A Machine-Checked Model for a Java-like Language, Virtual Machine and Compiler

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Contents

Chapter 1

Preface

This document contains the automatically generated listings of the Isabelle sources for the theories defining and analysing Jinja (a Java-like programming language), the Jinja Virtual Machine, and the compiler. To shorten the document, all proofs have been hidden. For a detailed exposition of these theories see the paper by Klein and Nipkow [1, 2].

1.1 Theory Dependencies

Figure 1.1 shows the dependencies between the Isabelle theories in the following sections.

A tabled implementation of the reflexive transitive closure **theory** *Transitive-Closure-Table*
imports *Main*
begin

```
inductive rtrancl-path :: ('a ⇒ 'a ⇒ bool) ⇒ 'a ⇒ 'a list ⇒ 'a ⇒ bool
  for r :: 'a ⇒ 'a ⇒ bool
where
  base: rtrancl-path r x [] x
| step: r x y ⟹ rtrancl-path r y ys z ⟹ rtrancl-path r x (y # ys) z

lemma rtranclp-eq-rtrancl-path: r** x y = (∃xs. rtrancl-path r x xs y)
proof
  assume r** x y
  then show ∃xs. rtrancl-path r x xs y
  proof (induct rule: converse-rtranclp-induct)
    case base
    have rtrancl-path r y [] y by (rule rtrancl-path.base)
    then show ?case ..
  next
    case (step x z)
    from ‹∃xs. rtrancl-path r z xs y›
    obtain xs where rtrancl-path r z xs y ..
    with ‹r x z› have rtrancl-path r x (z # xs) y
      by (rule rtrancl-path.step)
    then show ?case ..
  qed
next
  assume ∃xs. rtrancl-path r x xs y
  then obtain xs where rtrancl-path r x xs y ..
  then show r** x y
  proof induct
    case (base x)
    show ?case by (rule rtranclp.rtrancl-refl)
  next
    case (step x y ys z)
    from ‹r x y› ‹r** y z› show ?case
      by (rule converse-rtranclp-into-rtranclp)
  qed
qed

lemma rtrancl-path-trans:
  assumes xy: rtrancl-path r x xs y
  and yz: rtrancl-path r y ys z
  shows rtrancl-path r x (xs @ ys) z using xy yz
proof (induct arbitrary: z)
  case (base x)
  then show ?case by simp
next
  case (step x y xs)
  then have rtrancl-path r y (xs @ ys) z
    by simp
  with ‹r x y› have rtrancl-path r x (y # (xs @ ys)) z
    by (rule rtrancl-path.step)
  then show ?case by simp
```

```
qed

lemma rtrancl-path-appendE:
  assumes xz: rtrancl-path r x (xs @ y # ys) z
  obtains rtrancl-path r x (xs @ [y]) y and rtrancl-path r y ys z using xz
proof (induct xs arbitrary: x)
  case Nil
  then have rtrancl-path r x (y # ys) z by simp
  then obtain xy: r x y and yz: rtrancl-path r y ys z
    by cases auto
  from xy have rtrancl-path r x [y] y
    by (rule rtrancl-path.step [OF - rtrancl-path.base])
  then have rtrancl-path r x ([] @ [y]) y by simp
  then show ?thesis using yz by (rule Nil)
next
  case (Cons a as)
  then have rtrancl-path r x (a # (as @ y # ys)) z by simp
  then obtain xa: r x a and az: rtrancl-path r a (as @ y # ys) z
    by cases auto
  show ?thesis
  proof (rule Cons(1) [OF - az])
    assume rtrancl-path r y ys z
    assume rtrancl-path r a (as @ [y]) y
    with xa have rtrancl-path r x (a # (as @ [y])) y
      by (rule rtrancl-path.step)
    then have rtrancl-path r x ((a # as) @ [y]) y
      by simp
    then show ?thesis using ‹rtrancl-path r y ys z›
      by (rule Cons(2))
  qed
qed

lemma rtrancl-path-distinct:
  assumes xy: rtrancl-path r x xs y
  obtains xs' where rtrancl-path r x xs' y and distinct (x # xs') using xy
proof (induct xs rule: measure-induct-rule [of length])
  case (less xs)
  show ?case
  proof (cases distinct (x # xs))
    case True
    with ‹rtrancl-path r x xs y› show ?thesis by (rule less)
  next
    case False
    then have ∃ as bs cs a. x # xs = as @ [a] @ bs @ [a] @ cs
      by (rule not-distinct-decomp)
    then obtain as bs cs a where xxs: x # xs = as @ [a] @ bs @ [a] @ cs
      by iprover
    show ?thesis
    proof (cases as)
      case Nil
      with xxs have x: x = a and xs: xs = bs @ a # cs
        by auto
      from x xs ‹rtrancl-path r x xs y› have cs: rtrancl-path r x cs y
        by (auto elim: rtrancl-path-appendE)
```

```
    from xs have length cs < length xs by simp
    then show ?thesis
      by (rule less(1)) (iprover intro: cs less(2))+
  next
    case (Cons d ds)
    with xxs have xs: xs = ds @ a # (bs @ [a] @ cs)
      by auto
    with ‹rtrancl-path r x xs y› obtain xa: rtrancl-path r x (ds @ [a]) a
      and ay: rtrancl-path r a (bs @ a # cs) y
      by (auto elim: rtrancl-path-appendE)
    from ay have rtrancl-path r a cs y by (auto elim: rtrancl-path-appendE)
    with xa have xy: rtrancl-path r x ((ds @ [a]) @ cs) y
      by (rule rtrancl-path-trans)
    from xs have length ((ds @ [a]) @ cs) < length xs by simp
    then show ?thesis
      by (rule less(1)) (iprover intro: xy less(2))+
  qed
 qed
qed

inductive rtrancl-tab :: ('a ⇒ 'a ⇒ bool) ⇒ 'a list ⇒ 'a ⇒ 'a ⇒ bool
  for r :: 'a ⇒ 'a ⇒ bool
where
  base: rtrancl-tab r xs x x
| step: x ∉ set xs ⟹ r x y ⟹ rtrancl-tab r (x # xs) y z ⟹ rtrancl-tab r xs x z

lemma rtrancl-path-imp-rtrancl-tab:
  assumes path: rtrancl-path r x xs y
  and x: distinct (x # xs)
  and ys: ({x} ∪ set xs) ∩ set ys = {}
  shows rtrancl-tab r ys x y using path x ys
proof (induct arbitrary: ys)
  case base
  show ?case by (rule rtrancl-tab.base)
next
  case (step x y zs z)
  then have x ∉ set ys by auto
  from step have distinct (y # zs) by simp
  moreover from step have ({y} ∪ set zs) ∩ set (x # ys) = {}
    by auto
  ultimately have rtrancl-tab r (x # ys) y z
    by (rule step)
  with ‹x ∉ set ys› ‹r x y›
  show ?case by (rule rtrancl-tab.step)
qed

lemma rtrancl-tab-imp-rtrancl-path:
  assumes tab: rtrancl-tab r ys x y
  obtains xs where rtrancl-path r x xs y using tab
proof induct
  case base
  from rtrancl-path.base show ?case by (rule base)
next
  case step show ?case by (iprover intro: step rtrancl-path.step)
```

qed

lemma *rtranclp-eq-rtrancl-tab-nil*: $r^{**}\ x\ y = rtrancl\text{-}tab\ r\ []\ x\ y$
proof
  assume $r^{**}\ x\ y$
  then obtain *xs* **where** *rtrancl-path r x xs y*
    by (*auto simp add*: *rtranclp-eq-rtrancl-path*)
  then obtain *xs'* **where** *xs'*: *rtrancl-path r x xs' y*
    and *distinct*: *distinct* (x # *xs'*)
    by (*rule rtrancl-path-distinct*)
  have $(\{x\} \cup set\ xs') \cap set\ [] = \{\}$ **by** *simp*
  with *xs' distinct* **show** *rtrancl-tab r* [] *x y*
    by (*rule rtrancl-path-imp-rtrancl-tab*)
next
  assume *rtrancl-tab r* [] *x y*
  then obtain *xs* **where** *rtrancl-path r x xs y*
    by (*rule rtrancl-tab-imp-rtrancl-path*)
  then show $r^{**}\ x\ y$
    by (*auto simp add*: *rtranclp-eq-rtrancl-path*)
qed

declare *rtranclp-rtrancl-eq*[*code del*]
declare *rtranclp-eq-rtrancl-tab-nil*[*THEN iffD2*, *code-pred-intro*]

code-pred *rtranclp* **using** *rtranclp-eq-rtrancl-tab-nil* [*THEN iffD1*] **by** *fastforce*

1.1.1 A simple example

datatype *ty* = *A* | *B* | *C*

inductive *test* :: $ty \Rightarrow ty \Rightarrow bool$
where
  test A B
| *test B A*
| *test B C*

Invoking with the predicate compiler and the generic code generator

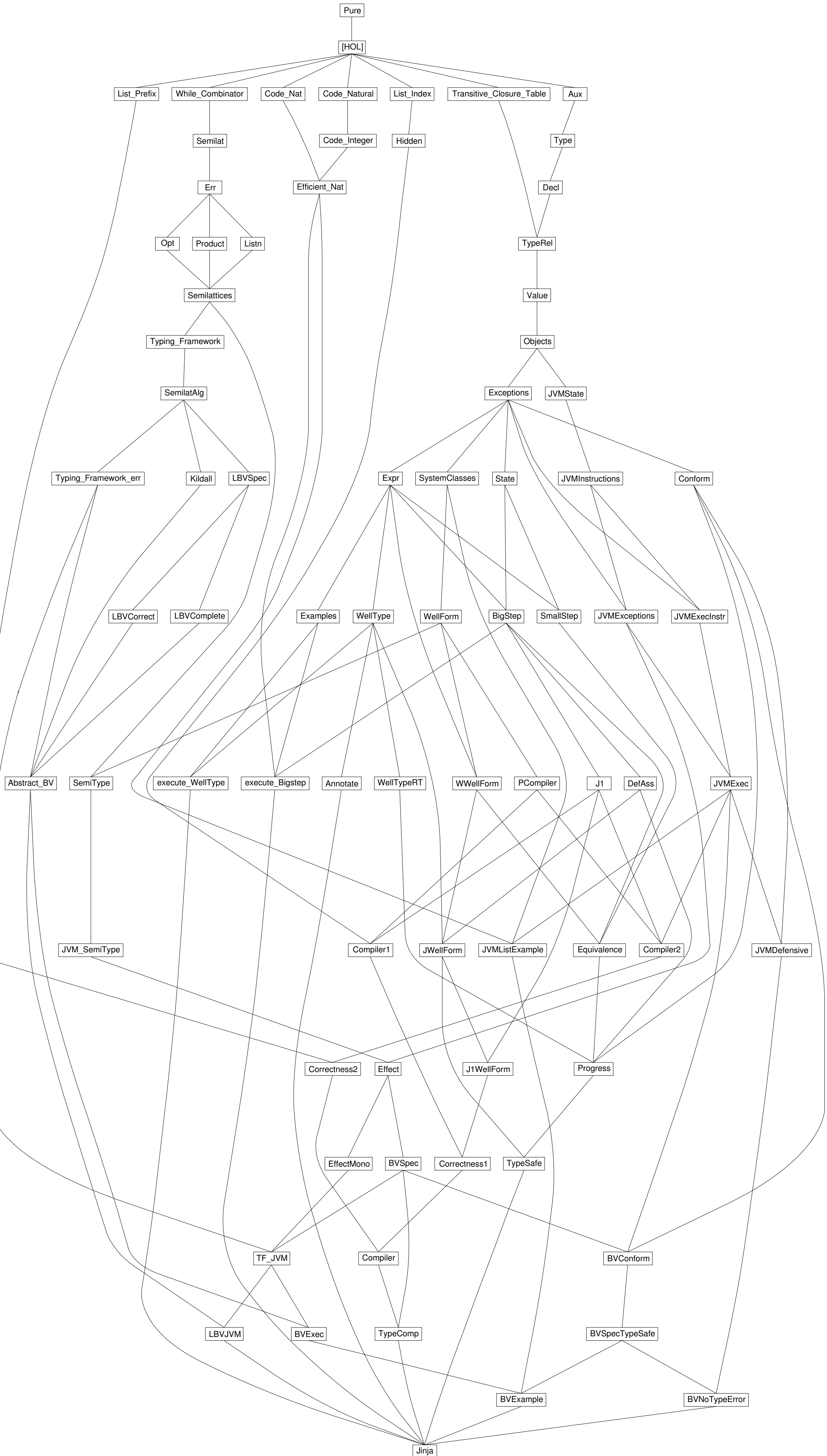
code-pred *test* **.**

values $\{x.\ test^{**}\ A\ C\}$
values $\{x.\ test^{**}\ C\ A\}$
values $\{x.\ test^{**}\ A\ x\}$
values $\{x.\ test^{**}\ x\ C\}$

value $test^{**}\ A\ C$
value $test^{**}\ C\ A$

hide-type *ty*
hide-const *test A B C*

end


Pure
[HOL]
List_Prefix
While_Combinator
Code_Nat
Code_Natural
List_Index
Transitive_Closure_Table
Aux
Semilat
Code_Integer
Hidden
Type
Err
Efficient_Nat
Decl
Opt
Product
Listn
TypeRel
Semilattices
Value
Typing_Framework
Objects
SemilatAlg
Exceptions
JVMState
Typing_Framework_err
Kildall
LBVSpec
Expr
SystemClasses
State
JVMInstructions
Conform
LBVCorrect
LBVComplete
Examples
WellType
WellForm
BigStep
SmallStep
JVMExceptions
JVMExecInstr
Abstract_BV
SemiType
execute_WellType
execute_Bigstep
Annotate
WellTypeRT
WWellForm
PCompiler
J1
DefAss
JVMExec
JVM_SemiType
Compiler1
JWellForm
JVMListExample
Equivalence
Compiler2
JVMDefensive
Correctness2
Effect
J1WellForm
Progress
EffectMono
BVSpec
Correctness1
TypeSafe
TF_JVM
Compiler
BVConform
LBVJVM
BVExec
TypeComp
BVSpecTypeSafe
BVExample
BVNoTypeError
Jinja

Chapter 2

Jinja Source Language

2.1 Auxiliary Definitions

theory *Aux* **imports** *Main* **begin**

lemma *nat-add-max-le*[*simp*]:
$((n::nat) + max\ i\ j \leq m) = (n + i \leq m \wedge n + j \leq m)$

lemma *Suc-add-max-le*[*simp*]:
$(Suc(n + max\ i\ j) \leq m) = (Suc(n + i) \leq m \wedge Suc(n + j) \leq m)$

notation *Some* $((\lfloor\text{-}\rfloor))$

2.1.1 *distinct-fst*

definition *distinct-fst* $::$ $('a \times 'b)$ *list* $\Rightarrow$ *bool*
where
distinct-fst $\equiv$ *distinct* $\circ$ *map fst*

lemma *distinct-fst-Nil* [*simp*]:
distinct-fst []

lemma *distinct-fst-Cons* [*simp*]:
distinct-fst $((k,x)\#kxs) = (\textit{distinct-fst}\ kxs \wedge (\forall y.\ (k,y) \notin \textit{set}\ kxs))$

lemma *map-of-SomeI*:
$[\![$ *distinct-fst kxs*; $(k,x) \in \textit{set}\ kxs$ $]\!] \Longrightarrow$ *map-of kxs k* $=$ *Some x*

2.1.2 Using *list-all2* for relations

definition *fun-of* $::$ $('a \times 'b)$ *set* $\Rightarrow 'a \Rightarrow 'b \Rightarrow$ *bool*
where
fun-of S $\equiv \lambda x\ y.\ (x,y) \in S$

Convenience lemmas

lemma *rel-list-all2-Cons* [*iff*]:
list-all2 (*fun-of S*) $(x\#xs)$ $(y\#ys)$ $=$
$((x,y) \in S \wedge$ *list-all2* (*fun-of S*) *xs ys*)

lemma *rel-list-all2-Cons1*:
list-all2 (*fun-of S*) $(x\#xs)$ *ys* $=$
$(\exists z\ zs.\ ys = z\#zs \wedge (x,z) \in S \wedge$ *list-all2* (*fun-of S*) *xs zs*)

lemma *rel-list-all2-Cons2*:
list-all2 (*fun-of S*) *xs* $(y\#ys)$ $=$
$(\exists z\ zs.\ xs = z\#zs \wedge (z,y) \in S \wedge$ *list-all2* (*fun-of S*) *zs ys*)

lemma *rel-list-all2-refl*:
$(\bigwedge x.\ (x,x) \in S) \Longrightarrow$ *list-all2* (*fun-of S*) *xs xs*

lemma *rel-list-all2-antisym*:
$[\![\ (\bigwedge x\ y.\ [\![(x,y) \in S;\ (y,x) \in T]\!] \Longrightarrow x = y);$
list-all2 (*fun-of S*) *xs ys*; *list-all2* (*fun-of T*) *ys xs* $]\!] \Longrightarrow xs = ys$

lemma *rel-list-all2-trans*:

$\llbracket \bigwedge a\ b\ c.\ \llbracket (a,b) \in R;\ (b,c) \in S \rrbracket \Longrightarrow (a,c) \in T;$
$\quad \textit{list-all2}\ (\textit{fun-of}\ R)\ \textit{as}\ \textit{bs};\ \textit{list-all2}\ (\textit{fun-of}\ S)\ \textit{bs}\ \textit{cs} \rrbracket$
$\Longrightarrow \textit{list-all2}\ (\textit{fun-of}\ T)\ \textit{as}\ \textit{cs}$

lemma *rel-list-all2-update-cong*:
$\llbracket\ i < \textit{size}\ \textit{xs};\ \textit{list-all2}\ (\textit{fun-of}\ S)\ \textit{xs}\ \textit{ys};\ (x,y) \in S\ \rrbracket$
$\Longrightarrow \textit{list-all2}\ (\textit{fun-of}\ S)\ (\textit{xs}[i:=x])\ (\textit{ys}[i:=y])$

lemma *rel-list-all2-nthD*:
$\llbracket\ \textit{list-all2}\ (\textit{fun-of}\ S)\ \textit{xs}\ \textit{ys};\ p < \textit{size}\ \textit{xs}\ \rrbracket \Longrightarrow (\textit{xs}!p,\textit{ys}!p) \in S$

lemma *rel-list-all2I*:
$\llbracket\ \textit{length}\ a = \textit{length}\ b;\ \bigwedge n.\ n < \textit{length}\ a \Longrightarrow (a!n,b!n) \in S\ \rrbracket \Longrightarrow \textit{list-all2}\ (\textit{fun-of}\ S)\ a\ b$

end

2.2 Jinja types

theory *Type* **imports** *Aux* **begin**

type-synonym *cname* = *string* — class names
type-synonym *mname* = *string* — method name
type-synonym *vname* = *string* — names for local/field variables

definition *Object* :: *cname*
where
Object ≡ ''*Object*''

definition *this* :: *vname*
where
this ≡ ''*this*''

— types
datatype *ty*
= *Void* — type of statements
| *Boolean*
| *Integer*
| *NT* — null type
| *Class cname* — class type

definition *is-refT* :: *ty* ⇒ *bool*
where
is-refT T ≡ $T = NT \vee (\exists C.\ T = Class\ C)$

lemma [*iff*]: *is-refT NT*
lemma [*iff*]: *is-refT*(*Class C*)
lemma *refTE*:
$[\![is\text{-}refT\ T;\ T = NT \Longrightarrow P;\ \bigwedge C.\ T = Class\ C \Longrightarrow P\ ]\!] \Longrightarrow P$
lemma *not-refTE*:
$[\![\ \neg is\text{-}refT\ T;\ T = Void \vee T = Boolean \vee T = Integer \Longrightarrow P\ ]\!] \Longrightarrow P$
end

2.3 Class Declarations and Programs

theory *Decl* **imports** *Type* **begin**

type-synonym
fdecl $=$ *vname* $\times$ *ty* — field declaration
type-synonym
'm mdecl $=$ *mname* $\times$ *ty list* $\times$ *ty* $\times$ *'m* — method = name, arg. types, return type, body
type-synonym
'm class $=$ *cname* $\times$ *fdecl list* $\times$ *'m mdecl list* — class = superclass, fields, methods
type-synonym
'm cdecl $=$ *cname* $\times$ *'m class* — class declaration
type-synonym
'm prog $=$ *'m cdecl list* — program

definition *class* :: *'m prog* $\Rightarrow$ *cname* $\rightharpoonup$ *'m class*
where
class $\equiv$ *map-of*

definition *is-class* :: *'m prog* $\Rightarrow$ *cname* $\Rightarrow$ *bool*
where
is-class P C $\equiv$ *class P C* $\neq$ *None*

lemma *finite-is-class*: *finite* $\{C.$ *is-class P C*$\}$

definition *is-type* :: *'m prog* $\Rightarrow$ *ty* $\Rightarrow$ *bool*
where
is-type P T $\equiv$
(*case T of Void* $\Rightarrow$ *True* | *Boolean* $\Rightarrow$ *True* | *Integer* $\Rightarrow$ *True* | *NT* $\Rightarrow$ *True*
| *Class C* $\Rightarrow$ *is-class P C*)

lemma *is-type-simps* [*simp*]:
is-type P Void $\wedge$ *is-type P Boolean* $\wedge$ *is-type P Integer* $\wedge$
is-type P NT $\wedge$ *is-type P* (*Class C*) $=$ *is-class P C*

abbreviation
types P $==$ *Collect* (*is-type P*)

end

2.4 Relations between Jinja Types

theory *TypeRel* **imports**
 ~~/src/HOL/Library/Transitive-Closure-Table
 Decl
begin

2.4.1 The subclass relations

inductive-set
 $subcls1 :: 'm\ prog \Rightarrow (cname \times cname)\ set$
 and $subcls1' :: 'm\ prog \Rightarrow [cname,\ cname] \Rightarrow bool\ (\text{-} \vdash \text{-} \prec^1 \text{-}\ [71,71,71]\ 70)$
 for $P :: 'm\ prog$
where
 $P \vdash C\ \prec^1\ D \equiv (C,D) \in subcls1\ P$
$|$ $subcls1I$: $\llbracket class\ P\ C = Some\ (D,rest);\ C \neq Object \rrbracket \Longrightarrow P \vdash C \prec^1 D$

abbreviation
 $subcls :: 'm\ prog \Rightarrow [cname,\ cname] \Rightarrow bool\ (\text{-} \vdash \text{-} \preceq^* \text{-}\ [71,71,71]\ 70)$
 where $P \vdash C\ \preceq^*\ D \equiv (C,D) \in (subcls1\ P)^*$

lemma $subcls1D$: $P \vdash C \prec^1 D \Longrightarrow C \neq Object \wedge (\exists fs\ ms.\ class\ P\ C = Some\ (D,fs,ms))$
lemma [*iff*]: $\neg\ P \vdash Object \prec^1 C$
lemma [*iff*]: $(P \vdash Object \preceq^* C) = (C = Object)$
lemma $subcls1\text{-}def2$:
 $subcls1\ P =$
  $(SIGMA\ C{:}\{C.\ is\text{-}class\ P\ C\}.\ \{D.\ C{\neq}Object \wedge fst\ (the\ (class\ P\ C)){=}D\})$
lemma $finite\text{-}subcls1$: $finite\ (subcls1\ P)$

2.4.2 The subtype relations

inductive
 $widen :: 'm\ prog \Rightarrow ty \Rightarrow ty \Rightarrow bool\ (\text{-} \vdash \text{-} \leq \text{-}\ [71,71,71]\ 70)$
 for $P :: 'm\ prog$
where
 $widen\text{-}refl[iff]$: $P \vdash T \leq T$
$|$ $widen\text{-}subcls$: $P \vdash C \preceq^* D \Longrightarrow P \vdash Class\ C \leq Class\ D$
$|$ $widen\text{-}null[iff]$: $P \vdash NT \leq Class\ C$

abbreviation (*xsymbols*)
 $widens :: 'm\ prog \Rightarrow ty\ list \Rightarrow ty\ list \Rightarrow bool$
  $(\text{-} \vdash \text{-}\ [\leq]\ \text{-}\ [71,71,71]\ 70)$ **where**
 $widens\ P\ Ts\ Ts' \equiv list\text{-}all2\ (widen\ P)\ Ts\ Ts'$

lemma [*iff*]: $(P \vdash T \leq Void) = (T = Void)$
lemma [*iff*]: $(P \vdash T \leq Boolean) = (T = Boolean)$
lemma [*iff*]: $(P \vdash T \leq Integer) = (T = Integer)$
lemma [*iff*]: $(P \vdash Void \leq T) = (T = Void)$
lemma [*iff*]: $(P \vdash Boolean \leq T) = (T = Boolean)$
lemma [*iff*]: $(P \vdash Integer \leq T) = (T = Integer)$

lemma $Class\text{-}widen$: $P \vdash Class\ C \leq T \Longrightarrow \exists D.\ T = Class\ D$
lemma [*iff*]: $(P \vdash T \leq NT) = (T = NT)$
lemma $Class\text{-}widen\text{-}Class$ [*iff*]: $(P \vdash Class\ C \leq Class\ D) = (P \vdash C \preceq^* D)$
lemma $widen\text{-}Class$: $(P \vdash T \leq Class\ C) = (T = NT \vee (\exists D.\ T = Class\ D \wedge P \vdash D \preceq^* C))$

lemma *widen-trans*[*trans*]: $[\![P \vdash S \leq U; P \vdash U \leq T]\!] \Longrightarrow P \vdash S \leq T$
lemma *widens-trans* [*trans*]: $[\![P \vdash Ss\ [\leq]\ Ts; P \vdash Ts\ [\leq]\ Us]\!] \Longrightarrow P \vdash Ss\ [\leq]\ Us$

2.4.3 Method lookup

inductive
 $Methods :: [{}'m\ prog,\ cname,\ mname \rightharpoonup (ty\ list \times ty \times {}'m) \times cname] \Rightarrow bool$
 $(\text{-} \vdash \text{-}\ sees'\text{-}methods\ \text{-}\ [51,51,51]\ 50)$
 for $P :: {}'m\ prog$
where
 sees-methods-Object:
 $[\![\ class\ P\ Object = Some(D,fs,ms);\ Mm = Option.map\ (\lambda m.\ (m,Object)) \circ map\text{-}of\ ms\ ]\!]$
 $\Longrightarrow P \vdash Object\ sees\text{-}methods\ Mm$
| *sees-methods-rec*:
 $[\![\ class\ P\ C = Some(D,fs,ms);\ C \neq Object;\ P \vdash D\ sees\text{-}methods\ Mm;$
 $Mm' = Mm\ {+}{+}\ (Option.map\ (\lambda m.\ (m,C)) \circ map\text{-}of\ ms)\ ]\!]$
 $\Longrightarrow P \vdash C\ sees\text{-}methods\ Mm'$

lemma *sees-methods-fun*:
assumes *1*: $P \vdash C\ sees\text{-}methods\ Mm$
shows $\bigwedge Mm'.\ P \vdash C\ sees\text{-}methods\ Mm' \Longrightarrow Mm' = Mm$

lemma *visible-methods-exist*:
 $P \vdash C\ sees\text{-}methods\ Mm \Longrightarrow Mm\ M = Some(m,D) \Longrightarrow$
 $(\exists D'\ fs\ ms.\ class\ P\ D = Some(D',fs,ms) \wedge map\text{-}of\ ms\ M = Some\ m)$

lemma *sees-methods-decl-above*:
assumes *Csees*: $P \vdash C\ sees\text{-}methods\ Mm$
shows $Mm\ M = Some(m,D) \Longrightarrow P \vdash C \preceq^* D$

lemma *sees-methods-idemp*:
assumes *Cmethods*: $P \vdash C\ sees\text{-}methods\ Mm$
shows $\bigwedge m\ D.\ Mm\ M = Some(m,D) \Longrightarrow$
 $\exists Mm'.\ (P \vdash D\ sees\text{-}methods\ Mm') \wedge Mm'\ M = Some(m,D)$

lemma *sees-methods-decl-mono*:
assumes *sub*: $P \vdash C' \preceq^* C$
shows $P \vdash C\ sees\text{-}methods\ Mm \Longrightarrow$
 $\exists Mm'\ Mm_2.\ P \vdash C'\ sees\text{-}methods\ Mm' \wedge Mm' = Mm\ {+}{+}\ Mm_2 \wedge$
 $(\forall M\ m\ D.\ Mm_2\ M = Some(m,D) \longrightarrow P \vdash D \preceq^* C)$

definition $Method :: {}'m\ prog \Rightarrow cname \Rightarrow mname \Rightarrow ty\ list \Rightarrow ty \Rightarrow {}'m \Rightarrow cname \Rightarrow bool$
 $(\text{-} \vdash \text{-}\ sees\ \text{-}:\ \text{-}{\rightarrow}\text{-} = \text{-}\ in\ \text{-}\ [51,51,51,51,51,51,51]\ 50)$
where
 $P \vdash C\ sees\ M:\ Ts{\rightarrow}T = m\ in\ D\ \equiv$
 $\exists Mm.\ P \vdash C\ sees\text{-}methods\ Mm \wedge Mm\ M = Some((Ts,T,m),D)$

definition $has\text{-}method :: {}'m\ prog \Rightarrow cname \Rightarrow mname \Rightarrow bool\ (\text{-} \vdash \text{-}\ has\ \text{-}\ [51,0,51]\ 50)$
where
 $P \vdash C\ has\ M \equiv \exists Ts\ T\ m\ D.\ P \vdash C\ sees\ M{:}Ts{\rightarrow}T = m\ in\ D$

lemma *sees-method-fun*:
$[\![P \vdash C \text{ sees } M{:}TS{\rightarrow}T = m \text{ in } D;\ P \vdash C \text{ sees } M{:}TS'{\rightarrow}T' = m' \text{ in } D']\!]$
$\Longrightarrow TS' = TS \wedge T' = T \wedge m' = m \wedge D' = D$

lemma *sees-method-decl-above*:
$P \vdash C \text{ sees } M{:}Ts{\rightarrow}T = m \text{ in } D \Longrightarrow P \vdash C \preceq^* D$

lemma *visible-method-exists*:
$P \vdash C \text{ sees } M{:}Ts{\rightarrow}T = m \text{ in } D \Longrightarrow$
$\exists D'\ fs\ ms.\ class\ P\ D = Some(D',fs,ms) \wedge map\text{-}of\ ms\ M = Some(Ts,T,m)$

lemma *sees-method-idemp*:
$P \vdash C \text{ sees } M{:}Ts{\rightarrow}T{=}m \text{ in } D \Longrightarrow P \vdash D \text{ sees } M{:}Ts{\rightarrow}T{=}m \text{ in } D$

lemma *sees-method-decl-mono*:
$[\![\ P \vdash C' \preceq^* C;\ P \vdash C \text{ sees } M{:}Ts{\rightarrow}T = m \text{ in } D;$
$P \vdash C' \text{ sees } M{:}Ts'{\rightarrow}T' = m' \text{ in } D'\ ]\!] \Longrightarrow P \vdash D' \preceq^* D$

lemma *sees-method-is-class*:
$[\![\ P \vdash C \text{ sees } M{:}Ts{\rightarrow}T = m \text{ in } D\ ]\!] \Longrightarrow is\text{-}class\ P\ C$

2.4.4 Field lookup

inductive
$Fields :: [{'}m\ prog,\ cname,\ ((vname \times cname) \times ty)\ list] \Rightarrow bool$
$(\text{-} \vdash \text{-}\ has'\text{-}fields\ \text{-}\ [51,51,51]\ 50)$
for $P :: {'}m\ prog$
where
has-fields-rec:
$[\![\ class\ P\ C = Some(D,fs,ms);\ C \neq Object;\ P \vdash D\ has\text{-}fields\ FDTs;$
$FDTs' = map\ (\lambda(F,T).\ ((F,C),T))\ fs\ @\ FDTs\ ]\!]$
$\Longrightarrow P \vdash C\ has\text{-}fields\ FDTs'$
| *has-fields-Object*:
$[\![\ class\ P\ Object = Some(D,fs,ms);\ FDTs = map\ (\lambda(F,T).\ ((F,Object),T))\ fs\ ]\!]$
$\Longrightarrow P \vdash Object\ has\text{-}fields\ FDTs$

lemma *has-fields-fun*:
assumes *1*: $P \vdash C\ has\text{-}fields\ FDTs$
shows $\bigwedge FDTs'.\ P \vdash C\ has\text{-}fields\ FDTs' \Longrightarrow FDTs' = FDTs$

lemma *all-fields-in-has-fields*:
assumes *sub*: $P \vdash C\ has\text{-}fields\ FDTs$
shows $[\![\ P \vdash C \preceq^* D;\ class\ P\ D = Some(D',fs,ms);\ (F,T) \in set\ fs\ ]\!]$
$\Longrightarrow ((F,D),T) \in set\ FDTs$

lemma *has-fields-decl-above*:
assumes *fields*: $P \vdash C\ has\text{-}fields\ FDTs$
shows $((F,D),T) \in set\ FDTs \Longrightarrow P \vdash C \preceq^* D$

lemma *subcls-notin-has-fields*:
assumes *fields*: $P \vdash C\ has\text{-}fields\ FDTs$
shows $((F,D),T) \in set\ FDTs \Longrightarrow (D,C) \notin (subcls1\ P)^+$

lemma *has-fields-mono-lem*:

assumes *sub*: $P \vdash D \preceq^* C$
shows $P \vdash C$ *has-fields FDTs*
$\Longrightarrow \exists pre.\ P \vdash D$ *has-fields pre@FDTs* $\wedge$ *dom*(*map-of pre*) $\cap$ *dom*(*map-of FDTs*) $= \{\}$

definition *has-field* :: $'m$ *prog* $\Rightarrow$ *cname* $\Rightarrow$ *vname* $\Rightarrow$ *ty* $\Rightarrow$ *cname* $\Rightarrow$ *bool*
(- $\vdash$ - *has* -:- *in* - [*51*,*51*,*51*,*51*,*51*] *50*)
where
$P \vdash C$ *has* F:T *in* $D \ \equiv$
$\exists$ *FDTs*. $P \vdash C$ *has-fields FDTs* $\wedge$ *map-of FDTs* $(F,D) =$ *Some* T

lemma *has-field-mono*:
$[\![P \vdash C$ *has* F:T *in* D; $P \vdash C' \preceq^* C\ ]\!] \Longrightarrow P \vdash C'$ *has* F:T *in* D

definition *sees-field* :: $'m$ *prog* $\Rightarrow$ *cname* $\Rightarrow$ *vname* $\Rightarrow$ *ty* $\Rightarrow$ *cname* $\Rightarrow$ *bool*
(- $\vdash$ - *sees* -:- *in* - [*51*,*51*,*51*,*51*,*51*] *50*)
where
$P \vdash C$ *sees* F:T *in* $D \ \equiv$
$\exists$ *FDTs*. $P \vdash C$ *has-fields FDTs* $\wedge$
map-of (*map* $(\lambda((F,D),T).\ (F,(D,T)))$ *FDTs*) F = *Some*(D,T)

lemma *map-of-remap-SomeD*:
map-of (*map* $(\lambda((k,k'),x).\ (k,(k',x)))$ *t*) k = *Some* $(k',x) \Longrightarrow$ *map-of t* $(k,\ k')$ = *Some x*

lemma *has-visible-field*:
$P \vdash C$ *sees* F:T *in* $D \Longrightarrow P \vdash C$ *has* F:T *in* D

lemma *sees-field-fun*:
$[\![P \vdash C$ *sees* F:T *in* D; $P \vdash C$ *sees* F:T' *in* $D']\!] \Longrightarrow T' = T \wedge D' = D$

lemma *sees-field-decl-above*:
$P \vdash C$ *sees* F:T *in* $D \Longrightarrow P \vdash C \preceq^* D$

lemma *sees-field-idemp*:
$P \vdash C$ *sees* F:T *in* $D \Longrightarrow P \vdash D$ *sees* F:T *in* D

2.4.5 Functional lookup

definition *method* :: $'m$ *prog* $\Rightarrow$ *cname* $\Rightarrow$ *mname* $\Rightarrow$ *cname* $\times$ *ty list* $\times$ *ty* $\times$ $'m$
where
method P C M $\equiv$ *THE* (D,Ts,T,m). $P \vdash C$ *sees* M:$Ts \rightarrow T = m$ *in* D

definition *field* :: $'m$ *prog* $\Rightarrow$ *cname* $\Rightarrow$ *vname* $\Rightarrow$ *cname* $\times$ *ty*
where
field P C F $\equiv$ *THE* (D,T). $P \vdash C$ *sees* F:T *in* D

definition *fields* :: $'m$ *prog* $\Rightarrow$ *cname* $\Rightarrow$ ((*vname* $\times$ *cname*) $\times$ *ty*) *list*
where
fields P C $\equiv$ *THE FDTs*. $P \vdash C$ *has-fields FDTs*

lemma *fields-def2* [*simp*]: $P \vdash C$ *has-fields FDTs* $\Longrightarrow$ *fields P C* = *FDTs*
lemma *field-def2* [*simp*]: $P \vdash C$ *sees* F:T *in* $D \Longrightarrow$ *field P C F* $= (D,T)$
lemma *method-def2* [*simp*]: $P \vdash C$ *sees* M: $Ts \rightarrow T = m$ *in* $D \Longrightarrow$ *method P C M* $= (D,Ts,T,m)$

2.4.6 Code generator setup

code-pred
  ($modes$: $i \Rightarrow i \Rightarrow i \Rightarrow bool$, $i \Rightarrow i \Rightarrow o \Rightarrow bool$)
  subcls1p
.
declare *subcls1-def* [*code-pred-def*]

code-pred
  ($modes$: $i \Rightarrow i \times o \Rightarrow bool$, $i \Rightarrow i \times i \Rightarrow bool$)
  [*inductify*]
  subcls1
.
definition $subcls'$ **where** $subcls'\ G = (subcls1p\ G)\hat{}{**}$
code-pred
  ($modes$: $i \Rightarrow i \Rightarrow i \Rightarrow bool$, $i \Rightarrow i \Rightarrow o \Rightarrow bool$)
  [*inductify*]
  $subcls'$
.

lemma *subcls-conv-subcls'* [*code-unfold*]:
  $(subcls1\ G)\hat{}{*} = \{(C, D).\ subcls'\ G\ C\ D\}$
  by (*simp add*: *subcls'-def subcls1-def rtrancl-def*)

code-pred
  ($modes$: $i \Rightarrow i \Rightarrow i \Rightarrow bool$)
  widen
.

code-pred
  ($modes$: $i \Rightarrow i \Rightarrow o \Rightarrow bool$)
  Fields
.

lemma *has-field-code* [*code-pred-intro*]:
  $[\![\ P \vdash C\ has\text{-}fields\ FDTs;\ map\text{-}of\ FDTs\ (F, D) = \lfloor T \rfloor\ ]\!]$
  $\Longrightarrow P \vdash C\ has\ F{:}T\ in\ D$
by(*auto simp add*: *has-field-def*)

code-pred
  ($modes$: $i \Rightarrow i \Rightarrow i \Rightarrow o \Rightarrow i \Rightarrow bool$, $i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow bool$)
  has-field
by(*auto simp add*: *has-field-def*)

lemma *sees-field-code* [*code-pred-intro*]:
  $[\![\ P \vdash C\ has\text{-}fields\ FDTs;\ map\text{-}of\ (map\ (\lambda((F, D), T).\ (F, D, T))\ FDTs)\ F = \lfloor (D, T) \rfloor\ ]\!]$
  $\Longrightarrow P \vdash C\ sees\ F{:}T\ in\ D$
by(*auto simp add*: *sees-field-def*)

code-pred
  ($modes$: $i \Rightarrow i \Rightarrow i \Rightarrow o \Rightarrow o \Rightarrow bool$, $i \Rightarrow i \Rightarrow i \Rightarrow o \Rightarrow i \Rightarrow bool$,
      $i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow o \Rightarrow bool$, $i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow bool$)
  sees-field
by(*auto simp add*: *sees-field-def*)

code-pred
  (*modes*: $i \Rightarrow i \Rightarrow o \Rightarrow bool$)
  Methods
.

lemma *Method-code* [*code-pred-intro*]:
  $\llbracket$ *P* ⊢ *C sees-methods Mm*; *Mm M* = ⌊((*Ts*, *T*, *m*), *D*)⌋ $\rrbracket$
  ⟹ *P* ⊢ *C sees M*: *Ts*→*T* = *m in D*
by(*auto simp add*: *Method-def*)

code-pred
  (*modes*: $i \Rightarrow i \Rightarrow i \Rightarrow o \Rightarrow o \Rightarrow o \Rightarrow o \Rightarrow bool$,
          $i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow bool$)
  Method
by(*auto simp add*: *Method-def*)

lemma *eval-Method-i-i-i-o-o-o-o-conv*:
  Predicate.eval (*Method-i-i-i-o-o-o-o P C M*) = (λ(*Ts*, *T*, *m*, *D*). *P* ⊢ *C sees M*:*Ts*→*T*=*m in D*)
by(*auto intro*: *Method-i-i-i-o-o-o-oI elim*: *Method-i-i-i-o-o-o-oE intro*!: *ext*)

lemma *method-code* [*code*]:
  method P C M =
  Predicate.the (*Predicate.bind* (*Method-i-i-i-o-o-o-o P C M*) (λ(*Ts*, *T*, *m*, *D*). *Predicate.single* (*D*, *Ts*, *T*, *m*)))
apply (*rule sym*, *rule the-eqI*)
apply (*simp add*: *method-def eval-Method-i-i-i-o-o-o-o-conv*)
apply (*rule arg-cong* [**where** *f*=*The*])
apply (*auto simp add*: *SUP-def Sup-fun-def Sup-bool-def fun-eq-iff*)
done

lemma *eval-Fields-conv*:
  Predicate.eval (*Fields-i-i-o P C*) = (λ*FDTs*. *P* ⊢ *C has-fields FDTs*)
by(*auto intro*: *Fields-i-i-oI elim*: *Fields-i-i-oE intro*!: *ext*)

lemma *fields-code* [*code*]:
  fields P C = *Predicate.the* (*Fields-i-i-o P C*)
by(*simp add*: *fields-def Predicate.the-def eval-Fields-conv*)

lemma *eval-sees-field-i-i-i-o-o-conv*:
  Predicate.eval (*sees-field-i-i-i-o-o P C F*) = (λ(*T*, *D*). *P* ⊢ *C sees F*:*T in D*)
by(*auto intro*!: *ext intro*: *sees-field-i-i-i-o-oI elim*: *sees-field-i-i-i-o-oE*)

lemma *eval-sees-field-i-i-i-o-i-conv*:
  Predicate.eval (*sees-field-i-i-i-o-i P C F D*) = (λ*T*. *P* ⊢ *C sees F*:*T in D*)
by(*auto intro*!: *ext intro*: *sees-field-i-i-i-o-iI elim*: *sees-field-i-i-i-o-iE*)

lemma *field-code* [*code*]:
  field P C F = *Predicate.the* (*Predicate.bind* (*sees-field-i-i-i-o-o P C F*) (λ(*T*, *D*). *Predicate.single* (*D*, *T*)))
apply (*rule sym*, *rule the-eqI*)
apply (*simp add*: *field-def eval-sees-field-i-i-i-o-o-conv*)
apply (*rule arg-cong* [**where** *f*=*The*])
apply (*auto simp add*: *SUP-def Sup-fun-def Sup-bool-def fun-eq-iff*)

done

2.5 Jinja Values

theory *Value* **imports** *TypeRel* **begin**

type-synonym *addr* = *nat*

datatype *val*
  = *Unit* — dummy result value of void expressions
  | *Null* — null reference
  | *Bool bool* — Boolean value
  | *Intg int* — integer value
  | *Addr addr* — addresses of objects in the heap

primrec *the-Intg* :: *val* ⇒ *int* **where**
  the-Intg (*Intg i*) = *i*

primrec *the-Addr* :: *val* ⇒ *addr* **where**
  the-Addr (*Addr a*) = *a*

primrec *default-val* :: *ty* ⇒ *val* — default value for all types **where**
  default-val Void = *Unit*
| *default-val Boolean* = *Bool False*
| *default-val Integer* = *Intg 0*
| *default-val NT* = *Null*
| *default-val* (*Class C*) = *Null*

end

2.6 Objects and the Heap

theory *Objects* **imports** *TypeRel Value* **begin**

2.6.1 Objects

type-synonym
$fields = vname \times cname \rightharpoonup val$ — field name, defining class, value
type-synonym
$obj = cname \times fields$ — class instance with class name and fields

definition $obj\text{-}ty :: obj \Rightarrow ty$
where
$obj\text{-}ty\ obj \equiv Class\ (fst\ obj)$

definition $init\text{-}fields :: ((vname \times cname) \times ty)\ list \Rightarrow fields$
where
$init\text{-}fields \equiv map\text{-}of \circ map\ (\lambda(F,T).\ (F,default\text{-}val\ T))$

— a new, blank object with default values in all fields:
definition $blank :: 'm\ prog \Rightarrow cname \Rightarrow obj$
where
$blank\ P\ C \equiv (C,init\text{-}fields\ (fields\ P\ C))$

lemma [*simp*]: $obj\text{-}ty\ (C,fs) = Class\ C$

2.6.2 Heap

type-synonym $heap = addr \rightharpoonup obj$

abbreviation
$cname\text{-}of :: heap \Rightarrow addr \Rightarrow cname$ **where**
$cname\text{-}of\ hp\ a == fst\ (the\ (hp\ a))$

definition $new\text{-}Addr :: heap \Rightarrow addr\ option$
where
$new\text{-}Addr\ h \equiv if\ \exists a.\ h\ a = None\ then\ Some(LEAST\ a.\ h\ a = None)\ else\ None$

definition $cast\text{-}ok :: 'm\ prog \Rightarrow cname \Rightarrow heap \Rightarrow val \Rightarrow bool$
where
$cast\text{-}ok\ P\ C\ h\ v \equiv v = Null \lor P \vdash cname\text{-}of\ h\ (the\text{-}Addr\ v) \preceq^* C$

definition $hext :: heap \Rightarrow heap \Rightarrow bool$ $(\text{-} \trianglelefteq \text{-}\ [51,51]\ 50)$
where
$h \trianglelefteq h' \equiv \forall a\ C\ fs.\ h\ a = Some(C,fs) \longrightarrow (\exists fs'.\ h'\ a = Some(C,fs'))$

primrec $typeof\text{-}h :: heap \Rightarrow val \Rightarrow ty\ option$ $(typeof_{\text{-}})$
where
$typeof_h\ Unit = Some\ Void$
$|\ typeof_h\ Null = Some\ NT$
$|\ typeof_h\ (Bool\ b) = Some\ Boolean$
$|\ typeof_h\ (Intg\ i) = Some\ Integer$
$|\ typeof_h\ (Addr\ a) = (case\ h\ a\ of\ None \Rightarrow None\ |\ Some(C,fs) \Rightarrow Some(Class\ C))$

lemma *new-Addr-SomeD*:

$new\text{-}Addr\ h = Some\ a \Longrightarrow h\ a = None$

lemma [*simp*]: $(typeof_h\ v = Some\ Boolean) = (\exists b.\ v = Bool\ b)$

lemma [*simp*]: $(typeof_h\ v = Some\ Integer) = (\exists i.\ v = Intg\ i)$
lemma [*simp*]: $(typeof_h\ v = Some\ NT) = (v = Null)$

lemma [*simp*]: $(typeof_h\ v = Some(Class\ C)) = (\exists a\ fs.\ v = Addr\ a \wedge h\ a = Some(C,fs))$

lemma [*simp*]: $h\ a = Some(C,fs) \Longrightarrow typeof_{(h(a \mapsto (C,fs')))}\ v = typeof_h\ v$

For literal values the first parameter of *typeof* can be set to *Map.empty* because they do not contain addresses:

abbreviation
$typeof :: val \Rightarrow ty\ option$ **where**
$typeof\ v == typeof\text{-}h\ empty\ v$

lemma *typeof-lit-typeof*:
$typeof\ v = Some\ T \Longrightarrow typeof_h\ v = Some\ T$

lemma *typeof-lit-is-type*:
$typeof\ v = Some\ T \Longrightarrow is\text{-}type\ P\ T$

2.6.3 Heap extension $\trianglelefteq$

lemma *hextI*: $\forall a\ C\ fs.\ h\ a = Some(C,fs) \longrightarrow (\exists fs'.\ h'\ a = Some(C,fs')) \Longrightarrow h \trianglelefteq h'$
lemma *hext-objD*: $[\![h \trianglelefteq h';\ h\ a = Some(C,fs)]\!] \Longrightarrow \exists fs'.\ h'\ a = Some(C,fs')$
lemma *hext-refl* [*iff*]: $h \trianglelefteq h$
lemma *hext-new* [*simp*]: $h\ a = None \Longrightarrow h \trianglelefteq h(a \mapsto x)$
lemma *hext-trans*: $[\![h \trianglelefteq h';\ h' \trianglelefteq h'']\!] \Longrightarrow h \trianglelefteq h''$
lemma *hext-upd-obj*: $h\ a = Some\ (C,fs) \Longrightarrow h \trianglelefteq h(a \mapsto (C,fs'))$
lemma *hext-typeof-mono*: $[\![h \trianglelefteq h';\ typeof_h\ v = Some\ T]\!] \Longrightarrow typeof_{h'}\ v = Some\ T$

Code generator setup for *new-Addr*

definition $gen\text{-}new\text{-}Addr :: heap \Rightarrow addr \Rightarrow addr\ option$
where $gen\text{-}new\text{-}Addr\ h\ n \equiv if\ \exists a.\ a \geq n \wedge h\ a = None\ then\ Some(LEAST\ a.\ a \geq n \wedge h\ a = None)\ else\ None$

lemma *new-Addr-code-code* [*code*]:
$new\text{-}Addr\ h = gen\text{-}new\text{-}Addr\ h\ 0$
by(*simp add*: *new-Addr-def gen-new-Addr-def split del*: *split-if cong*: *if-cong*)

lemma *gen-new-Addr-code* [*code*]:
$gen\text{-}new\text{-}Addr\ h\ n = (if\ h\ n = None\ then\ Some\ n\ else\ gen\text{-}new\text{-}Addr\ h\ (Suc\ n))$
apply(*simp add*: *gen-new-Addr-def*)
apply(*rule impI*)
apply(*rule conjI*)
apply *safe*[*1*]
apply(*fastforce intro*: *Least-equality*)
apply(*rule arg-cong*[**where** f=*Least*])
apply(*rule ext*)
apply(*case-tac* $n = ac$)

apply *simp*
apply(*auto*)[*1*]
apply *clarify*
apply(*subgoal-tac a = n*)
apply *simp*
apply(*rule Least-equality*)
apply *auto*[*2*]
apply(*rule ccontr*)
apply(*erule-tac x=a* **in** *allE*)
apply *simp*
done

end

2.7 Exceptions

theory *Exceptions* **imports** *Objects* **begin**

definition *NullPointer* :: *cname*
where
  $NullPointer \equiv ''NullPointer''$

definition *ClassCast* :: *cname*
where
  $ClassCast \equiv ''ClassCast''$

definition *OutOfMemory* :: *cname*
where
  $OutOfMemory \equiv ''OutOfMemory''$

definition *sys-xcpts* :: *cname set*
where
  $sys\text{-}xcpts \ \equiv\ \{NullPointer,\ ClassCast,\ OutOfMemory\}$

definition *addr-of-sys-xcpt* :: $cname \Rightarrow addr$
where
  $addr\text{-}of\text{-}sys\text{-}xcpt\ s \equiv if\ s = NullPointer\ then\ 0\ else$
  $if\ s = ClassCast\ then\ 1\ else$
  $if\ s = OutOfMemory\ then\ 2\ else\ undefined$

definition *start-heap* :: $'c\ prog \Rightarrow heap$
where
  $start\text{-}heap\ G \equiv empty\ (addr\text{-}of\text{-}sys\text{-}xcpt\ NullPointer \mapsto blank\ G\ NullPointer)$
  $(addr\text{-}of\text{-}sys\text{-}xcpt\ ClassCast \mapsto blank\ G\ ClassCast)$
  $(addr\text{-}of\text{-}sys\text{-}xcpt\ OutOfMemory \mapsto blank\ G\ OutOfMemory)$

definition *preallocated* :: $heap \Rightarrow bool$
where
  $preallocated\ h \equiv \forall\, C \in sys\text{-}xcpts.\ \exists fs.\ h(addr\text{-}of\text{-}sys\text{-}xcpt\ C) = Some\ (C,fs)$

2.7.1 System exceptions

lemma [*simp*]: $NullPointer \in sys\text{-}xcpts \wedge OutOfMemory \in sys\text{-}xcpts \wedge ClassCast \in sys\text{-}xcpts$

lemma *sys-xcpts-cases* [*consumes 1*, *cases set*]:
  $[\![\ C \in sys\text{-}xcpts;\ P\ NullPointer;\ P\ OutOfMemory;\ P\ ClassCast]\!] \Longrightarrow P\ C$

2.7.2 *preallocated*

lemma *preallocated-dom* [*simp*]:
  $[\![\ preallocated\ h;\ C \in sys\text{-}xcpts\ ]\!] \Longrightarrow addr\text{-}of\text{-}sys\text{-}xcpt\ C \in dom\ h$

lemma *preallocatedD*:
  $[\![\ preallocated\ h;\ C \in sys\text{-}xcpts\ ]\!] \Longrightarrow \exists fs.\ h(addr\text{-}of\text{-}sys\text{-}xcpt\ C) = Some\ (C,\ fs)$

lemma *preallocatedE* [*elim?*]:
  $[\![\ preallocated\ h;\ \ C \in sys\text{-}xcpts;\ \bigwedge fs.\ h(addr\text{-}of\text{-}sys\text{-}xcpt\ C) = Some(C,fs) \Longrightarrow P\ h\ C]\!]$
  $\Longrightarrow P\ h\ C$

lemma *cname-of-xcp* [*simp*]:
$[\![$ *preallocated* h; $C \in$ *sys-xcpts* $]\!] \Longrightarrow$ *cname-of* h (*addr-of-sys-xcpt* C) $= C$

lemma *typeof-ClassCast* [*simp*]:
preallocated $h \Longrightarrow$ typeof_h (*Addr*(*addr-of-sys-xcpt ClassCast*)) = *Some*(*Class ClassCast*)

lemma *typeof-OutOfMemory* [*simp*]:
preallocated $h \Longrightarrow$ typeof_h (*Addr*(*addr-of-sys-xcpt OutOfMemory*)) = *Some*(*Class OutOfMemory*)

lemma *typeof-NullPointer* [*simp*]:
preallocated $h \Longrightarrow$ typeof_h (*Addr*(*addr-of-sys-xcpt NullPointer*)) = *Some*(*Class NullPointer*)

lemma *preallocated-hext*:
$[\![$ *preallocated* h; $h \trianglelefteq h'$ $]\!] \Longrightarrow$ *preallocated* h'

lemma *preallocated-start*:
preallocated (*start-heap* P)

end

2.8 Expressions

theory *Expr*
imports *../Common/Exceptions*
begin

datatype *bop* = *Eq* | *Add* — names of binary operations

datatype *'a exp*
= *new cname* — class instance creation
| *Cast cname* (*'a exp*) — type cast
| *Val val* — value
| *BinOp* (*'a exp*) *bop* (*'a exp*) (- «-» - [*80*,*0*,*81*] *80*) — binary operation
| *Var 'a* — local variable (incl. parameter)
| *LAss 'a* (*'a exp*) (-:=- [*90*,*90*]*90*) — local assignment
| *FAcc* (*'a exp*) *vname cname* (-·-{-} [*10*,*90*,*99*]*90*) — field access
| *FAss* (*'a exp*) *vname cname* (*'a exp*) (-·-{-} := - [*10*,*90*,*99*,*90*]*90*) — field assignment
| *Call* (*'a exp*) *mname* (*'a exp list*) (-·-'(-') [*90*,*99*,*0*] *90*) — method call
| *Block 'a ty* (*'a exp*) ('{-:-; -})
| *Seq* (*'a exp*) (*'a exp*) (-;;/ - [*61*,*60*]*60*)
| *Cond* (*'a exp*) (*'a exp*) (*'a exp*) (*if* '(-') -/ *else* - [*80*,*79*,*79*]*70*)
| *While* (*'a exp*) (*'a exp*) (*while* '(-') - [*80*,*79*]*70*)
| *throw* (*'a exp*)
| *TryCatch* (*'a exp*) *cname 'a* (*'a exp*) (*try* -/ *catch*'(- -') - [*0*,*99*,*80*,*79*] *70*)

type-synonym
expr = *vname exp* — Jinja expression
type-synonym
J-mb = *vname list* × *expr* — Jinja method body: parameter names and expression
type-synonym
J-prog = *J-mb prog* — Jinja program

The semantics of binary operators:

fun *binop* :: *bop* × *val* × *val* ⇒ *val option* **where**
$binop(Eq,v_1,v_2) = Some(Bool\ (v_1 = v_2))$
| $binop(Add,Intg\ i_1,Intg\ i_2) = Some(Intg(i_1+i_2))$
| $binop(bop,v_1,v_2) = None$

lemma [*simp*]:
$(binop(Add,v_1,v_2) = Some\ v) = (\exists\, i_1\ i_2.\ v_1 = Intg\ i_1 \wedge v_2 = Intg\ i_2 \wedge v = Intg(i_1+i_2))$

2.8.1 Syntactic sugar

abbreviation (*input*)
InitBlock:: *'a* ⇒ *ty* ⇒ *'a exp* ⇒ *'a exp* ⇒ *'a exp* ((*1*'{-:- := -;/ -})) **where**
InitBlock V T e1 e2 == {*V*:*T*; *V* := *e1*;; *e2*}

abbreviation *unit* **where** *unit* == *Val Unit*
abbreviation *null* **where** *null* == *Val Null*
abbreviation *addr a* == *Val*(*Addr a*)
abbreviation *true* == *Val*(*Bool True*)
abbreviation *false* == *Val*(*Bool False*)

abbreviation

$Throw :: addr \Rightarrow {}'a\ exp$ **where**
$Throw\ a == throw(Val(Addr\ a))$

abbreviation
$THROW :: cname \Rightarrow {}'a\ exp$ **where**
$THROW\ xc == Throw(addr\text{-}of\text{-}sys\text{-}xcpt\ xc)$

2.8.2 Free Variables

primrec $fv :: expr \Rightarrow vname\ set$ **and** $fvs :: expr\ list \Rightarrow vname\ set$ **where**
$fv(new\ C) = \{\}$
$|\ fv(Cast\ C\ e) = fv\ e$
$|\ fv(Val\ v) = \{\}$
$|\ fv(e_1 \ll bop \gg e_2) = fv\ e_1 \cup fv\ e_2$
$|\ fv(Var\ V) = \{V\}$
$|\ fv(LAss\ V\ e) = \{V\} \cup fv\ e$
$|\ fv(e{\bullet}F\{D\}) = fv\ e$
$|\ fv(e_1{\bullet}F\{D\}{:=}e_2) = fv\ e_1 \cup fv\ e_2$
$|\ fv(e{\bullet}M(es)) = fv\ e \cup fvs\ es$
$|\ fv(\{V{:}T;\ e\}) = fv\ e - \{V\}$
$|\ fv(e_1;;e_2) = fv\ e_1 \cup fv\ e_2$
$|\ fv(if\ (b)\ e_1\ else\ e_2) = fv\ b \cup fv\ e_1 \cup fv\ e_2$
$|\ fv(while\ (b)\ e) = fv\ b \cup fv\ e$
$|\ fv(throw\ e) = fv\ e$
$|\ fv(try\ e_1\ catch(C\ V)\ e_2) = fv\ e_1 \cup (fv\ e_2 - \{V\})$
$|\ fvs([]) = \{\}$
$|\ fvs(e\#es) = fv\ e \cup fvs\ es$

lemma [*simp*]: $fvs(es_1\ @\ es_2) = fvs\ es_1 \cup fvs\ es_2$
lemma [*simp*]: $fvs(map\ Val\ vs) = \{\}$
end

2.9 Program State

theory *State* **imports** ../*Common*/*Exceptions* **begin**

type-synonym
  $locals = vname \rightharpoonup val$ — local vars, incl. params and "this"
type-synonym
  $state = heap \times locals$

definition $hp :: state \Rightarrow heap$
where
  $hp \equiv fst$
definition $lcl :: state \Rightarrow locals$
where
  $lcl \equiv snd$
end

2.10 Big Step Semantics

theory *BigStep* **imports** *Expr State* **begin**

inductive
  eval :: *J-prog* ⇒ *expr* ⇒ *state* ⇒ *expr* ⇒ *state* ⇒ *bool*
  (- ⊢ ((*1*⟨-,/-⟩) ⇒/ (*1*⟨-,/-⟩)) [*51*,*0*,*0*,*0*,*0*] *81*)
  and *evals* :: *J-prog* ⇒ *expr list* ⇒ *state* ⇒ *expr list* ⇒ *state* ⇒ *bool*
  (- ⊢ ((*1*⟨-,/-⟩) [⇒]/ (*1*⟨-,/-⟩)) [*51*,*0*,*0*,*0*,*0*] *81*)
  for *P* :: *J-prog*
where

New:
$[\![$ *new-Addr* h = *Some* a; $P \vdash C$ *has-fields* *FDTs*; $h' = h(a \mapsto (C,$*init-fields FDTs*$))$ $]\!]$
$\Longrightarrow P \vdash \langle$*new* $C,(h,l)\rangle \Rightarrow \langle$*addr* $a,(h',l)\rangle$

| *NewFail*:
new-Addr h = *None* $\Longrightarrow$
$P \vdash \langle$*new* C, $(h,l)\rangle \Rightarrow \langle$*THROW OutOfMemory*$,(h,l)\rangle$

| *Cast*:
$[\![$ $P \vdash \langle e,s_0\rangle \Rightarrow \langle$*addr* $a,(h,l)\rangle$; $h\ a$ = *Some*(D,fs); $P \vdash D \preceq^* C$ $]\!]$
$\Longrightarrow P \vdash \langle$*Cast* $C\ e,s_0\rangle \Rightarrow \langle$*addr* $a,(h,l)\rangle$

| *CastNull*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle$*null*$,s_1\rangle \Longrightarrow$
$P \vdash \langle$*Cast* $C\ e,s_0\rangle \Rightarrow \langle$*null*$,s_1\rangle$

| *CastFail*:
$[\![$ $P \vdash \langle e,s_0\rangle \Rightarrow \langle$*addr* $a,(h,l)\rangle$; $h\ a$ = *Some*(D,fs); $\neg\ P \vdash D \preceq^* C$ $]\!]$
$\Longrightarrow P \vdash \langle$*Cast* $C\ e,s_0\rangle \Rightarrow \langle$*THROW ClassCast*$,(h,l)\rangle$

| *CastThrow*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle$*throw* $e',s_1\rangle \Longrightarrow$
$P \vdash \langle$*Cast* $C\ e,s_0\rangle \Rightarrow \langle$*throw* $e',s_1\rangle$

| *Val*:
$P \vdash \langle$*Val* $v,s\rangle \Rightarrow \langle$*Val* $v,s\rangle$

| *BinOp*:
$[\![$ $P \vdash \langle e_1,s_0\rangle \Rightarrow \langle$*Val* $v_1,s_1\rangle$; $P \vdash \langle e_2,s_1\rangle \Rightarrow \langle$*Val* $v_2,s_2\rangle$; *binop*(bop,v_1,v_2) = *Some* v $]\!]$
$\Longrightarrow P \vdash \langle e_1 \ll bop \gg e_2,s_0\rangle \Rightarrow \langle$*Val* $v,s_2\rangle$

| *BinOpThrow1*:
$P \vdash \langle e_1,s_0\rangle \Rightarrow \langle$*throw* $e,s_1\rangle \Longrightarrow$
$P \vdash \langle e_1 \ll bop \gg e_2,\ s_0\rangle \Rightarrow \langle$*throw* $e,s_1\rangle$

| *BinOpThrow2*:
$[\![$ $P \vdash \langle e_1,s_0\rangle \Rightarrow \langle$*Val* $v_1,s_1\rangle$; $P \vdash \langle e_2,s_1\rangle \Rightarrow \langle$*throw* $e,s_2\rangle$ $]\!]$
$\Longrightarrow P \vdash \langle e_1 \ll bop \gg e_2,s_0\rangle \Rightarrow \langle$*throw* $e,s_2\rangle$

| *Var*:
$l\ V$ = *Some* $v \Longrightarrow$
$P \vdash \langle$*Var* $V,(h,l)\rangle \Rightarrow \langle$*Val* $v,(h,l)\rangle$

| *LAss*:
$[\![P \vdash \langle e,s_0\rangle \Rightarrow \langle Val\ v,(h,l)\rangle;\ l' = l(V \mapsto v)\ ]\!]$
$\Longrightarrow P \vdash \langle V:=e,s_0\rangle \Rightarrow \langle unit,(h,l')\rangle$

| *LAssThrow*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash \langle V:=e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$

| *FAcc*:
$[\![P \vdash \langle e,s_0\rangle \Rightarrow \langle addr\ a,(h,l)\rangle;\ h\ a = Some(C,fs);\ fs(F,D) = Some\ v\ ]\!]$
$\Longrightarrow P \vdash \langle e\cdot F\{D\},s_0\rangle \Rightarrow \langle Val\ v,(h,l)\rangle$

| *FAccNull*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle null,s_1\rangle \Longrightarrow$
$P \vdash \langle e\cdot F\{D\},s_0\rangle \Rightarrow \langle THROW\ NullPointer,s_1\rangle$

| *FAccThrow*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash \langle e\cdot F\{D\},s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$

| *FAss*:
$[\![P \vdash \langle e_1,s_0\rangle \Rightarrow \langle addr\ a,s_1\rangle;\ P \vdash \langle e_2,s_1\rangle \Rightarrow \langle Val\ v,(h_2,l_2)\rangle;$
$h_2\ a = Some(C,fs);\ fs' = fs((F,D)\mapsto v);\ h_2' = h_2(a\mapsto(C,fs'))\ ]\!]$
$\Longrightarrow P \vdash \langle e_1\cdot F\{D\}:=e_2,s_0\rangle \Rightarrow \langle unit,(h_2',l_2)\rangle$

| *FAssNull*:
$[\![P \vdash \langle e_1,s_0\rangle \Rightarrow \langle null,s_1\rangle;\ \ P \vdash \langle e_2,s_1\rangle \Rightarrow \langle Val\ v,s_2\rangle\ ]\!] \Longrightarrow$
$P \vdash \langle e_1\cdot F\{D\}:=e_2,s_0\rangle \Rightarrow \langle THROW\ NullPointer,s_2\rangle$

| *FAssThrow1*:
$P \vdash \langle e_1,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash \langle e_1\cdot F\{D\}:=e_2,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$

| *FAssThrow2*:
$[\![P \vdash \langle e_1,s_0\rangle \Rightarrow \langle Val\ v,s_1\rangle;\ P \vdash \langle e_2,s_1\rangle \Rightarrow \langle throw\ e',s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle e_1\cdot F\{D\}:=e_2,s_0\rangle \Rightarrow \langle throw\ e',s_2\rangle$

| *CallObjThrow*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash \langle e\cdot M(ps),s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$

| *CallParamsThrow*:
$[\![P \vdash \langle e,s_0\rangle \Rightarrow \langle Val\ v,s_1\rangle;\ P \vdash \langle es,s_1\rangle\ [\Rightarrow]\ \langle map\ Val\ vs\ @\ throw\ ex\ \#\ es',s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle e\cdot M(es),s_0\rangle \Rightarrow \langle throw\ ex,s_2\rangle$

| *CallNull*:
$[\![P \vdash \langle e,s_0\rangle \Rightarrow \langle null,s_1\rangle;\ \ P \vdash \langle ps,s_1\rangle\ [\Rightarrow]\ \langle map\ Val\ vs,s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle e\cdot M(ps),s_0\rangle \Rightarrow \langle THROW\ NullPointer,s_2\rangle$

| *Call*:
$[\![P \vdash \langle e,s_0\rangle \Rightarrow \langle addr\ a,s_1\rangle;\ \ P \vdash \langle ps,s_1\rangle\ [\Rightarrow]\ \langle map\ Val\ vs,(h_2,l_2)\rangle;$
$h_2\ a = Some(C,fs);\ \ P \vdash C\ sees\ M{:}Ts\to T = (pns,body)\ in\ D;$
$length\ vs = length\ pns;\ \ l_2' = [this\mapsto Addr\ a,\ pns[\mapsto]vs];$

$P \vdash \langle body,(h_2,l_2')\rangle \Rightarrow \langle e',(h_3,l_3)\rangle \ \rrbracket$
$\Longrightarrow P \vdash \langle e{\cdot}M(ps),s_0\rangle \Rightarrow \langle e',(h_3,l_2)\rangle$

| *Block*:
$P \vdash \langle e_0,(h_0,l_0(V{:=}None))\rangle \Rightarrow \langle e_1,(h_1,l_1)\rangle \Longrightarrow$
$P \vdash \langle\{V{:}T;\ e_0\},(h_0,l_0)\rangle \Rightarrow \langle e_1,(h_1,l_1(V{:=}l_0\ \ V))\rangle$

| *Seq*:
$\llbracket\ P \vdash \langle e_0,s_0\rangle \Rightarrow \langle Val\ v,s_1\rangle;\ P \vdash \langle e_1,s_1\rangle \Rightarrow \langle e_2,s_2\rangle\ \rrbracket$
$\Longrightarrow P \vdash \langle e_0;;e_1,s_0\rangle \Rightarrow \langle e_2,s_2\rangle$

| *SeqThrow*:
$P \vdash \langle e_0,s_0\rangle \Rightarrow \langle throw\ e,s_1\rangle \Longrightarrow$
$P \vdash \langle e_0;;e_1,s_0\rangle{\Rightarrow}\langle throw\ e,s_1\rangle$

| *CondT*:
$\llbracket\ P \vdash \langle e,s_0\rangle \Rightarrow \langle true,s_1\rangle;\ P \vdash \langle e_1,s_1\rangle \Rightarrow \langle e',s_2\rangle\ \rrbracket$
$\Longrightarrow P \vdash \langle if\ \ (e)\ \ e_1\ \ else\ \ e_2,s_0\rangle \Rightarrow \langle e',s_2\rangle$

| *CondF*:
$\llbracket\ P \vdash \langle e,s_0\rangle \Rightarrow \langle false,s_1\rangle;\ P \vdash \langle e_2,s_1\rangle \Rightarrow \langle e',s_2\rangle\ \rrbracket$
$\Longrightarrow P \vdash \langle if\ \ (e)\ \ e_1\ \ else\ \ e_2,s_0\rangle \Rightarrow \langle e',s_2\rangle$

| *CondThrow*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash \langle if\ \ (e)\ \ e_1\ \ else\ \ e_2,\ s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$

| *WhileF*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle false,s_1\rangle \Longrightarrow$
$P \vdash \langle while\ \ (e)\ \ c,s_0\rangle \Rightarrow \langle unit,s_1\rangle$

| *WhileT*:
$\llbracket\ P \vdash \langle e,s_0\rangle \Rightarrow \langle true,s_1\rangle;\ P \vdash \langle c,s_1\rangle \Rightarrow \langle Val\ v_1,s_2\rangle;\ P \vdash \langle while\ \ (e)\ \ c,s_2\rangle \Rightarrow \langle e_3,s_3\rangle\ \rrbracket$
$\Longrightarrow P \vdash \langle while\ \ (e)\ \ c,s_0\rangle \Rightarrow \langle e_3,s_3\rangle$

| *WhileCondThrow*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle\ throw\ e',s_1\rangle \Longrightarrow$
$P \vdash \langle while\ \ (e)\ \ c,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$

| *WhileBodyThrow*:
$\llbracket\ P \vdash \langle e,s_0\rangle \Rightarrow \langle true,s_1\rangle;\ P \vdash \langle c,s_1\rangle \Rightarrow \langle throw\ e',s_2\rangle\rrbracket$
$\Longrightarrow P \vdash \langle while\ \ (e)\ \ c,s_0\rangle \Rightarrow \langle throw\ e',s_2\rangle$

| *Throw*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle addr\ a,s_1\rangle \Longrightarrow$
$P \vdash \langle throw\ e,s_0\rangle \Rightarrow \langle Throw\ a,s_1\rangle$

| *ThrowNull*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle null,s_1\rangle \Longrightarrow$
$P \vdash \langle throw\ e,s_0\rangle \Rightarrow \langle THROW\ NullPointer,s_1\rangle$

| *ThrowThrow*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash \langle throw\ e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$

| *Try*:
$P \vdash \langle e_1,s_0\rangle \Rightarrow \langle Val\ v_1,s_1\rangle \Longrightarrow$
$P \vdash \langle try\ e_1\ catch(C\ V)\ e_2,s_0\rangle \Rightarrow \langle Val\ v_1,s_1\rangle$

| *TryCatch*:
$[\![\ P \vdash \langle e_1,s_0\rangle \Rightarrow \langle Throw\ a,(h_1,l_1)\rangle;\ \ h_1\ a = Some(D,fs);\ \ P \vdash D \preceq^* C;$
$P \vdash \langle e_2,(h_1,l_1(V\mapsto Addr\ a))\rangle \Rightarrow \langle e_2{}',(h_2,l_2)\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle try\ e_1\ catch(C\ V)\ e_2,s_0\rangle \Rightarrow \langle e_2{}',(h_2,l_2(V:=l_1\ V))\rangle$

| *TryThrow*:
$[\![\ P \vdash \langle e_1,s_0\rangle \Rightarrow \langle Throw\ a,(h_1,l_1)\rangle;\ \ h_1\ a = Some(D,fs);\ \ \neg\ P \vdash D \preceq^* C\ ]\!]$
$\Longrightarrow P \vdash \langle try\ e_1\ catch(C\ V)\ e_2,s_0\rangle \Rightarrow \langle Throw\ a,(h_1,l_1)\rangle$

| *Nil*:
$P \vdash \langle[],s\rangle\ [\Rightarrow]\ \langle[],s\rangle$

| *Cons*:
$[\![\ P \vdash \langle e,s_0\rangle \Rightarrow \langle Val\ v,s_1\rangle;\ P \vdash \langle es,s_1\rangle\ [\Rightarrow]\ \langle es',s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle e\#es,s_0\rangle\ [\Rightarrow]\ \langle Val\ v\ \#\ es',s_2\rangle$

| *ConsThrow*:
$P \vdash \langle e,\ s_0\rangle \Rightarrow \langle throw\ e',\ s_1\rangle \Longrightarrow$
$P \vdash \langle e\#es,\ s_0\rangle\ [\Rightarrow]\ \langle throw\ e'\ \#\ es,\ s_1\rangle$

2.10.1 Final expressions

definition *final* :: $'a\ exp \Rightarrow bool$
where
$final\ e\ \equiv\ (\exists\, v.\ e = Val\ v) \vee (\exists\, a.\ e = Throw\ a)$

definition *finals*:: $'a\ exp\ list \Rightarrow bool$
where
$finals\ es\ \equiv\ (\exists\, vs.\ es = map\ Val\ vs) \vee (\exists\, vs\ a\ es'.\ es = map\ Val\ vs\ @\ Throw\ a\ \#\ es')$

lemma [*simp*]: $final(Val\ v)$
lemma [*simp*]: $final(throw\ e) = (\exists\, a.\ e = addr\ a)$
lemma *finalE*: $[\![\ final\ e;\ \ \bigwedge v.\ e = Val\ v \Longrightarrow R;\ \ \bigwedge a.\ e = Throw\ a \Longrightarrow R\ ]\!] \Longrightarrow R$
lemma [*iff*]: $finals\ []$
lemma [*iff*]: $finals\ (Val\ v\ \#\ es) = finals\ es$
lemma *finals-app-map*[*iff*]: $finals\ (map\ Val\ vs\ @\ es) = finals\ es$
lemma [*iff*]: $finals\ (map\ Val\ vs)$
lemma [*iff*]: $finals\ (throw\ e\ \#\ es) = (\exists\, a.\ e = addr\ a)$
lemma *not-finals-ConsI*: $\neg\ final\ e \Longrightarrow \neg\ finals(e\#es)$

lemma *eval-final*: $P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle \Longrightarrow final\ e'$
and *evals-final*: $P \vdash \langle es,s\rangle\ [\Rightarrow]\ \langle es',s'\rangle \Longrightarrow finals\ es'$

lemma *eval-lcl-incr*: $P \vdash \langle e,(h_0,l_0)\rangle \Rightarrow \langle e',(h_1,l_1)\rangle \Longrightarrow dom\ l_0 \subseteq dom\ l_1$
and *evals-lcl-incr*: $P \vdash \langle es,(h_0,l_0)\rangle\ [\Rightarrow]\ \langle es',(h_1,l_1)\rangle \Longrightarrow dom\ l_0 \subseteq dom\ l_1$

Only used later, in the small to big translation, but is already a good sanity check:

lemma *eval-finalId*: *final* $e \Longrightarrow P \vdash \langle e,s \rangle \Rightarrow \langle e,s \rangle$

lemma *eval-finalsId*:
assumes *finals*: *finals es* **shows** $P \vdash \langle es,s \rangle\ [\Rightarrow]\ \langle es,s \rangle$

theorem *eval-hext*: $P \vdash \langle e,(h,l) \rangle \Rightarrow \langle e',(h',l') \rangle \Longrightarrow h \trianglelefteq h'$
and *evals-hext*: $P \vdash \langle es,(h,l) \rangle\ [\Rightarrow]\ \langle es',(h',l') \rangle \Longrightarrow h \trianglelefteq h'$

end

2.11 Small Step Semantics

theory *SmallStep*
imports *Expr State*
begin

fun *blocks* :: *vname list* $*$ *ty list* $*$ *val list* $*$ *expr* $\Rightarrow$ *expr*
where
$blocks(V\#Vs,\ T\#Ts,\ v\#vs,\ e) = \{V{:}T := Val\ v;\ blocks(Vs,Ts,vs,e)\}$
$|blocks([],[],[],e) = e$

lemmas *blocks-induct* = *blocks.induct*[*split-format* (*complete*)]

lemma [*simp*]:
$[\![\ size\ vs = size\ Vs;\ size\ Ts = size\ Vs\ ]\!] \Longrightarrow fv(blocks(Vs,Ts,vs,e)) = fv\ e - set\ Vs$

definition *assigned* :: *vname* $\Rightarrow$ *expr* $\Rightarrow$ *bool*
where
$assigned\ V\ e \ \equiv\ \exists\, v\ e'.\ e = (V := Val\ v;;\ e')$

inductive-set
red :: *J-prog* $\Rightarrow$ ((*expr* $\times$ *state*) $\times$ (*expr* $\times$ *state*)) *set*
and *reds* :: *J-prog* $\Rightarrow$ ((*expr list* $\times$ *state*) $\times$ (*expr list* $\times$ *state*)) *set*
and *red'* :: *J-prog* $\Rightarrow$ *expr* $\Rightarrow$ *state* $\Rightarrow$ *expr* $\Rightarrow$ *state* $\Rightarrow$ *bool*
$(\text{-} \vdash ((1\langle\text{-},/\text{-}\rangle) \rightarrow/\ (1\langle\text{-},/\text{-}\rangle))\ [51,0,0,0,0]\ 81)$
and *reds'* :: *J-prog* $\Rightarrow$ *expr list* $\Rightarrow$ *state* $\Rightarrow$ *expr list* $\Rightarrow$ *state* $\Rightarrow$ *bool*
$(\text{-} \vdash ((1\langle\text{-},/\text{-}\rangle)\ [\rightarrow]/\ (1\langle\text{-},/\text{-}\rangle))\ [51,0,0,0,0]\ 81)$
for *P* :: *J-prog*
where

$P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle \equiv ((e,s),\ e',s') \in red\ P$
$|\ P \vdash \langle es,s\rangle\ [\rightarrow]\ \langle es',s'\rangle \equiv ((es,s),\ es',s') \in reds\ P$

| *RedNew*:
$[\![\ new\text{-}Addr\ h = Some\ a;\ P \vdash C\ has\text{-}fields\ FDTs;\ h' = h(a\mapsto(C,init\text{-}fields\ FDTs))\ ]\!]$
$\Longrightarrow P \vdash \langle new\ C,\ (h,l)\rangle \rightarrow \langle addr\ a,\ (h',l)\rangle$

| *RedNewFail*:
$new\text{-}Addr\ h = None \Longrightarrow$
$P \vdash \langle new\ C,\ (h,l)\rangle \rightarrow \langle THROW\ OutOfMemory,\ (h,l)\rangle$

| *CastRed*:
$P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle Cast\ C\ e,\ s\rangle \rightarrow \langle Cast\ C\ e',\ s'\rangle$

| *RedCastNull*:
$P \vdash \langle Cast\ C\ null,\ s\rangle \rightarrow \langle null,s\rangle$

| *RedCast*:
$[\![\ hp\ s\ a = Some(D,fs);\ P \vdash D \preceq^* C\ ]\!]$
$\Longrightarrow P \vdash \langle Cast\ C\ (addr\ a),\ s\rangle \rightarrow \langle addr\ a,\ s\rangle$

| *RedCastFail*:
$[\![\ hp\ s\ a = Some(D,fs);\ \neg\ P \vdash D \preceq^* C\ ]\!]$

$\Longrightarrow P \vdash \langle Cast\ C\ (addr\ a),\ s\rangle \to \langle THROW\ ClassCast,\ s\rangle$

| *BinOpRed1*:
$P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle e \ll bop \gg e_2,\ s\rangle \to \langle e' \ll bop \gg e_2,\ s'\rangle$

| *BinOpRed2*:
$P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle (Val\ v_1) \ll bop \gg e,\ s\rangle \to \langle (Val\ v_1) \ll bop \gg e',\ s'\rangle$

| *RedBinOp*:
$binop(bop,v_1,v_2) = Some\ v \Longrightarrow$
$P \vdash \langle (Val\ v_1) \ll bop \gg (Val\ v_2),\ s\rangle \to \langle Val\ v,s\rangle$

| *RedVar*:
$lcl\ s\ V = Some\ v \Longrightarrow$
$P \vdash \langle Var\ V,s\rangle \to \langle Val\ v,s\rangle$

| *LAssRed*:
$P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle V{:=}e,s\rangle \to \langle V{:=}e',s'\rangle$

| *RedLAss*:
$P \vdash \langle V{:=}(Val\ v),\ (h,l)\rangle \to \langle unit,\ (h,l(V \mapsto v))\rangle$

| *FAccRed*:
$P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle e{\cdot}F\{D\},\ s\rangle \to \langle e'{\cdot}F\{D\},\ s'\rangle$

| *RedFAcc*:
$[\![\ hp\ s\ a = Some(C,fs);\ fs(F,D) = Some\ v\ ]\!]$
$\Longrightarrow P \vdash \langle (addr\ a){\cdot}F\{D\},\ s\rangle \to \langle Val\ v,s\rangle$

| *RedFAccNull*:
$P \vdash \langle null{\cdot}F\{D\},\ s\rangle \to \langle THROW\ NullPointer,\ s\rangle$

| *FAssRed1*:
$P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle e{\cdot}F\{D\}{:=}e_2,\ s\rangle \to \langle e'{\cdot}F\{D\}{:=}e_2,\ s'\rangle$

| *FAssRed2*:
$P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle Val\ v{\cdot}F\{D\}{:=}e,\ s\rangle \to \langle Val\ v{\cdot}F\{D\}{:=}e',\ s'\rangle$

| *RedFAss*:
$h\ a = Some(C,fs) \Longrightarrow$
$P \vdash \langle (addr\ a){\cdot}F\{D\}{:=}(Val\ v),\ (h,l)\rangle \to \langle unit,\ (h(a \mapsto (C,fs((F,D) \mapsto v))),l)\rangle$

| *RedFAssNull*:
$P \vdash \langle null{\cdot}F\{D\}{:=}Val\ v,\ s\rangle \to \langle THROW\ NullPointer,\ s\rangle$

| *CallObj*:
$P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle e{\cdot}M(es),s\rangle \to \langle e'{\cdot}M(es),s'\rangle$

| *CallParams*:
$P \vdash \langle es,s\rangle \ [\rightarrow] \ \langle es',s'\rangle \Longrightarrow$
$P \vdash \langle (Val\ v)\cdot M(es),s\rangle \rightarrow \langle (Val\ v)\cdot M(es'),s'\rangle$

| *RedCall*:
$[\![\ hp\ s\ a = Some(C,fs);\ P \vdash C\ sees\ M{:}Ts{\rightarrow}T = (pns,body)\ in\ D;\ size\ vs = size\ pns;\ size\ Ts = size\ pns\ ]\!]$
$\Longrightarrow P \vdash \langle (addr\ a)\cdot M(map\ Val\ vs),\ s\rangle \rightarrow \langle blocks(this\#pns,\ Class\ D\#Ts,\ Addr\ a\#vs,\ body),\ s\rangle$

| *RedCallNull*:
$P \vdash \langle null\cdot M(map\ Val\ vs),s\rangle \rightarrow \langle THROW\ NullPointer,s\rangle$

| *BlockRedNone*:
$[\![\ P \vdash \langle e,\ (h,l(V{:=}None))\rangle \rightarrow \langle e',\ (h',l')\rangle;\ l'\ V = None;\ \neg\ assigned\ V\ e\ ]\!]$
$\Longrightarrow P \vdash \langle \{V{:}T;\ e\},\ (h,l)\rangle \rightarrow \langle \{V{:}T;\ e'\},\ (h',l'(V := l\ V))\rangle$

| *BlockRedSome*:
$[\![\ P \vdash \langle e,\ (h,l(V{:=}None))\rangle \rightarrow \langle e',\ (h',l')\rangle;\ l'\ V = Some\ v;\neg\ assigned\ V\ e\ ]\!]$
$\Longrightarrow P \vdash \langle \{V{:}T;\ e\},\ (h,l)\rangle \rightarrow \langle \{V{:}T := Val\ v;\ e'\},\ (h',l'(V := l\ V))\rangle$

| *InitBlockRed*:
$[\![\ P \vdash \langle e,\ (h,l(V{\mapsto}v))\rangle \rightarrow \langle e',\ (h',l')\rangle;\ l'\ V = Some\ v'\ ]\!]$
$\Longrightarrow P \vdash \langle \{V{:}T := Val\ v;\ e\},\ (h,l)\rangle \rightarrow \langle \{V{:}T := Val\ v';\ e'\},\ (h',l'(V := l\ V))\rangle$

| *RedBlock*:
$P \vdash \langle \{V{:}T;\ Val\ u\},\ s\rangle \rightarrow \langle Val\ u,\ s\rangle$

| *RedInitBlock*:
$P \vdash \langle \{V{:}T := Val\ v;\ Val\ u\},\ s\rangle \rightarrow \langle Val\ u,\ s\rangle$

| *SeqRed*:
$P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle e;;e_2,\ s\rangle \rightarrow \langle e';;e_2,\ s'\rangle$

| *RedSeq*:
$P \vdash \langle (Val\ v);;e_2,\ s\rangle \rightarrow \langle e_2,\ s\rangle$

| *CondRed*:
$P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle if\ (e)\ e_1\ else\ e_2,\ s\rangle \rightarrow \langle if\ (e')\ e_1\ else\ e_2,\ s'\rangle$

| *RedCondT*:
$P \vdash \langle if\ (true)\ e_1\ else\ e_2,\ s\rangle \rightarrow \langle e_1,\ s\rangle$

| *RedCondF*:
$P \vdash \langle if\ (false)\ e_1\ else\ e_2,\ s\rangle \rightarrow \langle e_2,\ s\rangle$

| *RedWhile*:
$P \vdash \langle while(b)\ c,\ s\rangle \rightarrow \langle if(b)\ (c;;while(b)\ c)\ else\ unit,\ s\rangle$

| *ThrowRed*:
$P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle throw\ e,\ s\rangle \rightarrow \langle throw\ e',\ s'\rangle$

| *RedThrowNull*:
  $P \vdash \langle throw\ null,\ s\rangle \to \langle THROW\ NullPointer,\ s\rangle$

| *TryRed*:
  $P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow$
  $P \vdash \langle try\ e\ catch(C\ V)\ e_2,\ s\rangle \to \langle try\ e'\ catch(C\ V)\ e_2,\ s'\rangle$

| *RedTry*:
  $P \vdash \langle try\ (Val\ v)\ catch(C\ V)\ e_2,\ s\rangle \to \langle Val\ v,\ s\rangle$

| *RedTryCatch*:
  $[\![\ hp\ s\ a = Some(D,fs);\ P \vdash D \preceq^* C\ ]\!]$
  $\Longrightarrow P \vdash \langle try\ (Throw\ a)\ catch(C\ V)\ e_2,\ s\rangle \to \langle\{V{:}Class\ C := addr\ a;\ e_2\},\ s\rangle$

| *RedTryFail*:
  $[\![\ hp\ s\ a = Some(D,fs);\ \neg\ P \vdash D \preceq^* C\ ]\!]$
  $\Longrightarrow P \vdash \langle try\ (Throw\ a)\ catch(C\ V)\ e_2,\ s\rangle \to \langle Throw\ a,\ s\rangle$

| *ListRed1*:
  $P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow$
  $P \vdash \langle e\#es,s\rangle\ [\to]\ \langle e'\#es,s'\rangle$

| *ListRed2*:
  $P \vdash \langle es,s\rangle\ [\to]\ \langle es',s'\rangle \Longrightarrow$
  $P \vdash \langle Val\ v\ \#\ es,s\rangle\ [\to]\ \langle Val\ v\ \#\ es',s'\rangle$

— Exception propagation

| *CastThrow*: $P \vdash \langle Cast\ C\ (throw\ e),\ s\rangle \to \langle throw\ e,\ s\rangle$
| *BinOpThrow1*: $P \vdash \langle (throw\ e)\ \ll bop \gg\ e_2,\ s\rangle \to \langle throw\ e,\ s\rangle$
| *BinOpThrow2*: $P \vdash \langle (Val\ v_1)\ \ll bop \gg\ (throw\ e),\ s\rangle \to \langle throw\ e,\ s\rangle$
| *LAssThrow*: $P \vdash \langle V{:=}(throw\ e),\ s\rangle \to \langle throw\ e,\ s\rangle$
| *FAccThrow*: $P \vdash \langle (throw\ e){\cdot}F\{D\},\ s\rangle \to \langle throw\ e,\ s\rangle$
| *FAssThrow1*: $P \vdash \langle (throw\ e){\cdot}F\{D\}{:=}e_2,\ s\rangle \to \langle throw\ e,s\rangle$
| *FAssThrow2*: $P \vdash \langle Val\ v{\cdot}F\{D\}{:=}(throw\ e),\ s\rangle \to \langle throw\ e,\ s\rangle$
| *CallThrowObj*: $P \vdash \langle (throw\ e){\cdot}M(es),\ s\rangle \to \langle throw\ e,\ s\rangle$
| *CallThrowParams*: $[\![\ es = map\ Val\ vs\ @\ throw\ e\ \#\ es'\ ]\!] \Longrightarrow P \vdash \langle (Val\ v){\cdot}M(es),\ s\rangle \to \langle throw\ e,\ s\rangle$
| *BlockThrow*: $P \vdash \langle\{V{:}T;\ Throw\ a\},\ s\rangle \to \langle Throw\ a,\ s\rangle$
| *InitBlockThrow*: $P \vdash \langle\{V{:}T := Val\ v;\ Throw\ a\},\ s\rangle \to \langle Throw\ a,\ s\rangle$
| *SeqThrow*: $P \vdash \langle (throw\ e);;e_2,\ s\rangle \to \langle throw\ e,\ s\rangle$
| *CondThrow*: $P \vdash \langle if\ (throw\ e)\ e_1\ else\ e_2,\ s\rangle \to \langle throw\ e,\ s\rangle$
| *ThrowThrow*: $P \vdash \langle throw(throw\ e),\ s\rangle \to \langle throw\ e,\ s\rangle$

2.11.1 The reflexive transitive closure

abbreviation
  Step :: *J-prog* ⇒ *expr* ⇒ *state* ⇒ *expr* ⇒ *state* ⇒ *bool*
    (- ⊢ ((*1*⟨-,/-⟩) →∗/ (*1*⟨-,/-⟩)) [*51*,*0*,*0*,*0*,*0*] *81*)
  where $P \vdash \langle e,s\rangle \to\!*\ \langle e',s'\rangle \equiv ((e,s),\ e',s') \in (red\ P)^*$

abbreviation
  Steps :: *J-prog* ⇒ *expr list* ⇒ *state* ⇒ *expr list* ⇒ *state* ⇒ *bool*
    (- ⊢ ((*1*⟨-,/-⟩) [→]∗/ (*1*⟨-,/-⟩)) [*51*,*0*,*0*,*0*,*0*] *81*)

where $P \vdash \langle es,s\rangle\ [\rightarrow]{*}\ \langle es',s'\rangle \equiv ((es,s),\ es',s') \in (reds\ P)^*$

lemma *converse-rtrancl-induct-red*[*consumes 1*]:
assumes $P \vdash \langle e,(h,l)\rangle \rightarrow{*}\ \langle e',(h',l')\rangle$
and $\bigwedge e\ h\ l.\ R\ e\ h\ l\ e\ h\ l$
and $\bigwedge e_0\ h_0\ l_0\ e_1\ h_1\ l_1\ e'\ h'\ l'.$
$[\![\ P \vdash \langle e_0,(h_0,l_0)\rangle \rightarrow \langle e_1,(h_1,l_1)\rangle;\ R\ e_1\ h_1\ l_1\ e'\ h'\ l'\ ]\!] \Longrightarrow R\ e_0\ h_0\ l_0\ e'\ h'\ l'$
shows $R\ e\ h\ l\ e'\ h'\ l'$

2.11.2 Some easy lemmas

lemma [*iff*]: $\neg\ P \vdash \langle [],s\rangle\ [\rightarrow]\ \langle es',s'\rangle$
lemma [*iff*]: $\neg\ P \vdash \langle Val\ v,s\rangle \rightarrow \langle e',s'\rangle$
lemma [*iff*]: $\neg\ P \vdash \langle Throw\ a,s\rangle \rightarrow \langle e',s'\rangle$

lemma *red-hext-incr*: $P \vdash \langle e,(h,l)\rangle \rightarrow \langle e',(h',l')\rangle \Longrightarrow h \trianglelefteq h'$
and *reds-hext-incr*: $P \vdash \langle es,(h,l)\rangle\ [\rightarrow]\ \langle es',(h',l')\rangle \Longrightarrow h \trianglelefteq h'$

lemma *red-lcl-incr*: $P \vdash \langle e,(h_0,l_0)\rangle \rightarrow \langle e',(h_1,l_1)\rangle \Longrightarrow dom\ l_0 \subseteq dom\ l_1$
and $P \vdash \langle es,(h_0,l_0)\rangle\ [\rightarrow]\ \langle es',(h_1,l_1)\rangle \Longrightarrow dom\ l_0 \subseteq dom\ l_1$

lemma *red-lcl-add*: $P \vdash \langle e,(h,l)\rangle \rightarrow \langle e',(h',l')\rangle \Longrightarrow (\bigwedge l_0.\ P \vdash \langle e,(h,l_0{+}{+}l)\rangle \rightarrow \langle e',(h',l_0{+}{+}l')\rangle)$
and $P \vdash \langle es,(h,l)\rangle\ [\rightarrow]\ \langle es',(h',l')\rangle \Longrightarrow (\bigwedge l_0.\ P \vdash \langle es,(h,l_0{+}{+}l)\rangle\ [\rightarrow]\ \langle es',(h',l_0{+}{+}l')\rangle)$

lemma *Red-lcl-add*:
assumes $P \vdash \langle e,(h,l)\rangle \rightarrow{*}\ \langle e',(h',l')\rangle$ **shows** $P \vdash \langle e,(h,l_0{+}{+}l)\rangle \rightarrow{*}\ \langle e',(h',l_0{+}{+}l')\rangle$

end

2.12 System Classes

theory *SystemClasses*
imports *Decl Exceptions*
begin

This theory provides definitions for the *Object* class, and the system exceptions.

definition $ObjectC :: 'm\ cdecl$
where
$ObjectC \equiv (Object,\ (undefined,[],[]))$

definition $NullPointerC :: 'm\ cdecl$
where
$NullPointerC \equiv (NullPointer,\ (Object,[],[]))$

definition $ClassCastC :: 'm\ cdecl$
where
$ClassCastC \equiv (ClassCast,\ (Object,[],[]))$

definition $OutOfMemoryC :: 'm\ cdecl$
where
$OutOfMemoryC \equiv (OutOfMemory,\ (Object,[],[]))$

definition $SystemClasses :: 'm\ cdecl\ list$
where
$SystemClasses \equiv [ObjectC,\ NullPointerC,\ ClassCastC,\ OutOfMemoryC]$

end

2.13 Generic Well-formedness of programs

theory *WellForm* **imports** *TypeRel SystemClasses* **begin**

This theory defines global well-formedness conditions for programs but does not look inside method bodies. Hence it works for both Jinja and JVM programs. Well-typing of expressions is defined elsewhere (in theory *WellType*).

Because Jinja does not have method overloading, its policy for method overriding is the classical one: *covariant in the result type but contravariant in the argument types.* This means the result type of the overriding method becomes more specific, the argument types become more general.

type-synonym $'m\ \textit{wf-mdecl-test} = {}'m\ \textit{prog} \Rightarrow \textit{cname} \Rightarrow {}'m\ \textit{mdecl} \Rightarrow \textit{bool}$

definition $\textit{wf-fdecl} :: {}'m\ \textit{prog} \Rightarrow \textit{fdecl} \Rightarrow \textit{bool}$
where
$\textit{wf-fdecl}\ P \equiv \lambda(F,T).\ \textit{is-type}\ P\ T$

definition $\textit{wf-mdecl} :: {}'m\ \textit{wf-mdecl-test} \Rightarrow {}'m\ \textit{wf-mdecl-test}$
where
$\textit{wf-mdecl wf-md}\ P\ C \equiv \lambda(M,Ts,T,mb).$
$(\forall T{\in}\textit{set}\ Ts.\ \textit{is-type}\ P\ T) \wedge \textit{is-type}\ P\ T \wedge \textit{wf-md}\ P\ C\ (M,Ts,T,mb)$

definition $\textit{wf-cdecl} :: {}'m\ \textit{wf-mdecl-test} \Rightarrow {}'m\ \textit{prog} \Rightarrow {}'m\ \textit{cdecl} \Rightarrow \textit{bool}$
where
$\textit{wf-cdecl wf-md}\ P \equiv \lambda(C,(D,fs,ms)).$
$(\forall f{\in}\textit{set}\ fs.\ \textit{wf-fdecl}\ P\ f) \wedge \textit{distinct-fst}\ fs \wedge$
$(\forall m{\in}\textit{set}\ ms.\ \textit{wf-mdecl wf-md}\ P\ C\ m) \wedge \textit{distinct-fst}\ ms \wedge$
$(C \neq \textit{Object} \longrightarrow$
$\textit{is-class}\ P\ D \wedge \neg\ P \vdash D \preceq^* C \wedge$
$(\forall (M,Ts,T,m){\in}\textit{set}\ ms.$
$\forall D'\ Ts'\ T'\ m'.\ P \vdash D\ \textit{sees}\ M{:}Ts' \rightarrow T' = m'\ \textit{in}\ D' \longrightarrow$
$P \vdash Ts'\ [\le]\ Ts \wedge P \vdash T \le T'))$

definition $\textit{wf-syscls} :: {}'m\ \textit{prog} \Rightarrow \textit{bool}$
where
$\textit{wf-syscls}\ P \equiv \{\textit{Object}\} \cup \textit{sys-xcpts} \subseteq \textit{set}(\textit{map fst}\ P)$

definition $\textit{wf-prog} :: {}'m\ \textit{wf-mdecl-test} \Rightarrow {}'m\ \textit{prog} \Rightarrow \textit{bool}$
where
$\textit{wf-prog wf-md}\ P \equiv \textit{wf-syscls}\ P \wedge (\forall c \in \textit{set}\ P.\ \textit{wf-cdecl wf-md}\ P\ c) \wedge \textit{distinct-fst}\ P$

2.13.1 Well-formedness lemmas

lemma *class-wf*:
$[\![\textit{class}\ P\ C = \textit{Some}\ c;\ \textit{wf-prog wf-md}\ P]\!] \Longrightarrow \textit{wf-cdecl wf-md}\ P\ (C,c)$

lemma *class-Object* [*simp*]:
$\textit{wf-prog wf-md}\ P \Longrightarrow \exists C\ fs\ ms.\ \textit{class}\ P\ \textit{Object} = \textit{Some}\ (C,fs,ms)$

lemma *is-class-Object* [*simp*]:
$\textit{wf-prog wf-md}\ P \Longrightarrow \textit{is-class}\ P\ \textit{Object}$

lemma *is-class-xcpt*:
$[\![\ C \in \textit{sys-xcpts};\ \textit{wf-prog wf-md}\ P\ ]\!] \Longrightarrow \textit{is-class}\ P\ C$

lemma *subcls1-wfD*:
$[\![P \vdash C \prec^1 D; \textit{wf-prog wf-md } P]\!] \Longrightarrow D \neq C \wedge (D,C) \notin (\textit{subcls1 } P)^+$

lemma *wf-cdecl-supD*:
$[\![\textit{wf-cdecl wf-md } P\ (C,D,r);\ C \neq \textit{Object}]\!] \Longrightarrow \textit{is-class } P\ D$

lemma *subcls-asym*:
$[\![\textit{wf-prog wf-md } P;\ (C,D) \in (\textit{subcls1 } P)^+]\!] \Longrightarrow (D,C) \notin (\textit{subcls1 } P)^+$

lemma *subcls-irrefl*:
$[\![\textit{wf-prog wf-md } P;\ (C,D) \in (\textit{subcls1 } P)^+]\!] \Longrightarrow C \neq D$

lemma *acyclic-subcls1*:
$\textit{wf-prog wf-md } P \Longrightarrow \textit{acyclic } (\textit{subcls1 } P)$

lemma *wf-subcls1*:
$\textit{wf-prog wf-md } P \Longrightarrow \textit{wf } ((\textit{subcls1 } P)^{-1})$

lemma *single-valued-subcls1*:
$\textit{wf-prog wf-md } G \Longrightarrow \textit{single-valued } (\textit{subcls1 } G)$

lemma *subcls-induct*:
$[\![\textit{wf-prog wf-md } P;\ \bigwedge C.\ \forall D.\ (C,D) \in (\textit{subcls1 } P)^+ \longrightarrow Q\ D \Longrightarrow Q\ C]\!] \Longrightarrow Q\ C$

lemma *subcls1-induct-aux*:
$[\![\textit{is-class } P\ C;\ \textit{wf-prog wf-md } P;\ Q\ \textit{Object};$
$\bigwedge C\ D\ \textit{fs ms}.$
$[\![C \neq \textit{Object};\ \textit{is-class } P\ C;\ \textit{class } P\ C = \textit{Some } (D,\textit{fs},\textit{ms}) \wedge$
$\textit{wf-cdecl wf-md } P\ (C,D,\textit{fs},\textit{ms}) \wedge P \vdash C \prec^1 D \wedge \textit{is-class } P\ D \wedge Q\ D]\!] \Longrightarrow Q\ C]\!]$
$\Longrightarrow Q\ C$

lemma *subcls1-induct* [*consumes 2*, *case-names Object Subcls*]:
$[\![\textit{wf-prog wf-md } P;\ \textit{is-class } P\ C;\ Q\ \textit{Object};$
$\bigwedge C\ D.\ [\![C \neq \textit{Object};\ P \vdash C \prec^1 D;\ \textit{is-class } P\ D;\ Q\ D]\!] \Longrightarrow Q\ C]\!]$
$\Longrightarrow Q\ C$

lemma *subcls-C-Object*:
$[\![\textit{is-class } P\ C;\ \textit{wf-prog wf-md } P]\!] \Longrightarrow P \vdash C \preceq^* \textit{Object}$

lemma *is-type-pTs*:
assumes *wf-prog wf-md P* **and** $(C,S,\textit{fs},\textit{ms}) \in \textit{set } P$ **and** $(M,\textit{Ts},T,m) \in \textit{set ms}$
shows $\textit{set Ts} \subseteq \textit{types } P$

2.13.2 Well-formedness and method lookup

lemma *sees-wf-mdecl*:
$[\![\textit{wf-prog wf-md } P;\ P \vdash C \textit{ sees } M{:}\textit{Ts} \rightarrow T = m \textit{ in } D]\!] \Longrightarrow \textit{wf-mdecl wf-md } P\ D\ (M,\textit{Ts},T,m)$

lemma *sees-method-mono* [*rule-format* (*no-asm*)]:
$[\![P \vdash C' \preceq^* C;\ \textit{wf-prog wf-md } P]\!] \Longrightarrow$
$\forall D\ \textit{Ts}\ T\ m.\ P \vdash C \textit{ sees } M{:}\textit{Ts} \rightarrow T = m \textit{ in } D \longrightarrow$
$(\exists D'\ \textit{Ts}'\ T'\ m'.\ P \vdash C' \textit{ sees } M{:}\textit{Ts}' \rightarrow T' = m' \textit{ in } D' \wedge P \vdash \textit{Ts}\ [\leq]\ \textit{Ts}' \wedge P \vdash T' \leq T)$

lemma *sees-method-mono2*:
$[\![P \vdash C' \preceq^* C;\ wf\text{-}prog\ wf\text{-}md\ P;$
$P \vdash C\ sees\ M{:}Ts{\rightarrow}T = m\ in\ D;\ P \vdash C'\ sees\ M{:}Ts'{\rightarrow}T' = m'\ in\ D']\!]$
$\Longrightarrow P \vdash Ts\ [\leq]\ Ts' \wedge P \vdash T' \leq T$

lemma *mdecls-visible*:
assumes *wf*: $wf\text{-}prog\ wf\text{-}md\ P$ **and** *class*: $is\text{-}class\ P\ C$
shows $\bigwedge D\ fs\ ms.\ class\ P\ C = Some(D,fs,ms)$
$\Longrightarrow \exists Mm.\ P \vdash C\ sees\text{-}methods\ Mm \wedge (\forall (M,Ts,T,m) \in set\ ms.\ Mm\ M = Some((Ts,T,m),C))$

lemma *mdecl-visible*:
assumes *wf*: $wf\text{-}prog\ wf\text{-}md\ P$ **and** *C*: $(C,S,fs,ms) \in set\ P$ **and** *m*: $(M,Ts,T,m) \in set\ ms$
shows $P \vdash C\ sees\ M{:}Ts{\rightarrow}T = m\ in\ C$

lemma *Call-lemma*:
$[\![P \vdash C\ sees\ M{:}Ts{\rightarrow}T = m\ in\ D;\ P \vdash C' \preceq^* C;\ wf\text{-}prog\ wf\text{-}md\ P]\!]$
$\Longrightarrow \exists D'\ Ts'\ T'\ m'.$
$P \vdash C'\ sees\ M{:}Ts'{\rightarrow}T' = m'\ in\ D' \wedge P \vdash Ts\ [\leq]\ Ts' \wedge P \vdash T' \leq T \wedge P \vdash C' \preceq^* D'$
$\wedge\ is\text{-}type\ P\ T' \wedge (\forall T{\in}set\ Ts'.\ is\text{-}type\ P\ T) \wedge wf\text{-}md\ P\ D'\ (M,Ts',T',m')$

lemma *wf-prog-lift*:
assumes *wf*: $wf\text{-}prog\ (\lambda P\ C\ bd.\ A\ P\ C\ bd)\ P$
and *rule*:
$\bigwedge wf\text{-}md\ C\ M\ Ts\ C\ T\ m\ bd.$
$wf\text{-}prog\ wf\text{-}md\ P \Longrightarrow$
$P \vdash C\ sees\ M{:}Ts{\rightarrow}T = m\ in\ C \Longrightarrow$
$set\ Ts \subseteq\ types\ P \Longrightarrow$
$bd = (M,Ts,T,m) \Longrightarrow$
$A\ P\ C\ bd \Longrightarrow$
$B\ P\ C\ bd$
shows $wf\text{-}prog\ (\lambda P\ C\ bd.\ B\ P\ C\ bd)\ P$

2.13.3 Well-formedness and field lookup

lemma *wf-Fields-Ex*:
$[\![wf\text{-}prog\ wf\text{-}md\ P;\ is\text{-}class\ P\ C]\!] \Longrightarrow \exists FDTs.\ P \vdash C\ has\text{-}fields\ FDTs$

lemma *has-fields-types*:
$[\![P \vdash C\ has\text{-}fields\ FDTs;\ (FD,T) \in set\ FDTs;\ wf\text{-}prog\ wf\text{-}md\ P]\!] \Longrightarrow is\text{-}type\ P\ T$

lemma *sees-field-is-type*:
$[\![P \vdash C\ sees\ F{:}T\ in\ D;\ wf\text{-}prog\ wf\text{-}md\ P]\!] \Longrightarrow is\text{-}type\ P\ T$
lemma *wf-syscls*:
$set\ SystemClasses \subseteq set\ P \Longrightarrow wf\text{-}syscls\ P$
end

2.14 Weak well-formedness of Jinja programs

theory *WWellForm* **imports** ../*Common*/*WellForm Expr* **begin**

definition *wwf-J-mdecl* :: *J-prog* ⇒ *cname* ⇒ *J-mb mdecl* ⇒ *bool*
where
wwf-J-mdecl P C ≡ λ(*M*,*Ts*,*T*,(*pns*,*body*)).
length Ts = *length pns* ∧ *distinct pns* ∧ *this* ∉ *set pns* ∧ *fv body* ⊆ {*this*} ∪ *set pns*

lemma *wwf-J-mdecl*[*simp*]:
wwf-J-mdecl P C (*M*,*Ts*,*T*,*pns*,*body*) =
(*length Ts* = *length pns* ∧ *distinct pns* ∧ *this* ∉ *set pns* ∧ *fv body* ⊆ {*this*} ∪ *set pns*)
abbreviation
wwf-J-prog :: *J-prog* ⇒ *bool* **where**
wwf-J-prog == *wf-prog wwf-J-mdecl*

end

2.15 Equivalence of Big Step and Small Step Semantics

theory *Equivalence* **imports** *BigStep SmallStep WWellForm* **begin**

2.15.1 Small steps simulate big step

Cast

lemma *CastReds*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle Cast\ C\ e,s\rangle \to* \langle Cast\ C\ e',s'\rangle$
lemma *CastRedsNull*:
$P \vdash \langle e,s\rangle \to* \langle null,s'\rangle \Longrightarrow P \vdash \langle Cast\ C\ e,s\rangle \to* \langle null,s'\rangle$
lemma *CastRedsAddr*:
$[\![\ P \vdash \langle e,s\rangle \to* \langle addr\ a,s'\rangle;\ hp\ s'\ a = Some(D,fs);\ P \vdash D \preceq^*\ C\ ]\!] \Longrightarrow$
$P \vdash \langle Cast\ C\ e,s\rangle \to* \langle addr\ a,s'\rangle$
lemma *CastRedsFail*:
$[\![\ P \vdash \langle e,s\rangle \to* \langle addr\ a,s'\rangle;\ hp\ s'\ a = Some(D,fs);\ \neg\ P \vdash D \preceq^*\ C\ ]\!] \Longrightarrow$
$P \vdash \langle Cast\ C\ e,s\rangle \to* \langle THROW\ ClassCast,s'\rangle$
lemma *CastRedsThrow*:
$[\![\ P \vdash \langle e,s\rangle \to* \langle throw\ a,s'\rangle\ ]\!] \Longrightarrow P \vdash \langle Cast\ C\ e,s\rangle \to* \langle throw\ a,s'\rangle$

LAss

lemma *LAssReds*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle\ V{:=}e,s\rangle \to* \langle\ V{:=}e',s'\rangle$
lemma *LAssRedsVal*:
$[\![\ P \vdash \langle e,s\rangle \to* \langle Val\ v,(h',l')\rangle\ ]\!] \Longrightarrow P \vdash \langle\ V{:=}e,s\rangle \to* \langle unit,(h',l'(V\mapsto v))\rangle$
lemma *LAssRedsThrow*:
$[\![\ P \vdash \langle e,s\rangle \to* \langle throw\ a,s'\rangle\ ]\!] \Longrightarrow P \vdash \langle\ V{:=}e,s\rangle \to* \langle throw\ a,s'\rangle$

BinOp

lemma *BinOp1Reds*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle\ e \ll bop \gg e_2,\ s\rangle \to* \langle e' \ll bop \gg e_2,\ s'\rangle$
lemma *BinOp2Reds*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle (Val\ v) \ll bop \gg e,\ s\rangle \to* \langle (Val\ v) \ll bop \gg e',\ s'\rangle$
lemma *BinOpRedsVal*:
$[\![\ P \vdash \langle e_1,s_0\rangle \to* \langle Val\ v_1,s_1\rangle;\ P \vdash \langle e_2,s_1\rangle \to* \langle Val\ v_2,s_2\rangle;\ binop(bop,v_1,v_2) = Some\ v\ ]\!]$
$\Longrightarrow P \vdash \langle e_1 \ll bop \gg e_2,\ s_0\rangle \to* \langle Val\ v,s_2\rangle$
lemma *BinOpRedsThrow1*:
$P \vdash \langle e,s\rangle \to* \langle throw\ e',s'\rangle \Longrightarrow P \vdash \langle e \ll bop \gg e_2,\ s\rangle \to* \langle throw\ e',\ s'\rangle$
lemma *BinOpRedsThrow2*:
$[\![\ P \vdash \langle e_1,s_0\rangle \to* \langle Val\ v_1,s_1\rangle;\ P \vdash \langle e_2,s_1\rangle \to* \langle throw\ e,s_2\rangle]\!]$
$\Longrightarrow P \vdash \langle e_1 \ll bop \gg e_2,\ s_0\rangle \to* \langle throw\ e,s_2\rangle$

FAcc

lemma *FAccReds*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle e{\cdot}F\{D\},\ s\rangle \to* \langle e'{\cdot}F\{D\},\ s'\rangle$
lemma *FAccRedsVal*:
$[\![P \vdash \langle e,s\rangle \to* \langle addr\ a,s'\rangle;\ hp\ s'\ a = Some(C,fs);\ fs(F,D) = Some\ v\ ]\!]$
$\Longrightarrow P \vdash \langle e{\cdot}F\{D\},s\rangle \to* \langle Val\ v,s'\rangle$
lemma *FAccRedsNull*:
$P \vdash \langle e,s\rangle \to* \langle null,s'\rangle \Longrightarrow P \vdash \langle e{\cdot}F\{D\},s\rangle \to* \langle THROW\ NullPointer,s'\rangle$
lemma *FAccRedsThrow*:

$P \vdash \langle e,s\rangle \rightarrow* \langle throw\ a,s'\rangle \Longrightarrow P \vdash \langle e{\cdot}F\{D\},s\rangle \rightarrow* \langle throw\ a,s'\rangle$

FAss

lemma *FAssReds1*:
$P \vdash \langle e,s\rangle \rightarrow* \langle e',s'\rangle \Longrightarrow P \vdash \langle e{\cdot}F\{D\}{:=}e_2,\ s\rangle \rightarrow* \langle e'{\cdot}F\{D\}{:=}e_2,\ s'\rangle$
lemma *FAssReds2*:
$P \vdash \langle e,s\rangle \rightarrow* \langle e',s'\rangle \Longrightarrow P \vdash \langle Val\ v{\cdot}F\{D\}{:=}e,\ s\rangle \rightarrow* \langle Val\ v{\cdot}F\{D\}{:=}e',\ s'\rangle$
lemma *FAssRedsVal*:
$[\![\ P \vdash \langle e_1,s_0\rangle \rightarrow* \langle addr\ a,s_1\rangle;\ P \vdash \langle e_2,s_1\rangle \rightarrow* \langle Val\ v,(h_2,l_2)\rangle;\ Some(C,fs) = h_2\ a\ ]\!] \Longrightarrow$
$P \vdash \langle e_1{\cdot}F\{D\}{:=}e_2,\ s_0\rangle \rightarrow* \langle unit,\ (h_2(a{\mapsto}(C,fs((F,D){\mapsto}v))),l_2)\rangle$
lemma *FAssRedsNull*:
$[\![\ P \vdash \langle e_1,s_0\rangle \rightarrow* \langle null,s_1\rangle;\ P \vdash \langle e_2,s_1\rangle \rightarrow* \langle Val\ v,s_2\rangle\ ]\!] \Longrightarrow$
$P \vdash \langle e_1{\cdot}F\{D\}{:=}e_2,\ s_0\rangle \rightarrow* \langle THROW\ NullPointer,\ s_2\rangle$
lemma *FAssRedsThrow1*:
$P \vdash \langle e,s\rangle \rightarrow* \langle throw\ e',s'\rangle \Longrightarrow P \vdash \langle e{\cdot}F\{D\}{:=}e_2,\ s\rangle \rightarrow* \langle throw\ e',\ s'\rangle$
lemma *FAssRedsThrow2*:
$[\![\ P \vdash \langle e_1,s_0\rangle \rightarrow* \langle Val\ v,s_1\rangle;\ P \vdash \langle e_2,s_1\rangle \rightarrow* \langle throw\ e,s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle e_1{\cdot}F\{D\}{:=}e_2,s_0\rangle \rightarrow* \langle throw\ e,s_2\rangle$

;;

lemma *SeqReds*:
$P \vdash \langle e,s\rangle \rightarrow* \langle e',s'\rangle \Longrightarrow P \vdash \langle e;;e_2,\ s\rangle \rightarrow* \langle e';;e_2,\ s'\rangle$
lemma *SeqRedsThrow*:
$P \vdash \langle e,s\rangle \rightarrow* \langle throw\ e',s'\rangle \Longrightarrow P \vdash \langle e;;e_2,\ s\rangle \rightarrow* \langle throw\ e',\ s'\rangle$
lemma *SeqReds2*:
$[\![\ P \vdash \langle e_1,s_0\rangle \rightarrow* \langle Val\ v_1,s_1\rangle;\ P \vdash \langle e_2,s_1\rangle \rightarrow* \langle e_2',s_2\rangle\ ]\!] \Longrightarrow P \vdash \langle e_1;;e_2,\ s_0\rangle \rightarrow* \langle e_2',s_2\rangle$

If

lemma *CondReds*:
$P \vdash \langle e,s\rangle \rightarrow* \langle e',s'\rangle \Longrightarrow P \vdash \langle if\ (e)\ e_1\ else\ e_2,s\rangle \rightarrow* \langle if\ (e')\ e_1\ else\ e_2,s'\rangle$
lemma *CondRedsThrow*:
$P \vdash \langle e,s\rangle \rightarrow* \langle throw\ a,s'\rangle \Longrightarrow P \vdash \langle if\ (e)\ e_1\ else\ e_2,\ s\rangle \rightarrow* \langle throw\ a,s'\rangle$
lemma *CondReds2T*:
$[\![P \vdash \langle e,s_0\rangle \rightarrow* \langle true,s_1\rangle;\ P \vdash \langle e_1,\ s_1\rangle \rightarrow* \langle e',s_2\rangle\ ]\!] \Longrightarrow P \vdash \langle if\ (e)\ e_1\ else\ e_2,\ s_0\rangle \rightarrow* \langle e',s_2\rangle$
lemma *CondReds2F*:
$[\![P \vdash \langle e,s_0\rangle \rightarrow* \langle false,s_1\rangle;\ P \vdash \langle e_2,\ s_1\rangle \rightarrow* \langle e',s_2\rangle\ ]\!] \Longrightarrow P \vdash \langle if\ (e)\ e_1\ else\ e_2,\ s_0\rangle \rightarrow* \langle e',s_2\rangle$

While

lemma *WhileFReds*:
$P \vdash \langle b,s\rangle \rightarrow* \langle false,s'\rangle \Longrightarrow P \vdash \langle while\ (b)\ c,s\rangle \rightarrow* \langle unit,s'\rangle$
lemma *WhileRedsThrow*:
$P \vdash \langle b,s\rangle \rightarrow* \langle throw\ e,s'\rangle \Longrightarrow P \vdash \langle while\ (b)\ c,s\rangle \rightarrow* \langle throw\ e,s'\rangle$
lemma *WhileTReds*:
$[\![\ P \vdash \langle b,s_0\rangle \rightarrow* \langle true,s_1\rangle;\ P \vdash \langle c,s_1\rangle \rightarrow* \langle Val\ v_1,s_2\rangle;\ P \vdash \langle while\ (b)\ c,s_2\rangle \rightarrow* \langle e,s_3\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle while\ (b)\ c,s_0\rangle \rightarrow* \langle e,s_3\rangle$
lemma *WhileTRedsThrow*:
$[\![\ P \vdash \langle b,s_0\rangle \rightarrow* \langle true,s_1\rangle;\ P \vdash \langle c,s_1\rangle \rightarrow* \langle throw\ e,s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle while\ (b)\ c,s_0\rangle \rightarrow* \langle throw\ e,s_2\rangle$

Throw

lemma *ThrowReds*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle throw\ e,s\rangle \to* \langle throw\ e',s'\rangle$
lemma *ThrowRedsNull*:
$P \vdash \langle e,s\rangle \to* \langle null,s'\rangle \Longrightarrow P \vdash \langle throw\ e,s\rangle \to* \langle THROW\ NullPointer,s'\rangle$
lemma *ThrowRedsThrow*:
$P \vdash \langle e,s\rangle \to* \langle throw\ a,s'\rangle \Longrightarrow P \vdash \langle throw\ e,s\rangle \to* \langle throw\ a,s'\rangle$

InitBlock

lemma *InitBlockReds-aux*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow$
$\forall h\ l\ h'\ l'\ v.\ s = (h,l(V\mapsto v)) \longrightarrow s' = (h',l') \longrightarrow$
$P \vdash \langle\{V{:}T := Val\ v;\ e\},(h,l)\rangle \to* \langle\{V{:}T := Val(the(l'\ V));\ e'\},(h',l'(V{:=}(l\ V)))\rangle$
lemma *InitBlockReds*:
$P \vdash \langle e,\ (h,l(V\mapsto v))\rangle \to* \langle e',\ (h',l')\rangle \Longrightarrow$
$P \vdash \langle\{V{:}T := Val\ v;\ e\},\ (h,l)\rangle \to* \langle\{V{:}T := Val(the(l'\ V));\ e'\},\ (h',l'(V{:=}(l\ V)))\rangle$
lemma *InitBlockRedsFinal*:
$[\![\ P \vdash \langle e,(h,l(V\mapsto v))\rangle \to* \langle e',(h',l')\rangle;\ final\ e'\ ]\!] \Longrightarrow$
$P \vdash \langle\{V{:}T := Val\ v;\ e\},(h,l)\rangle \to* \langle e',(h',\ l'(V := l\ V))\rangle$

Block

lemma *BlockRedsFinal*:
assumes *reds*: $P \vdash \langle e_0,s_0\rangle \to* \langle e_2,(h_2,l_2)\rangle$ **and** *fin*: $final\ e_2$
shows $\bigwedge h_0\ l_0.\ s_0 = (h_0,l_0(V{:=}None)) \Longrightarrow P \vdash \langle\{V{:}T;\ e_0\},(h_0,l_0)\rangle \to* \langle e_2,(h_2,l_2(V{:=}l_0\ V))\rangle$

try-catch

lemma *TryReds*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle try\ e\ catch(C\ V)\ e_2,s\rangle \to* \langle try\ e'\ catch(C\ V)\ e_2,s'\rangle$
lemma *TryRedsVal*:
$P \vdash \langle e,s\rangle \to* \langle Val\ v,s'\rangle \Longrightarrow P \vdash \langle try\ e\ catch(C\ V)\ e_2,s\rangle \to* \langle Val\ v,s'\rangle$
lemma *TryCatchRedsFinal*:
$[\![\ P \vdash \langle e_1,s_0\rangle \to* \langle Throw\ a,(h_1,l_1)\rangle;\ \ h_1\ a = Some(D,fs);\ P \vdash D \preceq^* C;$
$P \vdash \langle e_2,\ (h_1,\ l_1(V \mapsto Addr\ a))\rangle \to* \langle e_2',\ (h_2,l_2)\rangle;\ final\ e_2'\ ]\!]$
$\Longrightarrow P \vdash \langle try\ e_1\ catch(C\ V)\ e_2,\ s_0\rangle \to* \langle e_2',\ (h_2,\ l_2(V := l_1\ V))\rangle$
lemma *TryRedsFail*:
$[\![\ P \vdash \langle e_1,s\rangle \to* \langle Throw\ a,(h,l)\rangle;\ h\ a = Some(D,fs);\ \neg\ P \vdash D \preceq^* C\ ]\!]$
$\Longrightarrow P \vdash \langle try\ e_1\ catch(C\ V)\ e_2,s\rangle \to* \langle Throw\ a,(h,l)\rangle$

List

lemma *ListReds1*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle e\#es,s\rangle\ [\to]*\ \langle e'\ \#\ es,s'\rangle$
lemma *ListReds2*:
$P \vdash \langle es,s\rangle\ [\to]*\ \langle es',s'\rangle \Longrightarrow P \vdash \langle Val\ v\ \#\ es,s\rangle\ [\to]*\ \langle Val\ v\ \#\ es',s'\rangle$
lemma *ListRedsVal*:
$[\![\ P \vdash \langle e,s_0\rangle \to* \langle Val\ v,s_1\rangle;\ P \vdash \langle es,s_1\rangle\ [\to]*\ \langle es',s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle e\#es,s_0\rangle\ [\to]*\ \langle Val\ v\ \#\ es',s_2\rangle$

Call

First a few lemmas on what happens to free variables during redction.

lemma assumes *wf*: *wwf-J-prog P*
shows *Red-fv*: $P \vdash \langle e,(h,l)\rangle \to \langle e',(h',l')\rangle \Longrightarrow fv\ e' \subseteq fv\ e$
and $P \vdash \langle es,(h,l)\rangle\ [\to]\ \langle es',(h',l')\rangle \Longrightarrow fvs\ es' \subseteq fvs\ es$

lemma *Red-dom-lcl*:
$P \vdash \langle e,(h,l)\rangle \to \langle e',(h',l')\rangle \Longrightarrow dom\ l' \subseteq dom\ l \cup fv\ e$ **and**
$P \vdash \langle es,(h,l)\rangle\ [\to]\ \langle es',(h',l')\rangle \Longrightarrow dom\ l' \subseteq dom\ l \cup fvs\ es$
lemma *Reds-dom-lcl*:
$[\![\ wwf\text{-}J\text{-}prog\ P;\ P \vdash \langle e,(h,l)\rangle \to\!*\ \langle e',(h',l')\rangle\ ]\!] \Longrightarrow dom\ l' \subseteq dom\ l \cup fv\ e$

Now a few lemmas on the behaviour of blocks during reduction.

lemma *override-on-upd-lemma*:
$(override\text{-}on\ f\ (g(a \mapsto b))\ A)(a := g\ a) = override\text{-}on\ f\ g\ (insert\ a\ A)$

lemma *blocksReds*:
$\bigwedge l.\ [\![\ length\ Vs = length\ Ts;\ length\ vs = length\ Ts;\ distinct\ Vs;$
$P \vdash \langle e,\ (h,l(Vs\ [\mapsto]\ vs))\rangle \to\!*\ \langle e',\ (h',l')\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle blocks(Vs,Ts,vs,e),\ (h,l)\rangle \to\!*\ \langle blocks(Vs,Ts,map\ (the \circ l')\ Vs,e'),\ (h',override\text{-}on\ l'$
$l\ (set\ Vs))\rangle$

lemma *blocksFinal*:
$\bigwedge l.\ [\![\ length\ Vs = length\ Ts;\ length\ vs = length\ Ts;\ final\ e\ ]\!] \Longrightarrow$
$P \vdash \langle blocks(Vs,Ts,vs,e),\ (h,l)\rangle \to\!*\ \langle e,\ (h,l)\rangle$

lemma *blocksRedsFinal*:
assumes *wf*: $length\ Vs = length\ Ts\ length\ vs = length\ Ts\ distinct\ Vs$
and *reds*: $P \vdash \langle e,\ (h,l(Vs\ [\mapsto]\ vs))\rangle \to\!*\ \langle e',\ (h',l')\rangle$
and *fin*: *final e'* **and** l'': $l'' = override\text{-}on\ l'\ l\ (set\ Vs)$
shows $P \vdash \langle blocks(Vs,Ts,vs,e),\ (h,l)\rangle \to\!*\ \langle e',\ (h',l'')\rangle$

An now the actual method call reduction lemmas.

lemma *CallRedsObj*:
$P \vdash \langle e,s\rangle \to\!*\ \langle e',s'\rangle \Longrightarrow P \vdash \langle e\cdot M(es),s\rangle \to\!*\ \langle e'\cdot M(es),s'\rangle$

lemma *CallRedsParams*:
$P \vdash \langle es,s\rangle\ [\to]\!*\ \langle es',s'\rangle \Longrightarrow P \vdash \langle (Val\ v)\cdot M(es),s\rangle \to\!*\ \langle (Val\ v)\cdot M(es'),s'\rangle$

lemma *CallRedsFinal*:
assumes *wwf*: *wwf-J-prog P*
and $P \vdash \langle e,s_0\rangle \to\!*\ \langle addr\ a,s_1\rangle$
$P \vdash \langle es,s_1\rangle\ [\to]\!*\ \langle map\ Val\ vs,(h_2,l_2)\rangle$
$h_2\ a = Some(C,fs)\ P \vdash C\ sees\ M{:}Ts {\to} T = (pns,body)\ in\ D$
$size\ vs = size\ pns$
and l_2': $l_2' = [this \mapsto Addr\ a,\ pns[\mapsto]vs]$
and *body*: $P \vdash \langle body,(h_2,l_2')\rangle \to\!*\ \langle ef,(h_3,l_3)\rangle$
and *final ef*
shows $P \vdash \langle e\cdot M(es),\ s_0\rangle \to\!*\ \langle ef,(h_3,l_2)\rangle$

lemma *CallRedsThrowParams*:
$[\![\ P \vdash \langle e,s0\rangle \to\!*\ \langle Val\ v,s_1\rangle;\ \ P \vdash \langle es,s_1\rangle\ [\to]\!*\ \langle map\ Val\ vs_1\ @\ throw\ a\ \#\ es_2,s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle e\cdot M(es),s0\rangle \to\!*\ \langle throw\ a,s_2\rangle$

lemma *CallRedsThrowObj*:
$P \vdash \langle e,s0\rangle \to\!*\ \langle throw\ a,s_1\rangle \Longrightarrow P \vdash \langle e\cdot M(es),s0\rangle \to\!*\ \langle throw\ a,s_1\rangle$

lemma *CallRedsNull*:
$[\![P \vdash \langle e,s_0\rangle \to* \langle null,s_1\rangle;\ P \vdash \langle es,s_1\rangle\ [\to]*\ \langle map\ Val\ vs,s_2\rangle]\!]$
$\Longrightarrow P \vdash \langle e\bullet M(es),s_0\rangle \to* \langle THROW\ NullPointer,s_2\rangle$

The main Theorem

lemma assumes *wwf*: *wwf-J-prog P*
shows *big-by-small*: $P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle \Longrightarrow P \vdash \langle e,s\rangle \to* \langle e',s'\rangle$
and *bigs-by-smalls*: $P \vdash \langle es,s\rangle\ [\Rightarrow]\ \langle es',s'\rangle \Longrightarrow P \vdash \langle es,s\rangle\ [\to]*\ \langle es',s'\rangle$

2.15.2 Big steps simulates small step

This direction was carried out by Norbert Schirmer and Daniel Wasserrab.

The big step equivalent of *RedWhile*:

lemma *unfold-while*:
$P \vdash \langle while(b)\ c,s\rangle \Rightarrow \langle e',s'\rangle\ =\ P \vdash \langle if\,(b)\ (c;;while(b)\ c)\ else\ (unit),s\rangle \Rightarrow \langle e',s'\rangle$

lemma *blocksEval*:
$\bigwedge Ts\ vs\ l\ l'.\ [\![size\ ps = size\ Ts;\ size\ ps = size\ vs;\ P \vdash \langle blocks(ps,Ts,vs,e),(h,l)\rangle \Rightarrow \langle e',(h',l')\rangle]\!]$
$\Longrightarrow \exists\ l''.\ P \vdash \langle e,(h,l(ps[\mapsto]vs))\rangle \Rightarrow \langle e',(h',l'')\rangle$

lemma
assumes *wf*: *wwf-J-prog P*
shows *eval-restrict-lcl*:
$P \vdash \langle e,(h,l)\rangle \Rightarrow \langle e',(h',l')\rangle \Longrightarrow (\bigwedge W.\ fv\ e \subseteq W \Longrightarrow P \vdash \langle e,(h,l|`W)\rangle \Rightarrow \langle e',(h',l'|`W)\rangle)$
and $P \vdash \langle es,(h,l)\rangle\ [\Rightarrow]\ \langle es',(h',l')\rangle \Longrightarrow (\bigwedge W.\ fvs\ es \subseteq W \Longrightarrow P \vdash \langle es,(h,l|`W)\rangle\ [\Rightarrow]\ \langle es',(h',l'|`W)\rangle)$

lemma *eval-notfree-unchanged*:
$P \vdash \langle e,(h,l)\rangle \Rightarrow \langle e',(h',l')\rangle \Longrightarrow (\bigwedge V.\ V \notin fv\ e \Longrightarrow l'\ V = l\ V)$
and $P \vdash \langle es,(h,l)\rangle\ [\Rightarrow]\ \langle es',(h',l')\rangle \Longrightarrow (\bigwedge V.\ V \notin fvs\ es \Longrightarrow l'\ V = l\ V)$

lemma *eval-closed-lcl-unchanged*:
$[\![P \vdash \langle e,(h,l)\rangle \Rightarrow \langle e',(h',l')\rangle;\ fv\ e = \{\}]\!] \Longrightarrow l' = l$

lemma *list-eval-Throw*:
assumes *eval-e*: $P \vdash \langle throw\ x,s\rangle \Rightarrow \langle e',s'\rangle$
shows $P \vdash \langle map\ Val\ vs\ @\ throw\ x\ \#\ es',s\rangle\ [\Rightarrow]\ \langle map\ Val\ vs\ @\ e'\ \#\ es',s'\rangle$

The key lemma:

lemma
assumes *wf*: *wwf-J-prog P*
shows *extend-1-eval*:
$P \vdash \langle e,s\rangle \to \langle e'',s''\rangle \Longrightarrow\ (\bigwedge s'\ e'.\ P \vdash \langle e'',s''\rangle \Rightarrow \langle e',s'\rangle \Longrightarrow P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle)$
and *extend-1-evals*:
$P \vdash \langle es,t\rangle\ [\to]\ \langle es'',t''\rangle \Longrightarrow (\bigwedge t'\ es'.\ P \vdash \langle es'',t''\rangle\ [\Rightarrow]\ \langle es',t'\rangle \Longrightarrow P \vdash \langle es,t\rangle\ [\Rightarrow]\ \langle es',t'\rangle)$

Its extension to $\to*$:

lemma *extend-eval*:
assumes *wf*: *wwf-J-prog P*
and *reds*: $P \vdash \langle e,s\rangle \to* \langle e'',s''\rangle$ **and** *eval-rest*: $P \vdash \langle e'',s''\rangle \Rightarrow \langle e',s'\rangle$
shows $P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle$

lemma *extend-evals*:
assumes *wf*: *wwf-J-prog P*
and *reds*: $P \vdash \langle es,s\rangle$ $[\rightarrow]*$ $\langle es'',s''\rangle$ **and** *eval-rest*: $P \vdash \langle es'',s''\rangle$ $[\Rightarrow]$ $\langle es',s'\rangle$
shows $P \vdash \langle es,s\rangle$ $[\Rightarrow]$ $\langle es',s'\rangle$

Finally, small step semantics can be simulated by big step semantics:

theorem
assumes *wf*: *wwf-J-prog P*
shows *small-by-big*: $[\![P \vdash \langle e,s\rangle \rightarrow* \langle e',s'\rangle;\ \mathit{final}\ e']\!] \Longrightarrow P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle$
and $[\![P \vdash \langle es,s\rangle\ [\rightarrow]*\ \langle es',s'\rangle;\ \mathit{finals}\ es']\!] \Longrightarrow P \vdash \langle es,s\rangle\ [\Rightarrow]\ \langle es',s'\rangle$

2.15.3 Equivalence

And now, the crowning achievement:

corollary *big-iff-small*:
wwf-J-prog P $\Longrightarrow$
$P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle\ =\ (P \vdash \langle e,s\rangle \rightarrow* \langle e',s'\rangle \wedge \mathit{final}\ e')$

end

2.16 Well-typedness of Jinja expressions

theory *WellType*
imports *../Common/Objects Expr*
begin

type-synonym
  $env = vname \rightharpoonup ty$

inductive
  $WT :: [J\text{-}prog, env, expr \quad , ty \quad] \Rightarrow bool$
    $(\text{-,-} \vdash \text{-} :: \text{-} \quad [51,51,51]50)$
  and $WTs :: [J\text{-}prog, env, expr\ list, ty\ list] \Rightarrow bool$
    $(\text{-,-} \vdash \text{-} [::] \text{-} [51,51,51]50)$
  for $P :: J\text{-}prog$
where

  WTNew:
  $is\text{-}class\ P\ C \implies$
  $P,E \vdash new\ C :: Class\ C$

| *WTCast*:
  $[\![P,E \vdash e :: Class\ D;\ \ is\text{-}class\ P\ C;\ \ P \vdash C \preceq^* D \lor P \vdash D \preceq^* C]\!]$
  $\implies P,E \vdash Cast\ C\ e :: Class\ C$

| *WTVal*:
  $typeof\ v = Some\ T \implies$
  $P,E \vdash Val\ v :: T$

| *WTVar*:
  $E\ V = Some\ T \implies$
  $P,E \vdash Var\ V :: T$

| *WTBinOpEq*:
  $[\![P,E \vdash e_1 :: T_1;\ \ P,E \vdash e_2 :: T_2;\ P \vdash T_1 \le T_2 \lor P \vdash T_2 \le T_1]\!]$
  $\implies P,E \vdash e_1 \ll Eq \gg e_2 :: Boolean$

| *WTBinOpAdd*:
  $[\![P,E \vdash e_1 :: Integer;\ \ P,E \vdash e_2 :: Integer]\!]$
  $\implies P,E \vdash e_1 \ll Add \gg e_2 :: Integer$

| *WTLAss*:
  $[\![E\ V = Some\ T;\ \ P,E \vdash e :: T';\ \ P \vdash T' \le T;\ \ V \neq this]\!]$
  $\implies P,E \vdash V{:=}e :: Void$

| *WTFAcc*:
  $[\![P,E \vdash e :: Class\ C;\ \ P \vdash C\ sees\ F{:}T\ in\ D]\!]$
  $\implies P,E \vdash e{\cdot}F\{D\} :: T$

| *WTFAss*:
  $[\![P,E \vdash e_1 :: Class\ C;\ \ P \vdash C\ sees\ F{:}T\ in\ D;\ \ P,E \vdash e_2 :: T';\ \ P \vdash T' \le T]\!]$
  $\implies P,E \vdash e_1{\cdot}F\{D\}{:=}e_2 :: Void$

| *WTCall*:

$[\![$ $P,E \vdash e :: Class\ C$; $P \vdash C\ sees\ M{:}Ts \to T = (pns,body)\ in\ D$;
$P,E \vdash es\ [::]\ Ts'$; $P \vdash Ts'\ [\leq]\ Ts$ $]\!]$
$\Longrightarrow P,E \vdash e{\bullet}M(es) :: T$

| *WTBlock*:
$[\![$ $is\text{-}type\ P\ T$; $P,E(V \mapsto T) \vdash e :: T'$ $]\!]$
$\Longrightarrow$ $P,E \vdash \{V{:}T;\ e\} :: T'$

| *WTSeq*:
$[\![$ $P,E \vdash e_1{::}T_1$; $P,E \vdash e_2{::}T_2$ $]\!]$
$\Longrightarrow$ $P,E \vdash e_1;;e_2 :: T_2$
| *WTCond*:
$[\![$ $P,E \vdash e :: Boolean$; $P,E \vdash e_1{::}T_1$; $P,E \vdash e_2{::}T_2$;
$P \vdash T_1 \leq T_2 \lor P \vdash T_2 \leq T_1$; $P \vdash T_1 \leq T_2 \longrightarrow T = T_2$; $P \vdash T_2 \leq T_1 \longrightarrow T = T_1$ $]\!]$
$\Longrightarrow P,E \vdash if\ (e)\ e_1\ else\ e_2 :: T$

| *WTWhile*:
$[\![$ $P,E \vdash e :: Boolean$; $P,E \vdash c{::}T$ $]\!]$
$\Longrightarrow P,E \vdash while\ (e)\ c :: Void$

| *WTThrow*:
$P,E \vdash e :: Class\ C$ $\Longrightarrow$
$P,E \vdash throw\ e :: Void$

| *WTTry*:
$[\![$ $P,E \vdash e_1 :: T$; $P,E(V \mapsto Class\ C) \vdash e_2 :: T$; $is\text{-}class\ P\ C$ $]\!]$
$\Longrightarrow P,E \vdash try\ e_1\ catch(C\ V)\ e_2 :: T$

— well-typed expression lists

| *WTNil*:
$P,E \vdash []\ [::]\ []$

| *WTCons*:
$[\![$ $P,E \vdash e :: T$; $P,E \vdash es\ [::]\ Ts$ $]\!]$
$\Longrightarrow$ $P,E \vdash e\#es\ [::]\ T\#Ts$

lemma [*iff*]: $(P,E \vdash []\ [::]\ Ts) = (Ts = [])$
lemma [*iff*]: $(P,E \vdash e\#es\ [::]\ T\#Ts) = (P,E \vdash e :: T \land P,E \vdash es\ [::]\ Ts)$
lemma [*iff*]: $(P,E \vdash (e\#es)\ [::]\ Ts) =$
$(\exists\, U\ Us.\ Ts = U\#Us \land P,E \vdash e :: U \land P,E \vdash es\ [::]\ Us)$
lemma [*iff*]: $\bigwedge Ts.\ (P,E \vdash es_1\ @\ es_2\ [::]\ Ts) =$
$(\exists\, Ts_1\ Ts_2.\ Ts = Ts_1\ @\ Ts_2 \land P,E \vdash es_1\ [::]\ Ts_1 \land P,E \vdash es_2[::]Ts_2)$
lemma [*iff*]: $P,E \vdash Val\ v :: T = (typeof\ v = Some\ T)$
lemma [*iff*]: $P,E \vdash Var\ V :: T = (E\ V = Some\ T)$
lemma [*iff*]: $P,E \vdash e_1;;e_2 :: T_2 = (\exists\, T_1.\ P,E \vdash e_1{::}T_1 \land P,E \vdash e_2{::}T_2)$
lemma [*iff*]: $(P,E \vdash \{V{:}T;\ e\} :: T') = (is\text{-}type\ P\ T \land P,E(V{\mapsto}T) \vdash e :: T')$

lemma *wt-env-mono*:
$P,E \vdash e :: T \Longrightarrow (\bigwedge E'.\ E \subseteq_m E' \Longrightarrow P,E' \vdash e :: T)$ **and**
$P,E \vdash es\ [::]\ Ts \Longrightarrow (\bigwedge E'.\ E \subseteq_m E' \Longrightarrow P,E' \vdash es\ [::]\ Ts)$

lemma *WT-fv*: $P,E \vdash e :: T \Longrightarrow fv\ e \subseteq dom\ E$
and $P,E \vdash es\ [::]\ Ts \Longrightarrow fvs\ es \subseteq dom\ E$

2.17 Runtime Well-typedness

theory *WellTypeRT*
imports *WellType*
begin

inductive
  WTrt :: *J-prog* ⇒ *heap* ⇒ *env* ⇒ *expr* ⇒ *ty* ⇒ *bool*
  and *WTrts* :: *J-prog* ⇒ *heap* ⇒ *env* ⇒ *expr list* ⇒ *ty list* ⇒ *bool*
  and *WTrt2* :: [*J-prog*,*env*,*heap*,*expr*,*ty*] ⇒ *bool*
    (-,-,- ⊢ - : - [*51*,*51*,*51*]*50*)
  and *WTrts2* :: [*J-prog*,*env*,*heap*,*expr list*, *ty list*] ⇒ *bool*
    (-,-,- ⊢ - [:] - [*51*,*51*,*51*]*50*)
  for *P* :: *J-prog* **and** *h* :: *heap*
where

  $P,E,h \vdash e : T \equiv WTrt\ P\ h\ E\ e\ T$
| $P,E,h \vdash es[:]Ts \equiv WTrts\ P\ h\ E\ es\ Ts$

| *WTrtNew*:
  $is\text{-}class\ P\ C \implies$
  $P,E,h \vdash new\ C : Class\ C$

| *WTrtCast*:
  $[\![\ P,E,h \vdash e : T;\ is\text{-}refT\ T;\ is\text{-}class\ P\ C\ ]\!]$
  $\implies P,E,h \vdash Cast\ C\ e : Class\ C$

| *WTrtVal*:
  $typeof_h\ v = Some\ T \implies$
  $P,E,h \vdash Val\ v : T$

| *WTrtVar*:
  $E\ V = Some\ T \implies$
  $P,E,h \vdash Var\ V : T$

| *WTrtBinOpEq*:
  $[\![\ P,E,h \vdash e_1 : T_1;\ \ P,E,h \vdash e_2 : T_2\ ]\!]$
  $\implies P,E,h \vdash e_1 \ll Eq \gg e_2 : Boolean$

| *WTrtBinOpAdd*:
  $[\![\ P,E,h \vdash e_1 : Integer;\ \ P,E,h \vdash e_2 : Integer\ ]\!]$
  $\implies P,E,h \vdash e_1 \ll Add \gg e_2 : Integer$

| *WTrtLAss*:
  $[\![\ E\ V = Some\ T;\ \ P,E,h \vdash e : T';\ \ P \vdash T' \leq T\ ]\!]$
  $\implies P,E,h \vdash V{:=}e : Void$

| *WTrtFAcc*:
  $[\![\ P,E,h \vdash e : Class\ C;\ P \vdash C\ has\ F{:}T\ in\ D\ ]\!] \implies$
  $P,E,h \vdash e{\bullet}F\{D\} : T$

| *WTrtFAccNT*:
  $P,E,h \vdash e : NT \implies$
  $P,E,h \vdash e{\bullet}F\{D\} : T$

| *WTrtFAss*:
$[\![P,E,h \vdash e_1 : Class\ C;\ \ P \vdash C\ has\ F{:}T\ in\ D;\ P,E,h \vdash e_2 : T_2;\ \ P \vdash T_2 \leq T]\!]$
$\Longrightarrow P,E,h \vdash e_1{\cdot}F\{D\}{:=}e_2 : Void$

| *WTrtFAssNT*:
$[\![P,E,h \vdash e_1{:}NT;\ P,E,h \vdash e_2 : T_2]\!]$
$\Longrightarrow P,E,h \vdash e_1{\cdot}F\{D\}{:=}e_2 : Void$

| *WTrtCall*:
$[\![P,E,h \vdash e : Class\ C;\ P \vdash C\ sees\ M{:}Ts \rightarrow T = (pns,body)\ in\ D;$
$P,E,h \vdash es\ [:]\ Ts';\ P \vdash Ts'\ [\leq]\ Ts]\!]$
$\Longrightarrow P,E,h \vdash e{\cdot}M(es) : T$

| *WTrtCallNT*:
$[\![P,E,h \vdash e : NT;\ P,E,h \vdash es\ [:]\ Ts]\!]$
$\Longrightarrow P,E,h \vdash e{\cdot}M(es) : T$

| *WTrtBlock*:
$P,E(V \mapsto T),h \vdash e : T' \Longrightarrow$
$P,E,h \vdash \{V{:}T;\ e\} : T'$

| *WTrtSeq*:
$[\![P,E,h \vdash e_1{:}T_1;\ \ P,E,h \vdash e_2{:}T_2]\!]$
$\Longrightarrow P,E,h \vdash e_1;;e_2 : T_2$

| *WTrtCond*:
$[\![P,E,h \vdash e : Boolean;\ \ P,E,h \vdash e_1{:}T_1;\ \ P,E,h \vdash e_2{:}T_2;$
$P \vdash T_1 \leq T_2 \lor P \vdash T_2 \leq T_1;\ P \vdash T_1 \leq T_2 \longrightarrow T = T_2;\ P \vdash T_2 \leq T_1 \longrightarrow T = T_1]\!]$
$\Longrightarrow P,E,h \vdash if\ (e)\ e_1\ else\ e_2 : T$

| *WTrtWhile*:
$[\![P,E,h \vdash e : Boolean;\ \ P,E,h \vdash c{:}T]\!]$
$\Longrightarrow\ \ P,E,h \vdash while(e)\ c : Void$

| *WTrtThrow*:
$[\![P,E,h \vdash e : T_r;\ is\text{-}refT\ T_r]\!] \Longrightarrow$
$P,E,h \vdash throw\ e : T$

| *WTrtTry*:
$[\![P,E,h \vdash e_1 : T_1;\ \ P,E(V \mapsto Class\ C),h \vdash e_2 : T_2;\ P \vdash T_1 \leq T_2]\!]$
$\Longrightarrow P,E,h \vdash try\ e_1\ catch(C\ V)\ e_2 : T_2$

— well-typed expression lists

| *WTrtNil*:
$P,E,h \vdash []\ [:]\ []$

| *WTrtCons*:
$[\![P,E,h \vdash e : T;\ \ P,E,h \vdash es\ [:]\ Ts]\!]$
$\Longrightarrow\ \ P,E,h \vdash e\#es\ [:]\ T\#Ts$

2.17.1 Easy consequences

lemma [*iff*]: $(P,E,h \vdash [] \ [:] \ Ts) = (Ts = [])$
lemma [*iff*]: $(P,E,h \vdash e\#es \ [:] \ T\#Ts) = (P,E,h \vdash e : T \wedge P,E,h \vdash es \ [:] \ Ts)$
lemma [*iff*]: $(P,E,h \vdash (e\#es) \ [:] \ Ts) =$
$(\exists U \ Us. \ Ts = U\#Us \wedge P,E,h \vdash e : U \wedge P,E,h \vdash es \ [:] \ Us)$
lemma [*simp*]: $\forall Ts. \ (P,E,h \vdash es_1 \ @ \ es_2 \ [:] \ Ts) =$
$(\exists Ts_1 \ Ts_2. \ Ts = Ts_1 \ @ \ Ts_2 \wedge P,E,h \vdash es_1 \ [:] \ Ts_1 \ \& \ P,E,h \vdash es_2 [:] Ts_2)$
lemma [*iff*]: $P,E,h \vdash Val \ v : T = (typeof_h \ v = Some \ T)$
lemma [*iff*]: $P,E,h \vdash Var \ v : T = (E \ v = Some \ T)$
lemma [*iff*]: $P,E,h \vdash e_1;;e_2 : T_2 = (\exists T_1. \ P,E,h \vdash e_1{:}T_1 \wedge P,E,h \vdash e_2{:}T_2)$
lemma [*iff*]: $P,E,h \vdash \{V{:}T; \ e\} : T' \ = \ (P,E(V \mapsto T),h \vdash e : T')$

2.17.2 Some interesting lemmas

lemma *WTrts-Val*[*simp*]:
$\bigwedge Ts. \ (P,E,h \vdash map \ Val \ vs \ [:] \ Ts) = (map \ (typeof_h) \ vs = map \ Some \ Ts)$

lemma *WTrts-same-length*: $\bigwedge Ts. \ P,E,h \vdash es \ [:] \ Ts \Longrightarrow length \ es = length \ Ts$

lemma *WTrt-env-mono*:
$P,E,h \vdash e : T \Longrightarrow (\bigwedge E'. \ E \subseteq_m E' \Longrightarrow P,E',h \vdash e : T)$ **and**
$P,E,h \vdash es \ [:] \ Ts \Longrightarrow (\bigwedge E'. \ E \subseteq_m E' \Longrightarrow P,E',h \vdash es \ [:] \ Ts)$

lemma *WTrt-hext-mono*: $P,E,h \vdash e : T \Longrightarrow h \trianglelefteq h' \Longrightarrow P,E,h' \vdash e : T$
and *WTrts-hext-mono*: $P,E,h \vdash es \ [:] \ Ts \Longrightarrow h \trianglelefteq h' \Longrightarrow P,E,h' \vdash es \ [:] \ Ts$

lemma *WT-implies-WTrt*: $P,E \vdash e :: T \Longrightarrow P,E,h \vdash e : T$
and *WTs-implies-WTrts*: $P,E \vdash es \ [::] \ Ts \Longrightarrow P,E,h \vdash es \ [:] \ Ts$

end

2.18 Definite assignment

theory *DefAss* **imports** *BigStep* **begin**

2.18.1 Hypersets

type-synonym $'a\ hyperset = 'a\ set\ option$

definition $hyperUn :: 'a\ hyperset \Rightarrow 'a\ hyperset \Rightarrow 'a\ hyperset$ (**infixl** $\sqcup$ *65*)
where
$A \sqcup B \equiv case\ A\ of\ None \Rightarrow None$
$\quad | \lfloor A\rfloor \Rightarrow (case\ B\ of\ None \Rightarrow None \mid \lfloor B\rfloor \Rightarrow \lfloor A \cup B\rfloor)$

definition $hyperInt :: 'a\ hyperset \Rightarrow 'a\ hyperset \Rightarrow 'a\ hyperset$ (**infixl** $\sqcap$ *70*)
where
$A \sqcap B \equiv case\ A\ of\ None \Rightarrow B$
$\quad | \lfloor A\rfloor \Rightarrow (case\ B\ of\ None \Rightarrow \lfloor A\rfloor \mid \lfloor B\rfloor \Rightarrow \lfloor A \cap B\rfloor)$

definition $hyperDiff1 :: 'a\ hyperset \Rightarrow 'a \Rightarrow 'a\ hyperset$ (**infixl** $\ominus$ *65*)
where
$A \ominus a \equiv case\ A\ of\ None \Rightarrow None \mid \lfloor A\rfloor \Rightarrow \lfloor A - \{a\}\rfloor$

definition $hyper\text{-}isin :: 'a \Rightarrow 'a\ hyperset \Rightarrow bool$ (**infix** $\in\in$ *50*)
where
$a \in\in A \equiv case\ A\ of\ None \Rightarrow True \mid \lfloor A\rfloor \Rightarrow a \in A$

definition $hyper\text{-}subset :: 'a\ hyperset \Rightarrow 'a\ hyperset \Rightarrow bool$ (**infix** $\sqsubseteq$ *50*)
where
$A \sqsubseteq B \equiv case\ B\ of\ None \Rightarrow True$
$\quad | \lfloor B\rfloor \Rightarrow (case\ A\ of\ None \Rightarrow False \mid \lfloor A\rfloor \Rightarrow A \subseteq B)$

lemmas *hyperset-defs* =
hyperUn-def hyperInt-def hyperDiff1-def hyper-isin-def hyper-subset-def

lemma [*simp*]: $\lfloor\{\}\rfloor \sqcup A = A\ \wedge\ A \sqcup \lfloor\{\}\rfloor = A$
lemma [*simp*]: $\lfloor A\rfloor \sqcup \lfloor B\rfloor = \lfloor A \cup B\rfloor \wedge \lfloor A\rfloor \ominus a = \lfloor A - \{a\}\rfloor$
lemma [*simp*]: $None \sqcup A = None \wedge A \sqcup None = None$
lemma [*simp*]: $a \in\in None \wedge None \ominus a = None$
lemma *hyperUn-assoc*: $(A \sqcup B) \sqcup C = A \sqcup (B \sqcup C)$
lemma *hyper-insert-comm*: $A \sqcup \lfloor\{a\}\rfloor = \lfloor\{a\}\rfloor \sqcup A \wedge A \sqcup (\lfloor\{a\}\rfloor \sqcup B) = \lfloor\{a\}\rfloor \sqcup (A \sqcup B)$

2.18.2 Definite assignment

primrec
$\mathcal{A} :: 'a\ exp \Rightarrow 'a\ hyperset$
and $\mathcal{A}s :: 'a\ exp\ list \Rightarrow 'a\ hyperset$
where
$\mathcal{A}\ (new\ C) = \lfloor\{\}\rfloor$
$| \mathcal{A}\ (Cast\ C\ e) = \mathcal{A}\ e$
$| \mathcal{A}\ (Val\ v) = \lfloor\{\}\rfloor$
$| \mathcal{A}\ (e_1 \ll bop \gg e_2) = \mathcal{A}\ e_1 \sqcup \mathcal{A}\ e_2$
$| \mathcal{A}\ (Var\ V) = \lfloor\{\}\rfloor$
$| \mathcal{A}\ (LAss\ V\ e) = \lfloor\{V\}\rfloor \sqcup \mathcal{A}\ e$
$| \mathcal{A}\ (e{\cdot}F\{D\}) = \mathcal{A}\ e$
$| \mathcal{A}\ (e_1{\cdot}F\{D\}{:=}e_2) = \mathcal{A}\ e_1 \sqcup \mathcal{A}\ e_2$

| $\mathcal{A}$ $(e{\cdot}M(es)) = \mathcal{A}\ e \sqcup \mathcal{A}s\ es$
| $\mathcal{A}$ $(\{V{:}T;\ e\}) = \mathcal{A}\ e \ominus V$
| $\mathcal{A}$ $(e_1;;e_2) = \mathcal{A}\ e_1 \sqcup \mathcal{A}\ e_2$
| $\mathcal{A}$ $(\mathit{if}\ (e)\ e_1\ \mathit{else}\ e_2) = \mathcal{A}\ e \sqcup (\mathcal{A}\ e_1 \sqcap \mathcal{A}\ e_2)$
| $\mathcal{A}$ $(\mathit{while}\ (b)\ e) = \mathcal{A}\ b$
| $\mathcal{A}$ $(\mathit{throw}\ e) = \mathit{None}$
| $\mathcal{A}$ $(\mathit{try}\ e_1\ \mathit{catch}(C\ V)\ e_2) = \mathcal{A}\ e_1 \sqcap (\mathcal{A}\ e_2 \ominus V)$

| $\mathcal{A}s$ $([]) = \lfloor\{\}\rfloor$
| $\mathcal{A}s$ $(e\#es) = \mathcal{A}\ e \sqcup \mathcal{A}s\ es$

primrec
$\mathcal{D}$ :: $'a\ \mathit{exp} \Rightarrow 'a\ \mathit{hyperset} \Rightarrow \mathit{bool}$
and $\mathcal{D}s$ :: $'a\ \mathit{exp}\ \mathit{list} \Rightarrow 'a\ \mathit{hyperset} \Rightarrow \mathit{bool}$
where
$\mathcal{D}$ $(\mathit{new}\ C)\ A = \mathit{True}$
| $\mathcal{D}$ $(\mathit{Cast}\ C\ e)\ A = \mathcal{D}\ e\ A$
| $\mathcal{D}$ $(\mathit{Val}\ v)\ A = \mathit{True}$
| $\mathcal{D}$ $(e_1 \ll \mathit{bop} \gg e_2)\ A = (\mathcal{D}\ e_1\ A \wedge \mathcal{D}\ e_2\ (A \sqcup \mathcal{A}\ e_1))$
| $\mathcal{D}$ $(\mathit{Var}\ V)\ A = (V \in\in A)$
| $\mathcal{D}$ $(\mathit{LAss}\ V\ e)\ A = \mathcal{D}\ e\ A$
| $\mathcal{D}$ $(e{\cdot}F\{D\})\ A = \mathcal{D}\ e\ A$
| $\mathcal{D}$ $(e_1{\cdot}F\{D\}{:=}e_2)\ A = (\mathcal{D}\ e_1\ A \wedge \mathcal{D}\ e_2\ (A \sqcup \mathcal{A}\ e_1))$
| $\mathcal{D}$ $(e{\cdot}M(es))\ A = (\mathcal{D}\ e\ A \wedge \mathcal{D}s\ es\ (A \sqcup \mathcal{A}\ e))$
| $\mathcal{D}$ $(\{V{:}T;\ e\})\ A = \mathcal{D}\ e\ (A \ominus V)$
| $\mathcal{D}$ $(e_1;;e_2)\ A = (\mathcal{D}\ e_1\ A \wedge \mathcal{D}\ e_2\ (A \sqcup \mathcal{A}\ e_1))$
| $\mathcal{D}$ $(\mathit{if}\ (e)\ e_1\ \mathit{else}\ e_2)\ A =$
$(\mathcal{D}\ e\ A \wedge \mathcal{D}\ e_1\ (A \sqcup \mathcal{A}\ e) \wedge \mathcal{D}\ e_2\ (A \sqcup \mathcal{A}\ e))$
| $\mathcal{D}$ $(\mathit{while}\ (e)\ c)\ A = (\mathcal{D}\ e\ A \wedge \mathcal{D}\ c\ (A \sqcup \mathcal{A}\ e))$
| $\mathcal{D}$ $(\mathit{throw}\ e)\ A = \mathcal{D}\ e\ A$
| $\mathcal{D}$ $(\mathit{try}\ e_1\ \mathit{catch}(C\ V)\ e_2)\ A = (\mathcal{D}\ e_1\ A \wedge \mathcal{D}\ e_2\ (A \sqcup \lfloor\{V\}\rfloor))$

| $\mathcal{D}s$ $([])\ A = \mathit{True}$
| $\mathcal{D}s$ $(e\#es)\ A = (\mathcal{D}\ e\ A \wedge \mathcal{D}s\ es\ (A \sqcup \mathcal{A}\ e))$

lemma *As-map-Val*[*simp*]: $\mathcal{A}s\ (\mathit{map}\ \mathit{Val}\ vs) = \lfloor\{\}\rfloor$
lemma *D-append*[*iff*]: $\bigwedge A.\ \mathcal{D}s\ (es\ @\ es')\ A = (\mathcal{D}s\ es\ A \wedge \mathcal{D}s\ es'\ (A \sqcup \mathcal{A}s\ es))$

lemma *A-fv*: $\bigwedge A.\ \mathcal{A}\ e = \lfloor A\rfloor \Longrightarrow A \subseteq \mathit{fv}\ e$
and $\bigwedge A.\ \mathcal{A}s\ es = \lfloor A\rfloor \Longrightarrow A \subseteq \mathit{fvs}\ es$

lemma *sqUn-lem*: $A \sqsubseteq A' \Longrightarrow A \sqcup B \sqsubseteq A' \sqcup B$
lemma *diff-lem*: $A \sqsubseteq A' \Longrightarrow A \ominus b \sqsubseteq A' \ominus b$

lemma *D-mono*: $\bigwedge A\ A'.\ A \sqsubseteq A' \Longrightarrow \mathcal{D}\ e\ A \Longrightarrow \mathcal{D}\ (e{::}'a\ \mathit{exp})\ A'$
and *Ds-mono*: $\bigwedge A\ A'.\ A \sqsubseteq A' \Longrightarrow \mathcal{D}s\ es\ A \Longrightarrow \mathcal{D}s\ (es{::}'a\ \mathit{exp}\ \mathit{list})\ A'$

lemma *D-mono'*: $\mathcal{D}\ e\ A \Longrightarrow A \sqsubseteq A' \Longrightarrow \mathcal{D}\ e\ A'$
and *Ds-mono'*: $\mathcal{D}s\ es\ A \Longrightarrow A \sqsubseteq A' \Longrightarrow \mathcal{D}s\ es\ A'$

end

2.19 Conformance Relations for Type Soundness Proofs

theory *Conform*
imports *Exceptions*
begin

definition $conf :: {}'m\ prog \Rightarrow heap \Rightarrow val \Rightarrow ty \Rightarrow bool$ $(\text{-,-} \vdash \text{-} :\leq \text{-}\ [51,51,51,51]\ 50)$
where
$P,h \vdash v :\leq T \equiv$
$\exists T'.\ typeof_h\ v = Some\ T' \wedge P \vdash T' \leq T$

definition $oconf :: {}'m\ prog \Rightarrow heap \Rightarrow obj \Rightarrow bool$ $(\text{-,-} \vdash \text{-} \surd\ [51,51,51]\ 50)$
where
$P,h \vdash obj \surd \equiv$
$let\ (C,fs) = obj\ in\ \forall F\ D\ T.\ P \vdash C\ has\ F{:}T\ in\ D \longrightarrow$
$(\exists v.\ fs(F,D) = Some\ v \wedge P,h \vdash v :\leq T)$

definition $hconf :: {}'m\ prog \Rightarrow heap \Rightarrow bool$ $(\text{-} \vdash \text{-} \surd\ [51,51]\ 50)$
where
$P \vdash h \surd \equiv$
$(\forall a\ obj.\ h\ a = Some\ obj \longrightarrow P,h \vdash obj \surd) \wedge preallocated\ h$

definition $lconf :: {}'m\ prog \Rightarrow heap \Rightarrow (vname \rightharpoonup val) \Rightarrow (vname \rightharpoonup ty) \Rightarrow bool$ $(\text{-,-} \vdash \text{-}\ '(:\leq')\ \text{-}$ $[51,51,51,51]\ 50)$
where
$P,h \vdash l\ (:\leq)\ E \equiv$
$\forall V\ v.\ l\ V = Some\ v \longrightarrow (\exists T.\ E\ V = Some\ T \wedge P,h \vdash v :\leq T)$

abbreviation
$confs :: {}'m\ prog \Rightarrow heap \Rightarrow val\ list \Rightarrow ty\ list \Rightarrow bool$
$(\text{-,-} \vdash \text{-}\ [:\leq]\ \text{-}\ [51,51,51,51]\ 50)$ **where**
$P,h \vdash vs\ [:\leq]\ Ts \equiv list\text{-}all2\ (conf\ P\ h)\ vs\ Ts$

2.19.1 Value conformance $:\leq$

lemma *conf-Null* [*simp*]: $P,h \vdash Null :\leq T = P \vdash NT \leq T$
lemma *typeof-conf*[*simp*]: $typeof_h\ v = Some\ T \Longrightarrow P,h \vdash v :\leq T$
lemma *typeof-lit-conf*[*simp*]: $typeof\ v = Some\ T \Longrightarrow P,h \vdash v :\leq T$
lemma *defval-conf*[*simp*]: $P,h \vdash default\text{-}val\ T :\leq T$
lemma *conf-upd-obj*: $h\ a = Some(C,fs) \Longrightarrow (P,h(a\mapsto(C,fs'))) \vdash x :\leq T) = (P,h \vdash x :\leq T)$
lemma *conf-widen*: $P,h \vdash v :\leq T \Longrightarrow P \vdash T \leq T' \Longrightarrow P,h \vdash v :\leq T'$
lemma *conf-hext*: $h \trianglelefteq h' \Longrightarrow P,h \vdash v :\leq T \Longrightarrow P,h' \vdash v :\leq T$
lemma *conf-ClassD*: $P,h \vdash v :\leq Class\ C \Longrightarrow$
$v = Null \vee (\exists a\ obj\ T.\ v = Addr\ a \wedge\ h\ a = Some\ obj \wedge obj\text{-}ty\ obj = T \wedge\ P \vdash T \leq Class\ C)$
lemma *conf-NT* [*iff*]: $P,h \vdash v :\leq NT = (v = Null)$
lemma *non-npD*: $[\![\ v \neq Null;\ P,h \vdash v :\leq Class\ C\ ]\!]$
$\Longrightarrow \exists a\ C'\ fs.\ v = Addr\ a \wedge h\ a = Some(C',fs) \wedge P \vdash C' \preceq^* C$

2.19.2 Value list conformance $[:\leq]$

lemma *confs-widens* [*trans*]: $[\![P,h \vdash vs\ [:\leq]\ Ts;\ P \vdash Ts\ [\leq]\ Ts']\!] \Longrightarrow P,h \vdash vs\ [:\leq]\ Ts'$
lemma *confs-rev*: $P,h \vdash rev\ s\ [:\leq]\ t = (P,h \vdash s\ [:\leq]\ rev\ t)$
lemma *confs-conv-map*:
$\bigwedge Ts'.\ P,h \vdash vs\ [:\leq]\ Ts' = (\exists Ts.\ map\ typeof_h\ vs = map\ Some\ Ts \wedge P \vdash Ts\ [\leq]\ Ts')$
lemma *confs-hext*: $P,h \vdash vs\ [:\leq]\ Ts \Longrightarrow h \trianglelefteq h' \Longrightarrow P,h' \vdash vs\ [:\leq]\ Ts$

lemma *confs-Cons2*: $P,h \vdash xs\ [:\leq]\ y\#ys = (\exists\, z\ zs.\ xs = z\#zs \wedge P,h \vdash z :\leq y \wedge P,h \vdash zs\ [:\leq]\ ys)$

2.19.3 Object conformance

lemma *oconf-hext*: $P,h \vdash obj\ \surd \Longrightarrow h \trianglelefteq h' \Longrightarrow P,h' \vdash obj\ \surd$
lemma *oconf-init-fields*:
$P \vdash C\ \mathit{has\text{-}fields}\ FDTs \Longrightarrow P,h \vdash (C,\ \mathit{init\text{-}fields}\ FDTs)\ \surd$
by(*fastforce simp add*: *has-field-def oconf-def init-fields-def map-of-map*
dest: *has-fields-fun*)

lemma *oconf-fupd* [*intro?*]:
$[\![\ P \vdash C\ \mathit{has}\ F{:}T\ \mathit{in}\ D;\ P,h \vdash v :\leq T;\ P,h \vdash (C,fs)\ \surd\ ]\!]$
$\Longrightarrow P,h \vdash (C,\ fs((F,D)\mapsto v))\ \surd$

2.19.4 Heap conformance

lemma *hconfD*: $[\![\ P \vdash h\ \surd;\ h\ a = Some\ obj\ ]\!] \Longrightarrow P,h \vdash obj\ \surd$
lemma *hconf-new*: $[\![\ P \vdash h\ \surd;\ h\ a = None;\ P,h \vdash obj\ \surd\ ]\!] \Longrightarrow P \vdash h(a\mapsto obj)\ \surd$
lemma *hconf-upd-obj*: $[\![\ P \vdash h\surd;\ h\ a = Some(C,fs);\ P,h \vdash (C,fs')\surd\ ]\!] \Longrightarrow P \vdash h(a\mapsto(C,fs'))\surd$

2.19.5 Local variable conformance

lemma *lconf-hext*: $[\![\ P,h \vdash l\ (:\leq)\ E;\ h \trianglelefteq h'\ ]\!] \Longrightarrow P,h' \vdash l\ (:\leq)\ E$
lemma *lconf-upd*:
$[\![\ P,h \vdash l\ (:\leq)\ E;\ P,h \vdash v :\leq T;\ E\ V = Some\ T\ ]\!] \Longrightarrow P,h \vdash l(V\mapsto v)\ (:\leq)\ E$
lemma *lconf-empty*[*iff*]: $P,h \vdash empty\ (:\leq)\ E$
lemma *lconf-upd2*: $[\![P,h \vdash l\ (:\leq)\ E;\ P,h \vdash v :\leq T]\!] \Longrightarrow P,h \vdash l(V\mapsto v)\ (:\leq)\ E(V\mapsto T)$

end

2.20 Progress of Small Step Semantics

theory *Progress*
imports *Equivalence WellTypeRT DefAss ../Common/Conform*
begin

lemma *final-addrE*:
$[\![\ P,E,h \vdash e : \mathit{Class}\ C;\ \mathit{final}\ e;$
$\bigwedge a.\ e = \mathit{addr}\ a \Longrightarrow R;$
$\bigwedge a.\ e = \mathit{Throw}\ a \Longrightarrow R\ ]\!] \Longrightarrow R$

lemma *finalRefE*:
$[\![\ P,E,h \vdash e : T;\ \mathit{is\text{-}refT}\ T;\ \mathit{final}\ e;$
$e = \mathit{null} \Longrightarrow R;$
$\bigwedge a\ C.\ [\![\ e = \mathit{addr}\ a;\ T = \mathit{Class}\ C\ ]\!] \Longrightarrow R;$
$\bigwedge a.\ e = \mathit{Throw}\ a \Longrightarrow R\ ]\!] \Longrightarrow R$

Derivation of new induction scheme for well typing:

inductive
$\mathit{WTrt}' :: [\mathit{J\text{-}prog},\mathit{heap},\mathit{env},\mathit{expr},\mathit{ty}] \Rightarrow \mathit{bool}$
and $\mathit{WTrts}' :: [\mathit{J\text{-}prog},\mathit{heap},\mathit{env},\mathit{expr\ list},\ \mathit{ty\ list}] \Rightarrow \mathit{bool}$
and $\mathit{WTrt2}' :: [\mathit{J\text{-}prog},\mathit{env},\mathit{heap},\mathit{expr},\mathit{ty}] \Rightarrow \mathit{bool}$
$(\text{-},\text{-},\text{-} \vdash \text{-} :' \text{-}\quad [51,51,51]50)$
and $\mathit{WTrts2}' :: [\mathit{J\text{-}prog},\mathit{env},\mathit{heap},\mathit{expr\ list},\ \mathit{ty\ list}] \Rightarrow \mathit{bool}$
$(\text{-},\text{-},\text{-} \vdash \text{-}\ [:'']\ \text{-}\ [51,51,51]50)$
for $P :: \mathit{J\text{-}prog}$ **and** $h :: \mathit{heap}$
where
$P,E,h \vdash e :' T \equiv \mathit{WTrt}'\ P\ h\ E\ e\ T$
$|\ P,E,h \vdash es\ [:']\ Ts \equiv \mathit{WTrts}'\ P\ h\ E\ es\ Ts$

$|\ \mathit{is\text{-}class}\ P\ C \implies P,E,h \vdash \mathit{new}\ C :' \mathit{Class}\ C$
$|\ [\![\ P,E,h \vdash e :' T;\ \mathit{is\text{-}refT}\ T;\ \mathit{is\text{-}class}\ P\ C\ ]\!]$
$\implies P,E,h \vdash \mathit{Cast}\ C\ e :' \mathit{Class}\ C$
$|\ \mathit{typeof}_h\ v = \mathit{Some}\ T \Longrightarrow P,E,h \vdash \mathit{Val}\ v :' T$
$|\ E\ v = \mathit{Some}\ T \implies P,E,h \vdash \mathit{Var}\ v :' T$
$|\ [\![\ P,E,h \vdash e_1 :' T_1;\ \ P,E,h \vdash e_2 :' T_2\ ]\!]$
$\implies P,E,h \vdash e_1 \ll Eq \gg e_2 :' \mathit{Boolean}$
$|\ [\![\ P,E,h \vdash e_1 :' \mathit{Integer};\ \ P,E,h \vdash e_2 :' \mathit{Integer}\ ]\!]$
$\implies P,E,h \vdash e_1 \ll Add \gg e_2 :' \mathit{Integer}$
$|\ [\![\ P,E,h \vdash \mathit{Var}\ V :' T;\ \ P,E,h \vdash e :' T';\ \ P \vdash T' \le T\ (*\ V \neq \mathit{This}*)\ ]\!]$
$\implies P,E,h \vdash V{:=}e :' \mathit{Void}$
$|\ [\![\ P,E,h \vdash e :' \mathit{Class}\ C;\ P \vdash C\ \mathit{has}\ F{:}T\ \mathit{in}\ D\ ]\!] \Longrightarrow P,E,h \vdash e{\cdot}F\{D\} :' T$
$|\ P,E,h \vdash e :' NT \Longrightarrow P,E,h \vdash e{\cdot}F\{D\} :' T$
$|\ [\![\ P,E,h \vdash e_1 :' \mathit{Class}\ C;\ \ P \vdash C\ \mathit{has}\ F{:}T\ \mathit{in}\ D;$
$P,E,h \vdash e_2 :' T_2;\ \ P \vdash T_2 \le T\ ]\!]$
$\implies P,E,h \vdash e_1{\cdot}F\{D\}{:=}e_2 :' \mathit{Void}$
$|\ [\![\ P,E,h \vdash e_1{:}'NT;\ P,E,h \vdash e_2 :' T_2\ ]\!] \Longrightarrow P,E,h \vdash e_1{\cdot}F\{D\}{:=}e_2 :' \mathit{Void}$
$|\ [\![\ P,E,h \vdash e :' \mathit{Class}\ C;\ P \vdash C\ \mathit{sees}\ M{:}Ts \to T = (\mathit{pns},\mathit{body})\ \mathit{in}\ D;$
$P,E,h \vdash es\ [:']\ Ts';\ P \vdash Ts'\ [\le]\ Ts\ ]\!]$
$\implies P,E,h \vdash e{\cdot}M(es) :' T$
$|\ [\![\ P,E,h \vdash e :' NT;\ P,E,h \vdash es\ [:']\ Ts\ ]\!] \Longrightarrow P,E,h \vdash e{\cdot}M(es) :' T$
$|\ P,E,h \vdash []\ [:']\ []$
$|\ [\![\ P,E,h \vdash e :' T;\ \ P,E,h \vdash es\ [:']\ Ts\ ]\!] \Longrightarrow\ \ P,E,h \vdash e\#es\ [:']\ T\#Ts$
$|\ [\![\ \mathit{typeof}_h\ v = \mathit{Some}\ T_1;\ P \vdash T_1 \le T;\ P,E(V{\mapsto}T),h \vdash e_2 :' T_2\ ]\!]$

$\Longrightarrow\ P,E,h \vdash \{V{:}T := Val\ v;\ e_2\} :' T_2$
$\mid [\![\ P,E(V{\mapsto}T),h \vdash e :' T';\ \neg\ assigned\ V\ e\ ]\!] \Longrightarrow\ P,E,h \vdash \{V{:}T;\ e\} :' T'$
$\mid [\![\ P,E,h \vdash e_1{:}'\ T_1;\ \ P,E,h \vdash e_2{:}'T_2\ ]\!]\ \Longrightarrow\ P,E,h \vdash e_1;;e_2 :' T_2$
$\mid [\![\ P,E,h \vdash e :' Boolean;\ \ P,E,h \vdash e_1{:}'\ T_1;\ \ P,E,h \vdash e_2{:}'\ T_2;$
$P \vdash T_1 \le T_2 \lor P \vdash T_2 \le T_1;$
$P \vdash T_1 \le T_2 \longrightarrow T = T_2;\ P \vdash T_2 \le T_1 \longrightarrow T = T_1\ ]\!]$
$\Longrightarrow P,E,h \vdash if\ (e)\ e_1\ else\ e_2 :' T$

$\mid [\![\ P,E,h \vdash e :' Boolean;\ \ P,E,h \vdash c{:}'\ T\ ]\!]$
$\Longrightarrow\ P,E,h \vdash while(e)\ c :' Void$
$\mid [\![\ P,E,h \vdash e :' T_r;\ is\text{-}refT\ T_r\ ]\!]\ \Longrightarrow\ P,E,h \vdash throw\ e :' T$
$\mid [\![\ P,E,h \vdash e_1 :' T_1;\ \ P,E(V \mapsto Class\ C),h \vdash e_2 :' T_2;\ P \vdash T_1 \le T_2\ ]\!]$
$\Longrightarrow P,E,h \vdash try\ e_1\ catch(C\ V)\ e_2 :' T_2$

lemma [*iff*]: $P,E,h \vdash e_1;;e_2 :' T_2 = (\exists\, T_1.\ P,E,h \vdash e_1{:}'\ T_1 \land P,E,h \vdash e_2{:}'\ T_2)$
lemma [*iff*]: $P,E,h \vdash Val\ v :' T = (typeof_h\ v = Some\ T)$
lemma [*iff*]: $P,E,h \vdash Var\ v :' T = (E\ v = Some\ T)$

lemma *wt-wt'*: $P,E,h \vdash e : T \Longrightarrow P,E,h \vdash e :' T$
and *wts-wts'*: $P,E,h \vdash es\ [:]\ Ts \Longrightarrow P,E,h \vdash es\ [:']\ Ts$

lemma *wt'-wt*: $P,E,h \vdash e :' T \Longrightarrow P,E,h \vdash e : T$
and *wts'-wts*: $P,E,h \vdash es\ [:']\ Ts \Longrightarrow P,E,h \vdash es\ [:]\ Ts$

corollary *wt'-iff-wt*: $(P,E,h \vdash e :' T) = (P,E,h \vdash e : T)$

corollary *wts'-iff-wts*: $(P,E,h \vdash es\ [:']\ Ts) = (P,E,h \vdash es\ [:]\ Ts)$
theorem assumes *wf*: *wwf-J-prog P* **and** *hconf*: $P \vdash h\ \surd$
shows *progress*: $P,E,h \vdash e : T \Longrightarrow$
$(\bigwedge l.\ [\![\ \mathcal{D}\ e\ \lfloor dom\ l\rfloor;\ \neg\ final\ e\ ]\!] \Longrightarrow \exists\, e'\ s'.\ P \vdash \langle e,(h,l)\rangle \to \langle e',s'\rangle)$
and $P,E,h \vdash es\ [:]\ Ts \Longrightarrow$
$(\bigwedge l.\ [\![\ \mathcal{D}s\ es\ \lfloor dom\ l\rfloor;\ \neg\ finals\ es\ ]\!] \Longrightarrow \exists\, es'\ s'.\ P \vdash \langle es,(h,l)\rangle\ [\to]\ \langle es',s'\rangle)$

end

2.21 Well-formedness Constraints

theory *JWellForm*
imports *../Common/WellForm WWellForm WellType DefAss*
begin

definition *wf-J-mdecl* :: *J-prog* $\Rightarrow$ *cname* $\Rightarrow$ *J-mb mdecl* $\Rightarrow$ *bool*
where
  $wf\text{-}J\text{-}mdecl\ P\ C\ \equiv\ \lambda(M,Ts,T,(pns,body)).$
  $length\ Ts = length\ pns\ \wedge$
  $distinct\ pns\ \wedge$
  $this \notin set\ pns\ \wedge$
  $(\exists\, T'.\ P,[this{\mapsto}Class\ C,pns[\mapsto]Ts] \vdash body :: T' \wedge P \vdash T' \le T)\ \wedge$
  $\mathcal{D}\ body\ \lfloor\{this\} \cup set\ pns\rfloor$

lemma *wf-J-mdecl[simp]*:
  $wf\text{-}J\text{-}mdecl\ P\ C\ (M,Ts,T,pns,body) \equiv$
  $(length\ Ts = length\ pns\ \wedge$
  $distinct\ pns\ \wedge$
  $this \notin set\ pns\ \wedge$
  $(\exists\, T'.\ P,[this{\mapsto}Class\ C,pns[\mapsto]Ts] \vdash body :: T' \wedge P \vdash T' \le T)\ \wedge$
  $\mathcal{D}\ body\ \lfloor\{this\} \cup set\ pns\rfloor)$

abbreviation
  wf-J-prog :: *J-prog* $\Rightarrow$ *bool* **where**
  wf-J-prog == *wf-prog wf-J-mdecl*

lemma *wf-J-prog-wf-J-mdecl*:
  $\llbracket$ *wf-J-prog P*; $(C,\ D,\ fds,\ mths) \in set\ P$; $jmdcl \in set\ mths$ $\rrbracket$
  $\Longrightarrow$ *wf-J-mdecl P C jmdcl*

lemma *wf-mdecl-wwf-mdecl*: *wf-J-mdecl P C Md* $\Longrightarrow$ *wwf-J-mdecl P C Md*

lemma *wf-prog-wwf-prog*: *wf-J-prog P* $\Longrightarrow$ *wwf-J-prog P*

end

2.22 Type Safety Proof

theory *TypeSafe*
imports *Progress JWellForm*
begin

2.22.1 Basic preservation lemmas

First two easy preservation lemmas.

theorem *red-preserves-hconf*:
$P \vdash \langle e,(h,l)\rangle \rightarrow \langle e',(h',l')\rangle \Longrightarrow (\bigwedge T\ E.\ [\![\ P,E,h \vdash e : T;\ P \vdash h\ \surd\ ]\!] \Longrightarrow P \vdash h'\ \surd)$
and *reds-preserves-hconf*:
$P \vdash \langle es,(h,l)\rangle\ [\rightarrow]\ \langle es',(h',l')\rangle \Longrightarrow (\bigwedge Ts\ E.\ [\![\ P,E,h \vdash es\ [:]\ Ts;\ P \vdash h\ \surd\ ]\!] \Longrightarrow P \vdash h'\ \surd)$

theorem *red-preserves-lconf*:
$P \vdash \langle e,(h,l)\rangle \rightarrow \langle e',(h',l')\rangle \Longrightarrow$
$(\bigwedge T\ E.\ [\![\ P,E,h \vdash e{:}T;\ P,h \vdash l\ (:\leq)\ E\ ]\!] \Longrightarrow P,h' \vdash l'\ (:\leq)\ E)$
and *reds-preserves-lconf*:
$P \vdash \langle es,(h,l)\rangle\ [\rightarrow]\ \langle es',(h',l')\rangle \Longrightarrow$
$(\bigwedge Ts\ E.\ [\![\ P,E,h \vdash es[:]Ts;\ P,h \vdash l\ (:\leq)\ E\ ]\!] \Longrightarrow P,h' \vdash l'\ (:\leq)\ E)$

Preservation of definite assignment more complex and requires a few lemmas first.

lemma [*iff*]: $\bigwedge A.\ [\![\ length\ Vs = length\ Ts;\ length\ vs = length\ Ts]\!] \Longrightarrow$
$\mathcal{D}\ (blocks\ (Vs,Ts,vs,e))\ A = \mathcal{D}\ e\ (A \sqcup \lfloor set\ Vs\rfloor)$

lemma *red-lA-incr*: $P \vdash \langle e,(h,l)\rangle \rightarrow \langle e',(h',l')\rangle \Longrightarrow \lfloor dom\ l\rfloor \sqcup \mathcal{A}\ e \sqsubseteq\ \lfloor dom\ l'\rfloor \sqcup \mathcal{A}\ e'$
and *reds-lA-incr*: $P \vdash \langle es,(h,l)\rangle\ [\rightarrow]\ \langle es',(h',l')\rangle \Longrightarrow \lfloor dom\ l\rfloor \sqcup \mathcal{A}s\ es \sqsubseteq\ \lfloor dom\ l'\rfloor \sqcup \mathcal{A}s\ es'$

Now preservation of definite assignment.

lemma assumes *wf*: *wf-J-prog P*
shows *red-preserves-defass*:
$P \vdash \langle e,(h,l)\rangle \rightarrow \langle e',(h',l')\rangle \Longrightarrow \mathcal{D}\ e\ \lfloor dom\ l\rfloor \Longrightarrow \mathcal{D}\ e'\ \lfloor dom\ l'\rfloor$
and $P \vdash \langle es,(h,l)\rangle\ [\rightarrow]\ \langle es',(h',l')\rangle \Longrightarrow \mathcal{D}s\ es\ \lfloor dom\ l\rfloor \Longrightarrow \mathcal{D}s\ es'\ \lfloor dom\ l'\rfloor$

Combining conformance of heap and local variables:

definition *sconf* :: *J-prog* ⇒ *env* ⇒ *state* ⇒ *bool* (-,- ⊢ - √ [*51*,*51*,*51*]*50*)
where
$P,E \vdash s\ \surd\ \equiv\ let\ (h,l) = s\ in\ P \vdash h\ \surd \wedge P,h \vdash l\ (:\leq)\ E$

lemma *red-preserves-sconf*:
$[\![\ P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle;\ P,E,hp\ s \vdash e : T;\ P,E \vdash s\ \surd\ ]\!] \Longrightarrow P,E \vdash s'\ \surd$
lemma *reds-preserves-sconf*:
$[\![\ P \vdash \langle es,s\rangle\ [\rightarrow]\ \langle es',s'\rangle;\ P,E,hp\ s \vdash es\ [:]\ Ts;\ P,E \vdash s\ \surd\ ]\!] \Longrightarrow P,E \vdash s'\ \surd$

2.22.2 Subject reduction

lemma *wt-blocks*:
$\bigwedge E.\ [\![\ length\ Vs = length\ Ts;\ length\ vs = length\ Ts\ ]\!] \Longrightarrow$
$(P,E,h \vdash blocks(Vs,Ts,vs,e) : T) =$
$(P,E(Vs[\mapsto]Ts),h \vdash e{:}T \wedge (\exists\ Ts'.\ map\ (typeof_h)\ vs = map\ Some\ Ts' \wedge P \vdash Ts'\ [\leq]\ Ts))$

theorem assumes *wf*: *wf-J-prog P*
shows *subject-reduction2*: $P \vdash \langle e,(h,l)\rangle \rightarrow \langle e',(h',l')\rangle \Longrightarrow$
$(\bigwedge E\ T.\ [\![\ P,E \vdash (h,l)\ \surd;\ P,E,h \vdash e{:}T\ ]\!]$

$\Longrightarrow \exists\, T'.\ P,E,h' \vdash e':T' \wedge P \vdash T' \leq T$)
and *subjects-reduction2*: $P \vdash \langle es,(h,l)\rangle\ [\to]\ \langle es',(h',l')\rangle \Longrightarrow$
($\bigwedge E\ Ts.\ [\![\ P,E \vdash (h,l)\ \surd;\ P,E,h \vdash es\ [:]\ Ts\ ]\!]$
$\Longrightarrow \exists\, Ts'.\ P,E,h' \vdash es'\ [:]\ Ts' \wedge P \vdash Ts'\ [\leq]\ Ts$)

corollary *subject-reduction*:
$[\![\ wf\text{-}J\text{-}prog\ P;\ P \vdash \langle e,s\rangle \to \langle e',s'\rangle;\ P,E \vdash s\ \surd;\ P,E,hp\ s \vdash e:T\ ]\!]$
$\Longrightarrow \exists\, T'.\ P,E,hp\ s' \vdash e':T' \wedge P \vdash T' \leq T$
corollary *subjects-reduction*:
$[\![\ wf\text{-}J\text{-}prog\ P;\ P \vdash \langle es,s\rangle\ [\to]\ \langle es',s'\rangle;\ P,E \vdash s\ \surd;\ P,E,hp\ s \vdash es[:]Ts\ ]\!]$
$\Longrightarrow \exists\, Ts'.\ P,E,hp\ s' \vdash es'[:]Ts' \wedge P \vdash Ts'\ [\leq]\ Ts$

2.22.3 Lifting to $\to*$

Now all these preservation lemmas are first lifted to the transitive closure ...

lemma *Red-preserves-sconf*:
assumes *wf*: *wf-J-prog P* **and** *Red*: $P \vdash \langle e,s\rangle \to* \langle e',s'\rangle$
shows $\bigwedge T.\ [\![\ P,E,hp\ s \vdash e : T;\ P,E \vdash s\ \surd\ ]\!] \Longrightarrow P,E \vdash s'\ \surd$

lemma *Red-preserves-defass*:
assumes *wf*: *wf-J-prog P* **and** *reds*: $P \vdash \langle e,s\rangle \to* \langle e',s'\rangle$
shows $\mathcal{D}\ e\ \lfloor dom(lcl\ s)\rfloor \Longrightarrow \mathcal{D}\ e'\ \lfloor dom(lcl\ s')\rfloor$
using *reds*
proof (*induct rule*:*converse-rtrancl-induct2*)
  case *refl* **thus** *?case* **.**
next
  case (*step e s e' s'*) **thus** *?case*
    by(*cases s*,*cases s'*)(*auto dest*:*red-preserves-defass*[*OF wf*])
qed

lemma *Red-preserves-type*:
assumes *wf*: *wf-J-prog P* **and** *Red*: $P \vdash \langle e,s\rangle \to* \langle e',s'\rangle$
shows $!!T.\ [\![\ P,E \vdash s\surd;\ P,E,hp\ s \vdash e:T\ ]\!]$
$\Longrightarrow \exists\, T'.\ P \vdash T' \leq T \wedge P,E,hp\ s' \vdash e':T'$

2.22.4 Lifting to $\Rightarrow$

... and now to the big step semantics, just for fun.

lemma *eval-preserves-sconf*:
$[\![\ wf\text{-}J\text{-}prog\ P;\ P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle;\ P,E \vdash e::T;\ P,E \vdash s\surd\ ]\!] \Longrightarrow P,E \vdash s'\surd$

lemma *eval-preserves-type*: **assumes** *wf*: *wf-J-prog P*
shows $[\![\ P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle;\ P,E \vdash s\surd;\ P,E \vdash e::T\ ]\!]$
$\Longrightarrow \exists\, T'.\ P \vdash T' \leq T \wedge P,E,hp\ s' \vdash e':T'$

2.22.5 The final polish

The above preservation lemmas are now combined and packed nicely.

definition *wf-config* :: *J-prog* $\Rightarrow$ *env* $\Rightarrow$ *state* $\Rightarrow$ *expr* $\Rightarrow$ *ty* $\Rightarrow$ *bool* (-,-,- $\vdash$ - : - $\surd$ [*51*,*0*,*0*,*0*,*0*]*50*)
where
$P,E,s \vdash e:T\ \surd\ \equiv\ P,E \vdash s\ \surd \wedge P,E,hp\ s \vdash e:T$

theorem *Subject-reduction*: **assumes** *wf*: *wf-J-prog P*
shows $P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow P,E,s \vdash e : T \surd$
$\Longrightarrow \exists T'.\ P,E,s' \vdash e' : T' \surd \wedge P \vdash T' \le T$

theorem *Subject-reductions*:
assumes *wf*: *wf-J-prog P* **and** *reds*: $P \vdash \langle e,s\rangle \to\!\ast \langle e',s'\rangle$
shows $\bigwedge T.\ P,E,s \vdash e{:}T \surd \Longrightarrow \exists T'.\ P,E,s' \vdash e'{:}T' \surd \wedge P \vdash T' \le T$

corollary *Progress*: **assumes** *wf*: *wf-J-prog P*
shows $[\![\ P,E,s\ \vdash e : T \surd;\ \mathcal{D}\ e\ \lfloor dom(lcl\ s)\rfloor;\ \neg\ final\ e\ ]\!] \Longrightarrow \exists e'\ s'.\ P \vdash \langle e,s\rangle \to \langle e',s'\rangle$

corollary *TypeSafety*:
$[\![\ wf\text{-}J\text{-}prog\ P;\ P,E \vdash s \surd;\ P,E \vdash e{::}T;\ \mathcal{D}\ e\ \lfloor dom(lcl\ s)\rfloor;$
$P \vdash \langle e,s\rangle \to\!\ast \langle e',s'\rangle;\ \neg(\exists e''\ s''.\ P \vdash \langle e',s'\rangle \to \langle e'',s''\rangle)\ ]\!]$
$\Longrightarrow (\exists v.\ e' = Val\ v \wedge P,hp\ s' \vdash v :\le T) \vee$
$(\exists a.\ e' = Throw\ a \wedge a \in dom(hp\ s'))$

end

2.23 Program annotation

theory *Annotate* **imports** *WellType* **begin**

inductive
Anno :: [*J-prog*,*env*, *expr* , *expr*] ⇒ *bool*
(-,- ⊢ - $\leadsto$ - [51,0,0,51]50)
and *Annos* :: [*J-prog*,*env*, *expr list*, *expr list*] ⇒ *bool*
(-,- ⊢ - [$\leadsto$] - [51,0,0,51]50)
for *P* :: *J-prog*
where

$AnnoNew$: $P,E \vdash new\ C \leadsto new\ C$
| $AnnoCast$: $P,E \vdash e \leadsto e' \Longrightarrow P,E \vdash Cast\ C\ e \leadsto Cast\ C\ e'$
| $AnnoVal$: $P,E \vdash Val\ v \leadsto Val\ v$
| $AnnoVarVar$: $E\ V = \lfloor T \rfloor \Longrightarrow P,E \vdash Var\ V \leadsto Var\ V$
| $AnnoVarField$: $[\![\ E\ V = None;\ E\ this = \lfloor Class\ C \rfloor;\ P \vdash C\ sees\ V{:}T\ in\ D\ ]\!]$
$\Longrightarrow P,E \vdash Var\ V \leadsto Var\ this{\cdot}V\{D\}$
| $AnnoBinOp$:
$[\![\ P,E \vdash e1 \leadsto e1';\ \ P,E \vdash e2 \leadsto e2'\ ]\!]$
$\Longrightarrow P,E \vdash e1 \ll bop \gg e2 \leadsto e1' \ll bop \gg e2'$
| $AnnoLAssVar$:
$[\![\ E\ V = \lfloor T \rfloor;\ P,E \vdash e \leadsto e'\ ]\!] \Longrightarrow P,E \vdash V{:=}e \leadsto V{:=}e'$
| $AnnoLAssField$:
$[\![\ E\ V = None;\ E\ this = \lfloor Class\ C \rfloor;\ P \vdash C\ sees\ V{:}T\ in\ D;\ P,E \vdash e \leadsto e'\ ]\!]$
$\Longrightarrow P,E \vdash V{:=}e \leadsto Var\ this{\cdot}V\{D\} := e'$
| $AnnoFAcc$:
$[\![\ P,E \vdash e \leadsto e';\ \ P,E \vdash e' :: Class\ C;\ \ P \vdash C\ sees\ F{:}T\ in\ D\ ]\!]$
$\Longrightarrow P,E \vdash e{\cdot}F\{[]\} \leadsto e'{\cdot}F\{D\}$
| $AnnoFAss$: $[\![\ P,E \vdash e1 \leadsto e1';\ \ P,E \vdash e2 \leadsto e2';$
$P,E \vdash e1' :: Class\ C;\ P \vdash C\ sees\ F{:}T\ in\ D\ ]\!]$
$\Longrightarrow P,E \vdash e1{\cdot}F\{[]\} := e2 \leadsto e1'{\cdot}F\{D\} := e2'$
| $AnnoCall$:
$[\![\ P,E \vdash e \leadsto e';\ \ P,E \vdash es\ [\leadsto]\ es'\ ]\!]$
$\Longrightarrow P,E \vdash Call\ e\ M\ es \leadsto Call\ e'\ M\ es'$
| $AnnoBlock$:
$P,E(V \mapsto T) \vdash e \leadsto e' \ \Longrightarrow\ P,E \vdash \{V{:}T;\ e\} \leadsto \{V{:}T;\ e'\}$
| $AnnoComp$: $[\![\ P,E \vdash e1 \leadsto e1';\ \ P,E \vdash e2 \leadsto e2'\ ]\!]$
$\Longrightarrow\ P,E \vdash e1;;e2 \leadsto e1';;e2'$
| $AnnoCond$: $[\![\ P,E \vdash e \leadsto e';\ P,E \vdash e1 \leadsto e1';\ \ P,E \vdash e2 \leadsto e2'\ ]\!]$
$\Longrightarrow P,E \vdash if\ (e)\ e1\ else\ e2 \leadsto if\ (e')\ e1'\ else\ e2'$
| $AnnoLoop$: $[\![\ P,E \vdash e \leadsto e';\ \ P,E \vdash c \leadsto c'\ ]\!]$
$\Longrightarrow P,E \vdash while\ (e)\ c \leadsto while\ (e')\ c'$
| $AnnoThrow$: $P,E \vdash e \leadsto e' \ \Longrightarrow\ P,E \vdash throw\ e \leadsto throw\ e'$
| $AnnoTry$: $[\![\ P,E \vdash e1 \leadsto e1';\ \ P,E(V \mapsto Class\ C) \vdash e2 \leadsto e2'\ ]\!]$
$\Longrightarrow P,E \vdash try\ e1\ catch(C\ V)\ e2 \leadsto try\ e1'\ catch(C\ V)\ e2'$

| $AnnoNil$: $P,E \vdash []\ [\leadsto]\ []$
| $AnnoCons$: $[\![\ P,E \vdash e \leadsto e';\ \ P,E \vdash es\ [\leadsto]\ es'\ ]\!]$
$\Longrightarrow\ P,E \vdash e\#es\ [\leadsto]\ e'\#es'$

end

2.24 Example Expressions

```
theory Examples imports Expr begin

definition classObject::J-mb cdecl
where
  classObject == (''Object'','''',[],[])


definition classI :: J-mb cdecl
where
  classI ==
  (''I'', Object,
  [],
  [(''mult'',[Integer,Integer],Integer,[''i'',''j''],
   if (Var ''i'' «Eq» Val(Intg 0)) (Val(Intg 0))
   else Var ''j'' «Add»
       Var this · ''mult''([Var ''i'' «Add» Val(Intg −1),Var ''j'']))
  ])


definition classL :: J-mb cdecl
where
  classL ==
  (''L'', Object,
  [(''F'',Integer), (''N'',Class ''L'')],
  [(''app'',[Class ''L''],Void,[''l''],
   if (Var this · ''N''{''L''} «Eq» null)
      (Var this · ''N''{''L''} := Var ''l'')
   else (Var this · ''N''{''L''}) · ''app''([Var ''l'']))
  ])


definition testExpr-BuildList :: expr
where
  testExpr-BuildList ==
  {''l1'':Class ''L'' := new ''L'';
   Var ''l1''·''F''{''L''} := Val(Intg 1);;
  {''l2'':Class ''L'' := new ''L'';
   Var ''l2''· ''F''{''L''} := Val(Intg 2);;
  {''l3'':Class ''L'' := new ''L'';
   Var ''l3''· ''F''{''L''} := Val(Intg 3);;
  {''l4'':Class ''L'' := new ''L'';
   Var ''l4''· ''F''{''L''} := Val(Intg 4);;
   Var ''l1''·''app''([Var ''l2'']);;
   Var ''l1''·''app''([Var ''l3'']);;
   Var ''l1''·''app''([Var ''l4''])}}}}

definition testExpr1 ::expr
where
  testExpr1 == Val(Intg 5)
definition testExpr2 ::expr
where
  testExpr2 == BinOp (Val(Intg 5)) Add (Val(Intg 6))
```

definition *testExpr3* ::*expr*
where
testExpr3 == *BinOp* (*Var* ''*V*'') *Add* (*Val*(*Intg 6*))
definition *testExpr4* ::*expr*
where
testExpr4 == ''*V*'' := *Val*(*Intg 6*)
definition *testExpr5* ::*expr*
where
testExpr5 == *new* ''*Object*'';; {''*V*'':(*Class* ''*C*'') := *new* ''*C*''; *Var* ''*V*''•''*F*''{''*C*''} := *Val*(*Intg 42*)}
definition *testExpr6* ::*expr*
where
testExpr6 == {''*V*'':(*Class* ''*I*'') := *new* ''*I*''; *Var* ''*V*''•''*mult*''([*Val*(*Intg 40*),*Val*(*Intg 4*)])}

definition *mb-isNull*:: *expr*
where
mb-isNull == *Var this* • ''*test*''{''*A*''} ≪*Eq*≫ *null*

definition *mb-add*:: *expr*
where
mb-add == (*Var this* • ''*int*''{''*A*''} :=(*Var this* • ''*int*''{''*A*''} ≪*Add*≫ *Var* ''*i*''));; (*Var this* • ''*int*''{''*A*''})

definition *mb-mult-cond*:: *expr*
where
mb-mult-cond == (*Var* ''*j*'' ≪*Eq*≫ *Val* (*Intg 0*)) ≪*Eq*≫ *Val* (*Bool False*)

definition *mb-mult-block*:: *expr*
where
mb-mult-block == ''*temp*'':=(*Var* ''*temp*'' ≪*Add*≫ *Var* ''*i*'');;''*j*'':=(*Var* ''*j*'' ≪*Add*≫ *Val* (*Intg* −*1*))

definition *mb-mult*:: *expr*
where
mb-mult == {''*temp*'':*Integer*:=*Val* (*Intg 0*); *While* (*mb-mult-cond*) *mb-mult-block*;; (*Var this* • ''*int*''{''*A*''} := *Var* ''*temp*'';; *Var* ''*temp*'')}

definition *classA*:: *J-mb cdecl*
where
classA ==
(''*A*'', *Object*,
[(''*int*'',*Integer*),
(''*test*'',*Class* ''*A*'')],
[(''*isNull*'',[],*Boolean*,[], *mb-isNull*),
(''*add*'',[*Integer*],*Integer*,[''*i*''], *mb-add*),
(''*mult*'',[*Integer*,*Integer*],*Integer*,[''*i*'',''*j*''], *mb-mult*)])

definition *testExpr-ClassA*:: *expr*
where
testExpr-ClassA ==
{''*A1*'':*Class* ''*A*'':= *new* ''*A*'';
{''*A2*'':*Class* ''*A*'':= *new* ''*A*'';
{''*testint*'':*Integer*:= *Val* (*Intg 5*);
(*Var* ''*A2*''• ''*int*''{''*A*''} := (*Var* ''*A1*''• ''*add*''([*Var* ''*testint*''])));;

$(Var\ ''A2''\cdot\ ''int''\{''A''\} := (Var\ ''A1''\cdot\ ''add''([Var\ ''testint'']))$);;
$Var\ ''A2''\cdot\ ''mult''([Var\ ''A2''\cdot\ ''int''\{''A''\},\ Var\ ''testint''])\ \}\}\}$

end

2.25 Code Generation For BigStep

theory *execute-Bigstep*
imports
  BigStep Examples
  ~~/src/HOL/Library/Efficient-Nat
begin

inductive *map-val* :: *expr list* ⇒ *val list* ⇒ *bool*
where
  Nil: *map-val* [] []
| *Cons*: *map-val xs ys* ⟹ *map-val* (*Val y* # *xs*) (*y* # *ys*)

inductive *map-val2* :: *expr list* ⇒ *val list* ⇒ *expr list* ⇒ *bool*
where
  Nil: *map-val2* [] [] []
| *Cons*: *map-val2 xs ys zs* ⟹ *map-val2* (*Val y* # *xs*) (*y* # *ys*) *zs*
| *Throw*: *map-val2* (*throw e* # *xs*) [] (*throw e* # *xs*)

theorem *map-val-conv*: (*xs* = *map Val ys*) = *map-val xs ys*
theorem *map-val2-conv*:
 (*xs* = *map Val ys* @ *throw e* # *zs*) = *map-val2 xs ys* (*throw e* # *zs*)
lemma *CallNull2*:
  $[\![\ P \vdash \langle e,s_0\rangle \Rightarrow \langle null,s_1\rangle;\ \ P \vdash \langle ps,s_1\rangle\ [\Rightarrow]\ \langle evs,s_2\rangle;\ map\text{-}val\ evs\ vs\ ]\!]$
  $\Longrightarrow P \vdash \langle e{\cdot}M(ps),s_0\rangle \Rightarrow \langle THROW\ NullPointer,s_2\rangle$
apply(*rule CallNull*, *assumption*+)
apply(*simp add*: *map-val-conv*[*symmetric*])
done

lemma *CallParamsThrow2*:
  $[\![\ P \vdash \langle e,s_0\rangle \Rightarrow \langle Val\ v,s_1\rangle;\ P \vdash \langle es,s_1\rangle\ [\Rightarrow]\ \langle evs,s_2\rangle;$
    $map\text{-}val2\ evs\ vs\ (throw\ ex\ \#\ es'')\ ]\!]$
   $\Longrightarrow P \vdash \langle e{\cdot}M(es),s_0\rangle \Rightarrow \langle throw\ ex,s_2\rangle$
apply(*rule eval-evals.CallParamsThrow*, *assumption*+)
apply(*simp add*: *map-val2-conv*[*symmetric*])
done

lemma *Call2*:
  $[\![\ P \vdash \langle e,s_0\rangle \Rightarrow \langle addr\ a,s_1\rangle;\ \ P \vdash \langle ps,s_1\rangle\ [\Rightarrow]\ \langle evs,(h_2,l_2)\rangle;$
    $map\text{-}val\ evs\ vs;$
    $h_2\ a = Some(C,fs);\ \ P \vdash C\ sees\ M{:}Ts{\rightarrow}T = (pns,body)\ in\ D;$
    $length\ vs = length\ pns;\ \ l_2{}' = [this{\mapsto}Addr\ a,\ pns[{\mapsto}]vs];$
    $P \vdash \langle body,(h_2,l_2{}')\rangle \Rightarrow \langle e',(h_3,l_3)\rangle\ ]\!]$
  $\Longrightarrow P \vdash \langle e{\cdot}M(ps),s_0\rangle \Rightarrow \langle e',(h_3,l_2)\rangle$
apply(*rule Call*, *assumption*+)
apply(*simp add*: *map-val-conv*[*symmetric*])
apply *assumption*+
done

code-pred
  (*modes*: *i* ⇒ *o* ⇒ *bool*)
  map-val
.

code-pred
  (*modes*: *i* ⇒ *o* ⇒ *o* ⇒ *bool*)
  map-val2
.

lemmas [*code-pred-intro*] =
 eval-evals.New eval-evals.NewFail
 eval-evals.Cast eval-evals.CastNull eval-evals.CastFail eval-evals.CastThrow
 eval-evals.Val eval-evals.Var
 eval-evals.BinOp eval-evals.BinOpThrow1 eval-evals.BinOpThrow2
 eval-evals.LAss eval-evals.LAssThrow
 eval-evals.FAcc eval-evals.FAccNull eval-evals.FAccThrow
 eval-evals.FAss eval-evals.FAssNull
 eval-evals.FAssThrow1 eval-evals.FAssThrow2
 eval-evals.CallObjThrow

declare *CallNull2* [*code-pred-intro CallNull2*]
declare *CallParamsThrow2* [*code-pred-intro CallParamsThrow2*]
declare *Call2* [*code-pred-intro Call2*]

lemmas [*code-pred-intro*] =
 eval-evals.Block
 eval-evals.Seq eval-evals.SeqThrow
 eval-evals.CondT eval-evals.CondF eval-evals.CondThrow
 eval-evals.WhileF eval-evals.WhileT
 eval-evals.WhileCondThrow

declare *eval-evals.WhileBodyThrow* [*code-pred-intro WhileBodyThrow2*]

lemmas [*code-pred-intro*] =
 eval-evals.Throw eval-evals.ThrowNull
 eval-evals.ThrowThrow
 eval-evals.Try eval-evals.TryCatch eval-evals.TryThrow
 eval-evals.Nil eval-evals.Cons eval-evals.ConsThrow

code-pred
  (*modes*: *i* ⇒ *i* ⇒ *i* ⇒ *o* ⇒ *o* ⇒ *bool as execute*)
  eval
proof −
  case *eval*
  from *eval.prems* **show** *thesis*
  proof(*cases* (*no-simp*))
    case *CallNull* **thus** *?thesis*
      by(*rule CallNull2*[*OF refl*])(*simp add*: *map-val-conv*[*symmetric*])
  next
    case *CallParamsThrow* **thus** *?thesis*
      by(*rule CallParamsThrow2*[*OF refl*])(*simp add*: *map-val2-conv*[*symmetric*])
  next
    case *Call* **thus** *?thesis*
      by −(*rule Call2*[*OF refl*], *simp-all add*: *map-val-conv*[*symmetric*])
  next
    case *WhileBodyThrow* **thus** *?thesis* **by**(*rule WhileBodyThrow2*[*OF refl*])
  qed(*assumption*|*erule* (*4*) *that*[*OF refl*]|*erule* (*3*) *that*[*OF refl*])+

next
  case *evals*
  from *evals.prems* **show** *thesis*
    by(*cases* (*no-simp*))(*assumption*|*erule* (*3*) *that*[*OF refl*])+
qed

notation *execute* (- ⊢ ((*1*⟨-,/-⟩) ⇒/ ⟨′-, ′-⟩) [*51*,*0*,*0*] *81*)

definition *test1* = [] ⊢ ⟨*testExpr1*,(*empty*,*empty*)⟩ ⇒ ⟨-,-⟩
definition *test2* = [] ⊢ ⟨*testExpr2*,(*empty*,*empty*)⟩ ⇒ ⟨-,-⟩
definition *test3* = [] ⊢ ⟨*testExpr3*,(*empty*,*empty*(″*V*″↦*Intg 77*))⟩ ⇒ ⟨-,-⟩
definition *test4* = [] ⊢ ⟨*testExpr4*,(*empty*,*empty*)⟩ ⇒ ⟨-,-⟩
definition *test5* = [(″*Object*″,(″″,[],[])),(″*C*″,(″*Object*″,[(″*F*″,*Integer*)],[]))] ⊢ ⟨*testExpr5*,(*empty*,*empty*)⟩ ⇒ ⟨-,-⟩
definition *test6* = [(″*Object*″,(″″,[],[])), *classI*] ⊢ ⟨*testExpr6*,(*empty*,*empty*)⟩ ⇒ ⟨-,-⟩

definition *V* = ″*V*″
definition *C* = ″*C*″
definition *F* = ″*F*″

ML ⟪
  val SOME ((@{*code Val*} (@{*code Intg*} *5*), -), -) = *Predicate.yield* @{*code test1*};
  val SOME ((@{*code Val*} (@{*code Intg*} *11*), -), -) = *Predicate.yield* @{*code test2*};
  val SOME ((@{*code Val*} (@{*code Intg*} *83*), -), -) = *Predicate.yield* @{*code test3*};

  val SOME ((-, (-, *l*)), -) = *Predicate.yield* @{*code test4*};
  val SOME (@{*code Intg*} *6*) = *l* @{*code V*};

  val SOME ((-, (*h*, -)), -) = *Predicate.yield* @{*code test5*};
  val SOME (*c*, *fs*) = *h 1*;
  val SOME (*obj*, -) = *h 0*;
  val SOME (@{*code Intg*} *i*) = *fs* (@{*code F*}, @{*code C*});
  if c = @{*code C*} *andalso obj* = @{*code Object*} *andalso i* = *42*
    then () *else error* ;

  val SOME ((@{*code Val*} (@{*code Intg*} *160*), -), -) = *Predicate.yield* @{*code test6*};
⟫

definition *test7* = [*classObject*, *classL*] ⊢ ⟨*testExpr-BuildList*, (*empty*,*empty*)⟩ ⇒ ⟨-,-⟩

definition *L* = ″*L*″
definition *N* = ″*N*″

ML ⟪
  val SOME ((-, (*h*, -)), -) = *Predicate.yield* @{*code test7*};
  val SOME (-, *fs1*) = *h 0*;
  val SOME (-, *fs2*) = *h 1*;
  val SOME (-, *fs3*) = *h 2*;
  val SOME (-, *fs4*) = *h 3*;

  val F = @{*code F*};
  val L = @{*code L*};
  val N = @{*code N*};

```
  if fs1 (F, L) = SOME (@{code Intg} 1) andalso
     fs1 (N, L) = SOME (@{code Addr} 1) andalso
     fs2 (F, L) = SOME (@{code Intg} 2) andalso
     fs2 (N, L) = SOME (@{code Addr} 2) andalso
     fs3 (F, L) = SOME (@{code Intg} 3) andalso
     fs3 (N, L) = SOME (@{code Addr} 3) andalso
     fs4 (F, L) = SOME (@{code Intg} 4) andalso
     fs4 (N, L) = SOME @{code Null}
  then () else error ;
⟫
```

definition *test8* = [*classObject*, *classA*] ⊢ ⟨*testExpr-ClassA*, (*empty*,*empty*)⟩ ⇒ ⟨-,-⟩
definition *i* = ''*int*''
definition *t* = ''*test*''
definition *A* = ''*A*''

ML ⟪
```
  val SOME ((-, (h, l)), -) = Predicate.yield @{code test8};
  val SOME (-, fs1) = h 0;
  val SOME (-, fs2) = h 1;

  val i = @{code i};
  val t = @{code t};
  val A = @{code A};

  if fs1 (i, A) = SOME (@{code Intg} 10) andalso
     fs1 (t, A) = SOME @{code Null} andalso
     fs2 (i, A) = SOME (@{code Intg} 50) andalso
     fs2 (t, A) = SOME @{code Null}
  then () else error ;
⟫
```

end

2.26 Code Generation For WellType

theory *execute-WellType*
imports
WellType Examples
begin

lemma *WTCond1*:
$[\![P,E \vdash e :: Boolean;\ \ P,E \vdash e_1::T_1;\ \ P,E \vdash e_2::T_2;\ P \vdash T_1 \leq T_2;$
$P \vdash T_2 \leq T_1 \longrightarrow T_2 = T_1\ ]\!] \Longrightarrow P,E \vdash if\ (e)\ e_1\ else\ e_2 :: T_2$
by (*fastforce*)

lemma *WTCond2*:
$[\![P,E \vdash e :: Boolean;\ \ P,E \vdash e_1::T_1;\ \ P,E \vdash e_2::T_2;\ P \vdash T_2 \leq T_1;$
$P \vdash T_1 \leq T_2 \longrightarrow T_1 = T_2\ ]\!] \Longrightarrow P,E \vdash if\ (e)\ e_1\ else\ e_2 :: T_1$
by (*fastforce*)

lemmas [*code-pred-intro*] =
WT-WTs.WTNew
WT-WTs.WTCast
WT-WTs.WTVal
WT-WTs.WTVar
WT-WTs.WTBinOpEq
WT-WTs.WTBinOpAdd
WT-WTs.WTLAss
WT-WTs.WTFAcc
WT-WTs.WTFAss
WT-WTs.WTCall
WT-WTs.WTBlock
WT-WTs.WTSeq

declare
WTCond1 [*code-pred-intro WTCond1*]
WTCond2 [*code-pred-intro WTCond2*]

lemmas [*code-pred-intro*] =
WT-WTs.WTWhile
WT-WTs.WTThrow
WT-WTs.WTTry
WT-WTs.WTNil
WT-WTs.WTCons

code-pred
(*modes*: $i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow$ *bool as type-check*, $i \Rightarrow i \Rightarrow i \Rightarrow o \Rightarrow$ *bool as infer-type*)
WT
proof −
case *WT*
from *WT.prems* **show** *thesis*
proof(*cases* (*no-simp*))
case (*WTCond E e e1 T1 e2 T2 T*)
from ‹$x \vdash T1 \leq T2 \lor x \vdash T2 \leq T1$› **show** *thesis*
proof

assume $x \vdash T1 \leq T2$
with ‹$x \vdash T1 \leq T2 \longrightarrow T = T2$› **have** $T = T2$ **..**
from ‹$xa = E$› ‹$xb = if\ (e)\ e1\ else\ e2$› ‹$xc = T$› ‹$x,E \vdash e :: Boolean$›
‹$x,E \vdash e1 :: T1$› ‹$x,E \vdash e2 :: T2$› ‹$x \vdash T1 \leq T2$› ‹$x \vdash T2 \leq T1 \longrightarrow T = T1$›
show *?thesis* **unfolding** ‹$T = T2$› **by**(*rule WTCond1*[*OF refl*])
next
assume $x \vdash T2 \leq T1$
with ‹$x \vdash T2 \leq T1 \longrightarrow T = T1$› **have** $T = T1$ **..**
from ‹$xa = E$› ‹$xb = if\ (e)\ e1\ else\ e2$› ‹$xc = T$› ‹$x,E \vdash e :: Boolean$›
‹$x,E \vdash e1 :: T1$› ‹$x,E \vdash e2 :: T2$› ‹$x \vdash T2 \leq T1$› ‹$x \vdash T1 \leq T2 \longrightarrow T = T2$›
show *?thesis* **unfolding** ‹$T = T1$› **by**(*rule WTCond2*[*OF refl*])
qed
qed(*assumption*|*erule (2) that*[*OF refl*])+
next
case *WTs*
from *WTs.prems* **show** *thesis*
by(*cases (no-simp)*)(*assumption*|*erule (2) that*[*OF refl*])+
qed

notation *infer-type* (-,- ⊢ - :: '- [51,51,51]100)

definition *test1* **where** *test1* = [],*empty* ⊢ *testExpr1* :: -
definition *test2* **where** *test2* = [], *empty* ⊢ *testExpr2* :: -
definition *test3* **where** *test3* = [], *empty*(''V'' ↦ *Integer*) ⊢ *testExpr3* :: -
definition *test4* **where** *test4* = [], *empty*(''V'' ↦ *Integer*) ⊢ *testExpr4* :: -
definition *test5* **where** *test5* = [*classObject*, (''C'',(''Object'',[(''F'',*Integer*)],[]))], *empty* ⊢ *testExpr5* :: -
definition *test6* **where** *test6* = [*classObject*, *classI*], *empty* ⊢ *testExpr6* :: -

ML ⟪
val SOME(@{*code Integer*}, -) = *Predicate.yield* @{*code test1*};
val SOME(@{*code Integer*}, -) = *Predicate.yield* @{*code test2*};
val SOME(@{*code Integer*}, -) = *Predicate.yield* @{*code test3*};
val SOME(@{*code Void*}, -) = *Predicate.yield* @{*code test4*};
val SOME(@{*code Void*}, -) = *Predicate.yield* @{*code test5*};
val SOME(@{*code Integer*}, -) = *Predicate.yield* @{*code test6*};
⟫

definition *testmb-isNull* **where** *testmb-isNull* = [*classObject*, *classA*], *empty*([*this*] [↦] [*Class* ''A'']) ⊢ *mb-isNull* :: -
definition *testmb-add* **where** *testmb-add* = [*classObject*, *classA*], *empty*([*this*,''i''] [↦] [*Class* ''A'',*Integer*]) ⊢ *mb-add* :: -
definition *testmb-mult-cond* **where** *testmb-mult-cond* = [*classObject*, *classA*], *empty*([*this*,''j''] [↦] [*Class* ''A'',*Integer*]) ⊢ *mb-mult-cond* :: -
definition *testmb-mult-block* **where** *testmb-mult-block* = [*classObject*, *classA*], *empty*([*this*,''i'',''j'',''temp''] [↦] [*Class* ''A'',*Integer*,*Integer*,*Integer*]) ⊢ *mb-mult-block* :: -
definition *testmb-mult* **where** *testmb-mult* = [*classObject*, *classA*], *empty*([*this*,''i'',''j''] [↦] [*Class* ''A'',*Integer*,*Integer*]) ⊢ *mb-mult* :: -

ML ⟪
val SOME(@{*code Boolean*}, -) = *Predicate.yield* @{*code testmb-isNull*};
val SOME(@{*code Integer*}, -) = *Predicate.yield* @{*code testmb-add*};
val SOME(@{*code Boolean*}, -) = *Predicate.yield* @{*code testmb-mult-cond*};
val SOME(@{*code Void*}, -) = *Predicate.yield* @{*code testmb-mult-block*};

```
  val SOME(@{code Integer}, -) = Predicate.yield @{code testmb-mult};
⟫
```

definition *test* **where** *test* = [*classObject*, *classA*], *empty* ⊢ *testExpr-ClassA* :: -

ML ⟪
```
  val SOME(@{code Integer}, -) = Predicate.yield @{code test};
⟫
```

end

Chapter 3

Jinja Virtual Machine

3.1 State of the JVM

theory *JVMState* **imports** ../*Common*/*Objects* **begin**

3.1.1 Frame Stack

type-synonym
pc = *nat*

type-synonym
frame = *val list* × *val list* × *cname* × *mname* × *pc*
— operand stack
— registers (including this pointer, method parameters, and local variables)
— name of class where current method is defined
— parameter types
— program counter within frame

3.1.2 Runtime State

type-synonym
jvm-state = *addr option* × *heap* × *frame list*
— exception flag, heap, frames

end

3.2 Instructions of the JVM

theory *JVMInstructions* **imports** *JVMState* **begin**

datatype
instr = *Load nat* — load from local variable
| *Store nat* — store into local variable
| *Push val* — push a value (constant)
| *New cname* — create object
| *Getfield vname cname* — Fetch field from object
| *Putfield vname cname* — Set field in object
| *Checkcast cname* — Check whether object is of given type
| *Invoke mname nat* — inv. instance meth of an object
| *Return* — return from method
| *Pop* — pop top element from opstack
| *IAdd* — integer addition
| *Goto int* — goto relative address
| *CmpEq* — equality comparison
| *IfFalse int* — branch if top of stack false
| *Throw* — throw top of stack as exception

type-synonym
bytecode = *instr list*

type-synonym
ex-entry = *pc* × *pc* × *cname* × *pc* × *nat*
— start-pc, end-pc, exception type, handler-pc, remaining stack depth

type-synonym
ex-table = *ex-entry list*

type-synonym
jvm-method = *nat* × *nat* × *bytecode* × *ex-table*
— max stacksize
— number of local variables. Add 1 + no. of parameters to get no. of registers
— instruction sequence
— exception handler table

type-synonym
jvm-prog = *jvm-method prog*

end

3.3 JVM Instruction Semantics

```
theory JVMExecInstr
imports JVMInstructions JVMState ../Common/Exceptions
begin

primrec
  exec-instr :: [instr, jvm-prog, heap, val list, val list,
                 cname, mname, pc, frame list] => jvm-state
where
exec-instr-Load:
 exec-instr (Load n) P h stk loc C0 M0 pc frs =
     (None, h, ((loc ! n) # stk, loc, C0, M0, pc+1)#frs)

| exec-instr (Store n) P h stk loc C0 M0 pc frs =
     (None, h, (tl stk, loc[n:=hd stk], C0, M0, pc+1)#frs)

| exec-instr-Push:
 exec-instr (Push v) P h stk loc C0 M0 pc frs =
     (None, h, (v # stk, loc, C0, M0, pc+1)#frs)

| exec-instr-New:
 exec-instr (New C) P h stk loc C0 M0 pc frs =
  (case new-Addr h of
    None ⇒   (Some (addr-of-sys-xcpt OutOfMemory), h, (stk, loc, C0, M0, pc)#frs)
  | Some a ⇒ (None, h(a↦blank P C), (Addr a#stk, loc, C0, M0, pc+1)#frs))

| exec-instr (Getfield F C) P h stk loc C0 M0 pc frs =
  (let v      = hd stk;
       xp'    = if v=Null then ⌊addr-of-sys-xcpt NullPointer⌋ else None;
       (D,fs) = the(h(the-Addr v))
   in (xp', h, (the(fs(F,C))#(tl stk), loc, C0, M0, pc+1)#frs))

| exec-instr (Putfield F C) P h stk loc C0 M0 pc frs =
  (let v    = hd stk;
       r    = hd (tl stk);
       xp'  = if r=Null then ⌊addr-of-sys-xcpt NullPointer⌋ else None;
       a    = the-Addr r;
       (D,fs) = the (h a);
       h'   = h(a ↦ (D, fs((F,C) ↦ v)))
   in (xp', h', (tl (tl stk), loc, C0, M0, pc+1)#frs))

| exec-instr (Checkcast C) P h stk loc C0 M0 pc frs =
  (let v   = hd stk;
       xp' = if ¬cast-ok P C h v then ⌊addr-of-sys-xcpt ClassCast⌋ else None
   in (xp', h, (stk, loc, C0, M0, pc+1)#frs))

| exec-instr-Invoke:
 exec-instr (Invoke M n) P h stk loc C0 M0 pc frs =
  (let ps  = take n stk;
       r   = stk!n;
       xp' = if r=Null then ⌊addr-of-sys-xcpt NullPointer⌋ else None;
       C   = fst(the(h(the-Addr r)));
       (D,M',Ts,mxs,mxl0,ins,xt)= method P C M;
```

$f' = ([],[r]@(rev\ ps)@(replicate\ mxl_0\ undefined),D,M,0)$
in $(xp', h, f'\#(stk, loc, C_0, M_0, pc)\#frs))$

| *exec-instr Return P h* $stk_0\ loc_0\ C_0\ M_0$ *pc frs* =
(*if frs*=[] *then* (*None*, *h*, []) *else*
let $v = hd\ stk_0$;
$(stk,loc,C,m,pc) = hd\ frs$;
$n = length\ (fst\ (snd\ (method\ P\ C_0\ M_0)))$
in $(None, h, (v\#(drop\ (n+1)\ stk),loc,C,m,pc+1)\#tl\ frs))$

| *exec-instr Pop P h stk loc* $C_0\ M_0$ *pc frs* =
$(None, h, (tl\ stk, loc, C_0, M_0, pc+1)\#frs)$

| *exec-instr IAdd P h stk loc* $C_0\ M_0$ *pc frs* =
(*let* $i_2 = the\text{-}Intg\ (hd\ stk)$;
$i_1 = the\text{-}Intg\ (hd\ (tl\ stk))$
in $(None, h, (Intg\ (i_1+i_2)\#(tl\ (tl\ stk)), loc, C_0, M_0, pc+1)\#frs))$

| *exec-instr* (*IfFalse i*) *P h stk loc* $C_0\ M_0$ *pc frs* =
(*let* $pc' =$ *if hd stk* = *Bool False then nat*(*int pc*+*i*) *else pc*+*1*
in $(None, h, (tl\ stk, loc, C_0, M_0, pc')\#frs))$

| *exec-instr CmpEq P h stk loc* $C_0\ M_0$ *pc frs* =
(*let* $v_2 = hd\ stk$;
$v_1 = hd\ (tl\ stk)$
in $(None, h, (Bool\ (v_1=v_2)\ \#\ tl\ (tl\ stk), loc, C_0, M_0, pc+1)\#frs))$

| *exec-instr-Goto*:
exec-instr (*Goto i*) *P h stk loc* $C_0\ M_0$ *pc frs* =
$(None, h, (stk, loc, C_0, M_0, nat(int\ pc+i))\#frs)$

| *exec-instr Throw P h stk loc* $C_0\ M_0$ *pc frs* =
(*let* $xp' =$ *if hd stk* = *Null then* ⌊*addr-of-sys-xcpt NullPointer*⌋ *else* ⌊*the-Addr*(*hd stk*)⌋
in $(xp', h, (stk, loc, C_0, M_0, pc)\#frs))$

lemma *exec-instr-Store*:
exec-instr (*Store n*) *P h* (*v*#*stk*) *loc* $C_0\ M_0$ *pc frs* =
$(None, h, (stk, loc[n:=v], C_0, M_0, pc+1)\#frs)$
by *simp*

lemma *exec-instr-Getfield*:
exec-instr (*Getfield F C*) *P h* (*v*#*stk*) *loc* $C_0\ M_0$ *pc frs* =
(*let* xp' = *if v*=*Null then* ⌊*addr-of-sys-xcpt NullPointer*⌋ *else None*;
$(D,fs) = the(h(the\text{-}Addr\ v))$
in $(xp', h, (the(fs(F,C))\#stk, loc, C_0, M_0, pc+1)\#frs))$
by *simp*

lemma *exec-instr-Putfield*:
exec-instr (*Putfield F C*) *P h* (*v*#*r*#*stk*) *loc* $C_0\ M_0$ *pc frs* =
(*let* xp' = *if r*=*Null then* ⌊*addr-of-sys-xcpt NullPointer*⌋ *else None*;
a = *the-Addr r*;
$(D,fs) = the\ (h\ a)$;
$h' = h(a \mapsto (D, fs((F,C) \mapsto v)))$

in $(xp', h', (stk, loc, C_0, M_0, pc+1)\#frs))$
by *simp*

lemma *exec-instr-Checkcast*:
$exec\text{-}instr\ (Checkcast\ C)\ P\ h\ (v\#stk)\ loc\ C_0\ M_0\ pc\ frs =$
$(let\ xp' = if\ \neg cast\text{-}ok\ P\ C\ h\ v\ then\ \lfloor addr\text{-}of\text{-}sys\text{-}xcpt\ ClassCast \rfloor\ else\ None$
$in\ (xp', h, (v\#stk, loc, C_0, M_0, pc+1)\#frs))$
by *simp*

lemma *exec-instr-Return*:
$exec\text{-}instr\ Return\ P\ h\ (v\#stk_0)\ loc_0\ C_0\ M_0\ pc\ frs =$
$(if\ frs=[]\ then\ (None, h, [])\ else$
$let\ (stk,loc,C,m,pc) = hd\ frs;$
$n = length\ (fst\ (snd\ (method\ P\ C_0\ M_0)))$
$in\ (None, h, (v\#(drop\ (n+1)\ stk),loc,C,m,pc+1)\#tl\ frs))$
by *simp*

lemma *exec-instr-IPop*:
$exec\text{-}instr\ Pop\ P\ h\ (v\#stk)\ loc\ C_0\ M_0\ pc\ frs =$
$(None, h, (stk, loc, C_0, M_0, pc+1)\#frs)$
by *simp*

lemma *exec-instr-IAdd*:
$exec\text{-}instr\ IAdd\ P\ h\ (Intg\ i_2\ \#\ Intg\ i_1\ \#\ stk)\ loc\ C_0\ M_0\ pc\ frs =$
$(None, h, (Intg\ (i_1+i_2)\#stk, loc, C_0, M_0, pc+1)\#frs)$
by *simp*

lemma *exec-instr-IfFalse*:
$exec\text{-}instr\ (IfFalse\ i)\ P\ h\ (v\#stk)\ loc\ C_0\ M_0\ pc\ frs =$
$(let\ pc' = if\ v = Bool\ False\ then\ nat(int\ pc+i)\ else\ pc+1$
$in\ (None, h, (stk, loc, C_0, M_0, pc')\#frs))$
by *simp*

lemma *exec-instr-CmpEq*:
$exec\text{-}instr\ CmpEq\ P\ h\ (v_2\#v_1\#stk)\ loc\ C_0\ M_0\ pc\ frs =$
$(None, h, (Bool\ (v_1=v_2)\ \#\ stk, loc, C_0, M_0, pc+1)\#frs)$
by *simp*

lemma *exec-instr-Throw*:
$exec\text{-}instr\ Throw\ P\ h\ (v\#stk)\ loc\ C_0\ M_0\ pc\ frs =$
$(let\ xp' = if\ v = Null\ then\ \lfloor addr\text{-}of\text{-}sys\text{-}xcpt\ NullPointer \rfloor\ else\ \lfloor the\text{-}Addr\ v \rfloor$
$in\ (xp', h, (v\#stk, loc, C_0, M_0, pc)\#frs))$
by *simp*

end

3.4 Exception handling in the JVM

theory *JVMExceptions* **imports** *JVMInstructions ../Common/Exceptions* **begin**

definition *matches-ex-entry* :: *'m prog* ⇒ *cname* ⇒ *pc* ⇒ *ex-entry* ⇒ *bool*
where
  matches-ex-entry P C pc xcp ≡
    let $(s, e, C', h, d) = xcp$ *in*
    $s \le pc \wedge pc < e \wedge P \vdash C \preceq^* C'$

primrec *match-ex-table* :: *'m prog* ⇒ *cname* ⇒ *pc* ⇒ *ex-table* ⇒ (*pc* × *nat*) *option*
where
  match-ex-table P C pc [] = *None*
| *match-ex-table P C pc* (*e#es*) = (*if matches-ex-entry P C pc e*
    then Some (*snd*(*snd*(*snd e*)))
    else match-ex-table P C pc es)

abbreviation
  ex-table-of :: *jvm-prog* ⇒ *cname* ⇒ *mname* ⇒ *ex-table* **where**
  ex-table-of P C M == *snd* (*snd* (*snd* (*snd* (*snd* (*snd*(*method P C M*))))))

primrec *find-handler* :: *jvm-prog* ⇒ *addr* ⇒ *heap* ⇒ *frame list* ⇒ *jvm-state*
where
  find-handler P a h [] = (*Some a*, *h*, [])
| *find-handler P a h* (*fr#frs*) =
    (*let* (*stk*,*loc*,*C*,*M*,*pc*) = *fr in*
    case match-ex-table P (*cname-of h a*) *pc* (*ex-table-of P C M*) *of*
      None ⇒ *find-handler P a h frs*
    | *Some pc-d* ⇒ (*None*, *h*, (*Addr a # drop* (*size stk* − *snd pc-d*) *stk*, *loc*, *C*, *M*, *fst pc-d*)*#frs*))

end

3.5 Program Execution in the JVM

theory *JVMExec*
imports *JVMExecInstr JVMExceptions*
begin

abbreviation
  $instrs\text{-}of :: jvm\text{-}prog \Rightarrow cname \Rightarrow mname \Rightarrow instr\ list$ **where**
  $instrs\text{-}of\ P\ C\ M == fst(snd(snd(snd(snd(snd(method\ P\ C\ M))))))$

fun $exec :: jvm\text{-}prog \times jvm\text{-}state => jvm\text{-}state\ option$ **where** — single step execution
  $exec\ (P,\ xp,\ h,\ [])=None$

$|\ exec\ (P,\ None,\ h,\ (stk,loc,C,M,pc)\#frs) =$
  $(let$
    $i = instrs\text{-}of\ P\ C\ M\ !\ pc;$
    $(xcpt',\ h',\ frs') = exec\text{-}instr\ i\ P\ h\ stk\ loc\ C\ M\ pc\ frs$
   $in\ Some(case\ xcpt'\ of$
        $None \Rightarrow (None,h',frs')$
       $|\ Some\ a \Rightarrow find\text{-}handler\ P\ a\ h\ ((stk,loc,C,M,pc)\#frs)))$

$|\ exec\ (P,\ Some\ xa,\ h,\ frs) = None$

— relational view
inductive-set
  $exec\text{-}1 :: jvm\text{-}prog \Rightarrow (jvm\text{-}state \times jvm\text{-}state)\ set$
  and $exec\text{-}1' :: jvm\text{-}prog \Rightarrow jvm\text{-}state \Rightarrow jvm\text{-}state \Rightarrow bool$
   $(\text{-} \vdash / \text{ - } -jvm\rightarrow_1 / \text{ - } [61,61,61]\ 60)$
  for $P :: jvm\text{-}prog$
where
  $P \vdash \sigma\ -jvm\rightarrow_1\ \sigma' \equiv (\sigma,\sigma') \in exec\text{-}1\ P$
$|\ exec\text{-}1I\colon exec\ (P,\sigma) = Some\ \sigma' \Longrightarrow P \vdash \sigma\ -jvm\rightarrow_1\ \sigma'$

— reflexive transitive closure:
definition $exec\text{-}all :: jvm\text{-}prog \Rightarrow jvm\text{-}state \Rightarrow jvm\text{-}state \Rightarrow bool$
       $(\text{-}\ |-/\ \text{-}\ -jvm->/\ \text{-}\ [61,61,61]60)$ **where**

  $exec\text{-}all\text{-}def1\colon P\ |-\ \sigma\ -jvm->\ \sigma' \longleftrightarrow (\sigma,\sigma') \in (exec\text{-}1\ P)^*$

notation $(xsymbols)$
  $exec\text{-}all\ \ ((\text{-} \vdash / \text{ - } -jvm\rightarrow / \text{ -})\ [61,61,61]60)$

lemma $exec\text{-}1\text{-}eq$:
  $exec\text{-}1\ P = \{(\sigma,\sigma').\ exec\ (P,\sigma) = Some\ \sigma'\}$
lemma $exec\text{-}1\text{-}iff$:
  $P \vdash \sigma\ -jvm\rightarrow_1\ \sigma' = (exec\ (P,\sigma) = Some\ \sigma')$
lemma $exec\text{-}all\text{-}def$:
  $P \vdash \sigma\ -jvm\rightarrow\ \sigma' = ((\sigma,\sigma') \in \{(\sigma,\sigma').\ exec\ (P,\sigma) = Some\ \sigma'\}^*)$
lemma $jvm\text{-}refl[iff]\colon P \vdash \sigma\ -jvm\rightarrow\ \sigma$
lemma $jvm\text{-}trans[trans]$:
 $[\![\ P \vdash \sigma\ -jvm\rightarrow\ \sigma';\ P \vdash \sigma'\ -jvm\rightarrow\ \sigma''\ ]\!] \Longrightarrow P \vdash \sigma\ -jvm\rightarrow\ \sigma''$
lemma $jvm\text{-}one\text{-}step1[trans]$:
 $[\![\ P \vdash \sigma\ -jvm\rightarrow_1\ \sigma';\ P \vdash \sigma'\ -jvm\rightarrow\ \sigma''\ ]\!] \Longrightarrow P \vdash \sigma\ -jvm\rightarrow\ \sigma''$

lemma *jvm-one-step2*[*trans*]:
$[\![P \vdash \sigma -jvm\rightarrow \sigma'; P \vdash \sigma' -jvm\rightarrow_1 \sigma'']\!] \Longrightarrow P \vdash \sigma -jvm\rightarrow \sigma''$
lemma *exec-all-conf*:
$[\![P \vdash \sigma -jvm\rightarrow \sigma'; P \vdash \sigma -jvm\rightarrow \sigma'']\!]$
$\Longrightarrow P \vdash \sigma' -jvm\rightarrow \sigma'' \vee P \vdash \sigma'' -jvm\rightarrow \sigma'$

lemma *exec-all-finalD*: $P \vdash (x,\ h,\ []) -jvm\rightarrow \sigma \Longrightarrow \sigma = (x,\ h,\ [])$
lemma *exec-all-deterministic*:
$[\![P \vdash \sigma -jvm\rightarrow (x,h,[]); P \vdash \sigma -jvm\rightarrow \sigma']\!] \Longrightarrow P \vdash \sigma' -jvm\rightarrow (x,h,[])$

The start configuration of the JVM: in the start heap, we call a method m of class C in program P. The *this* pointer of the frame is set to *Null* to simulate a static method invokation.

definition *start-state* :: *jvm-prog* ⇒ *cname* ⇒ *mname* ⇒ *jvm-state* **where**
start-state P C M =
(*let* (D,Ts,T,mxs,mxl_0,b) = *method P C M in*
(*None*, *start-heap P*, [([], *Null* # *replicate* mxl_0 *undefined*, C, M, *0*)]))

end

3.6 A Defensive JVM

theory *JVMDefensive*
imports *JVMExec ../Common/Conform*
begin

Extend the state space by one element indicating a type error (or other abnormal termination)

datatype $'a$ *type-error* = *TypeError* | *Normal* $'a$

fun *is-Addr* :: *val* ⇒ *bool* **where**
  is-Addr (*Addr a*) ⟷ *True*
| *is-Addr v* ⟷ *False*

fun *is-Intg* :: *val* ⇒ *bool* **where**
  is-Intg (*Intg i*) ⟷ *True*
| *is-Intg v* ⟷ *False*

fun *is-Bool* :: *val* ⇒ *bool* **where**
  is-Bool (*Bool b*) ⟷ *True*
| *is-Bool v* ⟷ *False*

definition *is-Ref* :: *val* ⇒ *bool* **where**
  is-Ref v ⟷ *v* = *Null* ∨ *is-Addr v*

primrec *check-instr* :: [*instr*, *jvm-prog*, *heap*, *val list*, *val list*,
        cname, *mname*, *pc*, *frame list*] ⇒ *bool* **where**
  check-instr-Load:
    check-instr (*Load n*) *P h stk loc C* M_0 *pc frs* =
    (*n* < *length loc*)

| *check-instr-Store*:
    check-instr (*Store n*) *P h stk loc* C_0 M_0 *pc frs* =
    (*0* < *length stk* ∧ *n* < *length loc*)

| *check-instr-Push*:
    check-instr (*Push v*) *P h stk loc* C_0 M_0 *pc frs* =
    (¬*is-Addr v*)

| *check-instr-New*:
    check-instr (*New C*) *P h stk loc* C_0 M_0 *pc frs* =
    is-class P C

| *check-instr-Getfield*:
    check-instr (*Getfield F C*) *P h stk loc* C_0 M_0 *pc frs* =
    (*0* < *length stk* ∧ (∃ *C' T*. *P* ⊢ *C sees F*:*T in C'*) ∧
    (*let* (*C'*, *T*) = *field P C F*; *ref* = *hd stk in*
      C' = *C* ∧ *is-Ref ref* ∧ (*ref* ≠ *Null* ⟶
        h (*the-Addr ref*) ≠ *None* ∧
        (*let* (*D*,*vs*) = *the* (*h* (*the-Addr ref*)) *in*
          P ⊢ *D* $\preceq^*$ *C* ∧ *vs* (*F*,*C*) ≠ *None* ∧ *P*,*h* ⊢ *the* (*vs* (*F*,*C*)) :≤ *T*))))

| *check-instr-Putfield*:
    check-instr (*Putfield F C*) *P h stk loc* C_0 M_0 *pc frs* =

$(1 < length\ stk \wedge (\exists C'\ T.\ P \vdash C\ sees\ F{:}T\ in\ C') \wedge$
$(let\ (C',\ T) = field\ P\ C\ F;\ v = hd\ stk;\ ref = hd\ (tl\ stk)\ in$
$C' = C \wedge is\text{-}Ref\ ref \wedge (ref \neq Null \longrightarrow$
$h\ (the\text{-}Addr\ ref) \neq None \wedge$
$(let\ D = fst\ (the\ (h\ (the\text{-}Addr\ ref)))\ in$
$P \vdash D \preceq^* C \wedge P,h \vdash v :\leq T))))$

| *check-instr-Checkcast*:
$check\text{-}instr\ (Checkcast\ C)\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ frs =$
$(0 < length\ stk \wedge is\text{-}class\ P\ C \wedge is\text{-}Ref\ (hd\ stk))$

| *check-instr-Invoke*:
$check\text{-}instr\ (Invoke\ M\ n)\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ frs =$
$(n < length\ stk \wedge is\text{-}Ref\ (stk!n) \wedge$
$(stk!n \neq Null \longrightarrow$
$(let\ a = the\text{-}Addr\ (stk!n);$
$C = cname\text{-}of\ h\ a;$
$Ts = fst\ (snd\ (method\ P\ C\ M))$
$in\ h\ a \neq None \wedge P \vdash C\ has\ M \wedge$
$P,h \vdash rev\ (take\ n\ stk)\ [:\leq]\ Ts)))$

| *check-instr-Return*:
$check\text{-}instr\ Return\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ frs =$
$(0 < length\ stk \wedge ((0 < length\ frs) \longrightarrow$
$(P \vdash C_0\ has\ M_0) \wedge$
$(let\ v = hd\ stk;$
$T = fst\ (snd\ (snd\ (method\ P\ C_0\ M_0)))$
$in\ P,h \vdash v :\leq T)))$

| *check-instr-Pop*:
$check\text{-}instr\ Pop\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ frs =$
$(0 < length\ stk)$

| *check-instr-IAdd*:
$check\text{-}instr\ IAdd\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ frs =$
$(1 < length\ stk \wedge is\text{-}Intg\ (hd\ stk) \wedge is\text{-}Intg\ (hd\ (tl\ stk)))$

| *check-instr-IfFalse*:
$check\text{-}instr\ (IfFalse\ b)\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ frs =$
$(0 < length\ stk \wedge is\text{-}Bool\ (hd\ stk) \wedge 0 \leq int\ pc{+}b)$

| *check-instr-CmpEq*:
$check\text{-}instr\ CmpEq\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ frs =$
$(1 < length\ stk)$

| *check-instr-Goto*:
$check\text{-}instr\ (Goto\ b)\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ frs =$
$(0 \leq int\ pc{+}b)$

| *check-instr-Throw*:
$check\text{-}instr\ Throw\ P\ h\ stk\ loc\ C_0\ M_0\ pc\ frs =$
$(0 < length\ stk \wedge is\text{-}Ref\ (hd\ stk))$

definition *check* :: *jvm-prog* ⇒ *jvm-state* ⇒ *bool* **where**

$check\ P\ \sigma = (let\ (xcpt,\ h,\ frs) = \sigma\ in$
$\quad (case\ frs\ of\ [] \Rightarrow True \mid (stk,loc,C,M,pc)\#frs' \Rightarrow$
$\quad P \vdash C\ has\ M\ \wedge$
$\quad (let\ (C',Ts,T,mxs,mxl_0,ins,xt) = method\ P\ C\ M;\ i = ins!pc\ in$
$\quad pc < size\ ins \wedge size\ stk \leq mxs\ \wedge$
$\quad check\text{-}instr\ i\ P\ h\ stk\ loc\ C\ M\ pc\ frs')))$

definition $exec\text{-}d :: jvm\text{-}prog \Rightarrow jvm\text{-}state \Rightarrow jvm\text{-}state\ option\ type\text{-}error$ **where**
$exec\text{-}d\ P\ \sigma = (if\ check\ P\ \sigma\ then\ Normal\ (exec\ (P,\ \sigma))\ else\ TypeError)$

inductive-set
$exec\text{-}1\text{-}d :: jvm\text{-}prog \Rightarrow (jvm\text{-}state\ type\text{-}error \times jvm\text{-}state\ type\text{-}error)\ set$
and $exec\text{-}1\text{-}d' :: jvm\text{-}prog \Rightarrow jvm\text{-}state\ type\text{-}error \Rightarrow jvm\text{-}state\ type\text{-}error \Rightarrow bool$
$(\text{-} \vdash \text{-} -jvmd\rightarrow_1 \text{-}\ [61,61,61]60)$
for $P :: jvm\text{-}prog$
where
$P \vdash \sigma\ -jvmd\rightarrow_1\ \sigma' \equiv (\sigma,\sigma') \in exec\text{-}1\text{-}d\ P$
$\mid exec\text{-}1\text{-}d\text{-}ErrorI$: $exec\text{-}d\ P\ \sigma = TypeError \Longrightarrow P \vdash Normal\ \sigma\ -jvmd\rightarrow_1\ TypeError$
$\mid exec\text{-}1\text{-}d\text{-}NormalI$: $exec\text{-}d\ P\ \sigma = Normal\ (Some\ \sigma') \Longrightarrow P \vdash Normal\ \sigma\ -jvmd\rightarrow_1\ Normal\ \sigma'$

— reflexive transitive closure:
definition $exec\text{-}all\text{-}d :: jvm\text{-}prog \Rightarrow jvm\text{-}state\ type\text{-}error \Rightarrow jvm\text{-}state\ type\text{-}error \Rightarrow bool$
$(\text{-} \mid\!-\ \text{-} -jvmd-\!>\ \text{-}\ [61,61,61]60)$ **where**
$exec\text{-}all\text{-}d\text{-}def1$: $P \mid\!-\ \sigma\ -jvmd-\!>\ \sigma' \longleftrightarrow (\sigma,\sigma') \in (exec\text{-}1\text{-}d\ P)^*$

notation $(xsymbols)$
$exec\text{-}all\text{-}d\quad (\text{-} \vdash \text{-} -jvmd\rightarrow \text{-}\ [61,61,61]60)$

lemma $exec\text{-}1\text{-}d\text{-}eq$:
$exec\text{-}1\text{-}d\ P = \{(s,t).\ \exists\sigma.\ s = Normal\ \sigma \wedge t = TypeError \wedge exec\text{-}d\ P\ \sigma = TypeError\} \cup$
$\quad \{(s,t).\ \exists\sigma\ \sigma'.\ s = Normal\ \sigma \wedge t = Normal\ \sigma' \wedge exec\text{-}d\ P\ \sigma = Normal\ (Some\ \sigma')\}$
by $(auto\ elim!: exec\text{-}1\text{-}d.cases\ intro!: exec\text{-}1\text{-}d.intros)$

declare $split\text{-}paired\text{-}All\ [simp\ del]$
declare $split\text{-}paired\text{-}Ex\ [simp\ del]$

lemma $if\text{-}neq\ [dest!]$:
$(if\ P\ then\ A\ else\ B) \neq B \Longrightarrow P$
by $(cases\ P,\ auto)$

lemma $exec\text{-}d\text{-}no\text{-}errorI\ [intro]$:
$check\ P\ \sigma \Longrightarrow exec\text{-}d\ P\ \sigma \neq TypeError$
by $(unfold\ exec\text{-}d\text{-}def)\ simp$

theorem $no\text{-}type\text{-}error\text{-}commutes$:
$exec\text{-}d\ P\ \sigma \neq TypeError \Longrightarrow exec\text{-}d\ P\ \sigma = Normal\ (exec\ (P,\ \sigma))$
by $(unfold\ exec\text{-}d\text{-}def,\ auto)$

lemma $defensive\text{-}imp\text{-}aggressive$:
$P \vdash (Normal\ \sigma)\ -jvmd\rightarrow\ (Normal\ \sigma') \Longrightarrow P \vdash \sigma\ -jvm\rightarrow\ \sigma'$

end

3.7 Example for generating executable code from JVM semantics

```
theory JVMListExample
imports
  ../Common/SystemClasses
  JVMExec
  ~~/src/HOL/Library/Efficient-Nat
begin

definition list-name :: string
where
  list-name == ''list''

definition test-name :: string
where
  test-name == ''test''

definition val-name :: string
where
  val-name == ''val''

definition next-name :: string
where
  next-name == ''next''

definition append-name :: string
where
  append-name == ''append''

definition makelist-name :: string
where
  makelist-name == ''makelist''

definition append-ins :: bytecode
where
  append-ins ==
      [Load 0,
       Getfield next-name list-name,
       Load 0,
       Getfield next-name list-name,
       Push Null,
       CmpEq,
       IfFalse 7,
       Pop,
       Load 0,
       Load 1,
       Putfield next-name list-name,
       Push Unit,
       Return,
       Load 1,
       Invoke append-name 1,
       Return]
```

definition *list-class* :: *jvm-method class*
where
list-class ==
(*Object*,
[(*val-name*, *Integer*), (*next-name*, *Class list-name*)],
[(*append-name*, [*Class list-name*], *Void*,
(*3*, *0*, *append-ins*, [(*1*, *2*, *NullPointer*, *7*, *0*)]))])

definition *make-list-ins* :: *bytecode*
where
make-list-ins ==
[*New list-name*,
Store 0,
Load 0,
Push (*Intg 1*),
Putfield val-name list-name,
New list-name,
Store 1,
Load 1,
Push (*Intg 2*),
Putfield val-name list-name,
New list-name,
Store 2,
Load 2,
Push (*Intg 3*),
Putfield val-name list-name,
Load 0,
Load 1,
Invoke append-name 1,
Pop,
Load 0,
Load 2,
Invoke append-name 1,
Return]

definition *test-class* :: *jvm-method class*
where
test-class ==
(*Object*, [],
[(*makelist-name*, [], *Void*, (*3*, *2*, *make-list-ins*, []))])

definition *E* :: *jvm-prog*
where
E == *SystemClasses* @ [(*list-name*, *list-class*), (*test-name*, *test-class*)]

definition *undefined-cname* :: *cname*
where [*code del*]: *undefined-cname* = *undefined*
declare *undefined-cname-def*[*symmetric*, *code-unfold*]
code-const *undefined-cname*
(*SML object*)

definition *undefined-val* :: *val*
where [*code del*]: *undefined-val* = *undefined*
declare *undefined-val-def*[*symmetric*, *code-unfold*]

code-const *undefined-val*
(*SML Unit*)

lemmas [*code-unfold*] = *SystemClasses-def* [*unfolded ObjectC-def NullPointerC-def ClassCastC-def OutOfMemoryC-def*]

definition *test* = *exec* (*E*, *start-state E test-name makelist-name*)

ML ⟪
```
@{code test};
@{code exec} (@{code E}, @{code the} it);
@{code exec} (@{code E}, @{code the} it);
@{code exec} (@{code E}, @{code the} it);
@{code exec} (@{code E}, @{code the} it);
@{code exec} (@{code E}, @{code the} it);
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@{code exec} (@{code E}, @{code the} it);
@{code exec} (@{code E}, @{code the} it);
```

@{*code exec*} (@{*code E*}, @{*code the*} *it*);
@{*code exec*} (@{*code E*}, @{*code the*} *it*);
@{*code exec*} (@{*code E*}, @{*code the*} *it*);
@{*code exec*} (@{*code E*}, @{*code the*} *it*);
@{*code exec*} (@{*code E*}, @{*code the*} *it*);
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@{*code exec*} (@{*code E*}, @{*code the*} *it*);
@{*code exec*} (@{*code E*}, @{*code the*} *it*);
@{*code exec*} (@{*code E*}, @{*code the*} *it*);
@{*code exec*} (@{*code E*}, @{*code the*} *it*);

val SOME (-, (*h*, -)) = *it*;
if snd (@{*code the*} (*h 3*)) (@{*code val-name*}, @{*code list-name*}) = *SOME* (@{*code Intg*} *1*) *then* () *else error wrong result*;
if snd (@{*code the*} (*h 3*)) (@{*code next-name*}, @{*code list-name*}) = *SOME* (@{*code Addr*} *4*) *then* () *else error wrong result*;
if snd (@{*code the*} (*h 4*)) (@{*code val-name*}, @{*code list-name*}) = *SOME* (@{*code Intg*} *2*) *then* () *else error wrong result*;
if snd (@{*code the*} (*h 4*)) (@{*code next-name*}, @{*code list-name*}) = *SOME* (@{*code Addr*} *5*) *then* () *else error wrong result*;
if snd (@{*code the*} (*h 5*)) (@{*code val-name*}, @{*code list-name*}) = *SOME* (@{*code Intg*} *3*) *then* () *else error wrong result*;
if snd (@{*code the*} (*h 5*)) (@{*code next-name*}, @{*code list-name*}) = *SOME* @{*code Null*} *then* () *else error wrong result*;
⟫

end

Chapter 4

Bytecode Verifier

4.1 Semilattices

theory *Semilat*
imports *Main* ~~/*src*/*HOL*/*Library*/*While-Combinator*
begin

type-synonym $'a$ *ord* $= 'a \Rightarrow 'a \Rightarrow bool$
type-synonym $'a$ *binop* $= 'a \Rightarrow 'a \Rightarrow 'a$
type-synonym $'a$ *sl* $= 'a\ set \times 'a\ ord \times 'a\ binop$

consts
  $lesub :: 'a \Rightarrow 'a\ ord \Rightarrow 'a \Rightarrow bool$
  $lesssub :: 'a \Rightarrow 'a\ ord \Rightarrow 'a \Rightarrow bool$
  $plussub :: 'a \Rightarrow ('a \Rightarrow 'b \Rightarrow 'c) \Rightarrow 'b \Rightarrow 'c$**notation** (*xsymbols*)
  $lesub\ \ ((\text{-}\ /\sqsubseteq_{\text{-}}\ \text{-})\ [50,\ 0,\ 51]\ 50)$ **and**
  $lesssub\ \ ((\text{-}\ /\sqsubset_{\text{-}}\ \text{-})\ [50,\ 0,\ 51]\ 50)$ **and**
  $plussub\ \ ((\text{-}\ /\sqcup_{\text{-}}\ \text{-})\ [65,\ 0,\ 66]\ 65)$
defs
  lesub-def: $x \sqsubseteq_r y \equiv r\ x\ y$
  lesssub-def: $x \sqsubset_r y \equiv x \sqsubseteq_r y \wedge x \neq y$
  plussub-def: $x \sqcup_f y \equiv f\ x\ y$

definition $ord :: ('a \times 'a)\ set \Rightarrow 'a\ ord$
where
  $ord\ r = (\lambda x\ y.\ (x,y) \in r)$

definition $order :: 'a\ ord \Rightarrow bool$
where
  $order\ r \longleftrightarrow (\forall x.\ x \sqsubseteq_r x) \wedge (\forall x\ y.\ x \sqsubseteq_r y \wedge y \sqsubseteq_r x \longrightarrow x{=}y) \wedge (\forall x\ y\ z.\ x \sqsubseteq_r y \wedge y \sqsubseteq_r z \longrightarrow x \sqsubseteq_r z)$

definition $top :: 'a\ ord \Rightarrow 'a \Rightarrow bool$
where
  $top\ r\ T \longleftrightarrow (\forall x.\ x \sqsubseteq_r T)$

definition $acc :: 'a\ ord \Rightarrow bool$
where
  $acc\ r \longleftrightarrow wf\ \{(y,x).\ x \sqsubset_r y\}$

definition $closed :: 'a\ set \Rightarrow 'a\ binop \Rightarrow bool$
where
  $closed\ A\ f \longleftrightarrow (\forall x{\in}A.\ \forall y{\in}A.\ x \sqcup_f y \in A)$

definition $semilat :: 'a\ sl \Rightarrow bool$
where
  $semilat = (\lambda(A,r,f).\ order\ r \wedge closed\ A\ f \wedge$
    $(\forall x{\in}A.\ \forall y{\in}A.\ x \sqsubseteq_r x \sqcup_f y) \wedge$
    $(\forall x{\in}A.\ \forall y{\in}A.\ y \sqsubseteq_r x \sqcup_f y) \wedge$
    $(\forall x{\in}A.\ \forall y{\in}A.\ \forall z{\in}A.\ x \sqsubseteq_r z \wedge y \sqsubseteq_r z \longrightarrow x \sqcup_f y \sqsubseteq_r z))$

definition $is\text{-}ub :: ('a \times 'a)\ set \Rightarrow 'a \Rightarrow 'a \Rightarrow 'a \Rightarrow bool$
where
  $is\text{-}ub\ r\ x\ y\ u \longleftrightarrow (x,u){\in}r \wedge (y,u){\in}r$

definition *is-lub* :: $('a \times 'a)$ *set* $\Rightarrow 'a \Rightarrow 'a \Rightarrow 'a \Rightarrow$ *bool*
where
  is-lub r x y u $\longleftrightarrow$ *is-ub r x y u* $\wedge$ $(\forall z.$ *is-ub r x y z* $\longrightarrow (u,z) \in r)$

definition *some-lub* :: $('a \times 'a)$ *set* $\Rightarrow 'a \Rightarrow 'a \Rightarrow 'a$
where
  some-lub r x y = (*SOME z. is-lub r x y z*)

locale *Semilat* =
  fixes A :: *'a set*
  fixes r :: *'a ord*
  fixes f :: *'a binop*
  assumes *semilat*: *semilat* (A, r, f)

lemma *order-refl* [*simp*, *intro*]: *order r* $\Longrightarrow x \sqsubseteq_r x$

lemma *order-antisym*: $[\![$ *order r*; $x \sqsubseteq_r y$; $y \sqsubseteq_r x$ $]\!] \Longrightarrow x = y$

lemma *order-trans*: $[\![$ *order r*; $x \sqsubseteq_r y$; $y \sqsubseteq_r z$ $]\!] \Longrightarrow x \sqsubseteq_r z$

lemma *order-less-irrefl* [*intro*, *simp*]: *order r* $\Longrightarrow \neg\, x \sqsubset_r x$

lemma *order-less-trans*: $[\![$ *order r*; $x \sqsubset_r y$; $y \sqsubset_r z$ $]\!] \Longrightarrow x \sqsubset_r z$

lemma *topD* [*simp*, *intro*]: *top r T* $\Longrightarrow x \sqsubseteq_r T$

lemma *top-le-conv* [*simp*]: $[\![$ *order r*; *top r T* $]\!] \Longrightarrow (T \sqsubseteq_r x) = (x = T)$

lemma *semilat-Def*:
semilat$(A,r,f) \longleftrightarrow$ *order r* $\wedge$ *closed A f* $\wedge$
  $(\forall x{\in}A.\ \forall y{\in}A.\ x \sqsubseteq_r x \sqcup_f y) \wedge$
  $(\forall x{\in}A.\ \forall y{\in}A.\ y \sqsubseteq_r x \sqcup_f y) \wedge$
  $(\forall x{\in}A.\ \forall y{\in}A.\ \forall z{\in}A.\ x \sqsubseteq_r z \wedge y \sqsubseteq_r z \longrightarrow x \sqcup_f y \sqsubseteq_r z)$

lemma (**in** *Semilat*) *orderI* [*simp*, *intro*]: *order r*

lemma (**in** *Semilat*) *closedI* [*simp*, *intro*]: *closed A f*

lemma *closedD*: $[\![$ *closed A f*; $x{\in}A$; $y{\in}A$ $]\!] \Longrightarrow x \sqcup_f y \in A$

lemma *closed-UNIV* [*simp*]: *closed UNIV f*

lemma (**in** *Semilat*) *closed-f* [*simp*, *intro*]: $[\![x \in A;\ y \in A]\!] \Longrightarrow x \sqcup_f y \in A$

lemma (**in** *Semilat*) *refl-r* [*intro*, *simp*]: $x \sqsubseteq_r x$ **by** *simp*

lemma (**in** *Semilat*) *antisym-r* [*intro?*]: $[\![\ x \sqsubseteq_r y;\ y \sqsubseteq_r x\ ]\!] \Longrightarrow x = y$

lemma (**in** *Semilat*) *trans-r* [*trans*, *intro?*]: $[\![x \sqsubseteq_r y;\ y \sqsubseteq_r z]\!] \Longrightarrow x \sqsubseteq_r z$

lemma (**in** *Semilat*) *ub1* [*simp*, *intro?*]: $[\![\ x \in A;\ y \in A\ ]\!] \Longrightarrow x \sqsubseteq_r x \sqcup_f y$

lemma (**in** *Semilat*) *ub2* [*simp*, *intro?*]: $[\![\ x \in A;\ y \in A\ ]\!] \Longrightarrow y \sqsubseteq_r x \sqcup_f y$

lemma (**in** *Semilat*) *lub* [*simp*, *intro?*]:
$[\![\ x \sqsubseteq_r z;\ y \sqsubseteq_r z;\ x \in A;\ y \in A;\ z \in A\ ]\!] \Longrightarrow x \sqcup_f y \sqsubseteq_r z$

lemma (**in** *Semilat*) *plus-le-conv* [*simp*]:
$[\![\ x \in A;\ y \in A;\ z \in A\ ]\!] \Longrightarrow (x \sqcup_f y \sqsubseteq_r z) = (x \sqsubseteq_r z \wedge y \sqsubseteq_r z)$

lemma (**in** *Semilat*) *le-iff-plus-unchanged*:
assumes $x \in A$ **and** $y \in A$
shows $x \sqsubseteq_r y \longleftrightarrow x \sqcup_f y = y$ (**is** $?P \longleftrightarrow ?Q$)
lemma (**in** *Semilat*) *le-iff-plus-unchanged2*:
assumes $x \in A$ **and** $y \in A$
shows $x \sqsubseteq_r y \longleftrightarrow y \sqcup_f x = y$ (**is** $?P \longleftrightarrow ?Q$)
lemma (**in** *Semilat*) *plus-assoc* [*simp*]:
assumes a: $a \in A$ **and** b: $b \in A$ **and** c: $c \in A$
shows $a \sqcup_f (b \sqcup_f c) = a \sqcup_f b \sqcup_f c$
lemma (**in** *Semilat*) *plus-com-lemma*:
$[\![a \in A;\ b \in A]\!] \Longrightarrow a \sqcup_f b \sqsubseteq_r b \sqcup_f a$
lemma (**in** *Semilat*) *plus-commutative*:
$[\![a \in A;\ b \in A]\!] \Longrightarrow a \sqcup_f b = b \sqcup_f a$

lemma *is-lubD*:
$\textit{is-lub}\ r\ x\ y\ u \Longrightarrow \textit{is-ub}\ r\ x\ y\ u \wedge (\forall z.\ \textit{is-ub}\ r\ x\ y\ z \longrightarrow (u,z) \in r)$

lemma *is-ubI*:
$[\![\ (x,u) \in r;\ (y,u) \in r\ ]\!] \Longrightarrow \textit{is-ub}\ r\ x\ y\ u$

lemma *is-ubD*:
$\textit{is-ub}\ r\ x\ y\ u \Longrightarrow (x,u) \in r \wedge (y,u) \in r$

lemma *is-lub-bigger1* [*iff*]:
$\textit{is-lub}\ (r\hat{\ }*\ )\ x\ y\ y = ((x,y) \in r\hat{\ }*\ )$
lemma *is-lub-bigger2* [*iff*]:
$\textit{is-lub}\ (r\hat{\ }*\ )\ x\ y\ x = ((y,x) \in r\hat{\ }*\ )$
lemma *extend-lub*:
$[\![\ \textit{single-valued}\ r;\ \textit{is-lub}\ (r\hat{\ }*\ )\ x\ y\ u;\ (x',x) \in r\ ]\!]$
$\Longrightarrow EX\ v.\ \textit{is-lub}\ (r\hat{\ }*\ )\ x'\ y\ v$
lemma *single-valued-has-lubs* [*rule-format*]:
$[\![\ \textit{single-valued}\ r;\ (x,u) \in r\hat{\ }*\ ]\!] \Longrightarrow (\forall y.\ (y,u) \in r\hat{\ }* \longrightarrow$
$(EX\ z.\ \textit{is-lub}\ (r\hat{\ }*\ )\ x\ y\ z))$
lemma *some-lub-conv*:
$[\![\ \textit{acyclic}\ r;\ \textit{is-lub}\ (r\hat{\ }*\ )\ x\ y\ u\ ]\!] \Longrightarrow \textit{some-lub}\ (r\hat{\ }*\ )\ x\ y = u$
lemma *is-lub-some-lub*:
$[\![\ \textit{single-valued}\ r;\ \textit{acyclic}\ r;\ (x,u) \in r\hat{\ }*;\ (y,u) \in r\hat{\ }*\ ]\!]$
$\Longrightarrow \textit{is-lub}\ (r\hat{\ }*\ )\ x\ y\ (\textit{some-lub}\ (r\hat{\ }*\ )\ x\ y)$

4.1.1 An executable lub-finder

definition $\textit{exec-lub} :: ('a * 'a)\ set \Rightarrow ('a \Rightarrow 'a) \Rightarrow 'a\ binop$
where
$\textit{exec-lub}\ r\ f\ x\ y = \textit{while}\ (\lambda z.\ (x,z) \notin r^*)\ f\ y$

lemma *exec-lub-refl*: $\textit{exec-lub}\ r\ f\ T\ T = T$

by (*simp add*: *exec-lub-def while-unfold*)

lemma *acyclic-single-valued-finite*:
$[\![$*acyclic r*; *single-valued r*; $(x,y) \in r^*]\!]$
$\Longrightarrow$ *finite* $(r \cap \{a.\ (x,\ a) \in r^*\} \times \{b.\ (b,\ y) \in r^*\})$

lemma *exec-lub-conv*:
$[\![$ *acyclic r*; $\forall\, x\ y.\ (x,y) \in r \longrightarrow f\ x = y$; *is-lub* $(r^*)\ x\ y\ u\ ]\!] \Longrightarrow$
exec-lub $r\ f\ x\ y = u$

lemma *is-lub-exec-lub*:
$[\![$ *single-valued r*; *acyclic r*; $(x,u){:}r\hat{\ }{*}$; $(y,u){:}r\hat{\ }{*}$; $\forall\, x\ y.\ (x,y) \in r \longrightarrow f\ x = y\ ]\!]$
$\Longrightarrow$ *is-lub* $(r\hat{\ }{*}\ )\ x\ y$ (*exec-lub* $r\ f\ x\ y$)

end

4.2 The Error Type

theory *Err*
imports *Semilat*
begin

datatype $'a\ err = Err \mid OK\ 'a$

type-synonym $'a\ ebinop = 'a \Rightarrow 'a \Rightarrow 'a\ err$
type-synonym $'a\ esl = 'a\ set \times 'a\ ord \times 'a\ ebinop$

primrec $ok\text{-}val :: 'a\ err \Rightarrow 'a$
where
$ok\text{-}val\ (OK\ x) = x$

definition $lift :: ('a \Rightarrow 'b\ err) \Rightarrow ('a\ err \Rightarrow 'b\ err)$
where
$lift\ f\ e = (case\ e\ of\ Err \Rightarrow Err \mid OK\ x \Rightarrow f\ x)$

definition $lift2 :: ('a \Rightarrow 'b \Rightarrow 'c\ err) \Rightarrow 'a\ err \Rightarrow 'b\ err \Rightarrow 'c\ err$
where
$lift2\ f\ e_1\ e_2 =$
$(case\ e_1\ of\ Err \Rightarrow Err \mid OK\ x \Rightarrow (case\ e_2\ of\ Err \Rightarrow Err \mid OK\ y \Rightarrow f\ x\ y))$

definition $le :: 'a\ ord \Rightarrow 'a\ err\ ord$
where
$le\ r\ e_1\ e_2 =$
$(case\ e_2\ of\ Err \Rightarrow True \mid OK\ y \Rightarrow (case\ e_1\ of\ Err \Rightarrow False \mid OK\ x \Rightarrow x \sqsubseteq_r\ y))$

definition $sup :: ('a \Rightarrow 'b \Rightarrow 'c) \Rightarrow ('a\ err \Rightarrow 'b\ err \Rightarrow 'c\ err)$
where
$sup\ f = lift2\ (\lambda x\ y.\ OK\ (x \sqcup_f\ y))$

definition $err :: 'a\ set \Rightarrow 'a\ err\ set$
where
$err\ A = insert\ Err\ \{OK\ x|x.\ x{\in}A\}$

definition $esl :: 'a\ sl \Rightarrow 'a\ esl$
where
$esl = (\lambda(A,r,f).\ (A,\ r,\ \lambda x\ y.\ OK(f\ x\ y)))$

definition $sl :: 'a\ esl \Rightarrow 'a\ err\ sl$
where
$sl = (\lambda(A,r,f).\ (err\ A,\ le\ r,\ lift2\ f))$

abbreviation
$err\text{-}semilat :: 'a\ esl \Rightarrow bool$ **where**
$err\text{-}semilat\ L == semilat(sl\ L)$

primrec $strict :: ('a \Rightarrow 'b\ err) \Rightarrow ('a\ err \Rightarrow 'b\ err)$
where
$strict\ f\ Err \quad = Err$
$\mid strict\ f\ (OK\ x) = f\ x$

lemma *err-def'*:
$err\ A = insert\ Err\ \{x.\ \exists y{\in}A.\ x = OK\ y\}$
lemma *strict-Some* [*simp*]:
$(strict\ f\ x = OK\ y) = (\exists z.\ x = OK\ z \wedge f\ z = OK\ y)$
lemma *not-Err-eq*: $(x \neq Err) = (\exists a.\ x = OK\ a)$
lemma *not-OK-eq*: $(\forall y.\ x \neq OK\ y) = (x = Err)$
lemma *unfold-lesub-err*: $e1 \sqsubseteq_{le\ r} e2 = le\ r\ e1\ e2$
lemma *le-err-refl*: $\forall x.\ x \sqsubseteq_r x \Longrightarrow e \sqsubseteq_{le\ r} e$
lemma *le-err-trans* [*rule-format*]:
$order\ r \Longrightarrow e1 \sqsubseteq_{le\ r} e2 \longrightarrow e2 \sqsubseteq_{le\ r} e3 \longrightarrow e1 \sqsubseteq_{le\ r} e3$
lemma *le-err-antisym* [*rule-format*]:
$order\ r \Longrightarrow e1 \sqsubseteq_{le\ r} e2 \longrightarrow e2 \sqsubseteq_{le\ r} e1 \longrightarrow e1{=}e2$
lemma *OK-le-err-OK*: $(OK\ x \sqsubseteq_{le\ r} OK\ y) = (x \sqsubseteq_r y)$
lemma *order-le-err* [*iff*]: $order(le\ r) = order\ r$
lemma *le-Err* [*iff*]: $e \sqsubseteq_{le\ r} Err$
lemma *Err-le-conv* [*iff*]: $Err \sqsubseteq_{le\ r} e\ = (e = Err)$
lemma *le-OK-conv* [*iff*]: $e \sqsubseteq_{le\ r} OK\ x\ =\ (\exists y.\ e = OK\ y \wedge y \sqsubseteq_r x)$
lemma *OK-le-conv*: $OK\ x \sqsubseteq_{le\ r} e = (e = Err \vee (\exists y.\ e = OK\ y \wedge x \sqsubseteq_r y))$
lemma *top-Err* [*iff*]: $top\ (le\ r)\ Err$
lemma *OK-less-conv* [*rule-format*, *iff*]:
$OK\ x \sqsubset_{le\ r} e = (e{=}Err \vee (\exists y.\ e = OK\ y \wedge x \sqsubset_r y))$
lemma *not-Err-less* [*rule-format*, *iff*]: $\neg(Err \sqsubset_{le\ r} x)$
lemma *semilat-errI* [*intro*]: **assumes** *Semilat A r f*
shows $semilat(err\ A,\ le\ r,\ lift2(\lambda x\ y.\ OK(f\ x\ y)))$
lemma *err-semilat-eslI-aux*:
assumes *Semilat A r f* **shows** $err\text{-}semilat(esl(A,r,f))$
lemma *err-semilat-eslI* [*intro*, *simp*]:
$semilat\ L \Longrightarrow err\text{-}semilat\ (esl\ L)$
lemma *acc-err* [*simp*, *intro!*]: $acc\ r \Longrightarrow acc(le\ r)$
lemma *Err-in-err* [*iff*]: $Err : err\ A$
lemma *Ok-in-err* [*iff*]: $(OK\ x \in err\ A) = (x{\in}A)$

4.2.1 lift

lemma *lift-in-errI*:
$[\![\ e \in err\ S;\ \forall x{\in}S.\ e = OK\ x \longrightarrow f\ x \in err\ S\ ]\!] \Longrightarrow lift\ f\ e \in err\ S$
lemma *Err-lift2* [*simp*]: $Err \sqcup_{lift2\ f} x = Err$
lemma *lift2-Err* [*simp*]: $x \sqcup_{lift2\ f} Err = Err$
lemma *OK-lift2-OK* [*simp*]: $OK\ x \sqcup_{lift2\ f} OK\ y = x \sqcup_f y$

4.2.2 sup

lemma *Err-sup-Err* [*simp*]: $Err \sqcup_{sup\ f} x = Err$
lemma *Err-sup-Err2* [*simp*]: $x \sqcup_{sup\ f} Err = Err$
lemma *Err-sup-OK* [*simp*]: $OK\ x \sqcup_{sup\ f} OK\ y = OK\ (x \sqcup_f y)$
lemma *Err-sup-eq-OK-conv* [*iff*]:
$(sup\ f\ ex\ ey = OK\ z) = (\exists x\ y.\ ex = OK\ x \wedge ey = OK\ y \wedge f\ x\ y = z)$
lemma *Err-sup-eq-Err* [*iff*]: $(sup\ f\ ex\ ey = Err) = (ex{=}Err \vee ey{=}Err)$

4.2.3 semilat (err A) (le r) f

lemma *semilat-le-err-Err-plus* [*simp*]:
$[\![\ x{\in}\ err\ A;\ semilat(err\ A,\ le\ r,\ f)\ ]\!] \Longrightarrow Err \sqcup_f x = Err$
lemma *semilat-le-err-plus-Err* [*simp*]:

$[\![\ x\in\ err\ A;\ semilat(err\ A,\ le\ r,\ f)\ ]\!] \Longrightarrow x \sqcup_f\ Err = Err$
lemma *semilat-le-err-OK1*:
$[\![\ x\in A;\ y\in A;\ semilat(err\ A,\ le\ r,\ f);\ OK\ x \sqcup_f\ OK\ y = OK\ z\ ]\!]$
$\Longrightarrow x \sqsubseteq_r z$
lemma *semilat-le-err-OK2*:
$[\![\ x\in A;\ y\in A;\ semilat(err\ A,\ le\ r,\ f);\ OK\ x \sqcup_f\ OK\ y = OK\ z\ ]\!]$
$\Longrightarrow y \sqsubseteq_r z$
lemma *eq-order-le*:
$[\![\ x{=}y;\ order\ r\ ]\!] \Longrightarrow x \sqsubseteq_r y$
lemma *OK-plus-OK-eq-Err-conv* [*simp*]:
assumes $x\in A\quad y\in A\quad semilat(err\ A,\ le\ r,\ fe)$
shows $(OK\ x \sqcup_{fe}\ OK\ y = Err) = (\neg(\exists z\in A.\ x \sqsubseteq_r z \wedge y \sqsubseteq_r z))$

4.2.4 semilat (err(Union AS))

lemma *all-bex-swap-lemma* [*iff*]:
$(\forall x.\ (\exists y\in A.\ x = f\ y) \longrightarrow P\ x) = (\forall y\in A.\ P(f\ y))$
lemma *closed-err-Union-lift2I*:
$[\![\ \forall A\in AS.\ closed\ (err\ A)\ (lift2\ f);\ AS \neq \{\};$
$\forall A\in AS.\forall B\in AS.\ A{\neq}B \longrightarrow (\forall a\in A.\forall b\in B.\ a \sqcup_f\ b = Err)\ ]\!]$
$\Longrightarrow closed\ (err(Union\ AS))\ (lift2\ f)$

If $AS = \{\}$ the thm collapses to $order\ r \wedge closed\ \{Err\}\ f \wedge Err \sqcup_f Err = Err$ which may not hold

lemma *err-semilat-UnionI*:
$[\![\ \forall A\in AS.\ err\text{-}semilat(A,\ r,\ f);\ AS \neq \{\};$
$\forall A\in AS.\forall B\in AS.\ A{\neq}B \longrightarrow (\forall a\in A.\forall b\in B.\ \neg a \sqsubseteq_r b \wedge a \sqcup_f\ b = Err)\ ]\!]$
$\Longrightarrow err\text{-}semilat(Union\ AS,\ r,\ f)$
end

4.3 More about Options

theory *Opt* **imports** *Err* **begin**

definition *le* :: $'a\ ord \Rightarrow 'a\ option\ ord$
where
$le\ r\ o_1\ o_2 =$
$(case\ o_2\ of\ None \Rightarrow o_1{=}None \mid Some\ y \Rightarrow (case\ o_1\ of\ None \Rightarrow True \mid Some\ x \Rightarrow x \sqsubseteq_r y))$

definition *opt* :: $'a\ set \Rightarrow 'a\ option\ set$
where
$opt\ A = insert\ None\ \{Some\ y\ |y.\ y \in A\}$

definition *sup* :: $'a\ ebinop \Rightarrow 'a\ option\ ebinop$
where
$sup\ f\ o_1\ o_2 =$
$(case\ o_1\ of\ None \Rightarrow OK\ o_2$
$\mid Some\ x \Rightarrow (case\ o_2\ of\ None \Rightarrow OK\ o_1$
$\mid Some\ y \Rightarrow (case\ f\ x\ y\ of\ Err \Rightarrow Err \mid OK\ z \Rightarrow OK\ (Some\ z))))$

definition *esl* :: $'a\ esl \Rightarrow 'a\ option\ esl$
where
$esl = (\lambda(A,r,f).\ (opt\ A,\ le\ r,\ sup\ f))$

lemma *unfold-le-opt*:
$o_1 \sqsubseteq_{le\ r} o_2 =$
$(case\ o_2\ of\ None \Rightarrow o_1{=}None \mid$
$Some\ y \Rightarrow (case\ o_1\ of\ None \Rightarrow True \mid Some\ x \Rightarrow x \sqsubseteq_r y))$
lemma *le-opt-refl*: $order\ r \Longrightarrow x \sqsubseteq_{le\ r} x$

4.4 Products as Semilattices

theory *Product*
imports *Err*
begin

definition $le :: 'a\ ord \Rightarrow 'b\ ord \Rightarrow ('a \times 'b)\ ord$
where
$le\ r_A\ r_B = (\lambda(a_1,b_1)\ (a_2,b_2).\ a_1 \sqsubseteq_{r_A}\ a_2 \wedge b_1 \sqsubseteq_{r_B}\ b_2)$

definition $sup :: 'a\ ebinop \Rightarrow 'b\ ebinop \Rightarrow ('a \times 'b)\ ebinop$
where
$sup\ f\ g = (\lambda(a_1,b_1)(a_2,b_2).\ Err.sup\ Pair\ (a_1 \sqcup_f\ a_2)\ (b_1 \sqcup_g\ b_2))$

definition $esl :: 'a\ esl \Rightarrow 'b\ esl \Rightarrow ('a \times 'b\ )\ esl$
where
$esl = (\lambda(A,r_A,f_A)\ (B,r_B,f_B).\ (A \times B,\ le\ r_A\ r_B,\ sup\ f_A\ f_B))$

abbreviation (*xsymbols*)
$lesubprod :: 'a \times 'b \Rightarrow ('a \Rightarrow 'a \Rightarrow bool) \Rightarrow ('b \Rightarrow 'b \Rightarrow bool) \Rightarrow 'a \times 'b \Rightarrow bool$
((- /⊑′(-,-′) -) [*50*, *0*, *0*, *51*] *50*) **where**
$p \sqsubseteq(rA,rB)\ q == p \sqsubseteq_{Product.le\ rA\ rB}\ q$

lemma *unfold-lesub-prod*: $x \sqsubseteq(r_A,r_B)\ y = le\ r_A\ r_B\ x\ y$
lemma *le-prod-Pair-conv* [*iff*]: $((a_1,b_1) \sqsubseteq(r_A,r_B)\ (a_2,b_2)) = (a_1 \sqsubseteq_{r_A}\ a_2\ \&\ b_1 \sqsubseteq_{r_B}\ b_2)$
lemma *less-prod-Pair-conv*:
$((a_1,b_1) \sqsubset_{Product.le\ r_A\ r_B}\ (a_2,b_2)) =$
$(a_1 \sqsubset_{r_A}\ a_2\ \&\ b_1 \sqsubseteq_{r_B}\ b_2 \mid a_1 \sqsubseteq_{r_A}\ a_2\ \&\ b_1 \sqsubset_{r_B}\ b_2)$
lemma *order-le-prod* [*iff*]: $order(Product.le\ r_A\ r_B) = (order\ r_A\ \&\ order\ r_B)$

lemma *acc-le-prodI* [*intro*!]:
$[\![\ acc\ r_A;\ acc\ r_B\ ]\!] \Longrightarrow acc(Product.le\ r_A\ r_B)$

lemma *closed-lift2-sup*:
$[\![\ closed\ (err\ A)\ (lift2\ f);\ closed\ (err\ B)\ (lift2\ g)\ ]\!] \Longrightarrow$
$closed\ (err(A{\times}B))\ (lift2(sup\ f\ g))$
lemma *unfold-plussub-lift2*: $e_1 \sqcup_{lift2\ f}\ e_2 = lift2\ f\ e_1\ e_2$

lemma *plus-eq-Err-conv* [*simp*]:
assumes $x{\in}A \quad y{\in}A \quad semilat(err\ A,\ Err.le\ r,\ lift2\ f)$
shows $(x \sqcup_f\ y = Err) = (\neg(\exists\ z{\in}A.\ x \sqsubseteq_r\ z \wedge y \sqsubseteq_r\ z))$
lemma *err-semilat-Product-esl*:
$\bigwedge L_1\ L_2.\ [\![\ err\text{-}semilat\ L_1;\ err\text{-}semilat\ L_2\ ]\!] \Longrightarrow err\text{-}semilat(Product.esl\ L_1\ L_2)$
end

4.5 Fixed Length Lists

theory *Listn*
imports *Err*
begin

definition *list* :: *nat* ⇒ *'a set* ⇒ *'a list set*
where
list n A = {*xs*. *size xs* = *n* ∧ *set xs* ⊆ *A*}

definition *le* :: *'a ord* ⇒ (*'a list*)*ord*
where
le r = *list-all2* (λ*x y*. $x \sqsubseteq_r y$)

abbreviation (*xsymbols*)
lesublist :: *'a list* ⇒ *'a ord* ⇒ *'a list* ⇒ *bool* ((- /[⊑-] -) [*50*, *0*, *51*] *50*) **where**
$x\ [\sqsubseteq_r]\ y$ == *x* <=-(*Listn.le r*) *y*

abbreviation (*xsymbols*)
lesssublist :: *'a list* ⇒ *'a ord* ⇒ *'a list* ⇒ *bool* ((- /[⊏-] -) [*50*, *0*, *51*] *50*) **where**
$x\ [\sqsubset_r]\ y$ == *x* <-(*Listn.le r*) *y*

definition *map2* :: (*'a* ⇒ *'b* ⇒ *'c*) ⇒ *'a list* ⇒ *'b list* ⇒ *'c list*
where
map2 f = (λ*xs ys*. *map* (*split f*) (*zip xs ys*))

abbreviation (*xsymbols*)
plussublist :: *'a list* ⇒ (*'a* ⇒ *'b* ⇒ *'c*) ⇒ *'b list* ⇒ *'c list*
((- /[⊔-] -) [*65*, *0*, *66*] *65*) **where**
$x\ [\sqcup_f]\ y$ == $x \sqcup_{map2\ f} y$

primrec *coalesce* :: *'a err list* ⇒ *'a list err*
where
coalesce [] = *OK*[]
| *coalesce* (*ex*#*exs*) = *Err.sup* (*op* #) *ex* (*coalesce exs*)

definition *sl* :: *nat* ⇒ *'a sl* ⇒ *'a list sl*
where
sl n = (λ(*A*,*r*,*f*). (*list n A*, *le r*, *map2 f*))

definition *sup* :: (*'a* ⇒ *'b* ⇒ *'c err*) ⇒ *'a list* ⇒ *'b list* ⇒ *'c list err*
where
sup f = (λ*xs ys*. *if size xs* = *size ys then coalesce*(*xs* $[\sqcup_f]$ *ys*) *else Err*)

definition *upto-esl* :: *nat* ⇒ *'a esl* ⇒ *'a list esl*
where
upto-esl m = (λ(*A*,*r*,*f*). (*Union*{*list n A* |*n*. *n* ≤ *m*}, *le r*, *sup f*))

lemmas [*simp*] = *set-update-subsetI*

lemma *unfold-lesub-list*: *xs* $[\sqsubseteq_r]$ *ys* = *Listn.le r xs ys*
lemma *Nil-le-conv* [*iff*]: ([] $[\sqsubseteq_r]$ *ys*) = (*ys* = [])

lemma *Cons-notle-Nil* [*iff*]: $\neg\ x\#xs\ [\sqsubseteq_r]\ []$
lemma *Cons-le-Cons* [*iff*]: $x\#xs\ [\sqsubseteq_r]\ y\#ys = (x \sqsubseteq_r y \wedge xs\ [\sqsubseteq_r]\ ys)$
lemma *Cons-less-Conss* [*simp*]:
$order\ r \Longrightarrow\ x\#xs\ [\sqsubset_r]\ y\#ys = (x \sqsubset_r y \wedge xs\ [\sqsubseteq_r]\ ys \vee x = y \wedge xs\ [\sqsubset_r]\ ys)$
lemma *list-update-le-cong*:
$[\![\ i{<}size\ xs;\ xs\ [\sqsubseteq_r]\ ys;\ x \sqsubseteq_r y\ ]\!] \Longrightarrow xs[i{:=}x]\ [\sqsubseteq_r]\ ys[i{:=}y]$

lemma *le-listD*: $[\![\ xs\ [\sqsubseteq_r]\ ys;\ p < size\ xs\ ]\!] \Longrightarrow xs!p \sqsubseteq_r ys!p$
lemma *le-list-refl*: $\forall x.\ x \sqsubseteq_r x \Longrightarrow xs\ [\sqsubseteq_r]\ xs$
lemma *le-list-trans*: $[\![\ order\ r;\ xs\ [\sqsubseteq_r]\ ys;\ ys\ [\sqsubseteq_r]\ zs\ ]\!] \Longrightarrow xs\ [\sqsubseteq_r]\ zs$
lemma *le-list-antisym*: $[\![\ order\ r;\ xs\ [\sqsubseteq_r]\ ys;\ ys\ [\sqsubseteq_r]\ xs\ ]\!] \Longrightarrow xs = ys$
lemma *order-listI* [*simp*, *intro*!]: $order\ r \Longrightarrow order(Listn.le\ r)$
lemma *lesub-list-impl-same-size* [*simp*]: $xs\ [\sqsubseteq_r]\ ys \Longrightarrow size\ ys = size\ xs$
lemma *lesssub-lengthD*: $xs\ [\sqsubset_r]\ ys \Longrightarrow size\ ys = size\ xs$
lemma *le-list-appendI*: $a\ [\sqsubseteq_r]\ b \Longrightarrow c\ [\sqsubseteq_r]\ d \Longrightarrow a@c\ [\sqsubseteq_r]\ b@d$
lemma *le-listI*:
assumes $length\ a = length\ b$
assumes $\bigwedge n.\ n < length\ a \Longrightarrow a!n \sqsubseteq_r b!n$
shows $a\ [\sqsubseteq_r]\ b$
lemma *listI*: $[\![\ size\ xs = n;\ set\ xs \subseteq A\ ]\!] \Longrightarrow xs \in list\ n\ A$

lemma *listE-length* [*simp*]: $xs \in list\ n\ A \Longrightarrow size\ xs = n$
lemma *less-lengthI*: $[\![\ xs \in list\ n\ A;\ p < n\ ]\!] \Longrightarrow p < size\ xs$
lemma *listE-set* [*simp*]: $xs \in list\ n\ A \Longrightarrow set\ xs \subseteq A$
lemma *list-0* [*simp*]: $list\ 0\ A = \{[]\}$
lemma *in-list-Suc-iff*:
$(xs \in list\ (Suc\ n)\ A) = (\exists y{\in}A.\ \exists ys \in list\ n\ A.\ xs = y\#ys)$
lemma *Cons-in-list-Suc* [*iff*]:
$(x\#xs \in list\ (Suc\ n)\ A) = (x{\in}A \wedge xs \in list\ n\ A)$
lemma *list-not-empty*:
$\exists a.\ a{\in}A \Longrightarrow \exists xs.\ xs \in list\ n\ A$

lemma *nth-in* [*rule-format*, *simp*]:
$\forall i\ n.\ size\ xs = n \longrightarrow set\ xs \subseteq A \longrightarrow i < n \longrightarrow (xs!i) \in A$
lemma *listE-nth-in*: $[\![\ xs \in list\ n\ A;\ i < n\ ]\!] \Longrightarrow xs!i \in A$
lemma *listn-Cons-Suc* [*elim*!]:
$l\#xs \in list\ n\ A \Longrightarrow (\bigwedge n'.\ n = Suc\ n' \Longrightarrow l \in A \Longrightarrow xs \in list\ n'\ A \Longrightarrow P) \Longrightarrow P$
lemma *listn-appendE* [*elim*!]:
$a@b \in list\ n\ A \Longrightarrow (\bigwedge n1\ n2.\ n{=}n1{+}n2 \Longrightarrow a \in list\ n1\ A \Longrightarrow b \in list\ n2\ A \Longrightarrow P) \Longrightarrow P$

lemma *listt-update-in-list* [*simp*, *intro*!]:
$[\![\ xs \in list\ n\ A;\ x{\in}A\ ]\!] \Longrightarrow xs[i := x] \in list\ n\ A$
lemma *list-appendI* [*intro?*]:
$[\![\ a \in list\ n\ A;\ b \in list\ m\ A\ ]\!] \Longrightarrow a\ @\ b \in list\ (n{+}m)\ A$
lemma *list-map* [*simp*]: $(map\ f\ xs \in list\ (size\ xs)\ A) = (f\ `\ set\ xs \subseteq A)$
lemma *list-replicateI* [*intro*]: $x \in A \Longrightarrow replicate\ n\ x \in list\ n\ A$
lemma *plus-list-Nil* [*simp*]: $[]\ [\sqcup_f]\ xs = []$
lemma *plus-list-Cons* [*simp*]:
$(x\#xs)\ [\sqcup_f]\ ys = (case\ ys\ of\ [] \Rightarrow []\ |\ y\#ys \Rightarrow (x \sqcup_f y)\#(xs\ [\sqcup_f]\ ys))$
lemma *length-plus-list* [*rule-format*, *simp*]:
$\forall ys.\ size(xs\ [\sqcup_f]\ ys) = min(size\ xs)\ (size\ ys)$
lemma *nth-plus-list* [*rule-format*, *simp*]:
$\forall xs\ ys\ i.\ size\ xs = n \longrightarrow size\ ys = n \longrightarrow i{<}n \longrightarrow (xs\ [\sqcup_f]\ ys)!i = (xs!i) \sqcup_f (ys!i)$

lemma (**in** *Semilat*) *plus-list-ub1* [*rule-format*]:
$[\![$ *set xs* $\subseteq A$; *set ys* $\subseteq A$; *size xs* $=$ *size ys* $]\!]$
$\Longrightarrow xs\ [\sqsubseteq_r]\ xs\ [\sqcup_f]\ ys$
lemma (**in** *Semilat*) *plus-list-ub2*:
$[\![$*set xs* $\subseteq A$; *set ys* $\subseteq A$; *size xs* $=$ *size ys* $]\!] \Longrightarrow ys\ [\sqsubseteq_r]\ xs\ [\sqcup_f]\ ys$
lemma (**in** *Semilat*) *plus-list-lub* [*rule-format*]:
shows $\forall xs\ ys\ zs.$ *set xs* $\subseteq A \longrightarrow$ *set ys* $\subseteq A \longrightarrow$ *set zs* $\subseteq A$
$\longrightarrow$ *size xs* $= n \wedge$ *size ys* $= n \longrightarrow$
$xs\ [\sqsubseteq_r]\ zs \wedge ys\ [\sqsubseteq_r]\ zs \longrightarrow xs\ [\sqcup_f]\ ys\ [\sqsubseteq_r]\ zs$
lemma (**in** *Semilat*) *list-update-incr* [*rule-format*]:
$x{\in}A \Longrightarrow$ *set xs* $\subseteq A \longrightarrow$
$(\forall i.\ i{<}$*size xs* $\longrightarrow xs\ [\sqsubseteq_r]\ xs[i := x \sqcup_f xs!i])$
lemma *acc-le-listI* [*intro*!]:
$[\![$ *order r*; *acc r* $]\!] \Longrightarrow$ *acc*(*Listn.le r*)
lemma *closed-listI*:
closed S f $\Longrightarrow$ *closed* (*list n S*) (*map2 f*)

lemma *Listn-sl-aux*:
assumes *Semilat A r f* **shows** *semilat* (*Listn.sl n* (*A*,*r*,*f*))
lemma *Listn-sl*: *semilat L* $\Longrightarrow$ *semilat* (*Listn.sl n L*)
lemma *coalesce-in-err-list* [*rule-format*]:
$\forall xes.\ xes \in$ *list n* (*err A*) $\longrightarrow$ *coalesce xes* $\in$ *err*(*list n A*)
lemma *lem*: $\bigwedge x\ xs.\ x \sqcup_{op\ \#}\ xs = x\#xs$
lemma *coalesce-eq-OK1-D* [*rule-format*]:
semilat(*err A*, *Err.le r*, *lift2 f*) $\Longrightarrow$
$\forall xs.\ xs \in$ *list n A* $\longrightarrow (\forall ys.\ ys \in$ *list n A* $\longrightarrow$
$(\forall zs.$ *coalesce* $(xs\ [\sqcup_f]\ ys) =$ *OK zs* $\longrightarrow xs\ [\sqsubseteq_r]\ zs))$
lemma *coalesce-eq-OK2-D* [*rule-format*]:
semilat(*err A*, *Err.le r*, *lift2 f*) $\Longrightarrow$
$\forall xs.\ xs \in$ *list n A* $\longrightarrow (\forall ys.\ ys \in$ *list n A* $\longrightarrow$
$(\forall zs.$ *coalesce* $(xs\ [\sqcup_f]\ ys) =$ *OK zs* $\longrightarrow ys\ [\sqsubseteq_r]\ zs))$
lemma *lift2-le-ub*:
$[\![$ *semilat*(*err A*, *Err.le r*, *lift2 f*); $x{\in}A$; $y{\in}A$; $x \sqcup_f y =$ *OK z*;
$u{\in}A$; $x \sqsubseteq_r u$; $y \sqsubseteq_r u$ $]\!] \Longrightarrow z \sqsubseteq_r u$
lemma *coalesce-eq-OK-ub-D* [*rule-format*]:
semilat(*err A*, *Err.le r*, *lift2 f*) $\Longrightarrow$
$\forall xs.\ xs \in$ *list n A* $\longrightarrow (\forall ys.\ ys \in$ *list n A* $\longrightarrow$
$(\forall zs\ us.$ *coalesce* $(xs\ [\sqcup_f]\ ys) =$ *OK zs* $\wedge xs\ [\sqsubseteq_r]\ us \wedge ys\ [\sqsubseteq_r]\ us$
$\wedge\ us \in$ *list n A* $\longrightarrow zs\ [\sqsubseteq_r]\ us))$
lemma *lift2-eq-ErrD*:
$[\![$ $x \sqcup_f y =$ *Err*; *semilat*(*err A*, *Err.le r*, *lift2 f*); $x{\in}A$; $y{\in}A$ $]\!]$
$\Longrightarrow \neg(\exists u{\in}A.\ x \sqsubseteq_r u \wedge y \sqsubseteq_r u)$

lemma *coalesce-eq-Err-D* [*rule-format*]:
$[\![$ *semilat*(*err A*, *Err.le r*, *lift2 f*) $]\!]$
$\Longrightarrow \forall xs.\ xs \in$ *list n A* $\longrightarrow (\forall ys.\ ys \in$ *list n A* $\longrightarrow$
coalesce $(xs\ [\sqcup_f]\ ys) =$ *Err* $\longrightarrow$
$\neg(\exists zs \in$ *list n A*. $xs\ [\sqsubseteq_r]\ zs \wedge ys\ [\sqsubseteq_r]\ zs))$
lemma *closed-err-lift2-conv*:
closed (*err A*) (*lift2 f*) $= (\forall x{\in}A.\ \forall y{\in}A.\ x \sqcup_f y \in$ *err A*$)$
lemma *closed-map2-list* [*rule-format*]:
closed (*err A*) (*lift2 f*) $\Longrightarrow$
$\forall xs.\ xs \in$ *list n A* $\longrightarrow (\forall ys.\ ys \in$ *list n A* $\longrightarrow$

$map2\ f\ xs\ ys \in list\ n\ (err\ A))$
lemma *closed-lift2-sup*:
$closed\ (err\ A)\ (lift2\ f) \Longrightarrow$
$closed\ (err\ (list\ n\ A))\ (lift2\ (sup\ f))$
lemma *err-semilat-sup*:
$err\text{-}semilat\ (A,r,f) \Longrightarrow$
$err\text{-}semilat\ (list\ n\ A,\ Listn.le\ r,\ sup\ f)$
lemma *err-semilat-upto-esl*:
$\bigwedge L.\ err\text{-}semilat\ L \Longrightarrow err\text{-}semilat(upto\text{-}esl\ m\ L)$
end

4.6 Typing and Dataflow Analysis Framework

theory *Typing-Framework* **imports** *Semilattices* **begin**

The relationship between dataflow analysis and a welltyped-instruction predicate.

type-synonym
$'s$ *step-type* $= nat \Rightarrow {}'s \Rightarrow (nat \times {}'s)$ *list*

definition *stable* :: $'s$ *ord* $\Rightarrow {}'s$ *step-type* $\Rightarrow {}'s$ *list* $\Rightarrow$ *nat* $\Rightarrow$ *bool*
where
stable r step τs $p \longleftrightarrow (\forall (q,\tau) \in set\ (step\ p\ (\tau s!p)).\ \tau \sqsubseteq_r \tau s!q)$

definition *stables* :: $'s$ *ord* $\Rightarrow {}'s$ *step-type* $\Rightarrow {}'s$ *list* $\Rightarrow$ *bool*
where
stables r step $\tau s \longleftrightarrow (\forall p < size\ \tau s.\ stable\ r\ step\ \tau s\ p)$

definition *wt-step* :: $'s$ *ord* $\Rightarrow {}'s \Rightarrow {}'s$ *step-type* $\Rightarrow {}'s$ *list* $\Rightarrow$ *bool*
where
wt-step r T step $\tau s \longleftrightarrow (\forall p{<}size\ \tau s.\ \tau s!p \neq T \wedge stable\ r\ step\ \tau s\ p)$

definition *is-bcv* :: $'s$ *ord* $\Rightarrow {}'s \Rightarrow {}'s$ *step-type* $\Rightarrow$ *nat* $\Rightarrow {}'s$ *set* $\Rightarrow ({}'s\ list \Rightarrow {}'s\ list) \Rightarrow$ *bool*
where
is-bcv r T step n A bcv $\longleftrightarrow (\forall \tau s_0 \in list\ n\ A.$
$(\forall p{<}n.\ (bcv\ \tau s_0)!p \neq T) = (\exists \tau s \in list\ n\ A.\ \tau s_0\ [\sqsubseteq_r]\ \tau s \wedge wt\text{-}step\ r\ T\ step\ \tau s))$

end

4.7 More on Semilattices

theory *SemilatAlg*
imports *Typing-Framework*
begin

consts
  $\mathit{lesubstep\text{-}type} :: (\mathit{nat} \times {'s})\ \mathit{set} \Rightarrow {'s}\ \mathit{ord} \Rightarrow (\mathit{nat} \times {'s})\ \mathit{set} \Rightarrow \mathit{bool}$**notation** (*xsymbols*)
  $\mathit{lesubstep\text{-}type}\ \ ((\text{-}\ /\{\sqsubseteq_{\text{-}}\}\ \text{-})\ [50,\ 0,\ 51]\ 50)$
defs *lesubstep-type-def*:
  $A\ \{\sqsubseteq_r\}\ B \equiv \forall (p,\tau) \in A.\ \exists \tau'.\ (p,\tau') \in B \wedge \tau \sqsubseteq_r \tau'$

primrec *pluslussub* $:: {'a}\ \mathit{list} \Rightarrow ({'a} \Rightarrow {'a} \Rightarrow {'a}) \Rightarrow {'a} \Rightarrow {'a}$
where
  $\mathit{pluslussub}\ []\ f\ y = y$
| $\mathit{pluslussub}\ (x\#xs)\ f\ y = \mathit{pluslussub}\ xs\ f\ (x \sqcup_f y)$**notation** (*xsymbols*)
  $\mathit{pluslussub}\ \ ((\text{-}\ /\bigsqcup_{\text{-}}\ \text{-})\ [65,\ 0,\ 66]\ 65)$

definition *bounded* $:: {'s}\ \mathit{step\text{-}type} \Rightarrow \mathit{nat} \Rightarrow \mathit{bool}$
where
  $\mathit{bounded}\ \mathit{step}\ n \longleftrightarrow (\forall p{<}n.\ \forall \tau.\ \forall (q,\tau') \in \mathit{set}\ (\mathit{step}\ p\ \tau).\ q{<}n)$

definition *pres-type* $:: {'s}\ \mathit{step\text{-}type} \Rightarrow \mathit{nat} \Rightarrow {'s}\ \mathit{set} \Rightarrow \mathit{bool}$
where
  $\mathit{pres\text{-}type}\ \mathit{step}\ n\ A \longleftrightarrow (\forall \tau{\in}A.\ \forall p{<}n.\ \forall (q,\tau') \in \mathit{set}\ (\mathit{step}\ p\ \tau).\ \tau' \in A)$

definition *mono* $:: {'s}\ \mathit{ord} \Rightarrow {'s}\ \mathit{step\text{-}type} \Rightarrow \mathit{nat} \Rightarrow {'s}\ \mathit{set} \Rightarrow \mathit{bool}$
where
  $\mathit{mono}\ r\ \mathit{step}\ n\ A \longleftrightarrow$
    $(\forall \tau\ p\ \tau'.\ \tau \in A \wedge p{<}n \wedge \tau \sqsubseteq_r \tau' \longrightarrow \mathit{set}\ (\mathit{step}\ p\ \tau)\ \{\sqsubseteq_r\}\ \mathit{set}\ (\mathit{step}\ p\ \tau'))$

lemma [*iff*]: $\{\}\ \{\sqsubseteq_r\}\ B$

lemma [*iff*]: $(A\ \{\sqsubseteq_r\}\ \{\}) = (A = \{\})$

lemma *lesubstep-union*:
  $[\![\ A_1\ \{\sqsubseteq_r\}\ B_1;\ A_2\ \{\sqsubseteq_r\}\ B_2\ ]\!] \Longrightarrow A_1 \cup A_2\ \{\sqsubseteq_r\}\ B_1 \cup B_2$

lemma *pres-typeD*:
  $[\![\ \mathit{pres\text{-}type}\ \mathit{step}\ n\ A;\ s{\in}A;\ p{<}n;\ (q,s'){\in}\mathit{set}\ (\mathit{step}\ p\ s)\ ]\!] \Longrightarrow s' \in A$
lemma *monoD*:
  $[\![\ \mathit{mono}\ r\ \mathit{step}\ n\ A;\ p < n;\ s{\in}A;\ s \sqsubseteq_r t\ ]\!] \Longrightarrow \mathit{set}\ (\mathit{step}\ p\ s)\ \{\sqsubseteq_r\}\ \mathit{set}\ (\mathit{step}\ p\ t)$
lemma *boundedD*:
  $[\![\ \mathit{bounded}\ \mathit{step}\ n;\ p < n;\ (q,t) \in \mathit{set}\ (\mathit{step}\ p\ xs)\ ]\!] \Longrightarrow q < n$
lemma *lesubstep-type-refl* [*simp*, *intro*]:
  $(\bigwedge x.\ x \sqsubseteq_r x) \Longrightarrow A\ \{\sqsubseteq_r\}\ A$
lemma *lesub-step-typeD*:
  $A\ \{\sqsubseteq_r\}\ B \Longrightarrow (x,y) \in A \Longrightarrow \exists y'.\ (x,\ y') \in B \wedge y \sqsubseteq_r y'$

lemma *list-update-le-listI* [*rule-format*]:
  $\mathit{set}\ xs \subseteq A \longrightarrow \mathit{set}\ ys \subseteq A \longrightarrow xs\ [\sqsubseteq_r]\ ys \longrightarrow p < \mathit{size}\ xs \longrightarrow$
   $x \sqsubseteq_r ys!p \longrightarrow \mathit{semilat}(A,r,f) \longrightarrow x{\in}A \longrightarrow$
   $xs[p := x \sqcup_f xs!p]\ [\sqsubseteq_r]\ ys$
lemma *plusplus-closed*: **assumes** *Semilat A r f* **shows**

$\bigwedge y.\ [\![\ set\ x \subseteq A;\ y \in A]\!] \Longrightarrow x \bigsqcup_f y \in A$

lemma (**in** *Semilat*) *pp-ub2*:

$\bigwedge y.\ [\![\ set\ x \subseteq A;\ y \in A]\!] \Longrightarrow y \sqsubseteq_r x \bigsqcup_f y$

lemma (**in** *Semilat*) *pp-ub1*:

shows $\bigwedge y.\ [\![set\ ls \subseteq A;\ y \in A;\ x \in set\ ls]\!] \Longrightarrow x \sqsubseteq_r ls \bigsqcup_f y$

lemma (**in** *Semilat*) *pp-lub*:

assumes z: $z \in A$

shows

$\bigwedge y.\ y \in A \Longrightarrow set\ xs \subseteq A \Longrightarrow \forall x \in set\ xs.\ x \sqsubseteq_r z \Longrightarrow y \sqsubseteq_r z \Longrightarrow xs \bigsqcup_f y \sqsubseteq_r z$

lemma *ub1'*: **assumes** *Semilat A r f*

shows $[\![\forall (p,s) \in set\ S.\ s \in A;\ y \in A;\ (a,b) \in set\ S]\!]$
$\Longrightarrow b \sqsubseteq_r map\ snd\ [(p',\ t') \leftarrow S.\ p' = a] \bigsqcup_f y$

lemma *plusplus-empty*:

$\forall s'.\ (q,\ s') \in set\ S \longrightarrow s' \sqcup_f ss\ !\ q = ss\ !\ q \Longrightarrow$
$(map\ snd\ [(p',\ t') \leftarrow S.\ p' = q] \bigsqcup_f ss\ !\ q) = ss\ !\ q$

end

4.8 Lifting the Typing Framework to err, app, and eff

theory *Typing-Framework-err* **imports** *Typing-Framework SemilatAlg* **begin**

definition $wt\text{-}err\text{-}step :: {'s}\ ord \Rightarrow {'s}\ err\ step\text{-}type \Rightarrow {'s}\ err\ list \Rightarrow bool$
where
$wt\text{-}err\text{-}step\ r\ step\ \tau s \longleftrightarrow wt\text{-}step\ (Err.le\ r)\ Err\ step\ \tau s$

definition $wt\text{-}app\text{-}eff :: {'s}\ ord \Rightarrow (nat \Rightarrow {'s} \Rightarrow bool) \Rightarrow {'s}\ step\text{-}type \Rightarrow {'s}\ list \Rightarrow bool$
where
$wt\text{-}app\text{-}eff\ r\ app\ step\ \tau s \longleftrightarrow$
$(\forall p < size\ \tau s.\ app\ p\ (\tau s!p) \wedge (\forall (q,\tau) \in set\ (step\ p\ (\tau s!p)).\ \tau <\!=\!-r\ \tau s!q))$

definition $map\text{-}snd :: ({'b} \Rightarrow {'c}) \Rightarrow ({'a} \times {'b})\ list \Rightarrow ({'a} \times {'c})\ list$
where
$map\text{-}snd\ f = map\ (\lambda(x,y).\ (x,\ f\ y))$

definition $error :: nat \Rightarrow (nat \times {'a}\ err)\ list$
where
$error\ n = map\ (\lambda x.\ (x,Err))\ [0..<n]$

definition $err\text{-}step :: nat \Rightarrow (nat \Rightarrow {'s} \Rightarrow bool) \Rightarrow {'s}\ step\text{-}type \Rightarrow {'s}\ err\ step\text{-}type$
where
$err\text{-}step\ n\ app\ step\ p\ t =$
$(case\ t\ of$
$\quad Err \quad \Rightarrow error\ n$
$|\ OK\ \tau \Rightarrow if\ app\ p\ \tau\ then\ map\text{-}snd\ OK\ (step\ p\ \tau)\ else\ error\ n)$

definition $app\text{-}mono :: {'s}\ ord \Rightarrow (nat \Rightarrow {'s} \Rightarrow bool) \Rightarrow nat \Rightarrow {'s}\ set \Rightarrow bool$
where
$app\text{-}mono\ r\ app\ n\ A \longleftrightarrow$
$(\forall s\ p\ t.\ s \in A \wedge p < n \wedge s \sqsubseteq_r t \longrightarrow app\ p\ t \longrightarrow app\ p\ s)$

lemmas $err\text{-}step\text{-}defs = err\text{-}step\text{-}def\ map\text{-}snd\text{-}def\ error\text{-}def$

lemma *bounded-err-stepD*:
$[\![\ bounded\ (err\text{-}step\ n\ app\ step)\ n;$
$\quad p < n;\ app\ p\ a;\ (q,b) \in set\ (step\ p\ a)\ ]\!] \Longrightarrow q < n$

lemma *in-map-sndD*: $(a,b) \in set\ (map\text{-}snd\ f\ xs) \Longrightarrow \exists b'.\ (a,b') \in set\ xs$

lemma *bounded-err-stepI*:
$\forall p.\ p < n \longrightarrow (\forall s.\ ap\ p\ s \longrightarrow (\forall (q,s') \in set\ (step\ p\ s).\ q < n))$
$\Longrightarrow bounded\ (err\text{-}step\ n\ ap\ step)\ n$

lemma *bounded-lift*:
$bounded\ step\ n \Longrightarrow bounded\ (err\text{-}step\ n\ app\ step)\ n$

lemma *le-list-map-OK* [*simp*]:
$\bigwedge b.\ (map\ OK\ a\ [\sqsubseteq_{Err.le\ r}]\ map\ OK\ b) = (a\ [\sqsubseteq_r]\ b)$

lemma *map-snd-lessI*:

$set\ xs\ \{\sqsubseteq_r\}\ set\ ys \Longrightarrow set\ (map\text{-}snd\ OK\ xs)\ \{\sqsubseteq_{Err.le\ r}\}\ set\ (map\text{-}snd\ OK\ ys)$

lemma *mono-lift*:
$[\![$ *order r*; *app-mono r app n A*; *bounded* (*err-step n app step*) *n*;
$\forall s\ p\ t.\ s \in A \wedge p < n \wedge s \sqsubseteq_r t \longrightarrow app\ p\ t \longrightarrow set\ (step\ p\ s)\ \{\sqsubseteq_r\}\ set\ (step\ p\ t)\ ]\!]$
$\Longrightarrow$ *mono* (*Err.le r*) (*err-step n app step*) *n* (*err A*)
lemma *in-errorD*: $(x,y) \in set\ (error\ n) \Longrightarrow y = Err$
lemma *pres-type-lift*:
$\forall s{\in}A.\ \forall p.\ p < n \longrightarrow app\ p\ s \longrightarrow (\forall (q,\ s'){\in}set\ (step\ p\ s).\ s' \in A)$
$\Longrightarrow$ *pres-type* (*err-step n app step*) *n* (*err A*)

lemma *wt-err-imp-wt-app-eff*:
assumes *wt*: *wt-err-step r* (*err-step* (*size ts*) *app step*) *ts*
assumes *b*: *bounded* (*err-step* (*size ts*) *app step*) (*size ts*)
shows *wt-app-eff r app step* (*map ok-val ts*)

lemma *wt-app-eff-imp-wt-err*:
assumes *app-eff*: *wt-app-eff r app step ts*
assumes *bounded*: *bounded* (*err-step* (*size ts*) *app step*) (*size ts*)
shows *wt-err-step r* (*err-step* (*size ts*) *app step*) (*map OK ts*)
end

4.9 Kildall's Algorithm

theory *Kildall*
imports *SemilatAlg*
begin

primrec $propa :: 's\ binop \Rightarrow (nat \times 's)\ list \Rightarrow 's\ list \Rightarrow nat\ set \Rightarrow 's\ list * nat\ set$
where
$propa\ f\ []\quad \tau s\ w = (\tau s, w)$
$|\ propa\ f\ (q'\#qs)\ \tau s\ w = (let\ (q,\tau) = q';$
$\quad u = \tau \sqcup_f \tau s!q;$
$\quad w' = (if\ u = \tau s!q\ then\ w\ else\ insert\ q\ w)$
$\quad in\ propa\ f\ qs\ (\tau s[q := u])\ w')$

definition $iter :: 's\ binop \Rightarrow 's\ step\text{-}type \Rightarrow$
$'s\ list \Rightarrow nat\ set \Rightarrow 's\ list \times nat\ set$
where
$iter\ f\ step\ \tau s\ w =$
$while\ (\lambda(\tau s,w).\ w \neq \{\})$
$(\lambda(\tau s,w).\ let\ p = SOME\ p.\ p \in w$
$in\ propa\ f\ (step\ p\ (\tau s!p))\ \tau s\ (w-\{p\}))$
$(\tau s,w)$

definition $unstables :: 's\ ord \Rightarrow 's\ step\text{-}type \Rightarrow 's\ list \Rightarrow nat\ set$
where
$unstables\ r\ step\ \tau s = \{p.\ p < size\ \tau s \wedge \neg stable\ r\ step\ \tau s\ p\}$

definition $kildall :: 's\ ord \Rightarrow 's\ binop \Rightarrow 's\ step\text{-}type \Rightarrow 's\ list \Rightarrow 's\ list$
where
$kildall\ r\ f\ step\ \tau s = fst(iter\ f\ step\ \tau s\ (unstables\ r\ step\ \tau s))$

primrec $merges :: 's\ binop \Rightarrow (nat \times 's)\ list \Rightarrow 's\ list \Rightarrow 's\ list$
where
$merges\ f\ []\quad \tau s = \tau s$
$|\ merges\ f\ (p'\#ps)\ \tau s = (let\ (p,\tau) = p'\ in\ merges\ f\ ps\ (\tau s[p := \tau \sqcup_f \tau s!p]))$

lemmas [*simp*] = *Let-def Semilat.le-iff-plus-unchanged* [*OF Semilat.intro*, *symmetric*]

lemma (**in** *Semilat*) *nth-merges*:
$\bigwedge ss.\ [\![p < length\ ss;\ ss \in list\ n\ A;\ \forall (p,t) \in set\ ps.\ p<n \wedge t \in A\ ]\!] \Longrightarrow$
$(merges\ f\ ps\ ss)!p = map\ snd\ [(p',t') \leftarrow ps.\ p'=p]\ \bigsqcup_f\ ss!p$
(**is** $\bigwedge ss.\ [\![-;\ -;\ ?steptype\ ps]\!] \Longrightarrow\ ?P\ ss\ ps$)

lemma *length-merges* [*simp*]:
$\bigwedge ss.\ size(merges\ f\ ps\ ss) = size\ ss$
lemma (**in** *Semilat*) *merges-preserves-type-lemma*:
shows $\forall xs.\ xs \in list\ n\ A \longrightarrow (\forall (p,x) \in set\ ps.\ p<n \wedge x \in A)$
$\longrightarrow merges\ f\ ps\ xs \in list\ n\ A$

lemma (**in** *Semilat*) *merges-preserves-type* [*simp*]:
$[\![$ *xs* $\in$ *list n A*; $\forall (p,x) \in$ *set ps*. $p<n \wedge x\in A$ $]\!]$
$\Longrightarrow$ *merges f ps xs* $\in$ *list n A*
by (*simp add*: *merges-preserves-type-lemma*)

lemma (**in** *Semilat*) *merges-incr-lemma*:
$\forall$ *xs*. *xs* $\in$ *list n A* $\longrightarrow$ ($\forall (p,x)\in$*set ps*. $p<$*size xs* $\wedge x \in A$) $\longrightarrow$ *xs* $[\sqsubseteq_r]$ *merges f ps xs*
lemma (**in** *Semilat*) *merges-incr*:
$[\![$ *xs* $\in$ *list n A*; $\forall (p,x)\in$*set ps*. $p<$*size xs* $\wedge x \in A$ $]\!]$
$\Longrightarrow$ *xs* $[\sqsubseteq_r]$ *merges f ps xs*
by (*simp add*: *merges-incr-lemma*)

lemma (**in** *Semilat*) *merges-same-conv* [*rule-format*]:
($\forall$ *xs*. *xs* $\in$ *list n A* $\longrightarrow$ ($\forall (p,x)\in$*set ps*. $p<$*size xs* $\wedge x\in A$) $\longrightarrow$
(*merges f ps xs* = *xs*) = ($\forall (p,x)\in$*set ps*. $x \sqsubseteq_r$ *xs*!*p*))

lemma (**in** *Semilat*) *list-update-le-listI* [*rule-format*]:
set xs $\subseteq A \longrightarrow$ *set ys* $\subseteq A \longrightarrow$ *xs* $[\sqsubseteq_r]$ *ys* $\longrightarrow p <$ *size xs* $\longrightarrow$
$x \sqsubseteq_r$ *ys*!*p* $\longrightarrow x\in A \longrightarrow$ *xs*[$p := x \sqcup_f$ *xs*!*p*] $[\sqsubseteq_r]$ *ys*
lemma (**in** *Semilat*) *merges-pres-le-ub*:
assumes *set ts* $\subseteq A$ *set ss* $\subseteq A$
$\forall (p,t)\in$*set ps*. $t \sqsubseteq_r$ *ts*!*p* $\wedge t \in A \wedge p <$ *size ts* *ss* $[\sqsubseteq_r]$ *ts*
shows *merges f ps ss* $[\sqsubseteq_r]$ *ts*

lemma *decomp-propa*:
$\bigwedge$*ss w*. ($\forall (q,t)\in$*set qs*. $q <$ *size ss*) $\Longrightarrow$
propa f qs ss w =
(*merges f qs ss*, $\{q.\ \exists t.(q,t)\in$*set qs* $\wedge t \sqcup_f$ *ss*!$q \neq$ *ss*!$q\} \cup w$)

lemma (**in** *Semilat*) *stable-pres-lemma*:
shows $[\![$*pres-type step n A*; *bounded step n*;
ss $\in$ *list n A*; $p \in w$; $\forall q\in w.\ q < n$;
$\forall q.\ q < n \longrightarrow q \notin w \longrightarrow$ *stable r step ss q*; $q < n$;
$\forall s'.\ (q,s') \in$ *set* (*step p* (*ss*!*p*)) $\longrightarrow s' \sqcup_f$ *ss*!*q* = *ss*!*q*;
$q \notin w \vee q = p$ $]\!]$
$\Longrightarrow$ *stable r step* (*merges f* (*step p* (*ss*!*p*)) *ss*) *q*

lemma (**in** *Semilat*) *merges-bounded-lemma*:
$[\![$ *mono r step n A*; *bounded step n*;
$\forall (p',s') \in$ *set* (*step p* (*ss*!*p*)). $s' \in A$; *ss* $\in$ *list n A*; *ts* $\in$ *list n A*; $p < n$;
ss $[\sqsubseteq_r]$ *ts*; $\forall p.\ p < n \longrightarrow$ *stable r step ts p* $]\!]$
$\Longrightarrow$ *merges f* (*step p* (*ss*!*p*)) *ss* $[\sqsubseteq_r]$ *ts*
lemma *termination-lemma*: **assumes** *Semilat A r f*
shows $[\![$ *ss* $\in$ *list n A*; $\forall (q,t)\in$*set qs*. $q<n \wedge t\in A$; $p\in w$ $]\!] \Longrightarrow$
ss $[\sqsubset_r]$ *merges f qs ss* $\vee$
merges f qs ss = *ss* $\wedge$ $\{q.\ \exists t.\ (q,t)\in$*set qs* $\wedge t \sqcup_f$ *ss*!$q \neq$ *ss*!$q\} \cup (w-\{p\}) \subset w$
lemma *iter-properties*[*rule-format*]: **assumes** *Semilat A r f*
shows $[\![$ *acc r*; *pres-type step n A*; *mono r step n A*;
bounded step n; $\forall p\in w0.\ p < n$; *ss0* $\in$ *list n A*;
$\forall p<n.\ p \notin w0 \longrightarrow$ *stable r step ss0 p* $]\!] \Longrightarrow$

$\mathit{iter}\ f\ \mathit{step}\ \mathit{ss0}\ \mathit{w0} = (\mathit{ss}',w')$
$\longrightarrow$
$\mathit{ss}' \in \mathit{list}\ n\ A \wedge \mathit{stables}\ r\ \mathit{step}\ \mathit{ss}' \wedge \mathit{ss0}\ [\sqsubseteq_r]\ \mathit{ss}' \wedge$
$(\forall\, \mathit{ts}{\in}\mathit{list}\ n\ A.\ \mathit{ss0}\ [\sqsubseteq_r]\ \mathit{ts} \wedge \mathit{stables}\ r\ \mathit{step}\ \mathit{ts} \longrightarrow \mathit{ss}'\ [\sqsubseteq_r]\ \mathit{ts})$

lemma *kildall-properties*: **assumes** *Semilat A r f*
shows $[\![\ \mathit{acc}\ r;\ \mathit{pres\text{-}type}\ \mathit{step}\ n\ A;\ \mathit{mono}\ r\ \mathit{step}\ n\ A;$
$\mathit{bounded}\ \mathit{step}\ n;\ \mathit{ss0} \in \mathit{list}\ n\ A\ ]\!] \Longrightarrow$
$\mathit{kildall}\ r\ f\ \mathit{step}\ \mathit{ss0} \in \mathit{list}\ n\ A \wedge$
$\mathit{stables}\ r\ \mathit{step}\ (\mathit{kildall}\ r\ f\ \mathit{step}\ \mathit{ss0}) \wedge$
$\mathit{ss0}\ [\sqsubseteq_r]\ \mathit{kildall}\ r\ f\ \mathit{step}\ \mathit{ss0} \wedge$
$(\forall\, \mathit{ts}{\in}\mathit{list}\ n\ A.\ \mathit{ss0}\ [\sqsubseteq_r]\ \mathit{ts} \wedge \mathit{stables}\ r\ \mathit{step}\ \mathit{ts} \longrightarrow$
$\mathit{kildall}\ r\ f\ \mathit{step}\ \mathit{ss0}\ [\sqsubseteq_r]\ \mathit{ts})$
end

4.10 The Lightweight Bytecode Verifier

theory *LBVSpec*
imports *SemilatAlg Opt*
begin

type-synonym
  's certificate = *'s list*

primrec *merge* :: *'s certificate* ⇒ *'s binop* ⇒ *'s ord* ⇒ *'s* ⇒ *nat* ⇒ (*nat* × *'s*) *list* ⇒ *'s* ⇒ *'s*
where
  merge cert f r T pc [] *x* = *x*
| *merge cert f r T pc* (*s*#*ss*) *x* = *merge cert f r T pc ss* (*let* (*pc'*,*s'*) = *s in*
    if pc'=*pc*+*1 then* $s' \sqcup_f x$
    else if $s' \sqsubseteq_r$ *cert*!*pc' then x*
    else T)

definition *wtl-inst* :: *'s certificate* ⇒ *'s binop* ⇒ *'s ord* ⇒ *'s* ⇒
    's step-type ⇒ *nat* ⇒ *'s* ⇒ *'s*
where
  wtl-inst cert f r T step pc s = *merge cert f r T pc* (*step pc s*) (*cert*!(*pc*+*1*))

definition *wtl-cert* :: *'s certificate* ⇒ *'s binop* ⇒ *'s ord* ⇒ *'s* ⇒ *'s* ⇒
    's step-type ⇒ *nat* ⇒ *'s* ⇒ *'s*
where
  wtl-cert cert f r T B step pc s =
  (*if cert*!*pc* = *B then*
    wtl-inst cert f r T step pc s
  else
    if s $\sqsubseteq_r$ *cert*!*pc then wtl-inst cert f r T step pc* (*cert*!*pc*) *else T*)

primrec *wtl-inst-list* :: *'a list* ⇒ *'s certificate* ⇒ *'s binop* ⇒ *'s ord* ⇒ *'s* ⇒ *'s* ⇒
    's step-type ⇒ *nat* ⇒ *'s* ⇒ *'s*
where
  wtl-inst-list [] *cert f r T B step pc s* = *s*
| *wtl-inst-list* (*i*#*is*) *cert f r T B step pc s* =
    (*let s'* = *wtl-cert cert f r T B step pc s in*
      if s' = *T* ∨ *s* = *T then T else wtl-inst-list is cert f r T B step* (*pc*+*1*) *s'*)

definition *cert-ok* :: *'s certificate* ⇒ *nat* ⇒ *'s* ⇒ *'s* ⇒ *'s set* ⇒ *bool*
where
  cert-ok cert n T B A ⟷ (∀ *i* < *n*. *cert*!*i* ∈ *A* ∧ *cert*!*i* ≠ *T*) ∧ (*cert*!*n* = *B*)

definition *bottom* :: *'a ord* ⇒ *'a* ⇒ *bool*
where
  bottom r B ⟷ (∀ *x*. *B* $\sqsubseteq_r$ *x*)

locale *lbv* = *Semilat* +
  fixes *T* :: *'a* (⊤)
  fixes *B* :: *'a* (⊥)
  fixes *step* :: *'a step-type*
  assumes *top*: *top r* ⊤
  assumes *T-A*: ⊤ ∈ *A*

assumes *bot*: *bottom r* $\bot$
assumes *B-A*: $\bot \in A$

fixes *merge* :: *'a certificate* $\Rightarrow$ *nat* $\Rightarrow$ (*nat* $\times$ *'a*) *list* $\Rightarrow$ *'a* $\Rightarrow$ *'a*
defines *mrg-def*: *merge cert* $\equiv$ *LBVSpec.merge cert f r* $\top$

fixes *wti* :: *'a certificate* $\Rightarrow$ *nat* $\Rightarrow$ *'a* $\Rightarrow$ *'a*
defines *wti-def*: *wti cert* $\equiv$ *wtl-inst cert f r* $\top$ *step*

fixes *wtc* :: *'a certificate* $\Rightarrow$ *nat* $\Rightarrow$ *'a* $\Rightarrow$ *'a*
defines *wtc-def*: *wtc cert* $\equiv$ *wtl-cert cert f r* $\top$ $\bot$ *step*

fixes *wtl* :: *'b list* $\Rightarrow$ *'a certificate* $\Rightarrow$ *nat* $\Rightarrow$ *'a* $\Rightarrow$ *'a*
defines *wtl-def*: *wtl ins cert* $\equiv$ *wtl-inst-list ins cert f r* $\top$ $\bot$ *step*

lemma (**in** *lbv*) *wti*:
wti c pc s = *merge c pc* (*step pc s*) (*c*!(*pc*+*1*))

lemma (**in** *lbv*) *wtc*:
wtc c pc s = (*if c*!*pc* = $\bot$ *then wti c pc s else if s* $\sqsubseteq_r$ *c*!*pc then wti c pc* (*c*!*pc*) *else* $\top$)

lemma *cert-okD1* [*intro?*]:
cert-ok c n T B A $\Longrightarrow$ *pc* < *n* $\Longrightarrow$ *c*!*pc* $\in$ *A*

lemma *cert-okD2* [*intro?*]:
cert-ok c n T B A $\Longrightarrow$ *c*!*n* = *B*

lemma *cert-okD3* [*intro?*]:
cert-ok c n T B A $\Longrightarrow$ *B* $\in$ *A* $\Longrightarrow$ *pc* < *n* $\Longrightarrow$ *c*!*Suc pc* $\in$ *A*

lemma *cert-okD4* [*intro?*]:
cert-ok c n T B A $\Longrightarrow$ *pc* < *n* $\Longrightarrow$ *c*!*pc* $\neq$ *T*

declare *Let-def* [*simp*]

4.10.1 more semilattice lemmas

lemma (**in** *lbv*) *sup-top* [*simp*, *elim*]:
assumes *x*: *x* $\in$ *A*
shows *x* $\sqcup_f$ $\top$ = $\top$
lemma (**in** *lbv*) *plusplussup-top* [*simp*, *elim*]:
set xs $\subseteq$ *A* $\Longrightarrow$ *xs* $\bigsqcup_f$ $\top$ = $\top$
by (*induct xs*) *auto*

lemma (**in** *Semilat*) *pp-ub1'*:
assumes *S*: *snd`set S* $\subseteq$ *A*
assumes *y*: *y* $\in$ *A* **and** *ab*: (*a*, *b*) $\in$ *set S*
shows *b* $\sqsubseteq_r$ *map snd* [(p', t') $\leftarrow$ *S* . p' = *a*] $\bigsqcup_f$ *y*
lemma (**in** *lbv*) *bottom-le* [*simp*, *intro*!]: $\bot$ $\sqsubseteq_r$ *x*
by (*insert bot*) (*simp add*: *bottom-def*)

lemma (**in** *lbv*) *le-bottom* [*simp*]: *x* $\sqsubseteq_r$ $\bot$ = (*x* = $\bot$)

by (*blast intro*: *antisym-r*)

4.10.2 merge

lemma (**in** *lbv*) *merge-Nil* [*simp*]:
merge c pc [] $x = x$ **by** (*simp add*: *mrg-def*)

lemma (**in** *lbv*) *merge-Cons* [*simp*]:
merge c pc (*l*#*ls*) $x =$ *merge c pc ls* (*if fst l=pc+1 then snd l +-f x*
else if snd l $\sqsubseteq_r$ *c!fst l then x*
else $\top$)
by (*simp add*: *mrg-def split-beta*)

lemma (**in** *lbv*) *merge-Err* [*simp*]:
snd‘set ss $\subseteq A \Longrightarrow$ *merge c pc ss* $\top = \top$
by (*induct ss*) *auto*

lemma (**in** *lbv*) *merge-not-top*:
$\bigwedge x$. *snd‘set ss* $\subseteq A \Longrightarrow$ *merge c pc ss x* $\neq \top \Longrightarrow$
$\forall (pc',s') \in$ *set ss*. ($pc' \neq pc+1 \longrightarrow s' \sqsubseteq_r c!pc'$)
(**is** $\bigwedge x$. *?set ss* $\Longrightarrow$ *?merge ss x* $\Longrightarrow$ *?P ss*)

lemma (**in** *lbv*) *merge-def*:
shows
$\bigwedge x$. $x \in A \Longrightarrow$ *snd‘set ss* $\subseteq A \Longrightarrow$
merge c pc ss x $=$
(*if* $\forall (pc',s') \in$ *set ss*. $pc' \neq pc+1 \longrightarrow s' \sqsubseteq_r c!pc'$ *then*
map snd [$(p',t') \leftarrow$ *ss*. $p'=pc+1$] $\bigsqcup_f x$
else $\top$)
(**is** $\bigwedge x$. - $\Longrightarrow$ - $\Longrightarrow$ *?merge ss x* = *?if ss x* **is** $\bigwedge x$. - $\Longrightarrow$ - $\Longrightarrow$ *?P ss x*)
lemma (**in** *lbv*) *merge-not-top-s*:
assumes *x*: $x \in A$ **and** *ss*: *snd‘set ss* $\subseteq A$
assumes *m*: *merge c pc ss x* $\neq \top$
shows *merge c pc ss x* = (*map snd* [$(p',t') \leftarrow$ *ss*. $p'=pc+1$] $\bigsqcup_f x$)

4.10.3 wtl-inst-list

lemmas [*iff*] = *not-Err-eq*

lemma (**in** *lbv*) *wtl-Nil* [*simp*]: *wtl* [] *c pc s* = *s*
by (*simp add*: *wtl-def*)

lemma (**in** *lbv*) *wtl-Cons* [*simp*]:
wtl (*i*#*is*) *c pc s* =
(*let* s' = *wtc c pc s in if* $s' = \top \vee s = \top$ *then* $\top$ *else wtl is c* (*pc+1*) s')
by (*simp add*: *wtl-def wtc-def*)

lemma (**in** *lbv*) *wtl-Cons-not-top*:
wtl (*i*#*is*) *c pc s* $\neq \top$ =
(*wtc c pc s* $\neq \top \wedge s \neq T \wedge$ *wtl is c* (*pc+1*) (*wtc c pc s*) $\neq \top$)
by (*auto simp del*: *split-paired-Ex*)

lemma (**in** *lbv*) *wtl-top* [*simp*]: *wtl ls c pc* $\top = \top$
by (*cases ls*) *auto*

lemma (**in** *lbv*) *wtl-not-top*:
$wtl\ ls\ c\ pc\ s \neq \top \Longrightarrow s \neq \top$
by (*cases* $s{=}\top$) *auto*

lemma (**in** *lbv*) *wtl-append* [*simp*]:
$\bigwedge pc\ s.\ wtl\ (a@b)\ c\ pc\ s = wtl\ b\ c\ (pc{+}length\ a)\ (wtl\ a\ c\ pc\ s)$
by (*induct a*) *auto*

lemma (**in** *lbv*) *wtl-take*:
$wtl\ is\ c\ pc\ s \neq \top \Longrightarrow wtl\ (take\ pc'\ is)\ c\ pc\ s \neq \top$
(**is** *?wtl is* $\neq$ - $\Longrightarrow$ -)
lemma *take-Suc*:
$\forall n.\ n < length\ l \longrightarrow take\ (Suc\ n)\ l = (take\ n\ l)@[l!n]$ (**is** *?P l*)
lemma (**in** *lbv*) *wtl-Suc*:
assumes *suc*: $pc{+}1 < length\ is$
assumes *wtl*: $wtl\ (take\ pc\ is)\ c\ 0\ s \neq \top$
shows $wtl\ (take\ (pc{+}1)\ is)\ c\ 0\ s = wtc\ c\ pc\ (wtl\ (take\ pc\ is)\ c\ 0\ s)$
lemma (**in** *lbv*) *wtl-all*:
assumes *all*: $wtl\ is\ c\ 0\ s \neq \top$ (**is** *?wtl is* $\neq$ -)
assumes *pc*: $pc < length\ is$
shows $wtc\ c\ pc\ (wtl\ (take\ pc\ is)\ c\ 0\ s) \neq \top$

4.10.4 preserves-type

lemma (**in** *lbv*) *merge-pres*:
assumes *s0*: $snd`set\ ss \subseteq A$ **and** *x*: $x \in A$
shows $merge\ c\ pc\ ss\ x \in A$
lemma *pres-typeD2*:
$pres\text{-}type\ step\ n\ A \Longrightarrow s \in A \Longrightarrow p < n \Longrightarrow snd`set\ (step\ p\ s) \subseteq A$
by *auto* (*drule pres-typeD*)

lemma (**in** *lbv*) *wti-pres* [*intro?*]:
assumes *pres*: $pres\text{-}type\ step\ n\ A$
assumes *cert*: $c!(pc{+}1) \in A$
assumes *s-pc*: $s \in A\ pc < n$
shows $wti\ c\ pc\ s \in A$
lemma (**in** *lbv*) *wtc-pres*:
assumes $pres\text{-}type\ step\ n\ A$
assumes $c!pc \in A$ **and** $c!(pc{+}1) \in A$
assumes $s \in A$ **and** $pc < n$
shows $wtc\ c\ pc\ s \in A$
lemma (**in** *lbv*) *wtl-pres*:
assumes *pres*: $pres\text{-}type\ step\ (length\ is)\ A$
assumes *cert*: $cert\text{-}ok\ c\ (length\ is)\ \top\ \bot\ A$
assumes *s*: $s \in A$
assumes *all*: $wtl\ is\ c\ 0\ s \neq \top$
shows $pc < length\ is \Longrightarrow wtl\ (take\ pc\ is)\ c\ 0\ s \in A$
(**is** $?len\ pc \Longrightarrow ?wtl\ pc \in A$)
end

4.11 Correctness of the LBV

theory *LBVCorrect*
imports *LBVSpec Typing-Framework*
begin

locale *lbvs* = *lbv* +
  fixes s_0 :: *'a*
  fixes *c* :: *'a list*
  fixes *ins* :: *'b list*
  fixes τs :: *'a list*
  defines *phi-def*:
  $\tau s \equiv map\ (\lambda pc.\ if\ c!pc = \bot\ then\ wtl\ (take\ pc\ ins)\ c\ 0\ s_0\ else\ c!pc)$
    $[0..<size\ ins]$

  assumes *bounded*: *bounded step* (*size ins*)
  assumes *cert*: *cert-ok c* (*size ins*) $\top$ $\bot$ *A*
  assumes *pres*: *pres-type step* (*size ins*) *A*

lemma (**in** *lbvs*) *phi-None* [*intro?*]:
  $[\![\ pc < size\ ins;\ c!pc = \bot\ ]\!] \Longrightarrow \tau s!pc = wtl\ (take\ pc\ ins)\ c\ 0\ s_0$
lemma (**in** *lbvs*) *phi-Some* [*intro?*]:
  $[\![\ pc < size\ ins;\ c!pc \neq \bot\ ]\!] \Longrightarrow \tau s!pc = c!pc$
lemma (**in** *lbvs*) *phi-len* [*simp*]: $size\ \tau s = size\ ins$
lemma (**in** *lbvs*) *wtl-suc-pc*:
  assumes *all*: $wtl\ ins\ c\ 0\ s_0 \neq \top$
  assumes *pc*: $pc+1 < size\ ins$
  shows $wtl\ (take\ (pc+1)\ ins)\ c\ 0\ s_0 \sqsubseteq_r \tau s!(pc+1)$
lemma (**in** *lbvs*) *wtl-stable*:
  assumes *wtl*: $wtl\ ins\ c\ 0\ s_0 \neq \top$
  assumes s_0: $s_0 \in A$ **and** *pc*: $pc < size\ ins$
  shows *stable r step* τs *pc*
lemma (**in** *lbvs*) *phi-not-top*:
  assumes *wtl*: $wtl\ ins\ c\ 0\ s_0 \neq \top$ **and** *pc*: $pc < size\ ins$
  shows $\tau s!pc \neq \top$
lemma (**in** *lbvs*) *phi-in-A*:
  assumes *wtl*: $wtl\ ins\ c\ 0\ s_0 \neq \top$ **and** s_0: $s_0 \in A$
  shows $\tau s \in list\ (size\ ins)\ A$
lemma (**in** *lbvs*) *phi0*:
  assumes *wtl*: $wtl\ ins\ c\ 0\ s_0 \neq \top$ **and** *0*: $0 < size\ ins$
  shows $s_0 \sqsubseteq_r \tau s!0$

theorem (**in** *lbvs*) *wtl-sound*:
  assumes *wtl*: $wtl\ ins\ c\ 0\ s_0 \neq \top$ **and** s_0: $s_0 \in A$
  shows $\exists \tau s.\ wt\text{-}step\ r\ \top\ step\ \tau s$

theorem (**in** *lbvs*) *wtl-sound-strong*:
  assumes *wtl*: $wtl\ ins\ c\ 0\ s_0 \neq \top$
  assumes s_0: $s_0 \in A$ **and** *ins*: $0 < size\ ins$
  shows $\exists \tau s \in list\ (size\ ins)\ A.\ wt\text{-}step\ r\ \top\ step\ \tau s \wedge s_0 \sqsubseteq_r \tau s!0$
end

4.12 Completeness of the LBV

theory *LBVComplete*
imports *LBVSpec Typing-Framework*
begin

definition *is-target* :: *'s step-type* ⇒ *'s list* ⇒ *nat* ⇒ *bool* **where**
is-target step τs pc' ⟷ (∃ *pc s'*. *pc'* ≠ *pc+1* ∧ *pc* < *size τs* ∧ (*pc'*,*s'*) ∈ *set* (*step pc* (*τs*!*pc*)))

definition *make-cert* :: *'s step-type* ⇒ *'s list* ⇒ *'s* ⇒ *'s certificate* **where**
make-cert step τs B = *map* (λ*pc*. *if is-target step τs pc then τs*!*pc else B*) [*0*..<*size τs*] @ [*B*]

lemma [*code*]:
is-target step τs pc' =
list-ex (λ*pc*. *pc'* ≠ *pc+1* ∧ *List.member* (*map fst* (*step pc* (*τs*!*pc*))) *pc'*) [*0*..<*size τs*]
locale *lbvc* = *lbv* +
fixes *τs* :: *'a list*
fixes *c* :: *'a list*
defines *cert-def*: *c* ≡ *make-cert step τs* ⊥

assumes *mono*: *mono r step* (*size τs*) *A*
assumes *pres*: *pres-type step* (*size τs*) *A*
assumes *τs*: ∀ *pc* < *size τs*. *τs*!*pc* ∈ *A* ∧ *τs*!*pc* ≠ ⊤
assumes *bounded*: *bounded step* (*size τs*)

assumes *B-neq-T*: ⊥ ≠ ⊤

lemma (**in** *lbvc*) *cert*: *cert-ok c* (*size τs*) ⊤ ⊥ *A*
lemmas [*simp del*] = *split-paired-Ex*

lemma (**in** *lbvc*) *cert-target* [*intro?*]:
⟦ (*pc'*,*s'*) ∈ *set* (*step pc* (*τs*!*pc*));
pc' ≠ *pc+1*; *pc* < *size τs*; *pc'* < *size τs* ⟧
⟹ *c*!*pc'* = *τs*!*pc'*
lemma (**in** *lbvc*) *cert-approx* [*intro?*]:
⟦ *pc* < *size τs*; *c*!*pc* ≠ ⊥ ⟧ ⟹ *c*!*pc* = *τs*!*pc*
lemma (**in** *lbv*) *le-top* [*simp*, *intro*]: *x* <=-*r* ⊤
lemma (**in** *lbv*) *merge-mono*:
assumes *less*: *set* ss_2 {⊑$_r$} *set* ss_1
assumes *x*: *x* ∈ *A*
assumes ss_1: *snd*‘*set* ss_1 ⊆ *A*
assumes ss_2: *snd*‘*set* ss_2 ⊆ *A*
shows *merge c pc* ss_2 *x* ⊑$_r$ *merge c pc* ss_1 *x* (**is** ?s_2 ⊑$_r$?s_1)
lemma (**in** *lbvc*) *wti-mono*:
assumes *less*: s_2 ⊑$_r$ s_1
assumes *pc*: *pc* < *size τs* **and** s_1: s_1 ∈ *A* **and** s_2: s_2 ∈ *A*
shows *wti c pc* s_2 ⊑$_r$ *wti c pc* s_1 (**is** ?s_2' ⊑$_r$?s_1')
lemma (**in** *lbvc*) *wtc-mono*:
assumes *less*: s_2 ⊑$_r$ s_1
assumes *pc*: *pc* < *size τs* **and** s_1: s_1 ∈ *A* **and** s_2: s_2 ∈ *A*
shows *wtc c pc* s_2 ⊑$_r$ *wtc c pc* s_1 (**is** ?s_2' ⊑$_r$?s_1')
lemma (**in** *lbv*) *top-le-conv* [*simp*]: ⊤ ⊑$_r$ *x* = (*x* = ⊤)
lemma (**in** *lbv*) *neq-top* [*simp*, *elim*]: ⟦ *x* ⊑$_r$ *y*; *y* ≠ ⊤ ⟧ ⟹ *x* ≠ ⊤

lemma (**in** *lbvc*) *stable-wti*:
  assumes *stable*: *stable r step* τs *pc* **and** *pc*: $pc < size\ \tau s$
  shows *wti c pc* $(\tau s!pc) \neq \top$
lemma (**in** *lbvc*) *wti-less*:
  assumes *stable*: *stable r step* τs *pc* **and** *suc-pc*: $Suc\ pc < size\ \tau s$
  shows *wti c pc* $(\tau s!pc) \sqsubseteq_r \tau s!Suc\ pc$ (**is** $?wti \sqsubseteq_r$ -)
lemma (**in** *lbvc*) *stable-wtc*:
  assumes *stable*: *stable r step* τs *pc* **and** *pc*: $pc < size\ \tau s$
  shows *wtc c pc* $(\tau s!pc) \neq \top$
lemma (**in** *lbvc*) *wtc-less*:
  assumes *stable*: *stable r step* τs *pc* **and** *suc-pc*: $Suc\ pc < size\ \tau s$
  shows *wtc c pc* $(\tau s!pc) \sqsubseteq_r \tau s!Suc\ pc$ (**is** $?wtc \sqsubseteq_r$ -)
lemma (**in** *lbvc*) *wt-step-wtl-lemma*:
  assumes *wt-step*: *wt-step r* $\top$ *step* τs
  shows $\bigwedge pc\ s.\ pc{+}size\ ls = size\ \tau s \Longrightarrow s \sqsubseteq_r \tau s!pc \Longrightarrow s \in A \Longrightarrow s{\neq}\top \Longrightarrow$
    $wtl\ ls\ c\ pc\ s \neq \top$
  (**is** $\bigwedge pc\ s.$ - $\Longrightarrow$ - $\Longrightarrow$ - $\Longrightarrow$ - $\Longrightarrow$ $?wtl\ ls\ pc\ s \neq$ -)
theorem (**in** *lbvc*) *wtl-complete*:
  assumes *wt*: *wt-step r* $\top$ *step* τs
  assumes *s*: $s \sqsubseteq_r \tau s!0$ $s \in A$ $s \neq \top$ **and** *eq*: $size\ ins = size\ \tau s$
  shows *wtl ins c 0 s* $\neq \top$
end

4.13 The Jinja Type System as a Semilattice

theory *SemiType*
imports *../Common/WellForm ../DFA/Semilattices*
begin

definition $super :: {}'a\ prog \Rightarrow cname \Rightarrow cname$
where $super\ P\ C \equiv fst\ (the\ (class\ P\ C))$

lemma *superI*:
  $(C,D) \in subcls1\ P \Longrightarrow super\ P\ C = D$
  by (*unfold super-def*) (*auto dest*: *subcls1D*)

primrec $the\text{-}Class :: ty \Rightarrow cname$
where
  $the\text{-}Class\ (Class\ C) = C$

definition $sup :: {}'c\ prog \Rightarrow ty \Rightarrow ty \Rightarrow ty\ err$
where
  $sup\ P\ T_1\ T_2 \equiv$
  $if\ is\text{-}refT\ T_1 \wedge is\text{-}refT\ T_2\ then$
  $OK\ (if\ T_1 = NT\ then\ T_2\ else$
    $if\ T_2 = NT\ then\ T_1\ else$
    $(Class\ (exec\text{-}lub\ (subcls1\ P)\ (super\ P)\ (the\text{-}Class\ T_1)\ (the\text{-}Class\ T_2))))$
  $else$
  $(if\ T_1 = T_2\ then\ OK\ T_1\ else\ Err)$

lemma *sup-def'*:
  $sup\ P = (\lambda T_1\ T_2.$
  $if\ is\text{-}refT\ T_1 \wedge is\text{-}refT\ T_2\ then$
  $OK\ (if\ T_1 = NT\ then\ T_2\ else$
    $if\ T_2 = NT\ then\ T_1\ else$
    $(Class\ (exec\text{-}lub\ (subcls1\ P)\ (super\ P)\ (the\text{-}Class\ T_1)\ (the\text{-}Class\ T_2))))$
  $else$
  $(if\ T_1 = T_2\ then\ OK\ T_1\ else\ Err))$
  by (*simp add*: *sup-def fun-eq-iff*)

abbreviation
  $subtype :: {}'c\ prog \Rightarrow ty \Rightarrow ty \Rightarrow bool$
  where $subtype\ P \equiv widen\ P$

definition $esl :: {}'c\ prog \Rightarrow ty\ esl$
where
  $esl\ P \equiv (types\ P,\ subtype\ P,\ sup\ P)$

lemma *is-class-is-subcls*:
  $wf\text{-}prog\ m\ P \Longrightarrow is\text{-}class\ P\ C = P \vdash C \preceq^* Object$

lemma *subcls-antisym*:
  $[\![wf\text{-}prog\ m\ P;\ P \vdash C \preceq^* D;\ P \vdash D \preceq^* C]\!] \Longrightarrow C = D$

lemma *widen-antisym*:
$[\![$ *wf-prog m P*; $P \vdash T \leq U$; $P \vdash U \leq T$ $]\!] \Longrightarrow T = U$
lemma *order-widen* [*intro*,*simp*]:
wf-prog m P $\Longrightarrow$ *order* (*subtype P*)

lemma *NT-widen*:
$P \vdash NT \leq T = (T = NT \vee (\exists C.\ T = \mathit{Class}\ C))$

lemma *Class-widen2*: $P \vdash \mathit{Class}\ C \leq T = (\exists D.\ T = \mathit{Class}\ D \wedge P \vdash C \preceq^* D)$
lemma *wf-converse-subcls1-impl-acc-subtype*:
wf ((*subcls1 P*)ˆ−*1*) $\Longrightarrow$ *acc* (*subtype P*)
lemma *wf-subtype-acc* [*intro*, *simp*]:
wf-prog wf-mb P $\Longrightarrow$ *acc* (*subtype P*)
lemma *exec-lub-refl* [*simp*]: *exec-lub r f T T* = *T*
lemma *closed-err-types*:
wf-prog wf-mb P $\Longrightarrow$ *closed* (*err* (*types P*)) (*lift2* (*sup P*))

lemma *sup-subtype-greater*:
$[\![$ *wf-prog wf-mb P*; *is-type P t1*; *is-type P t2*; *sup P t1 t2* = *OK s* $]\!]$
$\Longrightarrow$ *subtype P t1 s* $\wedge$ *subtype P t2 s*
lemma *sup-subtype-smallest*:
$[\![$ *wf-prog wf-mb P*; *is-type P a*; *is-type P b*; *is-type P c*;
subtype P a c; *subtype P b c*; *sup P a b* = *OK d* $]\!]$
$\Longrightarrow$ *subtype P d c*
lemma *sup-exists*:
$[\![$ *subtype P a c*; *subtype P b c* $]\!] \Longrightarrow$ *EX T*. *sup P a b* = *OK T*
lemma *err-semilat-JType-esl*:
wf-prog wf-mb P $\Longrightarrow$ *err-semilat* (*esl P*)

end

4.14 The JVM Type System as Semilattice

theory *JVM-SemiType* **imports** *SemiType* **begin**

type-synonym $ty_l = ty\ err\ list$
type-synonym $ty_s = ty\ list$
type-synonym $ty_i = ty_s \times ty_l$
type-synonym $ty_i{}' = ty_i\ option$
type-synonym $ty_m = ty_i{}'\ list$
type-synonym $ty_P = mname \Rightarrow cname \Rightarrow ty_m$

definition $stk\text{-}esl :: {}'c\ prog \Rightarrow nat \Rightarrow ty_s\ esl$
where
$stk\text{-}esl\ P\ mxs \equiv upto\text{-}esl\ mxs\ (SemiType.esl\ P)$

definition $loc\text{-}sl :: {}'c\ prog \Rightarrow nat \Rightarrow ty_l\ sl$
where
$loc\text{-}sl\ P\ mxl \equiv Listn.sl\ mxl\ (Err.sl\ (SemiType.esl\ P))$

definition $sl :: {}'c\ prog \Rightarrow nat \Rightarrow nat \Rightarrow ty_i{}'\ err\ sl$
where
$sl\ P\ mxs\ mxl \equiv$
$Err.sl(Opt.esl(Product.esl\ (stk\text{-}esl\ P\ mxs)\ (Err.esl(loc\text{-}sl\ P\ mxl))))$

definition $states :: {}'c\ prog \Rightarrow nat \Rightarrow nat \Rightarrow ty_i{}'\ err\ set$
where $states\ P\ mxs\ mxl \equiv fst(sl\ P\ mxs\ mxl)$

definition $le :: {}'c\ prog \Rightarrow nat \Rightarrow nat \Rightarrow ty_i{}'\ err\ ord$
where
$le\ P\ mxs\ mxl \equiv fst(snd(sl\ P\ mxs\ mxl))$

definition $sup :: {}'c\ prog \Rightarrow nat \Rightarrow nat \Rightarrow ty_i{}'\ err\ binop$
where
$sup\ P\ mxs\ mxl \equiv snd(snd(sl\ P\ mxs\ mxl))$

definition $sup\text{-}ty\text{-}opt :: [{}'c\ prog,ty\ err,ty\ err] \Rightarrow bool$
$(\text{- }|\text{– - <=}T\text{ - }[71,71,71]\ 70)$
where
$sup\text{-}ty\text{-}opt\ P \equiv Err.le\ (subtype\ P)$

definition $sup\text{-}state :: [{}'c\ prog,ty_i,ty_i] \Rightarrow bool$
$(\text{- }|\text{– - <=}i\text{ - }[71,71,71]\ 70)$
where
$sup\text{-}state\ P \equiv Product.le\ (Listn.le\ (subtype\ P))\ (Listn.le\ (sup\text{-}ty\text{-}opt\ P))$

definition $sup\text{-}state\text{-}opt :: [{}'c\ prog,ty_i{}',ty_i{}'] \Rightarrow bool$
$(\text{- }|\text{– - <='}\text{ - }[71,71,71]\ 70)$
where
$sup\text{-}state\text{-}opt\ P \equiv Opt.le\ (sup\text{-}state\ P)$

abbreviation

sup-loc :: [$'c$ *prog*,ty_l,ty_l] ⇒ *bool* (- |− - [<=T] - [71,71,71] 70)
where P |− LT [<=T] LT' ≡ *list-all2* (*sup-ty-opt* P) LT LT'

notation (*xsymbols*)
sup-ty-opt (- ⊢ - $\leq_\top$ - [71,71,71] 70) **and**
sup-state (- ⊢ - $\leq_i$ - [71,71,71] 70) **and**
sup-state-opt (- ⊢ - $\leq'$ - [71,71,71] 70) **and**
sup-loc (- ⊢ - [$\leq_\top$] - [71,71,71] 70)

4.14.1 Unfolding

lemma *JVM-states-unfold*:
states P mxs mxl ≡ *err*(*opt*((*Union* {*list n* (*types P*) |*n*. *n* <= *mxs*}) <*>
list mxl (*err*(*types P*))))
lemma *JVM-le-unfold*:
le P m n ≡
Err.le(*Opt.le*(*Product.le*(*Listn.le*(*subtype P*))(*Listn.le*(*Err.le*(*subtype P*)))))
lemma *sl-def2*:
JVM-SemiType.sl P mxs mxl ≡
(*states P mxs mxl*, *JVM-SemiType.le P mxs mxl*, *JVM-SemiType.sup P mxs mxl*)

lemma *JVM-le-conv*:
le P m n (*OK t1*) (*OK t2*) = $P \vdash t1 \leq' t2$
lemma *JVM-le-Err-conv*:
le P m n = *Err.le* (*sup-state-opt P*)
lemma *err-le-unfold* [*iff*]:
Err.le r (*OK a*) (*OK b*) = *r a b*

4.14.2 Semilattice

lemma *order-sup-state-opt* [*intro*, *simp*]:
wf-prog wf-mb P ⟹ *order* (*sup-state-opt P*)
lemma *semilat-JVM* [*intro?*]:
wf-prog wf-mb P ⟹ *semilat* (*JVM-SemiType.sl P mxs mxl*)
lemma *acc-JVM* [*intro*]:
wf-prog wf-mb P ⟹ *acc* (*JVM-SemiType.le P mxs mxl*)

4.14.3 Widening with ⊤

lemma *subtype-refl*[*iff*]: *subtype P t t*
lemma *sup-ty-opt-refl* [*iff*]: $P \vdash T \leq_\top T$
lemma *Err-any-conv* [*iff*]: $P \vdash Err \leq_\top T = (T = Err)$
lemma *any-Err* [*iff*]: $P \vdash T \leq_\top Err$
lemma *OK-OK-conv* [*iff*]:
$P \vdash OK\ T \leq_\top OK\ T' = P \vdash T \leq T'$
lemma *any-OK-conv* [*iff*]:
$P \vdash X \leq_\top OK\ T' = (\exists T.\ X = OK\ T \wedge P \vdash T \leq T')$
lemma *OK-any-conv*:
$P \vdash OK\ T \leq_\top X = (X = Err \vee (\exists T'.\ X = OK\ T' \wedge P \vdash T \leq T'))$
lemma *sup-ty-opt-trans* [*intro?*, *trans*]:
$[\![P \vdash a \leq_\top b;\ P \vdash b \leq_\top c]\!] \Longrightarrow P \vdash a \leq_\top c$

4.14.4 Stack and Registers

lemma *stk-convert*:

$P \vdash ST\ [\le]\ ST' = Listn.le\ (subtype\ P)\ ST\ ST'$
lemma *sup-loc-refl* [*iff*]: $P \vdash LT\ [\le_\top]\ LT$
lemmas *sup-loc-Cons1* [*iff*] = *list-all2-Cons1* [*of sup-ty-opt P*] **for** P

lemma *sup-loc-def*:
$P \vdash LT\ [\le_\top]\ LT' \equiv Listn.le\ (sup\text{-}ty\text{-}opt\ P)\ LT\ LT'$
lemma *sup-loc-widens-conv* [*iff*]:
$P \vdash map\ OK\ Ts\ [\le_\top]\ map\ OK\ Ts' = P \vdash Ts\ [\le]\ Ts'$

lemma *sup-loc-trans* [*intro?*, *trans*]:
$[\![P \vdash a\ [\le_\top]\ b;\ P \vdash b\ [\le_\top]\ c]\!] \Longrightarrow P \vdash a\ [\le_\top]\ c$

4.14.5 State Type

lemma *sup-state-conv* [*iff*]:
$P \vdash (ST,LT) \le_i (ST',LT') = (P \vdash ST\ [\le]\ ST' \wedge P \vdash LT\ [\le_\top]\ LT')$
lemma *sup-state-conv2*:
$P \vdash s1 \le_i s2 = (P \vdash fst\ s1\ [\le]\ fst\ s2 \wedge P \vdash snd\ s1\ [\le_\top]\ snd\ s2)$
lemma *sup-state-refl* [*iff*]: $P \vdash s \le_i s$
lemma *sup-state-trans* [*intro?*, *trans*]:
$[\![P \vdash a \le_i b;\ P \vdash b \le_i c]\!] \Longrightarrow P \vdash a \le_i c$

lemma *sup-state-opt-None-any* [*iff*]:
$P \vdash None \le' s$
lemma *sup-state-opt-any-None* [*iff*]:
$P \vdash s \le' None = (s = None)$
lemma *sup-state-opt-Some-Some* [*iff*]:
$P \vdash Some\ a \le' Some\ b = P \vdash a \le_i b$
lemma *sup-state-opt-any-Some*:
$P \vdash (Some\ s) \le' X = (\exists s'.\ X = Some\ s' \wedge P \vdash s \le_i s')$
lemma *sup-state-opt-refl* [*iff*]: $P \vdash s \le' s$
lemma *sup-state-opt-trans* [*intro?*, *trans*]:
$[\![P \vdash a \le' b;\ P \vdash b \le' c]\!] \Longrightarrow P \vdash a \le' c$
end

4.15 Effect of Instructions on the State Type

theory *Effect*
imports *JVM-SemiType ../JVM/JVMExceptions*
begin

— FIXME
locale *prog* =
  fixes *P* :: *'a prog*

locale *jvm-method* = *prog* +
  fixes *mxs* :: *nat*
  fixes mxl_0 :: *nat*
  fixes *Ts* :: *ty list*
  fixes T_r :: *ty*
  fixes *is* :: *instr list*
  fixes *xt* :: *ex-table*

  fixes *mxl* :: *nat*
  defines *mxl-def*: $mxl \equiv 1+size\ Ts+mxl_0$

Program counter of successor instructions:

primrec *succs* :: $instr \Rightarrow ty_i \Rightarrow pc \Rightarrow pc\ list$ **where**
  $succs\ (Load\ idx)\ \tau\ pc\ = [pc+1]$
| $succs\ (Store\ idx)\ \tau\ pc\ = [pc+1]$
| $succs\ (Push\ v)\ \tau\ pc\ = [pc+1]$
| $succs\ (Getfield\ F\ C)\ \tau\ pc = [pc+1]$
| $succs\ (Putfield\ F\ C)\ \tau\ pc = [pc+1]$
| $succs\ (New\ C)\ \tau\ pc\ = [pc+1]$
| $succs\ (Checkcast\ C)\ \tau\ pc\ = [pc+1]$
| $succs\ Pop\ \tau\ pc\ = [pc+1]$
| $succs\ IAdd\ \tau\ pc\ = [pc+1]$
| $succs\ CmpEq\ \tau\ pc\ = [pc+1]$
| *succs-IfFalse*:
    $succs\ (IfFalse\ b)\ \tau\ pc\ = [pc+1,\ nat\ (int\ pc\ +\ b)]$
| *succs-Goto*:
    $succs\ (Goto\ b)\ \tau\ pc\ = [nat\ (int\ pc\ +\ b)]$
| *succs-Return*:
    $succs\ Return\ \tau\ pc\ = []$
| *succs-Invoke*:
    $succs\ (Invoke\ M\ n)\ \tau\ pc\ = (if\ (fst\ \tau)!n = NT\ then\ []\ else\ [pc+1])$
| *succs-Throw*:
    $succs\ Throw\ \tau\ pc\ = []$

Effect of instruction on the state type:

fun *the-class*:: $ty \Rightarrow cname$ **where**
  $the\text{-}class\ (Class\ C) = C$

fun eff_i :: $instr \times 'm\ prog \times ty_i \Rightarrow ty_i$ **where**
  eff_i*-Load*:
    $\mathit{eff}_i\ (Load\ n,\ \ P,\ (ST,\ LT))\ = (ok\text{-}val\ (LT\ !\ n)\ \#\ ST,\ LT)$
| eff_i*-Store*:
    $\mathit{eff}_i\ (Store\ n,\ P,\ (T\#ST,\ LT))\ = (ST,\ LT[n{:=}\ OK\ T])$
| eff_i*-Push*:

eff_i $(Push\ v,\ P,\ (ST,\ LT)) = (the\ (typeof\ v)\ \#\ ST,\ LT)$
| eff_i-$Getfield$:
eff_i $(Getfield\ F\ C,\ P,\ (T\#ST,\ LT)) = (snd\ (field\ P\ C\ F)\ \#\ ST,\ LT)$
| eff_i-$Putfield$:
eff_i $(Putfield\ F\ C,\ P,\ (T_1\#T_2\#ST,\ LT)) = (ST,LT)$
| eff_i-New:
eff_i $(New\ C,\ P,\ (ST,LT)) = (Class\ C\ \#\ ST,\ LT)$
| eff_i-$Checkcast$:
eff_i $(Checkcast\ C,\ P,\ (T\#ST,LT)) = (Class\ C\ \#\ ST,LT)$
| eff_i-Pop:
eff_i $(Pop,\ P,\ (T\#ST,LT)) = (ST,LT)$
| eff_i-$IAdd$:
eff_i $(IAdd,\ P,(T_1\#T_2\#ST,LT)) = (Integer\#ST,LT)$
| eff_i-$CmpEq$:
eff_i $(CmpEq,\ P,\ (T_1\#T_2\#ST,LT)) = (Boolean\#ST,LT)$
| eff_i-$IfFalse$:
eff_i $(IfFalse\ b,\ P,\ (T_1\#ST,LT)) = (ST,LT)$
| eff_i-$Invoke$:
eff_i $(Invoke\ M\ n,\ P,\ (ST,LT)) =$
$(let\ C = the\text{-}class\ (ST!n);\ (D,Ts,T_r,b) = method\ P\ C\ M$
$in\ (T_r\ \#\ drop\ (n+1)\ ST,\ LT))$
| eff_i-$Goto$:
eff_i $(Goto\ n,\ P,\ s) = s$

fun $is\text{-}relevant\text{-}class :: instr \Rightarrow {}'m\ prog \Rightarrow cname \Rightarrow bool$ **where**
$rel\text{-}Getfield$:
$is\text{-}relevant\text{-}class\ (Getfield\ F\ D) = (\lambda P\ C.\ P \vdash NullPointer \preceq^* C)$
| $rel\text{-}Putfield$:
$is\text{-}relevant\text{-}class\ (Putfield\ F\ D) = (\lambda P\ C.\ P \vdash NullPointer \preceq^* C)$
| $rel\text{-}Checcast$:
$is\text{-}relevant\text{-}class\ (Checkcast\ D) = (\lambda P\ C.\ P \vdash ClassCast \preceq^* C)$
| $rel\text{-}New$:
$is\text{-}relevant\text{-}class\ (New\ D) = (\lambda P\ C.\ P \vdash OutOfMemory \preceq^* C)$
| $rel\text{-}Throw$:
$is\text{-}relevant\text{-}class\ Throw = (\lambda P\ C.\ True)$
| $rel\text{-}Invoke$:
$is\text{-}relevant\text{-}class\ (Invoke\ M\ n) = (\lambda P\ C.\ True)$
| $rel\text{-}default$:
$is\text{-}relevant\text{-}class\ i = (\lambda P\ C.\ False)$

definition $is\text{-}relevant\text{-}entry :: {}'m\ prog \Rightarrow instr \Rightarrow pc \Rightarrow ex\text{-}entry \Rightarrow bool$ **where**
$is\text{-}relevant\text{-}entry\ P\ i\ pc\ e \longleftrightarrow (let\ (f,t,C,h,d) = e\ in\ is\text{-}relevant\text{-}class\ i\ P\ C \wedge pc \in \{f..<t\})$

definition $relevant\text{-}entries :: {}'m\ prog \Rightarrow instr \Rightarrow pc \Rightarrow ex\text{-}table \Rightarrow ex\text{-}table$ **where**
$relevant\text{-}entries\ P\ i\ pc = filter\ (is\text{-}relevant\text{-}entry\ P\ i\ pc)$

definition $xcpt\text{-}eff :: instr \Rightarrow {}'m\ prog \Rightarrow pc \Rightarrow ty_i$
$\Rightarrow ex\text{-}table \Rightarrow (pc \times ty_i{}')\ list$ **where**
$xcpt\text{-}eff\ i\ P\ pc\ \tau\ et = (let\ (ST,LT) = \tau\ in$
$map\ (\lambda(f,t,C,h,d).\ (h,\ Some\ (Class\ C\#drop\ (size\ ST - d)\ ST,\ LT)))\ (relevant\text{-}entries\ P\ i\ pc\ et))$

definition $norm\text{-}eff :: instr \Rightarrow {}'m\ prog \Rightarrow nat \Rightarrow ty_i \Rightarrow (pc \times ty_i{}')\ list$ **where**
$norm\text{-}eff\ i\ P\ pc\ \tau = map\ (\lambda pc'.\ (pc',Some\ (eff_i\ (i,P,\tau))))\ (succs\ i\ \tau\ pc)$

definition $\mathit{eff} :: \mathit{instr} \Rightarrow {'m}\ \mathit{prog} \Rightarrow \mathit{pc} \Rightarrow \mathit{ex\text{-}table} \Rightarrow \mathit{ty_i}' \Rightarrow (\mathit{pc} \times \mathit{ty_i}')\ \mathit{list}$ **where**
$\mathit{eff}\ i\ P\ pc\ et\ t = (\mathit{case}\ t\ \mathit{of}$
$\mathit{None} \Rightarrow []$
$\mid \mathit{Some}\ \tau \Rightarrow (\mathit{norm\text{-}eff}\ i\ P\ pc\ \tau)\ @\ (\mathit{xcpt\text{-}eff}\ i\ P\ pc\ \tau\ et))$

lemma *eff-None*:
$\mathit{eff}\ i\ P\ pc\ xt\ \mathit{None} = []$
by (*simp add*: *eff-def*)

lemma *eff-Some*:
$\mathit{eff}\ i\ P\ pc\ xt\ (\mathit{Some}\ \tau) = \mathit{norm\text{-}eff}\ i\ P\ pc\ \tau\ @\ \mathit{xcpt\text{-}eff}\ i\ P\ pc\ \tau\ xt$
by (*simp add*: *eff-def*)

Conditions under which eff is applicable:

fun $\mathit{app_i} :: \mathit{instr} \times {'m}\ \mathit{prog} \times \mathit{pc} \times \mathit{nat} \times \mathit{ty} \times \mathit{ty_i} \Rightarrow \mathit{bool}$ **where**
$\mathit{app_i\text{-}Load}$:
$\mathit{app_i}\ (\mathit{Load}\ n,\ P,\ pc,\ mxs,\ T_r,\ (ST,LT)) =$
$(n < \mathit{length}\ LT \wedge LT\ !\ n \neq \mathit{Err} \wedge \mathit{length}\ ST < mxs)$
$\mid \mathit{app_i\text{-}Store}$:
$\mathit{app_i}\ (\mathit{Store}\ n,\ P,\ pc,\ mxs,\ T_r,\ (T\#ST,\ LT)) =$
$(n < \mathit{length}\ LT)$
$\mid \mathit{app_i\text{-}Push}$:
$\mathit{app_i}\ (\mathit{Push}\ v,\ P,\ pc,\ mxs,\ T_r,\ (ST,LT)) =$
$(\mathit{length}\ ST < mxs \wedge \mathit{typeof}\ v \neq \mathit{None})$
$\mid \mathit{app_i\text{-}Getfield}$:
$\mathit{app_i}\ (\mathit{Getfield}\ F\ C,\ P,\ pc,\ mxs,\ T_r,\ (T\#ST,\ LT)) =$
$(\exists\, T_f.\ P \vdash C\ \mathit{sees}\ F{:}T_f\ \mathit{in}\ C \wedge P \vdash T \leq \mathit{Class}\ C)$
$\mid \mathit{app_i\text{-}Putfield}$:
$\mathit{app_i}\ (\mathit{Putfield}\ F\ C,\ P,\ pc,\ mxs,\ T_r,\ (T_1\#T_2\#ST,\ LT)) =$
$(\exists\, T_f.\ P \vdash C\ \mathit{sees}\ F{:}T_f\ \mathit{in}\ C \wedge P \vdash T_2 \leq (\mathit{Class}\ C) \wedge P \vdash T_1 \leq T_f)$
$\mid \mathit{app_i\text{-}New}$:
$\mathit{app_i}\ (\mathit{New}\ C,\ P,\ pc,\ mxs,\ T_r,\ (ST,LT)) =$
$(\mathit{is\text{-}class}\ P\ C \wedge \mathit{length}\ ST < mxs)$
$\mid \mathit{app_i\text{-}Checkcast}$:
$\mathit{app_i}\ (\mathit{Checkcast}\ C,\ P,\ pc,\ mxs,\ T_r,\ (T\#ST,LT)) =$
$(\mathit{is\text{-}class}\ P\ C \wedge \mathit{is\text{-}refT}\ T)$
$\mid \mathit{app_i\text{-}Pop}$:
$\mathit{app_i}\ (\mathit{Pop},\ P,\ pc,\ mxs,\ T_r,\ (T\#ST,LT)) =$
True
$\mid \mathit{app_i\text{-}IAdd}$:
$\mathit{app_i}\ (\mathit{IAdd},\ P,\ pc,\ mxs,\ T_r,\ (T_1\#T_2\#ST,LT)) = (T_1 = T_2 \wedge T_1 = \mathit{Integer})$
$\mid \mathit{app_i\text{-}CmpEq}$:
$\mathit{app_i}\ (\mathit{CmpEq},\ P,\ pc,\ mxs,\ T_r,\ (T_1\#T_2\#ST,LT)) =$
$(T_1 = T_2 \vee \mathit{is\text{-}refT}\ T_1 \wedge \mathit{is\text{-}refT}\ T_2)$
$\mid \mathit{app_i\text{-}IfFalse}$:
$\mathit{app_i}\ (\mathit{IfFalse}\ b,\ P,\ pc,\ mxs,\ T_r,\ (\mathit{Boolean}\#ST,LT)) =$
$(0 \leq \mathit{int}\ pc + b)$
$\mid \mathit{app_i\text{-}Goto}$:
$\mathit{app_i}\ (\mathit{Goto}\ b,\ P,\ pc,\ mxs,\ T_r,\ s) =$
$(0 \leq \mathit{int}\ pc + b)$
$\mid \mathit{app_i\text{-}Return}$:
$\mathit{app_i}\ (\mathit{Return},\ P,\ pc,\ mxs,\ T_r,\ (T\#ST,LT)) =$
$(P \vdash T \leq T_r)$

| app_i-*Throw*:
  app_i (*Throw*, P, pc, mxs, T_r, ($T\#ST$,LT)) =
  is-refT T
| app_i-*Invoke*:
  app_i (*Invoke* M n, P, pc, mxs, T_r, (ST,LT)) =
  (n < *length* ST ∧
  ($ST!n \neq NT \longrightarrow$
   (∃ C D Ts T m. $ST!n$ = *Class* C ∧ $P \vdash C$ *sees* M:$Ts \rightarrow T = m$ *in* D ∧
        $P \vdash$ *rev* (*take* n ST) [≤] Ts)))

| app_i-*default*:
  app_i (i,P, pc,mxs,T_r,s) = *False*

definition *xcpt-app* :: *instr* ⇒ ′*m prog* ⇒ *pc* ⇒ *nat* ⇒ *ex-table* ⇒ ty_i ⇒ *bool* **where**
 xcpt-app i P pc mxs xt τ ⟷ (∀(f,t,C,h,d) ∈ *set* (*relevant-entries P i pc xt*). *is-class P C* ∧ $d \leq$ *size* (*fst* τ) ∧ $d < mxs$)

definition *app* :: *instr* ⇒ ′*m prog* ⇒ *nat* ⇒ *ty* ⇒ *nat* ⇒ *nat* ⇒ *ex-table* ⇒ $ty_i{}'$ ⇒ *bool* **where**
 app i P mxs T_r *pc mpc xt t* = (*case t of None* ⇒ *True* | *Some* τ ⇒
 app_i (i,P,pc,mxs,T_r,τ) ∧ *xcpt-app i P pc mxs xt* τ ∧
 (∀(pc',τ') ∈ *set* (*eff i P pc xt t*). $pc' < mpc$))

lemma *app-Some*:
 app i P mxs T_r *pc mpc xt* (*Some* τ) =
 (app_i (i,P,pc,mxs,T_r,τ) ∧ *xcpt-app i P pc mxs xt* τ ∧
 (∀(pc',s') ∈ *set* (*eff i P pc xt* (*Some* τ)). $pc' < mpc$))
by (*simp add*: *app-def*)

locale *eff* = *jvm-method* +
 fixes eff_i **and** app_i **and** *eff* **and** *app*
 fixes *norm-eff* **and** *xcpt-app* **and** *xcpt-eff*

 fixes *mpc*
 defines *mpc* ≡ *size is*

 defines eff_i i τ ≡ *Effect*.eff_i (i,P,τ)
 notes eff_i-*simps* [*simp*] = *Effect*.eff_i.*simps* [**where** P = P, *folded* eff_i-*def*]

 defines app_i i pc τ ≡ *Effect*.app_i (i, P, pc, mxs, T_r, τ)
 notes app_i-*simps* [*simp*] = *Effect*.app_i.*simps* [**where** P=P **and** mxs=mxs **and** T_r=T_r, *folded* app_i-*def*]

 defines *xcpt-eff i pc* τ ≡ *Effect.xcpt-eff i P pc* τ *xt*
 notes *xcpt-eff* = *Effect.xcpt-eff-def* [*of - P - - xt*, *folded xcpt-eff-def*]

 defines *norm-eff i pc* τ ≡ *Effect.norm-eff i P pc* τ
 notes *norm-eff* = *Effect.norm-eff-def* [*of - P*, *folded norm-eff-def* eff_i-*def*]

 defines *eff i pc* ≡ *Effect.eff i P pc xt*
 notes *eff* = *Effect.eff-def* [*of - P - xt*, *folded eff-def norm-eff-def xcpt-eff-def*]

 defines *xcpt-app i pc* τ ≡ *Effect.xcpt-app i P pc mxs xt* τ

notes *xcpt-app* = *Effect.xcpt-app-def* [*of* - *P* - *mxs xt*, *folded xcpt-app-def*]

defines *app i pc* ≡ *Effect.app i P mxs* T_r *pc mpc xt*
notes *app* = *Effect.app-def* [*of* - *P mxs* T_r - *mpc xt*, *folded app-def xcpt-app-def* app_i*-def eff-def*]

lemma *length-cases2*:
assumes ⋀*LT*. *P* ([],*LT*)
assumes ⋀*l ST LT*. *P* (*l*#*ST*,*LT*)
shows *P s*
by (*cases s*, *cases fst s*) (*auto intro*!: *assms*)

lemma *length-cases3*:
assumes ⋀*LT*. *P* ([],*LT*)
assumes ⋀*l LT*. *P* ([*l*],*LT*)
assumes ⋀*l ST LT*. *P* (*l*#*ST*,*LT*)
shows *P s*
lemma *length-cases4*:
assumes ⋀*LT*. *P* ([],*LT*)
assumes ⋀*l LT*. *P* ([*l*],*LT*)
assumes ⋀*l l′ LT*. *P* ([*l*,*l′*],*LT*)
assumes ⋀*l l′ ST LT*. *P* (*l*#*l′*#*ST*,*LT*)
shows *P s*

simp rules for *app*

lemma *appNone*[*simp*]: *app i P mxs* T_r *pc mpc et None* = *True*
by (*simp add*: *app-def*)

lemma *appLoad*[*simp*]:
app_i (*Load idx*, *P*, T_r, *mxs*, *pc*, *s*) = (∃ *ST LT*. *s* = (*ST*,*LT*) ∧ *idx* < *length LT* ∧ *LT*!*idx* ≠ *Err* ∧ *length ST* < *mxs*)
by (*cases s*, *simp*)

lemma *appStore*[*simp*]:
app_i (*Store idx*,*P*,*pc*,*mxs*,T_r,*s*) = (∃ *ts ST LT*. *s* = (*ts*#*ST*,*LT*) ∧ *idx* < *length LT*)
by (*rule length-cases2*, *auto*)

lemma *appPush*[*simp*]:
app_i (*Push v*,*P*,*pc*,*mxs*,T_r,*s*) =
(∃ *ST LT*. *s* = (*ST*,*LT*) ∧ *length ST* < *mxs* ∧ *typeof v* ≠ *None*)
by (*cases s*, *simp*)

lemma *appGetField*[*simp*]:
app_i (*Getfield F C*,*P*,*pc*,*mxs*,T_r,*s*) =
(∃ *oT vT ST LT*. *s* = (*oT*#*ST*, *LT*) ∧
P ⊢ *C sees F*:*vT in C* ∧ *P* ⊢ *oT* ≤ (*Class C*))
by (*rule length-cases2* [*of* - *s*]) *auto*

lemma *appPutField*[*simp*]:
app_i (*Putfield F C*,*P*,*pc*,*mxs*,T_r,*s*) =
(∃ *vT vT′ oT ST LT*. *s* = (*vT*#*oT*#*ST*, *LT*) ∧
P ⊢ *C sees F*:*vT′ in C* ∧ *P* ⊢ *oT* ≤ (*Class C*) ∧ *P* ⊢ *vT* ≤ *vT′*)

by (*rule length-cases4* [*of* - *s*], *auto*)

lemma *appNew*[*simp*]:
app$_i$ (*New C*,*P*,*pc*,*mxs*,T_r,*s*) =
(∃ *ST LT*. *s*=(*ST*,*LT*) ∧ *is-class P C* ∧ *length ST* < *mxs*)
by (*cases s*, *simp*)

lemma *appCheckcast*[*simp*]:
app$_i$ (*Checkcast C*,*P*,*pc*,*mxs*,T_r,*s*) =
(∃ *T ST LT*. *s* = (*T*#*ST*,*LT*) ∧ *is-class P C* ∧ *is-refT T*)
by (*cases s*, *cases fst s*, *simp add*: *app-def*) (*cases hd* (*fst s*), *auto*)

lemma *app*$_i$*Pop*[*simp*]:
app$_i$ (*Pop*,*P*,*pc*,*mxs*,T_r,*s*) = (∃ *ts ST LT*. *s* = (*ts*#*ST*,*LT*))
by (*rule length-cases2*, *auto*)

lemma *appIAdd*[*simp*]:
app$_i$ (*IAdd*,*P*,*pc*,*mxs*,T_r,*s*) = (∃ *ST LT*. *s* = (*Integer*#*Integer*#*ST*,*LT*))

lemma *appIfFalse* [*simp*]:
app$_i$ (*IfFalse b*,*P*,*pc*,*mxs*,T_r,*s*) =
(∃ *ST LT*. *s* = (*Boolean*#*ST*,*LT*) ∧ *0* ≤ *int pc* + *b*)
lemma *appCmpEq*[*simp*]:
app$_i$ (*CmpEq*,*P*,*pc*,*mxs*,T_r,*s*) =
(∃ T_1 T_2 *ST LT*. *s* = (T_1#T_2#*ST*,*LT*) ∧ (¬*is-refT* T_1 ∧ T_2 = T_1 ∨ *is-refT* T_1 ∧ *is-refT* T_2))
by (*rule length-cases4*, *auto*)

lemma *appReturn*[*simp*]:
app$_i$ (*Return*,*P*,*pc*,*mxs*,T_r,*s*) = (∃ *T ST LT*. *s* = (*T*#*ST*,*LT*) ∧ *P* ⊢ *T* ≤ T_r)
by (*rule length-cases2*, *auto*)

lemma *appThrow*[*simp*]:
app$_i$ (*Throw*,*P*,*pc*,*mxs*,T_r,*s*) = (∃ *T ST LT*. *s*=(*T*#*ST*,*LT*) ∧ *is-refT T*)
by (*rule length-cases2*, *auto*)

lemma *effNone*:
(*pc*′, *s*′) ∈ *set* (*eff i P pc et None*) ⟹ *s*′ = *None*
by (*auto simp add*: *eff-def xcpt-eff-def norm-eff-def*)

some helpers to make the specification directly executable:

lemma *relevant-entries-append* [*simp*]:
relevant-entries P i pc (*xt* @ *xt*′) = *relevant-entries P i pc xt* @ *relevant-entries P i pc xt*′
by (*unfold relevant-entries-def*) *simp*

lemma *xcpt-app-append* [*iff*]:
xcpt-app i P pc mxs (*xt*@*xt*′) τ = (*xcpt-app i P pc mxs xt* τ ∧ *xcpt-app i P pc mxs xt*′ τ)
by (*unfold xcpt-app-def*) *fastforce*

lemma *xcpt-eff-append* [*simp*]:
xcpt-eff i P pc τ (*xt*@*xt*′) = *xcpt-eff i P pc* τ *xt* @ *xcpt-eff i P pc* τ *xt*′
by (*unfold xcpt-eff-def*, *cases* τ) *simp*

lemma *app-append* [*simp*]:
app i P pc T mxs mpc (*xt*@*xt*′) τ = (*app i P pc T mxs mpc xt* τ ∧ *app i P pc T mxs mpc xt*′ τ)

by (*unfold app-def eff-def*) *auto*

end

4.16 Monotonicity of eff and app

theory *EffectMono* **imports** *Effect* **begin**

declare *not-Err-eq* [*iff*]

lemma app_i*-mono*:
  assumes *wf*: *wf-prog p P*
  assumes *less*: $P \vdash \tau \leq_i \tau'$
  shows $app_i\ (i,P,mxs,mpc,rT,\tau') \Longrightarrow app_i\ (i,P,mxs,mpc,rT,\tau)$
lemma *succs-mono*:
  assumes *wf*: *wf-prog p P* **and** app_i: $app_i\ (i,P,mxs,mpc,rT,\tau')$
  shows $P \vdash \tau \leq_i \tau' \Longrightarrow set\ (succs\ i\ \tau\ pc) \subseteq set\ (succs\ i\ \tau'\ pc)$

lemma *app-mono*:
  assumes *wf*: *wf-prog p P*
  assumes *less'*: $P \vdash \tau \leq' \tau'$
  shows $app\ i\ P\ m\ rT\ pc\ mpc\ xt\ \tau' \Longrightarrow app\ i\ P\ m\ rT\ pc\ mpc\ xt\ \tau$

lemma eff_i*-mono*:
  assumes *wf*: *wf-prog p P*
  assumes *less*: $P \vdash \tau \leq_i \tau'$
  assumes app_i: $app\ i\ P\ m\ rT\ pc\ mpc\ xt\ (Some\ \tau')$
  assumes *succs*: $succs\ i\ \tau\ pc \neq []\quad succs\ i\ \tau'\ pc \neq []$
  shows $P \vdash eff_i\ (i,P,\tau) \leq_i eff_i\ (i,P,\tau')$
end

4.17 The Bytecode Verifier

theory *BVSpec*
imports *Effect*
begin

This theory contains a specification of the BV. The specification describes correct typings of method bodies; it corresponds to type *checking*.

definition
— The method type only contains declared classes:
$check\text{-}types :: 'm\ prog \Rightarrow nat \Rightarrow nat \Rightarrow ty_i'\ err\ list \Rightarrow bool$
where
$check\text{-}types\ P\ mxs\ mxl\ \tau s \equiv set\ \tau s \subseteq states\ P\ mxs\ mxl$

— An instruction is welltyped if it is applicable and its effect
— is compatible with the type at all successor instructions:
definition
$wt\text{-}instr :: ['m\ prog,ty,nat,pc,ex\text{-}table,instr,pc,ty_m] \Rightarrow bool$
$(\text{-},\text{-},\text{-},\text{-},\text{-} \vdash \text{-},\text{-} :: \text{-}\ [60,0,0,0,0,0,0,61]\ 60)$
where
$P,T,mxs,mpc,xt \vdash i,pc :: \tau s \equiv$
$app\ i\ P\ mxs\ T\ pc\ mpc\ xt\ (\tau s!pc) \wedge$
$(\forall (pc',\tau') \in set\ (eff\ i\ P\ pc\ xt\ (\tau s!pc)).\ P \vdash \tau' \leq' \tau s!pc')$

— The type at $pc=0$ conforms to the method calling convention:
definition $wt\text{-}start :: ['m\ prog,cname,ty\ list,nat,ty_m] \Rightarrow bool$
where
$wt\text{-}start\ P\ C\ Ts\ mxl_0\ \tau s \equiv$
$P \vdash Some\ ([],OK\ (Class\ C)\#map\ OK\ Ts@replicate\ mxl_0\ Err) \leq' \tau s!0$

— A method is welltyped if the body is not empty,
— if the method type covers all instructions and mentions
— declared classes only, if the method calling convention is respected, and
— if all instructions are welltyped.
definition $wt\text{-}method :: ['m\ prog,cname,ty\ list,ty,nat,nat,instr\ list,$
$ex\text{-}table,ty_m] \Rightarrow bool$
where
$wt\text{-}method\ P\ C\ Ts\ T_r\ mxs\ mxl_0\ is\ xt\ \tau s \equiv$
$0 < size\ is \wedge size\ \tau s = size\ is \wedge$
$check\text{-}types\ P\ mxs\ (1+size\ Ts+mxl_0)\ (map\ OK\ \tau s) \wedge$
$wt\text{-}start\ P\ C\ Ts\ mxl_0\ \tau s \wedge$
$(\forall pc < size\ is.\ P,T_r,mxs,size\ is,xt \vdash is!pc,pc :: \tau s)$

— A program is welltyped if it is wellformed and all methods are welltyped
definition $wf\text{-}jvm\text{-}prog\text{-}phi :: ty_P \Rightarrow jvm\text{-}prog \Rightarrow bool\ (wf'\text{-}jvm'\text{-}prog_{\text{-}})$
where
$wf\text{-}jvm\text{-}prog_\Phi \equiv$
$wf\text{-}prog\ (\lambda P\ C\ (M,Ts,T_r,(mxs,mxl_0,is,xt)).$
$wt\text{-}method\ P\ C\ Ts\ T_r\ mxs\ mxl_0\ is\ xt\ (\Phi\ C\ M))$

definition $wf\text{-}jvm\text{-}prog :: jvm\text{-}prog \Rightarrow bool$
where
$wf\text{-}jvm\text{-}prog\ P \equiv \exists \Phi.\ wf\text{-}jvm\text{-}prog_\Phi\ P$

notation (*input*)
  wf-jvm-prog-phi (*wf′-jvm′-prog*$_{\_}$ - [*0*,*999*] *1000*)

lemma *wt-jvm-progD*:
  wf-jvm-prog$_\Phi$ $P \Longrightarrow \exists\, wt.$ *wf-prog wt P*
lemma *wt-jvm-prog-impl-wt-instr*:
  $\llbracket$ *wf-jvm-prog*$_\Phi$ P;
    $P \vdash C$ *sees* M:$Ts \to T$ = (mxs,mxl_0,ins,xt) *in* C; $pc <$ *size ins* $\rrbracket$
  $\Longrightarrow P,T,mxs,$*size ins*$,xt \vdash ins!pc,pc :: \Phi\ C\ M$
lemma *wt-jvm-prog-impl-wt-start*:
  $\llbracket$ *wf-jvm-prog*$_\Phi$ P;
    $P \vdash C$ *sees* M:$Ts \to T$ = (mxs,mxl_0,ins,xt) *in* C $\rrbracket \Longrightarrow$
  0 $<$ *size ins* $\wedge$ *wt-start P C Ts* mxl_0 (Φ C M)
end

4.18 The Typing Framework for the JVM

theory *TF-JVM*
imports *../DFA/Typing-Framework-err EffectMono BVSpec*
begin

definition *exec* :: *jvm-prog* ⇒ *nat* ⇒ *ty* ⇒ *ex-table* ⇒ *instr list* ⇒ ty_i' *err step-type*
where
exec G maxs rT et bs ≡
err-step (*size bs*) (λ*pc*. *app* (*bs*!*pc*) *G maxs rT pc* (*size bs*) *et*)
(λ*pc*. *eff* (*bs*!*pc*) *G pc et*)

locale *JVM-sl* =
fixes *P* :: *jvm-prog* **and** *mxs* **and** mxl_0
fixes *Ts* :: *ty list* **and** *is* **and** *xt* **and** T_r

fixes *mxl* **and** *A* **and** *r* **and** *f* **and** *app* **and** *eff* **and** *step*
defines [*simp*]: *mxl* ≡ *1+size Ts+*mxl_0
defines [*simp*]: *A* ≡ *states P mxs mxl*
defines [*simp*]: *r* ≡ *JVM-SemiType.le P mxs mxl*
defines [*simp*]: *f* ≡ *JVM-SemiType.sup P mxs mxl*

defines [*simp*]: *app* ≡ λ*pc*. *Effect.app* (*is*!*pc*) *P mxs* T_r *pc* (*size is*) *xt*
defines [*simp*]: *eff* ≡ λ*pc*. *Effect.eff* (*is*!*pc*) *P pc xt*
defines [*simp*]: *step* ≡ *err-step* (*size is*) *app eff*

locale *start-context* = *JVM-sl* +
fixes *p* **and** *C*
assumes *wf*: *wf-prog p P*
assumes *C*: *is-class P C*
assumes *Ts*: *set Ts* ⊆ *types P*

fixes *first* :: ty_i' **and** *start*
defines [*simp*]:
first ≡ *Some* ([],*OK* (*Class C*) # *map OK Ts* @ *replicate* mxl_0 *Err*)
defines [*simp*]:
start ≡ *OK first* # *replicate* (*size is* − *1*) (*OK None*)

4.18.1 Connecting JVM and Framework

lemma (**in** *JVM-sl*) *step-def-exec*: *step* ≡ *exec P mxs* T_r *xt is*
by (*simp add*: *exec-def*)

lemma *special-ex-swap-lemma* [*iff*]:
(*? X*. (*? n*. *X* = *A n* & *P n*) & *Q X*) = (*? n*. *Q*(*A n*) & *P n*)
by *blast*

lemma *ex-in-list* [*iff*]:
(∃ *n*. *ST* ∈ *list n A* ∧ *n* ≤ *mxs*) = (*set ST* ⊆ *A* ∧ *size ST* ≤ *mxs*)
by (*unfold list-def*) *auto*

lemma *singleton-list*:
(∃ *n*. [*Class C*] ∈ *list n* (*types P*) ∧ *n* ≤ *mxs*) = (*is-class P C* ∧ *0* < *mxs*)

by *auto*

lemma *set-drop-subset*:
set xs $\subseteq$ *A* $\Longrightarrow$ *set* (*drop n xs*) $\subseteq$ *A*
by (*auto dest*: *in-set-dropD*)

lemma *Suc-minus-minus-le*:
$n < mxs \Longrightarrow Suc\ (n - (n - b)) \leq mxs$
by *arith*

lemma *in-listE*:
$\llbracket$ *xs* $\in$ *list n A*; $\llbracket$*size xs* = *n*; *set xs* $\subseteq$ *A*$\rrbracket$ $\Longrightarrow$ *P* $\rrbracket$ $\Longrightarrow$ *P*
by (*unfold list-def*) *blast*

declare *is-relevant-entry-def* [*simp*]
declare *set-drop-subset* [*simp*]

theorem (**in** *start-context*) *exec-pres-type*:
pres-type step (*size is*) *A*
declare *is-relevant-entry-def* [*simp del*]
declare *set-drop-subset* [*simp del*]

lemma *lesubstep-type-simple*:
xs $[\sqsubseteq_{Product.le\ (op\ =)\ r}]$ *ys* $\Longrightarrow$ *set xs* $\{\sqsubseteq_r\}$ *set ys*
declare *is-relevant-entry-def* [*simp del*]

lemma *conjI2*: $\llbracket$ *A*; *A* $\Longrightarrow$ *B* $\rrbracket$ $\Longrightarrow$ *A* $\wedge$ *B* **by** *blast*

lemma (**in** *JVM-sl*) *eff-mono*:
$\llbracket$*wf-prog p P*; *pc* < *length is*; *s* $\sqsubseteq_{sup\text{-}state\text{-}opt\ P}$ *t*; *app pc t*$\rrbracket$
$\Longrightarrow$ *set* (*eff pc s*) $\{\sqsubseteq_{sup\text{-}state\text{-}opt\ P}\}$ *set* (*eff pc t*)
lemma (**in** *JVM-sl*) *bounded-step*: *bounded step* (*size is*)
theorem (**in** *JVM-sl*) *step-mono*:
wf-prog wf-mb P $\Longrightarrow$ *mono r step* (*size is*) *A*

lemma (**in** *start-context*) *first-in-A* [*iff*]: *OK first* $\in$ *A*
using *Ts C* **by** (*force intro*!: *list-appendI simp add*: *JVM-states-unfold*)

lemma (**in** *JVM-sl*) *wt-method-def2*:
wt-method P C' Ts T_r *mxs* mxl_0 *is xt* τs =
(*is* $\neq$ [] $\wedge$
size τs = *size is* $\wedge$
OK ‘ *set* τs $\subseteq$ *states P mxs mxl* $\wedge$
wt-start P C' Ts mxl_0 τs $\wedge$
wt-app-eff (*sup-state-opt P*) *app eff* τs)

end

4.19 Kildall for the JVM

theory *BVExec*
imports *../DFA/Abstract-BV TF-JVM*
begin

definition *kiljvm* :: *jvm-prog* ⇒ *nat* ⇒ *nat* ⇒ *ty* ⇒
instr list ⇒ *ex-table* ⇒ ty_i*' err list* ⇒ ty_i*' err list*
where
kiljvm P mxs mxl T_r *is xt* ≡
kildall (*JVM-SemiType.le P mxs mxl*) (*JVM-SemiType.sup P mxs mxl*)
(*exec P mxs* T_r *xt is*)

definition *wt-kildall* :: *jvm-prog* ⇒ *cname* ⇒ *ty list* ⇒ *ty* ⇒ *nat* ⇒ *nat* ⇒
instr list ⇒ *ex-table* ⇒ *bool*
where
wt-kildall P C' Ts T_r *mxs* mxl_0 *is xt* ≡
0 < *size is* ∧
(*let first* = *Some* ([],[*OK* (*Class C'*)]@(*map OK Ts*)@(*replicate* mxl_0 *Err*));
start = *OK first*#(*replicate* (*size is* − *1*) (*OK None*));
result = *kiljvm P mxs* (*1*+*size Ts*+mxl_0) T_r *is xt start*
in ∀ *n* < *size is*. *result*!*n* ≠ *Err*)

definition *wf-jvm-prog*$_k$ :: *jvm-prog* ⇒ *bool*
where
wf-jvm-prog$_k$ *P* ≡
wf-prog (λ*P C'* (*M*,*Ts*,T_r,(*mxs*,mxl_0,*is*,*xt*)). *wt-kildall P C' Ts* T_r *mxs* mxl_0 *is xt*) *P*

theorem (**in** *start-context*) *is-bcv-kiljvm*:
is-bcv r Err step (*size is*) *A* (*kiljvm P mxs mxl* T_r *is xt*)

lemma *subset-replicate* [*intro?*]: *set* (*replicate n x*) ⊆ {*x*}
by (*induct n*) *auto*

lemma *in-set-replicate*:
assumes *x* ∈ *set* (*replicate n y*)
shows *x* = *y*
lemma (**in** *start-context*) *start-in-A* [*intro?*]:
0 < *size is* ⟹ *start* ∈ *list* (*size is*) *A*
using *Ts C*

theorem (**in** *start-context*) *wt-kil-correct*:
assumes *wtk*: *wt-kildall P C Ts* T_r *mxs* mxl_0 *is xt*
shows ∃τ*s*. *wt-method P C Ts* T_r *mxs* mxl_0 *is xt* τ*s*

theorem (**in** *start-context*) *wt-kil-complete*:
assumes *wtm*: *wt-method P C Ts* T_r *mxs* mxl_0 *is xt* τ*s*
shows *wt-kildall P C Ts* T_r *mxs* mxl_0 *is xt*

theorem *jvm-kildall-correct*:
wf-jvm-prog$_k$ *P* = *wf-jvm-prog P*
end

4.20 LBV for the JVM

theory *LBVJVM*
imports *../DFA/Abstract-BV TF-JVM*
begin

type-synonym *prog-cert* = *cname* ⇒ *mname* ⇒ $ty_i{}'$ *err list*

definition *check-cert* :: *jvm-prog* ⇒ *nat* ⇒ *nat* ⇒ *nat* ⇒ $ty_i{}'$ *err list* ⇒ *bool*
where
check-cert P mxs mxl n cert ≡ *check-types P mxs mxl cert* ∧ *size cert* = *n*+*1* ∧
(∀ *i*<*n*. *cert*!*i* ≠ *Err*) ∧ *cert*!*n* = *OK None*

definition *lbvjvm* :: *jvm-prog* ⇒ *nat* ⇒ *nat* ⇒ *ty* ⇒ *ex-table* ⇒
$ty_i{}'$ *err list* ⇒ *instr list* ⇒ $ty_i{}'$ *err* ⇒ $ty_i{}'$ *err*
where
lbvjvm P mxs maxr T_r *et cert bs* ≡
wtl-inst-list bs cert (*JVM-SemiType.sup P mxs maxr*) (*JVM-SemiType.le P mxs maxr*) *Err* (*OK None*) (*exec P mxs* T_r *et bs*) *0*

definition *wt-lbv* :: *jvm-prog* ⇒ *cname* ⇒ *ty list* ⇒ *ty* ⇒ *nat* ⇒ *nat* ⇒
ex-table ⇒ $ty_i{}'$ *err list* ⇒ *instr list* ⇒ *bool*
where
wt-lbv P C Ts T_r *mxs* mxl_0 *et cert ins* ≡
check-cert P mxs (*1*+*size Ts*+mxl_0) (*size ins*) *cert* ∧
0 < *size ins* ∧
(*let start* = *Some* ([],(*OK* (*Class C*))#((*map OK Ts*))@(*replicate* mxl_0 *Err*));
result = *lbvjvm P mxs* (*1*+*size Ts*+mxl_0) T_r *et cert ins* (*OK start*)
in result ≠ *Err*)

definition *wt-jvm-prog-lbv* :: *jvm-prog* ⇒ *prog-cert* ⇒ *bool*
where
wt-jvm-prog-lbv P cert ≡
wf-prog (λ*P C* (*mn*,*Ts*,T_r,(*mxs*,mxl_0,*b*,*et*)). *wt-lbv P C Ts* T_r *mxs* mxl_0 *et* (*cert C mn*) *b*) *P*

definition *mk-cert* :: *jvm-prog* ⇒ *nat* ⇒ *ty* ⇒ *ex-table* ⇒ *instr list*
⇒ ty_m ⇒ $ty_i{}'$ *err list*
where
mk-cert P mxs T_r *et bs phi* ≡ *make-cert* (*exec P mxs* T_r *et bs*) (*map OK phi*) (*OK None*)

definition *prg-cert* :: *jvm-prog* ⇒ ty_P ⇒ *prog-cert*
where
prg-cert P phi C mn ≡ *let* (*C*,*Ts*,T_r,(*mxs*,mxl_0,*ins*,*et*)) = *method P C mn*
in mk-cert P mxs T_r *et ins* (*phi C mn*)

lemma *check-certD* [*intro?*]:
check-cert P mxs mxl n cert ⟹ *cert-ok cert n Err* (*OK None*) (*states P mxs mxl*)
by (*unfold cert-ok-def check-cert-def check-types-def*) *auto*

lemma (**in** *start-context*) *wt-lbv-wt-step*:
assumes *lbv*: *wt-lbv P C Ts* T_r *mxs* mxl_0 *xt cert is*
shows ∃ τ*s* ∈ *list* (*size is*) *A*. *wt-step r Err step* τ*s* ∧ *OK first* $\sqsubseteq_r$ τ*s*!*0*

lemma (**in** *start-context*) *wt-lbv-wt-method*:
assumes *lbv*: *wt-lbv P C Ts* T_r *mxs* mxl_0 *xt cert is*
shows $\exists \tau s$. *wt-method P C Ts* T_r *mxs* mxl_0 *is xt* τs

lemma (**in** *start-context*) *wt-method-wt-lbv*:
assumes *wt*: *wt-method P C Ts* T_r *mxs* mxl_0 *is xt* τs
defines [*simp*]: *cert* $\equiv$ *mk-cert P mxs* T_r *xt is* τs

shows *wt-lbv P C Ts* T_r *mxs* mxl_0 *xt cert is*

theorem *jvm-lbv-correct*:
wt-jvm-prog-lbv P Cert $\Longrightarrow$ *wf-jvm-prog P*
theorem *jvm-lbv-complete*:
assumes *wt*: *wf-jvm-prog*$_\Phi$ *P*
shows *wt-jvm-prog-lbv P* (*prg-cert P* Φ)
end

4.21 BV Type Safety Invariant

```
theory BVConform
imports BVSpec ../JVM/JVMExec ../Common/Conform
begin


definition confT :: 'c prog ⇒ heap ⇒ val ⇒ ty err ⇒ bool
          (-,- |- - :<=T - [51,51,51,51] 50)
where
  P,h |- v :<=T E ≡ case E of Err ⇒ True | OK T ⇒ P,h ⊢ v :≤ T

notation (xsymbols)
  confT  (-,- ⊢ - :≤⊤ - [51,51,51,51] 50)

abbreviation
  confTs :: 'c prog ⇒ heap ⇒ val list ⇒ ty_l ⇒ bool
          (-,- |- - [:<=T] - [51,51,51,51] 50) where
  P,h |- vs [:<=T] Ts ≡ list-all2 (confT P h) vs Ts

notation (xsymbols)
  confTs  (-,- ⊢ - [:≤⊤] - [51,51,51,51] 50)

definition conf-f  :: jvm-prog ⇒ heap ⇒ ty_i ⇒ bytecode ⇒ frame ⇒ bool
where
  conf-f P h ≡ λ(ST,LT) is (stk,loc,C,M,pc).
  P,h ⊢ stk [:≤] ST ∧ P,h ⊢ loc [:≤⊤] LT ∧ pc < size is

lemma conf-f-def2:
  conf-f P h (ST,LT) is (stk,loc,C,M,pc) ≡
  P,h ⊢ stk [:≤] ST ∧ P,h ⊢ loc [:≤⊤] LT ∧ pc < size is
  by (simp add: conf-f-def)


primrec conf-fs :: [jvm-prog,heap,ty_P,mname,nat,ty,frame list] ⇒ bool
where
  conf-fs P h Φ M_0 n_0 T_0 [] = True
| conf-fs P h Φ M_0 n_0 T_0 (f#frs) =
  (let (stk,loc,C,M,pc) = f in
  (∃ ST LT Ts T mxs mxl_0 is xt.
    Φ C M ! pc = Some (ST,LT) ∧
    (P ⊢ C sees M:Ts → T = (mxs,mxl_0,is,xt) in C) ∧
    (∃ D Ts' T' m D'.
       is!pc = (Invoke M_0 n_0) ∧ ST!n_0 = Class D ∧
       P ⊢ D sees M_0:Ts' → T' = m in D' ∧ P ⊢ T_0 ≤ T') ∧
    conf-f P h (ST, LT) is f ∧ conf-fs P h Φ M (size Ts) T frs))


definition correct-state :: [jvm-prog,ty_P,jvm-state] ⇒ bool
                  (-,- |- - [ok]  [61,0,0] 61)
where
  correct-state P Φ ≡ λ(xp,h,frs).
  case xp of
     None ⇒ (case frs of
```

$[] \Rightarrow True$
$| (f\#fs) \Rightarrow P \vdash h \surd \wedge$
$(let\ (stk,loc,C,M,pc) = f$
$\ in\ \exists\ Ts\ T\ mxs\ mxl_0\ is\ xt\ \tau.$
$(P \vdash C\ sees\ M{:}Ts {\rightarrow} T = (mxs,mxl_0,is,xt)\ in\ C) \wedge$
$\Phi\ C\ M\ !\ pc = Some\ \tau \wedge$
$conf\text{-}f\ P\ h\ \tau\ is\ f \wedge conf\text{-}fs\ P\ h\ \Phi\ M\ (size\ Ts)\ T\ fs))$
$| Some\ x \Rightarrow frs = []$

notation (*xsymbols*)
correct-state (-,- ⊢ - √ [*61*,*0*,*0*] *61*)

4.21.1 Values and ⊤

lemma *confT-Err* [*iff*]: $P,h \vdash x :\leq_\top Err$
by (*simp add*: *confT-def*)

lemma *confT-OK* [*iff*]: $P,h \vdash x :\leq_\top OK\ T = (P,h \vdash x :\leq T)$
by (*simp add*: *confT-def*)

lemma *confT-cases*:
$P,h \vdash x :\leq_\top X = (X = Err \vee (\exists\, T.\ X = OK\ T \wedge P,h \vdash x :\leq T))$
by (*cases X*) *auto*

lemma *confT-hext* [*intro?*, *trans*]:
$[\![\ P,h \vdash x :\leq_\top T;\ h \trianglelefteq h'\ ]\!] \Longrightarrow P,h' \vdash x :\leq_\top T$
by (*cases T*) (*blast intro*: *conf-hext*)+

lemma *confT-widen* [*intro?*, *trans*]:
$[\![\ P,h \vdash x :\leq_\top T;\ P \vdash T \leq_\top T'\ ]\!] \Longrightarrow P,h \vdash x :\leq_\top T'$
by (*cases T′*, *auto intro*: *conf-widen*)

4.21.2 Stack and Registers

lemmas *confTs-Cons1* [*iff*] = *list-all2-Cons1* [*of confT P h*] **for** *P h*

lemma *confTs-confT-sup*:
$[\![\ P,h \vdash loc\ [:\leq_\top]\ LT;\ n < size\ LT;\ LT!n = OK\ T;\ P \vdash T \leq T'\ ]\!]$
$\Longrightarrow P,h \vdash (loc!n) :\leq T'$
lemma *confTs-hext* [*intro?*]:
$P,h \vdash loc\ [:\leq_\top]\ LT \Longrightarrow h \trianglelefteq h' \Longrightarrow P,h' \vdash loc\ [:\leq_\top]\ LT$
by (*fast elim*: *list-all2-mono confT-hext*)

lemma *confTs-widen* [*intro?*, *trans*]:
$P,h \vdash loc\ [:\leq_\top]\ LT \Longrightarrow P \vdash LT\ [\leq_\top]\ LT' \Longrightarrow P,h \vdash loc\ [:\leq_\top]\ LT'$
by (*rule list-all2-trans*, *rule confT-widen*)

lemma *confTs-map* [*iff*]:
$\bigwedge vs.\ (P,h \vdash vs\ [:\leq_\top]\ map\ OK\ Ts) = (P,h \vdash vs\ [:\leq]\ Ts)$
by (*induct Ts*) (*auto simp add*: *list-all2-Cons2*)

lemma *reg-widen-Err* [*iff*]:
$\bigwedge LT.\ (P \vdash replicate\ n\ Err\ [\leq_\top]\ LT) = (LT = replicate\ n\ Err)$
by (*induct n*) (*auto simp add*: *list-all2-Cons1*)

lemma *confTs-Err* [*iff*]:
$P,h \vdash$ *replicate n v* $[:\leq_\top]$ *replicate n Err*
by (*induct n*) *auto*

4.21.3 correct-frames

lemmas [*simp del*] = *fun-upd-apply*

lemma *conf-fs-hext*:
$\bigwedge M\ n\ T_r.$
$[\![$ *conf-fs* $P\ h\ \Phi\ M\ n\ T_r$ *frs*; $h \trianglelefteq h'$ $]\!] \Longrightarrow$ *conf-fs* $P\ h'\ \Phi\ M\ n\ T_r$ *frs*
end

4.22 BV Type Safety Proof

theory *BVSpecTypeSafe*
imports *BVConform*
begin

This theory contains proof that the specification of the bytecode verifier only admits type safe programs.

4.22.1 Preliminaries

Simp and intro setup for the type safety proof:

lemmas *defs1* = *correct-state-def conf-f-def wt-instr-def eff-def norm-eff-def app-def xcpt-app-def*

lemmas *widen-rules* [*intro*] = *conf-widen confT-widen confs-widens confTs-widen*

4.22.2 Exception Handling

For the *Invoke* instruction the BV has checked all handlers that guard the current *pc*.

lemma *Invoke-handlers*:
$match\text{-}ex\text{-}table\ P\ C\ pc\ xt = Some\ (pc',d') \Longrightarrow$
$\exists (f,t,D,h,d) \in set\ (relevant\text{-}entries\ P\ (Invoke\ n\ M)\ pc\ xt).$
$P \vdash C \preceq^* D \wedge pc \in \{f..<t\} \wedge pc' = h \wedge d' = d$
by (*induct xt*) (*auto simp add*: *relevant-entries-def matches-ex-entry-def*
is-relevant-entry-def split: *split-if-asm*)

We can prove separately that the recursive search for exception handlers (*find-handler*) in the frame stack results in a conforming state (if there was no matching exception handler in the current frame). We require that the exception is a valid heap address, and that the state before the exception occured conforms.

term *find-handler*
lemma *uncaught-xcpt-correct*:
assumes *wt*: $wf\text{-}jvm\text{-}prog_{\Phi}\ P$
assumes *h*: $h\ xcp = Some\ obj$
shows $\bigwedge f.\ P,\Phi \vdash (None,\ h,\ f\#frs)\surd \Longrightarrow P,\Phi \vdash (find\text{-}handler\ P\ xcp\ h\ frs)\ \surd$
(**is** $\bigwedge f.\ ?correct\ (None,\ h,\ f\#frs) \Longrightarrow ?correct\ (?find\ frs)$)

The requirement of lemma *uncaught-xcpt-correct* (that the exception is a valid reference on the heap) is always met for welltyped instructions and conformant states:

lemma *exec-instr-xcpt-h*:
$[\![\ fst\ (exec\text{-}instr\ (ins!pc)\ P\ h\ stk\ vars\ Cl\ M\ pc\ frs) = Some\ xcp;$
$P,T,mxs,size\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M;$
$P,\Phi \vdash (None,\ h,\ (stk,loc,C,M,pc)\#frs)\surd\ ]\!]$
$\Longrightarrow \exists\, obj.\ h\ xcp = Some\ obj$
(**is** $[\![\ ?xcpt;\ ?wt;\ ?correct\ ]\!] \Longrightarrow ?thesis$)
lemma *conf-sys-xcpt*:
$[\![preallocated\ h;\ C \in sys\text{-}xcpts]\!] \Longrightarrow P,h \vdash Addr\ (addr\text{-}of\text{-}sys\text{-}xcpt\ C) :\leq Class\ C$
by (*auto elim*: *preallocatedE*)

lemma *match-ex-table-SomeD*:
$match\text{-}ex\text{-}table\ P\ C\ pc\ xt = Some\ (pc',d') \Longrightarrow$
$\exists (f,t,D,h,d) \in set\ xt.\ matches\text{-}ex\text{-}entry\ P\ C\ pc\ (f,t,D,h,d) \wedge h = pc' \wedge d{=}d'$
by (*induct xt*) (*auto split*: *split-if-asm*)

Finally we can state that, whenever an exception occurs, the next state always conforms:

lemma *xcpt-correct*:
  fixes σ' :: *jvm-state*
  assumes *wtp*: $wf\text{-}jvm\text{-}prog_\Phi\ P$
  assumes *meth*: $P \vdash C\ sees\ M{:}Ts{\rightarrow}T{=}(mxs,mxl_0,ins,xt)\ in\ C$
  assumes *wt*: $P,T,mxs,size\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M$
  assumes *xp*: $fst\ (exec\text{-}instr\ (ins!pc)\ P\ h\ stk\ loc\ C\ M\ pc\ frs) = Some\ xcp$
  assumes *s'*: $Some\ \sigma' = exec\ (P,\ None,\ h,\ (stk,loc,C,M,pc)\#frs)$
  assumes *correct*: $P,\Phi \vdash (None,\ h,\ (stk,loc,C,M,pc)\#frs)\surd$
  shows $P,\Phi \vdash \sigma'\surd$

4.22.3 Single Instructions

In this section we prove for each single (welltyped) instruction that the state after execution of the instruction still conforms. Since we have already handled exceptions above, we can now assume that no exception occurs in this step.

declare *defs1* [*simp*]

lemma *Invoke-correct*:
  fixes σ' :: *jvm-state*
  assumes *wtprog*: $wf\text{-}jvm\text{-}prog_\Phi\ P$
  assumes *meth-C*: $P \vdash C\ sees\ M{:}Ts{\rightarrow}T{=}(mxs,mxl_0,ins,xt)\ in\ C$
  assumes *ins*: $ins\ !\ pc = Invoke\ M'\ n$
  assumes *wti*: $P,T,mxs,size\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M$
  assumes σ': $Some\ \sigma' = exec\ (P,\ None,\ h,\ (stk,loc,C,M,pc)\#frs)$
  assumes *approx*: $P,\Phi \vdash (None,\ h,\ (stk,loc,C,M,pc)\#frs)\surd$
  assumes *no-xcp*: $fst\ (exec\text{-}instr\ (ins!pc)\ P\ h\ stk\ loc\ C\ M\ pc\ frs) = None$
  shows $P,\Phi \vdash \sigma'\surd$
declare *list-all2-Cons2* [*iff*]

lemma *Return-correct*:
  fixes σ' :: *jvm-state*
  assumes *wt-prog*: $wf\text{-}jvm\text{-}prog_\Phi\ P$
  assumes *meth*: $P \vdash C\ sees\ M{:}Ts{\rightarrow}T{=}(mxs,mxl_0,ins,xt)\ in\ C$
  assumes *ins*: $ins\ !\ pc = Return$
  assumes *wt*: $P,T,mxs,size\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M$
  assumes *s'*: $Some\ \sigma' = exec\ (P,\ None,\ h,\ (stk,loc,C,M,pc)\#frs)$
  assumes *correct*: $P,\Phi \vdash (None,\ h,\ (stk,loc,C,M,pc)\#frs)\surd$

  shows $P,\Phi \vdash \sigma'\surd$
declare *sup-state-opt-any-Some* [*iff*]
declare *not-Err-eq* [*iff*]

lemma *Load-correct*:
$\llbracket\ wf\text{-}prog\ wt\ P;$
  $P \vdash C\ sees\ M{:}Ts{\rightarrow}T{=}(mxs,mxl_0,ins,xt)\ in\ C;$
  $ins!pc = Load\ idx;$
  $P,T,mxs,size\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M;$
  $Some\ \sigma' = exec\ (P,\ None,\ h,\ (stk,loc,C,M,pc)\#frs);$
  $P,\Phi \vdash (None,\ h,\ (stk,loc,C,M,pc)\#frs)\surd\ \rrbracket$
$\Longrightarrow P,\Phi \vdash \sigma'\surd$
  by (*fastforce dest*: *sees-method-fun* [*of - C*] *elim*!: *confTs-confT-sup*)

declare [[*simproc del*: *list-to-set-comprehension*]]

lemma *Store-correct*:
$[\![$ *wf-prog wt P*;
$P \vdash C$ *sees* $M{:}Ts{\to}T{=}(mxs,mxl_0,ins,xt)$ *in* C;
ins!*pc* = *Store idx*;
$P,T,mxs,size\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M$;
Some σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*);
$P,\Phi \vdash$ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)$\surd$ $]\!]$
$\Longrightarrow P,\Phi \vdash \sigma'\surd$

lemma *Push-correct*:
$[\![$ *wf-prog wt P*;
$P \vdash C$ *sees* $M{:}Ts{\to}T{=}(mxs,mxl_0,ins,xt)$ *in* C;
ins!*pc* = *Push v*;
$P,T,mxs,size\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M$;
Some σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*);
$P,\Phi \vdash$ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)$\surd$ $]\!]$
$\Longrightarrow P,\Phi \vdash \sigma'\surd$

lemma *Cast-conf2*:
$[\![$ *wf-prog ok P*; $P,h \vdash v :\leq T$; *is-refT T*; *cast-ok P C h v*;
$P \vdash$ *Class* $C \leq T'$; *is-class P C*$]\!]$
$\Longrightarrow P,h \vdash v :\leq T'$

lemma *Checkcast-correct*:
$[\![$ $wf\text{-}jvm\text{-}prog_\Phi$ *P*;
$P \vdash C$ *sees* $M{:}Ts{\to}T{=}(mxs,mxl_0,ins,xt)$ *in* C;
ins!*pc* = *Checkcast D*;
$P,T,mxs,size\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M$;
Some σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*) ;
$P,\Phi \vdash$ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)$\surd$;
fst (*exec-instr* (*ins*!*pc*) *P h stk loc C M pc frs*) = *None* $]\!]$
$\Longrightarrow P,\Phi \vdash \sigma'\surd$
declare *split-paired-All* [*simp del*]

lemmas *widens-Cons* [*iff*] = *list-all2-Cons1* [*of widen P*] **for** *P*

lemma *Getfield-correct*:
fixes σ' :: *jvm-state*
assumes *wf*: *wf-prog wt P*
assumes *mC*: $P \vdash C$ *sees* $M{:}Ts{\to}T{=}(mxs,mxl_0,ins,xt)$ *in* C
assumes *i*: *ins*!*pc* = *Getfield F D*
assumes *wt*: $P,T,mxs,size\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M$
assumes *s'*: *Some* σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)
assumes *cf*: $P,\Phi \vdash$ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)$\surd$
assumes *xc*: *fst* (*exec-instr* (*ins*!*pc*) *P h stk loc C M pc frs*) = *None*

shows $P,\Phi \vdash \sigma'\surd$
lemma *Putfield-correct*:
fixes σ' :: *jvm-state*
assumes *wf*: *wf-prog wt P*
assumes *mC*: $P \vdash C$ *sees* $M{:}Ts{\to}T{=}(mxs,mxl_0,ins,xt)$ *in* C
assumes *i*: *ins*!*pc* = *Putfield F D*

assumes *wt*: $P,T,mxs,size\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M$
assumes *s'*: *Some* σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)
assumes *cf*: $P,\Phi \vdash$ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)$\surd$
assumes *xc*: *fst* (*exec-instr* (*ins*!*pc*) *P h stk loc C M pc frs*) = *None*

shows $P,\Phi \vdash \sigma'\surd$

lemma *has-fields-b-fields*:
$P \vdash C$ *has-fields FDTs* $\Longrightarrow$ *fields P C* = *FDTs*

lemma *oconf-blank* [*intro*, *simp*]:
⟦*is-class P C*; *wf-prog wt P*⟧ $\Longrightarrow P,h \vdash$ *blank P C* $\surd$
lemma *obj-ty-blank* [*iff*]: *obj-ty* (*blank P C*) = *Class C*
by (*simp add*: *blank-def*)

lemma *New-correct*:
fixes σ' :: *jvm-state*
assumes *wf*: *wf-prog wt P*
assumes *meth*: $P \vdash C$ *sees* $M{:}Ts{\rightarrow}T{=}(mxs,mxl_0,ins,xt)$ *in C*
assumes *ins*: *ins*!*pc* = *New X*
assumes *wt*: $P,T,mxs,size\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M$
assumes *exec*: *Some* σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)
assumes *conf*: $P,\Phi \vdash$ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)$\surd$
assumes *no-x*: *fst* (*exec-instr* (*ins*!*pc*) *P h stk loc C M pc frs*) = *None*
shows $P,\Phi \vdash \sigma'\surd$

lemma *Goto-correct*:
⟦ *wf-prog wt P*;
$P \vdash C$ *sees* $M{:}Ts{\rightarrow}T{=}(mxs,mxl_0,ins,xt)$ *in C*;
ins ! *pc* = *Goto branch*;
$P,T,mxs,size\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M$;
Some σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*) ;
$P,\Phi \vdash$ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)$\surd$ ⟧
$\Longrightarrow P,\Phi \vdash \sigma'\surd$

lemma *IfFalse-correct*:
⟦ *wf-prog wt P*;
$P \vdash C$ *sees* $M{:}Ts{\rightarrow}T{=}(mxs,mxl_0,ins,xt)$ *in C*;
ins ! *pc* = *IfFalse branch*;
$P,T,mxs,size\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M$;
Some σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*) ;
$P,\Phi \vdash$ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)$\surd$ ⟧
$\Longrightarrow P,\Phi \vdash \sigma'\surd$
lemma *CmpEq-correct*:
⟦ *wf-prog wt P*;
$P \vdash C$ *sees* $M{:}Ts{\rightarrow}T{=}(mxs,mxl_0,ins,xt)$ *in C*;
ins ! *pc* = *CmpEq*;
$P,T,mxs,size\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M$;
Some σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*) ;
$P,\Phi \vdash$ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)$\surd$ ⟧
$\Longrightarrow P,\Phi \vdash \sigma'\surd$
lemma *Pop-correct*:
⟦ *wf-prog wt P*;
$P \vdash C$ *sees* $M{:}Ts{\rightarrow}T{=}(mxs,mxl_0,ins,xt)$ *in C*;

$ins\ !\ pc = Pop;$
$P,T,mxs,size\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M;$
$Some\ \sigma' = exec\ (P,\ None,\ h,\ (stk,loc,C,M,pc)\#frs)\ ;$
$P,\Phi \vdash (None,\ h,\ (stk,loc,C,M,pc)\#frs)\surd\ \rrbracket$
$\Longrightarrow P,\Phi \vdash \sigma'\surd$

lemma *IAdd-correct*:
$\llbracket\ wf\text{-}prog\ wt\ P;$
$P \vdash C\ sees\ M{:}Ts{\to}T{=}(mxs,mxl_0,ins,xt)\ in\ C;$
$ins\ !\ pc = IAdd;$
$P,T,mxs,size\ ins,xt \vdash ins!pc,pc :: \Phi\ C\ M;$
$Some\ \sigma' = exec\ (P,\ None,\ h,\ (stk,loc,C,M,pc)\#frs)\ ;$
$P,\Phi \vdash (None,\ h,\ (stk,loc,C,M,pc)\#frs)\surd\ \rrbracket$
$\Longrightarrow P,\Phi \vdash \sigma'\surd$

lemma *Throw-correct*:
$\llbracket\ wf\text{-}prog\ wt\ P;$
$P \vdash C\ sees\ M{:}Ts{\to}T{=}(mxs,mxl_0,ins,xt)\ in\ C;$
$ins\ !\ pc = Throw;$
$Some\ \sigma' = exec\ (P,\ None,\ h,\ (stk,loc,C,M,pc)\#frs)\ ;$
$P,\Phi \vdash (None,\ h,\ (stk,loc,C,M,pc)\#frs)\surd;$
$fst\ (exec\text{-}instr\ (ins!pc)\ P\ h\ stk\ loc\ C\ M\ pc\ frs) = None\ \rrbracket$
$\Longrightarrow P,\Phi \vdash \sigma'\surd$
by *simp*

The next theorem collects the results of the sections above, i.e. exception handling and the execution step for each instruction. It states type safety for single step execution: in welltyped programs, a conforming state is transformed into another conforming state when one instruction is executed.

theorem *instr-correct*:
$\llbracket\ wf\text{-}jvm\text{-}prog_\Phi\ P;$
$P \vdash C\ sees\ M{:}Ts{\to}T{=}(mxs,mxl_0,ins,xt)\ in\ C;$
$Some\ \sigma' = exec\ (P,\ None,\ h,\ (stk,loc,C,M,pc)\#frs);$
$P,\Phi \vdash (None,\ h,\ (stk,loc,C,M,pc)\#frs)\surd\ \rrbracket$
$\Longrightarrow P,\Phi \vdash \sigma'\surd$

4.22.4 Main

lemma *correct-state-impl-Some-method*:
$P,\Phi \vdash (None,\ h,\ (stk,loc,C,M,pc)\#frs)\surd$
$\Longrightarrow \exists\, m\ Ts\ T.\ P \vdash C\ sees\ M{:}Ts{\to}T = m\ in\ C$
by *fastforce*

lemma *BV-correct-1* [*rule-format*]:
$\bigwedge\sigma.\ \llbracket\ wf\text{-}jvm\text{-}prog_\Phi\ P;\ P,\Phi \vdash \sigma\surd\rrbracket \Longrightarrow P \vdash \sigma\ {-jvm\to_1}\ \sigma' \longrightarrow P,\Phi \vdash \sigma'\surd$

theorem *progress*:
$\llbracket\ xp{=}None;\ frs{\neq}[]\ \rrbracket \Longrightarrow \exists\,\sigma'.\ P \vdash (xp,h,frs)\ {-jvm\to_1}\ \sigma'$
by (*clarsimp simp add*: *exec-1-iff neq-Nil-conv split-beta*
simp del: *split-paired-Ex*)

lemma *progress-conform*:
$\llbracket wf\text{-}jvm\text{-}prog_\Phi\ P;\ P,\Phi \vdash (xp,h,frs)\surd;\ xp{=}None;\ frs{\neq}[]\rrbracket$
$\Longrightarrow \exists\,\sigma'.\ P \vdash (xp,h,frs)\ {-jvm\to_1}\ \sigma' \wedge P,\Phi \vdash \sigma'\surd$

theorem *BV-correct* [*rule-format*]:
$[\![$ *wf-jvm-prog*$_\Phi$ P; $P \vdash \sigma$ *−jvm→* σ' $]\!] \Longrightarrow P,\Phi \vdash \sigma\surd \longrightarrow P,\Phi \vdash \sigma'\surd$
lemma *hconf-start*:
  assumes *wf*: *wf-prog wf-mb P*
  shows $P \vdash$ (*start-heap* P) $\surd$
lemma *BV-correct-initial*:
  shows $[\![$ *wf-jvm-prog*$_\Phi$ P; $P \vdash C$ *sees* M:[]→T = m *in* C $]\!]$
  $\Longrightarrow P,\Phi \vdash$ *start-state* P C M $\surd$
theorem *typesafe*:
  assumes *welltyped*: *wf-jvm-prog*$_\Phi$ P
  assumes *main-method*: $P \vdash C$ *sees* M:[]→T = m *in* C
  shows $P \vdash$ *start-state* P C M *−jvm→* σ $\Longrightarrow$ $P,\Phi \vdash \sigma$ $\surd$
end

4.23 Welltyped Programs produce no Type Errors

theory *BVNoTypeError*
imports *../JVM/JVMDefensive BVSpecTypeSafe*
begin

lemma *has-methodI*:
  $P \vdash C$ *sees* $M{:}Ts{\rightarrow}T = m$ *in* $D \Longrightarrow P \vdash C$ *has* M
  by (*unfold has-method-def*) *blast*

Some simple lemmas about the type testing functions of the defensive JVM:

lemma *typeof-NoneD* [*simp*,*dest*]: *typeof* v = *Some* $x \Longrightarrow \neg$*is-Addr* v
  by (*cases* v) *auto*

lemma *is-Ref-def2*:
  is-Ref $v = (v =$ *Null* $\vee (\exists\, a.\ v =$ *Addr* $a))$
  by (*cases* v) (*auto simp add*: *is-Ref-def*)

lemma [*iff*]: *is-Ref Null* **by** (*simp add*: *is-Ref-def2*)

lemma *is-RefI* [*intro*, *simp*]: $P,h \vdash v :\leq T \Longrightarrow$ *is-refT* $T \Longrightarrow$ *is-Ref* v
lemma *is-IntgI* [*intro*, *simp*]: $P,h \vdash v :\leq$ *Integer* $\Longrightarrow$ *is-Intg* v
lemma *is-BoolI* [*intro*, *simp*]: $P,h \vdash v :\leq$ *Boolean* $\Longrightarrow$ *is-Bool* v
declare *defs1* [*simp del*]

lemma *wt-jvm-prog-states*:
  $\llbracket$ *wf-jvm-prog*$_\Phi$ P; $P \vdash C$ *sees* M: $Ts{\rightarrow}T = (mxs, mxl, ins, et)$ *in* C;
    $\Phi\ C\ M\ !\ pc = \tau$; $pc <$ *size ins* $\rrbracket$
  $\Longrightarrow$ *OK* $\tau \in$ *states* P *mxs* (*1*+*size* *Ts*+*mxl*)

The main theorem: welltyped programs do not produce type errors if they are started in a conformant state.

theorem *no-type-error*:
  fixes σ :: *jvm-state*
  assumes *welltyped*: *wf-jvm-prog*$_\Phi$ P **and** *conforms*: $P,\Phi \vdash \sigma \surd$
  shows *exec-d* $P\ \sigma \neq$ *TypeError*

The theorem above tells us that, in welltyped programs, the defensive machine reaches the same result as the aggressive one (after arbitrarily many steps).

theorem *welltyped-aggressive-imp-defensive*:
  wf-jvm-prog$_\Phi$ $P \Longrightarrow P,\Phi \vdash \sigma \surd \Longrightarrow P \vdash \sigma$ *−jvm→* σ'
  $\Longrightarrow P \vdash$ (*Normal* σ) *−jvmd→* (*Normal* σ')

As corollary we get that the aggressive and the defensive machine are equivalent for welltyped programs (if started in a conformant state or in the canonical start state)

corollary *welltyped-commutes*:
  fixes σ :: *jvm-state*
  assumes *wf*: *wf-jvm-prog*$_\Phi$ P **and** *conforms*: $P,\Phi \vdash \sigma \surd$
  shows $P \vdash$ (*Normal* σ) *−jvmd→* (*Normal* σ') = $P \vdash \sigma$ *−jvm→* σ'
  apply *rule*
  apply (*erule defensive-imp-aggressive*)
  apply (*erule welltyped-aggressive-imp-defensive* [*OF wf conforms*])
  done

```
corollary welltyped-initial-commutes:
  assumes wf: wf-jvm-prog P
  assumes meth: P ⊢ C sees M:[]→T = b in C
  defines start: σ ≡ start-state P C M
  shows P ⊢ (Normal σ) −jvmd→ (Normal σ') = P ⊢ σ −jvm→ σ'
proof −
  from wf obtain Φ where wf': wf-jvm-prog_Φ P by (auto simp: wf-jvm-prog-def)
  from this meth have P,Φ ⊢ σ √ unfolding start by (rule BV-correct-initial)
  with wf' show ?thesis by (rule welltyped-commutes)
qed


lemma not-TypeError-eq [iff]:
  x ≠ TypeError = (∃ t. x = Normal t)
  by (cases x) auto

locale cnf =
  fixes P and Φ and σ
  assumes wf: wf-jvm-prog_Φ P
  assumes cnf: correct-state P Φ σ

theorem (in cnf) no-type-errors:
  P ⊢ (Normal σ) −jvmd→ σ' ⟹ σ' ≠ TypeError
  apply (unfold exec-all-d-def1)
  apply (erule rtrancl-induct)
   apply simp
  apply (fold exec-all-d-def1)
  apply (insert cnf wf)
  apply clarsimp
  apply (drule defensive-imp-aggressive)
  apply (frule (2) BV-correct)
  apply (drule (1) no-type-error) back
  apply (auto simp add: exec-1-d-eq)
  done

locale start =
  fixes P and C and M and σ and T and b
  assumes wf: wf-jvm-prog P
  assumes sees: P ⊢ C sees M:[]→T = b in C
  defines σ ≡ Normal (start-state P C M)

corollary (in start) bv-no-type-error:
  shows P ⊢ σ −jvmd→ σ' ⟹ σ' ≠ TypeError
proof −
  from wf obtain Φ where wf-jvm-prog_Φ P by (auto simp: wf-jvm-prog-def)
  moreover
  with sees have correct-state P Φ (start-state P C M)
    by − (rule BV-correct-initial)
  ultimately have cnf P Φ (start-state P C M) by (rule cnf.intro)
  moreover assume P ⊢ σ −jvmd→ σ'
  ultimately show ?thesis by (unfold σ-def) (rule cnf.no-type-errors)
qed
```

end

4.24 Example Welltypings

theory *BVExample*
imports *../JVM/JVMListExample BVSpecTypeSafe BVExec*
  ~~/src/HOL/Library/Efficient-Nat
begin

This theory shows type correctness of the example program in section 3.7 (p. 92) by explicitly providing a welltyping. It also shows that the start state of the program conforms to the welltyping; hence type safe execution is guaranteed.

4.24.1 Setup

lemma *distinct-classes′*:
  list-name ≠ *test-name*
  list-name ≠ *Object*
  list-name ≠ *ClassCast*
  list-name ≠ *OutOfMemory*
  list-name ≠ *NullPointer*
  test-name ≠ *Object*
  test-name ≠ *OutOfMemory*
  test-name ≠ *ClassCast*
  test-name ≠ *NullPointer*
  ClassCast ≠ *NullPointer*
  ClassCast ≠ *Object*
  NullPointer ≠ *Object*
  OutOfMemory ≠ *ClassCast*
  OutOfMemory ≠ *NullPointer*
  OutOfMemory ≠ *Object*
  by (*simp-all add*: *list-name-def test-name-def Object-def NullPointer-def*
    OutOfMemory-def ClassCast-def)

lemmas *distinct-classes* = *distinct-classes′ distinct-classes′* [*symmetric*]

lemma *distinct-fields*:
  val-name ≠ *next-name*
  next-name ≠ *val-name*
  by (*simp-all add*: *val-name-def next-name-def*)

Abbreviations for definitions we will have to use often in the proofs below:

lemmas *system-defs* = *SystemClasses-def ObjectC-def NullPointerC-def*
  OutOfMemoryC-def ClassCastC-def
lemmas *class-defs* = *list-class-def test-class-def*

These auxiliary proofs are for efficiency: class lookup, subclass relation, method and field lookup are computed only once:

lemma *class-Object* [*simp*]:
  class E Object = *Some* (*undefined*, [],[])
  by (*simp add*: *class-def system-defs E-def*)

lemma *class-NullPointer* [*simp*]:
  class E NullPointer = *Some* (*Object*, [], [])
  by (*simp add*: *class-def system-defs E-def distinct-classes*)

lemma *class-OutOfMemory* [*simp*]:
  class E OutOfMemory = *Some* (*Object*, [], [])
  by (*simp add*: *class-def system-defs E-def distinct-classes*)

lemma *class-ClassCast* [*simp*]:
  class E ClassCast = *Some* (*Object*, [], [])
  by (*simp add*: *class-def system-defs E-def distinct-classes*)

lemma *class-list* [*simp*]:
  class E list-name = *Some list-class*
  by (*simp add*: *class-def system-defs E-def distinct-classes*)

lemma *class-test* [*simp*]:
  class E test-name = *Some test-class*
  by (*simp add*: *class-def system-defs E-def distinct-classes*)

lemma *E-classes* [*simp*]:
  {*C*. *is-class E C*} = {*list-name*, *test-name*, *NullPointer*,
  ClassCast, *OutOfMemory*, *Object*}
  by (*auto simp add*: *is-class-def class-def system-defs E-def class-defs*)

The subclass releation spelled out:

lemma *subcls1*:
  subcls1 E = {(*list-name*,*Object*), (*test-name*,*Object*), (*NullPointer*, *Object*),
  (*ClassCast*, *Object*), (*OutOfMemory*, *Object*)}

The subclass relation is acyclic; hence its converse is well founded:

lemma *notin-rtrancl*:
  $(a,b) \in r^* \Longrightarrow a \neq b \Longrightarrow (\bigwedge y.\ (a,y) \notin r) \Longrightarrow False$
  by (*auto elim*: *converse-rtranclE*)

lemma *acyclic-subcls1-E*: *acyclic* (*subcls1 E*)
lemma *wf-subcls1-E*: *wf* $((subcls1\ E)^{-1})$

Method and field lookup:

lemma *method-append* [*simp*]:
  method E list-name append-name =
  (*list-name*, [*Class list-name*], *Void*, *3*, *0*, *append-ins*, [(*1*, *2*, *NullPointer*, *7*, *0*)])
lemma *method-makelist* [*simp*]:
  method E test-name makelist-name =
  (*test-name*, [], *Void*, *3*, *2*, *make-list-ins*, [])
lemma *field-val* [*simp*]:
  field E list-name val-name = (*list-name*, *Integer*)
lemma *field-next* [*simp*]:
  field E list-name next-name = (*list-name*, *Class list-name*)
lemma [*simp*]: *fields E Object* = []
  by (*fastforce intro*: *fields-def2 Fields.intros*)

lemma [*simp*]: *fields E NullPointer* = []
  by (*fastforce simp add*: *distinct-classes intro*: *fields-def2 Fields.intros*)

lemma [*simp*]: *fields E ClassCast* = []
  by (*fastforce simp add*: *distinct-classes intro*: *fields-def2 Fields.intros*)

lemma [*simp*]: *fields E OutOfMemory* = []
  by (*fastforce simp add*: *distinct-classes intro*: *fields-def2 Fields.intros*)

lemma [*simp*]: *fields E test-name* = []
lemmas [*simp*] = *is-class-def*

4.24.2 Program structure

The program is structurally wellformed:

lemma *wf-struct*:
  wf-prog (λ*G C mb. True*) *E* (**is** *wf-prog ?mb E*)

4.24.3 Welltypings

We show welltypings of the methods *append-name* in class *list-name*, and *makelist-name* in class *test-name*:

lemmas *eff-simps* [*simp*] = *eff-def norm-eff-def xcpt-eff-def*

definition *phi-append* :: ty_m (φ_a)
where
  φ_a ≡ *map* (λ(*x*,*y*). *Some* (*x*, *map OK y*)) [
  ([], [*Class list-name*, *Class list-name*]),
  ([*Class list-name*], [*Class list-name*, *Class list-name*]),
  ([*Class list-name*], [*Class list-name*, *Class list-name*]),
  ([*Class list-name*, *Class list-name*], [*Class list-name*, *Class list-name*]),
  ([*Class list-name*, *Class list-name*], [*Class list-name*, *Class list-name*]),
  ([*NT*, *Class list-name*, *Class list-name*], [*Class list-name*, *Class list-name*]),
  ([*Boolean*, *Class list-name*], [*Class list-name*, *Class list-name*]),

  ([*Class Object*], [*Class list-name*, *Class list-name*]),
  ([], [*Class list-name*, *Class list-name*]),
  ([*Class list-name*], [*Class list-name*, *Class list-name*]),
  ([*Class list-name*, *Class list-name*], [*Class list-name*, *Class list-name*]),
  ([], [*Class list-name*, *Class list-name*]),
  ([*Void*], [*Class list-name*, *Class list-name*]),

  ([*Class list-name*], [*Class list-name*, *Class list-name*]),
  ([*Class list-name*, *Class list-name*], [*Class list-name*, *Class list-name*]),
  ([*Void*], [*Class list-name*, *Class list-name*])]

The next definition and three proof rules implement an algorithm to enumarate natural numbers. The command *apply* (*elim pc-end pc-next pc-0* transforms a goal of the form

pc < *n* ⟹ *P pc*

into a series of goals

P (*0*::'*a*)

P (*Suc 0*)

...

P n

definition *intervall* :: *nat* ⇒ *nat* ⇒ *nat* ⇒ *bool* (- ∈ [-, -'))
where
$x \in [a,\ b) \equiv a \leq x \wedge x < b$

lemma *pc-0*: $x < n \Longrightarrow (x \in [0,\ n) \Longrightarrow P\ x) \Longrightarrow P\ x$
by (*simp add*: *intervall-def*)

lemma *pc-next*: $x \in [n0,\ n) \Longrightarrow P\ n0 \Longrightarrow (x \in [Suc\ n0,\ n) \Longrightarrow P\ x) \Longrightarrow P\ x$
lemma *pc-end*: $x \in [n,n) \Longrightarrow P\ x$
by (*unfold intervall-def*) *arith*

lemma *types-append* [*simp*]: *check-types E 3* (*Suc* (*Suc 0*)) (*map OK* φ_a)
lemma *wt-append* [*simp*]:
wt-method E list-name [*Class list-name*] *Void 3 0 append-ins*
[(*Suc 0*, *2*, *NullPointer*, *7*, *0*)] φ_a

Some abbreviations for readability

abbreviation *Clist* == *Class list-name*
abbreviation *Ctest* == *Class test-name*

definition *phi-makelist* :: ty_m (φ_m)
where
φ_m ≡ *map* (λ(*x*,*y*). *Some* (*x*, *y*)) [
([], [*OK Ctest*, *Err* , *Err*]),
([*Clist*], [*OK Ctest*, *Err* , *Err*]),
([], [*OK Clist*, *Err* , *Err*]),
([*Clist*], [*OK Clist*, *Err* , *Err*]),
([*Integer*, *Clist*], [*OK Clist*, *Err* , *Err*]),

([], [*OK Clist*, *Err* , *Err*]),
([*Clist*], [*OK Clist*, *Err* , *Err*]),
([], [*OK Clist*, *OK Clist*, *Err*]),
([*Clist*], [*OK Clist*, *OK Clist*, *Err*]),
([*Integer*, *Clist*], [*OK Clist*, *OK Clist*, *Err*]),

([], [*OK Clist*, *OK Clist*, *Err*]),
([*Clist*], [*OK Clist*, *OK Clist*, *Err*]),
([], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Clist*], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Integer*, *Clist*], [*OK Clist*, *OK Clist*, *OK Clist*]),

([], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Clist*], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Clist*, *Clist*], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Void*], [*OK Clist*, *OK Clist*, *OK Clist*]),
([], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Clist*], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Clist*, *Clist*], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Void*], [*OK Clist*, *OK Clist*, *OK Clist*])]

lemma *types-makelist* [*simp*]: *check-types E 3* (*Suc* (*Suc* (*Suc 0*))) (*map OK* φ_m)
lemma *wt-makelist* [*simp*]:
wt-method E test-name [] *Void 3 2 make-list-ins* [] φ_m

lemma *wf-md'E*:
$\llbracket$ *wf-prog wf-md P*;
$\bigwedge C\ S\ fs\ ms\ m.\llbracket(C,S,fs,ms) \in set\ P;\ m \in set\ ms\rrbracket \Longrightarrow wf\text{-}md'\ P\ C\ m\ \rrbracket$
$\Longrightarrow wf\text{-}prog\ wf\text{-}md'\ P$

The whole program is welltyped:

definition $Phi :: ty_P\ (\Phi)$
where
$\Phi\ C\ mn \equiv if\ C = test\text{-}name \wedge mn = makelist\text{-}name\ then\ \varphi_m\ else$
$if\ C = list\text{-}name \wedge mn = append\text{-}name\ then\ \varphi_a\ else\ []$

lemma *wf-prog*:
$wf\text{-}jvm\text{-}prog_\Phi\ E$

4.24.4 Conformance

Execution of the program will be typesafe, because its start state conforms to the welltyping:

lemma $E,\Phi \vdash start\text{-}state\ E\ test\text{-}name\ makelist\text{-}name\ \surd$

4.24.5 Example for code generation: inferring method types

definition $test\text{-}kil :: jvm\text{-}prog \Rightarrow cname \Rightarrow ty\ list \Rightarrow ty \Rightarrow nat \Rightarrow nat \Rightarrow$
$ex\text{-}table \Rightarrow instr\ list \Rightarrow ty_i'\ err\ list$
where
$test\text{-}kil\ G\ C\ pTs\ rT\ mxs\ mxl\ et\ instr \equiv$
$(let\ first\ = Some\ ([],(OK\ (Class\ C))\#(map\ OK\ pTs)@(replicate\ mxl\ Err));$
$start\ = OK\ first\#(replicate\ (size\ instr - 1)\ (OK\ None))$
$in\ \ kiljvm\ G\ mxs\ (1+size\ pTs+mxl)\ rT\ instr\ et\ start)$

lemma [*code*]:
$unstables\ r\ step\ ss =$
$fold\ (\lambda p\ A.\ if\ \neg stable\ r\ step\ ss\ p\ then\ insert\ p\ A\ else\ A)\ [0..<size\ ss]\ \{\}$
proof –
have $unstables\ r\ step\ ss = (UN\ p{:}\{..<size\ ss\}.\ if\ \neg stable\ r\ step\ ss\ p\ then\ \{p\}\ else\ \{\})$
apply (*unfold unstables-def*)
apply (*rule equalityI*)
apply (*rule subsetI*)
apply (*erule CollectE*)
apply (*erule conjE*)
apply (*rule UN-I*)
apply *simp*
apply *simp*
apply (*rule subsetI*)
apply (*erule UN-E*)
apply (*case-tac* $\neg$ *stable r step ss p*)
apply *simp+*
done
also have $\bigwedge f.\ (UN\ p{:}\{..<size\ ss\}.\ f\ p) = Union\ (set\ (map\ f\ [0..<size\ ss]))$ **by** *auto*
also note *Sup-set-fold* **also note** *fold-map*
also have $op \cup \circ (\lambda p.\ if\ \neg\ stable\ r\ step\ ss\ p\ then\ \{p\}\ else\ \{\}) =$
$(\lambda p\ A.\ if\ \neg stable\ r\ step\ ss\ p\ then\ insert\ p\ A\ else\ A)$
by(*auto simp add*: *fun-eq-iff*)
finally show *?thesis* **.**

qed

definition *some-elem* :: *'a set* ⇒ *'a* **where** [*code del*]:
some-elem = (%*S*. *SOME x*. *x* : *S*)
code-const *some-elem*
(*SML* (*case*/ - *of*/ *Set*/ *xs*/ =>/ *hd*/ *xs*))

This code setup is just a demonstration and *not* sound!

notepad begin
have *some-elem* (*set* [*False*, *True*]) = *False* **by** *eval*
moreover have *some-elem* (*set* [*True*, *False*]) = *True* **by** *eval*
ultimately have *False* **by** (*simp add*: *some-elem-def*)
end

lemma [*code*]:
iter f step ss w = *while* (λ(*ss*, *w*). ¬ *Set.is-empty w*)
(λ(*ss*, *w*).
let p = *some-elem w in propa f* (*step p* (*ss* ! *p*)) *ss* (*w* − {*p*}))
(*ss*, *w*)
unfolding *iter-def Set.is-empty-def some-elem-def* **..**

lemma *JVM-sup-unfold* [*code*]:
JVM-SemiType.sup S m n = *lift2* (*Opt.sup*
(*Product.sup* (*Listn.sup* (*SemiType.sup S*))
(λ*x y*. *OK* (*map2* (*lift2* (*SemiType.sup S*)) *x y*))))
apply (*unfold JVM-SemiType.sup-def JVM-SemiType.sl-def Opt.esl-def Err.sl-def*
stk-esl-def loc-sl-def Product.esl-def
Listn.sl-def upto-esl-def SemiType.esl-def Err.esl-def)
by *simp*

lemmas [*code*] = *SemiType.sup-def* [*unfolded exec-lub-def*] *JVM-le-unfold*

lemmas [*code*] = *lesub-def plussub-def*

lemma [*code*]:
is-refT T = (*case T of NT* ⇒ *True* | *Class C* ⇒ *True* | - ⇒ *False*)
by (*simp add*: *is-refT-def split add*: *ty.split*)

declare app_i*.simps* [*code*]

lemma [*code*]:
app_i (*Getfield F C*, *P*, *pc*, *mxs*, T_r, (*T*#*ST*, *LT*)) =
Predicate.holds (*Predicate.bind* (*sees-field-i-i-i-o-i P C F C*) (λT_f. *if P* ⊢ *T* ≤ *Class C then Predicate.single* () *else bot*))
by(*auto simp add*: *Predicate.holds-eq intro*: *sees-field-i-i-i-o-iI elim*: *sees-field-i-i-i-o-iE*)

lemma [*code*]:
app_i (*Putfield F C*, *P*, *pc*, *mxs*, T_r, (T_1#T_2#*ST*, *LT*)) =
Predicate.holds (*Predicate.bind* (*sees-field-i-i-i-o-i P C F C*) (λT_f. *if P* ⊢ T_2 ≤ (*Class C*) ∧ *P* ⊢ T_1 ≤ T_f *then Predicate.single* () *else bot*))
by(*auto simp add*: *Predicate.holds-eq simp del*: *eval-bind split*: *split-if-asm elim*!: *sees-field-i-i-i-o-iE Predicate.bindE intro*: *Predicate.bindI sees-field-i-i-i-o-iI*)

lemma [*code*]:

app_i (*Invoke M n*, *P*, *pc*, *mxs*, T_r, (*ST*,*LT*)) =
  (*n* < *length ST* ∧
  (*ST*!*n* ≠ *NT* ⟶
    (*case ST*!*n of*
      Class C ⇒ *Predicate.holds* (*Predicate.bind* (*Method-i-i-i-o-o-o-o P C M*) (λ(*Ts*, *T*, *m*, *D*). *if* *P* ⊢ *rev* (*take n ST*) [≤] *Ts then Predicate.single* () *else bot*))
    | - ⇒ *False*)))
by (*fastforce simp add*: *Predicate.holds-eq simp del*: *eval-bind split*: *ty.split-asm split-if-asm intro*: *bindI Method-i-i-i-o-o-o-oI elim*!: *bindE Method-i-i-i-o-o-o-oE*)

lemmas [*code*] =
  SemiType.sup-def [*unfolded exec-lub-def*]
  widen.equation
  is-relevant-class.simps

definition *test1* **where**
  test1 = *test-kil E list-name* [*Class list-name*] *Void 3 0*
    [(*Suc 0*, *2*, *NullPointer*, *7*, *0*)] *append-ins*
definition *test2* **where**
  test2 = *test-kil E test-name* [] *Void 3 2* [] *make-list-ins*
definition *test3* **where** *test3* = φ_a
definition *test4* **where** *test4* = φ_m

ML ⟪
  if @{*code test1*} = @{*code map*} @{*code OK*} @{*code test3*} *then* () *else error wrong result*;
  if @{*code test2*} = @{*code map*} @{*code OK*} @{*code test4*} *then* () *else error wrong result*
⟫

end

Chapter 5

Compilation

5.1 An Intermediate Language

theory *J1* **imports** *../J/BigStep* **begin**

type-synonym $expr_1 = nat\ exp$
type-synonym $J_1\text{-}prog = expr_1\ prog$
type-synonym $state_1 = heap \times (val\ list)$

primrec
$max\text{-}vars :: 'a\ exp \Rightarrow nat$
and $max\text{-}varss :: 'a\ exp\ list \Rightarrow nat$
where
$max\text{-}vars(new\ C) = 0$
$|\ max\text{-}vars(Cast\ C\ e) = max\text{-}vars\ e$
$|\ max\text{-}vars(Val\ v) = 0$
$|\ max\text{-}vars(e_1 \ll bop \gg e_2) = max\ (max\text{-}vars\ e_1)\ (max\text{-}vars\ e_2)$
$|\ max\text{-}vars(Var\ V) = 0$
$|\ max\text{-}vars(V{:=}e) = max\text{-}vars\ e$
$|\ max\text{-}vars(e{\bullet}F\{D\}) = max\text{-}vars\ e$
$|\ max\text{-}vars(FAss\ e_1\ F\ D\ e_2) = max\ (max\text{-}vars\ e_1)\ (max\text{-}vars\ e_2)$
$|\ max\text{-}vars(e{\bullet}M(es)) = max\ (max\text{-}vars\ e)\ (max\text{-}varss\ es)$
$|\ max\text{-}vars(\{V{:}T;\ e\}) = max\text{-}vars\ e + 1$
$|\ max\text{-}vars(e_1;;e_2) = max\ (max\text{-}vars\ e_1)\ (max\text{-}vars\ e_2)$
$|\ max\text{-}vars(if\ (e)\ e_1\ else\ e_2) =$
$max\ (max\text{-}vars\ e)\ (max\ (max\text{-}vars\ e_1)\ (max\text{-}vars\ e_2))$
$|\ max\text{-}vars(while\ (b)\ e) = max\ (max\text{-}vars\ b)\ (max\text{-}vars\ e)$
$|\ max\text{-}vars(throw\ e) = max\text{-}vars\ e$
$|\ max\text{-}vars(try\ e_1\ catch(C\ V)\ e_2) = max\ (max\text{-}vars\ e_1)\ (max\text{-}vars\ e_2 + 1)$

$|\ max\text{-}varss\ [] = 0$
$|\ max\text{-}varss\ (e\#es) = max\ (max\text{-}vars\ e)\ (max\text{-}varss\ es)$

inductive
$eval_1 :: J_1\text{-}prog \Rightarrow expr_1 \Rightarrow state_1 \Rightarrow expr_1 \Rightarrow state_1 \Rightarrow bool$
$(\text{-} \vdash_1 ((1\langle\text{-},/\text{-}\rangle) \Rightarrow/\ (1\langle\text{-},/\text{-}\rangle))\ [51,0,0,0,0]\ 81)$
and $evals_1 :: J_1\text{-}prog \Rightarrow expr_1\ list \Rightarrow state_1 \Rightarrow expr_1\ list \Rightarrow state_1 \Rightarrow bool$
$(\text{-} \vdash_1 ((1\langle\text{-},/\text{-}\rangle)\ [\Rightarrow]/\ (1\langle\text{-},/\text{-}\rangle))\ [51,0,0,0,0]\ 81)$
for $P :: J_1\text{-}prog$
where

New_1:
$[\![\ new\text{-}Addr\ h = Some\ a;\ P \vdash C\ has\text{-}fields\ FDTs;\ h' = h(a{\mapsto}(C,init\text{-}fields\ FDTs))\ ]\!]$
$\Longrightarrow P \vdash_1 \langle new\ C,(h,l)\rangle \Rightarrow \langle addr\ a,(h',l)\rangle$
$|\ NewFail_1$:
$new\text{-}Addr\ h = None \Longrightarrow$
$P \vdash_1 \langle new\ C,\ (h,l)\rangle \Rightarrow \langle THROW\ OutOfMemory,(h,l)\rangle$

$|\ Cast_1$:
$[\![\ P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle addr\ a,(h,l)\rangle;\ h\ a = Some(D,fs);\ P \vdash D \preceq^* C\ ]\!]$
$\Longrightarrow P \vdash_1 \langle Cast\ C\ e,s_0\rangle \Rightarrow \langle addr\ a,(h,l)\rangle$
$|\ CastNull_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle null,s_1\rangle \Longrightarrow$
$P \vdash_1 \langle Cast\ C\ e,s_0\rangle \Rightarrow \langle null,s_1\rangle$
$|\ CastFail_1$:

$\llbracket\ P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle addr\ a,(h,l)\rangle;\ h\ a = Some(D,fs);\ \neg\ P \vdash D \preceq^* C\ \rrbracket$
$\Longrightarrow P \vdash_1 \langle Cast\ C\ e,s_0\rangle \Rightarrow \langle THROW\ ClassCast,(h,l)\rangle$
| $CastThrow_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash_1 \langle Cast\ C\ e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$

| Val_1:
$P \vdash_1 \langle Val\ v,s\rangle \Rightarrow \langle Val\ v,s\rangle$

| $BinOp_1$:
$\llbracket\ P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle Val\ v_1,s_1\rangle;\ P \vdash_1 \langle e_2,s_1\rangle \Rightarrow \langle Val\ v_2,s_2\rangle;\ binop(bop,v_1,v_2) = Some\ v\ \rrbracket$
$\Longrightarrow P \vdash_1 \langle e_1 \ll bop \gg e_2,s_0\rangle \Rightarrow \langle Val\ v,s_2\rangle$
| $BinOpThrow_{11}$:
$P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle throw\ e,s_1\rangle \Longrightarrow$
$P \vdash_1 \langle e_1 \ll bop \gg e_2,\ s_0\rangle \Rightarrow \langle throw\ e,s_1\rangle$
| $BinOpThrow_{21}$:
$\llbracket\ P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle Val\ v_1,s_1\rangle;\ P \vdash_1 \langle e_2,s_1\rangle \Rightarrow \langle throw\ e,s_2\rangle\ \rrbracket$
$\Longrightarrow P \vdash_1 \langle e_1 \ll bop \gg e_2,s_0\rangle \Rightarrow \langle throw\ e,s_2\rangle$

| Var_1:
$\llbracket\ ls!i = v;\ i < size\ ls\ \rrbracket \Longrightarrow$
$P \vdash_1 \langle Var\ i,(h,ls)\rangle \Rightarrow \langle Val\ v,(h,ls)\rangle$

| $LAss_1$:
$\llbracket\ P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle Val\ v,(h,ls)\rangle;\ i < size\ ls;\ ls' = ls[i := v]\ \rrbracket$
$\Longrightarrow P \vdash_1 \langle i{:=}\ e,s_0\rangle \Rightarrow \langle unit,(h,ls')\rangle$
| $LAssThrow_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash_1 \langle i{:=}\ e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$

| $FAcc_1$:
$\llbracket\ P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle addr\ a,(h,ls)\rangle;\ h\ a = Some(C,fs);\ fs(F,D) = Some\ v\ \rrbracket$
$\Longrightarrow P \vdash_1 \langle e{\cdot}F\{D\},s_0\rangle \Rightarrow \langle Val\ v,(h,ls)\rangle$
| $FAccNull_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle null,s_1\rangle \Longrightarrow$
$P \vdash_1 \langle e{\cdot}F\{D\},s_0\rangle \Rightarrow \langle THROW\ NullPointer,s_1\rangle$
| $FAccThrow_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash_1 \langle e{\cdot}F\{D\},s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$

| $FAss_1$:
$\llbracket\ P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle addr\ a,s_1\rangle;\ P \vdash_1 \langle e_2,s_1\rangle \Rightarrow \langle Val\ v,(h_2,l_2)\rangle;$
$h_2\ a = Some(C,fs);\ fs' = fs((F,D)\mapsto v);\ h_2' = h_2(a\mapsto(C,fs'))\ \rrbracket$
$\Longrightarrow P \vdash_1 \langle e_1{\cdot}F\{D\}{:=}\ e_2,s_0\rangle \Rightarrow \langle unit,(h_2',l_2)\rangle$
| $FAssNull_1$:
$\llbracket\ P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle null,s_1\rangle;\ \ P \vdash_1 \langle e_2,s_1\rangle \Rightarrow \langle Val\ v,s_2\rangle\ \rrbracket$
$\Longrightarrow P \vdash_1 \langle e_1{\cdot}F\{D\}{:=}\ e_2,s_0\rangle \Rightarrow \langle THROW\ NullPointer,s_2\rangle$
| $FAssThrow_{11}$:
$P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash_1 \langle e_1{\cdot}F\{D\}{:=}\ e_2,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$
| $FAssThrow_{21}$:
$\llbracket\ P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle Val\ v,s_1\rangle;\ P \vdash_1 \langle e_2,s_1\rangle \Rightarrow \langle throw\ e',s_2\rangle\ \rrbracket$
$\Longrightarrow P \vdash_1 \langle e_1{\cdot}F\{D\}{:=}\ e_2,s_0\rangle \Rightarrow \langle throw\ e',s_2\rangle$

| $CallObjThrow_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash_1 \langle e{\cdot}M(es),s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$
| $CallNull_1$:
$[\![\ P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle null,s_1\rangle;\ P \vdash_1 \langle es,s_1\rangle\ [\Rightarrow]\ \langle map\ Val\ vs,s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e{\cdot}M(es),s_0\rangle \Rightarrow \langle THROW\ NullPointer,s_2\rangle$
| $Call_1$:
$[\![\ P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle addr\ a,s_1\rangle;\ P \vdash_1 \langle es,s_1\rangle\ [\Rightarrow]\ \langle map\ Val\ vs,(h_2,ls_2)\rangle;$
$h_2\ a = Some(C,fs);\ P \vdash C\ sees\ M{:}Ts{\rightarrow}T = body\ in\ D;$
$size\ vs = size\ Ts;\ ls_2{}' = (Addr\ a)\ \#\ vs\ @\ replicate\ (max\text{-}vars\ body)\ undefined;$
$P \vdash_1 \langle body,(h_2,ls_2{}')\rangle \Rightarrow \langle e',(h_3,ls_3)\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e{\cdot}M(es),s_0\rangle \Rightarrow \langle e',(h_3,ls_2)\rangle$
| $CallParamsThrow_1$:
$[\![\ P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle Val\ v,s_1\rangle;\ P \vdash_1 \langle es,s_1\rangle\ [\Rightarrow]\ \langle es',s_2\rangle;$
$es' = map\ Val\ vs\ @\ throw\ ex\ \#\ es_2\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e{\cdot}M(es),s_0\rangle \Rightarrow \langle throw\ ex,s_2\rangle$

| $Block_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle e',s_1\rangle \Longrightarrow P \vdash_1 \langle Block\ i\ T\ e,s_0\rangle \Rightarrow \langle e',s_1\rangle$

| Seq_1:
$[\![\ P \vdash_1 \langle e_0,s_0\rangle \Rightarrow \langle Val\ v,s_1\rangle;\ P \vdash_1 \langle e_1,s_1\rangle \Rightarrow \langle e_2,s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e_0;;e_1,s_0\rangle \Rightarrow \langle e_2,s_2\rangle$
| $SeqThrow_1$:
$P \vdash_1 \langle e_0,s_0\rangle \Rightarrow \langle throw\ e,s_1\rangle \Longrightarrow$
$P \vdash_1 \langle e_0;;e_1,s_0\rangle \Rightarrow \langle throw\ e,s_1\rangle$

| $CondT_1$:
$[\![\ P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle true,s_1\rangle;\ P \vdash_1 \langle e_1,s_1\rangle \Rightarrow \langle e',s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle if\ (e)\ e_1\ else\ e_2,s_0\rangle \Rightarrow \langle e',s_2\rangle$
| $CondF_1$:
$[\![\ P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle false,s_1\rangle;\ P \vdash_1 \langle e_2,s_1\rangle \Rightarrow \langle e',s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle if\ (e)\ e_1\ else\ e_2,s_0\rangle \Rightarrow \langle e',s_2\rangle$
| $CondThrow_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash_1 \langle if\ (e)\ e_1\ else\ e_2,\ s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$

| $WhileF_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle false,s_1\rangle \Longrightarrow$
$P \vdash_1 \langle while\ (e)\ c,s_0\rangle \Rightarrow \langle unit,s_1\rangle$
| $WhileT_1$:
$[\![\ P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle true,s_1\rangle;\ P \vdash_1 \langle c,s_1\rangle \Rightarrow \langle Val\ v_1,s_2\rangle;$
$P \vdash_1 \langle while\ (e)\ c,s_2\rangle \Rightarrow \langle e_3,s_3\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle while\ (e)\ c,s_0\rangle \Rightarrow \langle e_3,s_3\rangle$
| $WhileCondThrow_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash_1 \langle while\ (e)\ c,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$
| $WhileBodyThrow_1$:
$[\![\ P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle true,s_1\rangle;\ P \vdash_1 \langle c,s_1\rangle \Rightarrow \langle throw\ e',s_2\rangle]\!]$
$\Longrightarrow P \vdash_1 \langle while\ (e)\ c,s_0\rangle \Rightarrow \langle throw\ e',s_2\rangle$

| $Throw_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle addr\ a,s_1\rangle \Longrightarrow$
$P \vdash_1 \langle throw\ e,s_0\rangle \Rightarrow \langle Throw\ a,s_1\rangle$

| *ThrowNull*$_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle null,s_1\rangle \Longrightarrow$
$P \vdash_1 \langle throw\ e,s_0\rangle \Rightarrow \langle THROW\ NullPointer,s_1\rangle$
| *ThrowThrow*$_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash_1 \langle throw\ e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$

| *Try*$_1$:
$P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle Val\ v_1,s_1\rangle \Longrightarrow$
$P \vdash_1 \langle try\ e_1\ catch(C\ i)\ e_2,s_0\rangle \Rightarrow \langle Val\ v_1,s_1\rangle$
| *TryCatch*$_1$:
$[\![\ P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle Throw\ a,(h_1,ls_1)\rangle;$
$h_1\ a = Some(D,fs);\ P \vdash D \preceq^* C;\ i < length\ ls_1;$
$P \vdash_1 \langle e_2,(h_1,ls_1[i{:=}Addr\ a])\rangle \Rightarrow \langle e_2',(h_2,ls_2)\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle try\ e_1\ catch(C\ i)\ e_2,s_0\rangle \Rightarrow \langle e_2',(h_2,ls_2)\rangle$
| *TryThrow*$_1$:
$[\![\ P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle Throw\ a,(h_1,ls_1)\rangle;\ h_1\ a = Some(D,fs);\ \neg\ P \vdash D \preceq^* C\ ]\!]$
$\Longrightarrow P \vdash_1 \langle try\ e_1\ catch(C\ i)\ e_2,s_0\rangle \Rightarrow \langle Throw\ a,(h_1,ls_1)\rangle$

| *Nil*$_1$:
$P \vdash_1 \langle [],s\rangle\ [\Rightarrow]\ \langle [],s\rangle$

| *Cons*$_1$:
$[\![\ P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle Val\ v,s_1\rangle;\ P \vdash_1 \langle es,s_1\rangle\ [\Rightarrow]\ \langle es',s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e\#es,s_0\rangle\ [\Rightarrow]\ \langle Val\ v\ \#\ es',s_2\rangle$
| *ConsThrow*$_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash_1 \langle e\#es,s_0\rangle\ [\Rightarrow]\ \langle throw\ e'\ \#\ es,\ s_1\rangle$

lemma *eval*$_1$*-preserves-len*:
$P \vdash_1 \langle e_0,(h_0,ls_0)\rangle \Rightarrow \langle e_1,(h_1,ls_1)\rangle \Longrightarrow length\ ls_0 = length\ ls_1$
and *evals*$_1$*-preserves-len*:
$P \vdash_1 \langle es_0,(h_0,ls_0)\rangle\ [\Rightarrow]\ \langle es_1,(h_1,ls_1)\rangle \Longrightarrow length\ ls_0 = length\ ls_1$

lemma *evals*$_1$*-preserves-elen*:
$\bigwedge es'\ s\ s'.\ P \vdash_1 \langle es,s\rangle\ [\Rightarrow]\ \langle es',s'\rangle \Longrightarrow length\ es = length\ es'$

lemma *eval*$_1$*-final*: $P \vdash_1 \langle e,s\rangle \Rightarrow \langle e',s'\rangle \Longrightarrow final\ e'$
and *evals*$_1$*-final*: $P \vdash_1 \langle es,s\rangle\ [\Rightarrow]\ \langle es',s'\rangle \Longrightarrow finals\ es'$

end

5.2 Well-Formedness of Intermediate Language

theory *J1WellForm*
imports *../J/JWellForm J1*
begin

5.2.1 Well-Typedness

type-synonym
env_1 = *ty list* — type environment indexed by variable number

inductive
WT_1 :: [J_1-*prog*,env_1, $expr_1$, *ty*] ⇒ *bool*
((-,- $\vdash_1$/ - :: -) [*51*,*51*,*51*]*50*)
and WTs_1 :: [J_1-*prog*,env_1, $expr_1$ *list*, *ty list*] ⇒ *bool*
((-,- $\vdash_1$/ - [::] -) [*51*,*51*,*51*]*50*)
for P :: J_1-*prog*
where

$WTNew_1$:
$is\text{-}class\ P\ C \Longrightarrow$
$P,E \vdash_1 new\ C :: Class\ C$

| $WTCast_1$:
$[\![\ P,E \vdash_1 e :: Class\ D;\ \ is\text{-}class\ P\ C;\ \ P \vdash C \preceq^* D \vee P \vdash D \preceq^* C\ ]\!]$
$\Longrightarrow P,E \vdash_1 Cast\ C\ e :: Class\ C$

| $WTVal_1$:
$typeof\ v = Some\ T \Longrightarrow$
$P,E \vdash_1 Val\ v :: T$

| $WTVar_1$:
$[\![\ E!i = T;\ i < size\ E\ ]\!]$
$\Longrightarrow P,E \vdash_1 Var\ i :: T$

| $WTBinOp_1$:
$[\![\ P,E \vdash_1 e_1 :: T_1;\ \ P,E \vdash_1 e_2 :: T_2;$
$case\ bop\ of\ Eq \Rightarrow (P \vdash T_1 \leq T_2 \vee P \vdash T_2 \leq T_1) \wedge T = Boolean$
$\mid Add \Rightarrow T_1 = Integer \wedge T_2 = Integer \wedge T = Integer\ ]\!]$
$\Longrightarrow P,E \vdash_1 e_1 \ll bop \gg e_2 :: T$

| $WTLAss_1$:
$[\![\ E!i = T;\ \ i < size\ E;\ P,E \vdash_1 e :: T';\ \ P \vdash T' \leq T\ ]\!]$
$\Longrightarrow P,E \vdash_1 i{:=}e :: Void$

| $WTFAcc_1$:
$[\![\ P,E \vdash_1 e :: Class\ C;\ \ P \vdash C\ sees\ F{:}T\ in\ D\ ]\!]$
$\Longrightarrow P,E \vdash_1 e{\cdot}F\{D\} :: T$

| $WTFAss_1$:
$[\![\ P,E \vdash_1 e_1 :: Class\ C;\ \ P \vdash C\ sees\ F{:}T\ in\ D;\ \ P,E \vdash_1 e_2 :: T';\ \ P \vdash T' \leq T\ ]\!]$
$\Longrightarrow P,E \vdash_1 e_1{\cdot}F\{D\} := e_2 :: Void$

| $WTCall_1$:

$[\![$ $P,E \vdash_1 e :: Class\ C$; $P \vdash C\ sees\ M{:}Ts' \rightarrow T = m\ in\ D$;
$P,E \vdash_1 es\ [::]\ Ts$; $P \vdash Ts\ [\leq]\ Ts'$ $]\!]$
$\Longrightarrow P,E \vdash_1 e{\bullet}M(es) :: T$

| $WTBlock_1$:
$[\![$ $is\text{-}type\ P\ T$; $P,E@[T] \vdash_1 e{::}T'$ $]\!]$
$\Longrightarrow$ $P,E \vdash_1 \{i{:}T;\ e\} :: T'$

| $WTSeq_1$:
$[\![$ $P,E \vdash_1 e_1{::}T_1$; $P,E \vdash_1 e_2{::}T_2$ $]\!]$
$\Longrightarrow$ $P,E \vdash_1 e_1;;e_2 :: T_2$

| $WTCond_1$:
$[\![$ $P,E \vdash_1 e :: Boolean$; $P,E \vdash_1 e_1{::}T_1$; $P,E \vdash_1 e_2{::}T_2$;
$P \vdash T_1 \leq T_2 \vee P \vdash T_2 \leq T_1$; $P \vdash T_1 \leq T_2 \longrightarrow T = T_2$; $P \vdash T_2 \leq T_1 \longrightarrow T = T_1$ $]\!]$
$\Longrightarrow P,E \vdash_1 if\ (e)\ e_1\ else\ e_2 :: T$

| $WTWhile_1$:
$[\![$ $P,E \vdash_1 e :: Boolean$; $P,E \vdash_1 c{::}T$ $]\!]$
$\Longrightarrow P,E \vdash_1 while\ (e)\ c :: Void$

| $WTThrow_1$:
$P,E \vdash_1 e :: Class\ C \Longrightarrow$
$P,E \vdash_1 throw\ e :: Void$

| $WTTry_1$:
$[\![$ $P,E \vdash_1 e_1 :: T$; $P,E@[Class\ C] \vdash_1 e_2 :: T$; $is\text{-}class\ P\ C$ $]\!]$
$\Longrightarrow P,E \vdash_1 try\ e_1\ catch(C\ i)\ e_2 :: T$

| $WTNil_1$:
$P,E \vdash_1 []\ [::]\ []$

| $WTCons_1$:
$[\![$ $P,E \vdash_1 e :: T$; $P,E \vdash_1 es\ [::]\ Ts$ $]\!]$
$\Longrightarrow$ $P,E \vdash_1 e\#es\ [::]\ T\#Ts$

lemma $WTs_1\text{-}same\text{-}size$: $\bigwedge Ts.\ P,E \vdash_1 es\ [::]\ Ts \Longrightarrow size\ es = size\ Ts$

lemma $WT_1\text{-}unique$:
$P,E \vdash_1 e :: T_1 \Longrightarrow (\bigwedge T_2.\ P,E \vdash_1 e :: T_2 \Longrightarrow T_1 = T_2)$ **and**
$P,E \vdash_1 es\ [::]\ Ts_1 \Longrightarrow (\bigwedge Ts_2.\ P,E \vdash_1 es\ [::]\ Ts_2 \Longrightarrow Ts_1 = Ts_2)$

lemma assumes wf: $wf\text{-}prog\ p\ P$
shows $WT_1\text{-}is\text{-}type$: $P,E \vdash_1 e :: T \Longrightarrow set\ E \subseteq types\ P \Longrightarrow is\text{-}type\ P\ T$
and $P,E \vdash_1 es\ [::]\ Ts \Longrightarrow True$

5.2.2 Well-formedness

— Indices in blocks increase by 1

primrec $\mathcal{B} :: expr_1 \Rightarrow nat \Rightarrow bool$
and $\mathcal{B}s :: expr_1\ list \Rightarrow nat \Rightarrow bool$ **where**
$\mathcal{B}\ (new\ C)\ i = True$ |
$\mathcal{B}\ (Cast\ C\ e)\ i = \mathcal{B}\ e\ i$ |

$\mathcal{B}$ (*Val v*) i = *True* |
$\mathcal{B}$ (e_1 «*bop*» e_2) i = ($\mathcal{B}$ e_1 i ∧ $\mathcal{B}$ e_2 i) |
$\mathcal{B}$ (*Var j*) i = *True* |
$\mathcal{B}$ (e•F{D}) i = $\mathcal{B}$ e i |
$\mathcal{B}$ (j:=e) i = $\mathcal{B}$ e i |
$\mathcal{B}$ (e_1•F{D} := e_2) i = ($\mathcal{B}$ e_1 i ∧ $\mathcal{B}$ e_2 i) |
$\mathcal{B}$ (e•M(es)) i = ($\mathcal{B}$ e i ∧ $\mathcal{B}s$ es i) |
$\mathcal{B}$ ({j:T ; e}) i = (i = j ∧ $\mathcal{B}$ e (i+*1*)) |
$\mathcal{B}$ (e_1;;e_2) i = ($\mathcal{B}$ e_1 i ∧ $\mathcal{B}$ e_2 i) |
$\mathcal{B}$ (*if* (e) e_1 *else* e_2) i = ($\mathcal{B}$ e i ∧ $\mathcal{B}$ e_1 i ∧ $\mathcal{B}$ e_2 i) |
$\mathcal{B}$ (*throw* e) i = $\mathcal{B}$ e i |
$\mathcal{B}$ (*while* (e) c) i = ($\mathcal{B}$ e i ∧ $\mathcal{B}$ c i) |
$\mathcal{B}$ (*try* e_1 *catch*(C j) e_2) i = ($\mathcal{B}$ e_1 i ∧ i=j ∧ $\mathcal{B}$ e_2 (i+*1*)) |

$\mathcal{B}s$ [] i = *True* |
$\mathcal{B}s$ (e#es) i = ($\mathcal{B}$ e i ∧ $\mathcal{B}s$ es i)

definition *wf-J*$_1$*-mdecl* :: *J*$_1$*-prog* ⇒ *cname* ⇒ *expr*$_1$ *mdecl* ⇒ *bool*
where
  wf-J$_1$*-mdecl P C* ≡ λ(M,Ts,T,*body*).
    (∃ T'. P,*Class* C#Ts ⊢$_1$ *body* :: T' ∧ P ⊢ T' ≤ T) ∧
    $\mathcal{D}$ *body* ⌊{..*size Ts*}⌋ ∧ $\mathcal{B}$ *body* (*size Ts* + *1*)

lemma *wf-J*$_1$*-mdecl*[*simp*]:
  wf-J$_1$*-mdecl P C* (M,Ts,T,*body*) ≡
    ((∃ T'. P,*Class* C#Ts ⊢$_1$ *body* :: T' ∧ P ⊢ T' ≤ T) ∧
     $\mathcal{D}$ *body* ⌊{..*size Ts*}⌋ ∧ $\mathcal{B}$ *body* (*size Ts* + *1*))
abbreviation *wf-J*$_1$*-prog* == *wf-prog wf-J*$_1$*-mdecl*

end

5.3 Program Compilation

theory *PCompiler*
imports ../*Common*/*WellForm*
begin

definition $compM$ :: $('a \Rightarrow 'b) \Rightarrow 'a\ mdecl \Rightarrow 'b\ mdecl$
where
$compM\ f \equiv \lambda(M,\ Ts,\ T,\ m).\ (M,\ Ts,\ T,\ f\ m)$

definition $compC$ :: $('a \Rightarrow 'b) \Rightarrow 'a\ cdecl \Rightarrow 'b\ cdecl$
where
$compC\ f \equiv \lambda(C,D,Fdecls,Mdecls).\ (C,D,Fdecls,\ map\ (compM\ f)\ Mdecls)$

definition $compP$ :: $('a \Rightarrow 'b) \Rightarrow 'a\ prog \Rightarrow 'b\ prog$
where
$compP\ f \equiv map\ (compC\ f)$

Compilation preserves the program structure. Therfore lookup functions either commute with compilation (like method lookup) or are preserved by it (like the subclass relation).

lemma *map-of-map4*:
$map\text{-}of\ (map\ (\lambda(x,a,b,c).(x,a,b,f\ c))\ ts) =$
$Option.map\ (\lambda(a,b,c).(a,b,f\ c)) \circ (map\text{-}of\ ts)$

lemma *class-compP*:
$class\ P\ C = Some\ (D,\ fs,\ ms)$
$\Longrightarrow class\ (compP\ f\ P)\ C = Some\ (D,\ fs,\ map\ (compM\ f)\ ms)$

lemma *class-compPD*:
$class\ (compP\ f\ P)\ C = Some\ (D,\ fs,\ cms)$
$\Longrightarrow \exists ms.\ class\ P\ C = Some(D,fs,ms) \wedge cms = map\ (compM\ f)\ ms$

lemma [*simp*]: $is\text{-}class\ (compP\ f\ P)\ C = is\text{-}class\ P\ C$

lemma [*simp*]: $class\ (compP\ f\ P)\ C = Option.map\ (\lambda c.\ snd(compC\ f\ (C,c)))\ (class\ P\ C)$

lemma *sees-methods-compP*:
$P \vdash C\ sees\text{-}methods\ Mm \Longrightarrow$
$compP\ f\ P \vdash C\ sees\text{-}methods\ (Option.map\ (\lambda((Ts,T,m),D).\ ((Ts,T,f\ m),D)) \circ Mm)$

lemma *sees-method-compP*:
$P \vdash C\ sees\ M\colon\ Ts \rightarrow T = m\ in\ D \Longrightarrow$
$compP\ f\ P \vdash C\ sees\ M\colon\ Ts \rightarrow T = (f\ m)\ in\ D$

lemma [*simp*]:
$P \vdash C\ sees\ M\colon\ Ts \rightarrow T = m\ in\ D \Longrightarrow$
$method\ (compP\ f\ P)\ C\ M = (D,Ts,T,f\ m)$

lemma *sees-methods-compPD*:
$[\![\ cP \vdash C\ sees\text{-}methods\ Mm';\ cP = compP\ f\ P\ ]\!] \Longrightarrow$
$\exists Mm.\ P \vdash C\ sees\text{-}methods\ Mm \wedge$
$\quad Mm' = (Option.map\ (\lambda((Ts,T,m),D).\ ((Ts,T,f\ m),D)) \circ Mm)$

lemma *sees-method-compPD*:

$compP\ f\ P \vdash C\ sees\ M\colon\ Ts \to T = fm\ in\ D \Longrightarrow$
$\exists\, m.\ P \vdash C\ sees\ M\colon\ Ts \to T = m\ in\ D \wedge f\ m = fm$

lemma [*simp*]: $subcls1(compP\ f\ P) = subcls1\ P$

lemma *compP-widen*[*simp*]: $(compP\ f\ P \vdash T \le T') = (P \vdash T \le T')$

lemma [*simp*]: $(compP\ f\ P \vdash Ts\ [\le]\ Ts') = (P \vdash Ts\ [\le]\ Ts')$

lemma [*simp*]: $is\text{-}type\ (compP\ f\ P)\ T = is\text{-}type\ P\ T$

lemma [*simp*]: $(compP\ (f{::}'a \Rightarrow 'b)\ P \vdash C\ has\text{-}fields\ FDTs) = (P \vdash C\ has\text{-}fields\ FDTs)$

lemma [*simp*]: $fields\ (compP\ f\ P)\ C = fields\ P\ C$

lemma [*simp*]: $(compP\ f\ P \vdash C\ sees\ F{:}T\ in\ D) = (P \vdash C\ sees\ F{:}T\ in\ D)$

lemma [*simp*]: $field\ (compP\ f\ P)\ F\ D = field\ P\ F\ D$

5.3.1 Invariance of *wf-prog* under compilation

lemma [*iff*]: $distinct\text{-}fst\ (compP\ f\ P) = distinct\text{-}fst\ P$

lemma [*iff*]: $distinct\text{-}fst\ (map\ (compM\ f)\ ms) = distinct\text{-}fst\ ms$

lemma [*iff*]: $wf\text{-}syscls\ (compP\ f\ P) = wf\text{-}syscls\ P$

lemma [*iff*]: $wf\text{-}fdecl\ (compP\ f\ P) = wf\text{-}fdecl\ P$

lemma *set-compP*:
$((C,D,fs,ms') \in set(compP\ f\ P)) =$
$(\exists\, ms.\ (C,D,fs,ms) \in set\ P \wedge ms' = map\ (compM\ f)\ ms)$

lemma *wf-cdecl-compPI*:
$[\![\ \bigwedge C\ M\ Ts\ T\ m.$
$[\![\ wf\text{-}mdecl\ wf_1\ P\ C\ (M,Ts,T,m);\ P \vdash C\ sees\ M{:}Ts \to T = m\ in\ C\ ]\!]$
$\Longrightarrow wf\text{-}mdecl\ wf_2\ (compP\ f\ P)\ C\ (M,Ts,T,\ f\ m);$
$\forall\, x{\in}set\ P.\ wf\text{-}cdecl\ wf_1\ P\ x;\ x \in set\ (compP\ f\ P);\ wf\text{-}prog\ p\ P\ ]\!]$
$\Longrightarrow wf\text{-}cdecl\ wf_2\ (compP\ f\ P)\ x$

lemma *wf-prog-compPI*:
assumes *lift*:
$\bigwedge C\ M\ Ts\ T\ m.$
$[\![\ P \vdash C\ sees\ M{:}Ts \to T = m\ in\ C;\ wf\text{-}mdecl\ wf_1\ P\ C\ (M,Ts,T,m)\ ]\!]$
$\Longrightarrow wf\text{-}mdecl\ wf_2\ (compP\ f\ P)\ C\ (M,Ts,T,\ f\ m)$
and *wf*: $wf\text{-}prog\ wf_1\ P$
shows $wf\text{-}prog\ wf_2\ (compP\ f\ P)$

end

theory *List-Index* **imports** *Main* **begin**

This theory defines three functions for finding the index of items in a list:

find-index P xs finds the index of the first element in *xs* that satisfies *P*.

index xs x finds the index of the first occurrence of *x* in *xs*.

last-index xs x finds the index of the last occurrence of *x* in *xs*.

All functions return *length xs* if *xs* does not contain a suitable element.

The argument order of *find-index* follows the function of the same name in the Haskell standard library. For *index* (and *last-index*) the order is intentionally reversed: *index* maps lists to a mapping from elements to their indices, almost the inverse of function *nth*.

primrec *find-index* :: $('a \Rightarrow bool) \Rightarrow 'a\ list \Rightarrow nat$ **where**
find-index - [] = *0* |
find-index P $(x\#xs)$ = (*if P x then 0 else find-index P xs* + *1*)

definition *index* :: $'a\ list \Rightarrow 'a \Rightarrow nat$ **where**
index xs = (λa. *find-index* (λx. $x{=}a$) *xs*)

definition *last-index* :: $'a\ list \Rightarrow 'a \Rightarrow nat$ **where**
last-index xs x =
(*let i* = *index* (*rev xs*) *x*; *n* = *size xs*
in if i = *n then i else n* − (*i*+*1*))

lemma *find-index-le-size*: *find-index P xs* <= *size xs*
by(*induct xs*) *simp-all*

lemma *index-le-size*: *index xs x* <= *size xs*
by(*simp add*: *index-def find-index-le-size*)

lemma *last-index-le-size*: *last-index xs x* <= *size xs*
by(*simp add*: *last-index-def Let-def index-le-size*)

lemma *index-Nil*[*simp*]: *index* [] *a* = *0*
by(*simp add*: *index-def*)

lemma *index-Cons*[*simp*]: *index* $(x\#xs)$ *a* = (*if* $x{=}a$ *then 0 else index xs a* + *1*)
by(*simp add*: *index-def*)

lemma *index-append*: *index* (*xs* @ *ys*) *x* =
(*if x* : *set xs then index xs x else size xs* + *index ys x*)
by (*induct xs*) *simp-all*

lemma *index-conv-size-if-notin*[*simp*]: $x \notin$ *set xs* $\Longrightarrow$ *index xs x* = *size xs*
by (*induct xs*) *auto*

lemma *find-index-eq-size-conv*:
size xs = *n* $\Longrightarrow$ (*find-index P xs* = *n*) = (*ALL x* : *set xs*. ~ *P x*)
by(*induct xs arbitrary*: *n*) *auto*

lemma *size-eq-find-index-conv*:
size xs = *n* $\Longrightarrow$ (*n* = *find-index P xs*) = (*ALL x* : *set xs*. ~ *P x*)
by(*metis find-index-eq-size-conv*)

lemma *index-size-conv*: *size xs* = *n* $\Longrightarrow$ (*index xs x* = *n*) = ($x \notin$ *set xs*)
by(*auto simp*: *index-def find-index-eq-size-conv*)

lemma *size-index-conv*: *size xs* = *n* $\Longrightarrow$ (*n* = *index xs x*) = (*x* $\notin$ *set xs*)
by (*metis index-size-conv*)

lemma *last-index-size-conv*:
  size xs = *n* $\Longrightarrow$ (*last-index xs x* = *n*) = (*x* $\notin$ *set xs*)
apply(*auto simp*: *last-index-def index-size-conv*)
apply(*drule length-pos-if-in-set*)
apply *arith*
done

lemma *size-last-index-conv*:
  size xs = *n* $\Longrightarrow$ (*n* = *last-index xs x*) = (*x* $\notin$ *set xs*)
by (*metis last-index-size-conv*)

lemma *find-index-less-size-conv*:
  (*find-index P xs* < *size xs*) = (*EX x* : *set xs*. *P x*)
by (*induct xs*) *auto*

lemma *index-less-size-conv*:
  (*index xs x* < *size xs*) = (*x* $\in$ *set xs*)
by(*auto simp*: *index-def find-index-less-size-conv*)

lemma *last-index-less-size-conv*:
  (*last-index xs x* < *size xs*) = (*x* : *set xs*)
by(*simp add*: *last-index-def Let-def index-size-conv length-pos-if-in-set*
        del:*length-greater-0-conv*)

lemma *index-less*[*simp*]:
  x : *set xs* $\Longrightarrow$ *size xs* <= *n* $\Longrightarrow$ *index xs x* < *n*
apply(*induct xs*) **apply** *auto*
apply (*metis index-less-size-conv less-eq-Suc-le less-trans-Suc*)
done

lemma *last-index-less*[*simp*]:
  x : *set xs* $\Longrightarrow$ *size xs* <= *n* $\Longrightarrow$ *last-index xs x* < *n*
by(*simp add*: *last-index-less-size-conv*[*symmetric*])

lemma *last-index-Cons*: *last-index* (*x#xs*) *y* =
  (*if x=y then*
      if x $\in$ *set xs then last-index xs y* + *1 else 0*
   else last-index xs y + *1*)
using *index-le-size*[*of rev xs y*]
apply(*auto simp add*: *last-index-def index-append Let-def*)
apply(*simp add*: *index-size-conv*)
done

lemma *last-index-append*: *last-index* (*xs* @ *ys*) *x* =
  (*if x* : *set ys then size xs* + *last-index ys x*
   else if x : *set xs then last-index xs x else size xs* + *size ys*)
by (*induct xs*) (*simp-all add*: *last-index-Cons last-index-size-conv*)

lemma *last-index-Snoc*[*simp*]:
  last-index (*xs* @ [*x*]) *y* =

(*if* $x{=}y$ *then size xs*
else if y : *set xs then last-index xs y else size xs* + *1*)
by(*simp add*: *last-index-append last-index-Cons*)

lemma *nth-find-index*: *find-index P xs* < *size xs* $\Longrightarrow$ *P*(*xs* ! *find-index P xs*)
by (*induct xs*) *auto*

lemma *nth-index*[*simp*]: $x \in$ *set xs* $\Longrightarrow$ *xs* ! *index xs x* = *x*
by (*induct xs*) *auto*

lemma *nth-last-index*[*simp*]: $x \in$ *set xs* $\Longrightarrow$ *xs* ! *last-index xs x* = *x*
by(*simp add*:*last-index-def index-size-conv Let-def rev-nth*[*symmetric*])

lemma *index-eq-index-conv*[*simp*]: $x \in$ *set xs* $\vee$ $y \in$ *set xs* $\Longrightarrow$
(*index xs x* = *index xs y*) = (x = y)
by (*induct xs*) *auto*

lemma *last-index-eq-index-conv*[*simp*]: $x \in$ *set xs* $\vee$ $y \in$ *set xs* $\Longrightarrow$
(*last-index xs x* = *last-index xs y*) = (x = y)
by (*induct xs*) (*auto simp*:*last-index-Cons*)

lemma *inj-on-index*: *inj-on* (*index xs*) (*set xs*)
by (*simp add*:*inj-on-def*)

lemma *inj-on-last-index*: *inj-on* (*last-index xs*) (*set xs*)
by (*simp add*:*inj-on-def*)

lemma *index-conv-takeWhile*: *index xs x* = *size*(*takeWhile* ($\lambda y.\ x{\neq}y$) *xs*)
by(*induct xs*) *auto*

lemma *index-take*: *index xs x* >= *i* $\Longrightarrow$ $x \notin$ *set*(*take i xs*)
apply(*subst* (*asm*) *index-conv-takeWhile*)
apply(*subgoal-tac set*(*take i xs*) <= *set*(*takeWhile* (*op* $\neq$ *x*) *xs*))
 apply(*blast dest*: *set-takeWhileD*)
apply(*metis set-take-subset-set-take takeWhile-eq-take*)
done

lemma *last-index-drop*:
last-index xs x < *i* $\Longrightarrow$ $x \notin$ *set*(*drop i xs*)
apply(*subgoal-tac set*(*drop i xs*) = *set*(*take* (*size xs* − *i*) (*rev xs*)))
 apply(*simp add*: *last-index-def index-take Let-def split*:*split-if-asm*)
apply (*metis rev-drop set-rev*)
done

end

theory *Hidden*
imports ../../*List−Index*/*List-Index*
begin

definition *hidden* :: *'a list* $\Rightarrow$ *nat* $\Rightarrow$ *bool* **where**
hidden xs i $\equiv$ $i <$ *size xs* $\wedge$ *xs*!*i* $\in$ *set*(*drop* (*i*+*1*) *xs*)

lemma *hidden-last-index*: $x \in set\ xs \Longrightarrow hidden\ (xs\ @\ [x])\ (last\text{-}index\ xs\ x)$
apply(*auto simp add*: *hidden-def nth-append rev-nth*[*symmetric*])
apply(*drule last-index-less*[*OF - le-refl*])
apply *simp*
done

lemma *hidden-inacc*: $hidden\ xs\ i \Longrightarrow last\text{-}index\ xs\ x \neq i$
by(*auto simp add*: *hidden-def last-index-drop last-index-less-size-conv*)

lemma [*simp*]: $hidden\ xs\ i \Longrightarrow hidden\ (xs@[x])\ i$
by(*auto simp add*:*hidden-def nth-append*)

lemma *fun-upds-apply*:
$(m(xs[\mapsto]ys))\ x =$
$(let\ xs' = take\ (size\ ys)\ xs$
$in\ if\ x \in set\ xs'\ then\ Some(ys\ !\ last\text{-}index\ xs'\ x)\ else\ m\ x)$
apply(*induct xs arbitrary*: *m ys*)
apply (*simp add*: *Let-def*)
apply(*case-tac ys*)
apply (*simp add*:*Let-def*)
apply (*simp add*: *Let-def last-index-Cons*)
done

lemma *map-upds-apply-eq-Some*:
$((m(xs[\mapsto]ys))\ x = Some\ y) =$
$(let\ xs' = take\ (size\ ys)\ xs$
$in\ if\ x \in set\ xs'\ then\ ys\ !\ last\text{-}index\ xs'\ x = y\ else\ m\ x = Some\ y)$
by(*simp add*:*fun-upds-apply Let-def*)

lemma *map-upds-upd-conv-last-index*:
$[\![x \in set\ xs;\ size\ xs \leq size\ ys\ ]\!]$
$\Longrightarrow m(xs[\mapsto]ys)(x\mapsto y) = m(xs[\mapsto]ys[last\text{-}index\ xs\ x := y])$
apply(*rule ext*)
apply(*simp add*:*fun-upds-apply eq-sym-conv Let-def*)
done

end

5.4 Compilation Stage 1

theory *Compiler1* **imports** *PCompiler J1 Hidden* **begin**

Replacing variable names by indices.

primrec $compE_1$ :: *vname list* ⇒ *expr* ⇒ $expr_1$
 and $compEs_1$ :: *vname list* ⇒ *expr list* ⇒ $expr_1$ *list* **where**
 $compE_1$ *Vs* (*new C*) = *new C*
| $compE_1$ *Vs* (*Cast C e*) = *Cast C* ($compE_1$ *Vs e*)
| $compE_1$ *Vs* (*Val v*) = *Val v*
| $compE_1$ *Vs* (e_1 ≪*bop*≫ e_2) = ($compE_1$ *Vs* e_1) ≪*bop*≫ ($compE_1$ *Vs* e_2)
| $compE_1$ *Vs* (*Var V*) = *Var*(*last-index Vs V*)
| $compE_1$ *Vs* (*V*:=*e*) = (*last-index Vs V*):= ($compE_1$ *Vs e*)
| $compE_1$ *Vs* (*e*•*F*{*D*}) = ($compE_1$ *Vs e*)•*F*{*D*}
| $compE_1$ *Vs* (e_1•*F*{*D*}:=e_2) = ($compE_1$ *Vs* e_1)•*F*{*D*} := ($compE_1$ *Vs* e_2)
| $compE_1$ *Vs* (*e*•*M*(*es*)) = ($compE_1$ *Vs e*)•*M*($compEs_1$ *Vs es*)
| $compE_1$ *Vs* {*V*:*T*; *e*} = {(*size Vs*):*T*; $compE_1$ (*Vs*@[*V*]) *e*}
| $compE_1$ *Vs* (e_1;;e_2) = ($compE_1$ *Vs* e_1);;($compE_1$ *Vs* e_2)
| $compE_1$ *Vs* (*if* (*e*) e_1 *else* e_2) = *if* ($compE_1$ *Vs e*) ($compE_1$ *Vs* e_1) *else* ($compE_1$ *Vs* e_2)
| $compE_1$ *Vs* (*while* (*e*) *c*) = *while* ($compE_1$ *Vs e*) ($compE_1$ *Vs c*)
| $compE_1$ *Vs* (*throw e*) = *throw* ($compE_1$ *Vs e*)
| $compE_1$ *Vs* (*try* e_1 *catch*(*C V*) e_2) =
  try($compE_1$ *Vs* e_1) *catch*(*C* (*size Vs*)) ($compE_1$ (*Vs*@[*V*]) e_2)

| $compEs_1$ *Vs* [] = []
| $compEs_1$ *Vs* (*e*#*es*) = $compE_1$ *Vs e* # $compEs_1$ *Vs es*

lemma [*simp*]: $compEs_1$ *Vs es* = *map* ($compE_1$ *Vs*) *es*

primrec fin_1:: *expr* ⇒ $expr_1$ **where**
 fin_1(*Val v*) = *Val v*
| fin_1(*throw e*) = *throw*(fin_1 *e*)

lemma *comp-final*: *final e* ⟹ $compE_1$ *Vs e* = fin_1 *e*

lemma [*simp*]:
  ⋀*Vs*. *max-vars* ($compE_1$ *Vs e*) = *max-vars e*
and ⋀*Vs*. *max-varss* ($compEs_1$ *Vs es*) = *max-varss es*

Compiling programs:

definition $compP_1$ :: *J-prog* ⇒ J_1*-prog*
where
 $compP_1$ ≡ *compP* (λ(*pns*,*body*). $compE_1$ (*this*#*pns*) *body*)

end

5.5 Correctness of Stage 1

theory *Correctness1*
imports *J1WellForm Compiler1*
begin

5.5.1 Correctness of program compilation

primrec *unmod* :: $expr_1 \Rightarrow nat \Rightarrow bool$
and *unmods* :: $expr_1\ list \Rightarrow nat \Rightarrow bool$ **where**
$unmod\ (new\ C)\ i = True\ |$
$unmod\ (Cast\ C\ e)\ i = unmod\ e\ i\ |$
$unmod\ (Val\ v)\ i = True\ |$
$unmod\ (e_1 \ll bop \gg e_2)\ i = (unmod\ e_1\ i \wedge unmod\ e_2\ i)\ |$
$unmod\ (Var\ i)\ j = True\ |$
$unmod\ (i{:=}e)\ j = (i \neq j \wedge unmod\ e\ j)\ |$
$unmod\ (e{\cdot}F\{D\})\ i = unmod\ e\ i\ |$
$unmod\ (e_1{\cdot}F\{D\}{:=}e_2)\ i = (unmod\ e_1\ i \wedge unmod\ e_2\ i)\ |$
$unmod\ (e{\cdot}M(es))\ i = (unmod\ e\ i \wedge unmods\ es\ i)\ |$
$unmod\ \{j{:}T;\ e\}\ i = unmod\ e\ i\ |$
$unmod\ (e_1;;e_2)\ i = (unmod\ e_1\ i \wedge\ \ unmod\ e_2\ i)\ |$
$unmod\ (if\ (e)\ e_1\ else\ e_2)\ i = (unmod\ e\ i \wedge unmod\ e_1\ i \wedge unmod\ e_2\ i)\ |$
$unmod\ (while\ (e)\ c)\ i = (unmod\ e\ i \wedge unmod\ c\ i)\ |$
$unmod\ (throw\ e)\ i = unmod\ e\ i\ |$
$unmod\ (try\ e_1\ catch(C\ i)\ e_2)\ j = (unmod\ e_1\ j \wedge (if\ i{=}j\ then\ False\ else\ unmod\ e_2\ j))\ |$

$unmods\ ([])\ i = True\ |$
$unmods\ (e\#es)\ i = (unmod\ e\ i \wedge unmods\ es\ i)$

lemma *hidden-unmod*: $\bigwedge Vs.\ hidden\ Vs\ i \Longrightarrow unmod\ (compE_1\ \ Vs\ e)\ i$ **and**
$\bigwedge Vs.\ hidden\ Vs\ i \Longrightarrow unmods\ (compEs_1\ \ Vs\ es)\ i$

lemma $eval_1$*-preserves-unmod*:
$[\![\ P \vdash_1 \langle e,(h,ls)\rangle \Rightarrow \langle e',(h',ls')\rangle;\ unmod\ e\ i;\ i < size\ ls\ ]\!]$
$\Longrightarrow ls\ !\ i = ls'\ !\ i$
and $[\![\ P \vdash_1 \langle es,(h,ls)\rangle\ [\Rightarrow]\ \langle es',(h',ls')\rangle;\ unmods\ es\ i;\ i < size\ ls\ ]\!]$
$\Longrightarrow ls\ !\ i = ls'\ !\ i$

lemma *LAss-lem*:
$[\![x \in set\ xs;\ size\ xs \leq size\ ys\ ]\!]$
$\Longrightarrow m_1 \subseteq_m m_2(xs[\mapsto]ys) \Longrightarrow m_1(x \mapsto y) \subseteq_m m_2(xs[\mapsto]ys[last\text{-}index\ xs\ x := y])$**lemma** *Block-lem*:
fixes $l :: 'a \rightharpoonup 'b$
assumes *0*: $l \subseteq_m [Vs\ [\mapsto]\ ls]$
and *1*: $l' \subseteq_m [Vs\ [\mapsto]\ ls',\ V \mapsto v]$
and *hidden*: $V \in set\ Vs \Longrightarrow ls\ !\ last\text{-}index\ Vs\ V = ls'\ !\ last\text{-}index\ Vs\ V$
and *size*: $size\ ls = size\ ls' \quad size\ Vs < size\ ls'$
shows $l'(V := l\ V) \subseteq_m [Vs\ [\mapsto]\ ls']$

The main theorem:

theorem assumes *wf*: *wwf-J-prog P*
shows $eval_1$*-eval*: $P \vdash \langle e,(h,l)\rangle \Rightarrow \langle e',(h',l')\rangle$
$\Longrightarrow (\bigwedge Vs\ ls.\ [\![\ fv\ e \subseteq set\ Vs;\ \ l \subseteq_m [Vs[\mapsto]ls];\ size\ Vs + max\text{-}vars\ e \leq size\ ls\ ]\!]$
$\Longrightarrow \exists\, ls'.\ compP_1\ P \vdash_1 \langle compE_1\ \ Vs\ e,(h,ls)\rangle \Rightarrow \langle fin_1\ \ e',(h',ls')\rangle \wedge l' \subseteq_m [Vs[\mapsto]ls'])$
and $evals_1$*-evals*: $P \vdash \langle es,(h,l)\rangle\ [\Rightarrow]\ \langle es',(h',l')\rangle$

$\Longrightarrow (\bigwedge Vs\ ls.\ [\![\ fvs\ es \subseteq set\ Vs;\ \ l \subseteq_m [Vs[\mapsto]ls];\ size\ Vs + max\text{-}varss\ es \leq size\ ls\ ]\!]$
$\Longrightarrow \exists\, ls'.\ compP_1\ P \vdash_1 \langle compEs_1\ Vs\ es,(h,ls)\rangle\ [\Rightarrow]\ \langle compEs_1\ Vs\ es',(h',ls')\rangle \wedge$
$l' \subseteq_m [Vs[\mapsto]ls'])$

5.5.2 Preservation of well-formedness

The compiler preserves well-formedness. Is less trivial than it may appear. We start with two simple properties: preservation of well-typedness

lemma $compE_1\text{-}pres\text{-}wt$: $\bigwedge Vs\ Ts\ U.$
$[\![\ P,[Vs[\mapsto]Ts] \vdash e :: U;\ size\ Ts = size\ Vs\ ]\!]$
$\Longrightarrow compP\ f\ P,Ts \vdash_1 compE_1\ Vs\ e :: U$
and $\bigwedge Vs\ Ts\ Us.$
$[\![\ P,[Vs[\mapsto]Ts] \vdash es\ [::]\ Us;\ size\ Ts = size\ Vs\ ]\!]$
$\Longrightarrow compP\ f\ P,Ts \vdash_1 compEs_1\ Vs\ es\ [::]\ Us$

and the correct block numbering:

lemma $\mathcal{B}$: $\bigwedge Vs\ n.\ size\ Vs = n \Longrightarrow \mathcal{B}\ (compE_1\ Vs\ e)\ n$
and $\mathcal{B}s$: $\bigwedge Vs\ n.\ size\ Vs = n \Longrightarrow \mathcal{B}s\ (compEs_1\ Vs\ es)\ n$

The main complication is preservation of definite assignment $\mathcal{D}$.

lemma $image\text{-}last\text{-}index$: $A \subseteq set(xs@[x]) \Longrightarrow last\text{-}index\ (xs\ @\ [x])\ `\ A =$
$(if\ x \in A\ then\ insert\ (size\ xs)\ (last\text{-}index\ xs\ `\ (A-\{x\}))\ else\ last\text{-}index\ xs\ `\ A)$

lemma $A\text{-}compE_1\text{-}None[simp]$:
$\bigwedge Vs.\ \mathcal{A}\ e = None \Longrightarrow \mathcal{A}\ (compE_1\ Vs\ e) = None$
and $\bigwedge Vs.\ \mathcal{A}s\ es = None \Longrightarrow \mathcal{A}s\ (compEs_1\ Vs\ es) = None$

lemma $A\text{-}compE_1$:
$\bigwedge A\ Vs.\ [\![\ \mathcal{A}\ e = \lfloor A\rfloor;\ fv\ e \subseteq set\ Vs\ ]\!] \Longrightarrow \mathcal{A}\ (compE_1\ Vs\ e) = \lfloor last\text{-}index\ Vs\ `\ A\rfloor$
and $\bigwedge A\ Vs.\ [\![\ \mathcal{A}s\ es = \lfloor A\rfloor;\ fvs\ es \subseteq set\ Vs\ ]\!] \Longrightarrow \mathcal{A}s\ (compEs_1\ Vs\ es) = \lfloor last\text{-}index\ Vs\ `\ A\rfloor$

lemma $D\text{-}None[iff]$: $\mathcal{D}\ (e::'a\ exp)\ None$ **and** $[iff]$: $\mathcal{D}s\ (es::'a\ exp\ list)\ None$

lemma $D\text{-}last\text{-}index\text{-}compE_1$:
$\bigwedge A\ Vs.\ [\![\ A \subseteq set\ Vs;\ fv\ e \subseteq set\ Vs\ ]\!] \Longrightarrow$
$\mathcal{D}\ e\ \lfloor A\rfloor \Longrightarrow \mathcal{D}\ (compE_1\ Vs\ e)\ \lfloor last\text{-}index\ Vs\ `\ A\rfloor$
and $\bigwedge A\ Vs.\ [\![\ A \subseteq set\ Vs;\ fvs\ es \subseteq set\ Vs\ ]\!] \Longrightarrow$
$\mathcal{D}s\ es\ \lfloor A\rfloor \Longrightarrow \mathcal{D}s\ (compEs_1\ Vs\ es)\ \lfloor last\text{-}index\ Vs\ `\ A\rfloor$

lemma $last\text{-}index\text{-}image\text{-}set$: $distinct\ xs \Longrightarrow last\text{-}index\ xs\ `\ set\ xs = \{..<size\ xs\}$

lemma $D\text{-}compE_1$:
$[\![\ \mathcal{D}\ e\ \lfloor set\ Vs\rfloor;\ fv\ e \subseteq set\ Vs;\ distinct\ Vs\ ]\!] \Longrightarrow \mathcal{D}\ (compE_1\ Vs\ e)\ \lfloor\{..<length\ Vs\}\rfloor$

lemma $D\text{-}compE_1'$:
assumes $\mathcal{D}\ e\ \lfloor set(V\#Vs)\rfloor$ **and** $fv\ e \subseteq set(V\#Vs)$ **and** $distinct(V\#Vs)$
shows $\mathcal{D}\ (compE_1\ (V\#Vs)\ e)\ \lfloor\{..length\ Vs\}\rfloor$

lemma $compP_1\text{-}pres\text{-}wf$: $wf\text{-}J\text{-}prog\ P \Longrightarrow wf\text{-}J_1\text{-}prog\ (compP_1\ P)$

end

5.6 Compilation Stage 2

theory *Compiler2*
imports *PCompiler J1 ../JVM/JVMExec*
begin

primrec $compE_2$ :: $expr_1 \Rightarrow instr\ list$
  and $compEs_2$ :: $expr_1\ list \Rightarrow instr\ list$ **where**
  $compE_2\ (new\ C) = [New\ C]$
$|\ compE_2\ (Cast\ C\ e) = compE_2\ e\ @\ [Checkcast\ C]$
$|\ compE_2\ (Val\ v) = [Push\ v]$
$|\ compE_2\ (e_1 \ll bop \gg e_2) = compE_2\ e_1\ @\ compE_2\ e_2\ @$
  $(case\ bop\ of\ Eq \Rightarrow [CmpEq]$
        $|\ Add \Rightarrow [IAdd])$
$|\ compE_2\ (Var\ i) = [Load\ i]$
$|\ compE_2\ (i{:=}e) = compE_2\ e\ @\ [Store\ i,\ Push\ Unit]$
$|\ compE_2\ (e{\cdot}F\{D\}) = compE_2\ e\ @\ [Getfield\ F\ D]$
$|\ compE_2\ (e_1{\cdot}F\{D\} := e_2) =$
    $compE_2\ e_1\ @\ compE_2\ e_2\ @\ [Putfield\ F\ D,\ Push\ Unit]$
$|\ compE_2\ (e{\cdot}M(es)) = compE_2\ e\ @\ compEs_2\ es\ @\ [Invoke\ M\ (size\ es)]$
$|\ compE_2\ (\{i{:}T;\ e\}) = compE_2\ e$
$|\ compE_2\ (e_1;;e_2) = compE_2\ e_1\ @\ [Pop]\ @\ compE_2\ e_2$
$|\ compE_2\ (if\ (e)\ e_1\ else\ e_2) =$
    $(let\ cnd\ \ = compE_2\ e;$
      $thn\ \ = compE_2\ e_1;$
      $els\ \ = compE_2\ e_2;$
      $test\ = IfFalse\ (int(size\ thn + 2));$
      $thnex = Goto\ (int(size\ els + 1))$
     $in\ cnd\ @\ [test]\ @\ thn\ @\ [thnex]\ @\ els)$
$|\ compE_2\ (while\ (e)\ c) =$
    $(let\ cnd\ \ = compE_2\ e;$
      $bdy\ \ = compE_2\ c;$
      $test\ = IfFalse\ (int(size\ bdy + 3));$
      $loop\ = Goto\ (-int(size\ bdy + size\ cnd + 2))$
     $in\ cnd\ @\ [test]\ @\ bdy\ @\ [Pop]\ @\ [loop]\ @\ [Push\ Unit])$
$|\ compE_2\ (throw\ e) = compE_2\ e\ @\ [instr.Throw]$
$|\ compE_2\ (try\ e_1\ catch(C\ i)\ e_2) =$
  $(let\ catch = compE_2\ e_2$
   $in\ compE_2\ e_1\ @\ [Goto\ (int(size\ catch){+}2),\ Store\ i]\ @\ catch)$

$|\ compEs_2\ []\ \ \ \ = []$
$|\ compEs_2\ (e\#es) = compE_2\ e\ @\ compEs_2\ es$

Compilation of exception table. Is given start address of code to compute absolute addresses necessary in exception table.

primrec $compxE_2$ :: $expr_1 \Rightarrow pc \Rightarrow nat \Rightarrow ex\text{-}table$
  and $compxEs_2$ :: $expr_1\ list \Rightarrow pc \Rightarrow nat \Rightarrow ex\text{-}table$ **where**
  $compxE_2\ (new\ C)\ pc\ d = []$
$|\ compxE_2\ (Cast\ C\ e)\ pc\ d = compxE_2\ e\ pc\ d$
$|\ compxE_2\ (Val\ v)\ pc\ d = []$
$|\ compxE_2\ (e_1 \ll bop \gg e_2)\ pc\ d =$
  $compxE_2\ e_1\ pc\ d\ @\ compxE_2\ e_2\ (pc + size(compE_2\ e_1))\ (d{+}1)$
$|\ compxE_2\ (Var\ i)\ pc\ d = []$
$|\ compxE_2\ (i{:=}e)\ pc\ d = compxE_2\ e\ pc\ d$

| $compxE_2$ $(e{\cdot}F\{D\})$ pc d = $compxE_2$ e pc d
| $compxE_2$ $(e_1{\cdot}F\{D\} := e_2)$ pc d =
  $compxE_2$ e_1 pc d @ $compxE_2$ e_2 $(pc + size(compE_2\ e_1))$ $(d+1)$
| $compxE_2$ $(e{\cdot}M(es))$ pc d =
  $compxE_2$ e pc d @ $compxEs_2$ es $(pc + size(compE_2\ e))$ $(d+1)$
| $compxE_2$ $(\{i{:}T;\ e\})$ pc d = $compxE_2$ e pc d
| $compxE_2$ $(e_1;;e_2)$ pc d =
  $compxE_2$ e_1 pc d @ $compxE_2$ e_2 $(pc+size(compE_2\ e_1)+1)$ d
| $compxE_2$ $(if\ (e)\ e_1\ else\ e_2)$ pc d =
  $(let\ pc_1 = pc + size(compE_2\ e) + 1;$
    $pc_2 = pc_1 + size(compE_2\ e_1) + 1$
   in $compxE_2$ e pc d @ $compxE_2$ e_1 pc_1 d @ $compxE_2$ e_2 pc_2 $d)$
| $compxE_2$ $(while\ (b)\ e)$ pc d =
  $compxE_2$ b pc d @ $compxE_2$ e $(pc+size(compE_2\ b)+1)$ d
| $compxE_2$ $(throw\ e)$ pc d = $compxE_2$ e pc d
| $compxE_2$ $(try\ e_1\ catch(C\ i)\ e_2)$ pc d =
  $(let\ pc_1 = pc + size(compE_2\ e_1)$
   in $compxE_2$ e_1 pc d @ $compxE_2$ e_2 (pc_1+2) d @ $[(pc,pc_1,C,pc_1+1,d)])$

| $compxEs_2$ [] pc d = []
| $compxEs_2$ $(e\#es)$ pc d = $compxE_2$ e pc d @ $compxEs_2$ es $(pc+size(compE_2\ e))$ $(d+1)$

primrec *max-stack* :: $expr_1 \Rightarrow nat$
  and *max-stacks* :: $expr_1\ list \Rightarrow nat$ **where**
  max-stack $(new\ C) = 1$
| *max-stack* $(Cast\ C\ e)$ = *max-stack* e
| *max-stack* $(Val\ v) = 1$
| *max-stack* $(e_1 \ll bop \gg e_2)$ = max (*max-stack* e_1) (*max-stack* e_2) + 1
| *max-stack* $(Var\ i) = 1$
| *max-stack* $(i{:=}e)$ = *max-stack* e
| *max-stack* $(e{\cdot}F\{D\})$ = *max-stack* e
| *max-stack* $(e_1{\cdot}F\{D\} := e_2)$ = max (*max-stack* e_1) (*max-stack* e_2) + 1
| *max-stack* $(e{\cdot}M(es))$ = max (*max-stack* e) (*max-stacks* es) + 1
| *max-stack* $(\{i{:}T;\ e\})$ = *max-stack* e
| *max-stack* $(e_1;;e_2)$ = max (*max-stack* e_1) (*max-stack* e_2)
| *max-stack* $(if\ (e)\ e_1\ else\ e_2)$ =
  max (*max-stack* e) (max (*max-stack* e_1) (*max-stack* e_2))
| *max-stack* $(while\ (e)\ c)$ = max (*max-stack* e) (*max-stack* c)
| *max-stack* $(throw\ e)$ = *max-stack* e
| *max-stack* $(try\ e_1\ catch(C\ i)\ e_2)$ = max (*max-stack* e_1) (*max-stack* e_2)

| *max-stacks* [] = 0
| *max-stacks* $(e\#es)$ = max (*max-stack* e) (1 + *max-stacks* es)

lemma *max-stack1*: $1 \leq$ *max-stack* e

definition $compMb_2$ :: $expr_1 \Rightarrow$ *jvm-method*
where
  $compMb_2 \equiv \lambda body.$
  let ins = $compE_2$ *body* @ [*Return*];
    xt = $compxE_2$ *body* 0 0
  in (*max-stack body*, *max-vars body*, *ins*, *xt*)

definition $compP_2$ :: J_1*-prog* $\Rightarrow$ *jvm-prog*

where
$compP_2 \equiv compP\ compMb_2$

lemma $compMb_2$ [$simp$]:
$compMb_2\ e = (max\text{-}stack\ e,\ max\text{-}vars\ e,\ compE_2\ e\ @\ [Return],\ compxE_2\ e\ 0\ 0)$

end

5.7 Correctness of Stage 2

theory *Correctness2*
imports ~~/src/HOL/Library/List-Prefix Compiler2
begin

5.7.1 Instruction sequences

How to select individual instructions and subsequences of instructions from a program given the class, method and program counter.

definition *before* :: *jvm-prog* ⇒ *cname* ⇒ *mname* ⇒ *nat* ⇒ *instr list* ⇒ *bool*
((-,-,-,-/ ⊳ -) [51,0,0,0,51] 50) **where**
$P,C,M,pc \rhd is \longleftrightarrow is \le drop\ pc\ (instrs\text{-}of\ P\ C\ M)$

definition *at* :: *jvm-prog* ⇒ *cname* ⇒ *mname* ⇒ *nat* ⇒ *instr* ⇒ *bool*
((-,-,-,-/ ▹ -) [51,0,0,0,51] 50) **where**
$P,C,M,pc \triangleright i \longleftrightarrow (\exists is.\ drop\ pc\ (instrs\text{-}of\ P\ C\ M) = i\#is)$

lemma [*simp*]: $P,C,M,pc \rhd []$

lemma [*simp*]: $P,C,M,pc \rhd (i\#is) = (P,C,M,pc \triangleright i \wedge P,C,M,pc + 1 \rhd is)$

lemma [*simp*]: $P,C,M,pc \rhd (is_1\ @\ is_2) = (P,C,M,pc \rhd is_1 \wedge P,C,M,pc + size\ is_1 \rhd is_2)$

lemma [*simp*]: $P,C,M,pc \triangleright i \Longrightarrow instrs\text{-}of\ P\ C\ M\ !\ pc = i$

lemma *beforeM*:
$P \vdash C\ sees\ M\colon Ts{\rightarrow}T = body\ in\ D \Longrightarrow$
$compP_2\ P,D,M,0 \rhd compE_2\ body\ @\ [Return]$

This lemma executes a single instruction by rewriting:

lemma [*simp*]:
$P,C,M,pc \triangleright instr \Longrightarrow$
$(P \vdash (None,\ h,\ (vs,ls,C,M,pc)\ \#\ frs)\ {-jvm\rightarrow}\ \sigma') =$
$((None,\ h,\ (vs,ls,C,M,pc)\ \#\ frs) = \sigma' \vee$
$(\exists \sigma.\ exec(P,(None,\ h,\ (vs,ls,C,M,pc)\ \#\ frs)) = Some\ \sigma \wedge P \vdash \sigma\ {-jvm\rightarrow}\ \sigma'))$

5.7.2 Exception tables

definition *pcs* :: *ex-table* ⇒ *nat set*
where
$pcs\ xt \equiv \bigcup (f,t,C,h,d) \in set\ xt.\ \{f\ ..<\ t\}$

lemma *pcs-subset*:
shows $\bigwedge pc\ d.\ pcs(compxE_2\ e\ pc\ d) \subseteq \{pc..<pc{+}size(compE_2\ e)\}$
and $\bigwedge pc\ d.\ pcs(compxEs_2\ es\ pc\ d) \subseteq \{pc..<pc{+}size(compEs_2\ es)\}$

lemma [*simp*]: $pcs\ [] = \{\}$

lemma [*simp*]: $pcs\ (x\#xt) = \{fst\ x\ ..<\ fst(snd\ x)\} \cup pcs\ xt$

lemma [*simp*]: $pcs(xt_1\ @\ xt_2) = pcs\ xt_1 \cup pcs\ xt_2$

lemma [*simp*]: $pc < pc_0 \lor pc_0{+}size(compE_2\ e) \le pc \Longrightarrow pc \notin pcs(compxE_2\ e\ pc_0\ d)$

lemma [*simp*]: $pc < pc_0 \lor pc_0{+}size(compEs_2\ es) \le pc \Longrightarrow pc \notin pcs(compxEs_2\ es\ pc_0\ d)$

lemma [*simp*]: $pc_1 + size(compE_2\ e_1) \le pc_2 \Longrightarrow pcs(compxE_2\ e_1\ pc_1\ d_1) \cap pcs(compxE_2\ e_2\ pc_2\ d_2) = \{\}$

lemma [*simp*]: $pc_1 + size(compE_2\ e) \le pc_2 \Longrightarrow pcs(compxE_2\ e\ pc_1\ d_1) \cap pcs(compxEs_2\ es\ pc_2\ d_2) = \{\}$

lemma [*simp*]:
$pc \notin pcs\ xt_0 \Longrightarrow match\text{-}ex\text{-}table\ P\ C\ pc\ (xt_0\ @\ xt_1) = match\text{-}ex\text{-}table\ P\ C\ pc\ xt_1$

lemma [*simp*]: $[\![\ x \in set\ xt;\ pc \notin pcs\ xt\ ]\!] \Longrightarrow \neg\ matches\text{-}ex\text{-}entry\ P\ D\ pc\ x$

lemma [*simp*]:
assumes *xe*: $xe \in set(compxE_2\ e\ pc\ d)$ **and** *outside*: $pc' < pc \lor pc{+}size(compE_2\ e) \le pc'$
shows $\neg\ matches\text{-}ex\text{-}entry\ P\ C\ pc'\ xe$

lemma [*simp*]:
assumes *xe*: $xe \in set(compxEs_2\ es\ pc\ d)$ **and** *outside*: $pc' < pc \lor pc{+}size(compEs_2\ es) \le pc'$
shows $\neg\ matches\text{-}ex\text{-}entry\ P\ C\ pc'\ xe$

lemma *match-ex-table-app*[*simp*]:
$\forall xte \in set\ xt_1.\ \neg\ matches\text{-}ex\text{-}entry\ P\ D\ pc\ xte \Longrightarrow$
$match\text{-}ex\text{-}table\ P\ D\ pc\ (xt_1\ @\ xt) = match\text{-}ex\text{-}table\ P\ D\ pc\ xt$

lemma [*simp*]:
$\forall x \in set\ xtab.\ \neg\ matches\text{-}ex\text{-}entry\ P\ C\ pc\ x \Longrightarrow$
$match\text{-}ex\text{-}table\ P\ C\ pc\ xtab = None$

lemma *match-ex-entry*:
$matches\text{-}ex\text{-}entry\ P\ C\ pc\ (start,\ end,\ catch\text{-}type,\ handler) =$
$(start \le pc \land pc < end \land\ \ P \vdash C \preceq^* catch\text{-}type)$

definition *caught* :: $jvm\text{-}prog \Rightarrow pc \Rightarrow heap \Rightarrow addr \Rightarrow ex\text{-}table \Rightarrow bool$ **where**
$caught\ P\ pc\ h\ a\ xt \longleftrightarrow$
$(\exists entry \in set\ xt.\ matches\text{-}ex\text{-}entry\ P\ (cname\text{-}of\ h\ a)\ pc\ entry)$

definition *beforex* :: $jvm\text{-}prog \Rightarrow cname \Rightarrow mname \Rightarrow ex\text{-}table \Rightarrow nat\ set \Rightarrow nat \Rightarrow bool$
$((2\text{-},/\text{-},/\text{-} \rhd/\ \text{-}\ /'/\ \text{-},/\text{-})\ [51,0,0,0,0,51]\ 50)$ **where**
$P,C,M \rhd xt\ /\ I,d \longleftrightarrow$
$(\exists xt_0\ xt_1.\ ex\text{-}table\text{-}of\ P\ C\ M = xt_0\ @\ xt\ @\ xt_1 \land pcs\ xt_0 \cap I = \{\} \land pcs\ xt \subseteq I \land$
$(\forall pc \in I.\ \forall C\ pc'\ d'.\ match\text{-}ex\text{-}table\ P\ C\ pc\ xt_1 = \lfloor(pc',d')\rfloor \longrightarrow d' \le d))$

definition *dummyx* :: $jvm\text{-}prog \Rightarrow cname \Rightarrow mname \Rightarrow ex\text{-}table \Rightarrow nat\ set \Rightarrow nat \Rightarrow bool$ $((2\text{-},\text{-},\text{-} \rhd/\ \text{-}\ '/\text{-},\text{-})\ [51,0,0,0,0,51]\ 50)$ **where**
$P,C,M \rhd xt/I,d \longleftrightarrow P,C,M \rhd xt/I,d$

lemma *beforexD1*: $P,C,M \rhd xt\ /\ I,d \Longrightarrow pcs\ xt \subseteq I$

lemma *beforex-mono*: $[\![\ P,C,M \rhd xt/I,d';\ d' \le d\ ]\!] \Longrightarrow P,C,M \rhd xt/I,d$

lemma [*simp*]: $P,C,M \rhd xt/I,d \Longrightarrow P,C,M \rhd xt/I,Suc\ d$

lemma *beforex-append*[*simp*]:
$pcs\ xt_1 \cap pcs\ xt_2 = \{\} \Longrightarrow$
$P,C,M \rhd xt_1\ @\ xt_2/I,d =$
$(P,C,M \rhd xt_1/I-pcs\ xt_2,d\ \wedge\ P,C,M \rhd xt_2/I-pcs\ xt_1,d \wedge P,C,M \rhd xt_1@xt_2/I,d)$

lemma *beforex-appendD1*:
$[\![\ P,C,M \rhd xt_1\ @\ xt_2\ @\ [(f,t,D,h,d)]\ /\ I,d;$
$pcs\ xt_1 \subseteq J;\ J \subseteq I;\ J \cap pcs\ xt_2 = \{\}\ ]\!]$
$\Longrightarrow P,C,M \rhd xt_1\ /\ J,d$

lemma *beforex-appendD2*:
$[\![\ P,C,M \rhd xt_1\ @\ xt_2\ @\ [(f,t,D,h,d)]\ /\ I,d;$
$pcs\ xt_2 \subseteq J;\ J \subseteq I;\ J \cap pcs\ xt_1 = \{\}\ ]\!]$
$\Longrightarrow P,C,M \rhd xt_2\ /\ J,d$

lemma *beforexM*:
$P \vdash C\ sees\ M\colon\ Ts{\rightarrow}T = body\ in\ D \Longrightarrow$
$compP_2\ P,D,M \rhd compxE_2\ body\ 0\ 0/\{..{<}size(compE_2\ body)\},0$

lemma *match-ex-table-SomeD2*:
$[\![\ match\text{-}ex\text{-}table\ P\ D\ pc\ (ex\text{-}table\text{-}of\ P\ C\ M) = \lfloor(pc',d')\rfloor;$
$P,C,M \rhd xt/I,d;\ \forall x \in set\ xt.\ \neg\ matches\text{-}ex\text{-}entry\ P\ D\ pc\ x;\ pc \in I\ ]\!]$
$\Longrightarrow d' \leq d$

lemma *match-ex-table-SomeD1*:
$[\![\ match\text{-}ex\text{-}table\ P\ D\ pc\ (ex\text{-}table\text{-}of\ P\ C\ M) = \lfloor(pc',d')\rfloor;$
$P,C,M \rhd xt\ /\ I,d;\ pc \in I;\ pc \notin pcs\ xt\ ]\!] \Longrightarrow d' \leq d$

5.7.3 The correctness proof

definition
$handle :: jvm\text{-}prog \Rightarrow cname \Rightarrow mname \Rightarrow addr \Rightarrow heap \Rightarrow val\ list \Rightarrow val\ list \Rightarrow nat \Rightarrow frame\ list$
$\Rightarrow jvm\text{-}state$ **where**
$handle\ P\ C\ M\ a\ h\ vs\ ls\ pc\ frs = find\text{-}handler\ P\ a\ h\ ((vs,ls,C,M,pc)\ \#\ frs)$

lemma *handle-Cons*:
$[\![\ P,C,M \rhd xt/I,d;\ d \leq size\ vs;\ pc \in I;$
$\forall x \in set\ xt.\ \neg\ matches\text{-}ex\text{-}entry\ P\ (cname\text{-}of\ h\ xa)\ pc\ x\ ]\!] \Longrightarrow$
$handle\ P\ C\ M\ xa\ h\ (v\ \#\ vs)\ ls\ pc\ frs = handle\ P\ C\ M\ xa\ h\ vs\ ls\ pc\ frs$
lemma *handle-append*:
$[\![\ P,C,M \rhd xt/I,d;\ d \leq size\ vs;\ pc \in I;\ pc \notin pcs\ xt\ ]\!] \Longrightarrow$
$handle\ P\ C\ M\ xa\ h\ (ws\ @\ vs)\ ls\ pc\ frs = handle\ P\ C\ M\ xa\ h\ vs\ ls\ pc\ frs$

lemma *aux-isin*[*simp*]: $[\![\ B \subseteq A;\ a \in B\ ]\!] \Longrightarrow a \in A$

lemma fixes P_1 **defines** [*simp*]: $P \equiv compP_2\ P_1$
shows *Jcc*:
$P_1 \vdash_1 \langle e,(h_0,ls_0)\rangle \Rightarrow \langle ef,(h_1,ls_1)\rangle \Longrightarrow$
$(\bigwedge C\ M\ pc\ v\ xa\ vs\ frs\ I.$
$[\![\ P,C,M,pc \rhd compE_2\ e;\ P,C,M \rhd compxE_2\ e\ pc\ (size\ vs)/I,size\ vs;$
$\{pc..{<}pc{+}size(compE_2\ e)\} \subseteq I\ ]\!] \Longrightarrow$

$(ef = Val\ v \longrightarrow$
$P \vdash (None,h_0,(vs,ls_0,C,M,pc)\#frs)\ {-jvm\rightarrow}$
$(None,h_1,(v\#vs,ls_1,C,M,pc{+}size(compE_2\ e))\#frs))$
$\wedge$
$(ef = Throw\ xa \longrightarrow$
$(\exists\, pc_1.\ pc \leq pc_1 \wedge pc_1 < pc + size(compE_2\ e) \wedge$
$\neg\ caught\ P\ pc_1\ h_1\ xa\ (compxE_2\ e\ pc\ (size\ vs)) \wedge$
$P \vdash (None,h_0,(vs,ls_0,C,M,pc)\#frs)\ {-jvm\rightarrow}\ handle\ P\ C\ M\ xa\ h_1\ vs\ ls_1\ pc_1\ frs)))$
and $P_1 \vdash_1 \langle es,(h_0,ls_0)\rangle\ [\Rightarrow]\ \langle fs,(h_1,ls_1)\rangle \Longrightarrow$
$(\bigwedge C\ M\ pc\ ws\ xa\ es'\ vs\ frs\ I.$
$[\![\ P,C,M,pc \rhd compEs_2\ es;\ P,C,M \rhd compxEs_2\ es\ pc\ (size\ vs)/I,size\ vs;$
$\{pc..{<}pc{+}size(compEs_2\ es)\} \subseteq I\ ]\!] \Longrightarrow$
$(fs = map\ Val\ ws \longrightarrow$
$P \vdash (None,h_0,(vs,ls_0,C,M,pc)\#frs)\ {-jvm\rightarrow}$
$(None,h_1,(rev\ ws\ @\ vs,ls_1,C,M,pc{+}size(compEs_2\ es))\#frs))$
$\wedge$
$(fs = map\ Val\ ws\ @\ Throw\ xa\ \#\ es' \longrightarrow$
$(\exists\, pc_1.\ pc \leq pc_1 \wedge pc_1 < pc + size(compEs_2\ es) \wedge$
$\neg\ caught\ P\ pc_1\ h_1\ xa\ (compxEs_2\ es\ pc\ (size\ vs)) \wedge$
$P \vdash (None,h_0,(vs,ls_0,C,M,pc)\#frs)\ {-jvm\rightarrow}\ handle\ P\ C\ M\ xa\ h_1\ vs\ ls_1\ pc_1\ frs)))$

lemma $atLeast0AtMost[simp]$: $\{0{::}nat..n\} = \{..n\}$
by *auto*

lemma $atLeast0LessThan[simp]$: $\{0{::}nat..{<}n\} = \{..{<}n\}$
by *auto*

fun $exception :: {'a}\ exp \Rightarrow addr\ option$ **where**
$exception\ (Throw\ a) = Some\ a$
$|\ exception\ e = None$

lemma $comp_2$*-correct*:
assumes *method*: $P_1 \vdash C\ sees\ M{:}Ts{\rightarrow}T = body\ in\ C$
and *eval*: $P_1 \vdash_1 \langle body,(h,ls)\rangle \Rightarrow \langle e',(h',ls')\rangle$
shows $compP_2\ P_1 \vdash (None,h,[([],ls,C,M,0)])\ {-jvm\rightarrow}\ (exception\ e',h',[])$
end

5.8 Combining Stages 1 and 2

theory *Compiler*
imports *Correctness1 Correctness2*
begin

definition *J2JVM* :: *J-prog* ⇒ *jvm-prog*
where
  J2JVM ≡ $compP_2 \circ compP_1$

theorem *comp-correct*:
assumes *wwf*: *wwf-J-prog P*
and *method*: *P* ⊢ *C sees M*:*Ts*→*T* = (*pns*,*body*) *in C*
and *eval*: *P* ⊢ ⟨*body*,(*h*,[*this*#*pns* [↦] *vs*])⟩ ⇒ ⟨*e*′,(*h*′,*l*′)⟩
and *sizes*: *size vs* = *size pns* + *1*   *size rest* = *max-vars body*
shows *J2JVM P* ⊢ (*None*,*h*,[([],*vs*@*rest*,*C*,*M*,*0*)]) −*jvm*→ (*exception e*′,*h*′,[])

end

5.9 Preservation of Well-Typedness

theory *TypeComp*
imports *Compiler ../BV/BVSpec*
begin

locale *TC0* =
 fixes P :: J_1-*prog* **and** *mxl* :: *nat*
begin

definition $ty\ E\ e = (THE\ T.\ P,E \vdash_1 e :: T)$

definition $ty_l\ E\ A' = map\ (\lambda i.\ if\ i \in A' \wedge i < size\ E\ then\ OK(E!i)\ else\ Err)\ [0..<mxl]$

definition $ty_i{}'\ ST\ E\ A = (case\ A\ of\ None \Rightarrow None \mid \lfloor A' \rfloor \Rightarrow Some(ST,\ ty_l\ E\ A'))$

definition $after\ E\ A\ ST\ e = ty_i{}'\ (ty\ E\ e\ \#\ ST)\ E\ (A \sqcup \mathcal{A}\ e)$

end

lemma (**in** *TC0*) *ty-def2* [*simp*]: $P,E \vdash_1 e :: T \Longrightarrow ty\ E\ e = T$
lemma (**in** *TC0*) [*simp*]: $ty_i{}'\ ST\ E\ None = None$
lemma (**in** *TC0*) ty_l-*app-diff* [*simp*]:
 $ty_l\ (E@[T])\ (A - \{size\ E\}) = ty_l\ E\ A$

lemma (**in** *TC0*) $ty_i{}'$-*app-diff* [*simp*]:
 $ty_i{}'\ ST\ (E\ @\ [T])\ (A \ominus size\ E) = ty_i{}'\ ST\ E\ A$

lemma (**in** *TC0*) ty_l-*antimono*:
 $A \subseteq A' \Longrightarrow P \vdash ty_l\ E\ A'\ [\leq_\top]\ ty_l\ E\ A$

lemma (**in** *TC0*) $ty_i{}'$-*antimono*:
 $A \subseteq A' \Longrightarrow P \vdash ty_i{}'\ ST\ E\ \lfloor A' \rfloor \leq' ty_i{}'\ ST\ E\ \lfloor A \rfloor$

lemma (**in** *TC0*) ty_l-*env-antimono*:
 $P \vdash ty_l\ (E@[T])\ A\ [\leq_\top]\ ty_l\ E\ A$

lemma (**in** *TC0*) $ty_i{}'$-*env-antimono*:
 $P \vdash ty_i{}'\ ST\ (E@[T])\ A \leq' ty_i{}'\ ST\ E\ A$

lemma (**in** *TC0*) $ty_i{}'$-*incr*:
 $P \vdash ty_i{}'\ ST\ (E\ @\ [T])\ \lfloor insert\ (size\ E)\ A \rfloor \leq' ty_i{}'\ ST\ E\ \lfloor A \rfloor$

lemma (**in** *TC0*) ty_l-*incr*:
 $P \vdash ty_l\ (E\ @\ [T])\ (insert\ (size\ E)\ A)\ [\leq_\top]\ ty_l\ E\ A$

lemma (**in** *TC0*) ty_l-*in-types*:
 $set\ E \subseteq types\ P \Longrightarrow ty_l\ E\ A \in list\ mxl\ (err\ (types\ P))$
locale *TC1* = *TC0*
begin

primrec $compT$ :: $ty\ list \Rightarrow nat\ hyperset \Rightarrow ty\ list \Rightarrow expr_1 \Rightarrow ty_i{}'\ list$ **and**
 $compTs$ :: $ty\ list \Rightarrow nat\ hyperset \Rightarrow ty\ list \Rightarrow expr_1\ list \Rightarrow ty_i{}'\ list$ **where**
$compT\ E\ A\ ST\ (new\ C) = []$

$|\ compT\ E\ A\ ST\ (Cast\ C\ e) =$
$\quad compT\ E\ A\ ST\ e\ @\ [after\ E\ A\ ST\ e]$
$|\ compT\ E\ A\ ST\ (Val\ v) = []$
$|\ compT\ E\ A\ ST\ (e_1 \ll bop \gg e_2) =$
$\ (let\ ST_1 = ty\ E\ e_1\#ST;\ A_1 = A \sqcup \mathcal{A}\ e_1\ in$
$\quad compT\ E\ A\ ST\ e_1\ @\ [after\ E\ A\ ST\ e_1]\ @$
$\quad compT\ E\ A_1\ ST_1\ e_2\ @\ [after\ E\ A_1\ ST_1\ e_2])$
$|\ compT\ E\ A\ ST\ (Var\ i) = []$
$|\ compT\ E\ A\ ST\ (i := e) = compT\ E\ A\ ST\ e\ @$
$\quad [after\ E\ A\ ST\ e,\ ty_i'\ ST\ E\ (A \sqcup \mathcal{A}\ e \sqcup \lfloor\{i\}\rfloor)]$
$|\ compT\ E\ A\ ST\ (e{\cdot}F\{D\}) =$
$\quad compT\ E\ A\ ST\ e\ @\ [after\ E\ A\ ST\ e]$
$|\ compT\ E\ A\ ST\ (e_1{\cdot}F\{D\} := e_2) =$
$\ (let\ ST_1 = ty\ \ \ E\ e_1\#ST;\ A_1 = A \sqcup \mathcal{A}\ e_1;\ A_2 = A_1 \sqcup \mathcal{A}\ e_2\ in$
$\quad compT\ E\ A\ ST\ e_1\ @\ [after\ E\ A\ ST\ e_1]\ @$
$\quad compT\ E\ A_1\ ST_1\ e_2\ @\ [after\ E\ A_1\ ST_1\ e_2]\ @$
$\quad [ty_i'\ ST\ E\ A_2])$
$|\ compT\ E\ A\ ST\ \{i{:}T;\ e\} = compT\ (E@[T])\ (A \ominus i)\ ST\ e$
$|\ compT\ E\ A\ ST\ (e_1;;e_2) =$
$\ (let\ A_1 = A \sqcup \mathcal{A}\ e_1\ in$
$\quad compT\ E\ A\ ST\ e_1\ @\ [after\ E\ A\ ST\ e_1,\ ty_i'\ ST\ E\ A_1]\ @$
$\quad compT\ E\ A_1\ ST\ e_2)$
$|\ compT\ E\ A\ ST\ (if\ (e)\ e_1\ else\ e_2) =$
$\quad (let\ A_0 = A \sqcup \mathcal{A}\ e;\ \tau = ty_i'\ ST\ E\ A_0\ in$
$\quad\ compT\ E\ A\ ST\ e\ @\ [after\ E\ A\ ST\ e,\ \tau]\ @$
$\quad\ compT\ E\ A_0\ ST\ e_1\ @\ [after\ E\ A_0\ ST\ e_1,\ \tau]\ @$
$\quad\ compT\ E\ A_0\ ST\ e_2)$
$|\ compT\ E\ A\ ST\ (while\ (e)\ c) =$
$\quad (let\ A_0 = A \sqcup \mathcal{A}\ e;\ \ A_1 = A_0 \sqcup \mathcal{A}\ c;\ \tau = ty_i'\ ST\ E\ A_0\ in$
$\quad\ compT\ E\ A\ ST\ e\ @\ [after\ E\ A\ ST\ e,\ \tau]\ @$
$\quad\ compT\ E\ A_0\ ST\ c\ @\ [after\ E\ A_0\ ST\ c,\ ty_i'\ ST\ E\ A_1,\ ty_i'\ ST\ E\ A_0])$
$|\ compT\ E\ A\ ST\ (throw\ e) = compT\ E\ A\ ST\ e\ @\ [after\ E\ A\ ST\ e]$
$|\ compT\ E\ A\ ST\ (e{\cdot}M(es)) =$
$\quad compT\ E\ A\ ST\ e\ @\ [after\ E\ A\ ST\ e]\ @$
$\quad compTs\ E\ (A \sqcup \mathcal{A}\ e)\ (ty\ \ \ E\ e\ \#\ ST)\ es$
$|\ compT\ E\ A\ ST\ (try\ e_1\ catch(C\ i)\ e_2) =$
$\quad compT\ E\ A\ ST\ e_1\ @\ [after\ E\ A\ ST\ e_1]\ @$
$\quad [ty_i'\ (Class\ C\#ST)\ E\ A,\ ty_i'\ ST\ (E@[Class\ C])\ (A \sqcup \lfloor\{i\}\rfloor)]\ @$
$\quad compT\ (E@[Class\ C])\ (A \sqcup \lfloor\{i\}\rfloor)\ ST\ e_2$
$|\ compTs\ E\ A\ ST\ [] = []$
$|\ compTs\ \ E\ A\ ST\ (e\#es) = compT\ E\ A\ ST\ e\ @\ [after\ E\ A\ ST\ e]\ @$
$\qquad\qquad\qquad\qquad compTs\ E\ (A \sqcup (\mathcal{A}\ e))\ (ty\ E\ e\ \#\ ST)\ es$

definition $compT_a :: ty\ list \Rightarrow nat\ hyperset \Rightarrow ty\ list \Rightarrow expr_1 \Rightarrow ty_i'\ list$ **where**
$\ compT_a\ E\ A\ ST\ e = compT\ E\ A\ ST\ e\ @\ [after\ E\ A\ ST\ e]$

end

lemma $compE_2\text{-}not\text{-}Nil[simp]$: $compE_2\ e \neq []$
lemma (**in** $TC1$) $compT\text{-}sizes[simp]$:
shows $\bigwedge E\ A\ ST.\ size(compT\ E\ A\ ST\ e) = size(compE_2\ e) - 1$
and $\bigwedge E\ A\ ST.\ size(compTs\ E\ A\ ST\ es) = size(compEs_2\ es)$

lemma (**in** $TC1$) $[simp]$: $\bigwedge ST\ E.\ \lfloor\tau\rfloor \notin set\ (compT\ E\ None\ ST\ e)$

and [*simp*]: $\bigwedge ST\ E.\ \lfloor\tau\rfloor \notin set\ (compTs\ E\ None\ ST\ es)$

lemma (**in** *TC0*) *pair-eq-ty$_i$'-conv*:
$(\lfloor(ST,\ LT)\rfloor = ty_i{}'\ ST_0\ E\ A) =$
$(case\ A\ of\ None \Rightarrow False \mid Some\ A \Rightarrow (ST = ST_0 \wedge LT = ty_l\ E\ A))$

lemma (**in** *TC0*) *pair-conv-ty$_i$'*:
$\lfloor(ST,\ ty_l\ E\ A)\rfloor = ty_i{}'\ ST\ E\ \lfloor A\rfloor$

lemma (**in** *TC1*) *compT-LT-prefix*:
$\bigwedge E\ A\ ST_0.\ [\![\ \lfloor(ST,LT)\rfloor \in set(compT\ E\ A\ ST_0\ e);\ \mathcal{B}\ e\ (size\ E)\ ]\!]$
$\Longrightarrow P \vdash \lfloor(ST,LT)\rfloor \le'\ ty_i{}'\ ST\ E\ A$
and
$\bigwedge E\ A\ ST_0.\ [\![\ \lfloor(ST,LT)\rfloor \in set(compTs\ E\ A\ ST_0\ es);\ \mathcal{B}s\ es\ (size\ E)\ ]\!]$
$\Longrightarrow P \vdash \lfloor(ST,LT)\rfloor \le'\ ty_i{}'\ ST\ E\ A$

lemma [*iff*]: $OK\ None \in states\ P\ mxs\ mxl$
lemma (**in** *TC0*) *after-in-states*:
$[\![\ wf\text{-}prog\ p\ P;\ P,E \vdash_1 e :: T;\ set\ E \subseteq types\ P;\ set\ ST \subseteq types\ P;$
$size\ ST + max\text{-}stack\ e \le mxs\ ]\!]$
$\Longrightarrow OK\ (after\ E\ A\ ST\ e) \in states\ P\ mxs\ mxl$

lemma (**in** *TC0*) *OK-ty$_i$'-in-statesI*[*simp*]:
$[\![\ set\ E \subseteq types\ P;\ set\ ST \subseteq types\ P;\ size\ ST \le mxs\ ]\!]$
$\Longrightarrow OK\ (ty_i{}'\ ST\ E\ A) \in states\ P\ mxs\ mxl$

lemma *is-class-type-aux*: $is\text{-}class\ P\ C \Longrightarrow is\text{-}type\ P\ (Class\ C)$
theorem (**in** *TC1*) *compT-states*:
assumes *wf*: *wf-prog p P*
shows $\bigwedge E\ T\ A\ ST.$
$[\![\ P,E \vdash_1 e :: T;\ set\ E \subseteq types\ P;\ set\ ST \subseteq types\ P;$
$size\ ST + max\text{-}stack\ e \le mxs;\ size\ E + max\text{-}vars\ e \le mxl\ ]\!]$
$\Longrightarrow OK\ `\ set(compT\ E\ A\ ST\ e) \subseteq states\ P\ mxs\ mxl$
and $\bigwedge E\ Ts\ A\ ST.$
$[\![\ P,E \vdash_1 es[::]Ts;\ \ set\ E \subseteq types\ P;\ set\ ST \subseteq types\ P;$
$size\ ST + max\text{-}stacks\ es \le mxs;\ size\ E + max\text{-}varss\ es \le mxl\ ]\!]$
$\Longrightarrow OK\ `\ set(compTs\ E\ A\ ST\ es) \subseteq states\ P\ mxs\ mxl$

definition $shift :: nat \Rightarrow ex\text{-}table \Rightarrow ex\text{-}table$
where
$shift\ n\ xt \equiv map\ (\lambda(from,to,C,handler,depth).\ (from{+}n,to{+}n,C,handler{+}n,depth))\ xt$

lemma [*simp*]: $shift\ 0\ xt = xt$
lemma [*simp*]: $shift\ n\ [] = []$
lemma [*simp*]: $shift\ n\ (xt_1\ @\ xt_2) = shift\ n\ xt_1\ @\ shift\ n\ xt_2$
lemma [*simp*]: $shift\ m\ (shift\ n\ xt) = shift\ (m{+}n)\ xt$
lemma [*simp*]: $pcs\ (shift\ n\ xt) = \{pc{+}n|pc.\ pc \in pcs\ xt\}$

lemma *shift-compxE$_2$*:
shows $\bigwedge pc\ pc'\ d.\ shift\ pc\ (compxE_2\ e\ pc'\ d) = compxE_2\ e\ (pc' + pc)\ d$
and $\bigwedge pc\ pc'\ d.\ shift\ pc\ (compxEs_2\ es\ pc'\ d) = compxEs_2\ es\ (pc' + pc)\ d$

lemma *compxE$_2$-size-convs*[*simp*]:

shows $n \neq 0 \Longrightarrow compxE_2\ e\ n\ d = shift\ n\ (compxE_2\ e\ 0\ d)$
and $n \neq 0 \Longrightarrow compxEs_2\ es\ n\ d = shift\ n\ (compxEs_2\ es\ 0\ d)$
locale *TC2* = *TC1* +
fixes T_r :: *ty* **and** *mxs* :: *pc*
begin

definition
wt-instrs :: *instr list* ⇒ *ex-table* ⇒ ty_i' *list* ⇒ *bool*
((⊢ -, - /[::]/ -) [*0*,*0*,*51*] *50*) **where**
⊢ *is*,*xt* [::] *τs* ⟷ *size is* < *size τs* ∧ *pcs xt* ⊆ {*0*..<*size is*} ∧
(∀ *pc*< *size is*. P,T_r,*mxs*,*size τs*,*xt* ⊢ *is*!*pc*,*pc* :: *τs*)

end

notation *TC2.wt-instrs* ((-,-,- ⊢/ -, - /[::]/ -) [*50*,*50*,*50*,*50*,*50*,*51*] *50*)

lemma (**in** *TC2*) [*simp*]: *τs* ≠ [] ⟹ ⊢ [],[] [::] *τs*
lemma [*simp*]: *eff i P pc et None* = []
lemma *wt-instr-appR*:
⟦ *P*,*T*,*m*,*mpc*,*xt* ⊢ *is*!*pc*,*pc* :: *τs*;
pc < *size is*; *size is* < *size τs*; *mpc* ≤ *size τs*; *mpc* ≤ *mpc'* ⟧
⟹ *P*,*T*,*m*,*mpc'*,*xt* ⊢ *is*!*pc*,*pc* :: *τs*@*τs'*

lemma *relevant-entries-shift* [*simp*]:
relevant-entries P i (*pc*+*n*) (*shift n xt*) = *shift n* (*relevant-entries P i pc xt*)

lemma [*simp*]:
xcpt-eff i P (*pc*+*n*) *τ* (*shift n xt*) =
map (λ(*pc*,*τ*). (*pc* + *n*, *τ*)) (*xcpt-eff i P pc τ xt*)

lemma [*simp*]:
app_i (*i*, *P*, *pc*, *m*, *T*, *τ*) ⟹
eff i P (*pc*+*n*) (*shift n xt*) (*Some τ*) =
map (λ(*pc*,*τ*). (*pc*+*n*,*τ*)) (*eff i P pc xt* (*Some τ*))

lemma [*simp*]:
xcpt-app i P (*pc*+*n*) *mxs* (*shift n xt*) *τ* = *xcpt-app i P pc mxs xt τ*

lemma *wt-instr-appL*:
⟦ *P*,*T*,*m*,*mpc*,*xt* ⊢ *i*,*pc* :: *τs*; *pc* < *size τs*; *mpc* ≤ *size τs* ⟧
⟹ *P*,*T*,*m*,*mpc* + *size τs'*,*shift* (*size τs'*) *xt* ⊢ *i*,*pc*+*size τs'* :: *τs'*@*τs*

lemma *wt-instr-Cons*:
⟦ *P*,*T*,*m*,*mpc* − *1*,[] ⊢ *i*,*pc* − *1* :: *τs*;
0 < *pc*; *0* < *mpc*; *pc* < *size τs* + *1*; *mpc* ≤ *size τs* + *1* ⟧
⟹ *P*,*T*,*m*,*mpc*,[] ⊢ *i*,*pc* :: *τ*#*τs*

lemma *wt-instr-append*:
⟦ *P*,*T*,*m*,*mpc* − *size τs'*,[] ⊢ *i*,*pc* − *size τs'* :: *τs*;
size τs' ≤ *pc*; *size τs'* ≤ *mpc*; *pc* < *size τs* + *size τs'*; *mpc* ≤ *size τs* + *size τs'* ⟧
⟹ *P*,*T*,*m*,*mpc*,[] ⊢ *i*,*pc* :: *τs'*@*τs*

lemma *xcpt-app-pcs*:
pc ∉ *pcs xt* ⟹ *xcpt-app i P pc mxs xt τ*

lemma *xcpt-eff-pcs*:
$pc \notin pcs\ xt \Longrightarrow xcpt\text{-}eff\ i\ P\ pc\ \tau\ xt = []$

lemma *pcs-shift*:
$pc < n \Longrightarrow pc \notin pcs\ (shift\ n\ xt)$

lemma *wt-instr-appRx*:
$[\![\ P,T,m,mpc,xt \vdash is!pc,pc :: \tau s;\ pc < size\ is;\ size\ is < size\ \tau s;\ mpc \leq size\ \tau s\ ]\!]$
$\Longrightarrow P,T,m,mpc,xt\ @\ shift\ (size\ is)\ xt' \vdash is!pc,pc :: \tau s$

lemma *wt-instr-appLx*:
$[\![\ P,T,m,mpc,xt \vdash i,pc :: \tau s;\ pc \notin pcs\ xt'\ ]\!]$
$\Longrightarrow P,T,m,mpc,xt'@xt \vdash i,pc :: \tau s$

lemma (**in** *TC2*) *wt-instrs-extR*:
$\vdash is,xt\ [::]\ \tau s \Longrightarrow \vdash is,xt\ [::]\ \tau s\ @\ \tau s'$

lemma (**in** *TC2*) *wt-instrs-ext*:
$[\![\ \vdash is_1,xt_1\ [::]\ \tau s_1@\tau s_2;\ \vdash is_2,xt_2\ [::]\ \tau s_2;\ size\ \tau s_1 = size\ is_1\ ]\!]$
$\Longrightarrow \vdash is_1@is_2,\ xt_1\ @\ shift\ (size\ is_1)\ xt_2\ [::]\ \tau s_1@\tau s_2$
corollary (**in** *TC2*) *wt-instrs-ext2*:
$[\![\ \vdash is_2,xt_2\ [::]\ \tau s_2;\ \vdash is_1,xt_1\ [::]\ \tau s_1@\tau s_2;\ size\ \tau s_1 = size\ is_1\ ]\!]$
$\Longrightarrow \vdash is_1@is_2,\ xt_1\ @\ shift\ (size\ is_1)\ xt_2\ [::]\ \tau s_1@\tau s_2$

corollary (**in** *TC2*) *wt-instrs-ext-prefix* [*trans*]:
$[\![\ \vdash is_1,xt_1\ [::]\ \tau s_1@\tau s_2;\ \vdash is_2,xt_2\ [::]\ \tau s_3;$
$size\ \tau s_1 = size\ is_1;\ \tau s_3 \leq \tau s_2\ ]\!]$
$\Longrightarrow \vdash is_1@is_2,\ xt_1\ @\ shift\ (size\ is_1)\ xt_2\ [::]\ \tau s_1@\tau s_2$

corollary (**in** *TC2*) *wt-instrs-app*:
assumes is_1: $\vdash is_1,xt_1\ [::]\ \tau s_1@[\tau]$
assumes is_2: $\vdash is_2,xt_2\ [::]\ \tau\#\tau s_2$
assumes s: $size\ \tau s_1 = size\ is_1$
shows $\vdash is_1@is_2,\ xt_1@shift\ (size\ is_1)\ xt_2\ [::]\ \tau s_1@\tau\#\tau s_2$

corollary (**in** *TC2*) *wt-instrs-app-last*[*trans*]:
$[\![\ \vdash is_2,xt_2\ [::]\ \tau\#\tau s_2;\ \vdash is_1,xt_1\ [::]\ \tau s_1;$
$last\ \tau s_1 = \tau;\ \ size\ \tau s_1 = size\ is_1{+}1\ ]\!]$
$\Longrightarrow \vdash is_1@is_2,\ xt_1@shift\ (size\ is_1)\ xt_2\ [::]\ \tau s_1@\tau s_2$

corollary (**in** *TC2*) *wt-instrs-append-last*[*trans*]:
$[\![\ \vdash is,xt\ [::]\ \tau s;\ P,T_r,mxs,mpc,[] \vdash i,pc :: \tau s;$
$pc = size\ is;\ mpc = size\ \tau s;\ size\ is + 1 < size\ \tau s\ ]\!]$
$\Longrightarrow \vdash is@[i],xt\ [::]\ \tau s$

corollary (**in** *TC2*) *wt-instrs-app2*:
$[\![\ \vdash is_2,xt_2\ [::]\ \tau'\#\tau s_2;\ \ \vdash is_1,xt_1\ [::]\ \tau\#\tau s_1@[\tau'];$
$xt' = xt_1\ @\ shift\ (size\ is_1)\ xt_2;\ \ size\ \tau s_1{+}1 = size\ is_1\ ]\!]$
$\Longrightarrow \vdash is_1@is_2,xt'\ [::]\ \tau\#\tau s_1@\tau'\#\tau s_2$

corollary (**in** *TC2*) *wt-instrs-app2-simp*[*trans*,*simp*]:
$[\![\ \vdash is_2,xt_2\ [::]\ \tau'\#\tau s_2;\ \ \vdash is_1,xt_1\ [::]\ \tau\#\tau s_1@[\tau'];\ size\ \tau s_1{+}1 = size\ is_1\ ]\!]$
$\Longrightarrow \vdash is_1@is_2,\ xt_1@shift\ (size\ is_1)\ xt_2\ [::]\ \tau\#\tau s_1@\tau'\#\tau s_2$

corollary (**in** *TC2*) *wt-instrs-Cons*[*simp*]:
$\llbracket\ \tau s \neq []; \vdash [i],[]\ [::]\ [\tau,\tau']; \vdash is,xt\ [::]\ \tau'\#\tau s\ \rrbracket$
$\Longrightarrow \vdash i\#is,shift\ 1\ xt\ [::]\ \tau\#\tau'\#\tau s$

theory *Jinja*
imports
J/TypeSafe
J/Annotate

J/execute-Bigstep
J/execute-WellType
JVM/JVMDefensive
JVM/JVMListExample
BV/BVExec
BV/LBVJVM
BV/BVNoTypeError
BV/BVExample
Compiler/TypeComp
begin

end

Bibliography

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