rangle \to* \langle e',s_2\rangle\ ]\!] \Longrightarrow P \vdash \langle if\ (e)\ e_1\ else\ e_2,\ s_0\rangle \to* \langle e',s_2\rangle\langle proof\rangle$

While

lemma *WhileFReds*:
$P \vdash \langle b,s\rangle \to* \langle false,s'\rangle \Longrightarrow P \vdash \langle while\ (b)\ c,s\rangle \to* \langle unit,s'\rangle\langle proof\rangle$
lemma *WhileRedsThrow*:
$P \vdash \langle b,s\rangle \to* \langle throw\ e,s'\rangle \Longrightarrow P \vdash \langle while\ (b)\ c,s\rangle \to* \langle throw\ e,s'\rangle\langle proof\rangle$
lemma *WhileTReds*:
$[\![\ P \vdash \langle b,s_0\rangle \to* \langle true,s_1\rangle;\ P \vdash \langle c,s_1\rangle \to* \langle Val\ v_1,s_2\rangle;\ P \vdash \langle while\ (b)\ c,s_2\rangle \to* \langle e,s_3\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle while\ (b)\ c,s_0\rangle \to* \langle e,s_3\rangle\langle proof\rangle$
lemma *WhileTRedsThrow*:
$[\![\ P \vdash \langle b,s_0\rangle \to* \langle true,s_1\rangle;\ P \vdash \langle c,s_1\rangle \to* \langle throw\ e,s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle while\ (b)\ c,s_0\rangle \to* \langle throw\ e,s_2\rangle\langle proof\rangle$

Throw

lemma *ThrowReds*:
$P \vdash \langle e,s\rangle \rightarrow* \langle e',s'\rangle \Longrightarrow P \vdash \langle throw\ e,s\rangle \rightarrow* \langle throw\ e',s'\rangle\langle proof\rangle$
lemma *ThrowRedsNull*:
$P \vdash \langle e,s\rangle \rightarrow* \langle null,s'\rangle \Longrightarrow P \vdash \langle throw\ e,s\rangle \rightarrow* \langle THROW\ NullPointer,s'\rangle\langle proof\rangle$
lemma *ThrowRedsThrow*:
$P \vdash \langle e,s\rangle \rightarrow* \langle throw\ a,s'\rangle \Longrightarrow P \vdash \langle throw\ e,s\rangle \rightarrow* \langle throw\ a,s'\rangle\langle proof\rangle$

InitBlock

lemma *InitBlockReds-aux*:
$P \vdash \langle e,s\rangle \rightarrow* \langle e',s'\rangle \Longrightarrow$
$\forall h\ l\ h'\ l'\ v.\ s = (h,l(V\mapsto v)) \longrightarrow s' = (h',l') \longrightarrow$
$P \vdash \langle\{V{:}T := Val\ v;\ e\},(h,l)\rangle \rightarrow* \langle\{V{:}T := Val(the(l'\ V));\ e'\},(h',l'(V{:=}(l\ V)))\rangle\langle proof\rangle$
lemma *InitBlockReds*:
$P \vdash \langle e,\ (h,l(V\mapsto v))\rangle \rightarrow* \langle e',\ (h',l')\rangle \Longrightarrow$
$P \vdash \langle\{V{:}T := Val\ v;\ e\},\ (h,l)\rangle \rightarrow* \langle\{V{:}T := Val(the(l'\ V));\ e'\},\ (h',l'(V{:=}(l\ V)))\rangle\langle proof\rangle$
lemma *InitBlockRedsFinal*:
$[\![\ P \vdash \langle e,(h,l(V\mapsto v))\rangle \rightarrow* \langle e',(h',l')\rangle;\ final\ e'\ ]\!] \Longrightarrow$
$P \vdash \langle\{V{:}T := Val\ v;\ e\},(h,l)\rangle \rightarrow* \langle e',(h',\ l'(V := l\ V))\rangle\langle proof\rangle$

Block

lemma *BlockRedsFinal*:
assumes *reds*: $P \vdash \langle e_0,s_0\rangle \rightarrow* \langle e_2,(h_2,l_2)\rangle$ **and** *fin*: $final\ e_2$
shows $\bigwedge h_0\ l_0.\ s_0 = (h_0,l_0(V{:=}None)) \Longrightarrow P \vdash \langle\{V{:}T;\ e_0\},(h_0,l_0)\rangle \rightarrow* \langle e_2,(h_2,l_2(V{:=}l_0\ V))\rangle\langle proof\rangle$

try-catch

lemma *TryReds*:
$P \vdash \langle e,s\rangle \rightarrow* \langle e',s'\rangle \Longrightarrow P \vdash \langle try\ e\ catch(C\ V)\ e_2,s\rangle \rightarrow* \langle try\ e'\ catch(C\ V)\ e_2,s'\rangle\langle proof\rangle$
lemma *TryRedsVal*:
$P \vdash \langle e,s\rangle \rightarrow* \langle Val\ v,s'\rangle \Longrightarrow P \vdash \langle try\ e\ catch(C\ V)\ e_2,s\rangle \rightarrow* \langle Val\ v,s'\rangle\langle proof\rangle$
lemma *TryCatchRedsFinal*:
$[\![\ P \vdash \langle e_1,s_0\rangle \rightarrow* \langle Throw\ a,(h_1,l_1)\rangle;\ \ h_1\ a = Some(D,fs);\ P \vdash D \preceq^* C;$
$P \vdash \langle e_2,\ (h_1,\ l_1(V \mapsto Addr\ a))\rangle \rightarrow* \langle e_2',\ (h_2,l_2)\rangle;\ final\ e_2'\ ]\!]$
$\Longrightarrow P \vdash \langle try\ e_1\ catch(C\ V)\ e_2,\ s_0\rangle \rightarrow* \langle e_2',\ (h_2,\ l_2(V := l_1\ V))\rangle\langle proof\rangle$
lemma *TryRedsFail*:
$[\![\ P \vdash \langle e_1,s\rangle \rightarrow* \langle Throw\ a,(h,l)\rangle;\ h\ a = Some(D,fs);\ \neg\ P \vdash D \preceq^*\ C\ ]\!]$
$\Longrightarrow P \vdash \langle try\ e_1\ catch(C\ V)\ e_2,s\rangle \rightarrow* \langle Throw\ a,(h,l)\rangle\langle proof\rangle$

List

lemma *ListReds1*:
$P \vdash \langle e,s\rangle \rightarrow* \langle e',s'\rangle \Longrightarrow P \vdash \langle e\#es,s\rangle\ [\rightarrow]*\ \langle e'\ \#\ es,s'\rangle\langle proof\rangle$
lemma *ListReds2*:
$P \vdash \langle es,s\rangle\ [\rightarrow]*\ \langle es',s'\rangle \Longrightarrow P \vdash \langle Val\ v\ \#\ es,s\rangle\ [\rightarrow]*\ \langle Val\ v\ \#\ es',s'\rangle\langle proof\rangle$
lemma *ListRedsVal*:
$[\![\ P \vdash \langle e,s_0\rangle \rightarrow* \langle Val\ v,s_1\rangle;\ P \vdash \langle es,s_1\rangle\ [\rightarrow]*\ \langle es',s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle e\#es,s_0\rangle\ [\rightarrow]*\ \langle Val\ v\ \#\ es',s_2\rangle\langle proof\rangle$

Call

First a few lemmas on what happens to free variables during redction.

lemma assumes *wf*: $wwf\text{-}J\text{-}prog\ P$
shows *Red-fv*: $P \vdash \langle e,(h,l)\rangle \to \langle e',(h',l')\rangle \Longrightarrow fv\ e' \subseteq fv\ e$
and $P \vdash \langle es,(h,l)\rangle\ [\to]\ \langle es',(h',l')\rangle \Longrightarrow fvs\ es' \subseteq fvs\ es\langle proof\rangle$

lemma *Red-dom-lcl*:
$P \vdash \langle e,(h,l)\rangle \to \langle e',(h',l')\rangle \Longrightarrow dom\ l' \subseteq dom\ l \cup fv\ e$ **and**
$P \vdash \langle es,(h,l)\rangle\ [\to]\ \langle es',(h',l')\rangle \Longrightarrow dom\ l' \subseteq dom\ l \cup fvs\ es\langle proof\rangle$
lemma *Reds-dom-lcl*:
$[\![\ wwf\text{-}J\text{-}prog\ P;\ P \vdash \langle e,(h,l)\rangle \to\!\!*\ \langle e',(h',l')\rangle\ ]\!] \Longrightarrow dom\ l' \subseteq dom\ l \cup fv\ e\langle proof\rangle$

Now a few lemmas on the behaviour of blocks during reduction.

lemma *override-on-upd-lemma*:
$(override\text{-}on\ f\ (g(a\mapsto b))\ A)(a := g\ a) = override\text{-}on\ f\ g\ (insert\ a\ A)\langle proof\rangle$

lemma *blocksReds*:
$\bigwedge l.\ [\![\ length\ Vs = length\ Ts;\ length\ vs = length\ Ts;\ distinct\ Vs;$
$P \vdash \langle e,\ (h,l(Vs\ [\mapsto]\ vs))\rangle \to\!\!*\ \langle e',\ (h',l')\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle blocks(Vs,Ts,vs,e),\ (h,l)\rangle \to\!\!*\ \langle blocks(Vs,Ts,map\ (the \circ l')\ Vs,e'),\ (h',override\text{-}on\ l'\ l\ (set\ Vs))\rangle\langle proof\rangle$

lemma *blocksFinal*:
$\bigwedge l.\ [\![\ length\ Vs = length\ Ts;\ length\ vs = length\ Ts;\ final\ e\ ]\!] \Longrightarrow$
$P \vdash \langle blocks(Vs,Ts,vs,e),\ (h,l)\rangle \to\!\!*\ \langle e,\ (h,l)\rangle\langle proof\rangle$

lemma *blocksRedsFinal*:
assumes *wf*: $length\ Vs = length\ Ts\ length\ vs = length\ Ts\ distinct\ Vs$
and *reds*: $P \vdash \langle e,\ (h,l(Vs\ [\mapsto]\ vs))\rangle \to\!\!*\ \langle e',\ (h',l')\rangle$
and *fin*: $final\ e'$ **and** l'': $l'' = override\text{-}on\ l'\ l\ (set\ Vs)$
shows $P \vdash \langle blocks(Vs,Ts,vs,e),\ (h,l)\rangle \to\!\!*\ \langle e',\ (h',l'')\rangle\langle proof\rangle$

An now the actual method call reduction lemmas.

lemma *CallRedsObj*:
$P \vdash \langle e,s\rangle \to\!\!*\ \langle e',s'\rangle \Longrightarrow P \vdash \langle e\cdot M(es),s\rangle \to\!\!*\ \langle e'\cdot M(es),s'\rangle\langle proof\rangle$

lemma *CallRedsParams*:
$P \vdash \langle es,s\rangle\ [\to]*\ \langle es',s'\rangle \Longrightarrow P \vdash \langle (Val\ v)\cdot M(es),s\rangle \to\!\!*\ \langle (Val\ v)\cdot M(es'),s'\rangle\langle proof\rangle$

lemma *CallRedsFinal*:
assumes *wwf*: $wwf\text{-}J\text{-}prog\ P$
and $P \vdash \langle e,s_0\rangle \to\!\!*\ \langle addr\ a,s_1\rangle$
$P \vdash \langle es,s_1\rangle\ [\to]*\ \langle map\ Val\ vs,(h_2,l_2)\rangle$
$h_2\ a = Some(C,fs)\ P \vdash C\ sees\ M{:}Ts\to T = (pns,body)\ in\ D$
$size\ vs = size\ pns$
and l_2': $l_2' = [this \mapsto Addr\ a,\ pns[\mapsto]vs]$
and *body*: $P \vdash \langle body,(h_2,l_2')\rangle \to\!\!*\ \langle ef,(h_3,l_3)\rangle$
and $final\ ef$
shows $P \vdash \langle e\cdot M(es),\ s_0\rangle \to\!\!*\ \langle ef,(h_3,l_2)\rangle\langle proof\rangle$

lemma *CallRedsThrowParams*:
$[\![\ P \vdash \langle e,s0\rangle \to\!\!*\ \langle Val\ v,s_1\rangle;\ \ P \vdash \langle es,s_1\rangle\ [\to]*\ \langle map\ Val\ vs_1\ @\ throw\ a\ \#\ es_2,s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle e\cdot M(es),s0\rangle \to\!\!*\ \langle throw\ a,s_2\rangle\langle proof\rangle$

lemma *CallRedsThrowObj*:
$P \vdash \langle e,s0\rangle \to\!\!*\ \langle throw\ a,s_1\rangle \Longrightarrow P \vdash \langle e\cdot M(es),s0\rangle \to\!\!*\ \langle throw\ a,s_1\rangle\langle proof\rangle$

lemma *CallRedsNull*:
$[\![\ P \vdash \langle e,s_0\rangle \to\!* \langle null,s_1\rangle;\ P \vdash \langle es,s_1\rangle\ [\to]\!*\ \langle map\ Val\ vs,s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle e\!\cdot\! M(es),s_0\rangle \to\!* \langle THROW\ NullPointer,s_2\rangle \langle proof\rangle$

The main Theorem

lemma assumes *wwf*: *wwf-J-prog P*
shows *big-by-small*: $P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle \Longrightarrow P \vdash \langle e,s\rangle \to\!* \langle e',s'\rangle$
and *bigs-by-smalls*: $P \vdash \langle es,s\rangle\ [\Rightarrow]\ \langle es',s'\rangle \Longrightarrow P \vdash \langle es,s\rangle\ [\to]\!*\ \langle es',s'\rangle\langle proof\rangle$

2.15.2 Big steps simulates small step

This direction was carried out by Norbert Schirmer and Daniel Wasserrab.

The big step equivalent of *RedWhile*:

lemma *unfold-while*:
$P \vdash \langle while(b)\ c,s\rangle \Rightarrow \langle e',s'\rangle\ =\ P \vdash \langle if(b)\ (c;;while(b)\ c)\ else\ (unit),s\rangle \Rightarrow \langle e',s'\rangle\langle proof\rangle$

lemma *blocksEval*:
$\bigwedge Ts\ vs\ l\ l'.\ [\![size\ ps = size\ Ts;\ size\ ps = size\ vs;\ P \vdash \langle blocks(ps,Ts,vs,e),(h,l)\rangle \Rightarrow \langle e',(h',l')\rangle\ ]\!]$
$\Longrightarrow \exists\ l''.\ P \vdash \langle e,(h,l(ps[\mapsto]vs))\rangle \Rightarrow \langle e',(h',l'')\rangle\langle proof\rangle$

lemma
assumes *wf*: *wwf-J-prog P*
shows *eval-restrict-lcl*:
$P \vdash \langle e,(h,l)\rangle \Rightarrow \langle e',(h',l')\rangle \Longrightarrow (\bigwedge W.\ fv\ e \subseteq W \Longrightarrow P \vdash \langle e,(h,l|`W)\rangle \Rightarrow \langle e',(h',l'|`W)\rangle)$
and $P \vdash \langle es,(h,l)\rangle\ [\Rightarrow]\ \langle es',(h',l')\rangle \Longrightarrow (\bigwedge W.\ fvs\ es \subseteq W \Longrightarrow P \vdash \langle es,(h,l|`W)\rangle\ [\Rightarrow]\ \langle es',(h',l'|`W)\rangle)\langle proof\rangle$

lemma *eval-notfree-unchanged*:
$P \vdash \langle e,(h,l)\rangle \Rightarrow \langle e',(h',l')\rangle \Longrightarrow (\bigwedge V.\ V \notin fv\ e \Longrightarrow l'\ V = l\ V)$
and $P \vdash \langle es,(h,l)\rangle\ [\Rightarrow]\ \langle es',(h',l')\rangle \Longrightarrow (\bigwedge V.\ V \notin fvs\ es \Longrightarrow l'\ V = l\ V)\langle proof\rangle$

lemma *eval-closed-lcl-unchanged*:
$[\![\ P \vdash \langle e,(h,l)\rangle \Rightarrow \langle e',(h',l')\rangle;\ fv\ e = \{\}\ ]\!] \Longrightarrow l' = l\langle proof\rangle$

lemma *list-eval-Throw*:
assumes *eval-e*: $P \vdash \langle throw\ x,s\rangle \Rightarrow \langle e',s'\rangle$
shows $P \vdash \langle map\ Val\ vs\ @\ throw\ x\ \#\ es',s\rangle\ [\Rightarrow]\ \langle map\ Val\ vs\ @\ e'\ \#\ es',s'\rangle\langle proof\rangle$

The key lemma:

lemma
assumes *wf*: *wwf-J-prog P*
shows *extend-1-eval*:
$P \vdash \langle e,s\rangle \to \langle e'',s''\rangle \Longrightarrow\ (\bigwedge s'\ e'.\ P \vdash \langle e'',s''\rangle \Rightarrow \langle e',s'\rangle \Longrightarrow P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle)$
and *extend-1-evals*:
$P \vdash \langle es,t\rangle\ [\to]\ \langle es'',t''\rangle \Longrightarrow (\bigwedge t'\ es'.\ P \vdash \langle es'',t''\rangle\ [\Rightarrow]\ \langle es',t'\rangle \Longrightarrow P \vdash \langle es,t\rangle\ [\Rightarrow]\ \langle es',t'\rangle)\langle proof\rangle$

Its extension to $\to\!*$:

lemma *extend-eval*:
assumes *wf*: *wwf-J-prog P*
and *reds*: $P \vdash \langle e,s\rangle \to\!* \langle e'',s''\rangle$ **and** *eval-rest*: $P \vdash \langle e'',s''\rangle \Rightarrow \langle e',s'\rangle$
shows $P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle\langle proof\rangle$

lemma *extend-evals*:
assumes *wf*: *wwf-J-prog P*
and *reds*: $P \vdash \langle es,s\rangle \ [\rightarrow]{*}\ \langle es'',s''\rangle$ **and** *eval-rest*: $P \vdash \langle es'',s''\rangle\ [\Rightarrow]\ \langle es',s'\rangle$
shows $P \vdash \langle es,s\rangle\ [\Rightarrow]\ \langle es',s'\rangle \langle proof\rangle$

Finally, small step semantics can be simulated by big step semantics:

theorem
assumes *wf*: *wwf-J-prog P*
shows *small-by-big*: $[\![P \vdash \langle e,s\rangle \rightarrow{*}\ \langle e',s'\rangle;\ final\ e']\!] \Longrightarrow P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle$
and $[\![P \vdash \langle es,s\rangle\ [\rightarrow]{*}\ \langle es',s'\rangle;\ finals\ es']\!] \Longrightarrow P \vdash \langle es,s\rangle\ [\Rightarrow]\ \langle es',s'\rangle \langle proof\rangle$

2.15.3 Equivalence

And now, the crowning achievement:

corollary *big-iff-small*:
wwf-J-prog P $\Longrightarrow$
$P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle \ = \ (P \vdash \langle e,s\rangle \rightarrow{*}\ \langle e',s'\rangle \wedge final\ e')\langle proof\rangle$

end

2.16 Well-typedness of Jinja expressions

theory *WellType*
imports *../Common/Objects Expr*
begin

type-synonym
  env = *vname* ⇀ *ty*

inductive
  WT :: [*J-prog*,*env*, *expr*, *ty*] ⇒ *bool*
    (-,- ⊢ - :: - [*51*,*51*,*51*]*50*)
  and *WTs* :: [*J-prog*,*env*, *expr list*, *ty list*] ⇒ *bool*
    (-,- ⊢ - [::] - [*51*,*51*,*51*]*50*)
  for *P* :: *J-prog*
where

WTNew:
$\mathit{is\text{-}class}\ P\ C \implies$
$P,E \vdash \mathit{new}\ C :: \mathit{Class}\ C$

| *WTCast*:
$[\![\ P,E \vdash e :: \mathit{Class}\ D;\ \ \mathit{is\text{-}class}\ P\ C;\ \ P \vdash C \preceq^* D \lor P \vdash D \preceq^* C\ ]\!]$
$\implies P,E \vdash \mathit{Cast}\ C\ e :: \mathit{Class}\ C$

| *WTVal*:
$\mathit{typeof}\ v = \mathit{Some}\ T \implies$
$P,E \vdash \mathit{Val}\ v :: T$

| *WTVar*:
$E\ V = \mathit{Some}\ T \implies$
$P,E \vdash \mathit{Var}\ V :: T$

| *WTBinOpEq*:
$[\![\ P,E \vdash e_1 :: T_1;\ \ P,E \vdash e_2 :: T_2;\ P \vdash T_1 \le T_2 \lor P \vdash T_2 \le T_1\ ]\!]$
$\implies P,E \vdash e_1 \ll\!\mathit{Eq}\!\gg e_2 :: \mathit{Boolean}$

| *WTBinOpAdd*:
$[\![\ P,E \vdash e_1 :: \mathit{Integer};\ \ P,E \vdash e_2 :: \mathit{Integer}\ ]\!]$
$\implies P,E \vdash e_1 \ll\!\mathit{Add}\!\gg e_2 :: \mathit{Integer}$

| *WTLAss*:
$[\![\ E\ V = \mathit{Some}\ T;\ \ P,E \vdash e :: T';\ \ P \vdash T' \le T;\ \ V \ne \mathit{this}\ ]\!]$
$\implies P,E \vdash V{:=}e :: \mathit{Void}$

| *WTFAcc*:
$[\![\ P,E \vdash e :: \mathit{Class}\ C;\ \ P \vdash C\ \mathit{sees}\ F{:}T\ \mathit{in}\ D\ ]\!]$
$\implies P,E \vdash e{\cdot}F\{D\} :: T$

| *WTFAss*:
$[\![\ P,E \vdash e_1 :: \mathit{Class}\ C;\ \ P \vdash C\ \mathit{sees}\ F{:}T\ \mathit{in}\ D;\ \ P,E \vdash e_2 :: T';\ \ P \vdash T' \le T\ ]\!]$
$\implies P,E \vdash e_1{\cdot}F\{D\}{:=}e_2 :: \mathit{Void}$

| *WTCall*:

$[\![$ $P,E \vdash e :: Class\ C$; $P \vdash C\ sees\ M{:}Ts \rightarrow T = (pns,body)\ in\ D$;
$P,E \vdash es\ [::]\ Ts'$; $P \vdash Ts'\ [\leq]\ Ts$ $]\!]$
$\Longrightarrow P,E \vdash e{\cdot}M(es) :: T$

| *WTBlock*:
$[\![$ $is\text{-}type\ P\ T$; $P,E(V \mapsto T) \vdash e :: T'$ $]\!]$
$\Longrightarrow$ $P,E \vdash \{V{:}T;\ e\} :: T'$

| *WTSeq*:
$[\![$ $P,E \vdash e_1{::}T_1$; $P,E \vdash e_2{::}T_2$ $]\!]$
$\Longrightarrow$ $P,E \vdash e_1;;e_2 :: T_2$
| *WTCond*:
$[\![$ $P,E \vdash e :: Boolean$; $P,E \vdash e_1{::}T_1$; $P,E \vdash e_2{::}T_2$;
$P \vdash T_1 \leq T_2 \vee P \vdash T_2 \leq T_1$; $P \vdash T_1 \leq T_2 \longrightarrow T = T_2$; $P \vdash T_2 \leq T_1 \longrightarrow T = T_1$ $]\!]$
$\Longrightarrow P,E \vdash if\ (e)\ e_1\ else\ e_2 :: T$

| *WTWhile*:
$[\![$ $P,E \vdash e :: Boolean$; $P,E \vdash c{::}T$ $]\!]$
$\Longrightarrow P,E \vdash while\ (e)\ c :: Void$

| *WTThrow*:
$P,E \vdash e :: Class\ C \Longrightarrow$
$P,E \vdash throw\ e :: Void$

| *WTTry*:
$[\![$ $P,E \vdash e_1 :: T$; $P,E(V \mapsto Class\ C) \vdash e_2 :: T$; $is\text{-}class\ P\ C$ $]\!]$
$\Longrightarrow P,E \vdash try\ e_1\ catch(C\ V)\ e_2 :: T$

— well-typed expression lists

| *WTNil*:
$P,E \vdash []\ [::]\ []$

| *WTCons*:
$[\![$ $P,E \vdash e :: T$; $P,E \vdash es\ [::]\ Ts$ $]\!]$
$\Longrightarrow$ $P,E \vdash e\#es\ [::]\ T\#Ts$

lemma [*iff*]: $(P,E \vdash []\ [::]\ Ts) = (Ts = [])\langle proof\rangle$
lemma [*iff*]: $(P,E \vdash e\#es\ [::]\ T\#Ts) = (P,E \vdash e :: T \wedge P,E \vdash es\ [::]\ Ts)\langle proof\rangle$
lemma [*iff*]: $(P,E \vdash (e\#es)\ [::]\ Ts) =$
$(\exists\, U\ Us.\ Ts = U\#Us \wedge P,E \vdash e :: U \wedge P,E \vdash es\ [::]\ Us)\langle proof\rangle$
lemma [*iff*]: $\bigwedge Ts.\ (P,E \vdash es_1\ @\ es_2\ [::]\ Ts) =$
$(\exists\, Ts_1\ Ts_2.\ Ts = Ts_1\ @\ Ts_2 \wedge P,E \vdash es_1\ [::]\ Ts_1 \wedge P,E \vdash es_2 [::] Ts_2)\langle proof\rangle$
lemma [*iff*]: $P,E \vdash Val\ v :: T = (typeof\ v = Some\ T)\langle proof\rangle$
lemma [*iff*]: $P,E \vdash Var\ V :: T = (E\ V = Some\ T)\langle proof\rangle$
lemma [*iff*]: $P,E \vdash e_1;;e_2 :: T_2 = (\exists\, T_1.\ P,E \vdash e_1{::}T_1 \wedge P,E \vdash e_2{::}T_2)\langle proof\rangle$
lemma [*iff*]: $(P,E \vdash \{V{:}T;\ e\} :: T') = (is\text{-}type\ P\ T \wedge P,E(V{\mapsto}T) \vdash e :: T')\langle proof\rangle$

lemma *wt-env-mono*:
$P,E \vdash e :: T \Longrightarrow (\bigwedge E'.\ E \subseteq_m E' \Longrightarrow P,E' \vdash e :: T)$ **and**
$P,E \vdash es\ [::]\ Ts \Longrightarrow (\bigwedge E'.\ E \subseteq_m E' \Longrightarrow P,E' \vdash es\ [::]\ Ts)\langle proof\rangle$

lemma *WT-fv*: $P,E \vdash e :: T \Longrightarrow fv\ e \subseteq dom\ E$
and $P,E \vdash es\ [::]\ Ts \Longrightarrow fvs\ es \subseteq dom\ E\langle proof\rangle$

2.17 Runtime Well-typedness

theory *WellTypeRT*
imports *WellType*
begin

inductive
  WTrt :: *J-prog* ⇒ *heap* ⇒ *env* ⇒ *expr* ⇒ *ty* ⇒ *bool*
  and *WTrts* :: *J-prog* ⇒ *heap* ⇒ *env* ⇒ *expr list* ⇒ *ty list* ⇒ *bool*
  and *WTrt2* :: [*J-prog*,*env*,*heap*,*expr*,*ty*] ⇒ *bool*
    (-,-,- ⊢ - : - [*51*,*51*,*51*]*50*)
  and *WTrts2* :: [*J-prog*,*env*,*heap*,*expr list*, *ty list*] ⇒ *bool*
    (-,-,- ⊢ - [:] - [*51*,*51*,*51*]*50*)
  for *P* :: *J-prog* **and** *h* :: *heap*
where

  $P,E,h \vdash e : T \equiv WTrt\ P\ h\ E\ e\ T$
| $P,E,h \vdash es[:]Ts \equiv WTrts\ P\ h\ E\ es\ Ts$

| *WTrtNew*:
  $is\text{-}class\ P\ C \Longrightarrow$
  $P,E,h \vdash new\ C : Class\ C$

| *WTrtCast*:
  $[\![\ P,E,h \vdash e : T;\ is\text{-}refT\ T;\ is\text{-}class\ P\ C\ ]\!]$
  $\Longrightarrow P,E,h \vdash Cast\ C\ e : Class\ C$

| *WTrtVal*:
  $typeof_h\ v = Some\ T \Longrightarrow$
  $P,E,h \vdash Val\ v : T$

| *WTrtVar*:
  $E\ V = Some\ T \Longrightarrow$
  $P,E,h \vdash Var\ V : T$

| *WTrtBinOpEq*:
  $[\![\ P,E,h \vdash e_1 : T_1;\ \ P,E,h \vdash e_2 : T_2\ ]\!]$
  $\Longrightarrow P,E,h \vdash e_1 \ll Eq \gg e_2 : Boolean$

| *WTrtBinOpAdd*:
  $[\![\ P,E,h \vdash e_1 : Integer;\ \ P,E,h \vdash e_2 : Integer\ ]\!]$
  $\Longrightarrow P,E,h \vdash e_1 \ll Add \gg e_2 : Integer$

| *WTrtLAss*:
  $[\![\ E\ V = Some\ T;\ \ P,E,h \vdash e : T';\ \ P \vdash T' \le T\ ]\!]$
  $\Longrightarrow P,E,h \vdash V{:=}e : Void$

| *WTrtFAcc*:
  $[\![\ P,E,h \vdash e : Class\ C;\ P \vdash C\ has\ F{:}T\ in\ D\ ]\!] \Longrightarrow$
  $P,E,h \vdash e{\cdot}F\{D\} : T$

| *WTrtFAccNT*:
  $P,E,h \vdash e : NT \Longrightarrow$
  $P,E,h \vdash e{\cdot}F\{D\} : T$

| *WTrtFAss*:
$[\![\ P,E,h \vdash e_1 : \mathit{Class}\ C;\ \ P \vdash C\ \mathit{has}\ F{:}T\ \mathit{in}\ D;\ P,E,h \vdash e_2 : T_2;\ \ P \vdash T_2 \le T\ ]\!]$
$\Longrightarrow P,E,h \vdash e_1{\cdot}F\{D\}{:=}e_2 : \mathit{Void}$

| *WTrtFAssNT*:
$[\![\ P,E,h \vdash e_1{:}NT;\ P,E,h \vdash e_2 : T_2\ ]\!]$
$\Longrightarrow P,E,h \vdash e_1{\cdot}F\{D\}{:=}e_2 : \mathit{Void}$

| *WTrtCall*:
$[\![\ P,E,h \vdash e : \mathit{Class}\ C;\ P \vdash C\ \mathit{sees}\ M{:}Ts \to T = (\mathit{pns},\mathit{body})\ \mathit{in}\ D;$
$P,E,h \vdash es\ [:]\ Ts';\ P \vdash Ts'\ [\le]\ Ts\ ]\!]$
$\Longrightarrow P,E,h \vdash e{\cdot}M(es) : T$

| *WTrtCallNT*:
$[\![\ P,E,h \vdash e : NT;\ P,E,h \vdash es\ [:]\ Ts\ ]\!]$
$\Longrightarrow P,E,h \vdash e{\cdot}M(es) : T$

| *WTrtBlock*:
$P,E(V{\mapsto}T),h \vdash e : T' \Longrightarrow$
$P,E,h \vdash \{V{:}T;\ e\} : T'$

| *WTrtSeq*:
$[\![\ P,E,h \vdash e_1{:}T_1;\ \ P,E,h \vdash e_2{:}T_2\ ]\!]$
$\Longrightarrow P,E,h \vdash e_1;;e_2 : T_2$

| *WTrtCond*:
$[\![\ P,E,h \vdash e : \mathit{Boolean};\ \ P,E,h \vdash e_1{:}T_1;\ \ P,E,h \vdash e_2{:}T_2;$
$P \vdash T_1 \le T_2 \vee P \vdash T_2 \le T_1;\ P \vdash T_1 \le T_2 \longrightarrow T = T_2;\ P \vdash T_2 \le T_1 \longrightarrow T = T_1\ ]\!]$
$\Longrightarrow P,E,h \vdash \mathit{if}\ (e)\ e_1\ \mathit{else}\ e_2 : T$

| *WTrtWhile*:
$[\![\ P,E,h \vdash e : \mathit{Boolean};\ \ P,E,h \vdash c{:}T\ ]\!]$
$\Longrightarrow\ \ P,E,h \vdash \mathit{while}(e)\ c : \mathit{Void}$

| *WTrtThrow*:
$[\![\ P,E,h \vdash e : T_r;\ \mathit{is\text{-}refT}\ T_r\ ]\!] \Longrightarrow$
$P,E,h \vdash \mathit{throw}\ e : T$

| *WTrtTry*:
$[\![\ P,E,h \vdash e_1 : T_1;\ \ P,E(V \mapsto \mathit{Class}\ C),h \vdash e_2 : T_2;\ P \vdash T_1 \le T_2\ ]\!]$
$\Longrightarrow P,E,h \vdash \mathit{try}\ e_1\ \mathit{catch}(C\ V)\ e_2 : T_2$

— well-typed expression lists

| *WTrtNil*:
$P,E,h \vdash []\ [:]\ []$

| *WTrtCons*:
$[\![\ P,E,h \vdash e : T;\ \ P,E,h \vdash es\ [:]\ Ts\ ]\!]$
$\Longrightarrow\ \ P,E,h \vdash e\#es\ [:]\ T\#Ts$

2.17.1 Easy consequences

lemma [*iff*]: $(P,E,h \vdash [] \ [:] \ Ts) = (Ts = [])\langle proof\rangle$
lemma [*iff*]: $(P,E,h \vdash e\#es \ [:] \ T\#Ts) = (P,E,h \vdash e : T \wedge P,E,h \vdash es \ [:] \ Ts)\langle proof\rangle$
lemma [*iff*]: $(P,E,h \vdash (e\#es) \ [:] \ Ts) =$
$(\exists \, U \ Us. \ Ts = U\#Us \wedge P,E,h \vdash e : U \wedge P,E,h \vdash es \ [:] \ Us)\langle proof\rangle$
lemma [*simp*]: $\forall \, Ts. \ (P,E,h \vdash es_1 \ @ \ es_2 \ [:] \ Ts) =$
$(\exists \, Ts_1 \ Ts_2. \ Ts = Ts_1 \ @ \ Ts_2 \wedge P,E,h \vdash es_1 \ [:] \ Ts_1 \ \& \ P,E,h \vdash es_2 [:] Ts_2)\langle proof\rangle$
lemma [*iff*]: $P,E,h \vdash Val \ v : T = (typeof_h \ v = Some \ T)\langle proof\rangle$
lemma [*iff*]: $P,E,h \vdash Var \ v : T = (E \ v = Some \ T)\langle proof\rangle$
lemma [*iff*]: $P,E,h \vdash e_1;;e_2 : T_2 = (\exists \, T_1. \ P,E,h \vdash e_1{:}T_1 \wedge P,E,h \vdash e_2{:}T_2)\langle proof\rangle$
lemma [*iff*]: $P,E,h \vdash \{V{:}T; \ e\} : T' \ = \ (P,E(V \mapsto T),h \vdash e : T')\langle proof\rangle$

2.17.2 Some interesting lemmas

lemma *WTrts-Val*[*simp*]:
$\bigwedge Ts. \ (P,E,h \vdash map \ Val \ vs \ [:] \ Ts) = (map \ (typeof_h) \ vs = map \ Some \ Ts)\langle proof\rangle$

lemma *WTrts-same-length*: $\bigwedge Ts. \ P,E,h \vdash es \ [:] \ Ts \Longrightarrow length \ es = length \ Ts\langle proof\rangle$

lemma *WTrt-env-mono*:
$P,E,h \vdash e : T \Longrightarrow (\bigwedge E'. \ E \subseteq_m E' \Longrightarrow P,E',h \vdash e : T)$ **and**
$P,E,h \vdash es \ [:] \ Ts \Longrightarrow (\bigwedge E'. \ E \subseteq_m E' \Longrightarrow P,E',h \vdash es \ [:] \ Ts)\langle proof\rangle$

lemma *WTrt-hext-mono*: $P,E,h \vdash e : T \Longrightarrow h \trianglelefteq h' \Longrightarrow P,E,h' \vdash e : T$
and *WTrts-hext-mono*: $P,E,h \vdash es \ [:] \ Ts \Longrightarrow h \trianglelefteq h' \Longrightarrow P,E,h' \vdash es \ [:] \ Ts\langle proof\rangle$

lemma *WT-implies-WTrt*: $P,E \vdash e :: T \Longrightarrow P,E,h \vdash e : T$
and *WTs-implies-WTrts*: $P,E \vdash es \ [::] \ Ts \Longrightarrow P,E,h \vdash es \ [:] \ Ts\langle proof\rangle$

end

2.18 Definite assignment

theory *DefAss* **imports** *BigStep* **begin**

2.18.1 Hypersets

type-synonym $'a\ hyperset = 'a\ set\ option$

definition $hyperUn :: 'a\ hyperset \Rightarrow 'a\ hyperset \Rightarrow 'a\ hyperset$ (**infixl** $\sqcup$ *65*)
where
$A \sqcup B \ \equiv\ case\ A\ of\ None \Rightarrow None$
$\quad\quad | \lfloor A \rfloor \Rightarrow (case\ B\ of\ None \Rightarrow None\ |\ \lfloor B \rfloor \Rightarrow \lfloor A \cup B \rfloor)$

definition $hyperInt :: 'a\ hyperset \Rightarrow 'a\ hyperset \Rightarrow 'a\ hyperset$ (**infixl** $\sqcap$ *70*)
where
$A \sqcap B \ \equiv\ case\ A\ of\ None \Rightarrow B$
$\quad\quad | \lfloor A \rfloor \Rightarrow (case\ B\ of\ None \Rightarrow \lfloor A \rfloor\ |\ \lfloor B \rfloor \Rightarrow \lfloor A \cap B \rfloor)$

definition $hyperDiff1 :: 'a\ hyperset \Rightarrow 'a \Rightarrow 'a\ hyperset$ (**infixl** $\ominus$ *65*)
where
$A \ominus a \ \equiv\ case\ A\ of\ None \Rightarrow None\ |\ \lfloor A \rfloor \Rightarrow \lfloor A - \{a\} \rfloor$

definition $hyper\text{-}isin :: 'a \Rightarrow 'a\ hyperset \Rightarrow bool$ (**infix** $\in\in$ *50*)
where
$a \in\in A \ \equiv\ case\ A\ of\ None \Rightarrow True\ |\ \lfloor A \rfloor \Rightarrow a \in A$

definition $hyper\text{-}subset :: 'a\ hyperset \Rightarrow 'a\ hyperset \Rightarrow bool$ (**infix** $\sqsubseteq$ *50*)
where
$A \sqsubseteq B \ \equiv\ case\ B\ of\ None \Rightarrow True$
$\quad\quad | \lfloor B \rfloor \Rightarrow (case\ A\ of\ None \Rightarrow False\ |\ \lfloor A \rfloor \Rightarrow A \subseteq B)$

lemmas *hyperset-defs* =
hyperUn-def hyperInt-def hyperDiff1-def hyper-isin-def hyper-subset-def

lemma [*simp*]: $\lfloor\{\}\rfloor \sqcup A = A\ \wedge\ A \sqcup \lfloor\{\}\rfloor = A\langle proof\rangle$
lemma [*simp*]: $\lfloor A \rfloor \sqcup \lfloor B \rfloor = \lfloor A \cup B \rfloor \wedge \lfloor A \rfloor \ominus a = \lfloor A - \{a\} \rfloor\langle proof\rangle$
lemma [*simp*]: $None \sqcup A = None \wedge A \sqcup None = None\langle proof\rangle$
lemma [*simp*]: $a \in\in None \wedge None \ominus a = None\langle proof\rangle$
lemma *hyperUn-assoc*: $(A \sqcup B) \sqcup C = A \sqcup (B \sqcup C)\langle proof\rangle$
lemma *hyper-insert-comm*: $A \sqcup \lfloor\{a\}\rfloor = \lfloor\{a\}\rfloor \sqcup A \wedge A \sqcup (\lfloor\{a\}\rfloor \sqcup B) = \lfloor\{a\}\rfloor \sqcup (A \sqcup B)\langle proof\rangle$

2.18.2 Definite assignment

primrec
$\mathcal{A} :: 'a\ exp \Rightarrow 'a\ hyperset$
and $\mathcal{A}s :: 'a\ exp\ list \Rightarrow 'a\ hyperset$
where
$\mathcal{A}\ (new\ C) = \lfloor\{\}\rfloor$
$|\ \mathcal{A}\ (Cast\ C\ e) = \mathcal{A}\ e$
$|\ \mathcal{A}\ (Val\ v) = \lfloor\{\}\rfloor$
$|\ \mathcal{A}\ (e_1 \ll bop \gg e_2) = \mathcal{A}\ e_1 \sqcup \mathcal{A}\ e_2$
$|\ \mathcal{A}\ (Var\ V) = \lfloor\{\}\rfloor$
$|\ \mathcal{A}\ (LAss\ V\ e) = \lfloor\{V\}\rfloor \sqcup \mathcal{A}\ e$
$|\ \mathcal{A}\ (e{\cdot}F\{D\}) = \mathcal{A}\ e$
$|\ \mathcal{A}\ (e_1{\cdot}F\{D\}{:=}e_2) = \mathcal{A}\ e_1 \sqcup \mathcal{A}\ e_2$

| $\mathcal{A}$ $(e{\cdot}M(es)) = \mathcal{A}\ e \sqcup \mathcal{A}s\ es$
| $\mathcal{A}$ $(\{V{:}T;\ e\}) = \mathcal{A}\ e \ominus V$
| $\mathcal{A}$ $(e_1;;e_2) = \mathcal{A}\ e_1 \sqcup \mathcal{A}\ e_2$
| $\mathcal{A}$ $(\mathit{if}\ (e)\ e_1\ \mathit{else}\ e_2) = \mathcal{A}\ e \sqcup (\mathcal{A}\ e_1 \sqcap \mathcal{A}\ e_2)$
| $\mathcal{A}$ $(\mathit{while}\ (b)\ e) = \mathcal{A}\ b$
| $\mathcal{A}$ $(\mathit{throw}\ e) = \mathit{None}$
| $\mathcal{A}$ $(\mathit{try}\ e_1\ \mathit{catch}(C\ V)\ e_2) = \mathcal{A}\ e_1 \sqcap (\mathcal{A}\ e_2 \ominus V)$

| $\mathcal{A}s$ $([]) = \lfloor\{\}\rfloor$
| $\mathcal{A}s$ $(e\#es) = \mathcal{A}\ e \sqcup \mathcal{A}s\ es$

primrec
  $\mathcal{D}$ :: $'a\ \mathit{exp} \Rightarrow {'a}\ \mathit{hyperset} \Rightarrow \mathit{bool}$
  and $\mathcal{D}s$ :: $'a\ \mathit{exp\ list} \Rightarrow {'a}\ \mathit{hyperset} \Rightarrow \mathit{bool}$
where
  $\mathcal{D}\ (\mathit{new}\ C)\ A = \mathit{True}$
| $\mathcal{D}\ (\mathit{Cast}\ C\ e)\ A = \mathcal{D}\ e\ A$
| $\mathcal{D}\ (\mathit{Val}\ v)\ A = \mathit{True}$
| $\mathcal{D}\ (e_1 \ll \mathit{bop} \gg e_2)\ A = (\mathcal{D}\ e_1\ A \wedge \mathcal{D}\ e_2\ (A \sqcup \mathcal{A}\ e_1))$
| $\mathcal{D}\ (\mathit{Var}\ V)\ A = (V \in\in A)$
| $\mathcal{D}\ (\mathit{LAss}\ V\ e)\ A = \mathcal{D}\ e\ A$
| $\mathcal{D}\ (e{\cdot}F\{D\})\ A = \mathcal{D}\ e\ A$
| $\mathcal{D}\ (e_1{\cdot}F\{D\}{:=}e_2)\ A = (\mathcal{D}\ e_1\ A \wedge \mathcal{D}\ e_2\ (A \sqcup \mathcal{A}\ e_1))$
| $\mathcal{D}\ (e{\cdot}M(es))\ A = (\mathcal{D}\ e\ A \wedge \mathcal{D}s\ es\ (A \sqcup \mathcal{A}\ e))$
| $\mathcal{D}\ (\{V{:}T;\ e\})\ A = \mathcal{D}\ e\ (A \ominus V)$
| $\mathcal{D}\ (e_1;;e_2)\ A = (\mathcal{D}\ e_1\ A \wedge \mathcal{D}\ e_2\ (A \sqcup \mathcal{A}\ e_1))$
| $\mathcal{D}\ (\mathit{if}\ (e)\ e_1\ \mathit{else}\ e_2)\ A =$
  $(\mathcal{D}\ e\ A \wedge \mathcal{D}\ e_1\ (A \sqcup \mathcal{A}\ e) \wedge \mathcal{D}\ e_2\ (A \sqcup \mathcal{A}\ e))$
| $\mathcal{D}\ (\mathit{while}\ (e)\ c)\ A = (\mathcal{D}\ e\ A \wedge \mathcal{D}\ c\ (A \sqcup \mathcal{A}\ e))$
| $\mathcal{D}\ (\mathit{throw}\ e)\ A = \mathcal{D}\ e\ A$
| $\mathcal{D}\ (\mathit{try}\ e_1\ \mathit{catch}(C\ V)\ e_2)\ A = (\mathcal{D}\ e_1\ A \wedge \mathcal{D}\ e_2\ (A \sqcup \lfloor\{V\}\rfloor))$

| $\mathcal{D}s\ ([])\ A = \mathit{True}$
| $\mathcal{D}s\ (e\#es)\ A = (\mathcal{D}\ e\ A \wedge \mathcal{D}s\ es\ (A \sqcup \mathcal{A}\ e))$

lemma *As-map-Val*[*simp*]: $\mathcal{A}s\ (\mathit{map\ Val\ vs}) = \lfloor\{\}\rfloor\langle \mathit{proof}\rangle$
lemma *D-append*[*iff*]: $\bigwedge A.\ \mathcal{D}s\ (es\ @\ es')\ A = (\mathcal{D}s\ es\ A \wedge \mathcal{D}s\ es'\ (A \sqcup \mathcal{A}s\ es))\langle \mathit{proof}\rangle$

lemma *A-fv*: $\bigwedge A.\ \mathcal{A}\ e = \lfloor A\rfloor \Longrightarrow A \subseteq \mathit{fv}\ e$
and $\bigwedge A.\ \mathcal{A}s\ es = \lfloor A\rfloor \Longrightarrow A \subseteq \mathit{fvs}\ es\langle \mathit{proof}\rangle$

lemma *sqUn-lem*: $A \sqsubseteq A' \Longrightarrow A \sqcup B \sqsubseteq A' \sqcup B\langle \mathit{proof}\rangle$
lemma *diff-lem*: $A \sqsubseteq A' \Longrightarrow A \ominus b \sqsubseteq A' \ominus b\langle \mathit{proof}\rangle$

lemma *D-mono*: $\bigwedge A\ A'.\ A \sqsubseteq A' \Longrightarrow \mathcal{D}\ e\ A \Longrightarrow \mathcal{D}\ (e{::}'a\ \mathit{exp})\ A'$
and *Ds-mono*: $\bigwedge A\ A'.\ A \sqsubseteq A' \Longrightarrow \mathcal{D}s\ es\ A \Longrightarrow \mathcal{D}s\ (es{::}'a\ \mathit{exp\ list})\ A'\langle \mathit{proof}\rangle$

lemma *D-mono'*: $\mathcal{D}\ e\ A \Longrightarrow A \sqsubseteq A' \Longrightarrow \mathcal{D}\ e\ A'$
and *Ds-mono'*: $\mathcal{D}s\ es\ A \Longrightarrow A \sqsubseteq A' \Longrightarrow \mathcal{D}s\ es\ A'\langle \mathit{proof}\rangle$

end

2.19 Conformance Relations for Type Soundness Proofs

theory *Conform*
imports *Exceptions*
begin

definition $conf :: {}'m\ prog \Rightarrow heap \Rightarrow val \Rightarrow ty \Rightarrow bool$ $(\text{-,-} \vdash \text{-} :\leq \text{-}\ [51,51,51,51]\ 50)$
where
$P,h \vdash v :\leq T \equiv$
$\exists T'.\ typeof_h\ v = Some\ T' \wedge P \vdash T' \leq T$

definition $oconf :: {}'m\ prog \Rightarrow heap \Rightarrow obj \Rightarrow bool$ $(\text{-,-} \vdash \text{-} \surd\ [51,51,51]\ 50)$
where
$P,h \vdash obj \surd \equiv$
$let\ (C,fs) = obj\ in\ \forall F\ D\ T.\ P \vdash C\ has\ F{:}T\ in\ D \longrightarrow$
$(\exists v.\ fs(F,D) = Some\ v \wedge P,h \vdash v :\leq T)$

definition $hconf :: {}'m\ prog \Rightarrow heap \Rightarrow bool$ $(\text{-} \vdash \text{-} \surd\ [51,51]\ 50)$
where
$P \vdash h \surd \equiv$
$(\forall a\ obj.\ h\ a = Some\ obj \longrightarrow P,h \vdash obj \surd) \wedge preallocated\ h$

definition $lconf :: {}'m\ prog \Rightarrow heap \Rightarrow (vname \rightharpoonup val) \Rightarrow (vname \rightharpoonup ty) \Rightarrow bool$ $(\text{-,-} \vdash \text{-}\ '(:\leq')\ \text{-}$ $[51,51,51,51]\ 50)$
where
$P,h \vdash l\ (:\leq)\ E \equiv$
$\forall V\ v.\ l\ V = Some\ v \longrightarrow (\exists T.\ E\ V = Some\ T \wedge P,h \vdash v :\leq T)$

abbreviation
$confs :: {}'m\ prog \Rightarrow heap \Rightarrow val\ list \Rightarrow ty\ list \Rightarrow bool$
$(\text{-,-} \vdash \text{-}\ [:\leq]\ \text{-}\ [51,51,51,51]\ 50)$ **where**
$P,h \vdash vs\ [:\leq]\ Ts \equiv list\text{-}all2\ (conf\ P\ h)\ vs\ Ts$

2.19.1 Value conformance $:\leq$

lemma *conf-Null* [*simp*]: $P,h \vdash Null :\leq T = P \vdash NT \leq T\langle proof\rangle$
lemma *typeof-conf*[*simp*]: $typeof_h\ v = Some\ T \Longrightarrow P,h \vdash v :\leq T\langle proof\rangle$
lemma *typeof-lit-conf*[*simp*]: $typeof\ v = Some\ T \Longrightarrow P,h \vdash v :\leq T\langle proof\rangle$
lemma *defval-conf*[*simp*]: $P,h \vdash default\text{-}val\ T :\leq T\langle proof\rangle$
lemma *conf-upd-obj*: $h\ a = Some(C,fs) \Longrightarrow (P,h(a{\mapsto}(C,fs')) \vdash x :\leq T) = (P,h \vdash x :\leq T)\langle proof\rangle$
lemma *conf-widen*: $P,h \vdash v :\leq T \Longrightarrow P \vdash T \leq T' \Longrightarrow P,h \vdash v :\leq T'\langle proof\rangle$
lemma *conf-hext*: $h \trianglelefteq h' \Longrightarrow P,h \vdash v :\leq T \Longrightarrow P,h' \vdash v :\leq T\langle proof\rangle$
lemma *conf-ClassD*: $P,h \vdash v :\leq Class\ C \Longrightarrow$
$v = Null \vee (\exists a\ obj\ T.\ v = Addr\ a \wedge h\ a = Some\ obj \wedge obj\text{-}ty\ obj = T \wedge P \vdash T \leq Class\ C)\langle proof\rangle$
lemma *conf-NT* [*iff*]: $P,h \vdash v :\leq NT = (v = Null)\langle proof\rangle$
lemma *non-npD*: $[\![\ v \neq Null;\ P,h \vdash v :\leq Class\ C\ ]\!]$
$\Longrightarrow \exists a\ C'\ fs.\ v = Addr\ a \wedge h\ a = Some(C',fs) \wedge P \vdash C' \preceq^* C\langle proof\rangle$

2.19.2 Value list conformance $[:\leq]$

lemma *confs-widens* [*trans*]: $[\![P,h \vdash vs\ [:\leq]\ Ts;\ P \vdash Ts\ [\leq]\ Ts']\!] \Longrightarrow P,h \vdash vs\ [:\leq]\ Ts'\langle proof\rangle$
lemma *confs-rev*: $P,h \vdash rev\ s\ [:\leq]\ t = (P,h \vdash s\ [:\leq]\ rev\ t)\langle proof\rangle$
lemma *confs-conv-map*:
$\bigwedge Ts'.\ P,h \vdash vs\ [:\leq]\ Ts' = (\exists Ts.\ map\ typeof_h\ vs = map\ Some\ Ts \wedge P \vdash Ts\ [\leq]\ Ts')\langle proof\rangle$
lemma *confs-hext*: $P,h \vdash vs\ [:\leq]\ Ts \Longrightarrow h \trianglelefteq h' \Longrightarrow P,h' \vdash vs\ [:\leq]\ Ts\langle proof\rangle$

lemma *confs-Cons2*: $P,h \vdash xs\ [:\leq]\ y\#ys = (\exists z\ zs.\ xs = z\#zs \wedge P,h \vdash z :\leq y \wedge P,h \vdash zs\ [:\leq]\ ys)\langle proof\rangle$

2.19.3 Object conformance

lemma *oconf-hext*: $P,h \vdash obj \surd \Longrightarrow h \trianglelefteq h' \Longrightarrow P,h' \vdash obj \surd\langle proof\rangle$
lemma *oconf-init-fields*:
$P \vdash C\ has\text{-}fields\ FDTs \Longrightarrow P,h \vdash (C,\ init\text{-}fields\ FDTs) \surd$
$\langle proof\rangle$

lemma *oconf-fupd* [*intro?*]:
$[\![\ P \vdash C\ has\ F{:}T\ in\ D;\ P,h \vdash v :\leq T;\ P,h \vdash (C,fs) \surd\ ]\!]$
$\Longrightarrow P,h \vdash (C,\ fs((F,D)\mapsto v)) \surd\langle proof\rangle$

2.19.4 Heap conformance

lemma *hconfD*: $[\![\ P \vdash h \surd;\ h\ a = Some\ obj\ ]\!] \Longrightarrow P,h \vdash obj \surd\langle proof\rangle$
lemma *hconf-new*: $[\![\ P \vdash h \surd;\ h\ a = None;\ P,h \vdash obj \surd\ ]\!] \Longrightarrow P \vdash h(a\mapsto obj) \surd\langle proof\rangle$
lemma *hconf-upd-obj*: $[\![\ P \vdash h\surd;\ h\ a = Some(C,fs);\ P,h \vdash (C,fs')\surd\ ]\!] \Longrightarrow P \vdash h(a\mapsto(C,fs'))\surd\langle proof\rangle$

2.19.5 Local variable conformance

lemma *lconf-hext*: $[\![\ P,h \vdash l\ (:\leq)\ E;\ h \trianglelefteq h'\ ]\!] \Longrightarrow P,h' \vdash l\ (:\leq)\ E\langle proof\rangle$
lemma *lconf-upd*:
$[\![\ P,h \vdash l\ (:\leq)\ E;\ P,h \vdash v :\leq T;\ E\ V = Some\ T\ ]\!] \Longrightarrow P,h \vdash l(V\mapsto v)\ (:\leq)\ E\langle proof\rangle$
lemma *lconf-empty*[*iff*]: $P,h \vdash empty\ (:\leq)\ E\langle proof\rangle$
lemma *lconf-upd2*: $[\![P,h \vdash l\ (:\leq)\ E;\ P,h \vdash v :\leq T]\!] \Longrightarrow P,h \vdash l(V\mapsto v)\ (:\leq)\ E(V\mapsto T)\langle proof\rangle$

end

2.20 Progress of Small Step Semantics

theory *Progress*
imports *Equivalence WellTypeRT DefAss ../Common/Conform*
begin

lemma *final-addrE*:
$\llbracket\ P,E,h \vdash e : Class\ C;\ final\ e;$
$\bigwedge a.\ e = addr\ a \Longrightarrow R;$
$\bigwedge a.\ e = Throw\ a \Longrightarrow R\ \rrbracket \Longrightarrow R\langle proof\rangle$

lemma *finalRefE*:
$\llbracket\ P,E,h \vdash e : T;\ is\text{-}refT\ T;\ final\ e;$
$e = null \Longrightarrow R;$
$\bigwedge a\ C.\ \llbracket\ e = addr\ a;\ T = Class\ C\ \rrbracket \Longrightarrow R;$
$\bigwedge a.\ e = Throw\ a \Longrightarrow R\ \rrbracket \Longrightarrow R\langle proof\rangle$

Derivation of new induction scheme for well typing:

inductive
$WTrt' :: [J\text{-}prog,heap,env,expr,ty] \Rightarrow bool$
and $WTrts' :: [J\text{-}prog,heap,env,expr\ list,\ ty\ list] \Rightarrow bool$
and $WTrt2' :: [J\text{-}prog,env,heap,expr,ty] \Rightarrow bool$
$(\text{-},\text{-},\text{-} \vdash \text{-} :' \text{-}\quad [51,51,51]50)$
and $WTrts2' :: [J\text{-}prog,env,heap,expr\ list,\ ty\ list] \Rightarrow bool$
$(\text{-},\text{-},\text{-} \vdash \text{-}\ [:'] \text{-}\ [51,51,51]50)$
for $P :: J\text{-}prog$ **and** $h :: heap$
where
$P,E,h \vdash e :' T \equiv WTrt'\ P\ h\ E\ e\ T$
$|\ P,E,h \vdash es\ [:']\ Ts \equiv WTrts'\ P\ h\ E\ es\ Ts$

$|\ is\text{-}class\ P\ C \Longrightarrow P,E,h \vdash new\ C :' Class\ C$
$|\ \llbracket\ P,E,h \vdash e :' T;\ is\text{-}refT\ T;\ is\text{-}class\ P\ C\ \rrbracket$
$\Longrightarrow P,E,h \vdash Cast\ C\ e :' Class\ C$
$|\ typeof_h\ v = Some\ T \Longrightarrow P,E,h \vdash Val\ v :' T$
$|\ E\ v = Some\ T \Longrightarrow P,E,h \vdash Var\ v :' T$
$|\ \llbracket\ P,E,h \vdash e_1 :' T_1;\ P,E,h \vdash e_2 :' T_2\ \rrbracket$
$\Longrightarrow P,E,h \vdash e_1 \ll Eq \gg e_2 :' Boolean$
$|\ \llbracket\ P,E,h \vdash e_1 :' Integer;\ P,E,h \vdash e_2 :' Integer\ \rrbracket$
$\Longrightarrow P,E,h \vdash e_1 \ll Add \gg e_2 :' Integer$
$|\ \llbracket\ P,E,h \vdash Var\ V :' T;\ P,E,h \vdash e :' T';\ P \vdash T' \leq T\ (*\ V \neq This*)\ \rrbracket$
$\Longrightarrow P,E,h \vdash V{:=}e :' Void$
$|\ \llbracket\ P,E,h \vdash e :' Class\ C;\ P \vdash C\ has\ F{:}T\ in\ D\ \rrbracket \Longrightarrow P,E,h \vdash e{\bullet}F\{D\} :' T$
$|\ P,E,h \vdash e :' NT \Longrightarrow P,E,h \vdash e{\bullet}F\{D\} :' T$
$|\ \llbracket\ P,E,h \vdash e_1 :' Class\ C;\ P \vdash C\ has\ F{:}T\ in\ D;$
$P,E,h \vdash e_2 :' T_2;\ P \vdash T_2 \leq T\ \rrbracket$
$\Longrightarrow P,E,h \vdash e_1{\bullet}F\{D\}{:=}e_2 :' Void$
$|\ \llbracket\ P,E,h \vdash e_1{:'}NT;\ P,E,h \vdash e_2 :' T_2\ \rrbracket \Longrightarrow P,E,h \vdash e_1{\bullet}F\{D\}{:=}e_2 :' Void$
$|\ \llbracket\ P,E,h \vdash e :' Class\ C;\ P \vdash C\ sees\ M{:}Ts \rightarrow T = (pns,body)\ in\ D;$
$P,E,h \vdash es\ [:']\ Ts';\ P \vdash Ts'\ [\leq]\ Ts\ \rrbracket$
$\Longrightarrow P,E,h \vdash e{\bullet}M(es) :' T$
$|\ \llbracket\ P,E,h \vdash e :' NT;\ P,E,h \vdash es\ [:']\ Ts\ \rrbracket \Longrightarrow P,E,h \vdash e{\bullet}M(es) :' T$
$|\ P,E,h \vdash []\ [:']\ []$
$|\ \llbracket\ P,E,h \vdash e :' T;\ P,E,h \vdash es\ [:']\ Ts\ \rrbracket \Longrightarrow P,E,h \vdash e\#es\ [:']\ T\#Ts$
$|\ \llbracket\ typeof_h\ v = Some\ T_1;\ P \vdash T_1 \leq T;\ P,E(V{\mapsto}T),h \vdash e_2 :' T_2\ \rrbracket$

$\Longrightarrow\ P,E,h \vdash \{V{:}T := Val\ v;\ e_2\} :'\ T_2$
$|\ [\![\ P,E(V{\mapsto}T),h \vdash e :'\ T';\ \neg\ assigned\ V\ e\ ]\!] \Longrightarrow\ P,E,h \vdash \{V{:}T;\ e\} :'\ T'$
$|\ [\![\ P,E,h \vdash e_1{:}'\ T_1;\ \ P,E,h \vdash e_2{:}'T_2\ ]\!]\ \Longrightarrow\ P,E,h \vdash e_1;;e_2 :'\ T_2$
$|\ [\![\ P,E,h \vdash e :'\ Boolean;\ \ P,E,h \vdash e_1{:}'\ T_1;\ \ P,E,h \vdash e_2{:}'\ T_2;$
$P \vdash T_1 \leq T_2 \lor P \vdash T_2 \leq T_1;$
$P \vdash T_1 \leq T_2 \longrightarrow T = T_2;\ P \vdash T_2 \leq T_1 \longrightarrow T = T_1\ ]\!]$
$\Longrightarrow P,E,h \vdash if\ (e)\ e_1\ else\ e_2 :'\ T$

$|\ [\![\ P,E,h \vdash e :'\ Boolean;\ \ P,E,h \vdash c{:}'\ T\ ]\!]$
$\Longrightarrow\ P,E,h \vdash while(e)\ c :'\ Void$
$|\ [\![\ P,E,h \vdash e :'\ T_r;\ is\text{-}refT\ T_r\ ]\!]\ \Longrightarrow\ P,E,h \vdash throw\ e :'\ T$
$|\ [\![\ P,E,h \vdash e_1 :'\ T_1;\ \ P,E(V \mapsto Class\ C),h \vdash e_2 :'\ T_2;\ P \vdash T_1 \leq T_2\ ]\!]$
$\Longrightarrow P,E,h \vdash try\ e_1\ catch(C\ V)\ e_2 :'\ T_2$

lemma [*iff*]: $P,E,h \vdash e_1;;e_2 :'\ T_2 = (\exists\, T_1.\ P,E,h \vdash e_1{:}'\ T_1 \land P,E,h \vdash e_2{:}'\ T_2)\langle proof\rangle$
lemma [*iff*]: $P,E,h \vdash\ Val\ v :'\ T = (typeof_h\ v = Some\ T)\langle proof\rangle$
lemma [*iff*]: $P,E,h \vdash\ Var\ v :'\ T = (E\ v = Some\ T)\langle proof\rangle$

lemma *wt-wt'*: $P,E,h \vdash e : T \Longrightarrow P,E,h \vdash e :'\ T$
and *wts-wts'*: $P,E,h \vdash es\ [:]\ Ts \Longrightarrow P,E,h \vdash es\ [:']\ Ts\langle proof\rangle$

lemma *wt'-wt*: $P,E,h \vdash e :'\ T \Longrightarrow P,E,h \vdash e : T$
and *wts'-wts*: $P,E,h \vdash es\ [:']\ Ts \Longrightarrow P,E,h \vdash es\ [:]\ Ts\langle proof\rangle$

corollary *wt'-iff-wt*: $(P,E,h \vdash e :'\ T) = (P,E,h \vdash e : T)\langle proof\rangle$

corollary *wts'-iff-wts*: $(P,E,h \vdash es\ [:']\ Ts) = (P,E,h \vdash es\ [:]\ Ts)\langle proof\rangle$
theorem assumes *wf*: *wwf-J-prog* P **and** *hconf*: $P \vdash h \surd$
shows *progress*: $P,E,h \vdash e : T \Longrightarrow$
$(\bigwedge l.\ [\![\ \mathcal{D}\ e\ \lfloor dom\ l\rfloor;\ \neg\ final\ e\ ]\!] \Longrightarrow \exists\, e'\ s'.\ P \vdash \langle e,(h,l)\rangle \rightarrow \langle e',s'\rangle)$
and $P,E,h \vdash es\ [:]\ Ts \Longrightarrow$
$(\bigwedge l.\ [\![\ \mathcal{D}s\ es\ \lfloor dom\ l\rfloor;\ \neg\ finals\ es\ ]\!] \Longrightarrow \exists\, es'\ s'.\ P \vdash \langle es,(h,l)\rangle\ [\rightarrow]\ \langle es',s'\rangle)\langle proof\rangle$

end

2.21 Well-formedness Constraints

theory *JWellForm*
imports ../*Common*/*WellForm* *WWellForm* *WellType* *DefAss*
begin

definition *wf-J-mdecl* :: *J-prog* ⇒ *cname* ⇒ *J-mb mdecl* ⇒ *bool*
where
wf-J-mdecl P C ≡ λ(*M*,*Ts*,*T*,(*pns*,*body*)).
length Ts = *length pns* ∧
distinct pns ∧
this ∉ *set pns* ∧
(∃ *T*′. *P*,[*this*↦*Class C*,*pns*[↦]*Ts*] ⊢ *body* :: *T*′ ∧ *P* ⊢ *T*′ ≤ *T*) ∧
𝒟 *body* ⌊{*this*} ∪ *set pns*⌋

lemma *wf-J-mdecl*[*simp*]:
wf-J-mdecl P C (*M*,*Ts*,*T*,*pns*,*body*) ≡
(*length Ts* = *length pns* ∧
distinct pns ∧
this ∉ *set pns* ∧
(∃ *T*′. *P*,[*this*↦*Class C*,*pns*[↦]*Ts*] ⊢ *body* :: *T*′ ∧ *P* ⊢ *T*′ ≤ *T*) ∧
𝒟 *body* ⌊{*this*} ∪ *set pns*⌋)⟨*proof*⟩

abbreviation
wf-J-prog :: *J-prog* ⇒ *bool* **where**
wf-J-prog == *wf-prog wf-J-mdecl*

lemma *wf-J-prog-wf-J-mdecl*:
⟦ *wf-J-prog P*; (*C*, *D*, *fds*, *mths*) ∈ *set P*; *jmdcl* ∈ *set mths* ⟧
⟹ *wf-J-mdecl P C jmdcl*⟨*proof*⟩

lemma *wf-mdecl-wwf-mdecl*: *wf-J-mdecl P C Md* ⟹ *wwf-J-mdecl P C Md*⟨*proof*⟩

lemma *wf-prog-wwf-prog*: *wf-J-prog P* ⟹ *wwf-J-prog P*⟨*proof*⟩

end

2.22 Type Safety Proof

theory *TypeSafe*
imports *Progress JWellForm*
begin

2.22.1 Basic preservation lemmas

First two easy preservation lemmas.

theorem *red-preserves-hconf*:
$P \vdash \langle e,(h,l)\rangle \to \langle e',(h',l')\rangle \Longrightarrow (\bigwedge T\ E.\ [\![\ P,E,h \vdash e : T;\ P \vdash h \surd\ ]\!] \Longrightarrow P \vdash h' \surd)$
and *reds-preserves-hconf*:
$P \vdash \langle es,(h,l)\rangle\ [\to]\ \langle es',(h',l')\rangle \Longrightarrow (\bigwedge Ts\ E.\ [\![\ P,E,h \vdash es\ [:]\ Ts;\ P \vdash h \surd\ ]\!] \Longrightarrow P \vdash h' \surd)\langle proof\rangle$

theorem *red-preserves-lconf*:
$P \vdash \langle e,(h,l)\rangle \to \langle e',(h',l')\rangle \Longrightarrow$
$(\bigwedge T\ E.\ [\![\ P,E,h \vdash e{:}T;\ P,h \vdash l\ (:\le)\ E\ ]\!] \Longrightarrow P,h' \vdash l'\ (:\le)\ E)$
and *reds-preserves-lconf*:
$P \vdash \langle es,(h,l)\rangle\ [\to]\ \langle es',(h',l')\rangle \Longrightarrow$
$(\bigwedge Ts\ E.\ [\![\ P,E,h \vdash es[:]Ts;\ P,h \vdash l\ (:\le)\ E\ ]\!] \Longrightarrow P,h' \vdash l'\ (:\le)\ E)\langle proof\rangle$

Preservation of definite assignment more complex and requires a few lemmas first.

lemma [*iff*]: $\bigwedge A.\ [\![\ length\ Vs = length\ Ts;\ length\ vs = length\ Ts]\!] \Longrightarrow$
$\mathcal{D}\ (blocks\ (Vs,Ts,vs,e))\ A = \mathcal{D}\ e\ (A \sqcup \lfloor set\ Vs \rfloor)\langle proof\rangle$

lemma *red-lA-incr*: $P \vdash \langle e,(h,l)\rangle \to \langle e',(h',l')\rangle \Longrightarrow \lfloor dom\ l\rfloor \sqcup \mathcal{A}\ e \sqsubseteq\ \lfloor dom\ l'\rfloor \sqcup \mathcal{A}\ e'$
and *reds-lA-incr*: $P \vdash \langle es,(h,l)\rangle\ [\to]\ \langle es',(h',l')\rangle \Longrightarrow \lfloor dom\ l\rfloor \sqcup \mathcal{A}s\ es \sqsubseteq\ \lfloor dom\ l'\rfloor \sqcup \mathcal{A}s\ es'\langle proof\rangle$

Now preservation of definite assignment.

lemma assumes *wf*: *wf-J-prog P*
shows *red-preserves-defass*:
$P \vdash \langle e,(h,l)\rangle \to \langle e',(h',l')\rangle \Longrightarrow \mathcal{D}\ e\ \lfloor dom\ l\rfloor \Longrightarrow \mathcal{D}\ e'\ \lfloor dom\ l'\rfloor$
and $P \vdash \langle es,(h,l)\rangle\ [\to]\ \langle es',(h',l')\rangle \Longrightarrow \mathcal{D}s\ es\ \lfloor dom\ l\rfloor \Longrightarrow \mathcal{D}s\ es'\ \lfloor dom\ l'\rfloor\langle proof\rangle$

Combining conformance of heap and local variables:

definition *sconf* :: *J-prog* $\Rightarrow$ *env* $\Rightarrow$ *state* $\Rightarrow$ *bool* (-,- $\vdash$ - $\surd$ [51,51,51]50)
where
$P,E \vdash s \surd\ \equiv\ let\ (h,l) = s\ in\ P \vdash h \surd \wedge P,h \vdash l\ (:\le)\ E$

lemma *red-preserves-sconf*:
$[\![\ P \vdash \langle e,s\rangle \to \langle e',s'\rangle;\ P,E,hp\ s \vdash e : T;\ P,E \vdash s \surd\ ]\!] \Longrightarrow P,E \vdash s' \surd\langle proof\rangle$
lemma *reds-preserves-sconf*:
$[\![\ P \vdash \langle es,s\rangle\ [\to]\ \langle es',s'\rangle;\ P,E,hp\ s \vdash es\ [:]\ Ts;\ P,E \vdash s \surd\ ]\!] \Longrightarrow P,E \vdash s' \surd\langle proof\rangle$

2.22.2 Subject reduction

lemma *wt-blocks*:
$\bigwedge E.\ [\![\ length\ Vs = length\ Ts;\ length\ vs = length\ Ts\ ]\!] \Longrightarrow$
$(P,E,h \vdash blocks(Vs,Ts,vs,e) : T) =$
$(P,E(Vs[\mapsto]Ts),h \vdash e{:}T \wedge (\exists Ts'.\ map\ (typeof_h)\ vs = map\ Some\ Ts' \wedge P \vdash Ts'\ [\le]\ Ts))\langle proof\rangle$

theorem assumes *wf*: *wf-J-prog P*
shows *subject-reduction2*: $P \vdash \langle e,(h,l)\rangle \to \langle e',(h',l')\rangle \Longrightarrow$
$(\bigwedge E\ T.\ [\![\ P,E \vdash (h,l) \surd;\ P,E,h \vdash e{:}T\ ]\!]$

$\Longrightarrow \exists\, T'.\ P,E,h' \vdash e'{:}T' \wedge P \vdash T' \leq T)$
and *subjects-reduction2*: $P \vdash \langle es,(h,l)\rangle\ [\rightarrow]\ \langle es',(h',l')\rangle \Longrightarrow$
$(\bigwedge E\ Ts.\ [\![\ P,E \vdash (h,l)\ \surd;\ P,E,h \vdash es\ [:]\ Ts\ ]\!]$
$\Longrightarrow \exists\, Ts'.\ P,E,h' \vdash es'\ [:]\ Ts' \wedge P \vdash Ts'\ [\leq]\ Ts)\langle proof\rangle$

corollary *subject-reduction*:
$[\![\ wf\text{-}J\text{-}prog\ P;\ P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle;\ P,E \vdash s\ \surd;\ P,E,hp\ s \vdash e{:}T\ ]\!]$
$\Longrightarrow \exists\, T'.\ P,E,hp\ s' \vdash e'{:}T' \wedge P \vdash T' \leq T\langle proof\rangle$
corollary *subjects-reduction*:
$[\![\ wf\text{-}J\text{-}prog\ P;\ P \vdash \langle es,s\rangle\ [\rightarrow]\ \langle es',s'\rangle;\ P,E \vdash s\ \surd;\ P,E,hp\ s \vdash es[:]Ts\ ]\!]$
$\Longrightarrow \exists\, Ts'.\ P,E,hp\ s' \vdash es'[:]Ts' \wedge P \vdash Ts'\ [\leq]\ Ts\langle proof\rangle$

2.22.3 Lifting to $\rightarrow*$

Now all these preservation lemmas are first lifted to the transitive closure . . .

lemma *Red-preserves-sconf*:
assumes *wf*: *wf-J-prog P* **and** *Red*: $P \vdash \langle e,s\rangle \rightarrow* \langle e',s'\rangle$
shows $\bigwedge T.\ [\![\ P,E,hp\ s \vdash e : T;\ P,E \vdash s\ \surd\ ]\!] \Longrightarrow P,E \vdash s'\ \surd\langle proof\rangle$

lemma *Red-preserves-defass*:
assumes *wf*: *wf-J-prog P* **and** *reds*: $P \vdash \langle e,s\rangle \rightarrow* \langle e',s'\rangle$
shows $\mathcal{D}\ e\ \lfloor dom(lcl\ s)\rfloor \Longrightarrow \mathcal{D}\ e'\ \lfloor dom(lcl\ s')\rfloor$
$\langle proof\rangle$

lemma *Red-preserves-type*:
assumes *wf*: *wf-J-prog P* **and** *Red*: $P \vdash \langle e,s\rangle \rightarrow* \langle e',s'\rangle$
shows $!!T.\ [\![\ P,E \vdash s\surd;\ P,E,hp\ s \vdash e{:}T\ ]\!]$
$\Longrightarrow \exists\, T'.\ P \vdash T' \leq T \wedge P,E,hp\ s' \vdash e'{:}T'\langle proof\rangle$

2.22.4 Lifting to $\Rightarrow$

. . . and now to the big step semantics, just for fun.

lemma *eval-preserves-sconf*:
$[\![\ wf\text{-}J\text{-}prog\ P;\ P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle;\ P,E \vdash e{::}T;\ P,E \vdash s\surd\ ]\!] \Longrightarrow P,E \vdash s'\surd\langle proof\rangle$

lemma *eval-preserves-type*: **assumes** *wf*: *wf-J-prog P*
shows $[\![\ P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle;\ P,E \vdash s\surd;\ P,E \vdash e{::}T\ ]\!]$
$\Longrightarrow \exists\, T'.\ P \vdash T' \leq T \wedge P,E,hp\ s' \vdash e'{:}T'\langle proof\rangle$

2.22.5 The final polish

The above preservation lemmas are now combined and packed nicely.

definition *wf-config* :: *J-prog* $\Rightarrow$ *env* $\Rightarrow$ *state* $\Rightarrow$ *expr* $\Rightarrow$ *ty* $\Rightarrow$ *bool* (-,-,- $\vdash$ - : - $\surd$ [*51,0,0,0,0*]*50*)
where
$P,E,s \vdash e{:}T\ \surd\ \equiv\ P,E \vdash s\ \surd \wedge P,E,hp\ s \vdash e{:}T$

theorem *Subject-reduction*: **assumes** *wf*: *wf-J-prog P*
shows $P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle \Longrightarrow P,E,s \vdash e : T\ \surd$
$\Longrightarrow \exists\, T'.\ P,E,s' \vdash e' : T'\ \surd \wedge P \vdash T' \leq T\langle proof\rangle$

theorem *Subject-reductions*:
assumes *wf*: *wf-J-prog P* **and** *reds*: $P \vdash \langle e,s\rangle \rightarrow* \langle e',s'\rangle$

shows $\bigwedge T.\ P,E,s \vdash e{:}T \ \surd \Longrightarrow \exists\, T'.\ P,E,s' \vdash e'{:}T' \ \surd \wedge P \vdash T' \leq T \langle proof \rangle$

corollary *Progress*: **assumes** *wf*: *wf-J-prog P*
shows $[\![\ P,E,s\ \vdash e : T \ \surd;\ \mathcal{D}\ e\ \lfloor dom(lcl\ s)\rfloor;\ \neg\ final\ e\ ]\!] \Longrightarrow \exists\, e'\ s'.\ P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle \langle proof \rangle$

corollary *TypeSafety*:
$[\![\ wf\text{-}J\text{-}prog\ P;\ P,E \vdash s \ \surd;\ P,E \vdash e{::}T;\ \mathcal{D}\ e\ \lfloor dom(lcl\ s)\rfloor;$
$P \vdash \langle e,s\rangle \rightarrow\!* \langle e',s'\rangle;\ \neg(\exists\, e''\ s''.\ P \vdash \langle e',s'\rangle \rightarrow \langle e'',s''\rangle)\ ]\!]$
$\Longrightarrow (\exists\, v.\ e' = Val\ v \wedge P,hp\ s' \vdash v :\leq T) \vee$
$(\exists\, a.\ e' = Throw\ a \wedge a \in dom(hp\ s'))\langle proof \rangle$

end

2.23 Program annotation

theory *Annotate* **imports** *WellType* **begin**

inductive
  $Anno :: [J\text{-}prog, env, expr \quad , expr] \Rightarrow bool$
    $(\text{-},\text{-} \vdash \text{-} \rightsquigarrow \text{-} \quad [51,0,0,51]50)$
  and $Annos :: [J\text{-}prog, env, expr\ list, expr\ list] \Rightarrow bool$
    $(\text{-},\text{-} \vdash \text{-} [\rightsquigarrow] \text{-} [51,0,0,51]50)$
  for $P :: J\text{-}prog$
where

$AnnoNew$: $P,E \vdash new\ C \rightsquigarrow new\ C$
| $AnnoCast$: $P,E \vdash e \rightsquigarrow e' \Longrightarrow P,E \vdash Cast\ C\ e \rightsquigarrow Cast\ C\ e'$
| $AnnoVal$: $P,E \vdash Val\ v \rightsquigarrow Val\ v$
| $AnnoVarVar$: $E\ V = \lfloor T \rfloor \Longrightarrow P,E \vdash Var\ V \rightsquigarrow Var\ V$
| $AnnoVarField$: $[\![\ E\ V = None;\ E\ this = \lfloor Class\ C \rfloor;\ P \vdash C\ sees\ V{:}T\ in\ D\ ]\!]$
  $\Longrightarrow P,E \vdash Var\ V \rightsquigarrow Var\ this{\cdot}V\{D\}$
| $AnnoBinOp$:
  $[\![\ P,E \vdash e1 \rightsquigarrow e1';\ \ P,E \vdash e2 \rightsquigarrow e2'\ ]\!]$
  $\Longrightarrow P,E \vdash e1 \ll bop \gg e2 \rightsquigarrow e1' \ll bop \gg e2'$
| $AnnoLAssVar$:
  $[\![\ E\ V = \lfloor T \rfloor;\ P,E \vdash e \rightsquigarrow e'\ ]\!] \Longrightarrow P,E \vdash V{:=}e \rightsquigarrow V{:=}e'$
| $AnnoLAssField$:
  $[\![\ E\ V = None;\ E\ this = \lfloor Class\ C \rfloor;\ P \vdash C\ sees\ V{:}T\ in\ D;\ P,E \vdash e \rightsquigarrow e'\ ]\!]$
  $\Longrightarrow P,E \vdash V{:=}e \rightsquigarrow Var\ this{\cdot}V\{D\} := e'$
| $AnnoFAcc$:
  $[\![\ P,E \vdash e \rightsquigarrow e';\ \ P,E \vdash e' :: Class\ C;\ \ P \vdash C\ sees\ F{:}T\ in\ D\ ]\!]$
  $\Longrightarrow P,E \vdash e{\cdot}F\{[]\} \rightsquigarrow e'{\cdot}F\{D\}$
| $AnnoFAss$: $[\![\ P,E \vdash e1 \rightsquigarrow e1';\ \ P,E \vdash e2 \rightsquigarrow e2';$
  $P,E \vdash e1' :: Class\ C;\ P \vdash C\ sees\ F{:}T\ in\ D\ ]\!]$
  $\Longrightarrow P,E \vdash e1{\cdot}F\{[]\} := e2 \rightsquigarrow e1'{\cdot}F\{D\} := e2'$
| $AnnoCall$:
  $[\![\ P,E \vdash e \rightsquigarrow e';\ \ P,E \vdash es\ [\rightsquigarrow]\ es'\ ]\!]$
  $\Longrightarrow P,E \vdash Call\ e\ M\ es \rightsquigarrow Call\ e'\ M\ es'$
| $AnnoBlock$:
  $P,E(V \mapsto T) \vdash e \rightsquigarrow e' \ \Longrightarrow\ P,E \vdash \{V{:}T;\ e\} \rightsquigarrow \{V{:}T;\ e'\}$
| $AnnoComp$: $[\![\ P,E \vdash e1 \rightsquigarrow e1';\ \ P,E \vdash e2 \rightsquigarrow e2'\ ]\!]$
  $\Longrightarrow\ P,E \vdash e1;;e2 \rightsquigarrow e1';;e2'$
| $AnnoCond$: $[\![\ P,E \vdash e \rightsquigarrow e';\ P,E \vdash e1 \rightsquigarrow e1';\ \ P,E \vdash e2 \rightsquigarrow e2'\ ]\!]$
  $\Longrightarrow P,E \vdash if\ (e)\ e1\ else\ e2 \rightsquigarrow if\ (e')\ e1'\ else\ e2'$
| $AnnoLoop$: $[\![\ P,E \vdash e \rightsquigarrow e';\ \ P,E \vdash c \rightsquigarrow c'\ ]\!]$
  $\Longrightarrow P,E \vdash while\ (e)\ c \rightsquigarrow while\ (e')\ c'$
| $AnnoThrow$: $P,E \vdash e \rightsquigarrow e' \ \Longrightarrow\ P,E \vdash throw\ e \rightsquigarrow throw\ e'$
| $AnnoTry$: $[\![\ P,E \vdash e1 \rightsquigarrow e1';\ \ P,E(V \mapsto Class\ C) \vdash e2 \rightsquigarrow e2'\ ]\!]$
  $\Longrightarrow P,E \vdash try\ e1\ catch(C\ V)\ e2 \rightsquigarrow try\ e1'\ catch(C\ V)\ e2'$

| $AnnoNil$: $P,E \vdash []\ [\rightsquigarrow]\ []$
| $AnnoCons$: $[\![\ P,E \vdash e \rightsquigarrow e';\ \ P,E \vdash es\ [\rightsquigarrow]\ es'\ ]\!]$
  $\Longrightarrow\ P,E \vdash e\#es\ [\rightsquigarrow]\ e'\#es'$

end

2.24 Example Expressions

theory *Examples* **imports** *Expr* **begin**

```
definition classObject::J-mb cdecl
where
  classObject == (''Object'','''',[],[])
```

```
definition classI :: J-mb cdecl
where
  classI ==
  (''I'', Object,
  [],
  [(''mult'',[Integer,Integer],Integer,[''i'',''j''],
   if (Var ''i'' «Eq» Val(Intg 0)) (Val(Intg 0))
   else Var ''j'' «Add»
       Var this · ''mult''([Var ''i'' «Add» Val(Intg -1),Var ''j'']))
  ])
```

```
definition classL :: J-mb cdecl
where
  classL ==
  (''L'', Object,
  [(''F'',Integer), (''N'',Class ''L'')],
  [(''app'',[Class ''L''],Void,[''l''],
   if (Var this · ''N''{''L''} «Eq» null)
      (Var this · ''N''{''L''} := Var ''l'')
   else (Var this · ''N''{''L''}) · ''app''([Var ''l'']))
  ])
```

```
definition testExpr-BuildList :: expr
where
  testExpr-BuildList ==
  {''l1'':Class ''L'' := new ''L'';
   Var ''l1''·''F''{''L''} := Val(Intg 1);;
  {''l2'':Class ''L'' := new ''L'';
   Var ''l2''· ''F''{''L''} := Val(Intg 2);;
  {''l3'':Class ''L'' := new ''L'';
   Var ''l3''· ''F''{''L''} := Val(Intg 3);;
  {''l4'':Class ''L'' := new ''L'';
   Var ''l4''· ''F''{''L''} := Val(Intg 4);;
   Var ''l1''·''app''([Var ''l2'']);;
   Var ''l1''·''app''([Var ''l3'']);;
   Var ''l1''·''app''([Var ''l4''])}}}}
```

```
definition testExpr1 ::expr
where
  testExpr1 == Val(Intg 5)
definition testExpr2 ::expr
where
  testExpr2 == BinOp (Val(Intg 5)) Add (Val(Intg 6))
```

definition *testExpr3* ::*expr*
where
testExpr3 == *BinOp* (*Var* ''*V*'') *Add* (*Val*(*Intg* *6*))
definition *testExpr4* ::*expr*
where
testExpr4 == ''*V*'' := *Val*(*Intg* *6*)
definition *testExpr5* ::*expr*
where
testExpr5 == *new* ''*Object*'';; {''*V*'':(*Class* ''*C*'') := *new* ''*C*''; *Var* ''*V*''•''*F*''{''*C*''} := *Val*(*Intg* *42*)}
definition *testExpr6* ::*expr*
where
testExpr6 == {''*V*'':(*Class* ''*I*'') := *new* ''*I*''; *Var* ''*V*''•''*mult*''([*Val*(*Intg* *40*),*Val*(*Intg* *4*)])}

definition *mb-isNull*:: *expr*
where
mb-isNull == *Var this* • ''*test*''{''*A*''} ≪*Eq*≫ *null*

definition *mb-add*:: *expr*
where
mb-add == (*Var this* • ''*int*''{''*A*''} :=(*Var this* • ''*int*''{''*A*''} ≪*Add*≫ *Var* ''*i*''));; (*Var this* • ''*int*''{''*A*''})

definition *mb-mult-cond*:: *expr*
where
mb-mult-cond == (*Var* ''*j*'' ≪*Eq*≫ *Val* (*Intg* *0*)) ≪*Eq*≫ *Val* (*Bool False*)

definition *mb-mult-block*:: *expr*
where
mb-mult-block == ''*temp*'':=(*Var* ''*temp*'' ≪*Add*≫ *Var* ''*i*'');;''*j*'':=(*Var* ''*j*'' ≪*Add*≫ *Val* (*Intg* −*1*))

definition *mb-mult*:: *expr*
where
mb-mult == {''*temp*'':*Integer*:=*Val* (*Intg* *0*); *While* (*mb-mult-cond*) *mb-mult-block*;; (*Var this* • ''*int*''{''*A*''} := *Var* ''*temp*'';; *Var* ''*temp*'')}

definition *classA*:: *J-mb cdecl*
where
classA ==
(''*A*'', *Object*,
[(''*int*'',*Integer*),
(''*test*'',*Class* ''*A*'')],
[(''*isNull*'',[],*Boolean*,[], *mb-isNull*),
(''*add*'',[*Integer*],*Integer*,[''*i*''], *mb-add*),
(''*mult*'',[*Integer*,*Integer*],*Integer*,[''*i*'',''*j*''], *mb-mult*)])

definition *testExpr-ClassA*:: *expr*
where
testExpr-ClassA ==
{''*A1*'':*Class* ''*A*'':= *new* ''*A*'';
{''*A2*'':*Class* ''*A*'':= *new* ''*A*'';
{''*testint*'':*Integer*:= *Val* (*Intg* *5*);
(*Var* ''*A2*''• ''*int*''{''*A*''} := (*Var* ''*A1*''• ''*add*''([*Var* ''*testint*'']));;

$(Var\ ''A2''\cdot\ ''int''\{''A''\} := (Var\ ''A1''\cdot\ ''add''([Var\ ''testint''])));;$
$Var\ ''A2''\cdot\ ''mult''([Var\ ''A2''\cdot\ ''int''\{''A''\},\ Var\ ''testint''])\ \}\}\}$

end

2.25 Code Generation For BigStep

theory *execute-Bigstep*
imports
  BigStep Examples
  ~~/src/HOL/Library/Efficient-Nat
begin

inductive *map-val* :: *expr list* ⇒ *val list* ⇒ *bool*
where
  Nil: *map-val* [] []
| *Cons*: *map-val xs ys* ⟹ *map-val* (*Val y* # *xs*) (*y* # *ys*)

inductive *map-val2* :: *expr list* ⇒ *val list* ⇒ *expr list* ⇒ *bool*
where
  Nil: *map-val2* [] [] []
| *Cons*: *map-val2 xs ys zs* ⟹ *map-val2* (*Val y* # *xs*) (*y* # *ys*) *zs*
| *Throw*: *map-val2* (*throw e* # *xs*) [] (*throw e* # *xs*)

theorem *map-val-conv*: (*xs* = *map Val ys*) = *map-val xs ys*⟨*proof*⟩
theorem *map-val2-conv*:
 (*xs* = *map Val ys* @ *throw e* # *zs*) = *map-val2 xs ys* (*throw e* # *zs*)⟨*proof*⟩
lemma *CallNull2*:
  $[\![P \vdash \langle e,s_0\rangle \Rightarrow \langle null,s_1\rangle;\ \ P \vdash \langle ps,s_1\rangle\ [\Rightarrow]\ \langle evs,s_2\rangle;\ map\text{-}val\ evs\ vs\ ]\!]$
  $\Longrightarrow P \vdash \langle e\cdot M(ps),s_0\rangle \Rightarrow \langle THROW\ NullPointer,s_2\rangle$
⟨*proof*⟩

lemma *CallParamsThrow2*:
  $[\![P \vdash \langle e,s_0\rangle \Rightarrow \langle Val\ v,s_1\rangle;\ P \vdash \langle es,s_1\rangle\ [\Rightarrow]\ \langle evs,s_2\rangle;$
    $map\text{-}val2\ evs\ vs\ (throw\ ex\ \#\ es'')\ ]\!]$
   $\Longrightarrow P \vdash \langle e\cdot M(es),s_0\rangle \Rightarrow \langle throw\ ex,s_2\rangle$
⟨*proof*⟩

lemma *Call2*:
  $[\![P \vdash \langle e,s_0\rangle \Rightarrow \langle addr\ a,s_1\rangle;\ \ P \vdash \langle ps,s_1\rangle\ [\Rightarrow]\ \langle evs,(h_2,l_2)\rangle;$
    $map\text{-}val\ evs\ vs;$
    $h_2\ a = Some(C,fs);\ \ P \vdash C\ sees\ M{:}Ts{\rightarrow}T = (pns,body)\ in\ D;$
    $length\ vs = length\ pns;\ \ l_2' = [this{\mapsto}Addr\ a,\ pns[\mapsto]vs];$
    $P \vdash \langle body,(h_2,l_2')\rangle \Rightarrow \langle e',(h_3,l_3)\rangle\ ]\!]$
  $\Longrightarrow P \vdash \langle e\cdot M(ps),s_0\rangle \Rightarrow \langle e',(h_3,l_2)\rangle$
⟨*proof*⟩

code-pred
  (*modes*: *i* ⇒ *o* ⇒ *bool*)
  map-val
⟨*proof*⟩

code-pred
  (*modes*: *i* ⇒ *o* ⇒ *o* ⇒ *bool*)
  map-val2
⟨*proof*⟩

lemmas [*code-pred-intro*] =

eval-evals.New eval-evals.NewFail
eval-evals.Cast eval-evals.CastNull eval-evals.CastFail eval-evals.CastThrow
eval-evals.Val eval-evals.Var
eval-evals.BinOp eval-evals.BinOpThrow1 eval-evals.BinOpThrow2
eval-evals.LAss eval-evals.LAssThrow
eval-evals.FAcc eval-evals.FAccNull eval-evals.FAccThrow
eval-evals.FAss eval-evals.FAssNull
eval-evals.FAssThrow1 eval-evals.FAssThrow2
eval-evals.CallObjThrow

declare *CallNull2* [*code-pred-intro CallNull2*]
declare *CallParamsThrow2* [*code-pred-intro CallParamsThrow2*]
declare *Call2* [*code-pred-intro Call2*]

lemmas [*code-pred-intro*] =
eval-evals.Block
eval-evals.Seq eval-evals.SeqThrow
eval-evals.CondT eval-evals.CondF eval-evals.CondThrow
eval-evals.WhileF eval-evals.WhileT
eval-evals.WhileCondThrow

declare *eval-evals.WhileBodyThrow* [*code-pred-intro WhileBodyThrow2*]

lemmas [*code-pred-intro*] =
eval-evals.Throw eval-evals.ThrowNull
eval-evals.ThrowThrow
eval-evals.Try eval-evals.TryCatch eval-evals.TryThrow
eval-evals.Nil eval-evals.Cons eval-evals.ConsThrow

code-pred
(*modes*: $i \Rightarrow i \Rightarrow i \Rightarrow o \Rightarrow o \Rightarrow$ *bool as execute*)
eval
⟨*proof*⟩

notation *execute* (- ⊢ ((*1*⟨-,/-⟩) ⇒/ ⟨′-, ′-⟩) [*51*,*0*,*0*] *81*)

definition *test1* = [] ⊢ ⟨*testExpr1*,(*empty*,*empty*)⟩ ⇒ ⟨-,-⟩
definition *test2* = [] ⊢ ⟨*testExpr2*,(*empty*,*empty*)⟩ ⇒ ⟨-,-⟩
definition *test3* = [] ⊢ ⟨*testExpr3*,(*empty*,*empty*(′′*V*′′↦*Intg 77*))⟩ ⇒ ⟨-,-⟩
definition *test4* = [] ⊢ ⟨*testExpr4*,(*empty*,*empty*)⟩ ⇒ ⟨-,-⟩
definition *test5* = [(′′*Object*′′,(′′′′,[],[])),(′′*C*′′,(′′*Object*′′,[(′′*F*′′,*Integer*)],[]))] ⊢ ⟨*testExpr5*,(*empty*,*empty*)⟩ ⇒ ⟨-,-⟩
definition *test6* = [(′′*Object*′′,(′′′′,[],[])), *classI*] ⊢ ⟨*testExpr6*,(*empty*,*empty*)⟩ ⇒ ⟨-,-⟩

definition *V* = ′′*V*′′
definition *C* = ′′*C*′′
definition *F* = ′′*F*′′

⟨*ML*⟩

definition *test7* = [*classObject*, *classL*] ⊢ ⟨*testExpr-BuildList*, (*empty*,*empty*)⟩ ⇒ ⟨-,-⟩

definition *L* = ′′*L*′′
definition *N* = ′′*N*′′

$\langle ML \rangle$

definition $test8 = [classObject,\ classA] \vdash \langle testExpr\text{-}ClassA,\ (empty,empty)\rangle \Rightarrow \langle\text{-,-}\rangle$
definition $i = ''int''$
definition $t = ''test''$
definition $A = ''A''$

$\langle ML \rangle$

end

2.26 Code Generation For WellType

theory *execute-WellType*
imports
WellType Examples
begin

lemma *WTCond1*:
$[\![P,E \vdash e :: \mathit{Boolean};\ \ P,E \vdash e_1{::}T_1;\ \ P,E \vdash e_2{::}T_2;\ P \vdash T_1 \leq T_2;$
$P \vdash T_2 \leq T_1 \longrightarrow T_2 = T_1\ ]\!] \Longrightarrow P,E \vdash \mathit{if}\ (e)\ e_1\ \mathit{else}\ e_2 :: T_2$
⟨*proof*⟩

lemma *WTCond2*:
$[\![P,E \vdash e :: \mathit{Boolean};\ \ P,E \vdash e_1{::}T_1;\ \ P,E \vdash e_2{::}T_2;\ P \vdash T_2 \leq T_1;$
$P \vdash T_1 \leq T_2 \longrightarrow T_1 = T_2\ ]\!] \Longrightarrow P,E \vdash \mathit{if}\ (e)\ e_1\ \mathit{else}\ e_2 :: T_1$
⟨*proof*⟩

lemmas [*code-pred-intro*] =
WT-WTs.WTNew
WT-WTs.WTCast
WT-WTs.WTVal
WT-WTs.WTVar
WT-WTs.WTBinOpEq
WT-WTs.WTBinOpAdd
WT-WTs.WTLAss
WT-WTs.WTFAcc
WT-WTs.WTFAss
WT-WTs.WTCall
WT-WTs.WTBlock
WT-WTs.WTSeq

declare
WTCond1 [*code-pred-intro WTCond1*]
WTCond2 [*code-pred-intro WTCond2*]

lemmas [*code-pred-intro*] =
WT-WTs.WTWhile
WT-WTs.WTThrow
WT-WTs.WTTry
WT-WTs.WTNil
WT-WTs.WTCons

code-pred
(*modes*: $i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow$ *bool as type-check*, $i \Rightarrow i \Rightarrow i \Rightarrow o \Rightarrow$ *bool as infer-type*)
WT
⟨*proof*⟩

notation *infer-type* (-,- ⊢ - :: ′- [*51*,*51*,*51*]*100*)

definition *test1* **where** *test1* = [],*empty* ⊢ *testExpr1* :: -
definition *test2* **where** *test2* = [], *empty* ⊢ *testExpr2* :: -
definition *test3* **where** *test3* = [], *empty*(′′*V*′′ ↦ *Integer*) ⊢ *testExpr3* :: -

definition *test4* **where** *test4* $= []$, *empty*$(''V'' \mapsto Integer)$ $\vdash$ *testExpr4* :: -
definition *test5* **where** *test5* $=$ [*classObject*, $(''C'',(''Object'',[(''F'',Integer)],[]))$], *empty* $\vdash$ *testExpr5* :: -
definition *test6* **where** *test6* $=$ [*classObject*, *classI*], *empty* $\vdash$ *testExpr6* :: -

$\langle ML \rangle$

definition *testmb-isNull* **where** *testmb-isNull* $=$ [*classObject*, *classA*], *empty*([*this*] $[\mapsto]$ [*Class* $''A''$]) $\vdash$ *mb-isNull* :: -
definition *testmb-add* **where** *testmb-add* $=$ [*classObject*, *classA*], *empty*([*this*,$''i''$] $[\mapsto]$ [*Class* $''A''$,*Integer*]) $\vdash$ *mb-add* :: -
definition *testmb-mult-cond* **where** *testmb-mult-cond* $=$ [*classObject*, *classA*], *empty*([*this*,$''j''$] $[\mapsto]$ [*Class* $''A''$,*Integer*]) $\vdash$ *mb-mult-cond* :: -
definition *testmb-mult-block* **where** *testmb-mult-block* $=$ [*classObject*, *classA*], *empty*([*this*,$''i''$,$''j''$,$''temp''$] $[\mapsto]$ [*Class* $''A''$,*Integer*,*Integer*,*Integer*]) $\vdash$ *mb-mult-block* :: -
definition *testmb-mult* **where** *testmb-mult* $=$ [*classObject*, *classA*], *empty*([*this*,$''i''$,$''j''$] $[\mapsto]$ [*Class* $''A''$,*Integer*,*Integer*]) $\vdash$ *mb-mult* :: -

$\langle ML \rangle$

definition *test* **where** *test* $=$ [*classObject*, *classA*], *empty* $\vdash$ *testExpr-ClassA* :: -

$\langle ML \rangle$

end

Chapter 3

Jinja Virtual Machine

3.1 State of the JVM

theory *JVMState* **imports** ../*Common*/*Objects* **begin**

3.1.1 Frame Stack

type-synonym
pc = *nat*

type-synonym
frame = *val list* × *val list* × *cname* × *mname* × *pc*
— operand stack
— registers (including this pointer, method parameters, and local variables)
— name of class where current method is defined
— parameter types
— program counter within frame

3.1.2 Runtime State

type-synonym
jvm-state = *addr option* × *heap* × *frame list*
— exception flag, heap, frames

end

3.2 Instructions of the JVM

theory *JVMInstructions* **imports** *JVMState* **begin**

datatype
instr = *Load nat* — load from local variable
| *Store nat* — store into local variable
| *Push val* — push a value (constant)
| *New cname* — create object
| *Getfield vname cname* — Fetch field from object
| *Putfield vname cname* — Set field in object
| *Checkcast cname* — Check whether object is of given type
| *Invoke mname nat* — inv. instance meth of an object
| *Return* — return from method
| *Pop* — pop top element from opstack
| *IAdd* — integer addition
| *Goto int* — goto relative address
| *CmpEq* — equality comparison
| *IfFalse int* — branch if top of stack false
| *Throw* — throw top of stack as exception

type-synonym
bytecode = *instr list*

type-synonym
ex-entry = *pc* × *pc* × *cname* × *pc* × *nat*
— start-pc, end-pc, exception type, handler-pc, remaining stack depth

type-synonym
ex-table = *ex-entry list*

type-synonym
jvm-method = *nat* × *nat* × *bytecode* × *ex-table*
— max stacksize
— number of local variables. Add 1 + no. of parameters to get no. of registers
— instruction sequence
— exception handler table

type-synonym
jvm-prog = *jvm-method prog*

end

3.3 JVM Instruction Semantics

```
theory JVMExecInstr
imports JVMInstructions JVMState ../Common/Exceptions
begin

primrec
  exec-instr :: [instr, jvm-prog, heap, val list, val list,
                 cname, mname, pc, frame list] => jvm-state
where
exec-instr-Load:
 exec-instr (Load n) P h stk loc C0 M0 pc frs =
     (None, h, ((loc ! n) # stk, loc, C0, M0, pc+1)#frs)

| exec-instr (Store n) P h stk loc C0 M0 pc frs =
     (None, h, (tl stk, loc[n:=hd stk], C0, M0, pc+1)#frs)

| exec-instr-Push:
 exec-instr (Push v) P h stk loc C0 M0 pc frs =
     (None, h, (v # stk, loc, C0, M0, pc+1)#frs)

| exec-instr-New:
 exec-instr (New C) P h stk loc C0 M0 pc frs =
  (case new-Addr h of
    None ⇒   (Some (addr-of-sys-xcpt OutOfMemory), h, (stk, loc, C0, M0, pc)#frs)
  | Some a ⇒ (None, h(a↦blank P C), (Addr a#stk, loc, C0, M0, pc+1)#frs))

| exec-instr (Getfield F C) P h stk loc C0 M0 pc frs =
  (let v      = hd stk;
       xp'    = if v=Null then ⌊addr-of-sys-xcpt NullPointer⌋ else None;
       (D,fs) = the(h(the-Addr v))
   in (xp', h, (the(fs(F,C))#(tl stk), loc, C0, M0, pc+1)#frs))

| exec-instr (Putfield F C) P h stk loc C0 M0 pc frs =
  (let v    = hd stk;
       r    = hd (tl stk);
       xp'  = if r=Null then ⌊addr-of-sys-xcpt NullPointer⌋ else None;
       a    = the-Addr r;
       (D,fs) = the (h a);
       h'   = h(a ↦ (D, fs((F,C) ↦ v)))
   in (xp', h', (tl (tl stk), loc, C0, M0, pc+1)#frs))

| exec-instr (Checkcast C) P h stk loc C0 M0 pc frs =
  (let v   = hd stk;
       xp' = if ¬cast-ok P C h v then ⌊addr-of-sys-xcpt ClassCast⌋ else None
   in (xp', h, (stk, loc, C0, M0, pc+1)#frs))

| exec-instr-Invoke:
 exec-instr (Invoke M n) P h stk loc C0 M0 pc frs =
  (let ps  = take n stk;
       r   = stk!n;
       xp' = if r=Null then ⌊addr-of-sys-xcpt NullPointer⌋ else None;
       C   = fst(the(h(the-Addr r)));
       (D,M',Ts,mxs,mxl0,ins,xt)= method P C M;
```

f' = ([],[r]@(rev ps)@(replicate mxl_0 undefined),D,M,0)
in (xp', h, f'#(stk, loc, C_0, M_0, pc)#frs))

| exec-instr Return P h stk_0 loc_0 C_0 M_0 pc frs =
(if frs=[] then (None, h, []) else
let v = hd stk_0;
(stk,loc,C,m,pc) = hd frs;
n = length (fst (snd (method P C_0 M_0)))
in (None, h, (v#(drop (n+1) stk),loc,C,m,pc+1)#tl frs))

| exec-instr Pop P h stk loc C_0 M_0 pc frs =
(None, h, (tl stk, loc, C_0, M_0, pc+1)#frs)

| exec-instr IAdd P h stk loc C_0 M_0 pc frs =
(let i_2 = the-Intg (hd stk);
i_1 = the-Intg (hd (tl stk))
in (None, h, (Intg (i_1+i_2)#(tl (tl stk)), loc, C_0, M_0, pc+1)#frs))

| exec-instr (IfFalse i) P h stk loc C_0 M_0 pc frs =
(let pc' = if hd stk = Bool False then nat(int pc+i) else pc+1
in (None, h, (tl stk, loc, C_0, M_0, pc')#frs))

| exec-instr CmpEq P h stk loc C_0 M_0 pc frs =
(let v_2 = hd stk;
v_1 = hd (tl stk)
in (None, h, (Bool (v_1=v_2) # tl (tl stk), loc, C_0, M_0, pc+1)#frs))

| exec-instr-Goto:
exec-instr (Goto i) P h stk loc C_0 M_0 pc frs =
(None, h, (stk, loc, C_0, M_0, nat(int pc+i))#frs)

| exec-instr Throw P h stk loc C_0 M_0 pc frs =
(let xp' = if hd stk = Null then ⌊addr-of-sys-xcpt NullPointer⌋ else ⌊the-Addr(hd stk)⌋
in (xp', h, (stk, loc, C_0, M_0, pc)#frs))

lemma exec-instr-Store:
exec-instr (Store n) P h (v#stk) loc C_0 M_0 pc frs =
(None, h, (stk, loc[n:=v], C_0, M_0, pc+1)#frs)
⟨proof⟩

lemma exec-instr-Getfield:
exec-instr (Getfield F C) P h (v#stk) loc C_0 M_0 pc frs =
(let xp' = if v=Null then ⌊addr-of-sys-xcpt NullPointer⌋ else None;
(D,fs) = the(h(the-Addr v))
in (xp', h, (the(fs(F,C))#stk, loc, C_0, M_0, pc+1)#frs))
⟨proof⟩

lemma exec-instr-Putfield:
exec-instr (Putfield F C) P h (v#r#stk) loc C_0 M_0 pc frs =
(let xp' = if r=Null then ⌊addr-of-sys-xcpt NullPointer⌋ else None;
a = the-Addr r;
(D,fs) = the (h a);
h' = h(a ↦ (D, fs((F,C) ↦ v)))

$in\ (xp',\ h',\ (stk,\ loc,\ C_0,\ M_0,\ pc{+}1)\#frs))$
⟨*proof*⟩

lemma *exec-instr-Checkcast*:
$exec\text{-}instr\ (Checkcast\ C)\ P\ h\ (v\#stk)\ loc\ C_0\ M_0\ pc\ frs =$
$(let\ xp' = if\ \neg cast\text{-}ok\ P\ C\ h\ v\ then\ \lfloor addr\text{-}of\text{-}sys\text{-}xcpt\ ClassCast \rfloor\ else\ None$
$in\ (xp',\ h,\ (v\#stk,\ loc,\ C_0,\ M_0,\ pc{+}1)\#frs))$
⟨*proof*⟩

lemma *exec-instr-Return*:
$exec\text{-}instr\ Return\ P\ h\ (v\#stk_0)\ loc_0\ C_0\ M_0\ pc\ frs =$
$(if\ frs{=}[]\ then\ (None,\ h,\ [])\ else$
$let\ (stk,loc,C,m,pc) = hd\ frs;$
$n = length\ (fst\ (snd\ (method\ P\ C_0\ M_0)))$
$in\ (None,\ h,\ (v\#(drop\ (n{+}1)\ stk),loc,C,m,pc{+}1)\#tl\ frs))$
⟨*proof*⟩

lemma *exec-instr-IPop*:
$exec\text{-}instr\ Pop\ P\ h\ (v\#stk)\ loc\ C_0\ M_0\ pc\ frs =$
$(None,\ h,\ (stk,\ loc,\ C_0,\ M_0,\ pc{+}1)\#frs)$
⟨*proof*⟩

lemma *exec-instr-IAdd*:
$exec\text{-}instr\ IAdd\ P\ h\ (Intg\ i_2\ \#\ Intg\ i_1\ \#\ stk)\ loc\ C_0\ M_0\ pc\ frs =$
$(None,\ h,\ (Intg\ (i_1{+}i_2)\#stk,\ loc,\ C_0,\ M_0,\ pc{+}1)\#frs)$
⟨*proof*⟩

lemma *exec-instr-IfFalse*:
$exec\text{-}instr\ (IfFalse\ i)\ P\ h\ (v\#stk)\ loc\ C_0\ M_0\ pc\ frs =$
$(let\ pc' = if\ v = Bool\ False\ then\ nat(int\ pc{+}i)\ else\ pc{+}1$
$in\ (None,\ h,\ (stk,\ loc,\ C_0,\ M_0,\ pc')\#frs))$
⟨*proof*⟩

lemma *exec-instr-CmpEq*:
$exec\text{-}instr\ CmpEq\ P\ h\ (v_2\#v_1\#stk)\ loc\ C_0\ M_0\ pc\ frs =$
$(None,\ h,\ (Bool\ (v_1{=}v_2)\ \#\ stk,\ loc,\ C_0,\ M_0,\ pc{+}1)\#frs)$
⟨*proof*⟩

lemma *exec-instr-Throw*:
$exec\text{-}instr\ Throw\ P\ h\ (v\#stk)\ loc\ C_0\ M_0\ pc\ frs =$
$(let\ xp' = if\ v = Null\ then\ \lfloor addr\text{-}of\text{-}sys\text{-}xcpt\ NullPointer \rfloor\ else\ \lfloor the\text{-}Addr\ v \rfloor$
$in\ (xp',\ h,\ (v\#stk,\ loc,\ C_0,\ M_0,\ pc)\#frs))$
⟨*proof*⟩

end

3.4 Exception handling in the JVM

theory *JVMExceptions* **imports** *JVMInstructions ../Common/Exceptions* **begin**

definition *matches-ex-entry* :: $'m$ *prog* $\Rightarrow$ *cname* $\Rightarrow$ *pc* $\Rightarrow$ *ex-entry* $\Rightarrow$ *bool*
where
matches-ex-entry P C pc xcp $\equiv$
let $(s,\ e,\ C',\ h,\ d) = xcp$ *in*
$s \le pc \wedge pc < e \wedge P \vdash C \preceq^* C'$

primrec *match-ex-table* :: $'m$ *prog* $\Rightarrow$ *cname* $\Rightarrow$ *pc* $\Rightarrow$ *ex-table* $\Rightarrow$ $(pc \times nat)$ *option*
where
match-ex-table P C pc [] = *None*
| *match-ex-table P C pc* (*e*#*es*) = (*if matches-ex-entry P C pc e*
then Some (*snd*(*snd*(*snd e*)))
else match-ex-table P C pc es)

abbreviation
ex-table-of :: *jvm-prog* $\Rightarrow$ *cname* $\Rightarrow$ *mname* $\Rightarrow$ *ex-table* **where**
ex-table-of P C M == *snd* (*snd* (*snd* (*snd* (*snd* (*snd*(*method P C M*))))))

primrec *find-handler* :: *jvm-prog* $\Rightarrow$ *addr* $\Rightarrow$ *heap* $\Rightarrow$ *frame list* $\Rightarrow$ *jvm-state*
where
find-handler P a h [] = (*Some a*, *h*, [])
| *find-handler P a h* (*fr*#*frs*) =
(*let* (*stk*,*loc*,*C*,*M*,*pc*) = *fr in*
case match-ex-table P (*cname-of h a*) *pc* (*ex-table-of P C M*) *of*
None $\Rightarrow$ *find-handler P a h frs*
| *Some pc-d* $\Rightarrow$ (*None*, *h*, (*Addr a* # *drop* (*size stk* − *snd pc-d*) *stk*, *loc*, *C*, *M*, *fst pc-d*)#*frs*))

end

3.5 Program Execution in the JVM

theory *JVMExec*
imports *JVMExecInstr JVMExceptions*
begin

abbreviation
  instrs-of :: *jvm-prog* $\Rightarrow$ *cname* $\Rightarrow$ *mname* $\Rightarrow$ *instr list* **where**
  instrs-of P C M == *fst*(*snd*(*snd*(*snd*(*snd*(*snd*(*method P C M*))))))

fun *exec* :: *jvm-prog* $\times$ *jvm-state* => *jvm-state option* **where** — single step execution
  exec (*P*, *xp*, *h*, []) = *None*

| *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*) =
  (*let*
    i = *instrs-of P C M* ! *pc*;
    (*xcpt*$'$, *h*$'$, *frs*$'$) = *exec-instr i P h stk loc C M pc frs*
   in Some(*case xcpt*$'$ *of*
          None $\Rightarrow$ (*None*,*h*$'$,*frs*$'$)
        | *Some a* $\Rightarrow$ *find-handler P a h* ((*stk*,*loc*,*C*,*M*,*pc*)#*frs*)))

| *exec* (*P*, *Some xa*, *h*, *frs*) = *None*

— relational view
inductive-set
  exec-1 :: *jvm-prog* $\Rightarrow$ (*jvm-state* $\times$ *jvm-state*) *set*
  and *exec-1*$'$ :: *jvm-prog* $\Rightarrow$ *jvm-state* $\Rightarrow$ *jvm-state* $\Rightarrow$ *bool*
   (- $\vdash$/ - $-jvm\to_1$/ - [*61*,*61*,*61*] *60*)
  for *P* :: *jvm-prog*
where
  $P \vdash \sigma\ -jvm\to_1\ \sigma' \equiv (\sigma,\sigma') \in$ *exec-1 P*
| *exec-1I*: *exec* (P,σ) = *Some* $\sigma' \Longrightarrow P \vdash \sigma\ -jvm\to_1\ \sigma'$

— reflexive transitive closure:
definition *exec-all* :: *jvm-prog* $\Rightarrow$ *jvm-state* $\Rightarrow$ *jvm-state* $\Rightarrow$ *bool*
        (- |−/ - −*jvm*−>/ - [*61*,*61*,*61*]*60*) **where**

  exec-all-def1: *P* |− σ −*jvm*−> $\sigma' \longleftrightarrow (\sigma,\sigma') \in$ (*exec-1 P*)*

notation (*xsymbols*)
  exec-all ((- $\vdash$/ - $-jvm\to$/ -) [*61*,*61*,*61*]*60*)

lemma *exec-1-eq*:
  exec-1 P = $\{(\sigma,\sigma')$. *exec* (P,σ) = *Some* $\sigma'\}\langle$*proof*$\rangle$
lemma *exec-1-iff*:
  $P \vdash \sigma\ -jvm\to_1\ \sigma'$ = (*exec* (P,σ) = *Some* σ')$\langle$*proof*$\rangle$
lemma *exec-all-def*:
  $P \vdash \sigma\ -jvm\to\ \sigma'$ = $((\sigma,\sigma') \in \{(\sigma,\sigma')$. *exec* (P,σ) = *Some* $\sigma'\}^*)\langle$*proof*$\rangle$
lemma *jvm-refl*[*iff*]: $P \vdash \sigma\ -jvm\to\ \sigma\langle$*proof*$\rangle$
lemma *jvm-trans*[*trans*]:
$[\![\ P \vdash \sigma\ -jvm\to\ \sigma';\ P \vdash \sigma'\ -jvm\to\ \sigma''\ ]\!] \Longrightarrow P \vdash \sigma\ -jvm\to\ \sigma''\langle$*proof*$\rangle$
lemma *jvm-one-step1*[*trans*]:
$[\![\ P \vdash \sigma\ -jvm\to_1\ \sigma';\ P \vdash \sigma'\ -jvm\to\ \sigma''\ ]\!] \Longrightarrow P \vdash \sigma\ -jvm\to\ \sigma''\langle$*proof*$\rangle$

lemma *jvm-one-step2*[*trans*]:
$[\![\ P \vdash \sigma\ -jvm\rightarrow\ \sigma';\ P \vdash \sigma'\ -jvm\rightarrow_1\ \sigma''\ ]\!] \Longrightarrow P \vdash \sigma\ -jvm\rightarrow\ \sigma''\langle proof\rangle$
lemma *exec-all-conf*:
$[\![\ P \vdash \sigma\ -jvm\rightarrow\ \sigma';\ P \vdash \sigma\ -jvm\rightarrow\ \sigma''\ ]\!]$
$\Longrightarrow P \vdash \sigma'\ -jvm\rightarrow\ \sigma'' \vee P \vdash \sigma''\ -jvm\rightarrow\ \sigma'\langle proof\rangle$

lemma *exec-all-finalD*: $P \vdash (x,\ h,\ [])\ -jvm\rightarrow\ \sigma \Longrightarrow \sigma = (x,\ h,\ [])\langle proof\rangle$
lemma *exec-all-deterministic*:
$[\![\ P \vdash \sigma\ -jvm\rightarrow\ (x,h,[]);\ P \vdash \sigma\ -jvm\rightarrow\ \sigma'\ ]\!] \Longrightarrow P \vdash \sigma'\ -jvm\rightarrow\ (x,h,[])\langle proof\rangle$

The start configuration of the JVM: in the start heap, we call a method m of class C in program P. The *this* pointer of the frame is set to *Null* to simulate a static method invokation.

definition *start-state* :: *jvm-prog* ⇒ *cname* ⇒ *mname* ⇒ *jvm-state* **where**
start-state P C M =
(*let* (D,Ts,T,mxs,mxl_0,b) = *method P C M in*
(*None*, *start-heap P*, [([], *Null* # *replicate* mxl_0 *undefined*, *C*, *M*, *0*)]))

end

3.6 A Defensive JVM

theory *JVMDefensive*
imports *JVMExec ../Common/Conform*
begin

Extend the state space by one element indicating a type error (or other abnormal termination)

datatype $'a$ *type-error* $=$ *TypeError* $|$ *Normal* $'a$

fun *is-Addr* :: *val* $\Rightarrow$ *bool* **where**
  $\textit{is-Addr}\ (\textit{Addr}\ a) \longleftrightarrow \textit{True}$
$|\ \textit{is-Addr}\ v \longleftrightarrow \textit{False}$

fun *is-Intg* :: *val* $\Rightarrow$ *bool* **where**
  $\textit{is-Intg}\ (\textit{Intg}\ i) \longleftrightarrow \textit{True}$
$|\ \textit{is-Intg}\ v \longleftrightarrow \textit{False}$

fun *is-Bool* :: *val* $\Rightarrow$ *bool* **where**
  $\textit{is-Bool}\ (\textit{Bool}\ b) \longleftrightarrow \textit{True}$
$|\ \textit{is-Bool}\ v \longleftrightarrow \textit{False}$

definition *is-Ref* :: *val* $\Rightarrow$ *bool* **where**
  $\textit{is-Ref}\ v \longleftrightarrow v = \textit{Null} \lor \textit{is-Addr}\ v$

primrec *check-instr* :: [*instr*, *jvm-prog*, *heap*, *val list*, *val list*,
      cname, *mname*, *pc*, *frame list*] $\Rightarrow$ *bool* **where**
  check-instr-Load:
    $\textit{check-instr}\ (\textit{Load}\ n)\ P\ h\ \textit{stk}\ \textit{loc}\ C\ M_0\ \textit{pc}\ \textit{frs} =$
    $(n < \textit{length}\ \textit{loc})$

$|$ *check-instr-Store*:
    $\textit{check-instr}\ (\textit{Store}\ n)\ P\ h\ \textit{stk}\ \textit{loc}\ C_0\ M_0\ \textit{pc}\ \textit{frs} =$
    $(0 < \textit{length}\ \textit{stk} \land n < \textit{length}\ \textit{loc})$

$|$ *check-instr-Push*:
    $\textit{check-instr}\ (\textit{Push}\ v)\ P\ h\ \textit{stk}\ \textit{loc}\ C_0\ M_0\ \textit{pc}\ \textit{frs} =$
    $(\neg\textit{is-Addr}\ v)$

$|$ *check-instr-New*:
    $\textit{check-instr}\ (\textit{New}\ C)\ P\ h\ \textit{stk}\ \textit{loc}\ C_0\ M_0\ \textit{pc}\ \textit{frs} =$
    $\textit{is-class}\ P\ C$

$|$ *check-instr-Getfield*:
    $\textit{check-instr}\ (\textit{Getfield}\ F\ C)\ P\ h\ \textit{stk}\ \textit{loc}\ C_0\ M_0\ \textit{pc}\ \textit{frs} =$
    $(0 < \textit{length}\ \textit{stk} \land (\exists C'\ T.\ P \vdash C\ \textit{sees}\ F{:}T\ \textit{in}\ C') \land$
    $(\textit{let}\ (C',\ T) = \textit{field}\ P\ C\ F;\ \textit{ref} = \textit{hd}\ \textit{stk}\ \textit{in}$
      $C' = C \land \textit{is-Ref}\ \textit{ref} \land (\textit{ref} \neq \textit{Null} \longrightarrow$
        $h\ (\textit{the-Addr}\ \textit{ref}) \neq \textit{None} \land$
        $(\textit{let}\ (D,\textit{vs}) = \textit{the}\ (h\ (\textit{the-Addr}\ \textit{ref}))\ \textit{in}$
          $P \vdash D \preceq^* C \land \textit{vs}\ (F,C) \neq \textit{None} \land P,h \vdash \textit{the}\ (\textit{vs}\ (F,C)) :\leq T))))$

$|$ *check-instr-Putfield*:
    $\textit{check-instr}\ (\textit{Putfield}\ F\ C)\ P\ h\ \textit{stk}\ \textit{loc}\ C_0\ M_0\ \textit{pc}\ \textit{frs} =$

$(1 < length\ stk \wedge (\exists C'\ T.\ P \vdash C\ sees\ F{:}T\ in\ C') \wedge$
$(let\ (C',\ T) = field\ P\ C\ F;\ v = hd\ stk;\ ref = hd\ (tl\ stk)\ in$
$C' = C \wedge is\text{-}Ref\ ref \wedge (ref \neq Null \longrightarrow$
$h\ (the\text{-}Addr\ ref) \neq None \wedge$
$(let\ D = fst\ (the\ (h\ (the\text{-}Addr\ ref)))\ in$
$P \vdash D \preceq^* C \wedge P,h \vdash v :\leq T))))$

| *check-instr-Checkcast*:
$check\text{-}instr\ (Checkcast\ C)\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ frs =$
$(0 < length\ stk \wedge is\text{-}class\ P\ C \wedge is\text{-}Ref\ (hd\ stk))$

| *check-instr-Invoke*:
$check\text{-}instr\ (Invoke\ M\ n)\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ frs =$
$(n < length\ stk \wedge is\text{-}Ref\ (stk!n) \wedge$
$(stk!n \neq Null \longrightarrow$
$(let\ a = the\text{-}Addr\ (stk!n);$
$C = cname\text{-}of\ h\ a;$
$Ts = fst\ (snd\ (method\ P\ C\ M))$
$in\ h\ a \neq None \wedge P \vdash C\ has\ M \wedge$
$P,h \vdash rev\ (take\ n\ stk)\ [:\leq]\ Ts)))$

| *check-instr-Return*:
$check\text{-}instr\ Return\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ frs =$
$(0 < length\ stk \wedge ((0 < length\ frs) \longrightarrow$
$(P \vdash C_0\ has\ M_0) \wedge$
$(let\ v = hd\ stk;$
$T = fst\ (snd\ (snd\ (method\ P\ C_0\ M_0)))$
$in\ P,h \vdash v :\leq T)))$

| *check-instr-Pop*:
$check\text{-}instr\ Pop\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ frs =$
$(0 < length\ stk)$

| *check-instr-IAdd*:
$check\text{-}instr\ IAdd\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ frs =$
$(1 < length\ stk \wedge is\text{-}Intg\ (hd\ stk) \wedge is\text{-}Intg\ (hd\ (tl\ stk)))$

| *check-instr-IfFalse*:
$check\text{-}instr\ (IfFalse\ b)\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ frs =$
$(0 < length\ stk \wedge is\text{-}Bool\ (hd\ stk) \wedge 0 \leq int\ pc+b)$

| *check-instr-CmpEq*:
$check\text{-}instr\ CmpEq\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ frs =$
$(1 < length\ stk)$

| *check-instr-Goto*:
$check\text{-}instr\ (Goto\ b)\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ frs =$
$(0 \leq int\ pc+b)$

| *check-instr-Throw*:
$check\text{-}instr\ Throw\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ frs =$
$(0 < length\ stk \wedge is\text{-}Ref\ (hd\ stk))$

definition *check* :: *jvm-prog* $\Rightarrow$ *jvm-state* $\Rightarrow$ *bool* **where**

$check\ P\ \sigma = (let\ (xcpt,\ h,\ frs) = \sigma\ in$
$\quad (case\ frs\ of\ [] \Rightarrow True \mid (stk,loc,C,M,pc)\#frs' \Rightarrow$
$\quad P \vdash C\ has\ M\ \wedge$
$\quad (let\ (C',Ts,T,mxs,mxl_0,ins,xt) = method\ P\ C\ M;\ i = ins!pc\ in$
$\quad pc < size\ ins \wedge size\ stk \leq mxs\ \wedge$
$\quad check\text{-}instr\ i\ P\ h\ stk\ loc\ C\ M\ pc\ frs')))$

definition $exec\text{-}d :: jvm\text{-}prog \Rightarrow jvm\text{-}state \Rightarrow jvm\text{-}state\ option\ type\text{-}error$ **where**
$exec\text{-}d\ P\ \sigma = (if\ check\ P\ \sigma\ then\ Normal\ (exec\ (P,\ \sigma))\ else\ TypeError)$

inductive-set
$exec\text{-}1\text{-}d :: jvm\text{-}prog \Rightarrow (jvm\text{-}state\ type\text{-}error \times jvm\text{-}state\ type\text{-}error)\ set$
and $exec\text{-}1\text{-}d' :: jvm\text{-}prog \Rightarrow jvm\text{-}state\ type\text{-}error \Rightarrow jvm\text{-}state\ type\text{-}error \Rightarrow bool$
$(\text{-} \vdash \text{-}\ {-jvmd}{\rightarrow_1}\ \text{-}\ [61,61,61]60)$
for $P :: jvm\text{-}prog$
where
$P \vdash \sigma\ {-jvmd}{\rightarrow_1}\ \sigma' \equiv (\sigma,\sigma') \in exec\text{-}1\text{-}d\ P$
$\mid exec\text{-}1\text{-}d\text{-}ErrorI\text{: } exec\text{-}d\ P\ \sigma = TypeError \Longrightarrow P \vdash Normal\ \sigma\ {-jvmd}{\rightarrow_1}\ TypeError$
$\mid exec\text{-}1\text{-}d\text{-}NormalI\text{: } exec\text{-}d\ P\ \sigma = Normal\ (Some\ \sigma') \Longrightarrow P \vdash Normal\ \sigma\ {-jvmd}{\rightarrow_1}\ Normal\ \sigma'$

— reflexive transitive closure:
definition $exec\text{-}all\text{-}d :: jvm\text{-}prog \Rightarrow jvm\text{-}state\ type\text{-}error \Rightarrow jvm\text{-}state\ type\text{-}error \Rightarrow bool$
$(\text{-}\ |{-}\ \text{-}\ {-jvmd}{-}{>}\ \text{-}\ [61,61,61]60)$ **where**
$exec\text{-}all\text{-}d\text{-}def1\text{: } P\ |{-}\ \sigma\ {-jvmd}{-}{>}\ \sigma' \longleftrightarrow (\sigma,\sigma') \in (exec\text{-}1\text{-}d\ P)^*$

notation $(xsymbols)$
$exec\text{-}all\text{-}d \quad (\text{-} \vdash \text{-}\ {-jvmd}{\rightarrow}\ \text{-}\ [61,61,61]60)$

lemma $exec\text{-}1\text{-}d\text{-}eq$:
$exec\text{-}1\text{-}d\ P = \{(s,t).\ \exists\sigma.\ s = Normal\ \sigma \wedge t = TypeError \wedge exec\text{-}d\ P\ \sigma = TypeError\} \cup$
$\{(s,t).\ \exists\sigma\ \sigma'.\ s = Normal\ \sigma \wedge t = Normal\ \sigma' \wedge exec\text{-}d\ P\ \sigma = Normal\ (Some\ \sigma')\}$
$\langle proof \rangle$

declare $split\text{-}paired\text{-}All\ [simp\ del]$
declare $split\text{-}paired\text{-}Ex\ [simp\ del]$

lemma $if\text{-}neq\ [dest!]$:
$(if\ P\ then\ A\ else\ B) \neq B \Longrightarrow P$
$\langle proof \rangle$

lemma $exec\text{-}d\text{-}no\text{-}errorI\ [intro]$:
$check\ P\ \sigma \Longrightarrow exec\text{-}d\ P\ \sigma \neq TypeError$
$\langle proof \rangle$

theorem $no\text{-}type\text{-}error\text{-}commutes$:
$exec\text{-}d\ P\ \sigma \neq TypeError \Longrightarrow exec\text{-}d\ P\ \sigma = Normal\ (exec\ (P,\ \sigma))$
$\langle proof \rangle$

lemma $defensive\text{-}imp\text{-}aggressive$:
$P \vdash (Normal\ \sigma)\ {-jvmd}{\rightarrow}\ (Normal\ \sigma') \Longrightarrow P \vdash \sigma\ {-jvm}{\rightarrow}\ \sigma' \langle proof \rangle$

end

3.7 Example for generating executable code from JVM semantics

theory *JVMListExample*
imports
  ../Common/SystemClasses
  JVMExec
  ~~/src/HOL/Library/Efficient-Nat
begin

definition *list-name* :: *string*
where
  list-name == *''list''*

definition *test-name* :: *string*
where
  test-name == *''test''*

definition *val-name* :: *string*
where
  val-name == *''val''*

definition *next-name* :: *string*
where
  next-name == *''next''*

definition *append-name* :: *string*
where
  append-name == *''append''*

definition *makelist-name* :: *string*
where
  makelist-name == *''makelist''*

definition *append-ins* :: *bytecode*
where
  append-ins ==
    [*Load 0*,
     Getfield next-name list-name,
     Load 0,
     Getfield next-name list-name,
     Push Null,
     CmpEq,
     IfFalse 7,
     Pop,
     Load 0,
     Load 1,
     Putfield next-name list-name,
     Push Unit,
     Return,
     Load 1,
     Invoke append-name 1,
     Return]

definition *list-class* :: *jvm-method class*
where
list-class ==
(*Object*,
[(*val-name*, *Integer*), (*next-name*, *Class list-name*)],
[(*append-name*, [*Class list-name*], *Void*,
(*3*, *0*, *append-ins*, [(*1*, *2*, *NullPointer*, *7*, *0*)]))])

definition *make-list-ins* :: *bytecode*
where
make-list-ins ==
[*New list-name*,
Store 0,
Load 0,
Push (*Intg 1*),
Putfield val-name list-name,
New list-name,
Store 1,
Load 1,
Push (*Intg 2*),
Putfield val-name list-name,
New list-name,
Store 2,
Load 2,
Push (*Intg 3*),
Putfield val-name list-name,
Load 0,
Load 1,
Invoke append-name 1,
Pop,
Load 0,
Load 2,
Invoke append-name 1,
Return]

definition *test-class* :: *jvm-method class*
where
test-class ==
(*Object*, [],
[(*makelist-name*, [], *Void*, (*3*, *2*, *make-list-ins*, []))])

definition *E* :: *jvm-prog*
where
E == *SystemClasses* @ [(*list-name*, *list-class*), (*test-name*, *test-class*)]

definition *undefined-cname* :: *cname*
where [*code del*]: *undefined-cname* = *undefined*
declare *undefined-cname-def*[*symmetric*, *code-unfold*]
code-const *undefined-cname*
(*SML object*)

definition *undefined-val* :: *val*
where [*code del*]: *undefined-val* = *undefined*
declare *undefined-val-def*[*symmetric*, *code-unfold*]

code-const *undefined-val*
(*SML Unit*)

lemmas [*code-unfold*] = *SystemClasses-def* [*unfolded ObjectC-def NullPointerC-def ClassCastC-def OutOfMemoryC-def*]

definition *test* = *exec* (*E*, *start-state E test-name makelist-name*)

⟨*ML*⟩

end

Chapter 4

Bytecode Verifier

4.1 Semilattices

theory *Semilat*
imports *Main* ~~/*src*/*HOL*/*Library*/*While-Combinator*
begin

type-synonym $'a\ ord\ = 'a \Rightarrow 'a \Rightarrow bool$
type-synonym $'a\ binop\ = 'a \Rightarrow 'a \Rightarrow 'a$
type-synonym $'a\ sl\ = 'a\ set \times 'a\ ord \times 'a\ binop$

consts
$lesub :: 'a \Rightarrow 'a\ ord \Rightarrow 'a \Rightarrow bool$
$lesssub :: 'a \Rightarrow 'a\ ord \Rightarrow 'a \Rightarrow bool$
$plussub :: 'a \Rightarrow ('a \Rightarrow 'b \Rightarrow 'c) \Rightarrow 'b \Rightarrow 'c$**notation** (*xsymbols*)
lesub ((- /$\sqsubseteq_{-}$ -) [50, 0, 51] 50) **and**
lesssub ((- /$\sqsubset_{-}$ -) [50, 0, 51] 50) **and**
plussub ((- /$\sqcup_{-}$ -) [65, 0, 66] 65)
defs
lesub-def: $x \sqsubseteq_r y \equiv r\ x\ y$
lesssub-def: $x \sqsubset_r y \equiv x \sqsubseteq_r y \wedge x \neq y$
plussub-def: $x \sqcup_f y \equiv f\ x\ y$

definition $ord :: ('a \times 'a)\ set \Rightarrow 'a\ ord$
where
$ord\ r = (\lambda x\ y.\ (x,y) \in r)$

definition $order :: 'a\ ord \Rightarrow bool$
where
$order\ r \longleftrightarrow (\forall x.\ x \sqsubseteq_r x) \wedge (\forall x\ y.\ x \sqsubseteq_r y \wedge y \sqsubseteq_r x \longrightarrow x=y) \wedge (\forall x\ y\ z.\ x \sqsubseteq_r y \wedge y \sqsubseteq_r z \longrightarrow x \sqsubseteq_r z)$

definition $top :: 'a\ ord \Rightarrow 'a \Rightarrow bool$
where
$top\ r\ T \longleftrightarrow (\forall x.\ x \sqsubseteq_r T)$

definition $acc :: 'a\ ord \Rightarrow bool$
where
$acc\ r \longleftrightarrow wf\ \{(y,x).\ x \sqsubset_r y\}$

definition $closed :: 'a\ set \Rightarrow 'a\ binop \Rightarrow bool$
where
$closed\ A\ f \longleftrightarrow (\forall x{\in}A.\ \forall y{\in}A.\ x \sqcup_f y \in A)$

definition $semilat :: 'a\ sl \Rightarrow bool$
where
$semilat = (\lambda(A,r,f).\ order\ r \wedge closed\ A\ f \wedge$
$(\forall x{\in}A.\ \forall y{\in}A.\ x \sqsubseteq_r x \sqcup_f y) \wedge$
$(\forall x{\in}A.\ \forall y{\in}A.\ y \sqsubseteq_r x \sqcup_f y) \wedge$
$(\forall x{\in}A.\ \forall y{\in}A.\ \forall z{\in}A.\ x \sqsubseteq_r z \wedge y \sqsubseteq_r z \longrightarrow x \sqcup_f y \sqsubseteq_r z))$

definition $is\text{-}ub :: ('a \times 'a)\ set \Rightarrow 'a \Rightarrow 'a \Rightarrow 'a \Rightarrow bool$
where
$is\text{-}ub\ r\ x\ y\ u \longleftrightarrow (x,u){\in}r \wedge (y,u){\in}r$

definition *is-lub* :: $('a \times 'a)$ *set* $\Rightarrow 'a \Rightarrow 'a \Rightarrow 'a \Rightarrow$ *bool*
where
is-lub r x y u $\longleftrightarrow$ *is-ub r x y u* $\wedge$ $(\forall z.$ *is-ub r x y z* $\longrightarrow (u,z)\in r)$

definition *some-lub* :: $('a \times 'a)$ *set* $\Rightarrow 'a \Rightarrow 'a \Rightarrow 'a$
where
some-lub r x y = (*SOME z. is-lub r x y z*)

locale *Semilat* =
fixes A :: *'a set*
fixes r :: *'a ord*
fixes f :: *'a binop*
assumes *semilat*: *semilat* (A, r, f)

lemma *order-refl* [*simp*, *intro*]: *order r* $\Longrightarrow x \sqsubseteq_r x$
⟨*proof*⟩
lemma *order-antisym*: $[\![$ *order r*; $x \sqsubseteq_r y$; $y \sqsubseteq_r x$ $]\!] \Longrightarrow x = y$
⟨*proof*⟩
lemma *order-trans*: $[\![$ *order r*; $x \sqsubseteq_r y$; $y \sqsubseteq_r z$ $]\!] \Longrightarrow x \sqsubseteq_r z$
⟨*proof*⟩
lemma *order-less-irrefl* [*intro*, *simp*]: *order r* $\Longrightarrow \neg\, x \sqsubset_r x$
⟨*proof*⟩
lemma *order-less-trans*: $[\![$ *order r*; $x \sqsubset_r y$; $y \sqsubset_r z$ $]\!] \Longrightarrow x \sqsubset_r z$
⟨*proof*⟩
lemma *topD* [*simp*, *intro*]: *top r T* $\Longrightarrow x \sqsubseteq_r T$
⟨*proof*⟩
lemma *top-le-conv* [*simp*]: $[\![$ *order r*; *top r T* $]\!] \Longrightarrow (T \sqsubseteq_r x) = (x = T)$
⟨*proof*⟩
lemma *semilat-Def*:
semilat$(A,r,f) \longleftrightarrow$ *order r* $\wedge$ *closed A f* $\wedge$
$(\forall x\in A.\ \forall y\in A.\ x \sqsubseteq_r x \sqcup_f y) \wedge$
$(\forall x\in A.\ \forall y\in A.\ y \sqsubseteq_r x \sqcup_f y) \wedge$
$(\forall x\in A.\ \forall y\in A.\ \forall z\in A.\ x \sqsubseteq_r z \wedge y \sqsubseteq_r z \longrightarrow x \sqcup_f y \sqsubseteq_r z)$
⟨*proof*⟩
lemma (**in** *Semilat*) *orderI* [*simp*, *intro*]: *order r*
⟨*proof*⟩
lemma (**in** *Semilat*) *closedI* [*simp*, *intro*]: *closed A f*
⟨*proof*⟩
lemma *closedD*: $[\![$ *closed A f*; $x\in A$; $y\in A$ $]\!] \Longrightarrow x \sqcup_f y \in A$
⟨*proof*⟩
lemma *closed-UNIV* [*simp*]: *closed UNIV f*
⟨*proof*⟩
lemma (**in** *Semilat*) *closed-f* [*simp*, *intro*]: $[\![x \in A;\ y \in A]\!] \Longrightarrow x \sqcup_f y \in A$
⟨*proof*⟩
lemma (**in** *Semilat*) *refl-r* [*intro*, *simp*]: $x \sqsubseteq_r x$ ⟨*proof*⟩

lemma (**in** *Semilat*) *antisym-r* [*intro?*]: $[\![\ x \sqsubseteq_r y;\ y \sqsubseteq_r x\ ]\!] \Longrightarrow x = y$
⟨*proof*⟩
lemma (**in** *Semilat*) *trans-r* [*trans*, *intro?*]: $[\![x \sqsubseteq_r y;\ y \sqsubseteq_r z]\!] \Longrightarrow x \sqsubseteq_r z$
⟨*proof*⟩
lemma (**in** *Semilat*) *ub1* [*simp*, *intro?*]: $[\![\ x \in A;\ y \in A\ ]\!] \Longrightarrow x \sqsubseteq_r x \sqcup_f y$
⟨*proof*⟩
lemma (**in** *Semilat*) *ub2* [*simp*, *intro?*]: $[\![\ x \in A;\ y \in A\ ]\!] \Longrightarrow y \sqsubseteq_r x \sqcup_f y$
⟨*proof*⟩

lemma (**in** *Semilat*) *lub* [*simp*, *intro?*]:
$[\![x \sqsubseteq_r z; y \sqsubseteq_r z; x \in A; y \in A; z \in A]\!] \Longrightarrow x \sqcup_f y \sqsubseteq_r z$
⟨*proof*⟩
lemma (**in** *Semilat*) *plus-le-conv* [*simp*]:
$[\![x \in A; y \in A; z \in A]\!] \Longrightarrow (x \sqcup_f y \sqsubseteq_r z) = (x \sqsubseteq_r z \wedge y \sqsubseteq_r z)$
⟨*proof*⟩
lemma (**in** *Semilat*) *le-iff-plus-unchanged*:
assumes $x \in A$ **and** $y \in A$
shows $x \sqsubseteq_r y \longleftrightarrow x \sqcup_f y = y$ (**is** $?P \longleftrightarrow ?Q$)⟨*proof*⟩
lemma (**in** *Semilat*) *le-iff-plus-unchanged2*:
assumes $x \in A$ **and** $y \in A$
shows $x \sqsubseteq_r y \longleftrightarrow y \sqcup_f x = y$ (**is** $?P \longleftrightarrow ?Q$)⟨*proof*⟩
lemma (**in** *Semilat*) *plus-assoc* [*simp*]:
assumes a: $a \in A$ **and** b: $b \in A$ **and** c: $c \in A$
shows $a \sqcup_f (b \sqcup_f c) = a \sqcup_f b \sqcup_f c$⟨*proof*⟩
lemma (**in** *Semilat*) *plus-com-lemma*:
$[\![a \in A; b \in A]\!] \Longrightarrow a \sqcup_f b \sqsubseteq_r b \sqcup_f a$⟨*proof*⟩
lemma (**in** *Semilat*) *plus-commutative*:
$[\![a \in A; b \in A]\!] \Longrightarrow a \sqcup_f b = b \sqcup_f a$
⟨*proof*⟩
lemma *is-lubD*:
is-lub $r\ x\ y\ u \Longrightarrow$ *is-ub* $r\ x\ y\ u \wedge (\forall z.$ *is-ub* $r\ x\ y\ z \longrightarrow (u,z) \in r)$
⟨*proof*⟩
lemma *is-ubI*:
$[\![(x,u) \in r; (y,u) \in r]\!] \Longrightarrow$ *is-ub* $r\ x\ y\ u$
⟨*proof*⟩
lemma *is-ubD*:
is-ub $r\ x\ y\ u \Longrightarrow (x,u) \in r \wedge (y,u) \in r$
⟨*proof*⟩

lemma *is-lub-bigger1* [*iff*]:
is-lub $(r\hat{}*)\ x\ y\ y = ((x,y) \in r\hat{}*)$⟨*proof*⟩
lemma *is-lub-bigger2* [*iff*]:
is-lub $(r\hat{}*)\ x\ y\ x = ((y,x) \in r\hat{}*)$⟨*proof*⟩
lemma *extend-lub*:
$[\![$ *single-valued* r; *is-lub* $(r\hat{}*)\ x\ y\ u$; $(x',x) \in r]\!]$
$\Longrightarrow$ *EX* v. *is-lub* $(r\hat{}*)\ x'\ y\ v$⟨*proof*⟩
lemma *single-valued-has-lubs* [*rule-format*]:
$[\![$ *single-valued* r; $(x,u) \in r\hat{}*]\!] \Longrightarrow (\forall y.\ (y,u) \in r\hat{}* \longrightarrow$
$($*EX* z. *is-lub* $(r\hat{}*)\ x\ y\ z))$⟨*proof*⟩
lemma *some-lub-conv*:
$[\![$ *acyclic* r; *is-lub* $(r\hat{}*)\ x\ y\ u]\!] \Longrightarrow$ *some-lub* $(r\hat{}*)\ x\ y = u$⟨*proof*⟩
lemma *is-lub-some-lub*:
$[\![$ *single-valued* r; *acyclic* r; $(x,u) \in r\hat{}*$; $(y,u) \in r\hat{}*]\!]$
$\Longrightarrow$ *is-lub* $(r\hat{}*)\ x\ y$ (*some-lub* $(r\hat{}*)\ x\ y$)
⟨*proof*⟩

4.1.1 An executable lub-finder

definition *exec-lub* :: $('a * 'a)$ *set* $\Rightarrow ('a \Rightarrow 'a) \Rightarrow 'a$ *binop*
where
exec-lub $r\ f\ x\ y =$ *while* $(\lambda z.\ (x,z) \notin r^*)\ f\ y$

lemma *exec-lub-refl*: *exec-lub* $r\ f\ T\ T = T$

$\langle proof \rangle$

lemma *acyclic-single-valued-finite*:
$[\![acyclic\ r;\ single\text{-}valued\ r;\ (x,y) \in r^*]\!]$
$\Longrightarrow finite\ (r \cap \{a.\ (x,\ a) \in r^*\} \times \{b.\ (b,\ y) \in r^*\})\langle proof \rangle$

lemma *exec-lub-conv*:
$[\![\ acyclic\ r;\ \forall\, x\ y.\ (x,y) \in r \longrightarrow f\ x = y;\ is\text{-}lub\ (r^*)\ x\ y\ u\ ]\!] \Longrightarrow$
$exec\text{-}lub\ r\ f\ x\ y = u\langle proof \rangle$
lemma *is-lub-exec-lub*:
$[\![\ single\text{-}valued\ r;\ acyclic\ r;\ (x,u){:}r\hat{\ }*;\ (y,u){:}r\hat{\ }*;\ \forall\, x\ y.\ (x,y) \in r \longrightarrow f\ x = y\ ]\!]$
$\Longrightarrow is\text{-}lub\ (r\hat{\ }*\ )\ x\ y\ (exec\text{-}lub\ r\ f\ x\ y)$
$\langle proof \rangle$
end

4.2 The Error Type

theory *Err*
imports *Semilat*
begin

datatype $'a\ err = Err \mid OK\ 'a$

type-synonym $'a\ ebinop = 'a \Rightarrow 'a \Rightarrow 'a\ err$
type-synonym $'a\ esl = 'a\ set \times 'a\ ord \times 'a\ ebinop$

primrec $ok\text{-}val :: 'a\ err \Rightarrow 'a$
where
$ok\text{-}val\ (OK\ x) = x$

definition $lift :: ('a \Rightarrow 'b\ err) \Rightarrow ('a\ err \Rightarrow 'b\ err)$
where
$lift\ f\ e = (case\ e\ of\ Err \Rightarrow Err \mid OK\ x \Rightarrow f\ x)$

definition $lift2 :: ('a \Rightarrow 'b \Rightarrow 'c\ err) \Rightarrow 'a\ err \Rightarrow 'b\ err \Rightarrow 'c\ err$
where
$lift2\ f\ e_1\ e_2 =$
$(case\ e_1\ of\ Err \Rightarrow Err \mid OK\ x \Rightarrow (case\ e_2\ of\ Err \Rightarrow Err \mid OK\ y \Rightarrow f\ x\ y))$

definition $le :: 'a\ ord \Rightarrow 'a\ err\ ord$
where
$le\ r\ e_1\ e_2 =$
$(case\ e_2\ of\ Err \Rightarrow True \mid OK\ y \Rightarrow (case\ e_1\ of\ Err \Rightarrow False \mid OK\ x \Rightarrow x \sqsubseteq_r y))$

definition $sup :: ('a \Rightarrow 'b \Rightarrow 'c) \Rightarrow ('a\ err \Rightarrow 'b\ err \Rightarrow 'c\ err)$
where
$sup\ f = lift2\ (\lambda x\ y.\ OK\ (x \sqcup_f y))$

definition $err :: 'a\ set \Rightarrow 'a\ err\ set$
where
$err\ A = insert\ Err\ \{OK\ x|x.\ x{\in}A\}$

definition $esl :: 'a\ sl \Rightarrow 'a\ esl$
where
$esl = (\lambda(A,r,f).\ (A,\ r,\ \lambda x\ y.\ OK(f\ x\ y)))$

definition $sl :: 'a\ esl \Rightarrow 'a\ err\ sl$
where
$sl = (\lambda(A,r,f).\ (err\ A,\ le\ r,\ lift2\ f))$

abbreviation
$err\text{-}semilat :: 'a\ esl \Rightarrow bool$ **where**
$err\text{-}semilat\ L == semilat(sl\ L)$

primrec $strict :: ('a \Rightarrow 'b\ err) \Rightarrow ('a\ err \Rightarrow 'b\ err)$
where
$strict\ f\ Err = Err$
$\mid strict\ f\ (OK\ x) = f\ x$

lemma *err-def'*:
  $err\ A = insert\ Err\ \{x.\ \exists\, y{\in}A.\ x = OK\ y\}\langle proof\rangle$
lemma *strict-Some* [*simp*]:
  $(strict\ f\ x = OK\ y) = (\exists\, z.\ x = OK\ z \wedge f\ z = OK\ y)\langle proof\rangle$
lemma *not-Err-eq*: $(x \neq Err) = (\exists\, a.\ x = OK\ a)\langle proof\rangle$
lemma *not-OK-eq*: $(\forall\, y.\ x \neq OK\ y) = (x = Err)\langle proof\rangle$
lemma *unfold-lesub-err*: $e1 \sqsubseteq_{le\ r} e2 = le\ r\ e1\ e2\langle proof\rangle$
lemma *le-err-refl*: $\forall\, x.\ x \sqsubseteq_r x \Longrightarrow e \sqsubseteq_{le\ r} e\langle proof\rangle$
lemma *le-err-trans* [*rule-format*]:
  $order\ r \Longrightarrow e1 \sqsubseteq_{le\ r} e2 \longrightarrow e2 \sqsubseteq_{le\ r} e3 \longrightarrow e1 \sqsubseteq_{le\ r} e3\langle proof\rangle$
lemma *le-err-antisym* [*rule-format*]:
  $order\ r \Longrightarrow e1 \sqsubseteq_{le\ r} e2 \longrightarrow e2 \sqsubseteq_{le\ r} e1 \longrightarrow e1{=}e2\langle proof\rangle$
lemma *OK-le-err-OK*: $(OK\ x \sqsubseteq_{le\ r} OK\ y) = (x \sqsubseteq_r y)\langle proof\rangle$
lemma *order-le-err* [*iff*]: $order(le\ r) = order\ r\langle proof\rangle$
lemma *le-Err* [*iff*]: $e \sqsubseteq_{le\ r} Err\langle proof\rangle$
lemma *Err-le-conv* [*iff*]: $Err \sqsubseteq_{le\ r} e\ = (e = Err)\langle proof\rangle$
lemma *le-OK-conv* [*iff*]: $e \sqsubseteq_{le\ r} OK\ x\ =\ (\exists\, y.\ e = OK\ y \wedge y \sqsubseteq_r x)\langle proof\rangle$
lemma *OK-le-conv*: $OK\ x \sqsubseteq_{le\ r} e = (e = Err \vee (\exists\, y.\ e = OK\ y \wedge x \sqsubseteq_r y))\langle proof\rangle$
lemma *top-Err* [*iff*]: $top\ (le\ r)\ Err\langle proof\rangle$
lemma *OK-less-conv* [*rule-format*, *iff*]:
  $OK\ x \sqsubset_{le\ r} e = (e{=}Err \vee (\exists\, y.\ e = OK\ y \wedge x \sqsubset_r y))\langle proof\rangle$
lemma *not-Err-less* [*rule-format*, *iff*]: $\neg(Err \sqsubset_{le\ r} x)\langle proof\rangle$
lemma *semilat-errI* [*intro*]: **assumes** *Semilat A r f*
shows $semilat(err\ A,\ le\ r,\ lift2(\lambda x\ y.\ OK(f\ x\ y)))\langle proof\rangle$
lemma *err-semilat-eslI-aux*:
assumes *Semilat A r f* **shows** $err\text{-}semilat(esl(A,r,f))\langle proof\rangle$
lemma *err-semilat-eslI* [*intro*, *simp*]:
  $semilat\ L \Longrightarrow err\text{-}semilat\ (esl\ L)\langle proof\rangle$
lemma *acc-err* [*simp*, *intro*!]: $acc\ r \Longrightarrow acc(le\ r)\langle proof\rangle$
lemma *Err-in-err* [*iff*]: $Err : err\ A\langle proof\rangle$
lemma *Ok-in-err* [*iff*]: $(OK\ x \in err\ A) = (x{\in}A)\langle proof\rangle$

4.2.1 lift

lemma *lift-in-errI*:
  $[\![\ e \in err\ S;\ \forall\, x{\in}S.\ e = OK\ x \longrightarrow f\ x \in err\ S\ ]\!] \Longrightarrow lift\ f\ e \in err\ S\langle proof\rangle$
lemma *Err-lift2* [*simp*]: $Err \sqcup_{lift2\ f} x = Err\langle proof\rangle$
lemma *lift2-Err* [*simp*]: $x \sqcup_{lift2\ f} Err = Err\langle proof\rangle$
lemma *OK-lift2-OK* [*simp*]: $OK\ x \sqcup_{lift2\ f} OK\ y = x \sqcup_f y\langle proof\rangle$

4.2.2 sup

lemma *Err-sup-Err* [*simp*]: $Err \sqcup_{sup\ f} x = Err\langle proof\rangle$
lemma *Err-sup-Err2* [*simp*]: $x \sqcup_{sup\ f} Err = Err\langle proof\rangle$
lemma *Err-sup-OK* [*simp*]: $OK\ x \sqcup_{sup\ f} OK\ y = OK\ (x \sqcup_f y)\langle proof\rangle$
lemma *Err-sup-eq-OK-conv* [*iff*]:
  $(sup\ f\ ex\ ey = OK\ z) = (\exists\, x\ y.\ ex = OK\ x \wedge ey = OK\ y \wedge f\ x\ y = z)\langle proof\rangle$
lemma *Err-sup-eq-Err* [*iff*]: $(sup\ f\ ex\ ey = Err) = (ex{=}Err \vee ey{=}Err)\langle proof\rangle$

4.2.3 semilat (err A) (le r) f

lemma *semilat-le-err-Err-plus* [*simp*]:
  $[\![\ x{\in}\ err\ A;\ semilat(err\ A,\ le\ r,\ f)\ ]\!] \Longrightarrow Err \sqcup_f x = Err\langle proof\rangle$
lemma *semilat-le-err-plus-Err* [*simp*]:

$[\![x \in err\ A;\ semilat(err\ A,\ le\ r,\ f)\ ]\!] \Longrightarrow x \sqcup_f Err = Err \langle proof \rangle$

lemma *semilat-le-err-OK1*:

$[\![x \in A;\ y \in A;\ semilat(err\ A,\ le\ r,\ f);\ OK\ x \sqcup_f OK\ y = OK\ z\ ]\!]$

$\Longrightarrow x \sqsubseteq_r z \langle proof \rangle$

lemma *semilat-le-err-OK2*:

$[\![x \in A;\ y \in A;\ semilat(err\ A,\ le\ r,\ f);\ OK\ x \sqcup_f OK\ y = OK\ z\ ]\!]$

$\Longrightarrow y \sqsubseteq_r z \langle proof \rangle$

lemma *eq-order-le*:

$[\![x = y;\ order\ r\ ]\!] \Longrightarrow x \sqsubseteq_r y \langle proof \rangle$

lemma *OK-plus-OK-eq-Err-conv* [*simp*]:

assumes $x \in A \quad y \in A \quad semilat(err\ A,\ le\ r,\ fe)$

shows $(OK\ x \sqcup_{fe} OK\ y = Err) = (\neg(\exists z \in A.\ x \sqsubseteq_r z \wedge y \sqsubseteq_r z)) \langle proof \rangle$

4.2.4 semilat (err(Union AS))

lemma *all-bex-swap-lemma* [*iff*]:

$(\forall x.\ (\exists y \in A.\ x = f\ y) \longrightarrow P\ x) = (\forall y \in A.\ P(f\ y)) \langle proof \rangle$

lemma *closed-err-Union-lift2I*:

$[\![\forall A \in AS.\ closed\ (err\ A)\ (lift2\ f);\ AS \neq \{\};$

$\forall A \in AS. \forall B \in AS.\ A \neq B \longrightarrow (\forall a \in A. \forall b \in B.\ a \sqcup_f b = Err)\ ]\!]$

$\Longrightarrow closed\ (err(Union\ AS))\ (lift2\ f) \langle proof \rangle$

If $AS = \{\}$ the thm collapses to $order\ r \wedge closed\ \{Err\}\ f \wedge Err \sqcup_f Err = Err$ which may not hold

lemma *err-semilat-UnionI*:

$[\![\forall A \in AS.\ err\text{-}semilat(A,\ r,\ f);\ AS \neq \{\};$

$\forall A \in AS. \forall B \in AS.\ A \neq B \longrightarrow (\forall a \in A. \forall b \in B.\ \neg a \sqsubseteq_r b \wedge a \sqcup_f b = Err)\ ]\!]$

$\Longrightarrow err\text{-}semilat(Union\ AS,\ r,\ f) \langle proof \rangle$

end

4.3 More about Options

theory *Opt* **imports** *Err* **begin**

definition $le :: {}'a\ ord \Rightarrow {}'a\ option\ ord$
where
$le\ r\ o_1\ o_2 =$
$(case\ o_2\ of\ None \Rightarrow o_1{=}None \mid Some\ y \Rightarrow (case\ o_1\ of\ None \Rightarrow True \mid Some\ x \Rightarrow x \sqsubseteq_r y))$

definition $opt :: {}'a\ set \Rightarrow {}'a\ option\ set$
where
$opt\ A = insert\ None\ \{Some\ y\ |y.\ y \in A\}$

definition $sup :: {}'a\ ebinop \Rightarrow {}'a\ option\ ebinop$
where
$sup\ f\ o_1\ o_2 =$
$(case\ o_1\ of\ None \Rightarrow OK\ o_2$
$\mid Some\ x \Rightarrow (case\ o_2\ of\ None \Rightarrow OK\ o_1$
$\mid Some\ y \Rightarrow (case\ f\ x\ y\ of\ Err \Rightarrow Err \mid OK\ z \Rightarrow OK\ (Some\ z))))$

definition $esl :: {}'a\ esl \Rightarrow {}'a\ option\ esl$
where
$esl = (\lambda(A,r,f).\ (opt\ A,\ le\ r,\ sup\ f))$

lemma *unfold-le-opt*:
$o_1 \sqsubseteq_{le\ r} o_2 =$
$(case\ o_2\ of\ None \Rightarrow o_1{=}None \mid$
$Some\ y \Rightarrow (case\ o_1\ of\ None \Rightarrow True \mid Some\ x \Rightarrow x \sqsubseteq_r y))\langle proof\rangle$
lemma *le-opt-refl*: $order\ r \Longrightarrow x \sqsubseteq_{le\ r} x\langle proof\rangle\langle proof\rangle\langle proof\rangle\langle proof\rangle\langle proof\rangle\langle proof\rangle\langle proof\rangle\langle proof\rangle\langle proof\rangle\langle proof\rangle\langle proof\rangle\langle proo$

4.4 Products as Semilattices

theory *Product*
imports *Err*
begin

definition $le :: 'a\ ord \Rightarrow 'b\ ord \Rightarrow ('a \times 'b)\ ord$
where
$le\ r_A\ r_B = (\lambda(a_1,b_1)\ (a_2,b_2).\ a_1 \sqsubseteq_{r_A} a_2 \wedge b_1 \sqsubseteq_{r_B} b_2)$

definition $sup :: 'a\ ebinop \Rightarrow 'b\ ebinop \Rightarrow ('a \times 'b)\ ebinop$
where
$sup\ f\ g = (\lambda(a_1,b_1)(a_2,b_2).\ Err.sup\ Pair\ (a_1 \sqcup_f a_2)\ (b_1 \sqcup_g b_2))$

definition $esl :: 'a\ esl \Rightarrow 'b\ esl \Rightarrow ('a \times 'b\ )\ esl$
where
$esl = (\lambda(A,r_A,f_A)\ (B,r_B,f_B).\ (A \times B,\ le\ r_A\ r_B,\ sup\ f_A\ f_B))$

abbreviation (*xsymbols*)
$lesubprod :: 'a \times 'b \Rightarrow ('a \Rightarrow 'a \Rightarrow bool) \Rightarrow ('b \Rightarrow 'b \Rightarrow bool) \Rightarrow 'a \times 'b \Rightarrow bool$
((- /⊑'(-,-') -) [*50, 0, 0, 51*] *50*) **where**
$p \sqsubseteq(rA,rB)\ q == p \sqsubseteq_{Product.le\ rA\ rB}\ q$

lemma *unfold-lesub-prod*: $x \sqsubseteq(r_A,r_B)\ y = le\ r_A\ r_B\ x\ y\langle proof\rangle$
lemma *le-prod-Pair-conv* [*iff*]: $((a_1,b_1) \sqsubseteq(r_A,r_B)\ (a_2,b_2)) = (a_1 \sqsubseteq_{r_A} a_2\ \&\ b_1 \sqsubseteq_{r_B} b_2)\langle proof\rangle$
lemma *less-prod-Pair-conv*:
$((a_1,b_1) \sqsubset_{Product.le\ r_A\ r_B} (a_2,b_2)) =$
$(a_1 \sqsubset_{r_A} a_2\ \&\ b_1 \sqsubseteq_{r_B} b_2 \mid a_1 \sqsubseteq_{r_A} a_2\ \&\ b_1 \sqsubset_{r_B} b_2)\langle proof\rangle$
lemma *order-le-prod* [*iff*]: $order(Product.le\ r_A\ r_B) = (order\ r_A\ \&\ order\ r_B)\langle proof\rangle$

lemma *acc-le-prodI* [*intro*!]:
$[\![\ acc\ r_A;\ acc\ r_B\ ]\!] \Longrightarrow acc(Product.le\ r_A\ r_B)\langle proof\rangle$

lemma *closed-lift2-sup*:
$[\![\ closed\ (err\ A)\ (lift2\ f);\ closed\ (err\ B)\ (lift2\ g)\ ]\!] \Longrightarrow$
$closed\ (err(A{\times}B))\ (lift2(sup\ f\ g))\langle proof\rangle$
lemma *unfold-plussub-lift2*: $e_1 \sqcup_{lift2\ f} e_2 = lift2\ f\ e_1\ e_2\langle proof\rangle$

lemma *plus-eq-Err-conv* [*simp*]:
assumes $x{\in}A\quad y{\in}A\quad semilat(err\ A,\ Err.le\ r,\ lift2\ f)$
shows $(x \sqcup_f y = Err) = (\neg(\exists\, z{\in}A.\ x \sqsubseteq_r z \wedge y \sqsubseteq_r z))\langle proof\rangle$
lemma *err-semilat-Product-esl*:
$\bigwedge L_1\ L_2.\ [\![\ err\text{-}semilat\ L_1;\ err\text{-}semilat\ L_2\ ]\!] \Longrightarrow err\text{-}semilat(Product.esl\ L_1\ L_2)\langle proof\rangle$
end

4.5 Fixed Length Lists

theory *Listn*
imports *Err*
begin

definition *list* :: *nat* ⇒ *'a set* ⇒ *'a list set*
where
  list n A = {*xs*. *size xs* = *n* ∧ *set xs* ⊆ *A*}

definition *le* :: *'a ord* ⇒ (*'a list*)*ord*
where
  le r = *list-all2* (λ*x y*. $x \sqsubseteq_r y$)

abbreviation (*xsymbols*)
  lesublist :: *'a list* ⇒ *'a ord* ⇒ *'a list* ⇒ *bool* ((- /[⊑-] -) [*50*, *0*, *51*] *50*) **where**
  x [$\sqsubseteq_r$] y == x <=-(*Listn.le r*) y

abbreviation (*xsymbols*)
  lesssublist :: *'a list* ⇒ *'a ord* ⇒ *'a list* ⇒ *bool* ((- /[⊏-] -) [*50*, *0*, *51*] *50*) **where**
  x [$\sqsubset_r$] y == x <-(*Listn.le r*) y

definition *map2* :: (*'a* ⇒ *'b* ⇒ *'c*) ⇒ *'a list* ⇒ *'b list* ⇒ *'c list*
where
  map2 f = (λ*xs ys*. *map* (*split f*) (*zip xs ys*))

abbreviation (*xsymbols*)
  plussublist :: *'a list* ⇒ (*'a* ⇒ *'b* ⇒ *'c*) ⇒ *'b list* ⇒ *'c list*
    ((- /[⊔-] -) [*65*, *0*, *66*] *65*) **where**
  x [$\sqcup_f$] y == $x \sqcup_{map2\ f} y$

primrec *coalesce* :: *'a err list* ⇒ *'a list err*
where
  coalesce [] = *OK*[]
| *coalesce* (*ex*#*exs*) = *Err.sup* (*op* #) *ex* (*coalesce exs*)

definition *sl* :: *nat* ⇒ *'a sl* ⇒ *'a list sl*
where
  sl n = (λ(*A*,*r*,*f*). (*list n A*, *le r*, *map2 f*))

definition *sup* :: (*'a* ⇒ *'b* ⇒ *'c err*) ⇒ *'a list* ⇒ *'b list* ⇒ *'c list err*
where
  sup f = (λ*xs ys*. *if size xs* = *size ys then coalesce*(*xs* [$\sqcup_f$] *ys*) *else Err*)

definition *upto-esl* :: *nat* ⇒ *'a esl* ⇒ *'a list esl*
where
  upto-esl m = (λ(*A*,*r*,*f*). (*Union*{*list n A* |*n*. *n* ≤ *m*}, *le r*, *sup f*))

lemmas [*simp*] = *set-update-subsetI*

lemma *unfold-lesub-list*: *xs* [$\sqsubseteq_r$] *ys* = *Listn.le r xs ys*⟨*proof*⟩
lemma *Nil-le-conv* [*iff*]: ([] [$\sqsubseteq_r$] *ys*) = (*ys* = [])⟨*proof*⟩

lemma *Cons-notle-Nil* [*iff*]: $\neg\ x\#xs\ [\sqsubseteq_r]\ []\langle proof\rangle$
lemma *Cons-le-Cons* [*iff*]: $x\#xs\ [\sqsubseteq_r]\ y\#ys = (x \sqsubseteq_r y \wedge xs\ [\sqsubseteq_r]\ ys)\langle proof\rangle$
lemma *Cons-less-Conss* [*simp*]:
$order\ r \Longrightarrow\ x\#xs\ [\sqsubset_r]\ y\#ys = (x \sqsubset_r y \wedge xs\ [\sqsubseteq_r]\ ys \vee x = y \wedge xs\ [\sqsubset_r]\ ys)\langle proof\rangle$
lemma *list-update-le-cong*:
$[\![\ i{<}size\ xs;\ xs\ [\sqsubseteq_r]\ ys;\ x \sqsubseteq_r y\ ]\!] \Longrightarrow xs[i{:=}x]\ [\sqsubseteq_r]\ ys[i{:=}y]\langle proof\rangle$

lemma *le-listD*: $[\![\ xs\ [\sqsubseteq_r]\ ys;\ p < size\ xs\ ]\!] \Longrightarrow xs!p \sqsubseteq_r ys!p\langle proof\rangle$
lemma *le-list-refl*: $\forall x.\ x \sqsubseteq_r x \Longrightarrow xs\ [\sqsubseteq_r]\ xs\langle proof\rangle$
lemma *le-list-trans*: $[\![\ order\ r;\ xs\ [\sqsubseteq_r]\ ys;\ ys\ [\sqsubseteq_r]\ zs\ ]\!] \Longrightarrow xs\ [\sqsubseteq_r]\ zs\langle proof\rangle$
lemma *le-list-antisym*: $[\![\ order\ r;\ xs\ [\sqsubseteq_r]\ ys;\ ys\ [\sqsubseteq_r]\ xs\ ]\!] \Longrightarrow xs = ys\langle proof\rangle$
lemma *order-listI* [*simp*, *intro*!]: $order\ r \Longrightarrow order(Listn.le\ r)\langle proof\rangle$
lemma *lesub-list-impl-same-size* [*simp*]: $xs\ [\sqsubseteq_r]\ ys \Longrightarrow size\ ys = size\ xs\langle proof\rangle$
lemma *lesssub-lengthD*: $xs\ [\sqsubset_r]\ ys \Longrightarrow size\ ys = size\ xs\langle proof\rangle$
lemma *le-list-appendI*: $a\ [\sqsubseteq_r]\ b \Longrightarrow c\ [\sqsubseteq_r]\ d \Longrightarrow a@c\ [\sqsubseteq_r]\ b@d\langle proof\rangle$
lemma *le-listI*:
assumes $length\ a = length\ b$
assumes $\bigwedge n.\ n < length\ a \Longrightarrow a!n \sqsubseteq_r b!n$
shows $a\ [\sqsubseteq_r]\ b\langle proof\rangle$
lemma *listI*: $[\![\ size\ xs = n;\ set\ xs \subseteq A\ ]\!] \Longrightarrow xs \in list\ n\ A\langle proof\rangle$

lemma *listE-length* [*simp*]: $xs \in list\ n\ A \Longrightarrow size\ xs = n\langle proof\rangle$
lemma *less-lengthI*: $[\![\ xs \in list\ n\ A;\ p < n\ ]\!] \Longrightarrow p < size\ xs\langle proof\rangle$
lemma *listE-set* [*simp*]: $xs \in list\ n\ A \Longrightarrow set\ xs \subseteq A\langle proof\rangle$
lemma *list-0* [*simp*]: $list\ 0\ A = \{[]\}\langle proof\rangle$
lemma *in-list-Suc-iff*:
$(xs \in list\ (Suc\ n)\ A) = (\exists y{\in}A.\ \exists ys \in list\ n\ A.\ xs = y\#ys)\langle proof\rangle$
lemma *Cons-in-list-Suc* [*iff*]:
$(x\#xs \in list\ (Suc\ n)\ A) = (x{\in}A \wedge xs \in list\ n\ A)\langle proof\rangle$
lemma *list-not-empty*:
$\exists a.\ a{\in}A \Longrightarrow \exists xs.\ xs \in list\ n\ A\langle proof\rangle$

lemma *nth-in* [*rule-format*, *simp*]:
$\forall i\ n.\ size\ xs = n \longrightarrow set\ xs \subseteq A \longrightarrow i < n \longrightarrow (xs!i) \in A\langle proof\rangle$
lemma *listE-nth-in*: $[\![\ xs \in list\ n\ A;\ i < n\ ]\!] \Longrightarrow xs!i \in A\langle proof\rangle$
lemma *listn-Cons-Suc* [*elim*!]:
$l\#xs \in list\ n\ A \Longrightarrow (\bigwedge n'.\ n = Suc\ n' \Longrightarrow l \in A \Longrightarrow xs \in list\ n'\ A \Longrightarrow P) \Longrightarrow P\langle proof\rangle$
lemma *listn-appendE* [*elim*!]:
$a@b \in list\ n\ A \Longrightarrow (\bigwedge n1\ n2.\ n{=}n1{+}n2 \Longrightarrow a \in list\ n1\ A \Longrightarrow b \in list\ n2\ A \Longrightarrow P) \Longrightarrow P\langle proof\rangle$

lemma *listt-update-in-list* [*simp*, *intro*!]:
$[\![\ xs \in list\ n\ A;\ x{\in}A\ ]\!] \Longrightarrow xs[i := x] \in list\ n\ A\langle proof\rangle$
lemma *list-appendI* [*intro?*]:
$[\![\ a \in list\ n\ A;\ b \in list\ m\ A\ ]\!] \Longrightarrow a\ @\ b \in list\ (n{+}m)\ A\langle proof\rangle$
lemma *list-map* [*simp*]: $(map\ f\ xs \in list\ (size\ xs)\ A) = (f\ `\ set\ xs \subseteq A)\langle proof\rangle$
lemma *list-replicateI* [*intro*]: $x \in A \Longrightarrow replicate\ n\ x \in list\ n\ A\langle proof\rangle$
lemma *plus-list-Nil* [*simp*]: $[]\ [\sqcup_f]\ xs = []\langle proof\rangle$
lemma *plus-list-Cons* [*simp*]:
$(x\#xs)\ [\sqcup_f]\ ys = (case\ ys\ of\ [] \Rightarrow []\ |\ y\#ys \Rightarrow (x \sqcup_f y)\#(xs\ [\sqcup_f]\ ys))\langle proof\rangle$
lemma *length-plus-list* [*rule-format*, *simp*]:
$\forall ys.\ size(xs\ [\sqcup_f]\ ys) = min(size\ xs)\ (size\ ys)\langle proof\rangle$
lemma *nth-plus-list* [*rule-format*, *simp*]:
$\forall xs\ ys\ i.\ size\ xs = n \longrightarrow size\ ys = n \longrightarrow i{<}n \longrightarrow (xs\ [\sqcup_f]\ ys)!i = (xs!i) \sqcup_f (ys!i)\langle proof\rangle$

lemma (**in** *Semilat*) *plus-list-ub1* [*rule-format*]:
$\llbracket$ *set xs* $\subseteq A$; *set ys* $\subseteq A$; *size xs* $=$ *size ys* $\rrbracket$
$\Longrightarrow xs\ [\sqsubseteq_r]\ xs\ [\sqcup_f]\ ys\langle proof\rangle$
lemma (**in** *Semilat*) *plus-list-ub2*:
$\llbracket$*set xs* $\subseteq A$; *set ys* $\subseteq A$; *size xs* $=$ *size ys* $\rrbracket \Longrightarrow ys\ [\sqsubseteq_r]\ xs\ [\sqcup_f]\ ys\langle proof\rangle$
lemma (**in** *Semilat*) *plus-list-lub* [*rule-format*]:
shows $\forall\, xs\ ys\ zs.$ *set xs* $\subseteq A \longrightarrow$ *set ys* $\subseteq A \longrightarrow$ *set zs* $\subseteq A$
$\longrightarrow$ *size xs* $= n \wedge$ *size ys* $= n \longrightarrow$
$xs\ [\sqsubseteq_r]\ zs \wedge ys\ [\sqsubseteq_r]\ zs \longrightarrow xs\ [\sqcup_f]\ ys\ [\sqsubseteq_r]\ zs\langle proof\rangle$
lemma (**in** *Semilat*) *list-update-incr* [*rule-format*]:
$x{\in}A \Longrightarrow$ *set xs* $\subseteq A \longrightarrow$
$(\forall\, i.\ i{<}size\ xs \longrightarrow xs\ [\sqsubseteq_r]\ xs[i := x \sqcup_f xs!i])\langle proof\rangle$
lemma *acc-le-listI* [*intro*!]:
$\llbracket$ *order r*; *acc r* $\rrbracket \Longrightarrow acc(Listn.le\ r)\langle proof\rangle$
lemma *closed-listI*:
closed S f $\Longrightarrow$ *closed* (*list n S*) (*map2 f*)$\langle proof\rangle$

lemma *Listn-sl-aux*:
assumes *Semilat A r f* **shows** *semilat* (*Listn.sl n* (*A*,*r*,*f*))$\langle proof\rangle$
lemma *Listn-sl*: *semilat L* $\Longrightarrow$ *semilat* (*Listn.sl n L*)$\langle proof\rangle$
lemma *coalesce-in-err-list* [*rule-format*]:
$\forall\, xes.\ xes \in list\ n\ (err\ A) \longrightarrow coalesce\ xes \in err(list\ n\ A)\langle proof\rangle$
lemma *lem*: $\bigwedge x\ xs.\ x \sqcup_{op\ \#}\ xs = x\#xs\langle proof\rangle$
lemma *coalesce-eq-OK1-D* [*rule-format*]:
semilat(*err A*, *Err.le r*, *lift2 f*) $\Longrightarrow$
$\forall\, xs.\ xs \in list\ n\ A \longrightarrow (\forall\, ys.\ ys \in list\ n\ A \longrightarrow$
$(\forall\, zs.\ coalesce\ (xs\ [\sqcup_f]\ ys) = OK\ zs \longrightarrow xs\ [\sqsubseteq_r]\ zs))\langle proof\rangle$
lemma *coalesce-eq-OK2-D* [*rule-format*]:
semilat(*err A*, *Err.le r*, *lift2 f*) $\Longrightarrow$
$\forall\, xs.\ xs \in list\ n\ A \longrightarrow (\forall\, ys.\ ys \in list\ n\ A \longrightarrow$
$(\forall\, zs.\ coalesce\ (xs\ [\sqcup_f]\ ys) = OK\ zs \longrightarrow ys\ [\sqsubseteq_r]\ zs))\langle proof\rangle$
lemma *lift2-le-ub*:
$\llbracket$ *semilat*(*err A*, *Err.le r*, *lift2 f*); $x{\in}A$; $y{\in}A$; $x \sqcup_f y = OK\ z$;
$u{\in}A$; $x \sqsubseteq_r u$; $y \sqsubseteq_r u$ $\rrbracket \Longrightarrow z \sqsubseteq_r u\langle proof\rangle$
lemma *coalesce-eq-OK-ub-D* [*rule-format*]:
semilat(*err A*, *Err.le r*, *lift2 f*) $\Longrightarrow$
$\forall\, xs.\ xs \in list\ n\ A \longrightarrow (\forall\, ys.\ ys \in list\ n\ A \longrightarrow$
$(\forall\, zs\ us.\ coalesce\ (xs\ [\sqcup_f]\ ys) = OK\ zs \wedge xs\ [\sqsubseteq_r]\ us \wedge ys\ [\sqsubseteq_r]\ us$
$\wedge\ us \in list\ n\ A \longrightarrow zs\ [\sqsubseteq_r]\ us))\langle proof\rangle$
lemma *lift2-eq-ErrD*:
$\llbracket$ $x \sqcup_f y = Err$; *semilat*(*err A*, *Err.le r*, *lift2 f*); $x{\in}A$; $y{\in}A$ $\rrbracket$
$\Longrightarrow \neg(\exists\, u{\in}A.\ x \sqsubseteq_r u \wedge y \sqsubseteq_r u)\langle proof\rangle$

lemma *coalesce-eq-Err-D* [*rule-format*]:
$\llbracket$ *semilat*(*err A*, *Err.le r*, *lift2 f*) $\rrbracket$
$\Longrightarrow \forall\, xs.\ xs \in list\ n\ A \longrightarrow (\forall\, ys.\ ys \in list\ n\ A \longrightarrow$
$coalesce\ (xs\ [\sqcup_f]\ ys) = Err \longrightarrow$
$\neg(\exists\, zs \in list\ n\ A.\ xs\ [\sqsubseteq_r]\ zs \wedge ys\ [\sqsubseteq_r]\ zs))\langle proof\rangle$
lemma *closed-err-lift2-conv*:
closed (*err A*) (*lift2 f*) $= (\forall\, x{\in}A.\ \forall\, y{\in}A.\ x \sqcup_f y \in err\ A)\langle proof\rangle$
lemma *closed-map2-list* [*rule-format*]:
closed (*err A*) (*lift2 f*) $\Longrightarrow$
$\forall\, xs.\ xs \in list\ n\ A \longrightarrow (\forall\, ys.\ ys \in list\ n\ A \longrightarrow$

$map2\ f\ xs\ ys \in list\ n\ (err\ A))\langle proof\rangle$
lemma *closed-lift2-sup*:
$closed\ (err\ A)\ (lift2\ f) \Longrightarrow$
$closed\ (err\ (list\ n\ A))\ (lift2\ (sup\ f))\langle proof\rangle$
lemma *err-semilat-sup*:
$err\text{-}semilat\ (A,r,f) \Longrightarrow$
$err\text{-}semilat\ (list\ n\ A,\ Listn.le\ r,\ sup\ f)\langle proof\rangle$
lemma *err-semilat-upto-esl*:
$\bigwedge L.\ err\text{-}semilat\ L \Longrightarrow err\text{-}semilat(upto\text{-}esl\ m\ L)\langle proof\rangle$
end

4.6 Typing and Dataflow Analysis Framework

theory *Typing-Framework* **imports** *Semilattices* **begin**

The relationship between dataflow analysis and a welltyped-instruction predicate.

type-synonym
$'s\ step\text{-}type = nat \Rightarrow 's \Rightarrow (nat \times 's)\ list$

definition $stable :: 's\ ord \Rightarrow 's\ step\text{-}type \Rightarrow 's\ list \Rightarrow nat \Rightarrow bool$
where
$stable\ r\ step\ \tau s\ p \longleftrightarrow (\forall (q,\tau) \in set\ (step\ p\ (\tau s!p)).\ \tau \sqsubseteq_r \tau s!q)$

definition $stables :: 's\ ord \Rightarrow 's\ step\text{-}type \Rightarrow 's\ list \Rightarrow bool$
where
$stables\ r\ step\ \tau s \longleftrightarrow (\forall p < size\ \tau s.\ stable\ r\ step\ \tau s\ p)$

definition $wt\text{-}step :: 's\ ord \Rightarrow 's \Rightarrow 's\ step\text{-}type \Rightarrow 's\ list \Rightarrow bool$
where
$wt\text{-}step\ r\ T\ step\ \tau s \longleftrightarrow (\forall p{<}size\ \tau s.\ \tau s!p \neq T \wedge stable\ r\ step\ \tau s\ p)$

definition $is\text{-}bcv :: 's\ ord \Rightarrow 's \Rightarrow 's\ step\text{-}type \Rightarrow nat \Rightarrow 's\ set \Rightarrow ('s\ list \Rightarrow 's\ list) \Rightarrow bool$
where
$is\text{-}bcv\ r\ T\ step\ n\ A\ bcv \longleftrightarrow (\forall \tau s_0 \in list\ n\ A.$
$(\forall p{<}n.\ (bcv\ \tau s_0)!p \neq T) = (\exists \tau s \in list\ n\ A.\ \tau s_0\ [\sqsubseteq_r]\ \tau s \wedge wt\text{-}step\ r\ T\ step\ \tau s))$

end

4.7 More on Semilattices

theory *SemilatAlg*
imports *Typing-Framework*
begin

consts
  $lesubstep\text{-}type :: (nat \times 's)\ set \Rightarrow 's\ ord \Rightarrow (nat \times 's)\ set \Rightarrow bool$**notation** (*xsymbols*)
  $lesubstep\text{-}type\ \ ((\text{-}\ /\{\sqsubseteq_{\text{-}}\}\ \text{-})\ [50,\ 0,\ 51]\ 50)$
defs *lesubstep-type-def*:
  $A\ \{\sqsubseteq_r\}\ B \equiv \forall (p,\tau) \in A.\ \exists \tau'.\ (p,\tau') \in B \wedge \tau \sqsubseteq_r \tau'$

primrec $pluslussub :: 'a\ list \Rightarrow ('a \Rightarrow 'a \Rightarrow 'a) \Rightarrow 'a \Rightarrow 'a$
where
  $pluslussub\ []\ f\ y = y$
| $pluslussub\ (x\#xs)\ f\ y = pluslussub\ xs\ f\ (x \sqcup_f y)$**notation** (*xsymbols*)
  $pluslussub\ \ ((\text{-}\ /\bigsqcup_{\text{-}}\ \text{-})\ [65,\ 0,\ 66]\ 65)$

definition $bounded :: 's\ step\text{-}type \Rightarrow nat \Rightarrow bool$
where
  $bounded\ step\ n \longleftrightarrow (\forall p{<}n.\ \forall \tau.\ \forall (q,\tau') \in set\ (step\ p\ \tau).\ q{<}n)$

definition $pres\text{-}type :: 's\ step\text{-}type \Rightarrow nat \Rightarrow 's\ set \Rightarrow bool$
where
  $pres\text{-}type\ step\ n\ A \longleftrightarrow (\forall \tau{\in}A.\ \forall p{<}n.\ \forall (q,\tau') \in set\ (step\ p\ \tau).\ \tau' \in A)$

definition $mono :: 's\ ord \Rightarrow 's\ step\text{-}type \Rightarrow nat \Rightarrow 's\ set \Rightarrow bool$
where
  $mono\ r\ step\ n\ A \longleftrightarrow$
    $(\forall \tau\ p\ \tau'.\ \tau \in A \wedge p{<}n \wedge \tau \sqsubseteq_r \tau' \longrightarrow set\ (step\ p\ \tau)\ \{\sqsubseteq_r\}\ set\ (step\ p\ \tau'))$

lemma [*iff*]: $\{\}\ \{\sqsubseteq_r\}\ B$
  ⟨*proof*⟩
lemma [*iff*]: $(A\ \{\sqsubseteq_r\}\ \{\}) = (A = \{\})$
  ⟨*proof*⟩
lemma *lesubstep-union*:
  $[\![\ A_1\ \{\sqsubseteq_r\}\ B_1;\ A_2\ \{\sqsubseteq_r\}\ B_2\ ]\!] \Longrightarrow A_1 \cup A_2\ \{\sqsubseteq_r\}\ B_1 \cup B_2$
  ⟨*proof*⟩
lemma *pres-typeD*:
  $[\![\ pres\text{-}type\ step\ n\ A;\ s{\in}A;\ p{<}n;\ (q,s'){\in}set\ (step\ p\ s)\ ]\!] \Longrightarrow s' \in A$⟨*proof*⟩
lemma *monoD*:
  $[\![\ mono\ r\ step\ n\ A;\ p < n;\ s{\in}A;\ s \sqsubseteq_r t\ ]\!] \Longrightarrow set\ (step\ p\ s)\ \{\sqsubseteq_r\}\ set\ (step\ p\ t)$⟨*proof*⟩
lemma *boundedD*:
  $[\![\ bounded\ step\ n;\ p < n;\ (q,t) \in set\ (step\ p\ xs)\ ]\!] \Longrightarrow q < n$⟨*proof*⟩
lemma *lesubstep-type-refl* [*simp*, *intro*]:
  $(\bigwedge x.\ x \sqsubseteq_r x) \Longrightarrow A\ \{\sqsubseteq_r\}\ A$⟨*proof*⟩
lemma *lesub-step-typeD*:
  $A\ \{\sqsubseteq_r\}\ B \Longrightarrow (x,y) \in A \Longrightarrow \exists y'.\ (x,\ y') \in B \wedge y \sqsubseteq_r y'$⟨*proof*⟩

lemma *list-update-le-listI* [*rule-format*]:
  $set\ xs \subseteq A \longrightarrow set\ ys \subseteq A \longrightarrow xs\ [\sqsubseteq_r]\ ys \longrightarrow p < size\ xs \longrightarrow$
   $x \sqsubseteq_r ys!p \longrightarrow semilat(A,r,f) \longrightarrow x{\in}A \longrightarrow$
   $xs[p := x \sqcup_f xs!p]\ [\sqsubseteq_r]\ ys$⟨*proof*⟩
lemma *plusplus-closed*: **assumes** *Semilat A r f* **shows**

$\bigwedge y.$ ⟦ *set* $x \subseteq A$; $y \in A$⟧ $\Longrightarrow x \bigsqcup_f y \in A$⟨*proof*⟩

lemma (**in** *Semilat*) *pp-ub2*:
$\bigwedge y.$ ⟦ *set* $x \subseteq A$; $y \in A$⟧ $\Longrightarrow y \sqsubseteq_r x \bigsqcup_f y$⟨*proof*⟩

lemma (**in** *Semilat*) *pp-ub1*:
shows $\bigwedge y.$ ⟦*set ls* $\subseteq A$; $y \in A$; $x \in$ *set ls*⟧ $\Longrightarrow x \sqsubseteq_r$ *ls* $\bigsqcup_f y$⟨*proof*⟩

lemma (**in** *Semilat*) *pp-lub*:
assumes z: $z \in A$
shows
$\bigwedge y.\ y \in A \Longrightarrow$ *set xs* $\subseteq A \Longrightarrow \forall x \in$ *set xs*. $x \sqsubseteq_r z \Longrightarrow y \sqsubseteq_r z \Longrightarrow$ *xs* $\bigsqcup_f y \sqsubseteq_r z$⟨*proof*⟩

lemma *ub1'*: **assumes** *Semilat A r f*
shows ⟦$\forall (p,s) \in$ *set S*. $s \in A$; $y \in A$; $(a,b) \in$ *set S*⟧
$\Longrightarrow b \sqsubseteq_r$ *map snd* $[(p', t') \leftarrow S.\ p' = a] \bigsqcup_f y$⟨*proof*⟩

lemma *plusplus-empty*:
$\forall s'.\ (q, s') \in$ *set S* $\longrightarrow s' \sqcup_f$ *ss* ! q = *ss* ! $q \Longrightarrow$
(*map snd* $[(p', t') \leftarrow S.\ p' = q] \bigsqcup_f$ *ss* ! q) = *ss* ! q⟨*proof*⟩

end

4.8 Lifting the Typing Framework to err, app, and eff

theory *Typing-Framework-err* **imports** *Typing-Framework SemilatAlg* **begin**

definition $wt\text{-}err\text{-}step :: 's\ ord \Rightarrow 's\ err\ step\text{-}type \Rightarrow 's\ err\ list \Rightarrow bool$
where
$wt\text{-}err\text{-}step\ r\ step\ \tau s \longleftrightarrow wt\text{-}step\ (Err.le\ r)\ Err\ step\ \tau s$

definition $wt\text{-}app\text{-}eff :: 's\ ord \Rightarrow (nat \Rightarrow 's \Rightarrow bool) \Rightarrow 's\ step\text{-}type \Rightarrow 's\ list \Rightarrow bool$
where
$wt\text{-}app\text{-}eff\ r\ app\ step\ \tau s \longleftrightarrow$
$(\forall\, p < size\ \tau s.\ app\ p\ (\tau s!p) \wedge (\forall\,(q,\tau) \in set\ (step\ p\ (\tau s!p)).\ \tau <=\text{-}r\ \tau s!q))$

definition $map\text{-}snd :: ('b \Rightarrow 'c) \Rightarrow ('a \times 'b)\ list \Rightarrow ('a \times 'c)\ list$
where
$map\text{-}snd\ f = map\ (\lambda(x,y).\ (x,\ f\ y))$

definition $error :: nat \Rightarrow (nat \times 'a\ err)\ list$
where
$error\ n = map\ (\lambda x.\ (x,Err))\ [0..<n]$

definition $err\text{-}step :: nat \Rightarrow (nat \Rightarrow 's \Rightarrow bool) \Rightarrow 's\ step\text{-}type \Rightarrow 's\ err\ step\text{-}type$
where
$err\text{-}step\ n\ app\ step\ p\ t =$
$(case\ t\ of$
$\quad Err \quad \Rightarrow error\ n$
$\mid OK\ \tau \Rightarrow if\ app\ p\ \tau\ then\ map\text{-}snd\ OK\ (step\ p\ \tau)\ else\ error\ n)$

definition $app\text{-}mono :: 's\ ord \Rightarrow (nat \Rightarrow 's \Rightarrow bool) \Rightarrow nat \Rightarrow 's\ set \Rightarrow bool$
where
$app\text{-}mono\ r\ app\ n\ A \longleftrightarrow$
$(\forall\, s\ p\ t.\ s \in A \wedge p < n \wedge s \sqsubseteq_r t \longrightarrow app\ p\ t \longrightarrow app\ p\ s)$

lemmas $err\text{-}step\text{-}defs = err\text{-}step\text{-}def\ map\text{-}snd\text{-}def\ error\text{-}def$

lemma *bounded-err-stepD*:
$[\![\ bounded\ (err\text{-}step\ n\ app\ step)\ n;$
$p < n;\ app\ p\ a;\ (q,b) \in set\ (step\ p\ a)\ ]\!] \Longrightarrow q < n\langle proof\rangle$

lemma *in-map-sndD*: $(a,b) \in set\ (map\text{-}snd\ f\ xs) \Longrightarrow \exists\, b'.\ (a,b') \in set\ xs\langle proof\rangle$

lemma *bounded-err-stepI*:
$\forall\, p.\ p < n \longrightarrow (\forall\, s.\ ap\ p\ s \longrightarrow (\forall\,(q,s') \in set\ (step\ p\ s).\ q < n))$
$\Longrightarrow bounded\ (err\text{-}step\ n\ ap\ step)\ n\langle proof\rangle$

lemma *bounded-lift*:
$bounded\ step\ n \Longrightarrow bounded\ (err\text{-}step\ n\ app\ step)\ n\langle proof\rangle$

lemma *le-list-map-OK* [*simp*]:
$\bigwedge b.\ (map\ OK\ a\ [\sqsubseteq_{Err.le\ r}]\ map\ OK\ b) = (a\ [\sqsubseteq_r]\ b)\langle proof\rangle$

lemma *map-snd-lessI*:

set xs $\{\sqsubseteq_r\}$ *set ys* $\Longrightarrow$ *set* (*map-snd OK xs*) $\{\sqsubseteq_{Err.le\ r}\}$ *set* (*map-snd OK ys*)⟨*proof*⟩

lemma *mono-lift*:
$[\![$ *order r*; *app-mono r app n A*; *bounded* (*err-step n app step*) *n*;
$\forall s\ p\ t.\ s \in A \land p < n \land s \sqsubseteq_r t \longrightarrow app\ p\ t \longrightarrow set\ (step\ p\ s)\ \{\sqsubseteq_r\}\ set\ (step\ p\ t)$ $]\!]$
$\Longrightarrow$ *mono* (*Err.le r*) (*err-step n app step*) *n* (*err A*)⟨*proof*⟩
lemma *in-errorD*: $(x,y) \in set\ (error\ n) \Longrightarrow y = Err$⟨*proof*⟩
lemma *pres-type-lift*:
$\forall s{\in}A.\ \forall p.\ p < n \longrightarrow app\ p\ s \longrightarrow (\forall (q,\ s'){\in}set\ (step\ p\ s).\ s' \in A)$
$\Longrightarrow$ *pres-type* (*err-step n app step*) *n* (*err A*)⟨*proof*⟩

lemma *wt-err-imp-wt-app-eff*:
assumes *wt*: *wt-err-step r* (*err-step* (*size ts*) *app step*) *ts*
assumes *b*: *bounded* (*err-step* (*size ts*) *app step*) (*size ts*)
shows *wt-app-eff r app step* (*map ok-val ts*)⟨*proof*⟩

lemma *wt-app-eff-imp-wt-err*:
assumes *app-eff*: *wt-app-eff r app step ts*
assumes *bounded*: *bounded* (*err-step* (*size ts*) *app step*) (*size ts*)
shows *wt-err-step r* (*err-step* (*size ts*) *app step*) (*map OK ts*)⟨*proof*⟩
end

4.9 Kildall's Algorithm

theory *Kildall*
imports *SemilatAlg*
begin

primrec $propa :: {'s\ binop} \Rightarrow (nat \times {'s})\ list \Rightarrow {'s}\ list \Rightarrow nat\ set \Rightarrow {'s}\ list * nat\ set$
where
$propa\ f\ []\ \ \ \ \tau s\ w = (\tau s, w)$
$|\ propa\ f\ (q'\#qs)\ \tau s\ w = (let\ (q,\tau) = q';$
$\quad u = \tau \sqcup_f \tau s!q;$
$\quad w' = (if\ u = \tau s!q\ then\ w\ else\ insert\ q\ w)$
$\quad in\ propa\ f\ qs\ (\tau s[q := u])\ w')$

definition $iter :: {'s\ binop} \Rightarrow {'s\ step\text{-}type} \Rightarrow$
$\quad {'s}\ list \Rightarrow nat\ set \Rightarrow {'s}\ list \times nat\ set$
where
$iter\ f\ step\ \tau s\ w =$
$\ while\ (\lambda(\tau s,w).\ w \neq \{\})$
$\quad (\lambda(\tau s,w).\ let\ p = SOME\ p.\ p \in w$
$\qquad in\ propa\ f\ (step\ p\ (\tau s!p))\ \tau s\ (w - \{p\}))$
$\quad (\tau s,w)$

definition $unstables :: {'s\ ord} \Rightarrow {'s\ step\text{-}type} \Rightarrow {'s}\ list \Rightarrow nat\ set$
where
$unstables\ r\ step\ \tau s = \{p.\ p < size\ \tau s \wedge \neg stable\ r\ step\ \tau s\ p\}$

definition $kildall :: {'s\ ord} \Rightarrow {'s\ binop} \Rightarrow {'s\ step\text{-}type} \Rightarrow {'s}\ list \Rightarrow {'s}\ list$
where
$kildall\ r\ f\ step\ \tau s = fst(iter\ f\ step\ \tau s\ (unstables\ r\ step\ \tau s))$

primrec $merges :: {'s\ binop} \Rightarrow (nat \times {'s})\ list \Rightarrow {'s}\ list \Rightarrow {'s}\ list$
where
$merges\ f\ []\ \ \ \ \tau s = \tau s$
$|\ merges\ f\ (p'\#ps)\ \tau s = (let\ (p,\tau) = p'\ in\ merges\ f\ ps\ (\tau s[p := \tau \sqcup_f \tau s!p]))$

lemmas [*simp*] = *Let-def Semilat.le-iff-plus-unchanged* [*OF Semilat.intro*, *symmetric*]

lemma (**in** *Semilat*) *nth-merges*:
$\bigwedge ss.\ [\![p < length\ ss;\ ss \in list\ n\ A;\ \forall (p,t) \in set\ ps.\ p<n \wedge t \in A\ ]\!] \Longrightarrow$
$(merges\ f\ ps\ ss)!p = map\ snd\ [(p',t') \leftarrow ps.\ p'=p]\ \bigsqcup_f ss!p$
(**is** $\bigwedge ss.\ [\![\text{-};\ \text{-};\ ?steptype\ ps]\!] \Longrightarrow ?P\ ss\ ps)\langle proof\rangle$

lemma *length-merges* [*simp*]:
$\bigwedge ss.\ size(merges\ f\ ps\ ss) = size\ ss \langle proof\rangle$
lemma (**in** *Semilat*) *merges-preserves-type-lemma*:
shows $\forall xs.\ xs \in list\ n\ A \longrightarrow (\forall (p,x) \in set\ ps.\ p<n \wedge x \in A)$
$\quad \longrightarrow merges\ f\ ps\ xs \in list\ n\ A\langle proof\rangle$

lemma (**in** *Semilat*) *merges-preserves-type* [*simp*]:
$[\![\ xs \in list\ n\ A;\ \forall (p,x) \in set\ ps.\ p{<}n \wedge x{\in}A\ ]\!]$
$\Longrightarrow merges\ f\ ps\ xs \in list\ n\ A$
⟨*proof*⟩

lemma (**in** *Semilat*) *merges-incr-lemma*:
$\forall xs.\ xs \in list\ n\ A \longrightarrow (\forall (p,x){\in}set\ ps.\ p{<}size\ xs \wedge x \in A) \longrightarrow xs\ [\sqsubseteq_r]\ merges\ f\ ps\ xs$⟨*proof*⟩
lemma (**in** *Semilat*) *merges-incr*:
$[\![\ xs \in list\ n\ A;\ \forall (p,x){\in}set\ ps.\ p{<}size\ xs \wedge x \in A\ ]\!]$
$\Longrightarrow xs\ [\sqsubseteq_r]\ merges\ f\ ps\ xs$
⟨*proof*⟩

lemma (**in** *Semilat*) *merges-same-conv* [*rule-format*]:
$(\forall xs.\ xs \in list\ n\ A \longrightarrow (\forall (p,x){\in}set\ ps.\ p{<}size\ xs \wedge x{\in}A) \longrightarrow$
$(merges\ f\ ps\ xs = xs) = (\forall (p,x){\in}set\ ps.\ x \sqsubseteq_r xs!p))$⟨*proof*⟩

lemma (**in** *Semilat*) *list-update-le-listI* [*rule-format*]:
$set\ xs \subseteq A \longrightarrow set\ ys \subseteq A \longrightarrow xs\ [\sqsubseteq_r]\ ys \longrightarrow p < size\ xs \longrightarrow$
$x \sqsubseteq_r ys!p \longrightarrow x{\in}A \longrightarrow xs[p := x \sqcup_f xs!p]\ [\sqsubseteq_r]\ ys$⟨*proof*⟩
lemma (**in** *Semilat*) *merges-pres-le-ub*:
assumes $set\ ts \subseteq A\quad set\ ss \subseteq A$
$\forall (p,t){\in}set\ ps.\ t \sqsubseteq_r ts!p \wedge t \in A \wedge p < size\ ts\quad ss\ [\sqsubseteq_r]\ ts$
shows $merges\ f\ ps\ ss\ [\sqsubseteq_r]\ ts$⟨*proof*⟩

lemma *decomp-propa*:
$\bigwedge ss\ w.\ (\forall (q,t){\in}set\ qs.\ q < size\ ss) \Longrightarrow$
$propa\ f\ qs\ ss\ w =$
$(merges\ f\ qs\ ss,\ \{q.\ \exists t.(q,t){\in}set\ qs \wedge t \sqcup_f ss!q \neq ss!q\} \cup w)$⟨*proof*⟩

lemma (**in** *Semilat*) *stable-pres-lemma*:
shows $[\![pres\text{-}type\ step\ n\ A;\ bounded\ step\ n;$
$ss \in list\ n\ A;\ p \in w;\ \forall q{\in}w.\ q < n;$
$\forall q.\ q < n \longrightarrow q \notin w \longrightarrow stable\ r\ step\ ss\ q;\ q < n;$
$\forall s'.\ (q,s') \in set\ (step\ p\ (ss!p)) \longrightarrow s' \sqcup_f ss!q = ss!q;$
$q \notin w \vee q = p\ ]\!]$
$\Longrightarrow stable\ r\ step\ (merges\ f\ (step\ p\ (ss!p))\ ss)\ q$⟨*proof*⟩

lemma (**in** *Semilat*) *merges-bounded-lemma*:
$[\![\ mono\ r\ step\ n\ A;\ bounded\ step\ n;$
$\forall (p',s') \in set\ (step\ p\ (ss!p)).\ s' \in A;\ ss \in list\ n\ A;\ ts \in list\ n\ A;\ p < n;$
$ss\ [\sqsubseteq_r]\ ts;\ \forall p.\ p < n \longrightarrow stable\ r\ step\ ts\ p\ ]\!]$
$\Longrightarrow merges\ f\ (step\ p\ (ss!p))\ ss\ [\sqsubseteq_r]\ ts$⟨*proof*⟩
lemma *termination-lemma*: **assumes** *Semilat A r f*
shows $[\![\ ss \in list\ n\ A;\ \forall (q,t){\in}set\ qs.\ q{<}n \wedge t{\in}A;\ p{\in}w\ ]\!] \Longrightarrow$
$ss\ [\sqsubset_r]\ merges\ f\ qs\ ss\ \vee$
$merges\ f\ qs\ ss = ss \wedge \{q.\ \exists t.\ (q,t){\in}set\ qs \wedge t \sqcup_f ss!q \neq ss!q\} \cup (w{-}\{p\}) \subset w$⟨*proof*⟩
lemma *iter-properties*[*rule-format*]: **assumes** *Semilat A r f*
shows $[\![\ acc\ r;\ pres\text{-}type\ step\ n\ A;\ mono\ r\ step\ n\ A;$
$bounded\ step\ n;\ \forall p{\in}w0.\ p < n;\ ss0 \in list\ n\ A;$
$\forall p{<}n.\ p \notin w0 \longrightarrow stable\ r\ step\ ss0\ p\ ]\!] \Longrightarrow$

$iter\ f\ step\ ss0\ w0 = (ss',w')$
$\longrightarrow$
$ss' \in list\ n\ A \wedge stables\ r\ step\ ss' \wedge ss0\ [\sqsubseteq_r]\ ss' \wedge$
$(\forall ts\in list\ n\ A.\ ss0\ [\sqsubseteq_r]\ ts \wedge stables\ r\ step\ ts \longrightarrow ss'\ [\sqsubseteq_r]\ ts)\langle proof\rangle$

lemma *kildall-properties*: **assumes** *Semilat A r f*
shows $[\![\ acc\ r;\ pres\text{-}type\ step\ n\ A;\ mono\ r\ step\ n\ A;$
$bounded\ step\ n;\ ss0 \in list\ n\ A\ ]\!] \Longrightarrow$
$kildall\ r\ f\ step\ ss0 \in list\ n\ A \wedge$
$stables\ r\ step\ (kildall\ r\ f\ step\ ss0) \wedge$
$ss0\ [\sqsubseteq_r]\ kildall\ r\ f\ step\ ss0 \wedge$
$(\forall ts\in list\ n\ A.\ ss0\ [\sqsubseteq_r]\ ts \wedge stables\ r\ step\ ts \longrightarrow$
$kildall\ r\ f\ step\ ss0\ [\sqsubseteq_r]\ ts)\langle proof\rangle\langle proof\rangle$
end

4.10 The Lightweight Bytecode Verifier

theory *LBVSpec*
imports *SemilatAlg Opt*
begin

type-synonym
$'s$ *certificate* = $'s$ *list*

primrec *merge* :: $'s$ *certificate* ⇒ $'s$ *binop* ⇒ $'s$ *ord* ⇒ $'s$ ⇒ *nat* ⇒ (*nat* × $'s$) *list* ⇒ $'s$ ⇒ $'s$
where
merge cert f r T pc [] *x* = *x*
| *merge cert f r T pc* (*s*#*ss*) *x* = *merge cert f r T pc ss* (*let* (pc',s') = *s in*
if pc'=*pc*+*1 then* $s' \sqcup_f x$
else if $s' \sqsubseteq_r$ *cert*!pc' *then x*
else T)

definition *wtl-inst* :: $'s$ *certificate* ⇒ $'s$ *binop* ⇒ $'s$ *ord* ⇒ $'s$ ⇒
$'s$ *step-type* ⇒ *nat* ⇒ $'s$ ⇒ $'s$
where
wtl-inst cert f r T step pc s = *merge cert f r T pc* (*step pc s*) (*cert*!(*pc*+*1*))

definition *wtl-cert* :: $'s$ *certificate* ⇒ $'s$ *binop* ⇒ $'s$ *ord* ⇒ $'s$ ⇒ $'s$ ⇒
$'s$ *step-type* ⇒ *nat* ⇒ $'s$ ⇒ $'s$
where
wtl-cert cert f r T B step pc s =
(*if cert*!*pc* = *B then*
wtl-inst cert f r T step pc s
else
if s $\sqsubseteq_r$ *cert*!*pc then wtl-inst cert f r T step pc* (*cert*!*pc*) *else T*)

primrec *wtl-inst-list* :: $'a$ *list* ⇒ $'s$ *certificate* ⇒ $'s$ *binop* ⇒ $'s$ *ord* ⇒ $'s$ ⇒ $'s$ ⇒
$'s$ *step-type* ⇒ *nat* ⇒ $'s$ ⇒ $'s$
where
wtl-inst-list [] *cert f r T B step pc s* = *s*
| *wtl-inst-list* (*i*#*is*) *cert f r T B step pc s* =
(*let* s' = *wtl-cert cert f r T B step pc s in*
if s' = *T* ∨ *s* = *T then T else wtl-inst-list is cert f r T B step* (*pc*+*1*) s')

definition *cert-ok* :: $'s$ *certificate* ⇒ *nat* ⇒ $'s$ ⇒ $'s$ ⇒ $'s$ *set* ⇒ *bool*
where
cert-ok cert n T B A ⟷ (∀ *i* < *n*. *cert*!*i* ∈ *A* ∧ *cert*!*i* ≠ *T*) ∧ (*cert*!*n* = *B*)

definition *bottom* :: $'a$ *ord* ⇒ $'a$ ⇒ *bool*
where
bottom r B ⟷ (∀ *x*. *B* $\sqsubseteq_r$ *x*)

locale *lbv* = *Semilat* +
fixes *T* :: $'a$ (⊤)
fixes *B* :: $'a$ (⊥)
fixes *step* :: $'a$ *step-type*
assumes *top*: *top r* ⊤
assumes *T-A*: ⊤ ∈ *A*

assumes *bot*: *bottom r* $\bot$
assumes *B-A*: $\bot \in A$

fixes *merge* :: $'a$ *certificate* $\Rightarrow$ *nat* $\Rightarrow$ (*nat* $\times$ $'a$) *list* $\Rightarrow$ $'a \Rightarrow$ $'a$
defines *mrg-def*: *merge cert* $\equiv$ *LBVSpec.merge cert f r* $\top$

fixes *wti* :: $'a$ *certificate* $\Rightarrow$ *nat* $\Rightarrow$ $'a \Rightarrow$ $'a$
defines *wti-def*: *wti cert* $\equiv$ *wtl-inst cert f r* $\top$ *step*

fixes *wtc* :: $'a$ *certificate* $\Rightarrow$ *nat* $\Rightarrow$ $'a \Rightarrow$ $'a$
defines *wtc-def*: *wtc cert* $\equiv$ *wtl-cert cert f r* $\top$ $\bot$ *step*

fixes *wtl* :: $'b$ *list* $\Rightarrow$ $'a$ *certificate* $\Rightarrow$ *nat* $\Rightarrow$ $'a \Rightarrow$ $'a$
defines *wtl-def*: *wtl ins cert* $\equiv$ *wtl-inst-list ins cert f r* $\top$ $\bot$ *step*

lemma (**in** *lbv*) *wti*:
wti c pc s = *merge c pc* (*step pc s*) (*c*!(*pc*+*1*))
⟨*proof*⟩
lemma (**in** *lbv*) *wtc*:
wtc c pc s = (*if c*!*pc* = $\bot$ *then wti c pc s else if s* $\sqsubseteq_r$ *c*!*pc then wti c pc* (*c*!*pc*) *else* $\top$)
⟨*proof*⟩
lemma *cert-okD1* [*intro?*]:
cert-ok c n T B A $\Longrightarrow$ *pc* < *n* $\Longrightarrow$ *c*!*pc* $\in$ *A*
⟨*proof*⟩
lemma *cert-okD2* [*intro?*]:
cert-ok c n T B A $\Longrightarrow$ *c*!*n* = *B*
⟨*proof*⟩
lemma *cert-okD3* [*intro?*]:
cert-ok c n T B A $\Longrightarrow$ *B* $\in$ *A* $\Longrightarrow$ *pc* < *n* $\Longrightarrow$ *c*!*Suc pc* $\in$ *A*
⟨*proof*⟩
lemma *cert-okD4* [*intro?*]:
cert-ok c n T B A $\Longrightarrow$ *pc* < *n* $\Longrightarrow$ *c*!*pc* $\neq$ *T*
⟨*proof*⟩
declare *Let-def* [*simp*]

4.10.1 more semilattice lemmas

lemma (**in** *lbv*) *sup-top* [*simp*, *elim*]:
assumes *x*: $x \in A$
shows $x \sqcup_f \top = \top$⟨*proof*⟩
lemma (**in** *lbv*) *plusplussup-top* [*simp*, *elim*]:
set xs $\subseteq$ *A* $\Longrightarrow$ *xs* $\bigsqcup_f \top = \top$
⟨*proof*⟩

lemma (**in** *Semilat*) *pp-ub1'*:
assumes *S*: *snd'set S* $\subseteq$ *A*
assumes *y*: $y \in A$ **and** *ab*: (*a*, *b*) $\in$ *set S*
shows *b* $\sqsubseteq_r$ *map snd* [(p', t') $\leftarrow$ *S* . p' = *a*] $\bigsqcup_f$ *y*⟨*proof*⟩
lemma (**in** *lbv*) *bottom-le* [*simp*, *intro*!]: $\bot \sqsubseteq_r x$
⟨*proof*⟩

lemma (**in** *lbv*) *le-bottom* [*simp*]: $x \sqsubseteq_r \bot = (x = \bot)$

$\langle proof \rangle$

4.10.2 merge

lemma (**in** *lbv*) *merge-Nil* [*simp*]:
$merge\ c\ pc\ []\ x = x$ $\langle proof \rangle$

lemma (**in** *lbv*) *merge-Cons* [*simp*]:
$merge\ c\ pc\ (l\#ls)\ x = merge\ c\ pc\ ls\ (if\ fst\ l{=}pc{+}1\ then\ snd\ l\ {+}{-}f\ x$
$else\ if\ snd\ l \sqsubseteq_r c!fst\ l\ then\ x$
$else\ \top)$
$\langle proof \rangle$

lemma (**in** *lbv*) *merge-Err* [*simp*]:
$snd`set\ ss \subseteq A \Longrightarrow merge\ c\ pc\ ss\ \top = \top$
$\langle proof \rangle$

lemma (**in** *lbv*) *merge-not-top*:
$\bigwedge x.\ snd`set\ ss \subseteq A \Longrightarrow merge\ c\ pc\ ss\ x \neq \top \Longrightarrow$
$\forall (pc',s') \in set\ ss.\ (pc' \neq pc{+}1 \longrightarrow s' \sqsubseteq_r c!pc')$
(**is** $\bigwedge x.\ ?set\ ss \Longrightarrow ?merge\ ss\ x \Longrightarrow ?P\ ss$)$\langle proof \rangle$

lemma (**in** *lbv*) *merge-def*:
shows
$\bigwedge x.\ x \in A \Longrightarrow snd`set\ ss \subseteq A \Longrightarrow$
$merge\ c\ pc\ ss\ x =$
$(if\ \forall (pc',s') \in set\ ss.\ pc'{\neq}pc{+}1 \longrightarrow s' \sqsubseteq_r c!pc'\ then$
$map\ snd\ [(p',t') \leftarrow ss.\ p'{=}pc{+}1]\ \bigsqcup_f x$
$else\ \top)$
(**is** $\bigwedge x.\ \text{-} \Longrightarrow \text{-} \Longrightarrow ?merge\ ss\ x = ?if\ ss\ x$ **is** $\bigwedge x.\ \text{-} \Longrightarrow \text{-} \Longrightarrow ?P\ ss\ x$)$\langle proof \rangle$
lemma (**in** *lbv*) *merge-not-top-s*:
assumes x: $x \in A$ **and** ss: $snd`set\ ss \subseteq A$
assumes m: $merge\ c\ pc\ ss\ x \neq \top$
shows $merge\ c\ pc\ ss\ x = (map\ snd\ [(p',t') \leftarrow ss.\ p'{=}pc{+}1]\ \bigsqcup_f x)$$\langle proof \rangle$

4.10.3 wtl-inst-list

lemmas [*iff*] = *not-Err-eq*

lemma (**in** *lbv*) *wtl-Nil* [*simp*]: $wtl\ []\ c\ pc\ s = s$
$\langle proof \rangle$

lemma (**in** *lbv*) *wtl-Cons* [*simp*]:
$wtl\ (i\#is)\ c\ pc\ s =$
$(let\ s' = wtc\ c\ pc\ s\ in\ if\ s' = \top \lor s = \top\ then\ \top\ else\ wtl\ is\ c\ (pc{+}1)\ s')$
$\langle proof \rangle$

lemma (**in** *lbv*) *wtl-Cons-not-top*:
$wtl\ (i\#is)\ c\ pc\ s \neq \top =$
$(wtc\ c\ pc\ s \neq \top \land s \neq T \land wtl\ is\ c\ (pc{+}1)\ (wtc\ c\ pc\ s) \neq \top)$
$\langle proof \rangle$

lemma (**in** *lbv*) *wtl-top* [*simp*]: $wtl\ ls\ c\ pc\ \top = \top$
$\langle proof \rangle$

lemma (**in** *lbv*) *wtl-not-top*:
$wtl\ ls\ c\ pc\ s \neq \top \Longrightarrow s \neq \top$
$\langle proof \rangle$

lemma (**in** *lbv*) *wtl-append* [*simp*]:
$\bigwedge pc\ s.\ wtl\ (a@b)\ c\ pc\ s = wtl\ b\ c\ (pc{+}length\ a)\ (wtl\ a\ c\ pc\ s)$
$\langle proof \rangle$

lemma (**in** *lbv*) *wtl-take*:
$wtl\ is\ c\ pc\ s \neq \top \Longrightarrow wtl\ (take\ pc'\ is)\ c\ pc\ s \neq \top$
(**is** $?wtl\ is \neq$ - $\Longrightarrow$ -)$\langle proof \rangle$
lemma *take-Suc*:
$\forall n.\ n < length\ l \longrightarrow take\ (Suc\ n)\ l = (take\ n\ l)@[l!n]$ (**is** $?P\ l$)$\langle proof \rangle$
lemma (**in** *lbv*) *wtl-Suc*:
assumes *suc*: $pc{+}1 < length\ is$
assumes *wtl*: $wtl\ (take\ pc\ is)\ c\ 0\ s \neq \top$
shows $wtl\ (take\ (pc{+}1)\ is)\ c\ 0\ s = wtc\ c\ pc\ (wtl\ (take\ pc\ is)\ c\ 0\ s)\langle proof \rangle$
lemma (**in** *lbv*) *wtl-all*:
assumes *all*: $wtl\ is\ c\ 0\ s \neq \top$ (**is** $?wtl\ is \neq$ -)
assumes *pc*: $pc < length\ is$
shows $wtc\ c\ pc\ (wtl\ (take\ pc\ is)\ c\ 0\ s) \neq \top\langle proof \rangle$

4.10.4 preserves-type

lemma (**in** *lbv*) *merge-pres*:
assumes *s0*: $snd`set\ ss \subseteq A$ **and** *x*: $x \in A$
shows $merge\ c\ pc\ ss\ x \in A\langle proof \rangle$
lemma *pres-typeD2*:
$pres\text{-}type\ step\ n\ A \Longrightarrow s \in A \Longrightarrow p < n \Longrightarrow snd`set\ (step\ p\ s) \subseteq A$
$\langle proof \rangle$

lemma (**in** *lbv*) *wti-pres* [*intro?*]:
assumes *pres*: $pres\text{-}type\ step\ n\ A$
assumes *cert*: $c!(pc{+}1) \in A$
assumes *s-pc*: $s \in A\ pc < n$
shows $wti\ c\ pc\ s \in A\langle proof \rangle$
lemma (**in** *lbv*) *wtc-pres*:
assumes $pres\text{-}type\ step\ n\ A$
assumes $c!pc \in A$ **and** $c!(pc{+}1) \in A$
assumes $s \in A$ **and** $pc < n$
shows $wtc\ c\ pc\ s \in A\langle proof \rangle$
lemma (**in** *lbv*) *wtl-pres*:
assumes *pres*: $pres\text{-}type\ step\ (length\ is)\ A$
assumes *cert*: $cert\text{-}ok\ c\ (length\ is)\ \top\ \bot\ A$
assumes *s*: $s \in A$
assumes *all*: $wtl\ is\ c\ 0\ s \neq \top$
shows $pc < length\ is \Longrightarrow wtl\ (take\ pc\ is)\ c\ 0\ s \in A$
(**is** $?len\ pc \Longrightarrow ?wtl\ pc \in A$)$\langle proof \rangle$
end

4.11 Correctness of the LBV

theory *LBVCorrect*
imports *LBVSpec Typing-Framework*
begin

locale *lbvs* = *lbv* +
  fixes s_0 :: *'a*
  fixes *c* :: *'a list*
  fixes *ins* :: *'b list*
  fixes τs :: *'a list*
  defines *phi-def*:
  $\tau s \equiv map\ (\lambda pc.\ if\ c!pc = \bot\ then\ wtl\ (take\ pc\ ins)\ c\ 0\ s_0\ else\ c!pc)$
    $[0..<size\ ins]$

  assumes *bounded*: *bounded step* (*size ins*)
  assumes *cert*: *cert-ok c* (*size ins*) $\top$ $\bot$ *A*
  assumes *pres*: *pres-type step* (*size ins*) *A*

lemma (**in** *lbvs*) *phi-None* [*intro?*]:
  $[\![\ pc < size\ ins;\ c!pc = \bot\ ]\!] \Longrightarrow \tau s!pc = wtl\ (take\ pc\ ins)\ c\ 0\ s_0\langle proof\rangle$
lemma (**in** *lbvs*) *phi-Some* [*intro?*]:
  $[\![\ pc < size\ ins;\ c!pc \neq \bot\ ]\!] \Longrightarrow \tau s!pc = c!pc\langle proof\rangle$
lemma (**in** *lbvs*) *phi-len* [*simp*]: $size\ \tau s = size\ ins\langle proof\rangle$
lemma (**in** *lbvs*) *wtl-suc-pc*:
  assumes *all*: $wtl\ ins\ c\ 0\ s_0 \neq \top$
  assumes *pc*: $pc{+}1 < size\ ins$
  shows $wtl\ (take\ (pc{+}1)\ ins)\ c\ 0\ s_0 \sqsubseteq_r \tau s!(pc{+}1)\langle proof\rangle$
lemma (**in** *lbvs*) *wtl-stable*:
  assumes *wtl*: $wtl\ ins\ c\ 0\ s_0 \neq \top$
  assumes s_0: $s_0 \in A$ **and** *pc*: $pc < size\ ins$
  shows $stable\ r\ step\ \tau s\ pc\langle proof\rangle$
lemma (**in** *lbvs*) *phi-not-top*:
  assumes *wtl*: $wtl\ ins\ c\ 0\ s_0 \neq \top$ **and** *pc*: $pc < size\ ins$
  shows $\tau s!pc \neq \top\langle proof\rangle$
lemma (**in** *lbvs*) *phi-in-A*:
  assumes *wtl*: $wtl\ ins\ c\ 0\ s_0 \neq \top$ **and** s_0: $s_0 \in A$
  shows $\tau s \in list\ (size\ ins)\ A\langle proof\rangle$
lemma (**in** *lbvs*) *phi0*:
  assumes *wtl*: $wtl\ ins\ c\ 0\ s_0 \neq \top$ **and** *0*: $0 < size\ ins$
  shows $s_0 \sqsubseteq_r \tau s!0\langle proof\rangle$

theorem (**in** *lbvs*) *wtl-sound*:
  assumes *wtl*: $wtl\ ins\ c\ 0\ s_0 \neq \top$ **and** s_0: $s_0 \in A$
  shows $\exists \tau s.\ wt\text{-}step\ r\ \top\ step\ \tau s\langle proof\rangle$

theorem (**in** *lbvs*) *wtl-sound-strong*:
  assumes *wtl*: $wtl\ ins\ c\ 0\ s_0 \neq \top$
  assumes s_0: $s_0 \in A$ **and** *ins*: $0 < size\ ins$
  shows $\exists \tau s \in list\ (size\ ins)\ A.\ wt\text{-}step\ r\ \top\ step\ \tau s \wedge s_0 \sqsubseteq_r \tau s!0\langle proof\rangle$
end

4.12 Completeness of the LBV

theory *LBVComplete*
imports *LBVSpec Typing-Framework*
begin

definition *is-target* :: $'s$ *step-type* $\Rightarrow$ $'s$ *list* $\Rightarrow$ *nat* $\Rightarrow$ *bool* **where**
is-target step τs $pc' \longleftrightarrow (\exists pc\ s'.\ pc' \neq pc{+}1 \land pc < size\ \tau s \land (pc',s') \in set\ (step\ pc\ (\tau s!pc)))$

definition *make-cert* :: $'s$ *step-type* $\Rightarrow$ $'s$ *list* $\Rightarrow$ $'s \Rightarrow$ $'s$ *certificate* **where**
make-cert step τs $B = map\ (\lambda pc.\ if\ is\text{-}target\ step\ \tau s\ pc\ then\ \tau s!pc\ else\ B)\ [0..<size\ \tau s]\ @\ [B]$

lemma [*code*]:
is-target step τs pc' =
list-ex $(\lambda pc.\ pc' \neq pc{+}1 \land List.member\ (map\ fst\ (step\ pc\ (\tau s!pc)))\ pc')\ [0..<size\ \tau s]\langle proof\rangle$
locale *lbvc* = *lbv* +
fixes τs :: $'a$ *list*
fixes c :: $'a$ *list*
defines *cert-def*: $c \equiv make\text{-}cert\ step\ \tau s\ \bot$

assumes *mono*: *mono r step* $(size\ \tau s)$ A
assumes *pres*: *pres-type step* $(size\ \tau s)$ A
assumes τs: $\forall pc < size\ \tau s.\ \tau s!pc \in A \land \tau s!pc \neq \top$
assumes *bounded*: *bounded step* $(size\ \tau s)$

assumes *B-neq-T*: $\bot \neq \top$

lemma (**in** *lbvc*) *cert*: *cert-ok* $c\ (size\ \tau s)\ \top\ \bot\ A\langle proof\rangle$
lemmas [*simp del*] = *split-paired-Ex*

lemma (**in** *lbvc*) *cert-target* [*intro?*]:
$[\![(pc',s') \in set\ (step\ pc\ (\tau s!pc));$
$pc' \neq pc{+}1;\ pc < size\ \tau s;\ pc' < size\ \tau s\ ]\!]$
$\Longrightarrow c!pc' = \tau s!pc'\langle proof\rangle$
lemma (**in** *lbvc*) *cert-approx* [*intro?*]:
$[\![pc < size\ \tau s;\ c!pc \neq \bot\ ]\!] \Longrightarrow c!pc = \tau s!pc\langle proof\rangle$
lemma (**in** *lbv*) *le-top* [*simp*, *intro*]: $x <=\!-r\ \top\langle proof\rangle$
lemma (**in** *lbv*) *merge-mono*:
assumes *less*: $set\ ss_2\ \{\sqsubseteq_r\}\ set\ ss_1$
assumes x: $x \in A$
assumes ss_1: $snd`set\ ss_1 \subseteq A$
assumes ss_2: $snd`set\ ss_2 \subseteq A$
shows $merge\ c\ pc\ ss_2\ x \sqsubseteq_r merge\ c\ pc\ ss_1\ x$ (**is** $?s_2 \sqsubseteq_r ?s_1)\langle proof\rangle$
lemma (**in** *lbvc*) *wti-mono*:
assumes *less*: $s_2 \sqsubseteq_r s_1$
assumes *pc*: $pc < size\ \tau s$ **and** s_1: $s_1 \in A$ **and** s_2: $s_2 \in A$
shows $wti\ c\ pc\ s_2 \sqsubseteq_r wti\ c\ pc\ s_1$ (**is** $?s_2' \sqsubseteq_r ?s_1')\langle proof\rangle$
lemma (**in** *lbvc*) *wtc-mono*:
assumes *less*: $s_2 \sqsubseteq_r s_1$
assumes *pc*: $pc < size\ \tau s$ **and** s_1: $s_1 \in A$ **and** s_2: $s_2 \in A$
shows $wtc\ c\ pc\ s_2 \sqsubseteq_r wtc\ c\ pc\ s_1$ (**is** $?s_2' \sqsubseteq_r ?s_1')\langle proof\rangle$
lemma (**in** *lbv*) *top-le-conv* [*simp*]: $\top \sqsubseteq_r x = (x = \top)\langle proof\rangle$
lemma (**in** *lbv*) *neq-top* [*simp*, *elim*]: $[\![x \sqsubseteq_r y;\ y \neq \top\ ]\!] \Longrightarrow x \neq \top\langle proof\rangle$

lemma (**in** *lbvc*) *stable-wti*:
assumes *stable*: *stable r step* τs *pc* **and** *pc*: $pc < size\ \tau s$
shows $wti\ c\ pc\ (\tau s!pc) \neq \top$⟨*proof*⟩
lemma (**in** *lbvc*) *wti-less*:
assumes *stable*: *stable r step* τs *pc* **and** *suc-pc*: $Suc\ pc < size\ \tau s$
shows $wti\ c\ pc\ (\tau s!pc) \sqsubseteq_r \tau s!Suc\ pc$ (**is** $?wti \sqsubseteq_r$ -)⟨*proof*⟩
lemma (**in** *lbvc*) *stable-wtc*:
assumes *stable*: *stable r step* τs *pc* **and** *pc*: $pc < size\ \tau s$
shows $wtc\ c\ pc\ (\tau s!pc) \neq \top$⟨*proof*⟩
lemma (**in** *lbvc*) *wtc-less*:
assumes *stable*: *stable r step* τs *pc* **and** *suc-pc*: $Suc\ pc < size\ \tau s$
shows $wtc\ c\ pc\ (\tau s!pc) \sqsubseteq_r \tau s!Suc\ pc$ (**is** $?wtc \sqsubseteq_r$ -)⟨*proof*⟩
lemma (**in** *lbvc*) *wt-step-wtl-lemma*:
assumes *wt-step*: *wt-step r* $\top$ *step* τs
shows $\bigwedge pc\ s.\ pc{+}size\ ls = size\ \tau s \Longrightarrow s \sqsubseteq_r \tau s!pc \Longrightarrow s \in A \Longrightarrow s{\neq}\top \Longrightarrow$
$wtl\ ls\ c\ pc\ s \neq \top$
(**is** $\bigwedge pc\ s.$ - $\Longrightarrow$ - $\Longrightarrow$ - $\Longrightarrow$ - $\Longrightarrow$ $?wtl\ ls\ pc\ s \neq$ -)⟨*proof*⟩
theorem (**in** *lbvc*) *wtl-complete*:
assumes *wt*: *wt-step r* $\top$ *step* τs
assumes *s*: $s \sqsubseteq_r \tau s!0$ $s \in A$ $s \neq \top$ **and** *eq*: $size\ ins = size\ \tau s$
shows $wtl\ ins\ c\ 0\ s \neq \top$⟨*proof*⟩
end

4.13 The Jinja Type System as a Semilattice

theory *SemiType*
imports *../Common/WellForm ../DFA/Semilattices*
begin

definition $super :: 'a\ prog \Rightarrow cname \Rightarrow cname$
where $super\ P\ C \equiv fst\ (the\ (class\ P\ C))$

lemma *superI*:
$(C,D) \in subcls1\ P \Longrightarrow super\ P\ C = D$
$\langle proof \rangle$

primrec $the\text{-}Class :: ty \Rightarrow cname$
where
$the\text{-}Class\ (Class\ C) = C$

definition $sup :: 'c\ prog \Rightarrow ty \Rightarrow ty \Rightarrow ty\ err$
where
$sup\ P\ T_1\ T_2 \equiv$
$if\ is\text{-}refT\ T_1 \wedge is\text{-}refT\ T_2\ then$
$OK\ (if\ T_1 = NT\ then\ T_2\ else$
$\quad if\ T_2 = NT\ then\ T_1\ else$
$\quad (Class\ (exec\text{-}lub\ (subcls1\ P)\ (super\ P)\ (the\text{-}Class\ T_1)\ (the\text{-}Class\ T_2))))$
$else$
$(if\ T_1 = T_2\ then\ OK\ T_1\ else\ Err)$

lemma *sup-def'*:
$sup\ P = (\lambda T_1\ T_2.$
$if\ is\text{-}refT\ T_1 \wedge is\text{-}refT\ T_2\ then$
$OK\ (if\ T_1 = NT\ then\ T_2\ else$
$\quad if\ T_2 = NT\ then\ T_1\ else$
$\quad (Class\ (exec\text{-}lub\ (subcls1\ P)\ (super\ P)\ (the\text{-}Class\ T_1)\ (the\text{-}Class\ T_2))))$
$else$
$(if\ T_1 = T_2\ then\ OK\ T_1\ else\ Err))$
$\langle proof \rangle$

abbreviation
$subtype :: 'c\ prog \Rightarrow ty \Rightarrow ty \Rightarrow bool$
where $subtype\ P \equiv widen\ P$

definition $esl :: 'c\ prog \Rightarrow ty\ esl$
where
$esl\ P \equiv (types\ P,\ subtype\ P,\ sup\ P)$

lemma *is-class-is-subcls*:
$wf\text{-}prog\ m\ P \Longrightarrow is\text{-}class\ P\ C = P \vdash C \preceq^* Object \langle proof \rangle$

lemma *subcls-antisym*:
$[\![wf\text{-}prog\ m\ P;\ P \vdash C \preceq^* D;\ P \vdash D \preceq^* C]\!] \Longrightarrow C = D$

$\langle proof\rangle$

lemma *widen-antisym*:
$[\![$ *wf-prog m P*; $P \vdash T \leq U$; $P \vdash U \leq T$ $]\!] \Longrightarrow T = U\langle proof\rangle$
lemma *order-widen* [*intro*,*simp*]:
wf-prog m P $\Longrightarrow$ *order* (*subtype P*)$\langle proof\rangle$

lemma *NT-widen*:
$P \vdash NT \leq T = (T = NT \vee (\exists C.\ T = Class\ C))\langle proof\rangle$

lemma *Class-widen2*: $P \vdash Class\ C \leq T = (\exists D.\ T = Class\ D \wedge P \vdash C \preceq^* D)\langle proof\rangle$
lemma *wf-converse-subcls1-impl-acc-subtype*:
wf ((*subcls1 P*) ^−*1*) $\Longrightarrow$ *acc* (*subtype P*)$\langle proof\rangle$
lemma *wf-subtype-acc* [*intro*, *simp*]:
wf-prog wf-mb P $\Longrightarrow$ *acc* (*subtype P*)$\langle proof\rangle$
lemma *exec-lub-refl* [*simp*]: *exec-lub r f T T* = $T\langle proof\rangle$
lemma *closed-err-types*:
wf-prog wf-mb P $\Longrightarrow$ *closed* (*err* (*types P*)) (*lift2* (*sup P*))$\langle proof\rangle$

lemma *sup-subtype-greater*:
$[\![$ *wf-prog wf-mb P*; *is-type P t1*; *is-type P t2*; *sup P t1 t2* = *OK s* $]\!]$
$\Longrightarrow$ *subtype P t1 s* $\wedge$ *subtype P t2 s*$\langle proof\rangle$
lemma *sup-subtype-smallest*:
$[\![$ *wf-prog wf-mb P*; *is-type P a*; *is-type P b*; *is-type P c*;
subtype P a c; *subtype P b c*; *sup P a b* = *OK d* $]\!]$
$\Longrightarrow$ *subtype P d c*$\langle proof\rangle$
lemma *sup-exists*:
$[\![$ *subtype P a c*; *subtype P b c* $]\!] \Longrightarrow$ *EX T*. *sup P a b* = *OK T*$\langle proof\rangle$
lemma *err-semilat-JType-esl*:
wf-prog wf-mb P $\Longrightarrow$ *err-semilat* (*esl P*)$\langle proof\rangle$

end

4.14 The JVM Type System as Semilattice

theory *JVM-SemiType* **imports** *SemiType* **begin**

type-synonym $ty_l = ty\ err\ list$
type-synonym $ty_s = ty\ list$
type-synonym $ty_i = ty_s \times ty_l$
type-synonym $ty_i{}' = ty_i\ option$
type-synonym $ty_m = ty_i{}'\ list$
type-synonym $ty_P = mname \Rightarrow cname \Rightarrow ty_m$

definition $stk\text{-}esl :: {}'c\ prog \Rightarrow nat \Rightarrow ty_s\ esl$
where
$stk\text{-}esl\ P\ mxs \equiv upto\text{-}esl\ mxs\ (SemiType.esl\ P)$

definition $loc\text{-}sl :: {}'c\ prog \Rightarrow nat \Rightarrow ty_l\ sl$
where
$loc\text{-}sl\ P\ mxl \equiv Listn.sl\ mxl\ (Err.sl\ (SemiType.esl\ P))$

definition $sl :: {}'c\ prog \Rightarrow nat \Rightarrow nat \Rightarrow ty_i{}'\ err\ sl$
where
$sl\ P\ mxs\ mxl \equiv$
$Err.sl(Opt.esl(Product.esl\ (stk\text{-}esl\ P\ mxs)\ (Err.esl(loc\text{-}sl\ P\ mxl))))$

definition $states :: {}'c\ prog \Rightarrow nat \Rightarrow nat \Rightarrow ty_i{}'\ err\ set$
where $states\ P\ mxs\ mxl \equiv fst(sl\ P\ mxs\ mxl)$

definition $le :: {}'c\ prog \Rightarrow nat \Rightarrow nat \Rightarrow ty_i{}'\ err\ ord$
where
$le\ P\ mxs\ mxl \equiv fst(snd(sl\ P\ mxs\ mxl))$

definition $sup :: {}'c\ prog \Rightarrow nat \Rightarrow nat \Rightarrow ty_i{}'\ err\ binop$
where
$sup\ P\ mxs\ mxl \equiv snd(snd(sl\ P\ mxs\ mxl))$

definition $sup\text{-}ty\text{-}opt :: [{}'c\ prog, ty\ err, ty\ err] \Rightarrow bool$
$(\text{- } |\text{– - <=}T\text{ - } [71,71,71]\ 70)$
where
$sup\text{-}ty\text{-}opt\ P \equiv Err.le\ (subtype\ P)$

definition $sup\text{-}state :: [{}'c\ prog, ty_i, ty_i] \Rightarrow bool$
$(\text{- } |\text{– - <=}i\text{ - } [71,71,71]\ 70)$
where
$sup\text{-}state\ P \equiv Product.le\ (Listn.le\ (subtype\ P))\ (Listn.le\ (sup\text{-}ty\text{-}opt\ P))$

definition $sup\text{-}state\text{-}opt :: [{}'c\ prog, ty_i{}', ty_i{}'] \Rightarrow bool$
$(\text{- } |\text{– - <=}'\text{ - } [71,71,71]\ 70)$
where
$sup\text{-}state\text{-}opt\ P \equiv Opt.le\ (sup\text{-}state\ P)$

abbreviation

$sup\text{-}loc :: [{'}c\ prog, ty_l, ty_l] \Rightarrow bool\ \ (\text{-}\ |\text{--}\ \text{-}\ [<=T]\ \text{-}\ \ [71,71,71]\ 70)$
where $P\ |\text{--}\ LT\ [<=T]\ LT' \equiv list\text{-}all2\ (sup\text{-}ty\text{-}opt\ P)\ LT\ LT'$

notation (*xsymbols*)
$sup\text{-}ty\text{-}opt\ \ (\text{-} \vdash \text{-} \leq_{\top} \text{-}\ [71,71,71]\ 70)$ **and**
$sup\text{-}state\ \ (\text{-} \vdash \text{-} \leq_i \text{-}\ [71,71,71]\ 70)$ **and**
$sup\text{-}state\text{-}opt\ \ (\text{-} \vdash \text{-} \leq' \text{-}\ [71,71,71]\ 70)$ **and**
$sup\text{-}loc\ \ (\text{-} \vdash \text{-}\ [\leq_{\top}]\ \text{-}\ \ [71,71,71]\ 70)$

4.14.1 Unfolding

lemma *JVM-states-unfold*:
$states\ P\ mxs\ mxl \equiv err(opt((Union\ \{list\ n\ (types\ P)\ |n.\ n <= mxs\})\ <*>$
$list\ mxl\ (err(types\ P))))\langle proof\rangle$
lemma *JVM-le-unfold*:
$le\ P\ m\ n \equiv$
$Err.le(Opt.le(Product.le(Listn.le(subtype\ P))(Listn.le(Err.le(subtype\ P)))))\langle proof\rangle$
lemma *sl-def2*:
$JVM\text{-}SemiType.sl\ P\ mxs\ mxl \equiv$
$(states\ P\ mxs\ mxl,\ JVM\text{-}SemiType.le\ P\ mxs\ mxl,\ JVM\text{-}SemiType.sup\ P\ mxs\ mxl)\langle proof\rangle$

lemma *JVM-le-conv*:
$le\ P\ m\ n\ (OK\ t1)\ (OK\ t2) = P \vdash t1\ \leq'\ t2\langle proof\rangle$
lemma *JVM-le-Err-conv*:
$le\ P\ m\ n = Err.le\ (sup\text{-}state\text{-}opt\ P)\langle proof\rangle$
lemma *err-le-unfold* [*iff*]:
$Err.le\ r\ (OK\ a)\ (OK\ b) = r\ a\ b\langle proof\rangle$

4.14.2 Semilattice

lemma *order-sup-state-opt* [*intro*, *simp*]:
$wf\text{-}prog\ wf\text{-}mb\ P \Longrightarrow order\ (sup\text{-}state\text{-}opt\ P)\langle proof\rangle$
lemma *semilat-JVM* [*intro?*]:
$wf\text{-}prog\ wf\text{-}mb\ P \Longrightarrow semilat\ (JVM\text{-}SemiType.sl\ P\ mxs\ mxl)\langle proof\rangle$
lemma *acc-JVM* [*intro*]:
$wf\text{-}prog\ wf\text{-}mb\ P \Longrightarrow acc\ (JVM\text{-}SemiType.le\ P\ mxs\ mxl)\langle proof\rangle$

4.14.3 Widening with $\top$

lemma *subtype-refl*[*iff*]: $subtype\ P\ t\ t\ \langle proof\rangle$
lemma *sup-ty-opt-refl* [*iff*]: $P \vdash T \leq_{\top} T\langle proof\rangle$
lemma *Err-any-conv* [*iff*]: $P \vdash Err \leq_{\top} T = (T = Err)\langle proof\rangle$
lemma *any-Err* [*iff*]: $P \vdash T \leq_{\top} Err\langle proof\rangle$
lemma *OK-OK-conv* [*iff*]:
$P \vdash OK\ T \leq_{\top} OK\ T' = P \vdash T \leq T'\langle proof\rangle$
lemma *any-OK-conv* [*iff*]:
$P \vdash X \leq_{\top} OK\ T' = (\exists\, T.\ X = OK\ T \wedge P \vdash T \leq T')\langle proof\rangle$
lemma *OK-any-conv*:
$P \vdash OK\ T \leq_{\top} X = (X = Err \vee (\exists\, T'.\ X = OK\ T' \wedge P \vdash T \leq T'))\langle proof\rangle$
lemma *sup-ty-opt-trans* [*intro?*, *trans*]:
$[\![P \vdash a \leq_{\top} b;\ P \vdash b \leq_{\top} c]\!] \Longrightarrow P \vdash a \leq_{\top} c\langle proof\rangle$

4.14.4 Stack and Registers

lemma *stk-convert*:

$P \vdash ST\ [\leq]\ ST' = Listn.le\ (subtype\ P)\ ST\ ST'\langle proof\rangle$
lemma *sup-loc-refl* [*iff*]: $P \vdash LT\ [\leq_\top]\ LT\langle proof\rangle$
lemmas *sup-loc-Cons1* [*iff*] = *list-all2-Cons1* [*of sup-ty-opt P*] **for** *P*

lemma *sup-loc-def*:
$P \vdash LT\ [\leq_\top]\ LT' \equiv Listn.le\ (sup\text{-}ty\text{-}opt\ P)\ LT\ LT'\langle proof\rangle$
lemma *sup-loc-widens-conv* [*iff*]:
$P \vdash map\ OK\ Ts\ [\leq_\top]\ map\ OK\ Ts' = P \vdash Ts\ [\leq]\ Ts'\langle proof\rangle$

lemma *sup-loc-trans* [*intro?*, *trans*]:
$[\![P \vdash a\ [\leq_\top]\ b;\ P \vdash b\ [\leq_\top]\ c]\!] \Longrightarrow P \vdash a\ [\leq_\top]\ c\langle proof\rangle$

4.14.5 State Type

lemma *sup-state-conv* [*iff*]:
$P \vdash (ST,LT) \leq_i (ST',LT') = (P \vdash ST\ [\leq]\ ST' \wedge P \vdash LT\ [\leq_\top]\ LT')\langle proof\rangle$
lemma *sup-state-conv2*:
$P \vdash s1 \leq_i s2 = (P \vdash fst\ s1\ [\leq]\ fst\ s2 \wedge P \vdash snd\ s1\ [\leq_\top]\ snd\ s2)\langle proof\rangle$
lemma *sup-state-refl* [*iff*]: $P \vdash s \leq_i s\langle proof\rangle$
lemma *sup-state-trans* [*intro?*, *trans*]:
$[\![P \vdash a \leq_i b;\ P \vdash b \leq_i c]\!] \Longrightarrow P \vdash a \leq_i c\langle proof\rangle$

lemma *sup-state-opt-None-any* [*iff*]:
$P \vdash None \leq' s\langle proof\rangle$
lemma *sup-state-opt-any-None* [*iff*]:
$P \vdash s \leq' None = (s = None)\langle proof\rangle$
lemma *sup-state-opt-Some-Some* [*iff*]:
$P \vdash Some\ a \leq' Some\ b = P \vdash a \leq_i b\langle proof\rangle$
lemma *sup-state-opt-any-Some*:
$P \vdash (Some\ s) \leq' X = (\exists s'.\ X = Some\ s' \wedge P \vdash s \leq_i s')\langle proof\rangle$
lemma *sup-state-opt-refl* [*iff*]: $P \vdash s \leq' s\langle proof\rangle$
lemma *sup-state-opt-trans* [*intro?*, *trans*]:
$[\![P \vdash a \leq' b;\ P \vdash b \leq' c]\!] \Longrightarrow P \vdash a \leq' c\langle proof\rangle$
end

4.15 Effect of Instructions on the State Type

theory *Effect*
imports *JVM-SemiType ../JVM/JVMExceptions*
begin

— FIXME
locale *prog* =
  fixes *P* :: *'a prog*

locale *jvm-method* = *prog* +
  fixes *mxs* :: *nat*
  fixes mxl_0 :: *nat*
  fixes *Ts* :: *ty list*
  fixes T_r :: *ty*
  fixes *is* :: *instr list*
  fixes *xt* :: *ex-table*

  fixes *mxl* :: *nat*
  defines *mxl-def*: *mxl* ≡ *1+size Ts*+mxl_0

Program counter of successor instructions:

primrec *succs* :: *instr* ⇒ ty_i ⇒ *pc* ⇒ *pc list* **where**
  succs (*Load idx*) τ *pc* = [*pc+1*]
| *succs* (*Store idx*) τ *pc* = [*pc+1*]
| *succs* (*Push v*) τ *pc* = [*pc+1*]
| *succs* (*Getfield F C*) τ *pc* = [*pc+1*]
| *succs* (*Putfield F C*) τ *pc* = [*pc+1*]
| *succs* (*New C*) τ *pc* = [*pc+1*]
| *succs* (*Checkcast C*) τ *pc* = [*pc+1*]
| *succs Pop* τ *pc* = [*pc+1*]
| *succs IAdd* τ *pc* = [*pc+1*]
| *succs CmpEq* τ *pc* = [*pc+1*]
| *succs-IfFalse*:
    succs (*IfFalse b*) τ *pc* = [*pc+1*, *nat* (*int pc* + *b*)]
| *succs-Goto*:
    succs (*Goto b*) τ *pc* = [*nat* (*int pc* + *b*)]
| *succs-Return*:
    succs Return τ *pc* = []
| *succs-Invoke*:
    succs (*Invoke M n*) τ *pc* = (*if* (*fst* τ)!*n* = *NT then* [] *else* [*pc+1*])
| *succs-Throw*:
    succs Throw τ *pc* = []

Effect of instruction on the state type:

fun *the-class*:: *ty* ⇒ *cname* **where**
  the-class (*Class C*) = *C*

fun eff_i :: *instr* × *'m prog* × ty_i ⇒ ty_i **where**
  eff_i*-Load*:
    eff_i (*Load n*, *P*, (*ST*, *LT*)) = (*ok-val* (*LT* ! *n*) # *ST*, *LT*)
| eff_i*-Store*:
    eff_i (*Store n*, *P*, (*T*#*ST*, *LT*)) = (*ST*, *LT*[*n*:= *OK T*])
| eff_i*-Push*:

eff_i (*Push v*, *P*, (*ST*, *LT*)) = (*the* (*typeof v*) # *ST*, *LT*)
| eff_i-*Getfield*:
eff_i (*Getfield F C*, *P*, (*T*#*ST*, *LT*)) = (*snd* (*field P C F*) # *ST*, *LT*)
| eff_i-*Putfield*:
eff_i (*Putfield F C*, *P*, (T_1#T_2#*ST*, *LT*)) = (*ST*,*LT*)
| eff_i-*New*:
eff_i (*New C*, *P*, (*ST*,*LT*)) = (*Class C* # *ST*, *LT*)
| eff_i-*Checkcast*:
eff_i (*Checkcast C*, *P*, (*T*#*ST*,*LT*)) = (*Class C* # *ST*,*LT*)
| eff_i-*Pop*:
eff_i (*Pop*, *P*, (*T*#*ST*,*LT*)) = (*ST*,*LT*)
| eff_i-*IAdd*:
eff_i (*IAdd*, *P*,(T_1#T_2#*ST*,*LT*)) = (*Integer*#*ST*,*LT*)
| eff_i-*CmpEq*:
eff_i (*CmpEq*, *P*, (T_1#T_2#*ST*,*LT*)) = (*Boolean*#*ST*,*LT*)
| eff_i-*IfFalse*:
eff_i (*IfFalse b*, *P*, (T_1#*ST*,*LT*)) = (*ST*,*LT*)
| eff_i-*Invoke*:
eff_i (*Invoke M n*, *P*, (*ST*,*LT*)) =
(*let C* = *the-class* (*ST*!*n*); (*D*,*Ts*,T_r,*b*) = *method P C M*
in (T_r # *drop* (*n*+*1*) *ST*, *LT*))
| eff_i-*Goto*:
eff_i (*Goto n*, *P*, *s*) = *s*

fun *is-relevant-class* :: *instr* ⇒ ′*m prog* ⇒ *cname* ⇒ *bool* **where**
rel-Getfield:
is-relevant-class (*Getfield F D*) = (λ*P C*. *P* ⊢ *NullPointer* $\preceq^*$ *C*)
| *rel-Putfield*:
is-relevant-class (*Putfield F D*) = (λ*P C*. *P* ⊢ *NullPointer* $\preceq^*$ *C*)
| *rel-Checcast*:
is-relevant-class (*Checkcast D*) = (λ*P C*. *P* ⊢ *ClassCast* $\preceq^*$ *C*)
| *rel-New*:
is-relevant-class (*New D*) = (λ*P C*. *P* ⊢ *OutOfMemory* $\preceq^*$ *C*)
| *rel-Throw*:
is-relevant-class Throw = (λ*P C*. *True*)
| *rel-Invoke*:
is-relevant-class (*Invoke M n*) = (λ*P C*. *True*)
| *rel-default*:
is-relevant-class i = (λ*P C*. *False*)

definition *is-relevant-entry* :: ′*m prog* ⇒ *instr* ⇒ *pc* ⇒ *ex-entry* ⇒ *bool* **where**
is-relevant-entry P i pc e ⟷ (*let* (*f*,*t*,*C*,*h*,*d*) = *e in is-relevant-class i P C* ∧ *pc* ∈ {*f*..<*t*})

definition *relevant-entries* :: ′*m prog* ⇒ *instr* ⇒ *pc* ⇒ *ex-table* ⇒ *ex-table* **where**
relevant-entries P i pc = *filter* (*is-relevant-entry P i pc*)

definition *xcpt-eff* :: *instr* ⇒ ′*m prog* ⇒ *pc* ⇒ ty_i
⇒ *ex-table* ⇒ (*pc* × $ty_i{}'$) *list* **where**
xcpt-eff i P pc τ *et* = (*let* (*ST*,*LT*) = τ *in*
map (λ(*f*,*t*,*C*,*h*,*d*). (*h*, *Some* (*Class C*#*drop* (*size ST* − *d*) *ST*, *LT*))) (*relevant-entries P i pc et*))

definition *norm-eff* :: *instr* ⇒ ′*m prog* ⇒ *nat* ⇒ ty_i ⇒ (*pc* × $ty_i{}'$) *list* **where**
norm-eff i P pc τ = *map* (λ*pc*′. (*pc*′,*Some* (eff_i (*i*,*P*,τ)))) (*succs i* τ *pc*)

definition $\mathit{eff} :: \mathit{instr} \Rightarrow {}'m\ \mathit{prog} \Rightarrow \mathit{pc} \Rightarrow \mathit{ex\text{-}table} \Rightarrow \mathit{ty_i}' \Rightarrow (\mathit{pc} \times \mathit{ty_i}')\ \mathit{list}$ **where**
$\mathit{eff}\ i\ P\ \mathit{pc}\ \mathit{et}\ t = (\mathit{case}\ t\ \mathit{of}$
$\quad \mathit{None} \Rightarrow []$
$\mid \mathit{Some}\ \tau \Rightarrow (\mathit{norm\text{-}eff}\ i\ P\ \mathit{pc}\ \tau)\ @\ (\mathit{xcpt\text{-}eff}\ i\ P\ \mathit{pc}\ \tau\ \mathit{et}))$

lemma *eff-None*:
$\mathit{eff}\ i\ P\ \mathit{pc}\ \mathit{xt}\ \mathit{None} = []$
$\langle \mathit{proof} \rangle$

lemma *eff-Some*:
$\mathit{eff}\ i\ P\ \mathit{pc}\ \mathit{xt}\ (\mathit{Some}\ \tau) = \mathit{norm\text{-}eff}\ i\ P\ \mathit{pc}\ \tau\ @\ \mathit{xcpt\text{-}eff}\ i\ P\ \mathit{pc}\ \tau\ \mathit{xt}$
$\langle \mathit{proof} \rangle$

Conditions under which eff is applicable:

fun $\mathit{app_i} :: \mathit{instr} \times {}'m\ \mathit{prog} \times \mathit{pc} \times \mathit{nat} \times \mathit{ty} \times \mathit{ty_i} \Rightarrow \mathit{bool}$ **where**
$\mathit{app_i\text{-}Load}$:
$\quad \mathit{app_i}\ (\mathit{Load}\ n,\ P,\ \mathit{pc},\ \mathit{mxs},\ T_r,\ (\mathit{ST},\mathit{LT})) =$
$\quad (n < \mathit{length}\ \mathit{LT} \wedge \mathit{LT}\ !\ n \neq \mathit{Err} \wedge \mathit{length}\ \mathit{ST} < \mathit{mxs})$
$\mid \mathit{app_i\text{-}Store}$:
$\quad \mathit{app_i}\ (\mathit{Store}\ n,\ P,\ \mathit{pc},\ \mathit{mxs},\ T_r,\ (T\#\mathit{ST},\ \mathit{LT})) =$
$\quad (n < \mathit{length}\ \mathit{LT})$
$\mid \mathit{app_i\text{-}Push}$:
$\quad \mathit{app_i}\ (\mathit{Push}\ v,\ P,\ \mathit{pc},\ \mathit{mxs},\ T_r,\ (\mathit{ST},\mathit{LT})) =$
$\quad (\mathit{length}\ \mathit{ST} < \mathit{mxs} \wedge \mathit{typeof}\ v \neq \mathit{None})$
$\mid \mathit{app_i\text{-}Getfield}$:
$\quad \mathit{app_i}\ (\mathit{Getfield}\ F\ C,\ P,\ \mathit{pc},\ \mathit{mxs},\ T_r,\ (T\#\mathit{ST},\ \mathit{LT})) =$
$\quad (\exists\, T_f.\ P \vdash C\ \mathit{sees}\ F{:}T_f\ \mathit{in}\ C \wedge P \vdash T \leq \mathit{Class}\ C)$
$\mid \mathit{app_i\text{-}Putfield}$:
$\quad \mathit{app_i}\ (\mathit{Putfield}\ F\ C,\ P,\ \mathit{pc},\ \mathit{mxs},\ T_r,\ (T_1\#T_2\#\mathit{ST},\ \mathit{LT})) =$
$\quad (\exists\, T_f.\ P \vdash C\ \mathit{sees}\ F{:}T_f\ \mathit{in}\ C \wedge P \vdash T_2 \leq (\mathit{Class}\ C) \wedge P \vdash T_1 \leq T_f)$
$\mid \mathit{app_i\text{-}New}$:
$\quad \mathit{app_i}\ (\mathit{New}\ C,\ P,\ \mathit{pc},\ \mathit{mxs},\ T_r,\ (\mathit{ST},\mathit{LT})) =$
$\quad (\mathit{is\text{-}class}\ P\ C \wedge \mathit{length}\ \mathit{ST} < \mathit{mxs})$
$\mid \mathit{app_i\text{-}Checkcast}$:
$\quad \mathit{app_i}\ (\mathit{Checkcast}\ C,\ P,\ \mathit{pc},\ \mathit{mxs},\ T_r,\ (T\#\mathit{ST},\mathit{LT})) =$
$\quad (\mathit{is\text{-}class}\ P\ C \wedge \mathit{is\text{-}refT}\ T)$
$\mid \mathit{app_i\text{-}Pop}$:
$\quad \mathit{app_i}\ (\mathit{Pop},\ P,\ \mathit{pc},\ \mathit{mxs},\ T_r,\ (T\#\mathit{ST},\mathit{LT})) =$
$\quad \mathit{True}$
$\mid \mathit{app_i\text{-}IAdd}$:
$\quad \mathit{app_i}\ (\mathit{IAdd},\ P,\ \mathit{pc},\ \mathit{mxs},\ T_r,\ (T_1\#T_2\#\mathit{ST},\mathit{LT})) = (T_1 = T_2 \wedge T_1 = \mathit{Integer})$
$\mid \mathit{app_i\text{-}CmpEq}$:
$\quad \mathit{app_i}\ (\mathit{CmpEq},\ P,\ \mathit{pc},\ \mathit{mxs},\ T_r,\ (T_1\#T_2\#\mathit{ST},\mathit{LT})) =$
$\quad (T_1 = T_2 \vee \mathit{is\text{-}refT}\ T_1 \wedge \mathit{is\text{-}refT}\ T_2)$
$\mid \mathit{app_i\text{-}IfFalse}$:
$\quad \mathit{app_i}\ (\mathit{IfFalse}\ b,\ P,\ \mathit{pc},\ \mathit{mxs},\ T_r,\ (\mathit{Boolean}\#\mathit{ST},\mathit{LT})) =$
$\quad (0 \leq \mathit{int}\ \mathit{pc} + b)$
$\mid \mathit{app_i\text{-}Goto}$:
$\quad \mathit{app_i}\ (\mathit{Goto}\ b,\ P,\ \mathit{pc},\ \mathit{mxs},\ T_r,\ s) =$
$\quad (0 \leq \mathit{int}\ \mathit{pc} + b)$
$\mid \mathit{app_i\text{-}Return}$:
$\quad \mathit{app_i}\ (\mathit{Return},\ P,\ \mathit{pc},\ \mathit{mxs},\ T_r,\ (T\#\mathit{ST},\mathit{LT})) =$
$\quad (P \vdash T \leq T_r)$

| app_i-*Throw*:
  app_i (*Throw*, P, pc, mxs, T_r, ($T\#ST$,LT)) =
  is-refT T
| app_i-*Invoke*:
  app_i (*Invoke* M n, P, pc, mxs, T_r, (ST,LT)) =
  (n < *length* ST ∧
  ($ST!n \neq NT \longrightarrow$
   ($\exists$ C D Ts T m. $ST!n$ = *Class* C ∧ $P \vdash C$ *sees* $M{:}Ts \rightarrow T = m$ *in* D ∧
     $P \vdash$ *rev* (*take* n ST) [≤] Ts)))

| app_i-*default*:
  app_i (i,P, pc,mxs,T_r,s) = *False*

definition *xcpt-app* :: *instr* ⇒ *'m prog* ⇒ *pc* ⇒ *nat* ⇒ *ex-table* ⇒ ty_i ⇒ *bool* **where**
xcpt-app i P pc mxs xt τ ⟷ (∀ (f,t,C,h,d) ∈ *set* (*relevant-entries P i pc xt*). *is-class P C* ∧ $d \leq$ *size* (*fst* τ) ∧ d < *mxs*)

definition *app* :: *instr* ⇒ *'m prog* ⇒ *nat* ⇒ *ty* ⇒ *nat* ⇒ *nat* ⇒ *ex-table* ⇒ ty_i' ⇒ *bool* **where**
  app i P mxs T_r *pc mpc xt t* = (*case t of None* ⇒ *True* | *Some* τ ⇒
  app_i (i,P,pc,mxs,T_r,τ) ∧ *xcpt-app i P pc mxs xt* τ ∧
  (∀ (pc',τ') ∈ *set* (*eff i P pc xt t*). $pc' < mpc$))

lemma *app-Some*:
  app i P mxs T_r *pc mpc xt* (*Some* τ) =
  (app_i (i,P,pc,mxs,T_r,τ) ∧ *xcpt-app i P pc mxs xt* τ ∧
  (∀ (pc',s') ∈ *set* (*eff i P pc xt* (*Some* τ)). $pc' < mpc$))
⟨*proof*⟩

locale *eff* = *jvm-method* +
  fixes eff_i **and** app_i **and** *eff* **and** *app*
  fixes *norm-eff* **and** *xcpt-app* **and** *xcpt-eff*

  fixes *mpc*
  defines *mpc* ≡ *size is*

  defines eff_i i τ ≡ *Effect*.eff_i (i,P,τ)
  notes eff_i-*simps* [*simp*] = *Effect*.eff_i.*simps* [**where** $P = P$, *folded* eff_i-*def*]

  defines app_i i pc τ ≡ *Effect*.app_i (i, P, pc, mxs, T_r, τ)
  notes app_i-*simps* [*simp*] = *Effect*.app_i.*simps* [**where** P=P **and** mxs=mxs **and** T_r=T_r, *folded* app_i-*def*]

  defines *xcpt-eff i pc* τ ≡ *Effect.xcpt-eff i P pc* τ *xt*
  notes *xcpt-eff* = *Effect.xcpt-eff-def* [*of - P - - xt*, *folded xcpt-eff-def*]

  defines *norm-eff i pc* τ ≡ *Effect.norm-eff i P pc* τ
  notes *norm-eff* = *Effect.norm-eff-def* [*of - P*, *folded norm-eff-def* eff_i-*def*]

  defines *eff i pc* ≡ *Effect.eff i P pc xt*
  notes *eff* = *Effect.eff-def* [*of - P - xt*, *folded eff-def norm-eff-def xcpt-eff-def*]

  defines *xcpt-app i pc* τ ≡ *Effect.xcpt-app i P pc mxs xt* τ

notes *xcpt-app* = *Effect.xcpt-app-def* [*of - P - mxs xt*, *folded xcpt-app-def*]

defines *app i pc* ≡ *Effect.app i P mxs* T_r *pc mpc xt*
notes *app* = *Effect.app-def* [*of - P mxs* T_r *- mpc xt*, *folded app-def xcpt-app-def* app_i*-def eff-def*]

lemma *length-cases2*:
assumes $\bigwedge LT.\ P\ ([],LT)$
assumes $\bigwedge l\ ST\ LT.\ P\ (l\#ST,LT)$
shows $P\ s$
⟨*proof*⟩

lemma *length-cases3*:
assumes $\bigwedge LT.\ P\ ([],LT)$
assumes $\bigwedge l\ LT.\ P\ ([l],LT)$
assumes $\bigwedge l\ ST\ LT.\ P\ (l\#ST,LT)$
shows $P\ s$⟨*proof*⟩
lemma *length-cases4*:
assumes $\bigwedge LT.\ P\ ([],LT)$
assumes $\bigwedge l\ LT.\ P\ ([l],LT)$
assumes $\bigwedge l\ l'\ LT.\ P\ ([l,l'],LT)$
assumes $\bigwedge l\ l'\ ST\ LT.\ P\ (l\#l'\#ST,LT)$
shows $P\ s$⟨*proof*⟩

simp rules for *app*

lemma *appNone*[*simp*]: *app i P mxs* T_r *pc mpc et None* = *True*
⟨*proof*⟩

lemma *appLoad*[*simp*]:
app_i (*Load idx*, *P*, T_r, *mxs*, *pc*, *s*) = (∃ *ST LT*. *s* = (*ST*,*LT*) ∧ *idx* < *length LT* ∧ *LT*!*idx* ≠ *Err* ∧ *length ST* < *mxs*)
⟨*proof*⟩

lemma *appStore*[*simp*]:
app_i (*Store idx*,*P*,*pc*,*mxs*,T_r,*s*) = (∃ *ts ST LT*. *s* = (*ts*#*ST*,*LT*) ∧ *idx* < *length LT*)
⟨*proof*⟩

lemma *appPush*[*simp*]:
app_i (*Push v*,*P*,*pc*,*mxs*,T_r,*s*) =
(∃ *ST LT*. *s* = (*ST*,*LT*) ∧ *length ST* < *mxs* ∧ *typeof v* ≠ *None*)
⟨*proof*⟩

lemma *appGetField*[*simp*]:
app_i (*Getfield F C*,*P*,*pc*,*mxs*,T_r,*s*) =
(∃ *oT vT ST LT*. *s* = (*oT*#*ST*, *LT*) ∧
P ⊢ *C sees F*:*vT in C* ∧ *P* ⊢ *oT* ≤ (*Class C*))
⟨*proof*⟩

lemma *appPutField*[*simp*]:
app_i (*Putfield F C*,*P*,*pc*,*mxs*,T_r,*s*) =
(∃ *vT vT' oT ST LT*. *s* = (*vT*#*oT*#*ST*, *LT*) ∧
P ⊢ *C sees F*:*vT' in C* ∧ *P* ⊢ *oT* ≤ (*Class C*) ∧ *P* ⊢ *vT* ≤ *vT'*)

$\langle proof \rangle$

lemma *appNew*[*simp*]:
app_i (*New C*,*P*,*pc*,*mxs*,T_r,*s*) =
($\exists$ *ST LT*. *s*=(*ST*,*LT*) $\wedge$ *is-class P C* $\wedge$ *length ST* < *mxs*)
$\langle proof \rangle$

lemma *appCheckcast*[*simp*]:
app_i (*Checkcast C*,*P*,*pc*,*mxs*,T_r,*s*) =
($\exists$ *T ST LT*. *s* = (*T*#*ST*,*LT*) $\wedge$ *is-class P C* $\wedge$ *is-refT T*)
$\langle proof \rangle$

lemma $app_i Pop$[*simp*]:
app_i (*Pop*,*P*,*pc*,*mxs*,T_r,*s*) = ($\exists$ *ts ST LT*. *s* = (*ts*#*ST*,*LT*))
$\langle proof \rangle$

lemma *appIAdd*[*simp*]:
app_i (*IAdd*,*P*,*pc*,*mxs*,T_r,*s*) = ($\exists$ *ST LT*. *s* = (*Integer*#*Integer*#*ST*,*LT*))$\langle proof \rangle$

lemma *appIfFalse* [*simp*]:
app_i (*IfFalse b*,*P*,*pc*,*mxs*,T_r,*s*) =
($\exists$ *ST LT*. *s* = (*Boolean*#*ST*,*LT*) $\wedge$ *0* $\leq$ *int pc* + *b*)$\langle proof \rangle$
lemma *appCmpEq*[*simp*]:
app_i (*CmpEq*,*P*,*pc*,*mxs*,T_r,*s*) =
($\exists$ T_1 T_2 *ST LT*. *s* = (T_1#T_2#*ST*,*LT*) $\wedge$ ($\neg$*is-refT* T_1 $\wedge$ T_2 = T_1 $\vee$ *is-refT* T_1 $\wedge$ *is-refT* T_2))
$\langle proof \rangle$

lemma *appReturn*[*simp*]:
app_i (*Return*,*P*,*pc*,*mxs*,T_r,*s*) = ($\exists$ *T ST LT*. *s* = (*T*#*ST*,*LT*) $\wedge$ *P* $\vdash$ *T* $\leq$ T_r)
$\langle proof \rangle$

lemma *appThrow*[*simp*]:
app_i (*Throw*,*P*,*pc*,*mxs*,T_r,*s*) = ($\exists$ *T ST LT*. *s*=(*T*#*ST*,*LT*) $\wedge$ *is-refT T*)
$\langle proof \rangle$

lemma *effNone*:
(pc', s') $\in$ *set* (*eff i P pc et None*) $\Longrightarrow$ s' = *None*
$\langle proof \rangle$

some helpers to make the specification directly executable:

lemma *relevant-entries-append* [*simp*]:
relevant-entries P i pc (*xt* @ xt') = *relevant-entries P i pc xt* @ *relevant-entries P i pc* xt'
$\langle proof \rangle$

lemma *xcpt-app-append* [*iff*]:
xcpt-app i P pc mxs (*xt*@xt') τ = (*xcpt-app i P pc mxs xt* τ $\wedge$ *xcpt-app i P pc mxs* xt' τ)
$\langle proof \rangle$

lemma *xcpt-eff-append* [*simp*]:
xcpt-eff i P pc τ (*xt*@xt') = *xcpt-eff i P pc* τ *xt* @ *xcpt-eff i P pc* τ xt'
$\langle proof \rangle$

lemma *app-append* [*simp*]:
app i P pc T mxs mpc (*xt*@xt') τ = (*app i P pc T mxs mpc xt* τ $\wedge$ *app i P pc T mxs mpc* xt' τ)

⟨*proof*⟩

end

4.16 Monotonicity of eff and app

theory *EffectMono* **imports** *Effect* **begin**

declare *not-Err-eq* [*iff*]

lemma app_i*-mono*:
  assumes *wf*: *wf-prog p P*
  assumes *less*: $P \vdash \tau \leq_i \tau'$
  shows $app_i\ (i,P,mxs,mpc,rT,\tau') \Longrightarrow app_i\ (i,P,mxs,mpc,rT,\tau)\langle proof\rangle$
lemma *succs-mono*:
  assumes *wf*: *wf-prog p P* **and** app_i: $app_i\ (i,P,mxs,mpc,rT,\tau')$
  shows $P \vdash \tau \leq_i \tau' \Longrightarrow set\ (succs\ i\ \tau\ pc) \subseteq set\ (succs\ i\ \tau'\ pc)\langle proof\rangle$

lemma *app-mono*:
  assumes *wf*: *wf-prog p P*
  assumes *less'*: $P \vdash \tau \leq' \tau'$
  shows $app\ i\ P\ m\ rT\ pc\ mpc\ xt\ \tau' \Longrightarrow app\ i\ P\ m\ rT\ pc\ mpc\ xt\ \tau\langle proof\rangle$

lemma eff_i*-mono*:
  assumes *wf*: *wf-prog p P*
  assumes *less*: $P \vdash \tau \leq_i \tau'$
  assumes app_i: $app\ i\ P\ m\ rT\ pc\ mpc\ xt\ (Some\ \tau')$
  assumes *succs*: $succs\ i\ \tau\ pc \neq []\quad succs\ i\ \tau'\ pc \neq []$
  shows $P \vdash eff_i\ (i,P,\tau) \leq_i eff_i\ (i,P,\tau')\langle proof\rangle$
end

4.17 The Bytecode Verifier

theory *BVSpec*
imports *Effect*
begin

This theory contains a specification of the BV. The specification describes correct typings of method bodies; it corresponds to type *checking*.

definition
— The method type only contains declared classes:
$check\text{-}types :: 'm\ prog \Rightarrow nat \Rightarrow nat \Rightarrow ty_i'\ err\ list \Rightarrow bool$
where
$check\text{-}types\ P\ mxs\ mxl\ \tau s \equiv set\ \tau s \subseteq states\ P\ mxs\ mxl$

— An instruction is welltyped if it is applicable and its effect
— is compatible with the type at all successor instructions:
definition
$wt\text{-}instr :: ['m\ prog, ty, nat, pc, ex\text{-}table, instr, pc, ty_m] \Rightarrow bool$
$(\text{-,-,-,-,-} \vdash \text{-,-} :: \text{-}\ [60,0,0,0,0,0,0,61]\ 60)$
where
$P, T, mxs, mpc, xt \vdash i, pc :: \tau s \equiv$
$app\ i\ P\ mxs\ T\ pc\ mpc\ xt\ (\tau s!pc) \wedge$
$(\forall (pc',\tau') \in set\ (eff\ i\ P\ pc\ xt\ (\tau s!pc)).\ P \vdash \tau' \le' \tau s!pc')$

— The type at $pc=0$ conforms to the method calling convention:
definition $wt\text{-}start :: ['m\ prog, cname, ty\ list, nat, ty_m] \Rightarrow bool$
where
$wt\text{-}start\ P\ C\ Ts\ mxl_0\ \tau s \equiv$
$P \vdash Some\ ([], OK\ (Class\ C) \# map\ OK\ Ts @ replicate\ mxl_0\ Err) \le' \tau s!0$

— A method is welltyped if the body is not empty,
— if the method type covers all instructions and mentions
— declared classes only, if the method calling convention is respected, and
— if all instructions are welltyped.
definition $wt\text{-}method :: ['m\ prog, cname, ty\ list, ty, nat, nat, instr\ list,$
$ex\text{-}table, ty_m] \Rightarrow bool$
where
$wt\text{-}method\ P\ C\ Ts\ T_r\ mxs\ mxl_0\ is\ xt\ \tau s \equiv$
$0 < size\ is \wedge size\ \tau s = size\ is \wedge$
$check\text{-}types\ P\ mxs\ (1+size\ Ts+mxl_0)\ (map\ OK\ \tau s) \wedge$
$wt\text{-}start\ P\ C\ Ts\ mxl_0\ \tau s \wedge$
$(\forall pc < size\ is.\ P, T_r, mxs, size\ is, xt \vdash is!pc, pc :: \tau s)$

— A program is welltyped if it is wellformed and all methods are welltyped
definition $wf\text{-}jvm\text{-}prog\text{-}phi :: ty_P \Rightarrow jvm\text{-}prog \Rightarrow bool\ (wf'\text{-}jvm'\text{-}prog_{\text{-}})$
where
$wf\text{-}jvm\text{-}prog_\Phi \equiv$
$wf\text{-}prog\ (\lambda P\ C\ (M, Ts, T_r, (mxs, mxl_0, is, xt)).$
$wt\text{-}method\ P\ C\ Ts\ T_r\ mxs\ mxl_0\ is\ xt\ (\Phi\ C\ M))$

definition $wf\text{-}jvm\text{-}prog :: jvm\text{-}prog \Rightarrow bool$
where
$wf\text{-}jvm\text{-}prog\ P \equiv \exists \Phi.\ wf\text{-}jvm\text{-}prog_\Phi\ P$

notation (*input*)
  $\mathit{wf\text{-}jvm\text{-}prog\text{-}phi}$ ($\mathit{wf'\text{-}jvm'\text{-}prog_{\_}}$ - [0,999] 1000)

lemma *wt-jvm-progD*:
  $\mathit{wf\text{-}jvm\text{-}prog}_{\Phi}\ P \Longrightarrow \exists\, wt.\ \mathit{wf\text{-}prog}\ wt\ P$⟨*proof*⟩
lemma *wt-jvm-prog-impl-wt-instr*:
  $[\![\ \mathit{wf\text{-}jvm\text{-}prog}_{\Phi}\ P;$
    $P \vdash C\ \mathit{sees}\ M{:}Ts \rightarrow T = (mxs,mxl_0,ins,xt)\ \mathit{in}\ C;\ pc < \mathit{size}\ ins\ ]\!]$
  $\Longrightarrow P,T,mxs,\mathit{size}\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M$⟨*proof*⟩
lemma *wt-jvm-prog-impl-wt-start*:
  $[\![\ \mathit{wf\text{-}jvm\text{-}prog}_{\Phi}\ P;$
    $P \vdash C\ \mathit{sees}\ M{:}Ts \rightarrow T = (mxs,mxl_0,ins,xt)\ \mathit{in}\ C\ ]\!] \Longrightarrow$
  $0 < \mathit{size}\ ins \wedge \mathit{wt\text{-}start}\ P\ C\ Ts\ mxl_0\ (\Phi\ C\ M)$⟨*proof*⟩
end

4.18 The Typing Framework for the JVM

theory *TF-JVM*
imports *../DFA/Typing-Framework-err EffectMono BVSpec*
begin

definition *exec* :: *jvm-prog* ⇒ *nat* ⇒ *ty* ⇒ *ex-table* ⇒ *instr list* ⇒ ty_i′ *err step-type*
where
exec G maxs rT et bs ≡
err-step (*size bs*) (λ*pc*. *app* (*bs*!*pc*) *G maxs rT pc* (*size bs*) *et*)
(λ*pc*. *eff* (*bs*!*pc*) *G pc et*)

locale *JVM-sl* =
fixes *P* :: *jvm-prog* **and** *mxs* **and** mxl_0
fixes *Ts* :: *ty list* **and** *is* **and** *xt* **and** T_r

fixes *mxl* **and** *A* **and** *r* **and** *f* **and** *app* **and** *eff* **and** *step*
defines [*simp*]: *mxl* ≡ *1*+*size Ts*+mxl_0
defines [*simp*]: *A* ≡ *states P mxs mxl*
defines [*simp*]: *r* ≡ *JVM-SemiType.le P mxs mxl*
defines [*simp*]: *f* ≡ *JVM-SemiType.sup P mxs mxl*

defines [*simp*]: *app* ≡ λ*pc*. *Effect.app* (*is*!*pc*) *P mxs* T_r *pc* (*size is*) *xt*
defines [*simp*]: *eff* ≡ λ*pc*. *Effect.eff* (*is*!*pc*) *P pc xt*
defines [*simp*]: *step* ≡ *err-step* (*size is*) *app eff*

locale *start-context* = *JVM-sl* +
fixes *p* **and** *C*
assumes *wf*: *wf-prog p P*
assumes *C*: *is-class P C*
assumes *Ts*: *set Ts* ⊆ *types P*

fixes *first* :: ty_i′ **and** *start*
defines [*simp*]:
first ≡ *Some* ([],*OK* (*Class C*) # *map OK Ts* @ *replicate* mxl_0 *Err*)
defines [*simp*]:
start ≡ *OK first* # *replicate* (*size is* − *1*) (*OK None*)

4.18.1 Connecting JVM and Framework

lemma (**in** *JVM-sl*) *step-def-exec*: *step* ≡ *exec P mxs* T_r *xt is*
⟨*proof*⟩

lemma *special-ex-swap-lemma* [*iff*]:
(*? X*. (*? n*. *X* = *A n* & *P n*) & *Q X*) = (*? n*. *Q*(*A n*) & *P n*)
⟨*proof*⟩

lemma *ex-in-list* [*iff*]:
(∃ *n*. *ST* ∈ *list n A* ∧ *n* ≤ *mxs*) = (*set ST* ⊆ *A* ∧ *size ST* ≤ *mxs*)
⟨*proof*⟩

lemma *singleton-list*:
(∃ *n*. [*Class C*] ∈ *list n* (*types P*) ∧ *n* ≤ *mxs*) = (*is-class P C* ∧ *0* < *mxs*)

⟨*proof*⟩

lemma *set-drop-subset*:
set xs ⊆ *A* ⟹ *set* (*drop n xs*) ⊆ *A*
⟨*proof*⟩

lemma *Suc-minus-minus-le*:
$n < mxs \Longrightarrow Suc\ (n - (n - b)) \leq mxs$
⟨*proof*⟩

lemma *in-listE*:
⟦ *xs* ∈ *list n A*; ⟦*size xs* = *n*; *set xs* ⊆ *A*⟧ ⟹ *P* ⟧ ⟹ *P*
⟨*proof*⟩

declare *is-relevant-entry-def* [*simp*]
declare *set-drop-subset* [*simp*]

theorem (**in** *start-context*) *exec-pres-type*:
pres-type step (*size is*) *A*⟨*proof*⟩
declare *is-relevant-entry-def* [*simp del*]
declare *set-drop-subset* [*simp del*]

lemma *lesubstep-type-simple*:
$xs\ [\sqsubseteq_{Product.le\ (op\ =)\ r}]\ ys \Longrightarrow set\ xs\ \{\sqsubseteq_r\}\ set\ ys$⟨*proof*⟩
declare *is-relevant-entry-def* [*simp del*]

lemma *conjI2*: ⟦ *A*; *A* ⟹ *B* ⟧ ⟹ *A* ∧ *B* ⟨*proof*⟩

lemma (**in** *JVM-sl*) *eff-mono*:
$[\![wf\text{-}prog\ p\ P;\ pc < length\ is;\ s \sqsubseteq_{sup\text{-}state\text{-}opt\ P}\ t;\ app\ pc\ t]\!]$
$\Longrightarrow set\ (eff\ pc\ s)\ \{\sqsubseteq_{sup\text{-}state\text{-}opt\ P}\}\ set\ (eff\ pc\ t)$⟨*proof*⟩
lemma (**in** *JVM-sl*) *bounded-step*: *bounded step* (*size is*)⟨*proof*⟩
theorem (**in** *JVM-sl*) *step-mono*:
wf-prog wf-mb P ⟹ *mono r step* (*size is*) *A*⟨*proof*⟩

lemma (**in** *start-context*) *first-in-A* [*iff*]: *OK first* ∈ *A*
⟨*proof*⟩

lemma (**in** *JVM-sl*) *wt-method-def2*:
$wt\text{-}method\ P\ C'\ Ts\ T_r\ mxs\ mxl_0\ is\ xt\ \tau s =$
$(is \neq [] \wedge$
$size\ \tau s = size\ is \wedge$
$OK\ `\ set\ \tau s \subseteq states\ P\ mxs\ mxl \wedge$
$wt\text{-}start\ P\ C'\ Ts\ mxl_0\ \tau s \wedge$
$wt\text{-}app\text{-}eff\ (sup\text{-}state\text{-}opt\ P)\ app\ eff\ \tau s)$⟨*proof*⟩

end

4.19 Kildall for the JVM

theory *BVExec*
imports *../DFA/Abstract-BV TF-JVM*
begin

definition *kiljvm* :: *jvm-prog* ⇒ *nat* ⇒ *nat* ⇒ *ty* ⇒
instr list ⇒ *ex-table* ⇒ ty_i′ *err list* ⇒ ty_i′ *err list*
where
kiljvm P mxs mxl T_r *is xt* ≡
kildall (*JVM-SemiType.le P mxs mxl*) (*JVM-SemiType.sup P mxs mxl*)
(*exec P mxs* T_r *xt is*)

definition *wt-kildall* :: *jvm-prog* ⇒ *cname* ⇒ *ty list* ⇒ *ty* ⇒ *nat* ⇒ *nat* ⇒
instr list ⇒ *ex-table* ⇒ *bool*
where
wt-kildall P C′ Ts T_r *mxs* mxl_0 *is xt* ≡
0 < *size is* ∧
(*let first* = *Some* ([],[*OK* (*Class C′*)]@(*map OK Ts*)@(*replicate* mxl_0 *Err*));
start = *OK first*#(*replicate* (*size is* − *1*) (*OK None*));
result = *kiljvm P mxs* (*1*+*size Ts*+mxl_0) T_r *is xt start*
in ∀ *n* < *size is*. *result*!*n* ≠ *Err*)

definition *wf-jvm-prog*$_k$ :: *jvm-prog* ⇒ *bool*
where
wf-jvm-prog$_k$ *P* ≡
wf-prog (λ*P C′* (*M*,*Ts*,T_r,(*mxs*,mxl_0,*is*,*xt*)). *wt-kildall P C′ Ts* T_r *mxs* mxl_0 *is xt*) *P*

theorem (**in** *start-context*) *is-bcv-kiljvm*:
is-bcv r Err step (*size is*) *A* (*kiljvm P mxs mxl* T_r *is xt*)⟨*proof*⟩

lemma *subset-replicate* [*intro?*]: *set* (*replicate n x*) ⊆ {*x*}
⟨*proof*⟩

lemma *in-set-replicate*:
assumes *x* ∈ *set* (*replicate n y*)
shows *x* = *y*⟨*proof*⟩
lemma (**in** *start-context*) *start-in-A* [*intro?*]:
0 < *size is* ⟹ *start* ∈ *list* (*size is*) *A*
⟨*proof*⟩

theorem (**in** *start-context*) *wt-kil-correct*:
assumes *wtk*: *wt-kildall P C Ts* T_r *mxs* mxl_0 *is xt*
shows ∃τ*s*. *wt-method P C Ts* T_r *mxs* mxl_0 *is xt* τ*s*⟨*proof*⟩

theorem (**in** *start-context*) *wt-kil-complete*:
assumes *wtm*: *wt-method P C Ts* T_r *mxs* mxl_0 *is xt* τ*s*
shows *wt-kildall P C Ts* T_r *mxs* mxl_0 *is xt*⟨*proof*⟩

theorem *jvm-kildall-correct*:
wf-jvm-prog$_k$ *P* = *wf-jvm-prog P*⟨*proof*⟩
end

4.20 LBV for the JVM

theory *LBVJVM*
imports *../DFA/Abstract-BV TF-JVM*
begin

type-synonym *prog-cert* = *cname* ⇒ *mname* ⇒ $ty_i{}'$ *err list*

definition *check-cert* :: *jvm-prog* ⇒ *nat* ⇒ *nat* ⇒ *nat* ⇒ $ty_i{}'$ *err list* ⇒ *bool*
where
check-cert P mxs mxl n cert ≡ *check-types P mxs mxl cert* ∧ *size cert* = *n+1* ∧
(∀ *i*<*n*. *cert*!*i* ≠ *Err*) ∧ *cert*!*n* = *OK None*

definition *lbvjvm* :: *jvm-prog* ⇒ *nat* ⇒ *nat* ⇒ *ty* ⇒ *ex-table* ⇒
$ty_i{}'$ *err list* ⇒ *instr list* ⇒ $ty_i{}'$ *err* ⇒ $ty_i{}'$ *err*
where
lbvjvm P mxs maxr T_r *et cert bs* ≡
wtl-inst-list bs cert (*JVM-SemiType.sup P mxs maxr*) (*JVM-SemiType.le P mxs maxr*) *Err* (*OK None*) (*exec P mxs* T_r *et bs*) *0*

definition *wt-lbv* :: *jvm-prog* ⇒ *cname* ⇒ *ty list* ⇒ *ty* ⇒ *nat* ⇒ *nat* ⇒
ex-table ⇒ $ty_i{}'$ *err list* ⇒ *instr list* ⇒ *bool*
where
wt-lbv P C Ts T_r *mxs* mxl_0 *et cert ins* ≡
check-cert P mxs (*1+size Ts*+mxl_0) (*size ins*) *cert* ∧
0 < *size ins* ∧
(*let start* = *Some* ([],(*OK* (*Class C*))#((*map OK Ts*))@(*replicate* mxl_0 *Err*));
result = *lbvjvm P mxs* (*1+size Ts*+mxl_0) T_r *et cert ins* (*OK start*)
in result ≠ *Err*)

definition *wt-jvm-prog-lbv* :: *jvm-prog* ⇒ *prog-cert* ⇒ *bool*
where
wt-jvm-prog-lbv P cert ≡
wf-prog (λ*P C* (*mn*,*Ts*,T_r,(*mxs*,mxl_0,*b*,*et*)). *wt-lbv P C Ts* T_r *mxs* mxl_0 *et* (*cert C mn*) *b*) *P*

definition *mk-cert* :: *jvm-prog* ⇒ *nat* ⇒ *ty* ⇒ *ex-table* ⇒ *instr list*
⇒ ty_m ⇒ $ty_i{}'$ *err list*
where
mk-cert P mxs T_r *et bs phi* ≡ *make-cert* (*exec P mxs* T_r *et bs*) (*map OK phi*) (*OK None*)

definition *prg-cert* :: *jvm-prog* ⇒ ty_P ⇒ *prog-cert*
where
prg-cert P phi C mn ≡ *let* (*C*,*Ts*,T_r,(*mxs*,mxl_0,*ins*,*et*)) = *method P C mn*
in mk-cert P mxs T_r *et ins* (*phi C mn*)

lemma *check-certD* [*intro?*]:
check-cert P mxs mxl n cert ⟹ *cert-ok cert n Err* (*OK None*) (*states P mxs mxl*)
⟨*proof*⟩

lemma (**in** *start-context*) *wt-lbv-wt-step*:
assumes *lbv*: *wt-lbv P C Ts* T_r *mxs* mxl_0 *xt cert is*
shows ∃ τ*s* ∈ *list* (*size is*) *A*. *wt-step r Err step* τ*s* ∧ *OK first* $\sqsubseteq_r$ τ*s*!*0*⟨*proof*⟩

lemma (**in** *start-context*) *wt-lbv-wt-method*:
  assumes *lbv*: *wt-lbv P C Ts* T_r *mxs* mxl_0 *xt cert is*
  shows $\exists \tau s$. *wt-method P C Ts* T_r *mxs* mxl_0 *is xt* τs⟨*proof*⟩

lemma (**in** *start-context*) *wt-method-wt-lbv*:
  assumes *wt*: *wt-method P C Ts* T_r *mxs* mxl_0 *is xt* τs
  defines [*simp*]: *cert* ≡ *mk-cert P mxs* T_r *xt is* τs

  shows *wt-lbv P C Ts* T_r *mxs* mxl_0 *xt cert is*⟨*proof*⟩

theorem *jvm-lbv-correct*:
  wt-jvm-prog-lbv P Cert $\Longrightarrow$ *wf-jvm-prog P*⟨*proof*⟩
theorem *jvm-lbv-complete*:
  assumes *wt*: *wf-jvm-prog*$_\Phi$ *P*
  shows *wt-jvm-prog-lbv P* (*prg-cert P* Φ)⟨*proof*⟩
end

4.21 BV Type Safety Invariant

```
theory BVConform
imports BVSpec ../JVM/JVMExec ../Common/Conform
begin


definition confT :: 'c prog ⇒ heap ⇒ val ⇒ ty err ⇒ bool
          (-,- |− - :<=T - [51,51,51,51] 50)
where
  P,h |− v :<=T E ≡ case E of Err ⇒ True | OK T ⇒ P,h ⊢ v :≤ T

notation (xsymbols)
  confT (-,- ⊢ - :≤⊤ - [51,51,51,51] 50)

abbreviation
  confTs :: 'c prog ⇒ heap ⇒ val list ⇒ ty_l ⇒ bool
          (-,- |− - [:<=T] - [51,51,51,51] 50) where
  P,h |− vs [:<=T] Ts ≡ list-all2 (confT P h) vs Ts

notation (xsymbols)
  confTs (-,- ⊢ - [:≤⊤] - [51,51,51,51] 50)

definition conf-f :: jvm-prog ⇒ heap ⇒ ty_i ⇒ bytecode ⇒ frame ⇒ bool
where
  conf-f P h ≡ λ(ST,LT) is (stk,loc,C,M,pc).
  P,h ⊢ stk [:≤] ST ∧ P,h ⊢ loc [:≤⊤] LT ∧ pc < size is

lemma conf-f-def2:
  conf-f P h (ST,LT) is (stk,loc,C,M,pc) ≡
  P,h ⊢ stk [:≤] ST ∧ P,h ⊢ loc [:≤⊤] LT ∧ pc < size is
  ⟨proof⟩


primrec conf-fs :: [jvm-prog,heap,ty_P,mname,nat,ty,frame list] ⇒ bool
where
  conf-fs P h Φ M_0 n_0 T_0 [] = True
| conf-fs P h Φ M_0 n_0 T_0 (f#frs) =
  (let (stk,loc,C,M,pc) = f in
  (∃ ST LT Ts T mxs mxl_0 is xt.
    Φ C M ! pc = Some (ST,LT) ∧
    (P ⊢ C sees M:Ts → T = (mxs,mxl_0,is,xt) in C) ∧
    (∃ D Ts' T' m D'.
       is!pc = (Invoke M_0 n_0) ∧ ST!n_0 = Class D ∧
       P ⊢ D sees M_0:Ts' → T' = m in D' ∧ P ⊢ T_0 ≤ T') ∧
    conf-f P h (ST, LT) is f ∧ conf-fs P h Φ M (size Ts) T frs))


definition correct-state :: [jvm-prog,ty_P,jvm-state] ⇒ bool
                (-,- |− - [ok] [61,0,0] 61)
where
  correct-state P Φ ≡ λ(xp,h,frs).
  case xp of
     None ⇒ (case frs of
```

$[] \Rightarrow True$
$\mid (f\#fs) \Rightarrow P\vdash h\surd \wedge$
$(let\ (stk,loc,C,M,pc) = f$
$in\ \exists\, Ts\ T\ mxs\ mxl_0\ is\ xt\ \tau.$
$(P \vdash C\ sees\ M{:}Ts{\rightarrow}T = (mxs,mxl_0,is,xt)\ in\ C) \wedge$
$\Phi\ C\ M\ !\ pc = Some\ \tau \wedge$
$conf\text{-}f\ P\ h\ \tau\ is\ f \wedge conf\text{-}fs\ P\ h\ \Phi\ M\ (size\ Ts)\ T\ fs))$
$\mid Some\ x \Rightarrow frs = []$

notation (*xsymbols*)
correct-state (-,- ⊢ - √ [*61*,*0*,*0*] *61*)

4.21.1 Values and ⊤

lemma *confT-Err* [*iff*]: $P,h \vdash x :\leq_\top Err$
⟨*proof*⟩

lemma *confT-OK* [*iff*]: $P,h \vdash x :\leq_\top OK\ T = (P,h \vdash x :\leq T)$
⟨*proof*⟩

lemma *confT-cases*:
$P,h \vdash x :\leq_\top X = (X = Err \vee (\exists\, T.\ X = OK\ T \wedge P,h \vdash x :\leq T))$
⟨*proof*⟩

lemma *confT-hext* [*intro?*, *trans*]:
$[\![\ P,h \vdash x :\leq_\top T;\ h \trianglelefteq h'\ ]\!] \Longrightarrow P,h' \vdash x :\leq_\top T$
⟨*proof*⟩

lemma *confT-widen* [*intro?*, *trans*]:
$[\![\ P,h \vdash x :\leq_\top T;\ P \vdash T \leq_\top T'\ ]\!] \Longrightarrow P,h \vdash x :\leq_\top T'$
⟨*proof*⟩

4.21.2 Stack and Registers

lemmas *confTs-Cons1* [*iff*] = *list-all2-Cons1* [*of confT P h*] **for** *P h*

lemma *confTs-confT-sup*:
$[\![\ P,h \vdash loc\ [{:}\leq_\top]\ LT;\ n < size\ LT;\ LT!n = OK\ T;\ P \vdash T \leq T'\ ]\!]$
$\Longrightarrow P,h \vdash (loc!n) :\leq T'$⟨*proof*⟩
lemma *confTs-hext* [*intro?*]:
$P,h \vdash loc\ [{:}\leq_\top]\ LT \Longrightarrow h \trianglelefteq h' \Longrightarrow P,h' \vdash loc\ [{:}\leq_\top]\ LT$
⟨*proof*⟩

lemma *confTs-widen* [*intro?*, *trans*]:
$P,h \vdash loc\ [{:}\leq_\top]\ LT \Longrightarrow P \vdash LT\ [\leq_\top]\ LT' \Longrightarrow P,h \vdash loc\ [{:}\leq_\top]\ LT'$
⟨*proof*⟩

lemma *confTs-map* [*iff*]:
$\bigwedge vs.\ (P,h \vdash vs\ [{:}\leq_\top]\ map\ OK\ Ts) = (P,h \vdash vs\ [{:}\leq]\ Ts)$
⟨*proof*⟩

lemma *reg-widen-Err* [*iff*]:
$\bigwedge LT.\ (P \vdash replicate\ n\ Err\ [\leq_\top]\ LT) = (LT = replicate\ n\ Err)$
⟨*proof*⟩

lemma *confTs-Err* [*iff*]:
$P,h \vdash$ *replicate n v* $[:\leq_\top]$ *replicate n Err*
$\langle proof \rangle$

4.21.3 correct-frames

lemmas [*simp del*] = *fun-upd-apply*

lemma *conf-fs-hext*:
$\bigwedge M\ n\ T_r.$
$[\![$ *conf-fs* $P\ h\ \Phi\ M\ n\ T_r$ *frs*; $h \trianglelefteq h'$ $]\!] \Longrightarrow$ *conf-fs* $P\ h'\ \Phi\ M\ n\ T_r$ *frs*$\langle proof \rangle$
end

4.22 BV Type Safety Proof

theory *BVSpecTypeSafe*
imports *BVConform*
begin

This theory contains proof that the specification of the bytecode verifier only admits type safe programs.

4.22.1 Preliminaries

Simp and intro setup for the type safety proof:

lemmas *defs1* = *correct-state-def conf-f-def wt-instr-def eff-def norm-eff-def app-def xcpt-app-def*

lemmas *widen-rules* [*intro*] = *conf-widen confT-widen confs-widens confTs-widen*

4.22.2 Exception Handling

For the *Invoke* instruction the BV has checked all handlers that guard the current *pc*.

lemma *Invoke-handlers*:
$match\text{-}ex\text{-}table\ P\ C\ pc\ xt = Some\ (pc',d') \Longrightarrow$
$\exists (f,t,D,h,d) \in set\ (relevant\text{-}entries\ P\ (Invoke\ n\ M)\ pc\ xt).$
$P \vdash C \preceq^* D \wedge pc \in \{f..<t\} \wedge pc' = h \wedge d' = d$
$\langle proof \rangle$

We can prove separately that the recursive search for exception handlers (*find-handler*) in the frame stack results in a conforming state (if there was no matching exception handler in the current frame). We require that the exception is a valid heap address, and that the state before the exception occured conforms.

term *find-handler*
lemma *uncaught-xcpt-correct*:
assumes *wt*: $wf\text{-}jvm\text{-}prog_\Phi\ P$
assumes *h*: $h\ xcp = Some\ obj$
shows $\bigwedge f.\ P,\Phi \vdash (None,\ h,\ f\#frs)\surd \Longrightarrow P,\Phi \vdash (find\text{-}handler\ P\ xcp\ h\ frs)\ \surd$
(**is** $\bigwedge f.\ ?correct\ (None,\ h,\ f\#frs) \Longrightarrow ?correct\ (?find\ frs))\langle proof \rangle$

The requirement of lemma *uncaught-xcpt-correct* (that the exception is a valid reference on the heap) is always met for welltyped instructions and conformant states:

lemma *exec-instr-xcpt-h*:
$[\![\ fst\ (exec\text{-}instr\ (ins!pc)\ P\ h\ stk\ vars\ Cl\ M\ pc\ frs) = Some\ xcp;$
$P,T,mxs,size\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M;$
$P,\Phi \vdash (None,\ h,\ (stk,loc,C,M,pc)\#frs)\surd\ ]\!]$
$\Longrightarrow \exists\, obj.\ h\ xcp = Some\ obj$
(**is** $[\![\ ?xcpt;\ ?wt;\ ?correct\ ]\!] \Longrightarrow\ ?thesis)\langle proof \rangle$
lemma *conf-sys-xcpt*:
$[\![preallocated\ h;\ C \in sys\text{-}xcpts]\!] \Longrightarrow P,h \vdash Addr\ (addr\text{-}of\text{-}sys\text{-}xcpt\ C) :\leq Class\ C$
$\langle proof \rangle$

lemma *match-ex-table-SomeD*:
$match\text{-}ex\text{-}table\ P\ C\ pc\ xt = Some\ (pc',d') \Longrightarrow$
$\exists (f,t,D,h,d) \in set\ xt.\ matches\text{-}ex\text{-}entry\ P\ C\ pc\ (f,t,D,h,d) \wedge h = pc' \wedge d{=}d'$
$\langle proof \rangle$

Finally we can state that, whenever an exception occurs, the next state always conforms:

lemma *xcpt-correct*:
  fixes σ' :: *jvm-state*
  assumes *wtp*: *wf-jvm-prog*$_\Phi$ P
  assumes *meth*: $P \vdash C$ *sees* M:*Ts*$\rightarrow$$T$=(*mxs*,*mxl*$_0$,*ins*,*xt*) *in* C
  assumes *wt*: P,T,*mxs*,*size ins*,*xt* $\vdash$ *ins*!*pc*,*pc* :: Φ C M
  assumes *xp*: *fst* (*exec-instr* (*ins*!*pc*) P *h stk loc* C M *pc frs*) = *Some xcp*
  assumes *s'*: *Some* σ' = *exec* (P, *None*, *h*, (*stk*,*loc*,C,M,*pc*)#*frs*)
  assumes *correct*: P,Φ $\vdash$ (*None*, *h*, (*stk*,*loc*,C,M,*pc*)#*frs*)$\surd$
  shows P,Φ $\vdash$ $\sigma'$$\surd$⟨*proof*⟩

4.22.3 Single Instructions

In this section we prove for each single (welltyped) instruction that the state after execution of the instruction still conforms. Since we have already handled exceptions above, we can now assume that no exception occurs in this step.

declare *defs1* [*simp*]

lemma *Invoke-correct*:
  fixes σ' :: *jvm-state*
  assumes *wtprog*: *wf-jvm-prog*$_\Phi$ P
  assumes *meth-C*: $P \vdash C$ *sees* M:*Ts*$\rightarrow$$T$=(*mxs*,*mxl*$_0$,*ins*,*xt*) *in* C
  assumes *ins*: *ins* ! *pc* = *Invoke* M' *n*
  assumes *wti*: P,T,*mxs*,*size ins*,*xt* $\vdash$ *ins*!*pc*,*pc* :: Φ C M
  assumes σ': *Some* σ' = *exec* (P, *None*, *h*, (*stk*,*loc*,C,M,*pc*)#*frs*)
  assumes *approx*: P,Φ $\vdash$ (*None*, *h*, (*stk*,*loc*,C,M,*pc*)#*frs*)$\surd$
  assumes *no-xcp*: *fst* (*exec-instr* (*ins*!*pc*) P *h stk loc* C M *pc frs*) = *None*
  shows P,Φ $\vdash$ $\sigma'$$\surd$⟨*proof*⟩
declare *list-all2-Cons2* [*iff*]

lemma *Return-correct*:
  fixes σ' :: *jvm-state*
  assumes *wt-prog*: *wf-jvm-prog*$_\Phi$ P
  assumes *meth*: $P \vdash C$ *sees* M:*Ts*$\rightarrow$$T$=(*mxs*,*mxl*$_0$,*ins*,*xt*) *in* C
  assumes *ins*: *ins* ! *pc* = *Return*
  assumes *wt*: P,T,*mxs*,*size ins*,*xt* $\vdash$ *ins*!*pc*,*pc* :: Φ C M
  assumes *s'*: *Some* σ' = *exec* (P, *None*, *h*, (*stk*,*loc*,C,M,*pc*)#*frs*)
  assumes *correct*: P,Φ $\vdash$ (*None*, *h*, (*stk*,*loc*,C,M,*pc*)#*frs*)$\surd$

  shows P,Φ $\vdash$ $\sigma'$$\surd$⟨*proof*⟩
declare *sup-state-opt-any-Some* [*iff*]
declare *not-Err-eq* [*iff*]

lemma *Load-correct*:
⟦ *wf-prog wt* P;
    $P \vdash C$ *sees* M:*Ts*$\rightarrow$$T$=(*mxs*,*mxl*$_0$,*ins*,*xt*) *in* C;
    ins!*pc* = *Load idx*;
    P,T,*mxs*,*size ins*,*xt* $\vdash$ *ins*!*pc*,*pc* :: Φ C M;
    Some σ' = *exec* (P, *None*, *h*, (*stk*,*loc*,C,M,*pc*)#*frs*);
    P,Φ $\vdash$ (*None*, *h*, (*stk*,*loc*,C,M,*pc*)#*frs*)$\surd$ ⟧
$\Longrightarrow$ P,Φ $\vdash$ $\sigma'$$\surd$
  ⟨*proof*⟩

declare [[*simproc del*: *list-to-set-comprehension*]]

lemma *Store-correct*:
$[\![$ *wf-prog wt P*;
  $P \vdash C$ *sees* $M{:}Ts{\rightarrow}T{=}(mxs,mxl_0,ins,xt)$ *in* C;
  ins!*pc* = *Store idx*;
  $P,T,mxs,$*size ins*$,xt \vdash$ *ins*!*pc*,*pc* :: Φ C M;
  Some σ' = *exec* (P, *None*, h, (*stk*,*loc*,C,M,*pc*)#*frs*);
  $P,\Phi \vdash$ (*None*, h, (*stk*,*loc*,C,M,*pc*)#*frs*)$\surd$ $]\!]$
$\Longrightarrow P,\Phi \vdash \sigma'\surd\langle proof\rangle$

lemma *Push-correct*:
$[\![$ *wf-prog wt P*;
    $P \vdash C$ *sees* $M{:}Ts{\rightarrow}T{=}(mxs,mxl_0,ins,xt)$ *in* C;
    ins!*pc* = *Push v*;
    $P,T,mxs,$*size ins*$,xt \vdash$ *ins*!*pc*,*pc* :: Φ C M;
    Some σ' = *exec* (P, *None*, h, (*stk*,*loc*,C,M,*pc*)#*frs*);
    $P,\Phi \vdash$ (*None*, h, (*stk*,*loc*,C,M,*pc*)#*frs*)$\surd$ $]\!]$
$\Longrightarrow P,\Phi \vdash \sigma'\surd\langle proof\rangle$

lemma *Cast-conf2*:
  $[\![$ *wf-prog ok P*; $P,h \vdash v :\leq T$; *is-refT T*; *cast-ok P C h v*;
     $P \vdash$ *Class* $C \leq T'$; *is-class P C*$]\!]$
  $\Longrightarrow P,h \vdash v :\leq T'\langle proof\rangle$

lemma *Checkcast-correct*:
$[\![$ *wf-jvm-prog*$_\Phi$ *P*;
    $P \vdash C$ *sees* $M{:}Ts{\rightarrow}T{=}(mxs,mxl_0,ins,xt)$ *in* C;
    ins!*pc* = *Checkcast D*;
    $P,T,mxs,$*size ins*$,xt \vdash$ *ins*!*pc*,*pc* :: Φ C M;
    Some σ' = *exec* (P, *None*, h, (*stk*,*loc*,C,M,*pc*)#*frs*) ;
    $P,\Phi \vdash$ (*None*, h, (*stk*,*loc*,C,M,*pc*)#*frs*)$\surd$;
    fst (*exec-instr* (*ins*!*pc*) *P h stk loc C M pc frs*) = *None* $]\!]$
$\Longrightarrow P,\Phi \vdash \sigma'\surd\langle proof\rangle$
declare *split-paired-All* [*simp del*]

lemmas *widens-Cons* [*iff*] = *list-all2-Cons1* [*of widen P*] **for** *P*

lemma *Getfield-correct*:
  fixes σ' :: *jvm-state*
  assumes *wf*: *wf-prog wt P*
  assumes *mC*: $P \vdash C$ *sees* $M{:}Ts{\rightarrow}T{=}(mxs,mxl_0,ins,xt)$ *in* C
  assumes *i*: *ins*!*pc* = *Getfield F D*
  assumes *wt*: $P,T,mxs,$*size ins*$,xt \vdash$ *ins*!*pc*,*pc* :: Φ C M
  assumes s': *Some* σ' = *exec* (P, *None*, h, (*stk*,*loc*,C,M,*pc*)#*frs*)
  assumes *cf*: $P,\Phi \vdash$ (*None*, h, (*stk*,*loc*,C,M,*pc*)#*frs*)$\surd$
  assumes *xc*: *fst* (*exec-instr* (*ins*!*pc*) *P h stk loc C M pc frs*) = *None*

  shows $P,\Phi \vdash \sigma'\surd\langle proof\rangle$
lemma *Putfield-correct*:
  fixes σ' :: *jvm-state*
  assumes *wf*: *wf-prog wt P*
  assumes *mC*: $P \vdash C$ *sees* $M{:}Ts{\rightarrow}T{=}(mxs,mxl_0,ins,xt)$ *in* C
  assumes *i*: *ins*!*pc* = *Putfield F D*

assumes *wt*: $P,T,mxs,size\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M$
assumes *s'*: $Some\ \sigma' = exec\ (P,\ None,\ h,\ (stk,loc,C,M,pc)\#frs)$
assumes *cf*: $P,\Phi \vdash (None,\ h,\ (stk,loc,C,M,pc)\#frs)\surd$
assumes *xc*: $fst\ (exec\text{-}instr\ (ins!pc)\ P\ h\ stk\ loc\ C\ M\ pc\ frs) = None$

shows $P,\Phi \vdash \sigma'\surd\langle proof\rangle$

lemma *has-fields-b-fields*:
$P \vdash C\ has\text{-}fields\ FDTs \Longrightarrow fields\ P\ C = FDTs\langle proof\rangle$

lemma *oconf-blank* [*intro*, *simp*]:
$[\![is\text{-}class\ P\ C;\ wf\text{-}prog\ wt\ P]\!] \Longrightarrow P,h \vdash blank\ P\ C\ \surd\langle proof\rangle$
lemma *obj-ty-blank* [*iff*]: $obj\text{-}ty\ (blank\ P\ C) = Class\ C$
$\langle proof\rangle$

lemma *New-correct*:
fixes $\sigma' :: jvm\text{-}state$
assumes *wf*: $wf\text{-}prog\ wt\ P$
assumes *meth*: $P \vdash C\ sees\ M{:}Ts{\rightarrow}T{=}(mxs,mxl_0,ins,xt)\ in\ C$
assumes *ins*: $ins!pc = New\ X$
assumes *wt*: $P,T,mxs,size\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M$
assumes *exec*: $Some\ \sigma' = exec\ (P,\ None,\ h,\ (stk,loc,C,M,pc)\#frs)$
assumes *conf*: $P,\Phi \vdash (None,\ h,\ (stk,loc,C,M,pc)\#frs)\surd$
assumes *no-x*: $fst\ (exec\text{-}instr\ (ins!pc)\ P\ h\ stk\ loc\ C\ M\ pc\ frs) = None$
shows $P,\Phi \vdash \sigma'\surd\langle proof\rangle$

lemma *Goto-correct*:
$[\![\ wf\text{-}prog\ wt\ P;$
$P \vdash C\ sees\ M{:}Ts{\rightarrow}T{=}(mxs,mxl_0,ins,xt)\ in\ C;$
$ins\ !\ pc = Goto\ branch;$
$P,T,mxs,size\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M;$
$Some\ \sigma' = exec\ (P,\ None,\ h,\ (stk,loc,C,M,pc)\#frs)\ ;$
$P,\Phi \vdash (None,\ h,\ (stk,loc,C,M,pc)\#frs)\surd\ ]\!]$
$\Longrightarrow P,\Phi \vdash \sigma'\surd\langle proof\rangle$

lemma *IfFalse-correct*:
$[\![\ wf\text{-}prog\ wt\ P;$
$P \vdash C\ sees\ M{:}Ts{\rightarrow}T{=}(mxs,mxl_0,ins,xt)\ in\ C;$
$ins\ !\ pc = IfFalse\ branch;$
$P,T,mxs,size\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M;$
$Some\ \sigma' = exec\ (P,\ None,\ h,\ (stk,loc,C,M,pc)\#frs)\ ;$
$P,\Phi \vdash (None,\ h,\ (stk,loc,C,M,pc)\#frs)\surd\ ]\!]$
$\Longrightarrow P,\Phi \vdash \sigma'\surd\langle proof\rangle$
lemma *CmpEq-correct*:
$[\![\ wf\text{-}prog\ wt\ P;$
$P \vdash C\ sees\ M{:}Ts{\rightarrow}T{=}(mxs,mxl_0,ins,xt)\ in\ C;$
$ins\ !\ pc = CmpEq;$
$P,T,mxs,size\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M;$
$Some\ \sigma' = exec\ (P,\ None,\ h,\ (stk,loc,C,M,pc)\#frs)\ ;$
$P,\Phi \vdash (None,\ h,\ (stk,loc,C,M,pc)\#frs)\surd\ ]\!]$
$\Longrightarrow P,\Phi \vdash \sigma'\surd\langle proof\rangle$
lemma *Pop-correct*:
$[\![\ wf\text{-}prog\ wt\ P;$
$P \vdash C\ sees\ M{:}Ts{\rightarrow}T{=}(mxs,mxl_0,ins,xt)\ in\ C;$

$ins\ !\ pc = Pop$;
$P,T,mxs,size\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M$;
$Some\ \sigma' = exec\ (P,\ None,\ h,\ (stk,loc,C,M,pc)\#frs)$;
$P,\Phi \vdash (None,\ h,\ (stk,loc,C,M,pc)\#frs)\surd\ \rrbracket$
$\Longrightarrow P,\Phi \vdash \sigma'\surd\langle proof\rangle$

lemma *IAdd-correct*:
$\llbracket$ $wf\text{-}prog\ wt\ P$;
$P \vdash C\ sees\ M{:}Ts{\rightarrow}T{=}(mxs,mxl_0,ins,xt)\ in\ C$;
$ins\ !\ pc = IAdd$;
$P,T,mxs,size\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M$;
$Some\ \sigma' = exec\ (P,\ None,\ h,\ (stk,loc,C,M,pc)\#frs)$;
$P,\Phi \vdash (None,\ h,\ (stk,loc,C,M,pc)\#frs)\surd\ \rrbracket$
$\Longrightarrow P,\Phi \vdash \sigma'\surd\langle proof\rangle$

lemma *Throw-correct*:
$\llbracket$ $wf\text{-}prog\ wt\ P$;
$P \vdash C\ sees\ M{:}Ts{\rightarrow}T{=}(mxs,mxl_0,ins,xt)\ in\ C$;
$ins\ !\ pc = Throw$;
$Some\ \sigma' = exec\ (P,\ None,\ h,\ (stk,loc,C,M,pc)\#frs)$;
$P,\Phi \vdash (None,\ h,\ (stk,loc,C,M,pc)\#frs)\surd$;
$fst\ (exec\text{-}instr\ (ins!pc)\ P\ h\ stk\ loc\ C\ M\ pc\ frs) = None\ \rrbracket$
$\Longrightarrow P,\Phi \vdash \sigma'\surd$
$\langle proof\rangle$

The next theorem collects the results of the sections above, i.e. exception handling and the execution step for each instruction. It states type safety for single step execution: in welltyped programs, a conforming state is transformed into another conforming state when one instruction is executed.

theorem *instr-correct*:
$\llbracket$ $wf\text{-}jvm\text{-}prog_\Phi\ P$;
$P \vdash C\ sees\ M{:}Ts{\rightarrow}T{=}(mxs,mxl_0,ins,xt)\ in\ C$;
$Some\ \sigma' = exec\ (P,\ None,\ h,\ (stk,loc,C,M,pc)\#frs)$;
$P,\Phi \vdash (None,\ h,\ (stk,loc,C,M,pc)\#frs)\surd\ \rrbracket$
$\Longrightarrow P,\Phi \vdash \sigma'\surd\langle proof\rangle$

4.22.4 Main

lemma *correct-state-impl-Some-method*:
$P,\Phi \vdash (None,\ h,\ (stk,loc,C,M,pc)\#frs)\surd$
$\Longrightarrow \exists m\ Ts\ T.\ P \vdash C\ sees\ M{:}Ts{\rightarrow}T = m\ in\ C$
$\langle proof\rangle$

lemma *BV-correct-1* [*rule-format*]:
$\bigwedge\sigma.\ \llbracket\ wf\text{-}jvm\text{-}prog_\Phi\ P;\ P,\Phi \vdash \sigma\surd\rrbracket \Longrightarrow P \vdash \sigma\ {-jvm\rightarrow_1}\ \sigma' \longrightarrow P,\Phi \vdash \sigma'\surd\langle proof\rangle$

theorem *progress*:
$\llbracket\ xp{=}None;\ frs{\neq}[]\ \rrbracket \Longrightarrow \exists\sigma'.\ P \vdash (xp,h,frs)\ {-jvm\rightarrow_1}\ \sigma'$
$\langle proof\rangle$

lemma *progress-conform*:
$\llbracket wf\text{-}jvm\text{-}prog_\Phi\ P;\ P,\Phi \vdash (xp,h,frs)\surd;\ xp{=}None;\ frs{\neq}[]\rrbracket$
$\Longrightarrow \exists\sigma'.\ P \vdash (xp,h,frs)\ {-jvm\rightarrow_1}\ \sigma' \land P,\Phi \vdash \sigma'\surd\langle proof\rangle$
theorem *BV-correct* [*rule-format*]:

$[\![$ *wf-jvm-prog*$_\Phi$ P; $P \vdash \sigma$ *−jvm→* σ' $]\!] \Longrightarrow P,\Phi \vdash \sigma\surd \longrightarrow P,\Phi \vdash \sigma'\surd\langle proof\rangle$

lemma *hconf-start*:
  assumes *wf*: *wf-prog wf-mb P*
  shows $P \vdash$ (*start-heap P*) $\surd\langle proof\rangle$

lemma *BV-correct-initial*:
  shows $[\![$ *wf-jvm-prog*$_\Phi$ P; $P \vdash C$ *sees* M:[]→T = *m in C* $]\!]$
  $\Longrightarrow P,\Phi \vdash$ *start-state P C M* $\surd\langle proof\rangle$

theorem *typesafe*:
  assumes *welltyped*: *wf-jvm-prog*$_\Phi$ P
  assumes *main-method*: $P \vdash C$ *sees* M:[]→T = *m in C*
  shows $P \vdash$ *start-state P C M* *−jvm→* $\sigma \Longrightarrow P,\Phi \vdash \sigma\ \surd\langle proof\rangle$

end

4.23 Welltyped Programs produce no Type Errors

theory *BVNoTypeError*
imports *../JVM/JVMDefensive BVSpecTypeSafe*
begin

lemma *has-methodI*:
$P \vdash C\ sees\ M{:}Ts{\rightarrow}T = m\ in\ D \Longrightarrow P \vdash C\ has\ M$
⟨*proof*⟩

Some simple lemmas about the type testing functions of the defensive JVM:

lemma *typeof-NoneD* [*simp*,*dest*]: $typeof\ v = Some\ x \Longrightarrow \neg is\text{-}Addr\ v$
⟨*proof*⟩

lemma *is-Ref-def2*:
$is\text{-}Ref\ v = (v = Null \vee (\exists\, a.\ v = Addr\ a))$
⟨*proof*⟩

lemma [*iff*]: *is-Ref Null* ⟨*proof*⟩

lemma *is-RefI* [*intro*, *simp*]: $P,h \vdash v :\leq T \Longrightarrow is\text{-}refT\ T \Longrightarrow is\text{-}Ref\ v$⟨*proof*⟩
lemma *is-IntgI* [*intro*, *simp*]: $P,h \vdash v :\leq Integer \Longrightarrow is\text{-}Intg\ v$⟨*proof*⟩
lemma *is-BoolI* [*intro*, *simp*]: $P,h \vdash v :\leq Boolean \Longrightarrow is\text{-}Bool\ v$⟨*proof*⟩
declare *defs1* [*simp del*]

lemma *wt-jvm-prog-states*:
$[\![\ wf\text{-}jvm\text{-}prog_{\Phi}\ P;\ P \vdash C\ sees\ M{:}\ Ts{\rightarrow}T = (mxs,\ mxl,\ ins,\ et)\ in\ C;$
$\Phi\ C\ M\ !\ pc = \tau;\ pc < size\ ins\ ]\!]$
$\Longrightarrow OK\ \tau \in states\ P\ mxs\ (1{+}size\ Ts{+}mxl)$⟨*proof*⟩

The main theorem: welltyped programs do not produce type errors if they are started in a conformant state.

theorem *no-type-error*:
fixes σ :: *jvm-state*
assumes *welltyped*: $wf\text{-}jvm\text{-}prog_{\Phi}\ P$ **and** *conforms*: $P,\Phi \vdash \sigma\ \surd$
shows $exec\text{-}d\ P\ \sigma \neq TypeError$⟨*proof*⟩

The theorem above tells us that, in welltyped programs, the defensive machine reaches the same result as the aggressive one (after arbitrarily many steps).

theorem *welltyped-aggressive-imp-defensive*:
$wf\text{-}jvm\text{-}prog_{\Phi}\ P \Longrightarrow P,\Phi \vdash \sigma\ \surd \Longrightarrow P \vdash \sigma\ {-jvm\rightarrow}\ \sigma'$
$\Longrightarrow P \vdash (Normal\ \sigma)\ {-jvmd\rightarrow}\ (Normal\ \sigma')$⟨*proof*⟩

As corollary we get that the aggressive and the defensive machine are equivalent for well-typed programs (if started in a conformant state or in the canonical start state)

corollary *welltyped-commutes*:
fixes σ :: *jvm-state*
assumes *wf*: $wf\text{-}jvm\text{-}prog_{\Phi}\ P$ **and** *conforms*: $P,\Phi \vdash \sigma\ \surd$
shows $P \vdash (Normal\ \sigma)\ {-jvmd\rightarrow}\ (Normal\ \sigma') = P \vdash \sigma\ {-jvm\rightarrow}\ \sigma'$
⟨*proof*⟩

corollary *welltyped-initial-commutes*:
assumes *wf*: *wf-jvm-prog P*
assumes *meth*: $P \vdash C\ sees\ M{:}[]{\rightarrow}T = b\ in\ C$

defines *start*: $\sigma \equiv$ *start-state P C M*
shows $P \vdash (Normal\ \sigma)\ {-jvmd\rightarrow}\ (Normal\ \sigma') = P \vdash \sigma\ {-jvm\rightarrow}\ \sigma'$
$\langle proof \rangle$

lemma *not-TypeError-eq* [*iff*]:
$x \neq TypeError = (\exists\, t.\ x = Normal\ t)$
$\langle proof \rangle$

locale *cnf* =
fixes P **and** Φ **and** σ
assumes *wf*: *wf-jvm-prog*$_\Phi$ P
assumes *cnf*: *correct-state* $P\ \Phi\ \sigma$

theorem (**in** *cnf*) *no-type-errors*:
$P \vdash (Normal\ \sigma)\ {-jvmd\rightarrow}\ \sigma' \Longrightarrow \sigma' \neq TypeError$
$\langle proof \rangle$

locale *start* =
fixes P **and** C **and** M **and** σ **and** T **and** b
assumes *wf*: *wf-jvm-prog P*
assumes *sees*: $P \vdash C$ *sees* M:[]$\rightarrow T = b$ *in* C
defines $\sigma \equiv Normal\ (start\text{-}state\ P\ C\ M)$

corollary (**in** *start*) *bv-no-type-error*:
shows $P \vdash \sigma\ {-jvmd\rightarrow}\ \sigma' \Longrightarrow \sigma' \neq TypeError$
$\langle proof \rangle$

end

4.24 Example Welltypings

theory *BVExample*
imports *../JVM/JVMListExample BVSpecTypeSafe BVExec*
  ~~/src/HOL/Library/Efficient-Nat
begin

This theory shows type correctness of the example program in section 3.7 (p. 86) by explicitly providing a welltyping. It also shows that the start state of the program conforms to the welltyping; hence type safe execution is guaranteed.

4.24.1 Setup

lemma *distinct-classes'*:
  list-name ≠ *test-name*
  list-name ≠ *Object*
  list-name ≠ *ClassCast*
  list-name ≠ *OutOfMemory*
  list-name ≠ *NullPointer*
  test-name ≠ *Object*
  test-name ≠ *OutOfMemory*
  test-name ≠ *ClassCast*
  test-name ≠ *NullPointer*
  ClassCast ≠ *NullPointer*
  ClassCast ≠ *Object*
  NullPointer ≠ *Object*
  OutOfMemory ≠ *ClassCast*
  OutOfMemory ≠ *NullPointer*
  OutOfMemory ≠ *Object*
  ⟨*proof*⟩

lemmas *distinct-classes* = *distinct-classes' distinct-classes'* [*symmetric*]

lemma *distinct-fields*:
  val-name ≠ *next-name*
  next-name ≠ *val-name*
  ⟨*proof*⟩

Abbreviations for definitions we will have to use often in the proofs below:

lemmas *system-defs* = *SystemClasses-def ObjectC-def NullPointerC-def*
                    OutOfMemoryC-def ClassCastC-def
lemmas *class-defs* = *list-class-def test-class-def*

These auxiliary proofs are for efficiency: class lookup, subclass relation, method and field lookup are computed only once:

lemma *class-Object* [*simp*]:
  class E Object = *Some* (*undefined*, [],[])
  ⟨*proof*⟩

lemma *class-NullPointer* [*simp*]:
  class E NullPointer = *Some* (*Object*, [], [])
  ⟨*proof*⟩

lemma *class-OutOfMemory* [*simp*]:

class E OutOfMemory = *Some* (*Object*, [], [])
⟨*proof*⟩

lemma *class-ClassCast* [*simp*]:
class E ClassCast = *Some* (*Object*, [], [])
⟨*proof*⟩

lemma *class-list* [*simp*]:
class E list-name = *Some list-class*
⟨*proof*⟩

lemma *class-test* [*simp*]:
class E test-name = *Some test-class*
⟨*proof*⟩

lemma *E-classes* [*simp*]:
{*C*. *is-class E C*} = {*list-name*, *test-name*, *NullPointer*,
ClassCast, *OutOfMemory*, *Object*}
⟨*proof*⟩

The subclass releation spelled out:

lemma *subcls1*:
subcls1 E = {(*list-name*,*Object*), (*test-name*,*Object*), (*NullPointer*, *Object*),
(*ClassCast*, *Object*), (*OutOfMemory*, *Object*)}⟨*proof*⟩

The subclass relation is acyclic; hence its converse is well founded:

lemma *notin-rtrancl*:
$(a,b) \in r^* \Longrightarrow a \neq b \Longrightarrow (\bigwedge y.\ (a,y) \notin r) \Longrightarrow False$
⟨*proof*⟩

lemma *acyclic-subcls1-E*: *acyclic* (*subcls1 E*)⟨*proof*⟩
lemma *wf-subcls1-E*: *wf* $((subcls1\ E)^{-1})$⟨*proof*⟩

Method and field lookup:

lemma *method-append* [*simp*]:
method E list-name append-name =
(*list-name*, [*Class list-name*], *Void*, *3*, *0*, *append-ins*, [(*1*, *2*, *NullPointer*, *7*, *0*)])⟨*proof*⟩
lemma *method-makelist* [*simp*]:
method E test-name makelist-name =
(*test-name*, [], *Void*, *3*, *2*, *make-list-ins*, [])⟨*proof*⟩
lemma *field-val* [*simp*]:
field E list-name val-name = (*list-name*, *Integer*)⟨*proof*⟩
lemma *field-next* [*simp*]:
field E list-name next-name = (*list-name*, *Class list-name*)⟨*proof*⟩
lemma [*simp*]: *fields E Object* = []
⟨*proof*⟩

lemma [*simp*]: *fields E NullPointer* = []
⟨*proof*⟩

lemma [*simp*]: *fields E ClassCast* = []
⟨*proof*⟩

lemma [*simp*]: *fields E OutOfMemory* = []

⟨*proof*⟩

lemma [*simp*]: *fields E test-name* = []⟨*proof*⟩
lemmas [*simp*] = *is-class-def*

4.24.2 Program structure

The program is structurally wellformed:

lemma *wf-struct*:
wf-prog (λ*G C mb. True*) *E* (**is** *wf-prog ?mb E*)⟨*proof*⟩

4.24.3 Welltypings

We show welltypings of the methods *append-name* in class *list-name*, and *makelist-name* in class *test-name*:

lemmas *eff-simps* [*simp*] = *eff-def norm-eff-def xcpt-eff-def*

definition *phi-append* :: ty_m (φ_a)
where
φ_a ≡ *map* (λ(*x*,*y*). *Some* (*x*, *map OK y*)) [
([], [*Class list-name*, *Class list-name*]),
([*Class list-name*], [*Class list-name*, *Class list-name*]),
([*Class list-name*], [*Class list-name*, *Class list-name*]),
([*Class list-name*, *Class list-name*], [*Class list-name*, *Class list-name*]),
([*Class list-name*, *Class list-name*], [*Class list-name*, *Class list-name*]),
([*NT*, *Class list-name*, *Class list-name*], [*Class list-name*, *Class list-name*]),
([*Boolean*, *Class list-name*], [*Class list-name*, *Class list-name*]),

([*Class Object*], [*Class list-name*, *Class list-name*]),
([], [*Class list-name*, *Class list-name*]),
([*Class list-name*], [*Class list-name*, *Class list-name*]),
([*Class list-name*, *Class list-name*], [*Class list-name*, *Class list-name*]),
([], [*Class list-name*, *Class list-name*]),
([*Void*], [*Class list-name*, *Class list-name*]),

([*Class list-name*], [*Class list-name*, *Class list-name*]),
([*Class list-name*, *Class list-name*], [*Class list-name*, *Class list-name*]),
([*Void*], [*Class list-name*, *Class list-name*])]

The next definition and three proof rules implement an algorithm to enumarate natural numbers. The command *apply* (*elim pc-end pc-next pc-0* transforms a goal of the form

$pc < n \Longrightarrow P\ pc$

into a series of goals

$P\ (0::'a)$

$P\ (Suc\ 0)$

$\ldots$

$P\ n$

definition *intervall* :: $nat \Rightarrow nat \Rightarrow nat \Rightarrow bool$ (- $\in$ [-, -'))
where
$x \in [a,\ b) \equiv a \leq x \wedge x < b$

lemma *pc-0*: $x < n \Longrightarrow (x \in [0,\ n) \Longrightarrow P\ x) \Longrightarrow P\ x$
⟨*proof*⟩

lemma *pc-next*: $x \in [n0,\ n) \Longrightarrow P\ n0 \Longrightarrow (x \in [Suc\ n0,\ n) \Longrightarrow P\ x) \Longrightarrow P\ x$⟨*proof*⟩
lemma *pc-end*: $x \in [n,n) \Longrightarrow P\ x$
⟨*proof*⟩

lemma *types-append* [*simp*]: *check-types E 3* (*Suc* (*Suc 0*)) (*map OK* φ_a)⟨*proof*⟩
lemma *wt-append* [*simp*]:
wt-method E list-name [*Class list-name*] *Void 3 0 append-ins*
[(*Suc 0*, *2*, *NullPointer*, *7*, *0*)] φ_a⟨*proof*⟩

Some abbreviations for readability

abbreviation *Clist* == *Class list-name*
abbreviation *Ctest* == *Class test-name*

definition *phi-makelist* :: ty_m (φ_m)
where
φ_m ≡ *map* (λ(*x*,*y*). *Some* (*x*, *y*)) [
([], [*OK Ctest*, *Err* , *Err*]),
([*Clist*], [*OK Ctest*, *Err* , *Err*]),
([], [*OK Clist*, *Err* , *Err*]),
([*Clist*], [*OK Clist*, *Err* , *Err*]),
([*Integer*, *Clist*], [*OK Clist*, *Err* , *Err*]),

([], [*OK Clist*, *Err* , *Err*]),
([*Clist*], [*OK Clist*, *Err* , *Err*]),
([], [*OK Clist*, *OK Clist*, *Err*]),
([*Clist*], [*OK Clist*, *OK Clist*, *Err*]),
([*Integer*, *Clist*], [*OK Clist*, *OK Clist*, *Err*]),

([], [*OK Clist*, *OK Clist*, *Err*]),
([*Clist*], [*OK Clist*, *OK Clist*, *Err*]),
([], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Clist*], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Integer*, *Clist*], [*OK Clist*, *OK Clist*, *OK Clist*]),

([], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Clist*], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Clist*, *Clist*], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Void*], [*OK Clist*, *OK Clist*, *OK Clist*]),
([], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Clist*], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Clist*, *Clist*], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Void*], [*OK Clist*, *OK Clist*, *OK Clist*])]

lemma *types-makelist* [*simp*]: *check-types E 3* (*Suc* (*Suc* (*Suc 0*))) (*map OK* φ_m)⟨*proof*⟩
lemma *wt-makelist* [*simp*]:
wt-method E test-name [] *Void 3 2 make-list-ins* [] φ_m⟨*proof*⟩

lemma *wf-md′E*:
$\llbracket$ *wf-prog wf-md P*;
$\bigwedge C\ S\ fs\ ms\ m.\llbracket(C,S,fs,ms) \in set\ P;\ m \in set\ ms\rrbracket \Longrightarrow wf\text{-}md'\ P\ C\ m$ $\rrbracket$
$\Longrightarrow$ *wf-prog wf-md′ P*⟨*proof*⟩

The whole program is welltyped:

definition *Phi* :: ty_P (Φ)
where
$\Phi\ C\ mn \equiv if\ C = test\text{-}name \wedge mn = makelist\text{-}name\ then\ \varphi_m\ else$
$if\ C = list\text{-}name \wedge mn = append\text{-}name\ then\ \varphi_a\ else\ []$

lemma *wf-prog*:
$wf\text{-}jvm\text{-}prog_\Phi\ E$⟨*proof*⟩

4.24.4 Conformance

Execution of the program will be typesafe, because its start state conforms to the welltyping:

lemma $E,\Phi \vdash$ *start-state E test-name makelist-name* $\surd$⟨*proof*⟩

4.24.5 Example for code generation: inferring method types

definition *test-kil* :: *jvm-prog* ⇒ *cname* ⇒ *ty list* ⇒ *ty* ⇒ *nat* ⇒ *nat* ⇒
ex-table ⇒ *instr list* ⇒ ty_i' *err list*
where
test-kil G C pTs rT mxs mxl et instr ≡
(*let first* = *Some* ([],(*OK* (*Class C*))#(*map OK pTs*)@(*replicate mxl Err*));
start = *OK first*#(*replicate* (*size instr* − *1*) (*OK None*))
in kiljvm G mxs (*1*+*size pTs*+*mxl*) *rT instr et start*)

lemma [*code*]:
unstables r step ss =
fold (λ*p A*. *if* ¬*stable r step ss p then insert p A else A*) [*0*..<*size ss*] {}
⟨*proof*⟩

definition *some-elem* :: ′*a set* ⇒ ′*a* **where** [*code del*]:
some-elem = (%*S*. *SOME x*. *x* : *S*)
code-const *some-elem*
(*SML* (*case/ - of/ Set/ xs/ =>/ hd/ xs*))

This code setup is just a demonstration and *not* sound!

notepad begin
⟨*proof*⟩
end

lemma [*code*]:
iter f step ss w = *while* (λ(*ss*, *w*). ¬ *Set.is-empty w*)
(λ(*ss*, *w*).
let p = *some-elem w in propa f* (*step p* (*ss* ! *p*)) *ss* (*w* − {*p*}))
(*ss*, *w*)
⟨*proof*⟩

lemma *JVM-sup-unfold* [*code*]:
JVM-SemiType.sup S m n = *lift2* (*Opt.sup*

$(Product.sup\ (Listn.sup\ (SemiType.sup\ S))$
$(\lambda x\ y.\ OK\ (map2\ (lift2\ (SemiType.sup\ S))\ x\ y))))$
⟨*proof*⟩

lemmas [*code*] = *SemiType.sup-def* [*unfolded exec-lub-def*] *JVM-le-unfold*

lemmas [*code*] = *lesub-def plussub-def*

lemma [*code*]:
is-refT T = (*case* T *of* *NT* ⇒ *True* | *Class* C ⇒ *True* | - ⇒ *False*)
⟨*proof*⟩

declare app_i*.simps* [*code*]

lemma [*code*]:
app_i (*Getfield* F C, P, *pc*, *mxs*, T_r, (T#*ST*, *LT*)) =
Predicate.holds (*Predicate.bind* (*sees-field-i-i-i-o-i* P C F C) (λT_f. *if* $P \vdash T \leq$ *Class* C *then* *Predicate.single* () *else bot*))
⟨*proof*⟩

lemma [*code*]:
app_i (*Putfield* F C, P, *pc*, *mxs*, T_r, (T_1#T_2#*ST*, *LT*)) =
Predicate.holds (*Predicate.bind* (*sees-field-i-i-i-o-i* P C F C) (λT_f. *if* $P \vdash T_2 \leq$ (*Class* C) ∧ $P \vdash T_1 \leq T_f$ *then* *Predicate.single* () *else bot*))
⟨*proof*⟩

lemma [*code*]:
app_i (*Invoke* M n, P, *pc*, *mxs*, T_r, (*ST*,*LT*)) =
(n < *length ST* ∧
(*ST*!n ≠ *NT* ⟶
(*case ST*!n *of*
Class C ⇒ *Predicate.holds* (*Predicate.bind* (*Method-i-i-i-o-o-o-o* P C M) (λ(*Ts*, T, m, D). *if* $P \vdash$ *rev* (*take* n *ST*) [≤] *Ts* *then* *Predicate.single* () *else bot*))
| - ⇒ *False*)))
⟨*proof*⟩

lemmas [*code*] =
SemiType.sup-def [*unfolded exec-lub-def*]
widen.equation
is-relevant-class.simps

definition *test1* **where**
test1 = *test-kil E list-name* [*Class list-name*] *Void 3 0*
[(*Suc 0*, *2*, *NullPointer*, *7*, *0*)] *append-ins*
definition *test2* **where**
test2 = *test-kil E test-name* [] *Void 3 2* [] *make-list-ins*
definition *test3* **where** *test3* = φ_a
definition *test4* **where** *test4* = φ_m

⟨*ML*⟩

end

Chapter 5

Compilation

5.1 An Intermediate Language

theory *J1* **imports** *../J/BigStep* **begin**

type-synonym $expr_1 = nat\ exp$
type-synonym $J_1\text{-}prog = expr_1\ prog$
type-synonym $state_1 = heap \times (val\ list)$

primrec
$max\text{-}vars :: {}'a\ exp \Rightarrow nat$
and $max\text{-}varss :: {}'a\ exp\ list \Rightarrow nat$
where
$max\text{-}vars(new\ C) = 0$
$\mid max\text{-}vars(Cast\ C\ e) = max\text{-}vars\ e$
$\mid max\text{-}vars(Val\ v) = 0$
$\mid max\text{-}vars(e_1 \ll bop \gg e_2) = max\ (max\text{-}vars\ e_1)\ (max\text{-}vars\ e_2)$
$\mid max\text{-}vars(Var\ V) = 0$
$\mid max\text{-}vars(V{:=}e) = max\text{-}vars\ e$
$\mid max\text{-}vars(e{\bullet}F\{D\}) = max\text{-}vars\ e$
$\mid max\text{-}vars(FAss\ e_1\ F\ D\ e_2) = max\ (max\text{-}vars\ e_1)\ (max\text{-}vars\ e_2)$
$\mid max\text{-}vars(e{\bullet}M(es)) = max\ (max\text{-}vars\ e)\ (max\text{-}varss\ es)$
$\mid max\text{-}vars(\{V{:}T;\ e\}) = max\text{-}vars\ e + 1$
$\mid max\text{-}vars(e_1;;e_2) = max\ (max\text{-}vars\ e_1)\ (max\text{-}vars\ e_2)$
$\mid max\text{-}vars(if\ (e)\ e_1\ else\ e_2) =$
$max\ (max\text{-}vars\ e)\ (max\ (max\text{-}vars\ e_1)\ (max\text{-}vars\ e_2))$
$\mid max\text{-}vars(while\ (b)\ e) = max\ (max\text{-}vars\ b)\ (max\text{-}vars\ e)$
$\mid max\text{-}vars(throw\ e) = max\text{-}vars\ e$
$\mid max\text{-}vars(try\ e_1\ catch(C\ V)\ e_2) = max\ (max\text{-}vars\ e_1)\ (max\text{-}vars\ e_2 + 1)$

$\mid max\text{-}varss\ [] = 0$
$\mid max\text{-}varss\ (e\#es) = max\ (max\text{-}vars\ e)\ (max\text{-}varss\ es)$

inductive
$eval_1 :: J_1\text{-}prog \Rightarrow expr_1 \Rightarrow state_1 \Rightarrow expr_1 \Rightarrow state_1 \Rightarrow bool$
$(\text{-} \vdash_1 ((1\langle\text{-},/\text{-}\rangle) \Rightarrow/\ (1\langle\text{-},/\text{-}\rangle))\ [51,0,0,0,0]\ 81)$
and $evals_1 :: J_1\text{-}prog \Rightarrow expr_1\ list \Rightarrow state_1 \Rightarrow expr_1\ list \Rightarrow state_1 \Rightarrow bool$
$(\text{-} \vdash_1 ((1\langle\text{-},/\text{-}\rangle)\ [\Rightarrow]/\ (1\langle\text{-},/\text{-}\rangle))\ [51,0,0,0,0]\ 81)$
for $P :: J_1\text{-}prog$
where

New_1:
$[\![\ new\text{-}Addr\ h = Some\ a;\ P \vdash C\ has\text{-}fields\ FDTs;\ h' = h(a \mapsto (C,init\text{-}fields\ FDTs))\ ]\!]$
$\Longrightarrow P \vdash_1 \langle new\ C,(h,l)\rangle \Rightarrow \langle addr\ a,(h',l)\rangle$
$\mid NewFail_1$:
$new\text{-}Addr\ h = None \Longrightarrow$
$P \vdash_1 \langle new\ C,\ (h,l)\rangle \Rightarrow \langle THROW\ OutOfMemory,(h,l)\rangle$

$\mid Cast_1$:
$[\![\ P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle addr\ a,(h,l)\rangle;\ h\ a = Some(D,fs);\ P \vdash D \preceq^* C\ ]\!]$
$\Longrightarrow P \vdash_1 \langle Cast\ C\ e,s_0\rangle \Rightarrow \langle addr\ a,(h,l)\rangle$
$\mid CastNull_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle null,s_1\rangle \Longrightarrow$
$P \vdash_1 \langle Cast\ C\ e,s_0\rangle \Rightarrow \langle null,s_1\rangle$
$\mid CastFail_1$:

$[\![\ P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle addr\ a,(h,l)\rangle;\ h\ a = Some(D,fs);\ \neg\ P \vdash D \preceq^* C\ ]\!]$
$\Longrightarrow P \vdash_1 \langle Cast\ C\ e,s_0\rangle \Rightarrow \langle THROW\ ClassCast,(h,l)\rangle$
| $CastThrow_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash_1 \langle Cast\ C\ e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$

| Val_1:
$P \vdash_1 \langle Val\ v,s\rangle \Rightarrow \langle Val\ v,s\rangle$

| $BinOp_1$:
$[\![\ P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle Val\ v_1,s_1\rangle;\ P \vdash_1 \langle e_2,s_1\rangle \Rightarrow \langle Val\ v_2,s_2\rangle;\ binop(bop,v_1,v_2) = Some\ v\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e_1 \ll bop \gg e_2,s_0\rangle \Rightarrow \langle Val\ v,s_2\rangle$
| $BinOpThrow_{11}$:
$P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle throw\ e,s_1\rangle \Longrightarrow$
$P \vdash_1 \langle e_1 \ll bop \gg e_2,\ s_0\rangle \Rightarrow \langle throw\ e,s_1\rangle$
| $BinOpThrow_{21}$:
$[\![\ P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle Val\ v_1,s_1\rangle;\ P \vdash_1 \langle e_2,s_1\rangle \Rightarrow \langle throw\ e,s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e_1 \ll bop \gg e_2,s_0\rangle \Rightarrow \langle throw\ e,s_2\rangle$

| Var_1:
$[\![\ ls!i = v;\ i < size\ ls\ ]\!] \Longrightarrow$
$P \vdash_1 \langle Var\ i,(h,ls)\rangle \Rightarrow \langle Val\ v,(h,ls)\rangle$

| $LAss_1$:
$[\![\ P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle Val\ v,(h,ls)\rangle;\ i < size\ ls;\ ls' = ls[i := v]\ ]\!]$
$\Longrightarrow P \vdash_1 \langle i{:=}\ e,s_0\rangle \Rightarrow \langle unit,(h,ls')\rangle$
| $LAssThrow_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash_1 \langle i{:=}\ e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$

| $FAcc_1$:
$[\![\ P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle addr\ a,(h,ls)\rangle;\ h\ a = Some(C,fs);\ fs(F,D) = Some\ v\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e{\cdot}F\{D\},s_0\rangle \Rightarrow \langle Val\ v,(h,ls)\rangle$
| $FAccNull_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle null,s_1\rangle \Longrightarrow$
$P \vdash_1 \langle e{\cdot}F\{D\},s_0\rangle \Rightarrow \langle THROW\ NullPointer,s_1\rangle$
| $FAccThrow_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash_1 \langle e{\cdot}F\{D\},s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$

| $FAss_1$:
$[\![\ P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle addr\ a,s_1\rangle;\ P \vdash_1 \langle e_2,s_1\rangle \Rightarrow \langle Val\ v,(h_2,l_2)\rangle;$
$h_2\ a = Some(C,fs);\ fs' = fs((F,D)\mapsto v);\ h_2' = h_2(a\mapsto(C,fs'))\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e_1{\cdot}F\{D\}{:=}\ e_2,s_0\rangle \Rightarrow \langle unit,(h_2',l_2)\rangle$
| $FAssNull_1$:
$[\![\ P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle null,s_1\rangle;\ \ P \vdash_1 \langle e_2,s_1\rangle \Rightarrow \langle Val\ v,s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e_1{\cdot}F\{D\}{:=}\ e_2,s_0\rangle \Rightarrow \langle THROW\ NullPointer,s_2\rangle$
| $FAssThrow_{11}$:
$P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash_1 \langle e_1{\cdot}F\{D\}{:=}\ e_2,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$
| $FAssThrow_{21}$:
$[\![\ P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle Val\ v,s_1\rangle;\ P \vdash_1 \langle e_2,s_1\rangle \Rightarrow \langle throw\ e',s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e_1{\cdot}F\{D\}{:=}\ e_2,s_0\rangle \Rightarrow \langle throw\ e',s_2\rangle$

$|$ *CallObjThrow*$_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash_1 \langle e\cdot M(es),s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$
$|$ *CallNull*$_1$:
$[\![\ P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle null,s_1\rangle;\ P \vdash_1 \langle es,s_1\rangle\ [\Rightarrow]\ \langle map\ Val\ vs,s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e\cdot M(es),s_0\rangle \Rightarrow \langle THROW\ NullPointer,s_2\rangle$
$|$ *Call*$_1$:
$[\![\ P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle addr\ a,s_1\rangle;\ P \vdash_1 \langle es,s_1\rangle\ [\Rightarrow]\ \langle map\ Val\ vs,(h_2,ls_2)\rangle;$
$h_2\ a = Some(C,fs);\ P \vdash C\ sees\ M{:}Ts{\rightarrow}T = body\ in\ D;$
$size\ vs = size\ Ts;\ ls_2{}' = (Addr\ a)\ \#\ vs\ @\ replicate\ (max\text{-}vars\ body)\ undefined;$
$P \vdash_1 \langle body,(h_2,ls_2{}')\rangle \Rightarrow \langle e',(h_3,ls_3)\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e\cdot M(es),s_0\rangle \Rightarrow \langle e',(h_3,ls_2)\rangle$
$|$ *CallParamsThrow*$_1$:
$[\![\ P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle Val\ v,s_1\rangle;\ P \vdash_1 \langle es,s_1\rangle\ [\Rightarrow]\ \langle es',s_2\rangle;$
$es' = map\ Val\ vs\ @\ throw\ ex\ \#\ es_2\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e\cdot M(es),s_0\rangle \Rightarrow \langle throw\ ex,s_2\rangle$

$|$ *Block*$_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle e',s_1\rangle \Longrightarrow P \vdash_1 \langle Block\ i\ T\ e,s_0\rangle \Rightarrow \langle e',s_1\rangle$

$|$ *Seq*$_1$:
$[\![\ P \vdash_1 \langle e_0,s_0\rangle \Rightarrow \langle Val\ v,s_1\rangle;\ P \vdash_1 \langle e_1,s_1\rangle \Rightarrow \langle e_2,s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e_0;;e_1,s_0\rangle \Rightarrow \langle e_2,s_2\rangle$
$|$ *SeqThrow*$_1$:
$P \vdash_1 \langle e_0,s_0\rangle \Rightarrow \langle throw\ e,s_1\rangle \Longrightarrow$
$P \vdash_1 \langle e_0;;e_1,s_0\rangle \Rightarrow \langle throw\ e,s_1\rangle$

$|$ *CondT*$_1$:
$[\![\ P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle true,s_1\rangle;\ P \vdash_1 \langle e_1,s_1\rangle \Rightarrow \langle e',s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle if\ (e)\ e_1\ else\ e_2,s_0\rangle \Rightarrow \langle e',s_2\rangle$
$|$ *CondF*$_1$:
$[\![\ P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle false,s_1\rangle;\ P \vdash_1 \langle e_2,s_1\rangle \Rightarrow \langle e',s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle if\ (e)\ e_1\ else\ e_2,s_0\rangle \Rightarrow \langle e',s_2\rangle$
$|$ *CondThrow*$_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash_1 \langle if\ (e)\ e_1\ else\ e_2,\ s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$

$|$ *WhileF*$_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle false,s_1\rangle \Longrightarrow$
$P \vdash_1 \langle while\ (e)\ c,s_0\rangle \Rightarrow \langle unit,s_1\rangle$
$|$ *WhileT*$_1$:
$[\![\ P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle true,s_1\rangle;\ P \vdash_1 \langle c,s_1\rangle \Rightarrow \langle Val\ v_1,s_2\rangle;$
$P \vdash_1 \langle while\ (e)\ c,s_2\rangle \Rightarrow \langle e_3,s_3\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle while\ (e)\ c,s_0\rangle \Rightarrow \langle e_3,s_3\rangle$
$|$ *WhileCondThrow*$_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash_1 \langle while\ (e)\ c,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$
$|$ *WhileBodyThrow*$_1$:
$[\![\ P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle true,s_1\rangle;\ P \vdash_1 \langle c,s_1\rangle \Rightarrow \langle throw\ e',s_2\rangle]\!]$
$\Longrightarrow P \vdash_1 \langle while\ (e)\ c,s_0\rangle \Rightarrow \langle throw\ e',s_2\rangle$

$|$ *Throw*$_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle addr\ a,s_1\rangle \Longrightarrow$
$P \vdash_1 \langle throw\ e,s_0\rangle \Rightarrow \langle Throw\ a,s_1\rangle$

| *ThrowNull*$_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle null,s_1\rangle \Longrightarrow$
$P \vdash_1 \langle throw\ e,s_0\rangle \Rightarrow \langle THROW\ NullPointer,s_1\rangle$
| *ThrowThrow*$_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash_1 \langle throw\ e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$

| *Try*$_1$:
$P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle Val\ v_1,s_1\rangle \Longrightarrow$
$P \vdash_1 \langle try\ e_1\ catch(C\ i)\ e_2,s_0\rangle \Rightarrow \langle Val\ v_1,s_1\rangle$
| *TryCatch*$_1$:
$[\![\ P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle Throw\ a,(h_1,ls_1)\rangle;$
$h_1\ a = Some(D,fs);\ P \vdash D \preceq^* C;\ i < length\ ls_1;$
$P \vdash_1 \langle e_2,(h_1,ls_1[i:=Addr\ a])\rangle \Rightarrow \langle e_2',(h_2,ls_2)\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle try\ e_1\ catch(C\ i)\ e_2,s_0\rangle \Rightarrow \langle e_2',(h_2,ls_2)\rangle$
| *TryThrow*$_1$:
$[\![\ P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle Throw\ a,(h_1,ls_1)\rangle;\ h_1\ a = Some(D,fs);\ \neg\ P \vdash D \preceq^* C\ ]\!]$
$\Longrightarrow P \vdash_1 \langle try\ e_1\ catch(C\ i)\ e_2,s_0\rangle \Rightarrow \langle Throw\ a,(h_1,ls_1)\rangle$

| *Nil*$_1$:
$P \vdash_1 \langle [],s\rangle\ [\Rightarrow]\ \langle [],s\rangle$

| *Cons*$_1$:
$[\![\ P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle Val\ v,s_1\rangle;\ P \vdash_1 \langle es,s_1\rangle\ [\Rightarrow]\ \langle es',s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e\#es,s_0\rangle\ [\Rightarrow]\ \langle Val\ v\ \#\ es',s_2\rangle$
| *ConsThrow*$_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash_1 \langle e\#es,s_0\rangle\ [\Rightarrow]\ \langle throw\ e'\ \#\ es,\ s_1\rangle$

lemma *eval*$_1$*-preserves-len*:
$P \vdash_1 \langle e_0,(h_0,ls_0)\rangle \Rightarrow \langle e_1,(h_1,ls_1)\rangle \Longrightarrow length\ ls_0 = length\ ls_1$
and *evals*$_1$*-preserves-len*:
$P \vdash_1 \langle es_0,(h_0,ls_0)\rangle\ [\Rightarrow]\ \langle es_1,(h_1,ls_1)\rangle \Longrightarrow length\ ls_0 = length\ ls_1 \langle proof\rangle$

lemma *evals*$_1$*-preserves-elen*:
$\bigwedge es'\ s\ s'.\ P \vdash_1 \langle es,s\rangle\ [\Rightarrow]\ \langle es',s'\rangle \Longrightarrow length\ es = length\ es' \langle proof\rangle$

lemma *eval*$_1$*-final*: $P \vdash_1 \langle e,s\rangle \Rightarrow \langle e',s'\rangle \Longrightarrow final\ e'$
and *evals*$_1$*-final*: $P \vdash_1 \langle es,s\rangle\ [\Rightarrow]\ \langle es',s'\rangle \Longrightarrow finals\ es' \langle proof\rangle$

end

5.2 Well-Formedness of Intermediate Language

theory *J1WellForm*
imports *../J/JWellForm J1*
begin

5.2.1 Well-Typedness

type-synonym
$env_1 = ty\ list$ — type environment indexed by variable number

inductive
$WT_1 :: [J_1\text{-}prog, env_1, expr_1, ty] \Rightarrow bool$
$((\text{-},\text{-} \vdash_1/ \text{-} :: \text{-})\ [51,51,51]50)$
and $WTs_1 :: [J_1\text{-}prog, env_1, expr_1\ list, ty\ list] \Rightarrow bool$
$((\text{-},\text{-} \vdash_1/ \text{-} [::] \text{-})\ [51,51,51]50)$
for $P :: J_1\text{-}prog$
where

$WTNew_1$:
$is\text{-}class\ P\ C \Longrightarrow$
$P,E \vdash_1 new\ C :: Class\ C$

| $WTCast_1$:
$[\![P,E \vdash_1 e :: Class\ D;\ is\text{-}class\ P\ C;\ P \vdash C \preceq^* D \vee P \vdash D \preceq^* C]\!]$
$\Longrightarrow P,E \vdash_1 Cast\ C\ e :: Class\ C$

| $WTVal_1$:
$typeof\ v = Some\ T \Longrightarrow$
$P,E \vdash_1 Val\ v :: T$

| $WTVar_1$:
$[\![E!i = T;\ i < size\ E]\!]$
$\Longrightarrow P,E \vdash_1 Var\ i :: T$

| $WTBinOp_1$:
$[\![P,E \vdash_1 e_1 :: T_1;\ P,E \vdash_1 e_2 :: T_2;$
$case\ bop\ of\ Eq \Rightarrow (P \vdash T_1 \leq T_2 \vee P \vdash T_2 \leq T_1) \wedge T = Boolean$
$\mid Add \Rightarrow T_1 = Integer \wedge T_2 = Integer \wedge T = Integer]\!]$
$\Longrightarrow P,E \vdash_1 e_1 \ll bop \gg e_2 :: T$

| $WTLAss_1$:
$[\![E!i = T;\ i < size\ E;\ P,E \vdash_1 e :: T';\ P \vdash T' \leq T]\!]$
$\Longrightarrow P,E \vdash_1 i{:=}e :: Void$

| $WTFAcc_1$:
$[\![P,E \vdash_1 e :: Class\ C;\ P \vdash C\ sees\ F{:}T\ in\ D]\!]$
$\Longrightarrow P,E \vdash_1 e{\cdot}F\{D\} :: T$

| $WTFAss_1$:
$[\![P,E \vdash_1 e_1 :: Class\ C;\ P \vdash C\ sees\ F{:}T\ in\ D;\ P,E \vdash_1 e_2 :: T';\ P \vdash T' \leq T]\!]$
$\Longrightarrow P,E \vdash_1 e_1{\cdot}F\{D\} := e_2 :: Void$

| $WTCall_1$:

$[\![P,E \vdash_1 e :: Class\ C;\ P \vdash C\ sees\ M{:}Ts' \to T = m\ in\ D;$
$P,E \vdash_1 es\ [::]\ Ts;\ \ P \vdash Ts\ [\le]\ Ts']\!]$
$\Longrightarrow P,E \vdash_1 e{\cdot}M(es) :: T$

| $WTBlock_1$:
$[\![is\text{-}type\ P\ T;\ P,E@[T] \vdash_1 e{::}T']\!]$
$\Longrightarrow\ P,E \vdash_1 \{i{:}T;\ e\} :: T'$

| $WTSeq_1$:
$[\![P,E \vdash_1 e_1{::}T_1;\ \ P,E \vdash_1 e_2{::}T_2]\!]$
$\Longrightarrow\ P,E \vdash_1 e_1;;e_2 :: T_2$

| $WTCond_1$:
$[\![P,E \vdash_1 e :: Boolean;\ \ P,E \vdash_1 e_1{::}T_1;\ \ P,E \vdash_1 e_2{::}T_2;$
$P \vdash T_1 \le T_2 \lor P \vdash T_2 \le T_1;\ \ P \vdash T_1 \le T_2 \longrightarrow T = T_2;\ P \vdash T_2 \le T_1 \longrightarrow T = T_1]\!]$
$\Longrightarrow P,E \vdash_1 if\ (e)\ e_1\ else\ e_2 :: T$

| $WTWhile_1$:
$[\![P,E \vdash_1 e :: Boolean;\ \ P,E \vdash_1 c{::}T]\!]$
$\Longrightarrow P,E \vdash_1 while\ (e)\ c :: Void$

| $WTThrow_1$:
$P,E \vdash_1 e :: Class\ C\ \Longrightarrow$
$P,E \vdash_1 throw\ e :: Void$

| $WTTry_1$:
$[\![P,E \vdash_1 e_1 :: T;\ \ P,E@[Class\ C] \vdash_1 e_2 :: T;\ is\text{-}class\ P\ C]\!]$
$\Longrightarrow P,E \vdash_1 try\ e_1\ catch(C\ i)\ e_2 :: T$

| $WTNil_1$:
$P,E \vdash_1 []\ [::]\ []$

| $WTCons_1$:
$[\![P,E \vdash_1 e :: T;\ \ P,E \vdash_1 es\ [::]\ Ts]\!]$
$\Longrightarrow\ P,E \vdash_1 e\#es\ [::]\ T\#Ts$

lemma $WTs_1\text{-}same\text{-}size$: $\bigwedge Ts.\ P,E \vdash_1 es\ [::]\ Ts \Longrightarrow size\ es = size\ Ts\langle proof\rangle$

lemma $WT_1\text{-}unique$:
$P,E \vdash_1 e :: T_1 \Longrightarrow (\bigwedge T_2.\ P,E \vdash_1 e :: T_2 \Longrightarrow T_1 = T_2)$ **and**
$P,E \vdash_1 es\ [::]\ Ts_1 \Longrightarrow (\bigwedge Ts_2.\ P,E \vdash_1 es\ [::]\ Ts_2 \Longrightarrow Ts_1 = Ts_2)\langle proof\rangle$

lemma assumes wf: $wf\text{-}prog\ p\ P$
shows $WT_1\text{-}is\text{-}type$: $P,E \vdash_1 e :: T \Longrightarrow set\ E \subseteq types\ P \Longrightarrow is\text{-}type\ P\ T$
and $P,E \vdash_1 es\ [::]\ Ts \Longrightarrow True\langle proof\rangle$

5.2.2 Well-formedness

— Indices in blocks increase by 1

primrec $\mathcal{B} :: expr_1 \Rightarrow nat \Rightarrow bool$
and $\mathcal{B}s :: expr_1\ list \Rightarrow nat \Rightarrow bool$ **where**
$\mathcal{B}\ (new\ C)\ i = True\ |$
$\mathcal{B}\ (Cast\ C\ e)\ i = \mathcal{B}\ e\ i\ |$

$\mathcal{B}$ (*Val v*) i = *True* |
$\mathcal{B}$ (e_1 «*bop*» e_2) i = ($\mathcal{B}$ e_1 i $\wedge$ $\mathcal{B}$ e_2 i) |
$\mathcal{B}$ (*Var j*) i = *True* |
$\mathcal{B}$ ($e{\cdot}F\{D\}$) i = $\mathcal{B}$ e i |
$\mathcal{B}$ (j:=e) i = $\mathcal{B}$ e i |
$\mathcal{B}$ ($e_1{\cdot}F\{D\}$:= e_2) i = ($\mathcal{B}$ e_1 i $\wedge$ $\mathcal{B}$ e_2 i) |
$\mathcal{B}$ ($e{\cdot}M(es)$) i = ($\mathcal{B}$ e i $\wedge$ $\mathcal{B}s$ es i) |
$\mathcal{B}$ ($\{j{:}T\ ;\ e\}$) i = (i = j $\wedge$ $\mathcal{B}$ e (i+*1*)) |
$\mathcal{B}$ (e_1;;e_2) i = ($\mathcal{B}$ e_1 i $\wedge$ $\mathcal{B}$ e_2 i) |
$\mathcal{B}$ (*if* (e) e_1 *else* e_2) i = ($\mathcal{B}$ e i $\wedge$ $\mathcal{B}$ e_1 i $\wedge$ $\mathcal{B}$ e_2 i) |
$\mathcal{B}$ (*throw e*) i = $\mathcal{B}$ e i |
$\mathcal{B}$ (*while* (e) c) i = ($\mathcal{B}$ e i $\wedge$ $\mathcal{B}$ c i) |
$\mathcal{B}$ (*try* e_1 *catch*(C j) e_2) i = ($\mathcal{B}$ e_1 i $\wedge$ i=j $\wedge$ $\mathcal{B}$ e_2 (i+*1*)) |

$\mathcal{B}s$ [] i = *True* |
$\mathcal{B}s$ (e#es) i = ($\mathcal{B}$ e i $\wedge$ $\mathcal{B}s$ es i)

definition *wf-J_1-mdecl* :: *J_1-prog* $\Rightarrow$ *cname* $\Rightarrow$ *$expr_1$ mdecl* $\Rightarrow$ *bool*
where
wf-J_1-mdecl P C $\equiv$ λ(*M*,*Ts*,*T*,*body*).
($\exists$ T'. *P*,*Class C*#*Ts* $\vdash_1$ *body* :: T' $\wedge$ $P \vdash T' \le T$) $\wedge$
$\mathcal{D}$ *body* $\lfloor$\{..*size Ts*\}$\rfloor$ $\wedge$ $\mathcal{B}$ *body* (*size Ts* + *1*)

lemma *wf-J_1-mdecl*[*simp*]:
wf-J_1-mdecl P C (*M*,*Ts*,*T*,*body*) $\equiv$
(($\exists$ T'. *P*,*Class C*#*Ts* $\vdash_1$ *body* :: T' $\wedge$ $P \vdash T' \le T$) $\wedge$
$\mathcal{D}$ *body* $\lfloor$\{..*size Ts*\}$\rfloor$ $\wedge$ $\mathcal{B}$ *body* (*size Ts* + *1*))$\langle$*proof*$\rangle$
abbreviation *wf-J_1-prog* == *wf-prog wf-J_1-mdecl*

end

5.3 Program Compilation

theory *PCompiler*
imports ../*Common*/*WellForm*
begin

definition $compM :: ('a \Rightarrow 'b) \Rightarrow 'a\ mdecl \Rightarrow 'b\ mdecl$
where
$compM\ f \equiv \lambda(M,\ Ts,\ T,\ m).\ (M,\ Ts,\ T,\ f\ m)$

definition $compC :: ('a \Rightarrow 'b) \Rightarrow 'a\ cdecl \Rightarrow 'b\ cdecl$
where
$compC\ f \equiv \lambda(C,D,Fdecls,Mdecls).\ (C,D,Fdecls,\ map\ (compM\ f)\ Mdecls)$

definition $compP :: ('a \Rightarrow 'b) \Rightarrow 'a\ prog \Rightarrow 'b\ prog$
where
$compP\ f \equiv map\ (compC\ f)$

Compilation preserves the program structure. Therfore lookup functions either commute with compilation (like method lookup) or are preserved by it (like the subclass relation).

lemma *map-of-map4*:
$map\text{-}of\ (map\ (\lambda(x,a,b,c).(x,a,b,f\ c))\ ts) =$
$Option.map\ (\lambda(a,b,c).(a,b,f\ c)) \circ (map\text{-}of\ ts)\langle proof\rangle$

lemma *class-compP*:
$class\ P\ C = Some\ (D,\ fs,\ ms)$
$\Longrightarrow class\ (compP\ f\ P)\ C = Some\ (D,\ fs,\ map\ (compM\ f)\ ms)\langle proof\rangle$

lemma *class-compPD*:
$class\ (compP\ f\ P)\ C = Some\ (D,\ fs,\ cms)$
$\Longrightarrow \exists ms.\ class\ P\ C = Some(D,fs,ms) \wedge cms = map\ (compM\ f)\ ms\langle proof\rangle$

lemma [*simp*]: $is\text{-}class\ (compP\ f\ P)\ C = is\text{-}class\ P\ C\langle proof\rangle$

lemma [*simp*]: $class\ (compP\ f\ P)\ C = Option.map\ (\lambda c.\ snd(compC\ f\ (C,c)))\ (class\ P\ C)\langle proof\rangle$

lemma *sees-methods-compP*:
$P \vdash C\ sees\text{-}methods\ Mm \Longrightarrow$
$compP\ f\ P \vdash C\ sees\text{-}methods\ (Option.map\ (\lambda((Ts,T,m),D).\ ((Ts,T,f\ m),D)) \circ Mm)\langle proof\rangle$

lemma *sees-method-compP*:
$P \vdash C\ sees\ M:\ Ts \rightarrow T = m\ in\ D \Longrightarrow$
$compP\ f\ P \vdash C\ sees\ M:\ Ts \rightarrow T = (f\ m)\ in\ D\langle proof\rangle$

lemma [*simp*]:
$P \vdash C\ sees\ M:\ Ts \rightarrow T = m\ in\ D \Longrightarrow$
$method\ (compP\ f\ P)\ C\ M = (D,Ts,T,f\ m)\langle proof\rangle$

lemma *sees-methods-compPD*:
$[\![\ cP \vdash C\ sees\text{-}methods\ Mm';\ cP = compP\ f\ P\ ]\!] \Longrightarrow$
$\exists Mm.\ P \vdash C\ sees\text{-}methods\ Mm \wedge$
$Mm' = (Option.map\ (\lambda((Ts,T,m),D).\ ((Ts,T,f\ m),D)) \circ Mm)\langle proof\rangle$

lemma *sees-method-compPD*:

$compP\ f\ P \vdash C\ sees\ M\colon\ Ts{\rightarrow}T\ =\ fm\ in\ D \Longrightarrow$
$\exists\, m.\ P \vdash C\ sees\ M\colon\ Ts{\rightarrow}T\ =\ m\ in\ D \wedge f\ m = fm\langle proof\rangle$

lemma [*simp*]: $subcls1(compP\ f\ P) = subcls1\ P\langle proof\rangle$

lemma *compP-widen*[*simp*]: $(compP\ f\ P \vdash T \le T') = (P \vdash T \le T')\langle proof\rangle$

lemma [*simp*]: $(compP\ f\ P \vdash Ts\ [\le]\ Ts') = (P \vdash Ts\ [\le]\ Ts')\langle proof\rangle$

lemma [*simp*]: $is\text{-}type\ (compP\ f\ P)\ T = is\text{-}type\ P\ T\langle proof\rangle$

lemma [*simp*]: $(compP\ (f{::}'a{\Rightarrow}'b)\ P \vdash C\ has\text{-}fields\ FDTs) = (P \vdash C\ has\text{-}fields\ FDTs)\langle proof\rangle$

lemma [*simp*]: $fields\ (compP\ f\ P)\ C = fields\ P\ C\langle proof\rangle$

lemma [*simp*]: $(compP\ f\ P \vdash C\ sees\ F{:}T\ in\ D) = (P \vdash C\ sees\ F{:}T\ in\ D)\langle proof\rangle$

lemma [*simp*]: $field\ (compP\ f\ P)\ F\ D = field\ P\ F\ D\langle proof\rangle$

5.3.1 Invariance of *wf-prog* under compilation

lemma [*iff*]: $distinct\text{-}fst\ (compP\ f\ P) = distinct\text{-}fst\ P\langle proof\rangle$

lemma [*iff*]: $distinct\text{-}fst\ (map\ (compM\ f)\ ms) = distinct\text{-}fst\ ms\langle proof\rangle$

lemma [*iff*]: $wf\text{-}syscls\ (compP\ f\ P) = wf\text{-}syscls\ P\langle proof\rangle$

lemma [*iff*]: $wf\text{-}fdecl\ (compP\ f\ P) = wf\text{-}fdecl\ P\langle proof\rangle$

lemma *set-compP*:
$((C,D,fs,ms') \in set(compP\ f\ P)) =$
$(\exists\, ms.\ (C,D,fs,ms) \in set\ P \wedge ms' = map\ (compM\ f)\ ms)\langle proof\rangle$

lemma *wf-cdecl-compPI*:
$[\![\ \bigwedge C\ M\ Ts\ T\ m.$
$[\![\ wf\text{-}mdecl\ wf_1\ P\ C\ (M,Ts,T,m);\ P \vdash C\ sees\ M{:}Ts{\rightarrow}T = m\ in\ C\ ]\!]$
$\Longrightarrow wf\text{-}mdecl\ wf_2\ (compP\ f\ P)\ C\ (M,Ts,T,\ f\ m);$
$\forall\, x{\in}set\ P.\ wf\text{-}cdecl\ wf_1\ P\ x;\ x \in set\ (compP\ f\ P);\ wf\text{-}prog\ p\ P\ ]\!]$
$\Longrightarrow wf\text{-}cdecl\ wf_2\ (compP\ f\ P)\ x\langle proof\rangle$

lemma *wf-prog-compPI*:
assumes *lift*:
$\bigwedge C\ M\ Ts\ T\ m.$
$[\![\ P \vdash C\ sees\ M{:}Ts{\rightarrow}T = m\ in\ C;\ wf\text{-}mdecl\ wf_1\ P\ C\ (M,Ts,T,m)\ ]\!]$
$\Longrightarrow wf\text{-}mdecl\ wf_2\ (compP\ f\ P)\ C\ (M,Ts,T,\ f\ m)$
and *wf*: $wf\text{-}prog\ wf_1\ P$
shows $wf\text{-}prog\ wf_2\ (compP\ f\ P)\langle proof\rangle$

end

theory *List-Index* **imports** *Main* **begin**

This theory defines three functions for finding the index of items in a list:

find-index P xs finds the index of the first element in *xs* that satisfies *P*.

index xs x finds the index of the first occurrence of *x* in *xs*.

last-index xs x finds the index of the last occurrence of *x* in *xs*.

All functions return *length xs* if *xs* does not contain a suitable element.

The argument order of *find-index* follows the function of the same name in the Haskell standard library. For *index* (and *last-index*) the order is intentionally reversed: *index* maps lists to a mapping from elements to their indices, almost the inverse of function *nth*.

primrec *find-index* :: $('a \Rightarrow bool) \Rightarrow 'a\ list \Rightarrow nat$ **where**
find-index - [] = *0* |
find-index P (*x#xs*) = (*if P x then 0 else find-index P xs* + *1*)

definition *index* :: $'a\ list \Rightarrow 'a \Rightarrow nat$ **where**
index xs = (λa. *find-index* (λx. *x*=*a*) *xs*)

definition *last-index* :: $'a\ list \Rightarrow 'a \Rightarrow nat$ **where**
last-index xs x =
(*let i* = *index* (*rev xs*) *x*; *n* = *size xs*
in if i = *n then i else n* − (*i*+*1*))

lemma *find-index-le-size*: *find-index P xs* <= *size xs*
⟨*proof*⟩

lemma *index-le-size*: *index xs x* <= *size xs*
⟨*proof*⟩

lemma *last-index-le-size*: *last-index xs x* <= *size xs*
⟨*proof*⟩

lemma *index-Nil*[*simp*]: *index* [] *a* = *0*
⟨*proof*⟩

lemma *index-Cons*[*simp*]: *index* (*x#xs*) *a* = (*if x*=*a then 0 else index xs a* + *1*)
⟨*proof*⟩

lemma *index-append*: *index* (*xs* @ *ys*) *x* =
(*if x* : *set xs then index xs x else size xs* + *index ys x*)
⟨*proof*⟩

lemma *index-conv-size-if-notin*[*simp*]: $x \notin set\ xs \Longrightarrow index\ xs\ x = size\ xs$
⟨*proof*⟩

lemma *find-index-eq-size-conv*:
$size\ xs = n \Longrightarrow (find\text{-}index\ P\ xs = n) = (ALL\ x : set\ xs.\ {\sim}\ P\ x)$
⟨*proof*⟩

lemma *size-eq-find-index-conv*:
$size\ xs = n \Longrightarrow (n = find\text{-}index\ P\ xs) = (ALL\ x : set\ xs.\ {\sim}\ P\ x)$
⟨*proof*⟩

lemma *index-size-conv*: $size\ xs = n \Longrightarrow (index\ xs\ x = n) = (x \notin set\ xs)$
⟨*proof*⟩

lemma *size-index-conv*: $size\ xs = n \Longrightarrow (n = index\ xs\ x) = (x \notin set\ xs)$
⟨*proof*⟩

lemma *last-index-size-conv*:
$size\ xs = n \Longrightarrow (last\text{-}index\ xs\ x = n) = (x \notin set\ xs)$
⟨*proof*⟩

lemma *size-last-index-conv*:
$size\ xs = n \Longrightarrow (n = last\text{-}index\ xs\ x) = (x \notin set\ xs)$
⟨*proof*⟩

lemma *find-index-less-size-conv*:
$(find\text{-}index\ P\ xs < size\ xs) = (EX\ x : set\ xs.\ P\ x)$
⟨*proof*⟩

lemma *index-less-size-conv*:
$(index\ xs\ x < size\ xs) = (x \in set\ xs)$
⟨*proof*⟩

lemma *last-index-less-size-conv*:
$(last\text{-}index\ xs\ x < size\ xs) = (x : set\ xs)$
⟨*proof*⟩

lemma *index-less*[*simp*]:
$x : set\ xs \Longrightarrow size\ xs <= n \Longrightarrow index\ xs\ x < n$
⟨*proof*⟩

lemma *last-index-less*[*simp*]:
$x : set\ xs \Longrightarrow size\ xs <= n \Longrightarrow last\text{-}index\ xs\ x < n$
⟨*proof*⟩

lemma *last-index-Cons*: $last\text{-}index\ (x\#xs)\ y =$
$(if\ x{=}y\ then$
$\quad if\ x \in set\ xs\ then\ last\text{-}index\ xs\ y + 1\ else\ 0$
$else\ last\text{-}index\ xs\ y + 1)$
⟨*proof*⟩

lemma *last-index-append*: $last\text{-}index\ (xs\ @\ ys)\ x =$
$(if\ x : set\ ys\ then\ size\ xs + last\text{-}index\ ys\ x$
$else\ if\ x : set\ xs\ then\ last\text{-}index\ xs\ x\ else\ size\ xs + size\ ys)$
⟨*proof*⟩

lemma *last-index-Snoc*[*simp*]:
$last\text{-}index\ (xs\ @\ [x])\ y =$
$(if\ x{=}y\ then\ size\ xs$
$else\ if\ y : set\ xs\ then\ last\text{-}index\ xs\ y\ else\ size\ xs + 1)$
⟨*proof*⟩

lemma *nth-find-index*: $find\text{-}index\ P\ xs < size\ xs \Longrightarrow P(xs\ !\ find\text{-}index\ P\ xs)$
⟨*proof*⟩

lemma *nth-index*[*simp*]: $x \in set\ xs \Longrightarrow xs\ !\ index\ xs\ x = x$
⟨*proof*⟩

lemma *nth-last-index*[*simp*]: $x \in set\ xs \Longrightarrow xs\ !\ last\text{-}index\ xs\ x = x$
⟨*proof*⟩

lemma *index-eq-index-conv*[*simp*]: $x \in set\ xs \lor y \in set\ xs \Longrightarrow$
$(index\ xs\ x = index\ xs\ y) = (x = y)$
⟨*proof*⟩

lemma *last-index-eq-index-conv*[*simp*]: $x \in set\ xs \lor y \in set\ xs \Longrightarrow$
$(last\text{-}index\ xs\ x = last\text{-}index\ xs\ y) = (x = y)$
⟨*proof*⟩

lemma *inj-on-index*: $inj\text{-}on\ (index\ xs)\ (set\ xs)$
⟨*proof*⟩

lemma *inj-on-last-index*: $inj\text{-}on\ (last\text{-}index\ xs)\ (set\ xs)$
⟨*proof*⟩

lemma *index-conv-takeWhile*: $index\ xs\ x = size(takeWhile\ (\lambda y.\ x \neq y)\ xs)$
⟨*proof*⟩

lemma *index-take*: $index\ xs\ x >= i \Longrightarrow x \notin set(take\ i\ xs)$
⟨*proof*⟩

lemma *last-index-drop*:
$last\text{-}index\ xs\ x < i \Longrightarrow x \notin set(drop\ i\ xs)$
⟨*proof*⟩

end

theory *Hidden*
imports *../../List−Index/List-Index*
begin

definition *hidden* :: $'a\ list \Rightarrow nat \Rightarrow bool$ **where**
$hidden\ xs\ i\ \equiv\ i < size\ xs \land xs!i \in set(drop\ (i+1)\ xs)$

lemma *hidden-last-index*: $x \in set\ xs \Longrightarrow hidden\ (xs\ @\ [x])\ (last\text{-}index\ xs\ x)$
⟨*proof*⟩

lemma *hidden-inacc*: $hidden\ xs\ i \Longrightarrow last\text{-}index\ xs\ x \neq i$
⟨*proof*⟩

lemma [*simp*]: $hidden\ xs\ i \Longrightarrow hidden\ (xs@[x])\ i$
⟨*proof*⟩

lemma *fun-upds-apply*:
$(m(xs[\mapsto]ys))\ x =$
$(let\ xs' = take\ (size\ ys)\ xs$
$in\ if\ x \in set\ xs'\ then\ Some(ys\ !\ last\text{-}index\ xs'\ x)\ else\ m\ x)$
⟨*proof*⟩

lemma *map-upds-apply-eq-Some*:
$((m(xs[\mapsto]ys))\ x = Some\ y) =$
$(let\ xs' = take\ (size\ ys)\ xs$
$in\ if\ x \in set\ xs'\ then\ ys\ !\ last\text{-}index\ xs'\ x = y\ else\ m\ x = Some\ y)$
$\langle proof \rangle$

lemma *map-upds-upd-conv-last-index*:
$[\![x \in set\ xs;\ size\ xs \leq size\ ys\ ]\!]$
$\Longrightarrow m(xs[\mapsto]ys)(x \mapsto y) = m(xs[\mapsto]ys[last\text{-}index\ xs\ x := y])$
$\langle proof \rangle$

end

5.4 Compilation Stage 1

theory *Compiler1* **imports** *PCompiler J1 Hidden* **begin**

Replacing variable names by indices.

primrec $compE_1$ :: $vname\ list \Rightarrow expr \Rightarrow expr_1$
and $compEs_1$ :: $vname\ list \Rightarrow expr\ list \Rightarrow expr_1\ list$ **where**
$compE_1\ Vs\ (new\ C) = new\ C$
$|\ compE_1\ Vs\ (Cast\ C\ e) = Cast\ C\ (compE_1\ Vs\ e)$
$|\ compE_1\ Vs\ (Val\ v) = Val\ v$
$|\ compE_1\ Vs\ (e_1 \ll bop \gg e_2) = (compE_1\ Vs\ e_1) \ll bop \gg (compE_1\ Vs\ e_2)$
$|\ compE_1\ Vs\ (Var\ V) = Var(last\text{-}index\ Vs\ V)$
$|\ compE_1\ Vs\ (V{:=}e) = (last\text{-}index\ Vs\ V){:=}\ (compE_1\ Vs\ e)$
$|\ compE_1\ Vs\ (e{\cdot}F\{D\}) = (compE_1\ Vs\ e){\cdot}F\{D\}$
$|\ compE_1\ Vs\ (e_1{\cdot}F\{D\}{:=}e_2) = (compE_1\ Vs\ e_1){\cdot}F\{D\} := (compE_1\ Vs\ e_2)$
$|\ compE_1\ Vs\ (e{\cdot}M(es)) = (compE_1\ Vs\ e){\cdot}M(compEs_1\ Vs\ es)$
$|\ compE_1\ Vs\ \{V{:}T;\ e\} = \{(size\ Vs){:}T;\ compE_1\ (Vs@[V])\ e\}$
$|\ compE_1\ Vs\ (e_1;;e_2) = (compE_1\ Vs\ e_1);;(compE_1\ Vs\ e_2)$
$|\ compE_1\ Vs\ (if\ (e)\ e_1\ else\ e_2) = if\ (compE_1\ Vs\ e)\ (compE_1\ Vs\ e_1)\ else\ (compE_1\ Vs\ e_2)$
$|\ compE_1\ Vs\ (while\ (e)\ c) = while\ (compE_1\ Vs\ e)\ (compE_1\ Vs\ c)$
$|\ compE_1\ Vs\ (throw\ e) = throw\ (compE_1\ Vs\ e)$
$|\ compE_1\ Vs\ (try\ e_1\ catch(C\ V)\ e_2) =$
$try(compE_1\ Vs\ e_1)\ catch(C\ (size\ Vs))\ (compE_1\ (Vs@[V])\ e_2)$

$|\ compEs_1\ Vs\ []\quad = []$
$|\ compEs_1\ Vs\ (e\#es) = compE_1\ Vs\ e\ \#\ compEs_1\ Vs\ es$

lemma [*simp*]: $compEs_1\ Vs\ es = map\ (compE_1\ Vs)\ es\langle proof\rangle$

primrec fin_1:: $expr \Rightarrow expr_1$ **where**
$fin_1(Val\ v) = Val\ v$
$|\ fin_1(throw\ e) = throw(fin_1\ e)$

lemma *comp-final*: $final\ e \Longrightarrow compE_1\ Vs\ e = fin_1\ e\langle proof\rangle$

lemma [*simp*]:
$\bigwedge Vs.\ max\text{-}vars\ (compE_1\ Vs\ e) = max\text{-}vars\ e$
and $\bigwedge Vs.\ max\text{-}varss\ (compEs_1\ Vs\ es) = max\text{-}varss\ es\langle proof\rangle$

Compiling programs:

definition $compP_1$ :: $J\text{-}prog \Rightarrow J_1\text{-}prog$
where
$compP_1\ \equiv\ compP\ (\lambda(pns,body).\ compE_1\ (this\#pns)\ body)$

end

5.5 Correctness of Stage 1

theory *Correctness1*
imports *J1WellForm Compiler1*
begin

5.5.1 Correctness of program compilation

primrec $unmod :: expr_1 \Rightarrow nat \Rightarrow bool$
and $unmods :: expr_1\ list \Rightarrow nat \Rightarrow bool$ **where**
$unmod\ (new\ C)\ i = True\ |$
$unmod\ (Cast\ C\ e)\ i = unmod\ e\ i\ |$
$unmod\ (Val\ v)\ i = True\ |$
$unmod\ (e_1 \ll bop \gg e_2)\ i = (unmod\ e_1\ i \wedge unmod\ e_2\ i)\ |$
$unmod\ (Var\ i)\ j = True\ |$
$unmod\ (i{:=}e)\ j = (i \neq j \wedge unmod\ e\ j)\ |$
$unmod\ (e{\cdot}F\{D\})\ i = unmod\ e\ i\ |$
$unmod\ (e_1{\cdot}F\{D\}{:=}e_2)\ i = (unmod\ e_1\ i \wedge unmod\ e_2\ i)\ |$
$unmod\ (e{\cdot}M(es))\ i = (unmod\ e\ i \wedge unmods\ es\ i)\ |$
$unmod\ \{j{:}T;\ e\}\ i = unmod\ e\ i\ |$
$unmod\ (e_1;;e_2)\ i = (unmod\ e_1\ i \wedge\ \ unmod\ e_2\ i)\ |$
$unmod\ (if\ (e)\ e_1\ else\ e_2)\ i = (unmod\ e\ i \wedge unmod\ e_1\ i \wedge unmod\ e_2\ i)\ |$
$unmod\ (while\ (e)\ c)\ i = (unmod\ e\ i \wedge unmod\ c\ i)\ |$
$unmod\ (throw\ e)\ i = unmod\ e\ i\ |$
$unmod\ (try\ e_1\ catch(C\ i)\ e_2)\ j = (unmod\ e_1\ j \wedge (if\ i{=}j\ then\ False\ else\ unmod\ e_2\ j))\ |$

$unmods\ ([])\ i = True\ |$
$unmods\ (e\#es)\ i = (unmod\ e\ i \wedge unmods\ es\ i)$

lemma *hidden-unmod*: $\bigwedge Vs.\ hidden\ Vs\ i \Longrightarrow unmod\ (compE_1\ \ Vs\ e)\ i$ **and**
$\bigwedge Vs.\ hidden\ Vs\ i \Longrightarrow unmods\ (compEs_1\ \ Vs\ es)\ i\langle proof\rangle$

lemma $eval_1$*-preserves-unmod*:
$[\![\ P \vdash_1 \langle e,(h,ls)\rangle \Rightarrow \langle e',(h',ls')\rangle;\ unmod\ e\ i;\ i < size\ ls\ ]\!]$
$\Longrightarrow ls\ !\ i = ls'\ !\ i$
and $[\![\ P \vdash_1 \langle es,(h,ls)\rangle\ [\Rightarrow]\ \langle es',(h',ls')\rangle;\ unmods\ es\ i;\ i < size\ ls\ ]\!]$
$\Longrightarrow ls\ !\ i = ls'\ !\ i\langle proof\rangle$

lemma *LAss-lem*:
$[\![x \in set\ xs;\ size\ xs \leq size\ ys\ ]\!]$
$\Longrightarrow m_1 \subseteq_m m_2(xs[\mapsto]ys) \Longrightarrow m_1(x{\mapsto}y) \subseteq_m m_2(xs[\mapsto]ys[last\text{-}index\ xs\ x := y])\langle proof\rangle$**lemma**
Block-lem:
fixes $l :: 'a \rightharpoonup 'b$
assumes *0*: $l \subseteq_m [Vs\ [\mapsto]\ ls]$
and *1*: $l' \subseteq_m [Vs\ [\mapsto]\ ls',\ V{\mapsto}v]$
and *hidden*: $V \in set\ Vs \Longrightarrow ls\ !\ last\text{-}index\ Vs\ V = ls'\ !\ last\text{-}index\ Vs\ V$
and *size*: $size\ ls = size\ ls' \quad size\ Vs < size\ ls'$
shows $l'(V := l\ V) \subseteq_m [Vs\ [\mapsto]\ ls']\langle proof\rangle$

The main theorem:

theorem assumes *wf*: *wwf-J-prog P*
shows $eval_1$*-eval*: $P \vdash \langle e,(h,l)\rangle \Rightarrow \langle e',(h',l')\rangle$
$\Longrightarrow (\bigwedge Vs\ ls.\ [\![\ fv\ e \subseteq set\ Vs;\ \ l \subseteq_m [Vs[\mapsto]ls];\ size\ Vs + max\text{-}vars\ e \leq size\ ls\ ]\!]$
$\Longrightarrow \exists\, ls'.\ compP_1\ P \vdash_1 \langle compE_1\ \ Vs\ e,(h,ls)\rangle \Rightarrow \langle fin_1\ \ e',(h',ls')\rangle \wedge l' \subseteq_m [Vs[\mapsto]ls'])$

and *evals*$_1$-*evals*: $P \vdash \langle es,(h,l)\rangle \; [\Rightarrow] \; \langle es',(h',l')\rangle$
$\Longrightarrow (\bigwedge Vs\ ls.\ [\![\ fvs\ es \subseteq set\ Vs;\ \ l \subseteq_m [Vs[\mapsto]ls];\ size\ Vs + max\text{-}varss\ es \leq size\ ls\ ]\!]$
$\Longrightarrow \exists\, ls'.\ compP_1\ P \vdash_1 \langle compEs_1\ \ Vs\ es,(h,ls)\rangle \; [\Rightarrow] \; \langle compEs_1\ \ Vs\ es',(h',ls')\rangle \wedge$
$l' \subseteq_m [Vs[\mapsto]ls'])\langle proof\rangle$

5.5.2 Preservation of well-formedness

The compiler preserves well-formedness. Is less trivial than it may appear. We start with two simple properties: preservation of well-typedness

lemma *compE*$_1$-*pres-wt*: $\bigwedge Vs\ Ts\ U.$
$[\![\ P,[Vs[\mapsto]Ts] \vdash e :: U;\ size\ Ts = size\ Vs\ ]\!]$
$\Longrightarrow compP\ f\ P,Ts \vdash_1 compE_1\ \ Vs\ e :: U$
and $\bigwedge Vs\ Ts\ Us.$
$[\![\ P,[Vs[\mapsto]Ts] \vdash es\ [::]\ Us;\ size\ Ts = size\ Vs\ ]\!]$
$\Longrightarrow compP\ f\ P,Ts \vdash_1 compEs_1\ \ Vs\ es\ [::]\ Us\langle proof\rangle$

and the correct block numbering:

lemma $\mathcal{B}$: $\bigwedge Vs\ n.\ size\ Vs = n \Longrightarrow \mathcal{B}\ (compE_1\ \ Vs\ e)\ n$
and $\mathcal{B}s$: $\bigwedge Vs\ n.\ size\ Vs = n \Longrightarrow \mathcal{B}s\ (compEs_1\ \ Vs\ es)\ n\langle proof\rangle$

The main complication is preservation of definite assignment $\mathcal{D}$.

lemma *image-last-index*: $A \subseteq set(xs@[x]) \Longrightarrow last\text{-}index\ (xs\ @\ [x])\ `\ A =$
$(if\ x \in A\ then\ insert\ (size\ xs)\ (last\text{-}index\ xs\ `\ (A-\{x\}))\ else\ last\text{-}index\ xs\ `\ A)\langle proof\rangle$

lemma *A-compE*$_1$*-None*[*simp*]:
$\bigwedge Vs.\ \mathcal{A}\ e = None \Longrightarrow \mathcal{A}\ (compE_1\ \ Vs\ e) = None$
and $\bigwedge Vs.\ \mathcal{A}s\ es = None \Longrightarrow \mathcal{A}s\ (compEs_1\ \ Vs\ es) = None\langle proof\rangle$

lemma *A-compE*$_1$:
$\bigwedge A\ \ Vs.\ [\![\ \mathcal{A}\ e = \lfloor A\rfloor;\ fv\ e \subseteq set\ Vs\ ]\!] \Longrightarrow \mathcal{A}\ (compE_1\ \ Vs\ e) = \lfloor last\text{-}index\ Vs\ `\ A\rfloor$
and $\bigwedge A\ \ Vs.\ [\![\ \mathcal{A}s\ es = \lfloor A\rfloor;\ fvs\ es \subseteq set\ Vs\ ]\!] \Longrightarrow \mathcal{A}s\ (compEs_1\ \ Vs\ es) = \lfloor last\text{-}index\ Vs\ `\ A\rfloor\langle proof\rangle$

lemma *D-None*[*iff*]: $\mathcal{D}\ (e::'a\ exp)\ None$ **and** [*iff*]: $\mathcal{D}s\ (es::'a\ exp\ list)\ None\langle proof\rangle$

lemma *D-last-index-compE*$_1$:
$\bigwedge A\ \ Vs.\ [\![\ A \subseteq set\ Vs;\ fv\ e \subseteq set\ Vs\ ]\!] \Longrightarrow$
$\mathcal{D}\ e\ \lfloor A\rfloor \Longrightarrow \mathcal{D}\ (compE_1\ \ Vs\ e)\ \lfloor last\text{-}index\ Vs\ `\ A\rfloor$
and $\bigwedge A\ \ Vs.\ [\![\ A \subseteq set\ Vs;\ fvs\ es \subseteq set\ Vs\ ]\!] \Longrightarrow$
$\mathcal{D}s\ es\ \lfloor A\rfloor \Longrightarrow \mathcal{D}s\ (compEs_1\ \ Vs\ es)\ \lfloor last\text{-}index\ Vs\ `\ A\rfloor\langle proof\rangle$

lemma *last-index-image-set*: $distinct\ xs \Longrightarrow last\text{-}index\ xs\ `\ set\ xs = \{..<size\ xs\}\langle proof\rangle$

lemma *D-compE*$_1$:
$[\![\ \mathcal{D}\ e\ \lfloor set\ Vs\rfloor;\ fv\ e \subseteq set\ Vs;\ distinct\ Vs\ ]\!] \Longrightarrow \mathcal{D}\ (compE_1\ \ Vs\ e)\ \lfloor\{..<length\ Vs\}\rfloor\langle proof\rangle$

lemma *D-compE*$_1$*'*:
assumes $\mathcal{D}\ e\ \lfloor set(V\#Vs)\rfloor$ **and** $fv\ e \subseteq set(V\#Vs)$ **and** $distinct(V\#Vs)$
shows $\mathcal{D}\ (compE_1\ \ (V\#Vs)\ e)\ \lfloor\{..length\ Vs\}\rfloor\langle proof\rangle$

lemma *compP*$_1$*-pres-wf*: $wf\text{-}J\text{-}prog\ P \Longrightarrow wf\text{-}J_1\text{-}prog\ (compP_1\ \ P)\langle proof\rangle$

end

5.6 Compilation Stage 2

```
theory Compiler2
imports PCompiler J1 ../JVM/JVMExec
begin

primrec compE2 :: expr1 ⇒ instr list
  and compEs2 :: expr1 list ⇒ instr list where
  compE2 (new C) = [New C]
| compE2 (Cast C e) = compE2 e @ [Checkcast C]
| compE2 (Val v) = [Push v]
| compE2 (e1 «bop» e2) = compE2 e1 @ compE2 e2 @
  (case bop of Eq ⇒ [CmpEq]
           | Add ⇒ [IAdd])
| compE2 (Var i) = [Load i]
| compE2 (i:=e) = compE2 e @ [Store i, Push Unit]
| compE2 (e•F{D}) = compE2 e @ [Getfield F D]
| compE2 (e1•F{D} := e2) =
      compE2 e1 @ compE2 e2 @ [Putfield F D, Push Unit]
| compE2 (e•M(es)) = compE2 e @ compEs2 es @ [Invoke M (size es)]
| compE2 ({i:T; e}) = compE2 e
| compE2 (e1;;e2) = compE2 e1 @ [Pop] @ compE2 e2
| compE2 (if (e) e1 else e2) =
       (let cnd   = compE2 e;
            thn   = compE2 e1;
            els   = compE2 e2;
            test  = IfFalse (int(size thn + 2));
            thnex = Goto (int(size els + 1))
        in cnd @ [test] @ thn @ [thnex] @ els)
| compE2 (while (e) c) =
       (let cnd   = compE2 e;
            bdy   = compE2 c;
            test  = IfFalse (int(size bdy + 3));
            loop  = Goto (-int(size bdy + size cnd + 2))
        in cnd @ [test] @ bdy @ [Pop] @ [loop] @ [Push Unit])
| compE2 (throw e) = compE2 e @ [instr.Throw]
| compE2 (try e1 catch(C i) e2) =
   (let catch = compE2 e2
    in compE2 e1 @ [Goto (int(size catch)+2), Store i] @ catch)

| compEs2 []     = []
| compEs2 (e#es) = compE2 e @ compEs2 es
```

Compilation of exception table. Is given start address of code to compute absolute addresses necessary in exception table.

```
primrec compxE2  :: expr1      ⇒ pc ⇒ nat ⇒ ex-table
  and compxEs2 :: expr1 list ⇒ pc ⇒ nat ⇒ ex-table where
  compxE2 (new C) pc d = []
| compxE2 (Cast C e) pc d = compxE2 e pc d
| compxE2 (Val v) pc d = []
| compxE2 (e1 «bop» e2) pc d =
   compxE2 e1 pc d @ compxE2 e2 (pc + size(compE2 e1)) (d+1)
| compxE2 (Var i) pc d = []
| compxE2 (i:=e) pc d = compxE2 e pc d
```

| *compxE*$_2$ (*e*·*F*{*D*}) *pc d* = *compxE*$_2$ *e pc d*
| *compxE*$_2$ (*e*$_1$·*F*{*D*} := *e*$_2$) *pc d* =
  compxE$_2$ *e*$_1$ *pc d* @ *compxE*$_2$ *e*$_2$ (*pc* + *size*(*compE*$_2$ *e*$_1$)) (*d*+*1*)
| *compxE*$_2$ (*e*·*M*(*es*)) *pc d* =
  compxE$_2$ *e pc d* @ *compxEs*$_2$ *es* (*pc* + *size*(*compE*$_2$ *e*)) (*d*+*1*)
| *compxE*$_2$ ({*i*:*T*; *e*}) *pc d* = *compxE*$_2$ *e pc d*
| *compxE*$_2$ (*e*$_1$;;*e*$_2$) *pc d* =
  compxE$_2$ *e*$_1$ *pc d* @ *compxE*$_2$ *e*$_2$ (*pc*+*size*(*compE*$_2$ *e*$_1$)+*1*) *d*
| *compxE*$_2$ (*if* (*e*) *e*$_1$ *else e*$_2$) *pc d* =
    (*let pc*$_1$ = *pc* + *size*(*compE*$_2$ *e*) + *1*;
      pc$_2$ = *pc*$_1$ + *size*(*compE*$_2$ *e*$_1$) + *1*
    in compxE$_2$ *e pc d* @ *compxE*$_2$ *e*$_1$ *pc*$_1$ *d* @ *compxE*$_2$ *e*$_2$ *pc*$_2$ *d*)
| *compxE*$_2$ (*while* (*b*) *e*) *pc d* =
  compxE$_2$ *b pc d* @ *compxE*$_2$ *e* (*pc*+*size*(*compE*$_2$ *b*)+*1*) *d*
| *compxE*$_2$ (*throw e*) *pc d* = *compxE*$_2$ *e pc d*
| *compxE*$_2$ (*try e*$_1$ *catch*(*C i*) *e*$_2$) *pc d* =
  (*let pc*$_1$ = *pc* + *size*(*compE*$_2$ *e*$_1$)
   in compxE$_2$ *e*$_1$ *pc d* @ *compxE*$_2$ *e*$_2$ (*pc*$_1$+*2*) *d* @ [(*pc*,*pc*$_1$,*C*,*pc*$_1$+*1*,*d*)])

| *compxEs*$_2$ [] *pc d* = []
| *compxEs*$_2$ (*e*#*es*) *pc d* = *compxE*$_2$ *e pc d* @ *compxEs*$_2$ *es* (*pc*+*size*(*compE*$_2$ *e*)) (*d*+*1*)

primrec *max-stack* :: *expr*$_1$ ⇒ *nat*
 and *max-stacks* :: *expr*$_1$ *list* ⇒ *nat* **where**
 max-stack (*new C*) = *1*
| *max-stack* (*Cast C e*) = *max-stack e*
| *max-stack* (*Val v*) = *1*
| *max-stack* (*e*$_1$ «*bop*» *e*$_2$) = *max* (*max-stack e*$_1$) (*max-stack e*$_2$) + *1*
| *max-stack* (*Var i*) = *1*
| *max-stack* (*i*:=*e*) = *max-stack e*
| *max-stack* (*e*·*F*{*D*}) = *max-stack e*
| *max-stack* (*e*$_1$·*F*{*D*} := *e*$_2$) = *max* (*max-stack e*$_1$) (*max-stack e*$_2$) + *1*
| *max-stack* (*e*·*M*(*es*)) = *max* (*max-stack e*) (*max-stacks es*) + *1*
| *max-stack* ({*i*:*T*; *e*}) = *max-stack e*
| *max-stack* (*e*$_1$;;*e*$_2$) = *max* (*max-stack e*$_1$) (*max-stack e*$_2$)
| *max-stack* (*if* (*e*) *e*$_1$ *else e*$_2$) =
 max (*max-stack e*) (*max* (*max-stack e*$_1$) (*max-stack e*$_2$))
| *max-stack* (*while* (*e*) *c*) = *max* (*max-stack e*) (*max-stack c*)
| *max-stack* (*throw e*) = *max-stack e*
| *max-stack* (*try e*$_1$ *catch*(*C i*) *e*$_2$) = *max* (*max-stack e*$_1$) (*max-stack e*$_2$)

| *max-stacks* [] = *0*
| *max-stacks* (*e*#*es*) = *max* (*max-stack e*) (*1* + *max-stacks es*)

lemma *max-stack1*: *1* ≤ *max-stack e*⟨*proof*⟩

definition *compMb*$_2$ :: *expr*$_1$ ⇒ *jvm-method*
where
 compMb$_2$ ≡ λ*body*.
 let ins = *compE*$_2$ *body* @ [*Return*];
   xt = *compxE*$_2$ *body 0 0*
 in (*max-stack body*, *max-vars body*, *ins*, *xt*)

definition *compP*$_2$ :: *J*$_1$-*prog* ⇒ *jvm-prog*

where
compP$_2$ $\equiv$ *compP compMb*$_2$

lemma *compMb*$_2$ [*simp*]:
compMb$_2$ *e* = (*max-stack e*, *max-vars e*, *compE*$_2$ *e* @ [*Return*], *compxE*$_2$ *e 0 0*)$\langle$*proof*$\rangle$

end

5.7 Correctness of Stage 2

theory *Correctness2*
imports ~~/*src*/*HOL*/*Library*/*List-Prefix Compiler2*
begin

5.7.1 Instruction sequences

How to select individual instructions and subsequences of instructions from a program given the class, method and program counter.

definition $before :: jvm\text{-}prog \Rightarrow cname \Rightarrow mname \Rightarrow nat \Rightarrow instr\ list \Rightarrow bool$
$((\text{-},\text{-},\text{-},\text{-}/ \rhd\ \text{-})\ [51,0,0,0,51]\ 50)$ **where**
$P,C,M,pc \rhd is \longleftrightarrow is \le drop\ pc\ (instrs\text{-}of\ P\ C\ M)$

definition $at :: jvm\text{-}prog \Rightarrow cname \Rightarrow mname \Rightarrow nat \Rightarrow instr \Rightarrow bool$
$((\text{-},\text{-},\text{-},\text{-}/ \triangleright\ \text{-})\ [51,0,0,0,51]\ 50)$ **where**
$P,C,M,pc \triangleright i \longleftrightarrow (\exists is.\ drop\ pc\ (instrs\text{-}of\ P\ C\ M) = i\#is)$

lemma [*simp*]: $P,C,M,pc \rhd []\langle proof\rangle$

lemma [*simp*]: $P,C,M,pc \rhd (i\#is) = (P,C,M,pc \triangleright i \wedge P,C,M,pc + 1 \rhd is)\langle proof\rangle$

lemma [*simp*]: $P,C,M,pc \rhd (is_1\ @\ is_2) = (P,C,M,pc \rhd is_1 \wedge P,C,M,pc + size\ is_1 \rhd is_2)\langle proof\rangle$

lemma [*simp*]: $P,C,M,pc \triangleright i \Longrightarrow instrs\text{-}of\ P\ C\ M\ !\ pc = i\langle proof\rangle$

lemma *beforeM*:
$P \vdash C\ sees\ M\colon Ts{\rightarrow}T = body\ in\ D \Longrightarrow$
$compP_2\ P,D,M,0 \rhd compE_2\ body\ @\ [Return]\langle proof\rangle$

This lemma executes a single instruction by rewriting:

lemma [*simp*]:
$P,C,M,pc \triangleright instr \Longrightarrow$
$(P \vdash (None,\ h,\ (vs,ls,C,M,pc)\ \#\ frs)\ {-jvm\rightarrow}\ \sigma') =$
$((None,\ h,\ (vs,ls,C,M,pc)\ \#\ frs) = \sigma' \vee$
$(\exists\sigma.\ exec(P,(None,\ h,\ (vs,ls,C,M,pc)\ \#\ frs)) = Some\ \sigma \wedge P \vdash \sigma\ {-jvm\rightarrow}\ \sigma'))\langle proof\rangle$

5.7.2 Exception tables

definition $pcs :: ex\text{-}table \Rightarrow nat\ set$
where
$pcs\ xt \equiv \bigcup(f,t,C,h,d) \in set\ xt.\ \{f\ ..< t\}$

lemma *pcs-subset*:
shows $\bigwedge pc\ d.\ pcs(compxE_2\ e\ pc\ d) \subseteq \{pc..<pc+size(compE_2\ e)\}$
and $\bigwedge pc\ d.\ pcs(compxEs_2\ es\ pc\ d) \subseteq \{pc..<pc+size(compEs_2\ es)\}\langle proof\rangle$

lemma [*simp*]: $pcs\ [] = \{\}\langle proof\rangle$

lemma [*simp*]: $pcs\ (x\#xt) = \{fst\ x\ ..< fst(snd\ x)\} \cup pcs\ xt\langle proof\rangle$

lemma [*simp*]: $pcs(xt_1\ @\ xt_2) = pcs\ xt_1 \cup pcs\ xt_2\langle proof\rangle$

lemma [*simp*]: $pc < pc_0 \vee pc_0{+}size(compE_2\ e) \leq pc \Longrightarrow pc \notin pcs(compxE_2\ e\ pc_0\ d)\langle proof\rangle$

lemma [*simp*]: $pc < pc_0 \vee pc_0{+}size(compEs_2\ es) \leq pc \Longrightarrow pc \notin pcs(compxEs_2\ es\ pc_0\ d)\langle proof\rangle$

lemma [*simp*]: $pc_1 + size(compE_2\ e_1) \leq pc_2 \Longrightarrow pcs(compxE_2\ e_1\ pc_1\ d_1) \cap pcs(compxE_2\ e_2\ pc_2\ d_2) = \{\}\langle proof\rangle$

lemma [*simp*]: $pc_1 + size(compE_2\ e) \leq pc_2 \Longrightarrow pcs(compxE_2\ e\ pc_1\ d_1) \cap pcs(compxEs_2\ es\ pc_2\ d_2) = \{\}\langle proof\rangle$

lemma [*simp*]:
$pc \notin pcs\ xt_0 \Longrightarrow match\text{-}ex\text{-}table\ P\ C\ pc\ (xt_0\ @\ xt_1) = match\text{-}ex\text{-}table\ P\ C\ pc\ xt_1\langle proof\rangle$

lemma [*simp*]: $[\![\ x \in set\ xt;\ pc \notin pcs\ xt\ ]\!] \Longrightarrow \neg\ matches\text{-}ex\text{-}entry\ P\ D\ pc\ x\langle proof\rangle$

lemma [*simp*]:
assumes *xe*: $xe \in set(compxE_2\ e\ pc\ d)$ **and** *outside*: $pc' < pc \vee pc{+}size(compE_2\ e) \leq pc'$
shows $\neg\ matches\text{-}ex\text{-}entry\ P\ C\ pc'\ xe\langle proof\rangle$

lemma [*simp*]:
assumes *xe*: $xe \in set(compxEs_2\ es\ pc\ d)$ **and** *outside*: $pc' < pc \vee pc{+}size(compEs_2\ es) \leq pc'$
shows $\neg\ matches\text{-}ex\text{-}entry\ P\ C\ pc'\ xe\langle proof\rangle$

lemma *match-ex-table-app*[*simp*]:
$\forall xte \in set\ xt_1.\ \neg\ matches\text{-}ex\text{-}entry\ P\ D\ pc\ xte \Longrightarrow$
$match\text{-}ex\text{-}table\ P\ D\ pc\ (xt_1\ @\ xt) = match\text{-}ex\text{-}table\ P\ D\ pc\ xt\langle proof\rangle$

lemma [*simp*]:
$\forall x \in set\ xtab.\ \neg\ matches\text{-}ex\text{-}entry\ P\ C\ pc\ x \Longrightarrow$
$match\text{-}ex\text{-}table\ P\ C\ pc\ xtab = None\langle proof\rangle$

lemma *match-ex-entry*:
$matches\text{-}ex\text{-}entry\ P\ C\ pc\ (start,\ end,\ catch\text{-}type,\ handler) =$
$(start \leq pc \wedge pc < end \wedge\ \ P \vdash C \preceq^* catch\text{-}type)\langle proof\rangle$

definition *caught* :: $jvm\text{-}prog \Rightarrow pc \Rightarrow heap \Rightarrow addr \Rightarrow ex\text{-}table \Rightarrow bool$ **where**
$caught\ P\ pc\ h\ a\ xt \longleftrightarrow$
$(\exists entry \in set\ xt.\ matches\text{-}ex\text{-}entry\ P\ (cname\text{-}of\ h\ a)\ pc\ entry)$

definition *beforex* :: $jvm\text{-}prog \Rightarrow cname \Rightarrow mname \Rightarrow ex\text{-}table \Rightarrow nat\ set \Rightarrow nat \Rightarrow bool$
((*2*-,/-,/- ▷/ - /′/ -,/-) [*51*,*0*,*0*,*0*,*0*,*51*] *50*) **where**
$P,C,M \rhd xt\ /\ I,d \longleftrightarrow$
$(\exists xt_0\ xt_1.\ ex\text{-}table\text{-}of\ P\ C\ M = xt_0\ @\ xt\ @\ xt_1 \wedge pcs\ xt_0 \cap I = \{\} \wedge pcs\ xt \subseteq I \wedge$
$(\forall pc \in I.\ \forall C\ pc'\ d'.\ match\text{-}ex\text{-}table\ P\ C\ pc\ xt_1 = \lfloor(pc',d')\rfloor \longrightarrow d' \leq d))$

definition *dummyx* :: $jvm\text{-}prog \Rightarrow cname \Rightarrow mname \Rightarrow ex\text{-}table \Rightarrow nat\ set \Rightarrow nat \Rightarrow bool$ ((*2*-,-,- ▷/ - ′/-,-) [*51*,*0*,*0*,*0*,*0*,*51*] *50*) **where**
$P,C,M \rhd xt/I,d \longleftrightarrow P,C,M \rhd xt/I,d$

lemma *beforexD1*: $P,C,M \rhd xt\ /\ I,d \Longrightarrow pcs\ xt \subseteq I\langle proof\rangle$

lemma *beforex-mono*: $[\![\ P,C,M \rhd xt/I,d';\ d' \leq d\ ]\!] \Longrightarrow P,C,M \rhd xt/I,d\langle proof\rangle$

lemma [*simp*]: $P,C,M \rhd xt/I,d \Longrightarrow P,C,M \rhd xt/I,Suc\ d\langle proof\rangle$

lemma *beforex-append*[*simp*]:
$pcs\ xt_1 \cap pcs\ xt_2 = \{\} \Longrightarrow$
$P,C,M \rhd xt_1\ @\ xt_2/I,d =$
$(P,C,M \rhd xt_1/I{-}pcs\ xt_2,d\ \wedge\ P,C,M \rhd xt_2/I{-}pcs\ xt_1,d \wedge P,C,M \rhd xt_1@xt_2/I,d)\langle proof\rangle$

lemma *beforex-appendD1*:
$[\![\ P,C,M \rhd xt_1\ @\ xt_2\ @\ [(f,t,D,h,d)]\ /\ I,d;$
$pcs\ xt_1 \subseteq J;\ J \subseteq I;\ J \cap pcs\ xt_2 = \{\}\ ]\!]$
$\Longrightarrow P,C,M \rhd xt_1\ /\ J,d\langle proof\rangle$

lemma *beforex-appendD2*:
$[\![\ P,C,M \rhd xt_1\ @\ xt_2\ @\ [(f,t,D,h,d)]\ /\ I,d;$
$pcs\ xt_2 \subseteq J;\ J \subseteq I;\ J \cap pcs\ xt_1 = \{\}\ ]\!]$
$\Longrightarrow P,C,M \rhd xt_2\ /\ J,d\langle proof\rangle$

lemma *beforexM*:
$P \vdash C\ sees\ M{:}\ Ts{\rightarrow}T = body\ in\ D \Longrightarrow$
$compP_2\ P,D,M \rhd compxE_2\ body\ 0\ 0/\{..{<}size(compE_2\ body)\},0\langle proof\rangle$

lemma *match-ex-table-SomeD2*:
$[\![\ match\text{-}ex\text{-}table\ P\ D\ pc\ (ex\text{-}table\text{-}of\ P\ C\ M) = \lfloor(pc',d')\rfloor;$
$P,C,M \rhd xt/I,d;\ \forall x \in set\ xt.\ \neg\ matches\text{-}ex\text{-}entry\ P\ D\ pc\ x;\ pc \in I\ ]\!]$
$\Longrightarrow d' \le d\langle proof\rangle$

lemma *match-ex-table-SomeD1*:
$[\![\ match\text{-}ex\text{-}table\ P\ D\ pc\ (ex\text{-}table\text{-}of\ P\ C\ M) = \lfloor(pc',d')\rfloor;$
$P,C,M \rhd xt\ /\ I,d;\ pc \in I;\ pc \notin pcs\ xt\ ]\!] \Longrightarrow d' \le d\langle proof\rangle$

5.7.3 The correctness proof

definition
$handle :: jvm\text{-}prog \Rightarrow cname \Rightarrow mname \Rightarrow addr \Rightarrow heap \Rightarrow val\ list \Rightarrow val\ list \Rightarrow nat \Rightarrow frame\ list$
$\Rightarrow jvm\text{-}state$ **where**
$handle\ P\ C\ M\ a\ h\ vs\ ls\ pc\ frs = find\text{-}handler\ P\ a\ h\ ((vs,ls,C,M,pc)\ \#\ frs)$

lemma *handle-Cons*:
$[\![\ P,C,M \rhd xt/I,d;\ d \le size\ vs;\ pc \in I;$
$\forall x \in set\ xt.\ \neg\ matches\text{-}ex\text{-}entry\ P\ (cname\text{-}of\ h\ xa)\ pc\ x\ ]\!] \Longrightarrow$
$handle\ P\ C\ M\ xa\ h\ (v\ \#\ vs)\ ls\ pc\ frs = handle\ P\ C\ M\ xa\ h\ vs\ ls\ pc\ frs\langle proof\rangle$
lemma *handle-append*:
$[\![\ P,C,M \rhd xt/I,d;\ d \le size\ vs;\ pc \in I;\ pc \notin pcs\ xt\ ]\!] \Longrightarrow$
$handle\ P\ C\ M\ xa\ h\ (ws\ @\ vs)\ ls\ pc\ frs = handle\ P\ C\ M\ xa\ h\ vs\ ls\ pc\ frs\langle proof\rangle$

lemma *aux-isin*[*simp*]: $[\![\ B \subseteq A;\ a \in B\ ]\!] \Longrightarrow a \in A\langle proof\rangle$

lemma fixes P_1 **defines** [*simp*]: $P \equiv compP_2\ P_1$
shows *Jcc*:
$P_1 \vdash_1 \langle e,(h_0,ls_0)\rangle \Rightarrow \langle ef,(h_1,ls_1)\rangle \Longrightarrow$
$(\bigwedge C\ M\ pc\ v\ xa\ vs\ frs\ I.$
$[\![\ P,C,M,pc \rhd compE_2\ e;\ P,C,M \rhd compxE_2\ e\ pc\ (size\ vs)/I,size\ vs;$
$\{pc..{<}pc{+}size(compE_2\ e)\} \subseteq I\ ]\!] \Longrightarrow$

$(ef = Val\ v \longrightarrow$
$P \vdash (None,h_0,(vs,ls_0,C,M,pc)\#frs)\ -jvm\rightarrow$
$(None,h_1,(v\#vs,ls_1,C,M,pc{+}size(compE_2\ e))\#frs))$
$\wedge$
$(ef = Throw\ xa \longrightarrow$
$(\exists\, pc_1.\ pc \le pc_1 \wedge pc_1 < pc + size(compE_2\ e) \wedge$
$\neg\ caught\ P\ pc_1\ h_1\ xa\ (compxE_2\ e\ pc\ (size\ vs)) \wedge$
$P \vdash (None,h_0,(vs,ls_0,C,M,pc)\#frs)\ -jvm\rightarrow\ handle\ P\ C\ M\ xa\ h_1\ vs\ ls_1\ pc_1\ frs)))$
and $P_1 \vdash_1 \langle es,(h_0,ls_0)\rangle\ [\Rightarrow]\ \langle fs,(h_1,ls_1)\rangle \Longrightarrow$
$(\bigwedge C\ M\ pc\ ws\ xa\ es'\ vs\ frs\ I.$
$[\![\ P,C,M,pc \rhd compEs_2\ es;\ P,C,M \rhd compxEs_2\ es\ pc\ (size\ vs)/I,size\ vs;$
$\{pc..{<}pc{+}size(compEs_2\ es)\} \subseteq I\ ]\!] \Longrightarrow$
$(fs = map\ Val\ ws \longrightarrow$
$P \vdash (None,h_0,(vs,ls_0,C,M,pc)\#frs)\ -jvm\rightarrow$
$(None,h_1,(rev\ ws\ @\ vs,ls_1,C,M,pc{+}size(compEs_2\ es))\#frs))$
$\wedge$
$(fs = map\ Val\ ws\ @\ Throw\ xa\ \#\ es' \longrightarrow$
$(\exists\, pc_1.\ pc \le pc_1 \wedge pc_1 < pc + size(compEs_2\ es) \wedge$
$\neg\ caught\ P\ pc_1\ h_1\ xa\ (compxEs_2\ es\ pc\ (size\ vs)) \wedge$
$P \vdash (None,h_0,(vs,ls_0,C,M,pc)\#frs)\ -jvm\rightarrow\ handle\ P\ C\ M\ xa\ h_1\ vs\ ls_1\ pc_1\ frs)))\langle proof\rangle$

lemma *atLeast0AtMost*[*simp*]: $\{0{::}nat..n\} = \{..n\}$
$\langle proof\rangle$

lemma *atLeast0LessThan*[*simp*]: $\{0{::}nat..{<}n\} = \{..{<}n\}$
$\langle proof\rangle$

fun *exception* :: *'a exp* ⇒ *addr option* **where**
exception (*Throw a*) = *Some a*
| *exception e* = *None*

lemma $comp_2$-*correct*:
assumes *method*: $P_1 \vdash C\ sees\ M{:}Ts{\rightarrow}T = body\ in\ C$
and *eval*: $P_1 \vdash_1 \langle body,(h,ls)\rangle \Rightarrow \langle e',(h',ls')\rangle$
shows $compP_2\ P_1 \vdash (None,h,[([],ls,C,M,0)])\ -jvm\rightarrow\ (exception\ e',h',[])\langle proof\rangle$
end

5.8 Combining Stages 1 and 2

theory *Compiler*
imports *Correctness1 Correctness2*
begin

definition *J2JVM* :: *J-prog* ⇒ *jvm-prog*
where
$J2JVM \equiv compP_2 \circ compP_1$

theorem *comp-correct*:
assumes *wwf*: *wwf-J-prog P*
and *method*: $P \vdash C$ *sees* $M{:}Ts{\rightarrow}T = (pns,body)$ *in* C
and *eval*: $P \vdash \langle body,(h,[this\#pns\ [\mapsto]\ vs])\rangle \Rightarrow \langle e',(h',l')\rangle$
and *sizes*: *size vs* = *size pns* + *1*   *size rest* = *max-vars body*
shows *J2JVM P* $\vdash$ $(None,h,[([],vs@rest,C,M,0)])$ $-jvm\rightarrow$ $(exception\ e',h',[])$⟨*proof*⟩

end

5.9 Preservation of Well-Typedness

theory *TypeComp*
imports *Compiler ../BV/BVSpec*
begin

locale *TC0* =
fixes P :: J_1*-prog* **and** *mxl* :: *nat*
begin

definition $ty\ E\ e = (THE\ T.\ P,E \vdash_1 e :: T)$

definition $ty_l\ E\ A' = map\ (\lambda i.\ if\ i \in A' \wedge i < size\ E\ then\ OK(E!i)\ else\ Err)\ [0..<mxl]$

definition $ty_i{}'\ ST\ E\ A = (case\ A\ of\ None \Rightarrow None \mid \lfloor A' \rfloor \Rightarrow Some(ST,\ ty_l\ E\ A'))$

definition $after\ E\ A\ ST\ e = ty_i{}'\ (ty\ E\ e\ \#\ ST)\ E\ (A \sqcup \mathcal{A}\ e)$

end

lemma (**in** *TC0*) *ty-def2* [*simp*]: $P,E \vdash_1 e :: T \Longrightarrow ty\ E\ e = T\langle proof\rangle$
lemma (**in** *TC0*) [*simp*]: $ty_i{}'\ ST\ E\ None = None\langle proof\rangle$
lemma (**in** *TC0*) ty_l*-app-diff* [*simp*]:
$ty_l\ (E@[T])\ (A - \{size\ E\}) = ty_l\ E\ A\langle proof\rangle$

lemma (**in** *TC0*) $ty_i{}'$*-app-diff* [*simp*]:
$ty_i{}'\ ST\ (E\ @\ [T])\ (A \ominus size\ E) = ty_i{}'\ ST\ E\ A\langle proof\rangle$

lemma (**in** *TC0*) ty_l*-antimono*:
$A \subseteq A' \Longrightarrow P \vdash ty_l\ E\ A'\ [\le_\top]\ ty_l\ E\ A\langle proof\rangle$

lemma (**in** *TC0*) $ty_i{}'$*-antimono*:
$A \subseteq A' \Longrightarrow P \vdash ty_i{}'\ ST\ E\ \lfloor A' \rfloor \le'\ ty_i{}'\ ST\ E\ \lfloor A \rfloor\langle proof\rangle$

lemma (**in** *TC0*) ty_l*-env-antimono*:
$P \vdash ty_l\ (E@[T])\ A\ [\le_\top]\ ty_l\ E\ A\langle proof\rangle$

lemma (**in** *TC0*) $ty_i{}'$*-env-antimono*:
$P \vdash ty_i{}'\ ST\ (E@[T])\ A \le'\ ty_i{}'\ ST\ E\ A\langle proof\rangle$

lemma (**in** *TC0*) $ty_i{}'$*-incr*:
$P \vdash ty_i{}'\ ST\ (E\ @\ [T])\ \lfloor insert\ (size\ E)\ A \rfloor \le'\ ty_i{}'\ ST\ E\ \lfloor A \rfloor\langle proof\rangle$

lemma (**in** *TC0*) ty_l*-incr*:
$P \vdash ty_l\ (E\ @\ [T])\ (insert\ (size\ E)\ A)\ [\le_\top]\ ty_l\ E\ A\langle proof\rangle$

lemma (**in** *TC0*) ty_l*-in-types*:
$set\ E \subseteq types\ P \Longrightarrow ty_l\ E\ A \in list\ mxl\ (err\ (types\ P))\langle proof\rangle$
locale *TC1* = *TC0*
begin

primrec $compT :: ty\ list \Rightarrow nat\ hyperset \Rightarrow ty\ list \Rightarrow expr_1 \Rightarrow ty_i{}'\ list$ **and**
$compTs :: ty\ list \Rightarrow nat\ hyperset \Rightarrow ty\ list \Rightarrow expr_1\ list \Rightarrow ty_i{}'\ list$ **where**
$compT\ E\ A\ ST\ (new\ C) = []$

$|$ $compT\ E\ A\ ST\ (Cast\ C\ e) =$
$\quad compT\ E\ A\ ST\ e\ @\ [after\ E\ A\ ST\ e]$
$|$ $compT\ E\ A\ ST\ (Val\ v) = []$
$|$ $compT\ E\ A\ ST\ (e_1 \ll bop \gg e_2) =$
$\ (let\ ST_1 = ty\ E\ e_1\#ST;\ A_1 = A \sqcup \mathcal{A}\ e_1\ in$
$\quad compT\ E\ A\ ST\ e_1\ @\ [after\ E\ A\ ST\ e_1]\ @$
$\quad compT\ E\ A_1\ ST_1\ e_2\ @\ [after\ E\ A_1\ ST_1\ e_2])$
$|$ $compT\ E\ A\ ST\ (Var\ i) = []$
$|$ $compT\ E\ A\ ST\ (i := e) = compT\ E\ A\ ST\ e\ @$
$\quad [after\ E\ A\ ST\ e,\ ty_i{}'\ ST\ E\ (A \sqcup \mathcal{A}\ e \sqcup \lfloor\{i\}\rfloor)]$
$|$ $compT\ E\ A\ ST\ (e{\cdot}F\{D\}) =$
$\quad compT\ E\ A\ ST\ e\ @\ [after\ E\ A\ ST\ e]$
$|$ $compT\ E\ A\ ST\ (e_1{\cdot}F\{D\} := e_2) =$
$\ (let\ ST_1 = ty\quad E\ e_1\#ST;\ A_1 = A \sqcup \mathcal{A}\ e_1;\ A_2 = A_1 \sqcup \mathcal{A}\ e_2\ in$
$\quad compT\ E\ A\ ST\ e_1\ @\ [after\ E\ A\ ST\ e_1]\ @$
$\quad compT\ E\ A_1\ ST_1\ e_2\ @\ [after\ E\ A_1\ ST_1\ e_2]\ @$
$\quad [ty_i{}'\ ST\ E\ A_2])$
$|$ $compT\ E\ A\ ST\ \{i{:}T;\ e\} = compT\ (E@[T])\ (A \ominus i)\ ST\ e$
$|$ $compT\ E\ A\ ST\ (e_1;;e_2) =$
$\ (let\ A_1 = A \sqcup \mathcal{A}\ e_1\ in$
$\quad compT\ E\ A\ ST\ e_1\ @\ [after\ E\ A\ ST\ e_1,\ ty_i{}'\ ST\ E\ A_1]\ @$
$\quad compT\ E\ A_1\ ST\ e_2)$
$|$ $compT\ E\ A\ ST\ (if\ (e)\ e_1\ else\ e_2) =$
$\quad (let\ A_0 = A \sqcup \mathcal{A}\ e;\ \tau = ty_i{}'\ ST\ E\ A_0\ in$
$\quad\ compT\ E\ A\ ST\ e\ @\ [after\ E\ A\ ST\ e,\ \tau]\ @$
$\quad\ compT\ E\ A_0\ ST\ e_1\ @\ [after\ E\ A_0\ ST\ e_1,\ \tau]\ @$
$\quad\ compT\ E\ A_0\ ST\ e_2)$
$|$ $compT\ E\ A\ ST\ (while\ (e)\ c) =$
$\quad (let\ A_0 = A \sqcup \mathcal{A}\ e;\ \ A_1 = A_0 \sqcup \mathcal{A}\ c;\ \tau = ty_i{}'\ ST\ E\ A_0\ in$
$\quad\ compT\ E\ A\ ST\ e\ @\ [after\ E\ A\ ST\ e,\ \tau]\ @$
$\quad\ compT\ E\ A_0\ ST\ c\ @\ [after\ E\ A_0\ ST\ c,\ ty_i{}'\ ST\ E\ A_1,\ ty_i{}'\ ST\ E\ A_0])$
$|$ $compT\ E\ A\ ST\ (throw\ e) = compT\ E\ A\ ST\ e\ @\ [after\ E\ A\ ST\ e]$
$|$ $compT\ E\ A\ ST\ (e{\cdot}M(es)) =$
$\quad compT\ E\ A\ ST\ e\ @\ [after\ E\ A\ ST\ e]\ @$
$\quad compTs\ E\ (A \sqcup \mathcal{A}\ e)\ (ty\quad E\ e\ \#\ ST)\ es$
$|$ $compT\ E\ A\ ST\ (try\ e_1\ catch(C\ i)\ e_2) =$
$\quad compT\ E\ A\ ST\ e_1\ @\ [after\ E\ A\ ST\ e_1]\ @$
$\quad [ty_i{}'\ (Class\ C\#ST)\ E\ A,\ ty_i{}'\ ST\ (E@[Class\ C])\ (A \sqcup \lfloor\{i\}\rfloor)]\ @$
$\quad compT\ (E@[Class\ C])\ (A \sqcup \lfloor\{i\}\rfloor)\ ST\ e_2$
$|$ $compTs\ E\ A\ ST\ [] = []$
$|$ $compTs\ \ E\ A\ ST\ (e\#es) = compT\ E\ A\ ST\ e\ @\ [after\ E\ A\ ST\ e]\ @$
$\qquad\qquad\qquad\qquad compTs\ E\ (A \sqcup (\mathcal{A}\ e))\ (ty\ E\ e\ \#\ ST)\ es$

definition $compT_a :: ty\ list \Rightarrow nat\ hyperset \Rightarrow ty\ list \Rightarrow expr_1 \Rightarrow ty_i{}'\ list$ **where**
$\ compT_a\ E\ A\ ST\ e = compT\ E\ A\ ST\ e\ @\ [after\ E\ A\ ST\ e]$

end

lemma $compE_2\text{-}not\text{-}Nil[simp]$: $compE_2\ e \neq []\langle proof\rangle$
lemma (**in** $TC1$) $compT\text{-}sizes[simp]$:
shows $\bigwedge E\ A\ ST.\ size(compT\ E\ A\ ST\ e) = size(compE_2\ e) - 1$
and $\bigwedge E\ A\ ST.\ size(compTs\ E\ A\ ST\ es) = size(compEs_2\ es)\langle proof\rangle$

lemma (**in** $TC1$) $[simp]$: $\bigwedge ST\ E.\ \lfloor\tau\rfloor \notin set\ (compT\ E\ None\ ST\ e)$

and [*simp*]: $\bigwedge ST\ E.\ \lfloor\tau\rfloor \notin set\ (compTs\ E\ None\ ST\ es)\langle proof\rangle$

lemma (**in** *TC0*) *pair-eq-ty$_i$'-conv*:
$(\lfloor(ST,\ LT)\rfloor = ty_i{}'\ ST_0\ E\ A) =$
$(case\ A\ of\ None \Rightarrow False \mid Some\ A \Rightarrow (ST = ST_0 \wedge LT = ty_l\ E\ A))\langle proof\rangle$

lemma (**in** *TC0*) *pair-conv-ty$_i$'*:
$\lfloor(ST,\ ty_l\ E\ A)\rfloor = ty_i{}'\ ST\ E\ \lfloor A\rfloor\langle proof\rangle$

lemma (**in** *TC1*) *compT-LT-prefix*:
$\bigwedge E\ A\ ST_0.\ \llbracket\ \lfloor(ST,LT)\rfloor \in set(compT\ E\ A\ ST_0\ e);\ \mathcal{B}\ e\ (size\ E)\ \rrbracket$
$\Longrightarrow P \vdash \lfloor(ST,LT)\rfloor \le'\ ty_i{}'\ ST\ E\ A$
and
$\bigwedge E\ A\ ST_0.\ \llbracket\ \lfloor(ST,LT)\rfloor \in set(compTs\ E\ A\ ST_0\ es);\ \mathcal{B}s\ es\ (size\ E)\ \rrbracket$
$\Longrightarrow P \vdash \lfloor(ST,LT)\rfloor \le'\ ty_i{}'\ ST\ E\ A\langle proof\rangle$

lemma [*iff*]: $OK\ None \in states\ P\ mxs\ mxl\langle proof\rangle$
lemma (**in** *TC0*) *after-in-states*:
$\llbracket\ wf\text{-}prog\ p\ P;\ P,E \vdash_1 e :: T;\ set\ E \subseteq types\ P;\ set\ ST \subseteq types\ P;$
$size\ ST + max\text{-}stack\ e \le mxs\ \rrbracket$
$\Longrightarrow OK\ (after\ E\ A\ ST\ e) \in states\ P\ mxs\ mxl\langle proof\rangle$

lemma (**in** *TC0*) *OK-ty$_i$'-in-statesI*[*simp*]:
$\llbracket\ set\ E \subseteq types\ P;\ set\ ST \subseteq types\ P;\ size\ ST \le mxs\ \rrbracket$
$\Longrightarrow OK\ (ty_i{}'\ ST\ E\ A) \in states\ P\ mxs\ mxl\langle proof\rangle$

lemma *is-class-type-aux*: $is\text{-}class\ P\ C \Longrightarrow is\text{-}type\ P\ (Class\ C)\langle proof\rangle$
theorem (**in** *TC1*) *compT-states*:
assumes *wf*: *wf-prog p P*
shows $\bigwedge E\ T\ A\ ST.$
$\llbracket\ P,E \vdash_1 e :: T;\ set\ E \subseteq types\ P;\ set\ ST \subseteq types\ P;$
$size\ ST + max\text{-}stack\ e \le mxs;\ size\ E + max\text{-}vars\ e \le mxl\ \rrbracket$
$\Longrightarrow OK\ `\ set(compT\ E\ A\ ST\ e) \subseteq states\ P\ mxs\ mxl$
and $\bigwedge E\ Ts\ A\ ST.$
$\llbracket\ P,E \vdash_1 es[::]Ts;\ \ set\ E \subseteq types\ P;\ set\ ST \subseteq types\ P;$
$size\ ST + max\text{-}stacks\ es \le mxs;\ size\ E + max\text{-}varss\ es \le mxl\ \rrbracket$
$\Longrightarrow OK\ `\ set(compTs\ E\ A\ ST\ es) \subseteq states\ P\ mxs\ mxl\langle proof\rangle$

definition $shift :: nat \Rightarrow ex\text{-}table \Rightarrow ex\text{-}table$
where
$shift\ n\ xt \equiv map\ (\lambda(from,to,C,handler,depth).\ (from+n,to+n,C,handler+n,depth))\ xt$

lemma [*simp*]: $shift\ 0\ xt = xt\langle proof\rangle$
lemma [*simp*]: $shift\ n\ [] = []\langle proof\rangle$
lemma [*simp*]: $shift\ n\ (xt_1\ @\ xt_2) = shift\ n\ xt_1\ @\ shift\ n\ xt_2\langle proof\rangle$
lemma [*simp*]: $shift\ m\ (shift\ n\ xt) = shift\ (m+n)\ xt\langle proof\rangle$
lemma [*simp*]: $pcs\ (shift\ n\ xt) = \{pc+n|pc.\ pc \in pcs\ xt\}\langle proof\rangle$

lemma *shift-compxE$_2$*:
shows $\bigwedge pc\ pc'\ d.\ shift\ pc\ (compxE_2\ e\ pc'\ d) = compxE_2\ e\ (pc' + pc)\ d$
and $\bigwedge pc\ pc'\ d.\ shift\ pc\ (compxEs_2\ es\ pc'\ d) = compxEs_2\ es\ (pc' + pc)\ d\langle proof\rangle$

lemma *compxE$_2$-size-convs*[*simp*]:

shows $n \neq 0 \Longrightarrow compxE_2\ e\ n\ d = shift\ n\ (compxE_2\ e\ 0\ d)$
and $n \neq 0 \Longrightarrow compxEs_2\ es\ n\ d = shift\ n\ (compxEs_2\ es\ 0\ d)\langle proof\rangle$
locale $TC2 = TC1\ +$
fixes $T_r :: ty$ **and** $mxs :: pc$
begin

definition
$wt\text{-}instrs :: instr\ list \Rightarrow ex\text{-}table \Rightarrow ty_i'\ list \Rightarrow bool$
$((\vdash\ \text{-},\ \text{-}\ /[::]/\ \text{-})\ [0,0,51]\ 50)$ **where**
$\vdash is,xt\ [::]\ \tau s \longleftrightarrow size\ is < size\ \tau s \wedge pcs\ xt \subseteq \{0..<size\ is\} \wedge$
$(\forall pc< size\ is.\ P,T_r,mxs,size\ \tau s,xt \vdash is!pc,pc :: \tau s)$

end

notation $TC2.wt\text{-}instrs\ ((\text{-},\text{-},\text{-} \vdash/\ \text{-},\ \text{-}\ /[::]/\ \text{-})\ [50,50,50,50,50,51]\ 50)$

lemma (**in** $TC2$) [$simp$]: $\tau s \neq [] \Longrightarrow \vdash [],[]\ [::]\ \tau s\langle proof\rangle$
lemma [$simp$]: $eff\ i\ P\ pc\ et\ None = []\langle proof\rangle$
lemma $wt\text{-}instr\text{-}appR$:
$[\![\ P,T,m,mpc,xt \vdash is!pc,pc :: \tau s;$
$pc < size\ is;\ size\ is < size\ \tau s;\ mpc \leq size\ \tau s;\ mpc \leq mpc'\ ]\!]$
$\Longrightarrow P,T,m,mpc',xt \vdash is!pc,pc :: \tau s@\tau s'\langle proof\rangle$

lemma $relevant\text{-}entries\text{-}shift$ [$simp$]:
$relevant\text{-}entries\ P\ i\ (pc+n)\ (shift\ n\ xt) = shift\ n\ (relevant\text{-}entries\ P\ i\ pc\ xt)\langle proof\rangle$

lemma [$simp$]:
$xcpt\text{-}eff\ i\ P\ (pc+n)\ \tau\ (shift\ n\ xt) =$
$map\ (\lambda(pc,\tau).\ (pc + n,\ \tau))\ (xcpt\text{-}eff\ i\ P\ pc\ \tau\ xt)\langle proof\rangle$

lemma [$simp$]:
$app_i\ (i,\ P,\ pc,\ m,\ T,\ \tau) \Longrightarrow$
$eff\ i\ P\ (pc+n)\ (shift\ n\ xt)\ (Some\ \tau) =$
$map\ (\lambda(pc,\tau).\ (pc+n,\tau))\ (eff\ i\ P\ pc\ xt\ (Some\ \tau))\langle proof\rangle$

lemma [$simp$]:
$xcpt\text{-}app\ i\ P\ (pc+n)\ mxs\ (shift\ n\ xt)\ \tau = xcpt\text{-}app\ i\ P\ pc\ mxs\ xt\ \tau\langle proof\rangle$

lemma $wt\text{-}instr\text{-}appL$:
$[\![\ P,T,m,mpc,xt \vdash i,pc :: \tau s;\ pc < size\ \tau s;\ mpc \leq size\ \tau s\ ]\!]$
$\Longrightarrow P,T,m,mpc + size\ \tau s',shift\ (size\ \tau s')\ xt \vdash i,pc+size\ \tau s' :: \tau s'@\tau s\langle proof\rangle$

lemma $wt\text{-}instr\text{-}Cons$:
$[\![\ P,T,m,mpc - 1,[] \vdash i,pc - 1 :: \tau s;$
$0 < pc;\ 0 < mpc;\ pc < size\ \tau s + 1;\ mpc \leq size\ \tau s + 1\ ]\!]$
$\Longrightarrow P,T,m,mpc,[] \vdash i,pc :: \tau\#\tau s\langle proof\rangle$

lemma $wt\text{-}instr\text{-}append$:
$[\![\ P,T,m,mpc - size\ \tau s',[] \vdash i,pc - size\ \tau s' :: \tau s;$
$size\ \tau s' \leq pc;\ size\ \tau s' \leq mpc;\ pc < size\ \tau s + size\ \tau s';\ mpc \leq size\ \tau s + size\ \tau s'\ ]\!]$
$\Longrightarrow P,T,m,mpc,[] \vdash i,pc :: \tau s'@\tau s\langle proof\rangle$

lemma $xcpt\text{-}app\text{-}pcs$:
$pc \notin pcs\ xt \Longrightarrow xcpt\text{-}app\ i\ P\ pc\ mxs\ xt\ \tau\langle proof\rangle$

lemma *xcpt-eff-pcs*:
$pc \notin pcs\ xt \Longrightarrow xcpt\text{-}eff\ i\ P\ pc\ \tau\ xt = []\langle proof\rangle$

lemma *pcs-shift*:
$pc < n \Longrightarrow pc \notin pcs\ (shift\ n\ xt)\langle proof\rangle$

lemma *wt-instr-appRx*:
$[\![\ P,T,m,mpc,xt \vdash is!pc,pc :: \tau s;\ pc < size\ is;\ size\ is < size\ \tau s;\ mpc \le size\ \tau s\ ]\!]$
$\Longrightarrow P,T,m,mpc,xt\ @\ shift\ (size\ is)\ xt' \vdash is!pc,pc :: \tau s\langle proof\rangle$

lemma *wt-instr-appLx*:
$[\![\ P,T,m,mpc,xt \vdash i,pc :: \tau s;\ pc \notin pcs\ xt'\ ]\!]$
$\Longrightarrow P,T,m,mpc,xt'@xt \vdash i,pc :: \tau s\langle proof\rangle$

lemma (**in** *TC2*) *wt-instrs-extR*:
$\vdash is,xt\ [::]\ \tau s \Longrightarrow \vdash is,xt\ [::]\ \tau s\ @\ \tau s'\langle proof\rangle$

lemma (**in** *TC2*) *wt-instrs-ext*:
$[\![\ \vdash is_1,xt_1\ [::]\ \tau s_1@\tau s_2;\ \vdash is_2,xt_2\ [::]\ \tau s_2;\ size\ \tau s_1 = size\ is_1\ ]\!]$
$\Longrightarrow \vdash is_1@is_2,\ xt_1\ @\ shift\ (size\ is_1)\ xt_2\ [::]\ \tau s_1@\tau s_2\langle proof\rangle$
corollary (**in** *TC2*) *wt-instrs-ext2*:
$[\![\ \vdash is_2,xt_2\ [::]\ \tau s_2;\ \vdash is_1,xt_1\ [::]\ \tau s_1@\tau s_2;\ size\ \tau s_1 = size\ is_1\ ]\!]$
$\Longrightarrow \vdash is_1@is_2,\ xt_1\ @\ shift\ (size\ is_1)\ xt_2\ [::]\ \tau s_1@\tau s_2\langle proof\rangle$

corollary (**in** *TC2*) *wt-instrs-ext-prefix* [*trans*]:
$[\![\ \vdash is_1,xt_1\ [::]\ \tau s_1@\tau s_2;\ \vdash is_2,xt_2\ [::]\ \tau s_3;$
$size\ \tau s_1 = size\ is_1;\ \tau s_3 \le \tau s_2\ ]\!]$
$\Longrightarrow \vdash is_1@is_2,\ xt_1\ @\ shift\ (size\ is_1)\ xt_2\ [::]\ \tau s_1@\tau s_2\langle proof\rangle$

corollary (**in** *TC2*) *wt-instrs-app*:
assumes is_1: $\vdash is_1,xt_1\ [::]\ \tau s_1@[\tau]$
assumes is_2: $\vdash is_2,xt_2\ [::]\ \tau\#\tau s_2$
assumes s: $size\ \tau s_1 = size\ is_1$
shows $\vdash is_1@is_2,\ xt_1@shift\ (size\ is_1)\ xt_2\ [::]\ \tau s_1@\tau\#\tau s_2\langle proof\rangle$

corollary (**in** *TC2*) *wt-instrs-app-last*[*trans*]:
$[\![\ \vdash is_2,xt_2\ [::]\ \tau\#\tau s_2;\ \vdash is_1,xt_1\ [::]\ \tau s_1;$
$last\ \tau s_1 = \tau;\ \ size\ \tau s_1 = size\ is_1{+}1\ ]\!]$
$\Longrightarrow \vdash is_1@is_2,\ xt_1@shift\ (size\ is_1)\ xt_2\ [::]\ \tau s_1@\tau s_2\langle proof\rangle$

corollary (**in** *TC2*) *wt-instrs-append-last*[*trans*]:
$[\![\ \vdash is,xt\ [::]\ \tau s;\ P,T_r,mxs,mpc,[] \vdash i,pc :: \tau s;$
$pc = size\ is;\ mpc = size\ \tau s;\ size\ is + 1 < size\ \tau s\ ]\!]$
$\Longrightarrow \vdash is@[i],xt\ [::]\ \tau s\langle proof\rangle$

corollary (**in** *TC2*) *wt-instrs-app2*:
$[\![\ \vdash is_2,xt_2\ [::]\ \tau'\#\tau s_2;\ \ \vdash is_1,xt_1\ [::]\ \tau\#\tau s_1@[\tau'];$
$xt' = xt_1\ @\ shift\ (size\ is_1)\ xt_2;\ \ size\ \tau s_1{+}1 = size\ is_1\ ]\!]$
$\Longrightarrow \vdash is_1@is_2,xt'\ [::]\ \tau\#\tau s_1@\tau'\#\tau s_2\langle proof\rangle$

corollary (**in** *TC2*) *wt-instrs-app2-simp*[*trans*,*simp*]:
$[\![\ \vdash is_2,xt_2\ [::]\ \tau'\#\tau s_2;\ \ \vdash is_1,xt_1\ [::]\ \tau\#\tau s_1@[\tau'];\ size\ \tau s_1{+}1 = size\ is_1\ ]\!]$
$\Longrightarrow \vdash is_1@is_2,\ xt_1@shift\ (size\ is_1)\ xt_2\ [::]\ \tau\#\tau s_1@\tau'\#\tau s_2\langle proof\rangle$

corollary (**in** *TC2*) *wt-instrs-Cons*[*simp*]:
  $\llbracket$ $\tau s \neq$ []; ⊢ [*i*],[] [::] [τ,τ']; ⊢ *is*,*xt* [::] τ'#τs $\rrbracket$
  $\Longrightarrow$ ⊢ *i*#*is*,*shift 1 xt* [::] τ#τ'#τs⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*pro*
theory *Jinja*
imports
  J/TypeSafe
  J/Annotate

  J/execute-Bigstep
  J/execute-WellType
  JVM/JVMDefensive
  JVM/JVMListExample
  BV/BVExec
  BV/LBVJVM
  BV/BVNoTypeError
  BV/BVExample
  Compiler/TypeComp
begin

end

Bibliography

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